

# Long-term and Short-term Targets: Conflict and Reconciliation

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**Abstract:** We address the issue of a tradeoff between long- and short-term interests in economic management. Starting with an obvious observation that the actions targeted to long- and short-term goals are generally in conflict, we pass on to a less obvious statement that it is not an exceptional situation that a smart decision maker can reduce or even eliminate the conflict. We illustrate the statement by an informal analysis of a stylized model of management of an enterprise. We show that if the managers put enough effort in identification of the current-value shadow prices of the enterprise, their current short-term-optimal actions become non-distiguishable from the long-term-optimal ones.

**Key Words:** Economic growth, Shadow price, Control theory

## 1. Introduction

Each of us has multiple objectives, and each of us realizes that there is no chance to achieve all of them simultaneously; current decisions are driven by tradeoffs of the objectives. A typical example is the tradeoff between the short-term and long-term objectives. As usual, the short-term interest dominates in current decision-making, and the long-term interest is kept on the background as a useful but rather abstract guideline. Does the repetition of short-term optimal actions move one away from one's long-term objective, or does it eventually allow one to reach that objective? From a systems analysis perspective these two options split the world of the agent-driven systems into two big clusters – the cluster of 'unsustainable' systems (in which the agents' short- and long-term objectives contradict each other) and that of 'sustainable' ones (in which the agents' short- and long-term objectives are, structurally, in agreement). In this paper we focus on an example of a decision making pattern arising in studies of long-term economic management.

Historically, rigorous studies of economic development were strongly motivated by the *mathematical theory of optimal control* (Pontryagin, et al, 1961). Accordingly, the issue of optimal long-term economic growth was put in the focus (Arrow and Kurz, 1970; Intriligator, 1971). The long-term optimization approach was rapidly developing, expanding the spectrum of economic models and entering, in many aspects, mathematics (see, e.g., Aubin and Clarke, 1979; Grossman and Helpman, 1991; Barro and Sala-i-Martin, 1995; Weitzmann, 2003; Aseev and Kryazhimskiy, 2007).

In recent years, an alternative approach, known as *behavioral economics* (see, e.g., Diamond and Vartiainen, 2007), was becoming more and more popular among the economists. Behav-

ioral economics aims at understanding motivations/actions of agents that drive economic processes. The agents (for example, companies), in their practice, follow, usually, short-term objectives (such as increase of the income in the next season), and consider long-term goals as strategic guidelines.

The 'behavioral' paradigm seems to be in conflict with the theory of optimal long-term economic growth. However, purely logically, the conflict is not that obvious. Why can't the 'myopic' agents act so as if they were instructed by the central planner?

If the 'myopic' and long-term-oriented actions are not in conflict indeed, we can claim that the economic development is *multi-optimal*, the optimal long-term scenario is implemented through the short-term-optimal actions. In mathematics, multi-optimality is exceptional. The multi-optimality of an economic model can therefore be a reflection of the exceptional fact that the economic system establishes 'double guarding' against moving away from a 'correct' economic trajectory.

## 2. Model

Below, we use a simplified model of an enterprise to show that, from the viewpoint of a long-term planner, short-term optimal decisions may range from counter-optimal to the optimal ones, depending on how smart are the managers in estimating the current shadow prices in the production process.

Let us imagine an enterprise, whose managers do not plan to close or sell it; based on that, we assume that the enterprise operates over a time interval expanding from zero to infinity.

Let  $x(t)$  denote the size of the enterprise's production capital, the number of the equipment units operating at the enterprise at a current time  $t$ . We assume  $x(t)$  to be a positive real (in reality, the production capital is subdivided in a number of categories and characterized by a vector). Below,  $x_0$  is a given size of the production capital at the initial time, zero,

$$x(0) = x_0 \quad (1)$$

The managers regulate the enterprise by fixing, at each time,  $t$ , the current investment,  $u(t)$ , in the development of the enterprise. Thus,  $u(t)$  varies over time, being chosen by the managers

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from some set representing all allowable sizes for the current investment.

