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FLOATER-TETHER SEMI-COUPLED DYNAMIC RESPONSE ANALYSIS OF TENSION LEG PLATFORMS

Abstract

Mathematical model for the uncoupled floater-tether dynamic response analysis of floating tethered bodies (e.g. Tension Leg Platforms, Floating Offshore Wind Turbines) is extended to account for the nonlinear restoring stiffness due to the influence of mooring lines on the floating body dynamics. Also, a simple approximation of the inertia and viscous effects of tethered mooring lines is also taken into account resulting in a computationally inexpensive semi-coupled floater-tether mathematical model. Model is validated in the case of ISSC TLP in medium range depths using the fully coupled FEM model (theory of rod) taking into account tethered mooring lines deformations. Validation analysis includes static displacements and free decay tests. Time domain simulation is done by mapping linear frequency domain hydrodynamics to time domain using the Cummins integral approach, utilizing the HYDROSTAR potential flow code to determine the hydrodynamic reaction forces. Some useful results are presented and discussed.

Key words: semi-coupled analysis, nonlinear restoring stiffness, time domain, tension leg platform

POLU-SPREGNUTA ANALIZA DINAMIČKOG ODZIVA PLUTAJUĆEG TIJELA I SIDRENIH LINIJA PRITEGNUTIH PUČINSKIH PLATFORMI

Sažetak

Matematički model nespregnutog dinamičkog odziva plutajućeg tijela i sidrenih linija pritegnutih pučinskih platformi (pritegnute plutajuće platforme, plutajuće pučinske vjetroturbine) unaprijeđen je uzimajući u obzir nelinearnu povratnu krutost kao posljedicu utjecaja sidrenih linija na dinamiku plutajućeg tijela. Osim nelinearne povratne krutosti model na jednostavan način uzima u obzir i inerciju i viskozno prigušenje prednapregnutih sidrenih linija što rezultira računalno efikasnim poluspregnutim matematičkim modelom. Točnost modela ocjenjena je na primjeru ISSC TLP-s usidrenog u moru srednje dubine usporedbom s potpuno spregnutim modelom konačnih elementa (teorija štapa) koji uzima u obzir deformacije prednapregnutih sidrenih linija. Analiza točnosti modela uključuje analizu statičkih pomaka i slobodnog njihanja plutajućeg tijela. Simulacija u vremenskoj domeni provedena je preslikavanjem linearnih hidrodinamičkih sila iz frekvencijske u vremensku domenu pomoću Cumminsovog konvolucijskog integrala, pri čemu je hidrodinamička reakcija određena programom HYDROSTAR. Neki važniji rezultati i zaključci navedeni su u radu.

Ključne riječi: poluspregnuta analiza, nelinearna povratna krutost, vremenska domena, pritegnuta pučinska platforma

1. Introduction

Tension Leg Platform (TLP) is a *hybrid* structure, incorporating features of both fixed and floating offshore structures. Its main economic advantage is a fact that the cost of a TLP in deep water oil developments is lower than the cost of fixed offshore structures (e.g. jackets, gravity platforms). In comparison to other floating platforms (e.g. semi-submersible platforms, SPAR) moored using spread catenary lines, a TLP is moored using vertical pretension lines (displacement of a TLP hull is significantly larger than its weight!) and is practically *fixed* in the vertical plane (heave, roll and pitch), thus providing a stable horizontal work area. Furthermore, motions of a TLP in the horizontal plane (surge, sway and yaw) are more compliant to the environmental forces of wind, waves and sea currents. Finally, one should note two equally complex subsystems of a TLP structure: hull and superstructure that are a visible part of a TLP and (almost invisible) subsea structures such as highly pretensioned mooring lines (tendons) and a bundle of top tensioned marine risers (TTR's).

