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Spatial Stochastic Model for Optimization Capacity of Insurance Networks Under Dependent Catastrophic Risks: Numerical Experiments

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Contents

Abstract

The paper proposes a general framework for the optimization capacity of an insurance industry in responding to catastrophic risks. Explicit geographical representation allows for sufficient differentiation of property values and insurance coverages in different parts of the region and for realistic modeling of catastrophes in space and time. Numerical experiments demonstrate the possibility of stochastic optimization techniques for optimal diversification of catastrophic exposure. This is important for increasing the stability of insurers, their profits and for the financial protection of the population.

Key words: Catastrophic risks, insurance, insolvency, stochastic optimization, quasigradient methods.

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1. Introduction

The severity of natural and human-made catastrophes depends on their geographical patterns, clustering of property values in the region, available mitigation measures and regulations, the spread of insurance coverages among different locations, etc. The occurrence of a catastrophe at any particular location is rare, and different locations may be subject to disasters. These disasters may be unlike anything that has been experienced in the past. Therefore, it is impossible to improve policies by relying only on historical databases of past experience. Probable high future losses make it also impossible to use learning by accident practices for adjusting default policies. For all these reasons models become essential for making decisions on a company's solvency, reinsurance requirements, premiums, effects of mitigation measures and diversification of coverages (see [1]-[3]). The occurrence of various episodes in the region can be simulated on a computer in the same way as it may happen in reality [3]. For this purpose the model must be geographically explicit. The geographical representation should allow for sufficient differentiation of property values and insurance coverages in different parts of the region. It must also allow for the realistic modeling of catastrophes in space and time.

Existing catastrophe modeling relies on straightforward Monte Carlo simulation of a large number of catastrophic events. If the results of such a simulation are not satisfactory, then the policy is changed, and , again, a large number of catastrophic events is simulated, and so on. Since the Monte Carlo evaluation of each combination of policy variables is time consuming and the number of combinations is usually infinite, such a "trial and error" approach can not produce consistent conclusions on the capacity of insurers to respond to catastrophes.

The aim of this paper is to demonstrate the possibilities of spatial stochastic catastrophe models and optimization procedures for improving the geographical diversification of insurance contracts, for improving the stability of an insurance industry, increasing insurance profits, and providing financial protection of the population. Catastrophes produce highly correlated risks, which cannot be diversified properly without cooperation between insurers. In general cases, dependencies between possible claims have a complex character defined by spatial patterns of events and feasible policy variables. The spatial stochastic model simulates explicitly these dependencies, and stochastic optimization procedures create robust policies without the exact evaluation of all the risks associated with the infinite combinations of feasible policy variables. It is important that this evaluation requires roughly the same number of simulations as the evaluation of a single combination of policy variables. The state insurance is considered as a possibility for financial protection of the population. Therefore, the optimal distribution of covereges shows the level of governmental intervention necessary for dealing with catastrophic risks.

We analyze two approaches: deterministic approximations of stochastic models and the use of stochastic search procedures (stochastic quasigradient procedures). The number of variables in the model is $M \times N$, where N is the number of insurers and M is the number of grid cells of the region (which may be rather large).

The deterministic approximation requires $N \times S$ additional constraints, where S is the number of scenarios. Since S is large for rare events, the number of variables for deterministic approximation, $MN + 2SN$, is very large. Besides the increase of dimensionality, this approach is impossible in general to use for dynamic models involving insolvency or "stopping times" of insurance companies, which implicitly depend on policy variables.

Stochastic quasigradient methods do not increase the size of the original model. They also allow us to bypass obstacles in solving dynamic models by intensive simulations of catastrophic events and adaptive adjustment of policies to random outcomes of these simulations. In a sense, we create a "laboratory world", where companies may easily adjust and even reverse default policies by learning from the simulated history of their operations.

Section 2 outlines the model. Section 3 illustrates the problem with dependent claims for two regions with identical distribution of damages causing insolvency of "local" insurers. The optimization procedures easily "learn" dependencies between damages and they "propose" insurers to take more catastrophic risks from other regions. Sections 4, 5 discuss deterministic and stochastic approaches for optimization insurance industry capacity. General example of spatial model is considered in Section 6. Section 7 concludes.

2. Model Description

The approach we adopted is based on subdividing the study region into subregions (compartments). Depending on scale compartments, these subregions may correspond to a collection of households, a zone with similar seismic activity, to a watershed, etc. In applied analysis spatial modeling is usually accomplished on the basis of spatial data sets organized by rectangular grids. Compartments can be identified with the collection of grid cells. The notion of compartment does not exclude internal heterogeneity: a compartment may itself be subdivided into smaller units for a meaningful representation of the simulated patterns of events in space and time.

In the existing model we assume that the region consists of squares (i,j) as is shown on Fig.1.

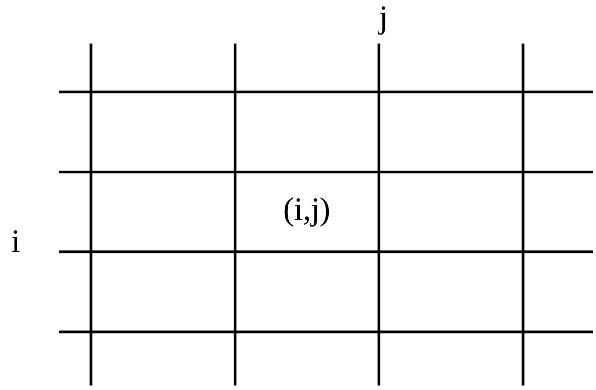

Fig.1. Units of region.

For each square (i,j) there exists an estimation $W(i, j)$ of the property value or "wealth". It includes values of houses, lands, and factories situated in this part of the region. For example, the initial property value of the square (i,j) is equal to 10 (Fig.2).

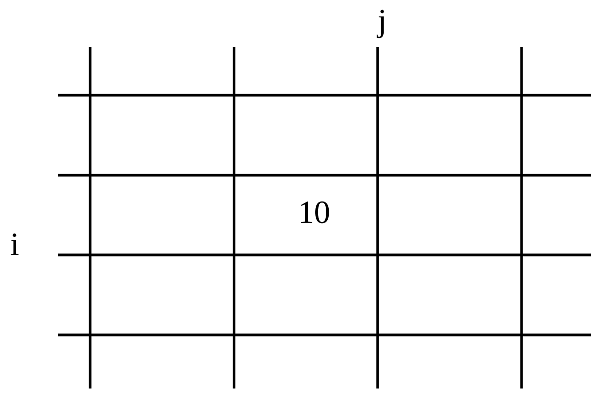

Fig.2. Property values.

We suppose that N insurance companies may have contracts with all the squares and partially cover their losses. Connections between a company k and the squares are shown in Fig.3 by arrows.

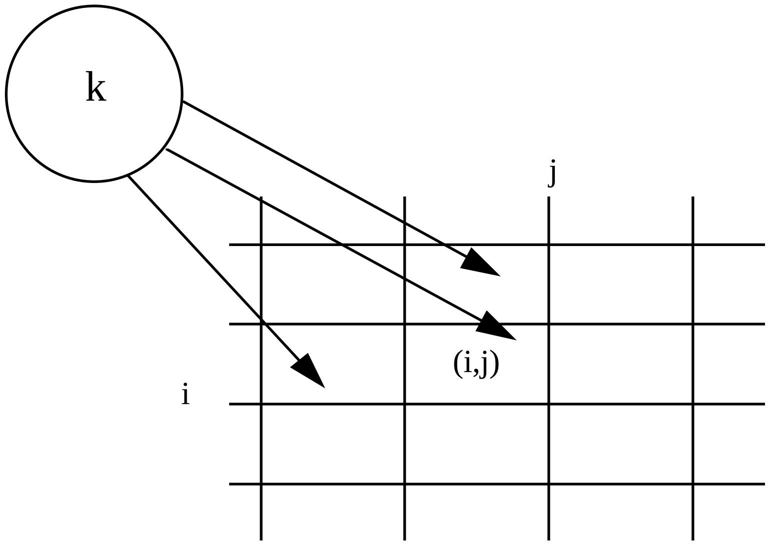

Fig.3. Connections of insurance companies.

Each company k has an initial risk reserve, R_k^o, which in general is characterized by a random vector. For example, on Fig.5 the initial reserve of each company is equal to 2 units. Each company k receives premiums $\pi_k(i,j)$ from all insured (i,j) according to certain agreements.

In general the risk reserve for company k has the following form:

$$R_k = R_k^0 - \sum_{(i,j)\in E(\omega)} D(i,j)q_k(i,j) + \sum_{(i,j)} \pi_k(i,j)q_k(i,j) - \sum_{(i,j)} C_k(i,j)q_k(i,j)$$

where $q_k(i,j)$ is the coverage of company k in (i,j), and $C_k(i,j)$ is the transaction costs, $D(i,j)$ is a random damage caused by the simulated catastrophe. This value depends on the pattern of random catastrophic events, their strength and decay. Thus we assume that random events may have random directions of propagation through the region, and they affect random numbers of squares. In general, an event is modeled by a random subset $\mathrm{E}(\omega)$ of squares and its strength in each (i,j). The damage $D(i,j)$ is a function of the strength, mitigation measures and type of constructions.

