

# IMPACT OF CORONARY FLOW ON PARAMETERS OF A LUMPED MODEL OF THE ARTERIAL TREE

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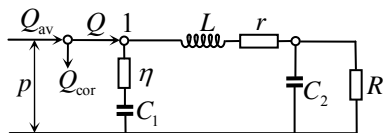
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## 1. Introduction

The arterial tree is a network of visco-elastic blood vessels, which delivers blood to the whole body. The simplest models of the arterial tree are lumped models, which describe the arterial tree as a system of compliant chambers. The lumped models are attractive for clinicians because they describe the function of the whole arterial tree in terms of simple parameters such as compliance, resistance, and inertance [1]. The main purpose of lumped models is in modeling the arterial input impedance (the pressure to flow ratio in the frequency domain) as an afterload to the left ventricle. The lumped models are used in defining the total arterial compliance and resistance, which can explain changes of arterial trees due to ageing and disease.

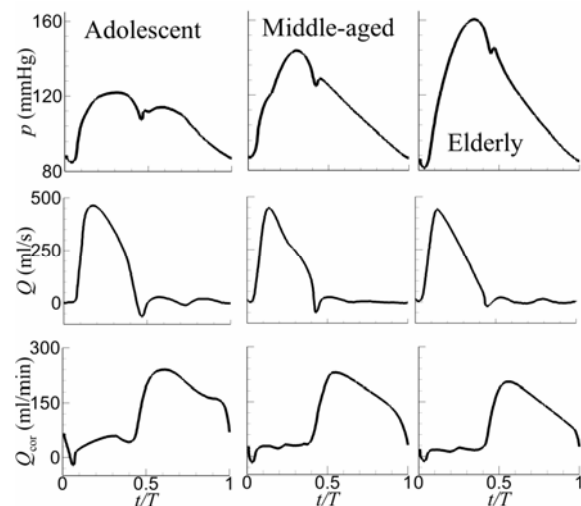
Fig.1 shows an electrical analogue scheme of the six-element lumped model of the arterial tree. At the inlet of the arterial tree the aortic valve flow  $Q_{av}$  is divided into the coronary flow  $Q_{cor}$  and ascending aorta flow  $Q$ .



**Fig. 1:** Schematic representation of the selected lumped model of the arterial tree

The concept of input impedance requires a linear model of the arterial tree. The model of the coronary circulation is complex and highly non-linear, therefore it should be excluded from the arterial tree. By doing this, the beginning of the arterial tree becomes the ascending aorta, denoted by point 1 in Fig. 1. During diastole  $Q_{av}$  is equal to zero, it is clear that  $Q_{cor}$  comes

from the compliant aorta (modeled by  $C_1$  in Fig. 1). Therefore, the measured coronary flow should be included in the arterial tree model in the point 1 in Fig. 1. The goal of this paper is to compare parameters of the arterial tree models with and without coronary flow at the inlet of the arterial tree. The models will be applied to three typical subjects (adolescent, middle-aged and elderly) with recorded data as shown in Fig 2.



**Fig. 2:** Recorded pressure (top) and flow (middle) in the ascending aorta and coronary flow (bottom), for the three subject, digitized from [2] ( $T$  is the heart period)

The model parameters are obtained by minimizing the pressure RMSE, with prescribed measured flows at the arterial tree inlet. RMSE is defined as

$$RMSE = \sqrt{\sum_1^n (p^{calc} - p)^2 / n} \quad (1)$$

where  $p^{calc}$  is the calculated and  $p$  measured aortic pressure. The impact of coronary flow will be quantified it terms of a changes of RMSE and model parameters.

