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Abstract

The paper describes how the MCDM modelling framework can be extended to account for a notion of regret. The non-regrettable decisions are generated in accordance with a DM's regret attitude which is established through an analysis of the trade-offs. Decisional validity of a proposed modeling framework is illustrated with a simple example.

Keywords: MCDM; Regret Theory; Non-regrettable Decisions; Trade-offs

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1 Introduction

A classical decision problem concerns a choice (from a given set) of "the best" decision according to decision making circumstances. A very common and frequently used interpretation of such a problem specifies (either by explicit enumeration or by implicit constraints) a set of feasible decisions which are evaluated by a decision maker (DM) with respect to their attributes. Since such an interpretation leads most likely to a partial ordering of decisions, it is often assumed that a DM is willing to provide some additional information concerning his/her preferences so complete ordering can be established. The multiple criteria decision making (MCDM) provides a framework for gathering and processing such information in a convenient interactive manner (Wierzbicki (1980); Chankong and Haimes (1983); Yu (1985); Steuer (1986)). However, the process of preference elicitation from a DM is often cast in a rigid and structured form, thus the resulting preferred decision becomes a subject of posterior regret associated with selecting it instead of some other one.

The purpose of this paper is to show how a notion of decisional *regret*, originating from extensions of the expected utility theory can be incorporated into the MCDM modelling framework, and how *non-regrettable* decisions conforming to a DM's regret attitude can be identified.

The paper is organized as follows. In the next section we give an overview of interactions between the behavioral research and the MCDM. In Section 3 we position the notion of regret in the MCDM modelling framework. In Section 4 we give basic theoretical foundations of trade-offs and trade-off bounding. In Section 5 we discuss a notion of DM's regret attitude. Section 6 deals with non-regrettable decisions, whereas Section 7 gives an example illustrating how regret attitude of a DM incorporated into the MCDM framework may influence a choice process. The paper concludes with a discussion.

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2 Background

Decision problems considered in the MCDM literature have their foundations in classical economic analysis of choice operationalized by von Neumann and Morgenstern (1944). According to such considerations a DM is perceived as a pure utility maximizer who makes choices which maximize an explicitly unknown but assumed utility function. Such an approach to problem solving is prevalant in the MCDM literature despite evidence supplied by the behavioral scientists that it is not always appropriate to relate decision rationality to utility maximization. For example, Allais (1988) or Ellsberg (1961) provided convincing examples demonstrating deviations from this principle. The limitations of explanatory powers of utility maximization were partially accepted in the literature, as illustrated by the papers presented at the Santa Cruz conference (Edwards (1992)). In order to consider the departures in a DM's choice behavior from a generalized expected utility framework, one should look for decisions in the MCDM problems which have properties other than just Pareto-optimality (Larichev and Moshkovich (1997)).

Within the MCDM formulation, one of the first attempts to broaden utility maximization paradigm was a modeling framework developed according to Simon's bounded rationality principle (Simon (1956)). This work was carried out by Wierzbicki (1980) who proposed the aspiration-based methodology. With this methodology, prospective decisions are identified by probing a feasible set with the help of a so called *scalarizing function* (see for example, Wierzbicki (1986) for a detailed explanation). Instead of identifying decisions which maximize utility function, Wierzbicki proposes to identify those which satisfy a DM's preference expressed through the setting of scalarizing parameters.

A bounded rationality principle is supported by experimental psychological research which questions the use of the expected utility as a descriptive tool of individual decision making. Prospect theory considered as a choice generating algorithm (Kahneman and Tversky (1990)) represents such a line of research. According to this theory, a DM must "edit" prospects (attributes of decisions) prior to selecting a decision in order to account for his/her (risk seeking or risk averse) attitude. Korhonen et al., (1990) attempted to incorporate findings of prospect theory into the MCDM framework by using it to justify the inconsistent results of an experimental study involving interactive programming methods where risk seeking and risk averse attitudes were considered. Another line of research attempting to incorporate behavioral aspects into a normative MCDM framework dealt with a role of intuition in decision making (Wierzbicki (1997)). Wierzbicki's argument was that intuition can be captured within the aspiration-based methodology developed earlier, when piecewise linear scalarizing function is used to probe a feasible decision set.

