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Network Properties of Economic Input-Output Networks

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Interim Report

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Network Properties of Economic Input-Output Networks

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Preface

Current DYN research focuses on methodological development and application within three broad areas: 1) environmental modelling and assessment (NEA), 2) economic modelling (ECG), and 3) analysis of critical infrastructures (FCI). This interim report presents the work completed by James McNerney during YSSP 2008 which combines aspects of all three of the above areas. The general objective was to investigate the information contained within economic input-output tables using network analysis in order to identify emergent patterns or properties within various economies and over time (the connection to ECG). This project incorporates methods developed in ecological network analysis (the connection to NEA) as well as those from the general network analysis literature including clustering, cycling, centrality, link weights and densities. For example, the analysis informs, among other things, about the amount of currency that cycles between sectors versus that which reaches a sector for the first time. Preliminary results indicate that centrally-planned economies appear to have higher cycling. Lastly, this work is the first step in understanding the intricate connections within economic networks and the vulnerability they are subject to based on sectoral flows (the connection to FCI). For example, the analysis also identifies those sectors that are more closely implicated together in tight "virtuous circles" (e.g., sectors tended to be grouped into four classifications: Energy, Agrochemical, Manufacturing, and soft or service industries). This could lead to further work that analyzes how perturbations to economic structures and flows could affect these patterns and shed light on important questions related to critical infrastructures.

Brian Fath Gerald Silverberg

Abstract

This paper investigates applications of network analysis to national input-output tables. This includes initial steps to become familiar with sources for input-output data and the assumptions that go into their compilation; traditional input-output analysis; ecological input-output metrics; the difficulties in the analysis of weighted, directed graphs; the overall structure of economic input-output networks; and possible bases for comparison of network metrics. Both quantitative and qualitative regularities were observed across the OECD economies. Specifically, flow sizes and industry sizes appear to follow the same distribution for all OECD countries; the overall structure of flows within the network, as characterized by the relative amount of cycled and first-passage flow, followed a similar pattern for most OECD countries; and similar groups of closely connected sectors were found. More work needs to be done to understand these results in depth. Directions for future research are outlined; in particular, exploring (1) the stability of these results to IO data with different levels of detail, (2) community structure within the IO networks, and (3) generative/dynamic models of IO networks.

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Network Properties of Economic Input-Output Networks

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1 Introduction

An economic input-output table is a table showing the transfers of goods between sectors of an economy. Each entry of the table shows the total value of goods going from some sector of the economy to some other sector.¹

An input-output table implies a network. The table itself has the form of a weighted, directed adjacency matrix, with flow of goods going from the sectors of the rows to the sectors of the columns. In addition, there is at least one additional column for the economy's output—goods for final consumption—and at least one additional row for input from labor.

1.1 IO Networks

Input-output networks, whether in economics or in ecology, are weighted, directed networks where the weights represent sizes of flows. In addition to flows between nodes, IO networks have boundary flows both into and out of the network. An IO network can be described by a matrix:

$$T \equiv \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1n} & y_1 \\ f_{21} & f_{22} & \cdots & f_{2n} & y_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ f_{n1} & f_{n2} & \cdots & f_{nn} & y_n \\ z_1 & z_2 & \cdots & z_n & 0 \end{bmatrix} \equiv \begin{bmatrix} F & \vec{y} \\ \vec{z}^T & 0 \end{bmatrix}$$

The f_{ij} represent the internal flows of the network, and the z_i and y_i represent the boundary flows. Each f_{ij} gives the flow from the *i*th node to *j*th node, z_i is the flow into node i, and y_i is the flow out of node i.²

(graphic of simple input-output network)

The flows through each node i of an IO network must balance, so that total flow in is equal to total flow out:

$$z_i + F_{\cdot i} = F_{i\cdot} + y_i. \tag{1}$$

¹A sector is an area of economic activity; e.g. primary metals production.

 $^{^{2}}$ One could record the information in an IO network in different ways; for example, one could treat the boundary flows as coming from/going to other nodes, thus treating these flows on equal terms with the other flows in the network. Here we follow the notation used in ecological IO analysis.

Thus, IO networks are a special kind of weighted, directed network that has boundary flows that must obey the restriction above.³

2 Data & Methods

2.1 Sources

All results presented here are based on data from the Organization for Economic Cooperation and Development (OECD). The OECD compiles IO tables for the OECD countries. The OECD obtains the data from national data agencies and puts them into a consistent partitioning of sectors, making the OECD tables useful for country comparisons. (This attempt at a consistent partitioning usually can not be done perfectly, as described in the next section.) The data is available for download from the web in excel spreadsheets.

One can also obtain IO tables from national data agencies directly. The United States tables are available from the Bureau of Economic Analysis (BEA) going back to 1938. The recent BEA tables exist in 3 levels of detail during benchmark years (years ending in 2 and 7.) This allows one to test the stability of the results with respect to the level of aggregation of sectors. The BEA tables are also available on the web as excel spreadsheets. The UN also publishes IO tables, but I have not studied them yet.

2.2 Data Issues

Conflation of Price and Quantity Changes In theoretical analysis, flow are often considered to have units of quantity, but in empirical work it is only possible to compile tables in units of currency. This means that changes in price and changes in quantity are conflated. An increase in the number of cars sold, for example, may have the same effect on the total size of a flow as an increase in the price of cars. Thus, flow sizes may change via two different mechanisms that cannot be distinguished from one another.

Flow Size Cutoff Most countries only record flows between sectors down to some minimum size. Any flows below this size are recorded as zeros or left blank, giving the false appearance of many links of zero weight and an incomplete graph topology. Since a large total flow may pass through small links, if they are numerous, an important sensitivity test of the results should be the dependence upon the cutoff flow size. Excluded links may represent as much as 30% of all n^2 links, where n is the number of sectors.

Make and Use Tables IO sector classifications are applied in two ways. One is to classify commodities, the other is to classify industries; the same list of labels is used for both. In the context of commodities, the label indicates the type of product made. In the context of industries, it indicates firms whose primary area of activity is in making products of the given commodity label. For ease of discussion, we will call the former *product sectors* and the latter *industry sectors*.

Economies do not fit neatly into a partitioning by sectors because firms often produce commodities belonging to multiple sectors. This also makes it hard to say where a firm

$$z_i + F_{\cdot i} = F_{i\cdot} + y_i + \dot{x}_i$$

³One may also consider the more general case that the flow in does not equal the flow out, and that a changing stock x(t) exists at each node. Then the above relation becomes

obtained its inputs from; i.e. the data lets one say that a firm used a primary metals *product*, but that does not mean it received it from the primary metals *industry*. For some product sectors, production may be spread across multiple industry sectors. Likewise, the output of some industry sectors may be spread across multiple product sectors.

Some agencies that produce IO table publish *make-* and *use-tables* to take these differences into account. The make table is a industry sector-by-commodity sector table, showing the amount of commodity type produced by each industry. It is a heavily diagonal matrix, because most of an industry's production is usually in its "own" area, as it should be. The use table is a commodity sector-by-industry sector table showing the types of commodities used by each industry.

The standard IO table is an industry-by-industry table, but agencies compiling the table cannot obtain data in this format directly. The data is obtained from tax returns, which allow one to say what kind of product was purchased, but not who it was purchased from. Thus, some assumptions are necessary to go from the make and use tables to an industry-by-industry table.

Defining Sectors Some arbitrary choices are necessary to classify either commodities or industries into sectors, and there are inevitably problems of definition coming about as a result. A different definition of health services, for example, may cause that industry in one country to appear larger than expected when compared with other countries. The same is true for comparisons across time. Changes in the definition used by compiling agencies may cause artificial discontinuities in the data.

For the OECD tables, which try to put data from different national tables into a consistent partitioning of sectors for all countries, equivalent sectors cannot always be found between the national partitioning and OECD partitioning. Two industries, A and B, may be merged in the national table, but split up in the OECD table. Usually this is resolved by assigning all output to either A or B, and recording the use/production of the other industry as zero. Such mergers lead to "artificially" large sectors.

2.3 Software

All analysis was done in Matlab. The code can be made available to IIASA. The algorithm used for finding communities was based on Mark Newman and Albert Leicht's spectral bisection algorithm for directed networks [1] (see appendix) and generalized for weighted networks. To perform future information-based community analysis, C++ code was obtained from Martin Rosvall and C. Bergstrom [2]. Code used to perform the ecological analysis was obtained from Brian Fath.

3 Network Properties

Most figures in this section are based on the US 1997 input-output tables. The US tables are used as examples to demonstrate points made in the text; the same figures for other OECD countries are available in Appendix C.

3.1 Topology

Economic input-output networks are weighted, directed graphs. Self-links, representing input of an industry to itself, are allowed. In general, a weighted graph can be approximated by a binary graph if the weights are sufficiently similar in size, and a directed graph can be approximated by an undirected graph if the adjacency matrix is sufficiently symmetric. However, as shown in the next section, neither of these conditions hold; the weights vary across several orders of magnitude, with little symmetry in the weights of links and reciprocating links (see Figure 3.)

At first glance, economic input-output networks are nearly-complete graphs. On the US 1997 table, for example, about 85% of links are nonzero. However, it is best to think of the graph as being complete for several reasons:

- The other 15% appear to be absent because of data limitations; links below a certain cutoff size are unlisted in the tables.
- It is unlikely that zero exchange of goods occured between any two industries.
- It avoids problems with certain metrics. For example, the average nearest-neighbor strength of node i is defined as

$$\langle s_{nn}(i) \rangle = \frac{1}{k_i} \sum_{\langle ij \rangle} w_{ij} s_j,$$

where k_i is the degree of i, w_{ij} the weight of the link joining i and j, and the sum is over neighbors of i. Normally k_i is assumed to count the number of non-zero weights joining i with other nodes. But then the addition of a small link from i that did not exist before would *lower* the average neighbor strength, by increasing the denominator k_i by 1 while leaving the summation almost unchanged.

