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Optical Extraction Efficiency for External Cavity Quantum Cascade Lasers

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In this paper, we present a simple method for calculation of optical extraction efficiency of mid-infrared quantum cascade laser coupled to external cavity. The approach is based on the three-level rate equation model describing the variation of the electron number in the states and the photon number present within the Fabry-Perot quantum cascade laser and the external cavity. The system shares the same active region and includes the dependence of the loss on external cavity parameters. We find in particular that the power coupled out through the external mirror varies linearly with current injection and depends strongly on the external cavity reflectivity. Moreover, a considerable increase in the threshold current of external cavity occurs when decreasing the values of the external cavity reflectivity and the slope efficiency decreases with decreasing external cavity reflectivity. We also derive an analytical formula for the optical extraction efficiency of external cavity and analyze the simultaneous effects of the current injection and the external cavity reflectivity on it. Results show that at laser threshold, the optical extraction efficiency is zero and it rises as the current injection increases. For high current injection, extraction efficiency of up to 11 % at $R_{ext} = 10$ % can be attained. In addition, the equations allowing the determination of the optimum reflectance of external cavity and the maximum optical extraction efficiency are also derived within the premises of our model in the general case.

Keywords: Quantum cascade laser, External cavity, Output power, Extraction efficiency.

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1. INTRODUCTION

In many applications of a quantum cascade (QC) laser [1], external cavity (EC) plays an important role in the performance of the system. Due to their vast potential for applications in industry, medicine, security and research, these sources enjoy increasing interest within the research community as well as in industry. Therefore, the characteristics of QC lasers with EC have attracted much attention from many researchers since their invention [2-10]. There are many possible techniques for the wavelength tunability of QC lasers. One of the practical approaches is tunable EC-QC lasers. Theoretical analysis shows that the tuning range of the EC-QC laser is proportional to the full width at half maximum of the gain spectrum and to the square root of the gain margin between EC defined lasing wavelength and peak gain [11].

The QC laser coupled with EC leads to the EC parameters dependence of the losses, photon lifetime and threshold current, and thus influences the optical performance of the system, for example, output power, duty cycle, operation temperature and spectral tuning characteristics [11-13]. Equally as important for the EC-QC laser operation is the optical extraction efficiency. This parameter is useful for designing EC-QC lasers and optimising their performance. In this paper, we calculate the optical extraction efficiency η_{extr} for the external cavity, using a rate equation model. To obtain more detail technical information about the structure used here we kindly refer the reader to the published literature [14].

2. THEORY

2.1 The Rate Equation Model

The system of rate equations for electron numbers N_1, N_2 and N_3 in levels 1, 2 and 3, and the photon numbers S^{FP} and S^{EC} in the Fabry Perot (FP) and external cavity (EC) can be written in the following form [14]:

$$\frac{dN_3}{dt} = \frac{I_{inj}}{e} - \frac{N_3}{\tau_3} - \left(G^{FP}S^{FP} + G^{EC}S^{EC}\right)\left(N_3 - N_2\right), \tag{1a}$$

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} + \left(G^{FP}S^{FP} + G^{EC}S^{EC}\right)\left(N_3 - N_2\right), \tag{1b}$$

$$\frac{dN_1}{dt} = \frac{N_3}{\tau_{31}} + \frac{N_2}{\tau_{21}} - \frac{N_1}{\tau_{out}}, \tag{1c}$$

$$\frac{dS^{FP}}{dt} = N_p G^{FP}(N_3 - N_2)S^{FP} + \frac{N_p \beta N_3}{\tau_{sp}} - \frac{S^{FP}}{\tau_p^{FP}}, \tag{1d}$$

$$\frac{dS^{EC}}{dt} = \frac{1}{\rho_{cav}}\left(N_p G^{EC}(N_3 - N_2)S^{EC} + \frac{N_p \beta N_3}{\tau_{sp}} - \frac{S^{EC}}{\tau_p^{EC}}\right), \tag{1e}$$

where I_{inj} is the injected current, e is the electron charge, τ_{32}, τ_{31}, and τ_{21} are the nonradiative scattering times between the corresponding levels due to LO-phonon emission, τ_{sp} is the radiative spontaneous relaxation time between levels 3 and 2, τ_3 is the lifetime of the upper level and defined as $\tau_3 = 1/(1/\tau_{32} + 1/\tau_{31})$, τ_{out} is the electron escape time between two adjacent stages [15], β defines the fraction of the spontaneous emission light emitted in the lasing mode [16], N_p is the number of stages, $\rho_{cav} = 1 + L_{ext}/(n_{eff}L)$ is the ratio of optical path

