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LOVE DYNAMICS BETWEEN SECURE INDIVIDUALS: A MODELLING APPROACH

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Abstract

A mathematical model that qualitatively describes the dynamics of love between secure individuals is presented in this paper with two goals. The general goal is to show how dynamic phenomena in the field of social psychology can be analyzed following the modelling approach, traditionally used in all other fields of sciences. The specific goal is to derive, from very general assumptions on the behavior of secure individuals, a series of rather detailed properties of the dynamics of their feelings. The analysis shows, in particular, why couples can be partitioned into fragile and robust couples, how the quality of romantic relationships is influenced by behavioral parameters and in which sense individual appeal can create order in a community structure.

Introduction

Dynamic phenomena are of basic interest in all fields of sciences. Population growth, economic recessions, cell degeneracies, forest fires, learning and adaptation processes are relevant examples. Dynamic phenomena are usually described by means of differential equations (Luenberger, 1979). These equations were introduced by Newton in the 17-*th* century for studying the dynamics of mechanical systems. Given a system characterized by n variables varying in time $x_1(t)$, $x_2(t)$, ..., $x_n(t)$, the dynamic behavior of the system is usually described by n differential equations of the form

$$\begin{aligned} \frac{dx_1(t)}{dt} &= f_1\left(x_1(t), x_2(t), \ldots, x_n(t)\right) \\ \frac{dx_2(t)}{dt} &= f_2\left(x_1(t), x_2(t), \ldots, x_n(t)\right) \\ &\vdots \\ \frac{dx_n(t)}{dt} &= f_n\left(x_1(t), x_2(t), \ldots, x_n(t)\right) \end{aligned} \tag{1}$$

where $dx_i(t)/dt$ is the derivative of $x_i(t)$ with respect to time. Thus, each function f_i specifies the instantaneous rate of change of x_i. Depending upon the field and upon the character of the investigation, the functions f_i can express exact physical laws, empirical laws or conjectures. Equations (1) are the *model* of the system and the variables x_i are called *state variables*. Actually, model (1) is often a family of models: this happens when the functions f_i depend also upon some constant parameters p_1, p_2, ..., p_m that identify some features of the system. Only in very rare cases, model (1) mimics exactly the behavior of the real system.

Given the state at a specified time, say $t = 0$, equations (1) can be solved analytically or numerically and the solution $x_1(t)$, $x_2(t)$, ..., $x_n(t)$, $t > 0$, describes completely the dynamic behavior of the system. Often one is interested in qualitative properties of the solution, like positivity and boundedness of all $x_i(t)$. In particular, when the solution is bounded, it is of interest to know if the state tends toward a cyclic behavior or toward an equilibrium value as time goes on. These asymptotic behaviors usually depend continuously upon parameters. On the contrary, they are often independent upon initial conditions, in which case the system is called stable.

In conclusion, a model is a set of *first level* properties of a system, *i.e.*, laws or conjectures, while, the analysis of the model, *i.e.*, the solution of equations (1), is the formal mathematical process of deriving *second level* properties of the dynamics of the system. The identification of this hierarchy is very important both conceptually and in practice, in particular, when the properties are discovered through experiments, data collection and statistical analysis. This is clarified by a very simple physical example in Appendix A.

As far as the knowledge of the authors goes, the above described modelling approach has never been used in social psychology. The only exception is a very recent study (Rinaldi, 1996) on the dynamics of love between Petrarch, an Italian poet of the 14-*th* century, and Laura, a beautiful and married lady. Their love story developed over 21 years and has been described in the *Canzoniere*, a collection of 366 poems addressed by the poet to his platonic mistress. In such a study, the main traits of Petrarch's and Laura's personalities are identified by analyzing the *Canzoniere*: he is very sensitive and transforms emotions into poetic inspiration; she protects her marriage by reacting negatively when he becomes more demanding and puts pressure on her, but at the same time, following her genuine Catholic ethic, she arrives at the point of overcoming her antagonism by strong feelings of pity. Then, these traits are encapsulated in a model composed by differential equations of the form (1) where x_1 and x_2 are the emotions of the two individuals. Finally, the model is analyzed and the result (second level property) is that its solution tends toward a cyclic behavior. In other words, the peculiarities of the personalities of the two lovers inevitably generate a never ending story of recurrent periods of ecstasy and despair. This is indeed what happened, as ascertained by Jones (1995) through a detailed stylistic and linguistic analysis of the dated poems. It is interesting to note, however, that with the modelling approach the existence of the emotional cycle is fully understood and proved to be inevitable, while with the empirical approach it is only discovered.

