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The IIASA Social Security Reform Project Multiregional Economic-Demographic Growth Model: Policy Background and Algebraic Structure

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Contents

Abstract

The model presented here is a neoclassical two-factor multiregional economic-demographic growth model. It is designed to assess the impacts of different

- demographic futures,
- labor-market scenarios,
- combinations of accumulation-based and transfer-based pension systems, and
- international portfolio allocation decisions

on a range variables which play an important role in the population aging and social security reform debates. Among these are

- the overall rate of economic growth,
- the relative incomes of retirees and workers,
- financial flows into and out of the public and private pension systems and their implications for capital formation, and
- international capital flows.

The model tracks income and outlay of households by single-year age groups, as well as intergenerational transfers of resources via bequests. Households accumulate assets during working years and then dissave in retirement, in addition to which, intergenerational transfers between the working and retired populations are mediated through the PAYG public pension system. Capital may be installed either at home or abroad.

In this paper, the policy background is briefly summarized and the algebraic structure of the model is elaborated. Other *Interim Reports* in this series describe the simulation and robustness characteristics of the model and present the results of model applications.

Acknowledgments

A number of collaborators have contributed to the evolving structure of the IIASA model. Among these are Helmut Reisen (OECD Development Centre), David Horlacher (Middlebury College), Roger Bird (American College), Gerry Adams (Northeastern University).

About the Authors

Landis MacKellar is the Leader of the Social Security Reform (SSR) Project and Tatiana Ermolieva is Research Assistant of the Project.

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1. Introduction

With population aging now rapidly underway, the future of social security regimes -- the broad spectrum of institutions, public and private, which provide for income in old age -- is being questioned (OECD 1998, World Bank 1994a). At their meeting in Denver in 1997, heads of state of the G7 countries called for international research into the global impacts of population aging. The IIASA Social Security Reform (SSR) project is a response to this call.

Long-term projections and simulation analyses of pension system revenues and expenditures, stocks of private pension savings, etc., play a large role in the policy debate. Somewhat surprisingly, the models used to address these issues in detail have been mostly actuarial, that is, contain detailed demographic projections but very little representation of the economy. Crucial variables such as gross domestic product (GDP) and the rate of return to capital represent exogenous assumptions in such models. This does not seem reasonable in view of the important links between the pension system and capital formation (Schieber and Shoven, 1994). On the other hand, when traditional macroeconomic models have been employed to analyze impacts of population aging or social security system reform, the level of demographic detail has been limited. This loss of resolution makes it difficult to study equity questions. Overlapping generations (OLG) computable general equilibrium (CGE) models integrate detailed demographic projections and multisectoral economic models. However, population aging is a global phenomenon and, while open-economy OLG CGEs have been constructed, size considerations have discouraged researchers from elaborating multiregional models which go beyond a highly simplified Rest of the World approach.

In this paper, we set forth the algebraic structure of a model which represents a compromise between the two extremes of a purely actuarial approach and a multiregional OLG CGE. The IIASA model has been developed to make a comprehensive global assessment of the macroeconomic impacts of population aging under different assumptions regarding the nature of the social security regime.

Purpose and organization of this paper

The goal of this paper is to set forth the algebraic structure of the model. A subsequent *Interim Report* will detail the model's unadjusted projection properties and non-stochastic simulation properties. Westlund *et al*. (1999) analyze the robustness of model solutions with respect to initial parameter assumptions and consider the cases of parameter time invariance, i.e. robustness of the model towards changing parameters. An earlier model version which lacked full demographic dynamics is described by MacKellar and Reisen (1998a and 1998b), who also describe an extended application of the model to analyzing the potential of global capital market integration to address the projected costs of population aging. Finally, MacKellar and Ermolieva (1999) employ the model described below to analyze the impacts of global capital market integration on the intergenerational distribution of income and wealth.

The next section of the paper contains some background information on the social security policy debate. The following section describes the IIASA model in general terms. The remainder of the paper elaborates the algebraic structure of the model.

2. The policy background[1]

Public pensions and health care comprise the bulk of spending in all social security systems; the residual is comprised mostly of unemployment benefits, disability, and family allowances. We concentrate in this paper on pension spending, keeping in mind that health care systems are just as important. While the issues involved in health care are conceptually similar, this area deserves a paper of its own.

As an individual ages, it is practically inevitable that he or she eventually becomes incapable of remunerated work. Pension regimes -- the broad spectrum of public and private institutions which provide income for the non-working elderly population -- exist to assist (or force) persons to deal with this contingency. Why should the state be involved? From an economic point of view, two main reasons underlie the need for public involvement in pensions (Barr, 1992). The first is incomplete information, in the form of the myopia that causes many to save too little for retirement needs and to underinsure against low-probability, high-impact events such as permanent disability. The second is adverse selection. For example, demand for annuities will be high among those persons who have reason to believe they will be long-lived but low among those who believe that they will die young. Financial institutions respond to this adverse selection by rationing supply.

The serious consequence of these market failures is that some individuals, through no fault of their own save perhaps lack of foresight, will become destitute in old age. This problem can be addressed by compulsory poverty insurance, coupled with the condition that in order to collect, the individual must be unable to work. Yet, few governments have been willing to limit social security policy to the strict goal of protecting against destitution. The inescapable consequence would be a large segment of the elderly population living on the edge of poverty. But generous government pension programs have been found to give rise to serious problems of moral hazard,

[1] This section is taken from MacKellar and McGreevey (1999).

especially early retirement, low personal saving rates, and payroll tax evasion. These problems, as well as fiscal pressures and distortions caused by the social security tax on labor, give rise to misallocation of resources and impair macroeconomic performance. Balancing old-age poverty against economic inefficiency is, in a nutshell, the tradeoff in social security policy.

Pension schemes may be of several types. The crucial difference is between "defined contribution" and "defined benefit" arrangements (Diamond, 1996). Defined-contribution pension schemes amount to saving schemes, which may be voluntary or compulsory. Upon becoming eligible for a pension, the individual receives what he or she put contributed, plus accumulated capital returns, either as a lump sum or, more commonly, in the form of an annuity. The most celebrated example of a public defined-contribution pension scheme is the Chilean system put in place in the early 1980s. Under defined-benefit pension schemes, beneficiaries receive a pension set not with reference to how much money they contributed, but with reference to some other criterion, such as last salary. The most common way of financing defined-benefit schemes is "Pay As You Go" (PAYG), in which system requirements are calculated as a function of committed benefits and the required payroll tax on system contributors is calculated. Germany, where the payroll tax rate is calculated on a quarterly basis, is an example of a classic PAYG system. A defined-benefit pension system may, if desired, accumulate a capital reserve by collecting contributions in excess of benefits paid out, in which case it is said to be "partially funded." The capital returns on this reserve can then be used to pay pensions, in addition to which, the reserve itself can be liquidated to meet system obligations. The accumulation of the U.S. Social Security Trust Fund, designed to cover the pension requirements of the Baby Boom generation, is an example of this approach. In the limit, a pension system can be "fully funded," meaning that the present value of its capital reserve is equal to the present value of its liabilities, i.e. benefits which much be paid out in the future. A conventional defined-contribution system is by definition fully funded. So-called "notional" defined-contribution systems, in which pension entitlement is calculated based on a hypothetical contributions history but the system is financed on a PAYG basis, are also in use in countries such as Sweden.

Reams have been written on the pros and cons of various approaches and combinations thereof. Defined-contribution systems offer the advantage of transparency, which should discourage evasion; they may also be conducive to high national saving rates and are likely to stimulate financial-sector development (James, 1998). On the other hand, redistributive transfers to low-income elderly are easier under a defined-benefit system, in addition to which, administration and management costs are typically lower.

Somewhat obscured in this complex analytical debate is the fact that social security contains a substantial ideological component. Few aspects of a society reflect more closely the nature of the relationship between individual, market, and state, or are more closely tied to the nature of work and the family, than the pension regime. Under the Chilean system, the individual is personally responsible for pre-financing his/her retirement; at the other end of the spectrum, in most OECD countries, an implicit intergenerational contract binds current workers to make transfer payments to current retirees in the belief that their children will do the same. Each approach has advantages and disadvantages, and ideology has an important role to play in sorting these out. On the other hand, some pension regimes may contain contractual arrangements which are

doomed to fail because of moral hazard and other fundamental flaws. Experience shows us that pension regimes can and do become dysfunctional, leading to high economic costs and widespread old-age poverty.

The global distribution of social security systems

High-income countries, as identified in the current approach of the *World Bank Economic and Social Indicators*, are home to only 15 percent of the world's population (see Table 1), but 38 percent of all contributors to social security systems. We do not have data on the distribution of social security beneficiaries, but given the relatively youthful age structure of populations outside the high-income countries, the distribution of beneficiaries must be even more geographically skewed than is the distribution of contributors.

Since its origins in the compulsory sickness, disability and old-age insurance program instituted in Germany by Bismarck in the 1880s, social security has grown in the high-income countries to the point that it is the cornerstone of social welfare policy. Parrott's Fundamental Law of Social Security (Parrott, 1992) sums up, tongue in cheek, the experience of high-income countries: social security spending in any country must invariably rise over time. Hudson (1993) cites research which documents that, between 1930 and 1980 -- the "golden age" of social security -- the proportion of workers covered by old-age insurance in eighteen industrial countries grew from twenty percent to eighty percent. The average pension expressed as a percentage of the average wage (the average replacement ratio) rose from 14 percent to 55 percent. *Pari passu*, the proportion of aged persons in the active labor force fell and the concept of a universal entitlement to leisure after a certain age took root and flourished.[2]

India and the other low-income countries apart from China, with 37 percent of world's population, have only 10 percent of social security system participants and, although this is not shown in Table 1, a far smaller share still of social security recipients. They have barely begun to offer government-mandated social security programs, having done so in most instances only for government employees. China, with 21 percent of world population and 13 percent of social security system participants, is an exception among the low-income countries because of the influence of the Soviet model.

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[2] According to the *Oxford English Dictionary*, the first use of the word *retirement* in the modern sense of a transition from work to leisure is in Pepys' *Diary* (1667), but it refers to the willingness of princes to step down. This usage is closely tied to the original French sense of the word, withdrawal from society in order to engage in rest and contemplation. In the early 19th century, the concept of retirement was broadened to include the pensioning-off of public officials and soldiers and the withdrawal of private citizens from the work force on the strength of their savings.

Table 1

World Population and Social Security Coverage by Country Income Groups ca. 1990

	Population (millions)	% Distribution of Population	Contributors Labor Force %	% Distribution of Contributors
Low Income Economies				
China	1,162	21.4	23.7	13.4
India	884	16.3	10.5	4.5
Other Low-Income Economies	1,146	21.1	6.4-15	5.6*
Lower middle income Economies				
Transition Economies	597	11.0	80	23.4
Market Economies	341	6.3	20	3.3
Upper-Middle Income Economies	478	8.8	40-60	11.7*
High-Income Economies	828	15.2	94	38.0
World Total	**5,436**	**100.0**	**--**	**100.0**

Sources: Population: World Bank (1994b), pp. 162-63; contributors as a percentage of total labor force estimated by the authors on the basis of Table A.4 (pp. 356-7) in World Bank (1994a).

* Percentage distribution of contributors calculated at mean of contribution/labor force range multiplied by population.

The lower-middle income economies divide fairly clearly into two classes. Many, such as the Philippines, Peru, Morocco, Ecuador, Colombia, Tunisia, Thailand, Turkey, and Iran, have been market economies for a long time. The population of countries in this group as a whole was 341 million in 1990. Others, the "transition" economies, formed part of the USSR and socialist Eastern Europe, and their population is about 600 million. In the long-time market-oriented economies, only a fifth of the labor force contributes to social security schemes; but in the former command economies, an estimated 80 percent of labor force participants are potential social security claimants. With 11 percent of the world's population, these transition countries are home to nearly a quarter of the world's social security system participants. Likewise, the upper-middle income economies, among the largest of which are South Africa, Brazil, Malaysia, Venezuela, Mexico, Argentina, and Korea, have a higher-than-average incidence of participation in public social security programs.

3. The IIASA model: general description[3]

The IIASA model, based on work originally presented by Blanchet and Kessler (1992), is a neoclassical two-factor multiregional economic-demographic model with a particular focus on social security. It incorporates population projections, saving, labor force participation, and tax rates. The wage rate and rate of return to capital are endogenously calculated as marginal products of labor and capital. The emphasis of the model is on tracking income and outlay of households by single-year age groups, as well as intergenerational transfers of resources via bequests. Households accumulate assets during working years and then dissave in retirement, in addition to which, intergeneration transfers between the working and retired populations are mediated through the PAYG public pension system. While the model is suited to a wide range of applications dealing with long-run economic growth, it is especially designed to simulate the effects of differing demographic futures and different mixes between accumulation-based and transfer-based pension systems.

The IIASA model is essentially an accounting model based on the UN System of National Accounts (SNA). Age-specific saving and labor force participation rates are exogenous. For a given population size, age structure has three effects on per capita income: first, through the labor force as it affects the number of workers relative to nonworkers; second, through capital formation, as it affects the number of savers relative to dissavers; third, and also through capital formation, as it affects the wage rate and rate of return to capital, which in turn determine the income streams out of which savings are drawn. In concentrating on relatively detailed age-structure effects, our work complements other analyses (e.g., Cutler *et al.*, 1990; Börsch-Supan, 1996), where the impact of population aging is mediated through the life-cycle hypothesis of household consumption. Closely related to these are linked international macroeconomic model-based analyses (e.g., Masson and Tryon, 1990; OECD, 1998), in which the impact of aging is mediated through the major macroeconomic functions, particularly the aggregate consumption/saving function. Given theoretical ambiguities, a simple accounting model with ample demographic detail provides a useful benchmark for work with more economically sophisticated, but demographically sparse, models.

