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Greenhouse Gases, Cooperation, and Exchange

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Abstract

Emission of uniformly dispersed greenhouse gases is construed here as a cooperative production game, featuring side-payments, quota exchange, uncertainty, and multi-period planning. Stochastic programming offers good instruments to analyze such games. *Absent* efficient markets for emissions, such programming may help to imitate market-like, price-based transfers among concerned parties. *Present* appropriate markets, it may predict equilibrium outcomes. In both cases, shadow values of aggregate emissions define side-payments or prices that yield core solutions.

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1 Introduction

The possibility of global climate change has been perceived - and partly understood - for more than two decades. Scientific controversy and uncertainty not withstanding, the evidence supporting that possibility, and associated dire consequences, now attracts much attention. Above all, global heating, driven mainly by CO_2 emissions, has fostered widespread concern (or greater risk perception) - and thereby induced many nations to sign the Kyoto treaty.

Regarding such emissions economic theory has, for at least three decades, declared marketable permits (and Pigouvian taxes) superior to conventional regulation of quantities (or approval of technologies); see [11], [12]. Yet, the largely political debate remains polarized around the two extreme alternatives: command-and-control versus economic instruments.

Quite generally, economics depends on - and amply demonstrates - theoretical and practical advantages stemming from exchange of private commodities (goods or bads). Similarly, insurance and finance thrives on mutual benefits derived from exchange of private risks or securities. So, why then does advocacy for emission trade generate, in many corners, moderate enthusiasm or reserved support? There may be several reasons, including first, absence of efficient market (or market-like mechanisms/ institutions) to mediate emission trade. Second, even if such markets were functioning well, the resulting cost-benefit distribution might be perceived as unfair or lacking in legitimacy. Third, traders of standard variety often face credit rationing, and they are rarely as foresighted or rational as those commonly accommodated by economic theory. Fourth, trade can hardly clear the market in one equilibrated, immediate shot. Transactions rather evolve, step by step, in contingent manner, and frequently take place out of equilibrium.

Given such features it is a challenge to come up with concepts and procedures that stand good chances of being understood, accepted, and implemented. The novelty - and modest object - of this paper is to advocate, for these purposes, the joint use of cooperative game theory and stochastic programming. Doing so, I depart from the literature which studies pollution within the frames of competitive equilibrium [9], [15] or noncooperative games [13], [14].

As is well known, programming - and notably duality theory - may help in assessing the value of relaxed constraints, i.e. of marginal emissions. Less known is that it can also single out transfers, accompanied by payments, these serving as surrogates for reasonable market transactions. In fact, by imitating price-taking equilibrium not only efficiency is ensured (via the first welfare theorem) but social stability as well. To wit, the outcomes considered below belong to the core; they are such that no party can gain by deviating from a suitably specified, enforceable treaty. Added to these desirable properties comes one good and one bad: On the positive side, Pareto efficiency, and stability against deviations, can be produced and upheld by decentralized barter among honest parties. On the negative side, a cooperative treaty admittedly appears vulnerable and naive in presuming perfect compliance.

Mäler [14], who studied an acid rain game with non-uniform precipitation, rightly doubted the possibility of construing such instances as cooperative games in characteristic form. In that regard he expressed two well founded worries: first, the characteristic function could be beyond practical reach; second, the external impact of outsiders on any coalition might be hard to predict. However, when it comes to uniformly dispersed greenhouse gases the situation changes radically. On the first account, as shown below, there is no need to generate the characteristic function. On the second account, since aggregate emission most likely will equal the sum of permits, the externalities are common and predictable. Such simplification not only facilitates modelling; it also opens up for use of (multi-stage stochastic) programming - and for decentralized, practical implementation by means of markets.

Arguments, showing this, are organized below as follows. For motivation, Section 2 reconsiders an important linear instance of production games, first studied by Owen [16], later extended in [22]. Section 3, being the heart of the paper, defines the transferable-utility cooperative game and shows that core solutions can come handy in terms of Lagrange multipliers, these supporting equilibrium in a competitive quota market. Section 4 spells out those insights in a multistage setting, Section 5 includes uncertainty, and Section 6 synthesizes all this. Finally, Section 7 concludes by briefly mentioning the prospects for iterative trading, taking place out of equilibrium. Since this paper mainly is conceptual, I gloss over minor technicalities - and relegate numerical illustrations to subsequent studies.

