International Institute for Applied Systems Analysis
IIASA www.iiasa.ac.at

Using Trade-off Information in Attributes' Investing

Jog, V.M., Kaliszewski, I. and Michalowski, W.

IIASA Interim Report
April 1998

IIASA

International Institute for Applied Systems Analysis • A-2361 Laxenburg • Austria
Tel: +43 2236 807 • Fax: +43 2236 71313 • E-mail: info@iiasa.ac.at • Web: www.iiasa.ac.at

INTERIM REPORT IR-98-19 / April 1998

Using Trade-off Information in Attributes' Investing

Vijay Jog(vjog@business.carleton.ca)
Ignacy Kaliszewski(kaliszew@ibspan.waw.pl)
Wojtek Michalowski(michalow@iiasa.ac.at)

Approved by
Pekka Korhonen (korhonen@iiasa.ac.at)
Leader, *Decision Analysis and Support Project (DAS)*

Abstract

The paper describes the use of trade-off information to create effective stock portfolio characterized by the desired values of selected stock attributes. The basic notions behind such a process of portfolio creation are discussed and related to multi attribute analysis which is done by evaluating compensations among the attributes' values. A framework to construct a portfolio using only compensatory information is presented and applied to the analysis of stocks traded on the Toronto Stock Exchange.
Keywords: trade-offs, portfolio creation, portfolio tilting, dominance.

About the Authors

Vijay Jog is Professor of Finance at the School of Business, Carleton University, Ottawa, Ontario, Canada.

Ignacy Kaliszewski is a Senior Research Scholar with the Systems Research Institute, Polish Academy of Sciences, Warsaw, Poland.

Wojtek Michalowski is a Senior Research Scholar with the Decision Analysis and Support Project at IIASA.

Contents

1 Introduction **1**

2 Attributes' Investing **2**

3 Basic Notions Behind Trade-off Analysis of Stocks' Attributes **4**

4 Creating and Tilting Effective Portfolios **7**

5 Discussion of the Results **9**

Using Trade-off Information in Attributes' Investing

Vijay Jog(vjog@business.carleton.ca)
Ignacy Kaliszewski(kaliszew@ibspan.waw.pl)
Wojtek Michalowski(michalow@iiasa.ac.at)

1 Introduction

The paper presents an approach to portfolio construction which allows one to characterize different classes of stocks with the help of trade-off information about their attributes. We assume the role of assisting a portfolio manager who is in charge of an all-equity portfolio for a pension fund or a mutual fund. The role of this manager is to search for stocks which would generate superior returns, given the client's risk preferences and profile. To fulfill this role, a manager typically evaluates a large number of stocks and chooses those which satisfy the clients' requirements. In selecting this preferred set, the manager relies on an analysis of a complex set of interrelated attributes and their values. The main goal of this paper is to introduce a framework which is grounded in an analysis of compensations between values of attributes of stocks and which can be used to assist the portfolio manager in selecting an effective portfolio.

We consider portfolio construction a challenge where the manager is often confronted with multiple criteria evaluations (see for example, Nijkamp and Spronk, 1981; Spronk, 1981,1990; Spronk and Hallerbach, 1997). A recently recommended approach to tackle this type of a challenge involves use of expert system technologies, which have been applied in the area of capital budgeting (Myers, 1988), commercial and consumer lending (Srinivasan and Kim, 1988), stock market prediction (Braun and Chandler, 1987), loan default (Shaw and Gentry, 1988), bankruptcy analysis (Messier and Hansen, 1988), and portfolio selection (Jog and Michalowski, 1994). In some cases the systems were designed to suggest or to recommend specific actions, in others they were designed to act in an advisory capacity.

We are proposing an approach based on a different philosophical principle than an expert system or traditional decision support system. Instead of evaluating the individual stocks on the grounds of their absolute performance, we argue that the relative importance of an attribute's value defined through trade-offs might provide additional insight into stocks' selection for a portfolio. Thus, establishment of some *boundary* values for trade-offs among values of stocks' attributes should allow for

*This research was supported by a grant from the Natural Sciences and Engineering Research Council of Canada

their better categorization. We apply this theoretical framework to the analysis of stocks listed on the Toronto Stock Exchange, as described in Section 4.

In proposing our framework we assume that *a priori* understanding and description of the preferences of a manager is an impossible task due to the information acquisition limits. In order to disclose implicit investment strategy, it is necessary to acquire some additional information about the manager's preferences. Previous studies (Fischoff et al., 1988) show that the most appropriate method to achieve this is through an interactive approach. Such an approach to stocks' selection proposed here *develops* an effective portfolio through an iterative and interactive process similar to that employed in interactive multiple criteria decision-making methods (Gardiner and Steuer, 1994; Korhonen, 1992), while at the same time conforming to the recommendations for interactive decision support outlined by Dyer et at. (1992).