The cutoff weight of a given IO table is generally apparent from the size of the smallest links present.

3.2 Flow Sizes

Since the topology is trivial, the flows contain all information about the structure of the network. The flow size distributions of 20 OECD countries are shown in Figure 1. To compare distributions from different economies, which obviously vary in size, we show the distribution of the *normalized flow size*, defined to be a flow divided by the sum of all flows in the table:

$$f_{ij}^{norm} \equiv rac{f_{ij}}{\sum_{mn} f_{mn}} = rac{f_{ij}}{w}.$$

The distributions of different countries lie approximately on top of one another, with some deviation at the lower end of the distribution. This suggests that a similar generative mechanism could be at work in all economies.

Roughly speaking, the distribution is somewhere between an exponential and a powerlaw. The distribution for the US 1997 network is shown in Figure 2 with a fit to a stretched exponential $p(w_{ij}) = A \exp(-bw_{ij}^a)$.

The scatterplot of flow sizes versus reciprocating flow sizes is shown in Figure 3. Such a plot shows the degree of symmetry in a weighted, directed network. Economic IO networks are rather asymetric. The size of a flow f_{ij} is only weakly correlated with the size of the reciprocating flow f_{ji} .⁴ Note that the x- and y-axes are both log scales. It is not hard to find points in the cloud of data representing i - j pairs with one flow 100 times larger than the reciprocating flow. Sometimes directed networks are approximated by undirected networks if links and reciprocating links are sufficiently similar in size, but one cannot make that approximation here.

⁴The correlation coefficient of the off-diagonal elements .25. The correlation may appear higher, but note the logarithmic scales. Also, since the diagonal elements of any matrix are automatically correlated, they bias the correlation upward while contributing nothing to the degree of symmetry of the network.

Node *strength* is a generalization of node *degree* for weighted graphs; it is the sum of the weights of the links surrounding a node. In a directed graph, a node i has both an "in-strength" and an "out-strength," representing the sum of the weights of links to i and the sum of the weights of links from i:

$$s_i^{in} \equiv \sum_j f_{ji} = F_{.i}$$
$$s_i^{out} \equiv \sum_j f_{ij} = F_{i.}$$

A sector's strength measures how much an industry either provides to the other sectors in the economy, or how much it uses from the other sectors.⁵ As for the flow sizes, we consider here the distribution of the *normalized in-* and *out-strengths*, defined as the fractional contribution of each sector's strength to the sum,

$$s_i^{norm} \equiv \frac{s_i}{\sum_j s_j} = \frac{s_i}{w},$$

where s_i is either an in-strength or an out-strength. The normalized in-strength and outstrength distributions of 20 OECD countries are shown in Figure 4, and for the US only in Figure 5. The distribution of both in-strength and out-strength appears to be exponential, with approximately the same slope for both, and with the slope approximately the same for all OECD countries.

As a test for how "surprising" this distribution is, the distribution of industry strengths was examined after shuffling the flows of the IO table. Random pairs of elements of the IO table were swapped until the whole table was shuffled, and the strength distributions were recalculated. The idea is to see how "random" the empirical distribution is; if shuffling flows leads to a distribution that is indistinguishable from the empirical one, then the empirical one is easy to obtain in some sense, and therefore not surprising. The result is shown in Figure 6. However, this is an extremely crude basis for comparison; an IO table generated by swapping flows cannot represent an economy. After the swapping, total in-flow will no longer equal total out-flow at each node, so the new IO table violates the basic accounting requirement in equation (1). Another procedure for generating a random IO network that preserves equation (1) is described in Section 4.

The scatterplot of in-strength versus out-strength for each sector is shown in Figure 7. One can see a strong correlation between the two, which is not surprising since each industry must individually obey equation (1), which constrains how different s_i^{in} and s_i^{out} can be. The difference $F_{i.} - F_{.i} = s_i^{out} - s_i^{in} = z_i - y_i$, so the plot tells us about the difference between total revenue \vec{y} and wages and salaries \vec{z} . The industries above the diagonal receive less in final consumption revenue than they pay in wages; the opposite is true for industries below the diagonal.

3.4 Clustering

The clustering coefficient in binary networks is meant to describe how "clumpy" the network is. Social networks, for example, tend to be clumpy because groups of people tend to be mutually acquainted. For an undirected binary network, the clustering coefficient of

⁵This is not the same as how much the sector provides/uses in total for the economy; such a measure would also include output for final consumption/input from labor. The in-strength and out-strength as defined here are sums over intermediate consumption only.

node i is the fraction of i's nearest neighbors thats are connected to each other. This can be written in terms of the elements of the adjacency matrix A as

$$C(i) = \frac{\sum_{jk} a_{ij} a_{jk} a_{ki}}{\sum_{jk} a_{ij} a_{ki}}.$$

The a_{ij} have values of 0 or 1. The denominator counts all combinations of pairs of neighbors of *i*, and has the value $2k_i(k_i - 1)$ where k_i is the degree of *i*. By including an extra a_{jk} term, the numerator counts up only those nearest-neighbor of *i* that are themselves linked, with an extra factor of 2 which cancels that in the denominator.

Clustering captures that chunkiness of the network at the level of triangles of nodes. Two generalizations need to be made to apply the concept to IO networks; one for weighted networks, and one for directed networks. A number of generalizations have been proposed for weighted networks (see [3]) I have chosen the simplest generalization, which simply replaces the binary elements a_{ij} with continuous elements f_{ij} in the definition above. Note that this weighted definition reduces to the original binary definition when the network weights all have the same value.

I have followed Fagiolo's generalization [3] of the clustering coefficient to directed graphs. I am not aware of any others. For a directed network, there are 6 links present among 3 nodes, and one must decide which to include and how. Note that the ordering of the indices now matters. Taking into account each possible ordering of indices leads to 5 different clustering coefficients:

$$C^{cyc}(i) \equiv \frac{\sum_{j,k} F_{ij}F_{jk}F_{ki}}{\sum_{j \neq k} f_{ki}f_{ij}} = \frac{(F^3)_{ii}}{s_i^{in}s_i^{out} - s_i^{\leftrightarrow}}$$

$$C^{mid}(i) \equiv \frac{\sum_{j,k} f_{ij}f_{kj}f_{ki}}{\sum_{j \neq k} f_{ki}f_{ij}} = \frac{(FF^TF)_{ii}}{s_i^{in}s_i^{out} - s_i^{\leftrightarrow}}$$

$$C^{in}(i) \equiv \frac{\sum_{j,k} f_{ji}f_{jk}f_{ki}}{\sum_{j \neq k} f_{ki}f_{ji}} = \frac{(F^TF^2)_{ii}}{(s_i^{in})^2 - s_i^{in}}$$

$$C^{out}(i) \equiv \frac{\sum_{j,k} f_{ij}f_{jk}f_{kk}}{\sum_{j \neq k} f_{ik}f_{ij}} = \frac{(F^2F^T)_{ii}}{(s_i^{out})^2 - s_i^{out}}$$

$$C^{all}(i) \equiv \frac{\sum_{j,k} (f_{ij} + f_{ji})(f_{jk} + f_{kj})(f_{ki} + f_{kk})}{\sum_{j \neq k} (f_{ij} + f_{ji})(f_{ik} + f_{ki})(2)} = \frac{(F + F^T)^3}{2[s_i^{tot}(s_i^{tot} - 1) - 2s_i^{\leftrightarrow}]}$$

Each of these expressions sums over particular linking patterns between the three nodes, as shown in Table 1.

The cumulative distribution of each type of clustering coefficient is shown in Figure 8. The cycle clustering coefficient is consistently lower than all the others. This suggests that economies tends to be acyclic at the small scale of 3-cycles.

3.5 Centrality

Centrality in network analysis is a measure of a node's importance. There are a number of measures of centrality in use. Three of the most commonly used are *betweenness centrality*, *closeness centrality*, and *eigenvector centrality*. The propriety of a given definition depends on the kind of network being considered. For example, betweenness centrality, which is based on counting shortest paths passing through each node, is useful in networks where information is carried across links (e.g. the internet.) It is not an obviously useful notion of centrality in the context of economies.

We have adapted eigenvector centrality to input-output networks. The idea is based on the image of a dollar of currency (or a random dollar of product-embodied "value") making a random walk around the economy. Define a matrix Q whose elements q_{ij} are the probabilities that the dollar moves from sector i to sector j. For a given country and year, the probability can be taken to be

$$q_{ij} = \frac{f_{ij}}{\sum_{j} f_{ij} + y_i} = \frac{f_{ij}}{T_i}$$

for the "out-centrality" of a randomly-walking dollar of value, and

$$q_{ij} = \frac{f_{ji}}{\sum_j f_{ji} + z_i} = \frac{f_{ji}}{T_i}$$

for the "in-centrality" of a randomly-walking dollar of currency.

First consider a closed network. Suppose we start the dollar out at some particular sector, and we allow it to be split up into fractions of a dollar, as would be expected by the transition probabilities. With each step, the dollar would get split up, with probabilities accumulating at all the nodes of the network. After a long time, the probabilities may reach some steady state. These steady state probabilities serve as a measure of centrality; they are a measure of the likelihood of finding the random dollar in each particular sector.