2 An Example

The following stylized example, first studied by Owen [16], illustrates some key issues well: Suppose each agent i, who belongs to a finite society I, faces a linear program

$$\pi^i(e^i) := \max\left\{\langle c, x\rangle \;\middle|\; x \geq 0, e^i - Ax \geq 0\right\}, \tag{1}$$

assumed feasible with attained finite value. Here and elsewhere $\langle \cdot, \cdot \rangle$ means an inner product; A is a $m \times n$ activity matrix; the vector $e^i \in \mathbb{R}^m$ denotes i's endowment of m different emission permits; and $c \in \mathbb{R}^n$ accounts for monetary contributions created by activity plans $x \in \mathbb{R}^n_+$.

Most likely individual emission permits would come in proportions that cause shortages, excesses, or bottlenecks. Then gains can be had by pooling private endowments. Specifically, coalition $S \subseteq I$, in controlling endowment $e^S := \sum_{i \in S} e^i$, could achieve an optimal value

$$\pi^S(e^S) := \max\left\{\langle c, x\rangle \;\middle|\; x \geq 0, e^S - Ax \geq 0\right\} \tag{2}$$

superior to the individually assembled outcome $\sum_{i\in S}\pi^i(e^i)$. So, given advantages in the aggregate, it is fitting to ask: *How can potential gains of cooperation be secured and split?*

For a quick and motivating answer, suppose there is an optimal dual solution (a Lagrange multiplier) $\bar{p}$ to problem (2) when $S = I$. That price vector $\bar{p}$ evaluates (marginal) permits for the grand coalition $S = I$. Therefore, quite naturally, let i be offered payment $u^i := \langle \bar{p}, e^i\rangle$ for handing his holding over to the cooperative enterprise - or for bringing his emission quota to an internal market. Will he accept that offer? Yes, most likely! In fact, as it turns out, since $\sum_{i\in S} u^i \geq \pi^S(e^S)$ for all $S \subset I$, and $\sum_{i\in I} u^i = \pi^I(e^I)$, nobody has economic incentive to object.

This story was coached in terms of cooperation. The parties need not sign or enforce a contract though. Implementation can better, and more easily, come via a market where emission permits are traded at the equilibrium price vector $\bar{p}$. More realistically, trading may develop, and a market could come to function, by iterated bilateral exchanges of quotas. As shown below that market can accommodate nonlinear preferences/technologies, several stages, and substantial uncertainty. For simplicity those features are next presented separately.

3 Nonlinear Emission Games

Suppose that each member of a fixed, finite society I owns an emission permit, clearly codified as a vector residing in a finite-dimensional vector space $\mathbb{E}$. What I have in mind are quantified "licences to pollute", these being privately held rights to discharge diverse greenhouse gases into the atmosphere. As spelled out later, elements in $\mathbb{E}$ can be construed as processes, incorporating contingent emissions indexed by time/location and event. The set I could comprise diverse industries within a region or the signataries of an international treaty.

If $i \in I$ contends with his permit $e^i \in \mathbb{E}$, he obtains payoff $\pi^i(e^i)$. Instead of him going alone, the situation invites coordination or joint undertakings [8]. Specifically, any coalition $S \subseteq I$ could consider its *stand-alone payoff*

$$\pi^S(e^S) := \sup\left\{\sum_{i\in S}\pi^i(q^i) \;\middle|\; \sum_{i\in S} q^i = \sum_{i\in S} e^i =: e^S\right\}, \tag{3}$$

the aim being to distribute proceeds and quotas among the members. It is tacitly assumed here that no i misrepresents privately held information about $\pi^i(\cdot)$ to own advantage. Granted such honesty, what we have is a cooperative production game with player set I, *characteristic function* $I \supseteq S \mapsto \pi^S(e^S) \in \mathbb{R}$, and potential side-payments. For this game a *payoff allocation* $u = (u^i) \in \mathbb{R}^I$ belongs to the *core* iff it entails

$$\begin{array}{lrcl} \textit{Pareto efficiency:} & \sum_{i\in I} u^i & = & \pi^I(e^I), \\ \textit{and social stability:} & \sum_{i\in S} u^i & \geq & \pi^S(e^S) \;\; \textit{for all coalitions } S \subset I. \end{array}$$

