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# **The Value of Preference Information in Agency Relationships**

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# The Value of Preference Information in Agency Relationships

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## Abstract

Standard models of agency theory typically assume that the principal has considerable information about the preferences of the agent. Once this assumption is relaxed, the question arises whether the principal should try to obtain additional information about the agent's preferences. In this paper we introduce the concept of a Value of Preference Information (VPI), which describes the benefits to the principal from obtaining additional information about the agent's preferences. We show analytically that the VPI is non-negative and that the VPI will not decrease when the principal's information structure is refined. Computational methods are used to study factors influencing the VPI. The results of these experiments show a strong relationship between the entropy of the principal's information structure and the VPI. It is thus possible to evaluate an information structure independently of the decision problem delegated to the agent.

#### Keywords

Agency theory, preference information, value of information

## About the Author

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## The Value of Preference Information in Agency Relationships

Rudolf Vetschera

### 1. Introduction

Agency models provide considerable insight into the consequences of asymmetric information in economic transactions. Typical problems studied in agency theory involve the hidden information problem, where an agent possesses private information about the environment, or the hidden action problem, in which the principal is not able to observe the actions taken by the agent (Grossman/Hart 1983; Spremann 1987; Levinthal 1988). To mitigate the effects of asymmetric information, the principal must design a compensation system, which induces to agent to act in the principal's interest. In designing a compensation system, the principal anticipates the agent's reactions to the incentive system, and chooses the incentive system which maximizes her net benefit (i.e. the benefit left to the principal after paying the incentives to the agent).

Agency models can be considered as a special case of hierarchical decision models (Schneeweiß 1995). Many important problems in hierarchical planning result from the fact that the upper level (the principal in an agency model) is not able to analyze the lower level's (the agent's) decision problem with the same precision as the lower level. Thus the upper level's anticipation of the lower level's reaction will only be approximate. In the hierarchical planning literature, considerable effort is made to study the effects of this approximation on decision quality.

This aspect is largely neglected in the agency literature. While asymmetry of information is considered with respect to the state of the environment or the actions taken by the agent, the agent's preferences are typically assumed to be known to the principal. In the terminology of hierarchical planning, the principal's anticipation of the agent's decision model is identical to the agent's decision model itself. Usually, these preferences are represented by the agent's utility function.

These assumptions about the principal's information on the agent's preferences can be regarded as a strong argument against the realism of agency models (Nilakant/Rao 1994). (Anthony 1988 p. 173-174) even calls agency models "worthless" because of their overly simplified assumptions. Even in models dealing with the hidden characteristics problem, where the principal lacks information about certain characteristics of the agent, it is usually assumed that the principal knows a probability distribution of utility functions across agents or at least the utility functions of several classes of agents, for which a self-selection contract is drawn up.

In (Vetschera 1998a; Vetschera 1998b) a model has been proposed which relaxes the stringent assumptions about the preference information available to the principal. This model is based on concepts from multi-criteria decision making under incomplete information (Weber 1987). Incomplete information models were originally developed for decision makers who are unable or unwilling to exactly articulate their preferences. The model uses these techniques to represent the situation of a principal who is not fully informed about the agent's preferences.

The amount of information a principal possesses about an agent's preferences can be of great importance in designing an incentive system. For example, a system of monetary incentives will work better with agents for whom income is an important goal, than with agents for whom it is not. We can therefore expect that a principal will benefit from obtaining more precise information about the agent's preferences. In the present paper, we extend the model of (Vetschera 1998a) to derive a measure for the value of preference information to the principal and to analyze the factors on which that value depends. As the resulting models are too complex to derive conclusions analytically, computational methods are used to obtain the results.

The remainder of the paper is structured as follows. In section two, we briefly review the agency model with incomplete preference information. In section three, we develop the concept of a value of preference information (VPI). Section four presents results of computational experiments to analyze how the VPI depends on various characteristics of the decision problem and the information system available to the principal. Section five concludes the paper by summarizing the results and identifying open questions for further research.