Let \vec{p} be the vector of steady-state probabilities, and Q the matrix of transition probabilities, where each element q_{ij} is the probability of a transition to j starting from i:

$$\vec{p} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \end{bmatrix} \qquad \qquad Q = \begin{bmatrix} q_{11} & q_{12} & \cdots \\ q_{21} & q_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

The probability of being at i at the steady state is a sum over all possible starting points multiplied by the appropriate transition probabilities:

$$p_{1} = p_{1}q_{11} + p_{2}q_{21} + \cdots$$
$$p_{2} = p_{1}q_{12} + p_{2}q_{22} + \cdots$$
$$\vdots$$
$$p_{n} = p_{1}p_{1n} + p_{2}q_{2n} + \cdots$$

This set of equations is equivalent to the matrix equation

$$\vec{p} = Q\vec{p}.$$

Thus, the goal is to find an eigenvector of Q with eigenvalue 1. There are as many eigenvectors of a matrix as the dimension of the matrix. The Perron-Frobenius theorem guarantees that at least one of them will have all real non-negative entries that sum to 1. This is necessary so that the entries can be interpreted as probabilities.

Now consider a network with some final output leaving the network. The transition probabilities of a given row no longer add up to 1, because there is a probability that the dollar leaves the internal network of sectors. We can use the same definition above for this case, and note that the eigenvalue corresponding to the eigenvector of centralities will be less than 1.

$$Q\vec{p} = \lambda \bar{p}$$

This eigenvalue may be interpreted as the probability at each "step" that the dollar has left the network.

3.6 Community Structure

Communities in networks are groups of nodes that share a close relation, usually as it is realized in their pattern of links. Knowing the communities of a network helps one develop a picture of the network. It may lead to the discovery of previously unknown connections between nodes, or confirm connections known to exist. The problem of finding communities is common in network analysis, and a number of methods for doing it have been devised. The method used here was a weighted generalization of the spectral-bisection algorithm for directed graphs, developed by E. Leicht and Mark Newman [1]. I am aware of only one other algorithm that is applicable to directed networks, described in [2]

In the unweighted version, the algorithm works by attempting to maximize *modularity* Q, defined as

 $Q \equiv$ (fraction of links within communities) - (expected fraction of links within communities).

The modularity is a measure of the "statistical surprise" of linkage within some arbitrary group of nodes. For any given partitioning of the network into groups, the modularity can be measured easily. If a particular partitioning yields a high value of Q, it suggests that the partitions correspond to real groups. For weighted networks, I simply replaced the binary values of the adjacency matrix with the corresponding weights.

The number of possible partitionings of even small networks (~ 50 nodes) is huge, so any practical algorithm for finding communities must search only a portion of the space of partitions. The algorithm by Leicht and Newman uses a greedy procedure that accepts changes which increase the modularity. More details are provided in Appendix A.

Results of Spectral Bisection Algorithm There are several things to say about the groups found by the algorithm. They are not the same for all countries, though there is a strong tendency for many of the same industries to appear grouped together in different countries. The groupings do not obey any logical scheme perfectly, but they show a strong tendency towards the following four groups (using the sector names of the OECD tables):

• Energy

Mining and Quarrying Coke, Petroleum, Petroleum Products, and Nuclear Fuel Electricity, Gas, and Water Supply

• Agro-Chemical

Agriculture, Hunting, Forestry, and Fishing Food Products, Beverages, and Tobacco Textiles, Textile Products, Leather, and Footwear Pulp, Paper, Paper Products, Printing, and Publishing Wood and Products of Wood and Cork Chemicals Excluding Pharmaceuticals Pharmaceuticals Rubber and Plastics Products Manufacturing NEC, Recycling

• Manufacturing

Iron and Steel

Non-Ferrous Metals Other Non-Metallic Minerals Fabricated Metal Products Machinery and Equipment NEC Electric Machinery and Apparatus NEC Motor Vehicles, Trailers and Semi-Trailers Building and Repairing of Ships and Boats Railroad Equipment and Trasnport Equipment NEC Construction Aircraft and Spacecraft Medical, Precision, and ptical Instruments Office, Accounting, and Computing Machinery Radio, Television, and Communication Equipment

• Services

Wholesale and Retail Trade, Repairs Post and Telecommunications Transport and Storage Finance, Insurance Real Estate Activities Renting of Machinery and Equipment Computer and Related Activities Research and Development Education Hotels and Restaurants Health and Social Work Other Business Activities Other Community, Social, and Personal Services

No country obeys this scheme perfectly. The U.S. tables perhaps follow it best; other countries somewhat less strictly. Results for each country are given in Appendix C.

Nesting A shortcoming of the spectral bisection algorithm is that it cannot deal easily with nested communities. Suppose some collection of sectors shows a tendency to form strong bonds with one another, suggesting that these sectors form a community. Suppose also there are subgroups of the community that are even more exclusive, with members forming even stronger bonds with one another than with other members of the larger community. If this is the case, the network has a nested community structure, with strong groups inside of weaker groups.

The present algorithm faces the following dilemma in this situation. Suppose there are two large groups, 1 and 2, and group 1 has subgroups 1a and 1b. Since the algorithm can only output one level of groups, it must produce groups

 $1 \text{ and } 2 \quad or \quad 1a, 1b, and 2.$

The former contains no information about the subgroups of group 1, and the latter misleadingly implies that 1a and 1b are as distinct from each other as 1a from 2 or 1b from 2. This theoretical problem appears to occur in the real IO tables. Figure 10 visually depicts an IO table, with colors representing the size of individual links in the table. The more red a cell is, the larger the flow of goods/money across the link. The task of finding groups in the network can be restated as the task of finding a reordering of the rows and columns of this matrix that brings it as close as possible to block-diagonal form. Blocks along the diagonal that contain many red and orange links represent good candidates for communities. The black lines in the matrix show boundaries between groups, and reveal where the groups ought to appear. One can see two groups that are particularly strong. However, the circled boxes show links between members of one group and members of the other, and there is a fairly strong connection between them. This indicates that the two groups may form part of some larger group, which itself stands apart from the rest of the economy, exactly as in the example above.

Information-Based Algorithm Rosvall and Bergstrom describe an algorithm for finding communities that is based on a different concept of communities [2]. Their algorithm is based on the idea of assigning labels to nodes such that a random walk on the network can be recorded in the most efficient way. Suppose each node is given an address, say a string of bits. The goal is to find addresses that allow a random walk of arbitrary length to be recorded with the fewest bits. The authors argue that finding an efficient system of addresses leads to the discovery of communities, as realized by the groups of nodes where the random walker "gets stuck."

Code was obtained from the authors which implements their algorithm, but I have not yet applied it to the IO tables. The algorithm may be adaptable to allow search for nested community structure.

3.7 Global Flow Structure

The total flow through a node is known as its *throughflow*. Specifically, for an economic IO table or an ecological network at steady-state, the total flow into each node i is equal to the total flow out, and either sum equals i's throughflow T_i :

$$T_i = z_i + \sum_j f_{ji} = \sum_j f_{ij} + y_i \,.$$

Total system throughflow is a measure of the total flow through an IO network, defined as the sum of the throughflows through each node:

$$TST \equiv \sum_{i} T_i \,.$$

Flow through the network can be divided into boundary flow, first-passage flow, and cycled flow [5, 6]. These three modes reflect different ways that flow can arrive at some node i. Boundary flow is simply the flow into i from outside the network. After flow from the outside reaches some node, it may be passed on to i before exiting the network; this contribution is known as first-passage flow. Since the IO network may contain cycles, it is also possible that some of the flow through i has passed through it before and was recycled by the network; this contribution is known as cycled flow. The difference between first-passage flow and cycled flow is that first-passage flow is flow that goes through i once and only once before exiting the network.

The mathematical definitions of these three flow modes will make the statements above more precise. First it is useful to define some notation. The description of the input-output network is contained in the matrix F and the boundary vectors \vec{z} and \vec{y} , as described in Section 1.1. We define the matrix G, whose elements g_{ij} are the fraction of i's thoughflow going to j, $g_{ij} = f_{ij}/T_i$. From G we derive N, defined to be $N = (I - G^T)^{-1}$. Table 2 summarizes these definitions.

A useful identity for the following discussion is

$$\vec{T} = N\vec{z}.$$

One way to derive expressions for the contributions coming from each of the three flow modes is to consider the power expansion of N in the expression above:

$$\vec{T} = N\vec{z} = (I - G)^{-1}\vec{z} = (I + G + G^2 + G^3 + \cdots)\vec{z} = \vec{z} + (G + G^2 + G^3 + \cdots)\vec{z}$$

The first term in the expansion is easily identified with the boundary flow. The remaining sum accounts for all subsequent flows inside the network. The matrix G, applied to \vec{z} once, "propagates" the currency at each node one step. Each additional power of G corresponds to an additional step taken by the flows. The remaining sum, $(N-I)\vec{z}$, is the sum of first-passage and cycled flows. The *i*th element of this represents the combined flow through node *i* from first-passage and cycled flows:

$$[(N-I)\vec{z}]_i = \sum_j (N-I)_{ij} z_j$$
$$= \sum_{j \neq i} (N-I)_{ij} z_j + (N-I)_{ii} z_i$$

The ijth term of $(N - I) = G + G^2 + \cdots$ accounts for all flow ultimately going from node i to node j via any number of steps through the network. The second term above therefore yields the cycled flow through i, since $(N - I)_{ii}$ accounts for all flow leaving iand returning to i via any number of steps. The remaining sum yields the first-passage flow. Thus we have decomposed the throughflow through a node i as

$$T_i = z_i + \sum_{j \neq i} (N - I)_{ij} z_j + (N - I)_{ii} z_i$$

and taken the three terms above to define the three flow modes:

$$\begin{split} T_i^{boun} &\equiv z_i \\ T_i^{fp} &\equiv \sum_{j \neq i} (N-I)_{ij} \\ T_i^{cyc} &\equiv (N-I)_{ii} z_i \\ T_i &= T_i^{boun} + T_i^{fp} + T_i^{cyc} \end{split}$$

Summing over all nodes yields the total flow through the network of each mode:

$$\begin{split} TST^{boun} &\equiv \sum_{i} T_{i}^{boun} \\ TST^{fp} &\equiv \sum_{i} T_{i}^{fp} \\ TST^{cyc} &\equiv \sum_{i} T_{i}^{cyc} \\ TST &= TST^{boun} + TST^{fp} + TST^{cyc} \end{split}$$

The final line summarizes the decomposition of total system throughflow into the three flow modes. Rather than look at the absolute size of the terms on the right hand side, it is more convenient to study their fractional contributions to the total. The results in Appendix C are presented this way, although the absolute size of the total system throughflow is also given, as well as the total system boundary flow; the latter is equal to the country's GDP.

