# Network Properties of Economic Input-Output Networks 

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International Institute for

# Network Properties of Economic Input-Output Networks 

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## Preface

Current DYN research focuses on methodological development and application within three broad areas: 1) environmental modelling and assessment (NEA), 2) economic modelling (ECG), and 3) analysis of critical infrastructures (FCI). This interim report presents the work completed by James McNerney during YSSP 2008 which combines aspects of all three of the above areas. The general objective was to investigate the information contained within economic input-output tables using network analysis in order to identify emergent patterns or properties within various economies and over time (the connection to ECG). This project incorporates methods developed in ecological network analysis (the connection to NEA) as well as those from the general network analysis literature including clustering, cycling, centrality, link weights and densities. For example, the analysis informs, among other things, about the amount of currency that cycles between sectors versus that which reaches a sector for the first time. Preliminary results indicate that centrally-planned economies appear to have higher cycling. Lastly, this work is the first step in understanding the intricate connections within economic networks and the vulnerability they are subject to based on sectoral flows (the connection to FCI). For example, the analysis also identifies those sectors that are more closely implicated together in tight "virtuous circles"(e.g., sectors tended to be grouped into four classifications: Energy, Agrochemical, Manufacturing, and soft or service industries). This could lead to further work that analyzes how perturbations to economic structures and flows could affect these patterns and shed light on important questions related to critical infrastructures.

Brian Fath<br>Gerald Silverberg


#### Abstract

This paper investigates applications of network analysis to national input-output tables. This includes initial steps to become familiar with sources for input-output data and the assumptions that go into their compilation; traditional input-output analysis; ecological input-output metrics; the difficulties in the analysis of weighted, directed graphs; the overall structure of economic input-output networks; and possible bases for comparison of network metrics. Both quantitative and qualitative regularities were observed across the OECD economies. Specifically, flow sizes and industry sizes appear to follow the same distribution for all OECD countries; the overall structure of flows within the network, as characterized by the relative amount of cycled and first-passage flow, followed a similar pattern for most OECD countries; and similar groups of closely connected sectors were found. More work needs to be done to understand these results in depth. Directions for future research are outlined; in particular, exploring (1) the stability of these results to IO data with different levels of detail, (2) community structure within the IO networks, and (3) generative/dynamic models of IO networks.


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# Network Properties of Economic Input-Output Networks 

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## 1 Introduction

An economic input-output table is a table showing the transfers of goods between sectors of an economy. Each entry of the table shows the total value of goods going from some sector of the economy to some other sector. ${ }^{1}$

An input-output table implies a network. The table itself has the form of a weighted, directed adjacency matrix, with flow of goods going from the sectors of the rows to the sectors of the columns. In addition, there is at least one additional column for the economy's output-goods for final consumption - and at least one additional row for input from labor.

### 1.1 IO Networks

Input-output networks, whether in economics or in ecology, are weighted, directed networks where the weights represent sizes of flows. In addition to flows between nodes, IO networks have boundary flows both into and out of the network. An IO network can be described by a matrix:

$$
T \equiv\left[\begin{array}{ccccc}
f_{11} & f_{12} & \cdots & f_{1 n} & y_{1} \\
f_{21} & f_{22} & \cdots & f_{2 n} & y_{2} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
f_{n 1} & f_{n 2} & \cdots & f_{n n} & y_{n} \\
z_{1} & z_{2} & \cdots & z_{n} & 0
\end{array}\right] \equiv\left[\begin{array}{cc}
F & \vec{y} \\
\vec{z}^{T} & 0
\end{array}\right] .
$$

The $f_{i j}$ represent the internal flows of the network, and the $z_{i}$ and $y_{i}$ represent the boundary flows. Each $f_{i j}$ gives the flow from the $i$ th node to $j$ th node, $z_{i}$ is the flow into node $i$, and $y_{i}$ is the flow out of node $i .^{2}$
(graphic of simple input-output network)
The flows through each node $i$ of an IO network must balance, so that total flow in is equal to total flow out:

$$
\begin{equation*}
z_{i}+F_{. i}=F_{i .}+y_{i} . \tag{1}
\end{equation*}
$$

[^1]Thus, IO networks are a special kind of weighted, directed network that has boundary flows that must obey the restriction above. ${ }^{3}$

## 2 Data \& Methods

### 2.1 Sources

All results presented here are based on data from the Organization for Economic Cooperation and Development (OECD). The OECD compiles IO tables for the OECD countries. The OECD obtains the data from national data agencies and puts them into a consistent partitioning of sectors, making the OECD tables useful for country comparisons. (This attempt at a consistent partitioning usually can not be done perfectly, as described in the next section.) The data is available for download from the web in excel spreadsheets.

One can also obtain IO tables from national data agencies directly. The United States tables are available from the Bureau of Economic Analysis (BEA) going back to 1938. The recent BEA tables exist in 3 levels of detail during benchmark years (years ending in 2 and 7.) This allows one to test the stability of the results with respect to the level of aggregation of sectors. The BEA tables are also available on the web as excel spreadsheets. The UN also publishes IO tables, but I have not studied them yet.

### 2.2 Data Issues

Conflation of Price and Quantity Changes In theoretical analysis, flow are often considered to have units of quantity, but in empirical work it is only possible to compile tables in units of currency. This means that changes in price and changes in quantity are conflated. An increase in the number of cars sold, for example, may have the same effect on the total size of a flow as an increase in the price of cars. Thus, flow sizes may change via two different mechanisms that cannot be distinguished from one another.

Flow Size Cutoff Most countries only record flows between sectors down to some minimum size. Any flows below this size are recorded as zeros or left blank, giving the false appearance of many links of zero weight and an incomplete graph topology. Since a large total flow may pass through small links, if they are numerous, an important sensitivity test of the results should be the dependence upon the cutoff flow size. Excluded links may represent as much as $30 \%$ of all $n^{2}$ links, where $n$ is the number of sectors.

Make and Use Tables IO sector classifications are applied in two ways. One is to classify commodities, the other is to classify industries; the same list of labels is used for both. In the context of commodities, the label indicates the type of product made. In the context of industries, it indicates firms whose primary area of activity is in making products of the given commodity label. For ease of discussion, we will call the former product sectors and the latter industry sectors.

Economies do not fit neatly into a partitioning by sectors because firms often produce commodities belonging to multiple sectors. This also makes it hard to say where a firm

[^2]obtained its inputs from; i.e. the data lets one say that a firm used a primary metals product, but that does not mean it received it from the primary metals industry. For some product sectors, production may be spread across multiple industry sectors. Likewise, the output of some industry sectors may be spread across multiple product sectors.

Some agencies that produce IO table publish make- and use-tables to take these differences into account. The make table is a industry sector-by-commodity sector table, showing the amount of commodity type produced by each industry. It is a heavily diagonal matrix, because most of an industry's production is usually in its "own" area, as it should be. The use table is a commodity sector-by-industry sector table showing the types of commodities used by each industry.

The standard IO table is an industry-by-industry table, but agencies compiling the table cannot obtain data in this format directly. The data is obtained from tax returns, which allow one to say what kind of product was purchased, but not who it was purchased from. Thus, some assumptions are necessary to go from the make and use tables to an industry-by-industry table.

Defining Sectors Some arbitrary choices are necessary to classify either commodities or industries into sectors, and there are inevitably problems of definition coming about as a result. A different definition of health services, for example, may cause that industry in one country to appear larger than expected when compared with other countries. The same is true for comparisons across time. Changes in the definition used by compiling agencies may cause artificial discontinuities in the data.

For the OECD tables, which try to put data from different national tables into a consistent partitioning of sectors for all countries, equivalent sectors cannot always be found between the national partitioning and OECD partitioning. Two industries, A and B, may be merged in the national table, but split up in the OECD table. Usually this is resolved by assigning all output to either A or B, and recording the use/production of the other industry as zero. Such mergers lead to "artificially" large sectors.

### 2.3 Software

All analysis was done in Matlab. The code can be made available to IIASA. The algorithm used for finding communities was based on Mark Newman and Albert Leicht's spectral bisection algorithm for directed networks [1] (see appendix) and generalized for weighted networks. To perform future information-based community analysis, C++ code was obtained from Martin Rosvall and C. Bergstrom [2]. Code used to perform the ecological analysis was obtained from Brian Fath.

## 3 Network Properties

Most figures in this section are based on the US 1997 input-output tables. The US tables are used as examples to demonstrate points made in the text; the same figures for other OECD countries are available in Appendix C.

### 3.1 Topology

Economic input-output networks are weighted, directed graphs. Self-links, representing input of an industry to itself, are allowed. In general, a weighted graph can be approximated by a binary graph if the weights are sufficiently similar in size, and a directed graph can be approximated by an undirected graph if the adjacency matrix is sufficiently
symmetric. However, as shown in the next section, neither of these conditions hold; the weights vary across several orders of magnitude, with little symmetry in the weights of links and reciprocating links (see Figure 3.)

At first glance, economic input-output networks are nearly-complete graphs. On the US 1997 table, for example, about $85 \%$ of links are nonzero. However, it is best to think of the graph as being complete for several reasons:

- The other $15 \%$ appear to be absent because of data limitations; links below a certain cutoff size are unlisted in the tables.
- It is unlikely that zero exchange of goods occured between any two industries.
- It avoids problems with certain metrics. For example, the average nearest-neighbor strength of node $i$ is defined as

$$
\left\langle s_{n n}(i)\right\rangle=\frac{1}{k_{i}} \sum_{\langle i j\rangle} w_{i j} s_{j},
$$

where $k_{i}$ is the degree of $i, w_{i j}$ the weight of the link joining $i$ and $j$, and the sum is over neighbors of $i$. Normally $k_{i}$ is assumed to count the number of non-zero weights joining $i$ with other nodes. But then the addition of a small link from $i$ that did not exist before would lower the average neighbor strength, by increasing the denominator $k_{i}$ by 1 while leaving the summation almost unchanged.
The cutoff weight of a given IO table is generally apparent from the size of the smallest links present.

### 3.2 Flow Sizes

Since the topology is trivial, the flows contain all information about the structure of the network. The flow size distributions of 20 OECD countries are shown in Figure 1. To compare distributions from different economies, which obviously vary in size, we show the distribution of the normalized flow size, defined to be a flow divided by the sum of all flows in the table:

$$
f_{i j}^{n o r m} \equiv \frac{f_{i j}}{\sum_{m n} f_{m n}}=\frac{f_{i j}}{w} .
$$

The distributions of different countries lie approximately on top of one another, with some deviation at the lower end of the distribution. This suggests that a similar generative mechanism could be at work in all economies.

Roughly speaking, the distribution is somewhere between an exponential and a powerlaw. The distribution for the US 1997 network is shown in Figure 2 with a fit to a stretched exponential $p\left(w_{i j}\right)=A \exp \left(-b w_{i j}^{a}\right)$.

