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Induced Discounting and Its Implications to Catastrophic Risk Management

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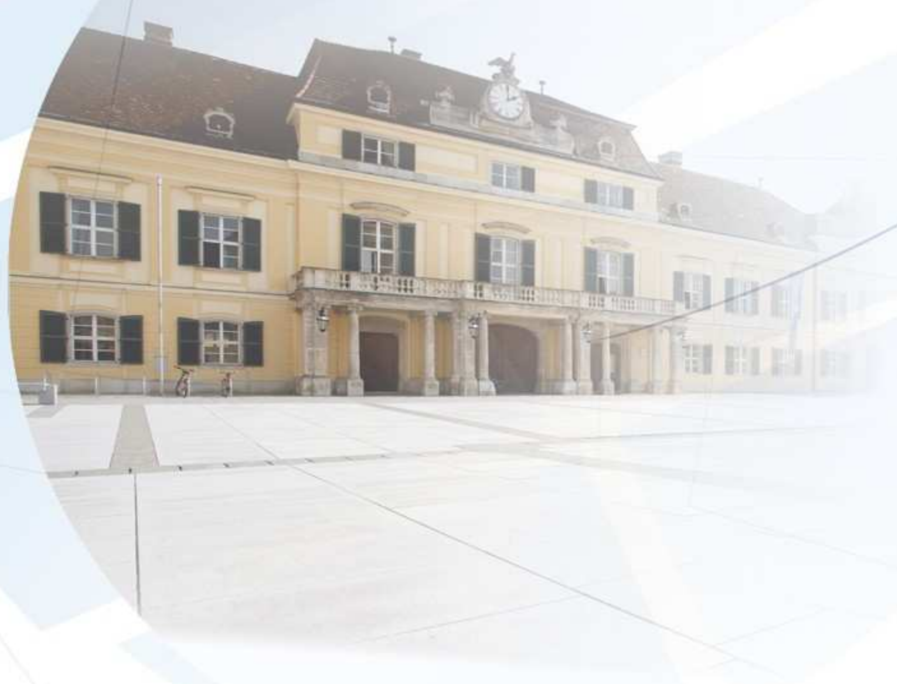

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Induced Discounting and Its Implications to Catastrophic Risk Management

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Contents

1 INTRODUCTION 1

2 INDUCED DISCOUNTING 2

2.1 Standard Discounting 2

2.2 Sets of Potential Events; Declining Discount Rates 5

2.3 Endogenous Discounting 6

3 LONG-TERM STRATEGIES: IMPLICATIONS OF INDUCED DISCOUNTING 7

4 MODEL 8

5 NUMERICAL EXPERIMENTS 10

5.1 The Naïve Society 11

5.2 The Sophisticated Society 12

5.3 The Committed Society 12

6 CONCLUDING REMARKS 13

REFERENCES 13

Abstract

The implication of risks for justifying long-term investment remains a controversial issue. For example, how can we justify mitigation efforts for a 200-year flood that may, in fact, occur in one year or in 300 years? Discount rates obtained from capital markets are linked to assets with lifespans of a few decades and, as such, may significantly underestimate the results of long-term mitigations. In this paper, we show that the explicit treatment of extreme catastrophic events and related uncertain time horizons and risks induce dynamically adjusted discount rates, conditional on the degree of social commitment to mitigate risk. In particular, the standard time consistent geometric (exponential) discount factors are induced by an event with time horizons characterized by a "memoryless" geometric (exponential) probability distribution. A set of such events induces declining time inconsistent discount rates that are dominated by least probable extreme events. In general, risk affects discount rates, which alter the optimal mitigation efforts that in turn, change the risk. We show that the induced discount factors can be analyzed by solving stochastic optimization problems. Our simulation results indicate that the misperception of time inconsistency associated with induced discounting may dramatically effect — delay or provoke — the possibility of a catastrophic collapse.

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Induced Discounting and Its Implications to Catastrophic Risk Management

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1 Introduction

The Intergovernmental Panel on Climate Change (IPCC) Third Assessment Report indicates the degree of extreme uncertainties underlying climate change policy assessment. In particular, uncertainties are inevitably large when policies involve very long time horizons of a century or more. Such time horizons pose serious challenges to standard ideas about investments and discounting.