A behavioral notion of *regret* (first formulated by Savage (1954)) associated with not achieving by a DM his/her expectations also plays an important role in the analysis of choice. So far, despite obvious implications of regret for choice process as considered in MCDM, there are no methodological approaches which attempt to incorporate this notion into a modelling framework. The research reported here presents an initial attempt to tackle this problem.

3 Regret Theory and the MCDM

A notion of regret has its source in what Bell (1985) terms as "... a comparison of an outcome with the payoff one could have had by making different choice". Diverging from a view taken by Savage (1954), Loomes and Sugden (1982) present a *regret theory* arguing that it allows for less restrictive treatment of rationality. Regret theory in such a form is based on two fundamental assumptions:

a) people experience the sensations of regret and rejoicing;

b) while making decisions people try to anticipate and take into account those sensations.

Munier (1989) observed that regret theory is, in a sense, a generalization of the expected utility, by considering that evaluations of decision not only depend on direct decision, but also on those which a DM could achieve if he/she had made a different choice.

Phenomenon of regret studied so far is firmly embedded in the expected utility framework, where probabilistic evaluations of prospective decisions are possible. An interesting extension of that framework was proposed by Acker (1997) who considered a choice situation under *total ignorance*. Such a consideration implies that despite a probabilistic nature of the decision process, a DM does not have sufficient information to make meaningful assumptions regarding the probabilities, thus making in fact a probabilistic information obsolete. Within the total ignorance framework Acker (1997) proposed a new notion of *tempered regret* where disappointment associated with not achieving "the best" is compensated by the joy of avoiding the worst.

A choice situation under total ignorance is similar to the deterministic choice problems considered in the MCDM. Despite a probabilistic character of a process being modelled, and in light of lack of the relevant probabilistic evaluations (situation of total ignorance), a DM is represented as assessing decisions within a deterministic framework.

In an expected utility maximization situation, incorporation of regret (for example, application of a minimax regret rule as proposed by Savage) usually leads to a construction of a regret matrix representing opportunity cost of choosing one action over another. Such a matrix is built assuming that a level of regret is being measured as a difference between "the best" decision, represented column-wise, for a given situation (state), represented row-wise, and all other decisions for that state. Under the assumption stated by Loomes and Sugden, combined with a state of total ignorance, and with a premise that every decision is evaluated with respect to multiple attributes, the notion of regret becomes meaningful in the MCDM modelling framework. Indeed, if a notion of *state* is equated to a notion of *attribute*, and a state/attribute matrix conveys regret type information (for example, the difference between *ideal* and actual values of attributes), the equivalence is complete.

As stated earlier, regret occurs when a given decision is compared with another one (having at least one better value of an attribute) which one could have selected under different circumstances. Since "the best" (ideal) decision where values of all attributes are maximal does not normally exist, then one would expect that regret should be naturally associated with a choice process in MCDM. We argue that this association can be expressed and operationalized by means of the trade-off analysis.

This allows one to address a problem common in the MCDM, that numerous decisions must be compared in order to complete a regret (or pay-off) assessment, and such a comparison is often infeasible. In the next section we discuss basic theoretical and methodological notions of the theory behind trade-offs evaluation and trade-off bounding which will be used later to identify non-regrettable decisions without a necessity of extensive evaluations of all possible decisions.

4 Trade-offs and Trade-off Bounding

It is natural for a DM to look for the choices which he/she is not going "to regret". In other words, a DM would expect from a preferred decision to have a limited "compensation" potential in some attributes' value gain, for a loss in value of the other attribute (a "robustness" in terms of attribute *gain to loss ratios*). Lack of limited (in the subjective terms) compensation potential might inflict a feeling of regret associated with not selecting another decision. With such an interpretation we should be able to analyze a DM's regret attitude using trade-offs which evaluate relative gains and losses in values of the attributes. Manipulation of bounds imposed on the trade-offs and their acceptance or rejection by a DM will reflect his/her sensitivity to changes in the attributes' values. In order to do so, we need to introduce some basic notions and definitions associated with trade-off theory.

