SIXTH FRAMEWORK PROGRAMME

Project no: **502687**

NEEDS

New Energy Externalities Developments for Sustainability

INTEGRATED PROJECT

Priority 6.1: Sustainable Energy Systems and, more specifically,
Sub-priority 6.1.3.2.5: Socio-economic tools and concepts for energy strategy.

Deliverable T9.2 - RS2b

Multicriteria methodology for the NEEDS project

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Abstract

This report begins with an overview of multicriteria analysis methods, and the basic principles of developing mathematical models for such analysis. An overview of various representation of user preferences is then presented, including methods based on pairwise comparisons of criteria and those based on scalarizing functions. This is followed by a summary of structures of criteria and alternatives. Next, basic properties of multi-criteria analysis are discussed, followed by a more detailed presentation of the similarities of and differences between the main methods based on scalarizing functions. This report concludes that existing methods do not best meet the needs of the NEEDS project, presents the reasons, and proposes a new methodology for development. Depending upon the development and testing of this new methodology, an existing method will also be chosen as a backup for comparative or alternate use.

Contents

1 Introduction

The objective of this paper is to report the results of the activities of WP9 of Stream 2b pertinent to the evaluation of methodology and a corresponding tool for multicriteria analysis best suited to the requirements of the NEEDS projects. The requirements are discussed in detail in [45], and it is assumed that a reader of this paper is familiar with this requirement analysis.

In any rational analysis of a complex problem the choice of a method is of critical importance because it predetermines to a large extent the scope (and in most cases the correctness) of analysis. Each analysis method is based on specific assumptions and supports only a certain type of analysis. A selected method must fit to the problem characteristics and the desired scope/features of analysis. Therefore it is critically important to specify the requirement analysis (composed of the specification of the problem to be analyzed, and a desired scope of analysis) before considering analysis methods and tools pertinent to the problem.

We use in this paper the widely used term MCDA (Multicriteria Decision Analysis) because it covers a well developed field of OR (Operational Research) that provides methods and tools applicable to our problem. However, we need to stress that our problem (described in detail in [45]) substantially differs from typical MCDA problems in which a decision-maker (conventionally called *a user*) analyzes a decision problem through a process of interactive modifications of his/her preferences upon the analysis of solutions obtained for previously defined preferences. It is commonly known that in the initial stage of problem analysis a user typically specifies preferences/goals that are far from being attainable; thus the essence of most MCDA methods is to help users to revise preferences in order to make them consistent with attainable/feasible solutions. This in turn implies that MCDA is actually a learning process about the analyzed problem during which a user modifies her/his original preferences (defined for a solution which is typically far from being feasible) towards preferences (trade-offs between values of criteria) for a feasible solution. The latter can be interpreted as defining attainable goals (a composition of attainable values for all criteria) that have trade-offs between criteria reflecting, in the best possible way, the preferences of the user. It is commonly known among researchers involved in non-trivial applications that defining such goals indeed requires an interactive learning procedure during which the user substantially changes his/her preferences. Therefore it is not practicable to attempt identification of preferences without an iterative process during which preferences can be modified upon analysis of solutions corresponding to previously specified preferences.

Generally, in order to effectively use MCDA for the analysis of efficient solutions corresponding best to various trade-offs between conflicting objectives specified by users, it is necessary to select a method (out of dozens of MCDA methods) and the corresponding software tool that best fit to the characteristics of:

- The substantive problem represented by the underlying mathematical model. The most common approach for problems with many (for discrete alternatives) or infinite (for continuous problems) solutions is to use an achievement scalarizing function for specification of a parametric optimization problem of the underlying model; the solution of such an optimization problem is Pareto-efficient, and corresponds to the preferences represented by the scalarizing function. Different MCDA methods use different scalarizing functions, parameters of which are defined by the preferences of the MCDA users. An alternative is to generate from the model a set of discrete alternatives, and apply MCDA to analyzing trade-offs between the generated alternatives.
- The users of MCDA. In particular, such a selection needs to be based on the ways in which users with different backgrounds and preferential structures can specify their preferences, which need to be translated in a transparent way into trade-offs between objectives. Trans-

parency and an appropriate representation of trade-offs are critically important for comparative analysis of various efficient solutions (each corresponding to a different structure of preferences) by stakeholders.

This paper summarizes the basic features of MCDA, and focuses on features of these MCDA methods that are relevant for the analysis problem described in [45]. This will form a basis for justifying the approach proposed in Section 7.

The requirement analysis [45] clearly shows that we have to deal with a problem that is far more challenging than a typical MCDA problem for which a user analyzes a problem with respect to her/his preferences. Our problem is composed of two stages of analysis:

- diversified stakeholders specify individual preferences, and
- analysts analyze the problem taking into account these preferences (expected to differ substantially amongst groups of stakeholders) in order to identify the characteristics of solutions (either technologies or scenarios) that can help in rational decision-making.

Thus, in fact, one needs two compatible MCDA methods:

- to support each stakeholder in a multicriteria analysis of the original problem, which will result in finding a solution corresponding best to his/her preferences; the other outcome of this analysis is a representation of a set of consistent preferences of various stakeholders;
- to support analysts in a consistent exploitation of the elicited preferences for a comprehensive analysis of the original problem.

The remaining part of this paper is organized as follows: We start with an overview of multicriteria analysis methods and basic principles of development of mathematical models for such an analysis; these topics are covered by Sections 2 and 3, respectively. Section 4 provides a more detailed discussion of various representations of user preferences pertinent to analysis sets of discrete alternatives, including methods based on pairwise comparison, and those based on scalarizing functions. Next, Section 5 summarizes structures of criteria, and of alternatives. Basic properties of multicriteria analysis together with a more detailed presentation of the similarities and differences of the main methods based on scalarizing functions are discussed in Section 6. Finally, multicriteria methodology for the NEEDS project is proposed and justified in Section 7.

2 Overview of MCDA

2.1 Context

Policy makers and almost all industrial companies, research, educational and other organizations are faced with problems of finding the best compromise between conflicting goals, such as: costs versus performance and reliability of products and technologies and the time to bring them to the market; life-time costs versus environmental impacts; economic growth versus inter-generation fairness of a pension system; or spatial and temporal allocation of costs of climate change mitigation versus ex-ante and/or ex-post risk management. Making rational decisions for any complex problem requires various analyses of trade-offs between the conflicting goals (objectives, outcomes) used for measuring the results of applying various decisions in a wide range of application domains. A typical decision problem has a large (or even an infinite) number of solutions, and users are interested in analyzing trade-offs between those that correspond to their preferences, which is often called the preferential structure of the user. Such preferences are typically expressed in terms of criteria, e.g., desired criteria values (or their ranges), trade-offs between improving/worsening criteria values. A preferential structure typically induces partial ordering of solutions (characterized by values of criteria) obtained for different combinations of values of decisions.

Mathematical models can potentially provide better solutions for such problems, if an appropriate modeling technology is applied. The classical OR (Operational Research) approach is to define a single goal function (performance criterion) and look for a solution that optimizes its value. The purpose of multicriteria problem analysis is to support users in exploring solutions that correspond best to his/her preferences. In other words, multicriteria methods fit to the situations in which users are not able to define a single goal function.

Complex problems do not have unique and easy-to-find solutions. However, as a result of decision-making processes a unique solution/decision must be determined. Therefore rational decision-making requires a diligent analysis of the decision problem which is aimed at finding a decision that is the best (in the sense of typically conflicting objectives of decision-makers). The role of mathematical modeling is to support the decision-making process (which for complex problems involves not only decision-makers, but also stakeholders, analysts, experts and advisors) by providing information about solutions which correspond best to the preferences of decision-makers and also stakeholders, if stakeholders are involved in the decision-making process.

2.2 Stages of multicriteria problem analysis

2.2.1 Purpose of the analysis

The purpose of analysis is to provide decision-makers (and optionally stakeholders) with a manageable number of solutions/alternatives for more detailed consideration. The set of solutions should be representative for the decision problem, in particular for the preferences of stakeholders. One should be aware that meeting the requirement of *representative* is difficult because non-trivial problems are characterized by:

- Preferences (i.e., trade-offs between criteria) that are substantially different not only amongst stakeholders but also for a single user/stakeholder who explores different types of solutions (e.g., trade-offs between costs and quality are clearly different for ”cheap” and ”expensive” solutions).
- Mathematical properties of the underlying problem, which typically has many substantially different solutions corresponding to preferences that are rather similar.

Thus a good analysis should involve a careful consideration of a consistency between:

- the mathematical properties of the model representing the problem,
- the selected method(s) of model-based problem analysis, and
- the composition of a set of solutions and their characteristics to be provided to the participants of the decision-making process (decision-makers, stakeholders, analysts, experts, advisors).

Consistency between the purpose of analysis and the proposed multicriteria methodology for the NEEDS project is discussed in Section 7.

2.2.2 Elements and stages of model-based problem analysis

Due to the space consideration we do not provide here any comprehensive description of model-based problem-solving methodology and the corresponding modeling process.[1]

We will give here an overview of the MCDA approaches focusing on the substantive model representation and preference specification by the stakeholders.

We concentrate on the following elements of MCDA which are important for selection and implementation of MCDA for the NEEDS project:

[1]Readers interested in this topic may want to consult e.g., [44, 47, 82].

- Requirement analysis (including specification of the problem), presented in detail in [45].
- Development of a substantive model of the decision situation, see Section 3.
- Representation of user/stakeholder preferences, see Section 4.
- Diverse structures of criteria and alternatives, see Section 5.
- Properties of multicriteria analysis methods pertinent to our problem, see Section 6.
- Analysis of the problem, see Section 7.

3 Substantive models

Although this report deals with the problem of discrete alternatives we briefly outline two types of substantive models of decision situation because several methods of multicriteria analysis originally developed for the first type are also pertinent to the second type:
- Algebraic models.
- Discrete sets of alternatives (objects).

3.1 Algebraic models

Because of the unquestionable success of modeling in problem solving, various modeling paradigms have been intensively developed over the last few decades. As a result, different types of models (characterized by types of variables and relations between them) were developed (e.g., static, dynamic, continuous, discrete, deterministic, stochastic, set-membership, fuzzy, soft constraints) with a view to best representing different problems by a selected type of model. Moreover, different methods of model analysis (e.g., simulation, optimization, soft simulation, multicriteria model analysis) have been developed as the best-possible support for various types of model analyses for different purposes and/or users. Finally, because of the growing complexity of various computational tasks, solvers have become more and more specialized, even for what was originally the same type of mathematical programming problem. Each modeling paradigm embodies a great deal of accumulated knowledge, expertise, methodology, and modeling tools specialized to solve various problems peculiar to each modeling paradigm.

A mathematical model describes the modeled problem by means of variables that are abstract representations of those elements of the problem which need to be considered in order to evaluate the consequences of implementing a decision (usually represented by a vector composed of many variables). More precisely, such a model is typically developed using the following concepts:
- Decisions (inputs) $\boldsymbol{x}$, which are controlled by the user;
- External decisions (inputs) $\boldsymbol{z}$, which are not controlled by the user;
- Outcomes (outputs) $\boldsymbol{y}$, used for measuring the consequences of the implementation of inputs;
- Auxiliary variables introduced for various reasons (e.g., to simplify model specification, or to allow for easier computational tasks); and
- Relations between decisions $\boldsymbol{x}$ and $\boldsymbol{z}$, and outcomes $\boldsymbol{y}$; such relations are typically presented in the form:

$$\boldsymbol{y} = \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{z}), \tag{1}$$

  where $\boldsymbol{F}(\cdot)$ is a vector of functions.

A structure of the use of a model for decision-making support is illustrated in Figure 1. The basic function of model-based problem-solving support is to help the user find values for his/her decision variables $\boldsymbol{x}$ which will result in a solution of the problem that best fits his/her preferences. To achieve this one needs to:
- Develop and maintain a model that adequately represents relations (1);

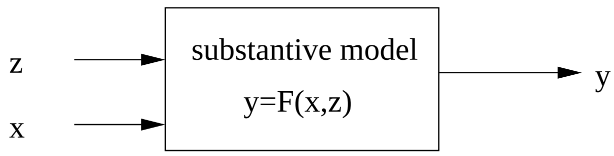


Figure 1: A substantive model of the decision situation

- Organize a process of the model analysis in which the user can specify and modify his/her preferences upon combining their own experience and intuition with learning about the problem from the analyses of various solutions.

Model development is a rather complex process. Discussion of the related issues is beyond the scope of this paper; interested readers may consult e.g., [44, 47, 82].

3.2 The discrete set of alternatives (objects) for selection

Each object is described by a set of numerical or non-numerical attributes.

criteria alternatives	c_1	c_2	...	c_n
o_1	$v_{1,1}$	$v_{1,2}$	...	$v_{1,n}$
o_2	$v_{2,1}$	$v_{2,2}$	...	$v_{2,n}$
...	...	...	...	...
o_m	$v_{m,1}$	$v_{m,2}$	...	$v_{m,n}$

In the process of problem analysis the user can select some of the attributes as criteria and the other as informative or selection attributes. The informative attribute is an attribute that is in use only to display additional information about an object. The selection attribute is an attribute that can be set for selection of the subsets of objects.

The table can be prepared in the following way:

- An expert (or group of experts) specify alternatives and decide about the attributes based on his/her knowledge of various types of supplementary information.

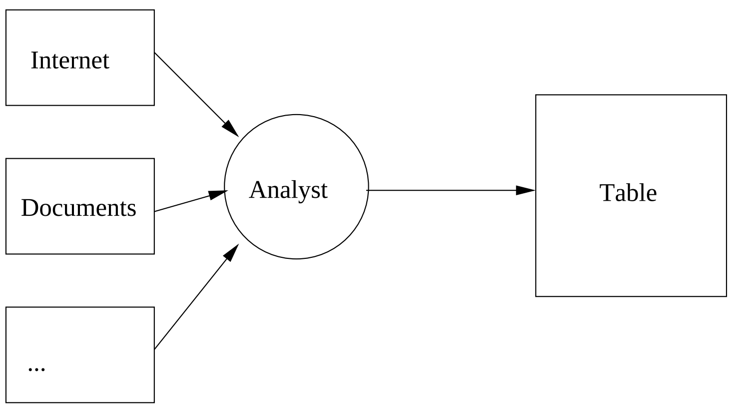


Figure 2: The definition of a table by an expert

- An expert (or group of experts) generates the alternatives and decides on the attributes. However, the values of attributes are calculated by software tools. The primary data for calculation of attributes are stored in the database.

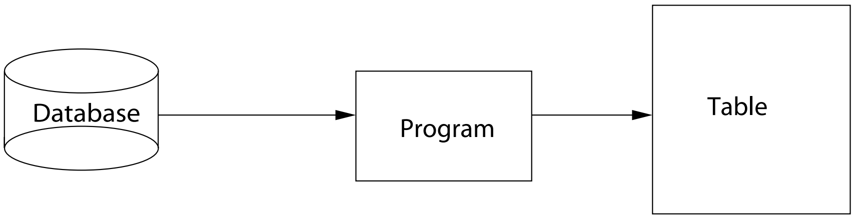


Figure 3: Software tools for table generation

- There is a simulation model of the decision situation. However, for various reasons (e.g., because of a long calculation time), the model cannot be used directly in the process of final selection. In such cases the simulation model is applied off-line for the generation of the set of objects and calculation of attributes.

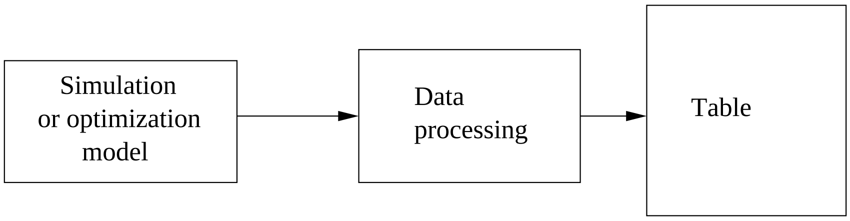


Figure 4: Alternatives generated by simulation

One should note that the discrete alternative choice problem can be represented as an algebraic model. This is particularly needed if values of criteria for (possibly many) alternatives must be computed from parameterized complex relations, see e.g., [46], and/or for problems with a large number of alternatives.

4 Representations of user preferences

Substantive models represent in various mathematical forms the objective part of the problem, i.e., the relationship between decisions and their outcomes measured by criteria. Development of a substantive model does not involve any actual representation of user preferences but a model specification and instantiation need to provide outcome variables which are used as criteria, which in turn are used for specification of user preferences. There exist several approaches to preference representation which are applicable to different classes of problems (e.g., discrete or continuous) and different information provided by users (e.g., pairwise comparisons of alternatives, using: (1) only dominance relation, (2) relative importance of criteria, (3) aspiration and reservation values for criteria). Before presenting below the approaches pertinent to our problem, we summarize the basic concepts used in multicriteria analysis.

4.1 Basic concepts

There are several variants for defining basic concepts of multiple criteria optimization. We recall here one of the simplest sets of definitions. Readers interested in more detailed and rigid definitions may consult e.g. [61, 66, 79, 84]. The following definitions will be used in the subsequent discussions.

In order to simplify the presentation we assume that we consider n criteria having real values denoted by $\boldsymbol{q} \in R^n$, where vector $\boldsymbol{q}$ is defined by:

$$\boldsymbol{q} = \{q_1, q_2, \ldots, q_n\}, \qquad q_i \in R, \;\; i = 1, 2, \ldots, n \tag{2}$$

Further we assume that all criteria are minimized. With such assumptions we also cover qualitative criteria, and criteria that are maximized; in both cases such criteria can be handled (after simple, commonly known transformations) within the discussed framework.

For numerical representations of criteria values various measurement scales can be used: nominal, ordinal, cardinal (interval, ratio). A nominal scale only labels the alternatives, i.e. no information is provided on the relationships between the alternatives; an ordinal scale provides information about the order of the alternatives, but there is no information about the interval (difference) between elements of the scale. A cardinal scale (also called a metric scale) attaches a number (measure) to each alternative; such measurement not only implies an order of the alternatives but also quantifies the differences between them. One distinguishes two kinds of cardinal scales: interval and ratio scale. On an interval scale a zero point is defined arbitrarily while on a ratio scale there exists a non-arbitrary zero point.