How can we justify investments into mitigation efforts, which may possibly turn into benefits over long and uncertain time horizons in the future? This is a key question in catastrophic risk management. The discounting is supposed to impose time preferences to answer this question. There are several possibilities for choosing discount rates (see, for example, the discussion in Akerlof (1991), Arrow and Lind (1970), Newell and Pizer (2001), and Portney and Weyant (1999)). One possibility is to use the rates obtained in capital markets, where investments are discounted with respect to both time and risk. The standard geometric discount factor $d(t)$ is usually connected with a constant rate r of returns from capital markets, i.e., $d(t)=1/(1+r)^t \approx e^{-rt}$. Since returns in capital markets are linked to assets with a lifespan of a few decades, this choice dramatically reduces the impacts that investments have beyond these intervals. Another serious problem (Newell and Pizer, 2001; Weitzman, 1999) arises from the use of the expected value Er and the discount factor e^{-Ert} that implies additional significant reduction of future values in contrast to the expected discount factor Ee^{-rt} since $Ee^{-rt} >> e^{-Ert}$.

Ramsey (1928) argued that to apply a positive rate to discount values across generations is “ethically indefensible”, which often leads to the tendency (see, e.g., the discussion in Ainslie (1992) and Newell and Pizer (2001)) of applying low “intergenerational discounting” over longer horizons. However, the use of discount factors other than geometric discount factors produces “time inconsistent” preferences affected by slowing down long-term projects. This inconsistency may lead to “unforeseen” collapses for a society (Hepburn, 2002) that does not anticipate time-inconsistent preferences.

In this paper, we deal with discounting and time preferences that are induced by the explicit treatment of long-term goals, uncertainties, extreme events and risks. Namely, we analyze the implication of uncertain time horizons. The concept of random time horizons associated with the occurrence of the most distractive catastrophic event, so-called stopping times, is a key feature of the catastrophic risk models proposed in (Ermolieva, 1997; Ermoliev *et al*., 2000a, b; Ermolieva *et al*., 2001). This concept is also strongly connected with "life chances" components of social time preferences (Akerlof, 1991), which attracted surprisingly little attention in recent research. For environmental problems, time horizons for balancing costs and benefits, i.e., the lifespans of investments are often linked to the lifespans of pollutants and related extreme events, e.g., such notions as 100-year, or 500-year floods exist. The explicit introduction of uncertainties associated with occurrences of extreme events implicitly induces a time preference, which may have a time inconsistent character defined by time dependent discount rates. Section 2 analyzes the implications of random time horizons on discounting. In particular, the standard geometric (exponential) discounting is induced by an event that is characterized by a random time horizon with a geometric (or exponential) "memoryless" probability distribution. The random time horizon associated with the first event from a set of possible events induces dynamically declining discount rates that are dominated by least probable extreme events. The explicit introduction of risk management decisions induces endogenous discounting that may, in a sense, equally emphasize the future and the present. Section 3 summarizes some implications of induced discounting and its time inconsistency on long-term strategic decisions. Section 4 describes a (stochastic optimization) catastrophic flood management model that is used to illustrate these implications by numerical experiments in Section 5. The long-term uncertain horizons of the model allow the evaluation of threats regarding misperceptions of the time inconsistency for three possible societies ("naïve", "sophisticated" and "committed") discussed in economic literature (Akerlof, 1991; Hepburn, 2002; Marglin, 1963). We illustrate how different types of societies effect — delay or provoke — the possibility of catastrophic collapse. The conclusions are presented in Section 6.

2 Induced Discounting

2.1 Standard Discounting

First of all, let us consider the simplest situation. Assume that an extreme event, such as a flood or an earthquake, may occur in time intervals $t = 0,1,...$ with probability p. This is often defined as a $(1/p)$-year event, say, a 100-year flood. In fact, $1/p$ is the expected "waiting" time until the event occurs although the event, for example a 100-year flood, may occur in two weeks or in 300 years. In this case we can speak of a random time horizon induced by the event. Besides the uncertainties concerning the occurrence time, there may also be uncertainties regarding the probability p, i.e., scenarios of other potential events. For example, for the case study in the upper Tisza river (Ermolieva *et al*., 2001) a catastrophe was associated with the break of one of nine existing dikes that may occur only after a 100-year, a 150-year, and a 1000-year flood situation characterized by different discharge curves.

