CONTROL OF NANOROBOT MOTION IN A QUANTUM ALPHA FIELD

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Abstract

An alpha field is the field that can be described by two dimensionless field parameters α and α'. A quantum alpha field is the alpha field in which quantum effects can be occurred. In classical mechanics, a nanorobot has, at every moment, an exact position and an exact momentum, following the Newton's laws of motion. In quantum mechanics, nanorobots do not have exactly determined properties and when they are measured, the result is randomly drawn from a probability distribution. The classical approaches to control of nanorobot motion in the multipotential field have been presented in the authors previous papers. In this paper we continue to investigate the related control algorithms for a nanorobot motion in a quantum alpha field. In that sense the nanorobot dynamics has been described by the Schrödinger equation in an alpha field that predicts what the probability distributions are. Meanwhile, it cannot fundamentally predict the exact result of each measurement. Thus, the problem is: how to control of a nanorobot motion in the case when quantum effects are occurred. Some approaches are discussed in this paper.

1. INTRODUCTION

As it is the well known, the nanorobotics is the multidisciplinary field that deals with the controlled manipulation with atomic and molecular-sized objects and therefore sometimes is called molecular robotics [1]. The state of the art in nanorobotics has been presented in reference [2]. The classical approaches to control of nanorobot motion in the multipotential field have been presented in the authors previous papers [2,3,4,5]. A relativistic Hamiltonian of the nanorobot motion in a multipotential field that includes external artificial control potential field has been derived and presented in [2]. Starting with the generalized relativistic Hamiltonian in an alpha field [3] the non-relativistic approximation of that Hamiltonian and the canonical differential equations of the nanorobot motion in a multipotential field has been derived in [4]. In that sense, the concept of the external linearization has been introduced in [5]. In this paper we continue to investigate the related control algorithms for a nanorobot motion in a quantum alpha field. As it is well known, in classical mechanics, a nanorobot has, at every moment, an exact position and an exact momentum, following the Newton's laws of motion. On the other side, in quantum mechanics, nanorobots do not have exactly determined properties and when they are measured, the result is randomly drawn from a probability distribution. In that sense the nanorobot dynamics has been described by the Schrödinger equation in an alpha field that predicts what the probability distributions are. Meanwhile, it cannot fundamentally predict the exact result of each measurement. Thus, the problem is: how to control of a nanorobot motion in the case when quantum effects are occurred. Some approaches are discussed in this paper.

Generally, the Schrödinger equation has been presented in the references [6-11]. The properties of the quantum mechanics are discussed in [12-15]. A gradient based approach to feedback control of quantum systems is presented in [16]. Control of observables in the finite level quantum systems is shown in [17]. Some properties of nonlinear systems are pointed out in [18]. The behavior of nanorobots as an analogous of the quantum harmonic oscillator has been described in [19]. Some versions of applications of fuzzy control algorithms to control of quantum systems have been proposed in [20-22]. Application of probabilistic self-stabilization algorithm to robot's control is presented in [23]. The quantum computation and quantum information are discussed in [24]. The coherent H^∞ control for a class of annihilation operator linear quantum systems is proposed in [25]. This control can be applied to the quantum systems that can be described by complex quantum stochastic differential equations in terms of annihilation operators only. For this class of quantum systems, the related control problem can be solved in terms of a pair of complex algebraic Riccati equations. In addition, the question of physical realizability of the resulting quantum controllers is related to a bounded real property. The finite controllability of

infinite dimensional quantum systems has been discussed in [26]. The quantum dissipative systems and feedback control design by interconnection are presented in [27]. Finally, the transfer function approaches to quantum control with presentations of dynamics of quantum feedback control and control concepts and applications are proposed in [28,29].

In this paper it has been proposed the control algorithm of the nanorobot motion in a quantum alpha field. This control algorithm is based on the inner quantum states of the nanorobot presented in [21] and on the gradient control approach that is discussed in [16]. The inner quantum states of the nanorobot are modeled by qubits.