In general, the framework described in this paper is applicable to a decision situation where choice can be justified by its posterior consistency. Such an assumption implies conformity to the posterior rationality principle (March, 1988) which advocates discovery of intentions of a decision maker instead of the interpretation of *a priori* position. Thus, such a posterior rationality framework provides additional flexibility in representing choices in comparison to the classical rationality frameworks (Bell et al., 1988; French, 1986). Basing this research on our earlier works (Kaliszewski and Michalowski, 1997a; 1997b), we propose to represent posterior preferences through the bounds imposed on trade-offs calculated for the values of stocks' attributes. Such an interpretation allows for *tilting* portfolio according to a manager's preferences in a process also called *attributes' investing* (i.e. choosing stocks with certain values of their attributes). The consequences of such investment decisions are observed and interpreted through the construction of a system of bounds which determine acceptable ranges of the trade-offs for selected attributes while considering portfolios which generate superior returns.

The paper is organized as follows. In the next section we discuss the attributes' investing problem. In Section 3 we present basic theoretical foundations of our framework. Methodology for creating effective portfolios used in the study is described in section 4. The application of a proposed framework for the analysis of stocks listed on the Toronto Stock Exchange and the corresponding results are described in Section 5.

2 Attributes' Investing

Since the advent of the Modern Portfolio Theory (MPT) stemming from the work of Markowitz (1959), the notion of diversified portfolios has become one of the most fundamental concepts of portfolio management. While developed as a financial economic theory in a normative framework, the MPT has spawned a variety of applications and provided background for further theoretical models. For the purpose of this paper, three aspects of the research arising from MPT are most relevant.

First, the MPT was derived using a representative investor belonging to the normative utility framework, which manifested in portfolio optimization techniques based on the mean-variance rule, and it proved to be sufficiently rich to provide the main theoretical background for the analysis of importance of diversification.

Second, MPT gave rise to a variety of asset pricing models for stock pricing, the most known among them being the Capital Asset Pricing Model (CAPM) (Mossin 1966) and the Arbitrage Pricing Model (APM) (Roll and Ross, 1984).

Third, it gave rise to performance measurement techniques designed to evaluate the performance of professional portfolio managers against a benchmark portfolio (Jensen, 1968).

All these aspects of the research led to the notion that the best managed portfolio is the one which is most widely diversified and such a portfolio may be created through passive buy-and-hold investment strategy. Subsequently, this led to a belief that investors should not expect professional portfolio managers to outperform a well-diversified benchmark, such as the S&P500 in the U.S. or the TSE300 in Canada. However, somewhat restrictive assumptions behind the MPT and CAPM have also resulted in a line of research which has criticized this way of reasoning for portfolio management and performance evaluation. The main thrust of this research is a notion that reliance on the MPT and CAPM for active portfolio management may result in ignoring both the firm (or stock) specific attributes as well as the multi-attribute nature of the investors preferences (Spronk and Hallerbach, 1997). In addition, a variety of anomalies in stocks returns have been reported in the literature, which further questions the viability of sole reliance on the MPT and CAPM for portfolio decisions. Since it is beyond the scope of this paper to summarize the vast amount of literature in this area, we recommend interesting reviews by Fama (1991) or Ziemba (1994).

It is important to stress that all these studies point to the fact that through attributes' investing, an investor may be able to achieve superior performance compared to the buy and hold portfolio strategy. Such a notion of *tilting* the portfolio defines a systematic approach to constructing a portfolio which has a higher (or lower) value of a particular set of attributes. Professional managers, as well as sophisticated investors, began evaluating the impact of tilting portfolio based on attributes other than the mean-variance and systematic risk. Thus, attributes' investing results in over- or under-investing in stocks which have the same (expected) mean-variance but which have certain values of other attributes deemed as desirable by an investor. The main thrust of frameworks for such portfolio construction has been to investigate the models for and consequences of moving away from benchmark portfolios or their proxies as represented by various stock indices. For example, if the investor believes that, all else being equal, he/she would prefer a low dividend paying stock over high dividend paying stock, it implies tilting the portfolio in such a way that it would either include a higher number of low dividend paying stocks or would have a higher investment in the few low dividend paying stocks. Similarly, tilting can be used not only for stocks but for any other asset categories such as bonds or international investments.

A single attribute tilting can be extended to multi attribute tilting by estimating the sensitivities of each attribute under consideration to the risk-return structure of the underlying stocks. A typical approach is to resort to goal programming which is applied to generate a portfolio achieving as close as possible aggregate target values (Spronk and Hallerbach, 1997).

