On Optimal Labor Allocation Policy for Technological Followers

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International Institute for Applied Systems Analysis

Viennese Vintage Workshop , 26-27 November 2007

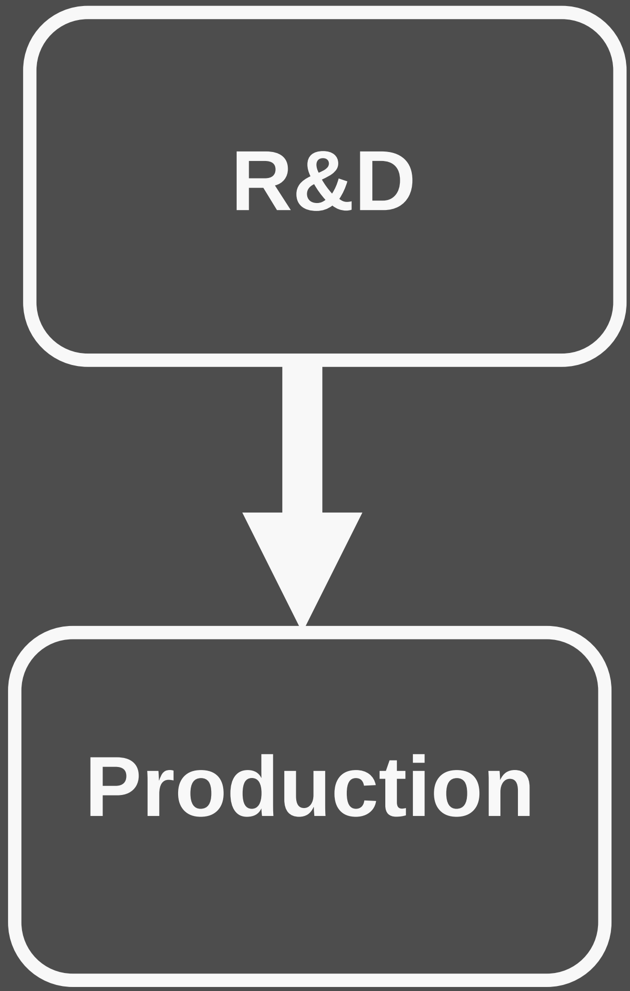
Knowledge absorption

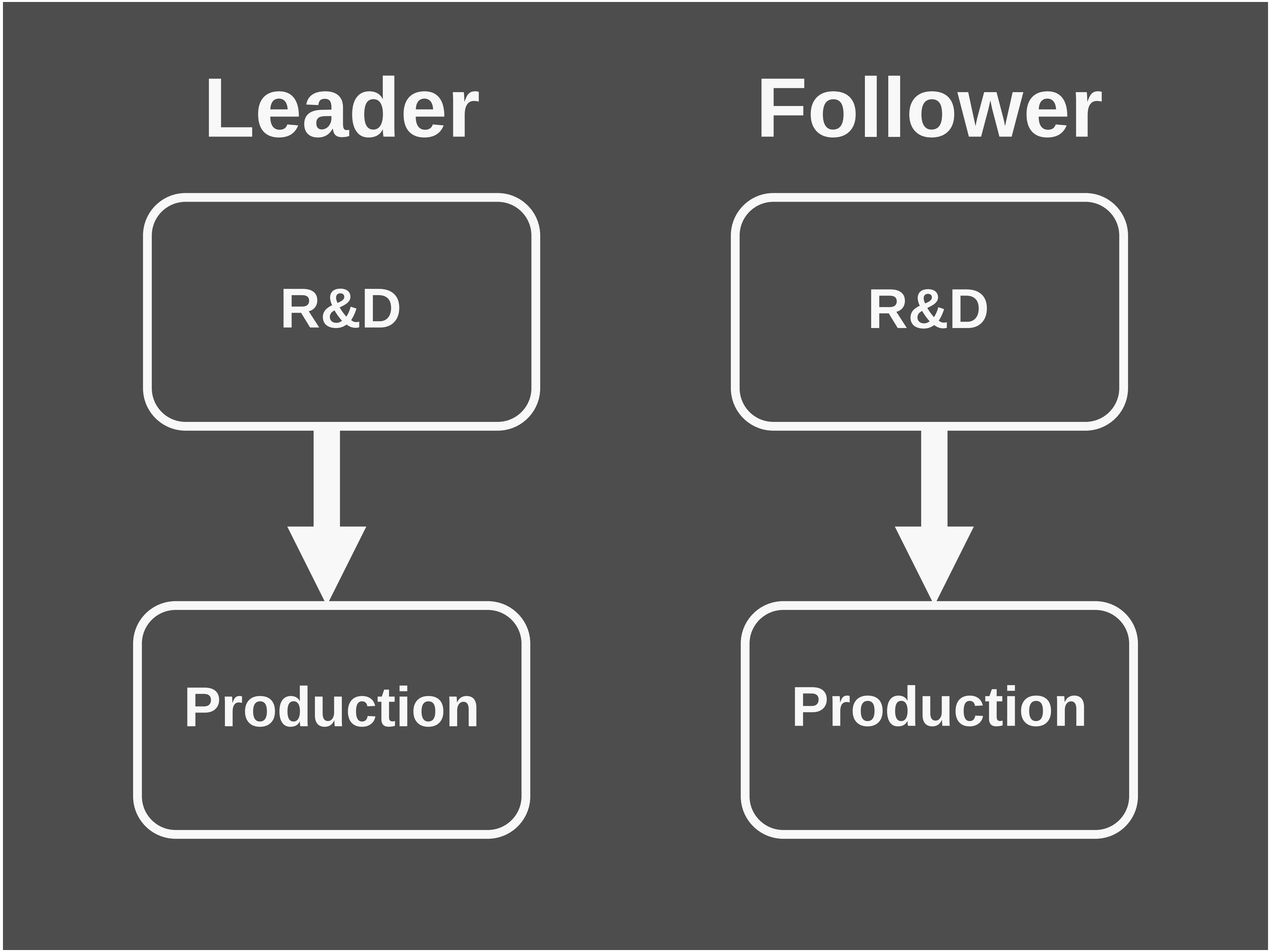
Optimal labor allocation

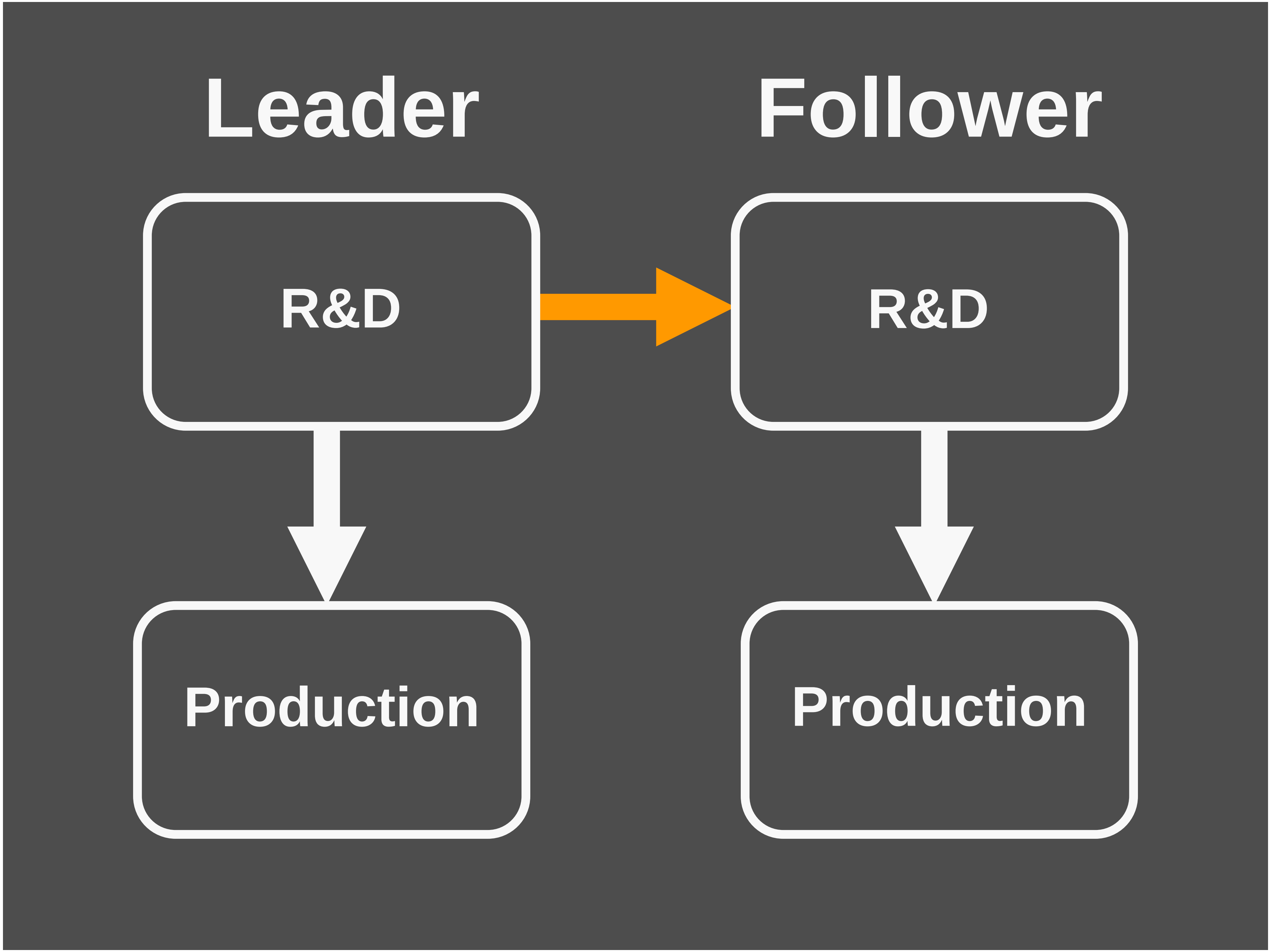
Catching up the leader

Overtaking the leader

Leader




Leader
R&D
Production
Follower
R&D
Production


Leader
Follower
R&D
R&D
Production
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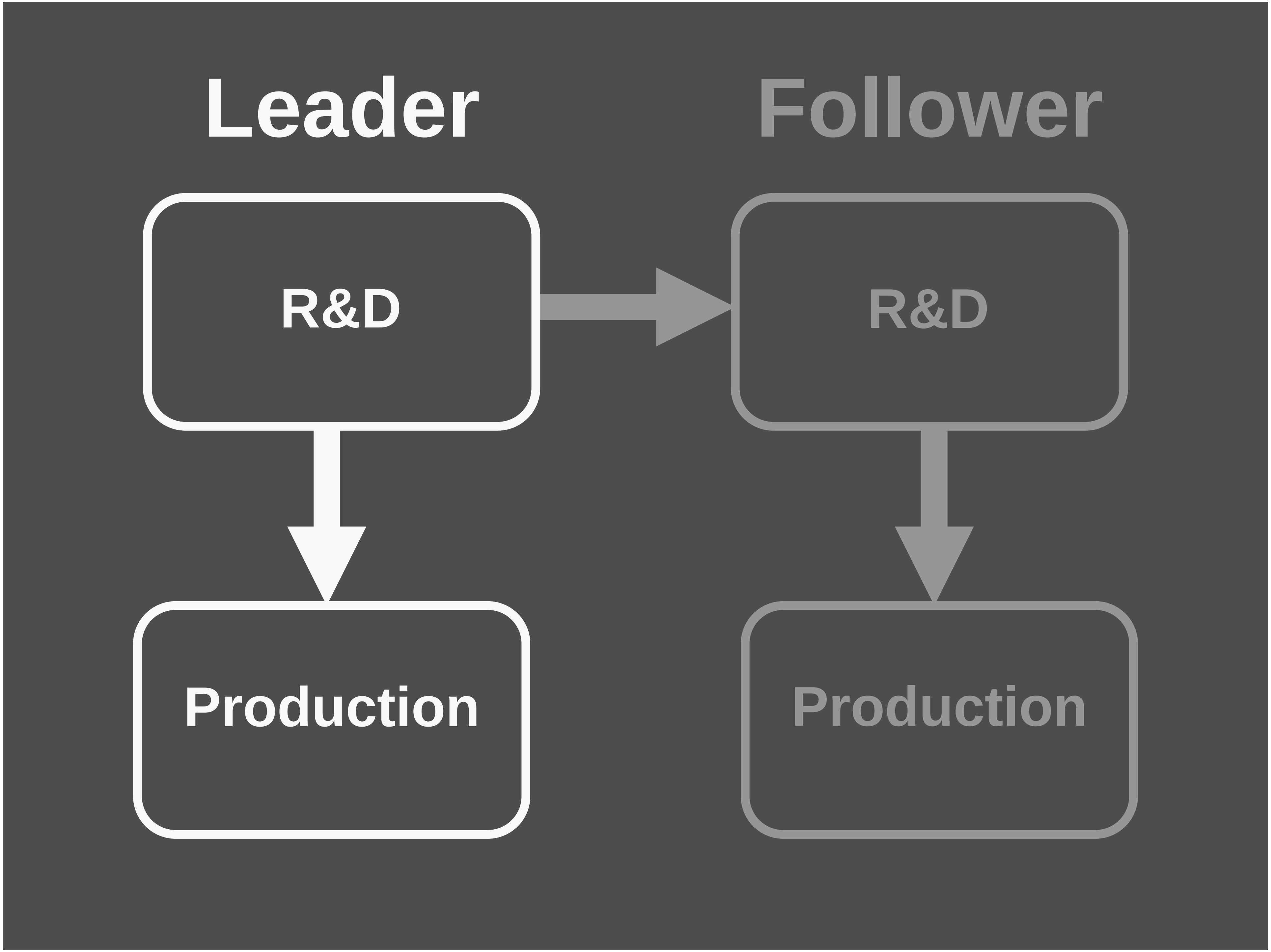
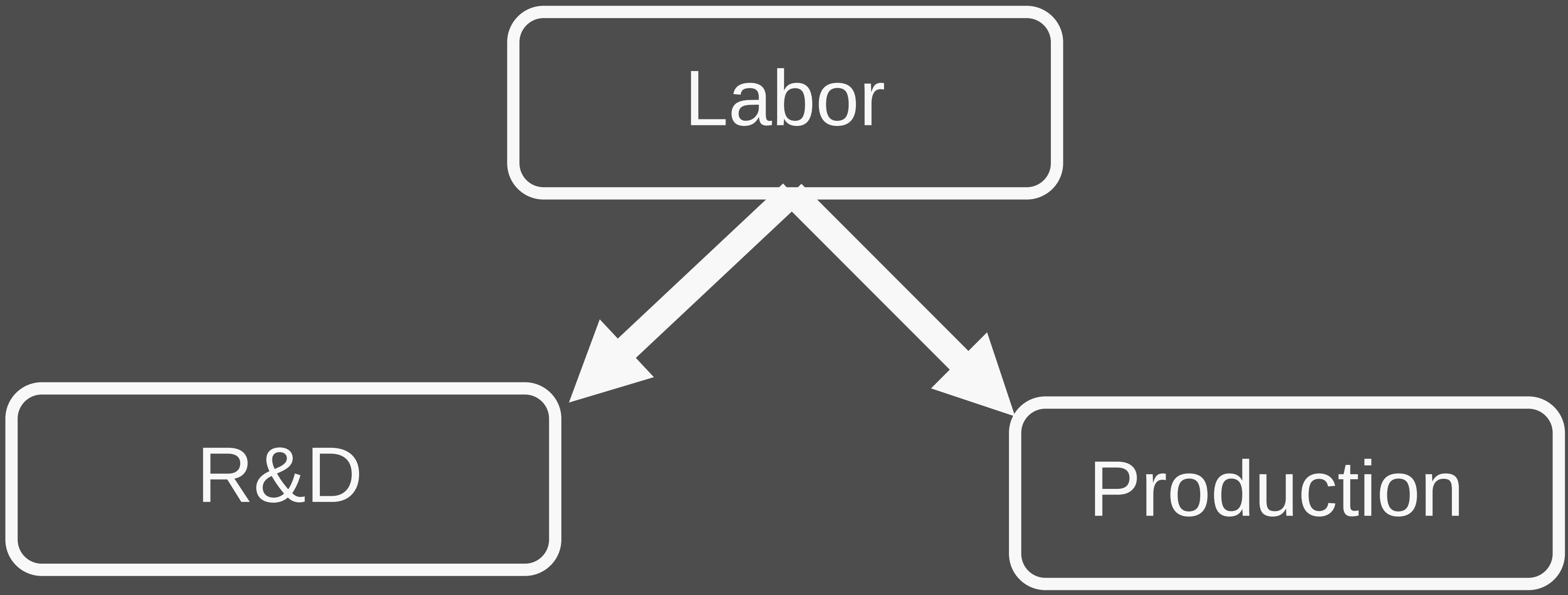