When designing and analysing a TLP (the same argument is valid for an arbitrary offshore structure), physically consistent model would be to consider both hull and subsea structures as an elastic continuum with infinite D.O.F. That is very expensive computationally indeed and a rational engineering approach is to take into consideration structural stiffness and natural frequencies of different structural components and to model:

(i) TLP hull as a rigid body with 6 D.O.F.,
(ii) Subsea structures as either rigid or elastic depending on the desired complexity of a mathematical model.

Applied mooring system geometrically constrains TLP motion to that of an inverted pendulum in the water and, assuming large motions in the horizontal plane, is a source of both nonlinearity in restoring stiffness and damping (TTR's have a similar effect). Consequently, analysis is constrained to the time domain complicating considerably both design and analyses. Additionally, there is an issue of coupling between the hull and subsea lines that needs to be accounted for.

Following the conclusions outlined in [1], an approximate mathematical model has been derived (semi-coupled model) with the main assumption of rigid subsea lines. Semi-coupled model is restricted to 6 D.O.F. and yet it incorporates much of the physics characterizing TLP responses thus enabling:

(i) Computationally inexpensive evaluation of different designs in the early design stage of a TLP,
(ii) Calculation of the fully coupled 2nd order hydrodynamic forces based on the consistent 1st order motion (taking the influence of mooring lines into account),
(iii) Optimization procedures, since obtaining Pareto front with the fully coupled dynamic analysis is practically impossible,
(iv) More reliable stochastic analysis, since the number of realisations can be considerably larger,
(v) Possibility of statistical linearization of mooring forces, thus enabling a very useful frequency domain analysis of TLP's.

Present paper validates the semi-coupled model against the fully coupled FEM model for a mid-range depth of 450 m of an ISSC TLP. It is to be expected that the differences will become larger as the depth increases since the main assumption of rigid lines becomes invalid due to the fact that both the stiffness and the natural frequencies of subsea lines become lower and thus neglecting deformation in no longer rational.

2. Mathematical model

When analysing the motions and stresses of an arbitrary offshore object in the time domain engineers usually distinct two different approaches: *uncoupled* and *fully-coupled* dynamic analysis. There is also a possibility of an extension of the uncoupled model called *the semi-coupled analysis*, which will be further elaborated in the present chapter. One should note that the equations of motion of an arbitrary offshore object are given by the system of 2nd order nonlinear ordinary integro-differential equations of differing complexity.

2.1. Semi-coupled dynamic analysis

Uncoupled analysis assumes no interaction between the rigid body (hull) dynamics and the dynamics of mooring lines/marine risers other than through the stiffness term in the equations of motion. As a result of that assumption mooring lines/marine risers are modelled as massless springs. These mathematical models are restrained to 6 D.O.F. and are computationally very inexpensive.

Uncoupled mathematical models can be further extended by:

(i) Consistent modelling of nonlinear restoring stiffness due to large motions,

(ii) Taking into account simplified inertia and damping influence of the subsea lines (mooring lines, marine risers).

Equations of motion of a TLP for the semi-coupled mathematical model can be written (in index notation) as, [2]:

$$\begin{aligned}&\left(M_{ij}+M_{ij}^{SL}+A_{ij}^{\infty}+A_{ij}^{SL}\right)\ddot{\delta}_j+\left(D_{ij}+D_{ij}^{SL}\right)\dot{\delta}_j+\int_0^t R_{ij}(t-\tau)\,\dot{\delta}_j(\tau)\,\mathrm{d}\tau+\\&+K_{ij}(\boldsymbol{\delta})\,\delta_j=F_i^E(t),\end{aligned}\tag{1}$$

where M_{ij} is the element of hull inertia matrix, M_{ij}^{SL} is the element of additional inertia matrix due to the action of subsea lines (mooring lines, marine risers), A_{ij}^{∞} is the element of hull added mass matrix at infinite frequency, A_{ij}^{SL} is the element of added mass matrix due to the action of subsea lines, D_{ij} is the element of hull viscous damping matrix, D_{ij}^{SL} is the element of additional viscous damping matrix due to action of subsea lines, $R_{ij}(t)$ is the element of hull memory function matrix, $K_{ij}(\boldsymbol{\delta})$ is the element of nonlinear restoring stiffness matrix (including both hull and subsea lines) and $F_i^E(t)$ is the element of environmental loads vector due to wind, waves and sea current.

