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ANALIZA ČVRSTOĆE VELIKIH KONTEJNERSKIH BRODOVA GREDNIM MODELOM PREMA KLASIFIKACIJSKIM PRAVILIMA

Sažetak

Osnivanje konstrukcije velikih kontejnerskih brodova, zbog njihove male krutosti na uvijanje, trebalo bi se temeljiti na hidroelastičnoj analizi. Budući da spomenuti problem nije dovoljno istražen još uvijek se koriste tzv. kvazi-statički proračuni čvrstoće. Za tu svrhu, može se koristiti gredni strukturni model ili 3D FEM model uz relativno jednostavne formule za određivanje presječnih sila prema Pravilima. U članku je opisan standardni postupak za osnivanje konstrukcije velikih kontejnerskih brodova koji se koristi danas. Zatim je primjena grednog modela, razvijenog za hidroelastičnu analizu, ilustrirana za analizu čvrstoće, kao glavnog dijela projektnog postupka. Presječne sile, određene prema Pravilima su korištene za određivanje valnog opterećenja po duljini grednog modela. Rezultati dobiveni programom DYANA za statičku analizu su kut uvijanja i njegov prirast, pomoću kojih se dalje mogu odrediti distribucije naprezanja i deformacije grotala važne za sigurnost kontejnera.

Ključne riječi: Kontejnerski brod, Gredni model, Analiza čvrstoće prema pravilima

STRENGTH ANALYSIS OF LARGE CONTAINER SHIPS BY USING A BEAM MODEL ACCORDING TO THE CLASSIFICATION RULES

Summary

The structural design of large container ships, due to their lower torsional stiffness, should be based on hydroelastic analysis. This problem is not yet completely investigated and therefore quasi static approach for strength analysis is still used. For that purpose beam structural model or 3D FEM model can be employed, with rather simple rule formulas for determining wave sectional forces. In this paper standard procedure for structural design of large container ships ordinary used nowadays is described. Then, application of an advanced beam model, developed for hydroelastic analysis, is illustrated for strength analysis as a main part of the design procedure. Sectional forces specified by classification rules are used for determining distribution of wave load along the beam model. The results obtained by the modified program DYANA for static analysis are twist angle and its variation, which may be further used for cross-section stress distributions calculation as well as for determining hatch deformations important for container safety.

Key words: Container ship, Beam model, Rule based strength assessment

1. Introduction

Container ships are characterised by a complex design problems where the torsional response in waves is considered to be one of the most important. From a structural design point of view large torsion gives rise to large diagonal shear deformations of the hatch openings and stress concentrations with corresponding fatigue risk in the hatch corners [1]. Nowadays, Ultra Large Container Ships (ULCS) with capacity up to 18000 TEU are being built and design and construction of such huge ships with large deck openings and high speed are at the margin of Classification Rules. It is already shown in a number of papers as for instance [2,3,4] that ULCS structural design should be based on direct calculations assuming hydroelastic mathematical model. However, methodology of ship hydroelastic analysis is not completely developed and validated yet, particularly in case of ULCS, and therefore quasi static approach for strength analysis is still used. For that purpose beam structural model or 3D FEM model can be employed, with rather simple rule formulas for determining wave sectional forces. The paper deals with the direct response analysis of a large container ship by a beam model subjected to the load distributions prescribed by the Bureau Veritas (BV) Classification Rules, [5].

2. Ship particulars

A large container ship of 11400 TEU is considered, Fig. 1. The main vessel particulars are the following:

Length overall	$L_{oa} = 363.44 \text{ m}$
Length between perpendiculars	$L_{pp} = 348.00 \text{ m}$
Breadth	B = 45.6 m
Depth	H = 29.74 m
Draught	T = 15.5 m
Displacement, full load	$\Delta_f = 171445 \text{ t}$
Displacement, ballast	$\Delta_b = 74977 \text{ t}$
Engine power	P = 72240 kW
Ship speed	v = 24.7 kn.