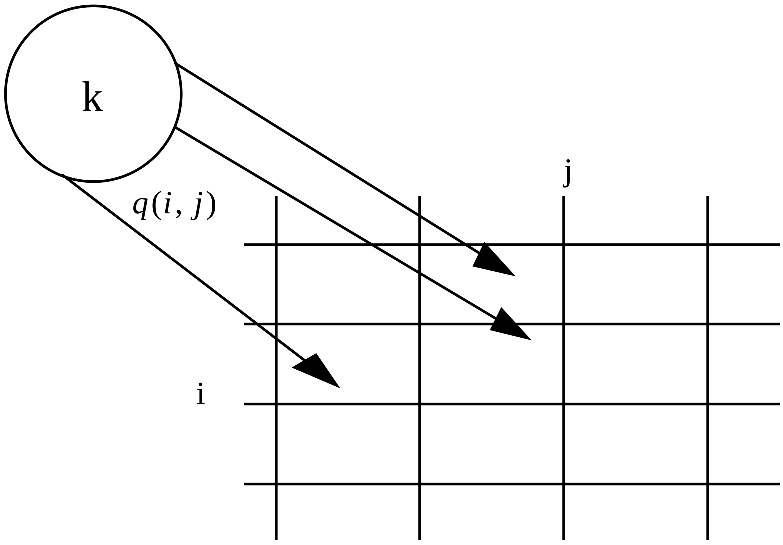

Fig.4.

We assume that damages of each square may be covered partially by all companies $k = 1,\dots,N$:

$$\sum_{k=1}^{N} q_k(i,j) \leq 1.$$

As performance indicators of the insurance industry, we use profit functions of insurers, risk of their insolvency, and loss functions of the insured. These indicators implicitly depend on the set of feasible policy variables. Their structure is analytically untractable because it is defined by the simulated patterns of the catastrophes. In particular, we use the following risks functions

$$I(q) = \sum_k W_k \mathrm{E}\min\{0, R_k\}$$

$$L(q) = \sum_{(i,j)} V(i,j) \mathrm{E} \min \left\{ 0, D(i,j) \sum_k q_k(i,j) - \sum_k \Pi_k(i,j) q_k(i,j) \right\}$$

with weights $W_k > 0$, $V(i,j) > 0$ which can be adjusted to satisfy additional constraints, for example, on fairness, equity, stability, profitability, etc. In particular, it can be proven that if weights W_k become large enough, the first risk function is equivalent to the so-called stability constraints requiring that the probability of insolvency for each insurer does not fall below a given level of "survival".

The optimal diversification requires that the insurers cover only a fraction of their catastrophic damages in each location. In this sence, catastrophic risks create a dynamic network of interdependent insurers.

3. Diversification of dependent claims

Consider a simple fragment of the regional model. This example explains effects which may be achieved by taking into account dependencies among damages.

Two companies insure property values in two remote squares or subregions 1, 2 (see Fig.5). With arrows we denote connections of the companies with insured. The property value of each square is evaluated equal to 10 monetary units, and initial risk reserves of insurers are the same and shown inside the circles for each insurer.

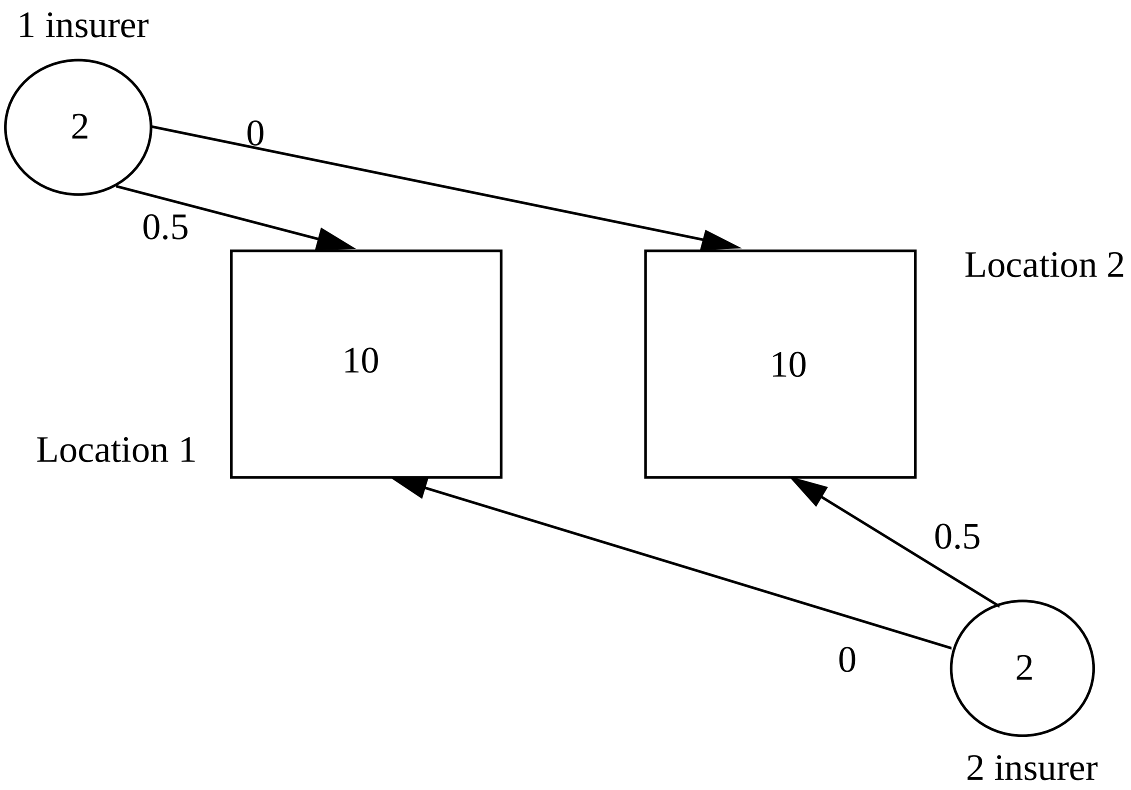

Fig.5. Initial coverages.

Initial coverages are shown by arrows on Fig.5. Thus, the first company covers 0.5 damages of the first square, and, symmetrically, the second company covers 0.5 damages of the second square.

The initial distribution of coverages shows that both insurers deal only with claims from the nearest location since there exist higher transaction costs for each insurer to deal with remote clients. It is easy to see that this policy is not appropriate in the case of dependent damages. Assume that catastrophic event with the same distribution of random strength "strikes" either square 1 or 2 with probability 1/2.

The histogram in Fig.6 shows the approximate distribution of damages in squares 1, 2 after 200 simulations (scenarios). Since the situation is symmetric for both squares, the distributions are identical. The identical distribution of damages for both locations, the lack of information on dependencies between damages and the existence of transaction costs may justify the policy of insurers dealing only with clients from an area where they leave.

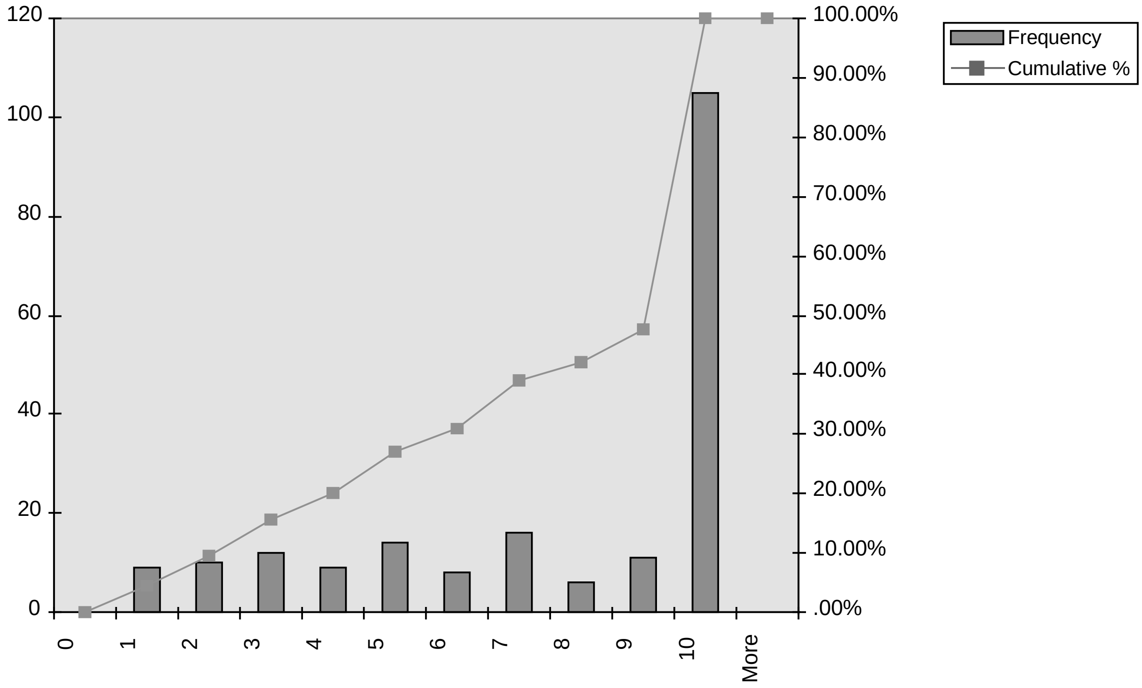

Fig.6. Histogram of damaged properties.