Under the convention that higher values of attributes are preferred to lower, a trade-off is defined for a particular decision (identified by a vector of its attributes y), and for a selected pair of attributes (identified by the components y_i of y, $i = 1, ..., k$). A trade-off specifies an amount by which the value of one decision attribute increases (*gain*) while the other decreases (*loss*) when moving away from a given decision along a feasible direction (i.e. a direction which leads to another feasible decision). Usually two types of trade-offs are considered: *point-to-point* trade-offs and *global* trade-offs.

A point-to-point trade-off is represented as a ratio of relative value increase in one attribute per one unit of value decrease in a reference attribute when a particular decision is replaced by another *given* decision.

A global trade-off for a decision $\bar{y}$ is defined as:

Definition 4.1 *Let Z be a set of feasible decisions and $\bar{y} \in Z$.* **Global trade-off** *$T_{ij}^G(\bar{y})$ involving attributes i and j, $i, j = 1, ..., k$, $i \neq j$, is defined as*

$$\sup_{y \in Z_j^<(\bar{y})} \frac{y_i - \bar{y}_i}{\bar{y}_j - y_j}.$$

where: $Z_j^<(\bar{y}) = \{y \in Z \mid y_j < \bar{y}_j, \ y_l \geq \bar{y}_l, \ l = 1, ..., k, \ l \neq j\}, \ j = 1, ..., k.$

A comprehensive review of a theory behind global trade-offs is given in (Kaliszewski (1993); (1994); Kaliszewski and Michalowski (1995),(1997a)). In this section we will present only those results which are relevant for a methodological framework discussed in Sections 5 and 6.

For finite Z, trade-offs (either global, or point-to-point) are calculated in a straightforward manner. For infinite Z we need to consider the following model of a decision problem:

$$\text{"max" f(x) s.t. } x \in X_0 \subseteq X \,, \tag{1}$$

where vector function $f : X \to R^k$, $f = (f_1, f_2, ..., f_k)$, is a vector of the objective functions $f_i : X \to R$, X_0 is the set of feasible decisions. As before, we can describe feasible decisions by vectors y , where $y = f(x)$, $x \in X_0$, with a clear correspondence between components y_i of y and an attribute i of a decision problem. It has been demonstrated in Kaliszewski and Michalowski (1997a) how to identify decisions with *a priori* specified upper bound(s) on global trade-offs while considering model (1).

Theorem 4.1 (Kaliszewski,Michalowski (1997a)) *Suppose $\bar{y}$ solves the following problem*

$$\min_{y \in Z} \max_i \lambda_i((y_i^* - y_i) + \sum_{t=1}^{k} \rho_t(y_t^* - y_t)) \,, \tag{2}$$

where $\lambda_i > 0$ and $\rho_i > 0$ for each i . Then,

$$T_{ti}^G(\bar{y}) \leq (1 + \rho_i)\rho_t^{-1}$$

for each $i, t = 1, ..., k$, $i \neq t$.

Theorem 4.2 (Kaliszewski,Michalowski (1997a)) *An element $\bar{y} \in Z$ is properly efficient[1] if and only if there exists a vector λ , $\lambda > 0$, and numbers ρ_i , $\rho_i > 0$, $i = 1, ..., k$, such that $\bar{y}$ solves*

$$\min_{y \in Z} \max_i \lambda_i((y_i^* - y_i) + \rho_i e^k(y^* - y)) \,, \tag{3}$$

5 Defining a DM's Regret Attitude

Ability to identify and select decisions with desired values of trade-offs (global - as shown above, or point-to-point), permits us to use this kind of information in describing DM's regret attitude. An analysis of bounds imposed by a DM on selected trade-offs has certain similarities with classical analysis of a DM's risk attitude within the expected utility framework. As a DM's risk profile is identified through the evaluation of his/her attitude towards different lotteries, a DM's regret attitude can be determined through an analysis of his/her preference of different bounds imposed on trade-offs.

