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THE GOLDEN RATIO IN PROBABLISTIC AND ARTIFICIAL INTELLIGENCE

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Subject review

The problem of the line section based on the golden ratio φ = 1,618033 has the analogy in probability. The solution of the elementary exponential distribution relies on the value 2lnφ in particular. This value also plays a key role to Riccati hyperbolic functions with Fibonacci and Lucas numbers in continuous domain. This establishes a close relationship between the constants e and φ. Two new theorems on the convergence between the constants φ and e were derived. The number e is the foundation of Markov processes, which find applications in probabilistic and artificial intelligence theory. The ratio between the constants φ and e, as well as many other natural phenomena based on the golden ratio, highlight the need to expand the field of probabilistic and artificial intelligence.

Keywords: *artificial intelligence, golden ratio, Markov processes*

Zlatni rez u vjerojatnosti i umjetnoj inteligenciji

Pregledni članak

Problem linijskog odsječka utemeljenog na omjeru zlatnog reza φ = 1,618033 ima analogiju u vjerojatnosti. Rješenje elementarne eksponencijalne raspodjele, posebice ističe vrijednost 2lnφ. Ova vrijednost također ima ključnu ulogu u odnosu Rikatijevih hiperboličnih funkcija s Fibonačijevim i Lukasovim brojevima u kontinuiranom području. Time se uspostavlja bliska veza između konstanti e i φ. Izvedena su dva nova teorema o konvergenciji konstanti φ i e . Broj e je temelj Markovskih procesa, koji su našli primjenu u teoriji vjerojatnosni i umjetne inteligencije. Omjer konstanti e i φ, kao i mnogi drugi prirodni fenomeni na temelju zlatnog reza, ističu potrebu za proširenjem područja vjerojatnosni i umjetne inteligencije.

Ključne riječi: *umjetna inteligencija, zlatni rez, Markovski procesi*

1 Introduction

The basis of a natural logarithm 2,71828182845…. is a real, irrational and transcedental number. The first application of this number was derived by a Scottish mathematician John Napier of Merchiston (15501617) in his study *Mirifici Logarithmorum Canonis Descriptio* (1614). The well known expression for the number:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 2{,}718281828459045...,$$

was discovered by the Swiss mathematician Jacob Bernoulli, (1654–1705). The first known use of the constant was in correspondence from Gottfried Leibniz (16461716) to Christiaan Huygens (1629–1695) in 1690 and 1691. The Swiss mathematician Leonhard Euler (1707–1783) was the first who used the letter e as the constant of a natural logarithm in 1727 or 1728, and the first use of letter e in a publication was in Euler's *Mechanica* (1736), 120 years after the first application of the constant. Sinergy of the number e "*significance*", Euler's work and authority, formed a standard that we consider today. The number e is one of the most significant numbers in modern mathematics.

The golden section constant has incomparably richer history dating from ancient times, from Egypt [1] to ancient Greece. Parthenon, whose construction started in 447 BC, was designed in the golden section proportions. Euclid (325–265 BC) gave the first recorded definition of the golden ratio: The total length (a+b) is to the length of the longer segment a as the length of a is to the length of the shorter segment b,

$$\frac{a+b}{a}=\frac{a}{b}\Leftrightarrow 1+\frac{b}{a}=\frac{a}{b}\Leftrightarrow$$

$$\Leftrightarrow 1+\frac{1}{\frac{a}{b}}=\frac{a}{b}\Leftrightarrow \frac{a}{b}+1=\left(\frac{a}{b}\right)^2\Leftrightarrow$$

$$\Leftrightarrow \left(\frac{a}{b}\right)^2-\frac{a}{b}-1=0.$$

Solution to this square equation is

$$\left(\frac{a}{b}\right)_{1,2}=\frac{1\pm\sqrt{1+4}}{2}=\frac{1\pm\sqrt{5}}{2}=\varphi=1{,}6180339...$$