Fig.7 shows the histogram of the initial risk reserve which is the same for both insurance companies since contracts are symmetrical. As we can see, the insolvency of companies occurs in more then 10% of simulations. It is intuitively clear that contracts from remote locations may stabilize the risk reserves of both insurers assuming that we know that these catastrophic events are mutually exclusive. The premium accumulation from the nearest location may not be enough to cover losses. Contracts from remote locations in the case of mutually exclusive events provide additional reserves to cover losses. Of course, in general cases dependencies between damages have a complex character defined by spatial patterns of events. The overall interplay between policies and risks are highly nonlinear and intuition cannot be helpful without explicit information about interdependencies. The spatial model simulates these interdependencies and spatial optimization techniques utilize them for designing better policies, for example, on premiums, spread of coverages, mitigation measures, governmental interventions and so on.

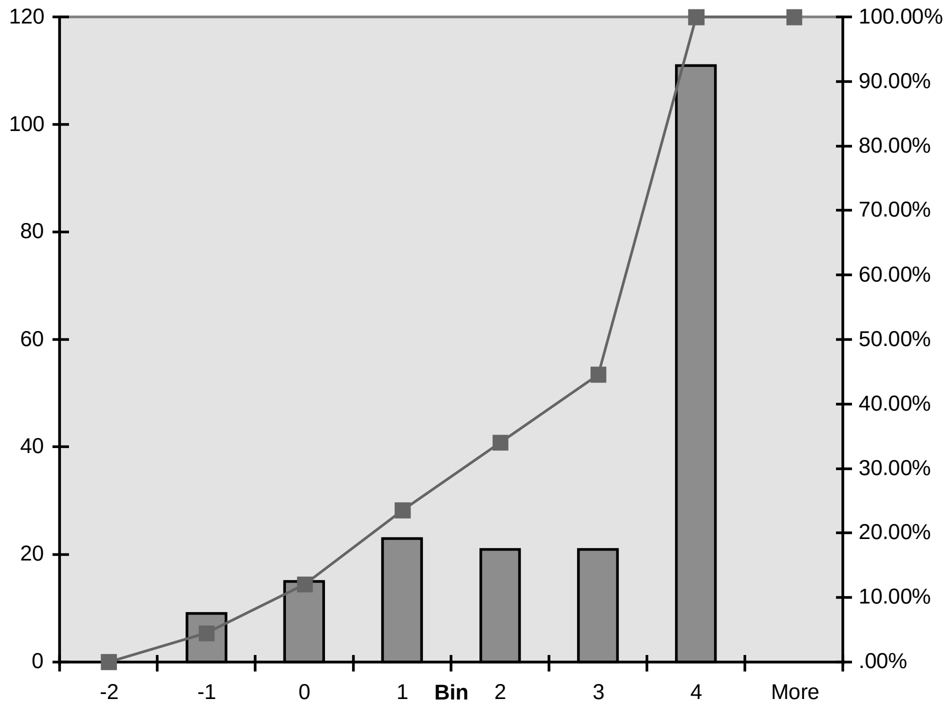

Fig.7. Histogram of initial risk reserve, insurers 1, 2.

In our example we are looking only for optimal spatial diversification of contracts. Premiums in a sense do not "overshoot" arising damages which generate demand for insurance. The current policies fail to provide stability of companies against catastrophic risk as it is shown on Fig.7. The model allows us to explore the "hidden" reserves of the companies by using all new feasible policies, for example, all possible combinations of covereges for both insurers in both regions (Fig.5): the first insurer instead of policy (0.5;0) may use (0.2;0.4) policy and at the same time the second insurer uses (0,3;0.6) and so on. It is obvious that in the case of more than two regions and insurers the number of such type of combinations may easily approach infinity (in fact it is infinity even in the simple case on Fig.5, if we don't use only discrete levels of possible coverages:0; 0.1; 0.2; ...). The stochastic optimization generates a final optimal solution which is robust against all possible scenarios of events. In contrast, the straightforward scenario analysis generates optimal solutions for each scenario without providing any clue as to the choice of the final robust solution against all scenarios.

There are two approaches (see[4]): 1) the use of deterministic, large-scale approximations of a stochastic model and special deterministic optimization technique; and 2) the use of stochastic quasigradient procedures to confront the complexity of spatial stochastic model directly without large-scale deterministic approximations.

4. Deterministic Approaches

This approach proceeds with the simulation of a finite number of scenarios (patterns of events) which are then used to approximate probabilistic functions of the stochastic problem by their mean values. It transfers the original stochastic model to a deterministic one with a large number of additional constraints. The number of constraints in the original model is equal to the number M of squares. The number of variables is $M \times N$, where N is the number of insurance companies. Thus for $30 \times 30 = 900$ squares in the region with $N = 5$, the number of variables is 3500. The deterministic approximation for S scenarios requires $N \times S$ additional constraints with $2NS$ additional variables. Thus in the case of 5 insurers and 1000 scenarios, the number of additional variables is 10000. In our simple example in Fig.5 the number of variables with $S = 200$ is 4+800=804; the number of constraints 2+400=402. The use of deterministic approximation for more realistic spatial models require special large scale optimization and scenario generation techniques, which is a main concern of stochastic optimization (see [4]).

The solution to the deterministic approximation with the number of scenarios $S = 200$ resulted in the new policies shown on Fig.8. The histogram of improved risk reserves for these policies (identical for both companies) is shown on Fig.9.

As we can see from Fig.8 the total amount of coverage is increased for both locations which guarantees improved financial protection of property values. At the same time the stability and profits of the companies are increased (Fig.9). The main drawback of this approach is the necessity to deal with large scale optimization problems. The computer memory may essentially limit the number of scenarios for providing consistent estimates.

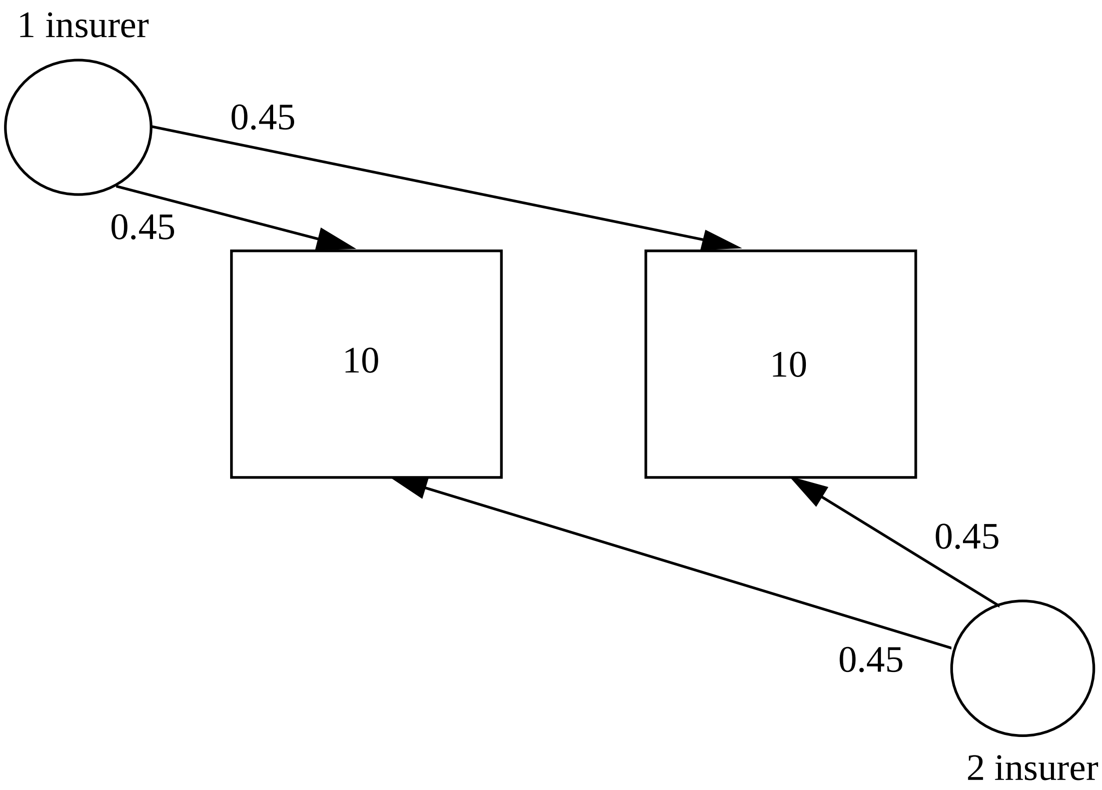

Fig.8. Improved coverages.