We follow in this paper the same modelling approach to discuss the dynamics of the feelings between two persons. But instead of concentrating our attention on a specific and well documented case, we deal with the most generic situation, namely that of couples composed of secure individuals (Bartholomew & Horowitz, 1991; Griffin & Bartholomew, 1994). For this we will first assume that secure individuals are characterized by a set of first level properties concerning their reactions to partner's love and appeal. The analysis of the corresponding model will produce a set of second level properties that describe how the feelings between two individuals evolve in time. Intentionally, we do not support these properties (neither the first nor the second level ones) with data, because we like only to highlight the power of the method, namely the possibility of discriminating between causes and effects. It is worth anticipating, however, that all the properties discussed in the paper are in agreement with common wisdom on the argument.

The paper is organized as follows. In the next section we assume that secure individuals follow precise rules of behavior (first level properties) and we build a mathematical model based on these rules. Then, we analyze the model and derive second level properties which formally describe the dynamic process of falling in love, *i.e.*, the transformation of the feelings, starting from complete indifference and tending toward a plateau. Other results are concerned with the influence that appeal and individual behavior have on the quality of romantic relationships. Some of these properties are used to derive the consequences on partner selection. Although the results are extreme, they explain to some extent facts observed in real life, such as the rarity of couples composed of individuals with very uneven appeal. Merits and weaknesses of the modelling approach as well as directions for further research are briefly discussed at the end of the paper.

The model

The model proposed in this paper is a *minimal* model, in the sense that it has the lowest possible number of state variables, namely one for each member of the couple. Such variables, indicated by x_1 and x_2, are a measure of the love of individual 1 and 2 for the partner. Positive values of x represent positive feelings, ranging from friendship to passion, while negative values are associated with antagonism and disdain. Complete indifference is identified by $x = 0$.

The model is a crude simplification of reality. Firstly, because love is a complex mixture of different feelings (esteem, friendship, sexual satisfaction, ...) and can be hardly captured by a single variable. Secondly, because the tensions and emotions involved in the social life of a person cannot be included in a minimal model. In other words, only the interactions between the two individuals are taken into account, while the rest of the world is kept frozen and does not participate explicitly in the formation of love dynamics. This means that rather than attempting to be complete, the aim is to check which part of the behaviors observed in real life can in theory be explained by the few ingredients included in the model.

It is important to state that the time scale to which we like to refer is an intermediate time scale. More precisely, we are not interested in fast fluctuations of the feelings, like those controlled by the daily or weekly activities or by the sexual rhythms of the couple, nor in the long term dynamics induced by life experiences. Thus, the model can only be used for relatively short periods of time (months/years), for example in predicting if a love story will be characterized by regular or stormy feelings. This implies that the present study can only weakly relate to *attachment theory* (Bowlby, 1969, 1973, 1980), which has been a main investigation tool in adult romantic relationships in the last decade (see, for instance, Hazan & Shaver, 1987; Collins & Read, 1990; Feeney & Noller, 1990, Simpson, 1990; Shaver & Brennan, 1992; Kirkpatrick & Davis, 1994).

Three basic processes, namely, *oblivion*, *return* and *instinct*, are assumed to be responsible of love dynamics. More precisely, the instantaneous rate of change $dx_i(t)/dt$ of individual's i love is assumed to be composed of three terms, *i.e.*,

$$\frac{dx_i}{dt} = O_i + R_i + I_i$$

where O_i, R_i and I_i must be further specified.

