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PHYSICAL REVIEW FLUIDS **00**, 003100 (2017)**Biopolymer dynamics driven by helical flagella**Andrew K. Balin,<sup>1</sup> Andreas Zöttl,<sup>1</sup> Julia M. Yeomans,<sup>1</sup> and Tyler Shendruk<sup>1,2,\*</sup><sup>1</sup>*Rudolf Peierls Centre for Theoretical Physics, 1 Keble Road, University of Oxford,  
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Microbial flagellates typically inhabit complex suspensions of polymeric material which can impact the swimming speed of motile microbes, filter feeding of sessile cells, and the generation of biofilms. There is currently a need to better understand how the fundamental dynamics of polymers near active cells or flagella impacts these various phenomena, in particular, the hydrodynamic and steric influence of a rotating helical filament on suspended polymers. Our Stokesian dynamics simulations show that as a stationary rotating helix pumps fluid along its long axis, polymers migrate radially inward while being elongated. We observe that the actuation of the helix tends to *increase* the probability of finding polymeric material within its pervaded volume. This accumulation of polymers within the vicinity of the helix is stronger for longer polymers. We further analyze the stochastic work performed by the helix on the polymers and show that this quantity is positive on average and increases with polymer contour length.

DOI: [10.1103/PhysRevFluids.00.003100](https://doi.org/10.1103/PhysRevFluids.00.003100)**I. INTRODUCTION**

While the physics of swimming microbes in Newtonian viscous fluids has been well characterized, attention has recently turned toward understanding how active microorganisms behave in more biologically relevant media where the presence of large biopolymers, elastic filaments, or exopolymer secretions can dictate dynamics. In such complex fluids, motility enhancement [1–6] and retardation [3,4,7–11] have both been reported for various biological swimmers. Theoretical studies exploring swimming in continuous viscoelastic media yield model- or parameter-dependent results [3–5]. Invariably, these studies concentrate on continuum models of viscoelasticity, and as such cannot provide a full insight into the specific microscopic mechanisms of interaction between single polymers and the flagella of the swimmers.

Swimming dynamics may indeed be affected by such interactions due in part to the comparatively similar length and time scales of cells and biopolymeric material *in vivo*. One hypothesis that has been applied to the swimming of *E. coli* in dilute polymer solutions is that a time-scale separation between the fast rotation of the bacterial flagellum and slow relaxation of the polymers effectively depletes the flagellum's local environment of polymeric material as it clears its surrounding volume [12]. Hence, the flagellum experiences an effective viscosity that can be markedly different from that perceived by the more slowly counter-rotating cell body. However, the microscopic assumptions underlying this hypothesis require more concrete justification. Another hypothesis posits that the curved streamlines that wind around the helical flagellum produce shear flows that stretch individual polymers, resulting in elastic stresses which stabilize—hence speeding up—the cell's swimming [6]. It is recognized that this hypothesis relies on the expectation that the swimmer generates sufficient strain rates to stretch polymer molecules and raises the question of how large biopolymers interact with actively actuated filaments such as flagella on the single-polymer level.

Additionally, there are separate phenomena in which the interactions between large polymers and active microorganisms are important but have yet to be studied on this microscopic level. For

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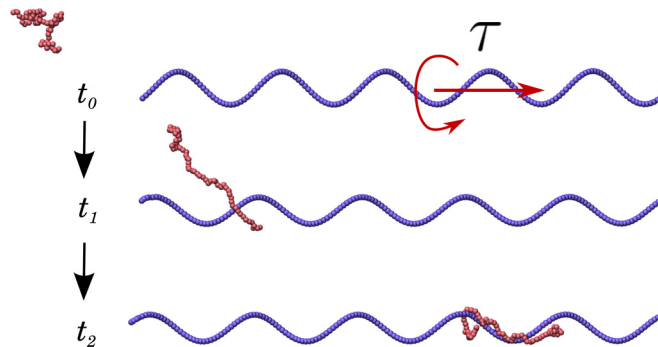


FIG. 1. Snapshots from a single simulation of a rotating helix and an advected polymer taken at times  $t_0 < t_1 < t_2$ . A torque  $\tau$  is distributed uniformly on the helix and, as it rotates, it generates a pumping flow. The polymer is drawn in radially and axially toward the helix, stretching in the process due to the induced shear flow. Once captured, it winds around the helix and is pumped axially in the direction of the average flow.

47 instance, various microbes have long been known to enhance filter feeding by employing their flagella  
 48 or cilia to generate feeding currents that carry detritus and nutrients toward the cell body [13]. While  
 49 motile neutrally buoyant planktonic bacteria [14] swim force free [15,16], these sessile microbes  
 50 tether themselves in order to exert non-negligible net forces on the surrounding medium [17,18].  
 51 Aggregations of such cells can collectively produce millimeter-scale fluid flows to actively combat  
 52 variations in the nutrient concentrations [19]. While hydrodynamic attraction of small nutrients has  
 53 been considered [20], the interaction of macromolecular polymers with sessile flagellates has yet to  
 54 be explored in detail. A question that remains open is whether the filter feeding of large polymers  
 55 progresses similarly to small tracer particles, or—as with swimming—whether the dynamics of  
 56 individual polymers enhances or reduces a tethered cell’s ability to draw in polymeric material  
 57 toward it.

58 In this work, we explore how macromolecular biopolymers hydrodynamically interact with  
 59 an active helical pump that transports fluid by external actuation using coarse-grained molecular  
 60 dynamics simulations. A snapshot from our simulations is shown in Fig. 1. We first construct a  
 61 simple *Stokesian dynamics* model of a rotating helix in a bulk fluid and justify the further use of this  
 62 simplified model by comparing the flow field in the vicinity of the helix to that of a more specific  
 63 biological model system comprised of a wall-tethered bacterium in a coarse-grained fluid. We show  
 64 that the Stokesian dynamics scheme captures the relevant near-field physics of flow around a helix  
 65 and proceed to study the dynamics of single polymers in its vicinity. Our results show that model  
 66 polymers are hydrodynamically drawn radially inward and are elongated by the high shear rate. The  
 67 polymers are then transported along the direction of the fluid flow while remaining elongated and  
 68 wrap around the helix before being deposited downstream.

69 We also study the stochastic energetics of the helix to show that the helix actively performs work  
 70 on the polymer, driving it to the higher free-energy state of elongation which we observe. This work  
 71 is positive on average, suggesting that the energy transferred to the polymer through hydrodynamic  
 72 interactions is dissipated and not elastically transferred back to the helix.

## 73 II. METHODS

### 74 A. Stokesian dynamics simulation

75 We employ a simulation scheme incorporating hydrodynamic interactions [21], referred to as  
 76 Stokesian dynamics (SD), to study the behavior of a polymer in response to a steadily rotating  
 77 helical filament. Both the polymer chain and the rotating helix are composed of sets of spherical

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78 particles whose positions  $\{\mathbf{r}_i\}$  are updated according to Langevin dynamics due to forces  $\{\mathbf{f}_i\}$  and  
79 thermal fluctuations  $\{\boldsymbol{\xi}_i\}$ :

$$\dot{\mathbf{r}}_i = \sum_j \left( k_B T \frac{\partial \boldsymbol{\mu}_{ij}}{\partial \mathbf{r}_j} + \boldsymbol{\mu}_{ij} \mathbf{f}_j \right) + \boldsymbol{\xi}_i(t), \quad (1)$$

$$\langle \boldsymbol{\xi}_i(t) \boldsymbol{\xi}_j(t') \rangle = 2k_B T \boldsymbol{\mu}_{ij} \delta(t - t'). \quad (2)$$

80 Equations (1) are coupled in two ways: (i) The total force  $\mathbf{f}_i(\{\mathbf{r}_j\})$  acting on particle  $i$  will in general  
81 contain a contribution due to pairwise interactions with nearby particles and (ii) hydrodynamic  
82 coupling between particles  $i$  and  $j$  is captured by the mobility tensor  $\boldsymbol{\mu}_{ij}(\mathbf{r}_i, \mathbf{r}_j)$ , which accounts  
83 for the advection of particle  $i$  due to the flow field created by forces  $\mathbf{f}_j$  acting on particle  $j$ .  
84 The fluid medium is responsible for dissipating the momentum of the particles, demanding the  
85 fluctuating forces obey the fluctuation-dissipation relation [Eq. (2)], which correlates the fluctuations  
86 experienced by two widely separated particles.

87 We use the Rotne-Prager-Yamakawa (RPY) tensor for the mobility [22,23],

$$\frac{\boldsymbol{\mu}_{ij}}{\mu_0} = \begin{cases} \frac{3a}{4r_{ij}} (\mathbf{I} + \hat{\mathbf{r}}_{ij} \hat{\mathbf{r}}_{ij}) + \frac{a^3}{2r_{ij}^3} (\mathbf{I} - 3\hat{\mathbf{r}}_{ij} \hat{\mathbf{r}}_{ij}) & \text{for } r_{ij} \geq 2a, \\ \left(1 - \frac{9r_{ij}}{32a}\right) \mathbf{I} + \frac{3}{32} \frac{r_{ij}}{a} \hat{\mathbf{r}}_{ij} \hat{\mathbf{r}}_{ij} & \text{otherwise,} \end{cases} \quad (3)$$

88 for  $i \neq j$  and where  $\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i$ ,  $\mu_0 = 1/6\pi\eta a$  is the Stokes mobility of a sphere with radius  $a$   
89 immersed in a fluid with viscosity  $\eta$  and  $\mathbf{I}$  is the identity matrix. The self-mobility of particle  $i$  is  
90 simply  $\boldsymbol{\mu}_{ii} = \mu_0 \mathbf{I}$ . The RPY tensor has the property that  $\partial \boldsymbol{\mu}_{ij} / \partial \mathbf{r}_j = 0$ , which simplifies the force  
91 balance equation (1).