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lengths of the FP and the EC [17] where n_{eff} is the effective refractive index of FP active region, L and L_{ext} are the FP active region and the EC lengths respectively, $G^{FP} = \Gamma c'\sigma_{32}^{FP} / V$ and $G^{EC} = \Gamma c'\sigma_{32}^{EC} / V$ are the gain coefficients per period in the FP and in the EC respectively where Γ is the mode confinement factor for wavelengths λ, c' is the speed of light in the medium, $V = N_pWLL_p$ is the whole volume of the active area where W is the lateral dimension of the cavity while L_p is the length of a single stage of the cascade laser structure. The parameters σ_{32}^{FP} and σ_{32}^{EC} are the stimulated emission cross-sections for FP and EC lasers respectively. The latter is defined as [14]

$$\sigma^{EC} = \sigma^{FP} \frac{\gamma_{32}^2}{\left(h\nu^{EC} - h\nu^{FP}\right)^2 + \gamma_{32}^2}, \tag{2}$$

where $2\gamma_{32}$ stands for the full width at half maximum of the electroluminescence spectrum, h is the Planck constant and ν is the lasing frequency.

Parameters τ_p^{FP} and τ_p^{EC} in Eqs. (1d), (1e) are, respectively, the photon lifetimes in the FP and in the EC [14]. The latter is given as a function of the waveguide loss of the cavity α_w and the mirror losses by

$$\tau_p^{EC} = \frac{2L}{c'} \frac{1}{\left(2L\alpha_w - \ln\left(R_1R_{eff}\right)\right)}, \tag{3}$$

where R_1 is the FP reflectivity and R_{eff} is the effective reflectivity of the equivalent EC-QC laser and can be written as [14, 18]

$$R_{eff} = \frac{R_2 + R_{ext} + 2\sqrt{R_2R_{ext}}\cos\left(\omega\tau\right)}{1 + R_2R_{ext} + 2\sqrt{R_2R_{ext}}\cos\left(\omega\tau\right)}, \tag{4}$$

where R_{ext} is that of the external reflector, ω is the laser angular frequency, $\tau = 2L_{ext}/c$ is the round-trip time of light in the EC, c is the speed of light in vacuum.

In Eq. (3), we have assumed for simplicity that the waveguide loss of the EC mode is the same as that for the FP mode, i.e. $\alpha_w^{EC} = \alpha_w^{FP} = \alpha_w$.

2.2 The Steady State Operation

Under steady state conditions, the population inversion ΔN and the nontrivial stable solution for photon number S^{EC} obey the following relations [14]:

$$\Delta N = \frac{I_{th}^{EC}}{e}\tau_3\left(1 - \frac{\tau_{21}}{\tau_{32}}\right), \tag{5}$$

$$S^{EC} \approx \left(\frac{I_{inj}}{I_{th}^{EC}} - 1 + \frac{1}{(1 + \frac{\tau_{21}}{\tau_{31}})} \frac{\beta}{\tau_{sp}} \frac{\tau_{21}}{\eta_r} \frac{I_{inj}}{I_{th}^{EC}}\right) S_{sat}^{EC}, \tag{6}$$

where we introduced the photon saturation number S_{sat}^{EC} given by

$$S_{sat}^{EC} = \frac{1}{\tau_3(1 + \frac{\tau_{21}}{\tau_{31}})G^{EC}}, \tag{7}$$

and the parameter $\eta_r = \left(1 - \tau_{21} / \tau_{32}\right) / \left(1 + \tau_{21} / \tau_{31}\right)$ is the radiative efficiency while I_{th}^{EC} is the threshold current under the effect of EC and given by

$$I_{th}^{EC} = \frac{eA\left(2L\alpha_w - \ln\left(R_1R_{eff}\right)\right)}{2N_p\Gamma\sigma_{32}^{EC}\tau_3(1 - \frac{\tau_{21}}{\tau_{32}})}, \tag{8}$$

where $A = N_pWL_p$ is the cross-sectional area of the active region.