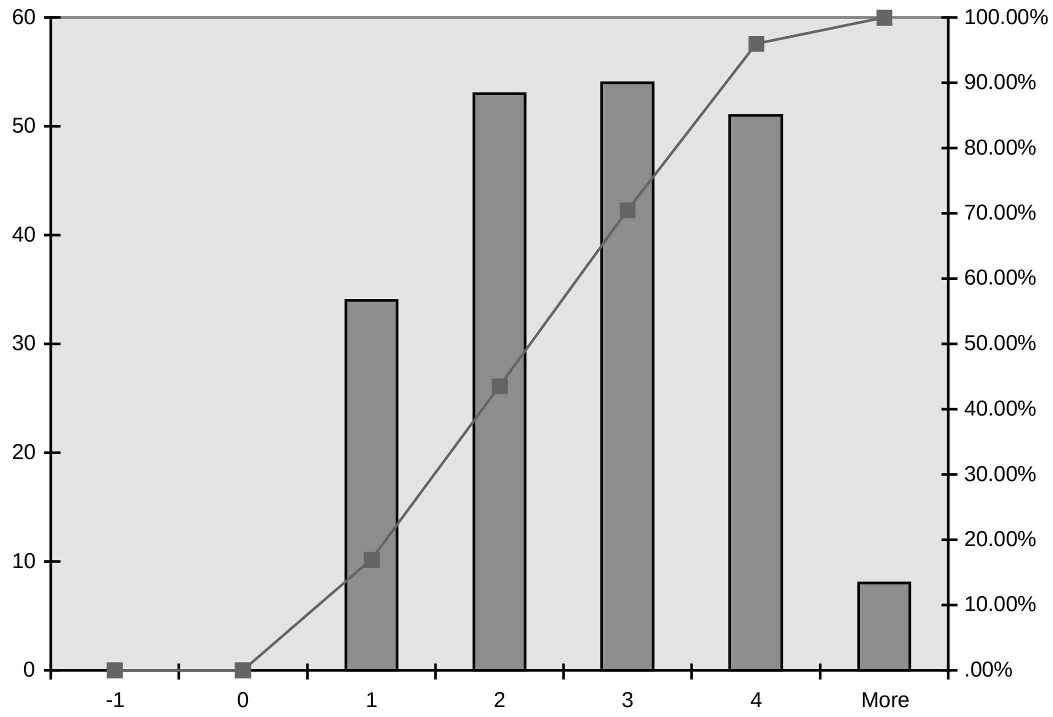

Fig.9. Histogram of improved risk reserves.

5. Stochastic Quasigradient Methods

These methods confront the complexity of arising nonsmooth stochastic optimization problems directly without deterministic approximations. The search for better policies starts with the current policies. A scenario is generated and promising improvements (stochastic quasigradient) are estimated. By using this random direction, the current policies are slightly adjusted; a new scenario is generated, and so on. This type of optimization techniques can be viewed as adaptive Monte-Carlo simulation, or adaptive scenario analysis. On Fig.10 a trajectory of adaptive improvements of the goal risk function is shown. The function includes expected profits, risk of underestimating insolvencies by insurers and damages by insured. This function forces the adaptation process towards better stability of insurance companies, their profits and better coverages of damages. As we can see the initial policy is sensitive to catastrophes and the goal function has considerable fluctuations towards insolvency at first simulations. New policy variables after 40 adjustments are less sensitive to catastrophic events, although the adaptive adjustments brings new improvements of goal function (steps 40-125) and catastrophes still may suddenly affect the stability (steps 125, 145,...). The final new policy is stabilized after 1000 scenarios around levels shown on Fig.11. The coverages of companies have increased, thus better financial protection of the population is achieved. At the same time the histograms of risk reserves (Fig.12) show better stability and profits of companies.

The difference in the results of deterministic approximation and stochastic solution techniques is due to the smaller number of scenarios used in the first case. The increase in the number of scenarios requires higher computational resources of computers.

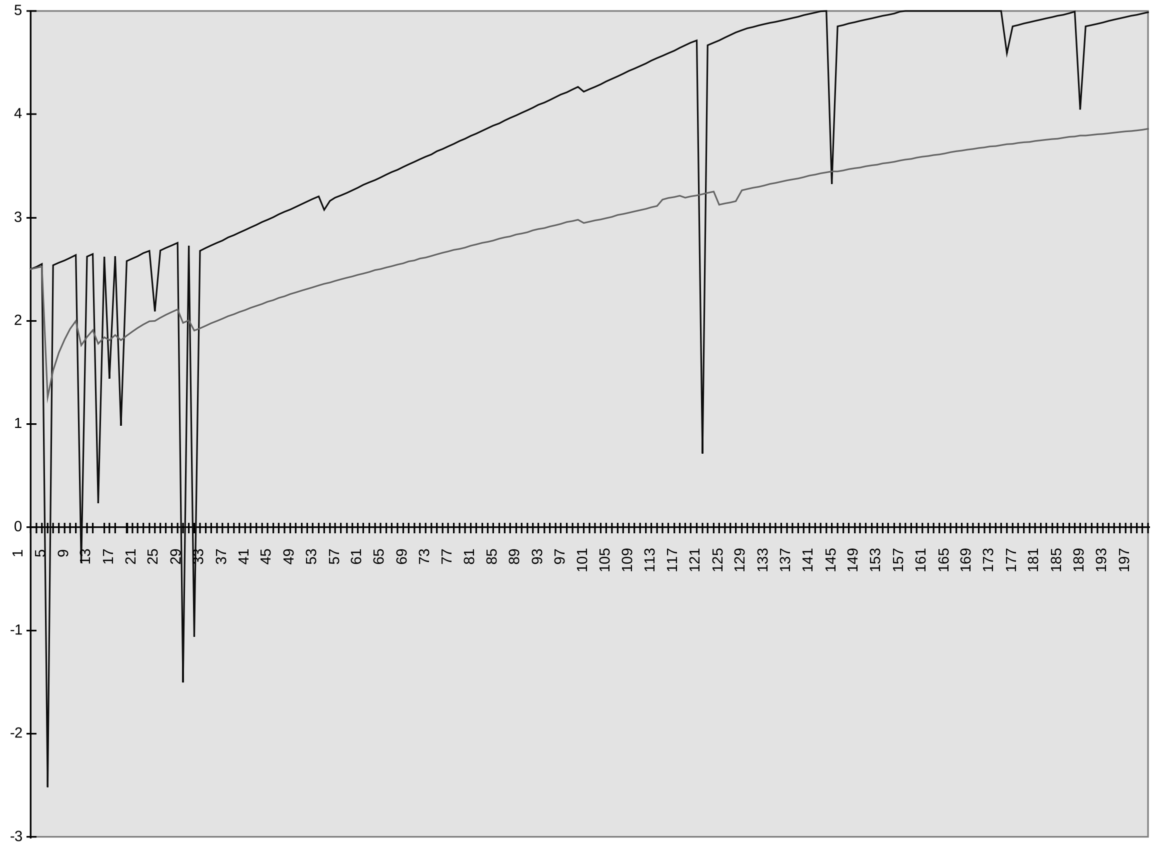

Fig.10. Dynamics of improvements: random goal function and moving average.

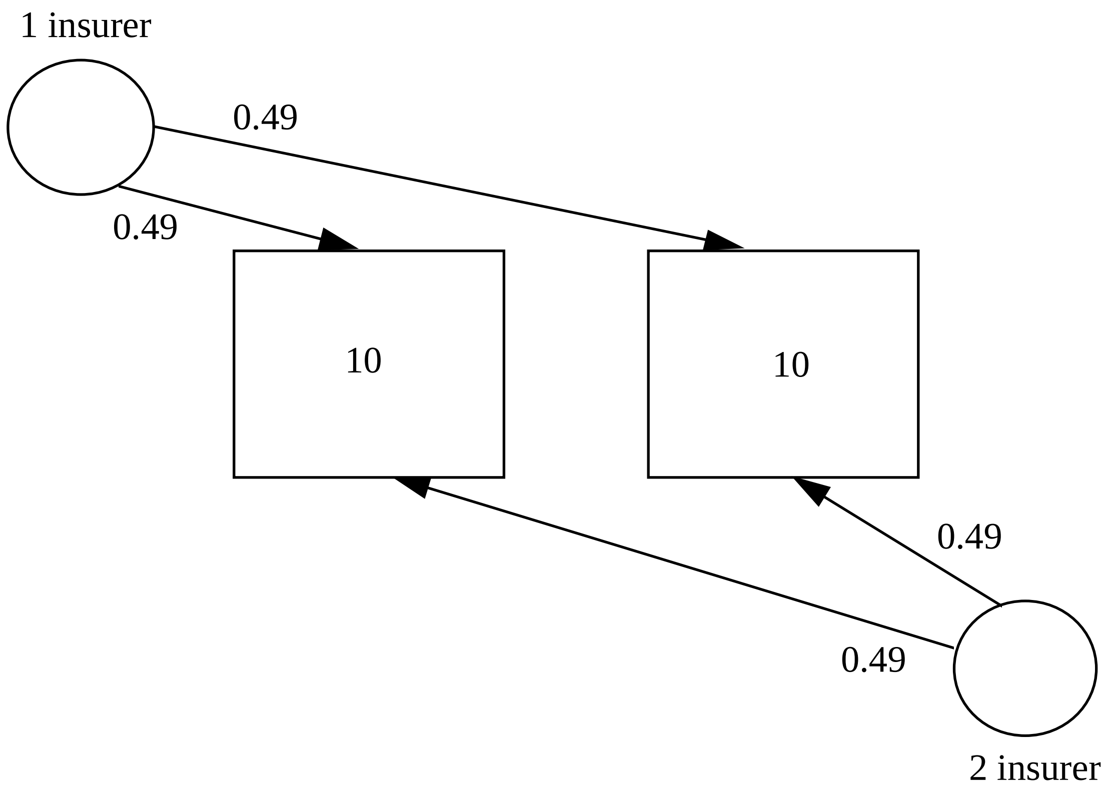

Fig.11. Improved allocation of contracts, quasigradient approach.

