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An Evolutionary Model for Spatial Location of Economic Facilities

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Preface

This new research project at IIASA is concerned with modeling technological and organisational change; the broader economic developments that are associated with technological change, both as cause and effect; the processes by which economic agents -- first of all, business firms -- acquire and develop the capabilities to generate, imitate and adopt technological and organisational innovations; and the aggregate dynamics -- at the levels of single industries and whole economies -- engendered by the interactions among agents which are heterogeneous in their innovative abilities, behavioural rules and expectations. The central purpose is to develop stronger theory and better modeling techniques. However, the basic philosophy is that such theoretical and modeling work is most fruitful when attention is paid to the known empirical details of the phenomena the work aims to address: therefore, a considerable effort is put into a better understanding of the `stylized facts' concerning corporate organisation routines and strategy; industrial evolution and the `demography' of firms; patterns of macroeconomic growth and trade.

From a modeling perspective, over the last decade considerable progress has been made on various techniques of dynamic modeling. Some of this work has employed ordinary differential and difference equations, and some of it stochastic equations. A number of efforts have taken advantage of the growing power of simulation techniques. Others have employed more traditional mathematics. As a result of this theoretical work, the toolkit for modeling technological and economic dynamics is significantly richer than it was a decade ago.

During the same period, there have been major advances in the empirical understanding. There are now many more detailed technological histories available. Much more is known about the similarities and differencers of technical advance in different fields and industries and there is some understanding of the key variables that lie behind those differences. A number of studies have provided rich information about how industry structure co-evolves with technology. In addition to empirical work at the technology or sector level, the last decade has also seen a great deal of empirical research on productivity growth and measured technical advance at the level of whole economies. A considerable body of empirical research now exists on the facts that seem associated with different rates of productivity growth across the range of nations, with the dynamics of convergence and divergence in the levels and rates of growth of income in different countries, with the diverse national institutional arrangements in which technological change is embedded.

As a result of this recent empirical work, the questions that successful theory and useful modeling techniques ought to address now are much more clearly defined. The theoretical work described above often has been undertaken in appreciation of certain stylized facts that needed to be explained. The list of these `facts' is indeed very long, ranging from the microeconomic evidence concerning for example dynamic increasing returns in learning activities or the persistence of particular sets of problem-solving routines within business firms; the industry-level evidence on entry, exit and size-distributions -- approximately log-normal; all the way to the evidence regarding the time-series properties of major economic aggregates. However, the connection between the theoretical work and the empirical phenomena has so far not been very close. The philosophy of this project is that the chances of developing powerful new theory and useful new analytical techniques can be greatly enhanced by performing the work in an environment where scholars who understand the empirical phenomena provide questions and challenges for the theorists and their work.

In particular, the project is meant to pursue an `evolutionary' interpretation of technological and economic dynamics modeling, first, the processes by which individual agents and organisations learn, search, adapt; second, the economic analogues of `natural selection' by which interactive environments -- often markets -- winnow out a population whose members have different attributes and behavioural traits; and, third, the collective emergence of statistical patterns, regularities and higher-level structures as the aggregate outcomes of the two former processes.

Together with a group of researchers located permanently at IIASA, the project coordinates multiple research efforts undertaken in several institutions around the world, organises workshops and provides a venue of scientific discussion among scholars working on evolutionary modeling, computer simulation and non-linear dynamical systems. The research will focus upon the following three major areas:

1. Learning Processes and Organisational Competence.

2. Technological and Industrial Dynamics

3. Innovation, Competition and Macrodynamics

Abstract

Locating an economic facility, warehouse, plant, retail store, etc., is one of the most important questions that a business company faces. In this paper we consider a normative model for a certain class of relocation processes. That is, when one location structure is gradually substituted by another one. This happens in response to external factors such as appearance of competitors or change of demand. Thus, we are facing with sequential decisions and the model and algorithm corresponding to them become endogenously dynamic. An evolutionary model for location of economic facilities is presented. Its application to an empirical case, namely changing locations of alcohol distribution stores, is briefly presented.

An Evolutionary Model for Spatial Location of Economic Facilities

Ulla Seppälä

1. Introduction

Locating an economic facility, warehouse, plant, retail store, etc., is one of the most important questions that a business company faces (see e.g. Harmon, 1992). This planning question is in the operations research literature referred as a facility location problem. It focuses on defining number and location of economic facilities. Due to complexity of the problem, it has gained very much interest among the operations researchers and management scientists during the last 30 years.

The traditional normative algorithms for facility location problem are based on an assumption of an equilibrium. However, it is widely approved that many location and transportation phenomena, such as congestion, saturation, etc., are non-linear. Furthermore, location decisions encounter feed-back effects that may change fundamentally the system under study.

Non-linearity and feed-back effects, in particular, imply that even small changes in some actions of agents at critical points may cause fundamental structural changes. Furthermore, the collective results of actions may lead to totally unexpected results. In other words, from the same initial conditions the system may evolve to various alternative structures depending on the actions of economic agents. Often no proof of convergence of the system to a certain equilibrium state can be found. Consequently, there are many alternative paths of development and evolution; and historical "accidents" can have a major effect on the development path adopted by the system.

Thus, it should be clear that there is no point to solve the facility location problem exactly, if the situation can immediately change after that. Making an optimal decision based on situation of one instant is not applicable when the situation changes. The only way to analyse the dynamic process of facility locations is to create corresponding algorithms of recursive nature.

Due to historical reasons, the algorithms and models for facility location problem, i.e. normative location models, have been developed independently from the so-called descriptive location models. The descriptive location models aim at explaining spatial behaviour of economic activities, e.g. describing residential housing or flow of goods to consumption. In the realm of descriptive location theories there have been a few scientists (see e.g. Allen & al. (1981)) that have realised the effects of non-linearity and feed-backs in the location pattern, and they have applied so-called evolutionary approach to location analysis.

The focus of this paper is to widen the normative location models to handle non-linearity and feedback effects. Consequently, this paper has a normative as opposed to interpretative emphasis on location of economic facilities. By understanding the evolving nature of the location pattern the business companies are able to build networks that are able to meet competition better while a facility location pattern that is built based on traditional equilibrium approach may encounter changes immediately after it has been implemented.

We begin this paper with a short overview of the normative models focusing on the facility location problems. Thereafter, we will look at descriptive location theories starting from the traditional models reaching to the state-or-the art in dynamic and disequilibrium modelling. This will lead to a discussion about the insufficiency of normative modelling to deal with endogenously dynamic environments and evolutionary approach that is focused to deal with evolving dynamic systems. Thereafter, “the evolutionary model for location of economic facilities” is presented. The end of the paper presents a study where the evolutionary model had been applied to a facility location problem of an alcohol retailer.

2. Location models

Normative models

The facility location problem involves decision over both number and location of the facilities. Facilities can be manufacturing sites, warehouses, retailer shops, etc. The facilities may be located with relation to demand points, supply points, and/or with respect to one another.

Beckmann (1968 and 1987) claims that the operations research methods for facility location problem are based on economic activity equilibrium approach derived from neo-classical economic theory, even though the development of these operation research techniques has taken place independently from the economic theories. In other words, these models are based on an assumption that the objective is to find an equilibrium where that firms (facilities, suppliers) maximise their profits (or minimise their costs). Most of these models are static and assume a static equilibrium. The dynamic facility location models follow the dynamics created outside the system (e.g. changing demand) while the system itself stays in equilibrium. No internal dynamics, feed-back, path-dependency, etc. are taken into account.

Mathematically formulated, facility location problem is following: Consider a network. Facilities, described by vector $Y = \{ y_1 , y_2 ,... y_n \}$, are to be located to the vertices of this network. Alternatively, there can be a continuos plane, to which the facilities are to be located. Values of Y describe the quantity of products flowing through a facility in a time unit. Values of Y can be either given, restricted or changing. Products refer to actual products or, for example, to services. Usually the facilities are located respective to demand points, described by vector X = { x_1 , x_2 ,... x_m }. The values of X describe the quantity of demand in demand points in a given time unit. Location of customers and location of facilities (coordinates) are represented by L(X) and L(Y).