The scatterplot of flow sizes versus reciprocating flow sizes is shown in Figure 3. Such a plot shows the degree of symmetry in a weighted, directed network. Economic IO networks are rather assymetric. The size of a flow $f_{i j}$ is only weakly correlated with the size of the reciprocating flow $f_{j i} .{ }^{4}$ Note that the $x$ - and $y$-axes are both log scales. It is not hard to find points in the cloud of data representing $i-j$ pairs with one flow 100 times larger than the reciprocating flow. Sometimes directed networks are approximated by undirected networks if links and reciprocating links are sufficiently similar in size, but one cannot make that approximation here.

[^3]
### 3.3 Sector Strengths

Node strength is a generalization of node degree for weighted graphs; it is the sum of the weights of the links surrounding a node. In a directed graph, a node $i$ has both an "in-strength" and an "out-strength," representing the sum of the weights of links to $i$ and the sum of the weights of links from $i$ :

$$
\begin{aligned}
s_{i}^{i n} & \equiv \sum_{j} f_{j i}=F_{. i} \\
s_{i}^{o u t} & \equiv \sum_{j} f_{i j}=F_{i .}
\end{aligned}
$$

A sector's strength measures how much an industry either provides to the other sectors in the economy, or how much it uses from the other sectors. ${ }^{5}$ As for the flow sizes, we consider here the distribution of the normalized in- and out-strengths, defined as the fractional contribution of each sector's strength to the sum,

$$
s_{i}^{n o r m} \equiv \frac{s_{i}}{\sum_{j} s_{j}}=\frac{s_{i}}{w}
$$

where $s_{i}$ is either an in-strength or an out-strength. The normalized in-strength and outstrength distributions of 20 OECD countries are shown in Figure 4, and for the US only in Figure 5. The distribution of both in-strength and out-strength appears to be exponential, with approximately the same slope for both, and with the slope approximately the same for all OECD countries.

As a test for how "surprising" this distribution is, the distribution of industry strengths was examined after shuffling the flows of the IO table. Random pairs of elements of the IO table were swapped until the whole table was shuffled, and the strength distributions were recalculated. The idea is to see how "random" the empirical distribution is; if shuffling flows leads to a distribution that is indistinguishable from the empirical one, then the empirical one is easy to obtain in some sense, and therefore not surprising. The result is shown in Figure 6. However, this is an extremely crude basis for comparison; an IO table generated by swapping flows cannot represent an economy. After the swapping, total in-flow will no longer equal total out-flow at each node, so the new IO table violates the basic accounting requirement in equation (1). Another procedure for generating a random IO network that preserves equation (1) is described in Section 4.

The scatterplot of in-strength versus out-strength for each sector is shown in Figure 7. One can see a strong correlation between the two, which is not surprising since each industry must individually obey equation (1), which constrains how different $s_{i}^{\text {in }}$ and $s_{i}^{\text {out }}$ can be. The difference $F_{i .}-F_{. i}=s_{i}^{\text {out }}-s_{i}^{i n}=z_{i}-y_{i}$, so the plot tells us about the difference between total revenue $\vec{y}$ and wages and salaries $\vec{z}$. The industries above the diagonal receive less in final consumption revenue than they pay in wages; the opposite is true for industries below the diagonal.

### 3.4 Clustering

The clustering coefficient in binary networks is meant to describe how "clumpy" the network is. Social networks, for example, tend to be clumpy because groups of people tend to be mutually acquainted. For an undirected binary network, the clustering coefficient of

[^4]node $i$ is the fraction of $i$ 's nearest neighbors thats are connected to each other. This can be written in terms of the elements of the adjacency matrix $A$ as
$$
C(i)=\frac{\sum_{j k} a_{i j} a_{j k} a_{k i}}{\sum_{j k} a_{i j} a_{k i}} .
$$

The $a_{i j}$ have values of 0 or 1 . The denominator counts all combinations of pairs of neighbors of $i$, and has the value $2 k_{i}\left(k_{i}-1\right)$ where $k_{i}$ is the degree of $i$. By including an extra $a_{j k}$ term, the numerator counts up only those nearest-neighbor of $i$ that are themselves linked, with an extra factor of 2 which cancels that in the denominator.

Clustering captures that chunkiness of the network at the level of triangles of nodes. Two generalizations need to be made to apply the concept to IO networks; one for weighted networks, and one for directed networks. A number of generalizations have been proposed for weighted networks (see [3]) I have chosen the simplest generalization, which simply replaces the binary elements $a_{i j}$ with continuous elements $f_{i j}$ in the definition above. Note that this weighted definition reduces to the original binary definition when the network weights all have the same value.

I have followed Fagiolo's generalization [3] of the clustering coefficient to directed graphs. I am not aware of any others. For a directed network, there are 6 links present among 3 nodes, and one must decide which to include and how. Note that the ordering of the indices now matters. Taking into account each possible ordering of indices leads to 5 different clustering coefficients:

$$
\begin{array}{rlrl}
C^{c y c}(i) & \equiv \frac{\sum_{j, k} F_{i j} F_{j k} F_{k i}}{\sum_{j \neq k} f_{k i} f_{i j}} & =\frac{\left(F^{3}\right)_{i i}}{s_{i}^{\text {in }} s_{i}^{\text {out }}-s_{i}^{\leftrightarrow}} \\
C^{\text {mid }}(i) \equiv \frac{\sum_{j, k} f_{i j} f_{k j} f_{k i}}{\sum_{j \neq k} f_{k i} f_{i j}} & =\frac{\left(F F^{T} F\right)_{i i}}{s_{i}^{\text {in }} s_{i}^{\text {out }}-s_{i}^{\leftrightarrow}} \\
C^{\text {in }}(i) \equiv \frac{\sum_{j, k} f_{j i} f_{j k} f_{k i}}{\sum_{j \neq k} f_{k i} f_{j i}} & =\frac{\left(F^{T} F^{2}\right)_{i i}}{\left(s_{i}^{\text {in }}\right)^{2}-s_{i}^{\text {in }}} \\
C^{\text {out }}(i) \equiv \frac{\sum_{j, k} f_{i j} f_{j k} f_{i k}}{\sum_{j \neq k} f_{i k} f_{i j}} & =\frac{\left(F^{2} F^{T}\right)_{i i}}{\left(s_{i}^{\text {out }}\right)^{2}-s_{i}^{\text {out }}} \\
C^{\text {all }}(i) \equiv \frac{\sum_{j, k}\left(f_{i j}+f_{j i}\right)\left(f_{j k}+f_{k j}\right)\left(f_{k i}+f_{i k}\right)}{\sum_{j \neq k}\left(f_{i j}+f_{j i}\right)\left(f_{i k}+f_{k i}\right)(2)} & =\frac{\left(F+F^{T}\right)^{3}}{2\left[s_{i}^{\text {tot } \left.\left(s_{i}^{\text {tot }}-1\right)-2 s_{i}^{\leftrightarrow}\right]}\right.}
\end{array}
$$

Each of these expressions sums over particular linking patterns between the three nodes, as shown in Table 1.

The cumulative distribution of each type of clustering coefficient is shown in Figure 8. The cycle clustering coefficient is consistently lower than all the others. This suggests that economies tends to be acyclic at the small scale of 3 -cycles.

### 3.5 Centrality

Centrality in network analysis is a measure of a node's importance. There are a number of measures of centrality in use. Three of the most commonly used are betweenness centrality, closeness centrality, and eigenvector centrality. The propriety of a given definition depends on the kind of network being considered. For example, betweenness centrality, which is based on counting shortest paths passing through each node, is useful in networks where information is carried across links (e.g. the internet.) It is not an obviously useful notion of centrality in the context of economies.

We have adapted eigenvector centrality to input-output networks. The idea is based on the image of a dollar of currency (or a random dollar of product-embodied "value") making a random walk around the economy. Define a matrix $Q$ whose elements $q_{i j}$ are the probabilities that the dollar moves from sector $i$ to sector $j$. For a given country and year, the probability can be taken to be

$$
q_{i j}=\frac{f_{i j}}{\sum_{j} f_{i j}+y_{i}}=\frac{f_{i j}}{T_{i}}
$$

for the "out-centrality" of a randomly-walking dollar of value, and

$$
q_{i j}=\frac{f_{j i}}{\sum_{j} f_{j i}+z_{i}}=\frac{f_{j i}}{T_{i}}
$$

for the "in-centrality" of a randomly-walking dollar of currency.
First consider a closed network. Suppose we start the dollar out at some particular sector, and we allow it to be split up into fractions of a dollar, as would be expected by the transition probabilities. With each step, the dollar would get split up, with probabilities accumulating at all the nodes of the network. After a long time, the probabilities may reach some steady state. These steady state probabilities serve as a measure of centrality; they are a measure of the likelihood of finding the random dollar in each particular sector.

Let $\vec{p}$ be the vector of steady-state probabilities, and $Q$ the matrix of transition probabilities, where each element $q_{i j}$ is the probability of a transition to $j$ starting from $i$ :

$$
\vec{p}=\left[\begin{array}{c}
p_{1} \\
p_{2} \\
\vdots
\end{array}\right] \quad Q=\left[\begin{array}{ccc}
q_{11} & q_{12} & \cdots \\
q_{21} & q_{22} & \cdots \\
\vdots & \vdots & \ddots
\end{array}\right]
$$

The probability of being at $i$ at the steady state is a sum over all possible starting points multiplied by the appropriate transition probabilities:

$$
\begin{aligned}
p_{1} & =p_{1} q_{11}+p_{2} q_{21}+\cdots \\
p_{2} & =p_{1} q_{12}+p_{2} q_{22}+\cdots \\
& \vdots \\
p_{n} & =p_{1} p_{1 n}+p_{2} q_{2 n}+\cdots
\end{aligned}
$$

This set of equations is equivalent to the matrix equation

$$
\vec{p}=Q \vec{p} .
$$

Thus, the goal is to find an eigenvector of $Q$ with eigenvalue 1 . There are as many eigenvectors of a matrix as the dimension of the matrix. The Perron-Frobenius theorem guarantees that at least one of them will have all real non-negative entries that sum to 1 . This is necessary so that the entries can be interpreted as probabilities.

Now consider a network with some final output leaving the network. The transition probabilities of a given row no longer add up to 1 , because there is a probability that the dollar leaves the internal network of sectors. We can use the same definition above for this case, and note that the eigenvalue corresponding to the eigenvector of centralities will be less than 1.

$$
Q \vec{p}=\lambda \vec{p}
$$

This eigenvalue may be interpreted as the probability at each "step" that the dollar has left the network.