Social stability means that no singleton or set $S \subset I$ of several players could improve their outcome by splitting away from the society. Note that mere stability is easy to achieve: Simply let the numbers u^i be so large that $\sum_{i\in S} u^i \geq \pi^S(e^S), \forall S \subset I$. So, to no surprise, the essential difficulty resides in the requirement that $\sum_{i\in I} u^i = \pi^I(e^I)$.

To regard sharing of quotas within cooperative game theory has the potential advantage of emphasizing equity issues. Besides, as it turns out, there is a direct connection to optimization and to exchange markets [23]. Indeed, by means of programming or a market, an explicit price-determined core allocation can be found under weak and natural assumptions. To show this let

$$L^S(p,q) := \sum_{i\in S} \left[\pi^i(q^i) + \langle p, e^i - q^i\rangle\right]$$

denote the standard Lagrangian of problem (3). Any price vector $\bar{p} \in \mathbb{E}$ satisfying $\pi^I(e^I) \geq \sup_q L^I(\bar{p}, q)$ will be named a *Lagrange multiplier for the grand coalition.* Clearly, given any constant price regime $p \in \mathbb{E}$ for permits, not necessarily a Lagrange multiplier, and also given the possibility to purchase any emission quota $q^i \in \mathbb{E}$, then agent i could secure himself production profit

$$\pi^i_*(p) := \sup\left\{\pi^i(q^i) - \langle p, q^i\rangle \;\middle|\; q^i \in \mathbb{E}\right\}.$$

Added to that profit comes the market value $\langle p, e^i\rangle$ of his initial endowment. Note for the subsequent argument that

$$\begin{array}{rl} \sup_q L^S(p,q) = & \sum_{i\in S}\{\langle p, e^i\rangle + \pi^i_*(p)\} \quad \text{and} \\ \inf_p L^S(p,q) = & \sum_{i\in S}\pi^i(q^i) \quad \text{if} \quad \sum_{i\in S} q^i = e^S, \quad -\infty \text{ otherwise.} \end{array} \tag{4}$$

In these terms a main result can now be stated forthwith:

Theorem 1 (Lagrange multipliers yield core solutions). *Suppose $\bar{p} \in \mathbb{E}$ is a Lagrange multiplier for the grand coalition. Then the payoff allocation $u^i := \langle \bar{p}, e^i\rangle + \pi^i_*(\bar{p}), i \in I$, belongs to the core.*

Proof. Social stability obtains via (4) because any coalition S receives $\sum_{i\in S} u^i =$

$$\sum_{i\in S}\left\{\langle \bar{p}, e^i\rangle + \pi^i_*(\bar{p})\right\} = \sup_q L^S(\bar{p},q) \geq \inf_p \sup_q L^S(p,q) \geq \sup_q \inf_p L^S(p,q) = \pi^S(e^S).$$

The very last inequality is often referred to as *weak duality.* The hypothesis concerning $\bar{p}$ ensures *strong duality.* To wit,

$$\pi^I(e^I) \geq \sup_q L^I(\bar{p},q) \geq \inf_p \sup_q L^I(p,q) \geq \sup_q \inf_p L^I(p,q) = \pi^I(e^I).$$

So, using $\sum_{i\in I} u^i = \sum_{i\in I}\{\langle \bar{p}, e^i\rangle + \pi^i_*(\bar{p})\} = \sup_q L^I(\bar{p},q)$, one sees that Pareto efficiency also prevails. $\Box$