### 92 B. Polymer model

93 All pairs of particles experience a mutual repulsion that acts over a characteristic length scale  $\sigma$   
94 and is given by the Weeks-Chandler-Andersen (WCA) potential,

$$\mathcal{H}_{\text{repel}}(\mathbf{r}_{ij}) = \begin{cases} 4\epsilon \left[ \left(\frac{\sigma}{r_{ij}}\right)^{12} - \left(\frac{\sigma}{r_{ij}}\right)^6 \right] + \epsilon, & \text{if } r_{ij} < 2^{1/6}\sigma, \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

95 The polymer is modeled as a chain of spherical beads, exerting pairwise attractive forces representing  
96 bonds between adjacent monomers. These bonding forces are calculated using the finitely extensible  
97 nonlinear elastic (FENE) potential [24],

$$\mathcal{H}_{\text{bond}}(\mathbf{r}_{ij}) = -\frac{1}{2} k_{\text{bond}} r_0^2 \ln \left[ 1 - \left( \frac{r_{ij}}{r_0} \right)^2 \right], \quad (5)$$

98 with Kremer-Grest parameters [25]  $k_{\text{bond}} = 30\epsilon/\sigma^2$  and  $r_0 = 1.5\sigma$ . We choose  $\epsilon = k_B T$  for the  
99 characteristic strength of the potentials,  $\sigma$  for the spatial unit, and  $\sigma^2/\mu_0 k_B T$  for the temporal unit.  
100 This allows us to set  $k_B T = 1$ ,  $\sigma = 1$ ,  $\mu_0 = 1$  hereafter. In these units, the bead diameter is  $2^{1/6}$ ;  
101 hence  $a = 2^{-5/6}$ , and the simulation time step is  $\delta t = 10^{-5}$ . The conservative Hamiltonians  $\mathcal{H}_{\text{repel}}$   
102 and  $\mathcal{H}_{\text{bond}}$  give rise to pairwise forces,  $\mathbf{f}_i = -\mathbf{f}_j = -\nabla[\mathcal{H}_{\text{repel}}(\mathbf{r}_{ij}) + \mathcal{H}_{\text{bond}}(\mathbf{r}_{ij})]$ , which are the  
103 equal and opposite forces acting on particles  $i$  and  $j$ .

104 For a three-dimensional system of  $N$  particles, Eqs. (1) can be rewritten in nondimensional form  
105 as a  $3N \times 3N$  matrix-vector difference equation involving time step  $\delta t$ ,

$$\delta \mathbf{r} = \boldsymbol{\mu} \mathbf{f} \delta t + \mathbf{b} \delta \mathbf{w}, \quad (6)$$

106 where  $\delta \mathbf{w}$  is a random Gaussian vector with the properties  $\langle \delta \mathbf{w} \rangle = \mathbf{0}$  and  $\langle \delta \mathbf{w} \delta \mathbf{w} \rangle = 2\delta t \mathbf{I}$ , and  $\mathbf{b}$  is  
107 any matrix which satisfies  $\mathbf{b} \mathbf{b}^T = \boldsymbol{\mu}$ . We find  $\mathbf{b}$  by computing the Cholesky decomposition of  $\boldsymbol{\mu}$  and  
108 note that Cholesky decomposition requires  $\boldsymbol{\mu}$  to be positive definite, which is ensured by the RPY

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109 tensor. Because the mobilities  $\mu_{ij}$  vary slowly with respect to the fastest time scales of the bond  
110 potentials, we update  $\mu$  once every 100 time steps.

### 111 C. Rotating helix model

112 The rotating helix comprises a set of particles whose individual positions are externally controlled  
113 by time-dependent forces that prescribe the shape, rotational frequency  $\omega$ , and translational velocity  
114  $v$  of the helix. This is done by applying a rectifying force that opposes displacements of a constituent  
115 particle from its prescribed location via a stronger harmonic potential,

$$\mathcal{H}_h(\mathbf{r}_i, t) = \frac{1}{2}k_h[\mathbf{r}_i - \mathbf{r}_i^0(t)]^2, \quad (7)$$

$$\mathbf{r}_i^0 = \begin{pmatrix} R_0 \cos(\kappa z_i^0 - \omega t) \\ R_0 \sin(\kappa z_i^0 - \omega t) \\ z_i^0 + vt \end{pmatrix}. \quad (8)$$

116 The prescribed positions  $\{\mathbf{r}_i^0\}$  trace a helix along  $\hat{\mathbf{z}}$ , with an imposed radius  $R_0$ , and pitch [26]  $\kappa$ .  
117 For the helix potential strength, we use  $k_h = 70\epsilon$ . By applying the constraint that the target  
118 positions must be separated by diameter  $2a$  in space, the required spacing in  $z$  is approximated  
119 by  $z_i^0 = 2a/\sqrt{1 + \kappa^2 R^2}$ . We impose a constant angular rotation rate  $\omega$  about the  $\hat{\mathbf{z}}$  axis, and  
120 enforce stationarity by setting  $v = 0$  along the  $\hat{\mathbf{z}}$  axis. This model is adequate for reproducing  
121 the hydrodynamics of a rigid helix. Alternatively, modeling the helix as a semiflexible polymer  
122 would require solving bonding angle and dihedral angle potentials with large stiffnesses [27], in turn  
123 requiring  $\delta t$  to be many orders of magnitude below the shortest time scale of interest, which for our  
124 purposes is the relaxation time  $\tau_0 \sim a^2/\mu_0 k_B T$  of the SD beads.

125 The helix particles (labeled by subscript  $i$ ) are initialized in their target positions at  $t = 0$ , and then  
126 the helix as a whole relaxes into a steady state after a short transient period. The steady state differs  
127 slightly from the target shape due to (i) a viscous, drag-induced phase lag behind their target position,  
128 causing a shrinkage in their radial coordinate, which can shown to be  $R = R_0/\sqrt{1 + (\mu_0\omega/k_h)^2}$  when  
129 no hydrodynamic interactions are present, and (ii) an additional collective displacement along  $z$  due  
130 to chiral asymmetry in the hydrodynamic interactions  $\sum_{j(\neq i)} \mu_{ij} \cdot \mathbf{f}_j$  with the other helix particles.  
131 For  $v = 0$ , these displacements  $\delta z_i$  are counteracted by a net force on the helix  $-\frac{\partial \mathcal{H}_h}{\partial z} = -N_h k_h \delta z$ ,  
132 which is imparted to the fluid in the  $\hat{\mathbf{z}}$  direction.

133 With this control over  $\omega$  and  $v$ , we can drag, rotate, or apply some combination of translation and  
134 rotation to the helix. The imposed rotation and translation implies an external axial force and torque  
135 via the linear mobility relation

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} \mu_{tt} & \mu_{tr} \\ \mu_{rt} & \mu_{rr} \end{pmatrix} \begin{pmatrix} f_z \\ \tau_z \end{pmatrix}, \quad (9)$$

136 where the components  $\mu_{tt}, \mu_{tr} = \mu_{rt}, \mu_{rr}$  are mobility coefficients for the helix as a whole and not  
137 to be confused with the Rotne-Prager tensor  $\mu$ . In general, the full mobility relation also includes  
138 off-axis force and torque components  $f_{x,y}$  and  $\tau_{x,y}$ ; however, these are small compared to  $f_z$  and  
139  $\tau_z$ , and because the potential  $\mathcal{H}_h$  keeps the helix centered and aligned along  $\hat{\mathbf{z}}$ , this results in small  
140 off-axis contributions to the fluid flow.

## 141 III. HELIX HYDRODYNAMICS

### 142 A. Flow field generated by rotating helix in SD simulations

143 We focus solely on a stationary helix ( $v = 0$ ) rotating at some angular speed  $\omega$ . This implies a  
144 nonzero force  $f_z = -\frac{\mu_{tr}}{\mu_{tt}}\tau_z$  must be imparted to the fluid, and the rotating helix acts as a pump. This  
145 model evokes the microscopic experiments on tethered bacteria [28,29] as well as many scaled-up  
146 experiments of tethered flagella-like filaments [30–34].

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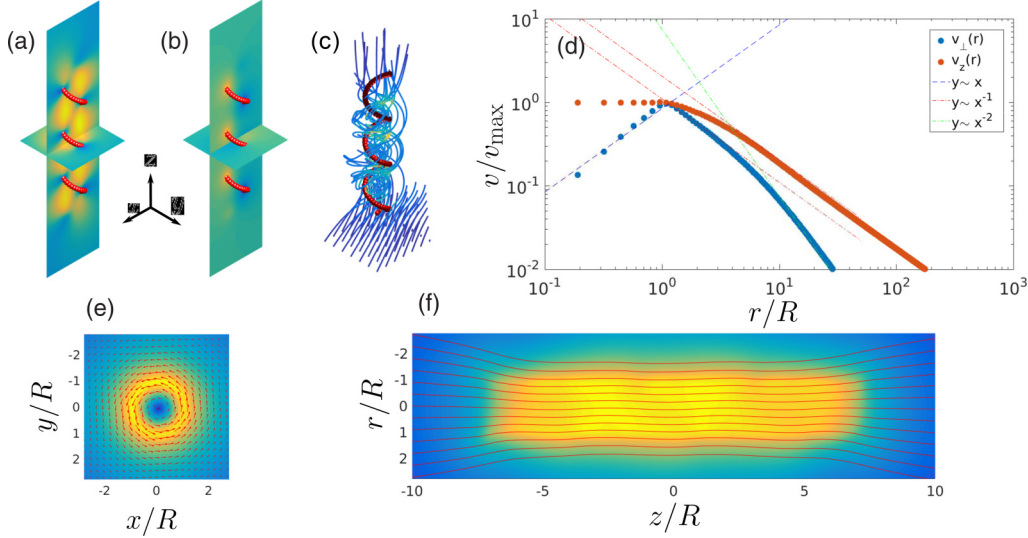


FIG. 2. [(a), (b)] Cross sections of the instantaneous flow field generated by a rotating helix, decomposed into components  $v_z(\mathbf{r}, t)$  and  $v_\perp(\mathbf{r}, t)$  respectively. (c) Streamlines of the full flow field  $\mathbf{v}(\mathbf{r}, t)$  show both the net flow along  $\hat{z}$  and the chiral winding caused by the rotation of the helix. (d) time-averaged fields as a function of radial distance from helix center line,  $\bar{v}_z(r, t)$  and  $\bar{v}_\perp(r, t)$ . Within the volume of the helical filament ( $r < R$ ), the fluid rotates like a rigid body,  $v_\perp \propto r$ , and flows axially at a constant rate  $v_r = \text{const}$ . (e) Slice in the  $xy$  plane of the mean rotational flow. (f) Slice in the  $xz$  plane of the mean axial flow.