We assume that at each time,  $t$ , the capital's current increment rate,  $\dot{x}(t)$ , is determined by the current capital size,  $x(t)$ , and current investment size,  $u(t)$ ,

$$\dot{x}(t) = f(x(t), u(t)) \quad (2)$$

here  $f$  is a function characterizing the effectiveness of investment; for example, if  $u(t)$  is used to purchase equipment for price  $r$ , (2) takes the form  $\dot{x}(t) = ru(t) - \delta x(t)$  where  $\delta$  is the capital obsolescence rate.

Now let us associate ourselves with the enterprise's managers who do their planning at the initial time, zero. Consider some investment scenario as a function of time,  $t \rightarrow u(t)$ . Substituting  $u(t)$ , for every  $t$ , in (2), we get an ordinary differential equation describing the process of capital development under the chosen investment scenario. Solving (2) subject to the initial condition (1), we construct a function of time,  $x \rightarrow x(t)$ , showing us the capital growth trajectory under the investment scenario  $t \rightarrow u(t)$ . Repeating this procedure for a large number of investment scenarios (ideally, for all investment scenarios), we construct the set of the corresponding capital growth trajectories. All those investment scenarios and capital growth trajectories are, potentially, feasible. We are now in a position to decide, which of those paths should be followed in the future. At this point, we need to use our strategic thinking and define our long-term goal. We define our long-term goal as follows:

*maximizing the enterprise's overall benefit for the entire period of its operation.*

Let  $g(x(t), u(t))$  be the enterprise's marginal benefit at state  $(x(t), u(t))$ , the marginal benefit gained through the sales of the products and other enterprise's activities is expressed in monetary terms. The function  $g$  characterizes the momentary efficiency of the enterprise's activities; for example, if the enterprise gains benefit through sales only,  $q$  is the number of products produced by one unit of equipment in one unit of time and  $c$  is the products' price, one can set  $g(x(t), u(t)) = cqx(t)$ .

Transition from time  $t$  to time  $t + \Delta t$  (where  $\Delta t$  is small) brings us benefit of size  $g(x(t), u(t))\Delta t$  (here and below we neglect additional second order terms). Taking into account inflation, with some rate  $\rho$ , we represent the above income in the monetary units operating at the initial time, zero, as  $e^{\rho t}g(x(t), u(t))$ . Summation over time with the vanishing time step  $\Delta t$  results in a standard integral form for the overall benefit,

$$J = \int_0^{\infty} e^{\rho t} g(x(t), u(t)) dt \quad (3)$$

Thus, prior to taking any management decisions, we declare that our strategic long-term goal is maximization of the enterprise's overall benefit,  $J$ . Our declaration is nothing else than a claim that we are going to find (and then follow) the solution to the *optimal control problem* for the control system (2) with the initial state (1) and objective functional (3),

$$\max J = \int_0^{\infty} e^{\rho t} g(x(t), u(t)) dt \quad (4)$$

Subject to

$$\dot{x}(t) = f(x(t), u(t))$$

$$x(0) = x_0.$$

Remembering that the optimal control problem (4) is our strategic principle, we, the enterprise's managers, do not regard it as a practical management recommendation. As usual, we make our current management decisions adaptively, as reactions to the current situation. More specifically, we follow an adaptation rule that seems most practical and reliable, *maximization of the current monetary capital growth rate*.

### 3. Adaptation rule

Let us consider our adaptation rule in more detail. Suppose  $x(t)$  is the current size of the enterprise's production capital and we are in a position to choose a current investment,  $u(t)$ . Let us take some hypothetical  $u(t)$  and estimate the increment in the enterprise's monetary capital in a small transition from  $t$  to  $t + \Delta t$ . The increment has two components, the income due to the sales, and the increment in the value of the production capital. As earlier, we estimate the income due to the sales as  $g(x(t), u(t))$ . An obvious estimate for the increment in the value of the production capital is  $p(t)\dot{x}(t)\Delta t$ , where  $p(t)$  is the current price for equipment. Thus, for the increment in the enterprise's monetary capital we have the estimate,

$$p(t)\dot{x}(t)\Delta t + g(x(t), u(t))\Delta t.$$

Dividing by  $\Delta t$  and letting  $\Delta t$  go to zero, we find that the current monetary capital growth rate is given by