Leader
Follower
R&D
R&D
Production
Production

Leader

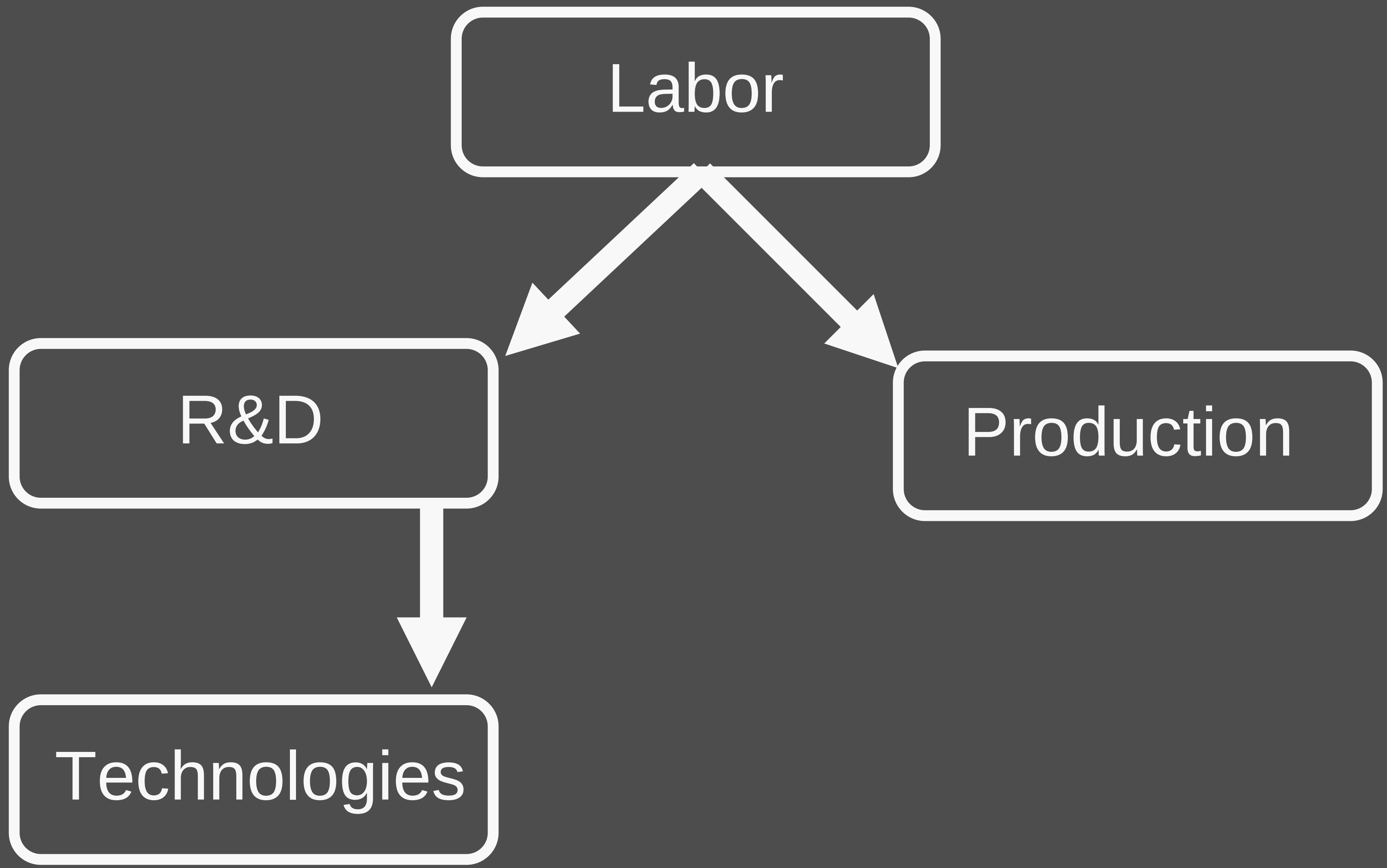
Leader

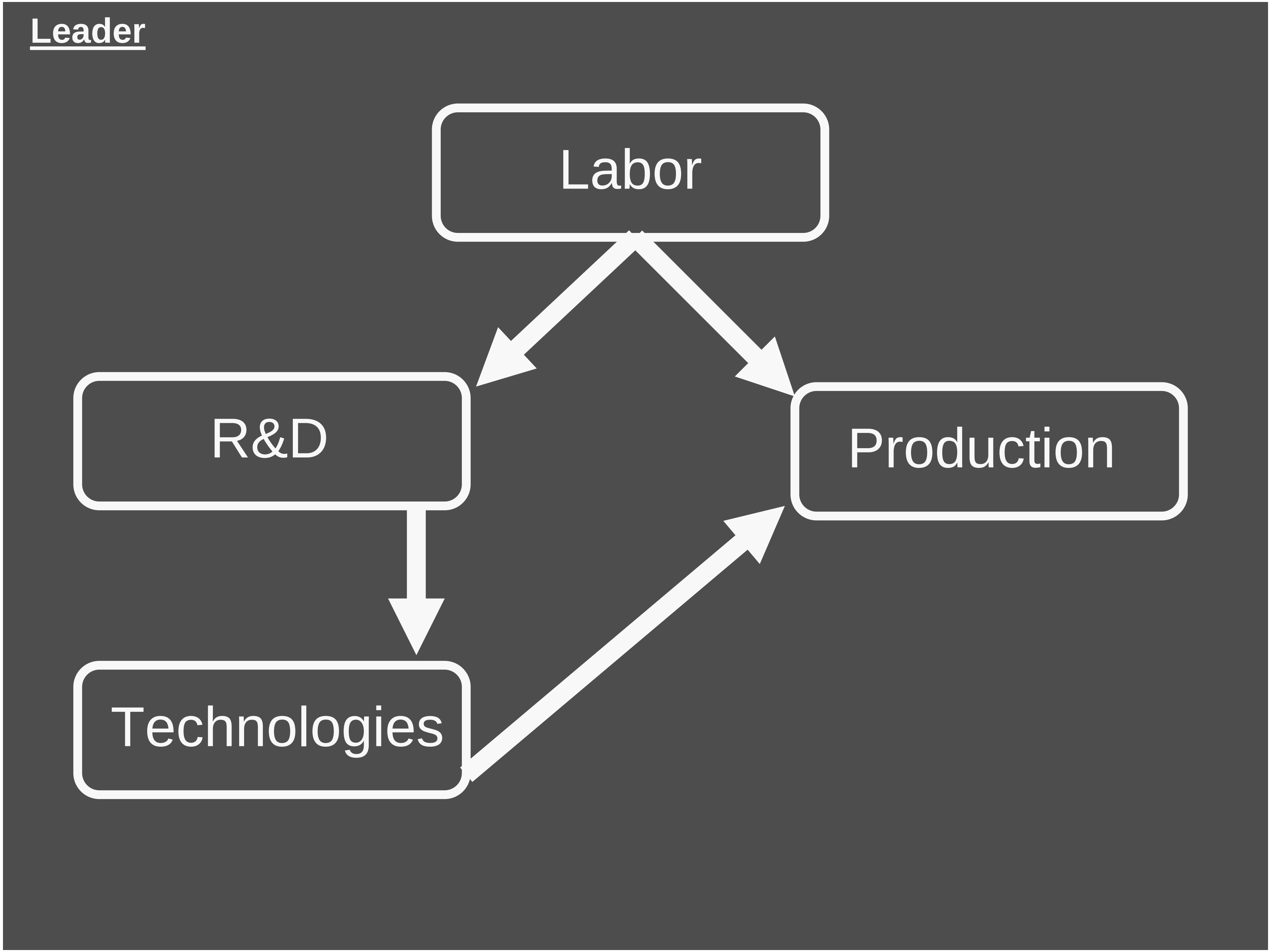
Labor

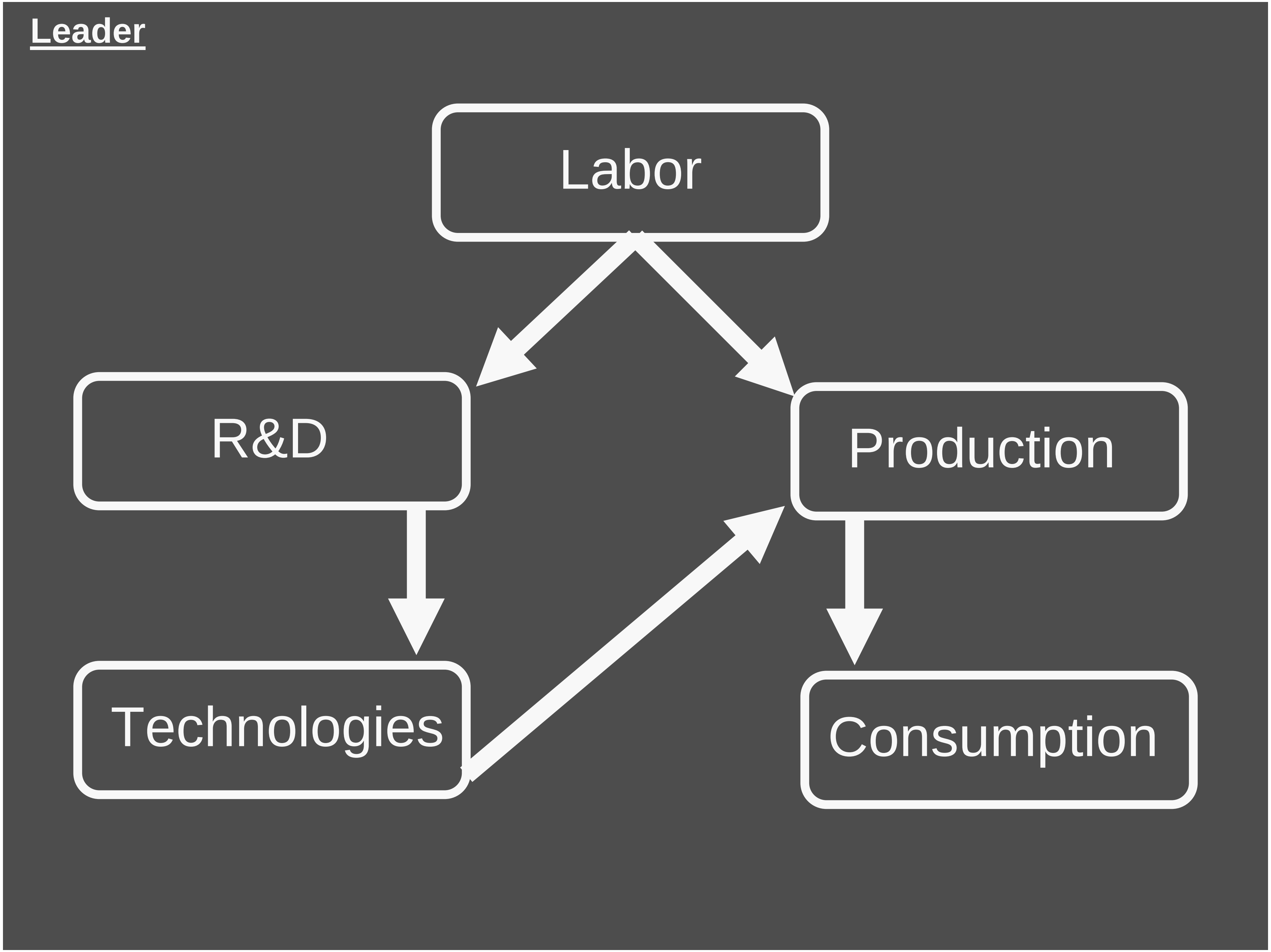
Leader



Leader




Leader
Labor
R&D
Production
Technologies



Leader
Labor
R&D
Production
Technologies
Consumption

Leader

Annual growth in technology $\approx \dfrac{\text{Labor in R\&D}}{\text{Technology stock}}$

$$T_{i+1} - T_i = L_i^{R\&D} T_i / a$$

Leader

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Annual consumption $\approx \dfrac{\text{Labor in production}}{\text{Technology stock}}$

$$C_i = (\overline{L} - L_i^{R\&D}) T_i^{1/\alpha - 1}$$

Leader

Annual growth in technology ≈ Labor in R&D Technology stock

$$T_{i+1} - T_i = L_i^{R\&D} T_i / a$$

Annual consumption ≈ Labor in production Technology stock

$$C_i = (\bar{L} - L_i^{R\&D}) T_i^{1/\alpha - 1}$$

Utility

$$U = \sum_i (1-\rho)^i [(1/\alpha - 1)\log T_i + \log(\bar{L} - L_i^{R\&D})]$$

Leader

Annual growth in technology ≈ Labor in R&D × Technology stock

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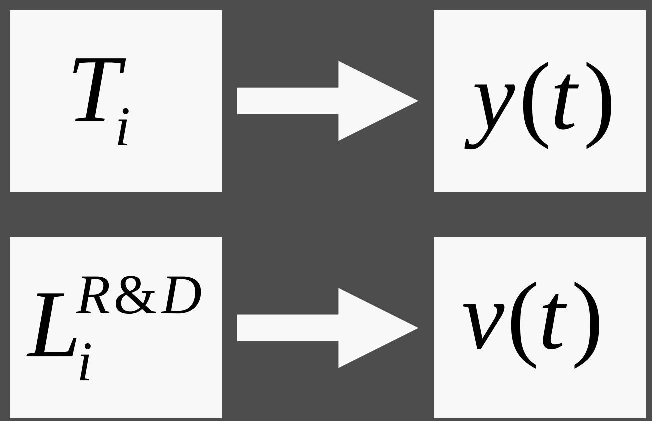
Leader



Leader

maximize

$$\bar{J} = \int_0^{\infty} e^{-\rho t}[\kappa \log y(t) + \log(\bar{L} - v(t))]dt$$

$$\dot{y}(t) = v(t)y(t)/a$$

$$y(0) = y^0$$

$$v(t) \in [0, \bar{L}]$$

$T_i \rightarrow y(t)$

$L_i^{R\&D} \rightarrow v(t)$

Leader

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$$v(t) = \bar{v} = \max[0, \bar{L} - a\alpha\rho/(1-\alpha)]$$

Leader

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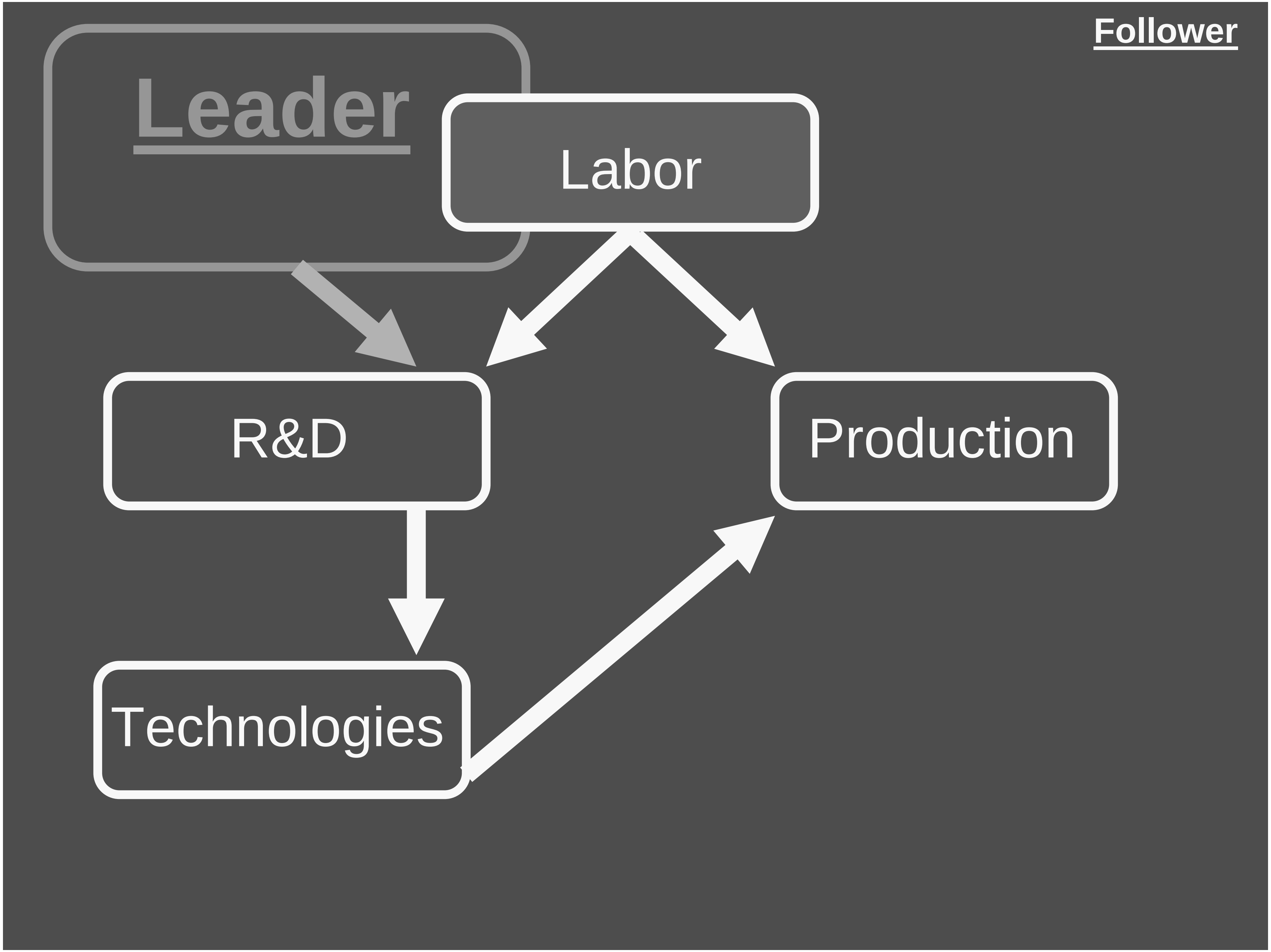
$$\dot{y}(t) = \nu y(t)$$