The breakdown of total system throughflow into these three modes offers a global characterization of an IO network. Theoretical work needs to be done to connect the contributions from these three modes to more micro-level properties of the network, such as the flow size or node strength distributions. Without this, it is very difficult to interpret results of the decomposition. Results for the OECD countries are shown in Table 3. The contributions from the three modes follow a consistent pattern across almost all countries. Anomalous values were observed for China and the Czech Republic. Excluding these two, a consistent pattern emerged. Boundary flow made up on average half of the network flow, first passage flow about 35%, and cycled flow about 13%. Individual countries showed modest deviation from these averages. The most variation was in the cycled percentage, which varied from 7.8% for Greece to 16.9% for Poland (excluding China and the Czech Republic, which both had very high values of cycling).

4 Generating Artifical IO Networks

Measuring properties of IO networks is the first step in their analysis. The next step will be to explain the measured values and identify features that are surprising or interesting. At the moment, however, there is little to compare the measurements of the previous section to, making the task of identifying interesting features difficult. A null model of IO networks is needed as a basis for comparison with the real world examples.

The model need not generate the features seen in the real network. For example, in the Erdos-Renyi random graph model, each pair of nodes is linked with probability p. For values of p below a critical value known as the percolation threshold, the degree distribution of the graph is binomial. Many real world networks, on the other hand, obey a power-law degree distribution. This indicates that this maximally simple rule—connecting each pair of nodes with a fixed probability—is probably not at work in these real world examples; a more sophisticated mechanism is needed.⁶ Although the Erdos-Renyi model does reproduce the observed property, it serves as a null model that helps reveal what

⁶For example, a rule known as "preferential attachment" posits a network that is growing, with new nodes attaching to old ones with a probability proportional to the current degree of the old node. Thus, nodes with many links tend to accumulate more links faster than nodes with few links. This model produces a power-law degree distribution, suggesting that a preferential attachment mechanism could be at work in some real world networks.

properties of the observed networks are interesting or surprising, aiding the interpretation of measured properties of real networks.

One simple basis of comparison for IO networks comes from shuffling flows; we take the whole list of n^2 flows in the network and reassign them to new i - j pairs at random. Although simple, this procedure does not preserve the input-output accounting of an IO network, $z_i + F_{i} = F_{i} + y_i$, so the result cannot represent a real economy.

One way to satisfy $z_i + F_{i} = F_{i} + y_i$ is to begin with a network in an initial condition that already satisfies this relation, and then take the network through many small changes that always preserve it. For example, consider the following procedure:

- 1. We initialize the network to a state satisfying $z_i + F_{\cdot i} = F_{i\cdot} + y_i$.
- 2. At each time step, we pick two nodes i and j at random with uniform probability. The two nodes may be the same.
- 3. With probability p, we decrease the strength of the link from i to j by a factor r, so that $f_{ij} \rightarrow f'_{ij} = f_{ij} rf_{ij}$. At the same time, we pick a third node k at random to "redirect" the lost flow through, and increase the flow to this third node by the amount lost from i to j: $f_{ik} \rightarrow f'_{ik} = f_{ik} + rf_{ij}$, $f_{kj} \rightarrow f'_{kj} = f_{kj} + rf_{ij}$.
- 4. With probability 1 p, the opposite happens: f_{ij} gets increased by drawing off flow from f_{ik} and f_{ij} . Let $f^* = \min(f_{ik}, f_{kj})$. With probability 1 - p, we have $f_{ij} \to f'_{ij} = f_{ij} + rf^*$, $f_{ik} \to f'_{ik} = f_{ik} - rf^*$, and $f_{kj} \to f'_{kj} = f_{kj} - rf^*$.

Any change will necessarily effect the strength of some connection between two nodes i and j. By weakening the ij link, we leave ourselves with excess output from i (which must go somewhere else) and a deficit of input to j (which must come from somewhere else.) Any change we make that preserves the accounting balance at each node must simply cause that same amount of flow to still go from i to j via some other path. The simplest modification to the ij flow involves a redirection through one other node k.

The final state is not sensitive to the initial condition. Since the empirical IO tables already satisfy (1), one can use any of them as the initial state. An alternative initial state can be obtained by the following procedure:

- 1. Construct the vector $\vec{\delta} = \vec{z} \vec{y}$. Each element of this vector is the net boundary flow into a node, and represents either an excess amount of flow which must be distributed to the rest of the network, or a deficit which must be obtained from the rest of the network. The sum of this vector is zero.
- 2. Sort δ from greatest element to least. The elements at the top of list will have large positive values, while those at the bottom will have large negative values.
- 3. Assign all excess flow from the first element to the second element: $f_{12} = \delta_1$. Node 1 now satisfies (1). Update $\vec{\delta}$ to include the internal flows as they are constructed, so that $\delta_1 \leftarrow 0$, and $\delta_2 \leftarrow \delta_2 + \delta_1$.
- 4. Repeat this for each node until the end of list is reached; i.e. distribute all of the excess flow at node 2, δ_2 , to node 3, so that $f_{23} = \delta_2$, $\delta_2 \leftarrow 0$, $\delta_3 \leftarrow \delta_3 + \delta_2$. Note that node 3 will receive flow accumulated from both nodes 1 and 2 in the list, so that as we move along the list the size of the flow passed along grows from zero at first, and then decreases once we enter the "negative part" of the list, finally reaching zero when we establish the final flow between the n 1th and nth nodes.

In addition to satisfying $z_i + F_{\cdot i} = F_{i\cdot} + y_i$, I believe this procedure generates, for a given \vec{z} and \vec{y} , the IO network with the largest ratio of first-passage flow to cycling flow. Using a \vec{z} , \vec{y} pair from real data, the algorithm reaches the same final state whether the internal flows were initialize using the above procedure, or using the original data in the input-output table.

Any algorithm for generating a random IO network must assume some properties of the generated network *a priori*. For example, in generating binary networks, one may assume a fixed degree sequence (the complete list of degrees for all the nodes of the network.) Given a degree sequence, one can randomly wire nodes together in many ways that satisfy the degree sequence. By randomly wiring nodes together, one can generate a random network from the ensemble of all networks having the same degree sequence. Likewise, we would like to have an algorithm that generates random IO networks from an ensemble with specific criteria. The algorithm above generates IO networks with a given set of input and output vectors, \vec{z} and \vec{y} . This means it confines itself to the ensemble of economies that use the same inputs to produce the same outputs. However, before using networks generated this way as a basis for comparison with real data, one must also know the probabilities associated with each configuration in the ensemble, which are probably not uniform. Determining these will require more analysis and simulation of the model.

5 Conclusions

These conclusions are tentative, contingent upon the sensitivity of these results various factors, which I discuss in the next section.

Possibility of Simple Mechanisms for Economic Organization The flow size and sector strength distributions appear to follow the same shape, at least among the 20 countries studied here. In addition, the breakdown of flows into boundary/first-passage/cycled flows follows a consistent pattern for most of the OECD countries. These results suggest that certain features of economies are universal, holding true regardless of other differences between economies at a finer scale. Furthermore, the simplicity of the results suggests that simple mechanisms may be at work to generate them. This opens up the possibility for parsimonious models for economies that give insight into how they organize themselves, and which would hold for all economies.

Non-Trivial Overall Flow Structure The in-strength and out-strength of a sector serve as one measure of its size, and the centrality a crude indication of where money in the network is most likely to end up. There is a striking difference between the list of largest sectors and the most central sectors, which suggests there is a non-trivial structure to flows that does not necessarily drive money towards the largest sectors.

Appearance of Consistent Communities Certain sectors showed strong tendencies to appear grouped together using the community-finding algorithm described in Section 3.6 and Appendix A. Even more interesting, the sectors tended to partition themselves into 4 main groups: one for energy sectors, one for manufacturing, one for agro-chemical sectors, and one for services.

The latter result is remarkable, in that sectors which appear together show *qualitative* similarities that are not and can not be represented in a table of numbers. It is worth emphasizing that the algorithm does not "know" the sector labels of the columns and rows of the IO table. It does not know, for example, to put "Mining and Quarrying" in

the same group as "Electricity, Gas, and Water Supply" because they are both energyrelated. The algorithm only takes as input the values of the table itself, and searches for communities implied by the pattern of weak and strong elements present in it. Thus, not only is it possible to make a connection between the numerical data of the IO table and economic intuition, but also this shows that quantitative network methods may reveal deep economic structure that is not otherwise apparent.