A strategy of tilting portfolios in growth and value stocks is well-known. For example, Sorensen and Thum (1992) claim that overweighing (tilting) the portfolio in value stocks can produce returns which are superior (on a risk-return basis) to a benchmark portfolio. On the other hand, Kritzman (1987) provides a three-step methodology to uncover the investor's style by identifying the attributes considered important for investment analysis and then analyzing the sensitivity of these attributes to changes in portfolio. He claims that such a methodology helps to distinguish between management style and discretionary investment judgments. The thrust of his approach is not to assist in portfolio tilting but to identify the implicit tilting associated with certain management styles. Berry et al (1988) propose a concept of *risk sterilization* where assets with different risk profiles are combined so as to negate, or *sterilize*, exposure to selected risk factors. Their strategy requires that the sensitivity of each stock in the portfolio to the individual risk factors is estimated and that these sensitivities remain constant during the time period under consideration. In that case, a portfolio can be constructed which matches the benchmark portfolio on all attributes except the one chosen for tilting.

Similar examples of attributes' investing can also be found in the management of bond portfolios and international investments. For example, Seix and Akhoury (1986) describe three approaches for constructing an optimal bond index portfolio with the desired characteristics, while Macedo (1995) extends the tilting concept to international investing. She claims that overweighting an investment towards countries with high relative value or with high relative strength generates higher returns than an equal-weighted benchmark.

Recent studies (Reinganum, 1988; Ziemba, 1994) indicate that attributes' investing has been established as a method to achieve either a superior risk-return characteristic in a portfolio or to achieve a portfolio with over or under weighted specific attribute(s). The proposed methodologies rely on identifying the sensitivities of various attributes to stock returns and then constructing the portfolio to achieve a desired set of characteristics. Despite the fact that attributes' investing generally results in gains in the values of tilting attributes, and loss in the values of other attributes, no attention is paid to the compensations (trade-offs) between the values of the tilting attributes.

We argue that tilted portfolio exhibits specific properties of trade-offs calculated for tilting attributes. Such information can be used to control *speed of change* between gains and losses related to attributes' investing. The framework proposed in this paper is designed to provide portfolio manager with a control instrument to regulate speed of exchange between gains and losses in tilting attributes' values. Such an instrument is based on a concept of trade-off and trade-off bounding. In the next section we describe theoretical foundations of trade-off analysis and demonstrate its usefulness for attributes' investing decisions.

3 Basic Notions Behind Trade-off Analysis of Stocks' Attributes

The methodology discussed here was presented in detail in Kaliszewski (1993, 1994) and Kaliszewski and Michalowski (1997b). In this section it is described in a format of an analysis of stocks. However, it can be abstracted to any problems which involve set representation and its mapping into the k-dimensional real space.

As stated in the previous section, our problem is to identify subsets of stocks with superior return profiles, using trade-off type of information about values of stock attributes. Here we propose a theoretical framework which allows one to identify stocks which follow rules based on composite indicators, as defined below.

Let N stocks be given, $|N| \geq 2$, with each described by a vector y composed of k numbers y_i, $i = 1, ..., k$. The number y_i is the value of the i-th attribute for stock y. Thus, each stock can be represented by a specific vector y. Without loss of generality we can assume that there are no two stocks y and y', such that $y = y'$, and that all stocks' attributes are of the type *the more the better.*

Let $\bar{y}$ be a given stock.

If $y_i \leq \bar{y}_i$, $i = 1, ..., k$, for all stocks, then we say that $\bar{y}$ *dominates* all other stocks. In this case the stock $\bar{y}$ is a more reasonable candidate for a portfolio than any other stock.

If $y_i \leq \bar{y}_i$, $i = 1, ..., k$, for some stock y, then we say that $\bar{y}$ *dominates* y. In this case the stock $\bar{y}$ is a more reasonable candidate for a portfolio than y.

Thus it can be stated that the most reasonable stocks for inclusion in a portfolio are those which do not have any other stock dominating it. We call such stocks *nondominated.* However, assuming that portfolios are composed exclusively of non-dominated stocks this may, in practice, be too restrictive and therefore it is not adopted here.

