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Do African Economies Grow Similarly?

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ABSTRACT

This paper examines economic growth in 52 African countries for 1961–2016 and seeks to find if there is common growth. As all African countries have their particular features, concerning climate, harvest, industry, size, politics, and infrastructure, and more, it seems best to rely on a non-parametric method. Dynamic Time Warping is such a convenient method, also as it allows leads and lags across countries to vary over time, and as it can easily be incorporated into a clustering technique. Five clusters are found, two of which concern Equatorial Guinea and Botswana, and the three other clusters have common growth rates of about 0, 2 and 4 over more than five decades.

Introduction

This short paper is about economic progress in 52 African countries for 1961–2016, as displayed in Figure 1 (all countries) and Figure 2 (all countries minus Equatorial Guinea and Botswana). The key question is whether there are similar growth patterns across the continent.

This simple question is not easy to answer for a variety of reasons. The dataset can be called a large dataset, as it involves 52 units of measurement (the countries) and 56 years of data. The data show a variety of patterns, and at first sight, there does not seem to be an overall African business cycle, nor is there a country with a leading economy. In fact, it can be that leads and lags change over time, that is, sometimes a country's economy can lead another one, and later on it can be the reverse. Also, African economies are very diverse in terms of infrastructure, dependence on agriculture, industry or services, and there are various political situations, regimes, and ethnic diversities. This all suggests that a single parametric

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Figure 1. Economic growth, 52 countries in Africa, 1961–2016.

econometric model, like for example a panel vector autoregression, would be difficult to calibrate for these data, let alone that it can be useful for disentangling common patterns.[1]

To alleviate these concerns I rely on a non-parametric method, called Dynamic Time Warping (DTW). This method is easy to apply, and it can give useful insights, while it is very flexible and resistant to all kinds of data features that could blur the view on common patterns. The method has been successfully introduced and used to study business cycles in US states in Franses and Wiemann (2018), where it also provided useful insights into which states show similar behavior.

The outline of this paper is as follows. Section "Dynamic Time Warping" briefly describes how a DTW method can be applied and how it can be implemented into a clustering algorithm. Section "The Results" then applies it to the data in Figures 1 and 2, and it is found that, additional to the two fast growing countries of Equatorial Guinea and Botswana, there are three clusters of countries with similar growth features. The three average growth rates for the three clusters for the period 1961–2016 are 0, 2, and about 4, respectively. Section "Conclusion" concludes with limitations and further research issues.

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[1]Potentially useful parametric methods (which are not considered here for the reasons indicated) are for example proposed in Bai and Ng (2004) who consider a common factor structure within a dynamic factor model, in Engle and Kozicki (1993) who consider common cyclical and other features, in Cubadda, Hecq, and Palm (2009) who consider common autoregressive components across individual series and Fröhwirth-Schnatter and Kaufmann (2008) who analyse a clustering method based on mixture models.
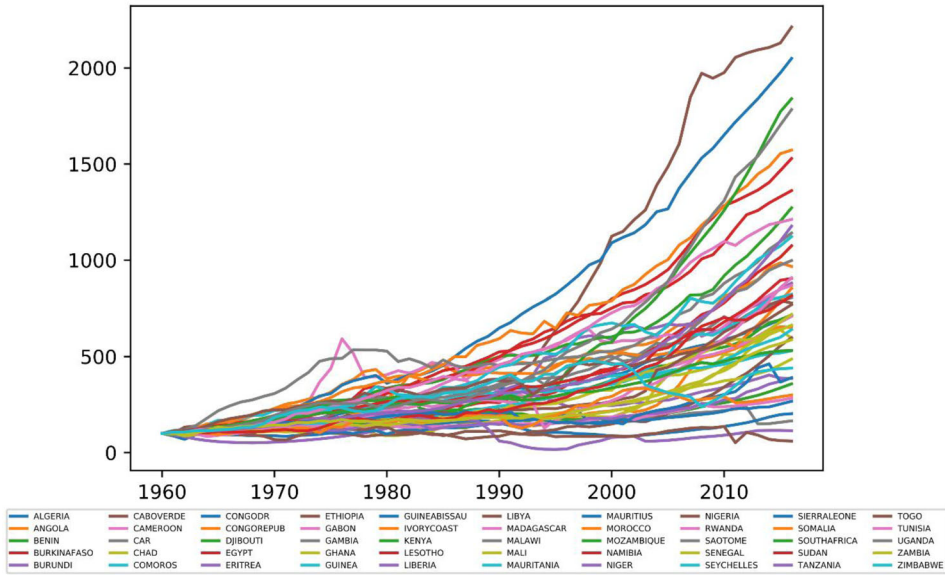
Figure 2. Economic growth in 50 counties in Africa 1961–2016, where Equatorial Guinea and Botswana are excluded.