The oblivion process can be easily studied by looking at the extreme case of an individual who has lost the partner. If we assume that in such conditions $x_i(t)$ vanishes exponentially at a rate α_i, we must write

$$\frac{dx_i(t)}{dt} = -\alpha_i x_i(t)$$

so that we can derive

$$O_i = -\alpha_i x_i(t)$$

The return R_i describes the reaction of individual i to the partner's love and can therefore be assumed to depend upon x_j, $j \neq i$. Loosely speaking, this term could be explained by saying that one "loves to be loved" and "hates to be hated". In order to deal with the most frequent situation, we restrict our attention to the class of secure individuals, who are known to react positively to the love of the partner. More precisely, we assume that the return function R_i is positive, increasing, concave and bounded for positive values of x_j and negative, increasing, convex and bounded for negative values of x_j. Fig. 1 shows the graph of a typical return function. The boundedness of the return function is a property that holds also for non secure individuals: it interprets the psycho-physical mechanisms that prevent people from reaching dangerously high stresses. By contrast, increasing return functions are typical of secure individuals, since non secure individuals react negatively to too high pressures and involvement (Griffin & Bartholomew, 1994).

Finally, the third term I_i describes the reaction of individual i to the partner's appeal A_j. Of course, it must be understood that appeal is not mere physical attractiveness, but, more properly and in accordance with evolutionary theory, a suitable combination of different attributes among which age, education, earning potential and social position. Moreover, there might be gender differences in the relative weights of the combination (Feingold, 1990; Sprecher et al., 1994). For reasons similar to those mentioned above, we assume that instinct functions $I_i(A_j)$ enjoy the same properties that hold for return functions. Nevertheless, for simplicity, we will restrict our attention to the case of individuals with positive appeals.

In conclusion, our model for secure individuals, is

$$\frac{dx_1(t)}{dt} = -\alpha_1 x_1(t) + R_1(x_2(t)) + I_1(A_2) \tag{2a}$$

$$\frac{dx_2(t)}{dt} = -\alpha_2 x_2(t) + R_2(x_1(t)) + I_2(A_1) \tag{2b}$$

where the functions R_i and I_i annihilate for $x_j = 0$ and $A_j = 0$ and satisfy the following properties (see Fig. 1)

$$\begin{array}{llll} \text{sign}\left[R_i\right] = \text{sign}\left[x_j\right] & \dfrac{dR_i}{dx_j} > 0 & \text{sign}\left[\dfrac{d^2R_i}{dx_j^2}\right] = -\text{sign}\left[x_j\right] & \lim\limits_{|x_j|\to\infty} |R_i| < \infty \\ \text{sign}\left[I_i\right] = \text{sign}\left[A_j\right] & \dfrac{dI_i}{dA_j} > 0 & \text{sign}\left[\dfrac{d^2I_i}{dA_j^2}\right] = -\text{sign}\left[A_j\right] & \lim\limits_{|A_j|\to\infty} |I_i| < \infty \end{array} \tag{3}$$

Each individual i is identified in the model by two parameters (the appeal A_i and the forgetting coefficient α_i) and two functions (the return function R_i and the instinct function I_i). Such parameters and functions are assumed to be constant in time: this rules out aging, learning and adaptation processes which are often important over a long range of time (Kobak & Hazan, 1991; Scharfe & Bartholomew, 1994) and sometimes even over relatively short periods of time (Fuller & Fincham, 1995). The estimate of these parameters and functions is undoubtedly a difficult task, although some studies on attachment styles (Bartholomew & Horowitz, 1991; Carnelly & Janoff-Bulman, 1992; Griffin & Bartholomew, 1994) might suggest ways for identifying categories of individuals with high or low reactiveness to love and appeal. This identification problem will not be considered in this paper, which is centered only on the derivation of the properties of the model.

Consequences at individual level

Following the approach outlined in the introduction, we have analyzed model (2) taking into account the first level properties (3). The result is a series of second level properties describing the dynamics of the feelings and the impact that individual behavior has on the quality of romantic relationships. In the following, we simply state and interpret these properties, while their formal derivations are reported in Appendix B. As already stated, we consider only the case of positive appeals.