Consider a stock $\bar{y}$ such that it does not dominate all other stock. Suppose another stock exists, say y such that $y_i \geq \bar{y}_i$ for some i. We may consider whether y is a better candidate for a portfolio than $\bar{y}$. If $y_i \geq \bar{y}_i$ for all $i = 1, ..., k$, then by the dominance relation given earlier, y dominates $\bar{y}$ (the possibility that $y = \bar{y}$ has been excluded), and therefore y is a more reasonable candidate for a portfolio than $\bar{y}$. Suppose that for some j, $j \neq i$, we have $y_j < \bar{y}_j$. Stock y offers an increase (or no change if $y_i = \bar{y}_i$) in the value of an attribute i with respect to $\bar{y}$, namely

$$y_i - \bar{y}_i \geq 0$$

at the expense of a loss (decrease) in the value of an attribute j, namely

$$\bar{y}_j - y_j > 0\,.$$

Now let us consider the attributes i and j only.

To decide whether $\bar{y}$ is more preferred than y we can use a relative measure of preference, such as

$$\frac{y_i - \bar{y}_i}{\bar{y}_j - y_j} \tag{1}$$

which describes how many units of the increase in value of the attribute i are gained for an unit of decrease in value of the attribute j.

If the ratio (1) is high (higher than a certain threshold), y is preferred to $\bar{y}$. If it is low (lower than a certain threshold), $\bar{y}$ is preferred to y. With intermediate values of (1), $\bar{y}$ is as attractive as y. Any projection of *high*, *low*, and *intermediate* values onto a cardinal scale is obviously subjective.

The ratio (1) is a convenient measure of attractiveness for a pair of stocks involving two selected attributes i and j.

A general measure of attractiveness of stock y is a *trade-off*:

$$T_{ij}(\bar{y}) = \sup_{\substack{y_i \geq \bar{y}_i \\ \bar{y}_j > y_j}} \frac{y_i - \bar{y}_i}{\bar{y}_j - \bar{y}_j} \qquad (2)$$

where the supremum is taken over all stocks being considered. Note that for a finite set, taking the supremum is equivalent to taking the maximum. If the trade-off is not defined (there is no such y that $y_i \geq \bar{y}_i$ and $\bar{y}_j > y_j$), then we put $T_{ij}(\bar{y}) = -\infty$.

If the trade-off is high (higher than a certain threshold), then at least one stock exists which is more attractive than $\bar{y}$. Thus, $\bar{y}$ should be discarded and not considered as a reasonable candidate for a portfolio. Consequently, reasonable candidates for a portfolio are stock with a trade-off below some threshold.

Trade-offs can be calculated for all stock. Given a threshold, *attractive* stocks are those for which $T_{ij}(y)$ is less than or equal to the threshold value. All the remaining stocks may be labelled as *nonattractive* for a portfolio.

We can take into account other pairs of attributes, calculating corresponding trade-offs (compensations) and setting for each pair the appropriate threshold. In this way the proposed measure of preference becomes multidimensional. One possibility of dealing with this is to neglect all nonattractive stocks. In other words, a stock is an attractive candidate for a portfolio if all its trade-offs are less or equal to the corresponding thresholds. If for a pair of attributes a threshold is not given (we may assume that it is equal to $+\infty$) the corresponding trade-off plays no role in the stock selection process and consequently stocks with arbitrary large trade-offs for that pair of attributes are attractive candidates for a portfolio.

Remark 3.1 *The word trade-off should be used with caution. Some authors use this word to describe the ratio (1) (Zionts and Wallenius, 1983), but the most common definition of trade-off is as in (2) with the additional constraint*

$$y_l \geq \bar{y}_l\,,\ l = 1, , ..., k\,,\ l \neq i, j\,, \qquad (3)$$

(Kaliszewski, 1994). This latter classical definition comes from early research on continuous mathematical programming. However, since in continuous mathematical programming sets for which trade-offs are defined are infinite, the definition of trade-off adopted in this paper would be in that case of limited use.

To illustrate that point let us consider the following example.

Let $Z = \{y \in R^k \mid \sum_{i=1}^{k} y_i = 1\,,\, y_i \geq 0\,,\ i = 1, ..., k\}$. *It is easy to see that for each efficient element* y *of* Z *such that* $y_i > 0\,,\ i = 1, ..., k$, *no trade-off as defined by (2) exists. However, adding additional constraints (3), i.e. for*

$$\sup_{\substack{y_l \geq \bar{y}_l \\ l = 1, ..., k\,, l \neq j \\ \bar{y}_j > y_j}} \frac{y_i - \bar{y}_i}{\bar{y}_j - y_j} \qquad (4)$$

guarantees that all suprema are finite.

In a finite case of a portfolio creation, both definitions can be applied, but it seems that the less restrictive definition (2) may be more appropriate, and therefore it was used in the experiment described in this paper.