In other words, the problem is to locate the facilities spatially in such a way that the demands and some other requirements, e.g. transportation costs, are satisfied. In summary, the facility location problem in this paper is referred as:

$L(Y) = f(X, L(X))$

Most of facility location problems can not be solved by any known polynomial-time algorithm. They are called NP-hard problems. NP stands for non-polynomial time. In other words, the time to solve a NP-hard problem grows faster than any polynom of the size of the input; for example, it grows exponentially or even faster (Harel, 1987). For mathematical presentation of NP-hardness, see Papadimitriou & al. (1982) and Hopcroft (1979).

For example, Hakimi (1983) proved that the following subclass of facility location problems, that is not even the most complicated, is NP-hard in nature and therefore polynomial time algorithms to solve it can not be found. The problem is as follows: suppose that we want to locate r new facilities on points of a graph G where there already exist p facilities. This is done so that the sum of weights of vertices from customers, located at vertices of the graph G, to r new facilities is maximised and so that the distance from customers to a new facility is smaller than to an existing facility.

Thus, most of the facility location problems are NP-hard in nature. This mathematical “difficulty” has made the facility location problem so attractive for operations researchers. Today, literature reviewing facility location algorithms is enormous: hundreds, maybe thousands, of papers presenting algorithms for various facility problems or their applications can be found in the scientific literature. (There exists several good reviews of the existing algorithms. See, for example, Brandeau and Chiu (1989) who present over 50 typical problems and related algorithms. Current & al. (1990) have reviewed over 45 facility location models. Other good presentations are available by Revelle & al. (1970), Krarup & al. (1983), Francis & al. (1983), Aikens (1985), Verter & al. (1992), Tansel & al. (1983a and 1983b), Chatterji & al. (1981) and Colorni (1987)).

The largest group of facility location problems are those with the objective to minimise the average travel time/average cost or maximise the net income. This so-called **“p-median“** problem aims at locating p servers to minimise average weighted or unweighted travel distance between facilities and customers (demand points). The simplest p-median problem is the so-called “Weber problem” that assumes a single facility to be located optimally according to discrete demand points. The generalised Weber-problem, that is also-called “warehouse-location problem” or “location-allocation” problem, is to locate m facilities to an area with n demand points and determine service areas for all of these facilities.

The “**p-centre”** problem is concerned with minimising the maximum of the weighted distances, travel costs, or travel times between p servers and a set of clients. The “**set-**

covering” problem is concerned with defining the minimum number of facilities within a specified distance, or travel time or costs, from customers to the facilities.

The broadest categories of facility location problems are planar and network problems. The former ones typically assume that the distances between facilities and demand points, supply points or other facilities are given by Euclidean distance. Network problems, in contrast, assume that travel can only occur on an underlying network and that distances are the shortest distances between the particular points on the network. Consequently, most planar problems have infinite set of potential locations while most network problems have only a finite set (Revelle, 1996).

In addition, there are many other characteristics that vary in the models (see e.g. Brandeau and Chiu (1989) and Current et al. (1990)): problem can concern locating a single or multiple facilities, with multiple commodities or a single one; facilities can be capacitated or not; there can be deterministic and stochastic parameters; the models can be static or dynamic in nature having linear or non-linear cost functions; there can be various coverage assumptions; and the number of echelons formed by the facilities can vary.

In the following the algorithms to solve the facility location problems are divided to four classes: centre of gravity based models, linear-optimisation based models, heuristics, and simulation models. They are presented according to that classification. This may not be the most logical way to divide these models, but it reflects the historical evolution of these models. This presentation concentrates only on static models. There exist also dynamic models for facility location problem, but those are based on the some of these approaches and the dynamics is only exogenous, e.g. changing demand, while the system itself stays in equilibrium. For presentation of dynamic facility location models, see Erlenkotter (1981).

The *centre of gravity based models* are probably the oldest group of facility location models. The simplest centre of gravity model calculates the weighted average of locations of customers. The obtained point represents the best location for a facility. The formula for the centre of gravity is as follows:

$$X_0 = \frac{\sum X_j D_j}{\sum D_j}; \qquad Y_0 = \frac{\sum Y_j D_j}{\sum D_j}$$

where D_j - demand of customer j; X_j,Y_j - co-ordinates of customer j; X_0,Y_0 - co-ordinates of the centre of gravity.

There are many versions of the centre of gravity model that incorporate not only the distances but also travel times, travel costs, etc. but all of them concentrate on locating only a single facility. The following equation presents a model where transportation costs have been taken into account:

$$X_0 = \frac{\sum T_j X_j D_j}{\sum D_j}; \qquad Y_0 = \frac{\sum T_j Y_j D_j}{\sum D_j}$$

where D_j - demand of customer j; T_j - transportation costs per unit for link j; X_j,Y_j - co-ordinates of customer j; X_0,Y_0 - co-ordinates of the centre of gravity.

Another way is to use iterative procedure for finding the facility location. This procedure is heuristics in its nature. It starts from a single facility location and iteratively improves the location until | X_0(t+1) - X_0(t) | and | Y_0(t+1) - Y_0(t) | are sufficiently small numbers. The formulation of the model is as follows:

$$X_0(t+1) = \frac{\sum \frac{m_j(t)}{d_j(t)} X_j}{\sum \frac{m_j(t)}{d_j(t)}}; \qquad Y_0(t+1) = \frac{\sum \frac{m_j(t)}{d_j(t)} Y_j}{\sum \frac{m_j(t)}{d_j(t)}}$$

where m_j(t) - weight factor for the link from the facility located at (X_0(t), Y_0(t)) to the customer j; d_j(t) - distance from the facility located at (X_0(t), Y_0(t)) to the customer j; X_j,Y_j - co-ordinates of customer j.

The above procedure is suited only for a single location; it is so-called “Weber problem”. When there are multiple facilities to be located, we deal with so-called “generalised Weber problem” and the situation comes much more difficult. Miehle (1958) seems to have been the first to consider the problem of partitioning the full set of potential locations into sub-sets and finding several minimising points (one per set) simultaneously. This work was continued by Cooper (1963) that was the first to formally recognise the multiple facility location problem. The models using the approach of Cooper and Miehle are called “location-allocation” models. Several “location-allocation” algorithms with various kinds of cost structure, capacity restrictions etc. have been developed (see e.g. Scott (1971), Vergin & Rogers (1967)).

Only at the beginning of 1990s an exact algorithm for the generalised Weber problem was developed by Rosing (1992). He creates a set of convex hulls to cover the fixed points of the problem and finally uses linear programming to arrive at the optimal solution.

Baumol & al. (1958) were the first to address the warehouse location problem and use a *linear programming model* for it. The optimisation setting used for facility location problem require a finite number of potential locations.[1]

Most facility location problems cannot be handled directly by standard linear programming models due to the existence of fixed costs associated to facilities. The basic structure of mixed - integer programming models for warehouse location problem (p-median) is the following, first presented by Balinski (1965):

$$\sum_{ij} C_{ij} X_{ij} + \sum_i F_i Y_i \quad \to \text{min},$$

subject to $Y_i \in \{0,1\}$, $\sum X_{ij} = D_j$, $1 \le i \le n$; $1 \le j \le m$

where n - number of facilities, m - number of customers, F_i - fixed costs associated with facility i, X_{ij} - customer j’s demand supplied by facility i, C_{ij} - unit shipment cost

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[1] Presentation of linear programming algorithms is given by several authors e.g. Taha (1975).

through facility i to customer j, D_j - demand at customer j, Y_i - facility i open/close binary decision variable.

