APPENDICES to the article

Monthly Estimates of the Quantum of Fertility: Towards a Fertility Monitoring System in Austria*

Tomáš Sobotka, Maria Winkler-Dworak, Maria Rita Testa, Wolfgang Lutz, Dimiter Philipov, Henriette Engelhardt, and Richard Gisser

* The article has been published in the *Vienna Yearbook of Population Research* 2005: 109-141. These Appendices are available only in electronic form on the Yearbook's Internet page (www.oeaw.ac.at/vid/yearbook).

APPENDIX 1: Decomposition of the calendar adjustment factor

The denominator of the calendar adjustment factor is given by the sum of the products of the weekday coefficients and the number of Mondays, Tuesdays, …, and Sundays in month *m* in year *t*. As mentioned earlier, this sum can be decomposed into an effect, which can be directly linked to the length of the month and a net effect for each day of the week (Ladiray and Quenneville 2001). Let $\overline{a}$ denote the arithmetic mean of the weekday coefficients, i.e.,

$$\overline{a} = \sum_{k=1}^{7} \frac{a^k}{7} \tag{A1.1}$$

and N_m the number of days of month *m*, i.e.,

$$N_m = \sum_{k=1}^{7} n_m^k \tag{A1.2}$$

we may write:

$$\begin{aligned} \sum_{k=1}^{7} n_m^k a^k &= \overline{a} \sum_{k=1}^{7} n_m^k + \sum_{k=1}^{7} n_m^k \left(a^k - \overline{a} \right) \\ &= \overline{a} N_m + \sum_{k=1}^{7} n_m^k \left(a^k - \overline{a} \right) \end{aligned} \tag{A1.3}$$

Since every month contains four complete weeks, we define

$$n_m^k = 4 + \eta_m^k, \tag{A1.4}$$

where

$$\eta_m^k = \begin{cases} 0 & \text{if month } m \text{ contains 4 days of type } k, \\ 1 & \text{if month } m \text{ contains 5 days of type } k. \end{cases}$$

Substitution of Equation (A1.4) into Equation (A1.3) yields

$$\begin{aligned} \sum_{k=1}^{7} n_m^k a^k &= \overline{a} N_m + \sum_{k=1}^{7} \left(4 + \eta_m^k \right)\left(a^k - \overline{a} \right) \\ &= \overline{a} N_m + 4 \sum_{k=1}^{7} \left(a^k - \overline{a} \right) + \sum_{k=1}^{7} \eta_m^k \left(a^k - \overline{a} \right) \\ &= \overline{a} N_m + \sum_{k=1}^{7} \eta_m^k \left(a^k - \overline{a} \right), \end{aligned} \tag{A1.5}$$

where the middle expression in the second line of Equation (A1.5) equals zero by definition, which implies that the net effect of the four complete weeks cancels out. Hence, the first expression in the last line of Equation (A1.5) adjusts for the length of the month, while the second expression corrects for the type of the additional days.

APPENDIX 2: Specification of the estimates of age structure and 'at risk' population

A-2.1 Mid-month female population by single years of age (birth cohort)

Purpose: Serves as a basis for computing age- and order-specific incidence rates by age of mother and birth order in each calendar month; women aged 12 to 50 (age reached during the year) are considered.

Estimation procedure:
First, linear approximation is used to estimate the age structure of the female population on the first day of each month between 1st January of the years t and $t+1$. The number of days in any given month or period served for estimating the total share of this period on the change in the number of women by age. The number of women belonging to the birth cohort C (or, alternatively, aged $a=t-C$) at the beginning of a month m (expressed as M) in a year t was calculated as follows:

$$P_F(C,t,M) = P_F(C,t,1) + [P_F(C,t+1,1) - P_F(C,t,1)] \cdot Sd\,(t, m-1)\,, \qquad \text{(A2.1)}$$

where $Sd\,(t, m-1)$ is the share of the cumulated number of days in months 1 to $m-1$ on the total number of days in the year t. The mid-month female population by single years of age, denoted as $P_F(C,t,m)$ was then computed from the population at the beginning of two consecutive months (denoted as M and $M+1$):

$$P_F(C,t,m) = [P_F(C,t,M) + P_F(C,t,M+1)] \,/\, 2 \qquad \text{(A2.2)}$$

Note: All age-specific calculations are expressed in a cohort format (data sorted by age reached during the year). If the official age structure during the year pertains to the actual age (age in completed years), the data must be reorganised to estimate the age distribution by birth cohort.

A-2.2 Mid-month female population by single months of age (birth month cohort)

Note: These data are used to a limited extent only since our computations of the total fertility rates show that using the more detailed month birth cohort indicators does not cause any appreciable change in the indicators of fertility quantum and timing (see Appendix 4).

Specification: Number of women by single months of birth specified for the middle of each calendar month in the period of January 1984 to December 2003

Purpose: Serves as a basis for computing age- and order-specific incidence rates by birth month-cohort of mother and birth order in each calendar month

Additional data sources: Number of births by calendar month in 1950 to 2002: EUROSTAT New Cronos database, accessed in December 2004. Number of births by calendar month in 1930 to 1949: data from the 1981 Census (Statistics Austria 1989).

Estimation procedure:
Data on the mid-month population of women by single years of age (birth cohorts), as described above, served for estimating the mid-month number of women for every month-cohort in reproductive age. For every calendar month considered, the data initially referring to the year-birth cohorts (*C*) born in the year *t=C* were redistributed into month-birth cohorts (C_m) on the basis of the proportion of live births in each single month *m* on the total number of live births during the year *t=C*:

$$P_F(C_m,t,m) = P_F(C,t,M) \cdot [B_m(t=C) / B(t=C)] \qquad \text{(A2.3)}$$

where $B_m(t=C)$ denotes the total number of live births during the month when the birth cohort C_m was born and $B(t=C)$ is the total number of live births during that year.
To connect these month-cohort data with other indicators specified by birth month or calendar month, all data are subsequently expressed in century-month codes, which are calculated for any month since January 1900 as follows:

$$\text{CMC} = (t - 1900) \cdot 12 + m \qquad \text{(A2.4)}$$

The *CMCs* permit an easy computation of age-specific indicators, such as age of mother at childbearing etc.

A-2.3 Monthly age-parity structure of the female population by single years of age (birth cohort)

Purpose: Serves for computing age-parity birth probabilities (exposure-specific indicators) used in the computation of the *PATFR* index and the Kohler-Ortega *adjPATFR* indicator.

Specification: Estimated for the beginning (1st day) of each calendar month. Computed by combining monthly data on the total number of women by single years of age (birth cohorts) as specified in Eq. A2.1 above with continually updated monthly series of the age-parity distribution of the female population (specified by birth cohort). Except for the birth cohorts 1982-89, the latter is based on the 1991 Census data combined with the age and order-specific incidence rates in the subsequent period. For the more recent time series starting from January 2001 we updated our estimates with the 2001 Census results. The relative age-parity distribution among women born in 1982-89 was reconstructed on the basis of cumulative age and order-specific incidence rates calculated from the vital statistics data.