Property 1

If the feelings are non-negative at a given time, then they are positive at any future time.

This property implies that an undisturbed love story between secure individuals can never enter a phase of antagonism. In fact when two individuals first meet, say at time 0, they are completely indifferent one to each other. Thus, $x_1 = x_2 = 0$ and Property 1 implies that the feelings immediately become positive and remain positive forever. Actually it can be noted that model (2) implies that at time 0

$$\frac{dx_1}{dt} = I_1(A_2) \qquad \qquad \frac{dx_2}{dt} = I_2(A_1)$$

i.e., at the very beginning of the love story the instantaneous rates of change of the feelings are determined only by the appeals and are therefore positive if the appeals are such. The fact that antagonism can never be present in a couple of secure individuals is obviously against observations. Indeed, in real life the feelings between two persons are also influenced by facts that are not taken into account in the model. We can therefore imagine that most of the times model (2) describes correctly the dynamics of the feelings and this is what we have called "undisturbed" behavior. But from time to time unpredictable, and hence unmodelled, facts can act as disturbances on the system. A typical example is a temporary infatuation for another person giving rise to a sudden drop in interest for the partner. Of course, heavy disturbances can imply negative values of the feelings. It is therefore of interest to know what happens in such cases after the disturbance has ceased. The answer is given by the following property.

Property 2

Couples can be partitioned into robust and fragile couples. As time goes on, the feelings $x_1(t)$ and $x_2(t)$ of the individuals forming a robust couple tend toward two constant positive values *no matter what the initial conditions are. By contrast, in fragile couples, the feelings evolve toward two positive values only if the initial conditions are not too negative and toward two negative values, otherwise.*

Fig. 2 illustrates Property 2 by showing in state space the trajectories of the system starting from various initial conditions. Each trajectory represents the contemporary evolution of $x_1(t)$ and $x_2(t)$ and the arrows indicate the direction of evolution. Note that, in accordance with Property 1, no trajectory leaves the first quadrant. In Fig. 2a, corresponding to robust couples, all trajectories tend toward point $E^+ = (x_1^+, x_2^+)$, which is therefore a globally attracting equilibrium point. By contrast, fragile couples (Fig. 2b) have two alternative attractors (points E^+ and E^-) with basins of attraction delimited by the dotted line.

Fig. 2b points out an interesting fact. Suppose that the couple is at the positive equilibrium E^+, and that for some reason individual 2 has a drop in interest for the partner. If the drop is not too large the couple recovers to the positive equilibrium after the disturbance has ceased (see trajectory starting from point 1). But if the disturbance has brought the system into the other basin of attraction (see point 2), the couple will tend inevitably toward point E^-, characterized by pronounced and reciprocal antagonism. This is why such kind of couples have been called fragile. In conclusion, robust couples are capable of absorbing disturbances of any amplitude and sign, in the sense that they recover to a high quality romantic relationship after the disturbance has ceased. On the contrary, individuals forming a fragile couple can become permanently antagonist after a heavy disturbance and never recover to their original high quality mode of behavior.

Of course, it would be interesting to have a magic formula that could predict if a couple is robust or fragile. Unfortunately, this is not possible without specifying the functional form of the return functions, a choice that we prefer to avoid, at least for the moment. Nevertheless, we can in part satisfy our curiosity with the following property.

Property 3

If the reactions I_1 and I_2 to the partner's appeal are sufficiently high, the couple is robust.

This property is rather intuitive: it simply states that very attractive individuals find their way to reconciliate.

Fig. 2 shows that the trajectory starting from the origin tends toward the positive equilibrium point E^+ without spiraling around it. This is true in general, as pointed out by the following property.

Property 4

Two individuals, completely indifferent one to each other when they first meet, develop a love story characterized by smoothly increasing feelings tending toward two positive values.

This property implies that stormy relationships with more or less regular and pronounced ups and downs are typical of couples composed of non secure individuals. Thus, we can, for example, immediately conclude that Laura and/or Petrarch were not secure individuals.