$$M(t) = p(t)\dot{x}(t) + g(x(t), u(t)),$$

or (see (2))

$$M(t) = p(t)f(x(t), u(t)) + g(x(t), u(t)). \quad (5)$$

In economic applications of optimal control theory,  $M(t)$  is known as the *current-value Hamiltonian*, and  $p(t)$  as the *current-value adjoint variable*. Trying different values for  $u(t)$ , we get different values for the current monetary capital growth rate,  $M(t)$ . Remembering that our adaptation rule is maximization of the current monetary capital growth rate, we choose  $u(t)$  so as to maximize  $M(t)$ ,

$$\max M(t) \text{ over } u(t) \quad (6)$$

In order to find  $u(t)$  from (6), we need to know the current price for equipment,  $p(t)$  (see (5)). Let us note that  $p(t)$  is not the price, for which equipment can be sold on the market of products today;  $p(t)$  is the price of the enterprise on the 'market of enterprises'. Using  $p(t)$ , a potential buyer estimates the efficiency of the enterprise in bringing income in the long run. Identification of  $p(t)$  is a non-trivial task, which is reflected in its name, '*current-value shadow price*'.

Certainly, we can estimate the current-value shadow price,  $p(t)$ , empirically, based on experience or common sense. However, empirical estimates can lead us to counter-optimal decisions.

### 4. Example

Let model (2) have the form

$$\dot{x}(t) = ru(t) - \delta c(t),$$

implying that investment,  $u(t)$ , is used to purchase equipment for price  $r$  and the production capital depreciates at rate  $\delta$ . Assume that investment in development of the production capital,  $u(t)$ , is not allowed to exceed a given upper bound  $u^+ > \delta/r$ , and the enterprise gains benefit due to the sales of products and due to some other activity, for which the amount of  $u^+ - u(t)$  is invested at each time  $t$ . The marginal benefit due to the sales is given by  $cqx(t)$  where  $q$  is the number of products produced by one unit of equipment in one unit of time and  $c$  is the products' price. Letting the marginal benefit due to the alternative activities be proportional to the current investment in those activities, we estimate it as  $a(u^+ - u(t))$  with some proportionality coefficient  $a$ . Thus, for the marginal benefit we have

$$g(x(t), u(t)) = cqx(t) + au^+ - au(t).$$

Let us, rather rationally, assume that the current shadow price,  $p(t)$ , is proportional to the current market value for the equipment, i.e.,

$$p(t) = bx(t).$$

where  $b$  is some proportionality coefficient. Then the current-value Hamiltonian (5) takes the form

$$M(t) = bx(t)(ru(t) - \delta x(t)) + cqx(t) + au^+ - au(t).$$

Using the adaptation rule (6), we find that

$$u(t) = \begin{cases} u^+ & , \text{ if } bx(t)r - a > 0 \\ u(t) - 0 & , \text{ if } bx(t)r - a < 0 \end{cases}.$$

Based on these relations, we easily deduce that

$$u(t) = \begin{cases} u^+ & , \text{ if } x_0 > a/br \\ u(t) - 0 & , \text{ if } x_0 < a/br \end{cases}. \quad (7)$$

In the former case, we constantly invest the full amount in development of the production capital and invest nothing in the alternative activities, and in the latter case we do the opposite.

Depending on parameter values, this rational investment strategy can be both optimal and counter-optimal. Indeed, straightforward calculations (see Appendix 2) yield that the optimal investment policy is the following:

$$u(t) = \begin{cases} 0 & \text{if } \delta < \rho \\ 0 & \text{if } \delta > \rho, a < cqr \text{ and } t < t_* \\ u^+ & \text{if } \delta > \rho, a < cqr \text{ and } t > t_* \end{cases}; \quad (8)$$

here  $t_*$  is defined by

$$e^{-\rho t_*}(cqr - a) = (\rho + \delta)e^{-\delta t_*}.$$

We see that the adaptation rule (7) gives us the optimal investment size (8) for all times,  $t$ , if  $x_0 < a/br$  (see the latter case in (7) holds and the former case in (8) takes place (i.e., either  $a \geq cqr$  or  $a < cqr, \delta < \rho$ ). Otherwise, the latter case in (8) takes place and (7) gives us the non-optimal investment size either for  $t < t_*$  or for  $t > t_*$  (if, respectively, the former and latter cases in (7) take place).