The relative age-parity composition of the female population at the beginning of a month *m* (expressed as *M*) in a year *t* is derived from the age-parity composition at the beginning of month *m-1* (i.e., at time *M-1*) and order-specific incidence rates in month *m-1*. This estimation is performed for every single birth cohort and for each parity status (denoted as *i*) as follows:

$$w_i(C,t,M) = w_i(C,t, M\text{-}1) + f_i(C,t, m\text{-}1) - f_{i+1}(C,t, m\text{-}1), \qquad \text{(A2.5)}$$

where $w_i(C,t,M)$ denotes the proportion of women at parity *i* at the beginning of a month *m* among each birth cohort *C* and $f_i(C,t,m)$ represents cohort-specific incidence rates of order *i,* recorded during the month *m*. Parities 0, 1, 2, 3, and 4+ are distinguished. Note that for parity 0 (childless women), the equation simplifies to:

$$w_0(C,t,M) = w_0(C,t, M\text{-}1) - f_1(C,t, m\text{-}1) \quad \text{(A2.6)}$$

The relative proportion of women in the highest-parity category (4+) is computed as follows:

$$w_{4+}(C,t,M) = 1 - w_0(C,t,M) - w_1(C,t,M) - w_2(C,t,M) - w_3(C,t,M) \quad \text{(A2.7)}$$

For any age and parity category, the number of women at the beginning of each calendar month *m* is calculated by combining Eq. (A2.1) above with the Eq. (A2.5) (or A2.6 and A2.7, respectively):

$$P_{F,\,i}(C,t,M) = P_F(C,t,M) \cdot w_i(C,t,M) \quad \text{(A2.8)}$$

Appendix 5 features the table of age and parity composition of the female population as estimated for December 1, 2004 (Table A-5.1) and sensitivity analysis exploring the effect of differences between the two estimates of the age and parity composition of the female population (one based on the 1991 Census data and the other on the 2001 Census results) on the PATFR index.

A-2.4 Number of live births by biological (true) birth order in 1961-1992

Purpose: Serves for the computation of duration-specific ‘incidence rates,’ and the period parity progression ratios (*PPRdIR*).

Source data & estimations: Number of live births by birth order in 1984-2004 was derived from the individual birth records provided by Statistics Austria. Since Statistics Austria collects data on ‘true’ birth order only since 1984, the number of live births by birth order had to be estimated for the previous years, namely for 1961-1983. Composition of live births by birth order in 1961-1979 was derived from the retrospective data on the distribution of births by birth order as recorded in the 1981 Census combined with the total registered number of live births in that period (Statistics Austria 1989). Number of live births by birth order in 1980-1983 was estimated from the total number of births and the relative distribution of order-specific births in 1978-1979 and 1984-1985.

A-2.5 Number of women by parity and the year of giving last previous birth in 1999-2004

Purpose: Serves for the computation of duration-parity birth probabilities and the period parity progression ratios (*PPRd*).

Source data & estimations: Number of live births by birth order in 1984-2004 is based on the individual birth records provided by Statistics Austria. The number of women (P_F) who gave birth of order *i* in the year *y* and still remain at parity *i* at the beginning of month *m* in the year *t* is estimated as follows:

$$P_{F,\,i,d,}(t,M) = B_i\,(y) - \sum_{z=y}^{t,m-1} B_{i+1}\,(z,\, \theta{=}y)\ ,\ i \geq 2,\ \ t \geq y,\ \ t \geq z\ ; \qquad \text{(A2.9)}$$

where *d* represents 'duration,' estimated as the difference between the given year and the year when women reached the current parity *i* ($d = t - y$); $B_i\,(y)$ is the number of live births of order *i* in the year *y* and θ refers to the year of the last previous birth. In other words, the number of women $P_{F,i,d}(t,M)$ is estimated by subtracting the cumulative number of births of order $i{+}1$ between the year *y* and month $m\text{-}1$ among women who gave birth of order *i* in the year *y* from the total number of live births of order *i* in the year *y*.

For women giving the last previous birth between 1974 and 1983, rough estimates of the number still remaining at a given parity have been made, based on the data for births since 1984 and cohort parity progression ratios. These estimates concern only a minor portion of the exposure population in the analysed period.

Note: This approximation of the exposure population that does not account for any possible effects of migration and mortality. Because Austria experiences a considerable net migration increase, the estimate tends to underrate the exposure population. Furthermore, it does not account for multiple births and assumes that the annual total number of births by birth order defines the initial population of women at the given parity status. In contrast to the 'zero migration' assumption, this assumption may overestimate the exposure population.

APPENDIX 3: Specification of fertility indicators analysed in this study (including calendar and seasonality-trend adjustments)

A-3.1 Age- and order-specific incidence rates and the period *TFR*

All indicators were computed for birth orders 1, 2, 3, 4, 5+, and for all birth orders combined. For each birth cohort (*C*) and birth order (*i*), monthly incidence rates are calculated as follows:

$$f_i(C,t,m) = B_i(C,t,m) / P_F(C,t,m), \qquad \text{(A3.1)}$$

where $B_i(C,t,m)$ is a total number of live births of order *i* in a month *m* among women born in the year *C*. In our computations, birth orders 1, 2, 3, 4, and 5+ were considered separately.

We considered only cohorts reaching ages 12 to 50 in a given calendar year. In case of recorded births to women below age 12 or above age 50, they were grouped together with the births to women aged 12 and 50, respectively.

The crude (unadjusted) monthly period total fertility rate (denoted as *gTFR*), specified by birth order, is computed as a sum of age- and order-specific incidence rates, multiplied by 12:

$$gTFR_i(t,m) = \sum_{a=12}^{50} f_i(a,t,m) \cdot 12 \quad \text{and} \quad gTFR(t,m) = \sum_{i=1}^{5+} gTFR_i(t,m), \qquad \text{(A3.2)}$$

where *a* is cohort age (age reached during the calendar year), which is simply calculated as $a = t - C$ (recall that *C* denotes birth cohort, i.e., the year of birth of the mother).

Calendar adjustment is identical for all birth orders and can be used to adjust the overall gross total fertility rate:

$$TFR_C(t,m) = gTFR(t,m) \cdot I_C(t,m), \qquad \text{(A3.3)}$$

where I_C denotes the monthly index allowing an adjustment for calendar factor.

Seasonal adjustment is order-specific. For any calendar-adjusted TFR, the trend-season adjustment is computed as follows:

$$TFR_{CS,i}(t,m) = TFR_{C,i}(t,m) \cdot I_{S,i}(t,m), \qquad \text{(A3.4)}$$

where I_S denotes the monthly index allowing an adjustment for seasonality and trend fluctuations, net of calendar factor.

Note: All computations are presented here for the yearly birth cohorts, which was our usual data format. When we used data specified by month cohorts, all cohort-specific calculations (here denoted as *C*) were based on month birth cohorts (denoted as C_m). The age categories *a* were expressed in months, ranging from “ages” 132 (age 11.0 in completed years) to 612 months (age 51.0 in completed years).