Theorem 1 has a nice interpretation, already sketched above, and pointing to implementation: If emission permits were traded at constant unit prices $\bar{p}$, then i could envisage a profit $\langle \bar{p}, e^i\rangle + \pi^i_*(\bar{p})$ composed of sales revenue plus production profit. Such trading possibilities decentralize production planning and profit considerations. The price $\bar{p}$ ensures both market clearing and efficiency. This result hinges, of course, upon existence of at least one Lagrange multiplier. As is well known, such existence largely depends on each objective $q^i \mapsto \pi^i(q^i)$ being concave [17].[1]

When all π^i are differentiable, the marginal profit $\bar{p}$ is the same across all active producers, that is, such an agent i chooses a net emission $e^i - q^i$ satisfying $\bar{p} = (\pi^i)'(q^i)$. This feature implicitly confirms the usual justification for marketable permits: Emission abatement will be undertaken by agents/firms with lowest cost.[2] Gersbach and Glazer [5] offer additional justification. They recall that hold-up problems can produce predictable relief instead of necessary reform, and they show that governments can overcome such time-inconsistency by issuing tradeable permits.

The cooperative set-up used here does not allow any imperfections in the quota market: Monopoly or cartels are not admitted. Clearly, this assumption is questionable, and apparently more so if banking and borrowing allow emissions to the shifted across time periods [7]. (Thus a regime which treats emission permits as perishable goods might be more efficient.)

In any case, and as already illustrated in section 2, the function $\pi^i(\cdot)$ is a reduced, indirect object. It is predicated on agent i exploiting the possibilities to reallocate resources (say over time, locations, or production lines). So, in particular, a deposit-refund system could be at work. I stress that $\pi^i(\cdot)$ might stem from a regional or national model, incorporating various modes of competition, not necessarily perfect.

Emission permits are given data here. How they were acquired is not an issue - be it by auction, in grandfatherly manner, or via a distribution that reflects some proportionality constraints [9]. The advantage of being silent about such important issues here is that many and diverse scenarios fit within unifying frames. Given the long-term nature of global warming, one may imagine aggregate emission constraints of increasing severity. Such aggregates would stem from targets for accumulated

[1] Otherwise there might be a nonnegative duality gap/deficit

$$d := \inf_p \sup_q L^I(p,q) - \sup_q \inf_p L^I(p,q)$$

such that any dual optimal solution $\bar{p} \in \arg\min \left[\sup_q L^I(p,q)\right]$, while ensuring social stability, implies non-sustainable over-spending or budget deficit $d = \sum_{i \in I} u^i - \pi^I(e^I)$.

Thus, for the sake of having $d = 0$, emissions should yield decreasing returns to scale in every part of the considered economy. That hypothesis, although pervasive in general equilibrium analysis, is problematic and far from innocuous.

Also worth notice is the stability of problems like (3) with respect to aggregation/ disaggregation. Specifically, suppose i stands for a syndicate of agents $j \in J(i)$, the sets $J(i), i \in I$, being nonempty and disjoint. Then

$$\pi^I(e^I) := \sup \left\{ \sum_{i \in I} \sum_{j \in J(i)} \pi^{ij}(q^{ij}) \;\middle|\; \sum_{i \in I, j \in J(i)} q^{ij} = \sum_{i \in I, j \in J(i)} e^{ij} = e^I \right\}.$$

[2] As a technical note, suppose $\pi^I(e^I)$ is attained, this meaning that $\pi^I(e^I) = \sum_{i \in I} \pi^i(q^i)$ with $e^I = \sum_{i \in I} q^i$. Provided all $\pi^i(\cdot)$ are concave, two things hold in that case: *first,* if some function π^i is strictly concave, then the corresponding component q^i becomes unique; *second,* if all π^i are continuous at q^i, except maybe one, then π^I becomes continuous whence superdifferentiable at e^I. Granted concave payoffs, it is easy to see that $\bar{p}$ is a multiplier for the grand coalition iff it is a supergradient of $\pi^I(\cdot)$ at e^I; see [17]. One may reasonably assume that each function $q^i \mapsto \pi^i(q^i)$ be increasing, whence any Lagrange multiplier (or price) $\bar{p}$ must quite naturally be nonnegative.

carbon content in the atmosphere.