Due to NP-hardness of the above problem many researchers have used relaxation methods which reformulate the problem in order to obtain a more easily solved problem. The most widely used relaxation method is branch and bound method, first applied to this problem by Efroymson & al. (1966) (see also Khumawala (1972), Akinc & al. (1977) and Kaufman & al. (1977)). Another widely used group of relaxation algorithms is based on dualisation procedures and the use of Lagrangean functions. A very efficient example of this procedure is given by Erlenkotter (1978). See also Van Roy (1986) that used Benders decomposition and Lagrangean relaxation for a simple facility location problem.

Also other methods have been developed, e.g. Drezner & al. (1978) have used trajectory method for multi-facility location problem, and Van Roy & al. (1982) have used cross decomposition. In addition, enhancements to the above problem are multiple, e.g. Louveaux and al. (1992) have studies stochastic facility location by dual-based procedure, Hooker (1986) has studied non-linear objective functions, and Schulman (1990) has studied the dynamic facility location problem.

From obvious reasons the public service oriented modelling, i.e. set coverage models, have been under much less interest. The service is usually represented as a constraint that determines the maximum distance between the facility and customers. For examples of solutions for coverage problems see e.g. Toregas & al. (1971) who have studied emergency service locations and Hogan & al. (1986) who developed a method for backup-coverage in service locations. Leonardi (1981a and 1981b) gives a thorough overview of various public facility location problems and algorithms.

There is also a group of optimisation models that are based on spatial competition models presented in previous chapter. Most of these competitive location models fall to a group of so-called “p-medianoid problem” that has been stated and theoretically developed by Hakimi (1983). It deals with the location of facilities when a company A has already established a number of facilities at the nodes of a network at which a population of customers has been distributed, a second company B plans to enter the market by establishing p facilities at the nodes of this network with the objective to attract as many customers as possible from the company A. It is assumed that customers base their decision solely on distance considerations: they prefer the facility nearest to them. Another problem, which is quite similar to the one above, occurs when a company is planning to locate p facilities where at present (in the region under consideration) no other competing facilities exist. However, the company wishes to avoid the possibility of a major loss of customers in case another organisation would enter the market by introducing new facilities (Hakimi, 1983). This problem falls to a minimax location problem, that is called “p-centroid” problem.

The p-medianoid problem has been further enhanced by Karkazis (1989) who enhanced the early p-medianoid model by adding to distance criterion with quality criterion. He solves the problem by breaking the iteration to two stages: (i) in the first stage type of facilities is assigned to sites; (ii) in the second stage the facilities are assigned to sites by solving a regular location problem with linear programming. The type of facility is equivalent to this quality criterion.

The *heuristics* are “rules of thumb” that aim at not optimal but adequate solution. In this section the word “heuristics” applies both to the economic fundament of the model and the selection procedure. Most of the heuristics for facility location problem belong to the group of “add and drop” models. The idea is to start with an initial situation and to add one facility at a time, until no further cost reduction is possible. Alternatively, they start with all possible facilities and drop them one at a time. Allocation of customers is based on minimising total costs at each stage. Thereafter, the feasible solution is found by shifting the locations that have become uneconomical during the add process until no further cost reductions can be made.

One of the first and most famous heuristic was the method of Kuehn and Hamburger (1963), based on the “drop” procedure. Several similar heuristics have been developed for various problems. Manne (1964) developed the steepest ascent one point move algorithm. Feldman & al. (1966) present a heuristic program that has been developed for solving warehouse location problem when economies of scale is represented by continuous concave function. Erlenkotter (1981) has presented an overview of heuristics methods in dynamic location problems.

There are also heuristics based on “search” algorithms. For example, Lawrence & al. (1969) present a method that first divides the distribution area into sub-areas and thereafter finds a location for depots to serve each of the sub-areas. This is done by incrementally improving the initial location by moving the location with a certain step length to four different dimensions. After that these potential locations are compared with each other and a best one (least-cost) is taken as a basis of new search. A review and presentation of general heuristic methods and about their applicability are given by Pearl (1984).

The *simulation models* for facility location problem are used only to calculate the effects of various facility locations to the costs and service levels; they do not guide the researcher to find the optimum or near-optimum solution. Instead they can be used as “what-if” tools to test various numbers and locations of facilities, i.e. “scenarios”. Traditionally these calculations have been done by static simulation and much software has been built on this approach, e.g. CAST-dpm, Stradis and Locate (Ballou & al. 1993).

Shycon & al. (1960) presented one the first and the most classical reference to the use of dynamic simulation in facility location problems. They claim that simulation is able to take into account most of characteristics of real world problems, e.g. stochasticity, complicated network structures, feedbacks, etc. This is said against analytic, optimisation and heuristic methods that usually need many abbreviations and aggregations to model real world situations. There exists a vast literature also about using simulation models in facility location planning (see e.g. Robinson & al. (1995), Bowersox & al. (1972)).

Descriptive models

Location theory was first formally introduced in the beginning of last century by German geographical economist Von Thünen concentrating on location of different types of agriculture. Later at the beginning of this century Alfred Weber considered the problem of locating a single warehouse to minimise the total travel distance between the warehouse and a set of spatially distributed customers.

Thereafter, location theories have been following two routes. Along one route, economists are following Von Thünen and concentrating on explaining the spatial behaviour of economic activities, e.g. describing residential housing or flow of goods to consumption. Along the other route operations researchers follow Weber. These two routes can be regarded as descriptive and normative approaches to the location theory. Descriptive models explain why a certain kind of spatial behaviour takes place and normative models give guidelines to decision makers for their location decisions. This distinction is not totally exclusive, there are some models that are used in both senses. In the previous section, we have taken a short look at the normative models and now we will turn to the descriptive models.

While traditional economic theories have largely neglected the effect of distance to economic activities, there have been several economic geographers who built location models without linking it to the main corpus of economic theories. In practice, they have been working on both the areas of regional science and urban economics. The most widely used “independent” models of location theories are so-called “central place theory” and “gravity models”. On the other hand, the stream of location theories that is most closely linked to neo-classical economics is based on the general equilibrium approach and called “new urban economics”. There is also a group of scientists that has been focusing on the issue of competition within the location approach, based on work of Hotelling (1928). In the following a brief historical overview to these four main streams of location theories is given.

Following a work of Von Thünen a German economic geographer Christaller formulated the so-called “*Central place theory*” in 1930’s. The critical focus of Christaller’s theory is that the urban areas, or ‘central places’ exist to provide goods and services for the territory surrounding them. The producers locate so as to obtain at least the minimum level of demand - below which normal profits would not be earned and the business could not survive. On the other hand, there is a certain distance which consumers are willing to travel to obtain goods or services. From these two principles Christaller argued that the market areas would be hexagonal in shape and the increase in market sizes would be governed by geometrical rules. In addition, he developed a concept of hierarchy of market areas, where some centres provide goods also to the lower level centres (Cameron, 1987).

Later the work of Christaller was reformulated by August Lösch. He argued that the hierarchy of size could increase in number of possible ways. In addition, he showed that geographical extent of the trading areas for different goods and services vary and how low order centres provide limited ranges of goods to small trading areas whereas larger centres service much wider areas and contain all the goods of the lower centre as well as goods unique to their size (Lösch, 1940).

Central place theory has been subjected to constant criticism, because of the many assumptions it is based on: for example uniform distribution of population, linear transportation costs, the fact that consumers are willing to travel for longer distances for some goods, while for others they are not, etc. However, it is still used as a basis of several urban studies. Beavon (1977) gives an overview of various applications of central place models.

More recently, Eaton & al. (1982) have developed an economic model of central places. The model is based on maximising behaviour of economic agents in market equilibrium where the producers are located on a regular lattice of points, servicing identical hexagonal market areas and charging common price. Instead of resulting to only a single equilibrium this model results one of several possible equilibrium states. This theory has been enhanced to retail central places by Harwitz & al. (1995) who propose that any particular equilibrium geometry of central places can be produced by sets of behavioural rules for managers and for consumers, where the location and characteristics of the consumers are fixed by an exogenously specified geography of population with various characteristics.