The organization of this paper is as follows. The second section presents the derivation of the Schrödinger quantum wave equation in an alpha field. In the third section it has been proposed the control algorithm of the nanorobot motion in a quantum alpha field. Finally, the conclusion of the paper with some comments and the reference list are presented in the fourth and fifth sections, respectively.

2. DERIVATION OF SCHRÖDINGER QUANTUM WAVE EQUATION IN AN ALPHA FIELD

As it is well known, the Schrödinger equation [6,7,8,9] can be derived by using the Hamiltonian operator $\hat{H}$. This operator characterizes the total energy of any given wavefunction and takes different forms depending on the situation. Thus, including the Hamiltonian operator one can obtain the most general form of the time dependent Schrödinger equation [10]:

$$i\hbar \frac{\partial}{\partial t}\psi = \hat{H}\psi. \qquad (1)$$

Here Ψ is the wavefunction, $\hbar$ is the well known reduced Planck's constant and i-is an imaginary unit. This equation gives a description of a system evolving with time. In the case of Schrödinger equation for a single particle that is moving in an electric field, but without a magnetic field, the Hamiltonian operator has the form:

$$\hat{H} = -\frac{\hbar^2}{2m_0}\nabla^2 + V(r,t),$$

$$\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right). \qquad (2)$$

Here m_0 is a nanorobot mass, $V(r,t)$ is a nanorobot potential energy at position r and time t and ∇^2 is the Laplacian operator. Thus, combining the relations (1) and (2) one obtains the time

dependent Schrödinger equation for the mentioned situation:

$$i\hbar \frac{\partial}{\partial t}\psi(r,t) = \left[-\frac{\hbar^2}{2m_0}\nabla^2 + V(r,t)\right]\psi(r,t). \qquad (3)$$

Here Ψ is the wavefunction that in this context is called the position-space wavefunction. This is a linear partial differential equation and therefore the superposition principle can be applied. It is also a diffusion equation.

The term Schrödinger equation can refer to both the general equation (1), and to the specific version (3) and variations thereof. The general equation (1) is indeed quite general. It is used throughout the quantum mechanics for everything, like for application of the Dirac equation to quantum field theory, by employing the various expressions of the Hamiltonian. In that sense, the Schrödinger equation (1) can be applied to an alpha field by employing general relativistic Hamiltonian $\mathbb{H}_\alpha$, derived for that field in the reference [3]:

$$i\hbar \frac{\partial}{\partial t}\psi_\alpha = \overset{\wedge}{\mathbb{H}}_\alpha \psi_\alpha, \quad \overset{\wedge}{\mathbb{H}}_\alpha = f(\mathbb{H}_\alpha),$$

$$\mathbb{H}_\alpha = Hm_0\alpha\alpha'c^2 + \frac{Hm_0\kappa(\alpha - \alpha')c\,v}{2}, \qquad (4)$$

$$H = \left(1 - \frac{v_\alpha^2}{c^2}\right)^{-1/2} = \left(\alpha\alpha' - \frac{v^2}{c^2} + \frac{\kappa(\alpha - \alpha')c\,v}{c^2}\right)^{-1/2}.$$

Here α and α' are field parameters, H is relativistic parameter, v_α and v are a nanorobot velocities in an alpha field and in a vacuum, κ is observation parameter and c is a speed of the light in vacuum [2,3,4,5]. Further, the nonrelativistic version (3) is a simplified approximation to reality. This is quite accurate in many situations, but very inaccurate in the region of the relativistic quantum mechanics. The time independent Schrödinger equation is describing stationary states of the system. It can be only used when the related Hamiltonian is not dependent on time:

$$E\psi = \overset{\wedge}{\mathbb{H}}\psi. \qquad (5)$$

This relation is the equation states and can be interpreted as follows. When the Hamiltonian operator acts on the wavefunction Ψ, the result might be proportional to the same wavefunction Ψ. If this is happened, then Ψ is a stationary state, and the proportionality constant E is the energy of the state Ψ. In the terminology of linear algebra the relation (5) is an eigenvalue equation. As an example one can use the nonrelativistic time independent Schrödinger equation for a single

nanorobot in an electric field, but without a magnetic field:

$$E\psi(r) = -\frac{\hbar^2}{2m_0}\nabla^2\psi(r) + V(r)\psi(r). \qquad (6)$$

If certain properties of a system are measured, then the Schrödinger equation predicts that the result may be quantized. This means that only specific discrete values can occur. As an example one can use the energy quantization, where the energy of an electron in atom is always one of the quantized energy levels that is discovered by atomic spectroscopy. The next example is the quantization of an angular momentum. Although this was an assumption in the earlier Bohr model of the atom, it is also prediction of the Schrödinger equation. It should be pointed out that not every measurement gives a quantized result in quantum mechanics. This can be happened by measurement of position, momentum and in special situation energy, where we can have any value across a continuous range.

The general solutions of the Schrödinger equation can be understood from the following discussion. The plane wave is definitely a solution because this was used by Schrödinger in order to construct the equation. Further, due to linearity principle any linear combination of plane waves is also a solution of the equation. Thus, in the case of discrete k, the sum is a superposition of plane waves [11]:

$$\psi(r,t) = \sum_{n=1}^{\infty} A_n e^{i(k_n \cdot r - \omega_n t)}. \qquad (7)$$

Here A is a plane wave amplitude, ω is an angular frequency and k is a wave vector. On the other side, in the case of continuous k, the sum becomes an integral that is the Fourier transform of a momentum space wavefunction [12]:

$$\psi(r,t) = \frac{1}{\left(\sqrt{2\pi}\right)^3}\int \Phi(k) e^{i(k \cdot r - \omega t)} d^3k,$$

$$(8)$$

$$d^3k = dk_x dk_y dk_z.$$

Here d^3k is the differential volume element in k-space, and the integrals are taken over all k-space. Further, the momentum wavefunction $\Phi(k)$ arises in the integrand since the position and momentum space wavefunctions are Fourier transforms of each other. Because these satisfy the Schrödinger equation, the solutions to the Schrödinger equation, for a given situation, will be the plane waves used to obtain it. Also any wavefunctions, which satisfy the Schrödinger equation prescribed by the system, including the

relevant boundary conditions, will be the solutions to the Schrödinger equation. Thus, the Schrödinger equation is true for any non relativistic situation. Further, one can say that the Schrödinger equation is a differential equation of wave-particle duality. This means that particles can behave like waves because their corresponding wavefunctions satisfy the Schrödinger equation.

In the case of one spatial dimension of time independent and non relativistic Schrödinger equation for one nanorobot we can apply the following relations [11]:

$$\hat{H} = -\frac{\hbar^2}{2m_0}\frac{d^2}{dx^2} + V(x) \quad \rightarrow$$

$$E\psi = -\frac{\hbar^2}{2m_0}\frac{d^2}{dx^2}\psi + V(x)\psi, \qquad (9)$$

$$\psi(x,t) = \psi(x)e^{-iEt/\hbar}, \quad \|\psi\|^2 = \int |\psi(x)|^2 dx.$$

The solution in (9) is restricted in the sense that it must not grow at infinity. In the case of a bound state, it should have a finite L^2- norm. If it is a part of a continuum, then it should have a slowly diverging norm.

In the case of three spatial dimensions of time independent and non relativistic Schrödinger equation for one nanorobot we can apply the relations:

$$\hat{H} = -\frac{\hbar^2}{2m_0}\nabla^2 + V(r) \quad \rightarrow$$

$$E\psi = -\frac{\hbar^2}{2m_0}\nabla^2\psi + V(r)\psi, \qquad (10)$$

$$\psi = \psi(r)e^{-iEt/\hbar}, \quad r = (x, y, z).$$

Here r(x,y.z) is the position of the nanorobot.