A-3.2 Age-parity birth probabilities and the period fertility index *PATFR*

Note: These indicators are mostly used for parity 1, especially in combination with the parity-progression ratios specified below. As a result, all specifications here are illustrated for birth order 1.

The gross probability for a childless woman belonging to a birth cohort C to give birth to a first child during a month m is computed as follows:

$$q_1(C,t,m) = B_1(C,t,m) \,/\, P_{F,0}(C,t,M) = B_1(C,t,m) \,/\, [P_F(C,t,M) \cdot w_0(C,t,M)], \qquad \text{(A3.5)}$$

where $P_{F,0}(C,t,M)$ denotes the total number of childless women among the birth cohort C at the beginning of a month m (see Eq. A2.6 and A2.8 in Appendix 2).

Similarly to the incidence rates calculations, only birth cohorts reaching ages 12 to 50 in a given calendar year were considered. Births recorded among women below age 12 or above age 50 were coded as births to women aged 12 and 50, respectively.

Calendar and seasonal adjustment is performed for each age separately. The calendar and seasonal-adjusted first birth probability for a woman born in the year C is computed as

$$q_{CS,1}(C,t,m) = q_1(C,t,m) \cdot I_C(t,m) \cdot I_{S,1}(t,m) \qquad \text{(A3.6)}$$

The calendar and seasonal-adjusted total fertility index of parity 1 ($PATFR_{CS,1}$**)** is computed as follows:

$$PATFR_{CS,1}(t,m) = 1 - \prod_{a=12}^{50} [1 - q_{CS,1}(a,t,m)] \cdot 12 \qquad \text{(A3.7)}$$

Recall that a is the age reached during the calendar year, which is calculated as $a = t - C$

Note: More details on the age-parity model can be consulted in Rallu and Toulemon (1994; the original was published in French in 1993). We follow Rallu and Toulemon's notation of the total parity-specific index as *PATFR*.

A-3.3 Parity-progression ratios *PPRdIR* based on duration-specific incidence rates (birth interval data)

For birth orders 2 and higher, duration-specific gross 'incidence rates' of having a child of order i among women who had their (i-1)th child in a year y is computed as follows:

$$n_{i,d}(t,m) = B_{i,d}(t,m) \,/\, B_{i-1}(y) \;;\; i \geq 2 \;;\; t \geq y\,, \qquad \text{(A3.8)}$$

where d indicates 'duration,' which is in this case simplified as a difference between the years when births of order i-1 (year y) and i (year t) took place: $d = t - y$.

Thus, $n_{i,d}(t,m)$ expresses the (incidence) rate of having an i-th child during a month m in a year t among women who have given birth to their i-1th child in a year y and $B_{i,d}(t,m)$ is the total number of live-born children of order *i* during a month *m* in a year *t* among these women. $B_{i-1}(y)$ is the total number of live-born children of order *i-1* reached in a year *y*.

The 'duration' indicator *d* ranged from 0 to 25; i.e., the earliest year of giving birth to a previous child (birth order *i-1*) that was considered in our analysis was $y(min) = t-25$. In case some women had given birth to their previous child even earlier, they were considered as giving birth in a year *y(min)*.

Notes:
1) Although the birth order *i* refers to live-born children only, the coding of the date of the previous birth in the official vital statistics pertains to any previous birth, including stillbirths. Thus, the birth interval between two consecutive live births is slightly underestimated insofar as a small fraction of the registered birth intervals refers to the interval between the most recent live birth of order *i* and the preceding stillbirth, while the preceding live birth of order *i-1* had taken place at an unknown date before this stillbirth. Since the proportion of stillbirths in Austria is very small (0.39% of all births in 1984-2002) however, the influence of stillbirths on computing fertility rates by duration can be disregarded.
2) In the case of multiple births, the information on the date of the last previous birth provided in the birth database refers to the last birth before the current multiple delivery. For instance, when a woman gives birth to twins of order 2 and 3, the date of her last previous birth *y* refers in both cases to her first child. In this case, our analysis correctly computes duration indicators for the second birth, but incorrectly relates the third birth to the number of second births in the year *y* instead of assuming zero birth interval between them. This procedure tends to overestimate slightly the length of birth intervals, but should not have a large impact on the estimated parity progression ratios. If one of the multiple births, however, is a first-born child, there is no information provided in the birth database on the date of the last previous birth and these children are disregarded in our computations. Although we could link the individual data for multiple births and impute the 'correct' zero intervals between them, this would be a time-consuming procedure which we did not pursue.

Gross parity-progression ratios (*gPPRdIR*) were estimated for women at parities 1, 2, 3, 4, and the open-ended parity category 5+:

$$gPPRdIR_{i-1,i}(t,m) = \sum_{d=0}^{25} n_{i,d}(t,m) \cdot 12 \quad ; i \geq 2 \qquad \text{(A3.9)}$$

The highest birth order considered constitutes an open-ended parity progression to 6^{th}+ child among women having 5+ children.

The gross parity-progression ratios are then adjusted for calendar factor and seasonality in a similar way as the gross TFR (see Equations A3.3 and A3.4 above):

$$PPRdIR_{CS,i-1,i}(t,m) = gPPR_{i-1,i}(t,m) \cdot I_C(t,m) \cdot I_{S,i}(t,m) \qquad \text{(A3.10)}$$

A-3.4 Parity-progression ratios *PPRd* based on duration-parity birth probabilities

These indicators are computed for birth orders 2 and higher. The gross probability for a woman who reached her current parity status (i) in the year y to give birth to another child during the month m of the year t is estimated as follows:

$$q_{i+1,d}(t,m) = B_{i+1}(t,m,\ \theta=y)\ /\ P_{F,\ i,d}(t,M)\ , \qquad \text{(A3.11)}$$

where $P_{F,i,d}(t,M)$ is the number of women (P_F) who gave birth of order i in the year $y=t\text{-}d$ and still remain at parity i at the beginning of month m in the year t and $B_{i+1}(t,m,\theta=y)$ is the number of births of order $i+1$ during the month m in the year t to women who gave their previous birth in the year y (see Equation A2.9 in Appendix 2).

Calendar and seasonal adjustment is performed for each duration-parity probability separately. The calendar and season-adjusted probability is computed as

$$q_{CS,i,d}(t,m) = q_{i,d}(t,m) \cdot I_C(t,m) \cdot I_{S,i}(t,m)\ . \qquad \text{(A3.12)}$$

Parity-progression ratios (*PPRd*) were computed for women at parities 1, 2, 3, 4, and the open-ended parity category 5+:

$$PPRd_{i\text{-}1,i}(t,m) = 1 - \prod_{d=0}^{25} [1 - q_{CS,i,d}(t,m)] \cdot 12; \qquad \text{(A3.13)}$$

Recall that d represents 'duration' of stay at a given parity, estimated as $d = t - y$ (y is the year of the last birth). We estimated exposure for all durations up to 25 years (see Section A-3.3 above). The highest birth order considered constitutes an open-ended parity progression to 6^{th}+ child among women having 5+ children.