Considering the fact that the intensity inside the EC is doubled, the intensity of light traveling inside the EC of QC laser, calculated by using Eq. (6), in the absence of spontaneous emission and taking into account the relationship between the intensity in units of W/cm^2 and the photon number in the EC $2I^{EC} = I_+^{EC} + I_-^{EC} = c'\hbar\omega^{EC}S^{EC} / V'$ is given by

$$I^{EC} = \frac{I_{sat}^{EC}}{2}\left(\frac{I_{inj}}{I_{th}^{EC}} - 1\right), \tag{9}$$

where I_+^{EC} and I_-^{EC} are, respectively, the forward and backward traveling beams in the EC, $V' = L_{ext}A_b/3$ is the whole volume of the EC mode, A_b is the cross-sectional area of the beam at the external mirror, and $I_{sat}^{EC} = c'\hbar\omega^{EC}S_{sat}^{EC} / V'$ is the saturation intensity.

3. POWER AND OPTICAL EXTRACTION EFFICIENCY

In this section, we will first derive the power coupled out through the external mirror by using the rate equation model. After that, we exploit our results to derive the corresponding optical extraction efficiency of the EC mode, and finally, we derive the optimum external reflectivity and the maximum extraction efficiency.

3.1 Output Power and Power Without Losses

We will now use the results obtained above to derive the power coupled out through the external mirror P_{out}^{EC} and the power transferred into the upper laser level P_{UL}^{EC} i.e. the power without losses. The power output P_{out}^{EC} is related to the light intensity by $P_{out}^{EC} = A_b(1 - R_{ext})I^{EC}$. Using Eq. (9) for I^{EC} one gets

$$P_{out}^{EC} = \frac{3LN_p\hbar\omega^{EC}\eta_r(1 - R_{ext})}{L_{ext}e} \times \\ \times \left(\frac{I_{inj}}{\left(2L\alpha_w - \ln\left(R_1R_{eff}\right)\right)} - \frac{c'eS_{sat}^{EC}}{2N_pL\eta_r}\right). \tag{10}$$

The maximum of the power transferred into the upper laser level P_{UL}^{EC} is attained when the waveguide losses of the cavity are zero ($\alpha_w = 0$) and the output coupling R_1 and R_{ext} approach 100 % (i.e. $R_1 = R_{ext} = R \to 0$). Thus, the power transferred into the upper laser level is obtained from Eq. (10) by using the approximation $(1 - R) \approx |\ln R|$, i.e.

$$P_{UL}^{EC} = 3\frac{L}{L_{ext}}\frac{N_p\hbar\omega^{EC}}{e}\eta_r I_{inj}, \qquad (11)$$

of course, the output power will generally be lower than this maximal value.

3.2 Derivation of the Optical Extraction Efficiency

We now want to derive a general expression for the optical extraction efficiency η_{extr} which is an important parameter characterizing the optical performance of EC-QC lasers [19]. This quantity is defined as the ratio of the power coupled out through the external mirror and the power transferred into the upper laser level, i.e. $\eta_{extr} = P_{out}^{EC} / P_{UL}^{EC}$. Then, the optical extraction efficiency can be calculated using Eq. (10) and Eq. (11)

$$\eta_{extr} = (1 - R_{ext})\left(\frac{1}{2L\alpha_w - \ln\left(R_1R_{eff}\right)} - \frac{c'eS_{sat}^{EC}}{2N_pL\eta_r I_{inj}}\right). \qquad (12)$$

As we can see from Eq. (12), the optical extraction efficiency depends on the material parameters of QC laser and can be controlled either by the EC parameters through R_{eff} or by the current injection.

The slope efficiency η_{slope} is defined as an increase in the output power per unit current, i.e. $\eta_{slop} = dP_{out}^{EC} / dI_{inj}$. Using Eq. (10), we obtain

$$\eta_{slop} = 3\frac{L}{L_{ext}}\eta_r\frac{N_p\hbar\omega^{EC}}{e}\frac{(1 - R_{ext})}{\left(2L\alpha_w - \ln\left(R_1R_{eff}\right)\right)}. \qquad (13)$$

The slope efficiency is a function of the loss and the EC parameters.