In the case of one spatial dimension of time dependent and non relativistic Schrödinger equation for one nanorobot we can apply the following relations [11]:

$$\hat{H} = -\frac{\hbar^2}{2m_0}\frac{\partial^2}{dx^2} + V(x,t), \quad \rightarrow$$

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m_0}\frac{\partial^2}{\partial x^2}\psi + V(x,t)\psi, \quad \psi = \psi(x,t). \qquad (11)$$

In the case of three spatial dimensions of time dependent and non relativistic Schrödinger equation for one nanorobot we can apply the relations:

$$\hat{H} = -\frac{\hbar^2}{2m_0}\nabla^2 + V(r,t), \quad \rightarrow$$

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m_0}\nabla^2\psi + V(r,t)\psi, \quad \psi = \psi(r,t). \qquad (12)$$

For solution of the Schrödinger equations, generally, one can use the following techniques: perturbation theory, the variational method, quantum Monte Carlo methods, density functional theory and WKB approximation and semi-classical expansion. For the special cases it also can be used the list of quantum-mechanical systems with analytical solutions and Hartree-Fock method as well as post Hartree-Fock methods.

One of the important properties of the Schrödinger equation is linearity. Thus, if two wave functions ψ_1 and ψ_2 are solutions, then so is any linear combination of the two. This property allows superposition of quantum states to be solutions of the Schrödinger equation. Further, if two wave-functions ψ_1 and ψ_2 are solutions to the time independent equation with the same energy E, then so is any linear combination:

$$\hat{\mathcal{H}}(a\psi_1 + b\psi_2) = a\hat{\mathcal{H}}\psi_1 + b\hat{\mathcal{H}}\psi_2 = E(a\psi_1 + b\psi_2). \tag{13}$$

Here a and b are any complex numbers. If we have two different solutions with the same energy, then it is called degenerate solutions [12]. The time independent eigenvalue problem can be restricted to real valued wave function. In the time dependent equation the complex conjugate waves satisfy time-reversal symmetry. This means, if $\psi(x,t)$ is one solution, then so is $\psi(x,-t)$, and these waves are moving in the opposite directions in time.

Further, the Schrödinger equation is consistent with probability conservation. This is because it can directly derive the continuity equation for probability [15]:

$$\frac{\partial}{\partial t}\rho(r,t) + \nabla \cdot j = 0, \quad \rho = |\psi|^2 = \psi^*(r,t)\psi(r,t),$$
$$j = \frac{1}{2m_0}(\psi^*\hat{p}\psi - \psi\hat{p}\psi^*). \tag{14}$$

Here ρ is the probability density per unit volume, j is the probability current (flow per unit area), $\hat{p}$ is the first order momentum operator and $*$ denotes complex conjugate.

The next property of the Schrödinger equation is the positive energy. Thus, if the potential is bounded from below, meaning there is a minimum value of potential energy, then the eigenfunctions of the Schrödinger equation have energy which is also bounded from below. This property of positive definiteness of energy allows the analytic continuation of the Schrödinger equation to be identified as a stochastic process, which can be represented by a path integral. Further, it can be prove that the solutions to the Schrödinger equation are not Galilean invariant. Also the solutions to the Schrödinger equation are not Lorentz invariant and therefore are not consistent with the special relativity. In order to extend Schrödinger's formalism to include relativity, the physical picture must be transformed. Thus, the Klein-Gordon equation [13] and the Dirac equation [14] are built from the relativistic mass-energy relation. Therefore, these equations are relativistic invariant and replace the Schrödinger equation in relativistic quantum mechanics.

3. CONTROL OF NANOROBOT MOTION IN A QUANTUM ALPHA FIELD

Generally, there exist two approaches to control of quantum systems [16]. The first one is open loop control. The second approach is feedback control, which is based on the measurement processes. In this paper the feedback control, or closed loop control, has been discussed. In order to employ the nonrelativistic Schrödinger equation for description of the quantum system in an alpha field, the relativistic Hamiltonian (4) should be transformed into the nonrelativistic approximation. The first step is the transformation of the Hamiltonian into the nonlinear (quadratic) form [3]:

$$\frac{\mathcal{H}_\alpha^2}{c^2} - P^2 = m_0\alpha\alpha'c^2, \quad P = Hm_0v \rightarrow$$
$$\frac{\mathcal{H}_\alpha^2}{\alpha\alpha'c^2} - \frac{P^2}{\alpha\alpha'} = m_0c^2. \tag{15}$$