We can now focus on the influence of individual parameters and behavior on the quality of romantic relationships at equilibrium. The first property we point out specifies the role of the appeals.

Property 5

An increase of the appeal A_i of individual i gives rise to an increase of the feelings of both individuals at equilibrium. Moreover, the relative increase is higher for the partner of individual i.

The first part of this property is rather obvious, while the second part is more subtle. Indeed it states that there is a touch of altruism in a woman [man] who tries to improve her [his] appeal.

We conclude our analysis by considering perturbations of the behavioral characteristics of the individuals. A positive perturbation of the instinct function of individual i gives rise to a higher value of I_i (A_j) and is therefore equivalent to a suitable increase of the appeal A_j. Thus, one can rephrase Property 4 and state that an increase of the instinct function I_i of individual i gives rise to an increase of the feelings of both individuals at equilibrium and that the relative improvement is higher for individual i. A similar property holds for perturbations of the return functions as indicated below.

Property 6

An increase of the return function R_i of individual i gives rise to an increase of both feelings at equilibrium, but in relative terms such an improvement is more rewarding for individual i.

This is the last relevant second level property we have been able to extract from model (2). Although some of them are rather intuitive others are more intriguing. But what is certainly surprising is that they are all mere logical consequences of our first level assumptions (3).

Consequences at community level

Now that we have identified the individual consequences of our first level properties, we can use them to extend the study to a more aggregated level. For this we will develop a purely theoretical exercise, dealing with a hypothetical community composed only of couples of secure individuals. Nevertheless, since secure individuals are a relevant fraction of population, we can be relatively confident and hope that our results retain, at least qualitatively, some of the most significant features of real societies.

Consistently with model (2), an individual i is identified by appeal, forgetting coefficient, return function and instinct function, *i.e.*, by the quadruplet (A_i, α_i, R_i, I_i). Thus, a community composed of N women and N men structured in couples is identified by N couples $\left[A_1^n, \alpha_1^n, R_1^n, I_1^n; A_2^n, \alpha_2^n, R_2^n, I_2^n\right]$ where the integer n = 1, 2, ..., N is the ordering number of the couple. For simplicity, let us assume that individual 1 is a woman and 2 is a man and suppose that there are no women or men with the same appeal, *i.e.*, $A_i^h \neq A_i^k$ for all $h \neq k$. This means that the couples can be numbered, for example, in order of decreasing appeal of the women. Since we have assumed that all individuals are secure, properties (3) hold for all individuals of the community. Moreover, in order to maintain the mathematical difficulties within reasonable limits, we perform only an equilibrium analysis and assume that all fragile couples are at their positive equilibrium E^+. In conclusion, our idealized community is composed by couples of secure individuals in a steady and high quality romantic relationship.

Such a community is called *unstable* if a woman and a man of two distinct couples believe they could be personally advantaged by forming a new couple together. In the opposite case the community is called *stable*. Thus, practically speaking, unstable communities are those in which the separation and the formation of couples are quite frequent. Obviously, this definition must be further specified. The most natural way is to assume that individual i would have a real advantage in changing the partner, if this change is accompanied by an increase of the quality of the romantic relationship, *i.e.*, by an increase of x_i^+. However, in order to forecast the value x_1^+ [x_2^+] that a woman [man] will reach by forming a couple with a new partner, she [he] should know everything about him [her]. Generally, this is not the case and the forecast is performed with limited information. Here we assume that the only available information is the appeal of the potential future partner and that the forecast is performed by imagining that the forgetting coefficient and the return and instinct functions of the future partner are the same as those of the present partner. Thus, the actual quality x_1^+ of the romantic relationship for the woman of the *h-th* couple is $x_1^+\left(A_1^h,\alpha_1^h,R_1^h,I_1^h;A_2^h,\alpha_2^h,R_2^h,I_2^h\right)$ while the quality she forecasts by imagining to form a new couple with the man of the *k-th* couple is $x_1^+\left(A_1^h,\alpha_1^h,R_1^h,I_1^h;A_2^k,\alpha_2^h,R_2^h,I_2^h\right)$. This choice of forecasting the quality of new couples obviously emphasizes the role of appeal. Quite reasonably, however, because appeal is the only easily identifiable parameter in real life.