Leonardo Pisano Bigollo – Fibonacci (1170–1250) mentioned the famous numerical series with initial numbers 0 and 1, and $F(n)=F(n-1)+F(n-2)$:

$0; 1; 1; 2; 3; 5; 8; 13; 21; 34; 55; 89; \ldots.$

Johannes Kepler (1571–1630) first proves that the golden ratio is the limit of the ratio of consecutive Fibonacci numbers,

$$\lim_{n\to\infty}\frac{F(n+1)}{F(n)}=\varphi=1{,}6180339....$$

Charles Bonnet (1720–1793), the Swiss naturalist and philosophical writer, first points out that in the spiral *phyllotaxis* of plants were frequently two successive Fibonacci series.

It has not been determined who actually established the identity between the geometric proportion and Kepler's limit of consecutive Fibonacci [2] numbers. It is assumed that it was Martin Ohm (1792–1872), the German mathematician and a younger brother of famous physicist Georg Ohm. Martin Ohm was the first who introduced the term "golden section" (*goldener Schnitt*).

Jacques Philippe Marie Binet (1786–1856), the French mathematician, established recurrence formulae for the sequence of Fibonacci numbers.

Edouard Lucas (1842–1891), the French mathematician, gives the numerical sequence now known as the Fibonacci sequence, its present name. Also, Edouard Lucas established a special numerical sequence with initial numbers 2 and 1:

2; 1; 3; 4; 7; 11; 29; 47; 199; 521; 2,207; 3,571….

This sequence is formed by means of the identical principle as Fibonacci numbers, $L(n) = L(n-1) + L(n-2)$. The limit of the ratio of consecutive Lucas numbers is equal to φ = 1,6180339…. Lucas extended the numbers to negative integers and formed a double infinite sequence with recurrent relation $L(-n)(-1)^n L(n)$. Binet's formulae with simple transformations were also applied to Lucas numbers. In that way, the basis for the analysis of the mathematical phenomenon of the golden section in a continuous domain was formed.

2 The golden section in the theory of probability

The densities of probability of continuous random variables are predominantly the number e function. Those are exponential, Erlang's, Logistic, Gamma, Gomperc's, Veilbull's, Pareto's, etc. The central position in this group of distribution is defined with **central limit theorem** (**CLT**), with conditions under which the mean of a sufficiently large number of independent random variables with finite mean and variance, will be approximately normally distributed. The density function of normal distribution also has the obligatory functional participation of the number e. On account of that, the number e has a dominant part in the theory of probability.

The elementary function of distribution density of the number e is the exponential distribution, $f(x, \lambda) = \lambda e^{-\lambda x}$. The fact that arbitrary derivations of the exponential functions are identical ($e^x = (e^x)' = (e^x)'' = ... = (e^x)^{(n)}$), when every subsequent increment is equal to the base, also contains the memoryless characteristic of the exponential function. This unique characteristic is the base of Markovian process.

Application of Euclid's definition of the golden section in the theory of probability is analogous to the following relations, according to Kolmogorov's axiomatic of probability:

- The complete system of the probability event 1 is the total value of the segment $(a + b)$;
- Probability $P < 1$ is the segment a;
- The complete probability $(1 - P)$ is the segment b.

The equivalence of geometric and probabilistic definitions now has the form:

$$\frac{a+b}{a} = \frac{a}{b} \Leftrightarrow \frac{1}{P} = \frac{P}{1-P}.$$

The application of the analogous probabilistic definition to the elementary exponential distribution with the parameter $\lambda = 1$, gives the relations which are defined by the integral quotients within the limits of the constant c (Fig. 1).

