CONTROL OF NANOROBOT MOTION IN A MULTIPOTENTIAL FIELD

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Abstract

A relativistic Hamiltonian of the nanorobot motion in a multipotential field that includes external artificial control potential field has been presented by Novakovic et al. in 2009. Starting with the non-relativistic approximation of that Hamiltonian, the canonical differential equations of the nanorobot motion in a multipotential field has been derived (Novakovic et al., 2010). In this paper we continue to investigate the related control algorithms for a nanorobot motion in a multipotential field. In that sense the concept of the external linearization has been introduced to the canonical differential equations of the nanorobot motion in a multipotential field. We use the nonlinear control algorithm that in the closed loop with the nonlinear canonical differential equations of the nanorobot motion is resulting in the linear behavior of the whole system. In that case the well known procedures for control synthesis of the linear systems can be applied to the control of a nanorobot motion in a multipotential field.

1. INTRODUCTION

As it is the well known, the nanorobotics is the multidisciplinary field that deals with the controlled manipulation with atomic and molecular-sized objects and therefore sometimes is called molecular robotics (A. A. G. Requicha, 2008). The state of the art in nanorobotics has been presented by Novakovic et al. in 2009. Generally, there are two main approaches for building useful devices from nanoscale components. The first one is based on self-assembly, and is a natural evolution of traditional chemistry and bulk processing (Gómez-López *et al*. 1996). The second approach is based on control of the positions and velocities of nanoscale objects by direct application of mechanical forces, electromagnetic fields, and the other potential fields. The research in nanorobotics in the second approach has proceeded along two lines. The first one is devoted to the design and computational simulation of robots with nanoscale dimensions (Drexler 1992). These nanorobots have various mechanical components such as nanogears built primarily with carbon atoms in a diamondoid structure. A big problem is how to build these nanoscale devices.

The second line of nanorobotics research involves manipulation of nanoscale objects with macroscopic instruments and related potential fields. Here it is pointed out the nanorobotics research that involves controlled manipulation of nanoscale objects with macroscopic instruments and related control potential fields. It is being studied by researchers, who are focusing on techniques based on Scanning Probe Microscopy (SPM). Experimental work has been focused on this area, especially through the use of SPMs as robots. This experimental approach follows the technique of the Scanning Tunneling Microscope (STM) that was invented by Binnig and Rohrer at the IBM Zürich laboratory in the early 1980s. The STM is useful at Ångstrom-scale distances (1 Å = 0.1 nm = 10^{-10} m), where a quantum-mechanical effect, called tunneling, and the piezoelectric actuators can be employed for the position control. The major limitation of the STM is that it only worked with conducting materials such as metals or semiconductors, but not with insulators or biological structures such as DNA. To remedy this situation, Binning, Quate and Gerber developed in 1986 the Atomic Force Microscope (AFM) which is sensitive directly to the forces between the tip and the sample (particle), rather than a tunneling current. Therefore, the all instruments, based on the interatomic forces (Stroscio and Eigler 1991), are called the Atomic Force Microscope (AFM).

The AFM does not require conducting tips and samples, and therefore has wider applicability than the STM. An AFM can operate in at least three modes. In attractive or non-contact mode the tip is held some tens of nanometers above the sample surface where it experiences the attractive combination of van der Waals, electrostatic, or magnetostatic forces. In repulsive or contact mode the tip is pressed close enough to the surface for tip and sample electron clouds to overlap, generating a repulsive electrostatic force (about

10nN), much like the stylus riding a groove in record player. There is also intermittent - contact mode, which is sometimes called tapping mode. All of these instruments are collectively known as Scanning Probe Microscopes (SPMs). For more information on SPM technology one can see the references (Wiesendanger 1994 and Freitas Jr. 1999). Although the SPM is not even twenty years old, it has had a large scientific impact. There are a lot of references on SPM applications, scattered through many journals such as Science and the Journal of Vacuum Science and Technology, and also in proceedings of meetings such as the biennial conference on Scanning Tunneling Microscopy. It is expected that nanomanipulation will be coupled with self-assembly in order to build true nanorobots, that is devices with overall dimensions in the nanometer range and capable of sensing, thinking and acting (A. A. G. Requicha, 2008).