[3] This section is based on MacKellar and Reisen (1998a).

Savings are allocated to investment projects at home and abroad by means of exogenous capital-flow coefficients, and investment in each region is equal to domestic plus foreign savings. A rise in foreign savings is assumed to be mirrored by a corresponding rise in domestic capital formation: the possibility that additional foreign savings might merely inflate asset prices or fuel consumption is not allowed for and the current account is assumed to adjust passively to changes in capital inflows.[4] The exchange rate plays no explicit role, and all economic variables are expressed in 1995 US dollars.

The model tracks receipts and disbursements, and thus net savings, by institutional sector (persons by single-year age group, firms, government). Following the convention of the OECD national income accounts, net savings in each sector of the economy are defined as gross receipts minus depreciation minus current expenditure. The sum of net savings across sectors is equal to net saving for the economy as a whole (national disposable income minus private consumption minus government consumption), which is in turn equal to net capital formation, i.e. change in the capital stock. Savings of firms and government are imputed to households based on the population age distribution. Capital consists of residential capital (*KRes*), capital operated by private unincorporated enterprises (*KPvtUnincorpEnt*), and capital operated by firms (i.e., corporate enterprises). Residential capital and capital operated by private unincorporated enterprises are installed exclusively in the home region; capital operated by firms is installed both at home and abroad. Claims on capital operated by firms are held on behalf of households by two financial intermediaries: the private pension system (*PvtPenSys*) and other financial institutions (*OthFinIns*). All claims consist of equity. Imputed rents (in the case of residential capital) and the profits of capital operated by private unincorporated enterprises accrue directly to households. Firms earn profits, pay taxes and distribute dividends to holders of claims. Direct taxation follows the principle of taxation at the source, meaning that capital returns are taxed only once, when and where they are earned.[5]

The *PvtPenSys* represents fully-funded, defined-contribution pension plans; the model does not specify a private PAYG, defined-benefit component. The rationale for not including a private PAYG component is twofold. First, the role of private PAYG pension funds is shrinking rapidly, as few new workers are being offered such arrangements. Second, the obligations of this component of the pension system are essentially underwritten by public authorities (e.g., the Pension Benefits Guarantee Corporation in the US), as a result of which, the distinction between the private and public PAYG systems is blurred. Implicitly, the private PAYG pension system is subsumed under the public PAYG pension system in our model.

OthFinIns are a residual sector in our model, covering banks, insurance companies, mutual funds, and other financial intermediaries apart from pension funds. Implicitly, *OthFinIns* also include individual households, to the extent that the latter hold financial claims directly.

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[4] However, to the extent that foreign capital inflows depress the rate of return to capital and thus the rate of profit on existing capital, the model incorporates a second-round offset in the form of lower domestic savings. This is in line with empirical evidence which suggests that only about one-half of a given increment to foreign savings translates into added investment.

[5] Thus, households pay no taxes on dividends received, taxes have already been paid by firms when profits were earned. Elderly persons are also assumed to pay no capital gains tax when they divest themselves of accumulated assets.

The distinction between portfolio investment and foreign direct investment (FDI) is a significant one.[6] Investors who purchase shares of a domestically based multinational firm are effectively acquiring an international asset to the extent that the firm operates globally. FDI, consisting mainly of the acquisition of fully-owned foreign subsidiaries by multinational firms, is one of the principal corporate globalization strategies. Thus, in the IIASA model, we recognize that firms in both regions earn profits both at home and abroad. In the two-region case, domestic firms are credited with profits earned on that portion of the domestic region's capital stock that is owned by foreign portfolio investors, and are debited with taxes and dividends paid out of these profits (to the government of the domestic region in the first case, to the *PvtPenSys* and *OthFinIns* of the foreign region, in the second case). However, profits on that portion of the domestic region's capital stock that represent FDI from abroad are credited to foreign firms. Taxes paid out of these profits are debited to firms in the foreign region and credited to the government of the domestic region. Firms in the foreign region reinvest a given share of these profits in the domestic region; the remainder they repatriate to the foreign region, where dividends are paid out to claimants.

Who are these claimants? Historically, *PvtPenSys* portfolio managers have engaged almost exclusively in portfolio investment. Almost all FDI has originated in firms, largely in the form of the acquisition of fully owned foreign subsidiaries. Since firms in our model only operate, but do not own, capital, we make the simplifying assumption that FDI is undertaken by corporate holding companies who are implicitly subsumed under *OthFinIns*, and the share of *OthFinIns* foreign assets consisting of FDI is an exogenous variable. Dividends paid out of repatriated profits on FDI from abroad are credited to *OthFinIns* in the foreign region. Symmetrically, profits on FDI from the domestic region in the foreign region are credited to firms in the domestic region, and dividends paid out of repatriated earnings are credited to *OthFinIns* in the domestic region.

Perhaps the most important feature of the IIASA model is that it is able to track the downward pressure on household saving and capital accumulation that is expected as the baby boomers begin to retire (Schieber and Shoven, 1994). During working life, households accumulate savings through contributions to the *PvtPenSys*; after retirement, they receive pension benefits which represent the drawing-down of this capital. Savings not captured by the pension system are distributed between the three remaining asset classes (*KRes*, *KPvtUnincorpEnt* and *KOthFinIns*) by means of share coefficients which sum to unity. These assets, too, are drawn down after retirement. Any assets remaining upon death are distributed to the surviving population as bequests. Persons receiving bequests in the form of inheritance are assumed to convert the inherited assets to cash, some of which is allocated to consumption, the remainder being allocated among the three non-pension forms of wealth.

The public pension system is assumed to be a balanced PAYG system. Upon retirement, a public pension entitlement is calculated on the basis of past years of labor force participation and average wage earnings. During retirement, this entitlement is indexed to growth in average real wages using an assumed indexation factor. Thus, social security benefits for members of a given single-year age cohort are a weighted average over number of retirees, number of years of labor force participation (and average earnings) prior to retirement, and number of years elapsed since retirement.

[6] FDI is defined as the acquisition of 20% or more of the outstanding equity in a foreign corporation. Acquisition of less than 20% of the outstanding equity of a foreign firm is defined as portfolio investment.

The social security contribution rate required to meet total pension entitlements is then calculated and levied against wage income and income from private unincorporated enterprises. Pressures on the social security system are thus reflected in rising payroll tax rates. The other two possibilities, i.e. declining levels of benefit per member of the eligible population or higher government budget deficits, can easily be incorporated by means of minor modifications of model structure.

4. The IIASA model: algebraic structure

4.1 Notation

The model contains three dimensions: age, time, and location (domestic and foreign regions). Indices corresponding to the first two dimensions (*age* and *t*) are given in parentheses and indices corresponding to the spatial dimension (*dom* and *for*) are subscripted. The presentation below is made from the standpoint of the domestic region assuming a single foreign region; extension to the case of multiple foreign regions involves only obvious summations.

4.2 Population, labor force, and employment

Population

Population is divided into age groups $age = \overline{0, MaxAge_{dom}(t)}$. There are four demographic model solution options. In the simplest of these, a single deterministic demographic scenario consisting of population by age group is loaded from another source. In the second, stochastic population scenarios produced outside the model are loaded. In the third, a deterministic population scenario is produced within the model using standard cohort-component projection methodology. In this case, for $age > 0$

$$Pop_{dom}(sex, age, t) = Pop_{dom}(sex, age-1, t-1)\left[1 - MortRate(sex, age-1, t-1)\right] + NetMigr(sex, age, t)$$

where *MortRate* is the age-specific mortality rate and *NetMigr* is the number of net migrants. The mortality rate is calculated on the basis of life expectancy at birth, an exogenous assumption, using a model life table. Similarly, the number of net migrants by age is calculated by sharing down total net migration, also an exogenous assumption, using a Castro-Rogers model migration schedule. For $age = 0$,

$$Pop_{dom}(0,t) = \sum_{age=15}^{49} Pop_{dom}(female, age, t)\, FertRate_{dom}(age, t)$$

$$Pop_{dom}(male, 0, t) = SexRatio\; Pop_{dom}(0, t)$$

$$Pop_{dom}(female, 0, t) = (1 - SexRatio)\, Pop_{dom}(0, t)$$

where the age-specific fertility rate *FertRate* is calculated on the basis of an assumed total fertility rate. The last solution option is to define mortality and fertility rates (and,

if desired, number of net migrants as well) as random variables and produce a stochastic population projection within the model.

Total population is the sum over age groups

$$Pop_{dom}(t) = \sum_{age=0}^{MaxAge_{dom}(t)} Pop_{dom}(male, age, t) + \sum_{age=0}^{MaxAge_{dom}(t)} Pop_{dom}(female, age, t)$$

Labor force and employment

Total labor force is the sum over age groups

$$LabForce_{dom}(t) = \sum_{age=15}^{MaxAge_{dom}(t)} LabForce_{dom}(age, t)$$

where

$$LabForce_{dom}(t) = Pop_{dom}(age, t)\, LabForcePartRate_{dom}(age, t)$$

Age-specific labor force participation rates are exogenous assumptions, as are unemployment rates:

$$Emp_{dom}(t) = \sum_{age=15}^{MaxAge_{dom}(reg,t)} Emp_{dom}(age, t)$$

$$Emp_{dom}(t) = LabForce_{dom}(age, t)\left[1 - UnempRate_{dom}(age, t)\right]$$

4.3 Capital, its location, and the nature of claims

Summarizing the description in Section 3, capital is either residential (*Res*) or non-residential (*NonRes*). The latter is further subdivided into capital operated by private unincorporated enterprises (*PvtUnincorpEnt*) and capital operated by firms, i.e., corporate enterprises. Firms operate capital, either distributing or reinvesting earnings which accrue; they do not own shares in other firms. Residential capital and capital operated by *PvtUnincorpEnt* are installed entirely in the home region and are held by households directly. Capital operated by corporate enterprises is installed either at home or abroad.

Financial claims on this capital are held on behalf of households by institutions which collect and distribute dividends. Note that no distinction is made between equity and debt claims on corporate capital. These institutions are subdivided into those which comprise the private pensions system (*PvtPenSys*) and other financial institutions

(*OthFinIns*) such as banks and mutual funds. Also implicitly assigned to *OthFinIns* are households themselves to the extent that they individually hold claims on corporate assets. *PvtPenSys* corresponds to the fully funded, defined contribution component of the private pension system; PAYG private corporate pension funds are implicitly subsumed under the public PAYG system.

Indexing the home region by *dom* and the foreign region by *for*, and aggregating over all claimant age groups, total capital installed and operated in the home region is

$$KTot_{dom,dom}(t) = K\,\mathrm{Re}\,s_{dom}(t) + KPvtUnincorpEnt_{dom}(t) + KPvtPenSys_{dom,dom}(t) + KOthFinIns_{dom,dom}(t)$$

$$KTot_{for,dom}(t) = KPvtPenSys_{for,dom}(t) + KOthFinIns_{for,dom}(t)$$

$$KTot_{*,dom}(t) = KTot_{dom,dom}(t) + KTot_{for,dom}(t)$$

Foreign investment can consist either of portfolio claims (claims representing less than 20 percent of the outstanding value of a firm's stock) or foreign direct investment (claims amounting to 20 percent or more of a firm's outstanding shares). Following historically observed trends, all claims held by *PvtPenSys* are assumed to be portfolio claims. Foreign direct investment (FDI) is assumed to be undertaken on behalf of domestic firms by holding companies which are implicitly included in *OthFinIns*:

$$KOthFinIns_{for,dom}(t) = KOthFinInsPort_{for,dom}(t) + KOthFinInsFDI_{for,dom}(t)$$

The reason for including FDI claims under *OthFinIns* is that we wish to preserve a distinction between firms, which operate capital, and financial institutions, which hold claims on capital. The split of foreign investment between portfolio and FDI claims is described in Section 4.9 below.

Total assets of domestic financial institutions are

$$KPvtPenSys_{dom,*}(t) = KPvtPenSys_{dom,dom}(t) + KPvtPenSys_{dom,for}(t),$$

$$KOthFinIns_{dom,*}(t) = KOthFinIns_{dom,dom}(t) + KOthFinIns_{dom,for}(t)$$

where again,

$$KOthFinIns_{dom,for}(t) = KOthFinInsPort_{dom,for}(t) + KOthFinInsFDI_{dom,for}(t)$$

All capital ultimately belongs to households. Each single-year age-cohort is tracked as it accumulates capital during its working life and draws it down during retirement. In any year, then, *KRes*, *KPvtUnincorpEnt*, *KPvtPenSys*, and *KOthFinIns* are disaggregated by age of claimant. The simplifying assumption is made that the spatial distribution of capital is constant over age groups; similarly, in the case of overseas *KOthFinIns*, the distribution between portfolio and FDI claims is constant over age groups.