Dynamic Time Warping

Dynamic Time Warping (DTW) is a technique that calibrates the differences between two times series. DTW alignment can be calculated using dynamic programming, and Müller (2007) provides a description of an algorithm.

DTW concerns a distance-minimising temporal alignment between two time series $X = [x_1, x_2, \ldots, x_N]$ and $Y = [y_1, y_2, \ldots, y_M]$. First, we calculate the $N \times M$ distance matrix D, where the $(i, j)^{th}$ element is the distance between each of the points x_i and y_j. An optimal alignment is given by a *warping path p* which minimizes the cumulative distance of all point-pairs on the path, denoted by

$$\gamma(i,j) = d(x_i, y_j) + \min[\gamma(i-1, j-1), \gamma(i-1, j), \ \gamma(i, j-1)] \qquad (1)$$

where $d(x_i, y_j)$ is a distance measure between points x_i and y_j. The minimum *warping cost*, $\gamma(N, M)$ is a non-parametric similarity measure of time series.

In standard DTW, the function $d(x_i, y_j) \to \mathbb{R} \geq 0$ is either $|x_i - y_j|$ or $(x_i - y_j)^2$ (see, for example, Raihan (2017) and Wang et al. (2012)). In the context of comparing economic time series which experience business cycles, one would want to consider an observation relative to its earlier and subsequent neighbors to identify the features of the time series. We therefore consider the *feature*-based distance function of Xie and Wiltgen (2010)

as it is a measure that captures both the overall shape of the time series and the local trend around the points.

A local feature of a particular observation is defined by a 2-element vector that summarizes the slope to its left- and right-hand side neighbor. We consider the growth rate between the periods to capture local trends. The local feature of observation i of a series X is then given by

$$f_{local}(x_i) = \left[\frac{x_i - x_{i-1}}{x_{i-1}}, \frac{x_{i+1} - x_i}{x_i}\right] \tag{2}$$

Following Franses and Wiemann (2018), the global feature of observation i of a series X is given by

$$f_{global}(x_i) = \left[\frac{x_i - \sum_{k=1}^{i-1}\frac{x_k}{i-1}}{\sum_{k=1}^{i-1}\frac{x_k}{i-1}}, \frac{x_i - \sum_{k=i+1}^{N}\frac{x_k}{N-i}}{\sum_{k=i+1}^{N}\frac{x_k}{N-i}}\right] \tag{3}$$

So, a final feature-based distance function is given by

$$\begin{aligned}
d(x_i, y_j) &= d_{local}(x_i, y_j) + d_{global}(x_i, y_j) \\
&= |f_{local}(x_i)_1 - f_{local}(y_j)_1| + |f_{local}(x_i)_2 - f_{local}(y_j)_2| \\
&\quad + |f_{global}(x_i)_1 - f_{global}(y_j)_1| + |f_{global}(x_i)_2 - f_{global}(y_j)_2|
\end{aligned} \tag{4}$$

where the subscripts 1 and 2 concern the corresponding elements in the vector.