The spatial region in nanorobotics is the bionanorobotics. The main goal in this region is to develop novel and revolutionary biomolecular machine components that can be assembled and form multi-degree of freedom nanodevices (M. Hamdi, A. Fereira, G. Sharma and C. Mavroidis, 2007). These bionanodevices should be able to apply forces and manipulate objects in the nanoworld, transfer information from the nano to the macro world, receive the information from the macro world and also be able to travel in the nano environment. Such ultra-miniature robotic systems and nano-mechanical devices should be the biomolecular electro-mechanical hardware of future manufacturing and biomedical and planetary applications (A. Dubey, C. Mavroidis and S.M. Tomassone, 2006). It is also expected that the modern bionanomachines will be employed for space applications in the NASA space traveling advanced concepts (C. Mavroidis, 2006).

Potential applications of the nanorobots are expected in the tree important regions: nanomedicine, nanotechnology and space applications. In nanomedicine the nanorobots can be employed for surgery, early diagnoses, drug delivery at the right place (for destroy a cancer cell),biomedical instrumentation, pharmacokinetics, monitoring of diabetes and genome applications by reading and manipulating DNA (Freitas Jr. 1999). In nanotechnology the nanorobots can be utilized for creation of new materials, nanofabrics for different products, cell probes with small dimensions, computer memory, near field optics, x-ray fabrication, very small batteries and optical antennas. In the space applications it is expected that nanorobots replace of human being in the intergalactic space missions, be hardware and software to fly on satellites and have a high level of an artificial intelligence. The complex tasks of the future nanorobots are sensing, thinking, acting and working cooperatively with the other nanorobots.

In order to control nanorobots in mechanics, electronics, electromagnetic, photonics, chemical and biomaterials regions we have to have the ability to construct the related artificial control potential fields. At the nanoscale the control dynamics is very complex because there are very strong interaction between nano robots and nanoenvironment. Thus, the first step in designing the control dynamics for nanorobots is the development of the relativistic Hamiltonian that will include external artificial potential field. This Hamiltonian has been derived and presented by Novakovic et al. in 2009. Starting with the non-relativistic approximation of that Hamiltonian, the canonical differential equations of the nanorobot motion in a multipotential field have been derived (Novakovic et al., 2010).

In this paper we continue to investigate the related control algorithms for a nanorobot motion in a multipotential field. This paper has been written by consideration of the related contributions and propositions given in the references [1-44]. The problem is to design the nonlinear control of the nonlinear system that in the closed loop with the nonlinear canonical differential equations of the nanorobot motion is resulting in the linear behavior of the whole system. In that case the well known procedures for control synthesis of the linear systems can be applied to the control of a nanorobot motion in a multipotential field. The mentioned problem has been solved in this paper by employing the so called concept of the external linearization [35-41]. This concept is introduced and presented in the section 3. This has been done for a general case with applications to control of a nanorobot motion in a multipotential field. As en example the general approach is elaborated in detail in the section 4 by application to control of a nanorobot motion in the two potential electromagnetic and gravitational field.

The presented control procedures in this paper are related only to the control of nanorobots in the regions where the quantum effects are not present. In the case where the quantum effects are present one should apply dynamics of the quantum feedback systems and control concepts and applications presented by Yanagisawa and Kimura in 2003. Recently, it has been proposed coherent H^{∞} control for a class of annihilation operator linear quantum systems (Maalouf and Petersen, 2011). This control can be applied to the quantum systems that can be described by complex quantum stochastic differential equations in terms of annihilation operators only. For this class of quantum systems, the related control problem can be solved in terms of a pair of complex algebraic Riccati equations. In addition, the question of physical realizability of the resulting quantum controllers is related to a bounded real property.