Risk management decisions generate a stream of random values v_t, $t = 0,1,...$, which may be composed of uncertain costs, benefits and risks indicators. Let τ be the occurrence time of the first event (so-called stopping time, see Ermolieva, 1997; Ermoliev *et al*., 2000a, b), and let V_t be the expected (conditional) value function given that the event occurs at t. For a given probability p, $q = 1 - p$, the expected (unconditional) value function at random time τ is:

$$V := EV_\tau = pV_0 + pqV_1 + pq^2V_2 + ... = p\sum_{t=0}^{\infty} q^t V_t , \quad (1)$$

i.e., the explicit introduction of an uncertain time horizon induces standard geometric discounting. On the other hand, let U_t be another value function. The evaluation

$$\sum_{t=0}^{\infty} \beta^t U_t ,$$

$$\sum_{t=0}^{\infty} \beta^t U_t = p\sum_{t=0}^{\infty} q^t V_t = EV_\tau , \ V_t = \frac{1}{p} U_t , \ q = \beta , \ p = 1 - \beta , \quad (2)$$

with a geometric discounting 1, β, …, β^t, …, can be viewed as the expected value function EV_τ, $V_t = \frac{1}{p} U_t$ at the random stopping time τ associated with the first occurrence of a $(1/p)$-year event, $p = 1 - \beta$. For example, it may be associated with the expected $(1/p)$-year lifespan of an economic agent or the $(1/p)$-year lifespan of assets linked to investments with the constant stream of returns U_t, $t = 0,1,...$.

If we know that the event cannot occur at the initial time interval, e.g., the evaluation takes place at the end of the initial time period, then equation (1) is transformed into:

$$V = V_0 + pV_1 + pqV_2 + pq^2V_3 + ... \quad (3)$$

i.e., the induced discounting is similar in character to quasi-hyperbolic discounting. The discount factors in equation (1) are further modified by adding details of possible extreme events. For example, assume that a $(1/p)$-year event may trigger more severe catastrophic scenarios. Say, a 100-year flood situation itself may not cause significant losses unless one of the existing dikes breaks. As a result, the discounting is now induced by the event of probability $\delta_t p$, where δ_t is the probability of triggering a severe catastrophe — a dike break — once the event occurs at t. The probability δ_t may depend on the decisions to increase the reliability of dikes, e.g., on different maintenance schedules. In this paper we do not analyze these types of induced discounting, as this requires a lengthy discussion of feasible decisions and their effects.

Remark 1*:* Only geometric or exponential discounting, $q^t = e^{(\ln q)t} = e^{-\lambda t}$, $\lambda = -\ln q \approx p$, defines a homogeneous time consistent preference. This means that the evaluation of a project today will have the same discount factors as the evaluation of the same project after any time interval in the future:

$$\sum_{t=0}^{\infty} q^t V_t = V_0 + qV_1 + ... + q^{T-1}V_{T-1} + q^T [V_T + qV_{T+1} + ...]. \qquad (4)$$

As our discussion illustrates, this is the direct consequence of the "memoryless" feature of geometric and exponential probability distributions. For other discount factors with time dependent rates, the so-called time inconsistency arises requiring appropriate adjustments of discount factors for projects undertaken later rather then earlier. The time inconsistency for stochastic models is understood in a rather natural way as the effects of learning. For stochastic models, therefore, we can call this phenomenon as temporal heterogeneity.

An important case is when the random value v_t is defined as the sum of other random values f_0, f_1, …, f_t generated in periods $k = 0,1,...,t$, i.e., $v_t = \sum_{k=0}^{t} f_k$. Consider v_τ, i.e., a random sum of random values. For example, v_τ can represent the accumulated risk reserve of a catastrophe fund until the first catastrophe. The induced discount factor at time t for the evaluation $V = Ev_\tau$ is now equal to the probability of "tails" $P(\tau \geq t)$:

***<u>Proposition 1</u>**:* Let $E|v_\tau|$ exist and the event $\{\tau \leq t\}$ depends only on f_0, f_1, …, f_t. Then

$$Ev_\tau = \sum_{t=0}^{\infty} P(\tau \geq t) Ef_t \, ,$$

where Ef_t is the conditional expectation given that the event occurs at t.