### 3.6 Community Structure

Communities in networks are groups of nodes that share a close relation, usually as it is realized in their pattern of links. Knowing the communities of a network helps one develop a picture of the network. It may lead to the discovery of previously unknown connections between nodes, or confirm connections known to exist. The problem of finding communities is common in network analysis, and a number of methods for doing it have been devised. The method used here was a weighted generalization of the spectral-bisection algorithm for directed graphs, developed by E. Leicht and Mark Newman [1]. I am aware of only one other algorithm that is applicable to directed networks, described in [2]

In the unweighted version, the algorithm works by attempting to maximize modularity $Q$, defined as

$$
\begin{aligned}
Q \equiv & (\text { fraction of links within communitiies }) \\
& -(\text { expected fraction of links within communities }) .
\end{aligned}
$$

The modularity is a measure of the "statistical surprise" of linkage within some arbitrary group of nodes. For any given partitioning of the network into groups, the modularity can be measured easily. If a particular partitioning yields a high value of $Q$, it suggests that the partitions correspond to real groups. For weighted networks, I simply replaced the binary values of the adjacency matrix with the corresponding weights.

The number of possible partitionings of even small networks ( $\sim 50$ nodes) is huge, so any practical algorithm for finding communities must search only a portion of the space of partitions. The algorithm by Leicht and Newman uses a greedy procedure that accepts changes which increase the modularity. More details are provided in Appendix A.

Results of Spectral Bisection Algorithm There are several things to say about the groups found by the algorithm. They are not the same for all countries, though there is a strong tendency for many of the same industries to appear grouped together in different countries. The groupings do not obey any logical scheme perfectly, but they show a strong tendency towards the following four groups (using the sector names of the OECD tables):

## - Energy

Mining and Quarrying
Coke, Petroleum, Petroleum Products, and Nuclear Fuel
Electricity, Gas, and Water Supply

## - Agro-Chemical

Agriculture, Hunting, Forestry, and Fishing
Food Products, Beverages, and Tobacco
Textiles, Textile Products, Leather, and Footwear
Pulp, Paper, Paper Products, Printing, and Publishing
Wood and Products of Wood and Cork
Chemicals Excluding Pharmaceuticals
Pharmaceuticals
Rubber and Plastics Products
Manufacturing NEC, Recycling

- Manufacturing

Iron and Steel

Non-Ferrous Metals
Other Non-Metallic Minerals
Fabricated Metal Products
Machinery and Equipment NEC
Electric Machinery and Apparatus NEC
Motor Vehicles, Trailers and Semi-Trailers
Building and Repairing of Ships and Boats
Railroad Equipment and Trasnport Equipment NEC
Construction
Aircraft and Spacecraft
Medical, Precision, and ptical Instruments
Office, Accounting, and Computing Machinery
Radio, Television, and Communication Equipment

## - Services

Wholesale and Retail Trade, Repairs
Post and Telecommunications
Transport and Storage
Finance, Insurance
Real Estate Activities
Renting of Machinery and Equipment
Computer and Related Activities
Research and Development
Education
Hotels and Restaurants
Health and Social Work
Other Business Activities
Other Community, Social, and Personal Services

No country obeys this scheme perfectly. The U.S. tables perhaps follow it best; other countries somewhat less strictly. Results for each country are given in Appendix C.

Nesting A shortcoming of the spectral bisection algorithm is that it cannot deal easily with nested communities. Suppose some collection of sectors shows a tendency to form strong bonds with one another, suggesting that these sectors form a community. Suppose also there are subgroups of the community that are even more exclusive, with members forming even stronger bonds with one another than with other members of the larger community. If this is the case, the network has a nested community structure, with strong groups inside of weaker groups.

The present algorithm faces the following dilemma in this situation. Suppose there are two large groups, 1 and 2, and group 1 has subgroups 1a and 1b. Since the algorithm can only output one level of groups, it must produce groups

$$
1 \text { and } 2 \quad \text { or } \quad 1 \mathrm{a}, 1 \mathrm{~b}, \text { and } 2 .
$$

The former contains no information about the subgroups of group 1 , and the latter misleadingly implies that 1 a and 1 b are as distinct from each other as 1 a from 2 or 1 b from 2.

This theoretical problem appears to occur in the real IO tables. Figure 10 visually depicts an IO table, with colors representing the size of individual links in the table. The more red a cell is, the larger the flow of goods/money across the link. The task of finding groups in the network can be restated as the task of finding a reordering of the rows and columns of this matrix that brings it as close as possible to block-diagonal form. Blocks along the diagonal that contain many red and orange links represent good candidates for communities. The black lines in the matrix show boundaries between groups, and reveal where the groups ought to appear. One can see two groups that are particularly strong. However, the circled boxes show links between members of one group and members of the other, and there is a fairly strong connection between them. This indicates that the two groups may form part of some larger group, which itself stands apart from the rest of the economy, exactly as in the example above.

Information-Based Algorithm Rosvall and Bergstrom describe an algorithm for finding communities that is based on a different concept of communities [2]. Their algorithm is based on the idea of assigning labels to nodes such that a random walk on the network can be recorded in the most efficient way. Suppose each node is given an address, say a string of bits. The goal is to find addresses that allow a random walk of arbitrary length to be recorded with the fewest bits. The authors argue that finding an efficient system of addresses leads to the discovery of communities, as realized by the groups of nodes where the random walker "gets stuck."

Code was obtained from the authors which implements their algorithm, but I have not yet applied it to the IO tables. The algorithm may be adaptable to allow search for nested community structure.

### 3.7 Global Flow Structure

The total flow through a node is known as its throughflow. Specifically, for an economic IO table or an ecological network at steady-state, the total flow into each node $i$ is equal to the total flow out, and either sum equals $i$ 's throughflow $T_{i}$ :

$$
T_{i}=z_{i}+\sum_{j} f_{j i}=\sum_{j} f_{i j}+y_{i} .
$$

Total system throughflow is a measure of the total flow through an IO network, defined as the sum of the throughflows through each node:

$$
T S T \equiv \sum_{i} T_{i} .
$$

Flow through the network can be divided into boundary flow, first-passage flow, and cycled flow $[5,6]$. These three modes reflect different ways that flow can arrive at some node $i$. Boundary flow is simply the flow into $i$ from outside the network. After flow from the outside reaches some node, it may be passed on to $i$ before exiting the network; this contribution is known as first-passage flow. Since the IO network may contain cycles, it is also possible that some of the flow through $i$ has passed through it before and was recycled by the network; this contribution is known as cycled flow. The difference between first-passage flow and cycled flow is that first-passage flow is flow that goes through $i$ once and only once before exiting the network.

The mathematical definitions of these three flow modes will make the statements above more precise. First it is useful to define some notation. The description of the input-output
network is contained in the matrix $F$ and the boundary vectors $\vec{z}$ and $\vec{y}$, as described in Section 1.1. We define the matrix $G$, whose elements $g_{i j}$ are the fraction of $i$ 's thoughflow going to $j, g_{i j}=f_{i j} / T_{i}$. From $G$ we derive $N$, defined to be $N=\left(I-G^{T}\right)^{-1}$. Table 2 summarizes these definitions.

A useful identity for the following discussion is

$$
\vec{T}=N \vec{z}
$$

One way to derive expressions for the contributions coming from each of the three flow modes is to consider the power expansion of $N$ in the expression above:

$$
\begin{aligned}
\vec{T} & =N \vec{z} \\
& =(I-G)^{-1} \vec{z} \\
& =\left(I+G+G^{2}+G^{3}+\cdots\right) \vec{z} \\
& =\vec{z}+\left(G+G^{2}+G^{3}+\cdots\right) \vec{z}
\end{aligned}
$$

The first term in the expansion is easily identified with the boundary flow. The remaining sum accounts for all subsequent flows inside the network. The matrix $G$, applied to $\vec{z}$ once, "propagates" the currency at each node one step. Each additional power of $G$ corresponds to an additional step taken by the flows. The remaining sum, $(N-I) \vec{z}$, is the sum of firstpassage and cycled flows. The $i$ th element of this represents the combined flow through node $i$ from first-passage and cycled flows:

$$
\begin{aligned}
{[(N-I) \vec{z}]_{i} } & =\sum_{j}(N-I)_{i j} z_{j} \\
& =\sum_{j \neq i}(N-I)_{i j} z_{j}+(N-I)_{i i} z_{i}
\end{aligned}
$$

The $i j$ th term of $(N-I)=G+G^{2}+\cdots$ accounts for all flow ultimately going from node $i$ to node $j$ via any number of steps through the network. The second term above therefore yields the cycled flow through $i$, since $(N-I)_{i i}$ accounts for all flow leaving $i$ and returning to $i$ via any number of steps. The remaining sum yields the first-passage flow. Thus we have decomposed the throughflow through a node $i$ as

$$
T_{i}=z_{i}+\sum_{j \neq i}(N-I)_{i j} z_{j}+(N-I)_{i i} z_{i}
$$

and taken the three terms above to define the three flow modes:

$$
\begin{aligned}
T_{i}^{b o u n} & \equiv z_{i} \\
T_{i}^{f p} & \equiv \sum_{j \neq i}(N-I)_{i j} \\
T_{i}^{c y c} & \equiv(N-I)_{i i} z_{i} \\
T_{i} & =T_{i}^{\text {boun }}+T_{i}^{f p}+T_{i}^{c y c}
\end{aligned}
$$

Summing over all nodes yields the total flow through the network of each mode:

$$
\begin{aligned}
T S T^{b o u n} & \equiv \sum_{i} T_{i}^{\text {boun }} \\
T S T^{f p} & \equiv \sum_{i} T_{i}^{f p} \\
T S T^{c y c} & \equiv \sum_{i} T_{i}^{c y c} \\
T S T & =T S T^{\text {boun }}+T S T^{f p}+T S T^{c y c}
\end{aligned}
$$

The final line summarizes the decomposition of total system throughflow into the three flow modes. Rather than look at the absolute size of the terms on the right hand side, it is more convenient to study their fractional contributions to the total. The results in Appendix C are presented this way, although the absolute size of the total system throughflow is also given, as well as the total system boundary flow; the latter is equal to the country's GDP.

The breakdown of total system throughflow into these three modes offers a global characterization of an IO network. Theoretical work needs to be done to connect the contributions from these three modes to more micro-level properties of the network, such as the flow size or node strength distributions. Without this, it is very difficult to interpret results of the decomposition. Results for the OECD countries are shown in Table 3. The contributions from the three modes follow a consistent pattern across almost all countries. Anomalous values were observed for China and the Czech Republic. Excluding these two, a consistent pattern emerged. Boundary flow made up on average half of the network flow, first passage flow about $35 \%$, and cycled flow about $13 \%$. Individual countries showed modest deviation from these averages. The most variation was in the cycled percentage, which varied from $7.8 \%$ for Greece to $16.9 \%$ for Poland (excluding China and the Czech Republic, which both had very high values of cycling).