The organization of this paper is as follows. The second section presents system and problem statements. It is started with the non-relativistic approximation of the Hamiltonian for a nanorobot motion in a multipotential field. It follows the transformation of that Hamiltonian into the canonical differential equations of the nanorobot motion in a multipotential field. The mentioned control problem is defined related to these canonical differential equations. The third section shows the derivation of the concept of the external linearization and its application to control of nanorobot motion in multipotential field. In the fourth section the procedure of the external linearization has been applied to control of a nanorobot motion in the two potential electromagnetic and gravitational field. Finally, the conclusion of the paper with some comments and the reference list are presented in the fifth and sixth sections, respectively.

2. SYSTEM AND PROBLEM STATEMENTS

Let the non-relativistic approximation of the Hamiltonian $\mathcal{H}$ for a nanorobot motion in a multipotential field is given by the relation derived in [2]:

$$\mathcal{H} \cong m_0 c^2 + \frac{1}{2m_0}\left[\begin{matrix}\left(p_x - \frac{v_x U}{c^2}\right)^2 + \left(p_y - \frac{v_y U}{c^2}\right)^2 \\ + \left(p_z - \frac{v_z U}{c^2}\right)^2\end{matrix}\right] + U. \tag{1}$$

Here m_0 is a rest mass of a nanorobot, c is a speed of the light in a vacuum, p_x, p_y, and p_z, as well as v_x, v_y and v_z are momentums and velocities, respectively, in x, **y**, and z directions and U is a total potential energy of a nanorobot in a multipotential field. The momentums of the nanorobot motion can be calculated by the equations:

$$p_x = m_0 v_x, \qquad p_y = m_0 v_y, \qquad p_z = m_0 v_z. \tag{2}$$

At the nanoscale control of a nanorobot motion we usually have the multi-potential field with n-potentials, plus an artificial control potential field of the nanorobot that influents to the nanorobot with a potential energy U_c. Thus, the related total potential energy of a nanorobot in a multipotential field is described by the following relation:

$$U = U_1 + U_2 + .. + U_n + U_c = \Sigma U_j + U_c, \quad j = 1, 2, .., n. \tag{3}$$

In the relation (3) U_j is a potential energy of the nanorobot in the j-th potential field. In the case where there are no quantum mechanical effects one can employ classic Hamiltonian canonic forms for designing equations of the nanorobot motion [29]:

$$\dot{p}_x = -\frac{\partial \mathcal{H}}{\partial x}, \quad \dot{p}_y = -\frac{\partial \mathcal{H}}{\partial y}, \quad \dot{p}_z = -\frac{\partial \mathcal{H}}{\partial z}, \quad \dot{x} = \frac{\partial \mathcal{H}}{\partial p_x}, \quad \dot{y} = \frac{\partial \mathcal{H}}{\partial p_y}, \quad \dot{z} = \frac{\partial \mathcal{H}}{\partial p_z}. \tag{4}$$

Now, one can define the so called interaction terms of a nanorobot motion in a multipotential field:

$$I_x = \frac{v_x U}{c}, \quad I_y = \frac{v_y U}{c}, \quad I_z = \frac{v_z U}{c}. \tag{5}$$

It follows the definition of the interaction forces as functions of the interaction terms:

$$F_{I_x} = \frac{\partial I_z}{\partial y} - \frac{\partial I_y}{\partial z}, \; F_{I_y} = \frac{\partial I_x}{\partial z} - \frac{\partial I_z}{\partial x}, \; F_{I_z} = \frac{\partial I_y}{\partial x} - \frac{\partial I_x}{\partial y}. \tag{6}$$