147 In the SD simulations of a helix on its own, we can evaluate the instantaneous flow field at  
 148 any point  $\mathbf{r}$  in space by summing the contributions that each particle in the simulation makes:  
 149  $\mathbf{v}(\mathbf{r}) = \sum_i \boldsymbol{\mu}(\mathbf{r} - \mathbf{r}_i) \cdot \mathbf{f}_i$ , where  $\boldsymbol{\mu}(\mathbf{r}')$  is given by Eq. (3) with  $\mathbf{r}_{ij} = \mathbf{r}'$ . Figures 2(a)–2(c) offer  
 150 a visualization of the instantaneous flow field surrounding a rotating helix. In Fig. 2(a), the axial  
 151 component  $v_z$  shows that the fluid is most strongly pumped in the interstitial volume of the helix,  
 152 similar to the instantaneous axial flow field measured in experiments on a tethered rotating helix  
 153 [34]. As the axial velocities of the helix beads are 0,  $v_z$  must vanish at the helix surface. However,  
 154 the transverse flow field  $v_\perp$  is strongest at the helix surface as it must match the transverse velocity  
 155 of the beads [Fig. 2(b)]. An instantaneous snapshot of the streamlines originating from a square grid  
 156 in the  $xy$  plane beneath the helix gives a visual sense of the chiral nature of the flow field [Fig. 2(c)].

157 By taking a time average  $\bar{\mathbf{v}}(x, y, z)$  over a complete rotation of the helix, we can understand how  
 158 the flow field varies in space in more detail. Figure 2(d) shows how  $\bar{v}_z$  and  $\bar{v}_\perp$  decay as a function  
 159 of radial distance  $r$  from the  $z$  axis along which the helix lies. In the far field, we observe  $\bar{v}_z \sim 1/r$ ,  
 160 which is the characteristic scaling expected from a point-force (stokeslet) response of an unbounded  
 161 fluid. This is as expected, since we must apply a force  $f_z$  on the helix such that  $\mu_{tt}f_z + \mu_{tr}\tau_z = 0$   
 162 by Eq. (9) to ensure the rotating helix remains stationary ( $v = 0$ ). Hence, far away from the helix,  
 163 the fluid responds as if subject to a point force.

164 The far-field scaling of the transverse velocity is the characteristic scaling for a rotlet,  $\bar{v}_\perp \sim 1/r^2$ ,  
 165 which we expect to dominate the far-field flow created by an external torque rotating a body immersed  
 166 in the fluid. However, in the intermediate region ( $R < r < 10R$ ), the transverse forces on the beads  
 167 on the near side of the helix dominate over the oppositely directed forces on the far side, and therefore  
 168 a stokeslet-like scaling  $\bar{v}_\perp \sim 1/r$  is seen.

169 Within the interior of the helix, we see interesting scaling properties  $\bar{v}_z \sim \text{const}$ , and  $\bar{v}_\perp \sim r$ ,  
 170 which shows that on average the fluid inside the helix rotates about  $\hat{z}$  and translates along  $\hat{z}$  as a  
 171 rigid body—though the instantaneous dynamics are more complicated. Figures 2(e)–2(f) show the

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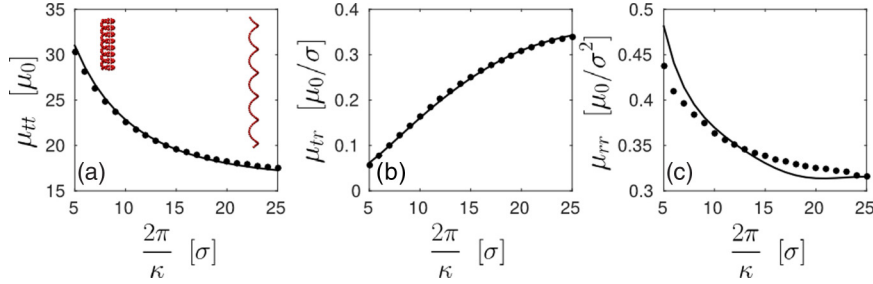


FIG. 3. The three components of the helix friction tensor as a function of pitch length  $2\pi/\kappa$ . The data (black dots) are determined by measuring  $f_z$ , and  $\tau_z$  separately as functions of  $v$  and  $\omega$  for different  $\kappa$  in order to determine the matrix components  $\mu_{tt}$ ,  $\mu_{tr}$ , and  $\mu_{rr}$ . The solid lines are calculated using analytical results from slender body theory (Eqs. (44) from Ref. [16]). Increasing the pitch length decreases the isotropic components of the friction tensor, while increasing the coupling between rotation and translation. Inset: The helix shapes for the extremal choices of pitch.

172 time-averaged flow fields. The transverse flow field is strongest in the annular region occupied by  
 173 the helical filament itself, while the axial flow field is uniformly strong across the whole volume.

### 174 B. Friction tensor of helical filament in SD simulations

175 In order to measure the helix mobility, we conducted SD simulations of a helix (with no polymer  
 176 present) while linearly ramping up either the velocity or angular velocity, while keeping the other  
 177 zero. In the first instance ( $v = v_0 t/T, 0$ ) was imposed and in the second instance ( $0, \omega = \omega_0 t/T$ )  
 178 was imposed, where  $v_0$  and  $\omega_0$  are the target final velocities and the length of the simulation,  $T$ ,  
 179 was sufficiently long to ensure that the system remained in a quasisteady state. In both cases, we  
 180 measured ( $f_z, \tau_z$ ) in order to solve the linear system Eq. (9). We conducted this for a range of  
 181  $\kappa \in (\frac{2\pi}{25}, \frac{2\pi}{5})$  to measure how the friction components changed as a function of helix shape. The  
 182 functional dependence of these coefficients on  $\kappa$  can be derived analytically using slender-body  
 183 theory [35,36]. The SD simulation results are in good agreement with theoretical predictions [16] as  
 184 shown in Fig. 3.

185 We observe that as the pitch length increases,  $\mu_{tt}$  and  $\mu_{rr}$  decrease, while  $\mu_{tr}$  increases. This  
 186 tells us that the coupling between axial force and rotation (or conversely between applied torque  
 187 and resulting translational speed) increases as the pitch length is increased over the range shown.  
 188 This behavior can be understood intuitively by considering the limiting case of small pitch length  
 189  $2\pi/\kappa \rightarrow \sigma$ , in which the filament resembles a cylinder which by symmetry must have a totally  
 190 decoupled mobility relation.

191 The data for  $\mu_{tt}$  and  $\mu_{tr}$  fit very well to the slender-body prediction. However, while  $\mu_{rr}$   
 192 qualitatively displays a similar dependence on pitch length to the analytic prediction, it appears to  
 193 systematically deviate from the theory. As previously discussed, the steady-state shape of the helix  
 194 realized in a simulation deviates slightly from its target shape [defined by Eq. (7)]. This effect of  
 195 this is generally small, but it appears that  $\mu_{rr}$  is the more sensitive to this dynamic remodeling than  
 196 the other components of the mobility matrix. However, in the remainder of this study, we will only  
 197 conduct simulations in which the helix parameters remain constant and so this discrepancy in  $\mu_{rr}$   
 198 as a function of  $\kappa$  does not affect our findings.