## 4 Generating Artifical IO Networks

Measuring properties of IO networks is the first step in their analysis. The next step will be to explain the measured values and identify features that are surprising or interesting. At the moment, however, there is little to compare the measurements of the previous section to, making the task of identifying interesting features difficult. A null model of IO networks is needed as a basis for comparison with the real world examples.

The model need not generate the features seen in the real network. For example, in the Erdos-Renyi random graph model, each pair of nodes is linked with probability $p$. For values of $p$ below a critical value known as the percolation threshold, the degree distribution of the graph is binomial. Many real world networks, on the other hand, obey a power-law degree distribution. This indicates that this maximally simple rule - connecting each pair of nodes with a fixed probability - is probably not at work in these real world examples; a more sophisticated mechanism is needed. ${ }^{6}$ Although the Erdos-Renyi model does reproduce the observed property, it serves as a null model that helps reveal what

[^5]properties of the observed networks are interesting or surprising, aiding the interpretation of measured properties of real networks.

One simple basis of comparison for IO networks comes from shuffling flows; we take the whole list of $n^{2}$ flows in the network and reassign them to new $i-j$ pairs at random. Although simple, this procedure does not preserve the input-output accounting of an IO network, $z_{i}+F_{. i}=F_{i .}+y_{i}$, so the result cannot represent a real economy.

One way to satisfy $z_{i}+F_{i}=F_{i}+y_{i}$ is to begin with a network in an initial condition that already satisfies this relation, and then take the network through many small changes that always preserve it. For example, consider the following procedure:

1. We initialize the network to a state satisfying $z_{i}+F_{\cdot i}=F_{i}+y_{i}$.
2. At each time step, we pick two nodes $i$ and $j$ at random with uniform probability. The two nodes may be the same.
3. With probability $p$, we decrease the strength of the link from $i$ to $j$ by a factor $r$, so that $f_{i j} \rightarrow f_{i j}^{\prime}=f_{i j}-r f_{i j}$. At the same time, we pick a third node $k$ at random to "redirect" the lost flow through, and increase the flow to this third node by the amount lost from $i$ to $j: f_{i k} \rightarrow f_{i k}^{\prime}=f_{i k}+r f_{i j}, \quad f_{k j} \rightarrow f_{k j}^{\prime}=f_{k j}+r f_{i j}$.
4. With probability $1-p$, the opposite happens: $f_{i j}$ gets increased by drawing off flow from $f_{i k}$ and $f_{i j}$. Let $f^{*}=\min \left(f_{i k}, f_{k j}\right)$. With probability $1-p$, we have $f_{i j} \rightarrow f_{i j}^{\prime}=f_{i j}+r f^{*}, \quad f_{i k} \rightarrow f_{i k}^{\prime}=f_{i k}-r f^{*}, \quad$ and $f_{k j} \rightarrow f_{k j}^{\prime}=f_{k j}-r f^{*}$.

Any change will necessarily effect the strength of some connection between two nodes $i$ and $j$. By weakening the $i j$ link, we leave ourselves with excess output from $i$ (which must go somewhere else) and a deficit of input to $j$ (which must come from somewhere else.) Any change we make that preserves the accounting balance at each node must simply cause that same amount of flow to still go from $i$ to $j$ via some other path. The simplest modification to the $i j$ flow involves a redirection through one other node $k$.

The final state is not sensitive to the initial condition. Since the empirical IO tables already satisfy (1), one can use any of them as the initial state. An alternative initial state can be obtained by the following procedure:

1. Construct the vector $\vec{\delta}=\vec{z}-\vec{y}$. Each element of this vector is the net boundary flow into a node, and represents either an excess amount of flow which must be distributed to the rest of the network, or a deficit which must be obtained from the rest of the network. The sum of this vector is zero.
2. Sort $\vec{\delta}$ from greatest element to least. The elements at the top of list will have large positive values, while those at the bottom will have large negative values.
3. Assign all excess flow from the first element to the second element: $f_{12}=\delta_{1}$. Node 1 now satisfies (1). Update $\vec{\delta}$ to include the internal flows as they are constructed, so that $\delta_{1} \leftarrow 0$, and $\delta_{2} \leftarrow \delta_{2}+\delta_{1}$.
4. Repeat this for each node until the end of list is reached; i.e. distribute all of the excess flow at node $2, \delta_{2}$, to node 3 , so that $f_{23}=\delta_{2}, \delta_{2} \leftarrow 0, \delta_{3} \leftarrow \delta_{3}+\delta_{2}$. Note that node 3 will receive flow accumulated from both nodes 1 and 2 in the list, so that as we move along the list the size of the flow passed along grows from zero at first, and then decreases once we enter the "negative part" of the list, finally reaching zero when we establish the final flow between the $n-1$ th and $n$th nodes.

In addition to satisfying $z_{i}+F_{. i}=F_{i} .+y_{i}$, I believe this procedure generates, for a given $\vec{z}$ and $\vec{y}$, the IO network with the largest ratio of first-passage flow to cycling flow. Using a $\vec{z}$, $\vec{y}$ pair from real data, the algorithm reaches the same final state whether the internal flows were initialize using the above procedure, or using the original data in the input-output table.

Any algorithm for generating a random IO network must assume some properties of the generated network a priori. For example, in generating binary networks, one may assume a fixed degree sequence (the complete list of degrees for all the nodes of the network.) Given a degree sequence, one can randomly wire nodes together in many ways that satisfy the degree sequence. By randomly wiring nodes together, one can generate a random network from the ensemble of all networks having the same degree sequence. Likewise, we would like to have an algorithm that generates random IO networks from an ensemble with specific criteria. The algorithm above generates IO networks with a given set of input and output vectors, $\vec{z}$ and $\vec{y}$. This means it confines itself to the ensemble of economies that use the same inputs to produce the same outputs. However, before using networks generated this way as a basis for comparison with real data, one must also know the probabilities associated with each configuration in the ensemble, which are probably not uniform. Determining these will require more analysis and simulation of the model.

## 5 Conclusions

These conclusions are tentative, contingent upon the sensitivity of these results various factors, which I discuss in the next section.

Possibility of Simple Mechanisms for Economic Organization The flow size and sector strength distributions appear to follow the same shape, at least among the 20 countries studied here. In addition, the breakdown of flows into boundary/first-passage/cycled flows follows a consistent pattern for most of the OECD countries. These results suggest that certain features of economies are universal, holding true regardless of other differences between economies at a finer scale. Furthermore, the simplicity of the results suggests that simple mechanisms may be at work to generate them. This opens up the possibility for parsimonious models for economies that give insight into how they organize themselves, and which would hold for all economies.

Non-Trivial Overall Flow Structure The in-strength and out-strength of a sector serve as one measure of its size, and the centrality a crude indication of where money in the network is most likely to end up. There is a striking difference between the list of largest sectors and the most central sectors, which suggests there is a non-trivial structure to flows that does not necessarily drive money towards the largest sectors.

Appearance of Consistent Communities Certain sectors showed strong tendencies to appear grouped together using the community-finding algorithm described in Section 3.6 and Appendix A. Even more interesting, the sectors tended to partition themselves into 4 main groups: one for energy sectors, one for manufacturing, one for agro-chemical sectors, and one for services.

The latter result is remarkable, in that sectors which appear together show qualitative similarities that are not and can not be represented in a table of numbers. It is worth emphasizing that the algorithm does not "know" the sector labels of the columns and rows of the IO table. It does not know, for example, to put "Mining and Quarrying" in
the same group as "Electricity, Gas, and Water Supply" because they are both energyrelated. The algorithm only takes as input the values of the table itself, and searches for communities implied by the pattern of weak and strong elements present in it. Thus, not only is it possible to make a connection between the numerical data of the IO table and economic intuition, but also this shows that quantitative network methods may reveal deep economic structure that is not otherwise apparent.

## 6 Future Work

Future works falls into 3 categories: checking the sensitivity of the current results, expanding the data examined, and understanding the data at a theoretical level. Goals include:

- Testing the stability of these results to IO data with different levels of detail.
- Testing the sensitivity of these results to the level of the cutoff weight used by the agency compiling the table.
- Using an alternative method of finding communities that can reveal nested groups.
- Performing a more thorough comparison of IO networks across countries.
- Performing a comparison of IO networks across time.
- Studying IO tables where the inputs are physical. (See for example [4].)
- Studying generative and/or dynamical network models of economies, along the lines described in section 4.


## References

[1] E. A. Leicht and M. E. J. Newman, "Community structure in directed networks," 2007
[2] M. Rosvall and C. T. Bergstrom, "Maps of random walks on complex networks reveal community structure," 2008
[3] Giorgio Fagiolo, "Clustering in Complex Directed Networks," 2006
[4] Nandan U. Ukidwe and Bhavik R. Bakshi, "Industrial and ecological cumulative exergy consumption of the United States via the 1997 input-output benchmark model," 2007
[5] M. Higashi, B. C. Patten, \& T. P. Burns, "Network trophic dynamics: the modes of energy utilization in ecosystems," 1993
[6] B. D. Fath, B. C. Patten, J. S. Choi, "Complementarity of ecological goal functions," 2001

## A Finding Communities by Spectral Bisection

I have included the following excerpt from [1] to help introduce the algorithm I used. For more detail, see the original publication. I make one modification to the algorithm presented in [1]; I allow the elements of the adjacency matrix to take on any value, not just binary values. The algorithm I used is applicable to any network-binary/weighted, directed/undirected.
"The premise of the modularity optimization method is that a good division of a network into communities will give high values of the benefit function $Q$, called the modularity, defined by
$Q=$ (fraction of edges within communities) - (expected fraction of such edges).
Large positive values of the modularity indicate when statistically surprising fraction of the edges in a network fall within the chosen communities; it tells us when there are more edges within communities than we would expect on the basis of chance.
"The expected fraction of edges is typically evaluated within the so-called configuration model, a random graph conditioned on the degree sequence of the original network, in which the probability of an edge beween two verticies $i$ and $j$ is $k_{i} k_{j} / 2 m$, where $k_{i}$ is the degree of vertex $i$ and $m$ the total number of edges in the network. The modularity can then be written

$$
Q=\frac{1}{2 m} \sum_{i j}\left[A_{i j}-\frac{k_{i} k_{j}}{2 m}\right] \delta_{C_{i} c_{j}}
$$

where $A_{i j}$ is an element of the adjacency matrix, $\delta_{i j}$ is the Kronecker delta symbol, and $c_{i}$ is the label of the community to which vertex $i$ is assigned. Then one maximizes $Q$ over possible divisions of the network into communities, that maximum begin taken as the best estimate of the true communities in the network. Neither the size nor the number of communities need be fixed; both can be varied freely in our attempt to find the maximum.
"In practice, the exhaustive optimization of modularity is comupationaly hard, known to be NP-complete over the set of all graphs of a given size, so practical methods based on modularity optimization make use of approximate schemes such as greedy algorithms, simulated annealing, spectral methods, and others."