Gravity models are probably the most widely used interaction models in locational analysis. In principle, gravity models are based on two basic elements: (i) scale, for example, cities with large population tend to attract more activities than cities with smaller population; (ii) distance, for example, the farther places of activities are apart, the less they interact. Gravity models resemble the Newtonian gravity formula to represent the interaction between population centres, but mass in the Newtonian formula is replaced by population.

Even though these models are fairly simple and they are regarded as very unconventional by many mainstream economists, they are still widely used in applied work. Applications of gravity models can be found in many areas, e.g. migration, facility location, market area analysis, etc.

Probably the most famous gravity model is based on work of Lowry in the 1960s, who built a model around two gravity model structures with residential and retail service feedbacks. It has had many successors who have widened and applied this basic model to predict the effect of an activity at change, such as the opening or closing of a large factory, to planning the urban structure and to explain the present state of an urban system (Webber, 1984). Its fundamental role in urban modelling history is linked to a basic “message”: the city is a system, made up of a set of different subsystems all interacting with each other through spatial and socio-economic interrelationships (Bertuglia & al., 1987).

An important enhancement to gravity models was given by Wilson (1970) who realised that gravity models could be derived on the basis of an analogy with statistical, rather than Newtonian, mechanics. The idea of these so-called entropy models is maximisation probability of the system states. The question can be, for example, finding the number of people using a certain service with highest probability. This is done on the basis of probabilities of microstates of the system, in this case probabilities of each individual to use the certain service. The message of entropy models was to steer away from the determinism and perfect rationality implied in neo-classic theory and to introduce stochastic aspects to location modelling.

For examples of more detailed gravity models and various applications, see e.g. Haynes & al. (1984). Sen & al. (1995) give a thorough overview of state-of-the-art gravity models. In addition, they give guidelines for defining estimates of gravity model parameters using maximum likelihood and least square methods (see also Yun & al. (1994)).

Another widely used location modelling approach concentrates on so-called spatial-allocation problem. Given location and size of demand and capacities of plants and transportation costs, it determines how the production of a commodity is spatially distributed with given capacity limits minimising the total transportation costs. Shipments from production to customers should be made if and only if the price difference equals transportation cost. Koopmans (see Koopmans (1949) and Koopmans & al. (1957)) was the one of the first who solved the above problem by means of linear programming. He characterises locations using a profitability matrix, whose elements show the profit received if an industrial unit was located there.

In the above models the demand is given. While this is in the spirit of the traditional location theory it is contrary to neo-classical economics since it neglects the effects of price on market demand. The stream of location theories that is the most close to neo-classical economics is called *"economic activity equilibrium approach"*. The basic assumption of the neo-classical economics is that there is an equilibrium state where firms (facilities, suppliers) maximise profits and users of services (customers) maximise utility. In the aspatial model, the demand and supply are balanced in an equilibrium state.

For the spatial setup, however, the existence of equilibrium can be only established by more elaborate methods. If the demand at each location is considered as a function of the commodity price at the location only, the resulting spatial market-equilibrium may be obtained as a solution to the following problem: maximise consumers' surplus in demand locations minus production and transportation costs. Equilibrium prices then appear different in each location depending on demand and supply curves and transportation costs in these locations (see Samuelson (1952)).

The neo-classical approach to location theories gave birth to a widely applied branch of urban economics, called *"New Urban Economics"*. The research question of New Urban Economics can be stated as to distribute the residential commuters of a circular city so as to achieve a locational equilibrium and to investigate the conditions under which this market solution might be covered in an optimum (Richardson, 1977). The main advantage of this school is its basis on equilibrium assumption that allows to use effective mathematical methods, in particular linear programming.

Today the general equilibrium approach is probably the most widely used approach in location theories. For current issues and applications in economic activity equilibrium approach, see Beckmann (1968 and 1987), or Schweitzer (1986). In addition, Takayama & al. (1986) give an extensive review to recent modelling developments based on general equilibrium in agricultural, energy and mineral modelling. For other examples of the state-of the-art, see Beladi & al. (1994) who study the implication of an exogenous shift in relative prices in the agricultural sector for an economy that suffers from urban unemployment and Mulligan & al. (1994) who study spatial pricing in equilibria in competition situations where firms anticipate reactions from their nearest rivals. Yang & al. (1991) present a review of spatial equilibrium methods applied to modelling regional mineral and energy issues. Worth mentioning is also the dispersed equilibrium model (see Roy (1987)) that proposes an entropy maximisation framework to handle dispersion around the profit-maximisation choice of markets and production levels by suppliers.

The existence of equilibrium is still a very much discussed topic in location theories, because only if the existence of equilibrium is proved the use of linear-optimisation based methods is justified. Macmillan (1995) presented a proof for the existence of general equilibrium in urban modelling when agents have dispersed preferences of discrete alternatives (see also Macmillan, 1993). Graves & al. (1993) have tested the existence of equilibrium in migration by building a model that allows both equilibrium and disequilibrium migration. They conclude that intertemporally systematic migration stems predominantly from equilibrium forces. Evans (1993) criticises this view because of its aggregated assumptions. In addition, De Fraja & al. (1993) show that price-location equilibrium exists under several pricing policies, namely f.o.b. mill, uniform deliveries and spatially discriminatory pricing.

Bertuglia & al. (1987) state several limitations of the economic activity equilibrium theory. The principal limitation of this theory is that it is necessarily founded on the idea of equilibrium. This is a condition rarely achieved in reality and even if it occurs, it gives only a static picture. It can be used to describe equilibrium situations and for comparative analysis of such situations but cannot explain how equilibrium is reached. Other disadvantages are the impossibility of dealing with indivisibilities, externalities and imperfect rationality of decision-makers. Service locations involve indivisibilities such as fixed costs of provision or capacity constraints of facilities. Externalities represent such factors as spill-over, diffusion, economies of agglomeration and environmental effects.

In his path-breaking paper, Hotelling (1928) provided a framework for the basic model of *spatial competition*. He formulated the following problem. Let’s think that buyers are uniformly distributed along a line, which may be Main street in a town. Two businesses A and B are located along this line and selling products at price p1 and p2. Each buyer transports his purchases home at a certain cost c per unit distance. The point of division between the regions served by the two entrepreneurs is determined by the condition that at this place it is a matter of indifference whether one buys from A or from B. Both A and B adjust their prices so that with the existing value of the other price, his own profit will be a maximum.

Interestingly, Hotelling believed that this problem of spatial differentiation yields to stability. It was later shown that no equilibrium price solution will exist when both sellers are not far enough from each other (d’Aspremont & al., 1979). However, today the spatial competition is encountering the wide interest among location theories (Eiselt & al., 1993). For presentation of state-of-the-art in competition modelling, see e.g. Gabszewicz & al. (1986). For example, Wendell & al. (1981), have enhanced the competitive location models by applying graph theory. De Palma & al. (1989) study the competitive location with random utilities by applying graph theory, as well. Economides (1993) has studied the Hotelling’s problem with more than two competitors. Drezner (1994) has enhanced the competitive location models to planar spaces, instead of networks, and Drezner & al. (1996) have used competitive location models to calculate the expected market share of companies when customer’s preferences are based on stochastic utility function.

The above presentation of descriptive location models above concentrated mostly on static models. The reason is the dominance of equilibrium approach and the “new urban economics” that mostly concentrates on static equilibrium situations. The already mentioned gravity based Lowry model was probably the first dynamic location model. The original version of the model was comparative static one: given the forecast input data for some future time, the equation system could be solved to predict the state of the city at that time. Later several enhancements to that model were made so that it was possible to follow the dynamic path of the system (Webber, 1984).