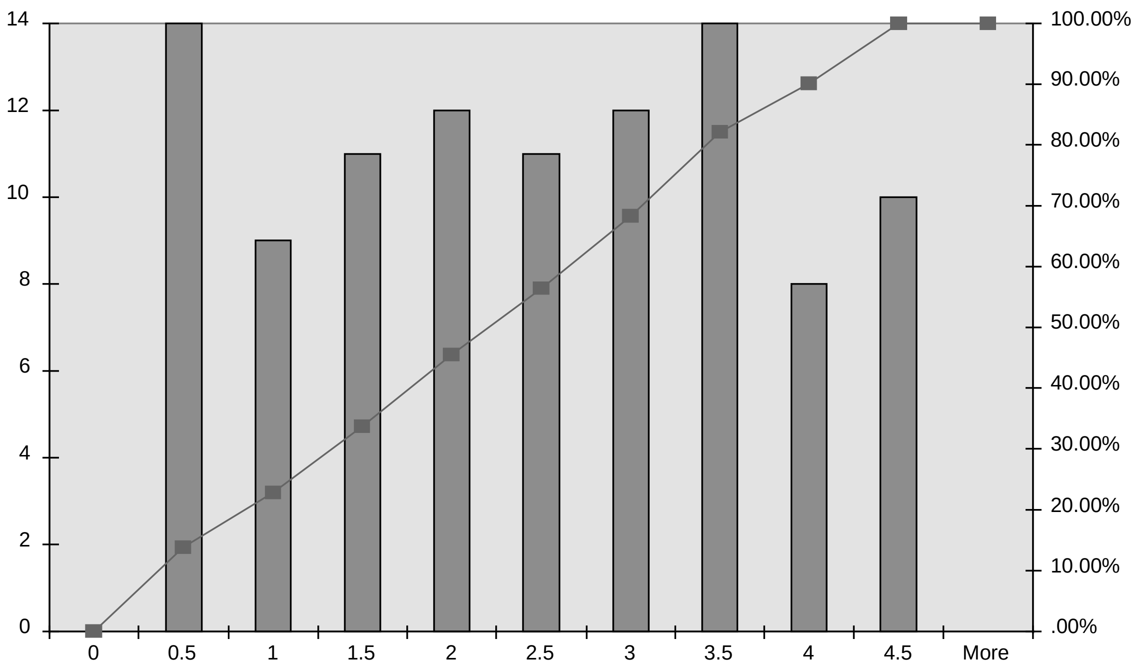

Fig.12. Histogram of improved risk reserves.

6. General case

Consider a more general fragment of the regional model with $10 \times 10 = 100$ squares shown in Fig.13 with property values in each of them. The "landscape" of initial regional property values is shown in Fig.14.

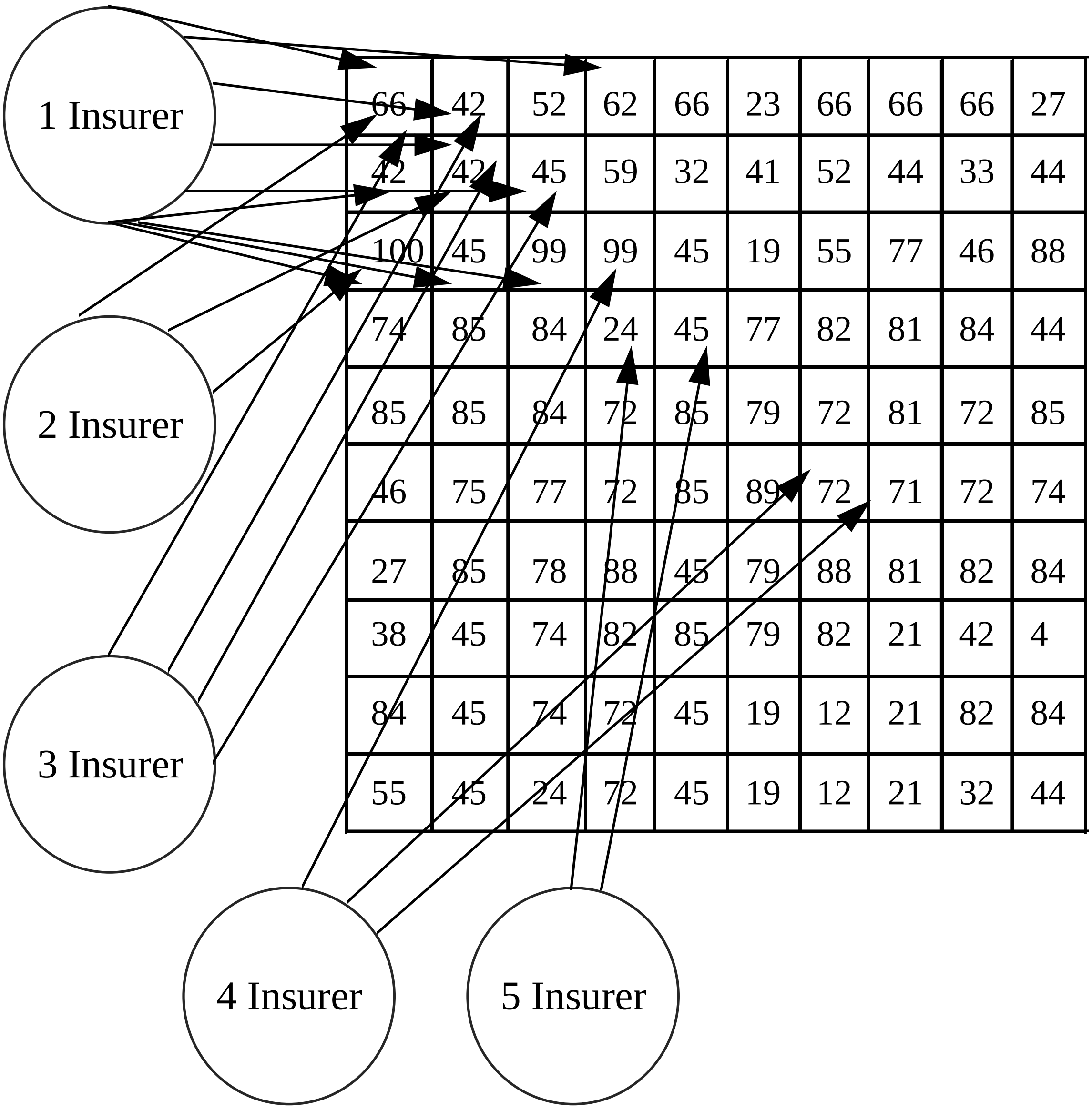

Fig.13. Initial allocations of contracts and property values.

There are five companies operating in the region. Current contracts of companies and values of coverages are shown by arrows.

A catastrophic event is simulated as an asymmetric path dependent random field with different probabilities to move to adjacent squares. Spatial realizations ("trajectories") of this field in a particular case may be random lines or trajectories of an asymmetric random walk, which has a random length and its random strength decreases with each step. A simulated pattern of the event allows us to calculate damages in each square which is shown on Fig.15, 16 in the form of landscape of damaged property value.

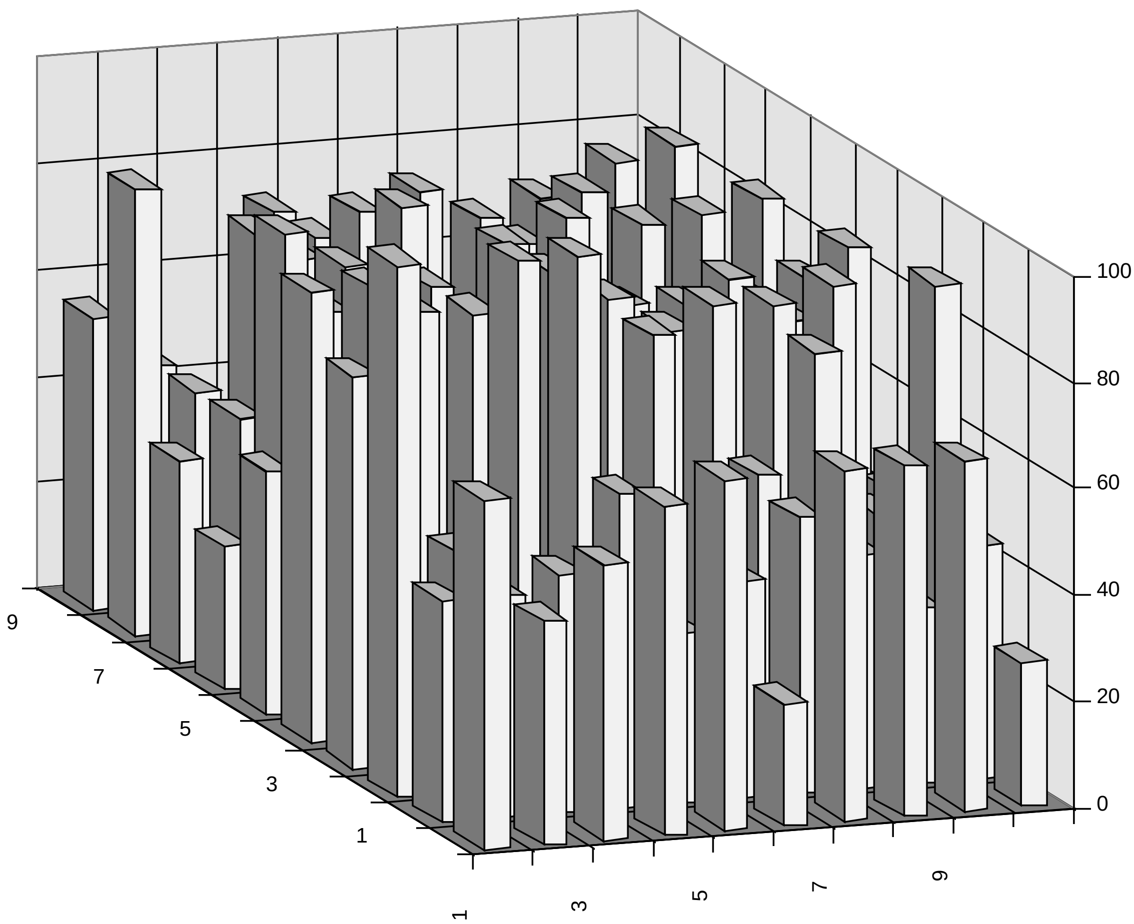

Fig.14. Landscape of property values.