## 2. An Agency Model with Incomplete Preference Information

In this section, we will review the underlying model only briefly. the reader is referred to (Vetschera 1998b; Vetschera 1998a) for a more detailed analysis.

To highlight the incomplete preference information aspect of the problem, the model only deals with a decision situation under certainty. This is in contrast to the usual assumptions of agency models, where uncertainty of the environment is an important factor. In fact, in the decision problem modeled here, it would be possible for the principal to design a first best contract, in which the agent is rewarded only if he chooses the alternative which is optimal for the principal. We will assume that this is not possible for some reason and the incentive system has to be designed in the way usually considered in agency models, i.e. incentives are specified as a linear function of the profit to the principal.

The decision situation could also be interpreted as a decision under risk when both the principal and the agent are risk neutral and have the same probabilistic information about the environment. Effects of asymmetric information about the environment or different risk attitudes could be incorporated into the model to bring it more in line with traditional agency models. But such modifications would further increase the complexity of the model without offering additional insight into the problem studied here.

The agent has to select his action among a set of discrete alternatives  $A_i$ : i=1,...,N. Each alternative is characterized by a vector of *K* attributes:  $A_i = (a_{i1},...,a_{iK})$ . The first attribute represents the profit each alternative yields for the principal, the other attributes represent characteristics of the alternative which are relevant to the agent. These attributes could relate, for example, to the effort the agent must spend in carrying out that alternative or the prestige associated with realizing the alternative or other "personal interests" (Lindstädt 1997) an agent might have in an alternative. All attributes are to be maximized.

The incentive system consists in paying the agent a fixed portion c of the profit of the selected alternative. Thus the principal retains a net profit of  $(1-c) \cdot a_{il}$  if the agent selects alternative  $A_i$ . A possible fixed payment, which is offered to the agent independently of the result obtained, would be irrelevant to the results obtained here and thus is not explicitly taken into account in the model.

The agent is assumed to have a linear utility function which, taking into account the fact that the agent receives only part of the profit of each alternative, can be written as

$$u(A_{i}) = w_{1} \cdot c \cdot a_{i1} + \sum_{k=2}^{K} w_{k} \cdot a_{ik}$$
(1)

The weights  $w_k$  (k=1,...,K) are known to the agent, but not to the principal. The principal only knows upper and lower bounds on the weights so that

$$w_k \le w_k \le \overline{w_k} \tag{2}$$

An extension of the model to more general (linear) conditions on the weights is straightforward, but in this paper we will only consider lower and upper bounds. Furthermore, without loss of generality, we assume that the weights are scaled so that

$$\sum_{k} w_{k} = 1 \tag{3}$$

The set

$$W = \left\{ (w_1, \dots, w_K) \middle| \underline{w_k} \le w_k \le \overline{w_k}; \sum_k w_k = 1 \right\}$$
(4)

is the set of all weight vectors the principal considers as possible. We denote the subset of weight vectors for which alternative  $A_i$  is optimal according to (1) by  $O(A_i) \subseteq W$ . Set  $O(A_i)$  is defined by the following linear constraints on the  $w_k$ 's:

$$w_{1} \cdot c \cdot a_{i1} + \sum_{k=2}^{K} w_{k} \cdot a_{ik} \ge w_{1} \cdot c \cdot a_{j1} + \sum_{k=2}^{K} w_{k} \cdot a_{jk} \quad \forall j \neq i$$

$$\frac{w_{k}}{\sum_{k} w_{k}} \le w_{k} \le \overline{w_{k}}$$
(5)

From (5), it is obvious that any fixed compensation to the agent that is paid independently of the alternative chosen would cancel out from the left and right hand sides of the first constraint and thus does not influence  $O(A_i)$ .