The value of the constant c is:

$$\frac{1}{P} = \frac{P}{1-P} \Leftrightarrow \frac{\int_0^\infty e^{-x}dx}{\int_0^c e^{-x}dx} = \frac{\int_0^c e^{-x}dx}{\int_c^\infty e^{-x}dx} \Leftrightarrow$$

$$\Leftrightarrow \int_c^\infty e^{-x}dx = \left[\int_0^c e^{-x}dx\right]^2 \Leftrightarrow$$

$$\Leftrightarrow e^{-c} - e^{-\infty} = \left[1 - e^{-c}\right]^2.$$

Relations of the defined integrals give the exponential equation:

$$e^{-c} = 1 - 2e^{-c} + e^{-2c} \Leftrightarrow e^{-2c} - 3e^{-c} + 1 = 0.$$

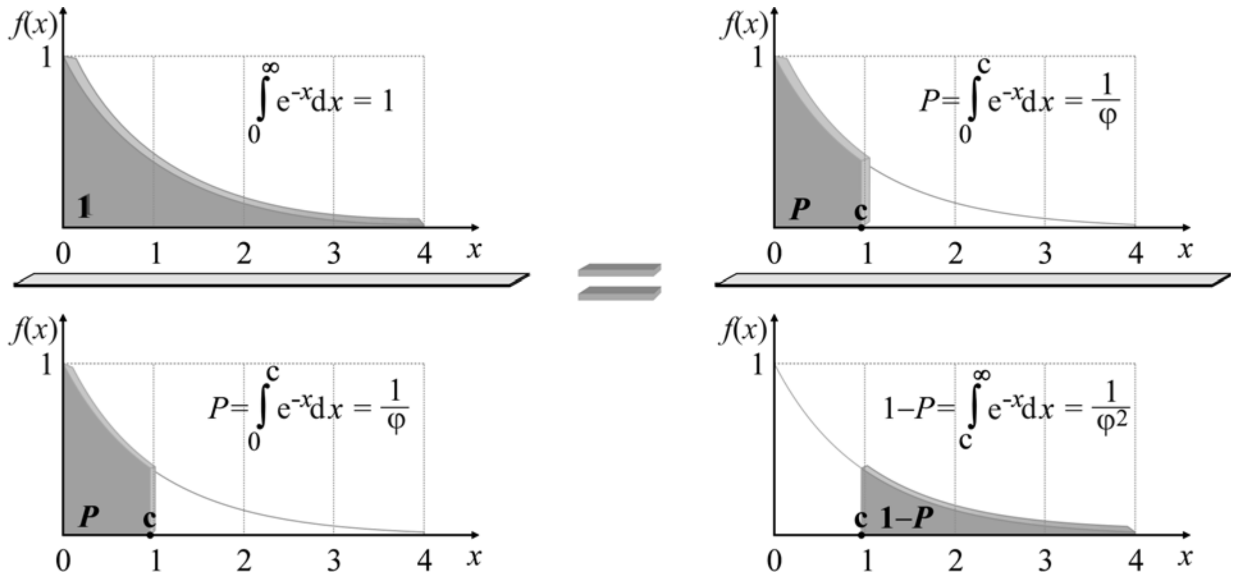

Figure 1 The golden section of the exponential distribution

With the change $t = \mathrm{e}^{-c}$, the exponential equation changes into a square one $t^2 - 3t + 1 = 0$ with the solutions:

$$t_{1,2} = \frac{3 \pm \sqrt{9-4}}{2} = 1 + \frac{1 \pm \sqrt{5}}{2},$$

$$\frac{1 \pm \sqrt{5}}{2} = 1{,}6180339... = \varphi$$

the number φ is the famous golden section constant, borderline value of subsequent numbers of Fibonacci and Lucas sequence. Apart from many special characteristics of the constant φ, the following is the most significant: $\varphi^n = \varphi^{n-1} + \varphi^{n-2}$. Now the values of the solution of the square equation of the exponential distribution are:

$$t_1 = 1 + \varphi = \varphi^2 \text{ and } t_2 = 1 - \frac{1}{\varphi} = \frac{1}{\varphi^2}.$$

If the change is returned, the solution t_1 gives a negative value for the constant c, and cannot be accepted, for c ≥ 0. From the solution t_2 it follows:

$$\mathrm{e}^{-c} = \frac{1}{\varphi^2} = \varphi^{-2} \Leftrightarrow -c = -\ln\varphi^2 \Leftrightarrow c = \ln\varphi^2 = 2\ln\varphi.$$

