

Introduction

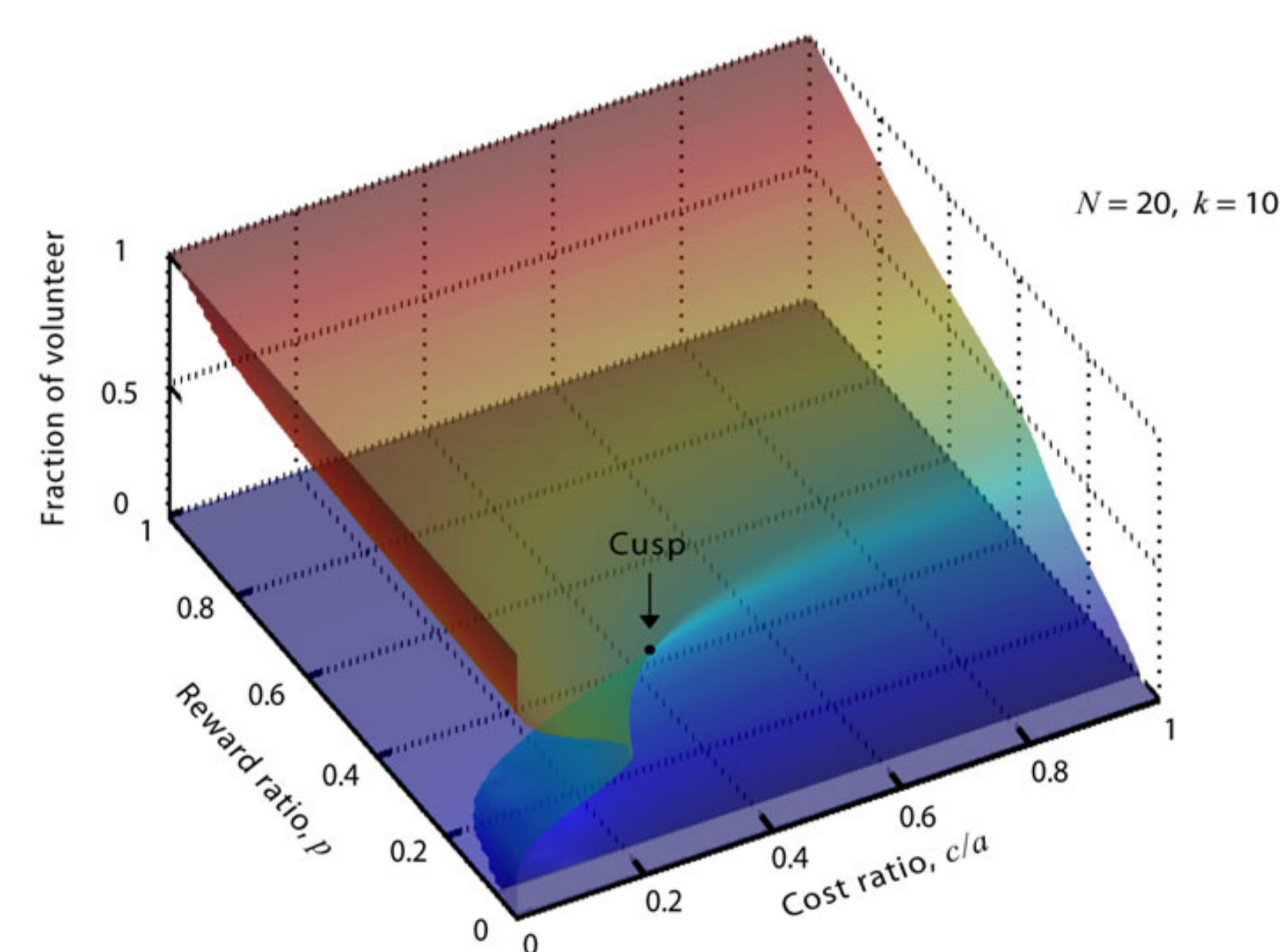
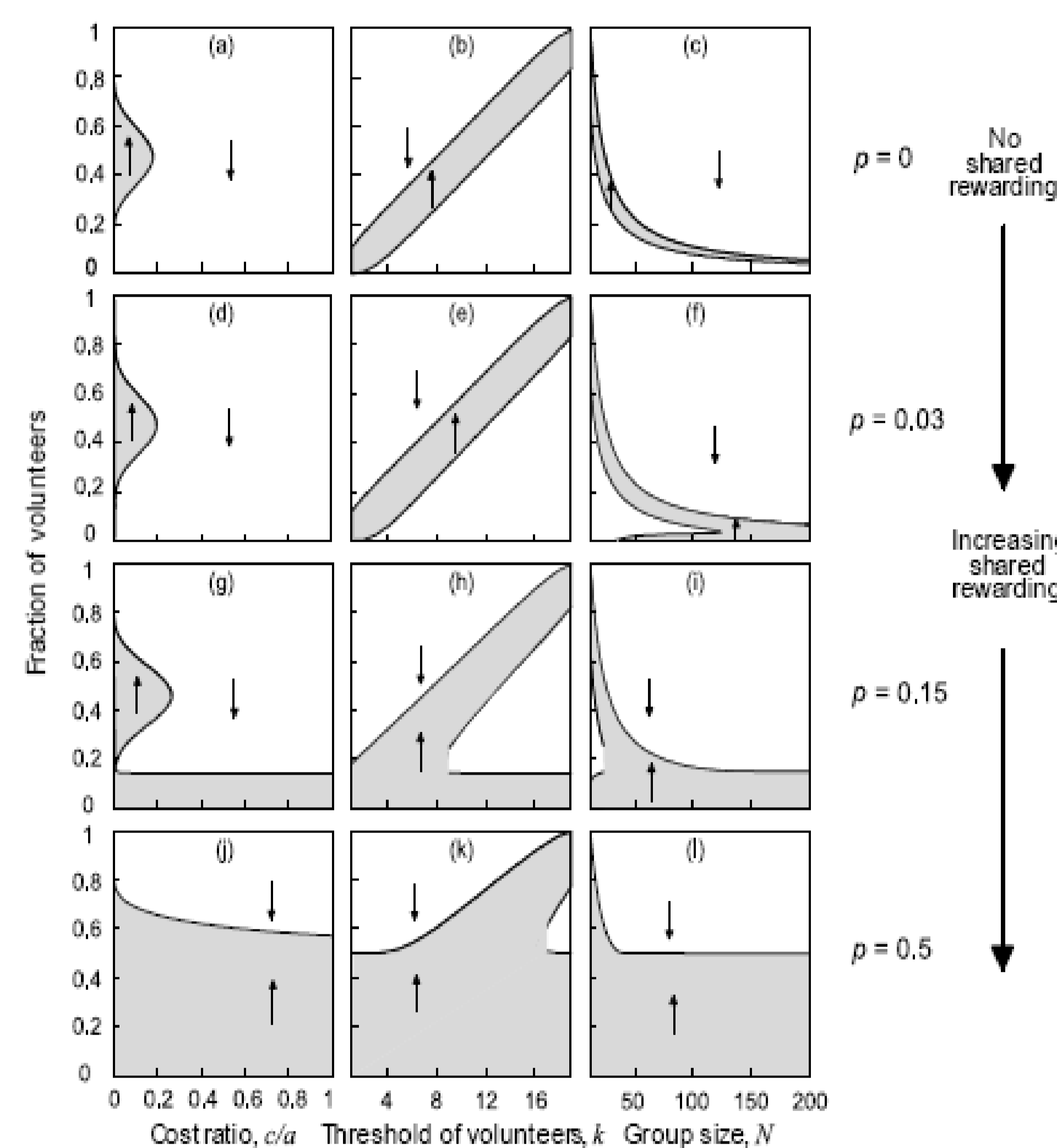
For societies to produce public goods, costly voluntary contributions are often required. From the perspective of each individual, however, it is advantageous not to volunteer such contributions, in the hope that other individuals will carry the associated costs. This conflict can be modeled as a volunteer's dilemma. To encourage rational individuals to make voluntary contributions, a government or other social planners can offer rewards, to be shared among the volunteers. Here, we apply such shared rewarding to the generalized N -person volunteer's dilemma, in which a threshold number of volunteers is required for producing a public good.