6 Future Work

Future works falls into 3 categories: checking the sensitivity of the current results, expanding the data examined, and understanding the data at a theoretical level. Goals include:

- Testing the stability of these results to IO data with different levels of detail.
- Testing the sensitivity of these results to the level of the cutoff weight used by the agency compiling the table.
- Using an alternative method of finding communities that can reveal nested groups.
- Performing a more thorough comparison of IO networks across countries.
- Performing a comparison of IO networks across time.
- Studying IO tables where the inputs are physical. (See for example [4].)
- Studying generative and/or dynamical network models of economies, along the lines described in section 4.

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A Finding Communities by Spectral Bisection

I have included the following excerpt from [1] to help introduce the algorithm I used. For more detail, see the original publication. I make one modification to the algorithm presented in [1]; I allow the elements of the adjacency matrix to take on any value, not just binary values. The algorithm I used is applicable to any network—binary/weighted, directed/undirected.

"The premise of the modularity optimization method is that a good division of a network into communities will give high values of the benefit function Q, called the modularity, defined by

Q = (fraction of edges within communities) - (expected fraction of such edges).

Large positive values of the modularity indicate when *statistically surprising* fraction of the edges in a network fall within the chosen communities; it tells us when there are more edges within communities than we would expect on the basis of chance.

"The expected fraction of edges is typically evaluated within the so-called configuration model, a random graph conditioned on the degree sequence of the original network, in which the probability of an edge between two vertices i and j is $k_i k_j/2m$, where k_i is the degree of vertex i and m the total number of edges in the network. The modularity can then be written

$$Q = \frac{1}{2m} \sum_{ij} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \delta_{c_i c_j}$$

where A_{ij} is an element of the adjacency matrix, δ_{ij} is the Kronecker delta symbol, and c_i is the label of the community to which vertex *i* is assigned. Then one maximizes Q over possible divisions of the network into communities, that maximum begin taken as the best estimate of the true communities in the network. Neither the size nor the number of communities need be fixed; both can be varied freely in our attempt to find the maximum.

"In practice, the exhaustive optimization of modularity is comupationaly hard, known to be NP-complete over the set of all graphs of a given size, so practical methods based on modularity optimization make use of approximate schemes such as greedy algorithms, simulated annealing, spectral methods, and others."

Algorithm:

 $\mathbf{B} \equiv \text{matrix with elements} B_{ij}$

$$B_{ij} = w_{ij} - \frac{s_i^{in} s_j^{out}}{m}$$
$$m \equiv \sum_{ij} a_{ij}$$

 $\mathbf{s} \equiv \text{sector whose elements are } s_i$

 $s_i = \pm 1$ (the assignment of a node to one of 2 communities

being considered in the given step)

$$\mathbf{B}_{ij}^{(g)} \equiv B_{ij} - \delta_{ij} \sum_{k \in g} B_{ik}$$

- 1. Calculate the eigenvector corresonding to the largest eigenvalue of $\mathbf{B} + \mathbf{B}^T$.
- 2. Divide the network into 2 communities. Assign each node *i* to one of the two communities based on the signs of the elements of the eigenvector. If the *i*th element is negative, assign $s_i = -1$; if positive, $s_i = +1$.

- 3. "Fine-tune" the assignment by moving individual nodes into the other community and measuring the effect on Q. Accept changes that increase Q until no more moves can be made.
- 4. For subdivision of communities, we maximize *changes* in Q by finding the eigenvector corresonding to the largest eigenvalue of $\mathbf{B}^{(g)} + \mathbf{B}^{(g)T}$ and assigning nodes into communities as before.
- 5. Repeat process of bisection and fine-tuning until no division can increase Q.

B Matlab Programs

The following files are Matlab scripts written to analyze the IO data and produce the results and figures presented here:

Main files:

- **processOECD.m** Loads the IO table of each country from a text file and processes it. The format of the data is slightly different for each country, requiring country-specific manipulations. Produces all variables needed as input for *analyze.m*.
- **analyze.m** Main file, which performs all analysis except for the community-finding algorithm.
- **IOcomm.m** Performs additional preprocessing of the IO data to allow application of *findcomm.m.*
- findcomm.m Searches for communities. Implements the algorithm described in Appendix A.
- **NEA.m** Measures various quantities from ecological input-output analysis. (Obtained from Brian Fath.)
- redirect.m Generates random IO networks via the procedure described in Appendix 4.

Support files:

printlargest.m

binpdf.m

see matrix.m

 $\mathbf{shuffle.m}$

C IO Table Results

All results come from the OECD-published IO tables.



Communities

Q = 0.205

Group 1

AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR CHEMICALS EXCLUDING PHARMACEUTICALS PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS MANUFACTURING NEC; RECYCLING HOTELS AND RESTAURANTS

Group 2

MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS AIRCRAFT AND SPACECRAFT WHOLESALE AND RETAIL TRADE; REPAIRS TRANSPORT AND STORAGE POST AND TELECOMMUNICATIONS FINANCE, INSURANCE REAL ESTATE ACTIVITIES COMPUTER AND RELATED ACTIVITIES OTHER BUSINESS ACTIVITIES PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Group 3

WOOD AND PRODUCTS OF WOOD AND CORK OTHER NON-METALLIC MINERAL PRODUCTS IRON & STEEL FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. OFFICE, ACCOUNTING AND COMPUTING MACHINERY ELECTRICAL MACHINERY AND APPARATUS, NEC BUILDING AND REPAIRING OF SHIPS AND BOATS RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. CONSTRUCTION

Group 4 MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL NON-FERROUS METALS ELECTRICITY, GAS AND WATER SUPPLY

Group 5 PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING MEDICAL, PRECISION AND OPTICAL INSTRUMENTS EDUCATION HEALTH AND SOCIAL WORK

PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

Ungrouped Sectors RADIO, TELEVISION AND COMMUNICATION EQUIPMENT RENTING OF MACHINERY AND EQUIPMENT RESEARCH AND DEVELOPMENT



Communities

Q = 0.245

Group 1 WOOD AND PRODUCTS OF WOOD AND CORK OTHER NON-METALLIC MINERAL PRODUCTS IRON & STEEL NON-FERROUS METALS FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. OFFICE, ACCOUNTING AND COMPUTING MACHINERY ELECTRICAL MACHINERY AND APPARATUS, NEC ELECTRICITY, GAS AND WATER SUPPLY CONSTRUCTION POST AND TELECOMMUNICATIONS FINANCE, INSURANCE REAL ESTATE ACTIVITIES

Group 2

TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING PHARMACEUTICALS MANUFACTURING NEC; RECYCLING WHOLESALE AND RETAIL TRADE; REPAIRS HOTELS AND RESTAURANTS OTHER BUSINESS ACTIVITIES PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY

Group 3

AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO CHEMICALS EXCLUDING PHARMACEUTICALS

Group 4

MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL RUBBER AND PLASTICS PRODUCTS TRANSPORT AND STORAGE

Group 5 MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.

Ungrouped Sectors RADIO, TELEVISION AND COMMUNICATION EQUIPMENT MEDICAL, PRECISION AND OPTICAL INSTRUMENTS BUILDING AND REPAIRING OF SHIPS AND BOATS AIRCRAFT AND SPACECRAFT RENTING OF MACHINERY AND EQUIPMENT COMPUTER AND RELATED ACTIVITIES RESEARCH AND DEVELOPMENT EDUCATION HEALTH AND SOCIAL WORK OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS



Communities

Q = 0.22

Group 1

AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR WOOD AND PRODUCTS OF WOOD AND CORK PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING CHEMICALS EXCLUDING PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS MANUFACTURING NEC; RECYCLING HOTELS AND RESTAURANTS

Group 2 MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY

Group 3 OTHER NON-METALLIC MINERAL PRODUCTS IRON & STEEL FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS AIRCRAFT AND SPACECRAFT RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. CONSTRUCTION

Group 4

BUILDING AND REPAIRING OF SHIPS AND BOATS PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY HEALTH AND SOCIAL WORK

Group 5 WHOLESALE AND RETAIL TRADE; REPAIRS TRANSPORT AND STORAGE POST AND TELECOMMUNICATIONS FINANCE, INSURANCE COMPUTER AND RELATED ACTIVITIES OTHER BUSINESS ACTIVITIES EDUCATION OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Group 6 OFFICE, ACCOUNTING AND COMPUTING MACHINERY ELECTRICAL MACHINERY AND APPARATUS, NEC RADIO, TELEVISION AND COMMUNICATION EQUIPMENT Ungrouped Sectors PHARMACEUTICALS NON-FERROUS METALS MEDICAL, PRECISION AND OPTICAL INSTRUMENTS REAL ESTATE ACTIVITIES RENTING OF MACHINERY AND EQUIPMENT RESEARCH AND DEVELOPMENT PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS



C.4 China

Communities

Q = 0.267

Group 1 AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR HOTELS AND RESTAURANTS REAL ESTATE ACTIVITIES OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Group 2

CHEMICALS EXCLUDING PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS

Group 3

MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL OTHER NON-METALLIC MINERAL PRODUCTS **IRON & STEEL** NON-FERROUS METALS FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. ELECTRICAL MACHINERY AND APPARATUS, NEC RADIO, TELEVISION AND COMMUNICATION EQUIPMENT MEDICAL, PRECISION AND OPTICAL INSTRUMENTS MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS BUILDING AND REPAIRING OF SHIPS AND BOATS AIRCRAFT AND SPACECRAFT RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. ELECTRICITY, GAS AND WATER SUPPLY CONSTRUCTION TRANSPORT AND STORAGE POST AND TELECOMMUNICATIONS **RESEARCH AND DEVELOPMENT**