Probability P is:

$$P = \int_0^c \mathrm{e}^{-x}\mathrm{d}x = \int_0^{\ln\varphi^2} \mathrm{e}^{-x}\mathrm{d}x = 1 - \mathrm{e}^{-\ln\varphi^2} = 1 - \frac{1}{\varphi^2} = \frac{\varphi^2 - 1}{\varphi^2} = \frac{1}{\varphi}.$$

The constant c = 2lnφ is the solution of the probabilistic golden section of the elementary exponential distribution.

3 New class of hyperbolic functions

Hyperbolic functions were introduced in the 1760s independently by Vincenzo Riccati (1707–1775) and Johann Heinrich Lambert (1728–1777):

$$\mathrm{sh}\, x = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2} = \frac{\mathrm{e}^{2x} - 1}{2\mathrm{e}^x}, \quad \mathrm{ch}\, x = \frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2} = \frac{\mathrm{e}^{2x} + 1}{2\mathrm{e}^x}. \quad (1)$$

A great part of the hyperbolic functions was understood later when the Russian mathematician Nikolay Lobachevsky (1792–1856) discovered nonEuclidean geometry and the German mathematician Herman Minkovsky (1864–1909) gave a geometric interpretation of Einstein's special theory of relativity. The similarity between hyperbolic functions and Binet's formulae for Fibonacci and Lucas numbers in the continuous domain:

$$F(n) = \frac{1}{\sqrt{5}} \cdot \frac{\varphi^{2n} - (-1)^n}{\varphi^n}, \quad L(n) = \frac{\varphi^{2n} + (-1)^n}{\varphi^n} \quad (1)$$

was first noticed by Stakhov and Tkachenko in 1993 [3], and Stakhov and Rozin in 2005 [4] gave a detailed explanation. Symetrical hyperbolic Fibonacci and Lucas functions are connected with classical hyperbolic functions by the following correlations:

$$\left.\begin{aligned} F(x) &= \frac{2}{\sqrt{5}}\mathrm{sh}((\ln\varphi)\cdot x) \\ L(x) &= \mathrm{ch}((\ln\varphi)\cdot x), \end{aligned}\right\} \text{ for } x = 2n$$

$$\left.\begin{aligned} F(x) &= \frac{2}{\sqrt{5}}\mathrm{ch}((\ln\varphi)\cdot x) \\ L(x) &= \mathrm{sh}((\ln\varphi)\cdot x) \end{aligned}\right\} \text{ for } x = 2n+1$$

The new class of hyperbolic functions emphasizes the characteristic relation between the constants e i φ. The main part has the constant lnφ, the same as with the probabilistic golden section.

4 Relation between the constants e and φ

The established relations between Fibonacci and Lucas numbers with hyperbolic functions, as well as the function of the golden section function in the probabilistic golden section of the elementary exponential distribution, intuitively leads us to the connection between these two significant constants. One of the possible relations is defined by this theorem.

Theorem 1: For sufficiently large *n* and *ch* Lucas numbers in continuous domain, exponential relationship between consecutive *ch* Lucas numbers and the ratio of *ch* Lucas numbers with derivative of *ch* Lucas numbers provide natural number e:

$$\lim_{n\to\infty}\left(1 + \frac{1}{\dfrac{L(n-1)_{ch}}{L(n-2)_{ch}}}\right)^{\frac{L(n)_{ch}}{L(n)_{ch}'}} = \mathrm{e}$$