Another famous example is dynamic urban model of Forrester (1969). Based on his work on industrial dynamics (Forrester, 1961), where delays in information flow of an industrial system lead to amplifications in product flow, Forrester developed similar kind of model to deal with complexity of urban dynamics. He focused on construction and population movement within a specific area. The urban area was represented as a social system setting and environment with which it communicates. The flows of people to and from the area depended on the relative attractiveness of the area compared to its surrounding environments. Life cycle of the urban area was generated by a simulation model. It showed how growth gives way to maturity and then stagnation. By the time the land area has become filled, new construction decreases, and the urban system stagnates into a high level of underemployed housing and declining industry. Despite the initial success of this model, it has had relatively few applications compered with the great number of models originating from the Lowry prototype (Bertuglia & al., 1987).

Today, the most rapidly expanding stream of location theories seems to be models based on non-linear dynamics. Good collections of fairly recent dynamic and especially non-linear location modelling are given by Andersson & al. (1989) and Crosby (1983). Probably the most influential attempt to tackle the non-linearity of regional systems was taken by Wilson and his colleagues who have applied *catastrophe theory* to urban modelling (Wilson, 1987, Beaumont & al., 1983). Models of Wilson reach to multiple equilibrium solutions arising from the presence of non-linearities and a high degree of interdependence between the economic agents. However, the catastrophe theory only represents switches from a postulated equilibrium state to another equilibrium state which represents a new dynamic regime. The switches are called “catastrophes”. There is no follow up of this dynamic process.

Peter Allen has used the theory of *self-organising systems* to follow the dynamic process of evolution of cities and regional development (see e.g. Allen 1982, Allen 1983). This theory implies some basic ideas from physical chemistry concerning disequilibrium systems (see Prigogine & al. (1984)). In the 1970's and 80's Allen and his colleagues focused on spatial self-organisation of cities and regions. They managed to account for spatial positive and negative feedbacks that emerge due to the interdependence of population and economic demand and their spatial distribution. These showed how spatial hierarchy of cities developed, and how, within these complex spatial structures of industrial, residential and commercial neighbourhoods are evolving over time, as the result of the interplay between the non-linear socio-economic interactions and the "noise" involved in detailed, local events.

More recently, Brian Arthur has been focusing on the *economies of agglomeration*. The economies of agglomeration implies that net benefits of being in a location together with other firms increase with the number of firms in the location. Economically it can be justified in the following way: access to services that companies share, use of the same labour markets, information and expertise, etc. Arthur showed by a simple model that where agglomeration economies are present, historical “accidents” select a limiting spatial pattern from a, possibly large, collection of candidates for limiting spatial patterns (Arthur, 1994). In other words, the firms benefit from the presence of other firms and they sequentially choose location in an order of choice that is subject to “historical accidents”. He claims that in reality historical events would have provided a location structure already in place; and this inherited structure combined with agglomeration tendencies would determine the future. Some locations selected early on by chance can become locked-in or “fixed” while some other locations are exercising “competitive exclusion” of each other.

Path-dependency has been also studied under equilibrium approach. Tellier (1992) presents so-called ‘topodynamic’ approach where he simulates a sequence of successive location optimisation problems that leads to an evolution of given locational system. Krugman (1991b) has studied a spatial structure where there is two factors of production: immobile agricultural workers and mobile manufacturing workers. Due to interaction among factor mobility, increasing returns, and transport costs firms tend to locate where other firms are concentrated. Working against these “centripetal” tendencies, however, is the “centrifugal” pull provided by the geographically dispersed agriculture. This complex interaction leads to possibility of multiple equilibrium when defining the location of the economic facilities (see also Krugman (1991a), (1993), (1994), (1996)).

3. An evolutionary model for location of economic facilities

Normative modelling and path-dependency

In the field of normative modelling, dynamics and non-linearity have received fairly mild interest, even though in descriptive modelling it has been reasonably well approved that small changes in actions of economic agents at critical points may lead to substantial structural changes in the system. The dynamic normative models have only concentrated on dynamics created by exogenous events, e.g. changing demand, while the system itself stays in equilibrium. The endogenous dynamics, effects of feedbacks, etc., have not received attention.

However, in reality when a location of facilities has been changed, correspondingly actions of customers will change in their turn, but not immediately. For example, it will take some time before customers will find their new ways of behaviour in this reorganised network. Thereafter the situation might not resemble any more the initial situation. In other words, increasing or decreasing the number facilities or relocating them is an endogenously dynamic process that may lead to several possible out-comes depending on the initial situation and actions of agents.

For example, let’s think about a situation where a company, working in retailing, adds a new retail shop to a new shopping centre. As this new facility is very attractive for the customers (there are plenty of parking place, other shops nearby etc.), the facility attracts very many customers. When its sales increase it can offer cheaper prices and a larger variety of products. This, of course, attracts even more customers. In a consequence, the company's other shops, situated further, will suffer from this competition and finally some of them are closed down. This eventually increases customers in other areas and the situation keeps changing.

Thus, it should be clear that there is no point to solve the facility location problem exactly, when the situation immediately changes after that. Making an optimal decision based on demands of one instant is not applicable when the situation changes. The only way to analyse the dynamic process of facility locations is to create corresponding algorithms of recursive nature.

An important remark here is, that the above applies only to industries or markets where the flow of demand has an effect to the attractiveness of facilities. In other words, it concerns the industries where an increase or decrease of the product flow through a facility has an effect to attractiveness of the facility; and customers are able to allocate their demand based on the attractiveness. Such industries include, for example, retail shops, that can offer a wider range of products and cheaper prices while the product flow increases. On the contrary, in a situation where a company relocates its own warehouses, this is not the case. The company will not start using any other facilities, so the flow of products will not change in the facilities due to the relocation.

In the following a normative non-linear dynamic model for spatial location of economic facilities is presented. The model is concentrated, in particular, to the spatial location of retail facilities. It is based on so-called evolutionary approach. Before going to the structure of the model, we shall have a brief look at the evolutionary modelling in general.

Evolutionary models

The term "evolutionary" is used to define a class of theories, or models, or arguments, that are, first of all, (i) dynamic. They are based on (ii) agents with different qualitative criteria, and (iii) with only a limited knowledge of their environment. The agents are (iv) in contact with each other, and (v) there is competition and selection among the agents (see definitions by Nelson (1995) and Lane (1993)). In addition, Coriat and Dosi (1995) claim that in truly evolutionary systems there is continuous appearance of various forms of novelty, e.g. new behaviour.

Thus, evolutionary models have a variety of agents interacting with each other. Different kind of agents, with different qualitative characteristics are competing with each other in "the battle of the fittest". It should be noticed that this fitness function is not fixed for once and forever. Due to the evolution of the system and its environment the qualities that are regarded as "fit" change. Some agents flourish and others disappear. Due to mutation or some other reasons new types of agents emerge to compete.

All agents in the evolutionary process are acting according to their characteristics and are trying to improve their performance the best way they can. However, they do not have any overall knowledge what would be the most profitable way of behaving. Therefore, any "optimising" characteristics of what exists must be understood as local and myopic (Nelson, 1995). In a consequence, the process of evolution is strongly path-dependent and there is not necessarily any unique selection equilibrium.

The main difference between evolutionary economics pioneered by Nelson and Winter (1982) and “traditional theories” in economics is that evolutionary theory focuses mainly on the processes of change in economies while “traditional theories” have mainly concerned in equilibrium situations.

Since the evolutionary model for location of economic facilities is based on evolutionary ideas, we cannot expect it to arrive at “optimum”. Therefore, it can be used to help the decision makers to learn more about the location of the facilities and to answer, for example, to the following question: which of the facilities are located so that they are very vulnerable to future competition and which of the facilities are located in robust locations?

Structure of the model

In the evolutionary model customers allocate their demand to facilities depending on the attractiveness of the facilities. The attractiveness describes how attractive a customer considers this facility. First of all, this dependency includes the size of flow of products flowing through the facility. Because of economies of scale when the number of products flowing through a facility increases, it allows the facility to offer cheaper prices and/or bigger variety of product, thus increasing its attractiveness. Also, other factors that affect the attractiveness, e.g. distance between customer and the facility, should be included to the attractiveness function. The exact form of the attractiveness function depends on the problem to which the model is applied.