The next definition is related to the time-varying forces as the functions of the interaction terms:

$$F_{t_x} = -\frac{1}{c}\frac{\partial I_x}{\partial t}, \quad F_{t_y} = -\frac{1}{c}\frac{\partial I_y}{\partial t}, \quad F_{t_z} = -\frac{1}{c}\frac{\partial I_z}{\partial t}. \tag{7}$$

Finally, one can define the potential forces as the function of the total potential energy of a nanorobot in a multipotential field:

$$F_{p_x} = -\frac{\partial U}{\partial x}, \quad F_{p_y} = -\frac{\partial U}{\partial y}, \quad F_{p_z} = -\frac{\partial U}{\partial z}. \tag{8}$$

Applying (1) to (4) and including the relations (5), (6), (7) and (8), one obtains the compact form of the canonical differential equations of the nanorobot motion in a multipotential field as the functions of the mentioned forces:

$$\begin{aligned} m_0\ddot{x} &= F_{p_x} + F_{t_x} + \frac{1}{c}\left(\dot{y}F_{I_z} - \dot{z}F_{I_y}\right), \\ m_0\ddot{y} &= F_{p_y} + F_{t_y} + \frac{1}{c}\left(\dot{z}F_{I_x} - \dot{x}F_{I_z}\right), \\ m_0\ddot{z} &= F_{p_z} + F_{t_z} + \frac{1}{c}\left(\dot{x}F_{I_y} - \dot{y}F_{I_x}\right). \end{aligned} \tag{9}$$

Following the previous consideration one can introduces the following vectors:

$$X=\left[x\ \ y\ \ z\right]^T,\quad \dot{X}=\left[\dot{x}\ \ \dot{y}\ \ \dot{z}\right]^T,\quad \ddot{X}=\left[\ddot{x}\ \ \ddot{y}\ \ \ddot{z}\right]^T,$$
$$F_I=\left[F_{I_x}\ F_{I_y}\ F_{I_z}\right]^T,\quad F_t=\left[F_{t_x}\ F_{t_y}\ F_{t_z}\right]^T, \tag{10}$$
$$F_p=\left[F_{p_x}\ F_{p_y}\ F_{p_z}\right]^T.$$

Including the vectors (10) into the relations (9) one generates the vector-matrix form of the canonical differential equations of the nanorobot motion in a multipotential field:

$$m_0\ddot{X}=F_p+F_t+\frac{1}{c}NF_I,\quad N=\begin{bmatrix}0 & -\dot{z} & \dot{y}\\ \dot{z} & 0 & -\dot{x}\\ -\dot{y} & \dot{x} & 0\end{bmatrix}. \tag{11}$$

As one can see from the relations (18) the matrix ***N*** is an anti-symmetric matrix.

Now, the problem is to design the nonlinear control of the nonlinear system (11) that in the closed loop with the nonlinear canonical differential equations of the nanorobot motion (11) is resulting in the linear behavior of the whole system. In that case the well known procedures for control synthesis of the linear systems can be applied to the control of a nanorobot motion in a multipotential field. The mentioned problem can be solved by employing the so called concept of the external linearization [35-41]. This concept is introduced and presented in the section 3.

3. CONTROL WITH EXTERNAL LINEARIZATION

In order to solve the control problem that is postulated in the section 2, the concept of the external linearization [35-41] can be employed. In that sense, let the position control error e(t) of a nanorobot motion in a multipotential field and the related derivatives are given by the relations:

$$e=X_w-X,\quad \dot{e}=\dot{X}_w-\dot{X},\quad \ddot{e}=\ddot{X}_w-\ddot{X}. \tag{12}$$

Here desired nanorobot motion is defined by the vector triplet:

$$(X_w,\dot{X}_w,\ddot{X}_w). \tag{13}$$

On the other side, real nanorobot motion is presented by the following vector triplet:

$$(X,\dot{X},\ddot{X}). \tag{14}$$

Applying (12) to the canonical differential equations of the nanorobot motion in a multipotential field (11), one obtains control error model of nanorobot motion in the form:

$$\begin{aligned}\ddot{e}(t)&=r(t)-\frac{1}{m_0}\left[F_p+F_t+\frac{1}{c}NF_I\right],\\ r(t)=\ddot{X}_w&=\frac{1}{m_0}\left[F_{p_w}+F_{t_w}+\frac{1}{c}N_wF_{I_w}\right].\end{aligned} \tag{15}$$

Here r(t) is a vector of desired (or nominal) nanorobot acceleration and subscript w denotes desired values of the related variables.