$$\nu = \bar{v}$$


Follower
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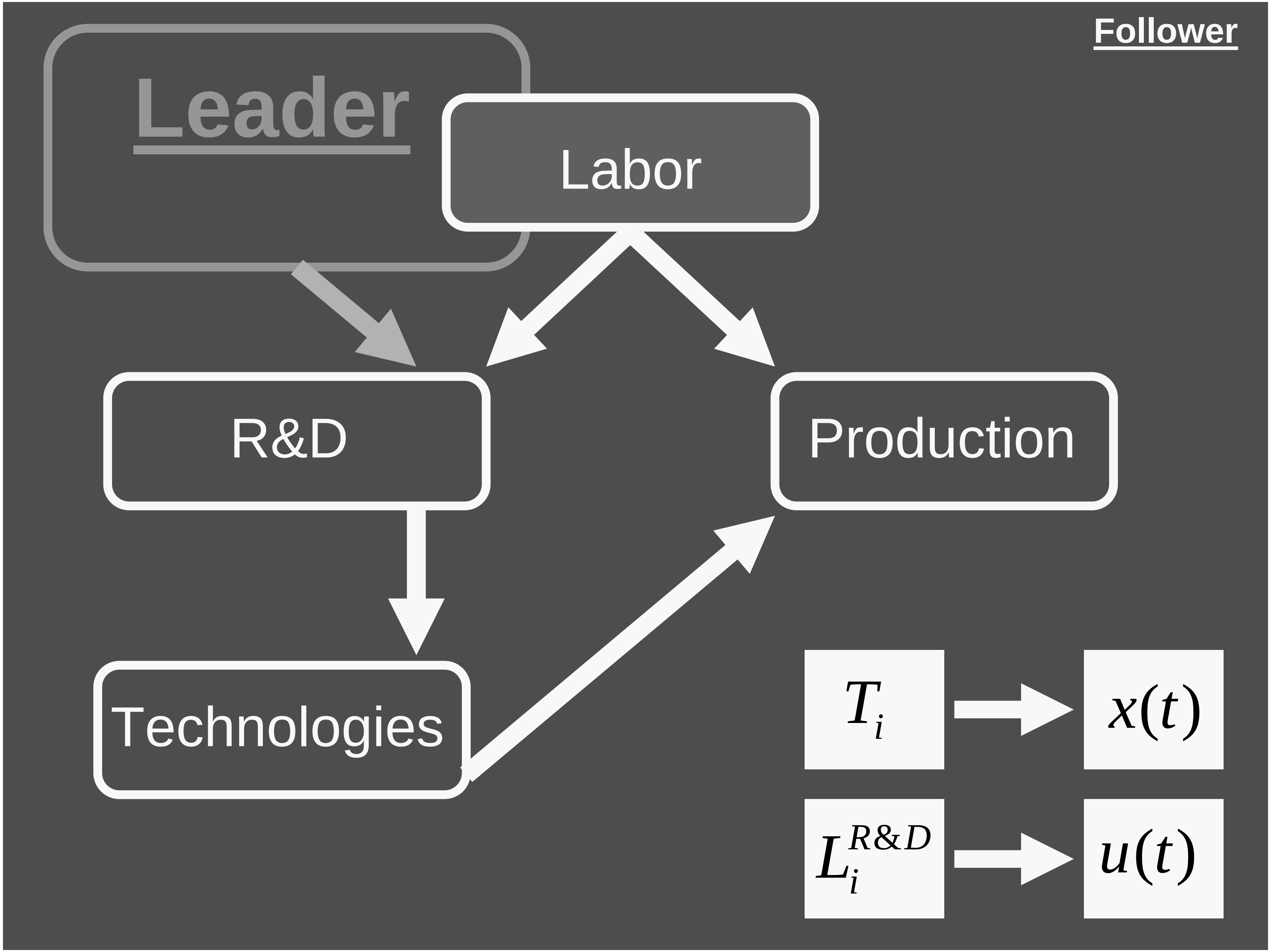
Follower
Leader
Labor
R&D
Production
Technologies
T_i
$x(t)$
$L_i^{R\&D}$
$u(t)$

Follower

maximize

$$J = \int_0^{\infty} e^{-\rho t}[\kappa \log y(t) + \log(b - u(t))]dt$$

$$\dot{x}(t) = u(t)[x(t) + \gamma y(t)]$$

$$x(0) = x^0$$

$$u(t) \in [0, b)$$

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absorption capacity

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absorption capacity

$$\dot{y}(t) = \nu y(t)$$

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Follower

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$$\dot{y}(t) = v y(t)$$

$$y(0) = y^0$$

$$z(t) = x(t) / y(t)$$

$$z(0) = x^0 / y^0$$

maximize

$$J = \int_0^\infty e^{-\rho t}[\kappa \log z(t) + \log(b - u(t))]dt$$

$$\dot{z}(t) = u(t)[z(t) + \gamma] - \nu z(t)$$

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$\tilde{M}(z(t), p(t), u)$ current Hamilton-Pontryagin function

Follower

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Follower

$$\dot{z}(t) = \varphi_1(z(t), p(t))$$
$$\dot{p}(t) = \varphi_2(z(t), p(t))$$

Hamiltonian system

$$\dot{z}(t) = u(t)[z(t) + \gamma] - \nu z(t)$$
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Hamiltonian system

Pontryagin maximum principle

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Hamiltonian system

Pontryagin maximum principle

If $z(t)$ is optimal, then there is a positive $p(t)$ such that $(z(t), p(t))$ solves the Hamiltonian system and

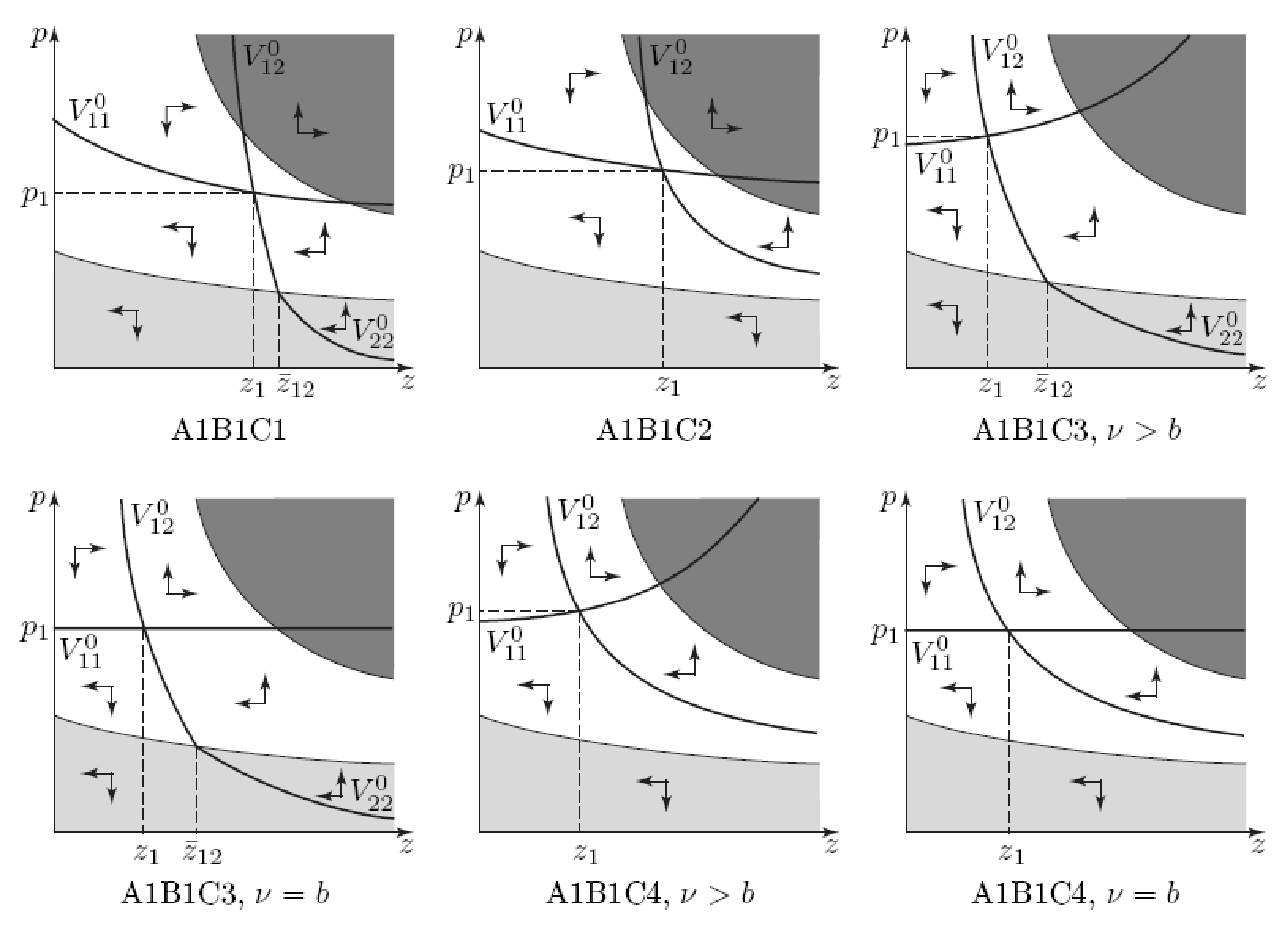
$$z(t)\,p(t) \leq \frac{\kappa}{\rho}$$

Vector field of the Hamiltonian system

Follower

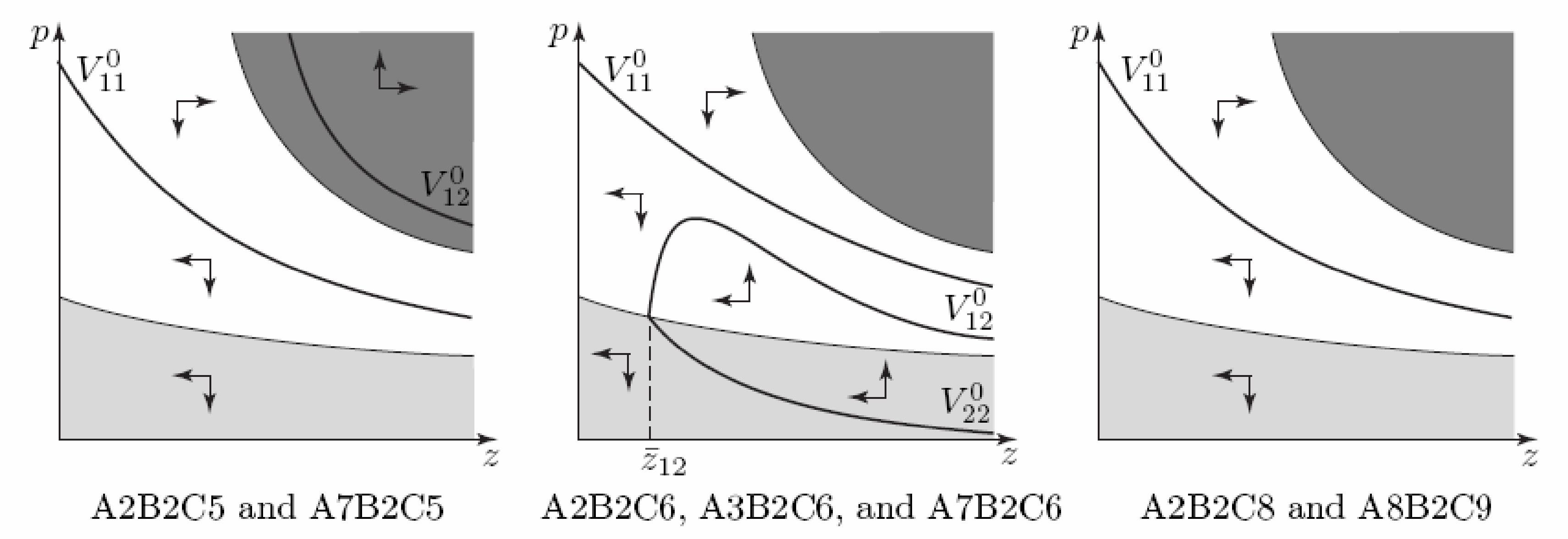
Follower

Vector field of the Hamiltonian system



Regular non-degenerate cases

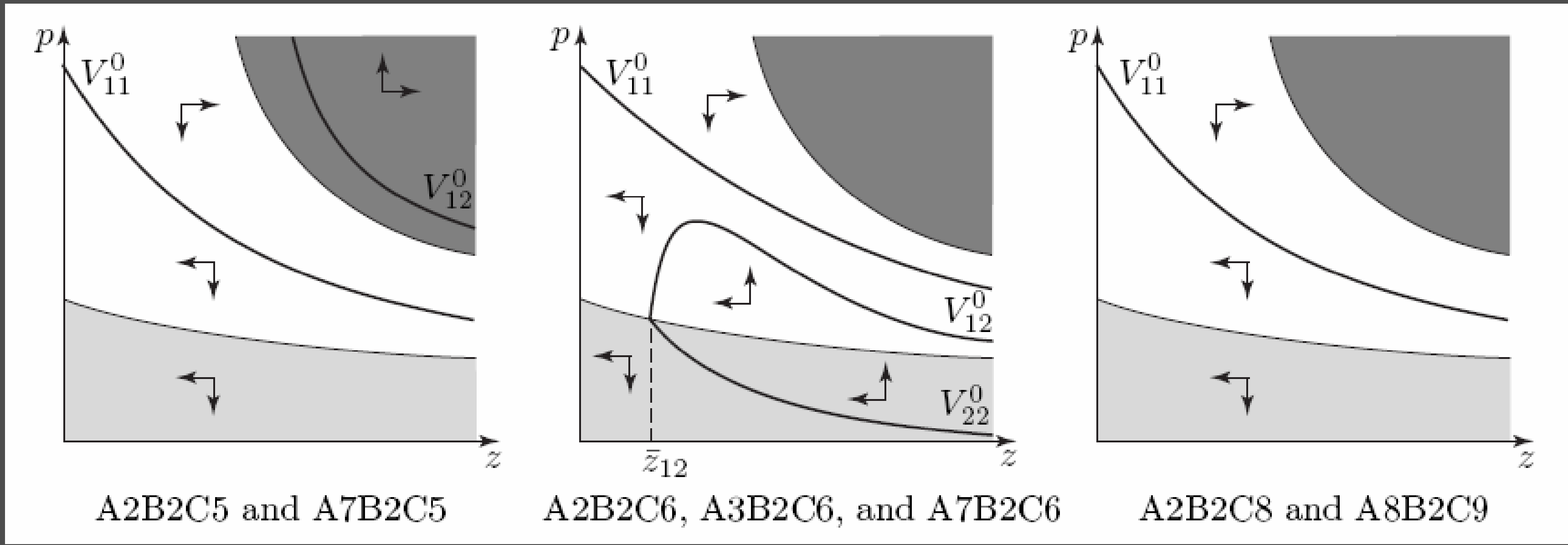
Vector field of the Hamiltonian system



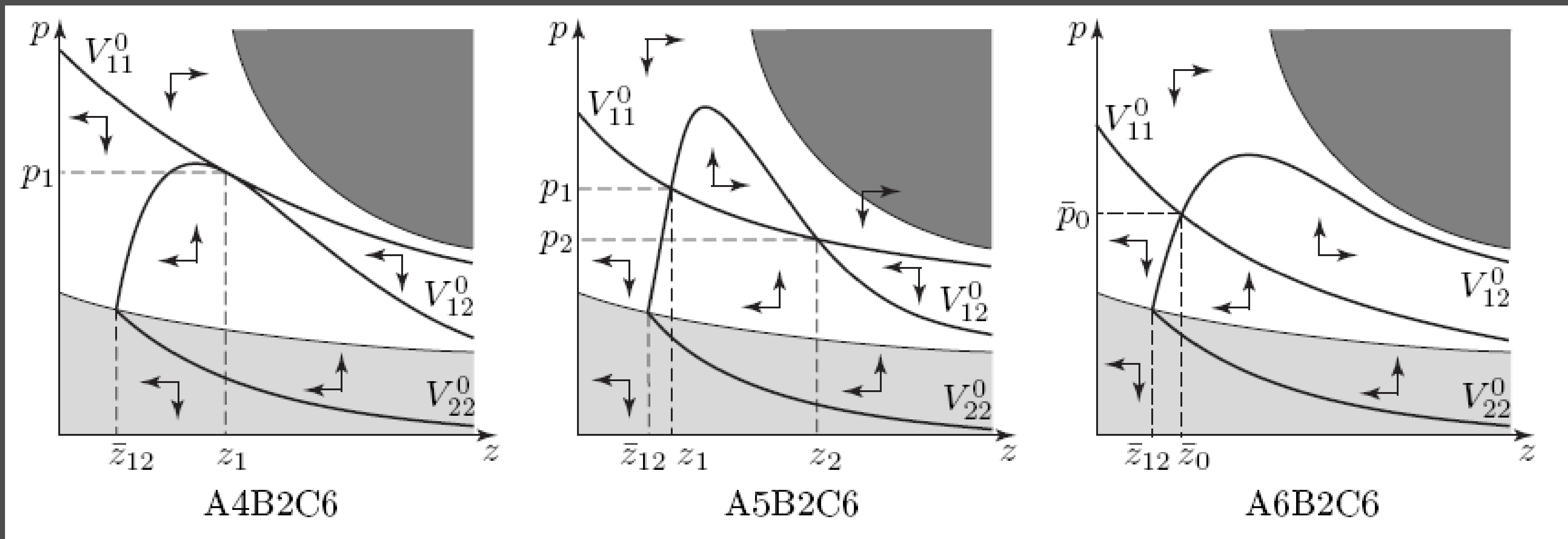
Degenerate cases

Vector field of the Hamiltonian system

Follower



Degenerate cases



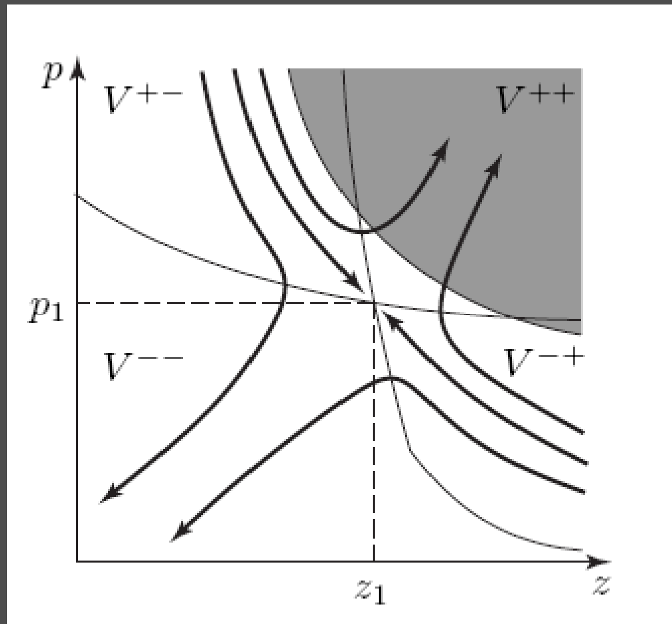
Singular non-degenerate cases

Follower

Solutions in regular non-degenerate cases

Follower

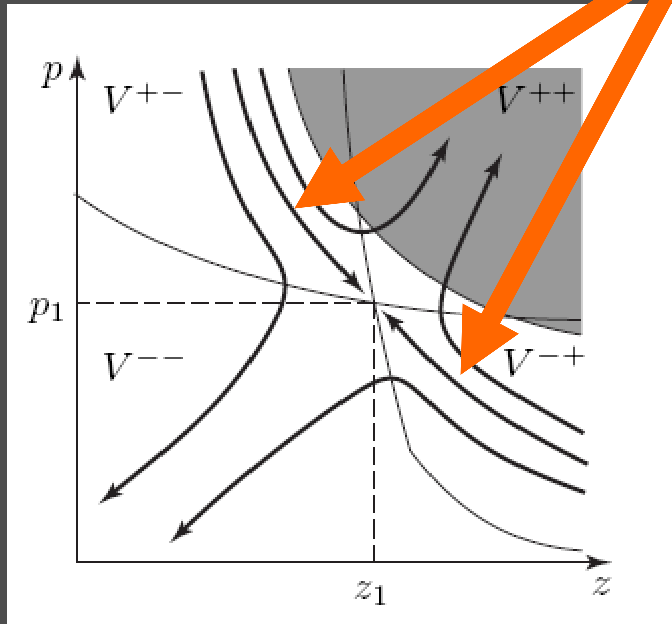
Solutions in regular non-degenerate cases