I also set aside emission taxes. These could be levied and included as endogenous parts of the payoff functions. Doing so can generate "double dividend" if taxes on unabated emissions are recycled as marginal cuts in taxes on other production factors, notably labor and capital [15]. Admittedly, the setting of appropriate tax rates is difficult since they induce technological change and affect progress that stems from learning-by-doing [6]. The present analysis abstracts from several, potentially significant considerations, such as capital accumulation, technological innovation, and "green mobility" [1]. Such considerations could, however, fit the approach outlined in Section 6.

4 Emissions over Time

Let now T be a time horizon, assumed finite for simplicity. Correspondingly, let $\mathbb{E} = \mathbb{E}_1 \times \cdots \times \mathbb{E}_T$ be a product of Euclidean spaces so that any emission permit $e \in \mathbb{E}$ has components $e_1, \ldots, e_T$, specified from the first period up to the last included. Sometimes one may posit that individual payoff is time separable, i.e. $\pi^i(e^i) = \sum_t \pi^i_t(e^i_t)$ for suitable single-period functions $\pi^i_t(\cdot)$. Then the results of Section 3 decompose across time. Specifically, let $\bar{p}_t$ be a Lagrange multiplier that applies to the aggregate emission constraint at time t. If each i receives the (present value) payoff $u^i_t := \langle \bar{p}_t, e^i_t \rangle + \pi^i_{t*}(\bar{p}_t)$ for his time t contribution, then that allocation belongs to the core of the game prevailing at that moment. Moreover, the numbers $u^i := \sum_{t=1}^T u^i_t, i \in I$, constitute an overall core allocation.

Admittedly, for large T, in letting the model focus on long-lived agents, concerns about intergenerational equity are likely to become more pronounced. Such concerns are not addressed here [21]. Also, if the far-distant, uncertain future is discounted, there are good reasons for using a most moderate interest rate [24].

5 Contingent Emissions

Suppose uncertainty is modelled by means of a finite probability space (Ω, P), commonly agreed upon by everybody. In our context this means that state $\omega \in \Omega$ happens with positive probability $P(\omega)$. We posit that each agent i holds a state contingent emission permit $\omega \mapsto e^i(\omega)$. Suppose also that his payoff function is separable across events, i.e. his preferences are of the von Neumann-Morgenstern expected payoff variety:

$$\pi^i(e^i) = \sum \pi^i(\omega, e^i(\omega))P(\omega).$$

Then again there will be decomposition. Indeed, let $\bar{p}(\omega)$ be the Lagrange multiplier that applies in state ω. If and when that state is realized, agent i receives a payoff $u^i(\omega) := \langle \bar{p}(\omega), e^i(\omega) \rangle + \pi^i_*(\omega, \bar{p}(\omega))$ which forms his part of a contingent core solution. The expected overall gain to him $u^i := \sum_\omega u^i(\omega)P(\omega)$ makes up what he gets in the overall core allocation.

6 Synthesis: Multi-stage Stochastic Programming

This section brings things together (and can be skipped). Consider planning over time $t = 1, \ldots, T < \infty$ - under imperfect knowledge about the state $\omega \in \Omega$ of the world. Although ω cannot be fully identified a priori, its probability distribution P is supposed commonly known, given exogenously, and defined on some *sigma-field* $\mathcal{F}_{T+1}$ over the finite set Ω.

Identification of ω improves over time. Specifically, there is an expanding family $\mathcal{F}_1 \subseteq \ldots \subseteq \mathcal{F}_T \subseteq \mathcal{F}_{T+1}$ of sigma-fields - or an unfolding scenario tree - which describes the information flow. At time t one may ascertain for any event in $\mathcal{F}_t$ - and such events only - whether it has happened or not. Since Ω is assumed finite, $\mathcal{F}_t$ will partition Ω into minimal events (atoms, information sets, decision nodes). The inclusion $\mathcal{F}_t \subseteq \mathcal{F}_{t+1}$, $t \leq T$, reflecting progressive acquisition of knowledge, says that the said partition becomes finer as time evolves; see [4].