Now, following the ideas of the external linearization [35-41], one can introduce the next substitution:

$$u(t)=\ddot{e}(t)=r(t)-\frac{1}{m_0}\left[F_p+F_t+\frac{1}{c}NF_I\right]. \tag{16}$$

In the relation (16) u(t) is an internal control vector. From the relation (16) one obtains the equivalent linear control error model of nanorobot motion.

$$\ddot{e}(t)=u(t),\qquad u(t)=(u_x\ u_y\ u_z)^T. \tag{17}$$

Here u_x, u_y and u_z are the related components of the internal control vector u(t). The phase state variables of the system (17) are determined by the following relations:

$$\begin{aligned}Z_I&=(z_1\ z_2\ z_3)^T=(e_x\ e_y\ e_z)^T=e,\\ Z_{II}&=(z_4\ z_5\ z_6)^T=(\dot{e}_x\ \dot{e}_y\ \dot{e}_z)^T=\dot{e}.\end{aligned} \tag{18}$$

The related state space model of a nanorobot motion is given by matrix form:

$$\begin{bmatrix}\dot{Z}_I\\ \dot{Z}_{II}\end{bmatrix}=\begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix}\begin{bmatrix}Z_I\\ Z_{II}\end{bmatrix}+\begin{bmatrix}0\\ I\end{bmatrix}u(t),\quad I=\mathrm{diag}\left[1,\ 1,\ 1\right]. \tag{19}$$

This model can be transformed into the well known form of the dynamical models in the state space:

$$\dot{Z}(t)=A\,Z(t)+B\,u(t),\qquad Z=\left[Z_I^T\ Z_{II}^T\right]^T. \tag{20}$$

Here A and B are constant matrices that are determined by (19) and have dimensions:

$$\dim.A=(6x6),\quad \dim.B=(6x3). \tag{21}$$

Now, one can suppose that the only disturbances to the system (20) are of the initial condition types. The other possibilities will be presented in the next paper. In order to eliminate the control error of a nanorobot motion, caused by disturbances of the initial condition types, one can introduced the following internal control [41]:

$$u(t)=-(K_IZ_I+K_{II}Z_{II}). \tag{22}$$

Here K_I and K_{II} are real control gain matrices of nanorobot position and velocities, respectively. Both of these matrices are of the dimension (3x3). In fact K_I and K_{II} are the components of the state controller K of a nanorobot motion in a multipotential field. Thus, the relation (22) can be transformed into the second form:

$$u(t) = -KZ, \quad K = \left[K_I \;\; K_{II} \right], \quad Z = \begin{bmatrix} Z_I \\ Z_{II} \end{bmatrix}. \tag{23}$$

Applying internal control relation (22) to (16) one obtains the following relation:

$$F_p = m_0 \left[r(t) + K_I Z_I + K_{II} Z_{II} \right] - \left[F_t + \frac{N}{c} F_I \right]. \tag{24}$$

From the equations (3) and (8), the next forms have been derived:

$$\begin{aligned} F_{p_x} &= -\left(\frac{\partial \Sigma U_j}{\partial x} + \frac{\partial U_c}{\partial x} \right) = F_{dp_x} + F_{cp_x}, \; j = 1, 2, .., n, \\ F_{p_y} &= -\left(\frac{\partial \Sigma U_j}{\partial y} + \frac{\partial U_c}{\partial y} \right) = F_{dp_y} + F_{cp_y}, \\ F_{p_z} &= -\left(\frac{\partial \Sigma U_j}{\partial z} + \frac{\partial U_c}{\partial z} \right) = F_{dp_z} + F_{cp_z}. \end{aligned} \tag{25}$$