Example 3.1 *This simple example shows how the process of selection of stocks using just trade-off information may work.*

A sample set of stocks represented by values of two selected attributes is presented in Figure 1. The values of the attributes are:

$$(16,1);(8,2);(11,3);(25,4);(19,5);$$
$$(22,6);(7,7);(24,8);(20,9);(10,10);$$
$$(16,11);(16,12);(20,13);(22,14);(15,15);$$
$$(16,16);(18,17);(5,18);(2,19);(19,20);$$
$$(15,21);(12,22);(9,23);(10,24);(4,25)\,.$$

Suppose that the predefined trade-off thresholds are: for $T_{12}(y)$ it is 2, and for $T_{21}(y)$ it is 3. The following stocks satisfy these thresholds (24,8);(22,14); (18,17);(19,20);(10,24) as shown in Figure 2.

4 Creating and Tilting Effective Portfolios

Tilting of a portfolio can be achieved in several different ways. One almost classical approach (Ziemba, 1994) involves development of a portfolio using stocks which guarantee the best values for tilting attributes. Such portfolio is normally created using top quartiles of stocks determined for each of the tilting attributes separately. An *intersection* of top quartiles establishes a portfolio (further labelled as TOP) which has potential to generate above average returns. In this study we attempt to establish whether it is possible to develop an equally good or better portfolio using information which can be extracted from the analysis of values of tilting attributes jointly. We argue that an additional analysis of compensations (trade-offs) between values of the tilting attributes can constitute a useful carrier of such information. Our argument is further reinforced by the fact that quite often it is difficult to decide about specific thresholds for tilting attributes, and also by a behavioral observation of investment decisions where it is important to analyze possible trade-offs between different investment opportunities. On this basis we put forward a hypothesis that examining and using a relationship between levels of compensations among tilting attributes can lead to better attributes' investing decisions. Testing of such a hypothesis is feasible due to the methodological developments in trade-off bounding outlined in Section 3.

A portfolio (labelled as TDOM) created entirely on the basis of the attributes' values compensatory information is developed according to the following procedure:

1. Identify a candidate set of N stocks.
2. Select the attributes for tilting and impose upper bounds on their trade-offs.

These upper bounds constitute the thresholds for investors' insensitivity to compensations between values of tilting attributes and are inferred from the reference TOP portfolio.

3. Identify those stocks with trade-offs satisfying imposed bounds. Label these stocks a *tilting set*.

4. If the cardinality of the tilting set is too small, then go to step 5; otherwise go to step 6.

5. Bounds defined in step 2 are too restrictive. Relax bounds on the trade-offs and go to step 3.

6. Find the nondominated elements of the tilting set and label the resulting set as the TDOM portfolio.

The procedure to create the TDOM portfolio was applied to stocks traded on the Toronto Stock Exchange. For each stock the following attributes were chosen as tilting attributes: *price per share, price to earnings ratio, price to cash flows ratio, and price to adjusted after-tax operating earnings ratio.* These are some of the attributes commonly used in the attributes' investing decisions. Values of these attributes were taken for the period 1991-1994 from version 4.6 of the Stock Guide database. The rate of return for the stocks under consideration was drawn from the return and index files of the TSE/Western database. To qualify for inclusion in the analysis for a given year, a firm had to meet the following requirements:

1) the firm must belong to an industry group other than Gold and Precious Metal; Mining Exploration without Production; Utilities; Financial Services including Insurance,

2) the firm must be listed on the Toronto Stock Exchange,

3) the firms common stock must be traded at a yearly closing price equal to or above CAD $1.50 in 1985 constant dollars,

4) the firm must have a total annual revenue higher than CAD $1.5 million in 1985 constant dollars,

5) the firm must have a year-end total asset base larger than CAD $3.0 million in 1985 constant dollars,

6) the firm must have defined total annual revenue, year-end price, year-end total assets and other relevant data available in the Stock Guide database as per requirements (2), (3), and (4) above and for calculating price-related stock attributes.

Requirement (1) excludes firms in those industry groups where price-related attributes may not be easily interpreted. Requirements (2), (3), and (4) impose size constraint on the firms, to ensure that the considered companies are large enough to be frequently researched and considered by investment professionals. In addition, these requirements control potential biases that characterize very small firms. Requirements (5) and (6) ensure that meaningful data are available. Although a number of stocks under consideration are reduced by enforcing the above requirements, a net benefit associated with clarity of the data set is considerable.

As a result of application of the above six criteria for inclusion in the sample, the considered number of firms averaged around 300 during each year of the four year period.