Group 4 WOOD AND PRODUCTS OF WOOD AND CORK PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING PHARMACEUTICALS OFFICE, ACCOUNTING AND COMPUTING MACHINERY MANUFACTURING NEC; RECYCLING WHOLESALE AND RETAIL TRADE; REPAIRS FINANCE, INSURANCE OTHER BUSINESS ACTIVITIES PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY EDUCATION Ungrouped Sectors

RENTING OF MACHINERY AND EQUIPMENTCOMPUTER AND RELATED AC-TIVITIESHEALTH AND SOCIAL WORKPRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS
Flow Mode Density of non-zero links = 0.919Boundary 36.4~%Boundary flow (GDP) = 1.285e+12 Koruna First-passage 42.1~%Total system through flow = 3.529e + 12 Koruna 21.5~%Cycled Link Weights Link Weight PDF Links vs. Reciprocating Links 10⁶ 10¹ 10⁵ 10¹ 10⁴ w_{ii} (nominal Koruna) 10³ 10⁹ کے 10² 10⁸ 10¹ 10⁰ 10 10 10⁻²[10 10⁶ 10⁻⁶ 10⁻⁵ 10⁻⁴ normalized weight w 10 10⁻² 10⁻¹ 10⁸ 10⁹ w_a (nominal Koruna) 10¹⁰ 10¹¹ 10 Sector Strengths Node Strength CDF In-strength vs. Out-strength 10⁰ 10¹² s_{in} 0 S 10¹¹ s_{out} (nominal Koruna) 10¹ ([']s<'_s)d 10⁹ 10⁸ c 10^{-≃∟}0 10 10⁹ 10¹⁰ s_{in} (nominal Koruna) 0.02 0.04 0.06 0.08 normalized strength s 0.1 0.12 10¹ 10¹² 10 10 Clustering Centrality Clustering Coefficients CDF Centrality CDF Forward+ Reverse cycling middlem 0 △ * 0.9 0.9 in out 0.8 0.8 all 0.7 0.7 0.6 p(P'; > P;) 0.5 0.4 0.4 0.3 0.3 0.2 0.2 0.1 0.1 0^L 0 10⁹ 0.02 0.04 0.06 Centrality P. 0.08 0.1 10¹⁰ Clustering Coefficient C (nominal Koruna) 10¹¹

0

0.12

Q = 0.198

Group 1

TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR WOOD AND PRODUCTS OF WOOD AND CORK PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING CHEMICALS EXCLUDING PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS MANUFACTURING NEC; RECYCLING

Group 2

MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY

Group 3

OTHER NON-METALLIC MINERAL PRODUCTS OFFICE, ACCOUNTING AND COMPUTING MACHINERY RADIO, TELEVISION AND COMMUNICATION EQUIPMENT RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. CONSTRUCTION WHOLESALE AND RETAIL TRADE; REPAIRS TRANSPORT AND STORAGE POST AND TELECOMMUNICATIONS FINANCE, INSURANCE **REAL ESTATE ACTIVITIES** RENTING OF MACHINERY AND EQUIPMENT COMPUTER AND RELATED ACTIVITIES OTHER BUSINESS ACTIVITIES PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY **EDUCATION** OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Group 4 IRON & STEEL NON-FERROUS METALS FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. ELECTRICAL MACHINERY AND APPARATUS, NEC MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS BUILDING AND REPAIRING OF SHIPS AND BOATS AIRCRAFT AND SPACECRAFT RESEARCH AND DEVELOPMENT

Group 5 AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO PHARMACEUTICALS

MEDICAL, PRECISION AND OPTICAL INSTRUMENTS HOTELS AND RESTAURANTS HEALTH AND SOCIAL WORK

Ungrouped Sectors PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS



Q = 0.253

Group 1 OFFICE, ACCOUNTING AND COMPUTING MACHINERY COMPUTER AND RELATED ACTIVITIES RESEARCH AND DEVELOPMENT EDUCATION

Group 2

PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING WHOLESALE AND RETAIL TRADE; REPAIRS TRANSPORT AND STORAGE POST AND TELECOMMUNICATIONS FINANCE, INSURANCE REAL ESTATE ACTIVITIES RENTING OF MACHINERY AND EQUIPMENT OTHER BUSINESS ACTIVITIES OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Group 3

TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR WOOD AND PRODUCTS OF WOOD AND CORK CHEMICALS EXCLUDING PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS OTHER NON-METALLIC MINERAL PRODUCTS IRON & STEEL FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. ELECTRICAL MACHINERY AND APPARATUS, NEC RADIO, TELEVISION AND COMMUNICATION EQUIPMENT MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS BUILDING AND REPAIRING OF SHIPS AND BOATS MANUFACTURING NEC; RECYCLING CONSTRUCTION

Group 4

AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO HOTELS AND RESTAURANTS

Group 5

MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY

Group 6 MEDICAL, PRECISION AND OPTICAL INSTRUMENTS PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY HEALTH AND SOCIAL WORK

Ungrouped Sectors PHARMACEUTICALS NON-FERROUS METALS AIRCRAFT AND SPACECRAFT RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS



Q = 0.238

Group 1

PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. POST AND TELECOMMUNICATIONS FINANCE, INSURANCE REAL ESTATE ACTIVITIES RENTING OF MACHINERY AND EQUIPMENT COMPUTER AND RELATED ACTIVITIES RESEARCH AND DEVELOPMENT OTHER BUSINESS ACTIVITIES PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY EDUCATION HEALTH AND SOCIAL WORK OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Group 2

AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO CHEMICALS EXCLUDING PHARMACEUTICALS PHARMACEUTICALS HOTELS AND RESTAURANTS

Group 3

MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY

Group 4

MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS WHOLESALE AND RETAIL TRADE; REPAIRS TRANSPORT AND STORAGE

Group 5 TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR WOOD AND PRODUCTS OF WOOD AND CORK RUBBER AND PLASTICS PRODUCTS OTHER NON-METALLIC MINERAL PRODUCTS IRON & STEEL NON-FERROUS METALS FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. OFFICE, ACCOUNTING AND COMPUTING MACHINERY ELECTRICAL MACHINERY AND APPARATUS, NEC RADIO, TELEVISION AND COMMUNICATION EQUIPMENT MEDICAL, PRECISION AND OPTICAL INSTRUMENTS BUILDING AND REPAIRING OF SHIPS AND BOATS MANUFACTURING NEC; RECYCLING CONSTRUCTION

Ungrouped Sectors AIRCRAFT AND SPACECRAFT PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS





-39-

Q = 0.265

Group 1

WOOD AND PRODUCTS OF WOOD AND CORK PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING OTHER NON-METALLIC MINERAL PRODUCTS OFFICE, ACCOUNTING AND COMPUTING MACHINERY MANUFACTURING NEC; RECYCLING CONSTRUCTION WHOLESALE AND RETAIL TRADE; REPAIRS TRANSPORT AND STORAGE POST AND TELECOMMUNICATIONS FINANCE, INSURANCE **REAL ESTATE ACTIVITIES** RENTING OF MACHINERY AND EQUIPMENT COMPUTER AND RELATED ACTIVITIES OTHER BUSINESS ACTIVITIES PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY EDUCATION OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Group 2

AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO HOTELS AND RESTAURANTS

Group 3

TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR CHEMICALS EXCLUDING PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS IRON & STEEL FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. ELECTRICAL MACHINERY AND APPARATUS, NEC RADIO, TELEVISION AND COMMUNICATION EQUIPMENT MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS BUILDING AND REPAIRING OF SHIPS AND BOATS AIRCRAFT AND SPACECRAFT RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. RESEARCH AND DEVELOPMENT

Group 4 PHARMACEUTICALS MEDICAL, PRECISION AND OPTICAL INSTRUMENTS HEALTH AND SOCIAL WORK

Group 5 MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY

Ungrouped Sectors NON-FERROUS METALS PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS



0.25

C.9 Finland

Q = 0.189

Group 1

AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR WOOD AND PRODUCTS OF WOOD AND CORK PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING CHEMICALS EXCLUDING PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS ELECTRICITY, GAS AND WATER SUPPLY HOTELS AND RESTAURANTS EDUCATION HEALTH AND SOCIAL WORK OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Group 2

OTHER NON-METALLIC MINERAL PRODUCTS CONSTRUCTION WHOLESALE AND RETAIL TRADE; REPAIRS TRANSPORT AND STORAGE POST AND TELECOMMUNICATIONS FINANCE, INSURANCE REAL ESTATE ACTIVITIES

Group 3 IRON & STEEL NON-FERROUS METALS FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. OFFICE, ACCOUNTING AND COMPUTING MACHINERY ELECTRICAL MACHINERY AND APPARATUS, NEC RADIO, TELEVISION AND COMMUNICATION EQUIPMENT MEDICAL, PRECISION AND OPTICAL INSTRUMENTS MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS BUILDING AND REPAIRING OF SHIPS AND BOATS RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. MANUFACTURING NEC; RECYCLING OTHER BUSINESS ACTIVITIES PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY

Group 4 MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL

Ungrouped Sectors PHARMACEUTICALS AIRCRAFT AND SPACECRAFT RENTING OF MACHINERY AND EQUIPMENT COMPUTER AND RELATED ACTIVITIES RESEARCH AND DEVELOPMENT PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS



C.10 France

Q = 0.155

Group 1 MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL OTHER NON-METALLIC MINERAL PRODUCTS **IRON & STEEL** NON-FERROUS METALS FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. ELECTRICAL MACHINERY AND APPARATUS, NEC ELECTRICITY, GAS AND WATER SUPPLY CONSTRUCTION POST AND TELECOMMUNICATIONS FINANCE, INSURANCE **REAL ESTATE ACTIVITIES** RENTING OF MACHINERY AND EQUIPMENT COMPUTER AND RELATED ACTIVITIES OTHER BUSINESS ACTIVITIES OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Group 2 TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR PHARMACEUTICALS MANUFACTURING NEC; RECYCLING HEALTH AND SOCIAL WORK