4.1.1 Concepts related to Pareto efficiency

Weakly Pareto-optimal solution: A solution $\hat{x} \in X_0$ (where X_0 is a set of considered/feasible solutions) is called a weakly Pareto-optimal solution, if no other feasible solution exists that has better values of all criteria. Weakly Pareto-optimal solutions are usually easier to be computed. Therefore a proper method (see the explanation of eq. (22) on page 19) should be implemented to avoid computing and reporting a weakly Pareto-optimal solution as an efficient solution. This is a purely technical problem and weakly Pareto-optimal solutions have no practical meaning for users of a properly implemented multicriteria analysis.

Pareto-optimal solution: A solution $\hat{x} \in X_0$ is called a Pareto-optimal solution, if there is no other feasible solution for which one can improve the value of any criterion without worsening the value of at least one other criterion. A Pareto-optimal solution is also called an *efficient* solution (some authors also call it a non-dominated solution) and it can be defined (for a minimized criterion q_i) as:[2]

$$\neg\exists x \in X_0 \neq \hat{x} : \;\; \{q_i(x) \leq q_i(\hat{x}) \;\; \forall i \in [1, ..., n] \;\text{ and } \\ \exists k \in [1, ..., n] : q_k(x) < q_k(\hat{x}) \; \} \tag{3}$$

Most practical in applications are properly Pareto-optimal solutions with a prior bound on trade-off coefficients (see [77] for more details). Further on, a properly Pareto-optimal solution will be simply called a Pareto solution.

Pareto-optimal point: Pareto-optimal point is composed of values of all criteria for a corresponding Pareto-optimal solution.

Pareto set: Pareto-optimal set (sometimes also called a Pareto frontier) is composed of all Pareto-optimal points.

[2]The relation (3) means that there is no other solution that has: (1) not worse value of any criterion, and (2) a better value of at least one criterion. In other words: one cannot improve value of at least one criterion without deteriorating value of another criterion.

Utopia point: A Utopia point $\boldsymbol{q}^U$ is composed of best values out of the set of all Pareto-solutions for each criterion. A Utopia point (often also called an ideal point) can be easily computed as a result of n single criterion optimization with each criterion in turn serving as an objective function.

Nadir point: A Nadir point $\boldsymbol{q}^N$ is composed of the worst values out of the set of all Pareto-solution for each criterion. Finding a Nadir point is typically difficult for problems that are either mixed-integer or continuous, and have more than two criteria, see e.g., [28].

Aspiration point: An Aspiration point (sometimes called a reference point) is composed of the desired values specified by a user for each criterion. In other words, the values that a user would like to achieve for each objective. The Aspiration point will be defined in this paper by $\bar{\boldsymbol{q}} \in R^n$.

Reservation point: A Reservation point is composed of the values still acceptable by a user for each criterion. The Reservation point will be defined in this paper by $\underline{\boldsymbol{q}} \in R^n$. Therefore, the pairs of aspiration and reservation levels define, for a corresponding criterion, a range of values between the desired and still acceptable levels.

Utopia and Nadir (or a good approximation of a Nadir) provide valuable information about the range of values (for all efficient solutions) for each criterion. Therefore those points outline for each criterion a range of reasonable values for aspiration and reservation levels.

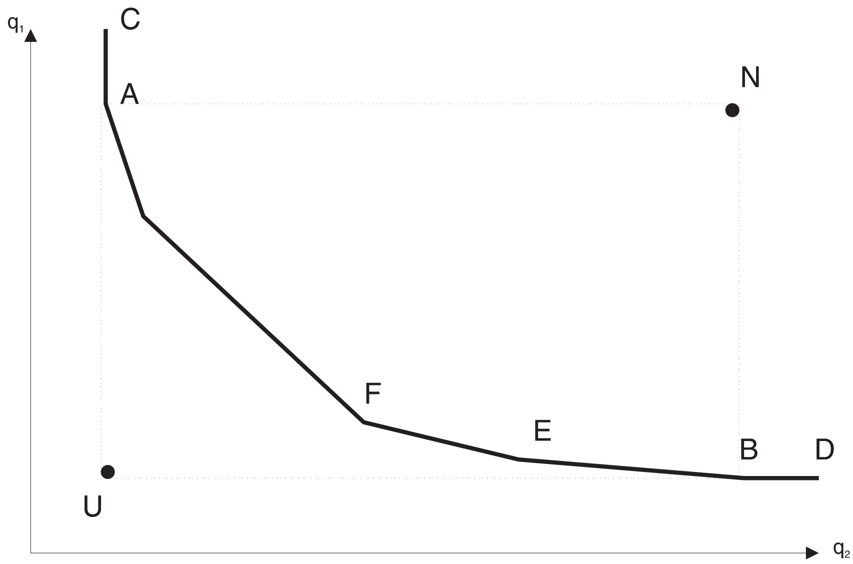


Figure 5: An illustration of basic concepts used in MCDA.

The above definitions are illustrated, for a problem with two minimized criteria (q_1 and q_2), in Figure 5. The Pareto set is contained between points **A** and **B**. Weakly Pareto points are located on the segments **AC** and **BD**, and non-properly optimal Pareto points are in the segment **BE**. Note, that the slope of segment **BE** corresponds to the trade-off coefficients and is usually very small.[3] If the bound on the trade-off coefficients is increased, then the set of properly Pareto-optimal solutions will be reduced to the two segments between points **A** and **F**. The Utopia and Nadir points are marked by **U** and **N**, respectively.

[3]A more detailed explanation of can be found in [41].

4.1.2 Scalarizing function

Achievements Scalarizing Functions (ASF) are used by methods which assume that it is possible to associate a real number with each solution, in order to measure the performance of that solution. Many of the discussed approaches do not use, in the corresponding original formulation, the achievement function concept introduced by Wierzbicki, see e.g., [74, 78, 82]. However, it is easy to formulate such functions for each approach in order to provide a consistent comparison.

Achievement scalarizing functions are sometimes called value functions (or utility functions, or scalarizing functions) and can be written in a compact form:

$$ASF = V(\boldsymbol{q}(\boldsymbol{x})) \tag{4}$$

where $\boldsymbol{q}$ is vector of criteria values corresponding to a solution $\boldsymbol{x}$ (for a discrete set of alternatives $\boldsymbol{x}$ can often be replaced by an identifier of an alternative), and $V(\cdot)$ is a function.

A more detailed discussion of approaches based on ASF is presented in Section 4.4.

4.1.3 Preference modeling

The preference model is a model that for each pair of alternatives (decisions) a b ($a \neq b$) assign one, two or three basic situations:
- strict preference a over b,
- weak preference a over b,
- indifference between a and b,
- incomparability between a and b,

Respectively, we can define relations, the set of ordered pairs (a,b) such that
- relation of strict preference a over b: $a \succ b$,
- relation of weak preference a over b: $a \succeq b$,
- relation of indifference between a and b: $a \sim b$,
- relation of incomparability between a and b: $a?b$.

A preference structure [53] is a collection of binary relations defined on the set A and such that:
- for each pair $a, b \in A$; at least one relation is satisfied, or
- for each pair $a, b \in A$; if one relation is satisfied, another one cannot be satisfied.

In other terms a preference structure defines a partition of the set$A \times A$. In general it is recommended to have two other hypotheses with this definition (also denoted as a fundamental relational system of preferences):
- Each preference relation in a preference structure is uniquely characterized by its properties (symmetry, transitivity, etc..)
- For each preference structure, there exists a unique relation from which the different relations making up the preference structure can be deduced. Any preference structure on the set A can thus be characterized by a unique binary relation R in the sense that the collection of the binary relations are to be defined through the combination of the epistemic states of this characteristic relation.

We will not go into the details of various preference structures (details can be found e.g., in [53]). We will focus on the most important issues.

The preference structure can be defined by the properties of binary relations of the relation set.

The most traditional preference model assumes that comparing two different elements of the set A we can distinguish only two situations: preference of one element over the other (relation

$\succ$), indifference of one element to the other (relation $\sim$). Therefore, we can define **preference structure**:

$$\langle \succ, \sim \rangle$$

as a pair of relations $\succ, \sim$ on A such that $\succ$ is asymmetric $\succ$ and $\sim$ is reflexive, symmetric. By adding additional properties to the binary relations we can define various, more specific structures called *orders*: total, weak, semi-order, interval order.

The classical preference structures do not consider incomparability between alternatives. In this case the partial preference structure is introduced:

$$\langle \succ, \succeq, \sim, ? \rangle$$

By defining specific properties of binary relations we can introduce structures called partial and quasi order. This structure is used by outranking methods.

From the point of view of practical applications we must have numerical representation of preference structures of the presented preference structures. Below we will present some of the numerical representations of preference structures.

If a value function $V(a)$ is defined for each alternative a then alternative a is preferred to b ($a \succ b$) if value function $V(a) > V(b)$, and a and b are indifferent ($a \sim b$ if and only if $V(a) = V(b)$. Value function $V(\cdot)$ must fulfill the following conditions:

- preferences are complete (i.e. for any pair of alternatives either $a \succ b$ or $b \succ a$, or $a \sim b$), and
- preferences and indifferences are transitive (for any three alternatives a, b, c if $a \succ b$ and $b \succ c$ then $a \succ c$ and for indifference if $a \sim b$ and $b \sim c$ then $a \sim c$).

4.2 Methods using pairwise comparison

For completeness we briefly outline here methods using pairwise comparisons. Such methods are practicable only for problems having a small (less than 10) number of alternatives and of criteria. Therefore these methods are not applicable to our problem. However, understanding of these methods may be helpful for comprehension of the other multicriteria methods.

4.2.1 AHP (Analytical Hierarchy Process)

AHP is a multicriteria decision analysis method developed by Saaty [59]. AHP can be considered as a method of elicitation of a value function.[4]

The AHP method is composed of the following steps:

1. Definition of hierarchy of criteria: The user selects m criteria and organizes them into a hierarchical structure.
2. Pairwise comparison: For each criterion and each pair of alternatives (denoted here as a_i and a_j) the user is requested to judge them by specifying a ratio w_i/w_j of the corresponding weights w_i and w_j. The estimate of this ratio is defined as:

$$a_{ij} = w_i/w_j \tag{5}$$

Thus to determine the complete set of relative priorities $n(n-1)/2$ pairwise comparisons are needed for each of m criteria. The resulting comparison matrix A has the form:

$$A = \begin{bmatrix} 1 & a_{12} & \dots & a_{1n} \\ 1/a_{12} & 1 & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ 1/a_{1n} & 1/a_{2n} & \dots & 1 \end{bmatrix} \tag{6}$$

[4] Although some of authors distinguish AHP and value functions as different methods [69].

3. Calculation of the relative priority vectors w: The standard AHP method finds the set of values $w_1, \ldots, w_n$, such that the elements of matrix A is approximated as closely as possible by the corresponding ratios w_i/w_j. It can be proved that values $w_1, \ldots, w_n$ can be found in the following way:

$$Aw = \lambda_{max} w \tag{7}$$

were λ_{max} is the maximal eigenvalue of matrix A, $w = (w_1, \ldots, w_n)$ is an eigenvector of matrix A ($\sum_i w_i = 1$).

4. Providing information about the importance of criteria: In the next step, information about the importance of the criteria should be provided. For criteria the pairwise comparison is also applied. However, because the criteria have a hierarchical structure the priority vector should be modified according to information provided on the upper level. For each level of the hierarchy l the sets of criteria are defined. For each set of the criteria on a given level of the hierarchy there is the priority function

$$w_{q_{i,l}} : Q_{q_{i,l}^-} \rightarrow [0, 1] \tag{8}$$

such that

$$\sum_{q_{i,l+1} \in Q_{q_{i,l}^-}} w_{q_{i,l}}(q_{i,l+1}) = 1 \tag{9}$$

This function should be modified by providing parameters from the upper level of the hierarchy. Considering the priority function of the $(l-1)$ level: $w_{q_{z,l-1}}$, the priority function $w(q_{i,l+1})$ is defined as:

$$w(q_{i,l+1}) = \sum_{q_{j,l} \in L_l} w_{q_{j,l}}(q_{i,l+1}) * w_{q_{z,l-1}}(q_{j,l}) \tag{10}$$

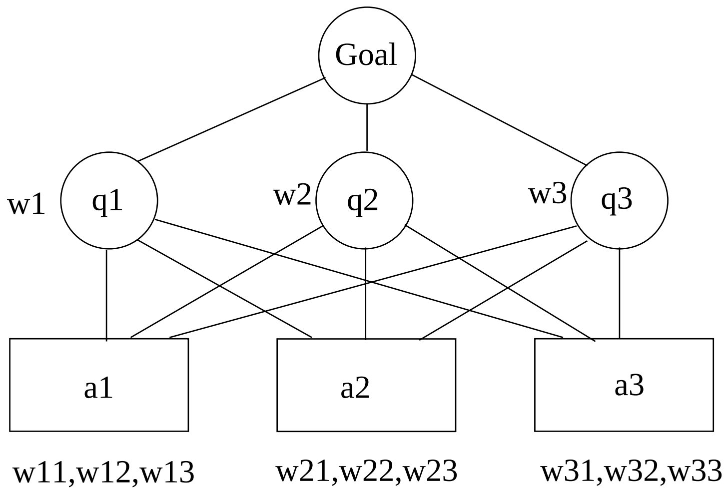


Figure 6: Hierarchy structure of AHP.

5. Synthesis: In this process the final vector of priorities is calculated $wa_1, wa_2, \ldots, wa_n$

$$wa_i = \sum_{j \in J} w_j * w_{i,j} \tag{11}$$

where J is the number of criteria, and i stands for the index of alternatives.

4.2.2 Outranking

The outranking approach accepts incomparabilities of alternatives, and does not impose transitivity properties. Therefore, the corresponding models of preferences cannot be expressed by a function. Vincke [71] points out the following situations where the outranking relation can be justified: at least criterion is not quantitative, units of different criteria are heterogeneous and finding a common scale is very difficult, compensation between gains on some criteria and losses on other criteria is not clear, preference or veto thresholds must be taken into account.

Let a and b be alternatives, and $q(\cdot)$ be a real valued criterion function defined in the following way [72]:

$$\begin{cases} q(a) > q(b) & \Longleftrightarrow \text{ a is preferred to b} \\ q(a) = q(b) & \Longleftrightarrow \text{ a is indifferent to b} \end{cases} \tag{12}$$

The preferences can be represented by a criterion iff relation $\mathcal{R}$ defined by:

$$a\mathcal{R}b \iff \text{ a is preferred or indifferent to b} \tag{13}$$

is a complete preorder.

- Electre I - multicriteria choice problem

  Concordance index:

  $$c(a,b) = \frac{1}{P} \sum_{j:g_j(a)\,\mathrm{lg}\,g_j(b)} p_j, \quad \text{where } P = \sum_{j=1}^{n} p_j$$

  Discorcondance index:

  $$d(a,b) = \begin{cases} 0 & \text{if } g_j(a0 \geq g_j(b) \quad \forall j \\ \frac{1}{\delta}\max_j[g_j(b) - g_j(a)] & \text{otherwise} \end{cases}$$

  where

  $$\delta = \max_{c,d,j}[g_j(c) - g_j(d)]$$

  If there are qualitative criteria the disconcordance set for each criterion D_j is a set of ordered pairs (x_j, y_j) such that if $g_j(a) = x_j$ and $g_j(b) = y_j$ then outranking b by a is refused.

  a outranks b (aSb) if:

  $$\begin{cases} c(a,b) \geq \hat{c} \\ d(a,b) \leq \hat{d} \end{cases}$$

  or

  $$\begin{cases} c(a,b) \geq \hat{c} \\ (g_j(a), g_j(b)) \notin D_j \quad \forall j \end{cases}$$

  where:

  $\hat{c}$ - concordance threshold
  $\hat{d}$ - disconcordance threshold.

- Electre II - multicriteria rank of alternatives (actions)

  a strongly outranks b ($aS^F b$, strong outranking relation) if:

$$\begin{cases} c(a,b) \geq \hat{c}_1 \\ \sum_{j:g_j(a)>g_j(b)} p_j > \sum_{j:g_j(a)<g_j(b)p_j} p_j \\ (g_j(a), g_j(b)) \notin D_j \;\; \forall j \end{cases}$$

- Electre III - multicriteria rank of alternatives (actions).
  This method uses a pseudo-criterion (g,q,p) - a triplet of real valued functions representing preferences

$$\begin{cases} q(a) > q(b) + p(g(b)) & \text{iff a is strictly preferred to b} \\ g(b) + p(g(b)) \geq q(a) > g(b) + q(g(b)) & \text{iff a is weakly preferred to b} \\ \text{a is indifferent to b} & \text{iff there is no strict or weak preference between them} \end{cases}$$

  where functions

  q() - is an indifference threshold
  p() - is a preference threshold

  The underlying preference structure is called a pseudo order.

  Concordance index:

$$c(a,b) = \frac{1}{P}\sum_{j=1}^{n} p_j c_j(a,b) \quad \text{where} \;\; P = \sum_{j=1}^{n} p_j$$

  where

$$c_j(a,b) \begin{cases} 1 & \text{if } g_j(a) + q_j(g_j(a)) \geq g_j(b) \\ 0 & \text{if } g_j(a) + p_j(g_j(a)) \geq g_j(b) \\ \text{linear between the two} & \end{cases}$$

  The discordance index is defined by

$$D_j(a,b) \begin{cases} 0 & \text{if } g_j(b) \leq p_j(g_j(a)) + p_j(g_j(a)) \\ 1 & \text{if } g_j(b) \geq g_j(a) + v_j(g_j(a)) \\ \text{linear between the two} & \end{cases}$$

  where $v_j(g_j(a))$ is a veto threshold. This is a function of $g_j(a)$ for each criterion such that any credibility for the outranking of b by a is refused if

$$g_j(b) \geq g_j(a) + v_j(g_j(a))$$

We finish this short overview of the outranking methods with a brief summary of the Promethee method. This method (as the Electre method)uses as the starting point the decision matrix of evaluations of alternatives against the given set of criteria. The next step in the Promethee method is the definition of a preference function for each criterion. Thus rather than the specification of indifference and preference thresholds (as used in the Electre III) the *intensities* of preferences for pairs of alternatives must be defined as a function of the differences between the corresponding criteria values.