We set that the value y_i (t+1) is calculated as weighted sum of all those shares of demands x_j. The value of y_i (t+1) represents the quantity of products flowing through a facility at the time t+1 (or, equivalently at the t+1:th instant of the adjustment process). The weights are so-called “relative attractivenesses” at t. The relative attractiveness of a facility to a demand node x_j is calculated by diving its attractiveness to x_j by the total attractiveness that x_j is encountering. The more relatively attractive a facility is the larger demand is allocated to it. Formally, what we have said looks as follows:

$$y_i(t+1) = \sum_j x_j \frac{Att(x_j, y_i(t))}{\sum_k Att(x_j, y_k(t))} \qquad (1)$$

Due to practical considerations, the minimum size of the facilities (representing the minimum flow of products through a facility) should be limited to a predefined level. For example, a facility becomes uneconomical if less than a certain amount of products flow though it. This imposes a “death rule” on the above location process. That is a facility is called dead and it does not participate in the dynamic process from now on if the quantity of products flowing through the facility becomes smaller than the “death rule”.

The maximum size of the facilities derive from the fact that the total sum of facilities can not grow bigger than the total sum of demand. On the other hand, the distance criteria or customers, that they are not willing to travel too long distances to a facility, implies that several facilities in various locations will remain over the “death rule”.

The model is simulated iteratively. Typically it stabilises at a situation where one or several facilities have succeeded in competition and their values are larger than the “death rule”. In some cases, the model may result in oscillation between several values. In any case, the stability of the outcome should be tested by making sensitivity analysis, which implies changing slightly demand and looking at if there are any changes in the resulting locations of facilities.

To make this model truly evolutionary, we can allow along with “killing” the most inefficient facilities emergence of new ones. The new facilities would take part in the competition of attractiveness. The allocation of demand, of course, would change due to emergence of each new facility and only facilities located to the most robust locations (if any) could survive in the competition. This modification would better reflect the continuously evolving nature of industry. For example, in restaurant business, or in food retailing, new facilities emerge all the time. Some succeed in competition, others try for a while and then disappear.

Model with several markets

It is also possible to take into account several markets, so that the facilities of one market are customers for another one. This is done by introducing a new market with facilities, e.g. there can be three markets: customers buying from retailers that are buying from gross-retailers. This modification is very simple, vector Z representing a new market is introduced. $L(z_i)$ represents the possible locations for the new facilities. During each iteration the values of Y are calculated according to (1) and simultaneously the values of Z are adjusted according to following (2), almost exactly similar equation.

$$z_i(t+1) = \sum_j y_j(t) \frac{Att(y_j(t), z_i(t))}{\sum_k Att(y_j(t), z_k(t))} \tag{2}$$

In the above formula, the Y-facilities are competing with each other for the demand of customers and Z-facilities are competing with each other for the demand of Y-facilities.

However, it may be necessary that both Y and Z are competing with each other. In reality, this kind of situation may occur when customers (X) can choose to buy either from a retailer (Y) or from a gross-retailer (Z) and gross-retailers (Z) are supplying both directly customers (X) and retailers (Y). In other words, the customers divide their demand among retailers (Y) and gross-retailers (Z). In that case, the (1) and (2) should be replaced by the following

$$y_i(t+1) = \sum_j x_j \frac{Att(x_j, y_i(t))}{\sum_k Att(x_j, y_k(t)) + \sum_k Att(x_j, z_k(t))} \tag{3}$$

$$z_i(t+1) = \sum_j x_j \frac{Att(x_j, z_i(t))}{\sum_k Att(x_j, y_k(t)) + \sum_k Att(x_j, z_k(t))} + \sum_j y_j(t) \frac{Att(y_j(t), z_i(t))}{\sum_k Att(y_j(t), z_k(t))} \quad (4)$$

If the customers prefer gross-retailers to retailers, it may result in situation where none of the retailers receive any demand from the customers. Similarly to (3) and (4), the model can be enlarged to take more and more markets into account.

4. Evolution of retail-stores of Alko

Background

The above model has been applied to a real world case, namely the distribution of alcohol in Finland. Due to confidentiality reasons, not all the data concerning the case can be presented. Therefore, the case is presented here only briefly, as an illustration of use of the evolutionary model.

Finnish government has had monopoly for production and distribution of alcohol since the fall of the Prohibition in Finland in 1932. On behalf of Finnish government, a separate company, Alko Ltd., has handled the production and distribution of alcohol in Finland. In 1995 Finland became part of the European Union and the monopoly of alcohol had to be broken. In 1995 Alko Ltd had 253 retail stores selling beer, wine and spirits. The distribution of retail stores around Finland is presented in Figure 1.

Several scenarios of future behaviour in Finnish alcohol markets and related effects of sales in Alko’s stores were formulated. These scenarios consisted of various percentile decreases of sales that would take place evenly in Alko’s stores. It would have been very easy for Alko’s management to decrease the sales of each shop according to the scenarios; and then close all shops that are under a minimum acceptable sales level. However, today it is apparent that Alko’s largest shops attract more customers than the smaller ones as they have a higher variety of products. Therefore, this static procedure would not take into account the dynamic aspects of the problem, i.e. customers continuing to change shops depending on the service provided by the stores. Because of the reasons presented above, an evolutionary model for location of economic facilities was applied to this problem.

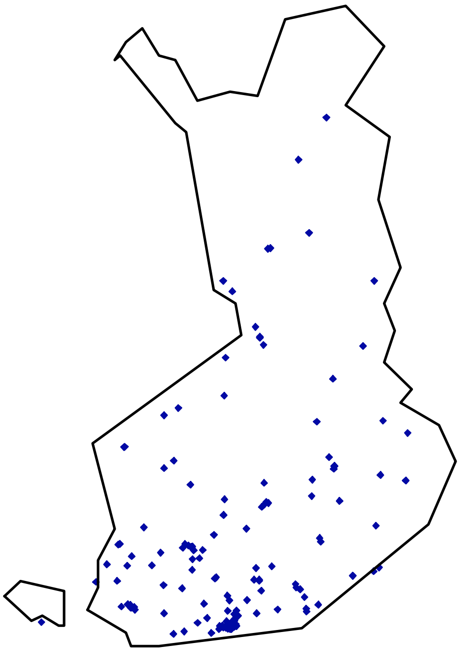

Figure 1. Present location of Alko's stores

Model structure

The basic structure of the model was similar to the one presented above. The attractiveness function comprised of the sales of a retail store divided by the costs of the facility that are incurred by the customer (5). The costs include fixed and variable cost of running the facility and customer's transportation costs. In other words, the attractiveness function represents the effect of economies of scale. Even though in this case the company is not able to decrease the price of produces when unit cost decreases, it is able to offer bigger variety in the retail store. The structure of the attractiveness function was as follows:

$$Att(x_j, y_i(t)) = \frac{y_i(t)}{a + bx_j + cx_j dist(L(x_j), L(y_i))} \tag{5}$$

where a - fixed costs of retail store; b - variable costs of retail store; c - calibration factor representing the transportation costs of customers; dist(L(x),L(y)) - the Euclidean distance between the present store the customer is using and an alternative store.

All of this data (location of Y and X, initial values of Y and X, "death rate", and fixed and variable costs of the retail stores) were received directly from the company's confidential data and therefore they cannot be presented here.

The model was calibrated by an inertia of customers to change the store they are using. In other words, calibration factor represents the transportation costs of customers per kilometre. The calibration factor was set so that the output of the model gave similar sales at each store as at the present situation.

The “death rule” used in the model represented a minimum acceptable level of sales in one store. Under this minimum level the retail stores are not profitable and they would be closed down.