Since the principal has only incomplete information on the agent's preferences, she can anticipate the agent's reaction to an incentive system only stochastically. Assuming that weight vectors are uniformly distributed over  $W_{\downarrow}$  the probability that the agent chooses alternative  $A_i$  is given by

$$p(A_i|c,W) = \frac{Vol(O(A_i))}{Vol(W)}$$
(6)

where Vol(.) denotes the volume of the respective polyhedron. We use the notation  $p(A_i|c, W)$  to denote that this probability depends on the level of compensation *c* as well as the information *W*. The expected net profit of the principal is therefore given by

$$G(c,W) = (1-c) \cdot \sum_{i=1}^{N} p(A_i | c, W) \cdot a_{i1}$$
(7)

We assume that the principal is risk neutral and thus wishes to maximize G(c, W). We define

$$c^{*}(W) = \arg \max G(c, W) \tag{8}$$

as the optimal level of compensation for a given set of preference information W.

#### 3. The Value of Preference Information

The question now arises whether the principal is able to increase her net profit by obtaining additional information on the agent's preferences, i.e. by reducing the set W. In (Vetschera 1998a), we have analyzed the effects of changes in the upper and lower bounds for a single weight  $w_k$ . In this paper, we extend this approach in two directions: firstly, we consider more general information structures. Secondly, we establish the value of information in an ex ante situation, before additional information on the agent's preferences is actually obtained.

The approach taken here to determine the value of preference information is thus similar to the calculation of the value of information in decision problems under risk (Lindley 1985). The entire framework is outlined in figure 1.

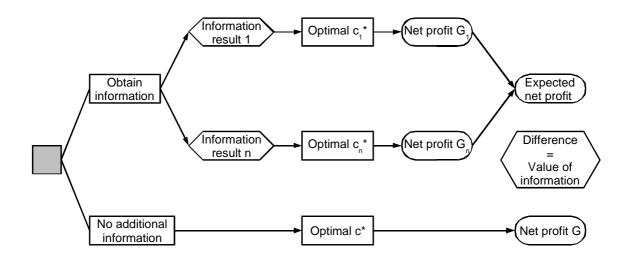


Figure 1: Framework for determining the VPI

We consider the principal's decision situation before information is received. If the principal does not receive additional information on the agent's preferences, she can determine the optimal level of compensation  $c^*$ , leading to an expected net profit *G*. If she decides to obtain additional information, she might receive one of several possible information results. Each of these information results corresponds to a different set  $W_j$ , for which the corresponding optimal  $c_j$  can be determined, leading to an expected net profit  $G_j$ . The expected net profit after information can then be computed by taking the expectation over all possible information results. The value of information is the difference between the expected net profit after obtaining information and the expected net profit without information.

To formalize this approach, we introduce the concept of an *information structure I*. We define an information structure as a partitioning of set W into disjoint subsets  $W_j$  which fulfill the following two properties:

$$\bigcup_{j} W_{j} = W$$

$$W_{i} \cap W_{l} = \emptyset \quad \forall l \neq j$$
(9)

In order to compute the VPI for an information structure I, we need to calculate

- the probabilities  $p(W_i)$  of obtaining the different information results  $W_i$  and
- the a posteriori choice probabilities  $p(A_i | c, W_j)$  for each alternative after obtaining information result  $W_j$ .

Using again the assumption of uniform distribution, the probabilities  $p(W_j)$  can be computed as

$$p(W_j) = \frac{Vol(W_j)}{Vol(W)}$$
(10)

The a posteriori choice probabilities are

$$p(A_i|c, W_j) = \frac{Vol(O(A_i) \cap W_j)}{Vol(W_i)}$$
(11)

For a given information structure I, the value of preference information is then defined as

$$VPI = \sum_{j} p(W_{j}) \cdot G(c^{*}(W_{j}), W_{j}) - G(c^{*}(W), W)$$
(12)

An obvious question is whether this value is always non-negative, i.e. whether obtaining (costless) preference information is always beneficial for the principal. This property can indeed be shown.