## 5. Conclusion

We used a stylized model of development of an enterprise to show that 'myopic' management decisions aimed at improving the enterprise's performance in the short run (namely, at raising its monetary capital in each next time period) and 'forward looking' ones aimed at optimizing the enterprise's operation in the long run (namely, at maximizing its benefit flow over an infinite time period) may both strongly disagree and stay in good agreement. A key to reducing a conflict between 'myopic' and 'forward looking' decisions is identification of the current-value shadow prices for the enterprise. If the current-value shadow prices are understood as market prices for the enterprise's equipment or are based on management experience, the conflict can be strong. A careful on-line analysis of the future scenarios for the enterprise's operation in the market and financial environments opens up a way to accurate identification of the current-value shadow prices and to elimination of the conflict.

## Appendix

### A. Current-value shadow price

Let us outline an argument that leads to finding the current-value shadow price,  $p(t)$ , to be used in the adaptation rule (6). In our argument, we use a currency unit common for all times,  $t$ ; for that currency unit we take the one that operates at the initial time, 0. This currency unit is, due to inflation,  $e^{\rho t}$  times stronger than the one used at time  $t$ . Therefore, the current-value shadow price evaluated as  $p(t)$  in the units of the current currency, takes a  $e^{\rho t}$  times smaller value,  $\psi(t)$ , being expressed in terms of the initial currency units,

$$\psi(t) = e^{-\rho t} p(t),$$

we call  $\psi(t)$  the universal shadow price, or, shorter, the shadow price at time  $t$ . Multiplying the current monetary capital growth rate,  $M(t)$ , (see (5)) by  $e^{-\rho t}$ , we recalculate  $M(t)$  using the initial currency unit; we get

$$H(t) = e^{-\rho t} M(t) = \psi(t)f(x(t), u(t)) + e^{-\rho t} g(x(t), u(t)). \quad (\text{A. 1})$$

Now we rewrite the adaptation rule (6) using  $H(t)$  instead of  $M(t)$ :

$$\max H(t), \text{ over } u(t). \quad (\text{A. 2})$$

We assume that  $u(t)$  that solves the maximization problem (A. 1) does not lie on the boundary of the set of all values admissible for the current investment (note that choosing boundary, 'emergency' values is not typical for economic decisions). Then the derivative of the maximized function (A. 1) vanishes at point  $u(t)$ , i.e.,

$$\psi(t) \frac{\partial f(x(t), u(t))}{\partial u} + e^{-\rho t} \frac{\partial g(x(t), u(t))}{\partial u} = 0. \quad (\text{A. 3})$$

Note that (A. 3) allows us, in principle, to find  $u(t)$  as an explicit function of  $t$ ,  $x(t)$  and  $\psi(t)$ ,

$$u(t) = U(t, x(t), \psi(t)). \quad (\text{A. 4})$$

Let us introduce the enterprise's monetary capital (expressed in terms of the initial currency unit),  $C(t)$ . Note that the derivative of the monetary capital gives us the current monetary capital growth rate,

$$\dot{C}(t) = H(t). \quad (\text{A. 5})$$

Let us now look at  $\dot{C}(t)$  from a different perspective. Consider the enterprise's overall benefit subsequent to time  $t$ ,  $J(t)$ . Similarly to (3), we have

$$J(t) = \int_t^{\infty} e^{-\rho t} g(x(s), u(s)) ds. \quad (\text{A. 6})$$

Assume for the moment that at time  $t$  we are able to 'convert' the enterprise into the overall benefit subsequent to  $t$ , to 'exhaust' the enterprise and get a monetary benefit of size  $J(t)$ . In this virtual scenario, we put  $J(t)$  on a bank account at time  $t$ , and subsequently get our dividend from the bank. Ideally, the interest rate equals the inflation rate,  $\rho$ . Therefore, in our virtual scenario, at time  $t$  we get  $\rho J(t)$  from the bank. In an ideal economy, the optimal investment policy suggests that the monetary capital rate in the production process,  $\dot{C}(t)$ , is as profitable as that we get from the bank in the virtual scenario,  $\rho J(t)$ ,

$$\dot{C}(t) = \rho J(t)$$

Using (A. 5), we rewrite this as

$$H(t) = \rho J(t) \quad (\text{A. 7})$$

Let us assume that our investment scenario,  $t \rightarrow u(t)$ , is optimal, implying that (A. 7) holds for all times,  $t$ . This assumption leads us to two important features of the dynamics of the shadow price,  $\psi(t)$ .