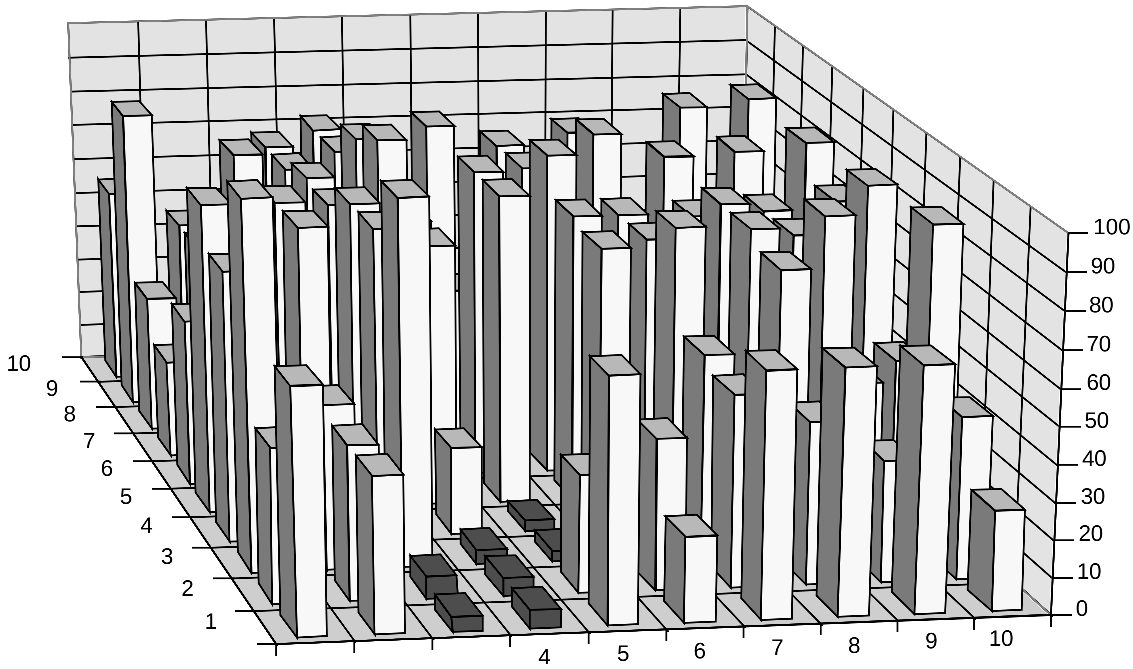

Fig.15. Landscape of damaged property values: Scenario 1.

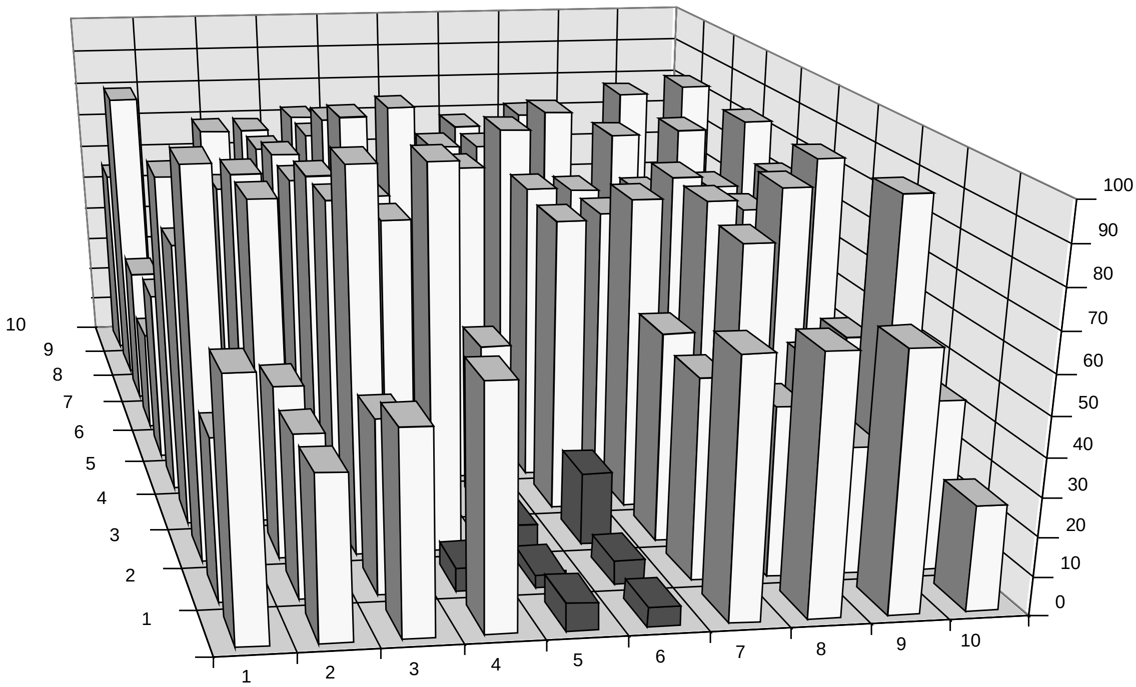

Fig.16. Landscape of damaged property values: Scenario 2.

On Fig.17, 18 we can see histograms of risk reserves for insurers 1, 2 .

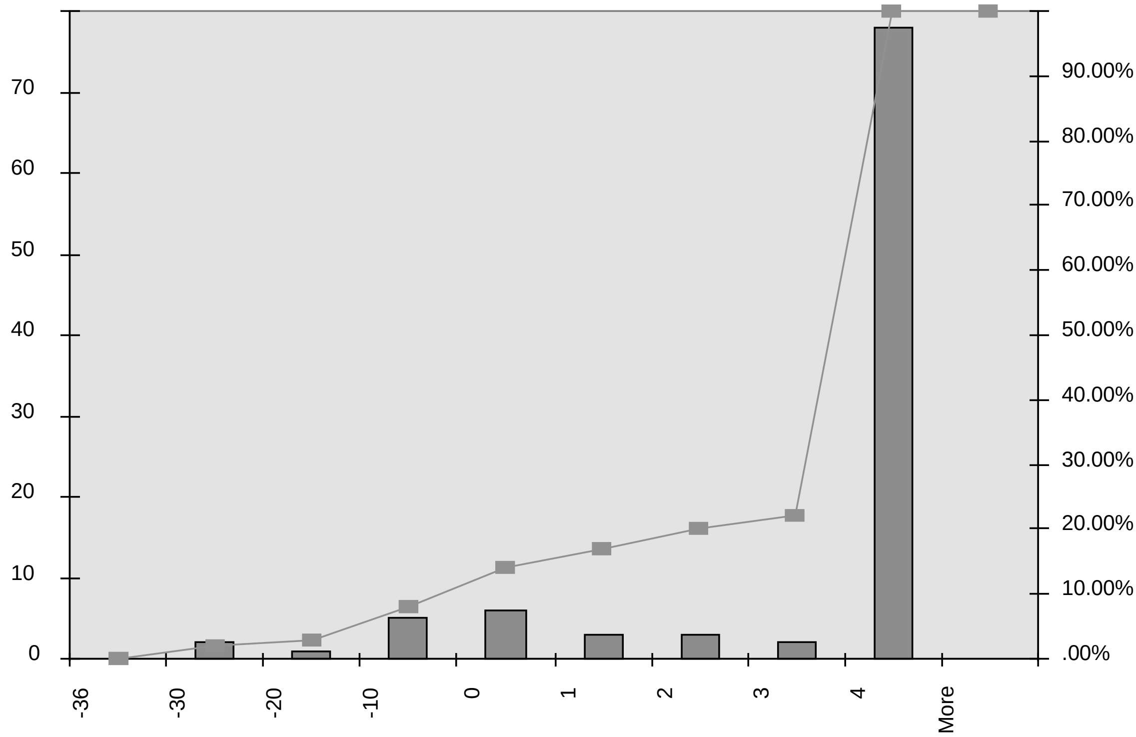

Fig.17. Histogram of initial risk reserve: Insurer 1.