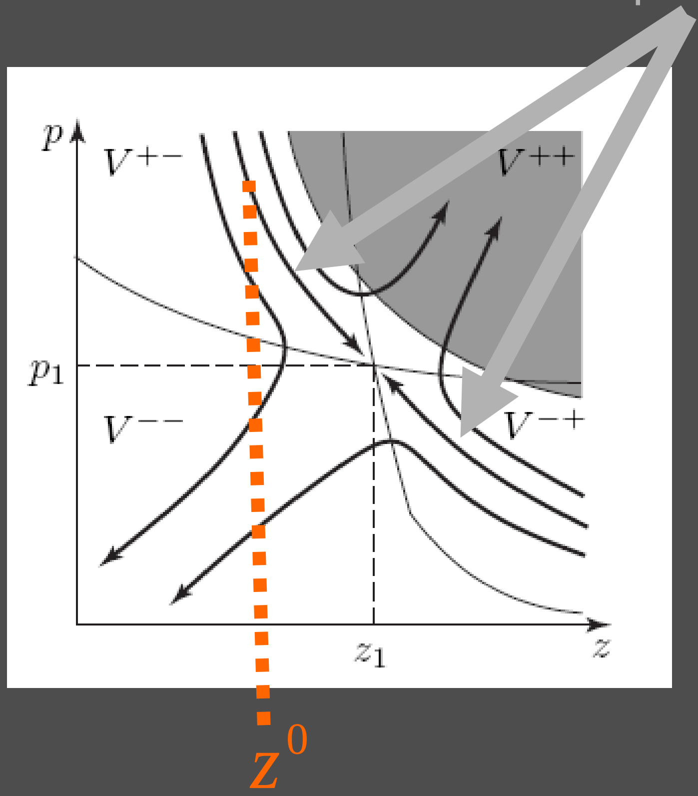
Solutions in regular non-degenerate cases

Follower

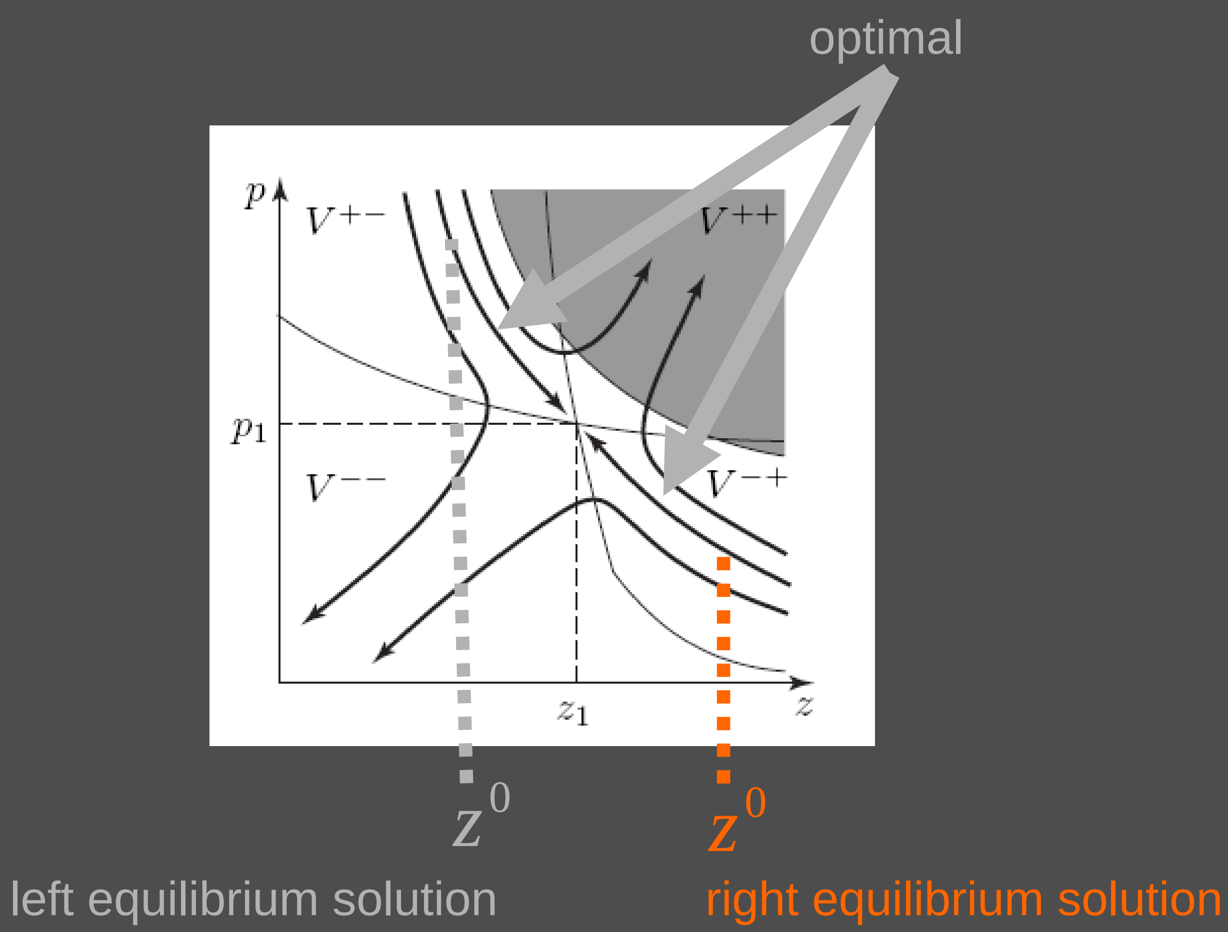
optimal



Solutions in regular non-degenerate cases



Solutions in regular non-degenerate cases

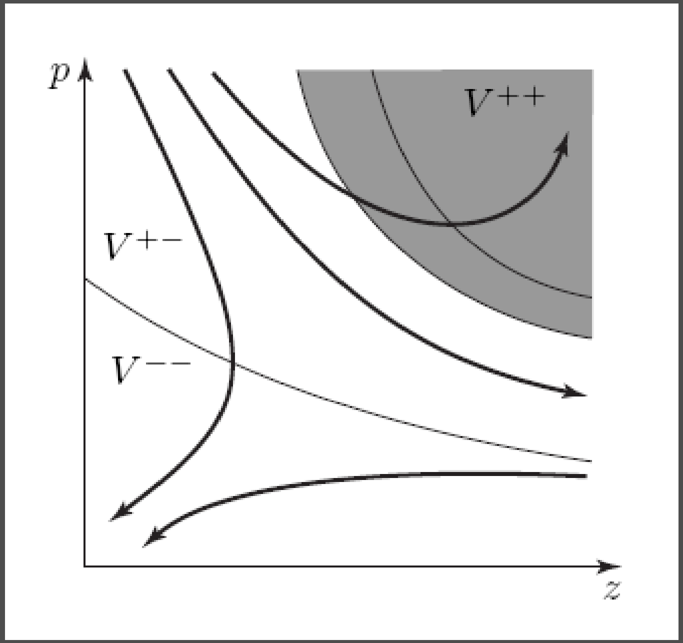


Solutions in degenerate cases

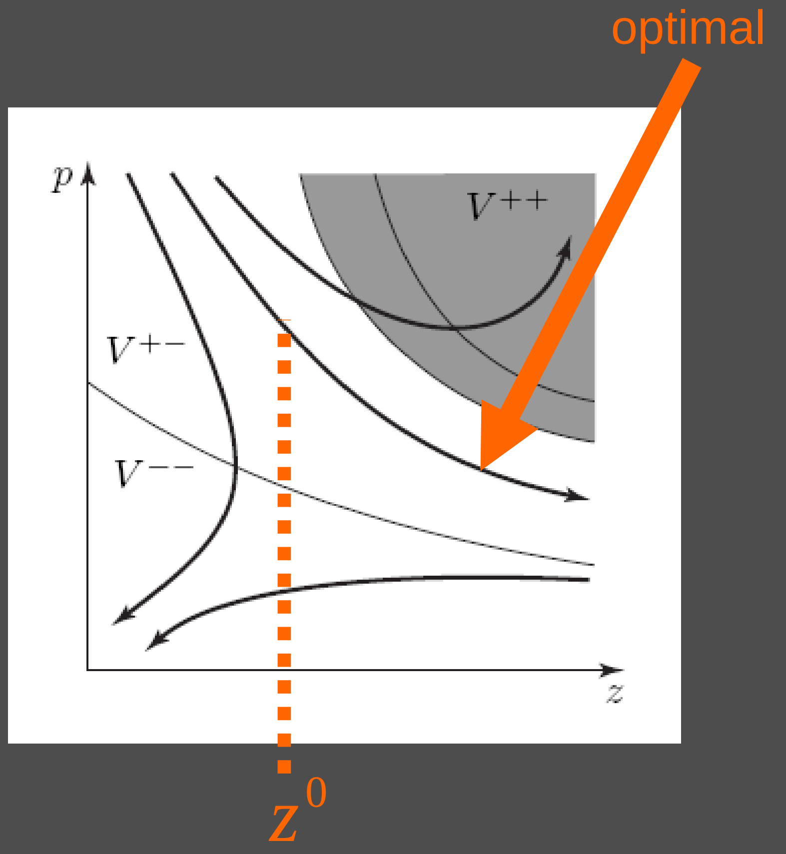
Follower

Solutions in degenerate cases

Follower



Solutions in degenerate cases

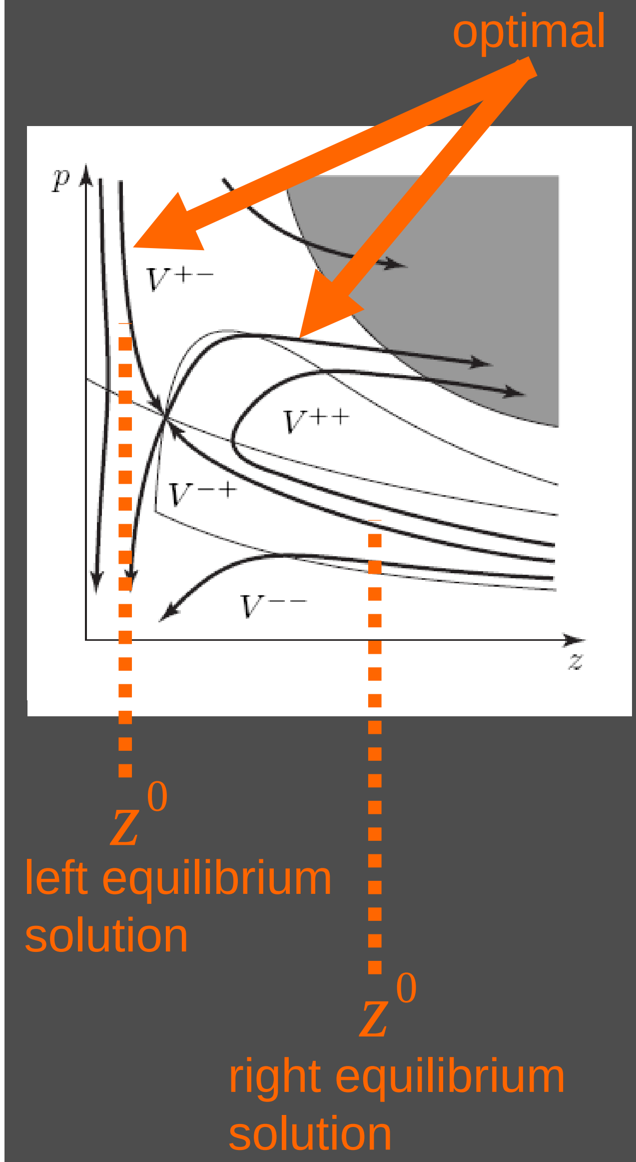


Solutions in singular non-degenerate cases

Follower

Solutions in singular non-degenerate cases

Follower



Solutions in singular non-degenerate cases

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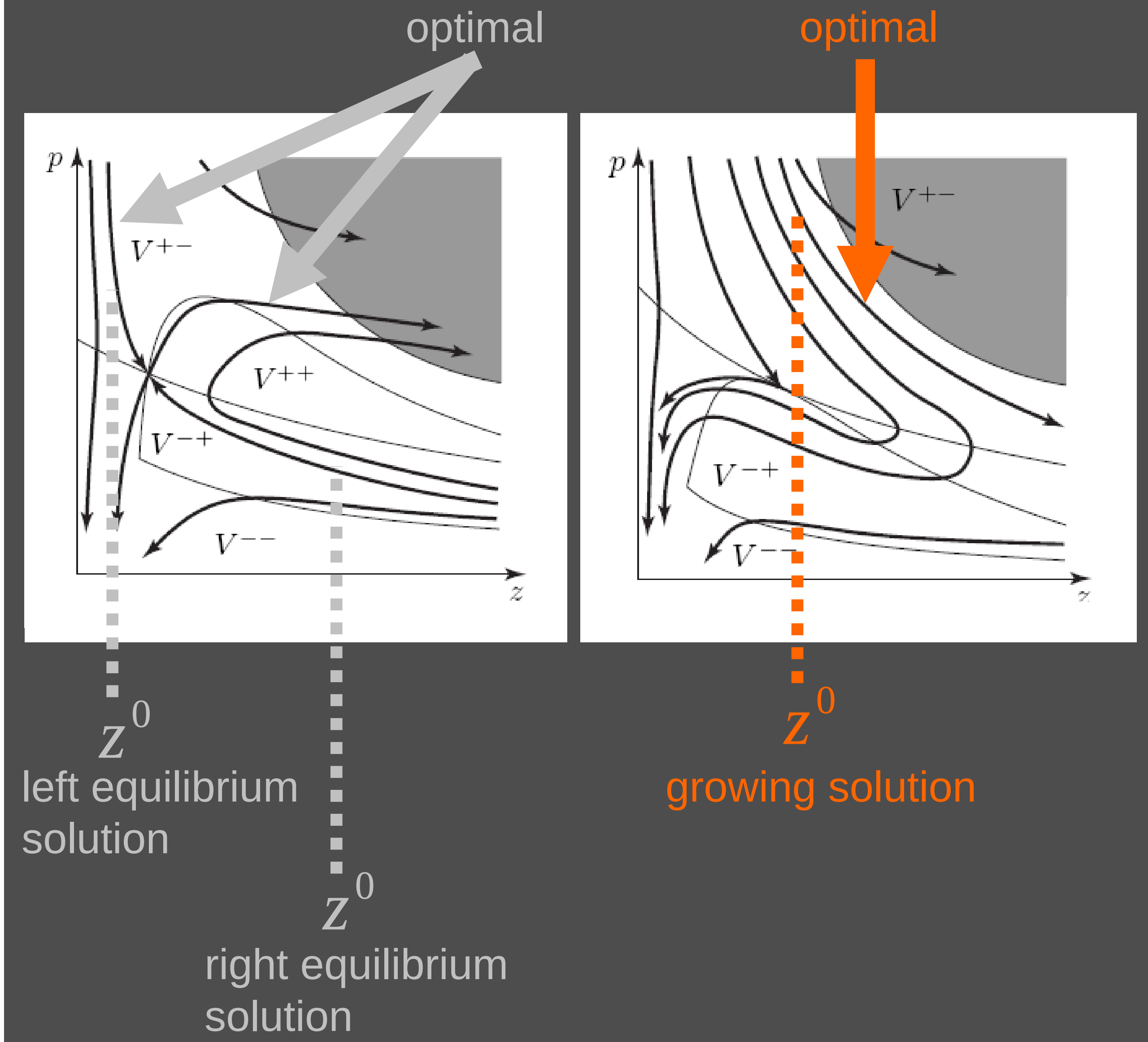