### 199 C. Scope of rotating helix model

200 Because our model helix remains in a fixed location in  $\hat{z}$ , it is neither force-free nor torque-free;  
 201 hence its flow field will differ to that generated by a swimming cell in the far field. Artificial  
 202 swimming magnetic ribbons [37,38] are arguably the most similar experimental realization of our  
 203 system due to the net torque they impart, but unless they are stalled by an external force, they

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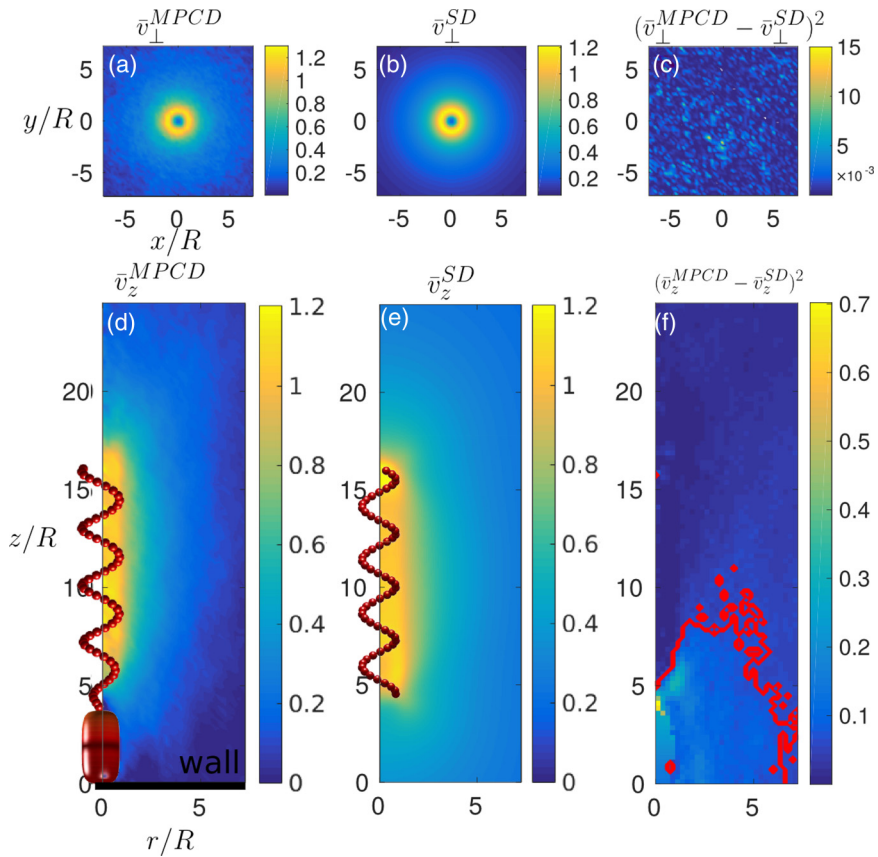


FIG. 4. [(a)–(c)] Comparison of the time-averaged flow fields generated by (a) MPCD fluid simulations of a cell attached to a wall vs (b) SD simulations of a helix rotating in a bulk fluid computed in the  $xy$  plane. These two measured fields differ only by Gaussian fluctuations inherent in the MPCD simulation as shown in (c). [(d)–(f)] Same as panels (a)–(c) except in  $xz$  plane. Panel (f) shows that there are systematic differences in the flow fields in this plane, mainly due to the presence of the wall. The contour-enclosed region in the lower quarter of the image represents where the flow field differs by  $|\bar{v}_z^{MPCD} - \bar{v}_z^{SD}| > 0.25$ . Everywhere else, the fields behave similarly.

204 remain force free too. Hence, the force and torque nature of our model is more akin to systems in  
 205 which flagellated cells are in some way stalled or tethered, e.g., by hydrodynamic accumulation at  
 206 boundaries [39], immobilization on microscope slides [28,29], or as part of their biological function  
 207 [17,18]. However, in these cases, hydrodynamic interactions with the boundaries and cell bodies  
 208 are a potential source of discrepancy between the results of our model and these experimental and  
 209 biological systems.

210 We wish to quantify this difference and demonstrate that our system is nevertheless a sufficiently  
 211 good model for studying the near-flagellum dynamics of a polymer. To achieve this, we compare  
 212 the flow fields measured from the SD simulations to multiparticle collision dynamics (MPCD)  
 213 simulations of a more experimentally realistic geometry with a cell body and neighboring wall,  
 214 as MPCD provides the appropriate machinery for incorporating boundaries and cell-specific body  
 215 geometry. Further details of the MPCD procedure can be found in the Appendix.

216 In Fig. 4, we compared the time-averaged flow fields  $\bar{v}^{MPCD}$  and  $\bar{v}^{SD}$  generated by the MPCD  
 217 and SD simulations respectively. Both quantities were normalized by dividing by the mean velocity



218 inside the volume occupied by the helix. By visual inspection, we observe that the coarse structure  
219 of the axial and transverse flows are qualitatively similar in both simulations.

220 To quantitatively compare the two fields, we analyzed the square differences  $(\bar{v}_{z,\perp}^{\text{MPCD}} - \bar{v}_{z,\perp}^{\text{SD}})^2$  to  
221 see how they decay relative to one another. In Fig. 4(c), we can see that  $(\bar{v}_{z,\perp}^{\text{MPCD}} - \bar{v}_{z,\perp}^{\text{SD}})^2$  appears  
222 to have the structure of uniform noise. A Kolmogorov-Smirnov test on the data for the unsquared  
223 difference  $\bar{v}_{z,\perp}^{\text{MPCD}} - \bar{v}_{z,\perp}^{\text{SD}}$  did not yield evidence for a non-Gaussian distribution, and so we conclude  
224 that  $\bar{v}_{z,\perp}^{\text{MPCD}}$  differs from  $\bar{v}_{z,\perp}^{\text{SD}}$  by the Gaussian noise present in the MPCD simulation only.

225 However, there are more significant differences in the radial flow fields due to the presence of  
226 the cell body and the wall in the MPCD simulations. The square difference  $(\bar{v}_z^{\text{MPCD}} - \bar{v}_z^{\text{SD}})^2$ , plotted  
227 in Fig. 4(f), reveals a systematic variation across the whole region that is roughly one order of  
228 magnitude greater than the noise in Fig. 4(c). The contour-enclosed area connected to the wall (at  
229  $z = 0$ ) shows the region in which the flow fields differ by  $|\bar{v}_z^{\text{MPCD}} - \bar{v}_z^{\text{SD}}| > 0.25$ . Evidently, the cell  
230 body and wall have some significant influence on the flow in this region but not in the immediate  
231 vicinity of the model flagellum.

232 The fact that the radial fields are in better agreement than the axial fields can be understood by  
233 noting that  $\bar{v}_{z,\perp} \sim 1/r^2$  decays over shorter distances than  $\bar{v}_z \sim 1/r$ , and hence the wall effects play  
234 a much larger role for the axial fields.

235 Since our main focus in this paper is the near-flagellum dynamics of polymers and helices  
236 interacting across length scales similar to and less than their own spatial dimensions, we take  
237 these MPCD results as evidence that far-field effects (such as those generated by counter-rotating  
238 cell bodies, solid boundaries, and indeed other nearby swimmers or filaments) do not contribute  
239 appreciably to the dynamics of polymers sufficiently close to the helical filament. In this region, we  
240 expect the chiral, high-shear, and geometry-specific flow of the helix to characterize the behavior of  
241 a nearby polymer, and far-field effects due to boundaries or other bodies to be secondary.

#### 242 IV. POLYMER CAPTURE

243 The central result of this paper is that a rotating helix transports a polymer along with the fluid  
244 it pumps, but in such a way that a polymer initially on the outside of the helix is drawn inward  
245 and “captured” by the helix. This is accompanied by an initial stretching out of the polymer as it  
246 migrates toward the helix, moving into a region of greater shear and greater flow as seen in Fig. 1. As  
247 it is transported along the interior of the helix, it occasionally wraps around the helical filament and  
248 rotates along with it. An instance of such wrapping is observed in Fig. 1. The polymer is deposited  
249 at the end of the helix, where a decaying axial current keeps moving it at a diminishing rate, while  
250 the lower shear results in the polymer collapsing back into its unstretched ground state. Two movies  
251 of a typical capturing process can be found in Ref. [40].

252 The polymer capture and transport is a stochastic, nonequilibrium transient process; however,  
253 by performing a large number of “scattering” simulations and averaging over these, we are able to  
254 quantify the typical nature of the interaction as a function of polymer size. We present batches of  
255 simulations for degrees of polymerization (number of beads)  $N_p = 1, 10, 30, 50$ —where a polymer  
256 of contour length 1 is simply a spherical monomer. In each of these simulations, we use helices  
257 with the same parameters:  $N_h = 200$  beads,  $\kappa = 2\pi/15\sigma$ ,  $R = 4\sigma$ . The helix is centered along the  
258  $z$  axis, with its two ends located at  $(z_0, z_{N_h-1}) = (-57\sigma, 57\sigma)$ . In each simulation, one polymer is  
259 initialized by placing its first monomer randomly on a disk of radius  $16\sigma$ , located at  $(z_0 - 30\sigma)$ , then  
260 performing a self-avoiding random walk to build the polymer bead by bead. We run each simulation  
261 for  $\mathcal{T} = 2 \times 10^6 \delta t$ , which is sufficient in all cases to advect the polymers beyond the rear of the helix.  
262 We measure the relaxation times of the polymers in separate simulations and found them to be  $\tau_p \approx$   
263  $\{1, 10, 20\} \times 10^6 \delta t$  for the  $N = \{1, 10, 30, 50\}$  polymers respectively. Taking the helix rotation speed  
264  $\omega = 2 \times 10^{-4} / \delta t$  as the characteristic shear rate for the fluid, we therefore estimate the respective  
265 Weissenberg numbers characterizing each set of simulations to be  $\text{Wi} \sim \omega \tau_p = \{200, 2000, 4000\}$ .  
266 As  $\text{Wi} \gg 1$  in each of these cases, the influence of shear flow dominates the polymers’ tendency to  
267 relax.