4.4 Output and rates of return to factors

Gross domestic product (*GDP*) is given by a Cobb-Douglas production function

$$GDP_{dom}(t) = \alpha_{dom}(t)\left[1 + g_{dom}(t)\right]^t KTot_{*,dom}(t)^{\beta_{dom}(t)} Emp_{dom}(t)^{1-\beta_{dom}(t)}$$

where g, the rate of total factor productivity growth, is exogenous. Rates of return to factors are neoclassical:

$$R_{dom}(t) = \beta_{dom}(t)\left[\frac{GDP_{dom}(t)}{K_{*,dom}(t)}\right]$$

$$\overline{WageRate}_{dom}(t) = \left[1 - \beta_{dom}(t)\right]\left[\frac{GDP_{dom}(t)}{EMP_{dom}(t)}\right]$$

where R is the gross profit rate, including depreciation and indirect taxes net of subsidies; and $\overline{WageRate}$ is average (over age groups) employee compensation, including social insurance contributions (contributions to public and private pension schemes).

In order to net depreciation and indirect taxes out of the rate of return to capital, we define

$$r_{dom}(t) = R_{dom}(t) - \frac{IndTaxRate_{dom}(t)GDP_{dom}(t)}{KTot_{*,dom}(t)} - DeprRate_{dom}(t),$$

where *IndTaxRate* is defined with respect to *GDP* and *DeprRate* is the depreciation rate. The advantage of netting out depreciation and indirect taxes is that we can ignore them in calculating income, outlay, and net savings. However, we shall need to add them when calculating net factor payments from abroad and gross national product (*GNP*).

Age-specific wage rates

In a model with age-structure detail, the age-profile of wage rates will be a significant variable. We require, therefore, a procedure to create an age-specific wage-rate profile consistent with the average wage rate calculated above as the marginal product of labor. Two approaches are possible, one based on age-specific human capital and the other based on years of experience in the labor force. We have implemented the second by defining a scale factor

$$\sigma_{dom}(age,t) = \sigma_{dom}\left[Exp(age,t)\right]$$

where $Exp_{dom}(age)$ is the number of years of work experience for persons aged *age* in year *t*:

$$Exp_{dom}(age,t) = \sum_{j=0}^{age-15} \frac{Emp_{dom}(age-j,t-j)}{Pop_{dom}(age-j,t-j)}$$

We then calculate age-specific wage rates as

$$WageRate_{dom}(age,t) = \sigma_{dom}(age,t)\,\overline{WageRate}_{dom}(t)$$

In practice, we have specified $\sigma(age,t)$ as logarithmic in age. Thus, wages rise rapidly in the twenties and thirties, the average wage over the life cycle is earned at approximately age 45, and there is little increase after 55. More complicated approaches to the wage profile could, of course, be easily implemented.

An adjustment is necessary to ensure that the average wage rate calculated from the economy-wide marginal productivity condition is equal to the average wage rate calculated by across age groups, each with its own number of years of experience in the workforce. In other words, we require:

$$\overline{WageRate}_{dom}(t) = \sum_{age=15}^{MaxAge}\left[\sigma_{dom}(age,t)\overline{WageRate}_{dom}(t)\frac{Emp_{dom}(age,t)}{\sum_{age=15}^{MaxAge} Emp_{dom}(age,t)}\right]$$

To do this, we define a second average wage rate over years of labor force experience:

$$\overline{\overline{WageRate}}_{dom}(t) = \sum_{Exp=0}^{MaxAge-15} \sum_{age=0}^{MaxAge} \left[WageRate_{dom}(age,t) \frac{\sum_{age=15}^{MaxAge} EMP_{dom}(age,Exp,t)}{\sum_{Exp=0}^{MaxAge-15} \sum_{age=15}^{MaxAge} EMP_{dom}(age,Exp,t)} \right],$$

Then we "squash" the experience scaling factor by $\overline{\overline{WageRate}}(t) / \overline{WageRate}(t)$, calculating a new factor

$$\sigma'_{dom}(age,t) = \sigma_{dom}(age,t) \frac{\overline{WageRate}(t)}{\overline{\overline{WageRate}}(t)}$$

If we take age-specific wage rates as

$$WageRate_{dom}(age,t) = \sigma'_{dom}(age,t) \overline{WageRate}_{dom}(age,t)$$

then age-specific wage rates averaged over age groups will equal experience-specific wage rates averaged over years of experience, both of which will, in turn, equal the economy-wide average wage rate calculated on the basis of the production function.

4.5 Income, consumption, and net saving of households

The articulation of income flows elaborated below has two main purposes. The first is to disaggregate income and consumption by age. The second is to break out the special role of the private pension system in saving and the allocation of capital.

Income

The sources of household income are wages, imputed rents from residential capital, profits which accrue to capital operated by unincorporated enterprises, dividends distributed from earnings on capital operated by firms, public social security system benefits, and private pension benefits.

A note on taxation.

The treatment of taxation in this model follows four simplifying principals. First, all taxation is assumed to occur at the level of factor incomes. There is no tax on transfer payments (such as social security benefits) on bequests, etc. Second, factor income is taxed once and only once, when and where it is earned. Thus, dividend income is not taxed because profits have already been taxed at the level of the firm; similarly, there is no capital gains tax when assets are sold because capital gains reflect profits which have already been taxed. Third, no distinction is made from a taxation

point of view between different types of capital: profits on capital operated by firms, capital operated by private unincorporated enterprises, and the imputed services of residential housing are all assumed to be taxed at the same rate. Finally, tax rates are not indexed by age or income.

Wage income

Disposable wage income is equal to gross wages minus direct taxes minus social insurance contributions to the public PAYG and private defined-contribution pension systems:

$$WageY_{dom}(age,t) = WageRate_{dom}(age,t)\ Emp_{dom}(age,t)$$

$$DispWageY_{dom}(age,t) = WageY_{dom}(age,t) - DirTaxWageY_{dom}(age,t) - PubPenSysContWageY_{dom}(age,t) - PvtPenSysContWageY_{dom}(age,t)$$

$$DirTaxWageY_{dom}(age,t) = DirTaxRate_{dom}(t)\,WageY_{dom}(age,t)$$

$$PubPenSysContWageY_{dom}(age,t) = PubPenSysContRate_{dom}(t)\,WageY_{dom}(age,t)$$

$$PvtPenSysContEntrY_{dom}(age,t) = PvtPenSysContRate_{dom}(age,t)\,WageY_{dom}(age,t),$$

The calculation of age-specific wage rates was discussed above in Section 4.4.

While the social security contribution rate is constant by age and equal for wages and entrepreneurial income across all age classes, the *PvtPenSys* contribution rate, which reflects in large part voluntary saving behavior (such as IRA and 401K plans in the US) is indexed by age. Note that, even though *PvtPenSys* contributions really represent the acquisition of a financial asset, rather than a current expenditure flow, the SNA nonetheless counts such transactions as a debit to disposable income. After the calculation of disposable income, however, an adjustment is made to ensure that the savings associated with such flows are credited to households.

Imputed rental income is assumed to be taxed like any other form of income; however, social contributions are assumed to be zero:

$$\mathrm{Re}ntalY_{dom}(age,t) = r_{dom}(t)\,K\,\mathrm{Re}s_{dom}(age,t)$$

$$Disp\,\mathrm{Re}ntalY_{dom}(age,t) = \mathrm{Re}ntalY_{dom}(age,t) - DirTax\,\mathrm{Re}ntalY_{dom}(age,t)$$

$$DirTax\,\mathrm{Re}ntalY_{dom}(age,t) = DirTaxRate_{dom}(t)\,\mathrm{Re}ntalY_{dom}(age,t)$$

Note that here, and in the sections which follow, capital returns are net of depreciation and indirect taxes.

Entrepreneurial income

Profits from capital operated by unincorporated enterprises are treated the same as wages:

$$EntrY_{dom}(age,t) = r_{dom}(t)\, KPvtUnincorpEnt_{dom}(age,t),$$

$$DispEntrY_{dom}(age,t) = EntrY_{dom}(age,t) - DirTaxEntrY_{dom}(age,t) - PubPenSysContEntrY_{dom}(age,t) - PvtPenSysContEntrY_{dom}(age,t),$$

$$DirTaxEntrY_{dom}(age,t) = DirTaxRate_{dom}(t)\, EntrY_{dom}(age,t),$$

$$PubPenSysContEntrY_{dom}(age,t) = PubPenSysContRate_{dom}(t)\, EntrY_{dom}(age,t),$$

$$PvtPenSysContEntrY_{dom}(age,t) = PvtPenSysContRate_{dom}(age,t)\, EntrY_{dom}(age,t),$$

Dividend income

The assets held on households' behalf by *PvtPenSys* and *OthFinIns* earn dividends. However, in the first case, dividends are not considered to be part of household income; rather, they are considered to be the acquisition of a financial asset. The subsequent adjustment to household income alluded to above incorporates these dividend earnings captured by the private pension system.

In the case of assets held by *OthFinIns*, special provision must be made for dividends distributed from earnings on FDI. As described in Section 4.6, an assumption is made regarding the share of after-tax earnings on FDI which is reinvested in the foreign region. Earnings not reinvested in the host region are repatriated to the parent firm, which retains some and distributes the remainder to *OthFinIns*. These earnings are, in turn, credited to households:

$$DividY_{dom}(age,t) = DivDistErngsFirmsKOthFinIns_{dom,dom}(age,t) + DivDistErngsFirmsKOthFinInsPort_{dom,for}(age,t) + DivDist\,\mathrm{Re}\,patrErngsFirmsKOthFinInsFDI_{dom,for}(age,t)$$

Social security benefits

Persons above the age of eligibility for social security benefits (*EligAge*) are entitled to public pension system benefits calculated on the basis of

- their years of employment,
- the number of years they have been retired,
- the degree of indexation of pension benefits to real wages, and

- the evolution of wages since their retirement.

The formula by which the pension entitlement for the average member of a cohort is calculated upon retirement and indexed thereafter is described below in Section 4.7. Total social security income for an age–group cohort, denoted *PubPenSysBen(age,t)*, is equal to the number of beneficiaries multiplied by average entitlement per beneficiary.

Private pension system benefits

Private pension system benefits represent the sale of financial assets, not current transactions; nonetheless, they are regarded as a current flow for purposes of calculating household disposable income. At *age* > *RetAge*, households are assumed to receive benefits at a rate calculated to deplete their private pension system assets at an assumed age *AssetDeplAge* ≤ *MaxAge* :

$$PvtPenSysBen_{dom}(age,t) = \frac{KPvtPenSys_{dom,*}(age,t)}{AssetDeplAge_{dom}(t) - age + 1}$$

There is no requirement that households consume these benefits in their entirety, so bequests are possible. The stock of remaining pension assets continues to earn dividends, and if pension claimants die before *AssetDeplAge*, remaining assets are bequeathed to younger persons.

Total income, disposable income, and adjusted disposable income

Total income. Total income of households is equal to the sum over all income sources:

$$TotYHH_{dom}(age,t) = WageY_{dom}(age,t) + \mathrm{Re}\,ntalY_{dom}(age,t) + EntrY_{dom}(age,t) + DividY_{dom}(age,t) + PubPenSysBen_{dom}(age,t) + PvtPenSysBen_{dom}(age,t)$$

Disposable income. Disposable income is analogous:

$$DispYHH_{dom}(age,t) = DispWageY_{dom}(age,t) + Disp\,\mathrm{Re}\,ntalY_{dom}(age,t) + DispEntrY_{dom}(age,t) + DividY_{dom}(age,t) + PubPenSysBen_{dom}(age,t) + PvtPenSysBen_{dom}(age,t)$$

Adjusted disposable income. Adjusted disposable income is equal to disposable income plus change in pension wealth. The latter consists of

- contributions to *PvtPenSys*, plus
- dividends earned on assets held by *PvtPenSys*, minus
- benefits received from *PvtPenSys*.

Thus,

$$AdjDispYHH_{dom}(age,t) = DispYHH_{dom}(age,t) + PvtPenSysCont_{dom}(age,t) + PvtPenSysDivid_{dom}(age,t) - PvtPenSysBen_{dom}(age,t)$$

Capital transfers

Resources available for household consumption take the form of disposable income and the proceeds of transferring claims to capital assets. In this section, the second of these is elaborated.

Dissaving in retirement

We have already described de-accumulation of pension assets in the form of benefits received. The treatment of the other three asset classes is identical:

$$AssetSalesK\,\mathrm{Re}s_{dom}(age,t) = \frac{K\,\mathrm{Re}s_{dom}(age,t)}{AssetDeplAge_{dom}(t) - age + 1} \quad , age \geq \mathrm{Re}tAge$$

$$AssetSalesKPvtUnincorpEnt_{dom}(age,t) = \frac{KPvtUnincorpEnt_{dom}(age,t)}{AssetDeplAge_{dom}(t) - age + 1} \quad , age \geq \mathrm{Re}tAge$$

$$AssetSalesKOthFinIns_{dom,*}(age,t) = \frac{KOthFinIns_{dom,*}(age,t)}{AssetDeplAge_{dom}(t) - age + 1} \quad , age \geq \mathrm{Re}tAge$$

As in the case of pension benefits, while we generally assume that the propensity to consume out of the proceeds of asset sales is unity, this assumption can be changed to incorporate a bequest motive. Note that it is assumed that no assets are sold prior to retirement, apart from the special case of assets received via inheritance, which we discuss in the next section.