Note: The notes on stillbirths and the treatment of multiple births in Section A-3.3 above apply here as well.

A-3.5 Period average parity (*PAP*)

The *period average parity* adjusted for calendar and seasonality factors (PAP_{CS}) is calculated for each parity category j by combining the adjusted *PATFR* index for parity 1 with the adjusted parity progression ratios for parities 2 to 6+:

$$PAP_{CS,j}(t,m) = PATFR_{CS,1}(t,m) \prod_{i=2}^{j} PPR_{CS,i\text{-}1,i}(t,m) \qquad \text{(A3.14)}$$

We computed the period average parity from both types of duration-based indicators described in sections A-3.3 and A-3.4 above. Therefore, no distinction is made between duration-specific incidence rates and duration-parity birth probabilities. All computations are equal for both types of indicators. For example, the *PAP* index for parity 4 is derived as follows:

$$PAP_{CS,4}(t,m) = PATFR_{CS,1}(t,m) \cdot PPR_{CS,1,2}(t,m) \cdot PPR_{CS,2,3}(t,m) \cdot PPR_{CS,3,4}(t,m) \qquad \text{(A3.15)}$$

The highest parity-progression category (5+ to 6+) was assumed to reflect the progression from 5^{th} to 6^{th} childbirth instead.

The progression from the sixth to higher parities was disregarded. Given the very small values of estimated *PAP* index for birth order 6 (the mean value was 0.005 for the whole 1984-2003 period), this procedure involves a systematic underestimation of the total *TFRI* index by about 0.002 in absolute terms (i.e., about 0.1% in relative terms).

The overall index of total fertility was calculated as follows:

$$PAP_{CS}(t,m) = PATFR_{CS,1}(t,m) + \sum_{j=2}^{6} PAP_{CS,j}(t,m) \quad \text{(A3.16)}$$

A-3.6 Kohler and Ortega's adjusted *PATFR* (*adjPATFR*)

Given that we use the Kohler-Ortega adjusted fertility index, the *adjPATFR*, only in the comparative section evaluating the aggregated annual results, and given that this method is relatively complex, a complete overview of all the equations would be beyond the scope of this report. Rather, we provide only a very brief characterisation of the method; a full description can be found in Kohler and Ortega (2002).

This method permits an estimation of period fertility measures that are free of the three distortions present in the *TFR*, namely distortions caused by (1) changes in the parity distribution of women, (2) changes in fertility timing and (3) changes in the variance of the fertility schedule. The authors employ a procedure that iteratively corrects the observed mean age and the inferred tempo for distortions caused by the variance effects (see also Kohler and Philipov 2001). For each parity and single age group, the Kohler-Ortega adjustment allows to derive the adjusted age-parity birth probability $q'_i(a) = q_i(a) / (1-r_i(a,t))$, where $q_i(a)$ is the observed probability that a woman aged *a*, who has *i-1* children at the beginning of the year *t* will give birth to another child during that year. The adjusted parity-specific tempo change $r_i(a,t)$ is computed following Kohler and Philipov (2001: 8, Eq. 11): $r_i(a) = \gamma_i + \delta_i (a - \bar{a}_i)$, where γ_i is the annual change in the mean age of the fertility schedule (here represented by birth probabilities) at parity *i*, δ is the annual increase in the standard deviation of the schedule, and $\bar{a}$ is the mean age of the schedule.

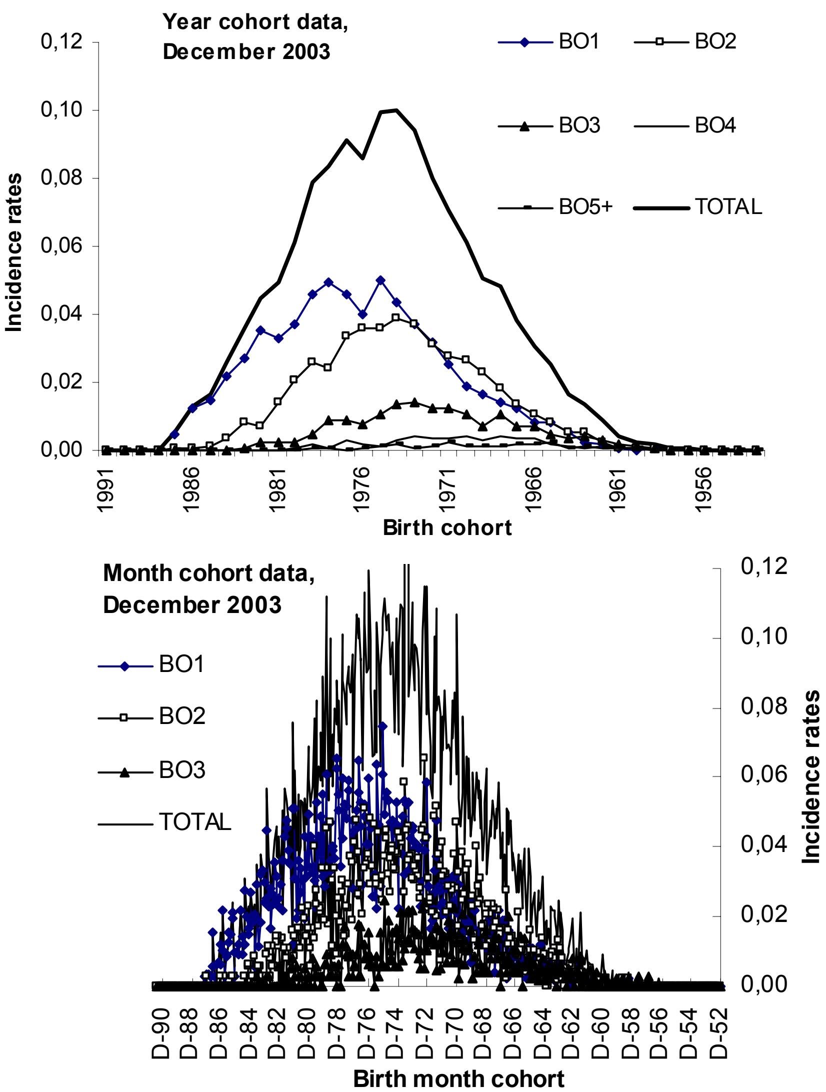
We used the Kohler-Ortega adjustment for the evaluation of tempo distortions performed on an annual basis (see Appendix 6) and we restrict its use for birth orders 1 and 2. Our adjustment differs somewhat from the original Kohler and Ortega (KO) application. First, we work with age-parity birth probabilities as contrasted with the occurrence-exposure rates (birth intensities) utilised by KO. Although the difference in results is small, birth probabilities are in our view methodologically better compatible with the life table framework. Second, we did not smooth the observed set of age-parity probabilities before the adjustment nor did we apply an iterative procedure aiming to provide a correction for variance effects. In order to reduce irregularities in the adjusted fertility index, we restricted the age range of birth probabilities to be used for inferring all the parameters necessary for the adjustment to ages 20 to 40 for birth order 1 and 22 to 40 for birth order 2. Although we computed the adjusted *PATFR* for the higher birth orders as well, we do not utilise these results in this study.