An important instrument in the analysis of risk attitude is provided by a notion of a *certainty equivalent* which establishes "a value" that would make a DM indifferent between an uncertain event and that value. This notion is used while determining a DM's risk attitude as either risk *neutral*, risk *seeking*, or risk *averse*. The knowledge of presence of risk seeking or risk averse behavior is crucial, as such behavior often signifies that the decisions are being made in discordance with the expected utility framework. This is because a DM's risk attitude introduces new dimension into the

[1] For the definition of proper efficiency see Geoffrion (1968), cf. also eg. Kaliszewski (1994), Kaliszewski, Michalowski (1997a).

analysis of the decisions. By the same token, knowledge of a DM's regret attitude is important for appropriate evaluation of "the best" decision.

According to a description of regret given earlier in this paper, it is possible to state that regret attitude reflects a DM's *proneness* to speed of change in attribute values associated with moving away from a given decision. In that sense, higher trade-off (global, point-to-point) associated with a decision y than with a decision $\bar{y}$ means that "more" regret can be experienced by a DM when selecting y instead of $\bar{y}$. Similarly, smaller trade-off implies less potential regret. This is because smaller value of a trade-off implies slower speed of changes in the attribute values.

A DM's preference of a "narrow bound" for trade-offs implies that he/she is willing to accept decisions with a relatively low potential regret. Such a DM is a *regret prone* person who would try to avoid experiencing a feeling of regret or disappointment. On the other hand, an acceptance of a "wider bound" for a trade-off implies that a DM is indifferent to a potential regret. Such a DM is a *regret neutral* person who is not influenced by regret or disappointment while making choices. There is no need for regret equivalent of a risk aversion attitude of a DM as regret aversion is included in a definition of a regret prone behavior. The identification of regret attitudes can be accomplished through an interactive establishment of patterns of bounds on trade-offs. This is further discussed in Section 6.

6 Non-regrettable Decisions

The MCDM literature usually considers that a search for "the best" decision is guided by some unique underlying principle (Gardiner and Steuer (1994); Lewandowski and Wierzbicki (1989); Michalowski and Szapiro (1992); Zeleny (1982)). Adherence to such a principle simplifies a search process, but also does not provide enough flexibility in exploring a DM's behavioral considerations. Satisfactory decisions (in a sense of Simon's bounded rationality and identified by Wierzbicki's scalarizing functions) and a desire to reach them, should guide an initial search for "the best" decision. However, when some satisfactory decisions are identified, values of their attributes no longer carry sufficient discriminatory information to distinguish among them. At that point, we propose to consider DM's regret attitude. Upon generating some satisfactory decision, further assessment should incorporate a DM's regret attitude expressed through bounding of the trade-offs. Such an approach allows one to "fine tune" the search for "the best" decision so that it is both satisfactory and it fits the DM's regret attitude. This leads us to the introduction of non-regrettable decisions into the MCDM modelling framework.

Definition 6.1 *A satisfactory decision y which has* **acceptable** *trade-offs is called* **non-regrettable**.

Observe that introduction of the non-regrettable decisions into the MCDM modelling framework may result in the elimination of some satisfactory decisions. Despite the fact that definition 6.1 is purely a qualitative one we should not propose any specific values for acceptable trade-offs to describe non-regrettable decisions, since they depend entirely on a DM's regret attitude.

Recently Kaliszewski and Michalowski (1997b) proposed an algorithm in which desired levels of objective functions and bounded trade-offs are two compensatory search principles in an interactive decision process. Psychologically stable solutions which are generated by this algorithm are according to definition 6.1 also non-regrettable.