The construction of TOP and TDOM portfolios was conducted as follows. Firms belonging to the TOP portfolio were those whose attribute values ranked them in the top 25 percentile in all rankings of the sample firms. Such rankings were conducted every year. On average, TOP portfolio contains seven to nine firms and it provides a reference for a successful investment strategy. Once the stocks in the TOP portfolio were identified, the TDOM portfolio was constructed using the procedure described earlier. The number of firms comprised in the TDOM portfolio ranged from two to six. Thus, compared to the traditional notion of a diversified portfolio containing about thirty stocks, none of these portfolios can be considered as well-diversified. The small number of stocks in these portfolios is strictly a function of a sample size and has nothing to do with the portfolio creation framework proposed here.

Once the portfolio creation process was completed for each year, monthly returns for the portfolio were constructed in the following manner. The monthly returns of individual stocks comprising a portfolio were used to construct an equally weighted monthly return series for the portfolio. Since the portfolio composition changed at the end of each year depending upon the values of stocks' attributes, the portfolio returns were constructed sequentially for each of the succeeding twelve months. Such a process was repeated for each of the four years. The portfolio cumulative wealth index (PCWI) used to evaluate performance of a portfolio, represents a value of one dollar of investment of this annually rebalanced portfolio return. Note, that for our data, the PCWI tracks the value of one dollar invested on January 1, 1992 with equal proportion in each stock comprising the respective portfolios and re-balanced annually to accommodate changes in the portfolio composition due to the changes in the values of the attributes. The PCWI values were calculated for each of the three portfolios. The relative performance of these portfolios over a 48 month period, and that of an additional portfolio (labelled TOPD and IND, and whose PCWI were calculated along the same lines) is discussed below.

5 Discussion of the Results

Figure 3 shows the performance of the two portfolios based on their PCWI values for the four years of the sample period. It also shows performance of the IND portfolio corresponding to the TSE300 market index. We decided to include IND portfolio in the analysis for purely comparative and reference purposes. Such inclusion helps also to reinforce an argument behind attributes' investing stating that it is possible to create a portfolio outperforming a market benchmark.

In general, it is clear that the TDOM portfolio performs as well or better than the TOP portfolio (with both of them outperforming the IND portfolio). This result indicates that the information on compensation among tilting attributes provides some useful insights into the construction of attractive portfolios and introduces relative stability into the portfolio tilting process.

We decided to extend the analysis by also comparing the TDOM portfolio with a modified TOP portfolio. A rationale behind such an extension is that just as TDOM consists of nondominated stocks only, the same is not the case for the TOP portfolio. Hence, we have created a modified TOP portfolio (labelled TOPD) consisting of the nondominated stocks. The PCWI for both TOPD and TDOM portfolios calculated for the 48 months period are graphically tracked on Figure 4. As can be seen, the TOPD outperforms TDOM portfolio. This is quite an interesting observation, but unfortunately of limited practical value because of a very small diversification of the TOPD portfolio. It consists of as little as one stock for 1994 and as many as three stocks for 1992.

In light of the above, and considering the good performance of the TDOM portfolio in comparison with the TOP portfolio, it is appropriate to state that the information about compensation among the tilting attributes can play an important role in addressing the attributes' investing. One should not forget here that the TDOM portfolio was created using just information about compensations (trade-offs) between tilting attributes, completely ignoring the actual values of these attributes. Such an approach to attributes' investing gives satisfactory results, but may generate less diversified portfolio. Moreover, it clearly demonstrates the importance of analyzing the nondominated stocks, as exemplified by the superior performance of the TOPD portfolio. If there is a restriction on the minimum number of stocks which must be held in the portfolio, then one could resort to relaxing the notion of dominance by introducing some form of ϵ-*dominance* measure which would allow to include ϵ-dominated stocks into a portfolio. Independently from a measure of dominance being applied while selecting attractive candidates for a portfolio, one can state that the results of our experiment confirm the merits of using relative information (represented by the trade-offs) in the attributes' investing. It is plausible to assume that experienced investors consider this kind of information at a cognitive level of their decision making process.

In this paper we have considered only four tilting attributes while constructing the TDOM portfolio. Obviously, from a methodological viewpoint, there is no restriction on the number of attributes. The framework proposed here provides sufficient guidelines for implementing the attributes' investing with due regard to the compensatory information about values of the attributes of interest.

References

Bell, D.E., H. Raiffa and A. Tversky (1988). "Descriptive, Normative, and Prescriptive Interactions in Decision Making", In D.E. Bell et al. (eds) *Decision Making. Descriptive, Normative, and Prescriptive Interactions*, New York, Cambridge University Press.

Berry, M.A., E. Burmeister and M.B. McElroy (1988). "Sorting Our Risks Using APT Factors", *Financial Analysts Journal*, 2, 29-42.

Braun, H. and J.S. Chandler (1987). "Predicting Stock Market Behavior Through Rule Induction: An Application of the Learning-From-Example Approach", *Decision Sciences*, 18, 415-429.