Shared Rewarding in Volunteer's Dilemma

We consider the generalized N -person volunteer's dilemma. In each round of the game, an interaction group is assembled by randomly drawing N individuals from a large (infinite) well-mixed population. Each player can choose between the strategies Volunteer or Ignore (a defective strategy). The public good is produced if at least k ($1 \leq k < N$) individuals choose to volunteer. The cost of volunteering is c , relative to a baseline payoff of 1; volunteers incur this cost irrespective of whether or not the public good is produced. A failure of producing the public good imposes a cost $a > c$ on each player in the group. Moreover, we assume that the volunteers in an interaction group in each round share a total reward δ ($0 < \delta < Nc$). We introduce the reward ratio $p = \delta/(Nc)$ to characterize the magnitude of the total reward δ relative to the maximal total volunteering cost Nc .

▶ Without shared rewarding only two evolutionary outcomes are possible: full defection or coexistence of volunteers and non-volunteers. Already small rewards destabilize full defection, stabilizing small fractions of volunteers instead. For even larger rewards, only a single social outcome remains, corresponding to the stable coexistence of volunteers and non-volunteers.

▶ At the intermediate reward levels, the highest level of volunteering is reached by first increasing and then decreasing the group size, leading to the emergence of hysteresis effect.



Cusp Bifurcation

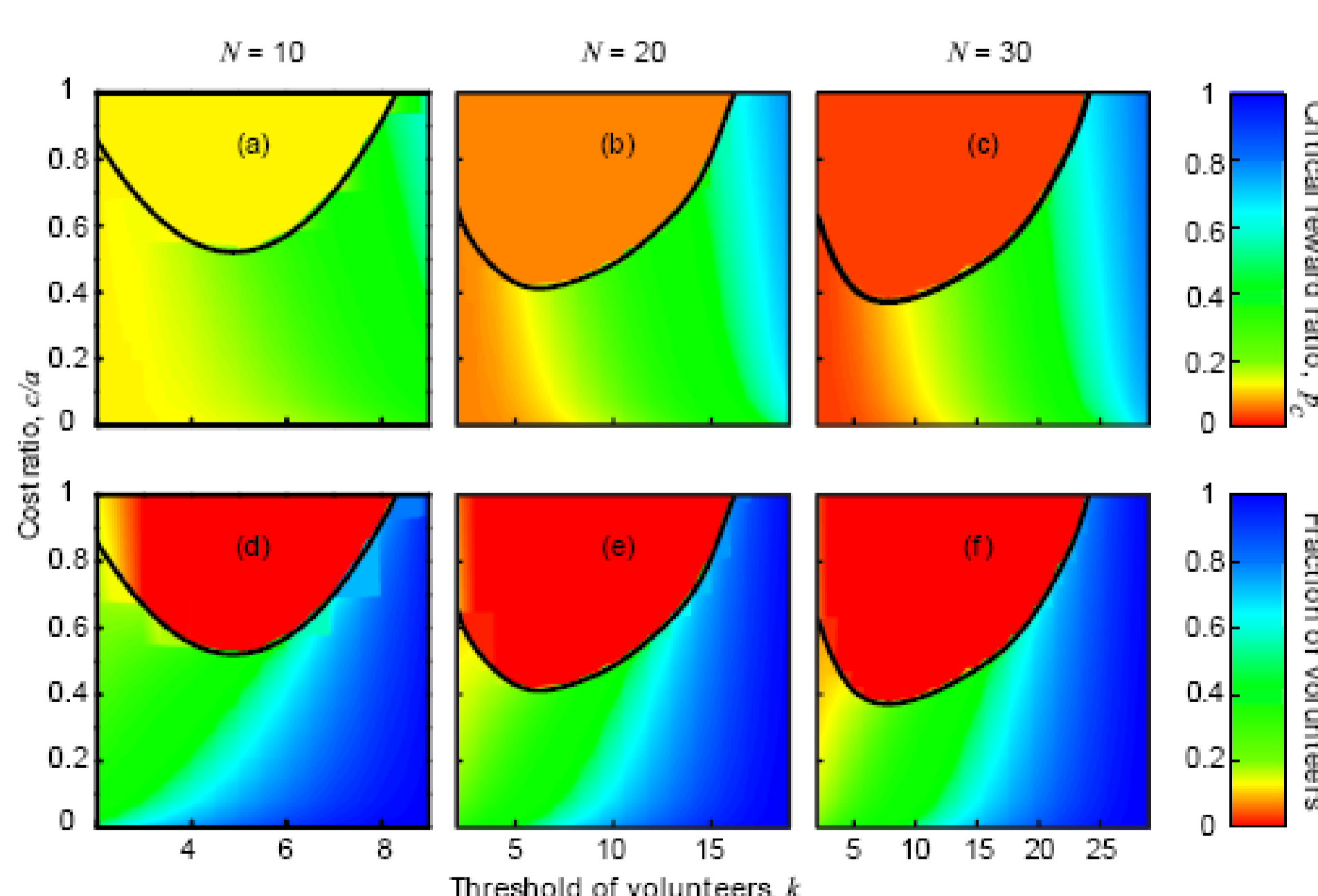
▶ With increasing the cost ratio c/a and the reward ratio p , the two branches of saddle-node bifurcation curves meet each other to form a cusp, where a cusp bifurcation exhibits.

▶ At the cusp point, the three equilibria collide and disappear pairwise via the saddle-node bifurcations, then only one stable equilibrium displays in the population.

Critical Reward Ratio

▶ Non-large reward ratios are needed to trigger the transition from the bistable state to the unique coexistence stable state, which shows that shared rewarding helps overcome defection traps in the generalized volunteer's dilemma.

▶ The critical level of shared rewarding is relatively small compared to the total cost of contributing to the public good.



Conclusions

- ▶ Shared rewarding is remarkably effective in overcoming defection traps in the generalized volunteer's dilemma.
- ▶ Shared rewarding leads to a hysteresis effect, under which the highest level of volunteering is reached by first increasing and then decreasing the group size. This could perhaps be translated into a practical recommendation for social planners.
- ▶ A simple model induces rich evolutionary dynamics of voluntary contributions, enhancing the beauty and complexity of social dynamics.