Slika 1. Kontejnerski brod nosivosti 11400 TEU

3. Outline of an advanced thin-walled girder theory

The beam model is based on the advanced thin-walled girder theory, i.e. it takes into account both shear influence on bending and torsion, contribution of transverse bulkheads and engine room structure to the ship hull global stiffness, in a reliable way. Total beam deflection and twist angle consist of pure bending and torsion, respectively, and shear contribution [6]

$$w = w_b + w_s = w_b - \frac{EI_b}{GA_s} \frac{\mathrm{d}^2 w_b}{\mathrm{d}x^2}, \qquad \psi = \psi_t + \psi_s = \psi_t - \frac{EI_w}{GI_s} \frac{\mathrm{d}^2 \psi_t}{\mathrm{d}x^2}, \tag{1}$$

where I_b is moment of inertia of cross-section, A_s is shear area, I_w is warping modulus and I_s is shear inertia modulus. Beam model presented here was originally developed for the needs of hydroelastic analysis of ULCS whose lowest natural frequencies belong to coupled horizontal and torsional vibrations. Matrix finite element equation for such vibrations yields [6]

$$\mathbf{f}^{\mathbf{e}} = \mathbf{k}^{\mathbf{e}} \boldsymbol{\delta}^{\mathbf{e}} + \mathbf{m}^{\mathbf{e}} \ddot{\boldsymbol{\delta}}^{\mathbf{e}}, \tag{2}$$

where \mathbf{f}^{e} is nodal forces vector, $\boldsymbol{\delta}^{e}$ is nodal displacements vector, \mathbf{k}^{e} is stiffness matrix, and \mathbf{m}^{e} is mass matrix. Since quasi static approach is used the inertial part in Eq. (2) is ignored, and according to [6] one can write:

$$\mathbf{f}^{\mathbf{e}} = \begin{cases} \mathbf{P} \\ \mathbf{R} \end{cases}, \qquad \mathbf{\delta}^{\mathbf{e}} = \begin{cases} \mathbf{U} \\ \mathbf{V} \end{cases}, \tag{3}$$

$$\mathbf{k}^{\mathbf{e}} = \begin{bmatrix} \mathbf{k}_{\mathbf{b}\mathbf{s}} & 0\\ 0 & \mathbf{k}_{\mathbf{w}\mathbf{s}} + \mathbf{k}_{\mathbf{t}} \end{bmatrix}.$$
 (4)

Vectors of nodal forces and displacements are:

$$\mathbf{P} = \begin{cases} -Q(0) \\ M(0) \\ Q(l) \\ -M(l) \end{cases}, \qquad \mathbf{R} = \begin{cases} -T(0) \\ -B_w(0) \\ T(l) \\ B_w(l) \end{cases}, \qquad \mathbf{U} = \begin{cases} w(0) \\ \varphi(0) \\ w(l) \\ \varphi(l) \end{cases}, \qquad \mathbf{V} = \begin{cases} \psi(0) \\ \vartheta(0) \\ \psi(l) \\ \vartheta(l) \end{cases}.$$
(5)

In the above formulae symbols Q, M, T and B_w denote shear force, bending moment, torque and warping bimoment, respectively. Also, w, φ , ψ and ϑ are deflection, rotation of cross-section, twist angle and its variation, respectively. The submatrices of \mathbf{k}^e , Eq. (4), which are specified in [6], have the following meanings:

 $\mathbf{k}_{\mathbf{bs}}$ – bending–shear stiffness matrix,

 \mathbf{k}_{ws} – warping–shear stiffness matrix,

 \mathbf{k}_{t} – torsion stiffness matrix.

Since coupling between horizontal and torsional vibrations is realized through the mass matrix due to eccentricity of the centre of gravity and shear centre, it is obvious that flexural and torsional responses in quasi static case can be analyzed independently.