## BIOPOLYMER DYNAMICS DRIVEN BY HELICAL FLAGELLA

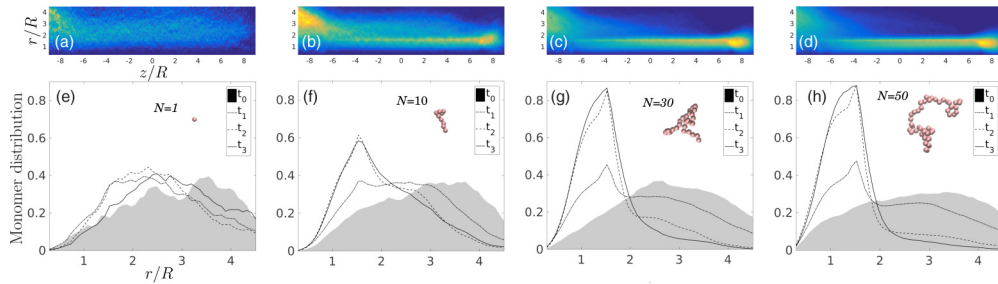


FIG. 5. [(a)–(d)] Polymer distributions averaged over all simulations and over all time show the net behavior of polymers being pumped in the positive- $z$  direction, given an initial distribution of polymers on a disk of radius  $r = 4R$ , located at  $z \approx -10R$  in cylindrical coordinates  $(r, z)$ . Each distribution is averaged over 200 simulations for polymers of size (a)  $N = 1$  (colloidal tracers), (b)  $N = 10$ , (c)  $N = 30$ , and (d)  $N = 50$ . The net behavior is a drift toward the right of each image, due to the helix (not shown) pumping the fluid. [(e)–(h)] The same data as in panels (a)–(c) respectively, but plotted as distributions over  $r$  only, with each curve representing contiguous quarter intervals of the simulation time. In each case, the initial quarter is the shaded region, and we observe that for increasing polymer size, a strong tendency for the polymers to migrate inward is observed. For larger polymers, this tendency is stronger and occurs faster.

268 To illustrate how the ensembles of polymers evolve in time, we plot the average monomer  
 269 distribution in cylindrical coordinates  $(r, z)$  taken over all simulations and at all times. These are  
 270 plotted for polymers of differing size in Figs. 5(a)–5(d). These images represent time- and ensemble-  
 271 averaged 2D histograms of the snapshots shown in Fig. 1. At the left edge of each of the images is  
 272 the initial distribution of polymeric material which in each case is smeared rightward in time. The  
 273 structure of this distribution gives a graphical indication that larger polymers [such as in Figs. 5(c)  
 274 and 5(d)] are much more strongly attracted to the helix than smaller polymers and are highly  
 275 concentrated in the region  $r < 2R$ . On the other hand, in Fig. 5(a) this effect is barely observed for  
 276  $N = 1$  monomers, which are simply advected along the streamlines shown in Fig. 2(f) like tracer  
 277 particles that cannot cross streamlines; polymers, on the other hand, are able to cross streamlines in  
 278 shear flow [41], and in this case do so strongly in a nontrivial manner.

279 Particularly for high Weissenberg numbers [42–44], we observe hotspots in the distributions  
 280 located at the downstream end of the helix. These indicate the accumulation of polymers in this  
 281 location when they are deposited at the end of the helix and collapse back into their equilibrium  
 282 conformation. Though they continue moving along the  $z$  axis, they do so at a lower rate than when  
 283 inside the helix. This is due to the fact that the axial flow  $v_z$  quickly decreases outside of the helix  
 284 [as can be seen in Fig. 2(f)].

285 In Figs. 5(e)–5(h), we segment each simulation into four equal and contiguous intervals in time,  
 286 and separately plot the marginal distributions over  $r$  only (i.e., with the  $z$  component integrated  
 287 out) for each time. The shaded region represents the interval  $t_0 = [0, T/4]$ , which is the first quarter  
 288 of each simulation and closely approximates the initial distribution. These figures show how the  
 289 initial distribution evolves with time for polymers of different size. For  $N_p = 30$  and  $N_p = 50$ , the  
 290 tendency to concentrate in and around the helix is markedly stronger than for shorter polymers. This  
 291 ensemble behavior shows that the actuation of the helix is responsible for a large density fluctuation  
 292 in the surrounding polymeric material that concentrates—rather than depletes—the polymers in the  
 293 immediately surrounding region. Animations of the densities as a function of time can be found in  
 294 Ref. [45].

295 This implies that the free energy of the polymers must be actively driven away from what we  
 296 would expect in equilibrium. We obtain an intuitive sense that this is occurring by considering the  
 297 snapshots in Fig. 1. Initially, the polymer is far away and its configuration is that of a self-avoiding  
 298 random walk. However, at intermediate times, the polymer is stretched out of equilibrium by the  
 299 shear flow and is transported radially inward as well as along  $z$  until it strongly interacts with the

300 helix, wrapping around it and continuing to move along  $z$ . In the vicinity of the helix, polymers  
 301 lose their equilibrium conformation, and we observe features in their dynamics similar to those  
 302 previously reported for polymers in shear flow due to a rotating microwire [46]. To gain further  
 303 insight into the energetic interplay within our system, we analyze the stochastic fluctuations in work  
 304 performed by the helix on the polymer.

### 305 V. FLUCTUATING WORK

306 Polymers tend to become stretched when immersed in a shear flow, and this agrees with the  
 307 current SD simulations. However, because the shear flow in this case is generated by external forces  
 308 acting on the helix, we expect these forces to be dependent on the proximity of a polymer in the  
 309 surrounding medium. The hydrodynamic origin of this is due to the effect of the polymer on the helix  
 310 particles: Hydrodynamic interactions displace the helix particles from their steady state positions,  
 311 resulting in a change in the forces acting on them due to Eq. (7). While these displacements are too  
 312 small to give rise to a perceptible change in helix shape, they should in general manifest themselves  
 313 as a change in work that the helix must do to maintain steady rotation.

314 The work applied to the helix by the external forces,  $w[t, \mathbf{r}(t)]$ , is a fluctuating quantity which is a  
 315 unique function for each realization of a stochastic dynamical process. Work is performed either by  
 316 the application of a nonconservative force or by a time-varying potential,  $\mathcal{H}(\mathbf{r}, \lambda(t))$  with an external  
 317 control parameter  $\lambda(t)$ . For the latter case, the work applied by a time  $t$  is defined by Ref. [47]:  
 318  $w[t, \mathbf{r}(t)] = \int_0^t dt' \dot{\lambda} \partial \mathcal{H} / \partial \lambda$ . The nonstochastic forces in our simulation are due to the potentials  
 319  $\mathcal{H}_{\text{repel}}$ ,  $\mathcal{H}_{\text{bond}}$ , and  $\mathcal{H}_h$  in Eqs. (4), (5), and (7). Of these, only the forces acting on the helix due to  
 320  $\mathcal{H}_h$  depend explicitly on time and it is these that are entirely responsible for the work done on the  
 321 system.

322 For each simulation, we calculate the incremental work performed by the helix at each time step  
 323 by

$$\delta w = \sum_i \mathbf{f}_i^h \cdot \delta \mathbf{r}_i^0, \quad (10)$$

324 where  $\mathbf{f}_i^h = -\nabla \mathcal{H}_h(\mathbf{r}_i, t)$  is time-varying force applied to particle  $i$ . Note the increment  $\delta \mathbf{r}_i^0$  is  
 325 the displacement of the bead target position, not the displacement of the bead itself. From these  
 326 increments  $\delta w$ , we build up an accumulated work trajectory  $w(t) = \sum_t \delta w$ .

327 We expect there to be two contributions to the work:  $w(t) = w_0(t) + w_{ex}(t)$ . The dominant  
 328 contribution  $w_0(t)$  is the deterministic work done by the rigid helix on the viscous fluid, which is  
 329 viscously dissipated. The second contribution is  $w_{ex}(t)$ , which is the stochastic excess work done on  
 330 the polymer. By conducting simulations without a polymer, we can measure the dominant viscous  
 331 contribution,  $w_0(t)$ , and use this to calculate the excess contribution in simulations that do contain a  
 332 polymer:  $w_{ex}(t) = w(t) - w_0(t)$ .

333 In Fig. 6, we plot three ensembles of trajectories  $w_{ex}(t)$  for the work done by a helix on three  
 334 sizes of polymer:  $N = (10, 20, 40)$ . In each simulation, the polymer is initialised by a self-avoiding  
 335 random walk starting at  $\mathbf{r}_0 = [0, 15\sigma, (z_0 - 15)\sigma]$ , where  $z_0$  is the  $z$  position of the negative-most  
 336 particle of the helix which pumps fluid in the positive- $z$  direction. We measure  $w_{ex}(t)$  for the entirety  
 337 of each simulation over a time interval of 80 full rotations. For all simulations, this is enough time to  
 338 allow for the polymer to relax back to its equilibrium conformation after it has exited the positive- $z$   
 339 end of the helix.

340 These sets of trajectories offer another way to look at the stretching effect: a set of stochastic  
 341 work trajectories  $\{w_i[t, \mathbf{r}(t)]\}$  drive the polymer to a higher free energy state though in each instance  
 342 requiring a different amount of work. The color of each curve denotes the radial point of closest  
 343 approach of the center-of-mass position of the polymer—i.e., the minimum of  $r_{cm}(t)/R$  over all  
 344  $t$ —from which we can see that more work was done on polymers which migrated further in.  
 345 This indicates that more work must be done on maintaining the stretched-out conformation which  
 346 polymers adopt in the high-shear region of the helix core.