4.2.3 Dominance relation

A simple but very useful basic concept in multicriteria analysis is a partial order of the Pareto type in the criteria space defined by the following dominance relations:

$$\boldsymbol{q}^a \succ \boldsymbol{q}^b \iff \boldsymbol{q}^b \in (\boldsymbol{q}^a + D \setminus \{0\}) \quad (14)$$

$$\boldsymbol{q}^a \succeq \boldsymbol{q}^b \iff \boldsymbol{q}^b \in (q^a + D) \quad (15)$$

where D is a positive cone in the criteria space. The dominance relations can be used for defining Pareto solutions, e.g., $\boldsymbol{q}^b$ is Pareto optimal (or weakly Pareto optimal), if there exists no $\boldsymbol{q}^a$ that dominates $\boldsymbol{q}^b$ in the sense of the relation (14) (or relation (15), respectively).

A more detailed discussion of dominance relations can be found in [82].

4.3 Ranking and sensitivity

Ranking deals with a given set $A = \{a_1, a_2, \dots, a_m\}$ of m alternatives, each characterized by n criteria $c_1, c_2, \dots c_n$. Ranking, see e.g., [6] provides an order for the alternatives from best to worst. This means that the complete and transitive relation should be built on A. However, this is not always possible, and often it is a difficult task. Therefore, some techniques assume that some of the alternatives are incomparable. In simple cases an aggregation function is defined $V(c_1(a_i), c_2(a_i), \dots, c_n(a_i))$ and based on values of this function, alternatives are rank-ordered.

4.4 Methods using scalarizing function

Achievement Scalarizing (value) Functions (ASF) map the R^n (n-dimensional space of criteria values) into R^1, which induces a complete order of solutions.[5] Moreover, the difference between the ASF values of two solutions may be interpreted as a similarity (in the sense of the quality of the solution) measure. Therefore, solutions which can only be partially ordered (e.g., by the dominance relation, see Section 4.2.3) in the n-dimensional criteria space can be ordered using an ASF, and the best solution is guaranteed to be a Pareto-efficient solution.

Parameters of a selected ASF are used to represent preferences of the user. Therefore a selection of the type of ASF implicitly determines the way in which the user can specify his/her preferences. The two most widely used ASFs are discussed in detail in Sections 4.4.1, and 4.4.2, respectively.

The key problem here is a selection of a particular Pareto-optimal solution out of a typically large set of such solutions. This selection is implicitly determined by the conversion of a multi-objective problem into a parametric single-objective problem. In the reference point approach the concept of Achievement Scalarizing Function (ASF) has been introduced by Wierzbicki [74].

4.4.1 Weighted Sum (WS) approach

The oldest, and still one of most popular multicriteria analysis methods, uses (for linear models) ASF in the form:

$$\sum_{i=1}^{n} w_i v_i(q_i) \quad (16)$$

[5] Actually, two (or more) solutions may have an equal value of an ASF. In such cases we consider them equally good/bad.

where n denotes number of criteria, q_i value of i-th criterion, w_i weighting coefficient, and v_i a linear transformation.[6]

Typically the following conditions are set:

- For weighting coefficients:

$$\sum_{i=1}^{n} w_i = \beta, \qquad 0 \leq w_i, \qquad i = 1, 2, \ldots, n \tag{17}$$

where β is usually equal to either 1 or 100.

- For linear transformation:

$$0 \leq v_i(q_i) \leq \gamma \qquad i = 1, 2, \ldots, n \tag{18}$$

where γ is usually equal to either 1 or 100.

Actually β and γ can be set to any positive number, thus their choice is a matter of convenience, or a desired interpretation (e.g., as fractions or percentages).

Weights have a clear interpretation in terms of a utility function $U(\boldsymbol{q})$ which transforms the multicriteria problem (defined in $\Re^n$) into $\Re^1$. That is to say, if we denote a ratio of partial derivatives (in respect to two criteria) of the utility function by:

$$\lambda_i = \frac{\partial U}{\partial q_i} / \frac{\partial U}{\partial q_1} \tag{19}$$

then the weighting coefficients are equal to normalized λ_i, i.e.

$$w_i = \frac{\lambda_i}{\sum_i^n \lambda_i} \tag{20}$$

Thus weights have a clear interpretation, namely, they are equal to the corresponding components of the utility function gradient, and thus to the change of the corresponding criterion value, if the function changes its value along its gradient.

Clearly, optimization of a utility function follows its gradient. Therefore, weights have also another obvious interpretation: namely, by accepting a certain ratio of two weights the user implicitly accepts that the proportion of changes of the corresponding criteria values are also equal to this ratio. This is equivalent to a full compensation of a change of one criterion by the corresponding change of the other criterion in the proportion determined by the ratio of the two criteria weights. Such a ratio is often referred to as a trade-off coefficient between the corresponding criteria.

Weights are typically defined in one of two ways:

- Values specified by the user, usually through a user-friendly interface, which provides on-line normalization of weights, and displays the resulting weights as percentages (of the sum of weights assumed to be equal to 1).
- Indirect specification by a user who defines the relative importance of the corresponding criterion, typically on a scale with seven degrees.[7] An integer number, say $r_i \in \{1, 2, ..., 7\}$, is associated with i-th importance level, and the weights w_i are defined by:

$$w_i = \frac{r_i}{\sum r_i} \tag{21}$$

This approach is based on the psychological aspect of human ability, namely that humans express preferences more easily on a nominal than on a cardinal scale.

[6] Some of the WS methods do not use any transformations of criteria values.

[7] The number of those degrees (seven) results from a series of surveys performed by psychologists, see e.g., [48] which revealed that this may be the average limit of accuracy of human judgment in a single dimensional space.

From a mathematical point of view weights can also be interpreted as a transformation of measurement units of the corresponding criteria.

Application of the ASF in the form of (16) implies that the user assumes constant (over the whole range of criteria values) trade-offs between criteria. Therefore, the main problem with using the original idea of weights is due to the fact that utility functions are typically highly nonlinear, and thus computed weights are only valid locally.

The linear transformations $v_i(q_i)$ have been introduced to simplify the process of determining weights w_i, especially for criteria having multimodal value distributions.[8] However, the introduction of linear transformation does not really solve the problem of handling criteria which have a large range of values. Although weights are formally easier to determine (because for the transformed criteria the weights are applied to quantities having the same range of values) by a transformation one loses information about actual units of the criteria, and thus the original interpretation of weights.

Summing-up: the ASF in the form of (16) transforms the original problem (defined in $\Re^n$ in actual units corresponding to the criteria) into a problem in $\Re^1$ with optional transformation of the criteria values into a relative scale.

Application of the WS approach to analysis of Pareto sets is discussed in Section 6.2.1. A more detailed discussion about the definition and interpretation of weights is available e.g., in [66].

4.4.2 Reference Point (RFP) methods

The selection of a particular Pareto-optimal point is determined by the definition of the ASF defined differently for various reference point (RFP) methods. We introduce the RFP method using one of the simplest approaches, i.e., the aspiration-led analysis which is built on the concept of an aspiration point. This approach uses ASF in the form:

$$s(q, \bar{q}, w) = \min_{1 \le i \le n} \{w_i(q_i - \bar{q}_i)\} + \epsilon \sum_{i=1}^{n} w_i(q_i - \bar{q}_i) \tag{22}$$

where $q(x) \in R^n$ is a vector of criteria, $x \in X_0$ are variables defined by the substantive (often referred to as a core) model, X_0 is set of feasible solutions implicitly defined by the core model, $\bar{q} \in R^n$ is an aspiration point, $w_i > 0$ are scaling coefficients (see the comment below) and ϵ is a given small positive number. Maximization of (22) for $x \in X_0$ generates a properly efficient solution with the trade-off coefficients (as recomputed in terms of u_i defined below) smaller than $(1 + 1/\epsilon)$. For a non-attainable $\bar{q}$, the resulting Pareto-optimal solution is the nearest – in the sense of a Chebyshev weighted norm – to the specified aspiration level $\bar{q}$. If $\bar{q}$ is attainable, then the Pareto-optimal solution is uniformly better. Setting a value of ϵ is itself a trade-off between getting an overly restricted set of properly Pareto-optimal solutions or an overly wide set that is practically equivalent to weakly Pareto-optimal optimal solutions. Assuming the ϵ parameter to be of a technical nature, the selection of efficient solutions is controlled by the two vector parameters: $\bar{q}$ and w.

There is a common agreement that the aspiration point is a very good controlling parameter for examining a Pareto-optimal set. Much less attention is given to the problem of defining the scaling coefficients w. Note that the coefficients w should not be confused with the weights

[8] Roughly speaking, multimodal distributions are characterized by values split into several disjointed subsets separated by empty subsets covering large ranges of values. Consider e.g., two subsets of values: the first composed of positive values smaller than 100, and the second composed of values larger than 100000. Typical statistical characteristics of sets of values may not be adequate. For example, the value of an average is often far away from the closest value of a member of the set.

used by some methods for conversion of a multi-criteria problem into a single-criterion problem with a weighted sum of original criteria. In the function (22), coefficients w play a different role than in a weighted sum of criteria.

In order to provide users with a more intuitive way of specification of the ASF a concept of Component Achievement Function (CAF) was introduced, see [22]. CAF are an extension of the concept of membership functions of the fuzzy sets, and thus have a similar intuitive interpretation. The ASF for the corresponding implementation is defined by:

$$\mathcal{S}(q, \bar{q}, \underline{q}) = \min_{1 \leq i \leq n} u_i(q_i, \bar{q}_i, \underline{q}_i) + \epsilon \sum_{i=1}^{n} u_i(q_i, \bar{q}_i, \underline{q}_i) \tag{23}$$

where $\bar{q}, \underline{q}$ are vectors (composed of $\bar{q}_i, \underline{q_i}$, respectively) of aspiration and reservation levels respectively, and $u_i(q_i, \bar{q}_i, \underline{q}_i)$ are the corresponding Component Achievement Functions, which can be simply interpreted as nonlinear monotone transformations of q_i taking into account the information represented by $\bar{q}_i$ and $\underline{q}_i$. Maximization of the function (23) over the set of feasible solutions X_0 defined by the corresponding core model provides a properly Pareto-optimal solution with the properties discussed above for the function (22).

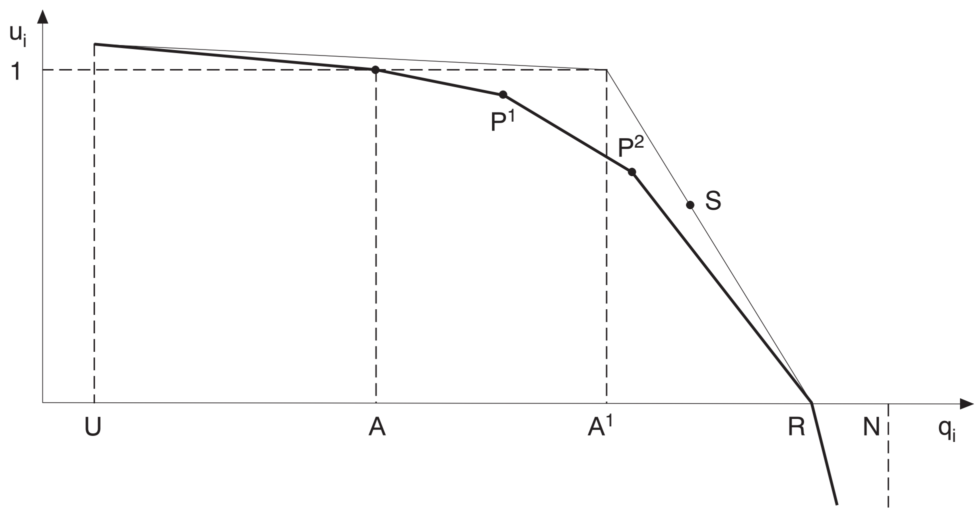


Figure 7: Component achievement scalarizing function.

Various graphical user interfaces can be used for specification of aspiration and reservation levels, as well as the interpretation of solutions. Thus CAF provides an easy and natural way for specification of the desired values of each criterion by a corresponding aspiration level, and to scale trade-offs between criteria by corresponding pairs of aspiration and reservation values.[9] Typically, initial aspiration values are far from being attainable, and the user must modify her/his preferences, which are expressed by pairs of aspiration and reservation levels, in order to achieve solutions that are not too far away from the realistic goals.

A graphical presentation of CAF not only supports users in the specification of preferences, but also helps them in interpreting the solutions. This analysis is done by projections of multidimensional criteria space into two dimensional spaces composed for each criterion of its values and the degree of satisfaction of meeting preferences expressed by aspiration and reservation levels.

[9] Note that this approach to scaling does not require any scaling of criteria values; therefore the user provides his/her preferences for aspiration/reservation levels expressed in original units of the criteria value.

Two examples of CAFs are illustrated in Fig. 7. The first CAF is defined by four points, with values of the criterion, **U, A**^{1}**, R,** and **N**, corresponding to the values of utopia, aspiration, reservation, and nadir, respectively. The second CAF is defined by a modification of the first CAF, where the previously defined aspiration level A^1 was moved to the point A and two more points – P^1 and P^2 – were interactively defined. Note that the utopia and nadir points are computed automatically, therefore the user must specify only two values (aspiration and reservation), and may optionally specify his/her preferences for values between the aspiration and reservation levels.

Values of CAF have a very easy and intuitive interpretation in terms of the degree of satisfaction from the corresponding value of the criterion. Values of 1 and 0 indicate that the value of the criterion exactly meets the aspiration and reservation values, respectively. Values of CAF between 0 and 1 can be interpreted as the degree of *goodness* of the criterion value, i.e., to what extent this value is close to the aspiration level and far away from the reservation level. These interpretations correspond to the interpretation of the membership function of the Fuzzy Sets, which is discussed in [23].

By using an interactive tool for specification of the CAF illustrated in Fig. 7 (and analysis of the corresponding solutions) such as MCMA [23] a user can analyze various parts of a Pareto set that best correspond to various preferences for trade-offs between criteria. These preferences are typically different for various stages of analysis, and are often modified substantially during the learning process, when aspiration and reservation levels for criteria values are confronted with the attainable solutions which correspond best to the aspiration and reservation levels. In such an interactive learning process, a user gradually comes to recognize attainable goals that correspond best to his/her trade-offs.

Application of the RFP approach to analysis of Pareto sets is discussed in Section 6.2.2. A more detailed description of the approach is available in [23].

4.5 Comments

We have outlined above the most representative methods for multicriteria problem analysis. Now we will comment on the applicability of each of these methods to the multicriteria analysis problem of the NEEDS project defined in [45]. Comments are rather short for methods that are clearly not suitable for the NEEDS project, and are more detailed for the two methods that can either be adapted or combined to support multicriteria analysis of the problem specified in [45].

AHP:

- Advantages: The main reasons for the popularity of the AHP method are its simplicity, flexibility, its intuitiveness and its ability to handle quantitative and qualitative criteria in the same framework [38].
- Disadvantages: In this method the concept of relative importance is applied, which means that the user should provide estimates w_i/w_j, where w_i is a value of criterion i and w_j is a value for criterion j. The procedure for determining the criteria weights in AHP independently of the units of single criterion variables is one of the main disadvantages of the method [3]. Another major disadvantage is the so-called rank-reversal.[10] Finally, the requirement of pairwise comparisons makes AHP impracticable for problems with many alternatives. Thus AHP is not applicable to the problem described in [45].

[10] The situation in which after removing from the analysis an alternative, the ranking of the remaining alternatives changes.

Outranking methods:

- Advantages: This method is widely used and has a strong theoretical background.
- Disadvantages: The pairwise comparisons must be done at the first stage of analysis which limits the application of the method to problems with a small number of alternatives. A major methodological problem is the rank reversal, see e.g., [73] for a discussion of this problem in the ELECTRE II and ELECTRE III methods. Moreover, the method is considered to be difficult to understand, especially by users without a mathematical background.

Multi-attribute value measurement theory:

- Advantages: After a value function is defined, the alternatives are automatically rank ordered.
- Disadvantages: There are difficulties in defining the value function, which is based on rather extensive inter-criteria information to be provided by the user. There are also methodological issues related to a proper specification of a value function. For example, the US Nuclear regulatory commission examined [27] the application of additive value functions; ten of thirteen applications of additive value functions violated theoretical requirements such as avoiding use of ordinal scales for single attribute value functions. Eight of thirteen violated the requirement for preference independence of attributes, and none acknowledged the existence of any requirements for validity.

Weighted sum approach:

- Advantages:
  - It is one of the most popular methods for analyzing multicriteria problems, and thus it is also widely applied to various energy problems, especially for energy planning problems.
  - Most users consider specification of preferences in terms of weights to be simple, intuitive, and robust.
  - It can be applied to problems with a large number of alternatives as well as to a hierarchy of criteria.
  - The method is not computationally intensive thus is suitable for interactive analysis.
  - Scalarizing (weighted sum) function can be used for ranking the alternatives.
- Disadvantages: The approach uses a linear aggregation function, which implies a number of consequences not recognized by users who are unfamiliar with the background of the method:
  - It implies a full compensation between all criteria with trade-off rates constant for the full range of criteria values; e.g., the trade-off between cost and quality is the same for the most expensive (and also high quality) alternatives as for the cheapest (and lowest quality) solutions.
  - The method assumes full independence of criteria (dependent criteria are (partially) double-counted).
  - For dependent criteria the method may be contra-intuitive, i.e., increasing a criterion weight does not necessarily improve its value, see e.g., [51].
  - The method will not find Pareto-efficient alternatives that do not belong to the convex hull of the Pareto-set.
  - Weights are difficult to specify for problems with many criteria.
  - Removing alternatives which have extreme values of criteria is likely to result in rank reversal.
  - Ranking based on values of the scalarizing function is likely to be sensitive for some ranges of weights, and insensitive for other ranges of weights. Moreover, alternatives

  with very different compositions of criteria values may have similar values of the scalarizing function, thus will be evaluated/ranked as the same.
  - Most approaches perform transformation of criteria values (say in the range [0, 100]). Thus the users must specify preferences for criteria values on relative scales. This is especially difficult for criteria having multimodal distributions of values.
  - Qualitative criteria must be mapped into a quantitative scale.