Here F_{dp} is a disturbance potential force that is caused by influences of n-potential fields to the nanorobot motion. On the other side, F_{cp} is a control force derived by the artificial control field with potential energy U_c. Now, including (25) into the relation (24) one obtains the nonlinear control of the nanorobot motion in a multipotential field:

$$F_{cp} = m_0 \left[r(t) + K_I Z_I + K_{II} Z_{II} \right] - \left[F_{dp} + F_t + \frac{N}{c} F_I \right]. \tag{26}$$

Taking into account the relations in (25), the canonical differential equations of the nanorobot motion in a multipotential field (11) can be rewritten into the form:

$$\ddot{X} = \frac{1}{m_0} \left[F_{cp} + F_{dp} + F_t + \frac{1}{c} N F_I \right]. \tag{27}$$

Applying the nonlinear control F_{cp} from (26) to the nonlinear dynamical model of the nanorobot motion (27) we obtain the closed loop system of the linear form:

$$\ddot{X} = r(t) + K_I Z_I + K_{II} Z_{II}. \tag{28}$$

The relation (26) is the nonlinear control that in the closed loop with the nonlinear canonical differential equations of the nanorobot motion (27) is resulting in the linear behavior of the whole system (28). Thus, the mentioned problem in the section 2 has been solved by employing the so called concept of the external linearization [35-41]. This is the general approach that can be applied to the special situation.

4. NANOROBOT CONTROL IN TWO POTENTIAL ELECTROMAGNETIC AND GRAVITATIONAL FIELD

In order to apply of the general approach given in the section 3, the derived general control (26) of a nanorobot motion in a multipotential field is applied to two-potential electromagnetic and gravitational field. Let a nanorobot is an electric charged particle with charge *q* and rest mass m_0 that is moving with a non-relativistic velocity ($v << c$) in a combined electromagnetic and gravitational potential field. It is also assumed that a gravitational potential field belongs to a spherically symmetric non-charged body with a mass *M*. In that case the total potential energy of a nanorobot in that two-potential field is determined by the following equation:

$$U = q V_e + m_0 V_g = q V_e + m_0 \left(-\frac{GM}{r} \right). \tag{29}$$

In the relation (29) V_e and V_g are the related scalar potentials of an electromagnetic and a gravitational field. Parameter *G* is a gravitational constant, *M* is gravitational mass and *r* is a gravitational radius between a nanorobot and a center of a mass *M*. Applying (29) and including the notations for an electromagnetic field (E_e,H_e) from [18] and a gravitomagnetic field (E_g,H_g) from [42,43], the relation (11) can be transformed into the related vector equation as the explicit function of the Lorentz forces [12]:

$$\begin{aligned} & m_0 \ddot{X} = q\left(E_e + \frac{1}{c} v \times H_e \right) + m_0 \left(E_g + \frac{1}{c} v \times H_g \right), \\ & m_0 \ddot{X} = F_{L_e} + F_{L_g}, \quad \ddot{X} = \left[\ddot{x} \; \ddot{y} \; \ddot{z} \right]^T, \quad v = \left[\dot{x} \; \dot{y} \; \dot{z} \right]^T, \\ & E_e = \left[E_{e_x} \; E_{e_y} \; E_{e_z} \right]^T, \quad E_g = \left[E_{g_x} \; E_{g_y} \; E_{g_z} \right]^T. \\ & H_e = \left[H_{e_x} \; H_{e_y} \; H_{e_z} \right]^T, \quad H_g = \left[H_{g_x} \; H_{g_y} \; H_{g_z} \right]^T. \end{aligned} \tag{30}$$