Formulation of the consistent nonlinear restoring stiffness is detailed in [3] and [4]. Some important findings are summarized here, using the force equilibrium approach. Resulting nonlinear secant restoring stiffness matrix is formulated for the kinematic pole *P* resulting in the uncoupled D.O.F.:

$$[K(\boldsymbol{\delta})]_P=\begin{bmatrix}K_{11}(\boldsymbol{\delta}) & & & \\ & K_{22}(\boldsymbol{\delta}) & & \\ & & \ddots & \\ & & & K_{66}(\boldsymbol{\delta})\end{bmatrix},\tag{2}$$

with the corresponding motions vector $\langle\delta\rangle_P=\langle\delta_1,\delta_2,\delta_3,\delta_4,\delta_5,\delta_6\rangle$

A TLP experiences *set-down* δ^s, which is a rather unique 2nd order vertical low-frequency motion, due to both translational and rotational motions in the horizontal plane and their coupling. Since the inertial forces and damping of the mooring lines are due to the total vertical motion (heave + set-down), as opposed to restoring stiffness which is due to heave only, one has to include set-down in the motions vector $\langle \delta \rangle_V = \langle \delta_1, \delta_2, \delta^V = \delta_3 - \delta^S, \delta_4, \delta_5, \delta_6 \rangle$ resulting in a new restoring stiffness matrix in which vertical and horizontal motions are coupled.

$$[K(\boldsymbol{\delta})]_P = \begin{bmatrix} K_{11}(\boldsymbol{\delta}) & & & & & \\ & K_{22}(\boldsymbol{\delta}) & & & & \\ K_{31}(\boldsymbol{\delta}) & K_{32}(\boldsymbol{\delta}) & K_{33}(\boldsymbol{\delta}) & & & K_{36}(\boldsymbol{\delta}) \\ & & & K_{44}(\boldsymbol{\delta}) & & \\ & & & & K_{55}(\boldsymbol{\delta}) & \\ & & & & & K_{66}(\boldsymbol{\delta}) \end{bmatrix} \tag{3}$$

Elements K_{31}, K_{32} and K_{36} account for the coupling between the vertical and horizontal motions. When solving the equations of motion (1) for the reference point in the centre of gravity one needs to transform the matrix $[K(\boldsymbol{\delta})]_P$ into:

$$[K(\boldsymbol{\delta})]_G = \begin{bmatrix} K_{11}(\boldsymbol{\delta}) & & & & K_{15}(\boldsymbol{\delta}) & \\ & K_{22}(\boldsymbol{\delta}) & & K_{24}(\boldsymbol{\delta}) & & \\ K_{31}(\boldsymbol{\delta}) & K_{32}(\boldsymbol{\delta}) & K_{33}(\boldsymbol{\delta}) & K_{34}(\boldsymbol{\delta}) & K_{35}(\boldsymbol{\delta}) & K_{36}(\boldsymbol{\delta}) \\ & K_{42}(\boldsymbol{\delta}) & & K_{44}(\boldsymbol{\delta}) & & \\ K_{51}(\boldsymbol{\delta}) & & & & K_{55}(\boldsymbol{\delta}) & \\ & & & & & K_{66}(\boldsymbol{\delta}) \end{bmatrix} \tag{4}$$