One feature characterizes the behavior of  $\psi(t)$  for large  $t$ . Let  $t$  go to infinity. Using (A. 6), we find that  $J(t)$  tends to zero. Then by (A. 7)  $H(t)$  tends to zero, too. Based on formula (A. 1) for  $H(t)$ , we get that

$$\psi(t)f(x(t), u(t)) + e^{-\rho t} g(x(t), u(t)) \rightarrow 0, \text{ as } t \rightarrow \infty. \quad (\text{A. 8})$$

Assuming  $g$  to be bounded, we find that the second term in (A. 8) tends to zero. Now (A. 8) tells us that the first term in (A. 8) tends to zero as well. The latter fact, under an additional assumption that the values of fare separated from zero, implies that

$$\psi(t) \rightarrow 0 \text{ as } t \rightarrow \infty. \quad (\text{A. 9})$$

Thus, the shadow price vanishes in the long run.

The other feature of the shadow price is its dynamics. Differentiation of (A. 7) yields

$$\dot{H}(t) = \rho \dot{J}(t). \quad (\text{A. 10})$$

Specify the right and left hand sides in this equality. Using formula (A. 1) for  $H(t)$ , we get

$$\begin{aligned} \dot{H}(t) &= \dot{\psi}(t)f(x(t), u(t)) + \psi(t) \frac{\partial f(x(t), u(t))}{\partial x} \dot{x}(t) \\ &+ \psi(t) \frac{\partial f(x(t), u(t))}{\partial u} \dot{u}(t) - \rho e^{-\rho t} g(x(t), u(t)) \\ &+ e^{-\rho t} \frac{\partial g(x(t), u(t))}{\partial x} \dot{x}(t) + e^{-\rho t} \frac{\partial g(x(t), u(t))}{\partial u} \dot{u}(t). \end{aligned}$$

By (A. 3), in the right hand side the sum of the last terms in the first and second lines is zero; the first term, using (2), we represent as  $\dot{\psi}(t)$ . Hence,

$$\begin{aligned} \dot{H}(t) &= \dot{\psi}(t)\dot{x}(t) + \psi(t) \frac{\partial f(x(t), u(t))}{\partial x} \dot{x}(t) \\ &- \rho e^{-\rho t} g(x(t), u(t)). \end{aligned} \quad (\text{A. 11})$$

Differentiation of  $J(t)$  (see (A. 5)) yields

$$\rho \dot{J}(t) = -\rho e^{-\rho t} g(x(t), u(t)). \quad (\text{A. 12})$$

Based on (A. 9), we equalize the right hand sides in (A. 11) and (A. 12). Canceling the common term,  $-\rho e^{-\rho t} g(x(t), u(t))$ , we get

$$\begin{aligned} \dot{\psi}(t)\dot{x}(t) + \psi(t) \frac{\partial f(x(t), u(t))}{\partial x} \dot{x}(t) \\ + e^{-\rho t} \frac{\partial g(x(t), u(t))}{\partial x} \dot{x}(t). \end{aligned}$$

Canceling the nonzero multiplier  $\dot{x}(t) = f(x(t), u(t))$ , we finally find that

$$\begin{aligned} \dot{\psi}(t) &= -\psi(t) \frac{\partial f(x(t), u(t))}{\partial x} \\ &- e^{-\rho t} \frac{\partial g(x(t), u(t))}{\partial x}. \end{aligned} \quad (\text{A. 13})$$