For K time series, one can assess $\frac{K(K-1)}{2}$ warping distances. As an alternative, clustering makes it possible to increase interpretation. This is achieved by allocating the series to homogeneous groups, which maximize intra-group similarity and maximize inter-group dissimilarity. Time series clustering based on DTW has proven to be a quite useful method, see Aghabozorgi, Shirkhorshidi and Wah (2015) and Liao (2005).

The K-means algorithm developed by MacQueen (1967) minimizes the total difference of the cluster's sequences to the respective cluster average by iteratively reassigning sequences to clusters with an average most similar to the sequence and then recalculating the cluster averages in each iteration. The K-means method can be adjusted to reflect time series similarity using growth rate-based dynamic time warping as follows.[2] The K-means algorithm requires a well-defined "average" of a set of series. Petitjean, Ketterlin, and Gancarski (2011) introduce a global averaging method, that is, DTW Barycenter Averaging (DBA). DBA iteratively refines an initial average sequence to minimize its squared warping distance to the set of the yet-to-be-averaged series. It calculates the temporal alignment of each series

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[2]An illustration of conventional K-means is provided by Liao (2005) or Aghabozorgi et al. (2015). For an application of (non-DTW) K-means in an economic context, see Sullivan and Miller (1996).

to the currently best average, and then defines the average of all time series' points that are temporally aligned with a particular point of the average as the updated version. Note that this again allows the DBA to reflect temporal shifts across time series. In fact, a specific point of the average might be associated with multiple points of a time series.[3] To adapt this method to the application of growth rates, we consider the average of associated growth rates rather than each point's absolute value. The DBA time series can be constructed by iterative multiplication of the growth rates and an initial index value (for example 100 as in Figures 1 and 2).

The cluster algorithm is optimized similar to the idea of the DTW-based K-means++ algorithm of Zhang and Hepner (2017). Each of the series has a probability to be assigned as the initial centroid to one of the k clusters and this is disproportional to its similarity with existing centroids. One of the time series is chosen at random to be the centroid of the first cluster. For the remaining time series, we calculate the probability p_i given by

$$p_i = \frac{\Gamma_i^2}{\sum_{j=1}^{G} \Gamma_j^2} \tag{5}$$

where $\Gamma_i = \sum_{c \in C} DTW(x_i, c)$ with C the set of existing centroids DTW the Dynamic Time Warping distance as defined before. A time series is selected as the centroid for the next cluster with probability p_i, and the step is repeated until all clusters are assigned to an initial centroid. The remaining time series are then grouped into the cluster with the "closest" centroid as assessed by the DTW distances.

Starting at this initial clustering, K-means then proceeds by averaging the time series of each cluster by means of DBA and reassigning each time series to the cluster with the lowest DTW distance. This is continued until no series is reassigned and the procedure ends.

The Results

This section concerns the application of the DTW DBA based clustering algorithm to the real GDP growth data in Figures 1 and 2.

The data source is Franses and Vasilev (2019) who compiled a complete data sets with real growth rates for 52 African countries. For various countries there are missing data, but the authors used imputation techniques and Principal Component Regressions to fill in the missing data. Setting all GDP levels in 1960 at 100, we can create GDP levels data for 1961 to 2016, as in Figures 1 and 2. Just looking at the patterns of the data in these figures is not informative about any potential growth clusters.

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[3]For an application in a non-economic context, see, for example, Gullo et al. (2012).