Algorithm:

$$
\begin{aligned}
& \mathbf{B} \equiv \text { matrix with elements } B_{i j} \\
& B_{i j}=w_{i j}-\frac{s_{i}^{i n} s_{j}^{o u t}}{m} \\
& m \equiv \sum_{i j} a_{i j} \\
& \mathbf{s} \equiv \text { sector whose elements are } s_{i} \\
& s_{i}= \pm 1 \text { (the assignment of a node to one of } 2 \text { communities } \\
& \text { being considered in the given step) } \\
& \mathbf{B}_{i j}^{(g)} \equiv B_{i j}-\delta_{i j} \sum_{k \in g} B_{i k}
\end{aligned}
$$

1. Calculate the eigenvector corresonding to the largest eigenvalue of $\mathbf{B}+\mathbf{B}^{T}$.
2. Divide the network into 2 communities. Assign each node $i$ to one of the two communities based on the signs of the elements of the eigenvector. If the $i$ th element is negative, assign $s_{i}=-1$; if positive, $s_{i}=+1$.
3. "Fine-tune" the assignment by moving individual nodes into the other community and measuring the effect on $Q$. Accept changes that increase $Q$ until no more moves can be made.
4. For subdivision of communities, we maximize changes in $Q$ by finding the eigenvector corresonding to the largest eigenvalue of $\mathbf{B}^{(g)}+\mathbf{B}^{(g) T}$ and assigning nodes into communities as before.
5. Repeat process of bisection and fine-tuning until no division can increase $Q$.

## B Matlab Programs

The following files are Matlab scripts written to analyze the IO data and produce the results and figures presented here:

Main files:
processOECD.m Loads the IO table of each country from a text file and processes it. The format of the data is slightly different for each country, requiring country-specific manipulations. Produces all variables needed as input for analyze.m.
analyze.m Main file, which performs all analysis except for the community-finding algorithm.

IOcomm.m Performs additional preprocessing of the IO data to allow application of findcomm.m.
findcomm.m Searches for communities. Implements the algorithm described in Appendix A.

NEA.m Measures various quantities from ecological input-output analysis. (Obtained from Brian Fath.)
redirect.m Generates random IO networks via the procedure described in Appendix 4.
Support files:
printlargest.m
binpdf.m
seematrix.m
shuffle.m

## C IO Table Results

All results come from the OECD-published IO tables.

## C. 1 Australia

| Flow Mode |  |
| :--- | :--- |
| Boundary | $50.3 \%$ |
| First-passage | $37.5 \%$ |
| Cycled | $12.2 \%$ |

Density of non-zero links $=0.858$
Boundary flow $($ GDP $)=4.485 \mathrm{e}+11$ Australian dollars
Total system throughflow $=8.921 \mathrm{e}+11$ Australian dollars Cycled

## Link Weights




## Sector Strengths




Clustering


Centrality


## Communities

$\mathrm{Q}=0.205$
Group 1
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR
CHEMICALS EXCLUDING PHARMACEUTICALS
PHARMACEUTICALS
RUBBER AND PLASTICS PRODUCTS
MANUFACTURING NEC; RECYCLING
HOTELS AND RESTAURANTS
Group 2
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS
AIRCRAFT AND SPACECRAFT
WHOLESALE AND RETAIL TRADE; REPAIRS
TRANSPORT AND STORAGE
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
REAL ESTATE ACTIVITIES
COMPUTER AND RELATED ACTIVITIES
OTHER BUSINESS ACTIVITIES
PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

## Group 3

WOOD AND PRODUCTS OF WOOD AND CORK
OTHER NON-METALLIC MINERAL PRODUCTS
IRON \& STEEL
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
ELECTRICAL MACHINERY AND APPARATUS, NEC
BUILDING AND REPAIRING OF SHIPS AND BOATS
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.
CONSTRUCTION

Group 4
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL NON-FERROUS METALS
ELECTRICITY, GAS AND WATER SUPPLY

## Group 5

PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
EDUCATION
HEALTH AND SOCIAL WORK

PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS
Ungrouped Sectors
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
RENTING OF MACHINERY AND EQUIPMENT
RESEARCH AND DEVELOPMENT

## C. 2 Brazil

| Flow Mode |  |
| :--- | ---: |
| Boundary | $55.1 \%$ |
| First-passage | $32.9 \%$ |
| Cycled | $12 \%$ |

Density of non-zero links $=0.552$
Boundary flow $(G D P)=7.285 \mathrm{e}+11$ novo cruzados
Total system throughflow $=1.323 \mathrm{e}+12$ novo cruzados
Cycled

## Link Weights




## Sector Strengths




Clustering


Centrality


## Communities

$\mathrm{Q}=0.245$
Group 1
WOOD AND PRODUCTS OF WOOD AND CORK
OTHER NON-METALLIC MINERAL PRODUCTS
IRON \& STEEL
NON-FERROUS METALS
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
ELECTRICAL MACHINERY AND APPARATUS, NEC
ELECTRICITY, GAS AND WATER SUPPLY
CONSTRUCTION
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
REAL ESTATE ACTIVITIES
Group 2
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING PHARMACEUTICALS
MANUFACTURING NEC; RECYCLING
WHOLESALE AND RETAIL TRADE; REPAIRS
HOTELS AND RESTAURANTS
OTHER BUSINESS ACTIVITIES
PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY
Group 3
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
CHEMICALS EXCLUDING PHARMACEUTICALS
Group 4
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL RUBBER AND PLASTICS PRODUCTS
TRANSPORT AND STORAGE
Group 5
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.

## Ungrouped Sectors

RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
BUILDING AND REPAIRING OF SHIPS AND BOATS
AIRCRAFT AND SPACECRAFT
RENTING OF MACHINERY AND EQUIPMENT

COMPUTER AND RELATED ACTIVITIES
RESEARCH AND DEVELOPMENT
EDUCATION
HEALTH AND SOCIAL WORK
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

## C. 3 Canada

| Flow Mode |  |
| :--- | :--- |
| Boundary | $53.5 \%$ |
| First-passage | $34.5 \%$ |
| Cycled | $11.9 \%$ |

Density of non-zero links $=0.681$
Boundary flow $(G D P)=8.256 \mathrm{e}+11$ Canadian dollars
Total system throughflow $=1.542 \mathrm{e}+12$ Canadian dollars Cycled

## Link Weights




## Sector Strengths




Clustering


Centrality


## Communities

$\mathrm{Q}=0.22$
Group 1
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR WOOD AND PRODUCTS OF WOOD AND CORK
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
CHEMICALS EXCLUDING PHARMACEUTICALS
RUBBER AND PLASTICS PRODUCTS
MANUFACTURING NEC; RECYCLING
HOTELS AND RESTAURANTS
Group 2
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY

Group 3
OTHER NON-METALLIC MINERAL PRODUCTS
IRON \& STEEL
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS AIRCRAFT AND SPACECRAFT
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. CONSTRUCTION

Group 4
BUILDING AND REPAIRING OF SHIPS AND BOATS PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY HEALTH AND SOCIAL WORK

Group 5
WHOLESALE AND RETAIL TRADE; REPAIRS
TRANSPORT AND STORAGE
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
COMPUTER AND RELATED ACTIVITIES
OTHER BUSINESS ACTIVITIES
EDUCATION
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Group 6
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
ELECTRICAL MACHINERY AND APPARATUS, NEC
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT

Ungrouped Sectors<br>PHARMACEUTICALS<br>NON-FERROUS METALS<br>MEDICAL, PRECISION AND OPTICAL INSTRUMENTS<br>REAL ESTATE ACTIVITIES<br>RENTING OF MACHINERY AND EQUIPMENT<br>RESEARCH AND DEVELOPMENT<br>PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

## C. 4 China

| Flow Mode |  |
| :--- | :--- |
| Boundary | $15.6 \%$ |
| First-passage | $63.2 \%$ |
| Cycled | $21.2 \%$ |

Density of non-zero links $=0.833$
Boundary flow $(G D P)=3.12 \mathrm{e}+13$ yuan
Total system throughflow $=1.998 \mathrm{e}+14$ yuan

## Link Weights




## Sector Strengths




## Clustering



Centrality


## Communities

$\mathrm{Q}=0.267$
Group 1
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR HOTELS AND RESTAURANTS
REAL ESTATE ACTIVITIES
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Group 2
CHEMICALS EXCLUDING PHARMACEUTICALS
RUBBER AND PLASTICS PRODUCTS
Group 3
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL
OTHER NON-METALLIC MINERAL PRODUCTS
IRON \& STEEL
NON-FERROUS METALS
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
ELECTRICAL MACHINERY AND APPARATUS, NEC
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS
BUILDING AND REPAIRING OF SHIPS AND BOATS
AIRCRAFT AND SPACECRAFT
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.
ELECTRICITY, GAS AND WATER SUPPLY
CONSTRUCTION
TRANSPORT AND STORAGE
POST AND TELECOMMUNICATIONS
RESEARCH AND DEVELOPMENT
Group 4
WOOD AND PRODUCTS OF WOOD AND CORK
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
PHARMACEUTICALS
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
MANUFACTURING NEC; RECYCLING
WHOLESALE AND RETAIL TRADE; REPAIRS
FINANCE, INSURANCE
OTHER BUSINESS ACTIVITIES
PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY
EDUCATION

Ungrouped Sectors
RENTING OF MACHINERY AND EQUIPMENTCOMPUTER AND RELATED ACTIVITIESHEALTH AND SOCIAL WORKPRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

## C. 5 Czech Republic

Flow Mode

| Boundary | $36.4 \%$ |
| :--- | :--- |
| First-passage | $42.1 \%$ |
| Cycled | $21.5 \%$ |

Density of non-zero links $=0.919$
Boundary flow $(G D P)=1.285 \mathrm{e}+12$ Koruna
Total system throughflow $=3.529 \mathrm{e}+12$ Koruna

## Link Weights




## Sector Strengths




Clustering


Centrality


## Communities

$\mathrm{Q}=0.198$
Group 1
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR WOOD AND PRODUCTS OF WOOD AND CORK
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
CHEMICALS EXCLUDING PHARMACEUTICALS
RUBBER AND PLASTICS PRODUCTS
MANUFACTURING NEC; RECYCLING
Group 2
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL
ELECTRICITY, GAS AND WATER SUPPLY
Group 3
OTHER NON-METALLIC MINERAL PRODUCTS
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.
CONSTRUCTION
WHOLESALE AND RETAIL TRADE; REPAIRS
TRANSPORT AND STORAGE
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
REAL ESTATE ACTIVITIES
RENTING OF MACHINERY AND EQUIPMENT
COMPUTER AND RELATED ACTIVITIES
OTHER BUSINESS ACTIVITIES
PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY
EDUCATION
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Group 4
IRON \& STEEL
NON-FERROUS METALS
FABRICATED METAL PRODUCTS, except machinery and equipment
MACHINERY AND EQUIPMENT, N.E.C.
ELECTRICAL MACHINERY AND APPARATUS, NEC
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS
BUILDING AND REPAIRING OF SHIPS AND BOATS
AIRCRAFT AND SPACECRAFT
RESEARCH AND DEVELOPMENT
Group 5
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
PHARMACEUTICALS