**Proposition 1:** The value of preference information defined in (12) is non-negative.

**Proof:** To prove this proposition, it is sufficient to show that, under any information structure, the principal can obtain at least the same expected net profit as without information. This can be achieved by selecting the same value  $c^*(W)$  for all information results. The expected net profit with information then becomes:

$$\sum_{j} p(W_{j}) \cdot G(c^{*}(W), W_{j}) =$$

$$\sum_{j} p(W_{j}) \cdot (1 - c^{*}(W)) \cdot \sum_{i} p(A_{i} | c^{*}(W), W_{j}) \cdot a_{i1} =$$

$$\sum_{j} \frac{Vol(W_{j})}{Vol(W)} \cdot (1 - c^{*}(W)) \cdot \sum_{i} \frac{Vol(O(A_{i}) \cap W_{j})}{Vol(W_{j})} \cdot a_{i1} =$$

$$(1 - c^{*}(W)) \cdot \sum_{i} a_{i1} \cdot \sum_{j} \frac{Vol(W_{j})}{Vol(W)} \cdot \frac{Vol(O(A_{i}) \cap W_{j})}{Vol(W_{j})} =$$

$$(1 - c^{*}(W)) \cdot \sum_{i} a_{i1} \cdot \sum_{j} \frac{Vol(O(A_{i}) \cap W_{j})}{Vol(W)} =$$

$$(1 - c^{*}(W)) \cdot \sum_{i} a_{i1} \cdot \frac{Vol(O(A_{i}) \cap W_{j})}{Vol(W)} = G(c^{*}(W), W)$$

The principal thus can at least obtain the same result as without information by leaving c unchanged. Since  $c^*(W_j)$  is the value of c maximizing the expected net profit for information result  $W_j$ , it will be different from  $c^*(W)$  only if  $c^*(W_j)$  yields a higher net profit. Therefore, the situation of the principal can only improve or remain unchanged by obtaining additional information. QED

In fact, proposition 1 is only a special case of a more general relationship. We call an information structure *I*' a *refinement* of information structure *I*, if

$$\forall W_k \in I' \exists W_i \in I: W_k \subseteq W_i \tag{14}$$

and for at least one  $W_k \in I$ :

$$W_k \subset W_i \in I \tag{15}$$

Thus, every information result in I' must be contained in only one result of I and at least one result of I' must be a proper subset of some result in I. I' is thus obtained from I by splitting up at least one result of I and not changing any existing boundaries between information results. By using the same argument as before, it can be shown that the VPI will never decrease when an information structure is refined.

## 4. Determinants of the VPI

The concept of refining information structures provides a ranking of information structures *independently of the actual decision problem* which is delegated to the agent. This is an extension to the framework depicted in figure 1, where the value of preference information depends on the decision problem and its solution obtained under different information sets.

But refinement only establishes a partial ordering of information structures. Changes in information structures which result from "shifting the borders" between different information results cannot be ranked using this concept. Consider for example an information structure  $I_1$ , which indicates that the agent's weight  $w_1$  is either high or low, i.e. that the agent is more or less responsive to monetary incentives. The concept of refinement states that an information structure  $I_2$ , which splits the weight space further along  $w_1$  and distinguishes between very highly, highly, moderately and weakly responsive agents is preferable to  $I_1$ . It also indicates that an information structure  $I_3$ , which not only differentiates for  $w_1$  but also for  $w_2$  and indicates whether the agent is strongly or weakly influenced not only by money but also by e.g. prestige, is preferable to  $I_1$ . But refinement does not allow us to determine whether  $I_2$  or  $I_3$  is better for the principal.

In this section, we will analyze problems of this kind. Such an analysis is not possible using analytical methods. In (Vetschera 1998a) it was shown that the optimal value  $c^*$  can be determined analytically only for problems with up to three alternatives, since calculating  $c^*$  for a problem with *n* alternatives involves solving a polynomial of degree 2(n-1). Therefore, we analyze different factors influencing the value of the VPI using computational methods.

#### 4.1 Entropy

Of particular interest are factors which are independent of the decision problem and involve only properties of the information structure. If it is possible to attribute differences in the VPI to differences in the information structure, more general recommendations can be given which information about an agent's preferences should be sought by the principal.