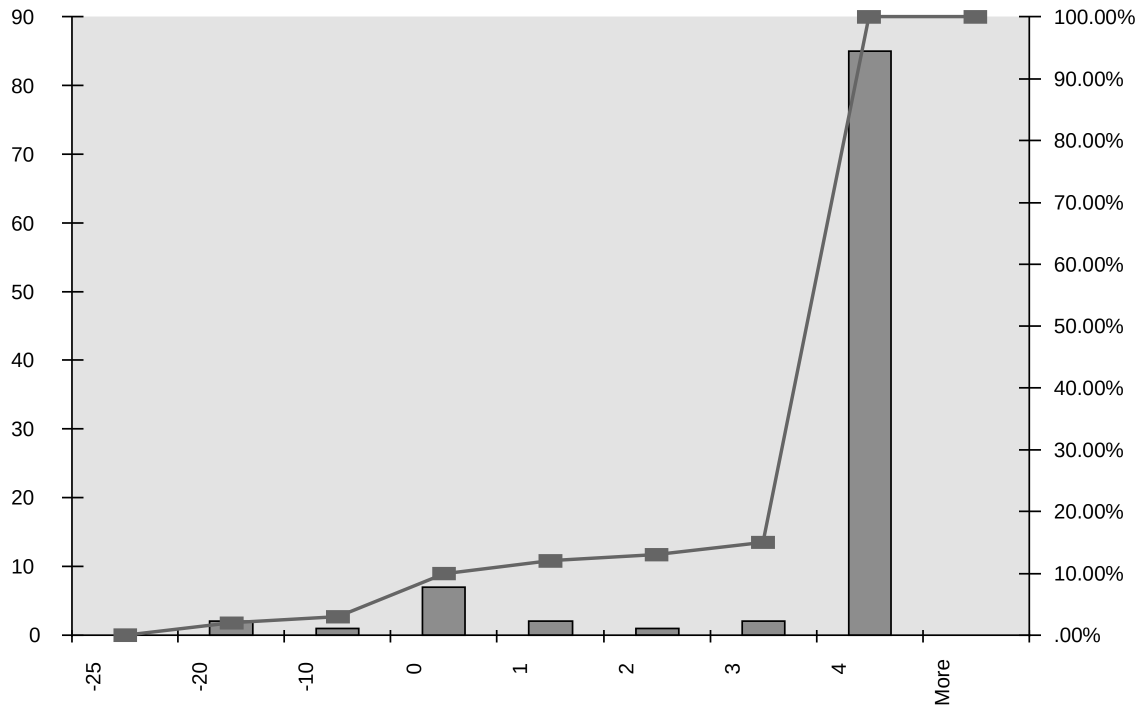

Fig.18. Histogram of initial risk reserve: Insurer 2.

The deterministic approximation with S=1000 scenarios has now $100 \times 5 + 2 \times 5 \times 1000 = 10500$ variables. Deterministic approximation approach generates new contracts given in Table 2:

	1 Company		2 Company		3 Company		4 Company		5 Company	
Initial reserve	2		2		2		2		2	
	Insured	Contracts	Insured	Contracts	Insured	Contracts	Insured	Contracts	Insured	Contracts
	1.1	0	3.1	0.04417	1.1	0.0162	3.4	1	4.4	1
	1.2	0.198	1.1	0.1	1.2	0.1491	6.7	1	4.5	1
	2.2	0.087	2.2	0.05	2.2	0.1257	6.8	1		
	2.3	0.0796	1.2	0	2.3	0.0981				
	1.3	0	1.3	0.1235						
	2.1	0	2.1	0						
	3.1	0	2.3	0						
	3.2	0	3.2	0						
	3.3	0.0354	3.3	0						

Table 2. Improved contracts: Deterministic approximation.

Histograms of improved risk reserves for companies 1, 2 are shown on Fig.19, 20.

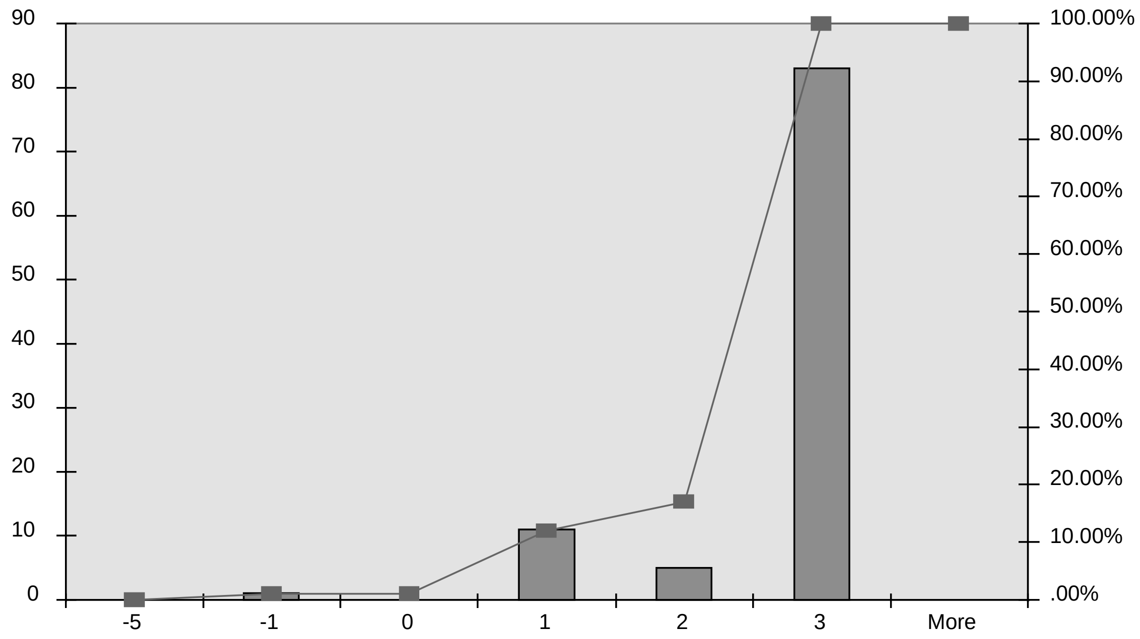

Fig.19. Histogram of improved risk reserve: Insurer 1.

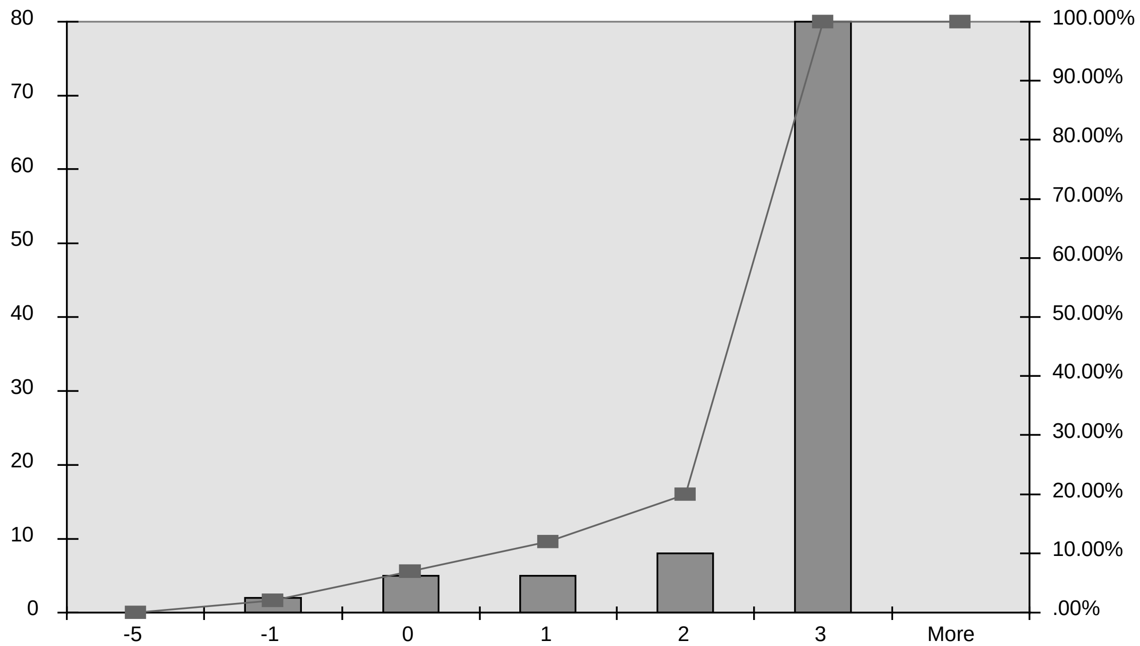

Fig.20. Histogram of improved risk reserve: Insurer 2.

A fragment of improvement for stochastic quasigradient methods is shown on Fig.23. It is interesting that the value of the goal function is stabilized very soon, after 200 scenarios. But variances still exists and the last 1000 steps of adjustments eliminate further the influence of rare catastrophes by making adjustments towards more robust policies. Allocation of improved contracts are shown in Table 3. Histograms of risk reserves at the improved contracts from Table 3 are shown on Fig.21, 22.

	1 Company		2 Company		3 Company		4 Company		5 Company	
Initial reserve	2		2		2		2		2	
	Insured	Contracts	Insured	Contracts	Insured	Contracts	Insured	Contracts	Insured	Contracts
	1.1	0.11	3.1	0.555	1.1	0.094	3.4	1	4.4	1
	1.2	0.046	1.1	0.0956	1.2	0.0378	6.7	1	4.5	1
	2.2	0.042	2.2	0	2.2	0.0042	6.8	1		
	2.3	0	1.2	0.029	2.3	0				
	1.3	0.0626	1.3	0.1308						
	2.1	0.0336	2.1	0.0126						
	3.1	0.445	2.3	0						
	3.2	0	3.2	0						
	3.3	0.5	3.3	0.49						

Table 3. Improved values of contracts.