Bequests / inheritance

In all four asset classes, age-specific bequests are equal to assets times the proportion of persons in the age group dying:

$$BeqK\,\mathrm{Re}s_{dom}(age,t) = K\,\mathrm{Re}s(age,t)\left[\frac{Deaths_{dom}(age,t)}{Pop_{dom}(age\text{-}1,t\text{-}1)}\right]$$

$$BeqKPvtUnincorpEnt_{dom}(age,t) = KPvtUnincorpEnt(age,t)\left[\frac{Deaths_{dom}(age,t)}{Pop_{dom}(age\text{-}1,t\text{-}1)}\right]$$

$$BeqKPvtPenSys_{dom,*}(age,t) = KPvtPenSys_{dom,*}(age,t)\left[\frac{Deaths_{dom}(age,t)}{Pop_{dom}(age\text{-}1,t\text{-}1)}\right]$$

$$BeqKOthFinIns_{dom,*}(age,t) = KOthFinIns_{dom,*}(age,t)\left[\frac{Deaths_{dom}(age,t)}{Pop_{dom}(age\text{-}1,t\text{-}1)}\right]$$

Bequests are received, in the form of inheritance, by the surviving population. For simplicity, we estimate age-specific inheritance simply by dividing total bequests by population age shares. We exclude the population under age 15 and the population in the terminal age group *MaxAge*. Total bequests in each asset class are the summation over age groups of persons dying:

$$BeqK\,\mathrm{Re}s_{dom}(t) = \sum_{age=15}^{MaxAge_{dom}(t)} BeqK\,\mathrm{Re}s_{dom}(age,t)$$

$$BeqKPvtUnincorpEnt_{dom}(t) = \sum_{age=15}^{MaxAge_{dom}(t)} BeqKPvtUnincorpEnt_{dom}(age,t)$$

$$BeqKPvtPenSys_{dom,*}(t) = \sum_{age=15}^{MaxAge_{dom}(t)} BeqKPvtPenSys_{dom,*}(age,t)$$

$$BeqKOthFinIns_{dom,*}(t) = \sum_{age=15}^{MaxAge_{dom}(t)} BeqKOthFinIns_{dom,*}(age,t)$$

and inheritance (for age groups over 15 and less than *MaxAge* only) is

$$InhK\,\mathrm{Re}s_{dom}(age,t) = \left[\frac{Pop_{dom}(age,t)}{\sum_{age=15}^{MaxAge_{dom}(t)-1} Pop_{dom}(age,t)}\right] \sum_{age=15}^{MaxAge_{dom}(t)} BeqK\,\mathrm{Re}s_{dom}(age,t)$$

$$InhKPvtUnincorpEnt_{dom}(age,t) = \left[\frac{Pop_{dom}(age,t)}{\sum_{age=15}^{MaxAge_{dom}(t)-1} Pop_{dom}(age,t)}\right] \sum_{age=15}^{MaxAge_{dom}(t)} BeqKPvtUnincorpEnt_{dom}(age,t)$$

$$InhKPvtPenSys_{dom,*}(age,t) = \left[\frac{Pop_{dom}(age,t)}{\sum_{age=15}^{MaxAge_{dom}(t)-1} Pop_{dom}(age,t)}\right] \sum_{age=15}^{MaxAge_{dom}(t)} BeqKPvtPenSys_{dom,*}(age,t)$$

$$InhKOthFinIns_{dom,*}(age,t) = \left[\frac{Pop_{dom}(age,t)}{\sum_{age=15}^{MaxAge_{dom}(t)-1} Pop_{dom}(age,t)} \right] \sum_{age=15}^{MaxAge_{dom}(t)} BeqKOthFinIns_{dom,*}(age,t)$$

Summing over age groups,

$$InhK\,\mathrm{Re}s_{dom}(t) = \sum_{age=15}^{MaxAge_{dom}(t)-1} InhK\,\mathrm{Re}s_{dom}(age,t)$$

$$InhKPvtUnincorpEnt_{dom}(t) = \sum_{age=15}^{MaxAge_{dom}(t)-1} InhKPvtUnincorpEnt_{dom}(age,t)$$

$$InhKPvtPenSys_{dom,*}(t) = \sum_{age=15}^{MaxAge_{dom}(t)-1} InhKPvtPenSys_{dom,*}(age,t)$$

$$InhKOthFinIns_{dom,*}(t) = \sum_{age=15}^{MaxAge_{dom}(t)-1} InhKOthFinIns_{dom,*}(age,t)$$

This simplification admittedly exaggerates the number of "backwards" bequests (elderly persons inheriting wealth from middle-aged persons, who are in fact more likely to bequeath assets to their children than to their parents).

The assumption is made that, when wealth is inherited, it is converted to cash, some of which is allocated to consumption and the remainder of which is allocated among $\Delta KInhFinIns$, $\Delta KRes$, and $\Delta KPvtUnincorpEnt$ using the same share coefficients applied to household net saving. Note, however, that the portion which is not consumed does not comprise new household savings; it represents rather the proceeds of transferring claims to capital formed as the result of past saving.

Implicit in this procedure is the assumption that remaining pension wealth is “paid out” to survivors and lost to the pension system upon death of the claimant. There is no provision made, in other words, for the transfer of pension rights to surviving spouses or children.

Under these assumptions, asset sales are

$$AssetSalesInhK\,\mathrm{Re}s_{dom}(age,t) = InhK\,\mathrm{Re}s_{dom}(age,t)$$

$$AssetSalesInhKPvtUnincorpEnt_{dom}(age,t) = InhKPvtUnincorpEnt_{dom}(age,t)$$

$$AssetSalesInhKPvtPenSys_{dom,*}(age,t) = InhKPvtPenSys_{dom,*}(age,t)$$

$$AssetSalesInhKOthFinIns_{dom,*}(age,t) = InhKOthFinIns_{dom,*}(age,t)$$

Consumption out of the proceeds of such sales is described below.

Consumption

Out of income

Average propensities to consume (*AvgPropCons*) out of disposable income streams are exogenous assumptions:

$$ConsWageY_{dom}(age,t) = DispWageY_{dom}(age,t)\ Avg\ \Pr opConsWageY_{dom}(age,t)$$

$$ConsEntrY_{dom}(age,t) = DispEntrY_{dom}(age,t)\ Avg\ \Pr opConsEntrY_{dom}(age,t)$$

$$ConsDividY_{dom}(age,t) = DividY_{dom}(age,t)\ Avg\ \Pr opConsDividY_{dom}(age,t)$$

$$ConsPubPenSysBen_{dom}(age,t) = PubPenSysBen_{dom}(age,t)\ Avg\ \Pr opConsSocSecY_{dom}(age,t)$$

$$ConsBenPvtPenSysY_{dom}(age,t) = BenPvtPenSys_{dom}(age,t)\ Avg\ \Pr opConsBenPvtPenSys_{dom}(age,t)$$

It is assumed that all imputed housing services are consumed:

$$Cons\ \mathrm{Re}ntalY_{dom}(age,t) = Disp\ \mathrm{Re}ntalY_{dom}(age,t)$$

Out of the proceeds of asset sales

It is assumed that consumption out of the proceeds of asset sales takes place in the year of the sale, i.e., households are not allowed to hold liquid balances. To assume otherwise would be to introduce a superfluous monetary dimension into the model.

Sales of inherited assets. Consumption is

$$ConsAssetSalesInhK\ \mathrm{Re}s_{dom}(age,t) = AssetSalesInhK\ \mathrm{Re}s_{dom}(age,t)\ ConsShareAssetSalesInhK\ \mathrm{Re}s_{dom}(age,t)$$

$$ConsAssetSalesInhKPvtUnincorpEnt_{dom}(age,t) = AssetSalesInhKPvtUnincorpEnt_{dom}(age,t)\ ConsShareAssetSalesInhKPvtUnincorpEnt_{dom}(age,t)$$

$$ConsAssetSalesInhKPvtPenSys_{dom}(age,t) = AssetSalesInhKPvtPenSys_{dom}(age,t)\ ConsShareAssetSalesInhKPvtPenSys_{dom}(age,t)$$

$$ConsAssetSalesInhKOthFinIns_{dom}(age,t) = AssetSalesInhKOthFinIns_{dom}(age,t)\ ConsShareAssetSalesInhKOthFinIns_{dom}(age,t)$$

and the sharing-out of what is not consumed between Δ *KOthFinIns*, Δ *KB*,

Δ *RtUnincorpEnt* is described in Section 4.8 below. We use a mnemonic corresponding to "consumption share" instead of *AvgPropCons* because average propensity to consume is properly considered with reference to income.

Retirement dissaving. Consumption during retirement financed by the sale of assets accumulated during working life is treated in the same way. Because private pension system benefits are classified as income, rather than capital transfers, this component has already been described above. The remaining components are:

$$ConsAssetSalesK\,\mathrm{Re}s_{dom}(age,t) =$$
$$AssetSalesK\,\mathrm{Re}s_{dom}(age,t)\,ConsShareAssetSalesK\,\mathrm{Re}s_{dom}(age,t)$$

$$ConsAssetSalesKPvtUnincorpEnt_{dom}(age,t) =$$
$$AssetSalesKPvtUnincorpEnt_{dom}(age,t)\,ConsShareAssetSalesKPvtUnincorpEnt_{dom}(age,t)$$

$$ConsAssetSalesKOthFinIns_{dom}(age,t) =$$
$$AssetSalesOthFinIns_{dom}(age,t)\,ConsShareAssetSalesKOthFinIns_{dom}(age,t)$$

If there is no bequest motive, the consumption shares (and the average propensity to consume out of *BenPvtPenSys*) are assumed to be unity. However, this assumption can be generalized to allow for bequests. In this case, the complement of the consumption share is simply the proportion of wealth which households wish to bequeath.

Net savings of households

Recapitulating, disposable and adjusted disposable household income are

$$DispYHH_{dom}(age,t) = DispWageY_{dom}(age,t) + Disp\,\mathrm{Re}ntalY_{dom}(age,t) + DispEntrY_{dom}(age,t)$$
$$+ DividY_{dom}(age,t) + PubPenSysBen_{dom}(age,t) + PvtPenSysBen_{dom}(age,t)$$

$$AdjDispYHH_{dom}(age,t) = DispYHH_{dom}(age,t)$$
$$+ PvtPenSysCont(age,t) + (age,t) + PvtPenSysDivid(age,t) - PvtPenSysBen_{dom}(age,t)$$

and total consumption is

$$Cons_{dom}(age,t) =$$
$$ConsDispWageY_{dom}(age,t) + ConsDisp\,\mathrm{Re}ntalY_{dom}(age,t) + ConsDispEntrY_{dom}(age,t)$$
$$+ ConsDividY_{dom}(age,t) + ConsPubPenSysBen_{dom}(age,t) + ConsPvtPenSysBen_{dom}(age,t)$$
$$+ ConsAssetSalesInhK\,\mathrm{Re}s_{dom}(age,t) + ConsAssetSalesInhKPvtUnincorpEnt_{dom}(age,t)$$
$$+ ConsAssetSalesInhKPvtPenSys_{dom}(age,t) + ConsAssetSalesInhKOthFinIns_{dom}(age,t)$$
$$+ ConsAssetSalesK\,\mathrm{Re}s_{dom}(age,t) + ConsAssetSalesKPvtUnincorpEnt_{dom}(age,t)$$
$$+ ConsAssetSalesKOthFinIns_{dom}(age,t)$$

Household net saving is the difference between disposable income and consumption:

$$NetSvngHH_{dom}(age,t) = DispYHH_{dom}(age,t) - Cons_{dom}(age,t)$$

and adjusted net savings includes savings captured by the private pension system:

$$AdjNetSvngHH_{dom}(age,t) = AdjDispYHH_{dom}(age,t) - Cons_{dom}(age,t)$$

4.6 Income, outlay, and net savings of firms

Firms operate capital installed at home and abroad; they earn profits and pay out direct taxes and dividends. The treatment of capital installed in the home region and owned by domestic firms is straightforward; however, investment abroad and investment from abroad both require special treatment. In the case of portfolio investment abroad, profits are credited to firms in the foreign, i.e. recipient, region; in the case of FDI abroad, earnings are credited to firms in the home, i.e. originating, region. The treatment of investment from abroad is symmetric: earnings on portfolio investment are credited to home-region firms while earnings on FDI from abroad are credited to foreign-region firms.

Note that in what follows, we make a distinction between FDI and portfolio investment only in the case of *OthFinIns*; all foreign claims of the *PvtPenSys* are assumed to be portfolio in nature.

Income

Earnings of firms located in the home region consist of earnings on:

- capital installed at home and owned by domestic investors ($KPvtPenSys_{dom,dom}$ and $KOthFinIns_{dom,dom}$),
- capital installed at home and owned by foreign portfolio investors ($KPvtPenSys_{for,dom}$ and $KOthFinInsPort_{for,dom}$), and
- capital installed abroad claims on which represent FDI originating in the home region ($KOthFinInsFDI_{dom,for}$)

In order, earnings on these types of capital are

$$ErngsFirmsKPvtPenSys_{dom,dom}(t) = r_{dom}(t)\, KPvtPenSys_{dom,dom}(t)$$

$$ErngsFirmsKOthFinIns_{dom,dom}(t) = r_{dom}(t)\, KOthFinIns_{dom,dom}(t)$$

$$ErngsFirmsKPvtPenSys_{for,dom}(t) = r_{dom}(t)\, KPvtPenSys_{for,dom}(t)$$

$$ErngsFirmsKOthFinInsPort_{for,dom}(t) = r_{dom}(t)\, KOthFinInsPort_{for,dom}(t)$$

$$ErngsFirmsKOthFinInsFDI_{dom,for}(t) = r_{for}(t)\, KOthFinInsFDI_{dom,for}(t)$$

Note that all of these flows are net of depreciation and indirect taxes.