Dyer, J.S., P. Fishburn, R. Steuer, J. Wallenius and S. Zionts (1992). "Multiple Criteria Decision Making, Multiattribute Utility Theory: The Next Ten Years", *Management Science*, 38, 645-655.

Fama, E.F. (1991). "Efficient Capital Markets", *Journal of Finance*, 46, 1575-1617.

Fischoff, B., P. Slovic and S. Lichtenstein (1988). "Knowing What You Want: Measuring Labile Values", In D.E. Bell et al. (eds) *Decision Making. Descriptive, Normative, and Prescriptive Interactions*, New York, Cambridge University Press.

French, S. (1986). *Decision Theory: An Introduction to the Mathematics of Rationality.* Chichester, Ellis Horwood.

Gardiner, L.R. and R.E. Steuer (1994). "Unified Interactive Multiple Objective Programming", *European Journal of Operational Research*, 74, 391-406.

Jensen, M.C. (1968). "The Performance of Mutual Funds in the Period 1945-1964", *Journal of Finance*, 33, 389-416.

Jog, V.M. and W. Michalowski (1994). "An Interactive Procedure for Learning about Preferences: Case Study of a Portfolio Manager", *Journal of Multi-Criteria Decision Analysis*, 3, 27-40.

Kaliszewski, I.

(1993). "Calculating Trade-offs by Two-step Parametric Programming", *Central European Journal of Operational Research and Economics*, 2, 291-305.

(1994). *Quantitative Pareto Analysis by Cone Separation Technique.* Dordrecht, Kluwer Academic Publishers.

Kaliszewski, I. and W. Michalowski

(1997a). "Psychological Stability of Solutions in the Multiple Criteria Decision Problems", International Institute for Applied Systems Analysis, IR-97-060

(1997b). "Efficient Solutions and Bounds on Trade-offs", *Journal of Optimization Theory and Applications*, 44, 381-394.

Korhonen, P. (1992). "Multiple Criteria Decision Support: The State of Research and Future Directions", *Computers and Operations Research*, 19, 549-551.

Kritzman, M. (1987). "How to Build a Normal Portfolio in Three Easy Steps", *The Journal of Portfolio Management*, 3, 21-23.

Macedo, R. (1995). "Value, Relative Strength, and Volatility in Global Equity Country Selection", *Financial Analysts Journal*, 2, 70-78.

March, J.G. (1988). "Bounded Rationality, Ambiguity, and the Engineering of Choice", In D.E. Bell et al. (eds) *Decision Making. Descriptive, Normative, and Prescriptive Interactions*, New York, Cambridge University Press.

Markowitz, H.M. (1959). *Portfolio Selection.* New York, J. Wiley.

Messier, W.F. and J.V. Hansen (1988). "Inducing Rules For Expert System Development: An Example Using Default and Bankruptcy Data", *Management Science*, 34, 1403-1415.

Mossin, J. (1966). "Equilibrium in a Capital Asset Market", *Econometrica*, 10, 768-783.

Myers, S.C. (1988). "Notes on an Expert System for Capital Budgeting", *Financial Management*, 17, 23-31.

Nijkamp, P. and J. Spronk (eds) (1981). *Multiple Criteria Analysis: Operational Methods*, London, Grover Press.

Reinganum, M.R. (1988). "The Anatomy of a Stock Market Winner", *Financial Analysts Journal*, 2, 17-28.

Roll, R. and S. Ross (1984). "The Arbitrage Pricing Theory Approach to Strategic Portfolio Planning", *Financial Analysts Journal*, 3, 14-26.

Seix, C. and R. Akhoury (1986). "Bond Indexation: The Optimal Quantitative Approach", *The Journal of Portfolio Management*, 2, 50-53.

Shaw, M.J. and J.A. Gentry (1988). "Using an Expert System with Inductive Learning to Evaluate Business Loans", *Financial Management*, 17, 45-56.

Sorensen, E.H. and C.Y. Thum (1992). "The Use and Misuse of Value Investing", *Financial Analysts Journal*, 2, 51-58.

Spronk, J. (1981). *Interactive Multiple Goal Programming: Applications to Financial Planning*, Boston, Martinus Nijhoff.

Spronk, J. (1990). "Interactive Multifactorial Planning: State of the Art", In C. Bana e Costa (ed). *Readings in Multiple Criteria Decision Aid.* Heidelberg, Springer Verlag.

Spronk, J. and W. Hallerbach (1997). "Financial Modelling: Where to Go? With an Illustration for Portfolio Management", *European Journal of Operational Research*, 99, 113-125.