Agent i seeks to maximize a monetary contribution $c^i(x^i) = c^i(x^i(\cdot))$ to himself over suitable trajectories $x^i = (x_1^i(\cdot), \ldots, x_T^i(\cdot))$ of random vectors $x_t^i(\omega) \in \mathbb{R}^{n_t^i}$. These vectors represent constrained choices made sequentially. At time t he implements the part x_t^i of his overall plan. That part is supposed to be a $\mathcal{F}_t$-measurable strategy (policy, behavioral rule) $x_t^i : \Omega \to \mathbb{R}^{n_t^i}$. Besides this insistence on measurability (nonanticipativity or adaptedness), there are other restrictions, one being that

$$x_t^i(\omega) \in X_t^i(\omega) \text{ almost surely for each } t. \tag{5}$$

Here $\omega \rightsquigarrow X_t^i(\omega) \subseteq \mathbb{R}^{n_t^i}$ is a nonempty closed $\mathcal{F}_t$-measurable random set. (For notational simplicity all inclusions, equalities, and inequalities that involve random objects are henceforth tacitly understood to hold almost surely and componentwise). Added to set-constraint (5) comes a family of explicit, functional constraints:

$$e_t^i(\omega) - A_t^i(\omega, x_1^i(\omega), \ldots, x_t^i(\omega)) \in \mathbb{R}_+^{m_t} \text{ for all } t, \tag{6}$$

this inclusion featuring an emission permit $e_t^i(\omega)$ and a vector-valued function A_t^i, both $\mathcal{F}_t$-measurable. Note that the basic decision spaces $\mathbb{R}^{n_t^i}$ can vary across agents (and time), but, most important, the emission permits e_t^i and functions $A_t^i, i \in I$, that come into effect at time t, all have the same image space $\mathbb{R}^{m_t}$.

Write $x^i \in X^i$ and $e^i - A^i(x^i) \geq 0$ for short to indicate satisfaction of (5) and (6), respectively. Agent i's planning under uncertainty can now be formalized succinctly as problem

$$\pi^i(e^i) := \sup\left\{c^i(x^i) \;\middle|\; x^i \in X^i \text{ and } e^i - A^i(x^i) \geq 0\right\},$$

much like (1). In this setting coalition $S \subseteq I$ could achieve stand-alone payoff

$$\begin{aligned}
\pi^S(e^S) &= \sup\left\{\sum_{i \in S} c^i(x^i) \;\middle|\; x^i \in X^i, \forall i \in S, \text{ and } \sum_{i \in S}\left[e^i - A^i(x^i)\right] \geq 0\right\} \\
&= \sup\left\{\sum_{i \in S} \pi^i(q^i) \;\middle|\; \sum_{i \in S} q^i = e^S\right\}.
\end{aligned}$$

Whether that optimal value is computed or not, once again it is tacitly assumed, somewhat heroically, that no agent i misrepresents privately held information to own advantage. On the emission space

$$\mathbb{E} := \{e = [e_t(\omega)] \ \mid e_t(\omega) \in \mathbb{R}^{m_t}, t = 1, \ldots, T, \omega \in \Omega\}$$

it is now natural to use the statistically motivated inner product $\langle p, q \rangle := \sum_t \sum_\omega p_t(\omega) \cdot q_t(\omega) P(\omega)$. For simplicity suppose that payoff is separable across time and events, i.e. $c^i(x^i) = \sum_\omega \sum_t c_t^i(\omega, x^i(\omega)) P(\omega)$. Let $\left[x_1^i, \ldots, x_{t-1}^i\right] =: x_{[1,t-1]}^i$ denote decisions which i has already made before time t. Given a $\mathcal{F}_t$-measurable Lagrangian price $\bar{p}_t(\omega)$, write

$$\begin{aligned}&u_t^i(\omega \left| \mathcal{F}_{t-1}, x_{[1,t-1]}^i \right.) := \\ &E\left[\sup\left\{\bar{p}_t(\omega) \cdot (e_t^i(\omega) - A_t^i(\omega, x_1^i, \ldots, x_t^i)) + c_t^i(\omega, x_t^i) \mid x_t^i \in X_t^i(\omega)\right\} \Big| \mathcal{F}_{t-1}, x_{[1,t-1]}^i\right]\end{aligned}$$

for the conditional expected (present) value of core payoff to agent i at time t.