The effect of large number of transverse watertight and support bulkheads can be incorporated into the hull torsional stiffness [7]:

$$I_{t}^{*} = \left[1 + \frac{a}{l_{1}} + \frac{4(1+\nu)C}{I_{t}l_{0}}\right]I_{t}, \qquad C = \frac{U}{E\psi_{t}^{\prime 2}},$$
(6)

where *a* is the web height of bulkhead girders, l_0 is the bulkhead spacing, $l_1 = l_0 - a$ is the net length, *C* is the energy coefficient, and *U* is the bulkhead grillage and stool strain energy

due to warping of cross-section. Warping shape function can be assumed in the following form:

$$\overline{u}(y,z) = -y\left\{\left(z-d\right) + \left[1-\left(\frac{y}{b}\right)^2\right]\frac{z^2}{H}\left(2-\frac{z}{H}\right)\right\}, \qquad u(y,z) = \overline{u}(y,z)\psi_t', \qquad (7)$$

where H is the ship height, b is one half of bulkhead breadth, d is the distance of warping centre from double bottom centroid, while y and z are transverse and vertical coordinates, respectively. The bulkhead grillage strain energy includes vertical and horizontal bending with contraction, and torsion [7]:

$$U_{g} = \frac{1}{1 - \nu^{2}} \left[\frac{116H^{3}}{35b} i_{y} + \frac{32b^{3}}{105H} i_{z} + \frac{8Hb}{75} \nu \left(i_{y} + i_{z} \right) + \frac{143Hb}{75} \left(1 - \nu \right) i_{t} \right] E \psi^{\prime 2}$$
(8)

where i_y , i_z and i_t are the average moments of inertia of cross-section and torsional modulus per unit breadth, respectively. The strain energy of the upper bulkhead stool, Fig. 2, is comprised of the bending, shear and torsional contributions

$$U_{s} = \left[\frac{12h^{2}I_{sb}}{b} + 72(1+\nu)\frac{h^{2}}{b^{3}}\frac{I_{sb}^{2}}{A_{s}} + \frac{9bI_{st}}{10(1+\nu)}\right]E\psi'^{2}$$
(9)

where I_{sb} , A_s and I_{st} are the moment of inertia of cross-section, shear area and torsional modulus, respectively. Quantity *h* is the stool distance from the inner bottom, Fig. 2.

In addition to large number of transverse bulkheads ULCS are also characterized by relatively short closed engine room structure with length of about a half of ship breadth, which doesn't behave like closed cross-section segment completely, and therefore the procedure for calculation its effective stiffness parameters is developed and presented in [8]. However, due to reason of simplicity the exact parameters calculated for closed cross-sections in the engine room area are used in this investigation.



Fig. 2 Longitudinal section of container ship holdSlika 2. Uzdužni presjek skladišta kontejnerskog broda

4. Load

Beam model of ship hull is divided into 47 finite elements. The model nodes are located at the transverse bulkheads and some chosen frames. The segments of closed cross-section (engine room structure and peaks) are modelled using 2 d.o.f. elements, i.e. nodal twist angles, while open segment FE includes warping and twist angle derivatives as additional degree of freedom.

Strength analysis of large container ships according to classification rules by using a beam model

According to the Classification Rules [5] wave load is given by sectional forces, i.e. torque, M_x , and horizontal shear force, Q_x .

CASE 1 – Pure torque

$$M_x = M_0 \left(1 - \cos \frac{2\pi x}{L} \right) \tag{10}$$

where M_0 is rule-based calculated amplitude.

Distributed torque to be imposed to the beam model:

$$\mu_x = \frac{\mathrm{d}M_x}{\mathrm{d}x} = M_0 \frac{2\pi}{L} \sin \frac{2\pi x}{L} \tag{11}$$



Fig. 3 Distributed pure torque, μ_x

Slika 3. Distribuirani čisti moment uvijanje, μ_x

CASE 2 – Horizontal shear force

$$Q = Q_0 \sin \frac{2\pi x}{L} \tag{12}$$

where Q_0 is rule-based calculated amplitude.