Group 3

AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO WOOD AND PRODUCTS OF WOOD AND CORK HOTELS AND RESTAURANTS

Group 4 CHEMICALS EXCLUDING PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS

Group 5

OFFICE, ACCOUNTING AND COMPUTING MACHINERY RADIO, TELEVISION AND COMMUNICATION EQUIPMENT MEDICAL, PRECISION AND OPTICAL INSTRUMENTS BUILDING AND REPAIRING OF SHIPS AND BOATS AIRCRAFT AND SPACECRAFT RESEARCH AND DEVELOPMENT

Group 6 PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS

RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. WHOLESALE AND RETAIL TRADE; REPAIRS TRANSPORT AND STORAGE EDUCATION

Ungrouped Sectors PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITYPRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS



Q = 0.335

Group 1 MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY

Group 2

AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR HOTELS AND RESTAURANTS

Group 3

WOOD AND PRODUCTS OF WOOD AND CORK OTHER NON-METALLIC MINERAL PRODUCTS IRON & STEEL FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. OFFICE, ACCOUNTING AND COMPUTING MACHINERY ELECTRICAL MACHINERY AND APPARATUS, NEC RADIO, TELEVISION AND COMMUNICATION EQUIPMENT BUILDING AND REPAIRING OF SHIPS AND BOATS MANUFACTURING NEC; RECYCLING CONSTRUCTION RESEARCH AND DEVELOPMENT PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY

Group 4

PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING CHEMICALS EXCLUDING PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS MEDICAL, PRECISION AND OPTICAL INSTRUMENTS MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS WHOLESALE AND RETAIL TRADE; REPAIRS TRANSPORT AND STORAGE POST AND TELECOMMUNICATIONS FINANCE, INSURANCE **REAL ESTATE ACTIVITIES** RENTING OF MACHINERY AND EQUIPMENT COMPUTER AND RELATED ACTIVITIES OTHER BUSINESS ACTIVITIES EDUCATION HEALTH AND SOCIAL WORK OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Ungrouped Sectors PHARMACEUTICALS NON-FERROUS METALS AIRCRAFT AND SPACECRAFT RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

C.12 Hungary



 $\mathbf{Q}=0.252$

Group 1 AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO HOTELS AND RESTAURANTS

Group 2 MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY

Group 3 WOOD AND PRODUCTS OF WOOD AND CORK OTHER NON-METALLIC MINERAL PRODUCTS MANUFACTURING NEC; RECYCLING CONSTRUCTION

Group 4 PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. WHOLESALE AND RETAIL TRADE; REPAIRS TRANSPORT AND STORAGE POST AND TELECOMMUNICATIONS FINANCE, INSURANCE REAL ESTATE ACTIVITIES RENTING OF MACHINERY AND EQUIPMENT COMPUTER AND RELATED ACTIVITIES OTHER BUSINESS ACTIVITIES PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY EDUCATION OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Group 5

IRON & STEEL FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. OFFICE, ACCOUNTING AND COMPUTING MACHINERY ELECTRICAL MACHINERY AND APPARATUS, NEC RADIO, TELEVISION AND COMMUNICATION EQUIPMENT MEDICAL, PRECISION AND OPTICAL INSTRUMENTS MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS

Group 6

TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR CHEMICALS EXCLUDING PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS RESEARCH AND DEVELOPMENT HEALTH AND SOCIAL WORK

Ungrouped Sectors PHARMACEUTICALS NON-FERROUS METALS BUILDING AND REPAIRING OF SHIPS AND BOATS AIRCRAFT AND SPACECRAFT PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS



Communities Q = 0.259

Group 1 MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY

Group 2 PHARMACEUTICALS HEALTH AND SOCIAL WORK

Group 3

WOOD AND PRODUCTS OF WOOD AND CORK OTHER NON-METALLIC MINERAL PRODUCTS IRON & STEEL FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. OFFICE, ACCOUNTING AND COMPUTING MACHINERY ELECTRICAL MACHINERY AND APPARATUS, NEC RADIO, TELEVISION AND COMMUNICATION EQUIPMENT MEDICAL, PRECISION AND OPTICAL INSTRUMENTS MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS BUILDING AND REPAIRING OF SHIPS AND BOATS RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. MANUFACTURING NEC; RECYCLING CONSTRUCTION

Group 4

PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING AIRCRAFT AND SPACECRAFT WHOLESALE AND RETAIL TRADE; REPAIRS TRANSPORT AND STORAGE POST AND TELECOMMUNICATIONS FINANCE, INSURANCE REAL ESTATE ACTIVITIES COMPUTER AND RELATED ACTIVITIES RESEARCH AND DEVELOPMENT OTHER BUSINESS ACTIVITIES PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Group 5

AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO HOTELS AND RESTAURANTS

Group 6 TEXTILE PRODUCTS, LEATHER AND FOOTWEAR

CHEMICALS EXCLUDING PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS

Ungrouped Sectors NON-FERROUS METALS RENTING OF MACHINERY AND EQUIPMENT EDUCATION PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

C.14 Japan



Q = 0.228

Group 1

WOOD AND PRODUCTS OF WOOD AND CORK OTHER NON-METALLIC MINERAL PRODUCTS IRON & STEEL FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. BUILDING AND REPAIRING OF SHIPS AND BOATS CONSTRUCTION

Group 2

TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR CHEMICALS EXCLUDING PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS MANUFACTURING NEC; RECYCLING

Group 3

PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING WHOLESALE AND RETAIL TRADE; REPAIRS TRANSPORT AND STORAGE POST AND TELECOMMUNICATIONS FINANCE, INSURANCE REAL ESTATE ACTIVITIES OTHER BUSINESS ACTIVITIES EDUCATION OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

Group 4 AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO PHARMACEUTICALS HEALTH AND SOCIAL WORK

Group 5 AIRCRAFT AND SPACECRAFT RENTING OF MACHINERY AND EQUIPMENT COMPUTER AND RELATED ACTIVITIES PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY

Group 6 NON-FERROUS METALS OFFICE, ACCOUNTING AND COMPUTING MACHINERY ELECTRICAL MACHINERY AND APPARATUS, NEC RADIO, TELEVISION AND COMMUNICATION EQUIPMENT MEDICAL, PRECISION AND OPTICAL INSTRUMENTS MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. RESEARCH AND DEVELOPMENT

Group 7 MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY

Ungrouped Sectors HOTELS AND RESTAURANTS



Q = 0.287

Group 1

AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO WOOD AND PRODUCTS OF WOOD AND CORK PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING PHARMACEUTICALS MANUFACTURING NEC; RECYCLING HOTELS AND RESTAURANTS EDUCATION HEALTH AND SOCIAL WORK OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Group 2

WHOLESALE AND RETAIL TRADE; REPAIRS POST AND TELECOMMUNICATIONS FINANCE, INSURANCE REAL ESTATE ACTIVITIES OTHER BUSINESS ACTIVITIES PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

Group 3 MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL AIRCRAFT AND SPACECRAFT ELECTRICITY, GAS AND WATER SUPPLY TRANSPORT AND STORAGE RENTING OF MACHINERY AND EQUIPMENT

Group 4 NON-FERROUS METALS OFFICE, ACCOUNTING AND COMPUTING MACHINERY ELECTRICAL MACHINERY AND APPARATUS, NEC RADIO, TELEVISION AND COMMUNICATION EQUIPMENT MEDICAL, PRECISION AND OPTICAL INSTRUMENTS RESEARCH AND DEVELOPMENT

Group 5 OTHER NON-METALLIC MINERAL PRODUCTS IRON & STEEL FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS BUILDING AND REPAIRING OF SHIPS AND BOATS RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. CONSTRUCTION Group 6 TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR CHEMICALS EXCLUDING PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS

Ungrouped Sectors COMPUTER AND RELATED ACTIVITIESPUBLIC ADMIN. AND DEFENCE; COM-PULSORY SOCIAL SECURITY



Q = 0.232

Group 1 AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO HOTELS AND RESTAURANTS

Group 2 WOOD AND PRODUCTS OF WOOD AND CORK RUBBER AND PLASTICS PRODUCTS OTHER NON-METALLIC MINERAL PRODUCTS IRON & STEEL FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. OFFICE, ACCOUNTING AND COMPUTING MACHINERY ELECTRICAL MACHINERY AND APPARATUS, NEC RADIO, TELEVISION AND COMMUNICATION EQUIPMENT MEDICAL, PRECISION AND OPTICAL INSTRUMENTS MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS BUILDING AND REPAIRING OF SHIPS AND BOATS AIRCRAFT AND SPACECRAFT RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. MANUFACTURING NEC; RECYCLING CONSTRUCTION **REAL ESTATE ACTIVITIES** RESEARCH AND DEVELOPMENT PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY EDUCATION

Group 3

TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING WHOLESALE AND RETAIL TRADE; REPAIRS TRANSPORT AND STORAGE POST AND TELECOMMUNICATIONS FINANCE, INSURANCE RENTING OF MACHINERY AND EQUIPMENT COMPUTER AND RELATED ACTIVITIES OTHER BUSINESS ACTIVITIES HEALTH AND SOCIAL WORK OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Group 4

MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL CHEMICALS EXCLUDING PHARMACEUTICALS ELECTRICITY, GAS AND WATER SUPPLY Ungrouped Sectors PHARMACEUTICALS NON-FERROUS METALS PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS
C.17 Norway