Theorem 2 (Core solutions in multi-stage, stochastic emission games). *Suppose* $(t, \omega) \mapsto \bar{p}_t(\omega) \in \mathbb{R}_+^{m_t}$ *are Lagrange multipliers for the grand coalition. Then* $\bar{p}_t$ *may be taken* $\mathcal{F}_t$*-measurable, and by giving* $u^i := \langle \bar{p}, e^i \rangle + \pi_*^i(\bar{p})$ *to* i *we get a core allocation. That allocation is re-negotiation proof in the following sense: If at some interim time* $\tau < T$*, the "agreed upon" decisions* $x_{[1,\tau]}^i$ *are already made, then the contingent payoff*

$$\sum_{t > \tau} E\left[u_t^i(\omega \left| \mathcal{F}_{t-1}, x_{[1,t-1]}^i \right.) \mid \mathcal{F}_\tau, x_{[1,\tau]})\right]$$

belongs to the core of the cooperative games which ensues from there onwards. In particular, when $A_t^i(\omega, x_1^i, \ldots, x_t^i) = A_t^i(\omega, x_t^i)$,

$$u^i = \sum_t \sum_\omega P(\omega) \sup\left\{\bar{p}_t(\omega) \cdot \left[e_t^i(\omega) - A_t^i(\omega, x_t^i)\right] + c_t^i(\omega, x_t^i) \ \mid x_t^i \in X_t^i(\omega)\right\}. \quad \square$$

[4] gives conditions ensuring existence of a Lagrange multiplier. For diverse sorts of relevant uncertainty, and their relative impact, see [19]. The opening up of this vista on stochastic programming invites use of computational approaches that use decomposition akin to what is effectuated by markets; see e.g. [18], [20]

7 Iterative Trading

The game introduced above reduces, in essence, to a widespread market in date-event goods, or so-called contingent commodities. More precisely, it fits within the frames of competitive equilibrium. That branch of economics, while presuming price-taking behavior, has failed to account for price formation, transactions out of equilibrium, and the role of money as medium of exchange.

The simplicity of our setting invites reconsideration of these issues. Since the emission market features constant supply - and since income effects are negligible or ignored - there should be good prospects for reaching a stable equilibrium over time. Indeed, Ermoliev et al. [2], [3] explore the convergence of repeated, bilateral exchange towards an efficient steady state. Their procedure, over stages $k = 0, 1, ...$, could be adapted to the present context broadly as follows:

• At the current stage k, pick randomly (or in cyclical manner) two agents i, j, these then having emissions q^i and q^j, respectively.

• Consider the difference $d := \nabla\pi^i(q^i) - \nabla\pi^j(q^j)$ between their marginal profit vectors (gradients) $\nabla\pi^i(q^i)$ and $\nabla\pi^j(q^j)$.

• If $d = 0$, or at least to a good approximation, then select anew two random agents.

• Otherwise, if necessary, scale d appropriately down so that the updated quotas

$$q^i \leftarrow q^i + d/(k+1) \quad \text{and} \quad q^j \leftarrow q^j - d/(k+1)$$

given respectively to i and j, both become non-negative.

• Increase k by 1 and continue to pick pairs of agents until convergence.

The stability analysis of such procedures is left for subsequent studies. Suffice it to emphasize here the following features: Trade is voluntary and driven by perceived prospects for mutual improvements. It happens out of equilibrium and uses money as medium of exchange. It requires no revealing of private information. While still away from equilibrium, the price - and the associated monetary compensation - that goes along with a bilateral quota could result from bargaining and would not easily be predicted. It depends on the divergence - as encapsulated in d - between willingness to accept and willingness to pay [10].