The proof follows from the Kolmogorov-Prochorov theorem, i.e., from the following rearrangements:

$$Ev_\tau = \sum_{t=0}^{\infty} E(v_t \mid \tau = t) = \sum_{t=0}^{\infty} \sum_{k=1}^{t} E(f_k \mid \tau = k) =$$
$$= \sum_{k=0}^{\infty} E(f_k \mid \tau \geq k) = \sum_{k=0}^{\infty} P(\tau \geq k) Ef_k ,$$

where symbol $E(\cdot \mid A)$ denotes the conditional expectation under given event A.

For geometric probability distribution $P(\tau = t) = pq^t$ we have

$$P(\tau \geq t) = pq^t + pq^{t+1} + ... = pq^t (1 + q + q^2 + ...) = pq^t \frac{1}{1-q} = q^t .$$

Thus, geometric (exponential) distribution of τ induces again the standard geometric (exponential) discounting $Ev_\tau = \sum_{t=0}^{\infty} q^t f_t$. For other distributions the misperception of induced discounting defined by tails of distributions according to equation (4) may lead to significant underestimations. This is evident from the following important situation.

2.2 Sets of Potential Events; Declining Discount Rates

Consider the case when stopping time τ is associated with a first event from a set of potential events, say floods, earthquakes, or windstorms, which may occur at different locations. Typically, extreme events are characterized by a finite set of scenarios, say, 50, 100 or 1000-year floods. In a more general case, they may also be characterized by infinite sets of scenarios, e.g., similarly to the Guttenberg-Richter law connecting the probability distribution of magnitudes with expected occurrence times of earthquakes. In other words, we have a set of $(1/p)$-year earthquakes, where p itself is characterized by scenarios with a given probability distribution.

Assume that there is a set of not necessarily mutually exclusive events $i = 1,...,n$, and stopping times (time horizons) τ_1, …, τ_n, associated with these events. Let τ be the moment of the first event, i.e., $\tau = \min_i \tau_i$. Assume also that the event i may occur for the first time at t with probability $p_i(t)$. For example, for the geometric distribution $p_i(t) = p_i q_i^t$, $q_i = 1 - p_i$, $t = 0,1,...$, where p_i is the probability for event i to occur at any time t. If p_i depends on t, p_{it}, then $p_i(t) = (1 - p_{i0})(1 - p_{i1})...(1 - p_{it-1})p_{it}$. Assuming the existence of $\lim \frac{1}{t} \sum_{k=0}^{t} p_{ik-1} = \overline{p}_i$ for $t \to \infty$, and that probabilities p_{it}, $t = 0,1,...$, are small enough, we can take approximately $p_i(t) = p_{it} e^{-\overline{p}_i t}$ for a large enough t since

$$\ln(1 - p_{i0})(1 - p_{i1})...(1 - p_{it-1}) \approx -\sum_{k=1}^{t} p_{ik-1}.$$

For a finite number n of events, the evaluation of equation (1) is transferred into

$$V = \sum_{t=1}^{\infty} \sum_{i=1}^{n} P\left(\tau_i < \min_{j \neq i} \tau_j\right) p_i(t) V_t. \tag{5}$$

Equation (5) essentially modifies the standard geometric discounting. Nevertheless, it is easy to show that the actual discounting tends to be defined by the smallest discount rates. The following proposition is similar to the main conclusions in (Weitzman, 1999).

Proposition 2: Assume that $p_i(t) = \alpha_i(t) e^{-\lambda_i t}$, $\underline{\alpha}_i \le \alpha_i(t) \le \overline{\alpha}_i$, $\lambda_i, \underline{\alpha}_i > 0$, e.g., for the geometric distribution $p_i(t) = p_i q_i^t = p_i e^{-\lambda_i t}$ we have $\lambda_i = -\ln q_i \approx p_i$ (for small

p_i). The induced discount factor $\sum_{i=1}^{n} \theta_i p_i(t)$, $\theta_i = P\left(\tau_i \le \min_{j \ne i} \tau_j\right)$ in equation (5) tends (for $t \to \infty$) to the standard exponential discounting with the smallest discount factor defined by such an extreme event $i^\bullet$ that $\lambda_{i^*} = \min_i \lambda_i$. For the geometric distribution ($\lambda_i = -\ln(1-p_i)$), the induced discount factor is dominated (for $t \to \infty$) by $\min_i p_i$.