Elements of the additional inertia matrix M_{ij}^{SL} due to the action of subsea lines are determined rather straight forward under the assumption of linear distribution of displacement, speed and acceleration, by using the moment equilibrium with respect to the connecting point to the sea bottom, [5]. Elements of the additional added mass and damping matrix due to the action of subsea lines are formulated using the Morison equation [6]:

$$\mathbf{q}^{SL} = -C_A \rho A \ddot{\boldsymbol{\delta}} + C_M \rho A \dot{\boldsymbol{\delta}} + \frac{1}{2} C_D \rho D \left\| \mathbf{v} - \dot{\boldsymbol{\delta}} \right\| (\mathbf{v} - \dot{\boldsymbol{\delta}}), \tag{5}$$

where $\mathbf{q}^{SL}$ is the distributed hydrodynamic load on the subsea lines, C_A, C_D and C_M are added mass, damping and inertia coefficients, respectively, *A* and *D* are cross-section area and diameter of the rigid subsea lines, respectively, $\dot{\boldsymbol{\delta}}$ is the velocity vector of hull motion and $\mathbf{v}$ is the velocity of water motion.

2.2. Fully-coupled dynamic analysis

Fully-coupled analysis assumes mooring lines and marine risers as an elastic continuum, utilizing the finite element method to constrain the arbitrary number D.O.F. to a finite value. Dynamic interaction between the subsea lines and the hull depends on the consistency of motion and force compatibility at the point of connection between the subsea structural elements and the hull. Subsea elements influence the rigid hull (and vice versa) through inertia, damping, restoring force and external loads. This is physically more consistent model but also computationally significantly more expensive. Equations of motion of a TLP for the fully-coupled mathematical model can be written (in index notation) as, [2]:

$$\left(M_{ij} + A_{ij}^{\infty}\right)\ddot{\delta}_j + \int_0^t R_{ij}(t-\tau)\dot{\delta}_j(\tau)\,\mathrm{d}\tau + C_{ij}\,\delta_j = F_i^E(t) + F_i^{SL}(t, \boldsymbol{\delta}, \dot{\boldsymbol{\delta}}, \ddot{\boldsymbol{\delta}}), \tag{6}$$

where C_{ij} is the element of hull hydrostatic stiffness and $F_i^{SL}(t, \boldsymbol{\delta}, \dot{\boldsymbol{\delta}}, \ddot{\boldsymbol{\delta}})$ is a force vector coupling the subsea lines dynamics to the rigid hull dynamics and its formulation is detailed in [7], along with the derivation of the applied beam finite elements.

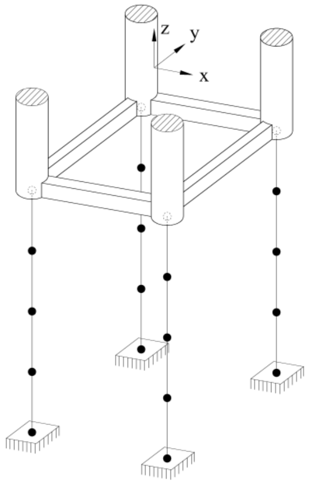

Fig. 1. ISSC TLP finite element model for the fully-coupled dynamic analysis

Slika 1. Model konačnih elemenata ISSC TLP-a za potpuno spregnutu dinamičku analizu

3. Time domain hydrodynamics

Hydrodynamic reaction forces are calculated in the time domain using the indirect method, where the linearized hydrodynamic boundary value problem is solved in the frequency domain (assuming the potential flow with free surface) using the HYDROSTAR software and then this linear solution is mapped to the time domain using the convolution integral, [2].

The hull memory function (retardation function, impulse response function) is calculated from the frequency dependant radiation damping coefficients $B_{ij}(\omega)$ using the inverse Fourier transform:

$$R_{ij}(t) = \frac{2}{\pi} \int_0^{\infty} B_{ij}(\omega) \cos \omega t \, \mathrm{d}\, \omega . \tag{7}$$

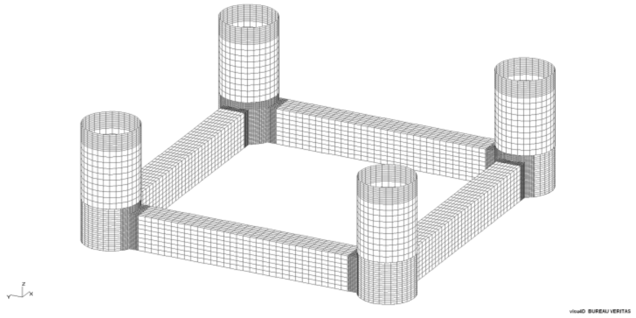

Fig. 2. ISSC TLP panel model (HYDROSTAR)