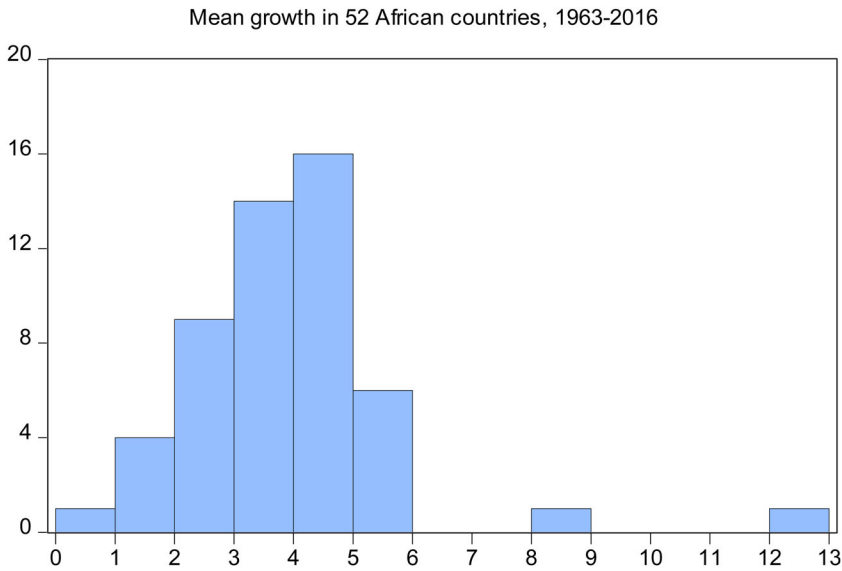Mean growth in 52 African countries, 1963-2016

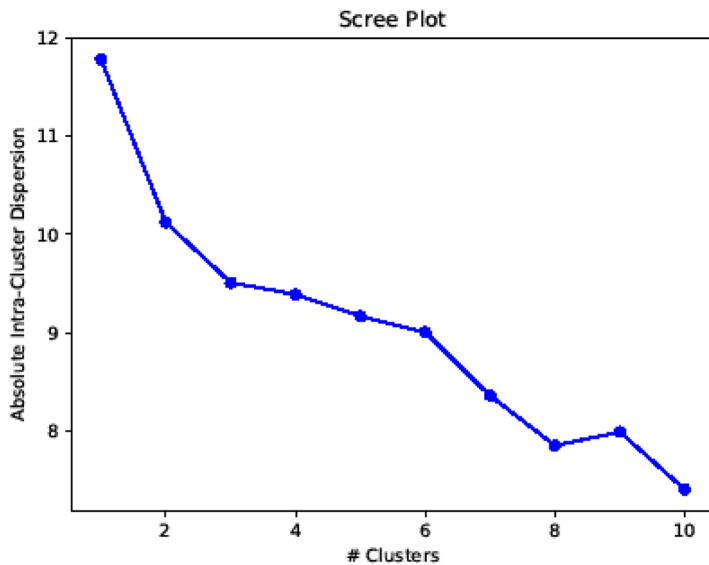Figure 3. Mean yearly growth, averaged over 1961–2016.

Figure 4. Deciding on the number of clusters using a scree plot.

Figure 3 presents the mean growth rates of the 52 countries, and there it is obvious that Equatorial Guinea (average growth 9.820) and Botswana (average growth 7.866) stand out. For the other 50 countries the histogram with the average growth rates in Figure 3 does not tell us much.

When we use the algorithm for the 50 countries, we can create 10 different cases, where case 1 has no clusters, case 2 has 2 clusters until case 10 with 10 clusters. The scree plot in Figure 4 suggests that an elbow appears
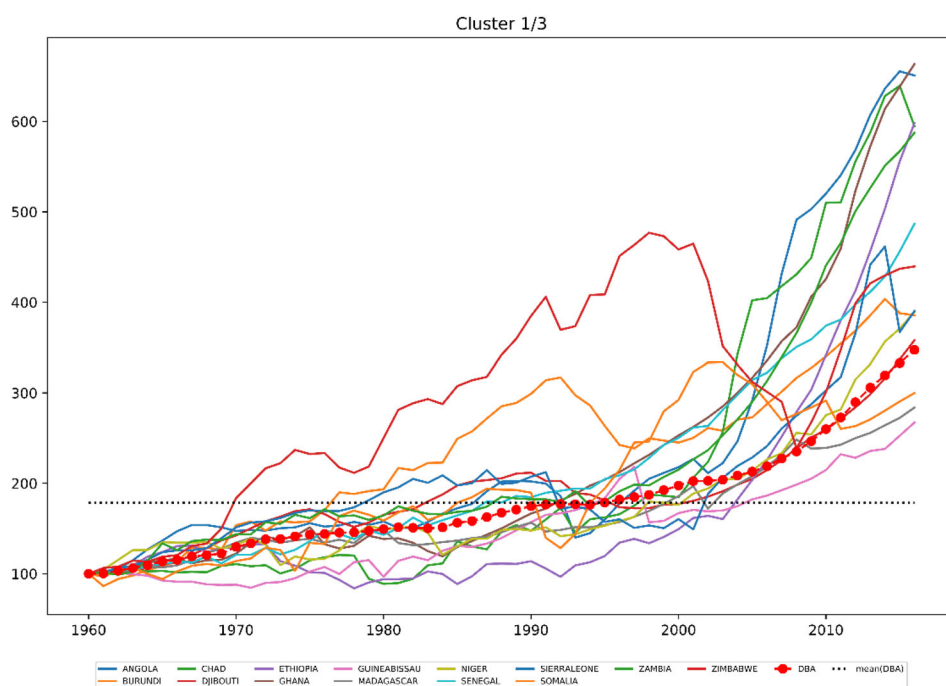
Figure 5. The countries in the first cluster and the associated DBA.