Reference point method:

- Advantages:
  - The basic method of preference specification is to specify reservation and aspiration criterion value, i.e., values that the user wants to avoid and achieve, respectively. Thus the interpretation of the preference is the easiest possibility.
  - The reservation/aspiration values imply trade-offs between criteria for the criteria ranges between the corresponding reservation/aspiration value. Thus the interpretation is the same as the interpretation of weights, but the trade-offs are different for values outside these ranges.
  - Specification of preferences in terms of values of the criteria; no scaling of criteria values is desired thus the method is immune to the rank reversal problem.
  - The component scalarizing functions (defined for each criterion) have an interpretation similar to the membership function of fuzzy sets.
  - The scalarizing function assures that the selected Pareto-solution is the best in respect to the criterion which has the worst (in terms of its reservation/aspiration) value.
  - The method can be applied to problems with a large number of alternatives.
  - It properly handles dependent criteria, and criteria with multimodal value distributions.
  - The method is not computationally intensive thus it is suitable for interactive analysis.
  - Scalarizing function can be used for ranking the alternatives.
- Disadvantages:
  - The method is less popular than other methods for multicriteria analysis of alternatives.
  - Preferences are specified for each criterion separately; this supports an easy and precise definition of preferences but less experienced users may have problems with correct interpretation of such preferences in terms of trade-offs between criteria.
  - Specification of preferences requires more information than required for the weighted criteria approach.
  - Modification of preferences aimed at examining certain regions of Pareto-sets might be difficult for problems with many criteria.
  - The known implementations do not deal with hierarchical criteria structures.
  - Qualitative criteria must be mapped into a quantitative scale.
  - Most known implementations have been done for continuous or mixed-integer problems. Although such implementations can be adapted for analysis of alternatives, a new interface and additional data processing should be developed for an efficient analysis of discrete problems with a large number of criteria.

5 Structures of criteria and alternatives

5.1 Hierarchical structure of criteria

The term hierarchy has different meanings. We can distinguish, see e.g., [35]:

- **Order hierarchy:** In this case hierarchy is equivalent to an ordering induced by the values of a variable defined on a set of elements. Order hierarchy does not refer to relationships and interactions among objects that comprise the hierarchy.

- **Inclusion hierarchy:** Recursive organization of objects. In this case an object can be treated as a container that contains other objects.

- **Control hierarchy:** In this context, hierarchy refers to a control system in which every entity has an assigned rank. Entities with a specified rank are entitled to give orders to entities with lower rank. It should be noted that entities that comprise a control hierarchy do not form an inclusion hierarchy.

- **Level hierarchy:** In this hierarchy entities exist on different levels. Entities at a given level may, through their interactions, construct and maintain entities at higher levels, and higher level entities may be composed of lower level entities. In this case we have *upward causation*. Through upward causations, level hierarchies may form inclusion hierarchies. Level hierarchies can be also characterized by *downward causation*: incorporation into a higher level entity can change the properties and interaction modalities of lower level entities.

The hierarchy can be defined in the following way [59]:

Let H be a finite partially ordered set with the largest element b. H is a hierarchy if it satisfies the following conditions:

1. There is a partition of H into sets $L_k, k = 1, \dots, h$, where $L_1 = \{b\}$.
2. $x \in L_k$ implies $x^- \subset L_{k+1}$ $k = 1, \dots, h-1$.
3. $x \in L_k$ implies $x^+ \subset L_{k-1}$ $k = 1, \dots, h$.

where sets x^- and x^+ are defined using the notion of covering[11] as follows:

$$x^- = \{y | x \text{ covers y}\} \tag{24}$$

$$x^+ = \{y | y \text{ covers } x\} \tag{25}$$

In the context of multicriteria decision analysis we can consider the hierarchy of criteria (level hierarchy and order hierarchy) and the hierarchy of alternatives (order hierarchy).

The hierarchy of criteria can be considered in the following ways:

- Hierarchy of the criteria comes from the structuring of the problem and it is used only for a better understanding of the problem, but in a mathematical model of preferences only the lower level of hierarchy is used. French [18] provides an example of this approach.

- Each level of the hierarchy contributes to a preferential model. The upper level of hierarchies influence the lower one.

- Each level of the hierarchy contributes to a preferential model. The lower level of hierarchies influence the upper one.

[11] x covers y if there is no z such that $x \leq z$ and $z \leq y$.

- Dynamic hierarchy. In this new approach one considers two hierarchies. The first one which has been built during the structuring of the decision problem. The second one is building dynamically during the problem analysis. The first hierarchy is used only for communicating the decision problem to the people. The second one is used actively in the analysis process. The analyst or decision maker selects the two tree as the most important criteria, that may belong to the different branches of the first hierarchy, and focuses his analysis on this most important criteria. In the next steps he/she extends the number of criteria which are analyzed. It is assumed that in the next steps the less important criteria are selected. If in the process of analysis he/she recognizes that a selected criterion should be more important, the next iteration of analysis can be done. What is important here is that the importance of the criteria is based on the subjective evaluation of the user and do not must be expressed quantitatively.

In modeling of decision problems it is useful to build the hierarchy of the criteria. However, it significantly complicates the process of building the mathematical model of preferences. Nevertheless, it is definitely important from the point of view of having a better insight into the problem being analyzed. The basic question is how to incorporate the hierarchy into the model of preferences.

There are methods that build a preference function in a structured way. The alternatives can be characterized by a set of attributes or criteria. The criteria to evaluate alternatives have various scales. The appropriate transformation of criteria values into preferences is one of the most important issues that form the foundation for further analysis. It is necessary to consider units. Assigning the values of the criteria to alternatives can be compared to the process of measurement which associates a numerical value with the object. It can be represented by a function $f : A \to C$, also called scale. Some of the common scales are numerical, ordinal, cardinal (ratio, interval). If it is a numerical scale having the lowest value does not always mean that it is the worst alternative e.g., the optimal temperature of the body is around 36.7, therefore we often need a transformation of real measurement into a preference scale $v_i(c_i)$.

Assuming that we have a well defined set of criteria, then the value function should be built. In building such a function the decision-maker, the stakeholder or the analyst should provide inter-criteria preference information.

The value function should have the following properties:

$$(c_1(A_k), \ldots, c_n(A_k)) \succeq (c_1(A_l)), \ldots, c_n(A_l)) \tag{26}$$

$$\Leftrightarrow \tag{27}$$

$$v(v_1(c_1(A_k)), \ldots, v_n(c_n(A_k))) \geq v(v_1(c_1(A_l)), \ldots, v_n(c_n(A_l))) \tag{28}$$

There are various approaches to building function $v(\cdot)$. One of them is an additive value function:

$$c(A_k) \succeq \boldsymbol{c}(A_l) \Leftrightarrow \sum_{i=1}^{n} v_i(c_i(A_k) \geq \sum_{i=1}^{n} v_i(c_i(A_l)) \tag{29}$$

The above function is defined under the following conditions:[12]

- weak ordering is defined;
- alternatives $A_1, \ldots, A_m$ are mutually preferentially independent;
- the weaker solvability (known also as restricted solvability) condition is accepted;
- Archimedian condition (every strictly bounded sequence is finite) holds;
- all criteria are essential, i.e., each has some effect on preference.

[12] See [18] for details.

The most popular is linear value function:

$$v(c) = \sum_{i=1}^{n} w_i * v_i(c_i) \tag{30}$$

For a hierarchy of criteria the value function should still have the property (33). However, it should depend on some parameters specified on the levels of the hierarchy. Moreover we can have upward and downward causation.

$$v(c(A)) = v(v_{11}(c_{11})), \ldots, v_{ih}(c_{ih}))) \text{ for } , i \in NC_h, h \in HL \tag{31}$$

One of the methods that deal with the hierarchy of criteria is Multi-attribute Value Tree Analysis. In this approach the attributes are organized as a value tree [54, 55, 56, 60, 62], also called an objectives hierarchy or criteria hierarchy.

In this case the value function:

$$v(A_k) = \sum_{i=1}^{M} \sum_{i \in L_I} w_i v_i(c_i(A_k)) \tag{32}$$

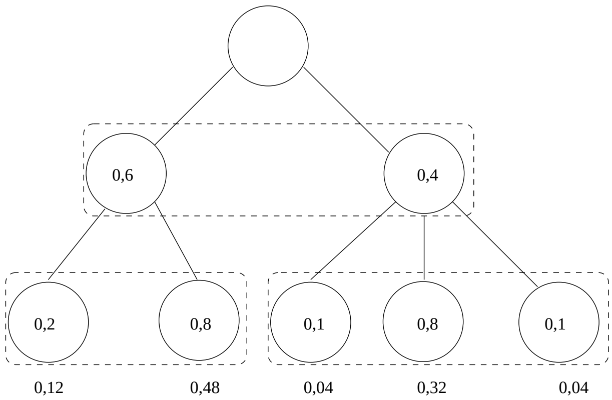


Figure 8: Hierarchical weighting

In the hierarchical weighting approach the analyst specifies weights for each hierarchical level separately, and then multiplies down to get the corresponding lower level weights, see Figure 8.

In the non-hierarchical weighting approach the analyst specifies simultaneously weights only for the lowest level. The weight of criteria at the upper level is by definition equal to the sum of the attribute weights on the lower level, see Figure 9.

We should also mention that the function decomposition method develops a hierarchical structure from class-labeled data [5]. There are also approaches to treat incomplete information within the framework of hierarchical structures, see e.g., [2, 36].

Let us show the main difficulties on the following, very simple, example. We have three levels of hierarchy; the main goal G, the two subgoals SG_1 and SG_2 , four criteria on the lower level c_1, c_2, c_3, c_4 and three alternatives to evaluate A_1, A_2, A_3. Let us assume that on the lower level we have the values of criteria shown in Table 1:

Let us denote the set of all alternatives in the example above as $A = \{A_1, A_2, A_3\}$ and $\succeq$ the decision maker's weak preference. Then $v(\cdot)$ is an ordinal value function representing these preferences if $v(\cdot)$ is a real value function on A such that

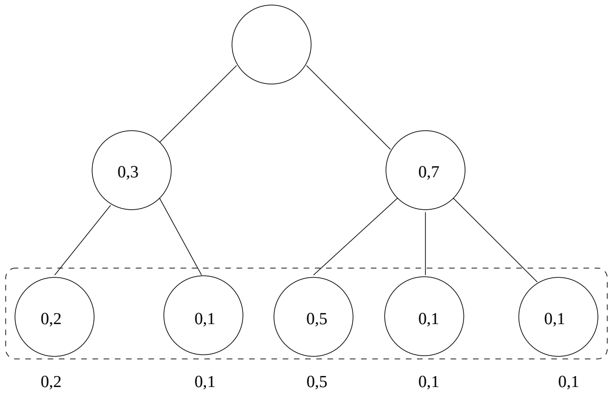


Figure 9: Non-hierarchical weighting

	c1	c2	c3	c4
A1	7 Euro	200 m2	good	5km
A2	5 Euro	1000 m2	very good	10km
A3	1 Euro	500 m2	bad	1km

Table 1: The values of the criteria.

$$v(A_i) \geq v(A_j) \Leftrightarrow A_i \succeq A_j \tag{33}$$

It should be stressed that ordinal value function $v(\cdot)$ encodes only the preference information; addition, subtraction, multiplication and division is meaningless. So, the $v(\cdot)$ does not encode the strength of the preference. If the function $v_m(\cdot)$ reflects the strength of preferences e.g. encoding that the decision maker prefers more A_1 to A_2 thenA_2 to A_3 and more general $v(\cdot)$ should have the properties that $v(A_i) - v(A_j) \geq v(A_k) - v(A_l)$. if the function $v(\cdot)$ is defined as follows $v(A_1) = 3$, $v(A_2) = 1$, $v(A_3) = 2$ then we can say that $A_2 \prec A_3 \prec A_1$. However, the identification of function $v(\cdot)$ is very challenging task and in many practical problems it is impossible to do it properly.

Assume that:

$$v_1(c1) = c1, \qquad v_2(c2) = c2, \qquad v_3(c3) \in \{1, 2, 3\}, \qquad v_4(c4) = c_4. \tag{34}$$

The aggregation function for c_1 and c_2 is defined by:

$$v_{G1}(v_1, v_2) \tag{35}$$

and for c_3 and c_4 by:

$$v_{G2}(v_3, v_4). \tag{36}$$

Then we need to define $v_G(v_{G1}, v_{G2})$. The main problems with such a definition are:

- What is an interpretation of the values v_{G1} and v_{G2}?
- Are the values v_{G1} and v_{G2} comparable?
- How is the trade-off between v_{G1} and v_{G2} interpreted?

Of course, one can use the weights as suggested in the Value Tree Analysis but the difficulties in specification and interpretation of the weights will remain.

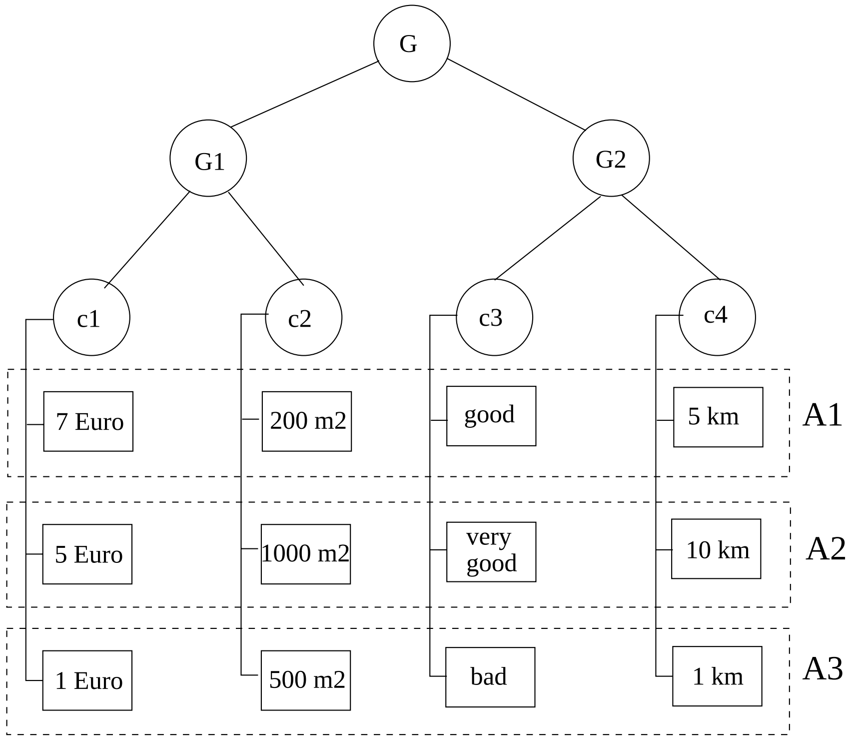


Figure 10: The structure of the problem.

5.2 Preference information about alternatives

Consider a set of alternatives $A = \{A_m, m \in M\}$ and a preference relation $\mathcal{R}(\mathcal{P})$ between the alternatives:

$$A_i \; \mathcal{R}(\mathcal{P}) \; A_j \qquad i \in M, j \in M \tag{37}$$

The preference relation $\mathcal{R}(\mathcal{P})$ induces a complete or partial order of all alternatives. Relationships between alternatives and the induced order might be represented by graphs. We can distinguish the following cases:

The order of alternatives by the value difference function: This is the result of aggregation of preferences that are the most preferred by the decision makers. There is information about the order of alternatives as well as about the distance between alternatives corresponding to the specified preferences, see Figure 11a. Although it is the most preferred result in many decision situations it is difficult to build such a value difference function.

The order of alternatives by the value function: The value function may by used for ordering the alternatives. However, it should be stressed that such a function cannot be used for providing information about the strength of the preferences, and it is typically very difficult to identify it, see Figure 11b.

Partially ordered sets: If the analyst cannot build any value function the partial orders might be applied. There is a method for multicriteria ranking built on the theory of partially ordered sets (called posets) [34]. In this method the alternatives can be only partially ranked because any two alternatives are only comparable if one of them has better values than all the other criteria; otherwise the alternatives are not comparable. The relationships between alternatives can be presented by the so-called Hasse-diagrams illustrated in Figure 12. In this case we can distinguish the hierarchy level and, with respect to preferences,

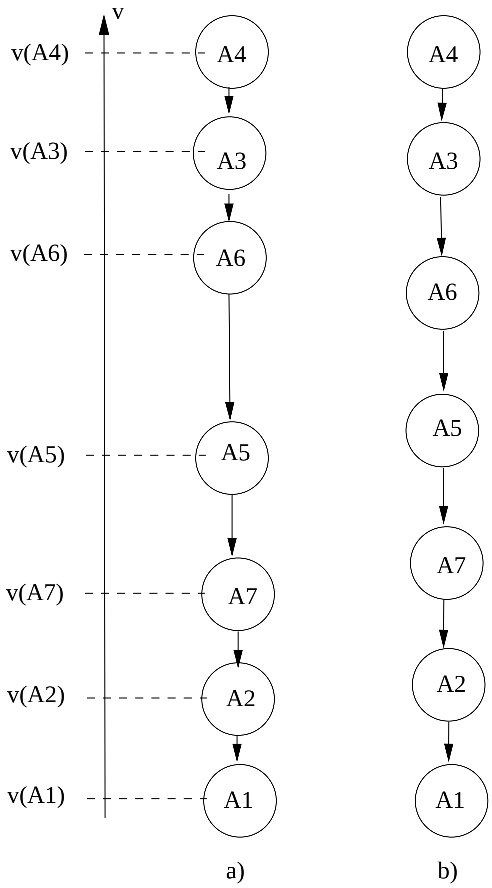


Figure 11: Order of alternatives with (left graph) and without (right graph) a measure of distance between alternatives.

if the alternative is in the upper hierarchy it is better than alternatives in the lower level of hierarchies.