This fact follows simply from the equation

$$\sum_{i=1}^{n} p_i(t) = e^{-\lambda_{i^\bullet} t} \sum_{i \ne i^*} \left(N + \chi_i(t)\right),$$

where $\chi_i(t) = \dfrac{\theta_i \alpha_i(t)}{\theta_{i^*} \alpha_{i^*}(t)} e^{-\left(\lambda_i - \lambda_{i^\bullet}\right)t}$ and N is the number of such i that $\lambda_i = \lambda_{i^*}$. Indeed, from $\lambda_{i^\bullet} < \lambda_i$, and $\alpha_{i^\bullet}(t) \ge \underline{\alpha}_{i^\bullet} > 0$, it follows that $\chi_i(t) \to 0$. Therefore, $\sum_{i \ne i^*} (N + \chi_i(t)) \to N$, and thus the induced discount factor in equation (5) decreases for $t \to \infty$, i.e., for large enough t it becomes close to exponential discounting defined by λ_{i^*}.

<u>Remark 2</u>: (Invariance of initial discounting.) The geometric discounting in equation (2) can be associated with $1/(1-\beta)$-year event. If the evaluation (2) is adjusted to a new $1/\delta$-year event, then from the proposition, it follows that the long-term discount factor is defined by random time horizon associated with $\min\{(1-\beta), \delta\}$-year event. If $\delta < 1-\beta$, then from the proposition, it follows that the evaluation (2) is dominated by δ-year extreme event, i.e., it is in a sense invariant with respect to the initial standard geometric discounting β^t.

2.3 Endogenous Discounting

The induced discounting becomes an especially complex issue when it is affected by decisions. We already briefly discussed this in Section 2.1 with respect to factors δ_t, the probability of a dike break. Let us return again to equation (1). The random time horizon τ often depends on the growth rate of different processes and the likelihood of these processes to abruptly pass certain thresholds. This is a typical situation for insurance, where the rate of growth is defined by the inflow of premiums and the thresholds are defined by uncertain losses. A similar situation arises in the analysis of environmental targets. Assume that a random process π_t represents the growth process and the threshold is defined by a random C_t. Let us define the stopping time τ as the first time moment t when π_t is below C_t. By introducing appropriate risk reduction decisions it is possible to regulate "survival" constraints, i.e., the probability

$P(\pi_t \geq C_t)$, $t = 0,1,2,...$, or the probability $P(\tau > T)$, (6)

e.g., such that the collapse may occur once in a 10000 years within the fixed planning time horizon T, i.e., $P(\pi_t \geq C_t) \geq 1-\gamma$, $t = 0,1,2,...,T$, $\gamma = 0.0001$.

Let us now consider catastrophic event $A_t = \{\pi_t < C_t\}$. Then equation (1) is transformed into:

$$V = Ev_\tau = \sum_{t=0}^{\infty} P(A_t)E(v_t \mid A_t), \quad (7)$$

where $E(v_t \mid A_t)$ is the conditional value function given that A_t occurs, which can be called a survival function. As we can see from equation (7), the induced discount factor at time t is the probability that the collapse occurs at time t. According to the goals defined by equation (6), this discounting can be regulated, e.g., within a constant level γ during the time horizon T. We will use this fact in Section 5 to illustrate the advantage of the so-called committed society to be aware of the probability defined by equation (6) despite a seemingly small γ.

3 Long-term Strategies: Implications of Induced Discounting

The justification of a particular investment (saving) strategy has usually been addressed within the utility maximization framework. A social planner chooses a saving plan for a future period of time so as to maximize the utility evaluated at the present moment. The most crucial issue is that the social planner has to choose time preferences weights or discount factors. Samuelson (1937) assumed time consistent geometric (exponential) discount factors that were dependent only on the *time distance* between the present and the future, not on the particular points in time.