The initial values of Y represented the present sales of each store and L(Y) represented their location. It was not known where the customers actually live who buy from a certain retail store, as the customers may use the retail store that is near their home, working place, etc. Therefore, customers were estimated to be located at the same location as the store they were presently using. The initial values of X were the same as initial values of Y in the same location.

Results of simulations

Several scenarios with varying decrease of demand were studied with the model. As an example of the outcomes of the model, Figure 2 presents the location of Alko’s stores in a scenario where decrease in sales of beer was estimated to be 80 %, in sales of wine 60 % and in sales of spirits 20 %.

As an another example of the results Figure 3 shows how decrease in total sales effects the number of stores. Non-linearity of the relationship can be noticed.

Ten different scenarios were simulated. By making comparisons between the scenarios, the results of the study showed that there were some locations where an Alko’s store was able to survive even in the strongest decrease of sales, and in contrast, there were other locations where a store was not able to survive any decrease in sales. In the worst scenario the sales would decrease in every store 80 %. Even in this case 46 stores out of 256 was able to survive. All these stores also survived also in all the other scenarios. They can be regarded as the most robust locations for the Alko.

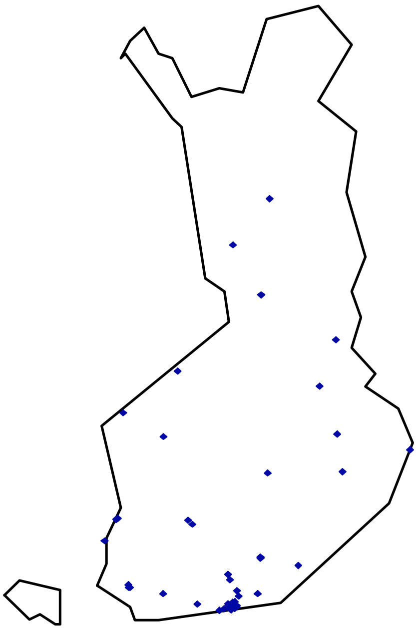

Figure 2. Output of a scenario where decrease in sales of beer was 80 %, wine 60 % and spirits 20 %

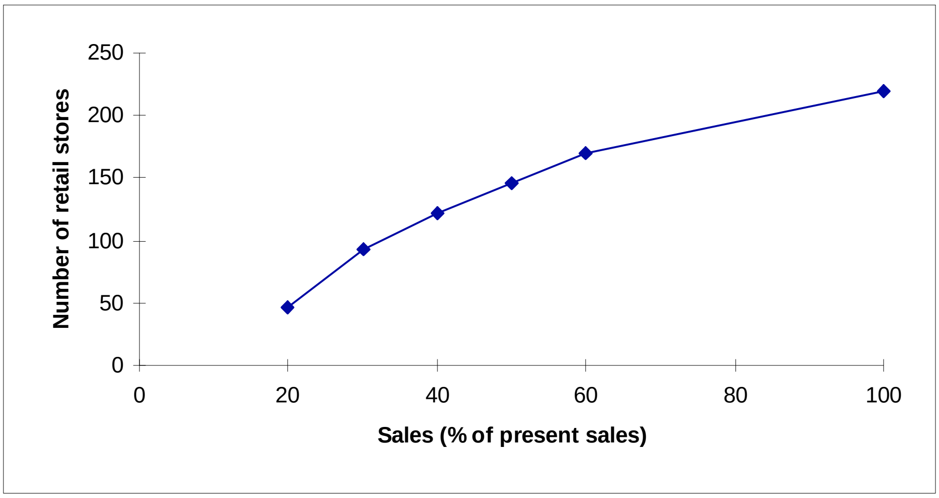

Figure 3. Relationship between number of stores and total sales

The most robust stores were usually the ones that were located quite centrally in a big customer area without any strong competitors around them. Actually, in areas where there were several small Alko's stores, all of them very small, the most profitably located of them was able to survive in the competition while the others were closed down.

This study did not lead directly to closing down of some stores, but based on this study the Alko's management was able to divide the shops to the ones that are likely to be profitable in future and to ones that are most likely to suffer so much from the competition that they will have to be closed down. This is very important decision for the allocation of resources and investments into different parts of the country.

Continuous competition

To be able to understand the dynamics in the evolutionary model, another study was carried out. In this study there were new stores emerging all the time to the market and competing with the Alko's stores. Because no data was available about the emerging competitors, about their costs, etc., it was assumed that the emerging competitors would have had the same parameter values as Alko's stores.

Location of a new store was randomly chosen among the locations of present stores by using a uniform distribution. The initial size of competing stores was chosen to be as large as the present store in that location was in the beginning of the simulation.

Several very interesting phenomena were observed. There were several stores that were not able to resist the competition from the neighbouring locations at all. In other words, they always disappeared after emergence of a competitor. Alternatively, some of the stores survived for thousands of time-steps. These stores were located so nicely in the middle of a neighbourhood that they were able to attract customers and compete with the new stores emerging to its own location and to neighbouring locations as well.

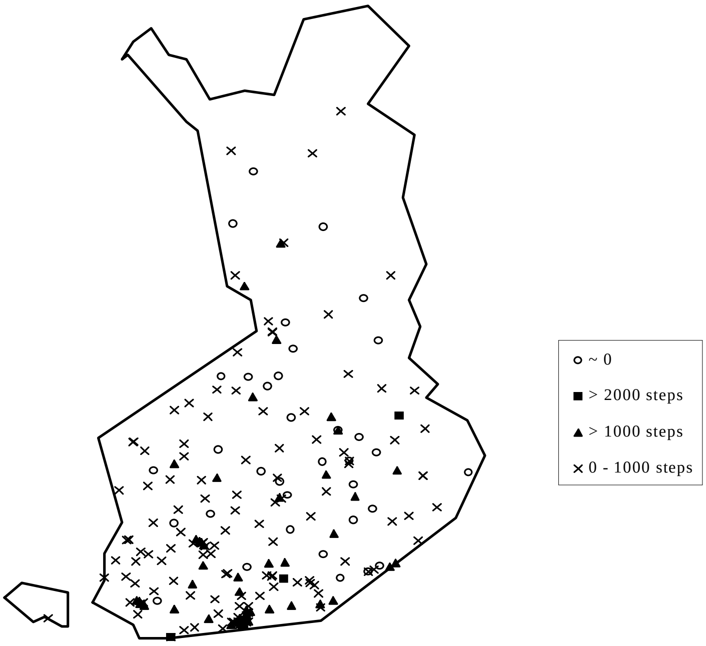

Figure 4. Classification of stores in evolutionary simulation run

Figure 4 shows four groups of locations of stores representing the longest time a store was able to survive in a location:

- location where stores could not survive (they survived approximately 0 time-steps);
- locations where stores were able to survive more than 2000 time-steps;
- locations where stores were able to survive 1000-2000 time-steps;
- locations where stores were replaced often (they survived less than 1000 time-steps).

The Figure 4 was constructed by studying a limited and finite time span of 10 000 time-steps. As new stores were all the time emerging to the area, the system changed all the time and no convergence could occur.

There were 54 locations that were able to survive more than 1000 time-steps. 23 of these locations belonged to the group of 46 the most robust locations of the previous simulation run where no new stores were emerging. In other words, half of the robust locations were the same in both of kinds of simulations. The difference between these two results can be explained by the path-dependency of the process. Depending on the initial situation, differing dynamic processes could take place in the simulations.

Figure 5 shows the sizes of sales in a Central Finnish town called Lahti. Each line in the figure represents a different store in the same location. At one instant of time there was often more than one store in the location. Stores are named by the length of their “life” in the side of the figure. The data collection was made every 50th time-step. Figure 6 shows the sales of other Finnish town called Nastola that is located about 20 kilometres from Lahti. The figure shows us that during the whole 10 000 time-step interval only three times the sales of the store decreased under “death rule”.