APPENDIX 4:
Comparing indicators derived from month-cohort and year-cohort data (sensitivity analysis)

In order to provide as precise estimates of fertility rates as possible, we computed the monthly series of order-specific incidence rates, total fertility rates, and the mean ages at childbearing from the data specified by month of birth of women as well as in the usual year-birth cohort format.

Figure A4-1 below shows that the resulting age and order-specific incidence rates become extremely erratic in the case of month-cohort data due to the small number of births in each

Figure A4-1
Age- and order-specific incidence rates in December 2003 based on year cohort and month cohort data



Notes: All incidence rates were multiplied by 12 to correspond with the usual annual data format. D denotes December.

monthly birth cohort. For higher birth orders, a typical number of births in most monthly age categories dropped to 0. By contrast, the incidence rates computed for women by single years of age (year cohorts) show considerably smoother trends.

However, when aggregating the incidence rates to obtain order-specific total fertility rates, the differences between these two approaches disappear. This is illustrated in Table A4-1, which presents gross monthly total fertility rates by birth order in January to June 1984. The estimates based on year-cohort data and month-cohort data are virtually identical and do not justify the use of detailed month-cohort computations. The differences are also very small in the case of the estimated mean ages at childbearing, presented in Table A4-2.

Table A4-1

Crude (unadjusted) monthly estimates of total fertility rates based on year-birth cohort and month-birth cohort data of the female population (January to June 1984)

	gTFR $_1$		***gTFR*** $_2$		***gTFR*** $_{3+}$		**Total** ***gTFR***	
	Year_BC	Month_BC	Year_BC	Month_BC	Year_BC	Month_BC	Year_BC	Month_BC
January 1984	0.660	0.660	0.514	0.513	0.222	0.221	1.532	1.532
February 1984	0.678	0.677	0.493	0.494	0.206	0.206	1.504	1.504
March 1984	0.692	0.692	0.529	0.528	0.215	0.215	1.561	1.561
April 1984	0.650	0.651	0.506	0.506	0.195	0.196	1.467	1.468
May 1984	0.709	0.708	0.510	0.510	0.213	0.213	1.557	1.555
June 1984	0.673	0.673	0.529	0.529	0.208	0.207	1.534	1.533

Table A4-2

Mean age at childbearing by birth order, based on year-birth cohort and month-birth cohort data of the female population (January to June 1984)

	MAB $_1$		***MAB*** $_2$		***MAB*** $_{3+}$		**Total** ***MAB***	
	Year_BC	Month_BC	Year_BC	Month_BC	Year_BC	Month_BC	Year_BC	Month_BC
January 1984	23.95	23.97	26.81	26.82	29.86	29.88	26.60	26.62
February 1984	24.16	24.19	26.82	26.84	29.86	29.88	26.56	26.58
March 1984	24.01	24.02	26.79	26.81	29.79	29.80	26.51	26.53
April 1984	24.12	24.13	26.86	26.88	29.76	29.78	26.53	26.55
May 1984	24.13	24.16	26.97	27.00	29.97	29.98	26.59	26.62
June 1984	23.97	23.98	26.86	26.88	29.58	29.61	26.48	26.49

APPENDIX 5: Estimating age and parity composition of the female population. Sensitivity analysis of the estimated age-parity composition on the computed PATFR index

In order to compute age-parity birth probabilities, we had to reconstruct the age and parity structure of the female population in reproductive age (age 12 to 50 was considered) for each calendar month since January 1984. Two possible approaches to derive the relative distribution for each birth cohort—(1) using the Census data on the age-parity distribution among women and (2) cumulating time series of age and order-specific incidence rates that cover the whole reproductive history of the birth cohorts under study—usually yield slightly different results. The second approach disregards possible effects of migration on the parity composition among women. However, the continuous recording of the age and parity composition always has to rely on the time series of incidence rates. The question is whether this database should be occasionally updated with the latest census data, whether such updating makes a significant difference for the age-parity composition records, and whether this difference is in turn translated into different values of the PATFR index.

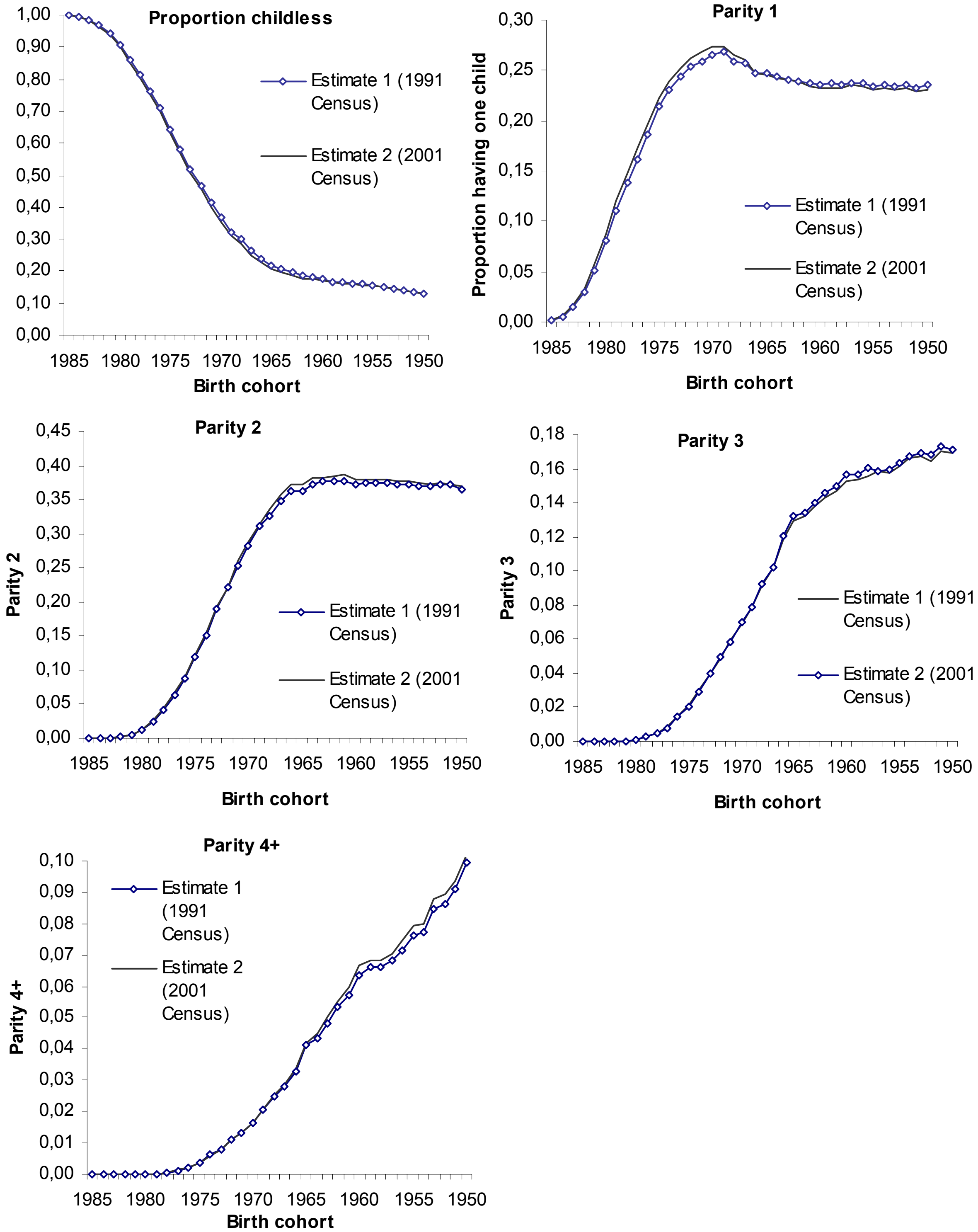
We deal with this topic in the first section (A5-1); the consequent section features the most recent estimate of the age and parity composition among women born between 1954 and 1990 as of December 1, 2004.