Slika 2. Model panela ISSC TLP-a (HYDROSTAR)

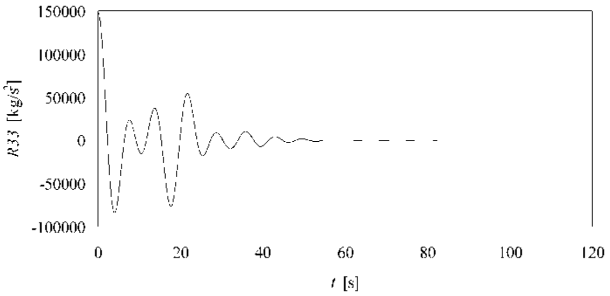

Fig. 3. Memory function for heave of ISSC TLP

Slika 3. Memorijska funkcija poniranja ISSC TLP-a

In order to verify the calculation of the hull memory functions the fact that added mass $A_{ij}(\omega)$ and radiation damping $B_{ij}(\omega)$ are a Fourier transform pair is used, [8]:

$$B_{ij}(\omega) = \int_0^{\infty} R_{ij}(t) \cos \omega t \, \mathrm{d}\, t , \tag{8}$$

$$A_{ij}(\omega) = A_{ij}^{\infty} - \frac{1}{\omega}\int_{0}^{\infty} R_{ij}(t)\sin\omega t\,\mathrm{d}t. \tag{9}$$

The equation (9) can also be utilized to indirectly calculate frequency independent added mass A_{ij}^{∞}.

As an example comparison of the values obtained using the Fourier transform of the memory function $R_{ij}(t)$ and those calculated directly using HYDROSTAR is shown in Fig. 4.

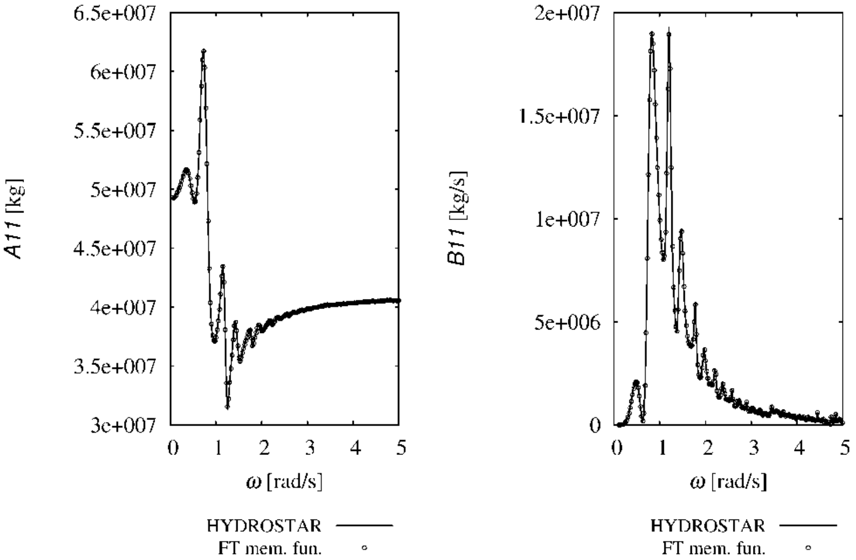

Fig. 4. Verfication of the mapping of linear hydrodynamic solution from frequency to time domain for surge

Slika 4. Verifikacija preslikavanja linearnog hidrodinamičkog rješenja iz frekvencijske u vremensku domenu

4. Numerical results

Table 1. contains the main particulars for the analysed ISSC TLP. That particular tension leg platform was researched in [9].