Outlay

Direct taxes

Direct taxes must be paid to the home-region government on each of the profit streams that comprise domestic income:

$$DirTaxErngsFirmsKPvtPenSys_{dom,dom}(t) = DirTaxRate_{dom}(t)\, ErngsFirmsKPvtPenSys_{dom,dom}(t)$$

$$DirTaxErngsFirmsKOthFinIns_{dom,dom}(t) = DirTaxRate_{dom}(t)\, ErngsFirmsKOthFinIns_{dom,dom}(t)$$

$$DirTaxErngsFirmsKPvtPenSys_{for,dom}(t) = DirTaxRate_{dom}(t)\, ErngsFirmsKPvtPenSys_{for,dom}(t)$$

$$DirTaxErngsFirmsKOthFinInsPort_{for,dom}(t) = \\ = DirTaxRate_{dom}(t)\, ErngsFirmsKOthFinInsPort_{for,dom}(t)$$

in addition to which home-region firms pay direct taxes abroad on FDI

$$DirTaxErngsFirmsKOthFinInsFDI_{dom,for}(t) = DirTaxRate_{for}(t)\, ErngsFirmsKOthFinInsFDI_{dom,for}(t)$$

No distinction is made between the tax rate on profits earned by capital owned by domestic and foreign investors and, in the latter case, the direct tax rate is assumed to be independent of the nature of the claim (portfolio versus FDI).

Dividends

Dividend distributions are made out of pre-tax earnings, and the proportion of earnings distributed is assumed to be independent of the nature of the claim:

$$DivDistErngsFirmsKPvtPenSys_{dom,dom}(t) = DivDistRate_{dom}(t)\, ErngsFirmsKPvtPenSys_{dom,dom}(t)$$

$$DivDistErngsFirmsKOthFinIns_{dom,dom}(t) = DivDistRate_{dom}(t)\, ErngsFirmsKOthFinIns_{dom,dom}(t)$$

$$DivDistErngsFirmsKPvtPenSys_{for,dom}(t) = DivDistRate_{dom}(t)\, ErngsFirmsKPvtPenSys_{for,dom}(t)$$

$$DivDistErngsFirmsKOthFinInsPort_{for,dom}(t) = \\ = DivDistRate_{dom}(t)\, ErngsFirmsKOthFinInsPort_{for,dom}(t)$$

In the case of FDI abroad, firms in the domestic (originating) region make a two-stage decision: (i) what proportion of earnings abroad to reinvest (and, the complementary decision, what proportion to repatriate); and (ii) what proportion of repatriated earnings to pay out as dividends to *OthFinIns*. Reinvestment is

$$\mathrm{Re}\, invErngsFirmsKOthFinInsFDI_{dom,for}(t) = \\ \mathrm{Re}\, InvShareErngsFirmsFDI_{dom,for}(t)\, ErngsFirmsKOthFinInsFDI_{dom,for}(t)$$

where the reinvestment share is an exogenous assumption and is assumed to be applied to pre-tax earnings. No account is taken of special arrangements for exempting reinvested earnings on FDI from taxation. The remainder of earnings on FDI is repatriated:

$$\mathrm{Re}\, patrErngsFirmsKOthFinInsFDI_{dom,for}(t) = \mathrm{Erngs}FirmsKOthFinInsFDI_{dom,for}(t) \\ - DirTaxErngsFirmsKOthFinInsFDI_{dom,for}(t) - \mathrm{Re}\, invErngsFirmsKOthFinInsFDI_{dom,for}(t)$$

and dividends are distributed

$$DivDist\, \mathrm{Re}\, patrErngsFirmsKOthFinInsFDI_{dom,for}(t) = \\ DivDistRate_{dom}(t)\, \mathrm{Re}\, patrErngsFirmsKOthFinInsFDI_{dom,for}(t)$$

Repatriated earnings represent neither a credit nor a debit to the firm: they represent a debit to the foreign subsidiary and a credit to the parent company; the two cancel out. Dividends distributed represent a debit to the firm and a credit to *OthFinIns*. It is assumed that, having been taxed in the region where they were earned, repatriated earnings are exempt from domestic taxation.

Net savings of firms

Net savings of firms are split into two components: savings out of domestic resources (including earnings repatriated from abroad), and savings in the form of reinvested earnings on FDI in the foreign region. Taking again the perspective of the home region, the streams that comprise domestic savings of firms are:

$$NetSvngErngsFirmsKPvtPenSys_{dom,dom}(t) = ErngsFirms\ KPvtPenSys_{dom,dom}(t) - DirTaxErngsFirmsKPvtPenSys_{dom,dom}(t) - DivDistErngsFirmsKPvtPenSys_{dom,dom}(t)$$

$$NetSvngErngsFirmsKOthFinIns_{dom,dom}(t) = ErngsFirmsKOthFinIns_{dom,dom}(t) - DirTaxErngsFirmsKOthFinIns_{dom,dom}(t) - DivDistErngsFirmsKOthFinIns_{dom,dom}(t)$$

$$NetSvngErngsFirmsKPvtPenSys_{for,dom}(t) = ErngsFirmsKPvtPenSys_{for,dom}(t) - DirTaxErngsFirmsKPvtPenSys_{for,dom}(t) - DivDistErngsFirmsKPvtPenSys_{for,dom}(t)$$

$$NetSvngErngsFirmsKOthFinInsPort_{for,dom}(t) = ErngsFirmsKOthFinInsPort_{for,dom}(t) - DirTaxErngsFirmsKOthFinInsPort_{for,dom}(t) - DivDistErngsFirmsKOthFinInsPort_{for,dom}(t)$$

$$NetSvng\ \mathrm{Re}\ patErngsFirmsKOthFinInsFDI_{dom,for}(t) = \mathrm{Re}\ patErngsFirmsKOthFinInsFDI_{dom,for}(t) - DivDist\ \mathrm{Re}\ patErngsFirmsKOthFinInsFDI_{dom,for}(t)$$

The sum over these is total net domestic savings of firms in the home region:

$$\begin{aligned} NetSvngFirms_{dom,dom}(t) = {} & NetSvngErngsFirmsKPvtPenSys_{dom,dom}(t) \\ & + NetSvngErngsFirmsKOthFinIns_{dom,dom}(t) \\ & + NetSvnErngsFirmsKPvtPenSys_{for,dom}(t) \\ & + NetSvngErngsFirmsKOthFinInsPort_{for,dom}(t) \\ & + NetSvng\ \mathrm{Re}\ patErngsFirmsKOthFinInsFDI_{dom,for}(t) \end{aligned}$$

Foreign savings are simply reinvested earnings on FDI, i.e.:

$$NetSavFirms_{dom,for}(t) = \mathrm{Re}\ InvErngsFirmsKOthFinInsFDI_{dom,for}(t),$$

and total net saving from corporate income is the sum of the two:

$$NetSvngFirms_{dom,*}(t) = NetSvngFirms_{dom,dom}(t) + NetForSvngFirms_{dom,for}(t).$$

4.7 Income, outlay, and net savings of government

The government sector is rudimentary. Government consumes an exogenous share of GDP, makes interest payments on public-sector debt, collects taxes and social security contributions and pays social security benefits. Government expenditure is:

$$GovExp_{dom}(t) = GovCons_{dom}(t) + r_{dom}(t-1)\, GovDebt_{dom}(t-1) + PubPenSysBen_{dom}(t)$$

where

$$GovCons_{dom}(t) = GovConsShare_{dom}(t)\, GDP_{dom}(t)$$

$$GovDebt_{dom}(t) = GovDebt_{dom}(t-1) + NetSavGov_{dom}(t)$$

Government revenues are

$$Gov\,\mathrm{Re}v_{dom}(t) = IndTax_{dom}(t) + DirTax_{dom}(t) + PubPenSysCont_{dom}(t)$$

where

$$IndTax_{dom}(t) = IndTaxRate_{dom}(t)\, GDP_{dom}(t)$$

$$\begin{aligned} DirTax_{dom}(t) = & \sum_{age=15}^{MaxAge} DirTaxWageY_{dom}(age,t) + \sum_{age=15}^{MaxAge} DirTaxEntrY_{dom}(age,t) + \sum_{age=15}^{MaxAge} DirTax\,\mathrm{Re}ntalY_{dom}(age,t) \\ & + DirTaxErngsFirmsKPvtPenSys_{dom,dom}(t) + DirTaxErngsFirmsKOthFinIns_{dom,dom}(t) \\ & + DirTaxErngsFirmsKPvtPensSys_{for,dom}(t) \\ & + DirTaxYFirmsKOthFinInsPort_{for,dom}(t) + DirTaxErngsFirmsKOthFinInsFDI_{for,dom}(t) \end{aligned}$$

The first line sums taxes on income earned by households across age of recipient, the second line represents taxes on profits earned by capital owned by domestic firms and the third and fourth lines represent taxes on profits earned by capital owned by foreign firms. Social security system revenues are described in the next section.

Public pension system (*PubPenSys*)

Expenditures

The public pension system is assumed to be a defined benefit system financed on a Pay As You Go (PAYG) basis. Let

$$PubPenSysEnt_{dom}(age,t,\mathrm{Re}tDur_{dom}) \ , \ \mathrm{Re}tDur_{dom}(t) = \overline{0:MaxAge_{dom}(t) - EligAge_{dom}(t)}$$

be the social security benefit entitlement for the average person aged *age* who retired *RetDur* years ago, where *EligAge* is the age of eligibility for a pension and we assume *PubPenSysEnt(age,t,0) = 0*. The pension for persons entering retirement is computed according to the formula:

$$PubPenSysEnt_{dom}(age,t,1) =$$

$$\mathrm{Re}\, plRate_{dom}(t) \sum_{j=1}^{age-15} LabForcePartRate_{dom}(age-j,t-j,1) \frac{\left(\sum_{k=1}^{D} W_{dom}(age-k,t-k)\right)}{\mathrm{Re}\, fPeriod_{dom}(t)}$$

where we assume $age \geq EligAge$. *RefPeriod* is the reference period over which past age-specific wages are averaged to compute the reference wage employed in calculation of the initial pension and *ReplRate* is the replacement rate, i.e. the ratio of the pension to the reference wage. Survivors' pensions, (i.e., pensions paid to the surviving spouses of social security system participants) are implicitly included in the replacement rate, which must therefore not be interpreted too literally. The first summation term on the right-hand side is the average lifetime years of labor force participation, computed as the sum of age-specific labor force participation rates. For example, if age=70 and t=100, we would have

$$\sum_{j=1}^{A-15} LabForcePartRate_{dom}(age-j,t-j,1) =$$

$$LabForcePartRate_{dom}(69,99,1) + LabForcePartRate_{dom}(68,98,1) + \ldots$$
$$+ LabForcePartRate_{dom}(15,45,1)$$

which gives average number of years spent in the labor force by members of the cohort who retire aged 70 in year 100. Assuming *RefPeriod*=10, the rest of the formula is

$$\frac{\left(\sum_{k=1}^{D} W_{dom}(age-k,t-k)\right)}{\mathrm{Re}\, fPeriod_{dom}(t)} = \frac{\left[W_{dom}(69,99) + W_{dom}(68,98) + \ldots + W_{dom}(60,90)\right]}{10}$$

which gives average wage over the last ten years for members of the cohort who retire aged 70 in year 100.

Once persons have retired, their pension is indexed to average wages. For people who were already retired at *(t-1)*, the pension is

$$PubPenSysEnt_{dom}(age,t,\mathrm{Re}tDur_{dom}) =$$

$$PubPenSysEnt_{dom}(age-1,t-1,\mathrm{Re}tDur_{dom}-1)\left[1+IndexRate_{dom(t)}\frac{\overline{Wage}_{dom}(t)}{\overline{Wage}_{dom}(t-1)}\right]$$

where *IndexRate* is the rate of indexation of pensions to the average wage rate $\overline{Wage}(\mathrm{Re}g,t)$ and $0 \le IndexRate \le 1$.

Social security system benefits paid out by age group of beneficiary are equal to the age- and retirement-duration specific entitlement times the number of recipients:

$$PubPenSysBen_{dom}(age,t) =$$

$$= \sum_{\mathrm{Re}tDur_{dom}(t)=0}^{MaxAge_{dom}(t)-EligAge_{dom}(t)} PubPenSysEnt_{dom}(age,t,\mathrm{Re}tDur_{dom})\, Pop_{dom}(age,t,\mathrm{Re}tDur_{dom})$$

and system-wide expenditures are equal to the summation over age groups

$$PubPenSysBen_{dom}(t) = \sum_{age=EligAge_{dom}(t)}^{MaxAge_{dom}(t)} PubPenSysBen_{dom}(age,t)\, Pop_{dom}(age,t)$$

For example, taking t=10, *EligAge*=65, and *MaxAge*=100

$$PubPenSysBen(reg,65,10) =$$
$$Pop(reg,65,10,0)\, PubPenSysEnt(reg,65,10,0)$$
$$+ Pop(reg,65,10,1)\, PubPenSysEnt(reg,65,10,1)$$
$$PubPenSysBen(reg,66,10) =$$
$$Pop(reg,66,10,0)\, PubPenSysEnt(reg,66,10,0)$$
$$+ Pop(reg,66,10,1)\, PubPenSysEnt(reg,66,10,1)$$
$$+ Pop(reg,66,10,2)\, PubPenSysEnt(reg,66,10,2)$$
$$\vdots$$
$$PubPenSysBen(reg,100,10) =$$
$$Pop(reg,100,10,0)\, PubPenSysEnt(reg,100,10,0)$$
$$+ Pop(reg,100,10,1)\, PubPenSysEnt(reg,100,10,1)$$
$$+ Pop(reg,100,10,2)\, PubPenSysEnt(reg,100,10,2)$$
$$\vdots$$
$$+ Pop(reg,100,10,35)\, PubPenSysEnt(reg,100,10,35)$$

and

$$PubPenSysBen(reg,10) = PubPenSysBen(reg,65,10) + PubPenSysBen(reg,66,10) + \cdots + PubPenSysBen(reg,100,10)$$

Revenues

Contributions to the public pension system out of wages are

$$PubPenSysContWageY_{dom}(t) = \sum_{age=15}^{MaxAge_{dom}(t)} PubPenSysContRate_{dom}(t)\, WageY_{dom}(age,t),$$

and out of entrepreneurial income are

$$PubPenSysContEntrY_{dom}(t) = \sum_{age=15}^{MaxAge_{dom}(t)} PubPenSysContRate_{dom}(t)\, EntrY_{dom}(age,t)$$

The social security contribution rate is assumed to be equal for all types of income and is independent of the age of the contributor. No distinction is made, in the case of contributions out of wage income, between employees' and employers' contributions.