Pairwise preference information: An example of this approach is an outranking relation. In [70] the outranking relation is represented by a graph of the type illustrated in Figure 13. It can be observed that there is no order hierarchy. We cannot conclude which alternative is the best or which is the worst.

Each of the orderings described above depends on preference information. It is necessary to analyze the stability of the solution if the decision maker slightly changes his/her preferences. The computerized method must be equipped with a tool for sensitivity analysis of the solutions.

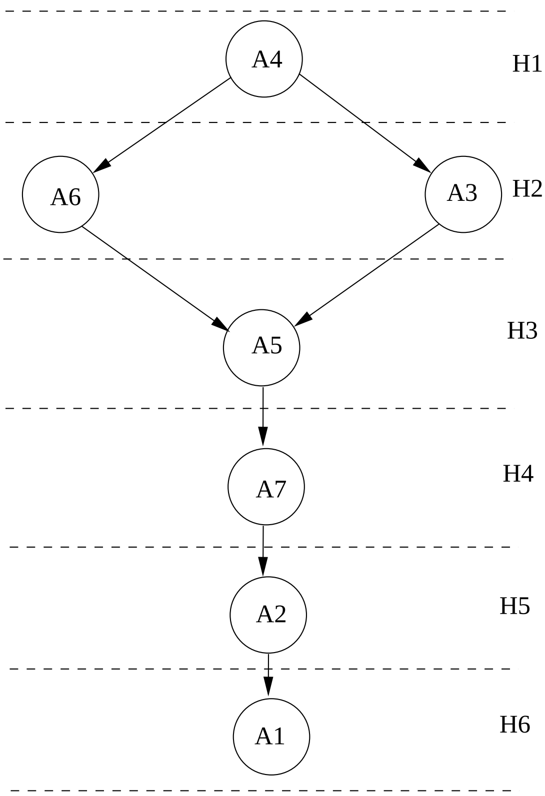


Figure 12: Hasse diagram.

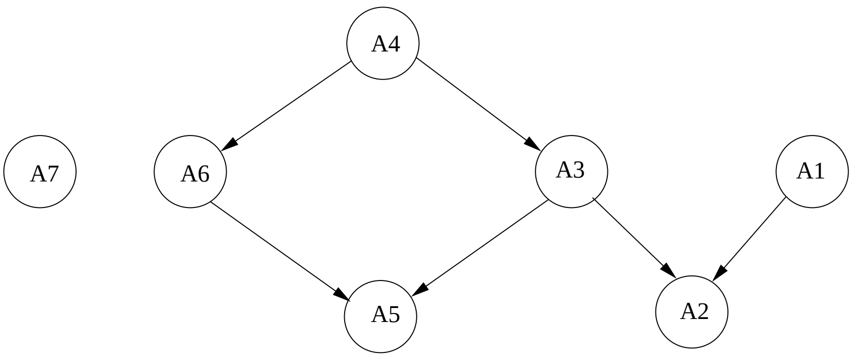


Figure 13: Outranking relation represented by a graph

6 Multicriteria analysis

6.1 Why multicriteria analysis is needed

Traditional OR approaches are based on the assumption that the best solution of a decision problem is the one that maximizes a selected criterion. However, this assumption is true only for a specific class of well structured problems. More than 50 years ago Simon [65] demonstrated that such an assumption is wrong for most actual decision-making problems. Recent studies, see e.g., [64] confirm Simon's results.

A treatment of a decision-making problem as a single criterion optimization seems to be very attractive because it offers a unique solution based on solid mathematical foundations; especially, if one considers that an abundant choice (even among discrete alternatives) typically creates problems, such as dissatisfaction or regret, see [63]. In reality, however, almost all actual decision problems have a large (or infinite) number of solutions typically evaluated with the help of conflicting criteria. Pareto-optimal solutions are not comparable in a mathematical programming sense, i.e., one can not formally decide which is better than the other. Thus, a choice of a solution depends on the preferences of the user that implicitly defines the properties of the corresponding solution. Thus, in order to find a Pareto-efficient solution that corresponds best to a user's preferences one needs to support the user in the analysis of trade-offs between criteria.

The (traditional) OR routine of representing a decision problem as a mathematical programming problem in the form:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x} \in X_0} \mathcal{P}(\boldsymbol{q}(\boldsymbol{x})), \tag{38}$$

which provides optimal solution $\hat{\boldsymbol{x}}$.[13] The optimality is defined in the sense of preferences defined for vector of criteria $\boldsymbol{q}$ values of which are defined (either implicitly by a model or explicitly by a set of alternatives) for each $\boldsymbol{x} \in X_0$ (where X_0 denotes a set of feasible/considered solutions).

However, this approach does not work in practice because there is no unique representation of preferences $\mathcal{P}(\cdot)$ that can be specified in a robust way. Thus, optimization in supporting decision-making for solving complex problems has quite a different role from its function in some engineering applications (especially real-time control problems) or in very early implementations of OR for solving well-structured military or production planning problems.

This point has already been clearly made e.g., by Ackoff [1], and by Chapman [8], who characterized the traditional way of using OR methods for solving problems as being composed of the following five stages: describe the problem; formulate a model of the problem; solve the model; test the solution; and implement the solution. The shortcomings of such an approach are discussed in many other publications, see e.g., [47] and [82] for more details, and have been the main driving force for developing methods of model analysis that better serve the needs of decision makers.

6.2 Basic properties of multicriteria analysis

The purpose of multicriteria analysis is to examine various areas of the Pareto-set that correspond to various preferences $\mathcal{P}(\cdot)$ Since Pareto-optimal solutions are not comparable in a mathematical programming sense, each of them can therefore be considered *the best*, and the choice depends on the preference of the user.

[13] By $\hat{\boldsymbol{x}}$ we denote a solution of the corresponding problem. For problems of discrete alternative choice $\hat{\boldsymbol{x}}$ denotes a selected alternative, and $\boldsymbol{q}(\hat{\boldsymbol{x}})$ denotes a vector of criteria values corresponding to $\hat{\boldsymbol{x}}$.

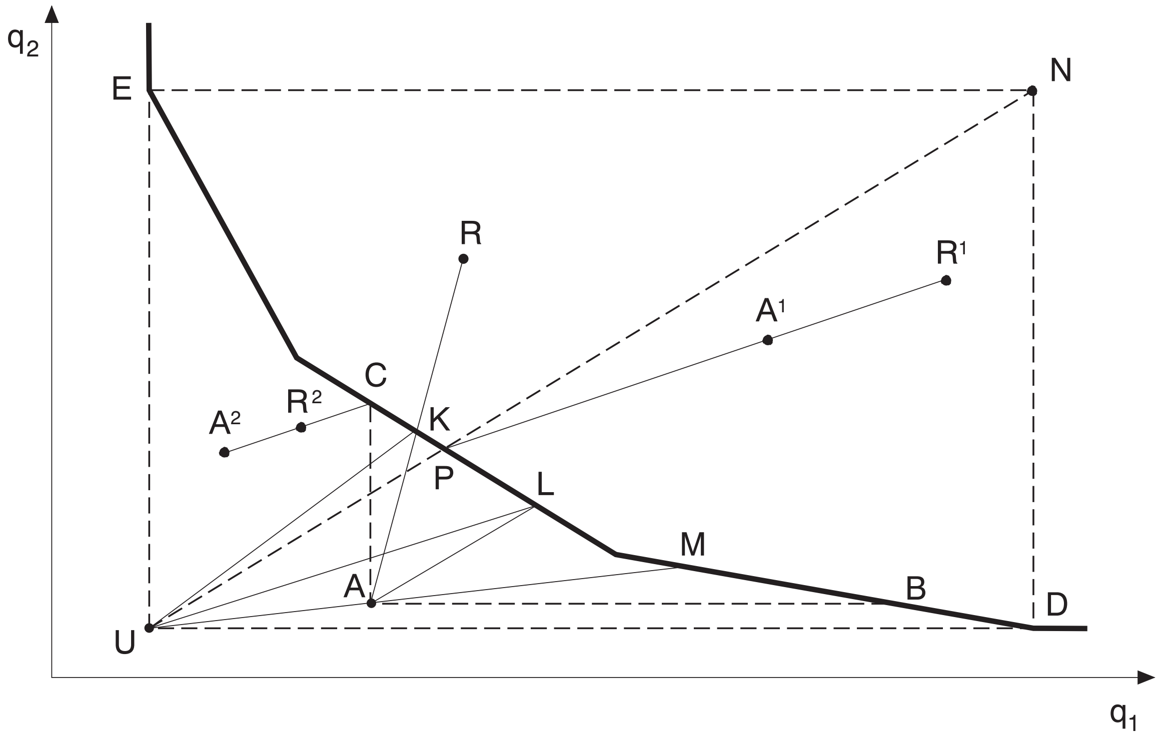


Figure 14: Trade-offs between criteria.

To illustrate this point let us consider a Pareto set shown in Figure 14 for two minimized criteria:
- q_1, costs of emission reduction, and
- q_2, a measure of a concentration of pollution,

Pareto-solutions are located on the thick line between the points marked by **D** and **E**. Clearly,[14] there are no solutions between the Pareto set and the Utopia point **U**, and any solution between the Pareto set and the Nadir point **N** is not efficient.[15] The solution denoted by **D** has the lowest (with the Pareto-set of solutions) concentration of pollution and is most expensive, and the solution denoted by **E** is the cheapest one but has the highest concentration of pollution. Solutions along the Pareto-set have different trade-offs between criteria.

A trade-off here is understood as the ratio of change of criteria values. For the illustrative case it can be interpreted e.g., as *How many Euros one needs to pay for decreasing the concentration of pollution by one unit.* Such a ratio is clearly related to the slope of the corresponding segment of the Pareto set. For our example the higher the ratio, the flatter the corresponding segment, e.g., for solutions denoted by **M, D, M** decreasing the pollution concentration is much more expensive than for solutions **C, K, L**; the cheapest improvements can be obtained for solutions located on the segment that starts at point **E**.

One should be aware that while the illustrative example presented in Figure 14 is easy to analyze, actual cases are not, because real problems typically require analysis of several criteria often having wide ranges of values, or multimodal distributions of values. A clear illustration of Pareto sets is possible for only two criteria, and the comprehension of trade-offs between more than two criteria over the whole Pareto set is practically impossible. Therefore, one needs to ex-

[14]By the definition of the Pareto set, see Section 4.1.1.

[15]Because there exist solutions which have better values of one criterion (with at least the same value of the second criterion).

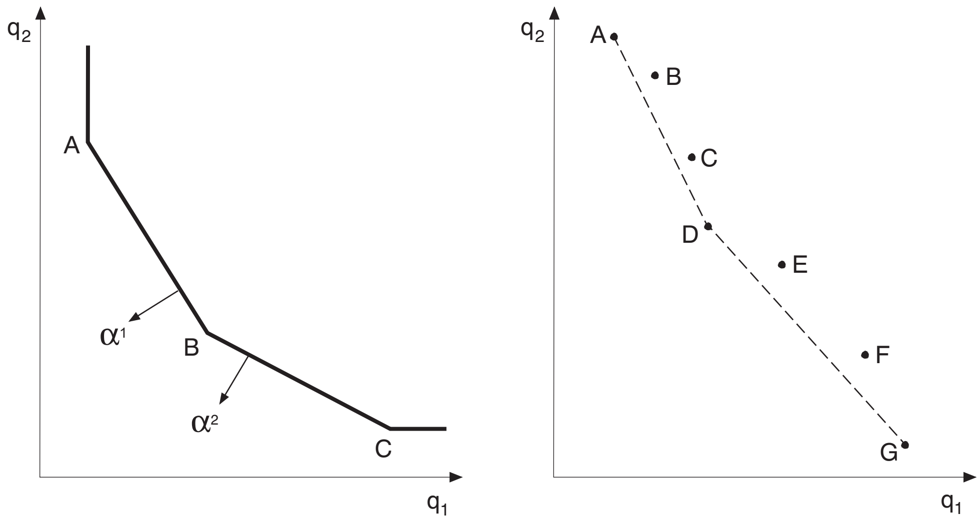
ploit the analytical properties of mathematical representations of preferences in order to support users interactively examining those areas of the Pareto set that have trade-offs corresponding to the user preferences. In our example such areas can be identified by either ranges of cost, or ranges of pollution levels, or tradeoffs between cost and pollution concentration.

The essence of multicriteria analysis is to support the user in specifying his/her preferences (which are expressed in terms corresponding to the criteria); to analyze the corresponding Pareto solution; and to redefine the preferences until the corresponding solution will be considered *the best* by the user. It must be stressed that *the best* solutions are not only very different for different users, but often the same user changes her/his preferences when analyzing the same problem after a few hours/days.

Thus the most important feature of any multicriteria analysis method should be to respect the user's sovereignty, i.e., to provide the user with full control of the analysis process (e.g., by making sure that no solution is excluded from the analysis). It should also support users in the specification of preferences in a way that is transparent and understandable for the user, who is typically unfamiliar with the mathematical representation of his/her preferences in the underlying optimization problem solution, presented as the Pareto-solution that corresponds to the specified preferences.

In the multicriteria analysis process each specification of preferences defines an instance of the multicriteria problem which is converted into an auxiliary parametric single-objective problem, the solution of which provides a Pareto-optimal point with the properties which correspond best to the specified preferences. Different methods apply different conversions, but all commonly known methods can be interpreted in terms of the Achievement Scalarizing Function (ASF),[16] see [41] for details. We provide below short interpretations of the ASF corresponding to the weighted sum and the reference point methods, outlined in Sections 4.4.1, and 4.4.2, respectively.

6.2.1 Exploring Pareto sets using the weighted sum method



(a) Continuous LP (b) Discrete LP

Figure 15: Pareto sets analyzed by the weighted sum method.

[16]The concept of ASF was introduced by Wierzbicki see, e.g., [74, 82].

Let us consider two examples of Pareto sets illustrated in Figure 15, with the same meaning of criteria as above, i.e.
• q_1, costs of emission reduction, and
• q_2, a measure of a concentration of pollution,
and the corresponding weights w_1 and w_2. The ASF is defined by

$$ASF = w_1 q_1 + w_2 q_2 \tag{39}$$

For the analysis of two-criteria examples it is enough to consider the trade-off ratio between pollution concentration and costs:

$$\alpha = w_1 / w_2. \tag{40}$$

Minimization of the ASF defined as:

$$ASF = \alpha q_1 + q_2 \tag{41}$$

will result in one of three solutions (denoted by **A, B, C**, respectively) depending on the trade-off (represented by value of α) between the two criteria. Typically, when q_1 attains its best value (which corresponds to a minimum cost solution and the corresponding high value of q_2, the solution denoted by point **A**) the value of α will be rather high, indicating a much lower weight attached to the environmental criterion than that of the cost criterion, which implies an unwillingness to accept costs for the reduction of pollution. Such preferences imply a large α (which is equivalent to a steep slope of the corresponding segment of the Pareto set) for the example presented in Figure 15. Actually, for any value of α larger than α^1, the resulting solution will be in point **A**. Conversely, for a best available purification technology the q_2 will attain a minimum, which also corresponds to the highest costs. In such a situation α will take a rather low value which corresponds to a much higher weight attached to the environmental criterion and the selected solution will be at point **C**.

One should note that the weighted sum approach provides (for linear problems) only Pareto-optimal solutions corresponding to vertices **A**, **B**, and **C**. For any weighting coefficients, vector α with a slope flatter than the slope of the vector α^1, a solution will be in the vertex **A**. For a weighting coefficient vector that is parallel to α^1, there is no unique solution,[17] and a very small increase of the slope of α will cause the solution to jump to the vertex **B**. A further increase in the slope of α will not cause any changes in the Pareto solution until the slope becomes greater than α^2 (which will cause another jump to the vertex **C**). This explains the experience known to everyone who has tried to use weights to analyze multiple-criteria LP models, namely that, often, a relatively large change of weights does not cause any changes to the solution, but for some combinations of weights, a small modification creates in the same model a substantially (in practice the distances between vertices are often large) different solution.

Third, a weighted aggregation of criteria does not allow us to find all Pareto solutions. For a discrete model, a surface spanning the Pareto set (that is composed of points) is usually non-convex. Therefore, for the example depicted in Figure 15 only some efficient solutions, namely, **A**, **D**, **G** will be found while possibly many other efficient solutions (e.g., **B**, **C**, **E**, **F**) will never be found.

6.2.2 Exploring Pareto sets using the reference point method

Reference point methods (RFP) are based on the concept of the reference (aspiration) point, which is composed of the desired values of all criteria. Typically such a point is infeasible,

[17] Therefore the corresponding problem will be degenerated and any solution from the edge **AB** is optimal. Hence, the reported solution will differ, depending not only on the applied solver but also on the parameters used for a solver, including the possibly defined starting point for optimization.

thus one looks for a Pareto solution that is closest to this point. Obviously, for the Utopia point (composed of best values of all criteria, and marked by the letter U in Fig. 14), any of the Pareto-optimal points between points E and D can be obtained for various definitions of the distance between the aspiration point U and the Pareto set. Thus, for a unique selection of a Pareto solution one needs to define either another point (which together with an aspiration point defines a direction) or an ASF that provides a unique selection of solutions.

We illustrate the RFP method by outlining the Aspiration-Reservation Based Decision Support (ARBDS) method, which requires a specification of two points, called aspiration and reservation, composed of the most desired and the worst acceptable values of criteria, respectively. A well implemented ARBDS does not impose any restrictions on the feasibility of the aspiration nor of the reservation values. E.g., in Fig. 14 there are three pairs of aspiration and reservation points, denoted by {A, R}, {A^1, R^1}, and {A^2, R^2}, respectively. The corresponding Pareto-solutions are marked by K, P, and C, respectively. A selection of a pair like {A, R} (i.e., an unattainable aspiration and a feasible reservation level) is typical for users who have learned the properties of the problem and have a good feeling about the attainable ranges of criteria values. Selection of an unattainable reservation level (e.g., {A^1, R^1}) is typical for early stages of the model analysis, when unrealistic reservation levels are specified. However, specifications of attainable aspiration levels (e.g., A^2) are not as rare as one would expect; especially, if some criteria are interdependent.