Table 1. ISSC TLP main particulars

Tablica 1. Osnovne značajke ISSC TLP-a

Parameters	Symbol	Value
Column spacing	$2a=2b$	86.25 m
Column diameter	D_C	16.87 m
Pontoon width	w	7.50 m
Pontoon height	d	10.50 m
Draft	$-z_T$	35.00 m
Waterplane area	A_{wl}	894 m^2
Displacement	U	$5.346 \cdot 10^5$ kN
Weight	Q	$3.973 \cdot 10^5$ kN
Vertical position of COG above keel	$\overline{PG}$	38.0 m
Vertical position of COB above keel	$\overline{PB}$	22.3 m
Platform mass	m	$40.5 \cdot 10^3$ t

Parameters	Symbol	Value
Total tether pretension	T	$1.373 \cdot 10^5$ kN
Longitudinal metacentric height	$\overline{M_L G}$	6.0 m
Transverse metacentric height	$\overline{M_T G}$	6.0 m
Length of mooring tethers	L	415.0 m
Tethers cross-section area	A	1.3 m^2
Roll mass moment of inertia	J_x^G	$82.37 \cdot 10^6$ t m^2
Pitch mass moment of inertia	J_y^G	$82.37 \cdot 10^6$ t m^2
Yaw mass moment of inertia	J_z^G	$98.07 \cdot 10^6$ t m^2
Vertical stiffness of combined tethers	EA/L	$0.813 \cdot 10^6$ kN/m
Roll and pitch effective stiffness	EI_x/L, EI_y/L	$1.501 \cdot 10^9$ kNm/rad

One should note that the height of Eiffel tower is around 324 m, and the largest jacket structure is Shell's Bullwinkle, installed in 412 m water depth.

Table 2. contains the main particulars for the analysed tendons (mooring lines).

Table 2. Tendon main particulars

Tablica 2. Osnovne značajke pripona

Parameters	Symbol	Value
Outside diameter	D_o	1.0 m
Inside diameter	D_i	0.686 m
Cross section area	A_t	0.415 m^2
Young's modulus	E	$2.1 \cdot 10^{11}$ N/m^2
Length	L_t	415 m
Drag coefficient	C_D	1.2
Added mass coefficient	C_A	1.0
Distance between tendons	B_t	86.25 m

In order to assess the contribution of different force components, due to coupling between the floating body and subsea lines, on the equations of motion, tendons are modelled using mathematical models of different complexity, as stated in Table 3. Damping of the tendons was assumed to be of viscous origin only. First, the static analysis is performed to obtain the force-displacement curves, by comparing semi-coupled and fully-coupled models. Secondly, dynamic analysis is performed utilizing free decay tests, again comparing semi-coupled and fully-coupled models. Only response in the vertical plane is analysed, and further calculations are needed to validate the free decay response for the coupled horizontal and vertical D.O.F.

Table 3. Different tendon models

Tablica 3. Različiti modeli pripona

	inertia	added mass	damping	elasticity	displacement
Model 1				+	
Model 2	+	+	+	+	+

Table 4. gives the comparison between the semi-coupled and fully-coupled static response for the most consistent mathematical model. The same is shown in Figs 5 and 6.

Table 4 Static analysis of ISSC TLP

Tablica 4. Statička analiza ISSC TLP-a

	semi-coupled		fully-coupled	
F_h	δ_l	δ_v	δ_l	δ_v
kN	m	m	m	m
0	0.000	0.000	0.000	0.000
3126	11.006	-0.146	10.948	-0.144
6252	21.350	-0.549	21.230	-0.542
9378	30.682	-1.134	30.498	-1.118
12504	38.964	-1.829	38.717	-1.802
15630	46.306	-2.583	45.999	-2.544
18756	52.856	-3.366	52.492	-3.313
21882	58.750	-4.159	58.334	-4.094