MEDICAL, PRECISION AND OPTICAL INSTRUMENTS HOTELS AND RESTAURANTS
HEALTH AND SOCIAL WORK
Ungrouped Sectors
PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

## C. 6 Germany

| Flow Mode |  |
| :--- | :--- |
| Boundary | $51.2 \%$ |
| First-passage | $36.6 \%$ |
| Cycled | $12.2 \%$ |

Density of non-zero links $=0.767$
Boundary flow $(G D P)=3.258 \mathrm{e}+12$ deutchemarks
Total system throughflow $=6.357 \mathrm{e}+12$ deutchemarks

## Link Weights




## Sector Strengths




Clustering


Centrality


## Communities

$\mathrm{Q}=0.253$
Group 1
OFFICE, ACCOUNTING AND COMPUTING MACHINERY COMPUTER AND RELATED ACTIVITIES
RESEARCH AND DEVELOPMENT
EDUCATION
Group 2
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
WHOLESALE AND RETAIL TRADE; REPAIRS
TRANSPORT AND STORAGE
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
REAL ESTATE ACTIVITIES
RENTING OF MACHINERY AND EQUIPMENT
OTHER BUSINESS ACTIVITIES
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Group 3
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR WOOD AND PRODUCTS OF WOOD AND CORK CHEMICALS EXCLUDING PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS OTHER NON-METALLIC MINERAL PRODUCTS IRON \& STEEL
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
ELECTRICAL MACHINERY AND APPARATUS, NEC
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS
BUILDING AND REPAIRING OF SHIPS AND BOATS
MANUFACTURING NEC; RECYCLING
CONSTRUCTION
Group 4
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
HOTELS AND RESTAURANTS
Group 5
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY

Group 6
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY

HEALTH AND SOCIAL WORK

Ungrouped Sectors
PHARMACEUTICALS
NON-FERROUS METALS
AIRCRAFT AND SPACECRAFT
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.
PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

## C. 7 Denmark

| Flow Mode |  |
| :--- | :--- |
| Boundary | $56.5 \%$ |
| First-passage | $34.4 \%$ |
| Cycled | $9.16 \%$ |

Density of non-zero links $=0.866$
Boundary flow $(G D P)=1.033 \mathrm{e}+12$ krones
Total system throughflow $=1.829 \mathrm{e}+12$ krones

## Link Weights




## Sector Strengths




## Clustering



Centrality


## Communities

$\mathrm{Q}=0.238$
Group 1
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
REAL ESTATE ACTIVITIES
RENTING OF MACHINERY AND EQUIPMENT
COMPUTER AND RELATED ACTIVITIES
RESEARCH AND DEVELOPMENT
OTHER BUSINESS ACTIVITIES
PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY
EDUCATION
HEALTH AND SOCIAL WORK
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Group 2
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
CHEMICALS EXCLUDING PHARMACEUTICALS
PHARMACEUTICALS
HOTELS AND RESTAURANTS

## Group 3

MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY

Group 4
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS
WHOLESALE AND RETAIL TRADE; REPAIRS
TRANSPORT AND STORAGE
Group 5
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR
WOOD AND PRODUCTS OF WOOD AND CORK
RUBBER AND PLASTICS PRODUCTS
OTHER NON-METALLIC MINERAL PRODUCTS
IRON \& STEEL
NON-FERROUS METALS
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
ELECTRICAL MACHINERY AND APPARATUS, NEC
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
BUILDING AND REPAIRING OF SHIPS AND BOATS

MANUFACTURING NEC; RECYCLING CONSTRUCTION

Ungrouped Sectors
AIRCRAFT AND SPACECRAFT
PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

## C. 8 Spain

| Flow Mode |  |  | Density of non-zero links $=0.869$ |
| :--- | :--- | :--- | :--- |
| Boundary | $51.3 \%$ |  | Boundary flow $($ GDP $)=6.815 \mathrm{e}+13$ pesetas |
| First-passage | $36.1 \%$ |  | Total system throughflow $=1.329 \mathrm{e}+14$ pesetas |
| Cycled | $12.6 \%$ |  |  |$\quad$| Clo |
| :--- |

## Link Weights




Sector Strengths


Clustering


Centrality


## Communities

$\mathrm{Q}=0.265$
Group 1
WOOD AND PRODUCTS OF WOOD AND CORK
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
OTHER NON-METALLIC MINERAL PRODUCTS
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
MANUFACTURING NEC; RECYCLING
CONSTRUCTION
WHOLESALE AND RETAIL TRADE; REPAIRS
TRANSPORT AND STORAGE
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
REAL ESTATE ACTIVITIES
RENTING OF MACHINERY AND EQUIPMENT
COMPUTER AND RELATED ACTIVITIES
OTHER BUSINESS ACTIVITIES
PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY
EDUCATION
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Group 2
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
HOTELS AND RESTAURANTS

## Group 3

TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR CHEMICALS EXCLUDING PHARMACEUTICALS
RUBBER AND PLASTICS PRODUCTS
IRON \& STEEL
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
ELECTRICAL MACHINERY AND APPARATUS, NEC
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS
BUILDING AND REPAIRING OF SHIPS AND BOATS
AIRCRAFT AND SPACECRAFT
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.
RESEARCH AND DEVELOPMENT

## Group 4

PHARMACEUTICALS
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
HEALTH AND SOCIAL WORK
Group 5
MINING AND QUARRYING

COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY

Ungrouped Sectors
NON-FERROUS METALS
PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

## C. 9 Finland

| Flow Mode |  |
| :--- | :--- |
| Boundary | $48.6 \%$ |
| First-passage | $35.8 \%$ |
| Cycled | $15.5 \%$ |

Density of non-zero links $=0.712$
Boundary flow $(G D P)=5.066 \mathrm{e}+11$ markkas
Total system throughflow $=1.042 \mathrm{e}+12$ markkas Cycled

## Link Weights




## Sector Strengths




Clustering


Centrality


## Communities

$\mathrm{Q}=0.189$
Group 1
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR
WOOD AND PRODUCTS OF WOOD AND CORK
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
CHEMICALS EXCLUDING PHARMACEUTICALS
RUBBER AND PLASTICS PRODUCTS
ELECTRICITY, GAS AND WATER SUPPLY
HOTELS AND RESTAURANTS
EDUCATION
HEALTH AND SOCIAL WORK
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Group 2
OTHER NON-METALLIC MINERAL PRODUCTS
CONSTRUCTION
WHOLESALE AND RETAIL TRADE; REPAIRS
TRANSPORT AND STORAGE
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
REAL ESTATE ACTIVITIES

## Group 3

IRON \& STEEL
NON-FERROUS METALS
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
ELECTRICAL MACHINERY AND APPARATUS, NEC
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS
BUILDING AND REPAIRING OF SHIPS AND BOATS
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.
MANUFACTURING NEC; RECYCLING
OTHER BUSINESS ACTIVITIES
PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY
Group 4
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL

RENTING OF MACHINERY AND EQUIPMENT COMPUTER AND RELATED ACTIVITIES
RESEARCH AND DEVELOPMENT
PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

## C. 10 France

| Flow Mode |  |
| :--- | :--- |
| Boundary | $55.2 \%$ |
| First-passage | $31.2 \%$ |
| Cycled | $13.6 \%$ |

Density of non-zero links $=0.724$
Boundary flow $(G D P)=7.752 \mathrm{e}+12$ French francs
Total system throughflow $=1.406 \mathrm{e}+13$ French francs Cycled

## Link Weights




## Sector Strengths



Clustering



Centrality


## Communities

$\mathrm{Q}=0.155$
Group 1
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL OTHER NON-METALLIC MINERAL PRODUCTS
IRON \& STEEL
NON-FERROUS METALS
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
ELECTRICAL MACHINERY AND APPARATUS, NEC
ELECTRICITY, GAS AND WATER SUPPLY
CONSTRUCTION
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
REAL ESTATE ACTIVITIES
RENTING OF MACHINERY AND EQUIPMENT
COMPUTER AND RELATED ACTIVITIES
OTHER BUSINESS ACTIVITIES
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Group 2
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR PHARMACEUTICALS
MANUFACTURING NEC; RECYCLING
HEALTH AND SOCIAL WORK
Group 3
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
WOOD AND PRODUCTS OF WOOD AND CORK
HOTELS AND RESTAURANTS
Group 4
CHEMICALS EXCLUDING PHARMACEUTICALS
RUBBER AND PLASTICS PRODUCTS
Group 5
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
BUILDING AND REPAIRING OF SHIPS AND BOATS
AIRCRAFT AND SPACECRAFT
RESEARCH AND DEVELOPMENT
Group 6
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS

RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.
WHOLESALE AND RETAIL TRADE; REPAIRS
TRANSPORT AND STORAGE
EDUCATION

Ungrouped Sectors
PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITYPRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

## C. 11 Greece

| Flow Mode |  |
| :--- | :--- |
| Boundary | $59.6 \%$ |
| First-passage | $32.7 \%$ |
| Cycled | $7.75 \%$ |

Density of non-zero links $=0.716$
Boundary flow $(G D P)=2.204 \mathrm{e}+13$ drachmas
Total system throughflow $=3.699 \mathrm{e}+13$ drachmas Cycled

## Link Weights




## Sector Strengths



Clustering



Centrality


## Communities

$\mathrm{Q}=0.335$
Group 1
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY

Group 2
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR
HOTELS AND RESTAURANTS
Group 3
WOOD AND PRODUCTS OF WOOD AND CORK
OTHER NON-METALLIC MINERAL PRODUCTS
IRON \& STEEL
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
ELECTRICAL MACHINERY AND APPARATUS, NEC
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
BUILDING AND REPAIRING OF SHIPS AND BOATS
MANUFACTURING NEC; RECYCLING
CONSTRUCTION
RESEARCH AND DEVELOPMENT
PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY
Group 4
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
CHEMICALS EXCLUDING PHARMACEUTICALS
RUBBER AND PLASTICS PRODUCTS
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS
WHOLESALE AND RETAIL TRADE; REPAIRS
TRANSPORT AND STORAGE
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
REAL ESTATE ACTIVITIES
RENTING OF MACHINERY AND EQUIPMENT
COMPUTER AND RELATED ACTIVITIES
OTHER BUSINESS ACTIVITIES
EDUCATION
HEALTH AND SOCIAL WORK
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Ungrouped Sectors
PHARMACEUTICALS

NON-FERROUS METALS
AIRCRAFT AND SPACECRAFT
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.
PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