Total public pension system contributions are the summation over contributions out of wages and entrepreneurial income:

$$PubPenSysCont_{dom}(t) = PubPenSysContWageY_{dom}(t) + PubPenSysContEntrY_{dom}(t)$$

In the classic PAYG system design (for example, the German system), total contributions equal total benefits; there is neither accumulation of a surplus nor a deficit to be financed out of general government revenue. The default model solution option is one in which the required contribution rate is calculated by setting contributions equal to expenditures. However, there are cases (for example, the USA), where nominally PAYG systems are currently running surpluses in order to accumulate resources to deal with the retirement of the baby boom generation. In other cases, deficits in the PAYG pension system are covered out of general tax revenue. To cover such cases, an alternative solution option is to set the contribution rate independent of benefits, in which case the model solves for the implied surplus or deficit. The balance of the social security system, say in year T, is then given by:

$$PubPensSysBal_{dom}(T) = PubPenSysCont_{dom}(T) - PubPensSysExp_{dom}(T) + r_{dom}(T-1)\sum_{t=0}^{T-1} PubPensSysBal_{dom}(T-1)$$

In the event of an unreasonable deficit in the social security system, assumptions on retirement age and benefit calculation must be examined.

4.8 The life-cycle dynamics of capital accumulation

Corresponding to each of the four types of capital is an age-specific capital accumulation identity which tracks assets as the population ages. The major structural difference is between *KPvtPensSys* and the other three asset classes. Funds flow into *PvtPenSys* only through payroll deductions (including deductions from entrepreneurial income) on behalf of system participants. Dividends earned on assets held by the *PvtPenSys* remain within the system until the worker retires. By contrast, savings of all origins, not just captive retirement-related savings, flow into *KOthFinIns, KRes, and KPvtUnincorpEnt*. Dividends earned on assets held by *OthFinIns* may be allocated to consumption at any point during the life cycle, as may profits accruing to *KPvtUnincorpEnt* (rents on *KRes* are assumed to be consumed in their entirety). If saved, dividends earned on assets held by *OthFinIns* may remain within *OthFinIns*, or be allocated to residential investment or investment in capital operated by *PvtUnincorpEnt.*

Private pension system (*PvtPenSys*)

The private pension system is assumed to be a fully-funded defined contribution system. No distinction is made between workers' and employers' contributions and the contribution rates out of wages and entrepreneurial income are assumed to be identical. Contributions are

$$PvtPenSysContWageY_{dom}(age,t) = PvtPenSysContRate_{dom}(age,t)\,WageY_{dom}(age,t)$$

$$PvtPenSysContEntrY_{dom}(age,t) = PvtPenSysContRate_{dom}(age,t)\;EntrY_{dom}(age,t)$$

The age-specific accumulation identity for the private pension wealth is

$$KPvtPenSys_{dom}(age,t) = KPvtPenSys_{dom}(t-1,age-1) + \Delta KPvtPensSys_{dom}(age,t)$$

where

$$\begin{aligned}\Delta KPvtPenSys_{dom}(age,t) &= ContPvtPenSys_{dom}(age,t)\\ &+ DividPvtPenSys_{dom}(age,t)\\ &- PvtPenSysBen_{dom}(age,t) - BeqKPvtPenSys_{dom}(age,t)\end{aligned}$$

In order, the components of change are:

- current contributions (zero for persons who have retired),
- dividend earnings,
- dissaving via the conversion of retirees' accumulated assets into consumption, and finally,
- outflow of funds via death of claimants and ensuing pay-out of their accumulation.

Note that, for an individual cohort born in year $t = 0$ whose last members dies out in year $t = MaxAge$, lifetime pension contributions plus lifetime earnings on pension assets plus lifetime pension benefits received equals bequest of pension wealth; i.e.,

$$\sum_{t=0}^{MaxAge}\sum_{age=0}^{MaxAge}\Delta KPvtPenSys_{dom}(age,t)$$
$$= \sum_{t=0}^{MaxAge}\sum_{age=0}^{MaxAge}\left[\begin{array}{l} ContPvtPenSys_{dom}(age,t) \\ \quad + DividPvtPenSys_{dom}(age,t) \\ \quad - PvtPenSysBen_{dom}(age,t) - BeqKPvtPenSys_{dom}(age,t)\end{array}\right] = 0$$

Note also that the first three terms of $\Delta KPvtPenSys$ equal the wedge between household disposable income and adjusted disposable income; therefore, we may write adjusted net household savings in terms of unadjusted net household savings and $\Delta KPvtPenSys$:

$$AdjNetSvngHH_{dom}(age,t) = NetSvngHH_{dom}(age,t)$$
$$+ \Delta KPvtPenSys_{dom}(age,t) + BeqKPvtPenSys_{dom}(age,t)$$

This identity expression is useful in the accounting consistency check presented in the Annex.

Other asset classes (*KOthFinIns, KRes, KPvtUnincorpEnt*)

For *KRes*, the age-specific accumulation identity is

$$\Delta K\,\mathrm{Res}(age,t) =$$
$$K\,\mathrm{Res}Share_{dom}(t)\left[NetSvngHH_{dom}(age,t) + NetSvngFirms_{dom}(age,t) + NetSvngGovt_{dom}(age,t)\right]$$
$$- AssetSalesK\,\mathrm{Res}(age,t)$$
$$+ K\,\mathrm{Res}Share_{dom}(t)\left[\begin{array}{l} AssetSalesK\,\mathrm{Res}(age,t) \\ + AssetSalesKPvtUnincorpEnt(age,t) \\ + AssetSalesKOthFinIns_{dom,*}(age,t)\end{array}\right]$$
$$- BeqK\,\mathrm{Res}(age,t)$$
$$+ InhK\,\mathrm{Res}(age,t) - AssetSalesInhK\,\mathrm{Res}(age,t)$$
$$+ K\,\mathrm{Res}Share_{dom}(t)\left[\begin{array}{l} AssetSalesInhK\,\mathrm{Res}(age,t) \\ + AssetSalesInhKPvtUnincorpEnt(age,t) \\ + AssetSalesInhKPvtPenSys_{dom,*}(age,t) \\ + AssetSalesInhKOthFinIns_{dom,*}(age,t)\end{array}\right]$$

The components of change are, in order:

- Unadjusted household net savings (i.e., not including savings captured by the private pension system) plus the imputed savings of firms and government. A share variable, which sums to unity across the three forms of non-pension wealth, is used to apportion savings between •*KRes*, •*KPvtUnincorpEnt*, and •*KOthFinIns*. Note that allocation shares are not indexed by age.
- The second line, of relevance only to retired households, subtracts sales of residential assets. The third line, also of relevance only for retired households, reflects the allocation of the proceeds of asset sales among the three forms of non-pension wealth. Consumption from the proceeds of asset sales is not subtracted because this consumption has already been factored into adjusted household net saving in the first line.
- The fourth line subtracts bequests, which represent a leakage of wealth out of the age group.
- The fifth line adds inheritance, an injection of wealth, and subtracts asset sales which occur in consequence of inheritance.
- The sixth and last line is analogous to the third line, but applies to households everywhere in the age spectrum and includes the disposition of inherited pension-, as well as non-pension, wealth.

The accumulation identities for the other two forms of non-pension wealth are analogous:

$$
\begin{aligned}
&\Delta KPvtUnincorpEnt(age,t) = \\
&\quad KPvtUnincorpEntShare_{dom}(t) \\
&\quad \left[NetSvngHH_{dom}(age,t) + NetSvngFirms_{dom}(age,t) + NetSvngGovt_{dom}(age,t)\right] \\
&\quad - AssetSalesKPvtUnincorpEnt(age,t) \\
&\quad + KPvtUnincorpEntShare_{dom}(t)\begin{bmatrix} AssetSalesK\,\mathrm{Re}s(age,t) \\ + AssetSalesKPvtUnincorpEnt(age,t) \\ + AssetSalesKOthFinIns_{dom,*}(age,t) \end{bmatrix} \\
&\quad - BeqKPvtUnincorpEnt(age,t) \\
&\quad + InhKPvtUnincorpEnt(age,t) - AssetSalesInhKPvtUnincorpEnt(age,t) \\
&\quad + KPvtUnincorpEntShare_{dom}(t)\begin{bmatrix} AssetSalesInhK\,\mathrm{Re}s(age,t) \\ + AssetSalesInhKPvtUnincorpEnt(age,t) \\ + AssetSalesInhKPvtPenSys_{dom,*}(age,t) \\ + AssetSalesInhKOthFinIns_{dom,*}(age,t) \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&\Delta KOthFinIns_{dom,*}(age,t) = \\
&\quad KOthFinInsShare_{dom}(t)\left[NetSvngHH_{dom}(age,t) + NetSvngFirms_{dom}(age,t) + NetSvngGovt_{dom}(age,t)\right] \\
&\quad - AssetSalesKOthFinIns_{dom,*}(age,t) \\
&\quad + KOthFinInShare_{dom}(t)\begin{bmatrix} AssetSalesK\,\mathrm{Re}s(age,t) \\ + AssetSalesKPvtUnincorpEnt(age,t) \\ + AssetSalesKOthFinIns_{dom,*}(age,t) \end{bmatrix} \\
&\quad - BeqKOthFinIns(age,t) \\
&\quad + InhKOthFinIns_{dom,*}(age,t) - AssetSalesInhKOhFinIns_{dom,*}(age,t) \\
&\quad + KOthFinInsShare_{dom}(t)\begin{bmatrix} AssetSalesInhK\,\mathrm{Re}s(age,t) \\ + AssetSalesInhKPvtUnincorpEnt(age,t) \\ + AssetSalesInhKPvtPenSys_{dom,*}(age,t) \\ + AssetSalesInhKOthFinIns_{dom,*}(age,t) \end{bmatrix}
\end{aligned}
$$

where

$$K\,\mathrm{Re}sShare + KPvtUnincorpEntShare + KOthFinInsShare = 1$$

Wealth

Total assets (*KTot*) for each age group are equal to pension wealth (*KPvtPenSys)* plus the three forms of non-pension wealth (*KOthFinIns*, *KRes*, and *KPvtUnincorpEnt*):

$$
\begin{aligned}
&KTot_{dom,*}(age,t) = KPvtPenSys_{dom,*}(age,t) + KOthFinIns_{dom,*}(age,t) \\
&\quad + K\,\mathrm{Re}s_{dom}(age,t) + KPvtUnincorpEnt_{dom}(age,t)
\end{aligned}
$$

4.9 Regional allocation of investment

•*KPvtPenSys*

$\Delta KPvtPenSys(age,t)$ • is divided into domestic and foreign components using exogenous flow coefficients:

$$KPvtPenSys_{dom,dom}(age,t) = KPvtPenSys_{dom,dom}(age-1,t-1) + \Delta KPvtPenSys_{dom,dom}(age,t)$$

$$KPvtPenSys_{dom,for}(age,t) = KPvtPenSys_{dom,for}(age-1,t-1) + \Delta KPvtPenSys_{dom,for}(age,t),$$

$$\Delta KPvtPenSys_{dom,dom}(age,t) = SharePvtPenSys_{dom,dom}(t)\,\Delta KPvtPenSys_{dom,*}(age,t)$$

$$\Delta KPvtPenSys_{dom,for}(age,t) = SharePvtPenSys_{dom,for}(t)\,\Delta KPvtPenSys_{dom,*}(age,t)$$

where the geographical investment shares sum to unity:

$$SharePvtPenSys_{dom,dom}(t) + SharePvtPenSys_{dom,for}(t) = 1$$

The flow coefficients are assumed to be identical over all age groups.