Proof: in compliance with formulae (1), (2) and (3), *ch* Lucas numbers are obtained for even values:

$$L(n) = \frac{\varphi^{2x} + (-1)^x}{\varphi^x}, \ x = 2n \Leftrightarrow L(n)_{ch} = \frac{\varphi^{4n} + 1}{\varphi^{2n}},$$

$$\lim_{n\to\infty}\left(1 + \frac{1}{\dfrac{L(n-1)_{ch}}{L(n-2)_{ch}}}\right)^{\frac{L(n)_{ch}}{L(n)_{ch}'}} =$$

$$= \lim_{n\to\infty}\left[\left(\frac{L(n-1)_{ch} + L(n-2)_{ch}}{L(n-1)_{ch}}\right)^{\frac{L(n)_{ch}}{L(n)_{ch}'}}\right] =$$

$$= \lim_{n\to\infty}\left[\left(\frac{L(n)_{ch}}{L(n-1)_{ch}}\right)^{\frac{L(n)_{ch}}{L(n)_{ch}'}}\right].$$

First derivation of *ch* Lucas numbers in continuous domain is:

Table 1 Convergence of the relation from Theorem 1 of ch Lucas numbers

n	Limes	n	Limes	n	Limes	n	Limes
0	1,7598484918225	5	2,7181747967700	10	2,7182818213831	15	2,7182818284586
1	2,5059662351366	6	2,7182662120511	11	2,7182818274267	**16**	**2,7182818284590**
2	2,6843831565611	7	2,7182795500410	12	2,7182818283084	17	2,7182818284590
3	2,7132656285100	8	2,7182814960420	13	2,7182818284371	18	2,7182818284590
4	2,7175484460193	9	2,7182817799601	14	2,7182818284558	19	2,7182818284590

$$L(n)_{ch}{}' = \left[\frac{\varphi^{4n}+1}{\varphi^{2n}}\right]' =$$

$$= \frac{4\varphi^{4n}\varphi^{2n}\ln\varphi - 2\varphi^{2n}\ln\varphi(\varphi^{4n}+1)}{\varphi^{4n}} =$$

$$= \frac{2\varphi^{4n}\ln\varphi - 2\ln\varphi}{\varphi^{2n}} = \frac{\varphi^{4n}-1}{\varphi^{2n}}\ln\varphi^2.$$

The ratio between derivation of *ch* Lucas numbers and *ch* Lucas numbers converges to $2\ln\varphi = \ln\varphi^2$:

$$\lim_{n\to\infty}\frac{L(n)_{ch}{}'}{L(n)_{ch}} = \lim_{n\to\infty}\frac{\dfrac{2(\varphi^{4n}-1)\ln\varphi}{\varphi^{2n}}}{\dfrac{(\varphi^{4n}+1)}{\varphi^{2n}}} =$$

$$2\ln\varphi\lim_{n\to\infty}\frac{(\varphi^{4n}-1)}{(\varphi^{4n}+1)} = 2\ln\varphi \Leftrightarrow \lim_{n\to\infty}\frac{L(n)_{ch}}{L(n)_{ch}{}'} = \frac{1}{\ln\varphi^2}.$$

Now it is:

$$\lim_{n\to\infty}\left[\left(\frac{L(n)_{ch}}{L(n-1)_{ch}}\right)^{\frac{L(n)_{ch}}{L(n)_{ch}{}'}}\right] =$$

$$= \lim_{n\to\infty}\left[\frac{L(n)_{ch}}{L(n-1)_{ch}}\right]^{\lim\limits_{n\to\infty}\frac{L(n)_{ch}}{L(n)_{ch}{}'}} = \mathrm{e} \Leftrightarrow$$

$$\Leftrightarrow \lim_{n\to\infty}\left[\frac{L(n)_{ch}}{L(n-1)_{ch}}\right] = \mathrm{e}^{\lim\limits_{n\to\infty}\frac{L(n)_{ch}{}'}{L(n)_{ch}}}.$$

Finding the logarithms of the left and right side of the previous expression results in obtaining their identity:

$$\ln\left(\lim_{n\to\infty}\left[\frac{L(n)_{ch}}{L(n-1)_{ch}}\right]\right) = \ln\mathrm{e}^{\lim\limits_{n\to\infty}\frac{L(n)_{ch}{}'}{L(n)_{ch}}} \Leftrightarrow$$

$$\Leftrightarrow \ln\varphi^2 = \lim_{n\to\infty}\frac{L(n)_{ch}{}'}{L(n)_{ch}}\ln\mathrm{e} \Leftrightarrow$$

$$\Leftrightarrow \ln\varphi^2 = \ln\varphi^2\cdot\ln\mathrm{e} \Leftrightarrow \ln\varphi^2 = \ln\varphi^2.$$

which proves the theorem. The expression (4) has an extremely fast convergence. The sixteenth member reaches the accuracy of 10^{-12}, incomparably faster than the well known Bernoulli expression for the number e (Tab. 1).