Solutions in singular non-degenerate cases

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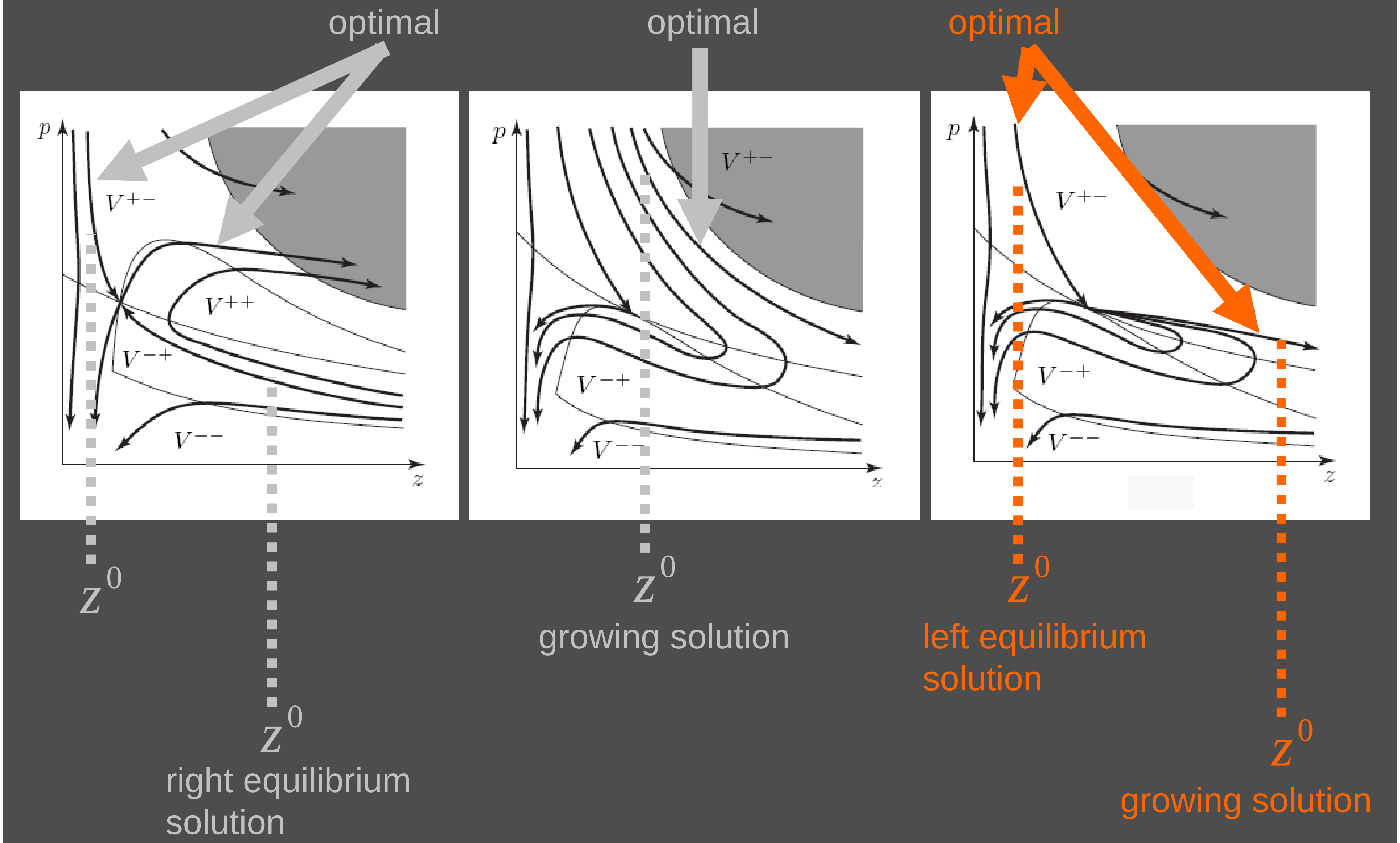

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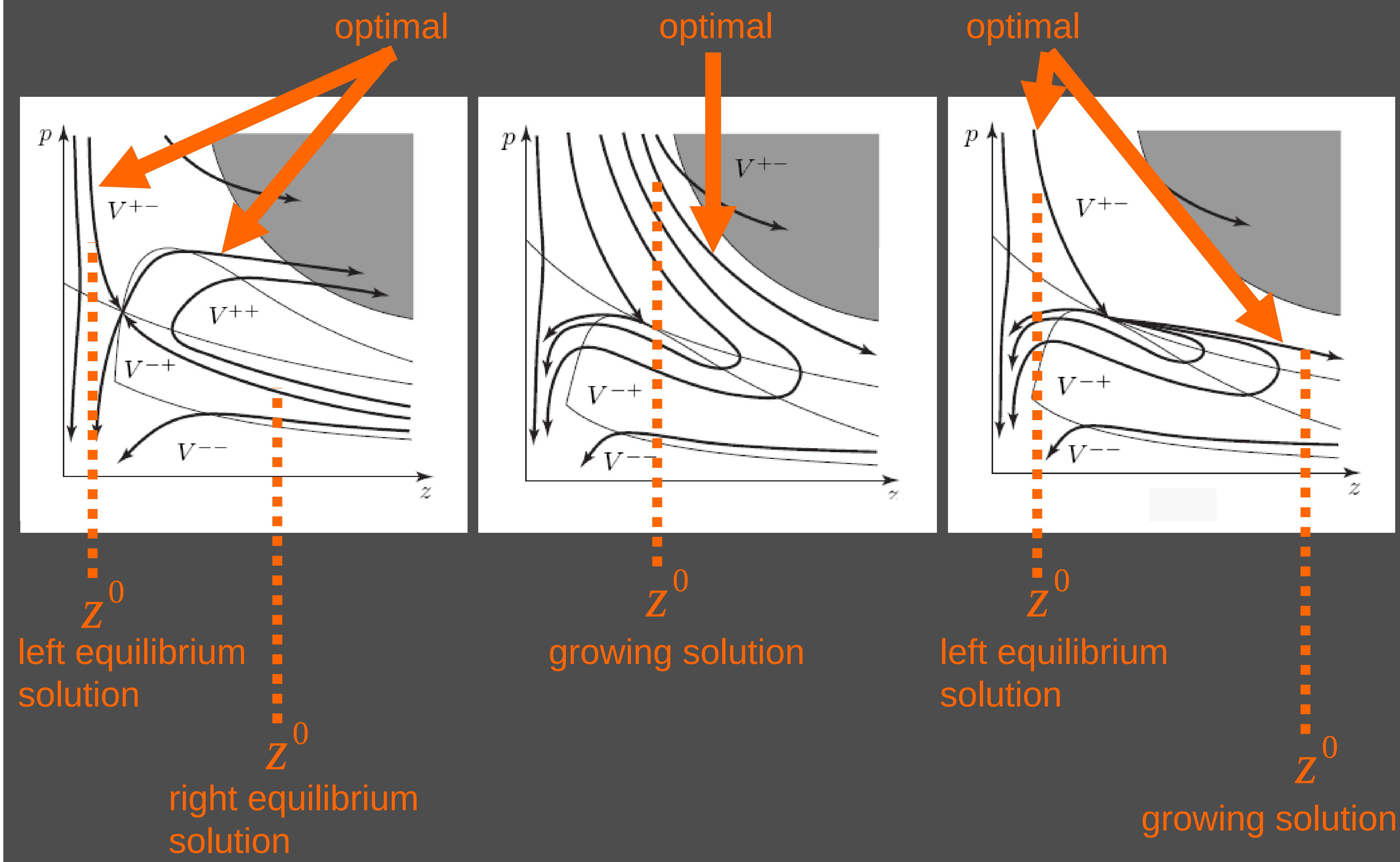
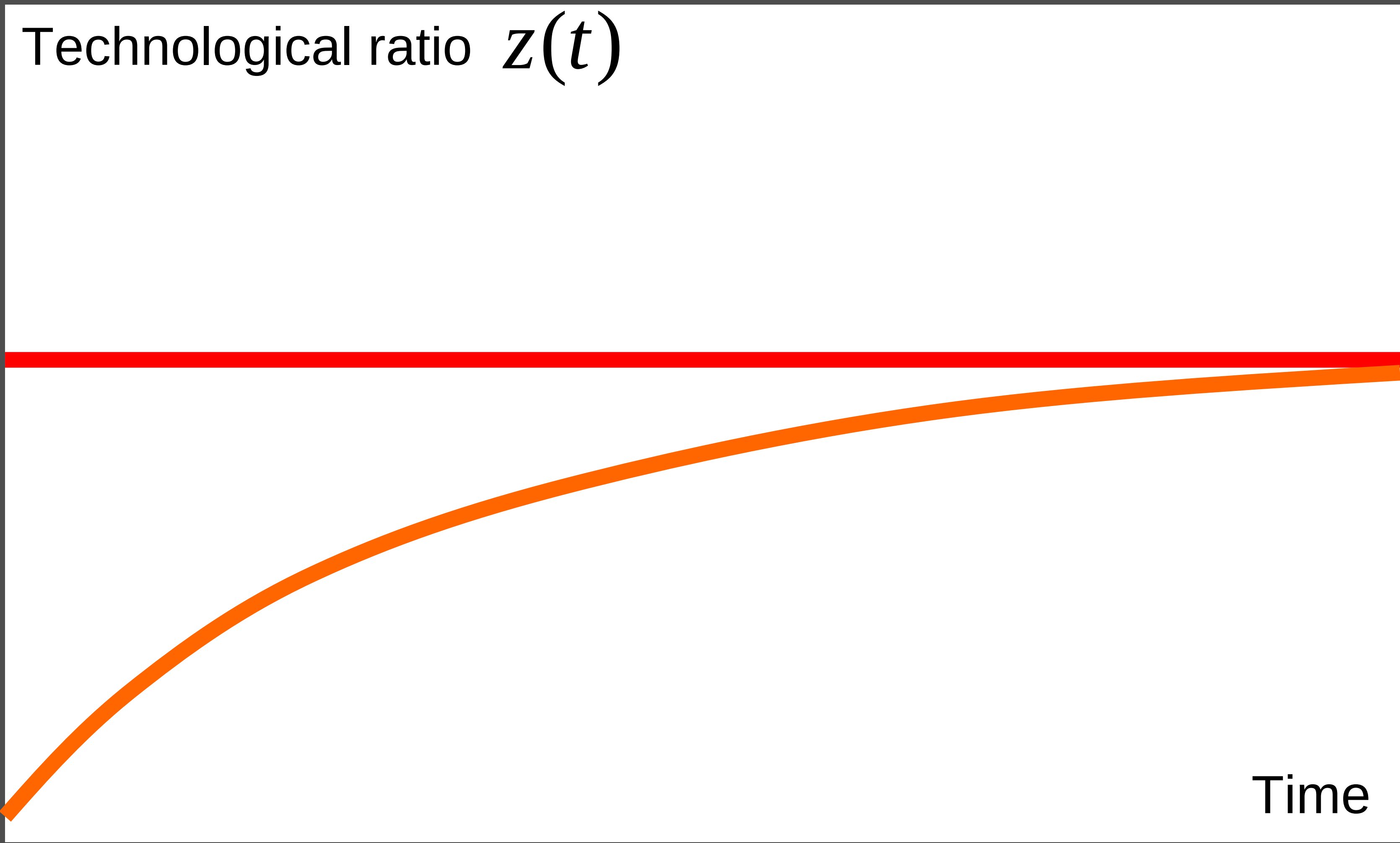