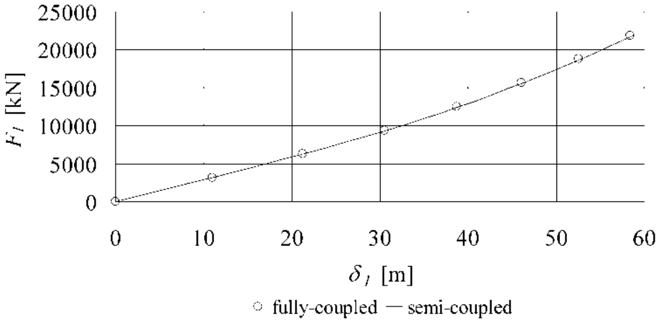

Fig. 5. Horizontal restoring force - displacement curve (mooring system characteristic)

Slika 5. Krivulja odnosa horizontalne povratne sile i pomaka (karakteristika sidrenog sustava)

Set-down is a feature of a TLP, and is modelled consistently by the semi-coupled method, as shown in Fig. 6.

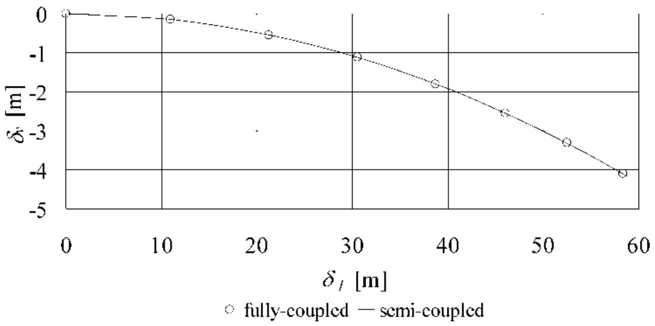

Fig. 6. Vertical displacement - horizontal displacement curve

Slika 6. Krivulja odnosa vertikalnog i horizontalnog pomaka

Evaluation of (dynamic) free decay response of ISSC TLP in the vertical plane is given in Figs. 7, 8 and 9. Evaluated mathematical model is *almost* a fully consistent one, since the TLP hull radiation damping is omitted due to simplicity (generally, for smaller TLP's it is a non-dominating member of the equation, and is the same for both semi-coupled and fully-coupled model, since it is independent from the influence of subsea lines, at least in the applied formulation).