One should note that a direction in the criteria space implies trade-offs between the corresponding criteria. Thus a specification of either:
- aspiration and reservation values, or
- an aspiration value and a direction, or
- a reservation value and a direction

implies trade-offs between criteria, which has exactly the same interpretation as weights in the weighted sum (WS) methods.

Actual implementations of the RFP method, see, e.g., the MultiCriteria Model Analysis (MCMA) [23], exploit ASF defined as:

$$ASF = \min_{1\leq i\leq n} u_i(q_i, \bar{q}_i, \underline{q}_i) + \epsilon \sum_{i=1}^{n} u_i(q_i, \bar{q}_i, \underline{q}_i) \tag{42}$$

where $u_i(\cdot)$ denotes i-th Component Achievement Function (CAF), $q_i, \bar{q}_i, \underline{q_i}$, are the value, aspiration and reservation levels of i-th criterion, respectively; n is the number of criteria, and ϵ is a small positive number, $\bar{q}, \underline{q}$ are vectors (composed of $\bar{q}_i, \underline{q_i}$, respectively) of aspiration and reservation levels, respectively, and $u_i(q_i, \bar{q}_i, \underline{q}_i)$ are the corresponding Component Achievement Functions (CAFs). A CAF $u_i(\cdot)$ can be simply interpreted as a nonlinear monotone transformation of the i-th criterion value q_i, which reflects the degree of satisfaction of the user.

ASF defined as (42) provides a very good way of expressing preferences for users comfortable with considering the problem in terms of satisfaction levels for each criterion separately. Such ASFs may be difficult to interpret in terms of trade-offs between criteria, especially for inexperienced users. However, one can adapt the RFP method to the discrete alternative choice by either:
- Providing additional (to the above discussed interpretation of the ASF) information, e.g., about trade-offs between criteria corresponding to the ASF, or about trade-offs leading to neighboring solutions.
- Using ASF defined by a reservation (or by an aspiration) point and a direction, the latter implicitly defined by explicit specification of trade-offs between criteria.

6.3 Similarities and differences between the weighted sum and the reference point methods

The two most widely used ASFs are discussed in detail in Sections 4.4.1, and 4.4.2, respectively. The difference between these two methods is due to the form of the corresponding ASFs. The ASF of the weighted sum (WS) method implies that the trade-offs are valid everywhere while the ASF of the reference point (RFP) methods apply trade-offs along the line defined by the aspiration (or reservation) point and the direction corresponding to the trade-offs. Here we summarize the basic similarities and differences between these two methods:

- Both methods are widely used, and are also easy to use for inexperienced users.
- The computational complexity of both methods is practically the same.
- The WS (Weighted Sum) is probably the most established multicriteria method while the RFP (Reference Point) method was introduced about 20 years ago and is still less popular than the WS approach.
- The WS method defines ASF as a weighted sum of criteria, which implies a full compensation between criteria with substitution rates that are constant for the full range of criteria values.
- The RFP method defines ASF as a nonlinear operator on Components Achievement Functions (CAF) defined for each criterion by a specification of aspiration and reservation levels (see Section 4.1.1); this implies that always the worst (in the sense of aspiration/reservation values) criterion is improved first.
- The WS uses ASF parameters (weights) specified for the whole ranges of criteria values, which imply constant trade-offs between criteria. The parameters of the ASF of the RFP method are defined implicitly for at least three subsets of values of each criterion, thus the trade-offs between criteria change.
- The WS method provides solutions that optimize the corresponding ASF on the given set of feasible solutions. Solutions in the RFP approach optimize the corresponding ASF along the line that crosses the selected reference point and has the direction defined by the parameters of the ASF.
- The WS requires scaling/normalization of criteria and weights values (typically to the [0,1] or [0,100] interval); the weights have an interpretation of trade-off coefficients between values of normalized criteria. The RFP uses original values of criteria, and the CAF for each criterion has an interpretation similar to the Membership Function of the Fuzzy Sets.
- The WS method is likely to cause the so-called "rank-reversal" problem (i.e., a change of ranking after removing an alternative from consideration). The RFP method is not exposed to such problems.
- Both methods are difficult to use for problems having multimodal distributions of criteria values, especially if the ranges of values differ by several orders of magnitude. However, customized solutions for such cases can be implemented.

6.3.1 Methodological roots of the WS and RFP methods

There are essentially two main methods of parameterizing Pareto-optimal decisions:

- By using weighting coefficients, i.e., specifying how much relative importance we assign to various objectives. Mathematically, the method corresponds to, e.g., maximizing the weighted sum of all objective functions over the set of admissible decisions. When the weighting coefficients are all positive, the maximization of the weighted sum results in Pareto-optimal decisions. However, more important is the issue of whether we could produce all Pareto-optimal decisions (which is called a complete parametric characterization of the Pareto frontier). When using the maximization of a weighted sum, we can sometimes produce all Pareto-optimal decisions and outcomes by changing weighting coefficients, but only under restrictive assumptions – e.g., the set of attainable objectives must be convex (or even strictly convex).

- By using goals or reference objectives in decision space, i.e., specifying what objective outcomes we would like to achieve. This method might work in a much more general setting than the method of using weighting coefficients, but it is more complicated mathematically. At first glance, an appealing mathematical method would be to minimize a distance measure or simply a norm of the difference between the goal and the attainable objective vector. Such techniques of norm minimization were first used historically, either in the displaced ideal method of [85] or in the broad family of goal programming techniques starting with the work of [10]. However, simple examples show that norm minimization might produce decisions that are not Pareto-optimal, thus additional assumptions are necessary. They amount, generally, to limiting the specification of goals to values that are highly unrealistic.[18] This motivated the development of a different approach – the reference point approach – that uses reference objectives that can be realistic, but avoids norm minimization and instead uses more complicated functions to be optimized (usually, maximized), called order-consistent achievement functions.

Thus, the reference point methodology could be considered as a generalization of goal programming, aiming at using arbitrary (not only unrealistic) goals or reference objectives and obtaining only efficient outcomes, at the cost of avoiding norm minimization and replacing it by optimization of a more complicated function. We shall discuss now the relations between these methods in more detail.

The main advantages of goal programming are related to the psychologically appealing idea that we can set a goal in objective space and try to come close to it. Coming close to a goal suggests minimizing a distance measure (usually a norm of the difference) between an attainable objective vector (decision outcome) and the goal vector.

The basic disadvantage relates to the fact that this idea is mathematically inconsistent with the concept of Pareto-optimality or efficiency. One of the basic requirements – a general sufficient condition for efficiency – for a function to produce a Pareto-optimal or vector-optimal outcome (when minimized or maximized) is an appropriate monotonicity of this function. However, any distance measure is obviously not monotone when its argument crosses zero. Therefore, distance minimization cannot, without additional assumptions, result in Pareto-optimal solutions.

6.4 Pareto solutions and rankings

There is a common temptation among analysts and software designers to exploit information gathered during the process of Pareto-set analysis for ranking of (possibly all) other (than the one Pareto solution finally selected) solutions. In particular there is a common belief that the values of ASF are a good measure of goodness for all solutions and thus can be used for ordering the whole set of alternatives. One should be aware that ranking based on the values of ASFs is likely to be different than a ranking which results from a sequence of $m - 1$ (where m denotes the number of alternatives) multicriteria analysis of sets of alternatives, where each analysis (except the first one) will be done on a set composed of alternatives that will remain after removing the selected Pareto-optimal one.

The reason for not using the values of an ASF for ranking comes directly from the meaning of ASF. That is, ASFs are designed for identifying a Pareto-optimal solution that corresponds

[18] The specification of attainable goals results in finding solutions that are not Pareto-efficient. Since attainable goals are often difficult to determine, to be on the safe side users typically specify unrealistic goals.

best to a given representation of the user preferences, and the user substantially changes parameters of the used ASF during the analysis process. Thus there are at least four mutually linked reasons for not using ASFs for ranking:

- The main reason for interactive (as opposed to a *”one-shot”*) multicriteria analysis is the commonly known fact that not only inexpert users but also experienced analysts substantially change preferences while learning about the problem properties during its multicriteria analysis. Modifications of ASFs are driven by unsatisfactory trade-offs between criteria values of a current solution; while ASFs are helpful for analyzing Pareto-solutions they are not suitable for ranking all solutions.
- ASF provides a local (i.e., for the current state of analysis) representation of the user preferences.
- Many (a typically infinite number of) ASFs correspond to a given Pareto solution (e,g., for the problem illustrated in Figure 15 on page 33 the solution at point D will be selected for any weight with a value between the two values corresponding to the slopes of segments AD and DG, respectively).
- Often even small changes of ASF result in a rather qualitative change not only of the corresponding solution but even bigger changes in ranking induced by ASFs, see Section 6.2.1 for an example.

Another argument for avoiding making a ranking on the basis of ASF values comes from a quick analysis of the discrete case example in Figure 15. Assume that the user has selected the Pareto solution at point D (which corresponds to a compromise between costs and the pollution concentration). If this solution was selected through ASF with a slope close to the slope of segment DG, then the second (in terms of the values of the ASF) solution would be an extreme (highest cost and best pollution level) solution at point G. However, most likely the user would prefer as the second choice either the solution at point C (slightly cheaper than solution D but with a higher pollution level) or at point E (substantially more expensive but with a lower pollution concentration). Also solutions B and F are likely to be more preferable than solution G.

One more illustration of the problems related to using ASFs for ranking of alternatives is provided in Section 7.4.1.

7 Multicriteria methodology for the NEEDS project

7.1 Characteristics of the problem

We briefly restate here a summary of the basic characteristics of the NEEDS problem (presented in detail in [45]) that the proposed multicriteria methodology must satisfy:

- The analysis is to be done in two stages. First, individual stakeholder preferences are to be elicited in an interactive and iterative process during which each stakeholder will make individual multicriteria analysis of the sets of technologies and scenarios. Second, individual stakeholder preferences and the corresponding solutions will be analyzed for group similarities and contrasts, and compared to a total cost ranking of the respective alternatives.
- The sizes of alternative and of criteria sets exclude methods using pairwise comparison.
- The group of stakeholders is expected to be widely diversified, with correspondingly different preferences.
- Both technology and scenario alternatives are expected to have at least some criteria that exhibit multi-modal value distributions.

7.2 Why none of the standard MCDA is suitable

The characteristics of the NEEDS problem and our previous discussion of the relative advantages and disadvantages of existing methods lead us to the conclusion that there is no existing method that is suited to the requirements defined in [45]. This is true for three primary reasons.

First, we need a two-stage analysis: (1) individual multicriteria analysis of alternatives by up to approximately 1500 stakeholders from four countries, and (2) analysis of eight sets (two sets of alternatives, each pair for four countries) of solutions corresponding to individual preferences. Different methods should be implemented for each stage of the analysis, and the methods must be capable of producing consistent and useful final results.

Second, there is no existing multicriteria analysis method and the corresponding tool that can be used for the analysis in the first stage. Most of the widely used methods for discrete alternatives use pairwise comparisons. However, the numbers of the alternatives and of the criteria in our problem implies that multicriteria analysis methods that use pairwise comparisons of alternatives/criteria are not practical. Moreover, the two main existing methods that can be used for problems with large numbers of alternatives/criteria (i.e., the classical weighted sum and reference point approaches) have key disadvantages that have been described earlier. These disadvantages include rank reversal and unsuitability for hierarchical structure of criteria. Rank reversal is a problem for any method that requires scaling/transformation of criteria values, including the classical weighted sum approach (see Section 7.4.1). The classical reference point method does not deal with hierarchical structure of criteria.

Third, the second stage of the analysis also requires a problem-specific method. Given the number and the diversity of stakeholders (who will make only individual multicriteria analysis) one needs rather advanced data analysis method for a comprehensive analysis of the problem by the analysts who will analyze the stakeholder preferences and the corresponding solutions. An approach to such an analysis using a clustering and sensitivity approach is suggested in Section 7.3.4.

The analytic team in WP9 believes that the strong possibility exists to develop a new multicriteria analysis method that will fit the requirements of the NEEDS problem. Such a method can be based on elements of the WS and RFP approaches. A draft of such a method is outlined in Section 7.4.2 of this document and fulfills the modified objective of WP9, which is to propose a multicriteria analysis methodology and its implementation. In addition to developing a new multicriteria analysis methodology, it will be necessary to develop an online (web-based) application for iterative elicitation of stakeholder preferences, with interactive use of a graphical presentation of preferences and the corresponding solutions. An approach for such a method is outline in Section 7.3.3.

It is acknowledged that such new developments are beyond the original scope of the planned NEEDS MCDA application, and will require some significant design and testing within the time frame of the rest of the project. For these reasons, it is considered that an existing MCDA method (or combination of methods) will be chosen, and this less theoretically suitable method may be used as an alternative or in parallel to the new proposed methodology as seems necessary or useful.

The WP9 team has extensive experience in a diverse range of methods of model and data analysis, and in adapting or developing the corresponding software tools, including multicriteria analysis and web-based applications. This experience has led to our conclusions that for the NEEDS problem a much better (than any existing) multicriteria analysis method is possible, and that such a method can be developed within the available time framework.

7.3 Proposed methodology

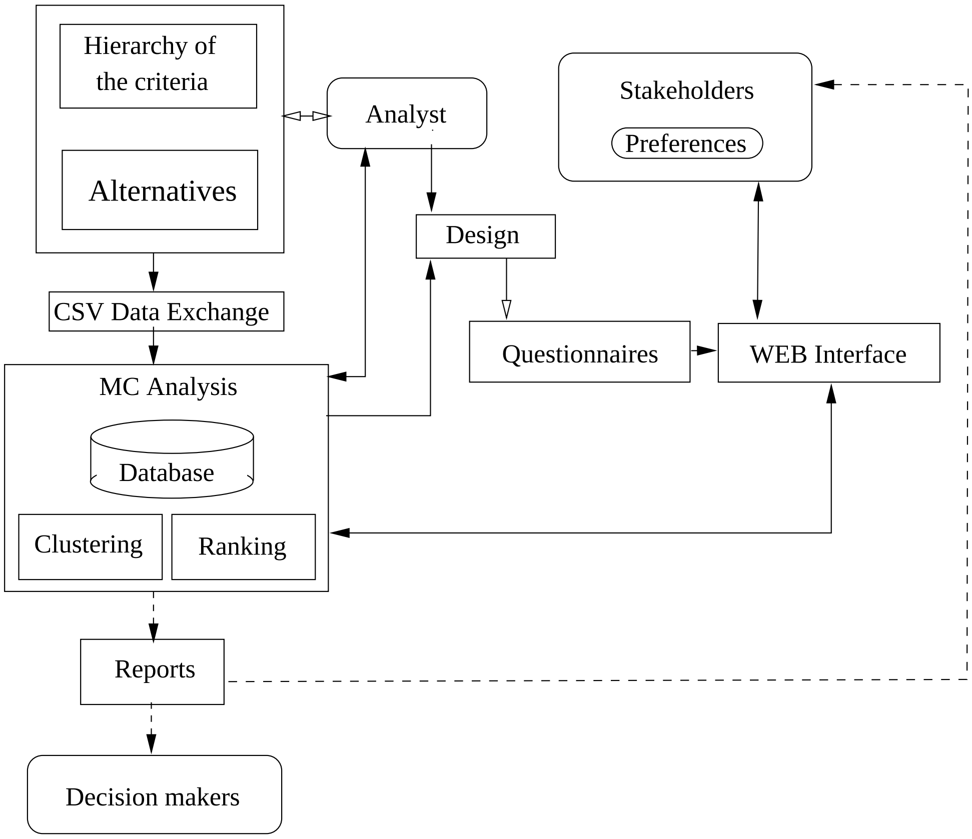


Figure 16: The main components of the process of analysis of alternatives

The structure of the multicriteria analysis process illustrated in Figure 16 is discussed in detail in Section on *Use cases* of [45]. Here we discuss methodological issues related to the following (interrelated but methodologically distinct) elements of the analysis process:
- Elicitation of stakeholder preferences, in Section 7.3.2.
- Multicriteria analysis by individual stakeholders, in Section 7.3.3.
- Multicriteria analysis by analysts, in Section 7.3.4.

7.3.1 Preparation of the MC analysis

Before turning to the methodological issues we briefly summarize the characteristics of the problem, and the necessary preparatory steps.

The problem to be subjected to multicriteria analysis has the following features:[19]:
- set of up to 50 discrete alternatives,
- approximately 50 criteria organized into an up to four-level hierarchy (branches of the hierarchy tree have different depths),
- each alternative is defined by values of all criteria,
- criteria are of both quantitative and qualitative types,
- value distributions for some criteria are multimodal,
- the first stage of the analysis is done by/for each stakeholder individually using a Web-based interactive tool for multicriteria analysis,

[19]Detailed specification of the problem is available in [45].

• the second stage of analysis will be done by the analysts, who will apply data analysis methods to the results of the first stage.

This report focuses on methodological issues. Therefore we assume that the following elements of the analysis process will be completed according to the approach proposed in [45]. This in particular includes:

• availability of a Web-based data-server,
• a proper preparation of data pertinent to the specification of criteria, and of alternatives,
• preliminary data analysis,
• development a of Web-based survey combined with an interactive multicriteria analysis of alternatives.

7.3.2 Elicitation of stakeholder preferences

One of the most important tasks of multicriteria analysis is the preference elicitation process. This will be done through an interactive Web-based survey directly linked with an application supporting a multicriteria analysis of alternatives outlined in Section 7.3.3. Here we summarize the basic methodological issues that should be considered during the design and implementation of the survey.

In the multicriteria preference modeling one distinguishes the phase of definition of the set of criteria (possibly organized in a hierarchy). Due to the conflicting/competing nature of criteria, stakeholders typically cannot order alternatives on the basis of criteria values. Therefore, elicitation of preferences involves gathering and analysis of diversified types of information. Typically in terms related to criteria but also related to the stakeholder preferences in the solution (alternatives) space.