A-5.1 Comparing the estimates of the age-parity composition of the female population based on the 1991 and 2001 Census data

We compared two different estimates of the age-parity composition among women born between 1950 and 1985 computed for the date of the 2001 Census (May 15). The first estimate, denoted as “Estimate 1,” is based on the 1991 Census results and the subsequent time series of age (cohort) and order-specific incidence rates computed up until May 15, 2001. The second estimate, “Estimate 2,” is based on the age-parity composition as reported in the 2001 Census. We also compared these data with the third estimate, derived for women born since 1970 solely from the cumulated series of age and order-specific incidence rates. We do not present these results here.

Overall, the differences between the two estimates were not wide. The 2001 census results indicated a lower proportion of childless women and a higher proportion of women having one, two, three and four or more children. These differences implied that Estimate 2 indicates slightly higher levels of actual cohort fertility than Estimate 1 based on the 1991 Census data. In absolute terms, the largest differences were recorded for the proportion of childless women—the 2001 Census (Estimate 2) recorded childlessness by 0.9-1.8% lower among women born in 1962-1979 than the Estimate 1 data (see Figure A5-1). These women were distributed across all parity distributions. For instance, among women born in 1968, where the difference in the estimated proportion childless has been largest (-1.8% in the Estimate 2), the proportion of women with one child in Estimate 2 was by 0.7% higher, with 2 children by 0.9% higher, and with 3 children by 0.1% higher than in Estimate 1 (Figure A5-1). The relative differences were most pronounced among younger birth cohorts (especially for parity 1), and, in the case of parity 4+, also among the ‘older’ women of reproductive age born between 1952 and 1964 (relative difference up to 5%). Combined together, these parity differences resulted in higher actual cohort fertility rates in Estimate 2, with the highest absolute differences (0.021 to 0.038) among women born before 1969. Keeping aside the possibility of incomplete or incorrect reporting in the 2001 Census, these differences are most likely attributable to higher fertility

Figure A5-1
A comparison of the two estimates on the distribution of women by birth cohort and parity on May 15, 2001



among immigrant women prior to their arrival to Austria when compared with the same generations of Austrian-born women.

The differences in the estimated age-parity composition affected the results of the PATFR only to a very little extent, and more its parity-specific components than the overall index for all parities. As a result of the lower numbers of childless women in Estimate 2, the PATFR for first birth order increased slightly, while the PATFR for the third and higher orders declined. These changes are very small, however. For instance for January 2002, the PATFR of parity 1 using the age-parity structure of the Estimate 2 was 0.749 as compared with the value of 0.734 produced by the data of the Estimate 1 structure and the PATFR for all parities was 1.390 (Estimate 2) and 1.383 (Estimate 1), respectively. Figure A5-2 plots the differences in the overall PATFR index in 2001-2003. We see these small differences as a sign of the general stability and low sensitivity of the PATFR indicators to the precision of the age-parity composition estimates. At the same time, we prefer to update the age-parity composition data whenever the detailed and reliable information on the actual parity composition will be available, thus keeping track of possible parity changes related to migration.

Figure A5-2

A comparison of two estimates of the PATFR index (all parities and parity 1) in 2001-2003

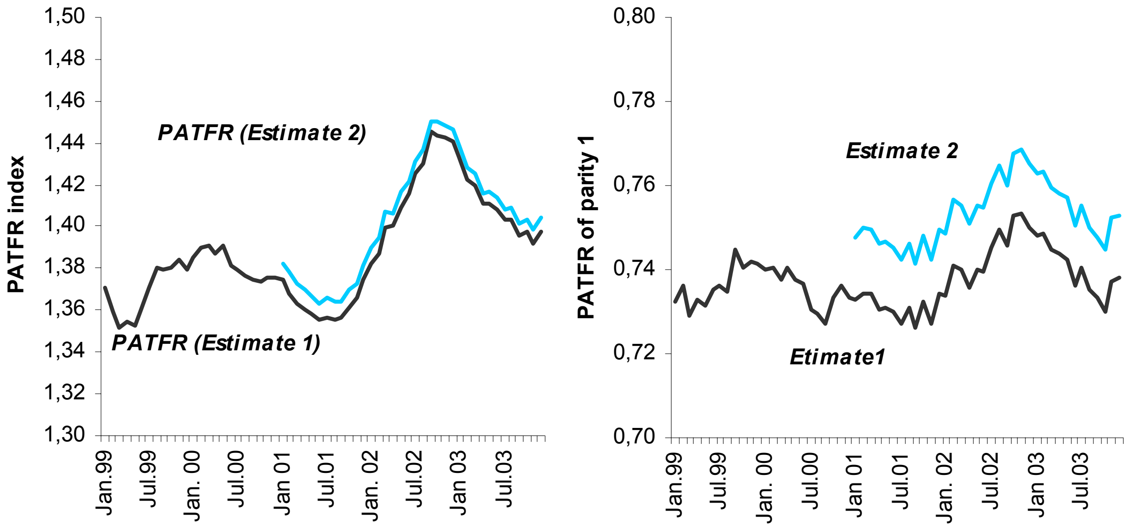


Table A5-1

Estimated age and parity composition of the female population as of December 1, 2004