Theorem 2: Expression

$$\varphi^{\frac{1}{\frac{1}{2}\sum\limits_{n=1}^{\infty}\frac{1}{n\varphi^n}}} \text{ converge to number e } \quad \mathrm{e} = \varphi^{\frac{1}{\frac{1}{2}\sum\limits_{n=1}^{\infty}\frac{1}{n\varphi^n}}}.$$

Proof: From the geometric series sum

$$\sum_{n=0}^{\infty}\frac{1}{\varphi} = \frac{1}{1-\dfrac{1}{\varphi}} = \frac{\varphi}{\varphi-1} = \varphi^2 \Leftrightarrow \frac{1}{\varphi^2}\cdot\frac{\varphi}{\varphi-1} = 1$$

and logarithm identity

$$\ln\left[\frac{1}{\varphi^2}\cdot\frac{\varphi}{\varphi-1}\right] = \ln 1 \Leftrightarrow$$

$$\Leftrightarrow 2\ln\frac{1}{\varphi} + \ln\frac{\varphi}{\varphi-1} = 0 \Leftrightarrow \ln\frac{\varphi}{\varphi-1} - 2\ln\varphi = 0$$

for $\varphi \neq 0 \wedge \varphi \neq 1$, exponential equation

$$\ln\frac{\varphi}{\varphi-1} = 2\ln\varphi / \mathrm{e}\uparrow \Leftrightarrow$$

$$\mathrm{e}^{\frac{1}{2}\ln\frac{\varphi}{\varphi-1}} = \mathrm{e}^{\ln\varphi} \Leftrightarrow \mathrm{e}^{\frac{1}{2}\ln\frac{\varphi}{\varphi-1}} = \varphi$$

by using the Euler transform on the Mercator series,

$$\ln\frac{a}{a-1} = \sum_{n=1}^{\infty}\frac{1}{na^n},$$

for value $1 < a = \varphi$ proves the Theorem 2:

$$\ln\frac{\varphi}{\varphi-1} = \sum_{n=1}^{\infty}\frac{1}{n\varphi^n} \Leftrightarrow \mathrm{e}^{\frac{1}{2}\sum\limits_{n=1}^{\infty}\frac{1}{n\varphi^n}} = \varphi \Leftrightarrow \mathrm{e} = \varphi^{\frac{1}{\frac{1}{2}\sum\limits_{n=1}^{\infty}\frac{1}{n\varphi^n}}}$$

Value

$$a^{\frac{1}{\frac{1}{2}\sum\limits_{n=1}^{\infty}\frac{1}{na^n}}}$$

is equal to number e only for value $1 < a = \varphi$ (based on unique properties of golden ratio constant:

$$\varphi^n = \varphi^{n-1} + \varphi^{n-2}.$$

Table 2 Convergence of the relation from Theorem 2, Mercator series

n	$\varphi^{\frac{1}{\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n\varphi^n}}}$	n	$\varphi^{\frac{1}{\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n\varphi^n}}}$	n	$\varphi^{\frac{1}{\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n\varphi^n}}}$	n	$\varphi^{\frac{1}{\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n\varphi^n}}}$
1	4,745677375304	11	2,720001147826	21	2,718289786303	31	2,718281873768
2	3,285835819393	12	2,719269092984	22	2,718286544546	32	2,718281855648
3	2,956981468008	13	2,718851725509	23	2,718284628145	33	2,718281844788
4	2,833124699745	14	2,718612289791	24	2,718283493096	34	2,718281838273
5	2,777213875649	15	2,718474204436	25	2,718282819658	35	2,718281834362
6	2,749660476893	16	2,718394206460	26	2,718282419458	36	2,718281832012
7	2,735389810234	17	2,718347676562	27	2,718282181282	37	2,718281830599
8	2,727764075604	18	2,718320518222	28	2,718282039338	38	2,718281829749
9	2,723601753552	19	2,718304617232	29	2,718281954637	39	2,718281829237
10	2,721294761752	20	2,718295281378	30	2,718281904034	40	2,718281828928

The established relations between the constants e and φ, shown in the new class of hyperbolic functions, in the probabilistic goldes section of the elementary exponential distribution and the conditions of *ch* Lucas numbers convergence, emphasize the possibility of the existence of a special class of Markovian processes related to φ.

5 Golden ratio in modern science

The golden ratio constant, probably the oldest mathematical constant, has not been considered in recent mathematical history. Probably the reason is the wide usage of the Golden ratio in socalled "esoteric sciences". There is a well known fact that the basic symbol of esoteric, the pentagram, is closely connected to the Golden ratio. However, in modern science, the attitude towards the Golden ratio is changing very quickly. The Golden ratio has a revolutionary importance for development in modern science.

In quantum mechanics, El Nashie is a follower of the Golden Ratio and shows in his works [5, 6, 7] that the Golden Ratio plays an outstanding role in physical researches. El Nashie's theory will lead to Nobel Prize if experimentally verified. New theoretical [8] and partially experimental results [9] confirm the correctness of his theories.

After quantum, the golden ratio constant was established in chemical reactions [10], as an individual value, and systematically, in linear and exponential combinations [11, 12] of the golden ratio constant. DNA [13, 14] complied with the golden ratio after the atom compliance with the golden ratio, and it was also transferred to other complex biological structures. The old observation of Charles Bonnet about *phyllotaxis* plants was confirmed [15, 16], then it was expanded to others, *nonphyllotaxis* species of plants [17].

The function of the golden ratio constant was established in human brain research [18, 19]. Alongside DNA structure, we get to the golden ratio constant research in human facial proportions and facial attractiveness assessment [20, 21]. In that way it can be concluded that the golden ratio constant has a significant part in aesthetics and art.

Butusov's resonance theory of the Solar system [22] based on the golden ratio, discovers the importance of this constant beyond the Earth, and takes us to the universe [23, 24]. Certainly, all listed phenomena are led and based on mathematics, where Alexey Stakhov work is distinguishable.

This short summary of the importance of the golden section constant in modern science first of all emphasizes the range from subatomic systems to the universe. The number of scientific studies and research referring to the Golden ratio keeps increasing. On the basis of that fact we can expect that the constant φ = 1,618033... will soon be classified by its importance alongside *e* and .

6 Conclusion

Markovian decision processes is the method of probabilistic and artificial intelligence which has been classified into the field of probabilistic methods for uncertain reasoning. It is based on exponential distribution. The exact relation between the constants e and φ , which are shown, state the golden ratio phenomena as a special case of Markovian processes. However, this hypothesis is only initially sustainable.

The importance of the golden ratio constant has been proved from subatomic systems, over atoms and chemistry, genes, neurology and brain waves, plants, human body proportions, the Solar system, to the universe. In this domain there is inspiration for most methods of artificial inelligence, like Genetic algorithms and Genetic programming, Neural networks, gravitational search algorithm, etc.

The golden ratio domain leads to the hypothesis that artificial intelligence methods are special analytical sectors of the golden ratio. In that way, the basis for the introduction of the phenomenon of the golden ratio in the area of artificial intelligence has been set.