7 Illustrative Example

Let us consider a water quality management problem formulated and solved in (Makowski et al., (1996)). In general terms, this problem deals with selecting affordable technologies for water treatment to satisfy water quality standards in the Nitra river basin in Slovakia. The problem was formulated as a mixed integer multiple criteria optimization problem. The set of efficient decisions was searched using Wierzbicki's scalarizing functions (Wierzbicki (1980)).

Two decision attributes were examined in greater detail, namely the minimum concentration of dissolved oxygen (DO) and the total investment cost (INV). A number of decisions were found with DO concentration around 5 mg/l, which is the level commonly considered while establishing goals for national water quality. A subset of efficient decisions with satisfactory levels of DO identified by Makowski et al. consisted of the following (efficient with respect to the whole original set of criteria) points:

	INV	DO
P1	11.3	4.93
P2	12.6	4.98
P3	14.5	5.05
P4	14.8	5.08
P5	13.1	5.10

(INV in millions of US$).

A straightforward observation was made that constraining DO to the level of 5 mg/l or greater would eliminate the slightly worse, in respect to DO concentration, but much less expensive decisions P1 and P2. This observation was used by the authors to illustrate differences between single and multiple criteria formulation of decision problems. It was observed that decisions P1 and P2 offered a good $\frac{\textit{gain in decreased INV}}{\textit{loss in decreased DO}}$ trade-off relative to decisions P3, P4, or P5 (see table).

	P1	P2	P3	P4	P5
P1	*	-	-	-	-
P2	26	*	-	-	-
P3	26.66	27.14	*	-	-
P4	23.33	22	10	*	-
P5	10.59	4.16	-	-	*

where "-" denotes that there is no trade-off associated with given pair of decisions.

Let us assume that all five decisions are satisfactory and thus lets temporarily omit all other decisions. It is possible to conduct a posterior analysis of those decisions in terms of the decisional regret. The maximal value of trade-offs involving gain in decreased INV and loss in decreased DO are:

for P1	-
for P2	26.00
for P3	27.14
for P4	23.33
for P5	10.59

For example, a regret prone DM (assuming his/her acceptance of a trade-off bound at the level of 20) would have a choice between either decision P1 or P5, as both are the non-regrettable ones.

An alternative and more cunning approach to posterior evaluation of decisions is to elicit from a DM his/her regret attitude a priori and encapsulate it in the form of bounds on trade-offs. Such an elicitation need not be a one step process but a result of some stepwise analysis. Having done so, one can make direct use of the elicited information to search for non-regrettable decisions fitting DM's regret attitude. In this example, assuming a DM's acceptance of trade-off threshold is at the level of 20 or less, the efficient frontier of the model of the problem can be searched (cf. Theorem 4.1 solving (2) by for various λ_i , $i = 1, ...k$, with parameters ρ_{INV} and ρ_{DO} satisfying $20 \leq \frac{1+\rho_{DO}}{\rho_{INV}}$.) As a result, only non-regrettable decisions with $T_{INV\ DO}(y) \leq 20$ will result from such a search.

8 Discussion

A notion of regret is naturally associated with problems of choice. It is manifested by a DM's disappointment which accompany a specific choice while the other one could be made. There are several proposals for expanding the expected utility framework to account for that phenomenon (see discussion in Section 3). However, none of these proposals were adopted within the MCDM modelling framework. In this paper we demonstrated that it is possible to expand the cognitive limits of the MCDM models by focusing on the non-regrettable decisions. These decisions are satisfactory for a DM, and at the same time, they fit his/her regret attitude. We argue that an analysis of a potential relative "speed of change" in attribute values (as measured by a trade-off) provides a convenient proxy for an assessment of such an attitude. Analysis of different bounds imposed on trade-offs allows one to classify a DM as either regret prone or neutral, and this information is incorporated into a search process. By employing an argument borrowed from the expected utility framework, we emphasize the benefits of establishing DMs' "regret attitude" within the MCDM modelling framework, as this kind of information contributes to behavioral validity of a process employed to reach "the best" decision.

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