Torque due to shear force:

$$M_t^{Q} = Q\Delta z = Q_0 \Delta z \sin \frac{2\pi x}{L}$$
(13)

Distributed torque due to shear force:

$$\mu_x^Q = Q_0 \Delta z \frac{2\pi}{L} \cos \frac{2\pi x}{L} + Q_0 \frac{d\Delta z}{dx} \sin \frac{2\pi x}{L}$$
(14)

where Δz is vertical distance from the shear centre to a point located at 0.6*T* above the baseline.

Due to reason of simplicity, it is assumed that the vertical coordinate of shear centre in the engine room structure correspond to that of the open section. This assumption does not influence the results significantly, because engine room structure is relatively short. Furthermore, the second term in (14) is neglected as a small quantity.



Fig. 4 Distance of shear centre from referent line 0.6TSlika 4. Udaljenost centra smicanja od 0.6T

Boundary condition (for both cases):



Strength analysis of large container ships by using a beam model according to classification rules

Fig. 5 Distributed torque due to shear force

Slika 5. Distribuirani moment uvijanja uslijed smične sile

$$x \approx \frac{L}{2}; \qquad \psi = 0 \tag{15}$$

Finite element equation for open cross-section, according to Section 3, reads

$$\left(\mathbf{k}_{ws} + \mathbf{k}_{t}\right)\boldsymbol{\delta} = \boldsymbol{\mu}_{\mathbf{x}},\tag{16}$$

where:

$$\boldsymbol{\delta} = \begin{cases} \boldsymbol{\psi}_1 \\ \boldsymbol{\vartheta}_1 \\ \boldsymbol{\psi}_2 \\ \boldsymbol{\vartheta}_2 \end{cases} \,. \tag{17}$$

The load vector $\boldsymbol{\mu}_{\mathbf{x}}$ can be given in a simplified form [6]:

$$\boldsymbol{\mu}_{\mathbf{x}} = \frac{l\bar{\mu}}{12} \begin{cases} 6\\l\\6\\-l \end{cases}, \quad \bar{\mu} = \frac{\mu_1 + \mu_2}{2}. \tag{18}$$

For closed cross-section the finite element equation reads:

$$\mathbf{k}_{t}\boldsymbol{\delta} = \boldsymbol{\mu}_{\mathbf{x}} \tag{19}$$

where:

$$\boldsymbol{\delta} = \begin{cases} \boldsymbol{\psi}_1 \\ \boldsymbol{\psi}_2 \end{cases}, \ \boldsymbol{\mu}_{\mathbf{x}} = \frac{\overline{\mu}l}{2} \begin{cases} 1 \\ 1 \end{cases}.$$

5. Calculation of ship stiffness properties

Stiffness parameters of the ship hull are calculated by using program STIFF [9], Fig. 6. The ship is designed with alternate watertight and support bulkheads, Figs. 2 and 7. The stiffness parameters of the bulkhead girders are listed in Tables 1 and 2, while the stool parameters are given in Table 3. The bulkhead dimensions are the following: H = 29.44 m, b = 20.45 m, $l_0 = 14.44$ m, a = 1.80 m.

Strength analysis of large container ships according to classification rules by using a beam model



Fig. 6 Warping of ship cross-section – program STIFF **Slika 6**. Vitoperenje poprečnog presjeka broda – program STIFF

The bulkhead strain energy, determined according to Eqs. (8) and (9), is summarized in Table 4, where also the energy coefficient is calculated as the average value of the watertight and support bulkhead strain energies. Most of the hull induced energy is absorbed by the stool. Thus, the equivalent torsional modulus for midship section yields $1.9 \cdot I_t$. This value is applied for all cross-sections as the first approximation.