•*OthFinIns*

The allocation of $\Delta KOthFinIns(age,t)$ is identical, the only difference being that reinvestment of earnings on FDI must be taken account of:

$$KOthFinIns_{dom,dom}(age,t) = KOthFinIns_{dom,dom}(age-1,t-1) + \Delta KOthFinIns_{dom,dom}(age,t)$$

$$KOthFinIns_{dom,for}(age,t) = KOthFinIns_{dom,for}(age-1,t-1) + \Delta KOthFinIns_{dom,for}(age,t),$$

$$\Delta KOthFinIns_{dom,dom}(age,t) = ShareOthFinIns_{dom,dom}(t)$$
$$\left[\Delta KOthFinIns_{dom,*}(age,t) - \mathrm{Re}\, invErngsYFirmsKOthFinInsFDI_{dom,for}(age,t)\right]$$

$$\Delta KOthFinIns_{dom,for}(age,t) = ShareOthFinIns_{dom,for}(t)$$
$$\left[\Delta KOthFinIns_{dom,*}(age,t) - \mathrm{Re}\, invErngsYFirmsKOthFinIns_{dom,for}(age,t)\right],$$
$$+ \mathrm{Re}\, invErngsYFirmsKOthFinIns_{dom,for}(t)$$

where

$$ShareOthFinIns_{dom,dom}(t) + ShareOthFinIns_{dom,for}(t) = 1$$

In allocating foreign investment between portfolio investment and FDI, the choice is between sharing out the flows $\Delta KOthFinIns(age,t)$ or the stock *KOthFinIns(age,t)*. The first approach is conceptually superior; on the other hand, it would require that share coefficients shift over time to reflect the composition of the stock. We adopt the simpler strategy, therefore, of sharing out the stock between the two forms of claims:

$$KOthFinInsFDI_{dom,for}(age,t) = KOthFinIns_{dom,for}(age,t)\, FDIShare_{dom,for}(t)$$

$$KOthFinInsPort_{dom,for}(age,t) = KOthFinIns_{dom,for}(age,t)\left[1 - FDIShare_{dom,for}(t)\right]$$

The FDI-portfolio split is assumed to be constant by age of claimant.

4.10 Macroeconomic identities

Gross domestic product

Gross domestic product (GDP) is the sum of wages, net profits (including earnings on capital corresponding to foreign investment in the region), indirect taxes, and depreciation:

$$
\begin{aligned}
&GDP_{dom}(t) = \\
&\quad WageY_{dom}(t) + \mathrm{Re}\,ntalY_{dom}(t) + EntrY_{dom}(t) \\
&\quad + ErngsFirms\,KPvtPenSys_{dom,dom}(t) + \; ErngsFirmsKOthFinIns_{dom,dom}(t) \\
&\quad + ErngsFirmsKPvtPenSys_{for,dom}(t) + ErngsFirmsKOthFinInsPort_{for,dom}(t) \\
&\quad + ErngsFirmsKOthFinInsFDI_{for,dom}(t) \\
&\quad + \left[\frac{IndTaxRate_{dom}(t)\,GDP_{dom}(t)}{KTot_{*,dom}(t)} + DeprRate_{dom}(t) \right] KTot_{*,dom}
\end{aligned}
$$

Since

$$
\begin{aligned}
&KTot_{*,dom}(t) = \\
&\quad K\,\mathrm{Re}\,s_{dom}(t) + KPvtUnincorpEnt_{dom}(t) \\
&\quad + KPvtPenSys_{dom,dom}(t) + \; KOthFinIns_{dom,dom}(t) \\
&\quad + KPvtPenSys_{for,dom}(t) + KOthFinInsPort_{for,dom}(t) \\
&\quad + KOthFinInsFDI_{for,dom}(t)
\end{aligned}
$$

it is clear without further checking that GDP thus expressed will be equal to GDP calculated using the production function in Section 4.4.

Gross national product

Gross national product (GNP) is equal to GDP adjusted for net (in the sense of receipts minus outlay) factor payments from abroad. These factor payments represent the sum of net dividend payments, net after-tax earnings on FDI (whether remitted or reinvested), net depreciation allowances, and net payments of indirect tax. Gross factor payments from the foreign to the domestic region are:

$$
\begin{aligned}
&GrossFactPay_{for,dom}(t) = \\
&\quad DivDistErngsFirmsKPvtPenSys_{dom,for}(t) + DivDistErngsFirmsKOthFinInsPort_{dom,for}(t) \\
&\quad + ErngsFirmsKOthFinInsFDI_{dom,for}(t) - DirTaxErngsFirmsKOthFinInsFDI_{dom,for}(t) \\
&\quad + DeprRate_{for}(t) KTot_{dom,for}(t) + IndTax_{dom,for(t)}
\end{aligned}
$$

where

$$
IndTax_{dom,for}(t) = \frac{IndTax_{*,for}(t)}{KTot_{*,for}(t)} KTot_{dom,for}(t)
$$

Symmetrically,

$$\begin{aligned} &GrossFactPay_{dom,for}(t) = \\ &\quad DivDistErngsFirmsKPvtPenSys_{for,dom}(t) + DivDistErngsFirmsKOthFinInsPort_{for,dom}(t) \\ &\quad + ErngsFirmsKOthFinInsFDI_{for,dom}(t) - DirTaxErngsFirmsKOthFinInsFDI_{for,dom}(t) \\ &\quad + DeprRate_{dom} KTot_{for,dom}(t) + IndTax_{for,dom}(t) \end{aligned}$$

Net factor payments from abroad, from the standpoint of the home region, are

$$NetFactPay_{for,dom}(t) = GrossFactPay_{for,dom}(t) - GrossFactPay_{dom,for}(t)$$

and GNP is

$$GNP_{dom}(t) = GDP_{dom}(t) + NetFactPay_{for,dom}(t)$$

or, fully broken out,

$$\begin{aligned} &GNP_{dom}(t) = \\ &\quad WageY_{dom}(t) + EntrY_{dom}(t) + \mathrm{Re}\,ntalY_{dom}(t) \\ &\quad + ErngsFirmsKPvtPenSys_{dom,dom}(t) + ErngsFirmsKOthFinIns_{dom,dom}(t) \\ &\quad + ErngsFirmsKPvtPenSys_{for,dom} + ErngsFirmsKOthFinInsPort_{for,dom}(t) \\ &\quad + IndTax_{dom,dom}(t) + IndTax_{for,dom}(t) + DeprRate_{dom}(t) KTot_{dom,dom} + DeprRate_{dom}(t) KTot_{for,dom} \\ &\quad + DivDistErngsFirmsKPvtPenSys_{dom,for}(t) + DivDistErngsFirmsKOthFinInsPort_{dom,for}(t) \\ &\quad + ErngsFirmsKOthFinInsFDI_{dom,for}(t) - DirTaxErngsFirmsKOthFinInsFDI_{dom,for}(t) \\ &\quad + DeprRate_{for}(t) KTot_{dom,for}(t) + IndTax_{dom,for}(t) \\ &\quad - DivDistErngsFirmsKPvtPenSys_{for,dom}(t) - DivDistErngsFirmsKOthFinInsPort_{for,dom}(t) \\ &\quad + DirTaxErngsFirmsKOthFinInsFDI_{for,dom}(t) - DeprRate_{dom}(t) KTot_{for,dom}(t) - IndTax_{for,dom}(t) \end{aligned}$$

National disposable income

National disposable income is GNP adjusted for depreciation and indirect taxes:

$$\begin{aligned} &NatDispY_{dom}(t) = GNP_{dom}(t) - DeprRate_{dom}(t) KTot_{dom,dom}(t) - DeprRate_{for}(t) KTot_{dom,for}(t) \\ &\quad + IndTax_{for,dom}(t) - IndTax_{dom,for}(t) \end{aligned}$$

Net national savings

Net national savings are equal to national disposable income minus consumption:

$$NetNatSvng_{dom}(t) = NatDispY_{dom}(t) - PvtCons_{dom}(t) - GovCons_{dom}(t)$$

We show in the Annex that net national savings thus calculated are equal to the sum of net savings of households, firms, and government derived in Sections 4.5, 4.6 and 4.7, respectively.

4.11 Directions for further model development

Concluding passages on directions for further model development tend to be largely hortatory. Therefore, we will mention only three areas which are currently (spring 1999) under investigation:

- Disaggregation of men and women. Males and females experience very different mortality and labor market histories; therefore, the social security policy debate contains a large equity component along lines of gender. It seems reasonable that, from the very start, the model should incorporate this dimension.
- Inclusion of health and disability / long-term care costs and financing. This is conceptually identical to the treatment of pensions and is necessary for a comprehensive assessment of the economic impacts of aging.
- Assignment of population to households. It is conceptually simple to translate population by age into number of households by age of head (using a headship rate approach) of head and then distribute the population among these households. Income and outlay per capita could then be re-expressed as income and outlay per household, with average propensities to consume reflecting household structure where appropriate.

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6. Annex: Accounting consistency checks

We apply two accounting consistency checks, first to confirm that net savings equals capital formation, and second to confirm that net savings calculated by summings across households, firms and government equal net savings calculated by subtracting consumption from GNP.

Net savings equals capital formation

Adding across the three non-pension forms of wealth,

$$
\begin{aligned}
&\Delta K\,\mathrm{Re}s(age,t) + \Delta KPvtUnincorpEnt(age,t) + \Delta KOthFinIns_{dom,*}(age,t) = \\
&\quad \left[NetSvngHH_{dom}(age,t) + NetSvngFirms_{dom}(age,t) + NetSvngGovt_{dom}(age,t)\right] \\
&\quad - AssetSalesK\,\mathrm{Re}s(age,t) - AssetSalesKPvtUnincorpEnt_{dom,*}(age,t) \\
&\quad - AssetSalesKOthFinIns_{dom,*}(age,t) \\
&\quad + AssetSalesK\,\mathrm{Re}s(age,t) + AssetSalesKPvtUnincorpEnt(age,t) + AssetSalesKOthFinIns_{dom,*}(age,t) \\
&\quad - BeqK\,\mathrm{Re}s(age,t) - BeqKPvtUnincorpEnt(age,t) - BeqKOthFinIns_{dom,*}(age,t) \\
&\quad + InhK\,\mathrm{Re}s_{dom,*}(age,t) - AssetSalesInhK\,\mathrm{Re}s_{dom,*}(age,t) \\
&\quad + InhKPvtUnincorpEnt_{dom,*}(age,t) - AssetSalesInhKPvtUnincorpEnt_{dom,*}(age,t) \\
&\quad + InhKOthFinIns_{dom,*}(age,t) - AssetSalesInhKOthFinIns_{dom,*}(age,t) \\
&\quad + AssetSalesInhK\,\mathrm{Re}s(age,t) + AssetSalesInhKPvtUnincorpEnt(age,t) \\
&\quad + AssetSalesInhKPvtPenSys_{dom,*}(age,t) + AssetSalesInhKOthFinIns_{dom,*}(age,t)
\end{aligned}
$$

Cancellations bring us to

$$
\begin{aligned}
&\Delta K\,\mathrm{Re}s(age,t) + \Delta KPvtUnincorpEnt(age,t) + \Delta KOthFinIns_{dom,*}(age,t) = \\
&\quad \left[NetSvngHH_{dom}(age,t) + NetSvngFirms_{dom}(age,t) + NetSvngGovt_{dom}(age,t)\right] \\
&\quad - BeqK\,\mathrm{Re}s(age,t) - BeqKPvtUnincorpEnt(age,t) - BeqKOthFinIns_{dom,*}(age,t) \\
&\quad + InhK\,\mathrm{Re}s(age,t) + InhKPvtUnincorpEnt(age,t) + InhKOthFinIns_{dom,*}(age,t) \\
&\quad + InhKPvtPenSys_{dom,*}(age,t)
\end{aligned}
$$

Adding pension wealth,

$$
\begin{aligned}
&\Delta KTot(age,t) = \\
&\quad \Delta KPvtPenSys_{dom,*}(age,t) + \left[NetSvngHH_{dom}(age,t) + NetSvngFirms_{dom}(age,t) + NetSvngGovt_{dom}(age,t)\right] \\
&\quad - BeqK\,\mathrm{Re}\,s(age,t) - BeqKPvtUnincorpEnt(age,t) - BeqKOthFinIns_{dom,*}(age,t) \\
&\quad + InhK\,\mathrm{Re}\,s(age,t) + InhKPvtUnincorpEnt(age,t) + InhKOthFinIns_{dom,*}(age,t) \\
&\quad + InhKPvtPenSys_{dom,*}(age,t)
\end{aligned}
$$

From above,

$$
\begin{aligned}
&NetSvngHH_{dom}(age,t) = AdjNetSvngHH_{dom}(age,t) - \Delta KPvtPenSys_{dom}(age,t) \\
&\quad - BeqKPvtPenSys_{dom}(age,t)
\end{aligned}
$$

so

$$
\begin{aligned}
&\Delta KTot(age,t) = \\
&\quad \Delta KPvtPenSys_{dom,*}(age,t) \\
&\quad + \left[\begin{array}{l} AdjNetSvgHH_{dom}(age,t) - \Delta KPvtPenSys_{dom}(age,t) - BeqKPvtPenSys(age,t) \\ \quad + NetSvngFirms_{dom}(age,t) + NetSvngGovt_{dom}(age,t) \end{array}\right] \\
&\quad - BeqK\,\mathrm{Re}\,s(age,t) - BeqKPvtUnincorpEnt(age,t) - BeqKOthFinIns_{dom,*}(age,t) \\
&\quad + InhK\,\mathrm{Re}\,s(age,t) + InhKPvtUnincorpEnt(age,t) + InhKOthFinIns_{dom,*}(age,t) \\
&\quad + InhKPvtPenSys_{dom,*}(age,t)
\end{aligned}
$$

- *KPvtPenSys* is cancelled out, leaving the result

$$
\begin{aligned}
&\Delta KTot(age,t) = \\
&\quad + \left[AdjNetSvgHH_{dom}(age,t) + NetSvngFirms_{dom}(age,t) + NetSvngGovt_{dom}(age,t)\right] \\
&\quad - BeqK\,\mathrm{Re}\,s(age,t) - BeqKPvtUnincorpEnt(age,t) - BeqKPvtPenSys(age,t) \\
&\qquad - BeqKOthFinIns_{dom,*}(age,t) \\
&\quad + InhK\,\mathrm{Re}\,s(age,t) + InhKPvtUnincorpEnt(age,t) + InhKPvtPenSys_{dom,*}(age,t) \\
&\qquad + InhKOthFinIns_{dom,*}(age,t)
\end{aligned}
$$

In other words, change in wealth for members of an age group in a given year is equal to

- their net saving, including net saving through the private pension system and their imputed share of the net savings of firms and government, plus
- the sum across all asset classes of inheritance minus bequests.