	Relative parity composition (number of children)					Absolute parity composition (number of children)				
Birth cohort	**0**	**1**	**2**	**3**	**4+**	**0**	**1**	**2**	**3**	**4+**
1990	1.000	0.000	0.000	0.000	0.000	47586	5	0	0	0
1989	0.999	0.001	0.000	0.000	0.000	47698	36	1	0	1
1988	0.996	0.004	0.000	0.000	0.000	47007	167	1	0	0
1987	0.989	0.010	0.000	0.000	0.000	46217	470	22	3	0
1986	0.977	0.021	0.001	0.000	0.000	46512	1004	70	1	0
1985	0.957	0.039	0.004	0.000	0.000	46194	1907	186	6	0
1984	0.929	0.062	0.008	0.001	0.000	46147	3084	396	30	2
1983	0.905	0.078	0.015	0.002	0.000	46043	3963	772	79	5
1982	0.849	0.121	0.027	0.003	0.000	44819	6399	1413	154	15
1981	0.808	0.142	0.044	0.005	0.001	43070	7581	2342	281	30
1980	0.754	0.171	0.064	0.009	0.001	38932	8820	3325	483	73
1979	0.698	0.192	0.093	0.015	0.002	34842	9561	4639	758	111
1978	0.641	0.213	0.120	0.022	0.004	31429	10463	5869	1072	215
1977	0.585	0.229	0.150	0.029	0.007	29064	11383	7450	1435	354
1976	0.529	0.245	0.176	0.041	0.009	26850	12405	8930	2073	450
1975	0.465	0.257	0.215	0.051	0.013	24918	13739	11489	2741	674
1974	0.413	0.257	0.252	0.062	0.016	22787	14160	13925	3422	897
1973	0.365	0.259	0.281	0.076	0.019	20601	14631	15888	4273	1081
1972	0.328	0.259	0.303	0.085	0.024	19508	15405	17991	5050	1444
1971	0.291	0.259	0.331	0.093	0.026	18077	16073	20596	5751	1640
1970	0.265	0.255	0.347	0.102	0.029	16974	16324	22217	6555	1880
1969	0.239	0.256	0.363	0.109	0.033	16136	17322	24508	7347	2226
1968	0.225	0.248	0.371	0.117	0.039	15603	17232	25731	8152	2678
1967	0.205	0.247	0.384	0.124	0.041	14131	17034	26562	8554	2807
1966	0.194	0.236	0.386	0.140	0.045	13466	16433	26841	9705	3095
1965	0.181	0.240	0.382	0.144	0.052	12737	16870	26892	10143	3664
1964	0.179	0.236	0.389	0.144	0.053	12709	16749	27656	10241	3757
1963	0.173	0.236	0.388	0.147	0.057	12309	16847	27664	10484	4030
1962	0.166	0.236	0.387	0.151	0.060	11514	16325	26811	10465	4120
1961	0.163	0.233	0.388	0.153	0.063	10857	15502	25854	10168	4173
1960	0.163	0.231	0.379	0.158	0.069	10731	15214	24932	10391	4533
1959	0.161	0.232	0.380	0.157	0.070	10237	14789	24202	9998	4430
1958	0.159	0.232	0.379	0.161	0.069	9679	14101	23005	9753	4188
1957	0.156	0.236	0.378	0.159	0.071	9250	13974	22432	9434	4198
1956	0.153	0.234	0.377	0.160	0.075	8981	13754	22142	9377	4403
1955	0.150	0.231	0.376	0.164	0.079	8324	12836	20884	9089	4412
1954	0.145	0.232	0.375	0.168	0.080	7659	12287	19859	8863	4230

APPENDIX 6:
Assessment of tempo effects and period fertility quantum in Austria

This appendix analyses period fertility indicators in an annual format and focuses on the total quantum of fertility. In addition to fertility indicators scrutinised in Section 6 we make use of the Kohler-Ortega adjusted *adjPATFR*, which is calculated from the time series of annual age-parity birth probabilities for women at parities 0 and 1. The inclusion of the Kohler-Ortega method allows us to compare the results provided by the PAP index with the indicator explicitly aimed at correcting the tempo effects. For birth order 1, the *adjPATFR* provides a benchmark to estimate the magnitude of the tempo effects in the (unadjusted) *PATFR* index, which is used in the *PAP* computations.

Figure A-6.1 below and Table AN-1 in the Annex summarise mean annual values of the *TFR* as compared with the *PATFR*, *PAP*, and the *adjPATFR* (Kohler-Ortega method). The table further features two estimates of tempo effects, the first based on the difference between the *PAP(rates)* and the *TFR*, and the second based on the difference between the *adjPATFR* and the *TFR*. The results presented here point out the persistence and relative stability of timing distortions in the period *TFR* in Austria. For most periods, the Kohler-Ortega adjustment suggests an extent of tempo effects and similar trends similar to the PAP index; both indicators have shown very close values since 1994. Only during the period of elevated fertility in the early 1990s, the *adjPATFR* indicated considerably less pronounced tempo effects and lower fertility level than the *PAP*. The ongoing fertility postponement is estimated to have depressed the recorded *TFR* by 0.19 on average when measured by the *PAP* as well as the Kohler-Ortega adjusted *PATFR* index. Given that the first-order component of the *PAP*, the *PATFR*, is also affected by tempo distortions, these values probably underestimate the negative influence of fertility postponement (see below). Overall, the mean *TFR* value in 1985-2003 was 1.43 as contrasted with the *PATFR* of 1.44 and *PAP* of 1.62. This may appear as a relatively small difference but in the context of low fertility, even minor differences in fertility quantum may have long-term implications for the eventual rates of population decline and the pace of population ageing.

Figure A-6.2 and Table A-6.1 below show the results of the mean annual values of the *TFR*, *PATFR*, the *adjPATFR* and the estimated tempo effects for first birth order, which accounts for almost half of all births (44-48% in 1984-2004) in Austria. The persistent difference between the *TFR* and the *PATFR* is clearly illustrated, indicating that the *TFR* has been strongly affected by the ongoing fertility postponement (see Section 6.2 of the article). Furthermore, the Kohler-Ortega adjusted *PATFR* index consistently indicates somewhat higher values of first-order fertility than the (unadjusted) PATFR. Although this difference cannot serve for a precise evaluation of tempo effects on a continuous basis, it gives a rough indication of the overall quantum of first birth in 1985-2003. In comparison with the mean *TFR* values of 0.67 and the mean *PATFR* of 0.76, the Kohler-Ortega *adjPATFR* reached 0.80.

Although the difference between the *PATFR* for birth order 1 and its ‘corrected’ version is relatively small, it becomes somewhat larger once we combine it with the subsequent parity progression ratio and estimate the tempo-adjusted *PAP* index. Applying the proportionality assumption, we arrive at the total fertility quantum in 1985-2003 of 1.69 as compared with the *PAP* index of 1.62 and the *TFR* of 1.43. These estimates suggest that the tempo effects have deflated the period *TFR* by about 0.26 in absolute terms during the whole analysed period. In relative terms, our estimates imply that the period fertility quantum in Austria since the mid-

1980 was on average about 19% below replacement level, while the *TFR* indicated a sub-replacement fertility of about 31%. We take this evidence as an indication that the eventual ending of fertility postponement would provide a considerable scope for a potential *TFR* increase to the levels of 1.65-1.70, provided that the 'underlying level' of fertility remains stable.