Fig. 7 Transverse bulkheads of the considered ship, a) watertight bulkhead, b) support bulkheadSlika 7. Pregrade analiziranog kontejnerskog broda, a) nepropusna, b) propusna

Table 1 Stiffness parameters of watertight bulkhead

Girder	Moment of inertia	Torsional modulus	Girder spacing	Moment of inertia per unit breadth	Torsional modulus per unit breadth
	$I(m^4)$	$I_t (\mathrm{m}^4)$	<i>c</i> (m)	$i (m^3)$	i_t (m ³)
Horizontal	0.0216	0.00905	5.184	0.004164	0.002843
Vertical	0.03094	0.023328	5.04	0.006139	

Tablica 1. Parametri krutosti nepropusne pregrade

Table 2 Stiffness parameters of support bulkhead

Tablica 2. Parametri krutosti propusne pregrade

Girder	Moment of inertia	Torsional modulus	Girder spacing	Moment of inertia per unit breadth	Torsional modulus per unit breadth
	$I(m^4)$	$I_t (m^4)$	<i>c</i> (m)	$i (m^3)$	$i_t (m^3)$
Horizontal	0.00972	0.00486	5.184	0.001875	0.002293
Vertical	0.02017	0.02827	5.04	0.004002	

Table 3 Stool stiffness parameters

Tablica 3. Parametri krutosti kutije pregrade

Shear area	Moment of	Torsional	
	inertia	modulus	
$A_s (\mathrm{m}^2)$	$I_s (\mathrm{m}^4)$	I_{ts} (m ⁴)	
0.045	0.12236	0.433	

Table 4 Bulkhead strain energy, $U / (E\psi'^2)$

Tablica 4. Energija deformacije pregrade, $U/(E\psi'^2)$

Water bulkh	Watertight bulkhead		oulkhead	Energy coefficient
Grillage	Stool	Grillage	Stool	<i>C</i> , Eq. (C5)
(1)	(2)	(3)	(4)	(5) = [(1)+(2)+(3)+(4)]/2
22.248	60.437	11.059	60.437	77.191

Longitudinal distribution of torsional modulus, warping modulus and shear inertia modulus are shown in Figs. 8, 9 and 10, respectively. Longitudinal distribution of vertical coordinate of shear centre is shown in Fig. 11.



Fig. 8 Torsional modulus Slika 8. Modul uvijanja



Fig. 9 Warping modulus Slika 9. Modul vitoperenja

Strength analysis of large container ships according to classification rules by using a beam model







6. Results and comments

Distributions of twist angle, ψ , and its derivative, $\vartheta = d\psi/dx$, for case of pure torsion and torsion due to horizontal shear force, are shown in Figures 12 and 13, respectively.



Fig. 12 Distribution of twist angle and twist angle derivative, Case 1Slika 12. Raspodjela kuta uvijanja i njegove derivacije, Slučaj 1



Fig. 13 Distribution of twist angle and twist angle derivative, Case 2Slika 13. Raspodjela kuta uvijanja i njegove derivacije, Slučaj 2



Fig. 14 Deflection and rotation angle of cross-section due to horizontal bendingSlika 14. Progib i kut zaokreta poprečnog presjeka uslijed horizontalnog savijanja

7. Conclusion

The structural design of Ultra Large Container Ships (ULCS) is driven by the economies of scale of transporting large numbers of containers in one ship and the commercial pressing of reducing the total production cost and steel weight through optimisation. Therefore, these ships are characterised by a complex design problems where the torsional response in waves is considered to be one of the most important. Structural design of ULCS should be based on hydroelastic analysis, and for the needs of such analysis a sophisticated beam model has been developed and further coupled to 3D potential flow hydrodynamic. This paper deals with direct response assessment of a 11400 TEU container ship by a beam model subjected to rule based load distributions, i.e. pure torque and horizontal shear force induced torque. Also, the case of pure horizontal bending is analyzed. The obtained results are twist angle and its variation, as well as deflection and rotation angle of cross-section due to horizontal bending, which may be further used for cross-section stress distributions calculation as well as for determining hatch deformations important for container safety.

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