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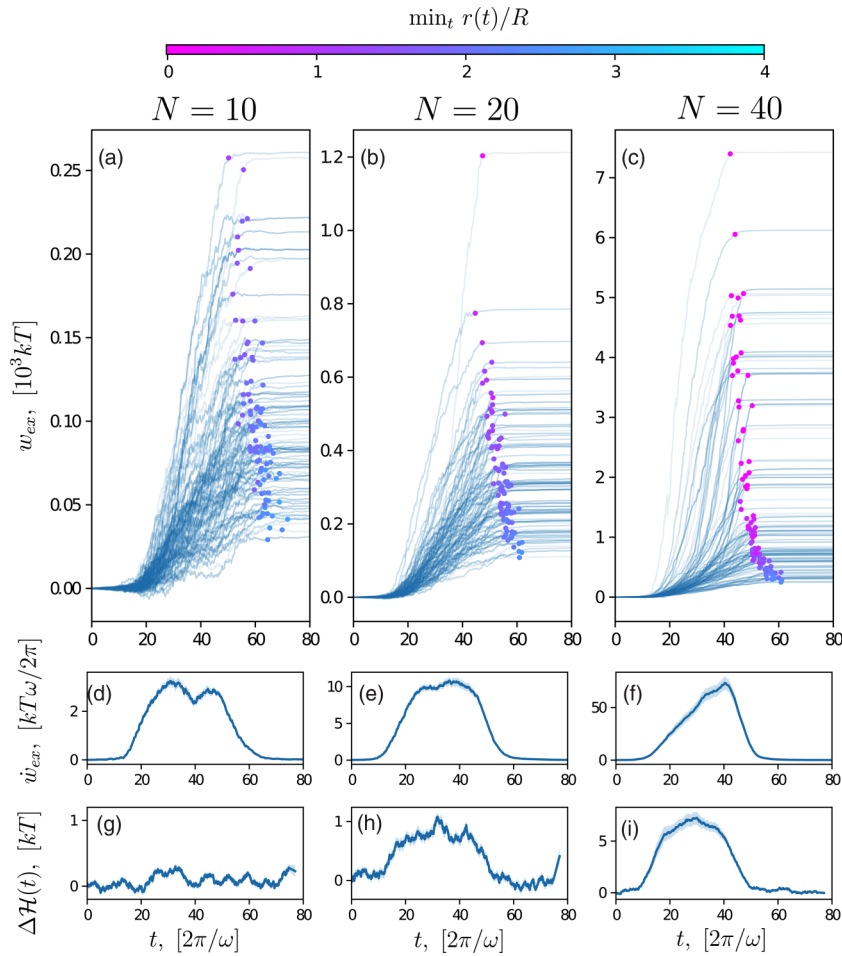


FIG. 6. [(a)–(c)] Stochastic excess work performed by a rotating helix in transporting polymers of size  $N = (10, 20, 40)$  respectively in 200 experiments performed for each case. The markers correspond to the point at which the center of mass of the polymer exits the negative end of the helix. Once this occurs, the polymers collapse and advect with the fluid; no more work is performed on them. The markers are color coded by the point of closest approach to the central long axis of the helix for the polymer in that experiment,  $\min_t(r)$ . More work on average was performed to transport polymers that migrated nearer (magenta) the central axis than on polymers which failed to become captured (cyan). [(d)–(f)] Mean excess power per revolution, averaged over 200 simulations. For each time step,  $dw_{ex}$  was calculated and smoothed with a window size of  $\sim 1/4$  revolution. The shaded region corresponds to the standard error on the mean. [(g)–(i)] Mean increase in polymer energy calculated using the potentials  $\mathcal{H}_{\text{repel}}$  and  $\mathcal{H}_{\text{bond}}$  with the same smoothing and averaging procedure employed in panels (d)–(f).

347 The trajectories are nonmonotonic, and some trajectories temporarily deviate into the negative  
 348 work region, which is a hallmark of the thermodynamics of stochastic systems. We further notice  
 349 that once the polymer has exited the helix,  $w_{ex}(t)$  flattens dramatically. This corresponds to the  
 350 observation that when the polymer is deposited at the rear end of the helix, it quickly collapses to its  
 351 equilibrium configuration and highlights that the rotation of the helix principally impacts polymers  
 352 in its immediate vicinity. Furthermore, the fact that  $w_{ex}(t)$  does not overshoot its final value tells  
 353 us that the helix does not regain any of the work it has supplied to the polymer when the polymer  
 354 relaxes. Indeed, the scale of  $w_{ex}$  is much larger than the stored energy in the polymer, suggesting that

355 this energy is being dissipated into the environment. This is in contrast to the observed enhancement  
 356 of swimming due to the energetics of noiseless elastic surroundings [48,49]. In such systems, elastic  
 357 networks or tubes which a swimmer swims through store elastic energy and transfer this energy  
 358 back to during relaxation. However, in our system, the heat bath to which the polymer is attached  
 359 robs the helix of any such energy storage mechanism.

360 To gain further insight, we measured the change in energy stored in the polymers,  $\Delta\mathcal{H}(t) =$   
 361  $\Delta\mathcal{H}_{\text{bond}}(t) + \Delta\mathcal{H}_{\text{repel}}(t)$  (where  $\Delta$  corresponds to a difference with respect to the quantities evaluated  
 362 at  $t = 0$ ). After smoothing these traces using a Savitzky-Golay filter with a window size of  $\sim 1/4$   
 363 helix revolutions, and then averaging over all simulations, we obtain an indication of how the energy  
 364 stored in the polymer increases on average. We also applied the same smoothing and averaging  
 365 procedure to the work increments  $dw_{\text{ex}}(t)$  to compare how the mean energy stored in the polymers  
 366 was related to the work rate of the helix. These results are plotted in Figs. 6(d)–6(i). We see no  
 367 measurable increase in stored energy for the  $N = 10$  polymer, but statistically significant increases  
 368 for the  $N = 20$  and  $N = 40$  polymers. These increases in energy are due to the elongation the larger  
 369 polymers suffer when they are captured by the helix—a phenomenon which we have shown varies  
 370 with polymer size. While the energy stored in the  $N = 10$  polymer does not significantly increase,  
 371 the helix still performs a significant amount of work in transporting it. For the  $N = 20$  and  $N = 50$   
 372 polymers, we can see that the helix performs roughly  $10k_B T$  of work per revolution for every  $1k_B T$   
 373 energy maintained in the polymer. Because for the  $N = 10$  polymer the power is on the order of  
 374  $1k_B T$  per revolution, this predicts the mean stored energy to be much less than the its fluctuations  
 375 and hence negligible.

376 In general, the work excess term is typically smaller than the viscous term by  $\sim 4$  orders of  
 377 magnitude. However, this is for a solitary polymer in the vicinity of the helix. In a suspension of  
 378 polymers, we hypothesize that the excess work takes the form of a sum over the work performed on  
 379 each polymer and so will be proportional to the local density of the solution at least in the dilute limit  
 380 where polymer-polymer interactions can be ignored. As we have shown separately, the effect of the  
 381 helix is to increase the local density of polymeric material so we expect this, combined with the work  
 382 done on stretching the polymers, to give rise to strongly nonlinear viscoelastic effects. This offers  
 383 some contrast to the hypothesis that bacterial flagella on their own deplete their local environment  
 384 of biopolymeric material and hence experience only the background Newtonian solvent [12].

## 385 VI. CONCLUSIONS

386 Microbes live in complex fluidic environments, often of their own making. Microbial extracellular  
 387 polymeric substances are continually secreted for a wide variety of purposes [50,51], including  
 388 anchoring to surfaces by long mucous stalks [18,19], bioaccumulation of contaminants [52], and to  
 389 serve as the polymeric matrix within biofilms [53], veils [54], and other collective structures [55].  
 390 Motile swimmers must move through these complex media, while sessile microorganisms drive the  
 391 transport of large high-conformational-entropy biopolymers. Previous work has focused on feeding  
 392 currents [13] entraining nutrients modelled as tracer particles (which can be well described by  
 393 hydrodynamic multipole expansion methods) [56,57] or on the continuum limit of a viscoelastic fluid  
 394 medium through which microbes must swim [1,7,58–60] or pump [61,62] fluid. Both approaches  
 395 average over the many internal degrees of freedom of long, flexible biopolymers.

396 We have shown how Stokesian dynamics simulations of a rotating helical filament can accurately  
 397 model the near-field fluid flow of a flagellated cell tethered to a wall and studied the effects of this  
 398 flow field on nearby coarse-grained polymers of various size. We have shown that long polymers are  
 399 strongly attracted to the model flagellum and undergo a nonequilibrium stretching process as they  
 400 are pulled toward it and pumped along it. This implies that a dilute suspension of polymers tend to  
 401 become locally concentrated in and around the flagellum rather than depleted.

402 Our results show that it is possible in simulations to measure the work applied to the polymer by  
 403 the helix and that this is on average positive. Moreover, all of the work supplied to the polymer is  
 404 dissipated, meaning that there is no elastic reclamation by the helix of the polymer’s free energy when

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405 it collapses upon exit of the helix such as that observed in noiseless systems [48,49]. Hence, our results  
 406 provide some fundamental phenomenological insights into activity in microscopic, viscoelastic  
 407 systems. In future work, we hope to tackle the question of a swimming helix (i.e., one in which  $v_z$   
 408 is set to a speed that ensures the force-free condition:  $f_z = 0$ ), as well as testing the hypothesis that  
 409 multiple polymers increase the excess work done by the helix by a proportionate amount.

## ACKNOWLEDGMENTS

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## APPENDIX: MULTIPARTICLE COLLISION DYNAMICS SIMULATION

415  
 416 To verify that our SD helix model well approximates the near-field flow of a sessile flagellated  
 417 microbe in the presence of environmental boundaries in a manner that does not depend significantly  
 418 on cell-body-induced or wall-induced hydrodynamic interactions, we construct a more biologically  
 419 accurate but computationally costly model of a wall-tethered bacterium and simulate it using  
 420 multiparticle collision dynamics (MPCD).

421 MPCD is a particle-based method to solve the Navier-Stokes equations on a coarse-grained level  
 422 where particle dynamics and interactions are solved in alternating streaming and collision steps  
 423 [63,64]. This method has been used successfully to model the hydrodynamics of microswimmers  
 424 near surfaces [65–72] and wall-tethered flagella [73,74]. The fluid is modeled by pointlike effective  
 425 particles with mass  $m$  at positions  $\mathbf{r}_i$  with velocities  $\mathbf{v}_i$ .