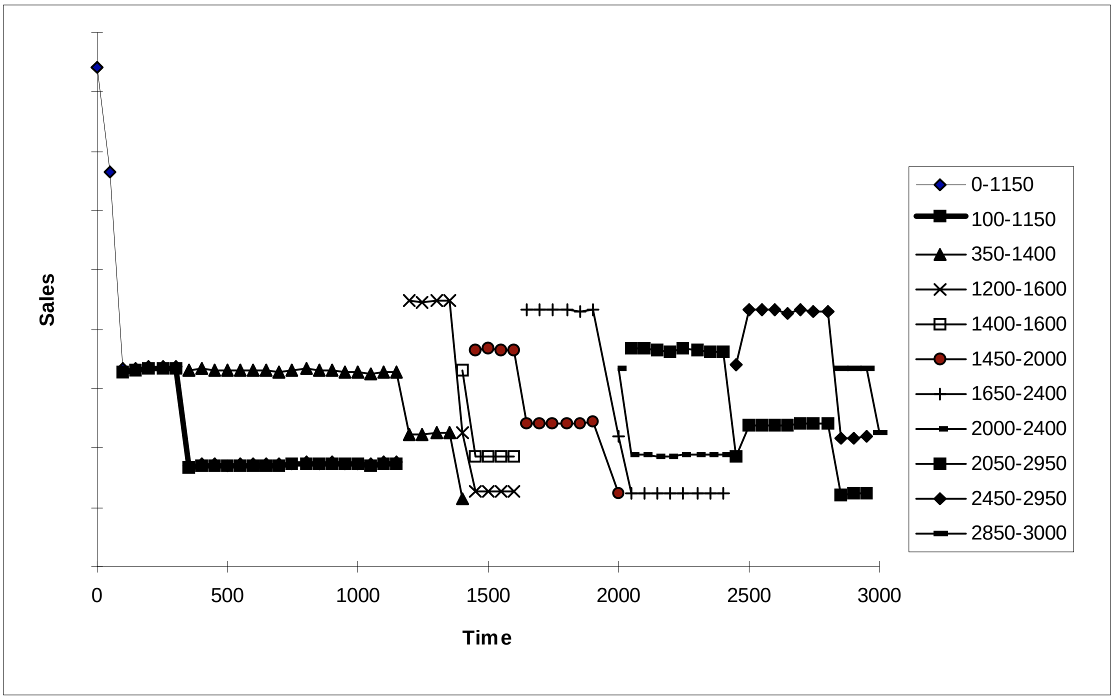

Figure 5. Size of stores in Lahti

Looking at Figure 5 we can notice that there are several intervals of stability and then suddenly there is a "catastrophe". During the stability the stores divide the consumption between each other. The competitors emerging to the market have no effect to the stable situation. Suddenly drastic changes in sales occur due to emergence of a competitor to this (or to a neighbouring) location. This emergence of "catastrophes" is similar to punctuated equilibrium found among populations, where long periods of stability are broken by sudden "catastrophes".

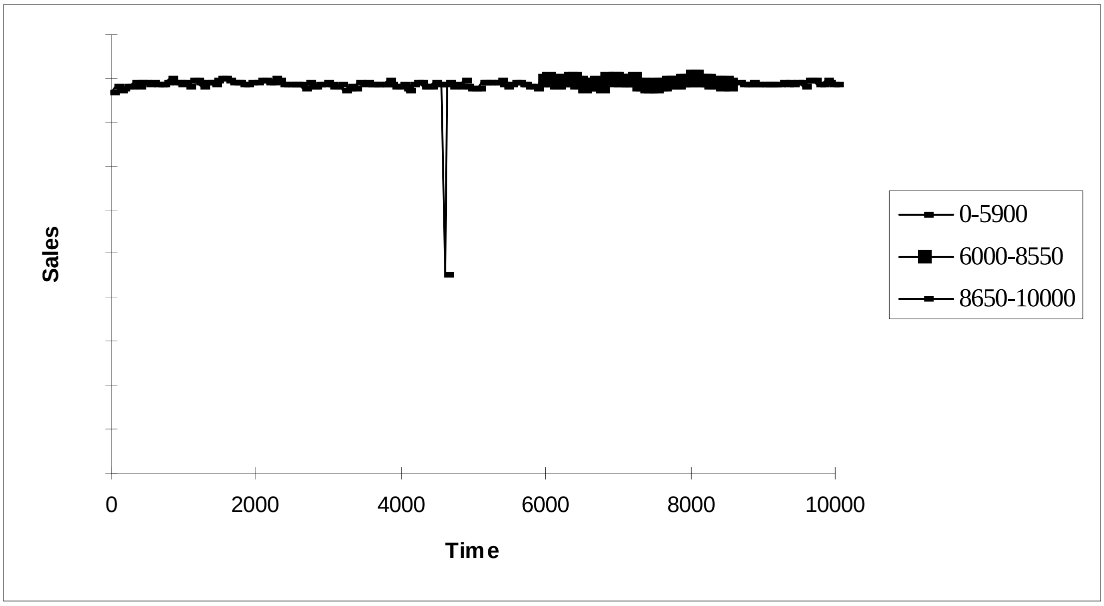

Figure 6. Size of stores in Nastola

5. Conclusions

Locating an economic facility, warehouse, plant, retail store, etc., is one of the most important questions that a business company faces. This planning question is in the operations research literature referred as a facility location problem that focuses on defining number and location of economic facilities. These so-called normative location models are focused only on equilibrium situations and they have neglected the effects of non-linearity and feedback to location decisions.

However, in reality the relocation of facilities may effect the behaviour of customers and this in turn effects the sales of facilities. Therefore, in these situations there is no point to study the situation in one point of time, and try to find an optimum solution in that instant, as the situation immediately changes after that. Making an optimal decision based on situation of one instant is not applicable when the situation changes.

There have been some descriptive location theories that have taken into account the non-linearity and path-dependency. However, the normative location models, that are used to help the decision making of companies, are all still based on the economic equilibrium approach. In this paper an evolutionary model for location of economic facilities was presented. The model is normative in its focus, it was designed to give guidelines for location decisions in a world that is changing. This model follows the endogenously dynamic properties of facility location decisions.

The evolutionary model for location of economic facilities can be seen as somewhere between the descriptive and normative approaches. Basically it is normative as it is designed for the help business companies. As it leaves the assumptions of stability and equilibrium away, it, in many cases, resembles the real world more than the present normative models. However, we cannot expect the evolutionary model for location of economic facilities to arrive at “optimum”. Therefore, it can be seen as a heuristic device to help the decision makers to learn more about the location of the facilities and to answer, for example, to the following question: which of the facilities are located so that they are very vulnerable to future competition and which of the facilities are located in robust locations?

Table 1 shows a rough picture how the evolutionary model for location of economic facilities can be compared with the other location theories and models.

	Descriptive	Normative
Disequilibrium models	Allen & Arthur	“Evolutionary model for location of economic facilities”
Static and equilibrium models	Traditional location theories	Traditional operation research models

Table 1. Classification of the descriptive and normative location models

The evolutionary model for location of economic facilities has been applied to relocation of retail shops of Finnish alcohol distributor called Alko. It was estimated that the overall sales will decrease, but how this would affect the network structure, was not known. By making several scenarios the most robust locations in the market were able to be identified. These locations could survive even the most severe decrease of sales.

In addition, an additional model was built where new competing stores were emerging all the time during simulation to the market to compete of demand of customers. Therefore, the allocation of demand changed continuously. The results showed that some of the locations were much more vulnerable to the competition than others.

The evolutionary model was able to show the management of Alko the robustness of locations of their stores in a dynamic environment. This could not have been possible with any of the traditional equilibrium based model. Even though the evolutionary model is path-dependent and several outcomes are possible, the results of the model can be used in decision making, when several scenarios are carried out and their results are compared with each other.

It should be noticed that the robust patterns in locations found by the model are only robust in this context. Therefore, they are slightly different from the concept of evolutionary stable strategies talked by evolutionary biologists, but metaphorically we can find some similarity between these two concepts.

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