The process of preference elicitation is actually equivalent to finding a link between a formal computerized model of preferences and the stakeholders. The main problem arises from the various languages/representations of formal methods and the way the stakeholders think about (and can express) his/her preferences. Therefore, we have different views of the information required by the algorithm and information presented to the stakeholder end retrieved from her/him. Examples of the method of communication with the stakeholders in comparison to mathematical languages can be found in [11].

The process of preference elicitation is well described e.g., in [50]. Moreover, a good overview of preference elicitation methods can be found in [11].

7.3.3 MC analysis by individual stakeholders

As shown in Section 7.2 no existing multicriteria analysis method is directly applicable to our case. However, it is possible to adapt (or to combine) for the needs of our case two well established methods, namely the weighted sum (WS) and the reference point (RFP). This work could not have been done during the period for which the work reported here was planned. The scope of the latter work was to specify the requirement analysis for the multicriteria analysis, and to propose an existing method (and the corresponding software tool) to meet the requirements.

After the requirement analysis was advanced it became clear that a new multicriteria analysis method needed to be developed. Thus we have explored this path and summarize here the current state of the corresponding research, which requires about 10 months more to be completed. The reasons for this time extension are as follows:

• The proposed methodology must be tested, and most likely modified based on the results of extensive tests.

- Implementation of the proposed methodology for the NEEDS project will certainly require some fine-tuning, and the latter will only be possible when a representative sample of actual data is available.

We propose that the methods will be transparent to the stakeholder, i.e., that the survey will be designed in such a way that it will work with either one or more methods:

- Preferences will be specified in such a way that complete information will be provided for either one or more analysis methods.
- The results presented to stakeholders will combine the results obtained with all implemented methods.
- The stakeholders will have the option to switch on/off a particular method; this may be desired by users with knowledge of multicriteria methodology who may want to make more advanced analysis.

The methodological background of the multicriteria analysis approach proposed to be explored is summarized in Section 7.4.2. The proposed methodology attempts to combine the advantages of two approaches, namely the WS and the RFP methods. It proposed alternative approaches that need to be tested to find out which one is most suitable for our problem.

Moreover, one needs to explore how to effectively and properly use the criteria hierarchy specified in the form of an unbalanced tree. At least two approaches need to be explored:

- asking stakeholders to specify preferences for each branch of the criteria tree,
- using nodes of the criteria tree for organizing the interaction with the user.

Another problem that needs to be solved is an appropriate treatment of criteria with multimodal distributions of values. Especially for the WS-based method this may require a combination of:

- a more sophisticated specification of weights, e.g., either in relative terms (instead of typically used fractions), or for ranges of criteria values (to be defined by the user),
- a more sophisticated scaling of criteria values (needed only for the WS method).

Finally, additional functionality dedicated to problems with a large number of criteria may be desired. This may include:

- introducing threshold levels for criteria values,
- easy (for the users) exploitation of the criteria hierarchy.

7.3.4 MC analysis by analysts

This analysis can be started after finalizing the process of elicitation of the stakeholder preferences. The analyst will start with preprocessing of the received data from the stakeholders.

The following types of analyses are proposed:

Preprocessing: This is a routine element of any data analysis process necessary for cleansing data and performing an initial data analysis, e.g., to identify missing data, outliers; a basic statistical data analysis is also part of the preprocessing task.

Clustering of the stakeholders according to their preferences: The analyst will explore the data using various clustering algorithm. Such analysis aims to find groups of the stakeholders with similar preferences. Next, intersections of the identified groups with the predefined categories of the stakeholders should be analyzed. Such analysis can answer a number of questions including:

- Do the stakeholders belonging to the same category have similar preferences?
- Are there sub-groups of stakeholders within the same category with similar patterns of preferences?

- Are there sub-groups of stakeholders belonging to different categories but expressing the same preferences?

Analysis of the results: The mathematical properties of the underlying problem imply that there is no way to obtain a robust "best" ranking of technologies or scenarios. Therefore, we propose to plan for the provision of a concise report based on various analyses of the problem. Such analysis may include identification of categories/sets of solutions (either technologies or scenarios) and/or stakeholders, each having certain (to be defined by analysts during the analysis process) properties. Such analysis may provide various sets of results, including:

- Sets of solutions corresponding to groups of stakeholders, either identified during the analysis, or defined a priori by predefined stakeholder categories.
- Clusters of solutions similar (according to various similarity measures to be defined by analysts).
- Clusters of stakeholders corresponding to clusters of solutions.
- Sensitivity analysis of the results. For example, analysis of possible changes of the clusters/classifications of technologies (or scenarios or stakeholders) by small changes of parameters of the algorithms.
- Identification of the most important factors of the preferences for each group of stakeholders.
- Comparison of results calculated by different algorithms.

The above are only examples of the possibilities offered by diversified data mining methods. It is practically impossible to specify in advance the data analysis because a complete specification is only possible after the characteristics of the data are available, and the latter will only be provided after the process of elicitation of stakeholder preferences is almost completed.

7.4 Methodological issues

7.4.1 Multimodal distribution of criteria values

We illustrate here the problems caused by criteria values having multimodal distribution.[20] In order to use a realistic example we have extracted the data from an energy case study [25]. It should, however, be stressed that the data analysis described here is qualitatively different from the analysis [25]. Therefore, the comparisons and conclusions from our illustrative example do not apply to the case study from which a sample of data was taken.

The example shows the rank reversal problem, i.e., a change in ranking after removing one of the alternatives. We use data summarized in Table 2. The problem is to rank 8 technologies evaluated by two criteria: production cost (c_1) and long term sustainability - energetic (c_2).[21]

For ranking technologies (from best to worst) we apply the scalarizing function:

$$s(w, c) = w_1 * c'_1 - (1 - w_1) * c'_2 \tag{43}$$

where the scaled values of criteria c'_i are computed in the usual way, i.e.:

$$c'_i = \frac{c_i - min(c_i)}{max(c_i) - min(c_i)} \tag{44}$$

[20] See the footnote on page 19 for the explanation of the multimodal distribution.

[21] Value of this criterion for Hydro, Wind and PV should be ∞ but since scaling of the problem of a problem requires a finite value we replace ∞ value by $1E6$.

	LABEL	Production cost [c/kWh]	Long term sustainability [Years]
		c_1	c_2
1	Lignite	3,3	400
2	Hard Coal	3	2000
3	Oil	3,1	100
4	NG	3,6	100
5	Nuclear	2,1	500
6	Hydro	7	1000000
7	Wind	9	1000000
8	PV	60	1000000

Table 2: Summary of the data used for the illustration of the rank reversal problem

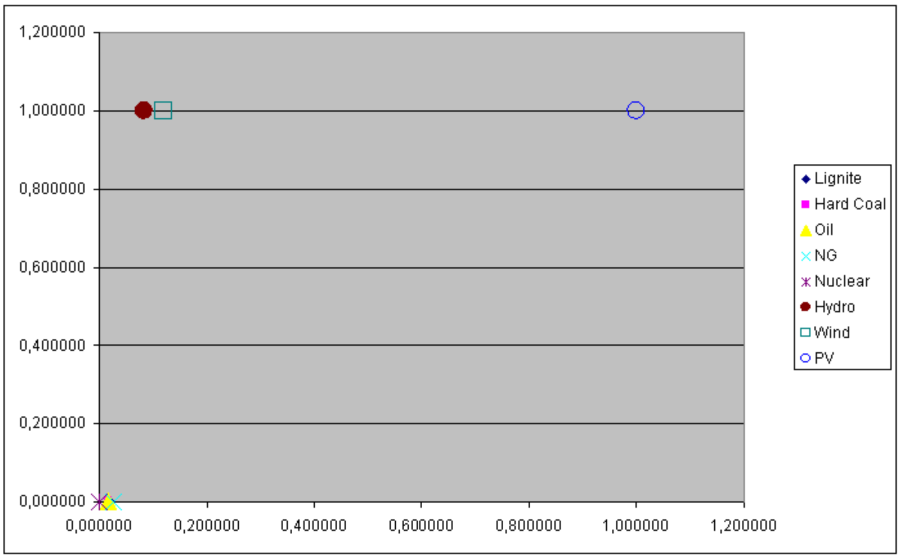


Figure 17: Plot of the scaled (for all alternatives) values of criteria: production cost on X-axis, and long-term sustainability on Y-axis.

The scatter plot of the scaled criteria is illustrated in Figure 17. It is easy to prove that the following ranking (illustrated in Figure 18) results from a minimization of the scalarizing function (43) for $0.9152 \leq w_1 < 1$:

1. Hydro,
2. Nuclear,
3. Hard Coal,
4. Oil,
5. Lignite,
6. NG,
7. Wind, and
8. PV.

Next we repeated (with the same weight) the analysis for the same data without the last alternative (PV), see the plot in Figure 19. The results presented in Figures 18 and 20, respectively show two problems. First, the technology (Hydro) which was the best in the first analysis

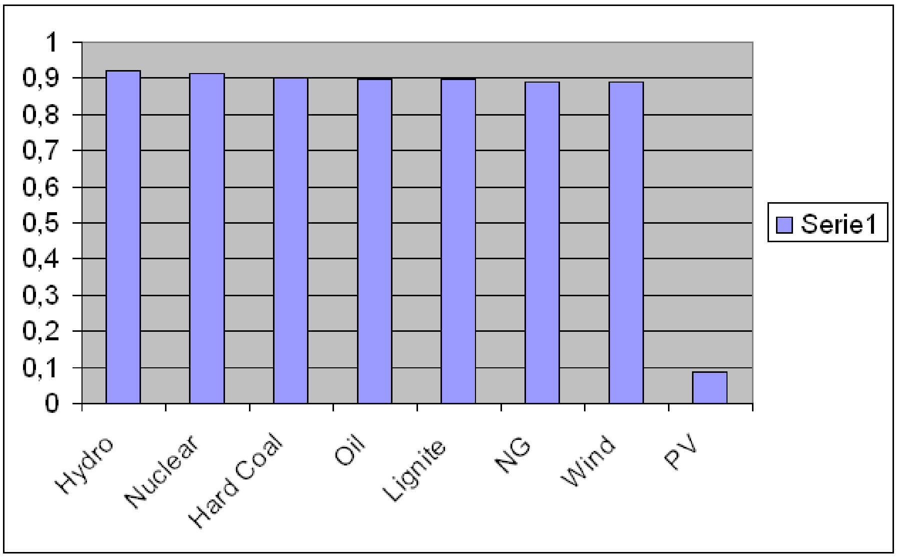


Figure 18: Ranking of eight alternatives.

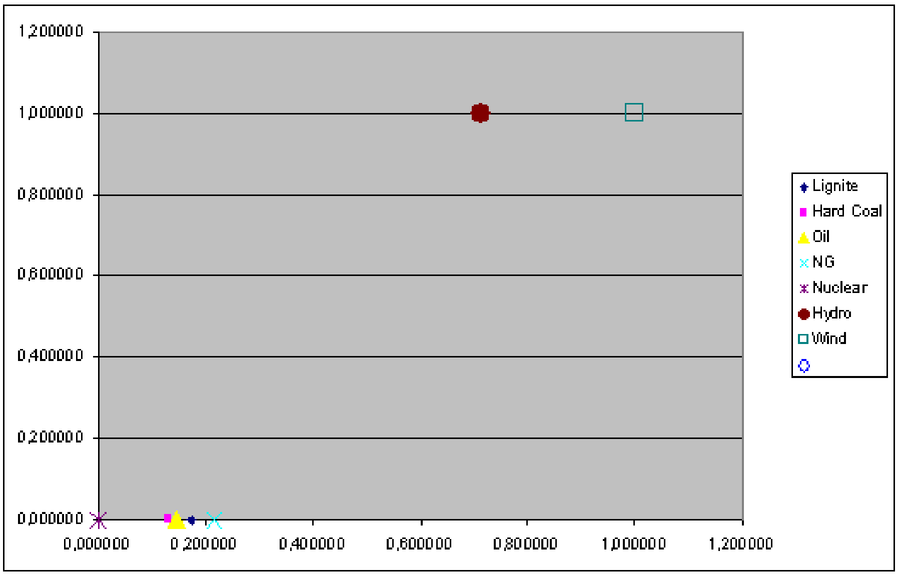


Figure 19: Plot of the scaled (for all alternatives but PV) values of criteria: production cost on X-axis, and long-term sustainability on Y-axis.

is now ranked the last but one. Second, the values of the scalarizing function (which are almost the same for all but two (Hydro and PV) technology in the first analysis, became diversified in the second analysis.

Another problem that can be demonstrated by these two sets of data (that differ by the PV technology) is the value of w_1 which causes the change of ranking. For the first data set the ranking is unchanged for $0.9152 \leq w_1 < 1$, while for the second data set the ranking is unchanged for a much wider range of w_1 values, namely $0.7104 \leq w_1 < 1$.

The example shows that the weights within the range $0.7104 \leq w_1 \leq 0.9152$ will result in very different rankings for two sets of alternatives that differ by only one alternative. This in turn illustrates the commonly known problem, namely that rankings based on a scalarizing

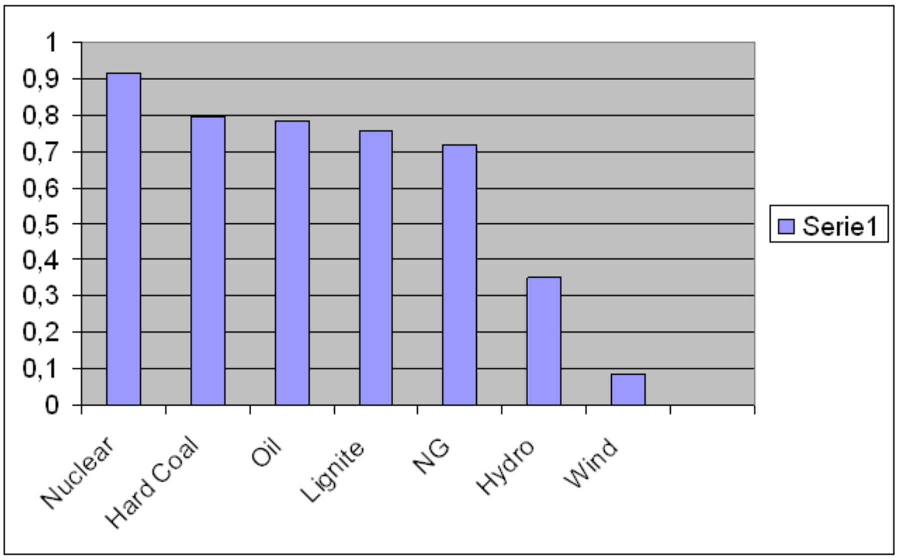


Figure 20: Ranking of seven alternatives (illustrates rank-reversal of the ranking shown in Figure 18).

function are likely to be unreliable.

7.4.2 A hierarchical reference approach to multicriteria analysis and objective ranking

This section summarizes a new approach to multicriteria analysis of problems dealing with a large number of criteria organized into a hierarchical structure. The approach explores the possibilities of combining the advantages of the RFP and WS methods. It must be stressed that research into new approaches has been motivated by the problem specified in [45] and, due to the short time and lack of data, the presented method has been tested only on a limited set of data which attempts to reflect the characteristics of the (not yet available) data to be used for the actual analysis.

The reference point approach can be combined with a weighted sum approach to multicriteria analysis of problems, where criteria are arranged in the hierarchical (tree-like) structure. Moreover, we can divide the criteria into two types:

- *Compensatory criteria* are such that a given improvement - increase of the value of one criterion by a given number in the relative percentage scale - can be rationally substantiated to compensate a deterioration of another quality criterion - its decrease by a unit or by one percent in the relative scale. The ratio of these changes can be used as a basis for determining the rationally substantiated weighting coefficients, called also compensatory or objective weighting coefficients. A basic example of such compensatory criteria is given by two financial quality indicators (both of minimized type): operational costs and investment costs. If we know the costs of a banking credit then we can rationally substantiate the trade-off, thus a weighting coefficient converts the investment costs into an addition to operating costs. The popularity of the use of weighting coefficients is based on an (erroneous) economic intuition that all criteria are of the compensatory character.

- *Non-compensatory criteria* are such that no rational substantiation exists for defining weighting coefficients. A basic example of non-compensatory criteria are costs and loss of human life; we can refuse on the grounds of principal to give a value for such a compensation, even if some economists try to define such values for insurance firms. Weighting coefficients defined for non-compensatory criteria are not substantiated rationally, they are a result of subjective, actually intuitive estimation. As such, they do not define trade-off coefficients that might be used in a weighted sum, they estimate only the relative importance of the increases of criteria. Therefore, we should actually find another way of using such subjective weighting coefficients; this will be proposed further in the description of the method.

In interactive decision analysis, we usually assume that the individual decision maker - the user of the decision support system - should be fully sovereign in the definition of his preferences, whatever form this definition takes (determination of weighting coefficients, or determination of aspiration and reservation levels, etc.). However, when it comes to ranking, the user could also propose her/his own subjective ranking list resulting, e.g., in forming intuitive pair-wise comparisons of subsequent alternatives. Thus, if the user wants a support of decision analysis in ranking, it is usually because she/he wants to have some objective grounds for this ranking.

The need of having some objective ranking is recognized. For an individual decision maker, this might mean that she/he needs some independent reasons for ranking, such as a Dean cannot rank the laboratories in her/his school fully subjectively, they must have some reasonable, objective grounds that can be explained to the entire faculty. For a ranking that expresses the preferences of a group, diverse methods of aggregating group preferences might be considered; but they must be accepted as fair - thus objective in the sense of inter-subjective fairness by the group. For this purpose, both weighting coefficients and/or aspiration and reservation levels should be determined in some objective or inter-subjectively fair fashion. We shall consider three possible ways of achieving this goal: neutral, statistical and voting.

Neutral means equal in relative terms, if we do not have any reasons for differentiating. This is a very poor method, if we use weighting coefficients alone: it means that we accept rationally substantiated, objective weighting coefficients for compensatory criteria (here we have reasons for differentiating), but use weighting coefficients equal in size for all non-compensatory criteria. This is actually a basic reason (there are also others) why we propose to use reference points and achievement functions for non-compensatory criteria. A neutral definition of reference points (say, all aspiration levels equal to 67% of criteria ranges, all reservation levels equal to 33% of these ranges) gives, as we shall show in further examples, far more reasonable rankings than a neutral definition of weighting coefficients (say, all weighting coefficients equal to 100%/ $|I|$).