426 In the streaming step, the fluid particles move ballistically for a time step  $\delta t$ , and their positions  
 427 are updated according to

$$\mathbf{r}_i(t + \delta t) = \mathbf{r}_i(t) + \mathbf{v}_i(t)\delta t. \quad (\text{A1})$$

428 In the collision step, particles are sorted into cubic cells of side length  $h$ , and all particles in a cell  
 429 stochastically exchange momentum according to

$$\mathbf{v}_i(t + \delta t) = \mathbf{u}_\xi + \mathbf{v}_r + \mathbf{v}_p + \mathbf{v}_L, \quad (\text{A2})$$

430 where  $\mathbf{u}_\xi$  is the mean velocity in the cell,  $\mathbf{v}_r$  is a random velocity drawn from a Maxwell-Boltzmann  
 431 distribution at temperature  $T$ , and  $\mathbf{v}_p$  and  $\mathbf{v}_L$  are correction factors to conserve momentum and  
 432 angular momentum in the cell [75]. All physical quantities are measured in units of cell length  $h$ ,  
 433 fluid mass  $m$ , and thermal energy  $k_B T$ . We use a time step  $\delta t = 0.02\sqrt{mh^2/k_B T}$  and a mean  
 434 number of fluid particles per cell  $\gamma = 10$ , resulting in high Schmidt and Mach numbers to reproduce  
 435 near-incompressible viscous Newtonian flows at low Reynolds number [76].

436 Figure 4(d) includes a representation of the bacterium model we use. The cell body itself  
 437 is modeled as a rigid superellipsoid [77] defined by the surface  $[(x/h_x)^{2/\epsilon_2} + (y/h_y)^{2/\epsilon_2}]^{\epsilon_2/\epsilon_1} +$   
 438  $(z/h_z)^{2/\epsilon_1} = 1$ , where we use  $h_x = h_y = 2h$ ,  $h_z = 4h$ ,  $\epsilon_1 = 0.5$ , and  $\epsilon_2 = 1$ . It is oriented  
 439 perpendicular to a wall (located at  $z = -50h$ ) and fixed at  $(x_0, y_0, z_0) = (0, 0, -46h)$ . We add a  
 440 second wall far away from the bacterium at  $z = 50h$ , and use periodic boundary conditions in the  
 441  $x$  and  $y$  directions with  $x, y, \in (-50h, 50h)$ . The flagellum is modeled as a rigid helical polymer  
 442 consisting of 57 pointlike beads of mass  $10m$ , which are separated by  $1h$ . The helix is given by the  
 443 curve

$$\mathbf{r}(z) = (R[1 - e^{-(\kappa z/l_s)^2}] \cos(\kappa z), R[1 - e^{-(\kappa z/l_s)^2}] \sin(\kappa z), z), \quad (\text{A3})$$

444 where we use  $R = 2h$  as the helix radius, pitch  $\kappa = 1/h$ , and  $\kappa z$  is the phase of the helix measured  
 445 from where it meets the body. A nonzero Higdon parameter [78]  $l_s = 3$  ensures that the helix is  
 446 attached at the center of the cell body surface.

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447 To model no-slip boundary conditions at the walls and the cell surface, a bounce-back rule for the  
 448 fluid particles is used [64]. The transfer of momentum between the flagellum and the fluid is achieved  
 449 by including the flagellum beads into the collision step [79]. While the cell body is kept fixed during  
 450 the simulation, the helix is rotated with a constant angular velocity  $\omega = (5.6 \times 10^{-4})/\delta t$ , and the  
 451 surrounding flow field is measured for a time  $t = 10^5 \delta t$  and averaged over 70 independent runs.

- 
- 2
- [1] B. Liu, T. R. Powers, and K. S. Breuer, Force-free swimming of a model helical flagellum in viscoelastic fluids, *Proc. Nat. Acad. Sci. USA* **108**, 19516 (2011).
  - [2] D. A. Gagnon, X. Shen, and P. E. Arratia, Undulatory swimming in fluids with polymer networks, *EPL* **104**, 14004 (2013).
  - [3] S. E. Spagnolie, B. Liu, and T. R. Powers, Locomotion of Helical Bodies in Viscoelastic Fluids: Enhanced Swimming at Large Helical Amplitudes, *Phys. Rev. Lett.* **111**, 068101 (2013).
  - [4] B. Thomases and R. D. Guy, Mechanisms of Elastic Enhancement and Hindrance for Finite-Length Undulatory Swimmers in Viscoelastic Fluids, *Phys. Rev. Lett.* **113**, 098102 (2014).
  - [5] E. E. Riley and E. Lauga, Enhanced active swimming in viscoelastic fluids, *EPL* **108**, 34003 (2014).
  - [6] A. E. Patteson, A. Gopinath, M. Goulian, and P. E. Arratia, Running and tumbling with *E. coli* in polymeric solutions, *Sci. Rep.* **5** (2015).
  - [7] E. Lauga, Propulsion in a viscoelastic fluid, *Phys. Fluids* **19**, 083104 (2007).
  - [8] H. C. Fu, T. R. Powers, and C. W. Wolgemuth, Theory of Swimming Filaments in Viscoelastic Media, *Phys. Rev. Lett.* **99**, 258101 (2007).
  - [9] X. N. Shen and P. E. Arratia, Undulatory Swimming in Viscoelastic Fluids, *Phys. Rev. Lett.* **106**, 208101 (2011).
  - [10] L. Zhu, E. Lauga, and L. Brandt, Self-propulsion in viscoelastic fluids: Pushers vs. pullers, *Phys. Fluids* **24**, 051902 (2012).
  - [11] B. Qin, A. Gopinath, J. Yang, J. P. Gollub, and P. E. Arratia, Flagellar kinematics and swimming of algal cells in viscoelastic fluids, *Sci. Rep.* **5**, 9190 (2015).
  - [12] V. A. Martinez, J. Schwarz-Linek, M. Reufer, L. G. Wilson, A. N. Morozov, and W. C. K. Poon, Flagellated bacterial motility in polymer solutions, *Proc. Nat. Acad. Sci. USA* **111**, 17771 (2014).
  - [13] J. J. L. Higdon, The generation of feeding currents by flagellar motions, *J. Fluid Mech.* **94**, 305 (1979).
  - [14] K. C. Marshall, Planktonic versus sessile life of prokaryotes, in *The Prokaryotes: Prokaryotic Communities and Ecophysiology*, edited by E. Rosenberg, E. F. DeLong, S. Lory, E. Stackebrandt, and F. Thompson (Springer, Berlin, 2013), pp. 191–201.
  - [15] E. Lauga and T. R. Powers, The hydrodynamics of swimming microorganisms, *Rep. Prog. Phys.* **72**, 096601 (2009).
  - [16] J. Elgeti, R. G. Winkler, and G. Gompper, Physics of microswimmers—single particle motion and collective behavior: A review, *Rep. Prog. Phys.* **78**, 056601 (2015).
  - [17] R. E. Pepper, M. Roper, S. Ryu, P. Matsudaira, and H. A. Stone, Nearby boundaries create eddies near microscopic filter feeders, *J. R. Soc. Interf.* **7**, 851 (2009).
  - [18] A. P. Petroff, A. L. Pasulka, N. Soplod, X.-L. Wu, and A. Libchaber, Biophysical basis for convergent evolution of two veil-forming microbes, *R. Soc. Open Sci.* **2**, 150437 (2015).
  - [19] A. P. Petroff and A. Libchaber, Hydrodynamics and collective behavior of the tethered bacterium *Thiovulum majus*, *Proc. Nat. Acad. Sci. USA* **111**, E537 (2014).
  - [20] J. S. Guasto, R. Rusconi, and R. Stocker, Fluid mechanics of planktonic microorganisms, *Annu. Rev. Fluid Mech.* **44**, 373 (2012).
  - [21] D. L. Ermak and J. A. McCammon, Brownian dynamics with hydrodynamic interactions, *J. Chem. Phys.* **69**, 1352 (1978).

3

  - [22] J. Rotne and S. Prager, *J. Chem. Phys.* **50**, 4831 (1969).
  - [23] H. Yamakawa, *J. Chem. Phys.* **53**, 436 (1970).
  - [24] G. W. Slater, C. Holm, M. V. Chubynsky, H. W. de Haan, A. Dubé, K. Grass, O. A. Hickey, C. Kingsburry, D. Sean, T. N. Shendruk, and L. Zhan, Modeling the separation of macromolecules: A review of current computer simulation methods, *Electrophoresis* **30**, 792 (2009).