The latter, holding for all times,  $t$ , describes the dynamics of the shadow price,  $\psi(t)$ . Recall that the optimal investment,  $u(t)$ , is an explicit function of  $t$ ,  $x(t)$  and  $\psi(t)$ ,  $u(t) = U(t, x(t), \psi(t))$  (see (12)). Substituting into (A. 13), we transform the latter into

$$\begin{aligned} \dot{H}(t) &= -\psi(t) \frac{\partial f(x(t), U(t, x(t), \psi(t)))}{\partial x} \\ &- e^{-\rho t} \frac{\partial g(x(t), U(t, x(t), \psi(t)))}{\partial x}. \end{aligned} \quad (\text{A. 14})$$

Substitution of  $u(t) = U(t, x(t), \psi(t))$  into (2) yields

$$\dot{x}(t) = f(x(t), U(t, x(t), \psi(t))). \quad (\text{A. 15})$$

We get the entire set of relations by adding the boundary conditions (1) and (A. 8) (in which we substitute

$$\begin{aligned} u(t) &= U(t, x(t), \psi(t)), \\ x(0) &= x_0, \end{aligned} \quad (\text{A. 16})$$

$$\begin{aligned} \Psi(t)f(x(t), U(t, x(t), \psi(t))) \\ + e^{-\rho t} g(x(t), U(t, x(t), \psi(t))) \rightarrow 0 \text{ as } t \rightarrow \infty. \end{aligned} \quad (\text{A. 17})$$

Thus, the coupled optimal dynamics of the production capital,  $x(t)$ , and the shadow price,  $\psi(t)$ , is described by the system of differential equations (A. 14), (A. 15) and the boundary conditions (A. 16), (A. 17). These relations follow from the Pontryagin maximum principle providing necessary optimality conditions for optimal control problems of type (4). We see that, in order to form the optimal investment policy,  $t \rightarrow u(t)$ , one needs to solve the system (A. 14), (A. 15) under the boundary conditions (A. 16), (A. 17) and set  $u(t) = U(t, x(t), \psi(t))$  at each time  $t$ . The latter equality agrees with our adaptation rule (maximization of the current monetary capital growth rate (see (A. 1)), in which the shadow price,  $\psi(t)$ , is computed by solving (A. 14)- (A. 17). Let us recall in conclusion that our non-mathematical argument is essentially based on an assumption that the economy, in which the enterprise operates, is a perfect environment for production and banking.

**B. Proof of formula (8)**

Here we derive formulas (8) for the optimal investment policy in the Example. Given an investment policy  $t \rightarrow u(t)$ , the corresponding trajectory for the production capital,  $t \rightarrow x(t)$ , for model  $\dot{x}(t) = ru(t) - \delta x(t)$ , is given by

$$x(t) = e^{-\delta t} x_0 + r \int_0^t e^{-\delta(t-s)} u(s) ds.$$

Hence, for the overall benefit  $J$ , (3), where  $g(x(t), u(t)) = cqx(t) + au^+ - au(t)$ , we have

$$J = \int_0^\infty e^{-\rho t} g(x(t), u(t)) dt = J_0 + J_1$$

where

$$J_0 = \int_0^\infty e^{-\rho t} [e^{-\delta t} x_0 + au^+] dt$$

is a constant and

$$\begin{aligned} J_1 &= cqr \int_0^\infty e^{(\rho+\delta)t} \int_0^t e^{\delta s} u(s) ds dt - a \int_0^\infty e^{\rho t} u(t) dt \\ &= \frac{1}{\rho + \delta} \int_0^\infty [e^{-\rho t} (cqr - a) - (\rho + \delta)e^{-\delta t}] u(t) dt \end{aligned}$$

The single  $t \rightarrow u(t)$  taking values between 0 and  $u^+$ , which maximizes  $J_1$  is given by

$$\begin{aligned} u(t) &= 0, \text{ if } a \geq cqr \text{ or } a < cqr, \delta < \rho; \\ u(t) &= \begin{cases} 0 & \text{if } t < t_*, a < cqr, \delta > \rho \\ u^+ & \text{if } t > t_*, a < cqr, \delta > \rho \end{cases} \end{aligned}$$

where  $t_*$  is defined by

$$e^{-\rho t_*} (cqr - a) = (\rho + \delta) e^{-\delta t_*}.$$

Formula (8) is stated.

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