The choice of discounting remains a controversial issue. As Section 2 illustrates, the key issue here is the explicit treatment of uncertainties and risks. In other words, instead of postulating exogenous time and risk preferences taken from capital markets, it is possible to impose implicit (induced) discounting by explicitly specifying goals, extreme events, and risks. This gives rise to a number of challenging problems, in particular, the need for the explicit treatment of catastrophic risks and long-term time preferences, which may go beyond the maturity of assets in existing capital markets. From *Remark 2,* it follows that the proper evaluation of a project may be dominated by time inconsistent discounting induced by extreme events, rather than initial standard geometric discounting. For example, a 4% discount rate can be linked in view of equation (2) to a 25-year event, i.e., to the time horizon, which is not matched with the lifespan of the investment that is linked, say to a 250-year flood. Therefore, in the long term the evaluation is dominated by the rate 0.004 rather than 0.04. The misperception of these effects may significantly underestimate the necessity of long-term mitigation efforts. As a result, it may provoke catastrophes (Section 6) and hence, lead to increasing vulnerability of the society. The increasing catastrophic losses is an alarming global tendency (Munich Re, 1999), which is primarily due to the misperception of rare catastrophic events and hence, movements of capital and people in risk-prone areas. The adequate perception is a challenging task requiring models that enable the explicit evaluation of risk profiles, induced discounting, its time inconsistency, and related long-term strategies. These models can be considered as a key mitigation measure to cope with increasing vulnerability.

A number of authors already distinguish between various types of so-called "imperfect altruism" resulting in the lack of social commitment to mitigate risks. For example, Akerlof (1991) and Strotz (1956) alluded definitions of a naïve, a sophisticated and a committed society (thrift). The main differences between these three societies are summarized in (Hepburn, 2002). Thus, the naïve society does not anticipate its time inconsistent preferences and actual risk profiles; it plans to consume less than it actually does at the expense of investments in mitigation efforts. The sophisticated society is aware of the time inconsistency and therefore chooses the strategy that is a best response to its later generations best response. Thus, the two societies are not committed to the strategy for the overall long horizon. The reasons for such policies lay simply in the misperception of risks and society's lack of power to lead a committed life. The third, committed society, has the ability to commit to mitigate risks. In Section 5 we discuss how different types of societies effect — delay or provoke — the possibility of a catastrophic collapse. The correct understanding of risk profiles and induced time inconsistent discounting, as we can see from Sections 4 and 5, requires appropriate stochastic optimization models. Consider a risk management model, which is used in Section 5 for numerical experiments.

4 Model

The model described in this Section has the structure outlined by equations (6) and (7). In fact, it is possible to formulate a simple analytical model, which incorporates the following three elements:

1. The risk of a catastrophe induces discounting;
2. The discount rate affects the optimal mitigation effort; and
3. Mitigation efforts affect the risk of a catastrophe (return to point 1).

This is evident from equation (7). In this way, we have a loop and the potential for positive feedback and branching (multiple equilibria). It also means that the discount rate will be time varying, so the implications of the three types of society (naïve, sophisticated, committed) can be illustrated analytically. However, this requires lengthy computations of solutions for arising stochastic optimization problems with an infinite horizon. Therefore, in what follows, we illustrate the implications of three societies by numerical experiments using a simplified version of a catastrophic risk management model that was developed in (Ermolieva, 1997; Ermoliev *et al*., 2000a, b). Namely, we deal with long uncertain time horizons embedded into the model that has been calibrated for the analyses of catastrophic flood (Ermolieva *et al*., 2001) risks. The main purpose is to evaluate the amount of precautionary financial resources needed in order to cope with a possible catastrophic flood. We assume that risk reserves are accumulated over years in a catastrophe fund through payments from the population through a mandatory insurance.

In our experiments, the system is modeled until the first catastrophic flood, which occurs at random within a given fixed time horizon $T=100$. We define this random moment as the stopping time. This event is associated with the break of one of nine existing dikes that may occur only after a 100, 150 or 1000-year flood. The timing of a first catastrophic flood significantly affects the accumulation of risk reserves by the insurance and total payments of individuals. For example, a 100-year flood with the break of a dike may occur in two years leading to considerable underpayments by individuals.