## BIOPOLYMER DYNAMICS DRIVEN BY HELICAL FLAGELLA

- [25] G. S. Grest and K. Kremer, Molecular dynamics simulation for polymers in the presence of a heat bath, *Phys. Rev. A* **33**, 3628 (1986).
- [26] The pitch as usually defined as the length of a single complete turn, which in our notation is  $2\pi/\kappa$ .
- [27] J. Hu, M. Yang, G. Gompper, and R. G. Winkler, Modelling the mechanics and hydrodynamics of swimming *E. coli*, *Soft Matter* **11**, 7867 (2015).
- [28] J. Yuan and H. C. Berg, Following the behavior of the flagellar rotary motor near zero load, *Exp. Mech.* **50**, 1263 (2010).
- [29] J. Xing, F. Bai, R. Berry, and G. Oster, Torque-speed relationship of the bacterial flagellar motor, *Proc. Nat. Acad. Sci. USA* **103**, 1260 (2006).
- [30] M. J. Kim, J. C. Bird, A. J. Van Parys, K. S. Breuer, and T. R. Powers, A macroscopic scale model of bacterial flagellar bundling, *Proc. Nat. Acad. Sci. USA* **100**, 15481 (2003).
- [31] M. J. Kim, M. J. Kim, J. C. Bird, J. Park, T. R. Powers, and K. S. Breuer, Particle image velocimetry experiments on a macro-scale model for bacterial flagellar bundling, *Exp. Fluids* **37**, 782 (2004).
- [32] T. S. Yu, E. Lauga, and A. E. Hosoi, Experimental investigations of elastic tail propulsion at low Reynolds number, *Phys. Fluids* **18**, 091701 (2006).
- [33] N. Coq, O. du Roure, M. Fermigier, and D. Bartolo, Helical beating of an actuated elastic filament, *J. Phys.: Condens. Matter* **21**, 204109 (2009).
- [34] S. Zhong, K. W. Moored, V. Pinedo, J. Garcia-Gonzalez, and A. J. Smits, The flow field and axial thrust generated by a rotating rigid helix at low Reynolds numbers, *Exp. Therm. Fluid Sci.* **46**, 1 (2013).
- [35] J. Lighthill, *Flagellar hydrodynamics*, (1976).
- [36] B. Rodenborn, C.-H. Chen, H. L. Swinney, B. Liu, and H. P. Zhang, Propulsion of microorganisms by a helical flagellum, *Proc. Nat. Acad. Sci. USA* **110**, E338 (2013).
- [37] W. Gao, X. Feng, A. Pei, C. R. Kane, R. Tam, C. Hennessy, and J. Wang, Bioinspired helical microswimmers based on vascular plants, *Nano Lett.* **14**, 305 (2014).
- [38] K. E. Peyer, L. Zhang, and B. J. Nelson, Bio-inspired magnetic swimming microrobots for biomedical applications, *Nanoscale* **5**, 1259 (2013).
- [39] S. E. Spagnolie and E. Lauga, Hydrodynamics of self-propulsion near a boundary: Predictions and accuracy of far-field approximations, *J. Fluid Mech.* **700**, 105 (2012).
- [40] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevF.xx.xxxxx> for two videos of a typical capture process.
- [41] M. D. Graham, Fluid dynamics of dissolved polymer molecules in confined geometries, *Annu. Rev. Fluid Mech.* **43**, 273 (2011).
- [42] T. N. Shendruk, D. Sean, D. J. Berard, J. Wolf, J. Dragoman, S. Battat, G. W. Slater, and S. R. Leslie, Rotation-induced macromolecular spooling of DNA, *Phys. Rev. X* **7**, 031005 (2017).
- [43] A. R. Klotz, V. Narsimhan, B. W. Soh, and P. S. Doyle, Dynamics of DNA knots during chain relaxation, *Macromolecules* **50**, 4074 (2017).
- [44] R. G. Winkler, Conformational and rheological properties of semiflexible polymers in shear flow, *J. Chem. Phys.* **133**, 164905 (2010).
- [45] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevF.xx.xxxxx> for animations of the density functions as a function of time.
- [46] T. N. Shendruk, D. Sean, D. Berard, J. Wolf, J. Dragoman, S. Battat, G. W. Slater, and S. R. Leslie, Spin-induced macromolecular spooling, *Phys. Rev. X* (submitted).
- [47] U. Seifert, Stochastic thermodynamics, fluctuation theorems and molecular machines, *Rep. Prog. Phys.* **75**, 126001 (2012).
- [48] R. Ledesma-Aguilar and J. M. Yeomans, Enhanced motility of a microswimmer in rigid and elastic confinement, *Phys. Rev. Lett.* **111**, 138101 (2013).
- [49] J. K. Wrobel, S. Lynch, A. Barrett, L. Fauci, and R. Cortez, Enhanced flagellar swimming through a compliant viscoelastic network in Stokes flow, *J. Fluid Mech.* **792**, 775 (2016).
- [50] A. W. Decho, Microbial exopolymer secretions in ocean environments: Their role(s) in food webs and marine processes, *Oceanogr. Mar. Biol. Annu. Rev.* **28**, 73 (1990).
- [51] G. M. Wolfaardt, J. R. Lawrence, and D. R. Korber, Function of EPS, in *Microbial Extracellular Polymeric Substances* (Springer, Berlin, 1999), pp. 171–200.

4

5

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- [52] G. M. Wolfaardt, J. R. Lawrence, J. V. Headley, R. D. Robarts, and D. E. Caldwell, Microbial exopolymers provide a mechanism for bioaccumulation of contaminants, *Microbial Ecol.* **27**, 279 (1994).
- [53] H.-C. Flemming, T. R. Neu, and D. J. Wozniak, The EPS matrix: The “house of biofilm cells,” *J. Bacteriol.* **189**, 7945 (2007).
- Q [54] C. O. Wirsen and H. W. Jannasch, Physiological and morphological observations on *Thiovulum* sp., *J. Bacteriol.* **136**, 765 (1978).
- Q [55] C. Schaudinn, P. Stoodley, A. Kainovic, T. O. Keeffe, B. Costerton, D. Robinson, M. Baum, G. Ehrlich, and P. Webster, Bacterial biofilms, other structures seen as mainstream concepts, *Microbe* **2**, 231 (2007).
- [56] A. J. T. M. Mathijssen, D. O. Pushkin, and J. M. Yeomans, Tracer trajectories and displacement due to a micro-swimmer near a surface, *J. Fluid Mech.* **773**, 498 (2015).
- [57] A. J. T. M. Mathijssen, A. Doostmohammadi, J. M. Yeomans, and T. N. Shendruk, Hydrodynamics of micro-swimmers in films, *J. Fluid Mech.* **806**, 35 (2016).
- [58] B. Liu, M. Gulino, M. Morse, J. X. Tang, T. R. Powers, and K. S. Breuer, Helical motion of the cell body enhances *Caulobacter crescentus* motility, *Proc. Nat. Acad. Sci. USA* **111**, 11252 (2014).
- [59] E. Lauga, Locomotion in complex fluids: Integral theorems, *Phys. Fluids* **26**, 081902 (2014).
- [60] A. E. Patteson, A. Gopinath, and P. E. Arratia, Active colloids in complex fluids, *Curr. Opin. Colloid Interface Sci.* **21**, 86 (2016).
- Q [61] M. A. Sleight, J. R. Blake, and N. Liron, The propulsion of mucus by cilia, *Am. J. Respir. Crit. Care Med.* **137**, 726 (1988).
- [62] J. R. Véllez-Cordero and E. Lauga, Waving transport and propulsion in a generalized Newtonian fluid, *J. Non-Newtonian Fluid Mech.* **199**, 37 (2013).
- Q [63] R. Kapral, Multiparticle collision dynamics: Simulation of complex systems on mesoscales, *Adv. Chem. Phys.* **140**, 89 (2008).
- Q [64] G. Gompper, T. Ihle, D. M. Kroll, and R. G. Winkler, Multi-particle collision dynamics: A particle-based mesoscale simulation approach to the hydrodynamics of complex fluids, *Adv. Polym. Sci.* **221**, 1 (2009).
- [65] J. Elgeti and G. Gompper, Self-propelled rods near surfaces, *Europhys. Lett. (EPL)* **85**, 38002 (2009).
- [66] J. Elgeti, U. B. Kaupp, and G. Gompper, Hydrodynamics of sperm cells near surfaces, *Biophys. J.* **99**, 1018 (2010).
- [67] A. Zöttl and H. Stark, Nonlinear Dynamics of a Microswimmer in Poiseuille Flow, *Phys. Rev. Lett.* **108**, 218104 (2012).
- [68] A. Zöttl and H. Stark, Hydrodynamics Determines Collective Motion and Phase Behavior of Active Colloids in Quasi-Two-Dimensional Confinement, *Phys. Rev. Lett.* **112**, 118101 (2014).
- [69] K. Schaar, A. Zöttl, and H. Stark, Detention Times of Microswimmers Close to Surfaces: Influence of Hydrodynamic Interactions and Noise, *Phys. Rev. Lett.* **115**, 038101 (2015).
- [70] J. Hu, A. Wysocki, R. G. Winkler, and G. Gompper, Physical sensing of surface properties by microswimmers—directing bacterial motion via wall slip, *Sci. Rep.* **5**, 9586 (2015).
- [71] M. Theers, E. Westphal, G. Gompper, and R. G. Winkler, Modeling a spheroidal microswimmer and cooperative swimming in a narrow slit, *Soft Matter* **12**, 7372 (2016).
- [72] T. Eisenstecken, J. Hu, and R. G. Winkler, Bacterial swarmer cells in confinement: A mesoscale hydrodynamic simulation study, *Soft Matter* **12**, 8316 (2016).
- [73] S. Y. Reigh, R. G. Winkler, and G. Gompper, Synchronization and bundling of anchored bacterial flagella, *Soft Matter* **8**, 4363 (2012).
- [74] S. Y. Reigh, R. G. Winkler, and G. Gompper, Synchronization, slippage, and unbundling of driven helical flagella, *PLoS ONE* **8**, e70868 (2013).
- [75] H. Noguchi, N. Kikuchi, and G. Gompper, Particle-based mesoscale hydrodynamic techniques, *Europhys. Lett. (EPL)* **78**, 10005 (2007).
- [76] J. T. Padding and A. A. Louis, Hydrodynamic interactions and Brownian forces in colloidal suspensions: Coarse-graining over time and length scales, *Phys. Rev. E* **74**, 031402 (2006).
- [77] A. H. Barr, Superquadrics and angle-preserving transformations, *IEEE Comput. Graph. Appl.* **1**, 11 (1981).
- [78] J. J. L. Higdon, A hydrodynamic analysis of flagellar propulsion, *J. Fluid Mech.* **90**, 685 (1979).
- [79] A. Malevanets and J. M. Yeomans, Dynamics of short polymer chains in solution, *EPL* **52**, 231 (2000).