## C. 12 Hungary

| Flow Mode |  |  |
| :--- | :--- | :--- |
| Boundary | $47.6 \%$ |  |
| Density of non-zero links $=0.771$ |  |  |
| First-passage | $38.1 \%$ |  |
| Coundary flow (GDP) $=9.515 \mathrm{e}+12$ forints |  |  |
| Cotal system throughflow $=2 \mathrm{e}+13$ forints |  |  |

## Link Weights




## Sector Strengths




## Clustering



Centrality


## Communities

$\mathrm{Q}=0.252$
Group 1
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
HOTELS AND RESTAURANTS
Group 2
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY

Group 3
WOOD AND PRODUCTS OF WOOD AND CORK
OTHER NON-METALLIC MINERAL PRODUCTS MANUFACTURING NEC; RECYCLING CONSTRUCTION

Group 4
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.
WHOLESALE AND RETAIL TRADE; REPAIRS
TRANSPORT AND STORAGE
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
REAL ESTATE ACTIVITIES
RENTING OF MACHINERY AND EQUIPMENT
COMPUTER AND RELATED ACTIVITIES
OTHER BUSINESS ACTIVITIES
PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY
EDUCATION
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Group 5
IRON \& STEEL
FABRICATED METAL PRODUCTS, except machinery and equipment
MACHINERY AND EQUIPMENT, N.E.C.
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
ELECTRICAL MACHINERY AND APPARATUS, NEC
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS
Group 6
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR CHEMICALS EXCLUDING PHARMACEUTICALS
RUBBER AND PLASTICS PRODUCTS
RESEARCH AND DEVELOPMENT

HEALTH AND SOCIAL WORK

Ungrouped Sectors
PHARMACEUTICALS
NON-FERROUS METALS
BUILDING AND REPAIRING OF SHIPS AND BOATS
AIRCRAFT AND SPACECRAFT
PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

## C. 13 Italy

| Flow Mode |  |
| :--- | :--- |
| Boundary | $52.4 \%$ |
| First-passage | $34.9 \%$ |
| Cycled | $12.8 \%$ |

Density of non-zero links $=0.714$
Boundary flow $(G D P)=1.507 \mathrm{e}+15$ lira
Total system throughflow $=2.878 \mathrm{e}+15$ lira

## Link Weights




## Sector Strengths




## Clustering



Centrality


## Communities

$\mathrm{Q}=0.259$
Group 1
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY

Group 2
PHARMACEUTICALS
HEALTH AND SOCIAL WORK
Group 3
WOOD AND PRODUCTS OF WOOD AND CORK
OTHER NON-METALLIC MINERAL PRODUCTS
IRON \& STEEL
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
ELECTRICAL MACHINERY AND APPARATUS, NEC
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS
BUILDING AND REPAIRING OF SHIPS AND BOATS
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.
MANUFACTURING NEC; RECYCLING
CONSTRUCTION
Group 4
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
AIRCRAFT AND SPACECRAFT
WHOLESALE AND RETAIL TRADE; REPAIRS
TRANSPORT AND STORAGE
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
REAL ESTATE ACTIVITIES
COMPUTER AND RELATED ACTIVITIES
RESEARCH AND DEVELOPMENT
OTHER BUSINESS ACTIVITIES
PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Group 5
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
HOTELS AND RESTAURANTS
Group 6
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR

CHEMICALS EXCLUDING PHARMACEUTICALS
RUBBER AND PLASTICS PRODUCTS

Ungrouped Sectors
NON-FERROUS METALS
RENTING OF MACHINERY AND EQUIPMENT
EDUCATION
PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

## C. 14 Japan

| Flow Mode |  |
| :--- | :--- |
| Boundary | $54.5 \%$ |
| First-passage | $34.8 \%$ |
| Cycled | $10.8 \%$ |

Density of non-zero links $=0.798$
Boundary flow $(G D P)=5.052 \mathrm{e}+14$ yen
Total system throughflow $=9.279 \mathrm{e}+14$ yen

## Link Weights




## Sector Strengths




Clustering


## Centrality



## Communities

$\mathrm{Q}=0.228$
Group 1
WOOD AND PRODUCTS OF WOOD AND CORK OTHER NON-METALLIC MINERAL PRODUCTS IRON \& STEEL
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
BUILDING AND REPAIRING OF SHIPS AND BOATS
CONSTRUCTION

Group 2
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR CHEMICALS EXCLUDING PHARMACEUTICALS
RUBBER AND PLASTICS PRODUCTS
MANUFACTURING NEC; RECYCLING

Group 3
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
WHOLESALE AND RETAIL TRADE; REPAIRS
TRANSPORT AND STORAGE
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
REAL ESTATE ACTIVITIES
OTHER BUSINESS ACTIVITIES
EDUCATION
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

Group 4
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
PHARMACEUTICALS
HEALTH AND SOCIAL WORK

Group 5
AIRCRAFT AND SPACECRAFT
RENTING OF MACHINERY AND EQUIPMENT
COMPUTER AND RELATED ACTIVITIES
PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY

Group 6
NON-FERROUS METALS
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
ELECTRICAL MACHINERY AND APPARATUS, NEC
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS

RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. RESEARCH AND DEVELOPMENT

Group 7
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL
ELECTRICITY, GAS AND WATER SUPPLY
Ungrouped Sectors
HOTELS AND RESTAURANTS

## C. 15 Korea

| Flow Mode |  |  |  |
| :--- | ---: | :--- | :--- |
| Boundary | $44.7 \%$ |  | Density of non-zero links $=0.826$ |
| First-passage | $40 \%$ |  | Bondary flow (GDP) $=3.758 \mathrm{e}+14$ won |
| Cycled | $15.3 \%$ |  |  |$\quad$| Total system throughflow $=8.415 \mathrm{e}+14$ won |
| :--- |

## Link Weights




## Sector Strengths




Clustering


Centrality


## Communities

$\mathrm{Q}=0.287$
Group 1
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
WOOD AND PRODUCTS OF WOOD AND CORK
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
PHARMACEUTICALS
MANUFACTURING NEC; RECYCLING
HOTELS AND RESTAURANTS
EDUCATION
HEALTH AND SOCIAL WORK
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Group 2
WHOLESALE AND RETAIL TRADE; REPAIRS
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
REAL ESTATE ACTIVITIES
OTHER BUSINESS ACTIVITIES
PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

Group 3
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL AIRCRAFT AND SPACECRAFT
ELECTRICITY, GAS AND WATER SUPPLY
TRANSPORT AND STORAGE
RENTING OF MACHINERY AND EQUIPMENT
Group 4
NON-FERROUS METALS
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
ELECTRICAL MACHINERY AND APPARATUS, NEC
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
RESEARCH AND DEVELOPMENT
Group 5
OTHER NON-METALLIC MINERAL PRODUCTS
IRON \& STEEL
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS
BUILDING AND REPAIRING OF SHIPS AND BOATS
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.
CONSTRUCTION

Group 6
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR CHEMICALS EXCLUDING PHARMACEUTICALS
RUBBER AND PLASTICS PRODUCTS

Ungrouped Sectors
COMPUTER AND RELATED ACTIVITIESPUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY

## C. 16 Netherlands

| Flow Mode |  |  |
| :--- | :--- | :--- |
| Boundary | $50.8 \%$ |  |
| First-passage | $34.8 \%$ |  |
| Coundary flow (GDP) $=6.181 \mathrm{e}+14$ guilder |  |  |
| Cycled | $14.4 \%$ |  |$\quad$| Total system throughflow $=1.217 \mathrm{e}+15$ guilder |
| :--- |

## Link Weights




## Sector Strengths




## Clustering



Centrality


## Communities

$\mathrm{Q}=0.232$
Group 1
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
HOTELS AND RESTAURANTS
Group 2
WOOD AND PRODUCTS OF WOOD AND CORK
RUBBER AND PLASTICS PRODUCTS
OTHER NON-METALLIC MINERAL PRODUCTS
IRON \& STEEL
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
ELECTRICAL MACHINERY AND APPARATUS, NEC
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS
BUILDING AND REPAIRING OF SHIPS AND BOATS
AIRCRAFT AND SPACECRAFT
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.
MANUFACTURING NEC; RECYCLING
CONSTRUCTION
REAL ESTATE ACTIVITIES
RESEARCH AND DEVELOPMENT
PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY
EDUCATION
Group 3
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
WHOLESALE AND RETAIL TRADE; REPAIRS
TRANSPORT AND STORAGE
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
RENTING OF MACHINERY AND EQUIPMENT
COMPUTER AND RELATED ACTIVITIES
OTHER BUSINESS ACTIVITIES
HEALTH AND SOCIAL WORK
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Group 4
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL CHEMICALS EXCLUDING PHARMACEUTICALS
ELECTRICITY, GAS AND WATER SUPPLY

Ungrouped Sectors
PHARMACEUTICALS
NON-FERROUS METALS
PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

## C. 17 Norway

| Flow Mode |  |
| :--- | ---: |
| Boundary | $54 \%$ |
| First-passage | $34.8 \%$ |
| Cycled | $11.2 \%$ |

Density of non-zero links $=0.951$
Boundary flow $(G D P)=1.021 \mathrm{e}+12$ krones
Total system throughflow $=1.891 \mathrm{e}+12$ krones

## Link Weights




## Sector Strengths




## Clustering



Centrality


## Communities

$\mathrm{Q}=0.218$
Group 1
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR PHARMACEUTICALS
HOTELS AND RESTAURANTS
HEALTH AND SOCIAL WORK
Group 2
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
COMPUTER AND RELATED ACTIVITIES
RESEARCH AND DEVELOPMENT
OTHER BUSINESS ACTIVITIES
EDUCATION
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Group 3
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL IRON \& STEEL
NON-FERROUS METALS
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
ELECTRICAL MACHINERY AND APPARATUS, NEC
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
BUILDING AND REPAIRING OF SHIPS AND BOATS
RENTING OF MACHINERY AND EQUIPMENT
Group 4
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS
AIRCRAFT AND SPACECRAFT
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.
ELECTRICITY, GAS AND WATER SUPPLY
WHOLESALE AND RETAIL TRADE; REPAIRS
TRANSPORT AND STORAGE
Group 5
WOOD AND PRODUCTS OF WOOD AND CORK CHEMICALS EXCLUDING PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS
OTHER NON-METALLIC MINERAL PRODUCTS MANUFACTURING NEC; RECYCLING

CONSTRUCTION
REAL ESTATE ACTIVITIES
PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY

Ungrouped Sectors
PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

## C. 18 Poland

| Flow Mode |  |  |
| :--- | :--- | :--- |
| Boundary | $45.2 \%$ |  |
| Density of non-zero links $=0.748$ |  |  |
| First-passage | $37.9 \%$ |  |
| Cycled | $16.9 \%$ |  |$\quad$ Total system throughflow $=6.267 \mathrm{e}+11$ zloty