Follower

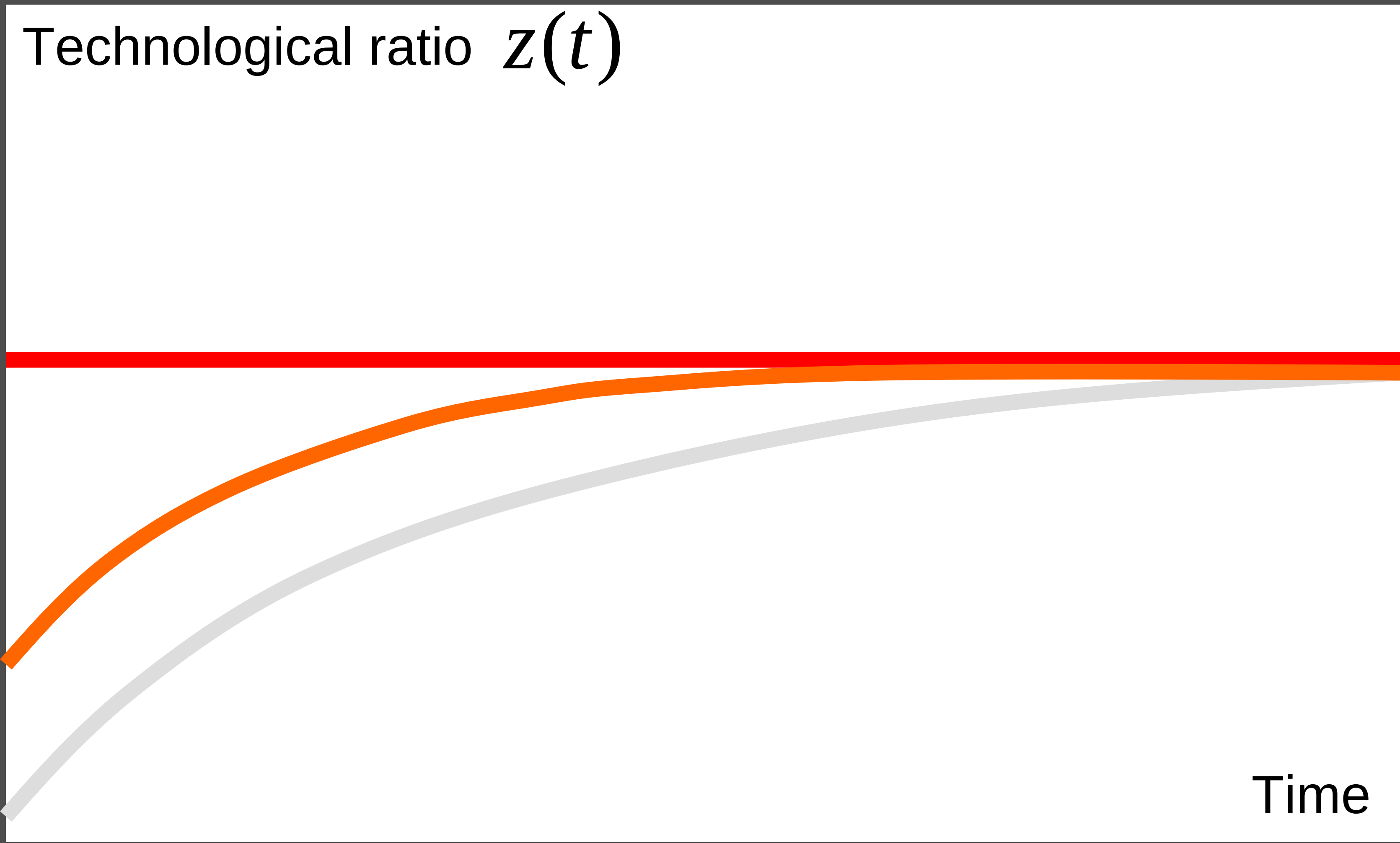
Equilibrium solution: catching up

Equilibrium solution: catching up



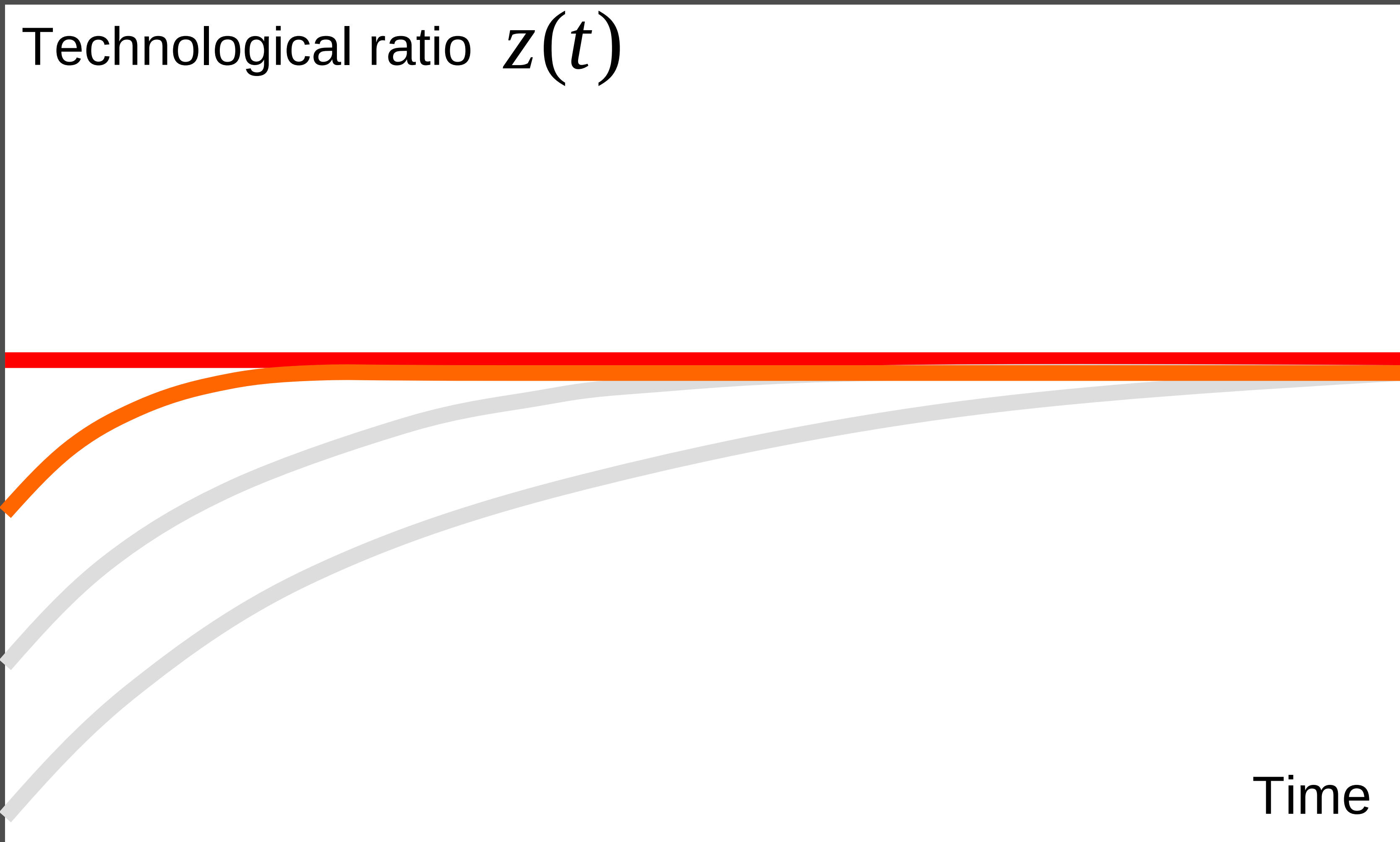
Equilibrium solution: catching up

Follower



Follower

Equilibrium solution: catching up



Equilibrium solution: catching up

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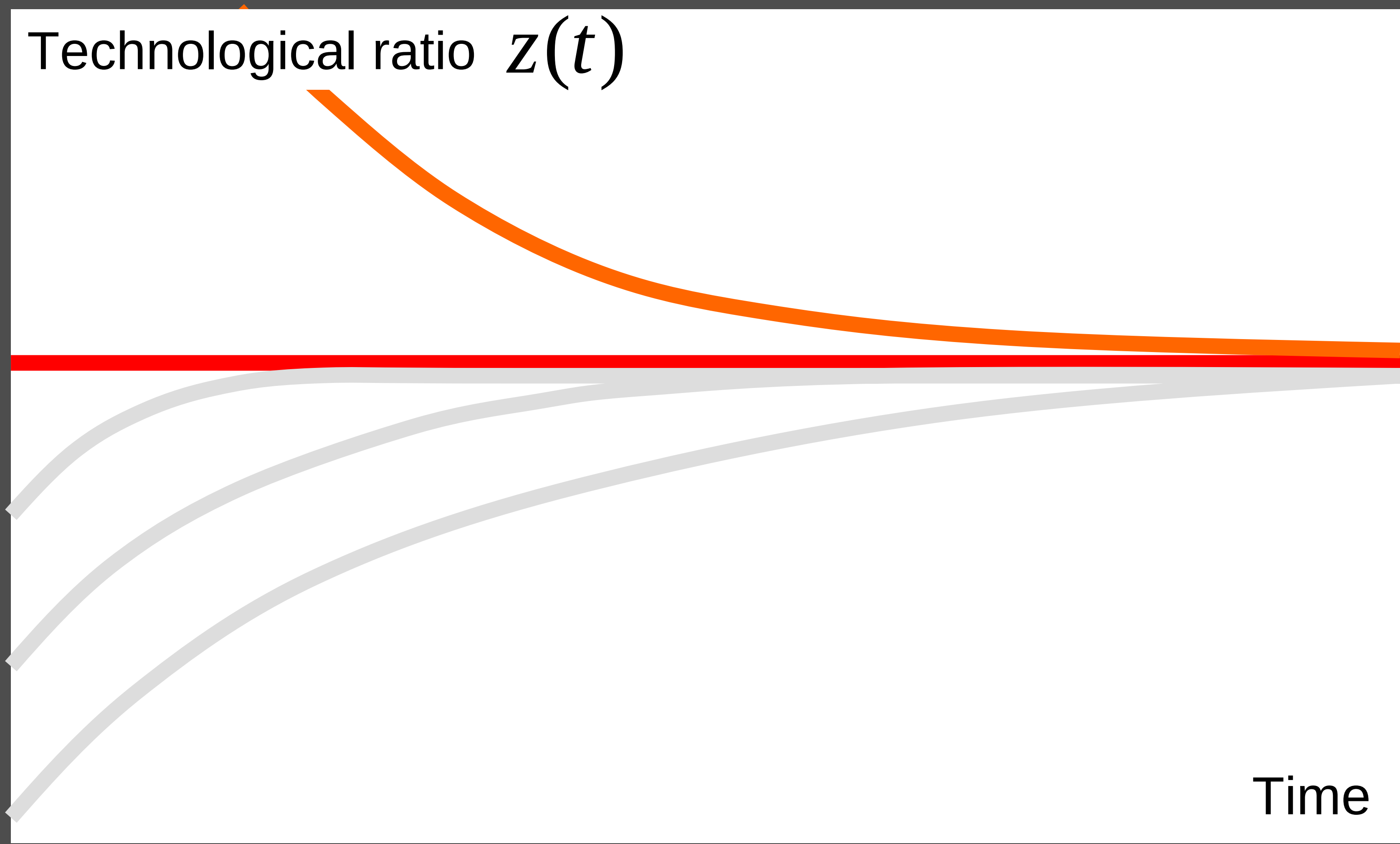
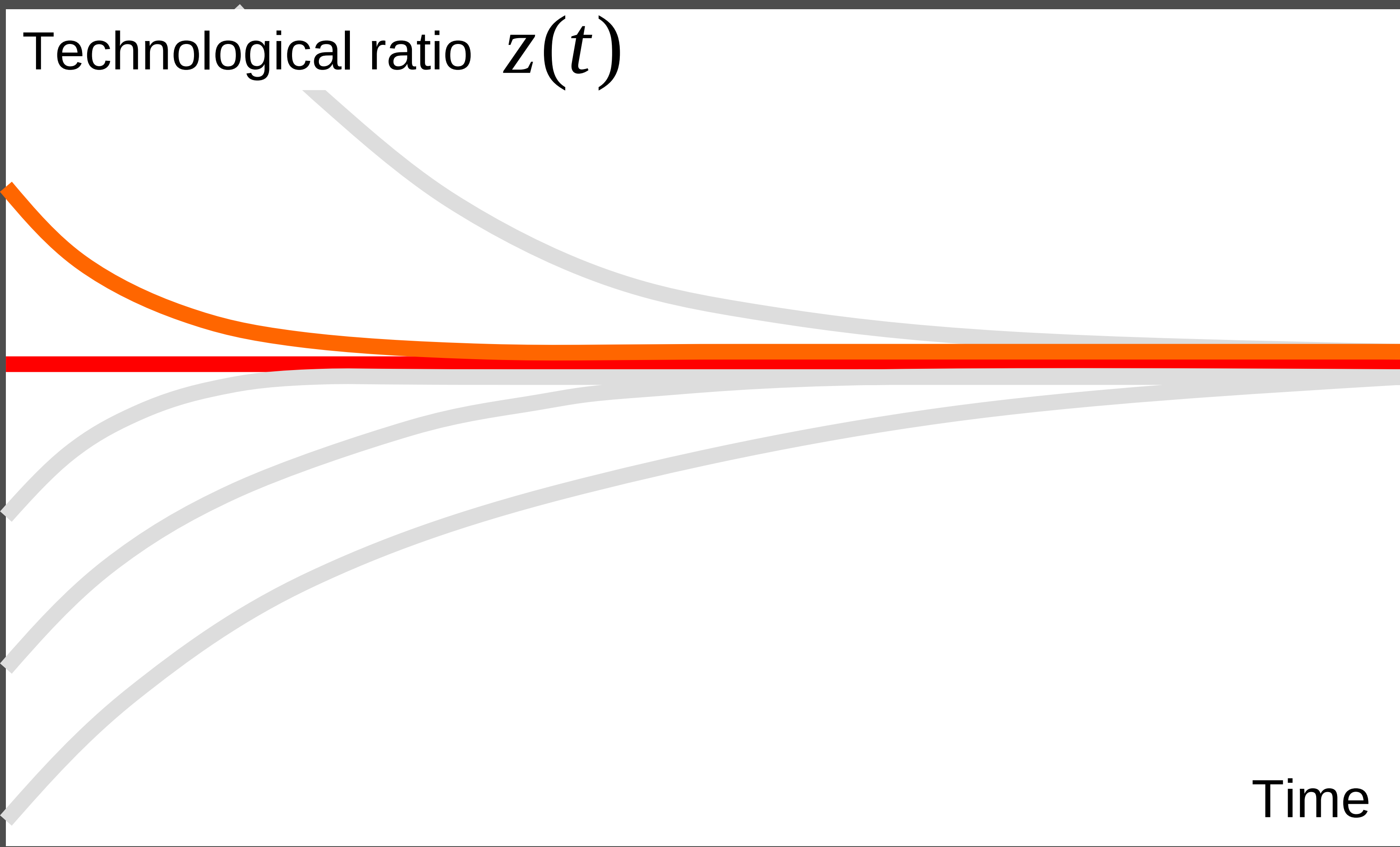


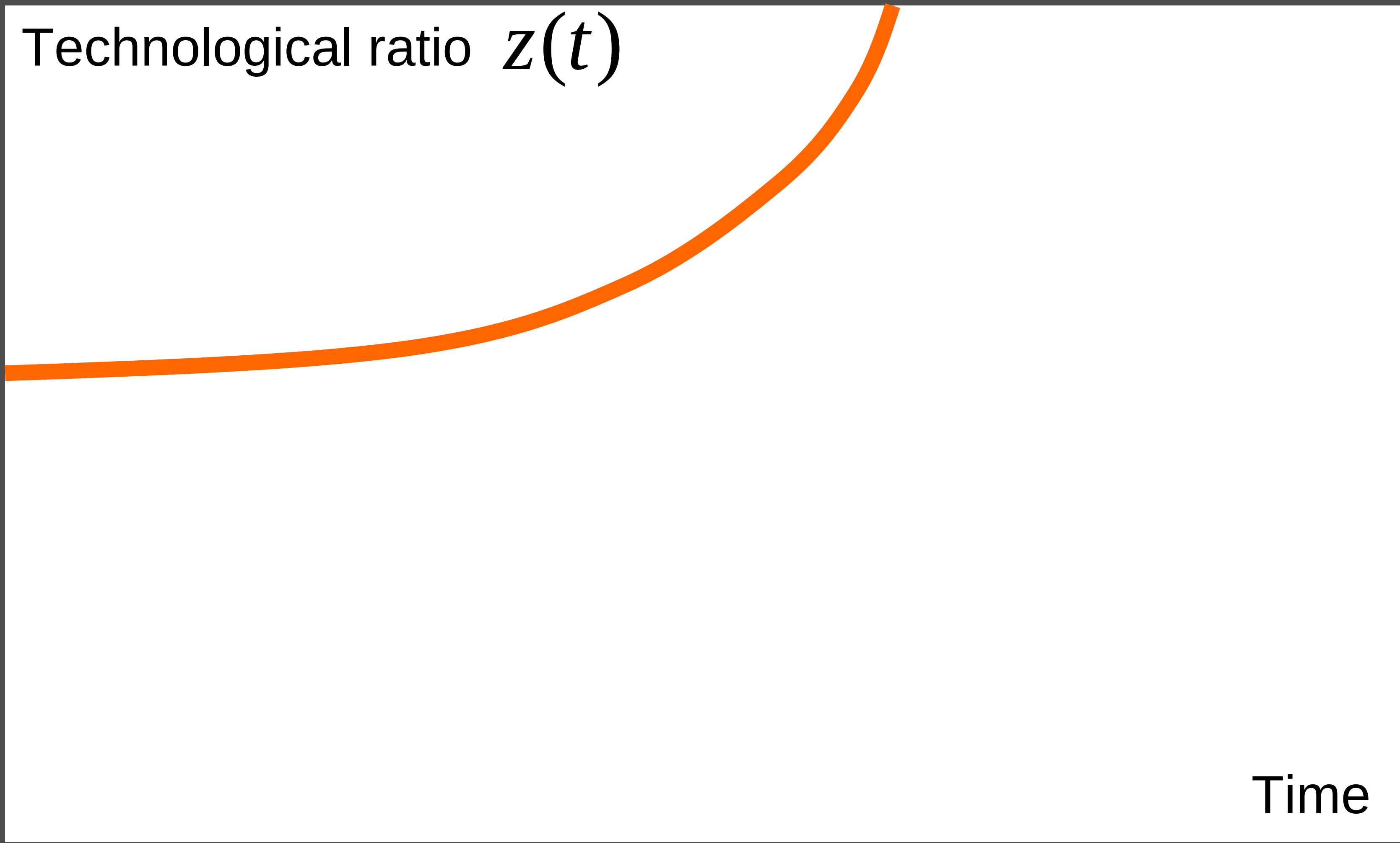
Equilibrium solution: catching up



Growing solution: overtaking

Growing solution: overtaking

Follower



Growing solution: overtaking

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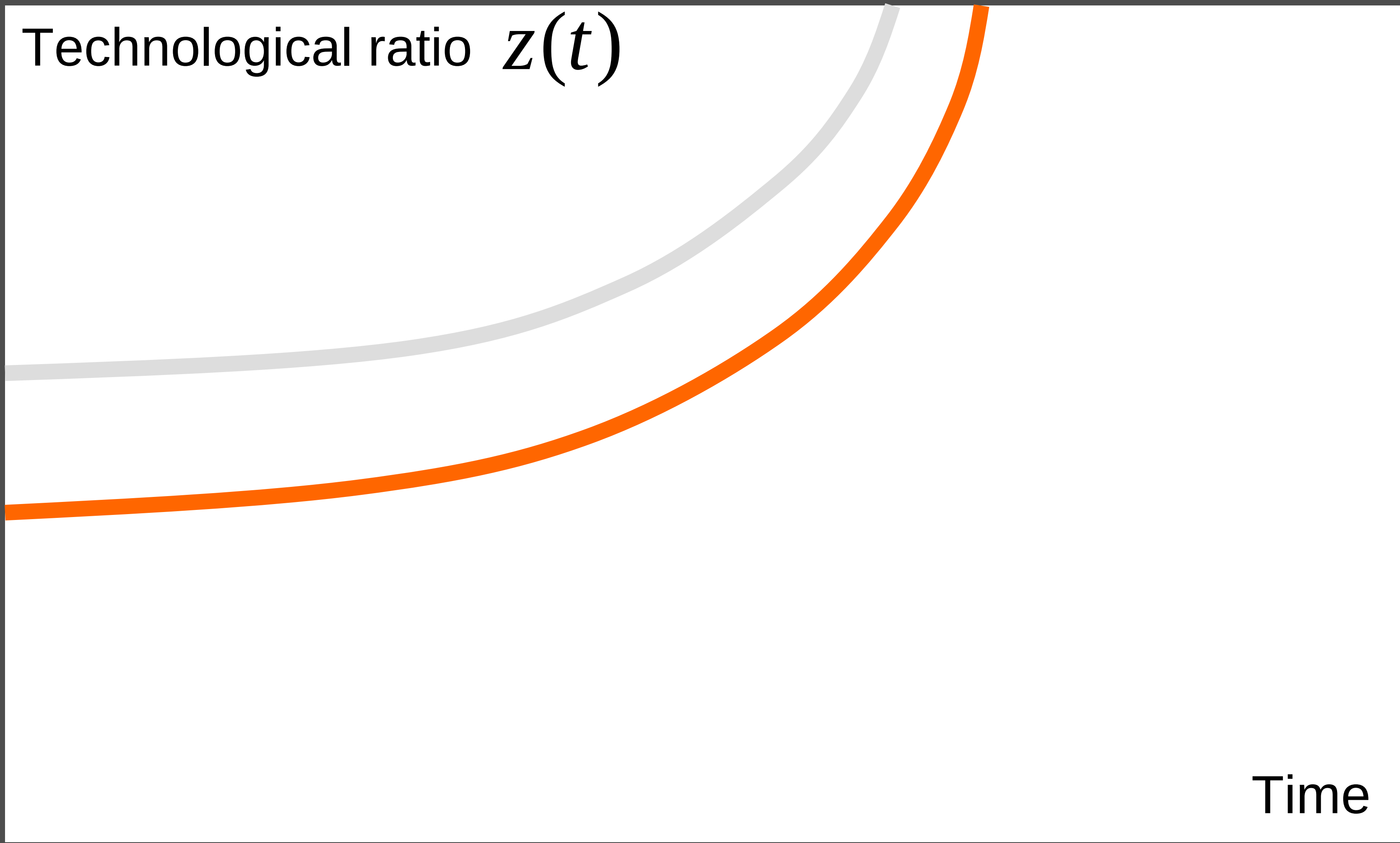