Summing over age groups, inheritance and bequests cancel out, leaving us with

$$\Delta KTot(age,t) = AdjNetSvngHH_{dom}(t) + NetSvngFirms_{dom}(t) + NetSvngGovt_{dom}(t)$$

Net savings calculated "bottom up" equal net savings calculated "top down"

We wish to confirm that net national savings as calculated “top down” as national disposable income minus consumption is equal to the net national savings as calculated "bottom up" by summing net savings across households, firms, and government. We start by summing across sectors:

Households

$$\begin{aligned} AdjDispYHH_{dom}(t) &= DispWageY_{dom}(t) + Disp\,\mathrm{Re}\,ntalY_{dom}(t) + DispEntrY_{dom}(t) + DividY_{dom}(t) \\ &+ PubPenSysBen_{dom}(t) + PvtPenSysBen_{*,dom}(t) \\ &+ PvtPenSysCont_{dom}(t) + DivDistErngsFirmsKPvtPenSys_{*,dom}(t) - PvtPenSysBen*,_{dom}(t) \end{aligned}$$

$$\begin{aligned} &AdjNetSvngHH_{dom}(t) = \\ &\quad WageY_{dom}(t) - DirTaxWageY_{dom}(t) - PubPenSysContWageY_{dom}(t) - PvtPenSysContWageY_{dom}(t) \\ &\quad + \mathrm{RentalY}_{\mathrm{dom}}(t) - \mathrm{DirTaxRentalY}_{\mathrm{dom}}(t) \\ &\quad + EntrY_{dom}(t) - DirTaxEntrY_{dom}(t) - PubPenSysContEntrY_{dom}(t) - PvtPenSysContEntrY_{dom}(t) \\ &\quad + DivDistErngsFirmsKOthFinIns_{dom,dom}(t) + DivDistErngsFirmsKOthFinInsPort_{dom,for}(t) \\ &\qquad + DivDist\,\mathrm{Re}\,patrErngsFirmsKOthFinInsFDI_{dom,for}(t) \\ &\quad + PubPenSysBen_{dom}(t) \\ &\quad + PvtPenSysCont_{dom}(t) + DivDistErngsFirmsKPvtPenSys_{dom,dom}(t) \\ &\quad + DivDistErngsFirmsKPvtPenSys_{dom,for}(t) \\ &\quad - Cons_{dom}(t) \end{aligned}$$

Firms

$$\begin{aligned} &NetSvngFirms_{dom,*}(t) = \\ &\quad NetSvngErngsFirmsKPvtPenSys_{dom,dom}(t) \\ &\quad + NetSvngErngsFirmsKOthFinIns_{dom,dom}(t) \\ &\quad + NetSvngErngsFirmsKPvtPenSys_{for,dom}(t) \\ &\quad + NetSvngErngsFirmsKOthFinInsPort_{for,dom}(t) \\ &\quad + NetSvngErngsFirmsKOthFinInsFDI_{dom,for}(t) \end{aligned}$$

$$NetSvngFirms_{dom,*}(t) =$$
$$ErngsFirms\ KPvtPenSys_{dom,dom}(t)$$
$$- DirTaxErngsFirmsKPvtPenSys_{dom,dom}(t) - DivDistErngsFirmsKPvtPenSys_{dom,dom}(t)$$
$$+ ErngsFirmsKOthFinIns_{dom,dom}(t)$$
$$- DirTaxErngsFirmsKOthFinIns_{dom,dom}(t) - DivDistErngsFirmsKOthFinIns_{dom,dom}(t)$$
$$+ ErngsFirmsKPvtPenSys_{for,dom}(t)$$
$$- DirTaxErngsFirmsKPvtPenSys_{for,dom}(t) - DivDistErngsFirmsKPvtPenSys_{for,dom}(t)$$
$$+ ErngsFirmsKOthFinInsPort_{for,dom}(t)$$
$$- DirTaxErngsFirmsKOthFinInsPort_{for,dom}(t) - DivDistErngsFirmsKOthFinInsPort_{for,dom}(t)$$
$$+ ErngsFirmsKOthFinInsFDI_{dom,for}(t)$$
$$- DirTaxErngsFirmsKOthFinInsFDI_{dom,for}(t)$$
$$- DivDist\,\mathrm{Re}\,patErngsFirmsKOthFinInsFDI_{dom,for}(t)$$

Government

$$NetSvngGov_{dom}(t) = IndTax_{*,dom}(t) + DirTax_{*,dom}(t) + PubPenSysCont_{dom}(t)$$
$$- GovCons_{dom}(t) - r_{dom}(t)\,GovDebt_{dom}(t) - PubPenSysBen_{dom}(t)$$

$$NetSvngGov_{dom}(t) = IndTax_{*,dom}(t)$$
$$+ DirTaxWageY_{dom}(age,t) + DirTaxEntrY_{dom}(age,t) + DirTax\,\mathrm{Re}\,nt_{dom}(age,t)$$
$$+ DirTaxErngsFirmsKPvtPenSys_{dom,dom}(t) + DirTaxErngsFirmsKOthFinIns_{dom,dom}(t)$$
$$+ DirTaxErngsFirmsKPvtPensSysPort_{for,dom}(t)$$
$$+ DirTaxYFirmsKOthFinInsPort_{for,dom}(t) + DirTaxErngsFirmsKOthFinInsFDI_{for,dom}(t)$$
$$+ PubPenSysContWageY_{dom}(t) + PubPenSysContEntrY_{dom}(t)$$
$$- GovCons_{dom}(t) - PubPenSysBen_{dom}(t)$$

Adding across sectors and making cancellations,

$$NetNatSvng_{dom}(t) =$$
$$WageY_{dom}(t) + \mathrm{RentalY_{dom}(t)} + EntrY_{dom}(t)$$
$$+ ErngsFirms\ KPvtPenSys_{dom,dom}(t) + ErngsFirmsKOthFinIns_{dom,dom}(t)$$
$$+ ErngsFirmsKPvtPenSys_{for,dom}(t) + ErngsFirmsKOthFinInsPort_{for,dom}(t)$$
$$+ DivDistErngsFirmsKPvtPenSys_{dom,for}(t) + DivDistErngsFirmsKOthFinInsPort_{dom,for}(t)$$
$$- DivDistErngsFirmsKPvtPenSys_{for,dom}(t) - DivDistErngsFirmsKOthFinInsPort_{for,dom}(t)$$
$$+ ErngsFirmsKOthFinInsFDI_{dom,for}(t) - DirTaxErngsFirmsKOthFinInsFDI_{dom,for}(t)$$
$$+ IndTax_{*,dom}(t) + DirTaxErngsFirmsKOthFinInsFDI_{for,dom}(t)$$
$$- Cons_{dom}(t) - GovCons_{dom}(t)$$

Now we want to arrive at the same expression working from the top down. Since

$$NetNatSvng_{dom}(t) = NatDispY_{dom}(t) - Cons_{dom}(t) - GovCons_{dom}(t)$$

the task can be translated into that of proving

$$\begin{aligned}
&NatDispY_{dom}(t) = \\
&\quad WageY_{dom}(t) + \mathrm{RentalY}_{\mathrm{dom}}(t) + EntrY_{dom}(t) \\
&\quad + ErngsFirms\ KPvtPenSys_{dom,dom}(t) + ErngsFirmsKOthFinIns_{dom,dom}(t) \\
&\quad + ErngsFirmsKPvtPenSys_{for,dom}(t) + ErngsFirmsKOthFinInsPort_{for,dom}(t) \\
&\quad + \mathrm{D}ivDistErngsFirmsKPvtPenSys_{dom,for}(t) + \mathrm{D}ivDistErngsFirmsKOthFinInsPort_{dom,for}(t) \\
&\qquad - \mathrm{D}ivDistErngsFirmsKPvtPenSys_{for,dom}(t) - \mathrm{D}ivDistErngsFirmsKOthFinInsPort_{for,dom}(t) \\
&\quad + ErngsFirmsKOthFinInsFDI_{dom,for}(t) - \mathrm{D}irTaxErngsFirmsKOthFinInsFDI_{dom,for}(t) \\
&\quad + IndTax_{*,dom}(t) + \mathrm{D}irTaxErngsFirmsKOthFinInsFDI_{for,dom}(t)
\end{aligned}$$

First, we express *NatDispY* in terms of *GNP*

$$\begin{aligned}
&NatDispY_{dom}(t) = GNP_{dom}(t) - DeprRate_{dom}(t)KTot_{dom,dom}(t) - DeprRate_{for}(t)K_{dom,for}(t) \\
&\quad + IndTax_{for,dom}(t) - IndTax_{dom,for}(t)
\end{aligned}$$

Using the expression for GNP from above,

$$\begin{aligned}
&NatDispY_{dom}(t) = \\
&\quad WageY_{dom}(t) + EntrY_{dom}(t) + \mathrm{Re}\,ntalY_{dom}(t) \\
&\quad + ErngsFirmsKPvtPenSys_{dom,dom}(t) + ErngsFirmsKOthFinIns_{dom,dom}(t) \\
&\quad + ErngsFirmsKPvtPenSys_{for,dom} + ErngsFirmsKOthFinInsPort_{for,dom}(t) \\
&\quad + DivDistErngsFirmsKPvtPenSys_{dom,for}(t) + DivDistErngsFirmsKOthFinInsPort_{dom,for}(t) \\
&\quad + ErngsFirmsKOthFinInsFDI_{dom,for}(t) - DirTaxErngsFirmsKOthFinInsFDI_{dom,for}(t) \\
&\quad - DivDistErngsFirmsKPvtPenSys_{for,dom}(t) - DivDistErngsFirmsKOthFinInsPort_{for,dom}(t) \\
&\quad + DirTaxErngsFirmsKOthFinInsFDI_{for,dom}(t) \\
&\quad + IndTax_{*,dom}(t) + DeprRate_{dom}(t)KTot_{*,dom} - DeprRate_{dom}(t)KTot_{for,dom}(t) - IndTax_{for,dom}(t) \\
&\quad - DeprRate_{dom}(t)KTot_{dom,dom}(t) - DeprRate_{for}(t)K_{dom,for}(t) + IndTax_{for,dom}(t) - IndTax_{dom,for}(t) \\
&\quad + DeprRate_{for}(t)KTot_{dom,for}(t) + IndTax_{dom,for}(t)
\end{aligned}$$

Making cancellations,

$$
\begin{aligned}
&NatDispY_{dom}(t) = \\
&\quad WageY_{dom}(t) + EntrY_{dom}(t) + \mathrm{Re}\,ntalY_{dom}(t) \\
&\quad + ErngsFirmsKPvtPenSys_{dom,dom}(t) + ErngsFirmsKOthFinIns_{dom,dom}(t) \\
&\quad + ErngsFirmsKPvtPenSys_{for,dom} + ErngsFirmsKOthFinInsPort_{for,dom}(t) \\
&\quad + IndTax_{*,dom}(t) \\
&\quad + DivDistErngsFirmsKPvtPenSys_{dom,for}(t) + DivDistErngsFirmsKOthFinInsPort_{dom,for}(t) \\
&\quad + ErngsFirmsKOthFinInsFDI_{dom,for}(t) - DirTaxErngsFirmsKOthFinInsFDI_{dom,for}(t) \\
&\quad - DivDistErngsFirmsKPvtPenSys_{for,dom}(t) - DivDistErngsFirmsKOthFinInsPort_{for,dom}(t) \\
&\quad + DirTaxErngsFirmsKOthFinInsFDI_{for,dom}(t)
\end{aligned}
$$

National disposable income calculated "bottom-up" as the sum across households, firms, and government was

$$
\begin{aligned}
&NatDispY_{dom}(t) = \\
&\quad WageY_{dom}(t) + \mathrm{RentalY_{dom}}(t) + EntrY_{dom}(t) \\
&\quad + ErngsFirms\ KPvtPenSys_{dom,dom}(t) + ErngsFirmsKOthFinIns_{dom,dom}(t) \\
&\quad + ErngsFirmsKPvtPenSys_{for,dom}(t) + ErngsFirmsKOthFinInsPort_{for,dom}(t) \\
&\quad + \mathrm{D}ivDistErngsFirmsKPvtPenSys_{dom,for}(t) + \mathrm{D}ivDistErngsFirmsKOthFinInsPort_{dom,for}(t) \\
&\quad - \mathrm{D}ivDistErngsFirmsKPvtPenSys_{for,dom}(t) - \mathrm{D}ivDistErngsFirmsKOthFinInsPort_{for,dom}(t) \\
&\quad + ErngsFirmsKOthFinInsFDI_{dom,for}(t) - \mathrm{D}irTaxErngsFirmsKOthFinInsFDI_{dom,for}(t) \\
&\quad + IndTax_{dom,dom}(t) + IndTax_{for,dom}(t) + \mathrm{D}irTaxErngsFirmsKOthFinInsFDI_{for,dom}(t)
\end{aligned}
$$

Rearrangement shows that the two expressions are identical.