The second step is the substitution of field parameters α and α' [3]:

$$\frac{1}{\sqrt{\alpha\alpha'}} = \frac{1}{\left(1 + \frac{U}{m_0c^2}\right)} \cong \left(1 - \frac{U}{m_0c^2}\right) \rightarrow$$

$$\frac{P}{\sqrt{\alpha\alpha'}} \cong \left(P - \frac{PU}{m_0c^2}\right) = P_e, \quad \frac{\mathcal{H}_\alpha}{\sqrt{\alpha\alpha'}} \cong \left(\mathcal{H}_\alpha - \frac{\mathcal{H}_\alpha}{m_0c^2}U\right),$$

$$\mathcal{H}_\alpha \cong m_0c^2 \rightarrow \frac{\mathcal{H}_\alpha}{\sqrt{\alpha\alpha'}} \cong \left(\mathcal{H}_\alpha - U\right) = \mathcal{H}_{\alpha e}. \tag{16}$$

Here P_e and $\mathcal{H}_{\alpha e}$ are the extended momentum and the extended Hamiltonian and U is the total potential energy of a nanorobot in an alpha field. Including (16) into the relation (15) we obtain the Hamiltonian in the following form:

$$\mathcal{H}_\alpha = c\sqrt{m_0^2c^2 + \left(P - \frac{PU}{m_0c^2}\right)^2} + U, \rightarrow \tag{17}$$

$$\mathcal{H}_\alpha = c\sqrt{m_0^2c^2 + P_e^2} + U.$$

The next step is the transformation of (17) into the nonrelativistic approximation:

$$\mathbb{H}_\alpha \cong c\left[\left(m_0 c + \frac{1}{2m_0 c}P_e^2\right)^2\right]^{1/2} + U, \rightarrow$$

$$\mathbb{H}_\alpha \cong m_0 c^2 + \frac{1}{2m_0}P_e^2 + U. \tag{18}$$

The final step is the exclusion of the rest mass energy $m_0 c^2$, what leads to the Hamiltonian that is consisting of kinetic and potential energy, only:

$$\mathbb{H}_\alpha \cong \frac{1}{2m_0}P_e^2 + U, \quad P_e = \left(P - \frac{PU}{m_0 c^2}\right), \quad P = m_0 v. \tag{19}$$

Now, the control algorithm should be developed for the quantum system described by nonrelativistic Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\psi_\alpha = \hat{\mathbb{H}}_\alpha\psi_\alpha, \quad \hat{\mathbb{H}}_\alpha = f(\mathbb{H}_\alpha),$$

$$\mathbb{H}_\alpha \cong \frac{1}{2m_0}P_e^2 + U. \tag{20}$$

Here m_0 is the nanorobot mass. The Schrödinger equation from (20) describes the quantum system in an alpha field without control. This is just a plant that should be controlled. In the case of the closed loop, or feedback, control system the dynamics of the controlled quantum system is described by the Schrödinger equation of the form [16]:

$$i\hbar\dot{\psi}_\alpha(t) = \left[\mathbb{H}_\alpha + u(t)\mathbb{H}_c\right]\psi_\alpha(t), \quad \psi_\alpha(t) \in C^n. \tag{21}$$

Here $\mathbb{H}_c$ is taken as the control Hamiltonian and $u(t)$ is external control input. Further, the desired states of the nanorobot motion on its trajectory should satisfy the following Schrödinger equation:

$$i\hbar\dot{\psi}_\alpha(t) = \mathbb{H}_d\psi_\alpha(t), \quad \psi_\alpha(t) \in C^n. \tag{22}$$