## Link Weights




## Sector Strengths




## Clustering



Centrality


## Communities

$\mathrm{Q}=0.175$
Group 1
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR WOOD AND PRODUCTS OF WOOD AND CORK
MANUFACTURING NEC; RECYCLING
HOTELS AND RESTAURANTS
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
HEALTH AND SOCIAL WORK
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Group 2
CHEMICALS EXCLUDING PHARMACEUTICALS
RUBBER AND PLASTICS PRODUCTS
OTHER NON-METALLIC MINERAL PRODUCTS
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
CONSTRUCTION
REAL ESTATE ACTIVITIES
RENTING OF MACHINERY AND EQUIPMENT
COMPUTER AND RELATED ACTIVITIES
OTHER BUSINESS ACTIVITIES
EDUCATION
Group 3
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL IRON \& STEEL
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
ELECTRICAL MACHINERY AND APPARATUS, NEC
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS
BUILDING AND REPAIRING OF SHIPS AND BOATS
ELECTRICITY, GAS AND WATER SUPPLY
TRANSPORT AND STORAGE
RESEARCH AND DEVELOPMENT
Group 4
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
WHOLESALE AND RETAIL TRADE; REPAIRS

AIRCRAFT AND SPACECRAFT RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

## C. 19 United Kingdom

| Flow Mode |  |
| :--- | :--- |
| Boundary | $50.2 \%$ |
| First-passage | $34.3 \%$ |
| Cycled | $15.5 \%$ |

Density of non-zero links $=0.904$
Boundary flow $(G D P)=8.517 \mathrm{e}+11$ pounds sterling
Total system throughflow $=1.698 \mathrm{e}+12$ pounds sterling

## Link Weights




## Sector Strengths




## Clustering



Centrality


## Communities

$\mathrm{Q}=0.186$
Group 1
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING
WHOLESALE AND RETAIL TRADE; REPAIRS
HOTELS AND RESTAURANTS
TRANSPORT AND STORAGE
POST AND TELECOMMUNICATIONS
FINANCE, INSURANCE
REAL ESTATE ACTIVITIES
RENTING OF MACHINERY AND EQUIPMENT
COMPUTER AND RELATED ACTIVITIES
RESEARCH AND DEVELOPMENT
OTHER BUSINESS ACTIVITIES
EDUCATION
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Group 2
WOOD AND PRODUCTS OF WOOD AND CORK
RUBBER AND PLASTICS PRODUCTS
OTHER NON-METALLIC MINERAL PRODUCTS
IRON \& STEEL
NON-FERROUS METALS
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
ELECTRICAL MACHINERY AND APPARATUS, NEC
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS
BUILDING AND REPAIRING OF SHIPS AND BOATS
AIRCRAFT AND SPACECRAFT
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.
MANUFACTURING NEC; RECYCLING
CONSTRUCTION
PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY
Group 3
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
PHARMACEUTICALS
HEALTH AND SOCIAL WORK
Group 4
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL
CHEMICALS EXCLUDING PHARMACEUTICALS

ELECTRICITY, GAS AND WATER SUPPLY
Ungrouped Sectors
PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

## C. 20 United States

| Flow Mode |  |
| :--- | :--- |
| Boundary | $57 \%$ |
| First-passage | $32 \%$ |
| Cycled | $11 \%$ |

Density of non-zero links $=0.868$
Boundary flow $($ GDP $)=8.385 \mathrm{e}+12$ US dollars
Total system throughflow $=1.47 \mathrm{e}+13$ US dollars

## Link Weights




## Sector Strengths




Clustering


Centrality


## Communities

$\mathrm{Q}=0.194$
Group 1
OTHER NON-METALLIC MINERAL PRODUCTS
IRON \& STEEL
NON-FERROUS METALS
FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C.
ELECTRICAL MACHINERY AND APPARATUS, NEC
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS
BUILDING AND REPAIRING OF SHIPS AND BOATS
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.
CONSTRUCTION
State and local government passenger transit
Other State and local government enterprises
Group 2
WOOD AND PRODUCTS OF WOOD AND CORK
MANUFACTURING NEC; RECYCLING
Group 3
AGRICULTURE, HUNTING, FORESTRY AND FISHING
FOOD PRODUCTS, BEVERAGES AND TOBACCO
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING CHEMICALS EXCLUDING PHARMACEUTICALS PHARMACEUTICALS
RUBBER AND PLASTICS PRODUCTS
HOTELS AND RESTAURANTS
HEALTH AND SOCIAL WORK
Group 4
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS
AIRCRAFT AND SPACECRAFT

Group 5
MINING AND QUARRYING
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY
TRANSPORT AND STORAGE
Federal electric utilities
State and local government electric utilities
Group 6
OFFICE, ACCOUNTING AND COMPUTING MACHINERY
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT
WHOLESALE AND RETAIL TRADE; REPAIRS
POST AND TELECOMMUNICATIONS

FINANCE, INSURANCE
REAL ESTATE ACTIVITIES
RENTING OF MACHINERY AND EQUIPMENT
COMPUTER AND RELATED ACTIVITIES
RESEARCH AND DEVELOPMENT
OTHER BUSINESS ACTIVITIES
EDUCATION
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES
Other Federal Government enterprises

## Ungrouped Sectors

PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY
PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS
SBFD including OTHER ADJUSTMENTS


Figure 1: The normalized flow size distributions of 20 OECD countries.

Table 1: Clustering Patterns for Directed Networks
Pattern Link Configurations



Figure 2: The distribution of flow sizes for the US 1997 OECD table.

| Description | Vector/Matrix | Elements |
| :--- | :--- | :--- |
| Internal flows matrix | $F$ | $f_{i j}$ |
| Inputs vector | $\vec{z}$ | $z_{i}$ |
| Outputs vector | $\vec{y}$ | $y_{i}$ |
| Throughflows vector | $\vec{T}=F^{T} \overrightarrow{1}+\vec{z}=F \overrightarrow{1}+\vec{y}$ | $T_{i}=F_{\cdot i}+z_{i}=F_{i \cdot}+y_{i}$ |
| $\hat{T}^{-1}$ | $g_{i j}=\frac{f_{i j}}{T_{j}}$ |  |
| Flow fractions matrix | $G \equiv \hat{T}^{-1} F$ | $g_{i j}=\frac{f_{i j}}{T_{i}}$ |
| Total requirements matrix | $N \equiv\left(I-G^{T}\right)^{-1}$ | $n_{i j}$ |

Table 2: Notation for the decomposition of flows through an IO network. $\overrightarrow{1}$ indicates a vector of ones. A hat ôver a vector's label indicates a matrix whose diagonal elements are the elements of the vector, with zeros elsewhere.


Figure 3: The scatterplot of size of a flow versus the size of the reciprocating flow, $f_{i j}$ versus $f_{j i}$.


Figure 4: The distribution of in-strength and out-strength for 20 OECD countries.


Figure 5: The distribution of in-strength and out-strength for the US 1997 OECD table.


Figure 6: The change in the strength-distribution resulting from shuffling the flows of the IO table. The two distributions with the lower slope are the original distributions, while the pair with the higher slope are the distributions after shuffling.


Figure 7: Scatterplot of in-strength versus out-strength for each sector for the US 1997 OECD table.


Figure 8: The cumulative distribution of clustering coefficients for the US 1997 OECD table.


Figure 9: The cumulative distribution of centrality for the US 1997 OECD table.


Figure 10: The US 1997 OECD table. The numerical values for the entries of the table have been mapped to colors, with red cells representing strong flows and blue cells representing weak flows. The color key at the right gives the mapping of colors to the logarithm of flows in base 10 , thus covering the range $10^{6}$ to $10^{11}$ dollars. Black lines indicate boundaries between groups (see text.)

| Country | \% Boundary | \% First Passage | \% cycled |
| :--- | :---: | :---: | :---: |
| Australia | 50.3 | 37.5 | 12.2 |
| Brazil | 55.1 | 32.9 | 12.0 |
| Canada | 53.5 | 34.5 | 11.9 |
| China | 15.6 | 63.2 | 21.2 |
| Czech Republic | 36.4 | 42.1 | 21.5 |
| Germany | 51.2 | 36.6 | 12.2 |
| Denmark | 56.5 | 34.4 | 9.2 |
| Spain | 51.3 | 36.1 | 12.6 |
| Finland | 48.6 | 35.8 | 15.5 |
| France | 55.2 | 31.2 | 13.6 |
| Greece | 59.6 | 32.7 | 7.8 |
| Hungary | 47.6 | 38.1 | 14.4 |
| Italy | 52.4 | 34.9 | 12.8 |
| Japan | 54.5 | 34.8 | 10.8 |
| Korea | 44.7 | 40.0 | 15.3 |
| Netherlands | 50.8 | 34.8 | 14.4 |
| Norway | 54.0 | 34.8 | 11.2 |
| Poland | 45.2 | 37.9 | 16.9 |
| United Kingdom | 50.2 | 34.3 | 15.5 |
| United States | 57.0 | 32.0 | 11.0 |
| Average | 49.5 | 36.9 | 13.6 |
| Average exc. China, Czech | 52.1 | 35.2 | 12.7 |

Table 3: The percent contribution of each flow mode to the total flow through the IO network for each OECD country. See text for a description of the flow modes.


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    January 2009

[^1]:    ${ }^{1} \mathrm{~A}$ sector is an area of economic activity; e.g. primary metals production.
    ${ }^{2}$ One could record the information in an IO network in different ways; for example, one could treat the boundary flows as coming from/going to other nodes, thus treating these flows on equal terms with the other flows in the network. Here we follow the notation used in ecological IO analysis.

[^2]:    ${ }^{3}$ One may also consider the more general case that the flow in does not equal the flow out, and that a changing stock $x(t)$ exists at each node. Then the above relation becomes

    $$
    z_{i}+F_{\cdot i}=F_{i \cdot}+y_{i}+\dot{x}_{i}
    $$

[^3]:    ${ }^{4}$ The correlation coefficient of the off-diagonal elements . 25 . The correlation may appear higher, but note the logarithmic scales. Also, since the diagonal elements of any matrix are automatically correlated, they bias the correlation upward while contributing nothing to the degree of symmetry of the network.

[^4]:    ${ }^{5}$ This is not the same as how much the sector provides/uses in total for the economy; such a measure would also include output for final consumption/input from labor. The in-strength and out-strength as defined here are sums over intermediate consumption only.

[^5]:    ${ }^{6}$ For example, a rule known as "preferential attachment" posits a network that is growing, with new nodes attaching to old ones with a probability proportional to the current degree of the old node. Thus, nodes with many links tend to accumulate more links faster than nodes with few links. This model produces a power-law degree distribution, suggesting that a preferential attachment mechanism could be at work in some real world networks.