3.3 Optimum External Reflectivity and Maximum Extraction Efficiency

If the optical extraction efficiency of EC is varied at a given current injection, the extraction efficiency exhibits a maximum at the optimum EC reflectivity. This behavior is easy to understand considering the fact that the extraction efficiency is zero at low reflectances (laser threshold is not reached) and at a reflectance of 100 % (no power is coupled out of the EC). Thus, a maximum of the extraction efficiency must exist for a certain value of the external reflectivity. The optimum external reflectivity $R_{ext,opt}$ and the maximum extraction efficiency $\eta_{extr,max}$ can be obtained by setting the derivative $d\eta_{extr}/dR_{ext}$ equal to zero. After easy algebra, we get the following expression for $R_{ext,opt}$:

$$\frac{I_{th}^{EC}}{I_{inj}} - 1 - \frac{(1 - R_{ext,opt})\left(\frac{R_2 + \sqrt{\frac{R_2}{R_{ext,opt}}}\cos(\omega\tau)}{1 + R_2R_{ext,opt} + 2\sqrt{R_2R_{ext,opt}}\cos(\omega\tau)} - \frac{1 + \sqrt{\frac{R_2}{R_{ext,opt}}}\cos(\omega\tau)}{R_2 + R_{ext,opt} + 2\sqrt{R_2R_{ext,opt}}\cos(\omega\tau)}\right)}{2L\alpha_w - \ln\left(R_1R_{eff,opt}\right)} = 0, \qquad (14)$$

where the parameter $R_{eff,opt}$ is the effective reflectance coefficient at optimum external reflectivity, obtained from Eq. (4) by making $R_{ext,opt}$ replace R_{ext}.

The corresponding maximum extraction efficiency is derived by using Eq. (12) and Eq. (14), i.e.

$$\eta_{extr,\max} = (1 - R_{ext,opt})\left(\frac{1}{2L\alpha_w - \ln\left(R_1R_{eff,opt}\right)} - \frac{c'eS_{sat}^{EC}}{2N_pL\eta_r I_{inj}}\right). \qquad (15)$$

4. RESULTS AND DISCUSSION

In the following discussion, we study the effects of the current injection and the EC reflectivity on the power coupled out through the external mirror, the optical extraction efficiency, the optimum external reflectivity and the maximum extraction efficiency for the structure of QC laser described in Refs [11, 14]. We use in our calculation the parameters taken from Refs [13, 14]: $L = 1.5$ mm, $R_1 = 1$, $R_2 = 0.01$, $\alpha_w = 14$ cm^{-1}, $N_p = 48$, $\tau_{32} = 2.4$ ps, $\tau_{31} = 3$ ps, $\tau_{21} = 0.4$ ps, $\lambda = 8$ μm, $\sigma_{32}^{FP} \approx 10^{-14}$ cm^{-2}, $W = 8$ μm, $L_p = 48$ nm, $\Gamma = 0.6$, $L_{ext} = 8$ cm, $\hbar\omega^{EC} \approx \hbar\omega^{FP}$, $I_{th}^{FP} \approx 0.39$ A. Our results are as follows: $\sigma_{32}^{EC} = 10^{-14}$ cm^{-2} and $S_{sat}^{EC} = 3.17\times10^{8}$.

Fig. 1 presents the normalized output power $P_{out}^{EC} / \left(A_bI_{sat}^{EC}\right)$ as a function of the current injection for different values of EC reflectivities. Starting at the threshold current I_{th}^{EC} the power increases linearly with the current injection and the slope of the curve becomes steeper as the EC reflectivity R_{ext} decreases. The output power cannot exceed the power P_{UL}^{EC} that is available in the EC in the form of inversion. Moreover, a considerable decrease in the threshold current occurs when increasing the values of the EC reflectivity as shown in Fig. 1. For completeness, the threshold values of the current at the EC reflectivity of 5 %, 10 %, 50 % and 90 % are given as 0.3, 0.275, 0.22, and 0.195 A, respectively.

The slope efficiency calculated from Eq. (13) is plotted in Fig. 2 as a function of EC reflectivity for different values of the EC lengths. Maximum values of η_{slope} of about 92, 46 and 31 mW/A at $L_{ext} = 4$, 8 and 12 cm, respectively, are obtained with an EC reflectivity of about 12 %. As the EC reflectivity increases beyond this value, the slope efficiency decreases dramatically.