Here $\mathbb{H}_d$ is the desired Hamiltonian that describes the desired dynamics of the nanorobot motion. Comparing (21) and (22) one can find the identity:

$$\mathbb{H}_\alpha + u(t)\mathbb{H}_c = \mathbb{H}_d, \rightarrow$$

$$u(t)\mathbb{H}_c = \left(\mathbb{H}_d - \mathbb{H}_\alpha\right). \tag{23}$$

From the last equation in (23) one should construct the control Hamiltonian $\mathbb{H}_c$. In order to obtain the stability for the quantum system (21) during the nanorobot motion one should introduce the related Lyapunov function. Following the approaches given in [16] and [17] one can employ the following Lyapunov function:

$$V(\psi_\alpha) = \left(\psi_\alpha^+ Z\psi_\alpha - Z_d\right)^2. \tag{24}$$

Here + is standing for the transposition and complex conjugation, Z is the observable of the quantum system that is associated to the energy of the nanorobot and Z_d is the desirable energy value. The term $\psi_\alpha^+ Z\psi_\alpha$ describes the observed mean energy of the nanorobot at time instant t. It follows the first derivative of the Lyapunov function (24) with respect to time:

$$\dot{V}(\psi_\alpha) = 2\left[\psi_\alpha^+ Z\psi_\alpha - Z_d\right]\left[\dot{\psi}_\alpha^+ Z\psi_\alpha + \psi_\alpha^+ Z\dot{\psi}_\alpha\right]. \tag{25}$$

The first time derivative $\dot{\psi}_\alpha$ can be obtained from (21) in the form:

$$\dot{\psi}_\alpha(t) = -\frac{i}{\hbar}\left[\mathbb{H}_\alpha + u(t)\mathbb{H}_c\right]\psi_\alpha(t), \quad \psi_\alpha(t) \in C^n. \tag{26}$$

Applying (26) to the relation (25) one obtains the following equation:

$$\dot{V}(\psi_\alpha) = \frac{2i}{\hbar}\left(\psi_\alpha^+ Z\psi_\alpha - Z_d\right)\cdot$$

$$\cdot\psi_\alpha^+\{\mathbb{H}_\alpha Z - Z\mathbb{H}_\alpha + u(t)\left(\mathbb{H}_c Z - Z\mathbb{H}_c\right)\}\psi_\alpha. \tag{27}$$

Now, one can use the quantum system observable Z in the form that satisfies the following condition:

$$\psi_\alpha^+ \mathbb{H}_\alpha Z\psi_\alpha = \psi_\alpha^+ Z\mathbb{H}_\alpha\psi_\alpha \rightarrow Z = \mathbb{H}_\alpha. \tag{28}$$

Taking into account (28), the relation (27) is transformed into the new one:

$$\dot{V}(\psi_\alpha) = \frac{2i}{\hbar}\left(\psi_\alpha^+ Z\psi_\alpha - Z_d\right)\psi_\alpha^+\{u(t)\left(\mathbb{H}_c Z - Z\mathbb{H}_c\right)\}\psi_\alpha. \tag{29}$$

Following a gradient based approach [16], one can choose the control variable u(t) in the form.

$$u(t) = k\nabla_u\{\dot{V}(\psi_\alpha)\}. \tag{30}$$

Here k is a proportional constant of the control algorithm in (30). The velocity gradient of the Lyapunov function (29) with respect to the control variable u(t) has the form:

$$\nabla_u\{\dot{V}(\psi_\alpha)\} = \frac{2i}{\hbar}\left(\psi_\alpha^+ Z\psi_\alpha - Z_d\right)\psi_\alpha^+\left(\mathbb{H}_c Z - Z\mathbb{H}_c\right)\psi_\alpha. \tag{31}$$

Applying (31) to the relation (30) one obtains the control variable u(t) in the final form:

$$u(t) = k\frac{2i}{\hbar}\left(\psi_\alpha^+ Z\psi_\alpha - Z_d\right)\psi_\alpha^+\left(\mathbb{H}_c Z - Z\mathbb{H}_c\right)\psi_\alpha. \tag{32}$$

After substitution of the control variable (32) into the relation (29) we obtain the first derivative of the Lyapunov function in the closed loop quantum system:

$$\dot{V}(\psi_\alpha) = -k\frac{4}{\hbar^2}\left(\psi_\alpha^+ Z\psi_\alpha - Z_d\right)^2 \psi_\alpha^{+^2}\left(\mathbb{H}_c Z - Z\mathbb{H}_c\right)^2 \psi_\alpha^2,$$

$$\dot{V}(\psi_\alpha) \le 0 \quad \text{if} \quad k \ge 0.$$

(33)