Growing solution: overtaking

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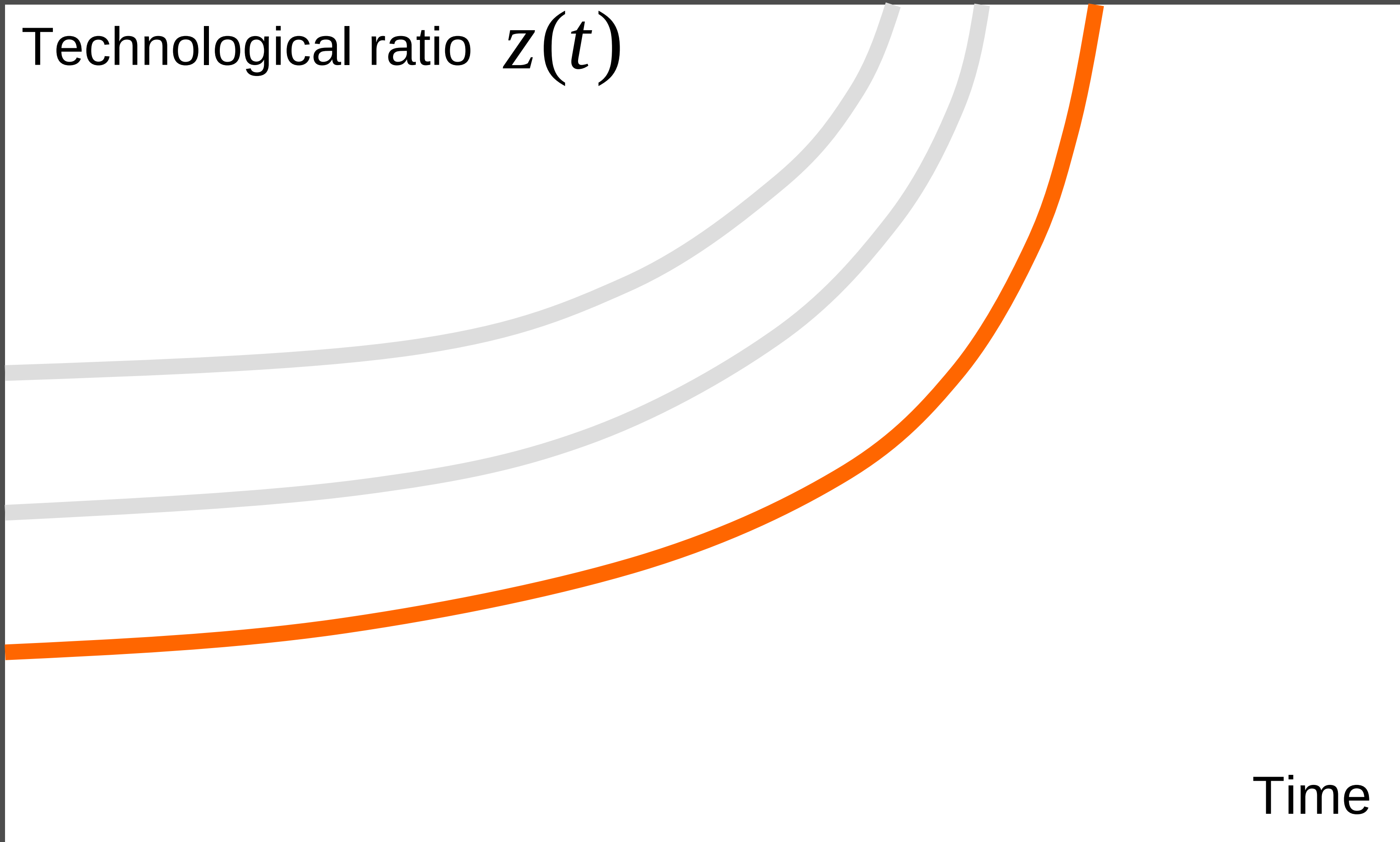
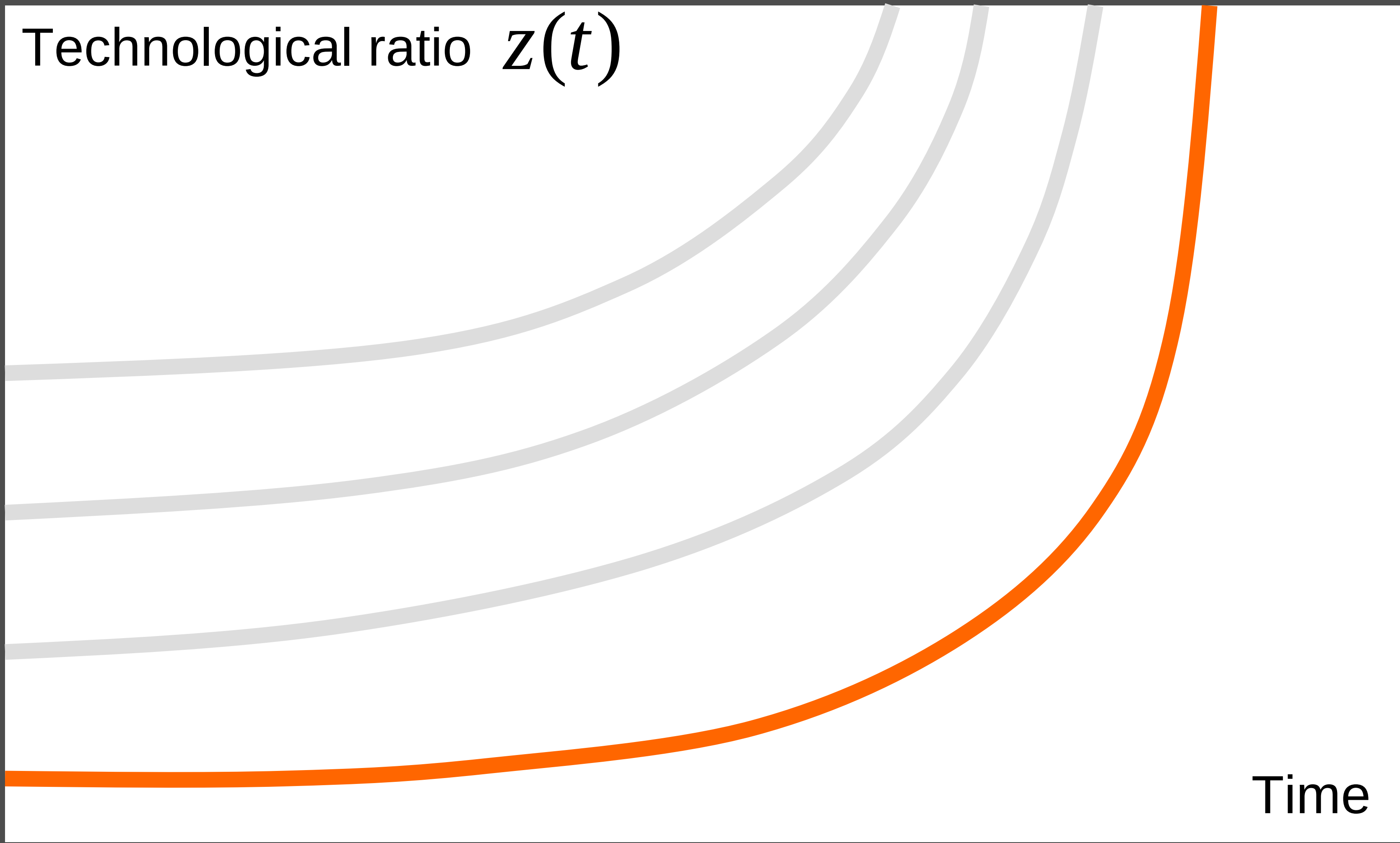


Follower

Growing solution: overtaking



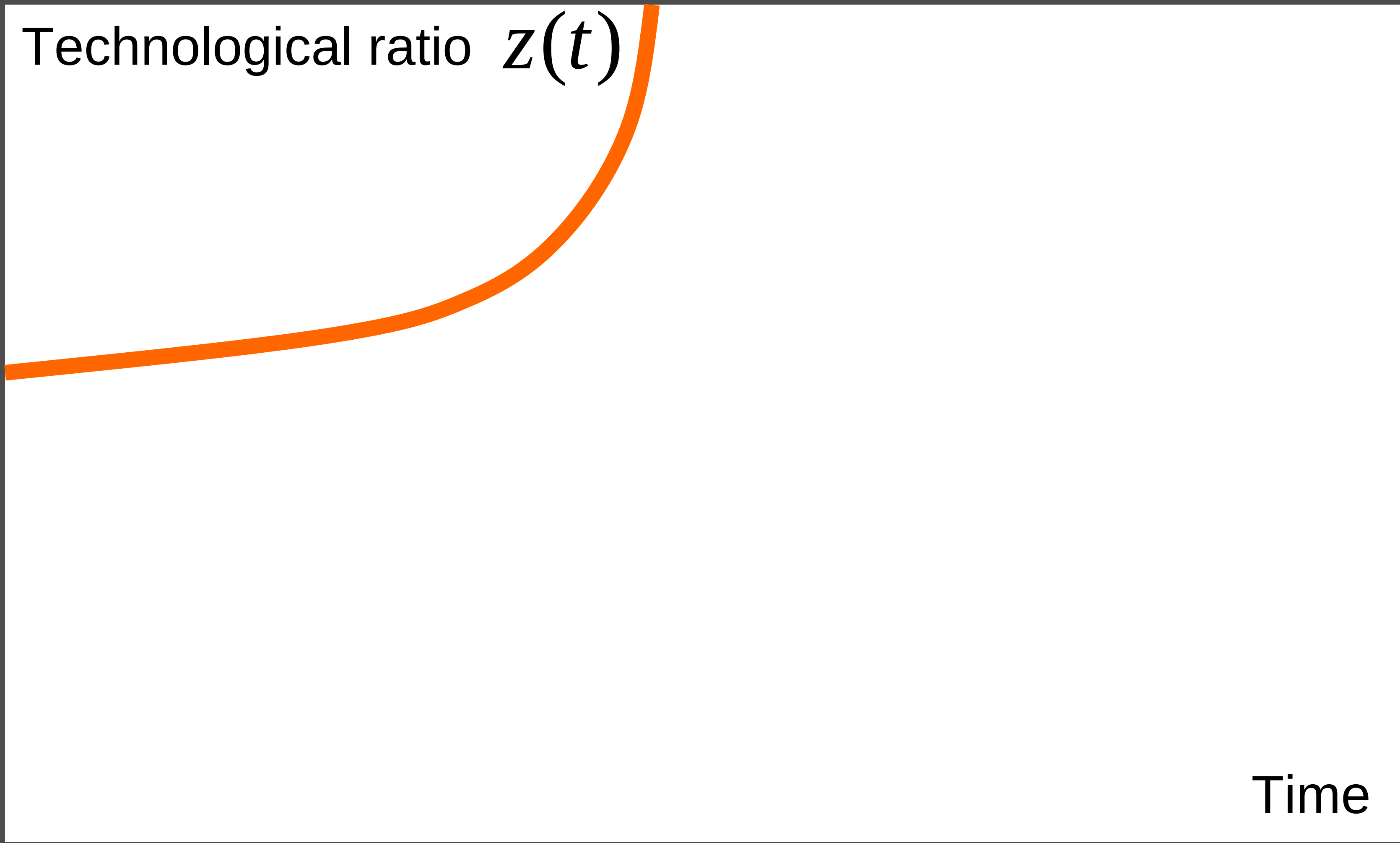
Follower

Sensitivity in ρ

$$J = \int_0^\infty e^{-\rho t}[\kappa \log y(t) + \log(b - u(t))]dt$$

Follower

Sensitivity in ρ



Follower

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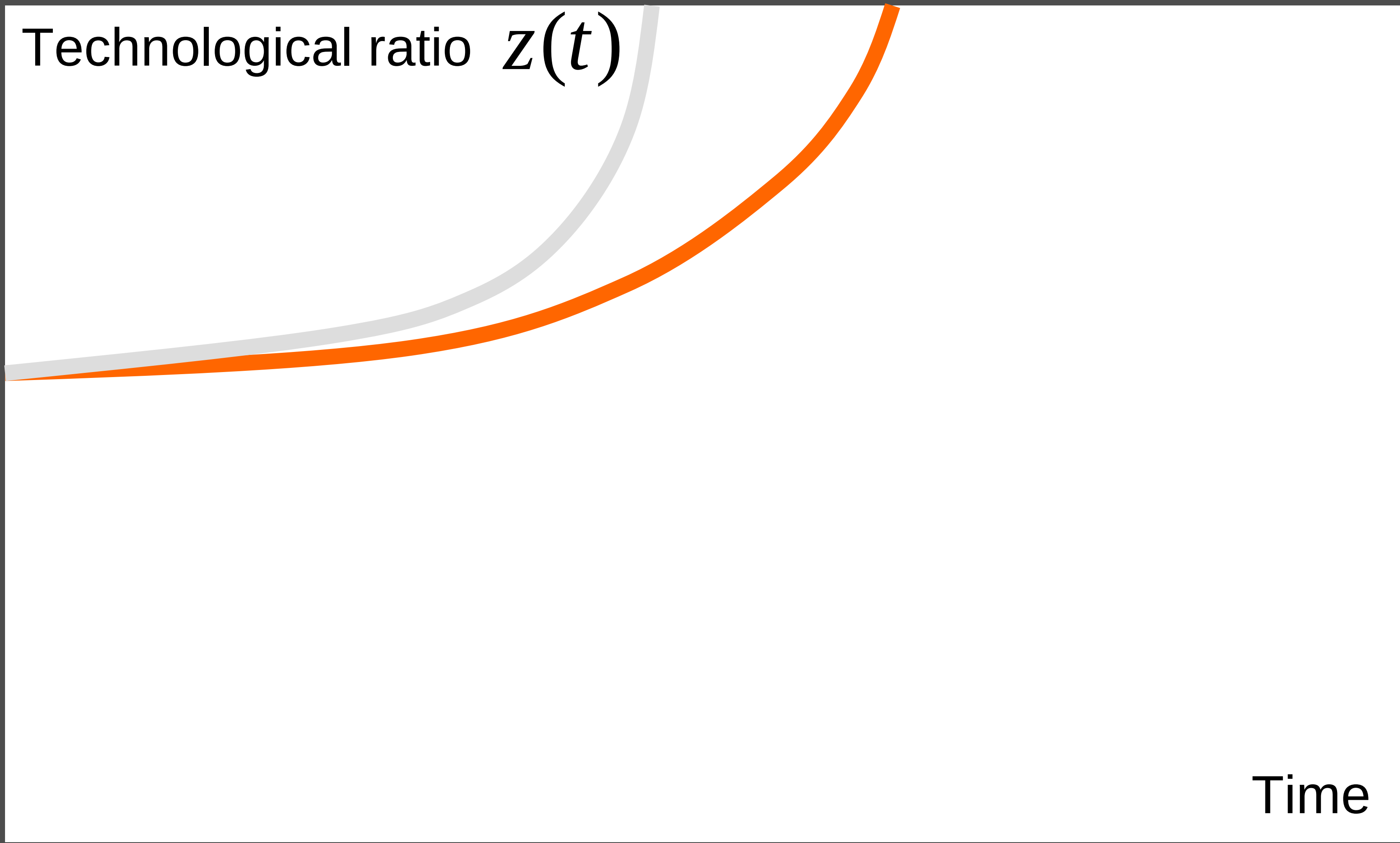

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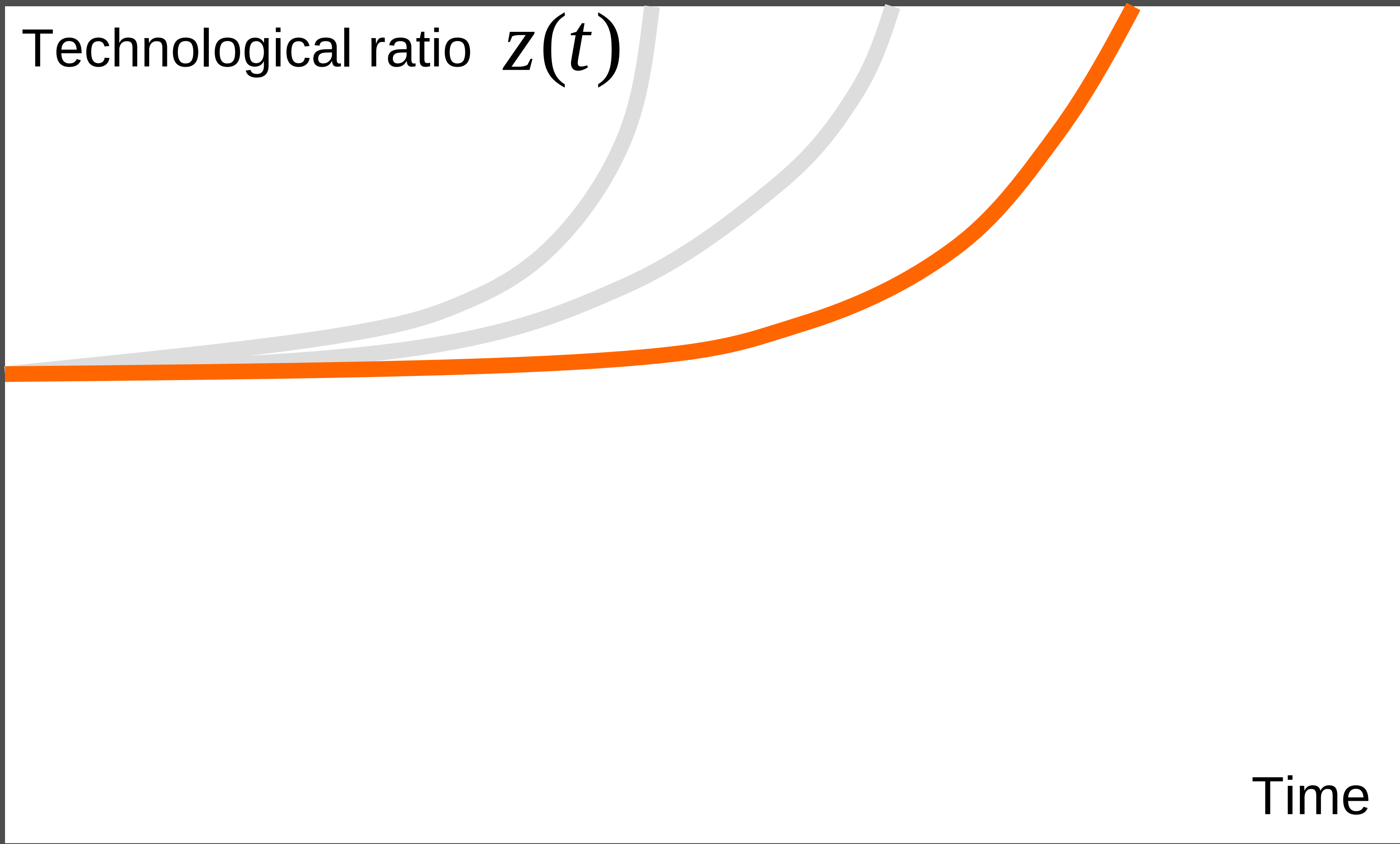