Figure A6-1:
Mean annual values of the *TFR, PATFR, PAP* (both methods), and the *adjPATFR* in 1984-2004

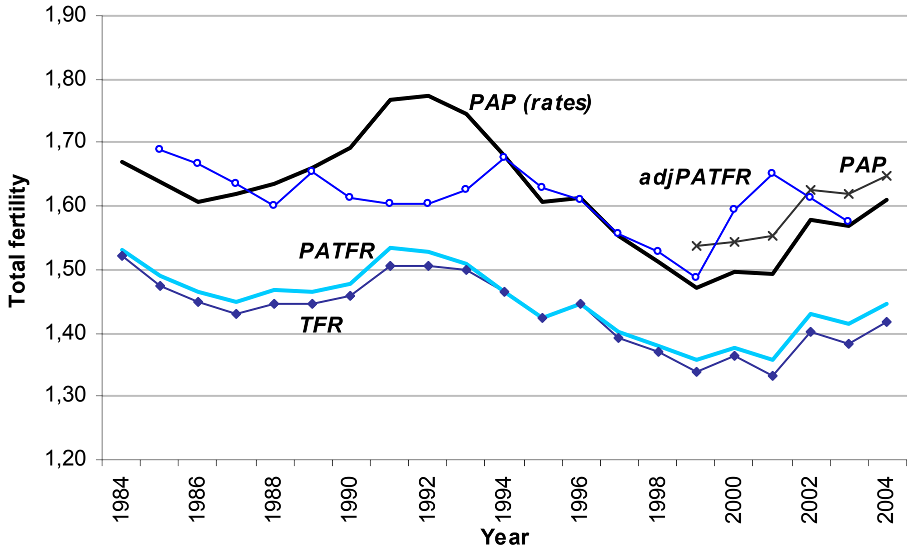


Figure A6-2:
Mean annual values of *TFR*, *PATFR*, *PAP*, and the *adjPATFR* for birth order 1

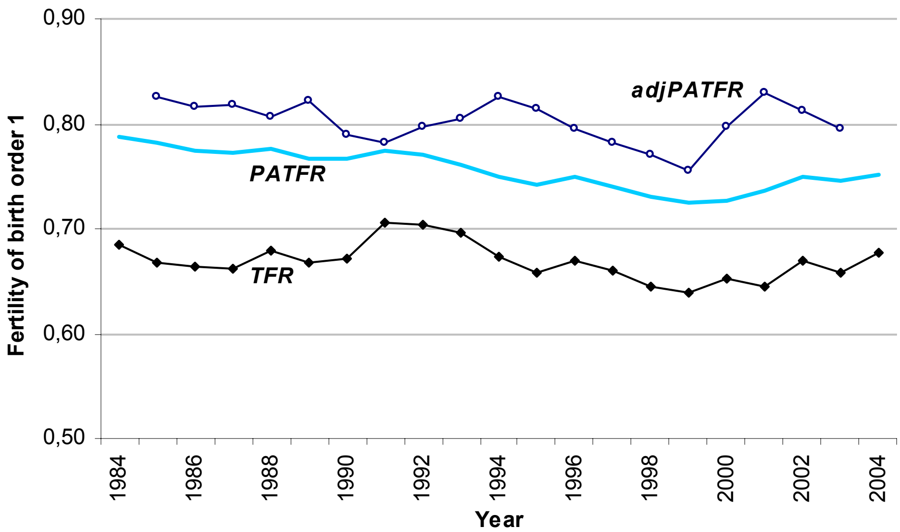


Table A6-1:
Annual values of *TFR*, *PATFR*, *PAP*, and the *adjPATFR* for birth order 1 and the estimated size of tempo effects, 1984-2004.

Year	*TFR* (1)	*PATFR* (2)	*KO adjPATFR* (3)	*Tempo effect (1)* (2)-(1)	*Tempo effect (2)* (3)-(1)
1984	0.685	0.788	..	0.104	..
1985	0.667	0.781	0.827	0.114	0.160
1986	0.664	0.774	0.817	0.110	0.153
1987	0.663	0.772	0.818	0.110	0.155
1988	0.679	0.776	0.807	0.097	0.128
1989	0.668	0.767	0.822	0.100	0.155
1990	0.672	0.767	0.790	0.095	0.119
1991	0.706	0.775	0.781	0.069	0.075
1992	0.705	0.771	0.796	0.066	0.092
1993	0.697	0.762	0.806	0.065	0.109
1994	0.674	0.750	0.827	0.076	0.153
1995	0.658	0.743	0.813	0.085	0.156
1996	0.670	0.750	0.796	0.079	0.125
1997	0.659	0.740	0.783	0.081	0.124
1998	0.646	0.730	0.770	0.084	0.125
1999	0.639	0.724	0.755	0.085	0.116
2000	0.651	0.726	0.798	0.075	0.146
2001	0.644	0.736	0.830	0.092	0.186
2002	0.669	0.750	0.812	0.080	0.142
2003	0.659	0.746	0.796	0.087	0.137
2004	0.677	0.752	..	0.074	..
Mean value 1985-2003	**0.668**	**0.755**	**0.802**	**0.087**	**0.134**

Table A6-2:
Annual values of *TFR*, *PATFR*, *PAP*, and the *adjPATFR* for birth order 2 and the estimated size of tempo effects, 1984-2004.

Year	**TFR** (1)	**PATFR** (2)	**PAP (rates)** (3)	**PAP** (4)	**KO adjPATFR** (5)	**Tempo effect (1)** (3)-(1)	**Tempo effect (2)** (5)-(1)
1984	0.506	0.506	0.561	..	..	0.055	..
1985	0.495	0.491	0.551	..	0.550	0.056	0.055
1986	0.485	0.484	0.542	..	0.551	0.057	0.066
1987	0.480	0.479	0.549	..	0.529	0.069	0.049
1988	0.483	0.484	0.558	..	0.510	0.075	0.027
1989	0.490	0.483	0.566	..	0.543	0.076	0.054
1990	0.500	0.490	0.582	..	0.536	0.082	0.036
1991	0.509	0.510	0.610	..	0.532	0.101	0.022
1992	0.512	0.507	0.616	..	0.518	0.104	0.006
1993	0.523	0.504	0.614	..	0.541	0.091	0.017
1994	0.519	0.491	0.589	..	0.578	0.070	0.059
1995	0.504	0.477	0.562	..	0.555	0.058	0.051
1996	0.515	0.488	0.562	..	0.553	0.047	0.038
1997	0.485	0.473	0.539	..	0.526	0.054	0.041
1998	0.485	0.465	0.530	..	0.519	0.045	0.034
1999	0.468	0.457	0.510	0.545	0.499	0.042	0.031
2000	0.481	0.467	0.527	0.552	0.565	0.046	0.084
2001	0.463	0.457	0.519	0.551	0.594	0.056	0.131
2002	0.488	0.479	0.551	0.575	0.556	0.063	0.068
2003	0.479	0.474	0.547	0.572	0.536	0.068	0.056
2004	0.492	0.486	0.567 1)	0.583 1)	..	0.075	..
Mean 1985-2003	**0.493**	**0.482**	**0.559**	..	**0.542**	**0.066**	**0.049**

NOTES:
TFR, PATFR, and KO adjPATFR are computed from the annual data.
PAP and PAP(rates) represent mean values of calendar and season-adjusted monthly time series.
1) data refer to the period of January to November 2004.