From the relation (33) one can see that the first derivative of Lyapunov function is non-positive if the proportional constant k is non-negative. This condition ensures the stability for the quantum system (21). In this case La Sale's principle [18] shows convergence not to equilibrium but to an area round this equilibrium. This is known as invariant set. Thus, following the previous consideration, any solution of the system, ψ_α, remains in the invariant set:

$$M = \{\psi_\alpha : \dot{V}(\psi_\alpha) = 0\}.$$

(34)

The objective of the quantum control (32) is to move a nanorobot from an initial eigenstate $\psi_{\alpha 0}$ (which is associated with a certain energy level) to a different eigenstates along the nanorobot trajectory, associated with the desirable energy levels. Also this quantum control should assure tracking of the desirable quantum states of the nanorobot motion within acceptable accuracy levels. In order to do it one should define the desirable nanorobot states along its trajectory of the motion. Since quantum effects can bi occurred during a nanorobot motion, the definition of the desirable nanorobot states should be in the quantum mechanics manner. Following the references [21,23], the desirable nanorobot states will be defined in the form of qubits. For the simplicity, it is assumed that a nanorobot is moving on a plane and has a set of inner states. On the mentioned sets one should apply the related actions, following the desired states along the trajectory of the nanorobot motion. Now, one can suppose that a nanorobot may be in one of the following inner states on a plane [21,23]:

$$S = \{\uparrow, \downarrow, \rightarrow, \leftarrow\}.$$

(35)

Here the orientations of the arrows on the plane are associated with the possible direction of the nanorobot motion. The related actions of the nanorobot motion are: step forward, step backward, turn left, turn right and stay still. The actions step forward and step backward are restricted by certain fixed distance. The inner states are modeled by qubits in the following forms:

$$s_i \in S: \quad \uparrow = |s_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \downarrow = |s_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$$\rightarrow = |s_3\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}, \quad \leftarrow = |s_4\rangle = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}.$$

(36)

The turn actions of the nanorobot are governed by the direct Hadamard and reverse Hadamard operators [24]:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \qquad H^r = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}.$$

(37)

Using the direct Hadamard operator one can have the following states transformations:

$$H|s_1\rangle = |s_3\rangle, \qquad H|s_3\rangle = |s_1\rangle,$$

$$H|s_2\rangle = |s_4\rangle, \qquad H|s_4\rangle = |s_2\rangle.$$

(38)

On the other hand, by using of the reverse Hadamard operator one can have the reverse states transformations:

$$H^r|s_1\rangle = |s_4\rangle, \qquad H^r|s_4\rangle = |s_1\rangle,$$

$$H^r|s_2\rangle = |s_3\rangle, \qquad H^r|s_3\rangle = |s_2\rangle.$$

(39)

It is very important to point out that the state $|s\rangle = (a,b)^T$ and the state $-|s\rangle = (-1)(a,b)^T$ are not distinguishable, as conventionally assumed. This means that the "positive" and "negative" quantum states correspond to the same orientation of the nanorobot.

Now, one can introduce the correspondence between the nanorobot states and its orientation on the plane:

$$-\text{nord,} \quad 0^0 = 360^0: \quad |s_1\rangle = (1,0)^T,$$

$$-\text{south,} \qquad 180^0: \quad |s_2\rangle = (0,1)^T,$$

$$-\text{east,} \quad 270^0: \quad |s_3\rangle = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)^T,$$

$$-\text{west,} \quad 90^0: \quad |s_4\rangle = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)^T.$$

(40)

In the process of planning the nanorobot motion the related trajectory should be presented by the desired nanorobot states. As an example, let the nanorobot trajectory has the following form:

$$
\begin{array}{ccc}
s_4 \leftarrow \cdots\cdots \leftarrow s_4 & & \\
\downarrow s_2 & & \uparrow s_1 \\
\bullet \leftarrow s_{4,f} & & \rightarrow s_3 \\
& & \uparrow s_1 \\
& & \rightarrow s_3 \\
& & \bullet \uparrow s_{1,0}
\end{array}
$$