References

[1] J. H. Ausubel, C. Marchetti, and P. S. Meyer, Toward green mobility: the evolution of transport, *European Review* 6,2, 137-156 81998).

[2] Y. Ermoliev, G. Klaassen, and A. Nentjes, Adaptive cost-effective ambient charges under incomplete information, *J. Environmental Economics and Management* 31, 37-48 (1996).

[3] Y. Ermoliev, M. Michalevich, and A. Nentjes, Markets for tradeable emission and ambient permits: A dynamic approach, *Environmental and Resource Economics* 15, 29-56 (2000).

[4] I. V. Evstigneev and S. D. Flåm, Stochastic programming: nonanticipativity and Lagrange multipliers, to appear in *Encyclopedia of Optimization*, Kluwer (2000).

[5] H. Gersbach and A. Glazer, Markets and regulatory hold-up problems, *J. Environmental Economics and Management.*

[6] L. H. Goulder and K. Mathai, Optimal CO_2 abatement in the presence of induced technological change, *J. Environmental Economics and Management* 39, 1-38 (2000).

[7] C. Hagem and H. Westskog, The design of a dynamic tradeable quota system under market imperfections, *J. Environmental Economics and Management* 36, 89-107 (1998).

[8] M. Hoel, Global environmental problems: the effect of unilateral actions taken by one country, *J. Environmental Economics and Management* 20, 55-70 (1991).

[9] J. Jensen and T. R. Rasmussen, Allocation of CO_2 emission permits: A general equilibrium analysis of policy instruments, *J. Environmental Economics and Management* 40, 111-136 (2000).

[10] C. D. Kolstad and R. M. Guzman, Information and the divergence between willingness to accept and willingness to pay, *J. Environmental Economics and Management* 38, 66-80 (1999).

[11] G. Klaassen, *Acid Rain and Environmental Degradation*, E. Elgar, Cheltenham (1996).

[12] G. Klaassen and F. R. Førsund, *Economic Instruments for Air Pollution Control*, Kluwer, Dordrecht (1994).

[13] A. Kryazhimskii, A. Nentjes, S. Shibayev, A. Tarasyev, Searching market equilibria under uncertain utilities, Interim report, IIASA 7 (1998).

[14] K.-G. Mäler, The acid rain game, in H. Folmer and E. van Ireland (eds.) *Valuation Methods and Policy Making in Environmental Economics*, Elsevier, Amsterdam (1989).

[15] I. W. H. Parry, R. C. Williams III, and L. H. Goulder, When can carbon abatement policies increase welfare? The fundamental role of distorted factor market prices, *J. Environmental Economics and Management* 37, 52-84 (1999).

[16] G. Owen, On the Core of Linear Production Games, *Mathematical Programming*, 9, 358-370 (1975).

[17] R. T. Rockafellar, *Convex Analysis*, Princeton University Press (1970).

[18] R. T. Rockafellar, Duality and optimality in multistage stochastic programming, *Annals of Operations Research* 85, 1-19 (1999).

[19] R. A. Roerhl and K. Riah, Technology dynamics and greenhouse gas emission mitigation: A cost assessment, *Technological Forecasting and Social Science* 63, 231-261 (2000).

[20] A. Ruszczynski, Decomposition methods in stochastic programming, *Mathematical Programming* 70, 333-353 (1997).

[21] T. F. Rutherford, C. Böhringer, and A. Pahlke, Carbon abatement, revenue recycling and intergenerational burden sharing, in P.J.J. Herings, G. van der Laan, and A.J.J. Talman (eds.) *The Theory of Markets*, North-Holland, Amsterdam (1999).

[22] M. Sandsmark, Production games under uncertainty, *Computational Economics* 14, 237-253 (1999).

[23] L. S. Shapley and M. Shubik, On market games, *Journal of Economic Theory* 1, 9-25 (1969).

[24] M. L. Weitzman, Discounting the far-distant future, *J. Environmental Economics and Management* 36, 201-208 (1998).