Fig. 3 shows the variation with current injection of the optical extraction efficiency η_{extr} for different values of EC reflectivities. At laser threshold, the extraction efficiency is zero and it rises as the current injection increases. For high current injection, extraction efficiencies

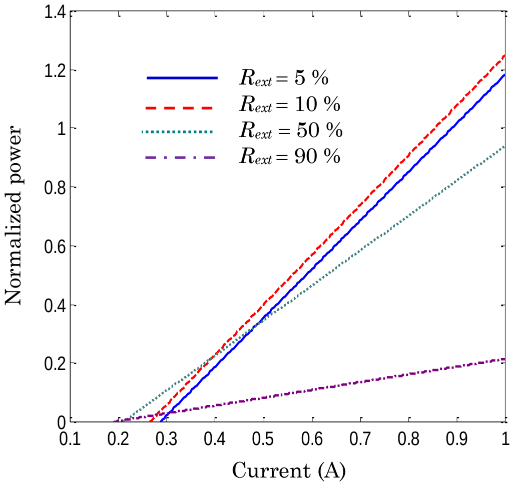

Fig. 1 – Normalized laser power coupled out through the external mirror as a function of the current injection for different values of EC reflectivities. The output power is normalized to the quantity $A_b I^{EC}_{sat} = 3c'\hbar\omega^{EC} S^{EC}_{sat} / L_{ext}$ which is the saturation power

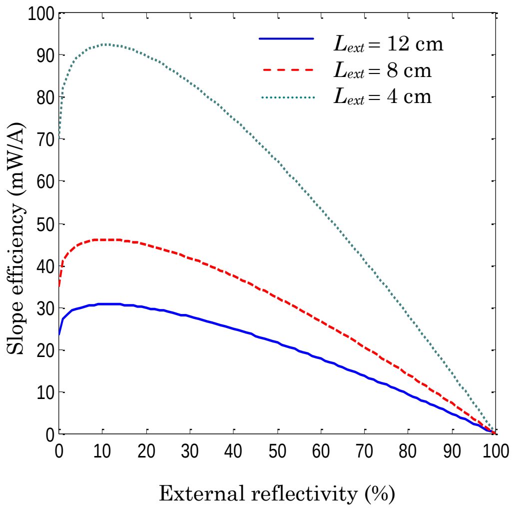

Fig. 2 – Slope efficiency η_{slope} as a function of EC reflectivity R_{ext} for different values of the EC lengths

of up to 11 % at R_{ext} = 10 % can be attained. It is difficult to achieve the extraction efficiencies higher than this value since the power coupled out through the external mirror is strongly affected by any loss inside the EC.

The optical extraction efficiency calculated from Eq. (12) for four values of the current injection is plotted in Fig. 4 as a function of the EC reflectivity. At small R_{ext}, the optical extraction efficiency η_{extr} is small. With increasing R_{ext}, the quantity η_{extr} increases showing a maximum corresponding to optimum R_{ext} at $R_{ext,opt}$ and then decreases to zero at R_{ext} = 100 %. Fig. 4 also shows that, as the current injection increases, the maximum optical extraction efficiency increases rapidly and the optimum EC reflectivity is shifted to lower values . We attribute this relatively low optical extraction efficiency to the large waveguide losses. At 0.6 A, maximum η_{extr} of 8.5 % was obtained around 18 % with the output power of 0.29 W. The maximum values of η_{extr} were still 4.5, 5.5 and 7.5 % at 0.35, 0.4 and 0.5 A, respectively.