Let τ be a random (stopping) time of a first catastrophic flood within a time interval $[0,T]$. If no catastrophe occurs, then $\tau=T$. Let L_j^τ be random losses at location j at time $t=\tau$. In the experiments we evaluate the capacity of the catastrophe insurance in the region only with respect to insurance decisions. Let π_j be the premium rate paid by location j to the mandatory insurance, then the accumulated mutual catastrophe fund at time τ together with the proportional compensation $\chi\sum_j L_j^\tau$ by the government is equal to $\tau\sum_j \pi_j + \chi\sum_j L_j^\tau - \sum_j \varphi_j L_j^\tau$, where $0 \le \varphi_j \le 1$, is the insurance coverage for cell j. Thus, in this model, we assume that the compensation to victims by the government is paid through the mandatory insurance.

The sustainability of the insurance program depends on whether the accumulated mutual fund together with the governmental compensation is able to cover claims, i.e., on the probability of insolvency defined by the event:

$$\tau\sum_j \pi_j + \chi\sum_j L_j^\tau - \sum_j \varphi_j L_j^\tau < 0 \ . \tag{8}$$

The sustainability also depends on the willingness of individuals to accept premiums, i.e., on the probability of overpayments:

$$\tau\pi_j - \varphi_j L_j^\tau > 0 , \ \ j=1,...,m \ . \tag{9}$$

This requirement can be written in the form:

$$\pi_j \le a\varphi_j , \ \ j=1,...,m , \tag{10}$$

where a is the minimal number satisfying the following equation $P\left(L_j^\tau \le a\tau\right)=0.04$, requiring that overpayment may occur only once in 25 years.

Inequalities (8) and (9) define events, which constrain the choice of the decision variables specifying the insurance program, i.e., the compensation rate χ by the government, coverages by the insurance company φ_j and premiums π_j. The likelihood of an event defined by equation (8), i.e., underpayments to the pool as well as equation (10), determine the resilience of the program. It can be expressed in terms of equation (10) and the probabilistic constraint:

$$P\left[\left(\tau\sum_j \pi_j + \chi\sum_j L_j^\tau - \sum_j \varphi_j L_j^\tau < 0\right)\right] \le \gamma, \tag{11}$$

where γ is a specified probability of the program's default, say a default that occurs only once in 1000 years, $\gamma = 0.001$. The constraint (11) is similar to the so-called insolvency constraint, a standard for regulations of the insurance business. In the stochastic optimization (Ermoliev and Wets, 1988), the constraint (11) is known as the so-called chance constraint. The main goal can now be formulated as the minimization of the expected total uncovered by insurance losses

$$F(x) = E\sum_j (1-\varphi_j)L_j^\tau \tag{12}$$

subject to equations (10) and (11), where vector x includes all decision variables. The solution procedure for this type of model can be found in Ermolieva (1997) and Ermoliev *et al*. (2000b).

5 Numerical Experiments

These experiments serve to demonstrate by how much the different (often erroneous and light-minded) risk perception, in other words, induced time inconsistency, may turn into a catastrophe.

Our model, defined by the maximization of value (12) subject to equations (10) and (11), is similar to the model outlined by equations (6) and (7). From equation (7), it follows that the induced discount factors for the value function (12) is related to the probability of ruin given that a catastrophe occurs at time t. This discount factor is subject to regulations according to equations (10)-(12), i.e., we have endogenously generated discount factors.

In what follows we use modified data from (Ermolieva *et al*., 2001). We assume that the dike system deteriorates over time, therefore after a passage of time the break may occur from less severe but more frequent rainfalls. The number of fast Monte Carlo simulations in a single experiment run equals 5000. The evaluation of risk management decisions accounts for only catastrophes that may occur within 100 years, i.e., $T = 100$. Hence, the stopping time $\tau \le 100$. The time period t of the model covers five overlapping generations and each generation acts as a social planner for 20 years. The parameter χ in the experiments is fixed, and we only simulate 150-year floods.

We consider the fixed 100-year horizon in which three societies, the naïve, the sophisticated, and the committed, live and plan for mitigating and coping with the catastrophic losses that may occur. They are able to mitigate the risks by laying aside money to be able to cover the losses. But, depending on their perception of risks, i.e., induced time preferences, the results are different.