Here $\ddot{X}$ is an acceleration vector, v is a velocity vector, F_{Le} and F_{Lg} are the related Lorentz forces and the vector pairs (E_e,H_e) and (E_g,H_g) determine an electromagnetic and a gravitomagnetic field, respectively [12,42]. In this example a nanorobot is a particle with charge q and rest mass m_0. Therefore this nanorobot has the interactions with both an electromagnetic and a gravitational field. Thus, the relations (30) describe the dynamics of the related nanorobot that is moving in the two-potential electromagnetic and gravitational field. The components of the vectors E_e and E_g can be calculated by using the following equations:

$$E_{e_x} = -\frac{\partial V_e}{\partial x} - \frac{1}{c}\frac{\partial A_{e_x}}{\partial t}, \quad E_{g_x} = -\frac{\partial V_g}{\partial x} - \frac{1}{c}\frac{\partial A_{g_x}}{\partial t},$$
$$E_{e_y} = -\frac{\partial V_e}{\partial y} - \frac{1}{c}\frac{\partial A_{e_y}}{\partial t}, \quad E_{g_y} = -\frac{\partial V_g}{\partial y} - \frac{1}{c}\frac{\partial A_{g_y}}{\partial t},$$
$$E_{e_z} = -\frac{\partial V_e}{\partial z} - \frac{1}{c}\frac{\partial A_{e_z}}{\partial t}, \quad E_{g_z} = -\frac{\partial V_g}{\partial z} - \frac{1}{c}\frac{\partial A_{g_z}}{\partial t}. \tag{31}$$

On the other side, the components of the vectors A_e and A_g can be calculated by the relations:

$$A_{e_x} = \left(\frac{v_x V_e}{c}\right), \quad A_{g_x} = \left(\frac{v_x V_g}{c}\right),$$
$$A_{e_y} = \left(\frac{v_y V_e}{c}\right), \quad A_{g_y} = \left(\frac{v_y V_g}{c}\right), \tag{32}$$
$$A_{e_z} = \left(\frac{v_z V_e}{c}\right), \quad A_{g_z} = \left(\frac{v_z V_g}{c}\right).$$

Finally, the components of the vectors H_e and H_g can be calculated by employing the equations:

$$H_{e_x} = \frac{\partial A_{e_z}}{\partial y} - \frac{\partial A_{e_y}}{\partial z}, \quad H_{g_x} = \frac{\partial A_{g_z}}{\partial y} - \frac{\partial A_{g_y}}{\partial z},$$
$$H_{e_y} = \frac{\partial A_{e_x}}{\partial z} - \frac{\partial A_{e_z}}{\partial x}, \quad H_{g_y} = \frac{\partial A_{g_x}}{\partial z} - \frac{\partial A_{g_z}}{\partial x}, \tag{33}$$
$$H_{e_z} = \frac{\partial A_{e_y}}{\partial x} - \frac{\partial A_{e_x}}{\partial y}, \quad H_{g_z} = \frac{\partial A_{g_y}}{\partial x} - \frac{\partial A_{g_x}}{\partial y}.$$

Now, for control synthesis of a nanorobot motion in a two potential electromagnetic and gravitational field one can follow the procedure from the section 3. Applying (12) to the canonical differential equations of the nanorobot motion in the mentioned two-potential field (30), one obtains control error model of nanorobot motion in the form:

$$\ddot{e}(t) = r(t) - \frac{q}{m_0}\left(E_e + \frac{1}{c} v \times H_e\right) - \left(E_g + \frac{1}{c} v \times H_g\right),$$
$$r(t) = \frac{q}{m_0}\left(E_{e_w} + \frac{1}{c} v_w \times H_{e_w}\right) - \left(E_{g_w} + \frac{1}{c} v_w \times H_{g_w}\right). \tag{34}$$

Here r(t) is a vector of desired (or nominal) nanorobot acceleration and subscript w denotes desired values of the related variables.