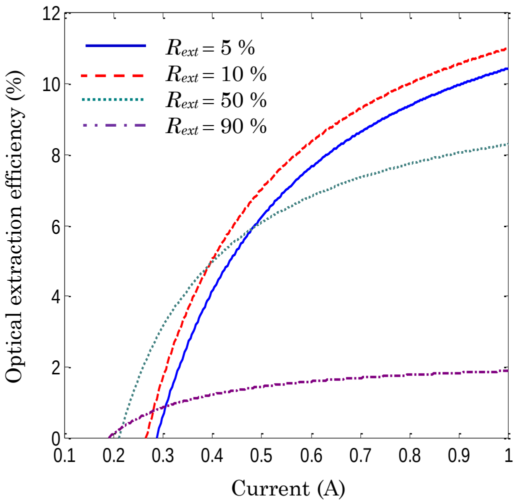

Fig. 3 – Optical extraction efficiency η_{extr} as a function of the current injection I_{inj} for different values of EC reflectivities

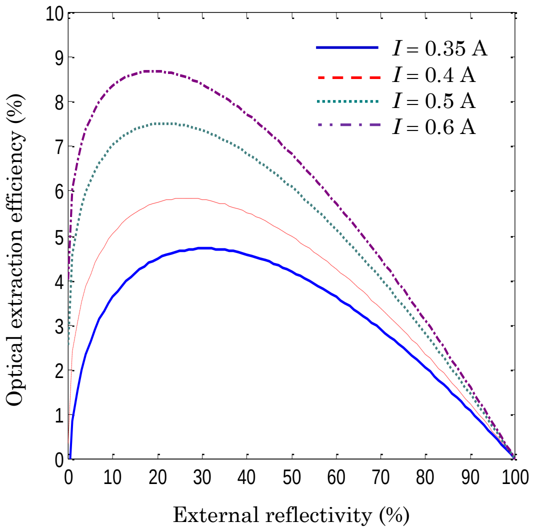

Fig. 4 – Dependence of the optical extraction efficiency η_{extr} on the EC reflectivity R_{ext} for different values of the currents injection

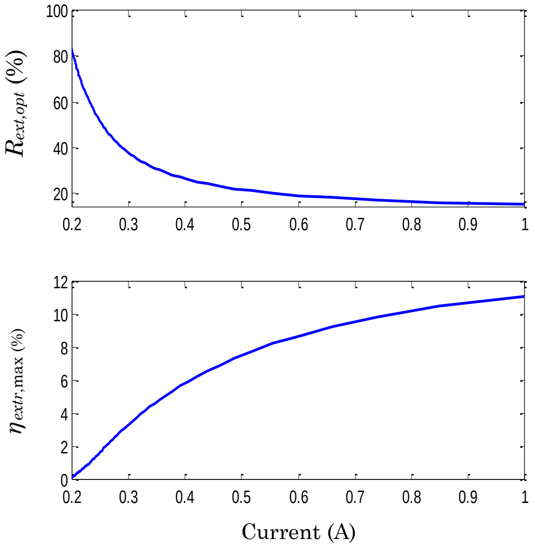

Fig. 5 – Optimum external reflectivity and maximum extraction efficiency as functions of the current

In Fig. 5, we show the variation of the optimum external reflectivity $R_{ext,opt}$ and the maximum extraction efficiency $\eta_{extr,\max}$ as a function of the current injection which varies from 0.2 to 1 A. Fig. 5 shows that the optimum external reflectivity decreases with increasing current injection that results in an increase of maximum extraction efficiency.

5. CONCLUSIONS

Using a coupled mode system, we studied the performance of the EC QC laser. The system is based on a three-level rate equation model. In particular, simple analytical formulas for the power, the slope efficiency and the optical extraction efficiency were derived. With the present design and physical parameters it is shown that the power coupled out through the external mirror increases linearly with current injection and the slope of the curves becomes steeper as the EC reflectivity R_{ext} decreases. This slope presents maxima of about 92, 46 and 31 mW/A at $L_{ext} = 4$, 8 and 12 cm, respectively, with an EC reflectivity of about 12 %. In addition, a considerable decrease in the threshold current occurs when increasing the values of R_{ext}. On the other hand, with increasing R_{ext}, the optical extraction efficiency η_{extr} increases, shows a maximum corresponding to optimum R_{ext} at $R_{ext,opt}$ and then decreases to zero at $R_{ext} = 100$ %. At 0.6 A, a maximum η_{extr} of 8.5 % was obtained around 18 % with the output power of 0.29 W. The maximum values of η_{extr} were still 4.5, 5.5 and 7.5 % at 0.35, 0.4 and 0.5 A, respectively. We also derived here simple analytical formulas for the optimum external reflectivity and the maximum extraction efficiency. Our numerical results show that the optimum external reflectivity decreases with increasing current injection which results in an increase of maximum extraction efficiency.

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