One potential approach to find such general factors is information theory. In information theory, the information content of a code is measured by its entropy, which is defined as (Conant 1990):

$$E = -\sum_{i} p_{i} \cdot \mathrm{Id}(p_{i}) \tag{16}$$

where  $p_i$  is the probability of occurrence of each symbol in the code and ld is the logarithm of base 2. The higher the entropy of a code, the higher is the information content of a message of given length written in that code.

We can interpret our information results as different symbols of a code. Receiving preference information is then equivalent to receiving a message of length 1 of that code. Information theory would then predict that a code (i.e. an information structure) with higher entropy would, on the average, provide more informative messages. This should be reflected in a higher VPI.

To test this assumption, computational experiments were performed in which different information structures (with different entropies) were applied to the same decision problem. To avoid overlaps with other effects (which were to be analyzed in later experiments), all information structures were generated by dividing the [0,1] interval for the weights  $w_1$  and  $w_2$  into two or three intervals at randomly chosen points. Using two or three intervals for the two weights resulted in four different types of information structures with 4, 6 and 9 information results respectively, which were analyzed separately. The other parameters are listed in table 1.

Number of attributes	3, 4
Number of alternatives	5, 10, 20
Number of decision problems	10
Information structures for each	20
decision problem	

Table 1: Parameter values for computational experiments

Table 2 gives an overview of the correlation coefficients between entropy of information structures and the value of information obtained in these experiments.

High correlations could be reached consistently across all parameter settings for at least some experiments. However, for some experiments the correlations were negative and the distribution is markedly skewed. The entropy of an information structure is thus not always a suitable predictor for the value of preference information across all decision problems.

The question now arises whether it is possible to identify those decision problems for which the entropy is a good predictor of the value of information, and those for which it is not. Table 2 indicates that both the average and the median correlation increase with the number of alternatives and decrease with the number of attributes. Another important factor is the structure of the problem. Figure 2 shows (for experiments with 3 attributes and 5 alternatives) the relationship between the optimal value of c without information and the correlation between entropy and VPI.

3 Attributes						
	Number of intervals				3	
Alternatives		2-2	2-3	3-2	3-3	
	Mean	0.4424	0.2525	0.3181	0.4682	
5	Min	-0.5210	-0.6390	-0.3540	-0.3481	
	Max	0.8413	0.7943	0.7311	0.8878	
	Median	0.5941	0.3034	0.3536	0.5288	
	Mean	0.4621	0.3768	0.4389	0.5661	
10	Min	-0.4524	-0.5227	-0.2999	-0.3754	
	Max	0.8473	0.7620	0.7105	0.8127	
	Median	0.6104	0.5726	0.5061	0.6964	
	Mean	0.7275	0.6888	0.5130	0.7734	
20	Min	0.3955	0.5123	0.3801	0.6356	
	Max	0.8950	0.8125	0.6529	0.8991	
	Median	0.7587	0.7157	0.5080	0.7757	

4 Attributes						
		Number of intervals				
Alternatives		2-2	2-3	3-2	3-3	
	Mean	0.0648	-0.0123	0.0561	0.1336	
5	Min	-0.5587	-0.6355	-0.4109	-0.5263	
	Max	0.8857	0.8272	0.7035	0.8357	
	Median	-0.2196	-0.3681	-0.2469	-0.1016	
	Mean	0.4430	0.3047	0.3510	0.5219	
10	Min	-0.4186	-0.5116	-0.2390	-0.0272	
	Max	0.8738	0.8584	0.7520	0.8700	
	Median	0.4655	0.2884	0.3231	0.5391	
	Mean	0.4879	0.4181	0.4408	0.5803	
20	Min	-0.1253	-0.3702	0.0224	0.1160	
	Max	0.8606	0.8383	0.7638	0.8948	
	Median	0.5898	0.5104	0.4907	0.6614	

Table 2: Correlations between entropy and value of preference information

The figure clearly indicates that low and even negative correlation coefficients occur only in problems where the optimal value of c is also low and in many instances zero. As soon as the optimal value of c reaches a threshold of about 0.05, high correlations are consistently obtained.