Now, following the ideas of the external linearization [35-41], the first equation in (34) can be transformed into the relation:

$$u(t) = r(t) - \frac{q}{m_0}\left(E_e + \frac{1}{c} v \times H_e\right) - \left(E_g + \frac{1}{c} v \times H_g\right). \tag{35}$$

In the relation (35) u(t) is an internal control vector. From the relations (34) and (35) one obtains the equivalent linear control error model of nanorobot motion, given by (17). The phase state variables of the system (17) are determined by (18). The related state space model of a nanorobot motion is given by matrix form in (19) and (20). In order to eliminate the control error of a nanorobot motion, caused by disturbances of the initial condition types, one can introduced the internal control in the form (22) or (23) [41]. Applying internal control relation (22) to (35) one obtains the following relation:

$$E_e = \frac{m_0}{q}\left[r(t) + K_I Z_I + K_{II} Z_{II}\right] - \left(\frac{1}{c} v \times H_e\right) - \frac{m_0}{q}\left(E_g + \frac{1}{c} v \times H_g\right). \tag{36}$$

Let the electric field E_e is consisting of the two electric fields:

$$E_e = E_{de} + E_{ce}. \tag{37}$$

Here E_{de} is a disturbance electric field that is caused by influences of two potential field to the nanorobot motion. On the other side, E_{ce} is the artificial electric control field that should be control the nanorobot motion in the two potential field. Now, including (37) into the relation (36) one obtains the nonlinear electric control of the nanorobot motion in the two potential field:

$$E_{ce} = \frac{m_0}{q}\left[r(t) + K_I Z_I + K_{II} Z_{II}\right] - \left(E_{de} + \frac{1}{c} v \times H_e\right) - \frac{m_0}{q}\left(E_g + \frac{1}{c} v \times H_g\right). \tag{38}$$

Taking into account the relation in (37), the canonical differential equations of the nanorobot motion in the two potential fields (30) can be rewritten into the form:

$$\ddot{X} = \frac{q}{m_0}\left(E_{de} + E_{ce} + \frac{1}{c} v \times H_e\right) + \left(E_g + \frac{1}{c} v \times H_g\right). \tag{39}$$

Applying the nonlinear control E_{ce} from (38) to the nonlinear dynamical model of the nanorobot motion (39) we obtain the closed loop system of the linear form that is the same as in the general case (28):

$$\ddot{X} = r(t) + K_I Z_I + K_{II} Z_{II}. \tag{40}$$

The relation (38) is the nonlinear control that in the closed loop with the nonlinear canonical differential equations of the nanorobot motion (39) is resulting in the linear behavior of the whole system (40). Thus, the mentioned problem in the section 2 has been solved also for the two potential field, i.e. for an electromagnetic and a gravitational field, by employing the so called concept of the external linearization [35-41].

5. CONCLUSION

The problem of designing of the nonlinear control that in the closed loop with the nonlinear dynamic model of the nanorobot motion is resulting in the linear behavior of the whole system is solved. For this purpose the concept of the external linearization [35-41] has been used. In that case the well known procedures for control synthesis of the linear systems can be applied to the control of a nanorobot motion in a multipotential field. This has been done for a general case with applications to control of a nanorobot motion in a multipotential field (section 3). As en example the general approach is elaborated in detail in the section 4 by application to control of a nanorobot motion in the two potential electromagnetic and gravitational field.

In the case where the quantum effects are present one should apply dynamics of the quantum feedback systems and control concepts and applications presented by Yanagisawa and Kimura in 2003. Recently, it has been proposed coherent H^{∞} control for a class of annihilation operator linear quantum systems (Maalouf and Petersen, 2011). This control can be applied to the quantum systems that can be described by complex quantum stochastic differential equations in terms of annihilation operators only. The presented approach to control of a nanorobot motion in a multipotential field will be continued in the next papers by synthesis of the related controllers K_I and K_{II} in the relations (28) and (40).

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