Srinivasan, V. and Y.H. Kim (1988). "Designing Expert Financial Systems: A Case Study of Corporate Credit Management", *Financial Management*, 17, 32-44.

Ziemba, W.T. (1994). "World Wide Security Market Regularities", *European Journal of Operational Research*, 74, 198-229.

Zionts, S. and J. Wallenius (1983). "An Interactive Multiple Objective Linear Programming Method for a Class of Underlying Nonlinear Utility Functions", *Management Science*, 29, 519-529.

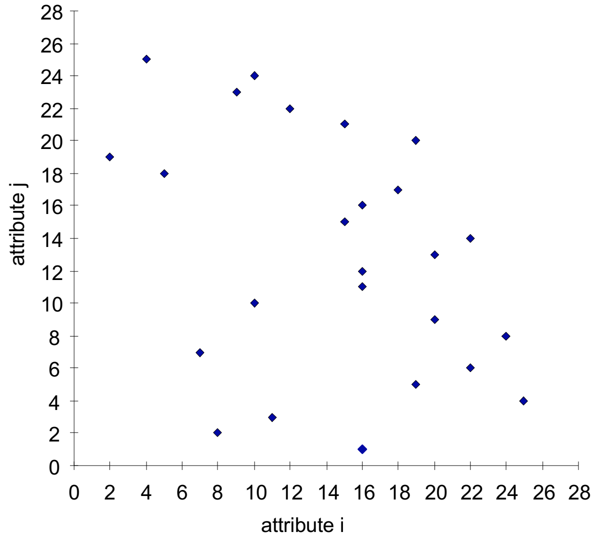

Figure 1

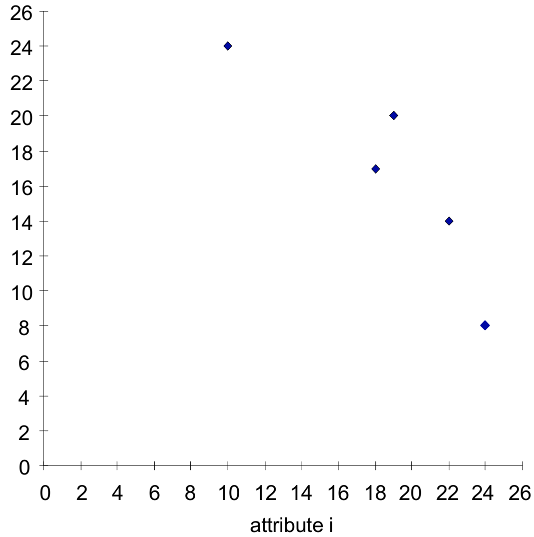

Figure 2

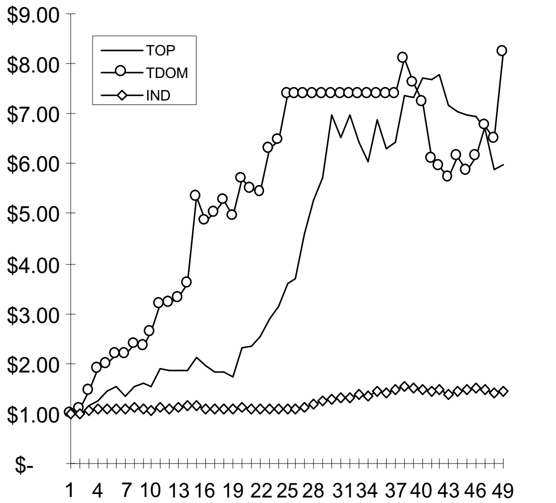

Figure 3. Portfolio Cumulative Wealth Index for TDOM, TOPD, and IND Portfolios

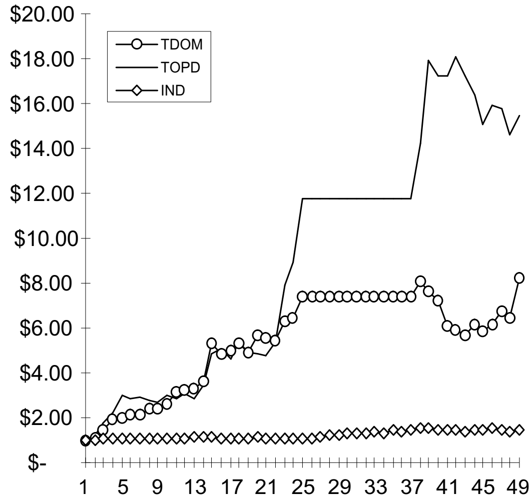

Figure 4. Portfolio Cumulative Wealth Index for TDOM, TOPD, and IND Portfolios