The above discussion is formally summarized by the following definition.

Definition 1

A community $\left[A_1^n,\alpha_1^n,R_1^n,I_1^n;A_2^n,\alpha_2^n,R_2^n,I_2^n\right]$, $n=1,2,\ldots,N$ *is unstable if and only if there exists at least one pair* (*h, k*) *of couples such that*

$$x_1^+\left(A_1^h,\alpha_1^h,R_1^h,I_1^h;A_2^k,\alpha_2^h,R_2^h,I_2^h\right)>x_1^+\left(A_1^h,\alpha_1^h,R_1^h,I_1^h;A_2^h,\alpha_2^h,R_2^h,I_2^h\right)$$
$$x_2^+\left(A_1^h,\alpha_1^k,R_1^k,I_1^k;A_2^k,\alpha_2^k,R_2^k,I_2^k\right)>x_2^+\left(A_1^k,\alpha_1^k,R_1^k,I_1^k;A_2^k,\alpha_2^k,R_2^k,I_2^k\right)$$

A community which is not unstable is called stable.

We can now show that stable communities are characterized by the following very simple but peculiar property involving only appeal.

Property 7

A community is stable if and only if the partner of the n-th most attractive woman ($n=1,2,\ldots,N$) *is the n-th most attractive man.*

In order to prove this property, note first that Property 5 implies that a community is unstable if and only if there exists at least one pair (*h, k*) of couples such that

$$A_2^k > A_2^h \qquad\qquad A_1^h > A_1^k \tag{4}$$

Condition (4) is illustrated in Fig. 3a in the appeal space, where each couple is represented by a point. Consider now a community in which the partner of the *n-th* most attractive woman is the *n-th* most attractive man ($n=1,2,\ldots,N$). Such a community is represented in Fig. 3b, which clearly shows that there is no pair (*h, k*) of couples satisfying inequalities (4). Thus, the community is stable. On the other hand, consider a stable community and assume that the couples have been numbered in order of decreasing appeal of the women, *i.e.*

$$A_1^1 > A_1^2 > \ldots > A_1^N \tag{5}$$

Then, connect the first point $\left(A_1^1, A_2^1\right)$ to the second point $\left(A_1^2, A_2^2\right)$ with a segment of a straight-line, and the second to the third, and so on until the last point $\left(A_1^N, A_2^N\right)$ is reached. Obviously, all connecting segments have positive slopes because, otherwise, there would be a pair of couples satisfying condition (4) and the community would be unstable (which would contradict the assumption). Thus, $A_2^1 > A_2^2 > \ldots > A_2^N$. This, together with (5), states that the partner of the *n-th* most attractive woman is the *n-th* most attractive man.

On the basis of Property 7, higher tensions should be expected in communities with couples in relevant conflict with the appeal ranking. This result, derived from purely theoretical arguments, is certainly in agreement with empirical evidence. Indeed, partners with very uneven appeals are rarely observed in our societies. Of course, in making these observations one must keep in mind that appeal is an aggregated measure of many different factors and that gender differences might be relevant. Thus, for example, the existence of couples composed of a beautiful young lady and an old but rich man does not contradict the theory, but, instead, confirms a classical stereotype.

Concluding remarks

A minimal model of love dynamics between secure individuals has been presented and discussed in this paper with two distinct goals, one generic and one specific. The generic goal was to show to social psychologists how one could possibly deal with dynamic phenomena by means of the modelling approach based on differential equations and traditionally used in other fields of sciences. This approach is quite powerful for establishing a hierarchy between different properties and distinguishing between causes and effects. The specific goal was to derive a series of properties concerning the quality and the dynamics of romantic relationships in couples composed of secure individuals.