Sensitivity in ρ

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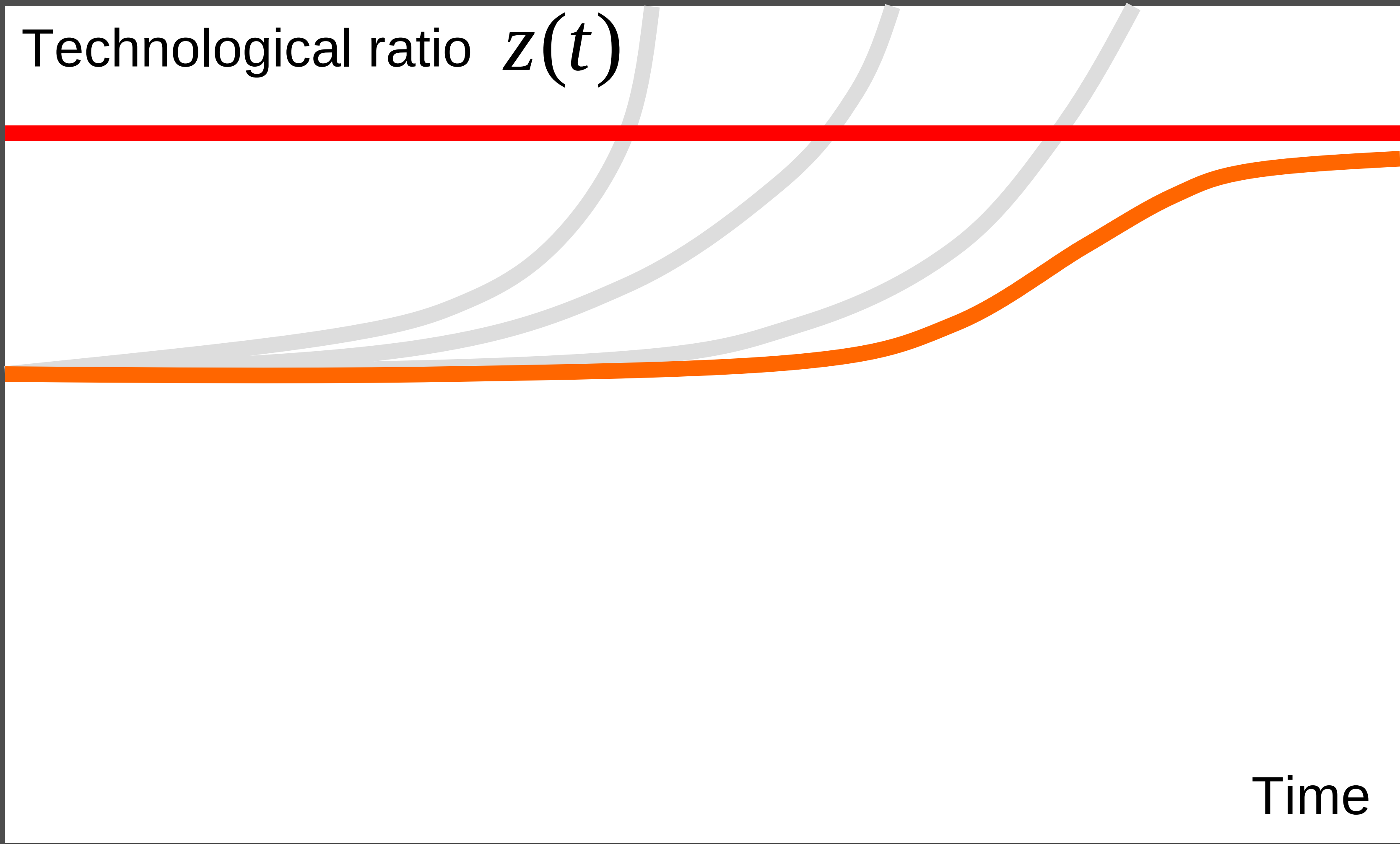
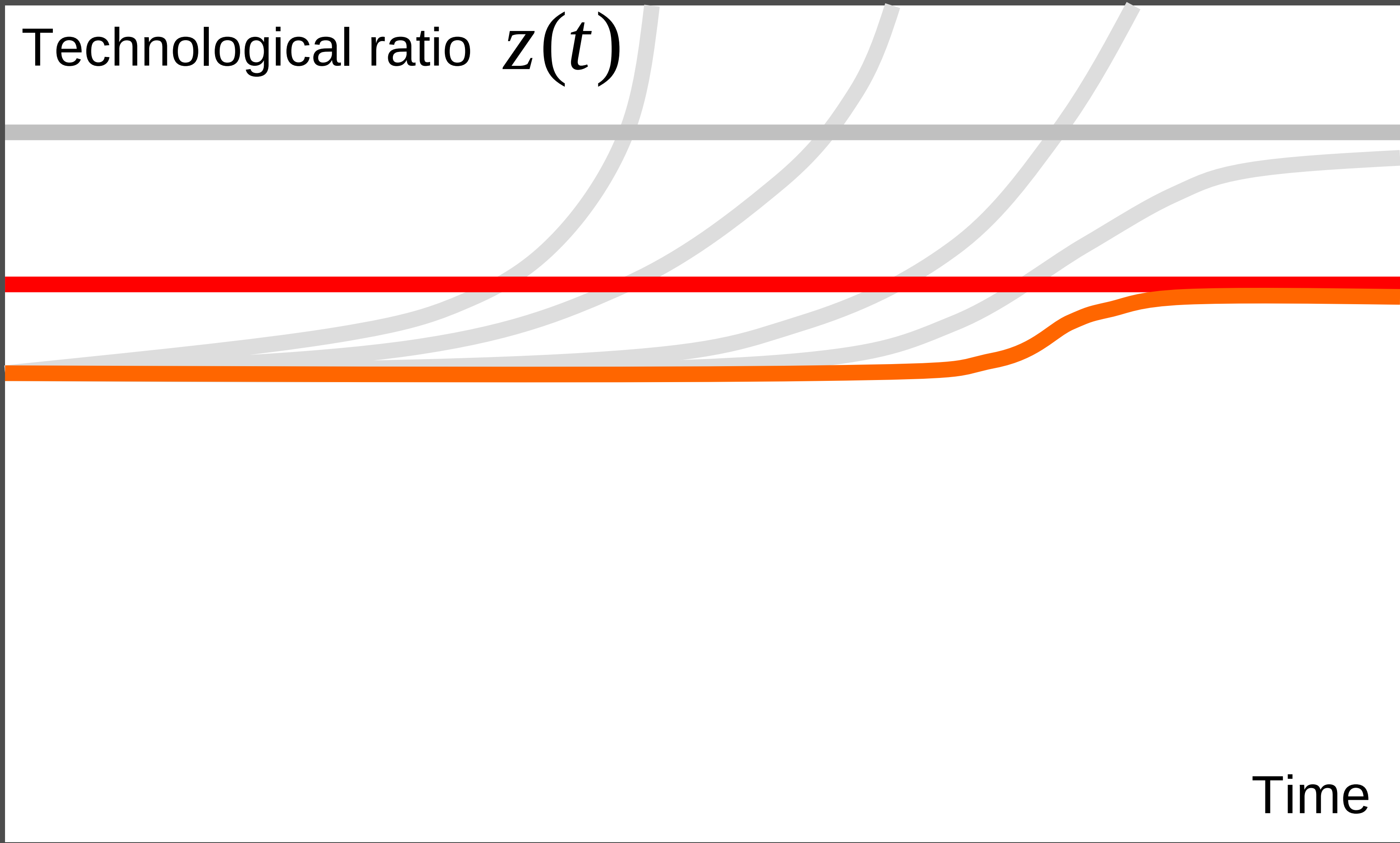


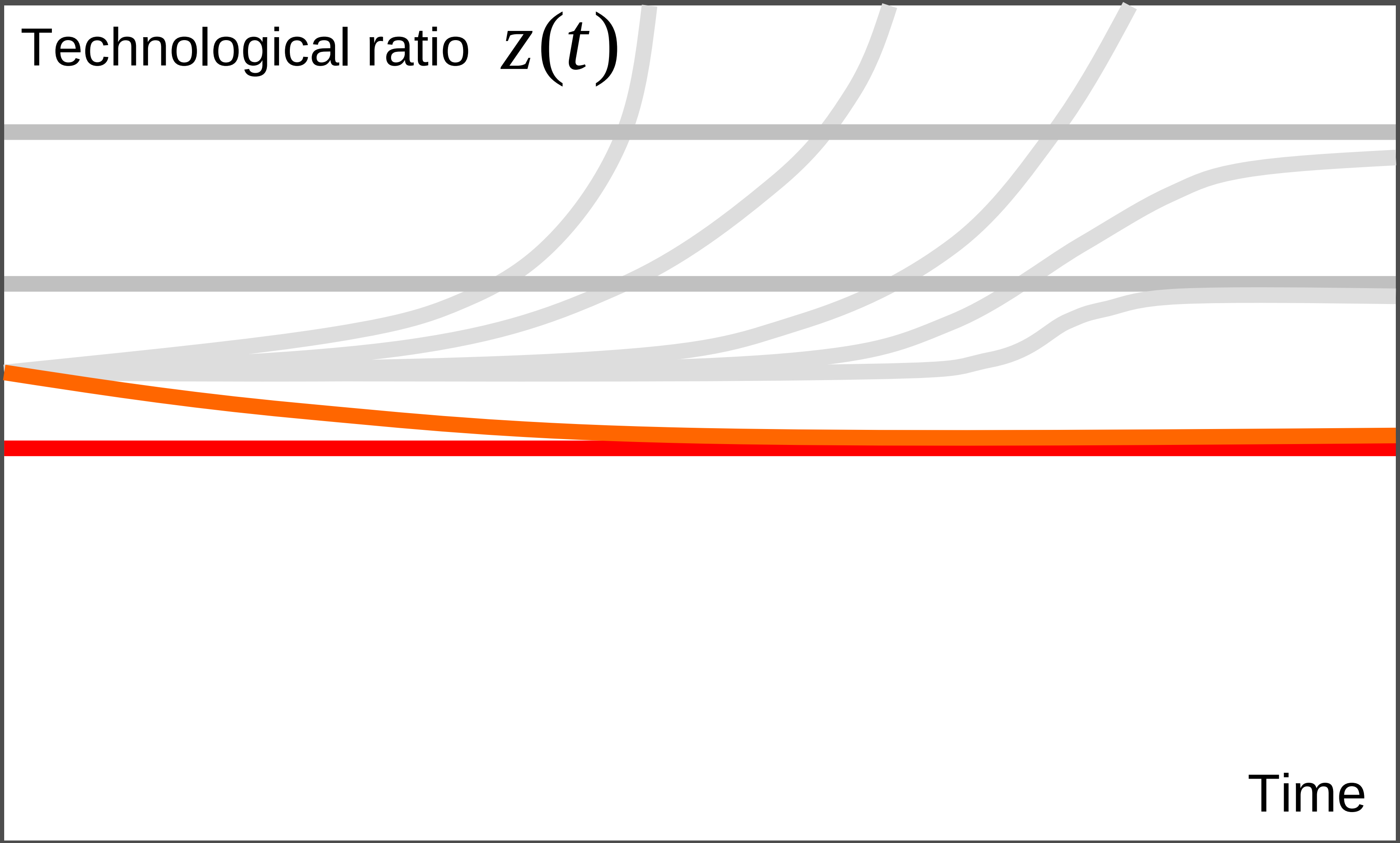
Follower

Sensitivity in ρ



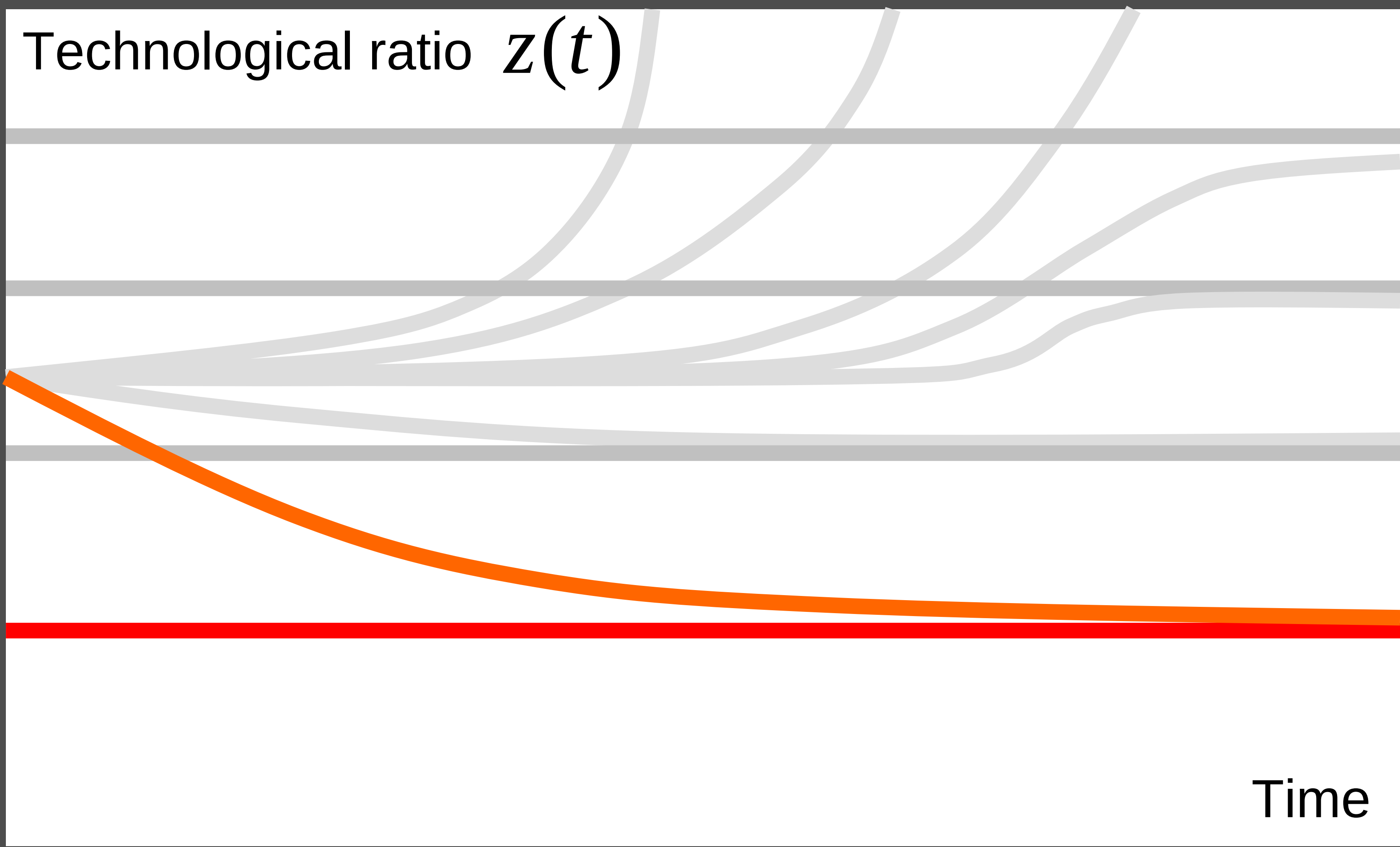
Follower

Sensitivity in ρ



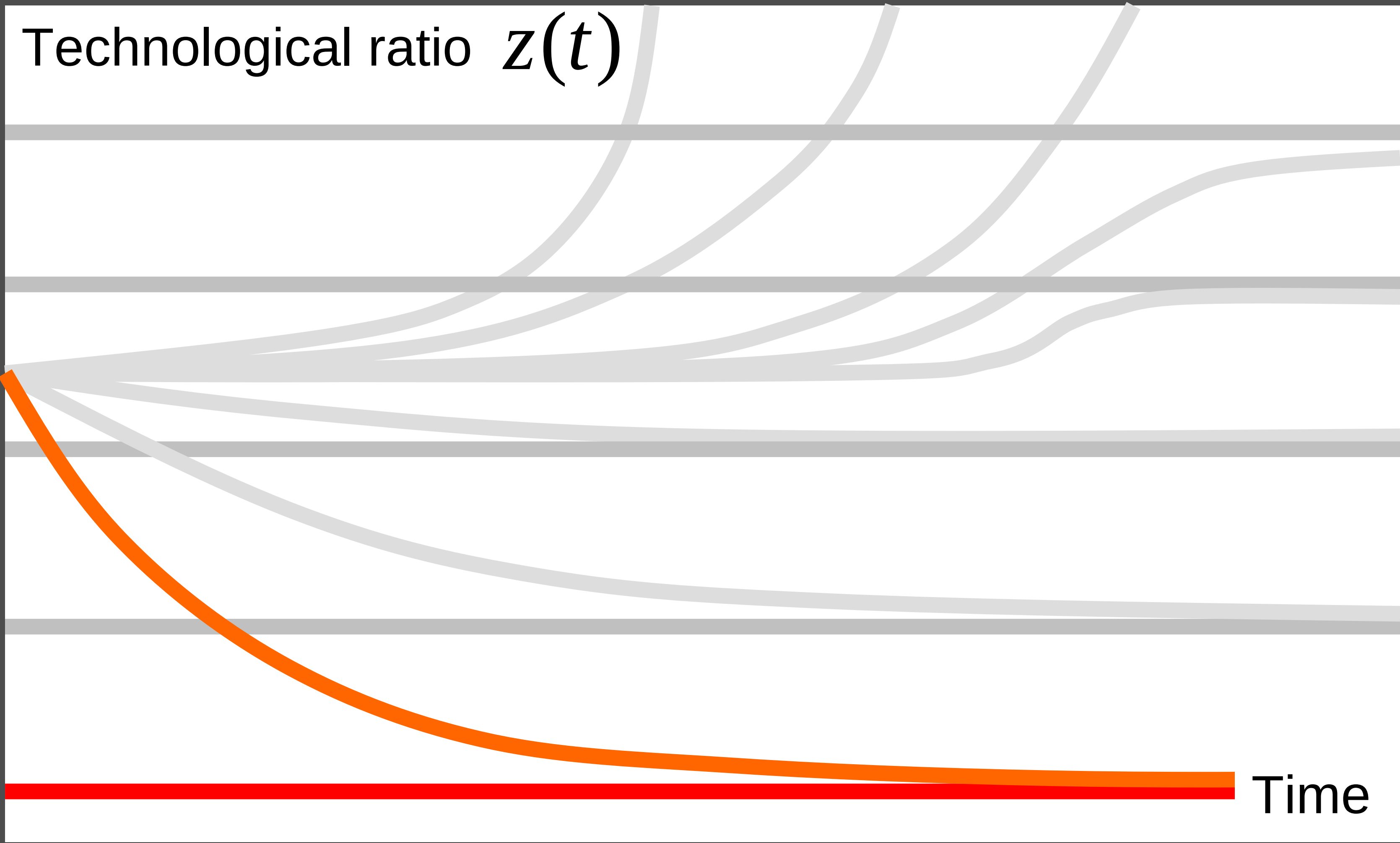
Follower

Sensitivity in ρ



Follower

Sensitivity in ρ



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