5.1 The Naïve Society

The current generation of social planners is aware of a possible catastrophe. It maximizes the value function (12) taking into account the potential need to save for the catastrophe by establishing a catastrophe fund and paying premiums. Unfortunately, the society postpones the implementation of decisions, i.e., let future generations take the lead. In this sense, the naïve society puts its preferences on consumption as the first priority, the first generation of the naïve society consumes at a higher rate than it actually plans.

For the next generation, the time is shifted forward by 20 years and the second generation, similar to the first, plans but does not implement saving actions essential for the catastrophe fund to function. It also has a misleading view on the catastrophe, namely, if the catastrophe has not occurred in the later generation the society believes that it will not occur within the current generation with the same probability, i.e., it fails to take into account the time inconsistency induced by increasing the probability of a dike break. Thus, the risk profiles, time preferences, and the actions are not adjusted towards the real risks. In a similar way, we simulate the other three generations, each time calculating how much insurance premiums they naïvely plan to save. The plans are never implemented and the view on a catastrophe is time invariant.

Now, what happens to the five generations of the society is shown in Table 1. The society believes that the ruin probability satisfies desirable level 0.05 calculated by using time consistent geometric discounting induced by 150-year flood. In fact, even if society implements its savings plan, the ruin would still increase ("Ruin probability under savings") due to the misunderstanding of the actual risk profiles — it keeps reducing the savings (premiums) despite the increasing actual threats in the remaining time intervals ("Ruin probability actual").

Table 1: Performance of the naïve society.

Planning Horizon	Probability of Ruin	Premium Per Location	Ruin Probability Under Savings	Ruin Probability Actual
0–100	0.05	1.61	0.05	0.32
20–100	0.05	1.32	0.06	0.46
40–100	0.05	0.97	0.07	0.61
60–100	0.05	0.63	0.09	0.76
80–100	0.05	0.35	0.12	0.89

5.2 The Sophisticated Society

The simulation scheme for the sophisticated society is similar to that of the naïve society. In contrast though, it implies a correct understanding of the time-inconsistencies induced by the deteriorating system of dikes. But in fact this society, similar to the naïve planners, also evaluates present consumption to be much higher than the future, i.e., they spend also more than plan. This leads to postponing the decisions made by each generation. If the catastrophe occurs, the procrastination may turn out to be very costly.

Table 2 shows that since the sophisticated society correctly understands its time inconsistency, it is able to keep the "Ruin probability" at a constant level. To do this, the sophisticated society plans for premiums that increase over time (higher savings), but the decisions are postponed to the next generation. Due to these delays, the risk burden is increasingly shifted to the next generation ("Ruin probability actual"). In any case, if a catastrophe occurs this society will also be not prepared to meet threats, as premiums are not accumulated.

Table 2: Performance of the sophisticated society.

Planning Horizon	Probability of Ruin	Premium Per Location	Ruin Probability Actual
0–100	0.05	1.92	0.32
20–100	0.05	2.49	0.46
40–100	0.05	3.06	0.61
60–100	0.05	3.63	0.76
80–100	0.05	4.19	0.89

The "pathologies" of the naïve and the sophisticated societies can be explained by their ignorance of risks, incorrect understanding of potential losses and, therefore, the lack of committed actions. The delays in actions may dramatically affect individuals and the growth of societies as a whole. Individuals could be better off if their consumption options were limited and their choices constrained by anticipating risks.

5.3 The Committed Society

The committed society evaluates savings plans by explicitly taking into account time dependent profiles of catastrophic risks and induced discounting. This society is able to implement decisions together with subsequent generations. As shown in Table 3, the premiums that the society saves for coping with catastrophes in 100 years time are much lower than those of the sophisticated, which is a direct consequence of their committed actions.

Table 3. Performance of the committed society.

Planning Horizon	Probability of Ruin	Premium Per Location
0–100	0.05	2.1

6 Concluding Remarks

The explicit treatment of extreme events, uncertain time horizons, social goals and risks leads to induced discounting, which may be significantly different from the standard discounting obtained from capital markets. Risk management decisions affect this discounting with the potential for positive feedbacks and locked-in “equilibriums”. The misperception of time inconsistent induced discounting may provoke catastrophic collapse. Stochastic optimization models enable us to deal with induced time-inconsistent discounting. It is important to analyze this with more analytical details.

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