An optimal value for c of zero indicates that, without additional preference information, it is not possible for the principal to set up an effective incentive system. This might be the case if the difference in profit between the best and the second best alternative from the principal's point of view is rather small, but the difference in the other attributes from the agent's point of view is rather large. If the principal can not induce the agent to chose the best alternative, even by transferring to him the entire additional profit, it is optimal for the principal not to pay the agent any compensation (i.e. to set c=0) and let the agent chose his most preferred alternative (which is not that bad from the principal's

point of view, too). However, if, in such a situation, the principal finds out that the agent is highly responsive to monetary incentives (i.e. the agent's weight for incentives  $w_i$  is very high), then the principal might be able to induce the agent to select the best alternative from the principal's point of view without awarding him the entire additional profit. In this situation, only an information structure that allows the principal to identify such agents will be of positive value. But such an information structure has a low entropy, because is splits the parameter space into uneven parts: a relatively small part containing the high values of  $w_i$  and a larger part containing other weight vectors. We can therefore expect the correlation between entropy and value of preference information to be negative for these problems, while it is positive for more regular problems.

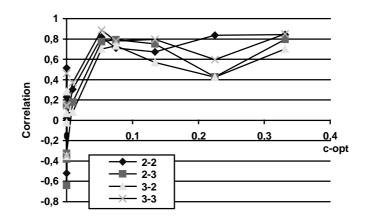


Figure 2: Relationship between optimal c and correlation between entropy and VPI

#### 4.2 Other Factors

While these results indicate that the entropy is an important factor influencing the VPI, it is certainly not the only factor. We can distinguish three levels of other factors: factors like the entropy, which are only related to the information structure, factors which are related to general properties of the decision problem and factors related to specific decision problems.

Information structures can have the same value of entropy, but relate to different attribute weights. The same probability distribution of information results (and thus the same entropy) can also be obtained by information structures that relate to one or to several weights. From an information theory point of view, this fact should make no difference. We thus can formulate the hypothesis that the VPI will not depend on the weights involved in defining the information results.

As second group of factors is related to the complexity of the decision problem which, in our framework, can be measured by the number of attributes and alternatives of the problem. Increasing the number of attributes makes it more likely that changes in one weight will be offset by changes in other weights and thus decrease the VPI. On the other hand, since a larger number of alternatives provides smaller steps by which the agent's behavior can change, we expect this variable to have a positive influence on the VPI.

The third group is related to characteristics of a specific decision problem. Two possible factors are the optimal level of c and the expected profit which can be obtained without preference information. Figure 2 already showed that optimal value of c without information interacts with the entropy of the information structure to determine the VPI, but it might also have a direct influence. Based on figure 2, we can formulate the hypothesis that the interaction of entropy and the optimal value of c will increase the VPI. A high initial value of c indicates that it is rather hard for the principal to provide an adequate incentive. Information will be particularly useful in this context, so we expect a high value of c to have also a positive effect on the VPI. On the other hand, if the expected profit without preference information is already high, it might be difficult to improve that result, so we expect this variable to have a negative effect on the VPI.

To analyze the potential effects of these variables, a combined regression analysis/analysis of variance was performed on the VPI obtained in the experiments using the SAS GLM procedure. Since significant correlations existed between the number of attributes and the other variables, the analysis was performed separately for the experiments with three and four attributes. Therefore the hypothesis on the number of attributes could not be tested. The main results are given in table 3.

3 Attributes ( $R^2 = 0.253566$ )						
Variable	Expected	F Value	Pr > F	Observed		
N. Alternatives	(+)	6.87	0.0088	(+)		
Type of Split	none	6.41	0.0003			
Entropy	(+)	56.41	0.0001	(+)		
Optimal <i>c</i>	(+)	78.86	0.0001	(-)		
Interaction	(+)	50.81	0.0001	(+)		
Expected Profit	(-)	273.11	0.0001	(-)		