(41)

Here $s_{1,0}$ is the initial nanorobot state showing the northern orientation of the nanorobot. The final nanorobot state is denoted by $s_{4,f}$ and is showing

the western orientation of the nanorobot. In order to change nanorobot states along the trajectory, as it is shown by (41), one should apply the following Hadamard operations:

$$H_{1,3} = s_{1,0} \text{ to } s_3, \quad H_{3,1} = s_3 \text{ to } s_1, \quad H_{1,3} = s_1 \text{ to } s_3,$$

$$H_{3,1} = s_3 \text{ to } s_1, \quad H^r_{1,4} = s_1 \text{ to } s_4, \quad \text{stay in } s_4, \quad (42)$$

$$H_{4,2} = s_4 \text{ to } s_2, \quad H_{2,4} = s_2 \text{ to } s_{4,f}.$$

According to the planned (desired) trajectory (41), the nanorobot should perform the following actions starting from its initial state $s_{1,0}$:

- a step forward, a direct Hadamard operator $H_{1,3}$ (turning on the right), one step forward, a direct Hadamard operator $H_{3,1}$ (turning on the left), one step forward, a direct Hadamard operator $H_{1,3}$ (turning on the right), one step forward, a direct Hadamard operator $H_{3,1}$ (turning on the left), one step forward, a reverse Hadamard operator $H^r_{1,4}$ (turning on the left), stay in s_4, six steps forward, a direct Hadamard operator $H_{4.2}$ (turning on the left), one step forward, a direct Hadamard operator $H_{2,4f}$ (turning on the right), one step forward, stop.

Meanwhile, in the realistic case in an alpha field we have the influences of the multipotential field to the nanorobot motion. As the consequence the inaccurate turns and/or the forward (backward) motions are occurred. Thus, in that case a nanorobot will not follow exactly the planed trajectory. In order to assure tracking of the desirable quantum states of the nanorobot motion within acceptable accuracy levels, the quantum control (32) should be applied. For realization of this quantum control one should determine the desired Hamiltonian $\mathbb{H}_d$ as the function of the desired states of the nanorobot motion (36) and related Hadamard operators (37), (38) and (39):

$$\mathbb{H}_d = f(s_i \in S, H, H^r). \qquad (43)$$

Including $\mathbb{H}_d$ from (43) into the second relation in (23), one can construct the control Hamiltonian $\mathbb{H}_c$. Further, starting with $\mathbb{H}_d$ from (43), one should determine the desirable energy value Z_d that is associated to the nanorobot motion. Now knowing $\mathbb{H}_c$, Z_d and the observable of the quantum system Z, the quantum nanorobot control (32) can be realized. The synthesis procedure for determination of $\mathbb{H}_d$, $\mathbb{H}_c$ and Z_d will be presented in the next paper.

4. CONCLUSION

The objective of the quantum control is to move a nanorobot from its initial eigenstate (associated to a certain energy level) to different eigenstates associated with the desirable energy levels on the trajectory in an alpha. Also this quantum control should assure tracking of the quantum states on the desired nanorobot trajectory within acceptable accuracy levels. In order to do it, the desirable nanorobot states along its trajectory of the motion have been defined. Since the quantum effects can bi occurred, the definition of the desirable nanorobot states is done in the form of qubits [21,23].

In order to employ the nonrelativistic Schrödinger equation the relativistic Hamiltonian has been transformed into the nonrelativistic and nonlinear (quadratic) form [3]. Using this Hamiltonian, the dynamics of the quantum system has been described by nonrelativistic Schrödinger equation. This equation is employed in the synthesis procedures of the control algorithm for a nanorobot motion in an alpha field. Following the gradient based approach [16], the related control algorithm valid in an alpha field has been employed. This algorithm is based on the Lyapunov function that ensures the stability for the quantum system with the convergence not to the equilibrium but to an area round this equilibrium, within acceptable accuracy levels. The synthesis of the control Hamiltonian $\mathbb{H}_c$ will be presented in the next paper.

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