Statistical means based on some meaningful statistics. It is very difficult to find statistical data to substantiate weighting coefficients, but it is easy in the case of reference points used for ranking. The average score of all alternatives on a given criterion is a good statistical basis for determining the reference points, for example, as in the equation below:

$$q_i^m = \sum_{j \in J} q_{ij}/|J|; q_i^a = 1.33 q_i^m; q_i^r = 0.67 q_i^m \qquad i \in L_l = \{L_{l,1}, \dots L_{l,g_l}\} \tag{45}$$

Voting means based on a voting procedure between a group of decision makers. Naturally, the members of this group could vote directly on the ranking of options. But this would make the results susceptible to various voting paradoxes and manipulations, see ([26]). Moreover, the result of such direct voting would only aggregate the subjective rankings of voting members - and they often perceive themselves the need for a more objective procedure. When voting first on the values of weighting coefficients or reference points, we might suggest the use of a voting procedure that is, according to ([26]), least susceptible to voting paradoxes. This procedure consists of assigning 100% points to every voting member of a group. Each member subdivides his allotted 100% points between voting options (in this case, between all criteria, because we want to establish either the weighting coefficients or the reference levels) in his vote; the vote is valid if she/he assigns no more than 100% in total. If the problem is politically and socially contentious, as for the choice of technologies of energy production, we can add a modification based on the procedure of ([83]): group members and criteria might be classified according to factions they represent and criteria ”owned” by the factions, then the votes on own criteria are not counted (or counted as not valid) when determining the results.

In the case of voting on weighting coefficients, the resulting weighting coefficients are just the voting averages (averages between valid votes of all voting members) of values of points obtained by each criterion. In the case of reference points, the middle, aspiration and reservation levels are placed on the intervals of change for each criterion according to the same voting average. This is more precisely specified by the following formulae:

$$w_i = v_i = \sum_{k \in K} v_{ik} / \sum_{i \in I} \sum_{k \in K} v_{ik} \qquad i \in L_l = \{L_{l,1}, \dots L_{l,g_l}\} \tag{46}$$

$$q_i^m = v_i; q_i^a = 1.33 v_i; q_i^r = 0.67 v_i \qquad i \in L_l = \{L_{l,1}, \dots L_{l,g_l}\} \tag{47}$$

where k denotes a voting member, K is the set of all voting members, v_{ik} is the number of percentage points given in a valid vote by the voting member k to the criterion i. The subdivision by the double sum in (46) is needed if not all voting members use fully their allotted 100% points; v_i in (47) are computed as in (46). Note that Eq. (47) is valid in relative terms, after the transformation ; in absolute terms, it means that $q_i^m = q_i^{lo} + v_i(q_i^{up} q_i^{lo})$, similarly for q_i^a and q_i^r.

We shall distinguish the following approaches to criteria aggregation in a hierarchical structure of criteria:

- Compensatory aggregation on lower level, non-compensatory analysis on upper level.
- Non-compensatory aggregation both on lower and upper level.
- Non-compensatory aggregation with weighting coefficients treated as importance factors.

There is a set A of alternatives and a set J of indexes $j \in J$, which identify alternatives. On each level of hierarchy we can distinguish groups of criteria $L_l = L_{l,1}, \dots L_{l,g_l}$ (g_l - is a number of groups on level l of the hierarchy). These groups of indicators can represent e.g. economic indicators, environmental indicators etc.

Compensatory aggregation on lower level, non-compensatory analysis on upper level. In this case we assume that all criteria are compensatory within their groups and thus weighting coefficients and weighted sums can be used to aggregate criteria in each group. This results in aggregated group quality indicators or aggregated criteria:

$$q_C = \sum_{i \in C} w_i q_i \qquad C = \{L_{l,1}, \dots, L_{l,g_l}\} \tag{48}$$

where w_i are compensatory weighting coefficients. We stress that these coefficients correspond to relative percentage scales of criteria changes, thus they are also dimension-free and should sum up to unity, $\sum_{i \in C} w_i = 1$ for all $C = L_{l,1}, \dots, L_{l,g_l}$. The values q_C of these aggregated criteria for all options $j \in J$ can be used to order the options into a group ranking list, starting with the highest value. It must be stressed that such a group ranking list orders the options only from the point of view of the given group of criteria, e.g., orders energy technology options from the point of view of environmental criteria.

On the other hand, it is difficult to substantiate the assumption that the aggregate criteria q_C will also be compensatory (for example, environmentalists would never agree to consider an aggregated environmental quality indicator as compensatory to an economic quality indicator). Therefore, we assume that a non-compensatory multiple criteria analysis is needed on the upper level, which is possible when applying the reference point approach. For this purpose, the upper and lower bounds for all aggregated criteria, q_C^{up} and q_C^{lo} for all $C = \{L_{l,1}, \ldots, L_{l,g_l}\}$ are first determined (by either computing the utopia point and an estimation of the nadir point, or simply by computing the aggregated criteria values q_{C_j} for all options $j \in J$ and then computing their bounds). Then we define a reference point or a reference interval for each aggregated criterion; we assume here the use of a reference interval composed of a reservation level q_C^r and an aspiration level q_C^a. The reservation and aspiration levels for all aggregated criteria might be thus determined either as neutral, or statistical, or by voting as explained above; we must only remember to convert the aggregated criteria to their relative scales.

These data help to define a partial achievement function for each aggregated criterion:

$$\sigma_C(q_C, q_C^a, q_C^r) = \begin{cases} 1 + \alpha(q_C - q_C^a)/(q_C^{up} - q_C^a) & \text{if } q_C^a \le q_C \le q_C^{up} \\ (q_C - q_C^r)/(q_C^a - q_C^r) & \text{if } q_C^r \le q_C < q_C^a \\ \beta(q_C - q_C^r)/(q_C^r - q_C^{lo}) & \text{if } q_C^{lo} \le q^C < q_C^r. \end{cases} \tag{49}$$

The coefficients α and β above are typically selected to assure the concavity of this function; but the concavity is needed only for problems with a continuous (non-empty interior) set of options for an easy transformation to a linear programming problem. In a ranking problem with a discrete and finite set of options, we can choose these coefficients to have a reasonable interpretation of the values of the partial achievement function. The value $\sigma_C(q_{C_j}, q_C^a, q_C^r)$ of this achievement function for a given option $j \in J$ signifies the satisfaction level with the aggregated quality indicator for this option. If we assign the values of satisfaction from -1 to 0 for $q_C^{lo} \le q_{C_j} < q_C^r$, values from 0 to 1 for $q_C^r \le q_{C_j} < q_C^a$, values from 1 to 2 for$q_C^a \le q_{C_j} \le q_C^{up}$, then we can just set $\alpha = \beta = 1$.

For a non-compensatory multiple criteria analysis on the upper level, we might then use the following form of the overall achievement function:

$$\sigma(q, q^a, q^r, \epsilon) = \min_C \sigma_C(q_C, q_C^a, q_C^r) + \epsilon \sum_{C=L_{l,1},\ldots L_{l,g_l}} \sigma_C(q_C, q_C^a, q_C^r) \tag{50}$$

where $q = (q_{L_{l,1}}, \ldots q_{L_{l,g_l}})$ is the vector of aggregated quality indicators and $q^a = (q^a_{L_{l,1}}, \ldots q^a_{L_{l,g_l}})$, $q^r = (q^r_{L_{l,1}}, \ldots q^r_{L_{l,g_l}})$ correspondingly, while $\epsilon > 0$ is a small regularizing coefficient. The values $\sigma(q_j, q^a, q^r, \epsilon)$ $j \in J$ can be used to order the options in an overall ranking list, starting with the highest achievement value.

Here we should comment more on the question of how to define the aspiration and reservation levels. A decision maker could define and modify them arbitrarily during an interactive session of decision analysis in order to investigate their impacts on the overall ranking list. However, as stressed in sections 2 and 3, there is also the question of possibly the most objective definition of aspiration and reservation levels. Even for a fully subjective interactive process, it is good to start from such an objective or neutral point. For a problem involving severe social debates, such as the choice of energy technologies, it is better to use principled negotiations (see, e.g. [16]): first to discuss with all factions involved the principles of reaching the agreement - in this case, of determining aspiration and reservation levels - then to implement these principles in order to propose the final ranking list. The representatives of all factions might thus first vote whether to vote directly on rankings, or to use neutral, or statistical, or a voting procedure to determine aspiration and reservation levels. Then they might determine the details of the procedure (e.g., whether or not to use the modification excluding the votes on owned criteria) and implement the procedure agreed upon.

Additional comment is necessary in relation to the neutral determination of aspiration and reservation levels. In order to obtain a reasonable scale of values of the overall achievement function, which will be useful especially in further considerations, we propose here a definition of neutral aspiration and reservation levels slightly different to that in ([82]). The neutral aspiration level q_C^{an} and the neutral reservation level q_C^{rn} should be determined as:

$$q_C^{an} = q_C^{lo} + (q_C^{up} - q_C^{lo})/|I_a|; q_C^{rn} = q_C^{lo} + 0.5(q_C^{up} - q_C^{lo})/|I_a| \qquad C = \{L_{l,1}, \dots, L_{l,g_l}\} \quad (51)$$

in absolute terms, or equivalently:

$$q_C^{an} = 100\%/|I_a|; q_C^{rn} = 50\%(q_C^{up} - q_C^{lo})/|I_a| \qquad C = \{L_{l,1}, \dots L_{l,g_l}\} \quad (52)$$

if the aggregated quality indicators were already converted to their relative percentage scale. The subdivision by the number of groups $|I_a|$ reflects the fact that the dimension of a space influences inversely the average achievement: if, e.g., the attainable values of q_C were constrained by an inequality $\sum_{C=L_{l,1},\dots L_{l,g_l}} q_C \leq 100\%$, then an equitable solution would be $q_C = 100\%/|Ia|$.

Using neutral aspiration and reservation levels defined as above, a neutral overall ranking list of all options $j \in J$ can be proposed; naturally, such a ranking list depends on the weighting coefficients used for the aggregation of lower level quality indicators, but if these coefficients represent a compensatory aggregation, they are in a sense objective, because they are rationally substantiated. Naturally, we can also use instead a statistical definition of objective aspiration and reservation levels for individual criteria, and the statistical reference overall ranking list might be different; finally, we can also use a voting procedure to determine inter-subjectively fair aspiration and reservation levels, and in this way obtain a fairly voted overall ranking list. All these lists might be different, but all represent attempts to secure some level of objectivity.

Non-compensatory aggregation on both lower and upper levels. If there are non-compensatory criteria in a group on the lower level, the use of weighting coefficients for aggregating them is most doubtful. But we can always use the reference point approach on the lower level as well and treat the resulting value of an achievement function as the aggregated (though somewhat transformed) quality indicator q_C for the group. In this case we use:

$$q_C = \sigma_C(q_C, q_C^a, q_C^r, \epsilon) = \min_{i \in C} \sigma_i(q_i, q_i^a, q_i^r) + \epsilon \sum_{i \in C} \sigma_i(q_i, q_i^a, q_i^r) \quad (53)$$

where partial achievement functions $\sigma_i(q_i, q_i^a, q_i^r)$ are determined similarly as in Eq. 49 (we must only change all C to i in this equation). Actually, we should establish the relative percentage values of q_{C_j} for all $j \in J$ after establishing their upper and lower bounds q_C^{up} and q_C^{lo}. Here we denote by q^C (different than the scalar aggregated group criterion q^C) the vector composed of q_i for $i \in C$, by q_C^a and q_C^r -the vectors composed of the aspiration levels q_i^a and reservation levels q_i^r for $i \in C$, by $\epsilon > 0$ - a small regularizing coefficient. If we use neutral aspiration and reservation levels for all criteria, defined in this case by:

$$q_i^{an} = q_i^{lo} + (q_i^{up} - q_i^{lo})/|C|; q_i^{rn} = q_i^{lo} + 0.5(q_i^{up} - q_i^{lo})/|C| \quad i \in C, \ C = \{L_{l,1}, \dots L_{l,g_l}\} \quad (54)$$

in absolute terms, or equivalently:

$$q_i^{an} = 100\%/|C|; q_i^{rn} = 50\%/|C|, \qquad i \in C, \ \ C = \{L_{l,1}, \dots L_{l,g_l}\} \quad (55)$$

in the relative percentage scale, then the resulting values of aggregated quality indicators q_C are defined, in a sense, objectively, because they are defined neutrally. Thus, neutral group ranking lists can be defined, starting with the highest values of $q_{C_j} = \sigma_C(q_{C_j}, q_C^a, q_C^r, \epsilon)$. It must be stressed again that these ranking lists order the options only from the point of view of the given group of criteria, thus they can differ considerably (e.g., technologies that are good from the point of view of environmental criteria need not be good from the point of view of economic criteria).

Another way made possible by the use of the reference approach (and not available when using the weighted sum approach) is to generate statistical reference group ranking lists: we determine statistical reference levels as in Eq. (45), use them in Eq. (52), determine $q_{C_j} = \sigma_C(q_{C_j}, q_C^a, q_C^r, \epsilon)$ and use them for ordering all options $j \in J$ - naturally, only with respect to criteria of the group C, we must repeat this for each $C = \{L_{l,1}, \dots L_{l,g_l}\}$. Yet another way is to let all the members of the group faction related to the criteria of each group C vote on the relative importance of their criteria; this is, however, equivalent to treating weighting coefficients as importance factors and will be discussed in more detail below.

The analysis on the upper level - the aggregation of aggregated group criteria q_C for all $C = \{L_{l,1}, \dots L_{l,g_l}\}$ can then be performed in the same way as in the previous case, because we do not assume the compensatory character of the aggregated quality indicators q_C. Using neutral aspiration and reservation levels on the upper level as well, we obtain a fully neutral overall ranking list of all options $j \in J$. Using statistical determination of aspiration and reservation levels on the upper level again, we obtain a statistical reference overall ranking list. Both these versions of aggregated ranking lists are in some sense objective and do not depend on any assumed weighting coefficients.

3. *Non-compensatory aggregation with weighting coefficients treated as importance factors.* Sometimes, however, we also need to take into account weighting coefficients, but treated differently in the case of non-compensatory aggregation. If the criteria are non-compensatory but weighting coefficients are given - by a voting procedure, or by an application of the AHP approach - we might interpret them as importance factors and use them in a modification of neutral aspiration and reservation levels, similar to the relationship between Eq. (46) and (47). Suppose we have relative weighting coefficients w_i for $i \in C$ such that $\sum_{i \in C} w_i = 1$ for all $C = \{L_{l,1}, \dots L_{l,g_l}\}$. Then the neutral aspiration and reservation levels can be modified as follows:

$$q_i^{anw} = q_i^{lo} + w_i(q_i^{up} - q_i^{lo}); q_i^{rnw} = q_i^{lo} + 0.5w_i(q_i^{up} - q_i^{lo}) \quad i \in C, \quad C = \{L_{l,1}, \dots L_{l,g_l}\} \tag{56}$$

in absolute terms, or equivalently:

$$q_i^{anw} = w_i 100\%; q_i^{rnw} = w_i 50\% \qquad i \in C, \quad C = \{L_{l,1}, \dots L_{l,g_l}\} \tag{57}$$

in the relative percentage scale. The results of ranking while using such modification can be interpreted as either objective - if the weighting coefficients are determined in some objective or at least inter-subjectively fair fashion - or subjective.

The difference between using weighting coefficients in a weighted sum and for the modification (12a, 12b) of neutral aspiration and reservation levels can be best illustrated by a very simple example.

Suppose, that we have a discrete problem with five options characterized by: (a) $q_{11} = 1, q_{21} = 0$; (b) $q_{12} = 0.63, q_{22} = 0.27$; (c) $q_{13} = 0.40, q_{23} = 0.40$; (d) $q_{14} = 0.27, q_{24} = 0.63$; (e) $q_{15} = 0, q_{25} = 1$. When using a weighted sum, we would never select options (b), (c), (d) as best, because their vectors of criteria values are contained in the interior of the convex cover of the vectors of criteria values for options (a), (e); a weighted sum would indicate as best either option (a) or option (b), as in the Korhonen paradox. When using the achievement function (10) with neutral aspiration and reservation levels (11a,b), the option (c) is selected as best - it is non-dominated and most balanced. When using the achievement function (10) with aspiration and reservation levels (12a,b) modified by weighting coefficients treated as importance factors, we select as best: option (b) with weights $w_1 = 0.7, w_2 = 0.3$; option (c) with weights $w_1 = 0.5, w_2 = 0.5$; option (d) with weights $w1 = 0.3, w2 = 0.7$. Thus, any (ϵ-properly) Pareto optimal option can be selected when using the achievement function (10) with aspiration levels modified by weights treated as importance factors.

The above example shows the theoretical advantages of using achievement functions with aspiration levels modified by weighting coefficients treated as importance factors. The advantages of using such an approach in a hierarchical structure of criteria will be shown later by a practical example; before turning to this example, however, we should comment on Pareto optimality of solutions in a hierarchical case.

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8 Summary

This report provides an extensive overview of methods pertinent to multicriteria analysis of sets of discrete alternatives, with a particular focus on large sets and large numbers of criteria. Although many methods for analysis of discrete sets of alternatives exist, none of them is best suited for analysis of the current problem considered in Stream 2b of the NEEDS project. This might be a surprising conclusion, therefore the report provides a detailed analysis of the features of such methods and compares the features with the characteristics of the problem defined in the requirement analysis presented in [45].

The authors of the report are aware that multicriteria analysis is an essential requirement for the whole NEEDS project, therefore an intensive research effort has been initiated to develop a consistent methodology for the whole multicriteria process which will be tailored to the requirements of the NEEDS project. Intermediate results of this research are summarized in the report.

The experienced project staff believe that this development of a new methodology will be successful, but an existing method of MCDA analysis will also be chosen as backup for a comparative or alternate use, consistent with using the same preference elicitation methods.

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