4 Attributes ( $R^2 = 0.249885$ )						
Variable	Expected	F Value	Pr > F	Observed		
N. Alternatives	(+)	13.61	0.0002	(+)		
Type of Split	none	4.58	0.0033			
Entropy	(+)	78.02	0.0001	(+)		
Optimal <i>c</i>	(+)	0.34	0.5623			
Interaction	(+)	86.95	0.0001	(+)		
<b>Expected Profit</b>	(-)	20.02	0.0001	(+)		

Table 3: Results of linear model estimation

As can be seen from table 3, most hypotheses were confirmed by the statistical analysis, although to a varying degree. The number of alternatives has indeed a significantly positive effect on the VPI, although this effect is not as marked as that of other variables. Contrary to our hypothesis, the type of split does have a significant influence, but this influence is rather weak when compared to the other variables. The entropy and the interaction effect between entropy and optimal c both exhibited the expected highly

significant positive influence. Interestingly, no significant effect could be attributed to the optimal *c* in the experiments with four attributes. In the experiments with three attributes, the observed effect worked in the opposite direction as expected. Profit without information exhibited the expected influence only in the experiments with three attributes, while the effect was considerably weaker (and in the opposite direction as expected) in the experiments with four attributes. Since the entropy was thus shown to be an important factor influencing the VPI, a second set of experiments was performed to analyze the effect of other factors more closely by keeping the entropy constant across experiments. In these experiments, information structures with maximum entropy were used. Since these are different for three and four attributes, separate experiments for these two conditions were performed.

In both cases, the information structures used consisted of four information results, which were generated by splitting the parameter space for the first attribute (profit) and the second attribute into two parts each or by splitting only the parameter space for the first or the second attributes into four parts. In the following figures, these different types of information structures are denoted by 41 (Splitting the parameter space for the first attribute into four parts), 22 and 14. The experiments of type 22 determine the maximum entropy that can be obtained, since it is not possible to split the parameter space into parts of the same size using only bounds on weights. For the case of three attributes, the maximum entropy of 1.01615 is obtained by splitting the parameter space for both weights into the intervals (0, 0.275) and (0.275,1). For experiments with four attributes, the maximum entropy of 1.36859 is obtained by splitting the parameter space for the first two weights into the intervals (0, 0.201) and (0.201,1).

3 Attributes (R <sup>2</sup> =0.086399)					
Variable	Expected	F Value	Pr > F	Observed	
N. Alternatives	(+)	23.49	0.0001	(+)	
Type of split	none	149.01	0.0001		
Expected profit	(-)	166.57	0.0001	(-)	
Optimal <i>c</i>	(+)	11.43	0.0007	(+)	

4 Attributes (R <sup>2</sup> =0.362513)					
N. Alternatives	(+)	393.34	0.0001	(+)	
Type of split	none	676.73	0.0001		
Expected profit	(-)	355.47	0.0001	(-)	
Optimal <i>c</i>	(+)	931.17	0.0001	(+)	

 Table 4: Results, constant entropy

Table 4 shows the results of a estimating a linear model on the remaining influence factors. In contrast to the experiments with different entropy, there are marked differences between the cases of three and four attributes. The overall fit of the model is rather poor for the case of three attributes, while in the experiments with four attributes, it is even better than in the experiments with different entropy values. Conforming to our hypothesis, the number of attributes had a positive influence. The expected profit without information had a consistent negative influence on the value of preference information. The optimal value of c, which in the previous experiments was found to influence the VPI mainly through its interaction with entropy, has a significant positive influence in the experiments with four attributes.