Parameter	Adolescent			Middle-aged			Elderly		
$100\bar{Q}_{cor} / \bar{Q}$	0	1.739	3.478 <sup>†</sup>	0	1.774	3.548 <sup>†</sup>	0	1.487	2.974 <sup>†</sup>
$C_1$	1.707	1.722	1.737	0.735	0.749	0.776	0.495	0.502	0.512
$10\eta$	0.423	0.421	0.420	1.843	1.795	1.700	0.927	0.931	0.926
$1000L$	19.49	19.45	19.40	2.857	2.860	2.933	3.361	3.280	3.238
$10r$	0.000	0.000	0.000	0.519	0.523	0.522	0.720	0.716	0.714
$C_2$	0.436	0.439	0.442	0.591	0.590	0.573	0.298	0.299	0.297
$R$	0.908	0.924	0.941	1.063	1.083	1.104	1.130	1.149	1.167
$R_{cor} = \bar{p} / \bar{Q}_{cor}$	-	52.20	26.10	-	62.87	31.44	-	80.85	40.42
$R_{tot}$	0.908	0.908	0.908	1.115	1.115	1.115	1.202	1.202	1.202
$C_{tot} = C_1 + C_2$	2.143	2.161	2.179	1.326	1.338	1.349	0.793	0.804	0.809
RMSE	1.894	1.874	1.854	1.528	1.545	1.565	1.222	1.182	1.146

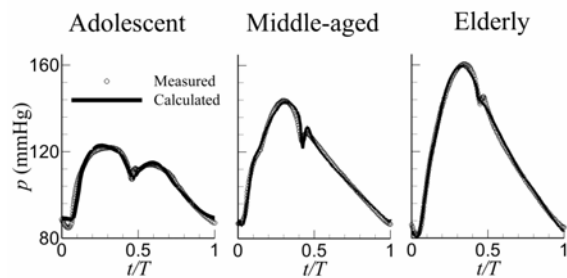
**Tab. 1:** Obtained values of the model parameters ( $C_1$ ,  $C_2$ , and  $C_{tot}$  are in ml/mmHg;  $L$  is in mmHg·s<sup>2</sup>/ml;  $r$ ,  $R$ ,  $R_{cor}$ , and  $R_{tot}$  are in mmHg·s/ml; RMSE is in mmHg; <sup>†</sup> $2Q_{cor}$ ; overline denotes the average value)

## 2. Mathematical model

The six-element model shown in Fig.1 was chosen because of its good ability to reproduce the pressure shape from the measured aortic flow in all three cases. In this model  $C_1$  and  $C_2$  denote proximal and distal compliance, respectively  $\eta$  models the arterial wall viscosity, while  $r$  and  $R$  model arterial and peripheral bed resistance, respectively. The coronary flow is prescribed in the point 1.

## 3. Results

The model parameters are identified from the measured data for the tree subjects from Fig. 2, for three cases: (a) with neglected coronary flow, (b) with prescribing the measured coronary flow, and (c) with prescribing the measured coronary flow doubled. Fig. 3 illustrates the ability of the selected model to reconstruct the measured pressure.



**Fig. 3:** Measured and calculated pressure for the three subjects (with no coronary flow in the model).

Tab. 1 summarizes the obtained values of the model parameters. Averaged coronary flow  $\bar{Q}_{cor}$  is in the range from 1.487 to 3.548 percent

of the average ascending aorta flow  $\bar{Q}$ . The coronary resistance  $R_{cor}$  is defined as the ratio of the average aortic pressure to average coronary flow. The total arterial resistance is defined by the following expression:

$$\frac{1}{R_{tot}} = \frac{1}{R_{cor}} + \frac{1}{R+r} \quad (2)$$

It remains constant regardless to the value of the coronary flow. The total arterial compliance  $C_{tot}$  increases with increasing the coronary flow. In all cases the relative increase of  $C_{tot}$  is less than the 2.1 %. The relative changes of  $\eta$  and  $L$  with increasing the coronary flow are negligible. The coronary flow has no significant impact on RMSE.

## 4. Conclusion

The inclusion of the coronary flow into the lumped model of the arterial tree, does not significantly improve the model because its impact on the model parameters is small.

## 5. Acknowledgements

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## 6. References

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- [2] Nichols W.W., O'Rourke M.F.: McDonald's Blood flow in Arteries. Theoretical, experimental and clinical principles, (2005) 5<sup>th</sup> ed., Arnold, London.