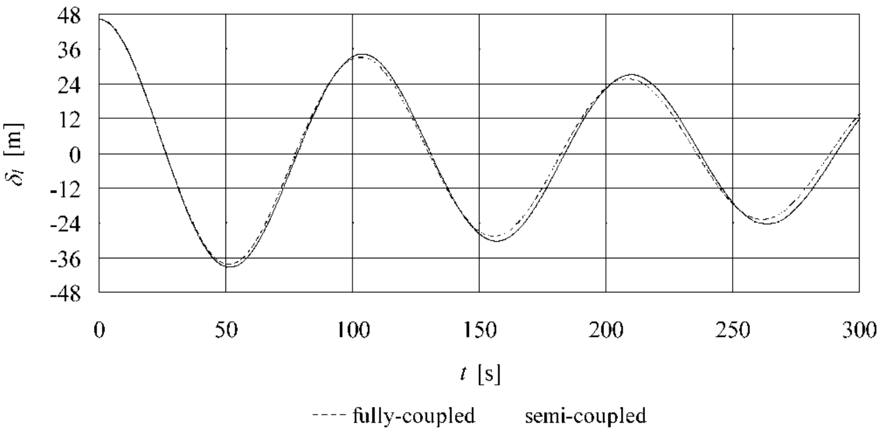

Fig. 7. Comparison of free decay in surge of ISSC TLP

Slika 7. Usporedba slobodnog zalijetanja ISSC TLP-a

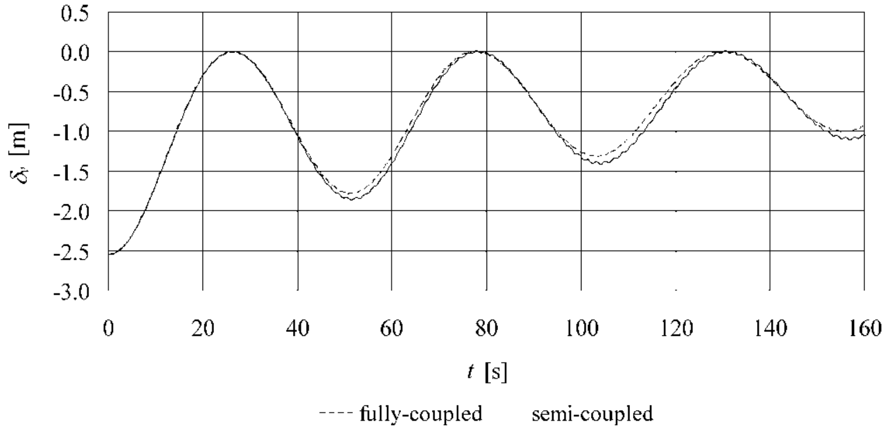

Fig. 8. Comparison of free decay in vertical D.O.F. (heave + set-down) of ISSC TLP

Slika 8. Usporedba slobodnog poniranja i sjedanja ISSC TLP-a

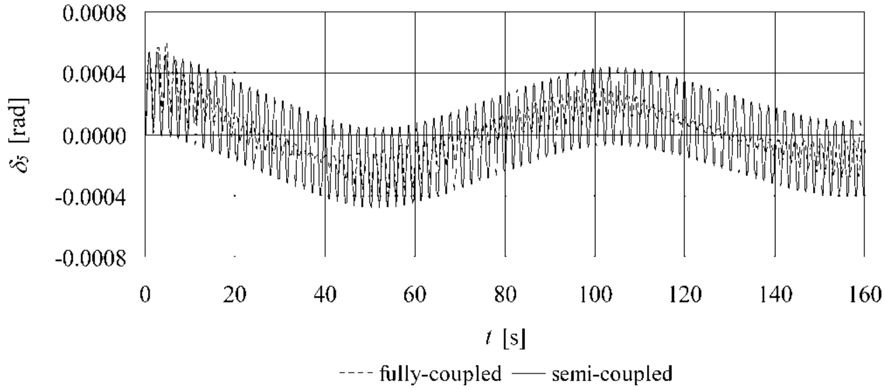

Fig. 9. Comparison of free decay in pitch of ISSC TLP

Slika 9. Usporedba slobodnog posrtanja ISSC TLP-a

Response in surge is characterized by quite good agreement between semi-coupled and fully-coupled model. In the vertical D.O.F. (and the same can be said for pitch) there are more pronounced differences due to the fact that the semi-coupled model tends to give more energy in higher frequencies due to the fact that this model has larger mooring stiffness than fully-coupled one. Amplitudes agree well and there is a slight difference in the period between two models. There is some difference in pitch, although the frequency content of the time signal is the same. This needs further clarification.

5. Conclusion

Based on the result of numerical comparison between the semi-coupled and fully-coupled dynamic analysis of floating tethered bodies, for medium sea depths of at least up to 450 m, it should be concluded that the semi-coupled method is reliable enough in the preliminary design stage, for optimization procedures and consistent 2nd order hydrodynamic force calculation (if the nonlinear 1st order motions of a TLP, due to nonlinear mooring forces, are taken into account). Also, it should be noted that the radiation damping of a TLP hull is almost negligible in terms of the response amplitudes, and that facilitates assumption that the convolution integral could be ignored in the equations of motion, thus significantly reducing the calculation time. It does however influence the period of a response. That conclusion is valid at least for ISSC TLP.

It is to be expected that the fully-coupled method will be more reliable, as the sea depth increases, due to the fact that the natural frequencies, as well as structural stiffness, of subsea lines decreases, invalidating the assumption of straight lines for the semi-coupled method. Therefore, further research is needed to establish the boundaries for the reliable dynamic analysis using the semi-coupled method.

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