The improvement of the suggestion starts from the artificial intelligence definition, which is determined as a capability of an artificial system to simulate the functioning of human thinking at the level of perception, learning, memory, reasoning and problem solving. The concept of artificial intelligence, at the moment, does not have at its disposal models for emotions and ideas simulation, and their transfer between intelligent agents. Artificial intelligence has not achieved an analogous method for creativity yet, which is one of dominant characteristics of human intelligence. Creativity is, first of all, necessary for the adaptation in a new, not previously learnt system of events. Creativity, at the same time, is a human need which is especially expressed through art. The golden ratio constant has been declared as an aesthetic constant through architecture, since Parthenon, and as a constant of harmony, and according to Pitagora, harmony is in the basis of the universe. First important step is the introduction of the Golden Mean and Mathematics of Harmony into university education.

Appendix: relation between the constants π and φ

Distribution of Hyperbolic secant has a special meaning in analysis of probabilistic concept of golden ratio. As an inverse function of hyperbolic sinus:

$$\mathrm{sh}\, x = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2}, \quad \mathrm{csch}\, x = \frac{2}{\mathrm{e}^x - \mathrm{e}^{-x}} = \frac{1}{\mathrm{sh}\, x}$$

density and function of distribution of two parametric Hyperbolic secant in a interval $(-\infty, +\infty)$ are

$$f(x) = \frac{1}{b\pi}\,\mathrm{csch}\,\frac{x-a}{b},$$

$$F(x) = \frac{2}{\pi}\arctan(\mathrm{e}^{\frac{x-a}{b}}).$$

Elementary distribution of Hyperbolic secant for parametar values $a = 0$ and $b = 1$ has a density and function of distribution:

$$f(x) = \frac{1}{\pi}\,\mathrm{csch}\, x,$$

$$F(x) = \frac{2}{\pi}\arctan(\mathrm{e}^x).$$

Euklid analogy for probabilistic golden ration of elementary distribution Hyperbolic secant is to be found in a result of integral equation:

$$\int_{-\infty}^{c_\varphi} \frac{2}{\pi}\,\mathrm{csch}\, x\, \mathrm{d}x = \frac{1}{\varphi}.$$

Result of integral of elementary Hyperbolic secant distribution density:

$$\frac{2}{\pi}\int_{-\infty}^{c_\varphi} \mathrm{csch}\, x\, \mathrm{d}x = \frac{2}{\pi}\arctan \mathrm{e}^x \Big|_{-\infty}^{c_\varphi} =$$

$$= \frac{2}{\pi}\arctan \mathrm{e}^{c_\varphi} - \frac{2}{\pi}\arctan \mathrm{e}^{-\infty} = \frac{2}{\pi}\arctan \mathrm{e}^{c_\varphi}.$$

Inversal function of posibility distribution gives the value of constant of golden ratio for elementary distributioin of Hiperbolic secant:

$$\frac{2}{\pi}\arctan \mathrm{e}^{c_\varphi} = \frac{1}{\varphi} \Leftrightarrow \arctan \mathrm{e}^{c_\varphi} = \frac{\pi}{2\varphi} \Leftrightarrow$$

$$\Leftrightarrow \mathrm{e}^{c_\varphi} = \tan\frac{\pi}{2\varphi} \Leftrightarrow c_\varphi = \ln\left(\tan\frac{\pi}{2\varphi}\right).$$

Proved relation of Hiperbolic sinus with new class of hyperbolic functions [4], gives inverse analogy for making connection between constants and with hyperbolic secant. The result of golden ratio of probablistic function is based on tangent function. Inverse tangent function could be developed in Maclaren series:

$$\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$$

From value [25]:

$$\tan\frac{2\pi}{10} = \frac{\sqrt{\varphi+2}}{\varphi^2}$$

follows

$$\arctan\frac{\sqrt{\varphi+2}}{\varphi^2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}\left(\frac{\sqrt{\varphi+2}}{\varphi^2}\right)^{2n+1} = \frac{2\pi}{10}$$

and give us final connection between costants φ and π:

$$\pi = 5\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}\left(\frac{\sqrt{\varphi+2}}{\varphi^2}\right)^{2n+1} = 3{,}1415926...$$

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