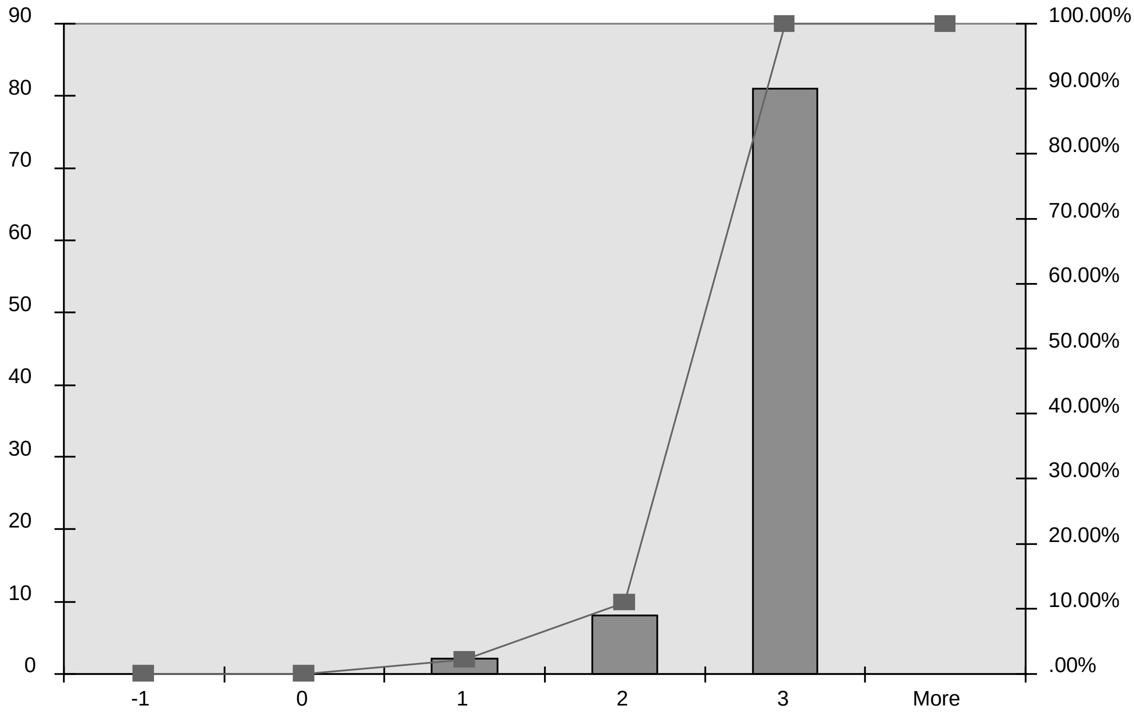

Fig.21. Histogram of improved risk reserve: Insurer 1.

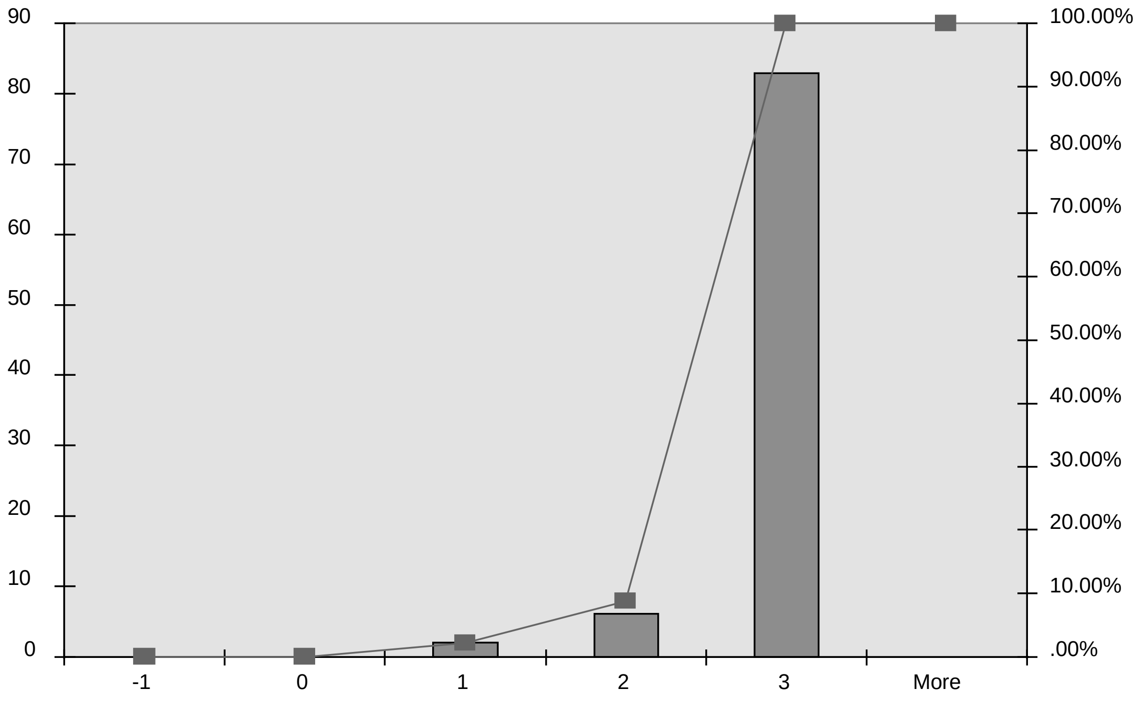

Fig.22. Histogram of improved risk reserve: Insurer 2.

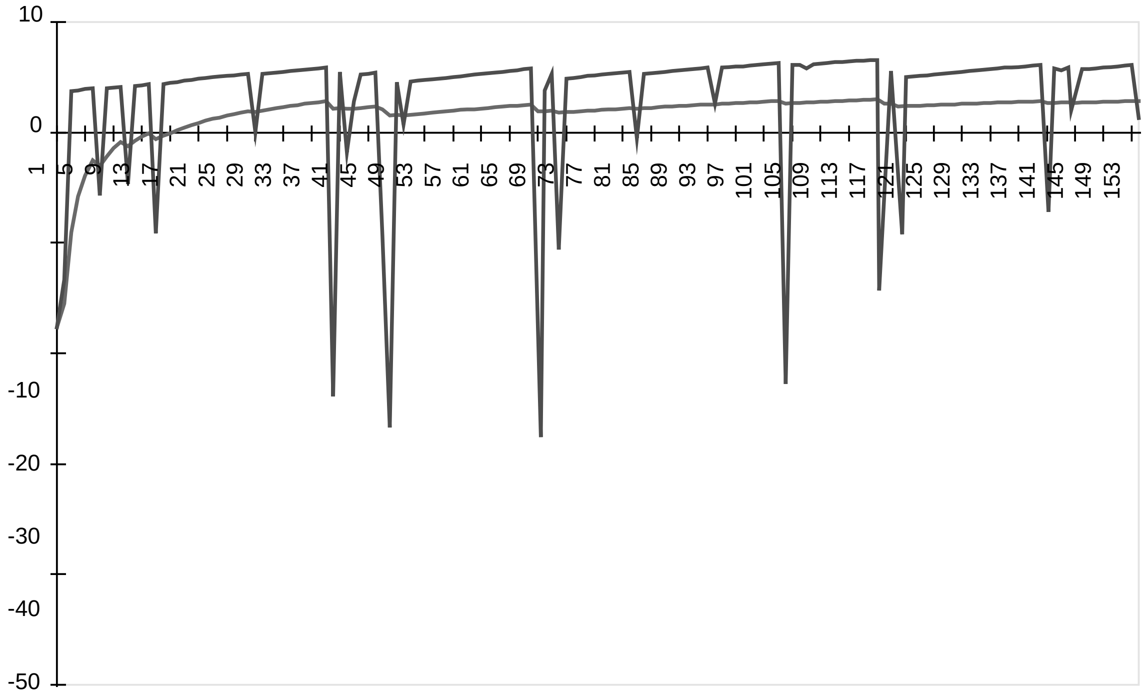

Fig.23. Moving average and values of random goal function.

Conclusions

The numerical experiments illustrate the importance of the geographical diversification of catastrophe coverages because of the dependencies between damages in different locations: The numerical experiments also show the possibility of different approaches for the design of new policies. The first approach is based on simulating a "basket" of scenarios in advance with further adjustment of policy variables to these scenarios by using large-scale, deterministic optimization techniques. This approach cannot be used for a dynamic model when the time of bankruptcy implicitly depends on the policy variables. In addition, it requires high computational resources of computers when the number of scenarios increases to achieve consistency in the case of rare events. The second approach is based on the sequential simulation of scenarios and adjustments of policy variables. The policy variables are adjusted after each simulation, which allows us to use this approach also for spatial and dynamic risk management problems.

Neither approach requires the exact evaluation of all the possible risks associated with different combinations of policy variables, which may be infinite. The search for a desirable combination of policy variables is made possibly by the use of special optimization techniques which avoid the need for an infinite number of evaluations. In the more general dynamic, case the analyzed problem is equivalent to the study of a multidimensional accumulation jumping process, which can be described also by systems of integro-differential equations. The discussed approaches avoid this attractable task by dealing directly with the stochasticity.

This general class of problems cannot be solved by standard optimization techniques. More practically oriented studies require the development of appropriate tools and decision support systems. In particular, the first approach requires the generation of sets of most representative scenarios.

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