Table 1. Average growth rates, 1961–2016.

	Mean	Standard Deviation	Maximum	Minimum
Equatorial Guinea	9.820	19.414	91.629	−45.889
Botswana	7.866	5.482	23.428	−8.012
Cluster 1	2.224	1.662	6.006	−0.425
Cluster 2	0.313	4.192	11.414	−11.796
Cluster 3	3.756	1.443	7.560	1.222

for 3 clusters. If we look in more details into the clusters when there are more than 4 clusters, we see that many of these new clusters concerns just a single country. Hence, it is decided to continue with 3 clusters, beyond the two fast-growing countries.

Figure 5 gives the data for the first cluster as well as the DBA over the years. This first cluster includes Angola, Burundi, Chad, Djibouti, Ethiopia, Ghana, Guinea Bissau, Madagascar, Niger, Senegal, Sierra Leone, Somalia, Zambia and Zimbabwe. Table 1 reports that the DBA real GDP growth of these countries is 2.224, with a maximum of about 6 and a minimum of −0.425.

The second cluster includes Central African Republic, the Democratic Republic of Congo, Liberia and Libya, as can be learned from Figure 6. Table 1 reports that the effective average growth rate of these four countries over all these years simply is about 0. The maximum of the DBA
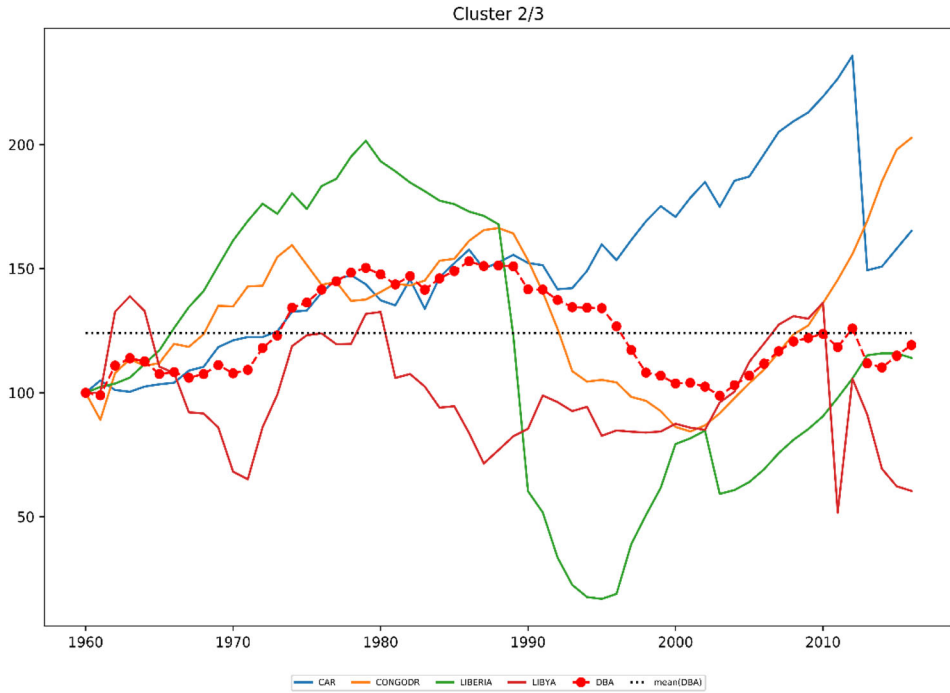
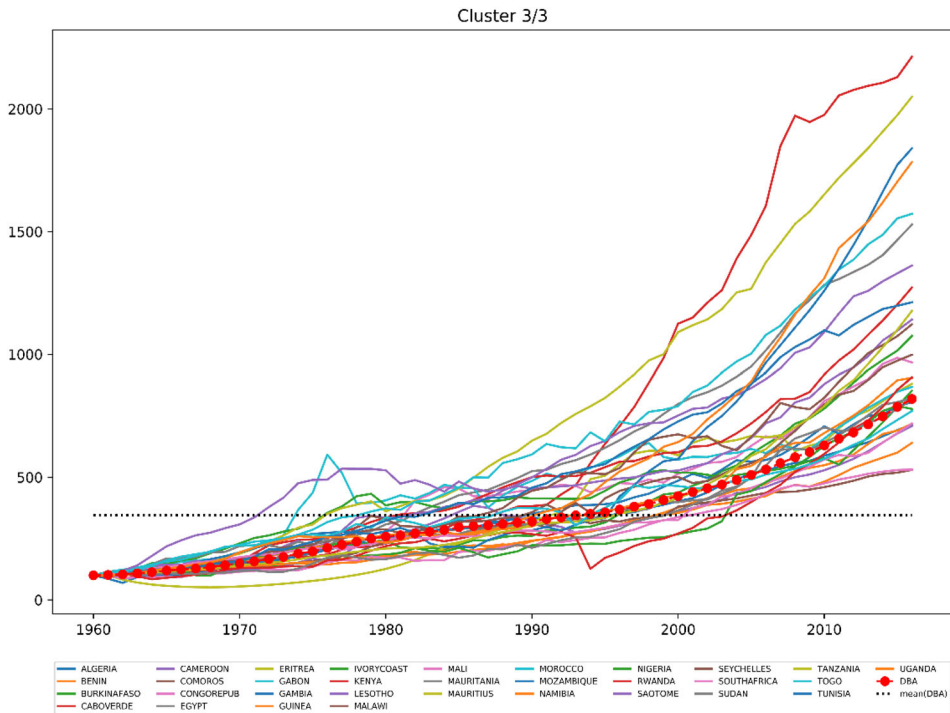**Figure 6.** The countries in the second cluster and the associated DBA.

Figure 7. The countries in the third cluster and the associated DBA.

Figure 8. The five clusters.

growth rates is 11.414, but unfortunately, the minimum is −11.796, which reinforces the average of 0.

Finally, the third cluster involves the other countries, as they are displayed in Figure 7. Table 1 shows that the growth rate across these countries is 3.756, with maximum of 7.560 and a minimum 0f 1.222. Note that an average growth rate of about 4 compares favorably with growth rates commonly found in western industrialized countries.

Figure 8 displays all clusters again, but now in a map of Africa.

Conclusion

This paper has examined real GDP based economic growth in 52 African countries for 1961–2016, with the main focus on finding clusters of

countries with common growth. As all African countries have their particular features, it was deemed best to rely on a non-parametric method. Dynamic Time Warping was found to be such a convenient method, also as it allows leads and lags across countries to vary over time, and as it can easily be incorporated into a clustering technique. Five clusters were found, two of which concern Equatorial Guinea and Botswana, and the three other clusters have common growth rates of about 0, 2 and 4 over more than five decades. For the 32 countries in the latter cluster, this means that they have an average growth rate that favors in comparison with many western industrialized countries in the same period.

Acknowledgements

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