Q = 0.218

Group 1 AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR PHARMACEUTICALS HOTELS AND RESTAURANTS HEALTH AND SOCIAL WORK

Group 2

PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING OFFICE, ACCOUNTING AND COMPUTING MACHINERY POST AND TELECOMMUNICATIONS FINANCE, INSURANCE COMPUTER AND RELATED ACTIVITIES RESEARCH AND DEVELOPMENT OTHER BUSINESS ACTIVITIES EDUCATION OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Group 3

MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL IRON & STEEL NON-FERROUS METALS FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. ELECTRICAL MACHINERY AND APPARATUS, NEC RADIO, TELEVISION AND COMMUNICATION EQUIPMENT MEDICAL, PRECISION AND OPTICAL INSTRUMENTS BUILDING AND REPAIRING OF SHIPS AND BOATS RENTING OF MACHINERY AND EQUIPMENT

Group 4

MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS AIRCRAFT AND SPACECRAFT RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. ELECTRICITY, GAS AND WATER SUPPLY WHOLESALE AND RETAIL TRADE; REPAIRS TRANSPORT AND STORAGE

Group 5 WOOD AND PRODUCTS OF WOOD AND CORK CHEMICALS EXCLUDING PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS OTHER NON-METALLIC MINERAL PRODUCTS MANUFACTURING NEC; RECYCLING

CONSTRUCTION REAL ESTATE ACTIVITIES PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY

Ungrouped Sectors PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS



Q = 0.175

Group 1

TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR WOOD AND PRODUCTS OF WOOD AND CORK MANUFACTURING NEC; RECYCLING HOTELS AND RESTAURANTS POST AND TELECOMMUNICATIONS FINANCE, INSURANCE HEALTH AND SOCIAL WORK OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Group 2

CHEMICALS EXCLUDING PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS OTHER NON-METALLIC MINERAL PRODUCTS OFFICE, ACCOUNTING AND COMPUTING MACHINERY CONSTRUCTION REAL ESTATE ACTIVITIES RENTING OF MACHINERY AND EQUIPMENT COMPUTER AND RELATED ACTIVITIES OTHER BUSINESS ACTIVITIES EDUCATION

Group 3

MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL IRON & STEEL FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. ELECTRICAL MACHINERY AND APPARATUS, NEC RADIO, TELEVISION AND COMMUNICATION EQUIPMENT MEDICAL, PRECISION AND OPTICAL INSTRUMENTS MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS BUILDING AND REPAIRING OF SHIPS AND BOATS ELECTRICITY, GAS AND WATER SUPPLY TRANSPORT AND STORAGE RESEARCH AND DEVELOPMENT

Group 4 AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING WHOLESALE AND RETAIL TRADE; REPAIRS

Ungrouped Sectors PHARMACEUTICALS NON-FERROUS METALS AIRCRAFT AND SPACECRAFT RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS

C.19 United Kingdom



Q = 0.186

Group 1

TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING WHOLESALE AND RETAIL TRADE; REPAIRS HOTELS AND RESTAURANTS TRANSPORT AND STORAGE POST AND TELECOMMUNICATIONS FINANCE, INSURANCE REAL ESTATE ACTIVITIES RENTING OF MACHINERY AND EQUIPMENT COMPUTER AND RELATED ACTIVITIES RESEARCH AND DEVELOPMENT OTHER BUSINESS ACTIVITIES EDUCATION OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES

Group 2 WOOD AND PRODUCTS OF WOOD AND CORK RUBBER AND PLASTICS PRODUCTS OTHER NON-METALLIC MINERAL PRODUCTS **IRON & STEEL** NON-FERROUS METALS FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. OFFICE, ACCOUNTING AND COMPUTING MACHINERY ELECTRICAL MACHINERY AND APPARATUS, NEC RADIO, TELEVISION AND COMMUNICATION EQUIPMENT MEDICAL, PRECISION AND OPTICAL INSTRUMENTS MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS BUILDING AND REPAIRING OF SHIPS AND BOATS AIRCRAFT AND SPACECRAFT RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. MANUFACTURING NEC; RECYCLING CONSTRUCTION PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY

Group 3 AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO PHARMACEUTICALS HEALTH AND SOCIAL WORK

Group 4 MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL CHEMICALS EXCLUDING PHARMACEUTICALS Ungrouped Sectors PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS



C.20 United States

Q = 0.194

Group 1 OTHER NON-METALLIC MINERAL PRODUCTS IRON & STEEL NON-FERROUS METALS FABRICATED METAL PRODUCTS, except machinery and equipment MACHINERY AND EQUIPMENT, N.E.C. ELECTRICAL MACHINERY AND APPARATUS, NEC MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS BUILDING AND REPAIRING OF SHIPS AND BOATS RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C. CONSTRUCTION State and local government passenger transit Other State and local government enterprises

Group 2 WOOD AND PRODUCTS OF WOOD AND CORK MANUFACTURING NEC; RECYCLING

Group 3

AGRICULTURE, HUNTING, FORESTRY AND FISHING FOOD PRODUCTS, BEVERAGES AND TOBACCO TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING CHEMICALS EXCLUDING PHARMACEUTICALS PHARMACEUTICALS RUBBER AND PLASTICS PRODUCTS HOTELS AND RESTAURANTS HEALTH AND SOCIAL WORK

Group 4 MEDICAL, PRECISION AND OPTICAL INSTRUMENTS AIRCRAFT AND SPACECRAFT

Group 5 MINING AND QUARRYING COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL ELECTRICITY, GAS AND WATER SUPPLY TRANSPORT AND STORAGE Federal electric utilities State and local government electric utilities

Group 6 OFFICE, ACCOUNTING AND COMPUTING MACHINERY RADIO, TELEVISION AND COMMUNICATION EQUIPMENT WHOLESALE AND RETAIL TRADE; REPAIRS POST AND TELECOMMUNICATIONS FINANCE, INSURANCE REAL ESTATE ACTIVITIES RENTING OF MACHINERY AND EQUIPMENT COMPUTER AND RELATED ACTIVITIES RESEARCH AND DEVELOPMENT OTHER BUSINESS ACTIVITIES EDUCATION OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES Other Federal Government enterprises

Ungrouped Sectors PUBLIC ADMIN. AND DEFENCE; COMPULSORY SOCIAL SECURITY PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS SBFD including OTHER ADJUSTMENTS



Figure 1: The normalized flow size distributions of 20 OECD countries.



Table 1: Clustering Patterns for Directed Networks



Figure 2: The distribution of flow sizes for the US 1997 OECD table.

Description	Vector/Matrix	Elements
Internal flows matrix	F	f_{ij}
Inputs vector	\vec{z}	z_i
Outputs vector	\vec{y}	y_i
Throughflows vector	$\vec{T} = F^T \vec{1} + \vec{z} = F \vec{1} + \vec{y}$	$T_i = F_{\cdot i} + z_i = F_{i\cdot} + y_i$
\hat{T}^{-1}	$g_{ij} = rac{f_{ij}}{T_i}$	
Flow fractions matrix	$G \equiv \hat{T}^{-1}F$	$g_{ij} = \frac{f_{ij}}{T_i}$
Total requirements matrix	$N \equiv (I - G^T)^{-1}$	n_{ij}

Table 2: Notation for the decomposition of flows through an IO network. $\vec{1}$ indicates a vector of ones. A hat ôver a vector's label indicates a matrix whose diagonal elements are the elements of the vector, with zeros elsewhere.



Figure 3: The scatter plot of size of a flow versus the size of the reciprocating flow, f_{ij} versus $f_{ji}.$



Figure 4: The distribution of in-strength and out-strength for 20 OECD countries.



Figure 5: The distribution of in-strength and out-strength for the US 1997 OECD table.



Figure 6: The change in the strength-distribution resulting from shuffling the flows of the IO table. The two distributions with the lower slope are the original distributions, while the pair with the higher slope are the distributions after shuffling.



Figure 7: Scatterplot of in-strength versus out-strength for each sector for the US 1997 OECD table.



Figure 8: The cumulative distribution of clustering coefficients for the US 1997 OECD table.



Figure 9: The cumulative distribution of centrality for the US 1997 OECD table.



Figure 10: The US 1997 OECD table. The numerical values for the entries of the table have been mapped to colors, with red cells representing strong flows and blue cells representing weak flows. The color key at the right gives the mapping of colors to the logarithm of flows in base 10, thus covering the range 10^6 to 10^{11} dollars. Black lines indicate boundaries between groups (see text.)

Country	% Boundary	% First Passage	% cycled
Australia	50.3	37.5	12.2
Brazil	55.1	32.9	12.0
Canada	53.5	34.5	11.9
China	15.6	63.2	21.2
Czech Republic	36.4	42.1	21.5
Germany	51.2	36.6	12.2
Denmark	56.5	34.4	9.2
Spain	51.3	36.1	12.6
Finland	48.6	35.8	15.5
France	55.2	31.2	13.6
Greece	59.6	32.7	7.8
Hungary	47.6	38.1	14.4
Italy	52.4	34.9	12.8
Japan	54.5	34.8	10.8
Korea	44.7	40.0	15.3
Netherlands	50.8	34.8	14.4
Norway	54.0	34.8	11.2
Poland	45.2	37.9	16.9
United Kingdom	50.2	34.3	15.5
United States	57.0	32.0	11.0
Average	49.5	36.9	13.6
Average exc. China, Czech	52.1	35.2	12.7

Table 3: The percent contribution of each flow mode to the total flow through the IO network for each OECD country. See text for a description of the flow modes.