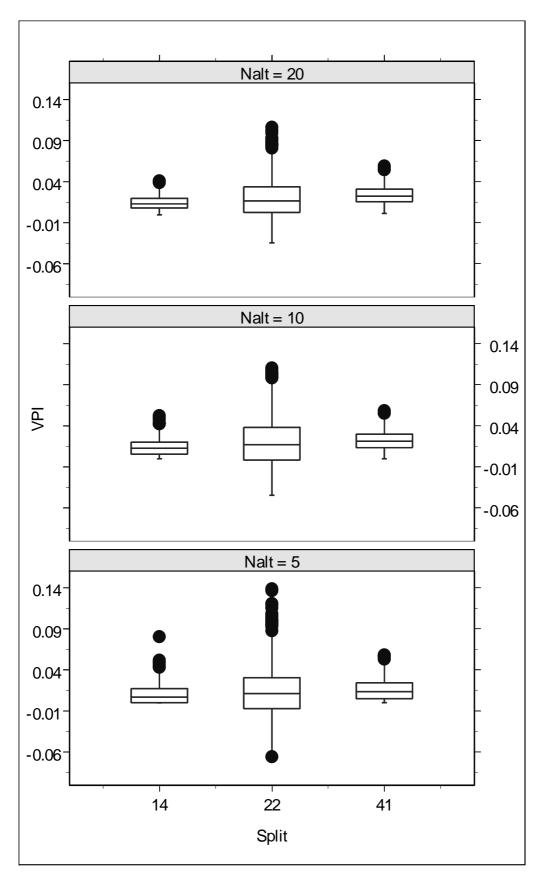


Figure 3: Box plot of VPI, 3 attributes

Nalt = 200.05 0.03 0.01 -0.01 Nalt = 10- 0.05 ١٩٧ 0.03 0.01 -0.01 Nalt = 50.05-0.03 0.01 -0.01-14 22 41 Split

Figure 4: Box plot of VPI, 4 attributes

The type of split also has a significant influence in both sets of experiments. The influence of this parameter is further demonstrated in figures 3 and 4. These figures contain a few instances in which a negative VPI was calculated due to the limited precision of numerical calculations when computing the volumes of polyhedra.

It can be noticed that for information structures which provide information on two weights (type 22) rather than only one weight (types 14 and 41), the variance of the VPI is considerably higher. Especially in the experiments with four attributes, the VPI is also higher when the information structure provides information about the value of the first weight. This effect can be readily explained. In order to select an optimal value of c, the principal should try to find out how strongly the agent will react to monetary incentives. If the alternatives are characterized only by few attributes, this information is implicitly also conveyed by information about other weights. But if there are several other weights, changes in one of those might easily be compensated by the weights not related to monetary incentives, so this information is of less value to the principal. As was already observed in the first set of experiments, this effect becomes stronger when the number of alternatives increases.

## 5. Conclusions and Topics for Further Research

In this paper, we have introduced the concept of the value of preference information in an agency context. We have shown that, in general, obtaining additional information on the agent's preferences is beneficial for the principal.

The simulation experiments enabled us to identify several factors that influence the value of preference information. One important result we have obtained is the fact that the VPI is to a large extent independent of the decision problem itself and can be predicted by the entropy of the information structure. This result means that it is possible to evaluate an information structure in general terms, without knowing which decision problems will be delegated to the agent.

Our simulation results also indicated that, contrary to intuition, in more complex decision problems with more alternatives, the principal will be able to make better use of information on the agent's preferences. However, the markedly different results obtained in the experiments with three and four attributes indicate that these results should be generalized only very cautiously.

The model underlying the analysis of this paper is a rather simple one and relies on some strong assumptions. The present approach can therefore be extended in several directions. First of all, the model used here represents a decision problem under certainty, while traditional agency models usually deal with decisions under uncertainty. While the model can also be thought of as representing a decision under risk, where both principal and agent are risk neutral, taking into account the risk aspect explicitly is certainly an important next step.

The model also relies on the assumption of a linear utility function, where only the weights are unknown to the principal. A more general model should also include the possibility that the principal is unaware of more fundamental, structural properties of the agent's utility function.

Finally, many decision problems in which uncertainty about the behavior of other actors plays an important role are not characterized by the distinct hierarchical roles of

principal and agent, but involve an exchange relationship between actors of more equal power and rank. Network structures of organizations are characterized at the same time by a more symmetric distribution of power and higher behavioral uncertainty than hierarchies. Extending the present model to a game theoretic model, in which players are uncertain about the benefits other players derive from different outcomes of the game, might therefore lead to more realistic models of such organization structures.

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