The model equations take into account three mechanisms of love growth and decay: the forgetting process, the pleasure of being loved and the reaction to the partner's appeal. For reasonable assumptions on the behavioral parameters of the individuals, the model turns out to enjoy a number of remarkable properties. It predicts, for example, that the feelings of two partners vary monotonically after they first meet, growing from zero (complete indifference) to a maximum. The value of this maximum, *i.e.*, the quality of the romantic relationship at equilibrium, is higher if appeal and reactiveness to love are higher. The model explains also why there are two kinds of couples, called robust and fragile. Robust couples are those that have only one stationary mode of behavior characterized by positive quality of the romantic relationship. By contrast, fragile couples can also be trapped in an unpleasant mode of behavior characterized by antagonism. All these properties are in agreement with common wisdom on the dynamics of love between two persons.

These properties have been used to derive the characteristics under which the couples of a given community have no tendency to separate (stability). The main result along this line is that the driving force that creates order in the community is the appeal of the individuals. In other words, couples with uneven appeals should be expected to have higher chances to brake off.

As for any minimal model, the extensions one could propose are innumerable. Aging, learning and adaptation processes could be taken into account allowing for some behavioral parameters to slowly vary in time, in accordance with the most recent developments of attachment theory. Particular nonlinearities could be introduced in order to develop theories for non secure individuals. Men and women could be distinguished by using two structurally different state equations. The dimension of the model could also be enlarged in order to consider individuals with more complex personalities or the dynamics of love in larger groups of individuals. Moreover, the process followed by each individual in forecasting the quality of the relationship with a potential new partner could be modelled more realistically, in order to attenuate the role of appeal, which has been somehow overemphasized in this paper. This could be done quite naturally by formulating a suitable differential game problem. Undoubtedly, all these problems deserve further attention.

Appendix A

Consider the system depicted in Figure A1, composed of two cylindrical reservoirs. Assume that a first investigator knows that the outflow rate $y(t)$ of a reservoir is uniquely determined by the pressure at the bottom and that such a pressure is proportional to the volume of water contained in the reservoir (storage $x(t)$). This means that the investigator knows that the outflow rate $y(t)$ is a function of the storage $x(t)$, *i.e.*, $y(t) = \Psi(x(t))$. On the basis of this knowledge, the investigator might decide to estimate the functions Ψ_1 and Ψ_2 by making a number of measures of storage and flow rate on both reservoirs. Assume that his conclusion is that storages and flow rates are practically proportional one to each other, *i.e.*,

$$y_1(t) = k_1\, x_1(t) \qquad\qquad y_2(t) = k_2\, x_2(t) \tag{A1}$$

Obviously, these relationships are structural properties of the two components of the system.

On the contrary, a second investigator might decide to perform a completely different experiment. He starts with the first reservoir containing some water $x_1(0)$ and the second reservoir empty ($x_2(0) = 0$) and then records the time pattern of the outflow rate $y_2(t)$. He repeats the experiments for many different values of $x_1(0)$ and discovers that the time t_{max} at which the outflow rate is maximum is the same for all experiments.

At this point, a natural question is: are the two properties equivalent, or independent, or one of them implies the other and not viceversa? The answer to this question is not obvious. Actually, it is impossible to give an answer if a model of the system is not developed and analyzed. For this, let us first notice that the mass conservation law applied for an infinitesimal time interval $(t, t+dt)$ implies that the variations $dx_i(t)$ of the storages during that interval are related to the flow rates by the equations

$$dx_1(t) = -y_1(t)\, dt$$
$$dx_2(t) = y_1(t)\, dt - y_2(t)\, dt$$

Dividing both relationships by dt, one obtains the two following differential equations

$$\frac{dx_1(t)}{dt} = -y_1(t)$$
$$\frac{dx_2(t)}{dt} = y_1(t) - y_2(t) \tag{A2}$$

This is not yet a model of the system because the two differential equations are not in the form (1). Nevertheless, the property discovered by the first investigator allows one to obtain the model. In fact, substituting equations (A1) into equations (A2), one obtains

$$\begin{aligned} \frac{dx_1(t)}{dt} &= -k_1\, x_1(t) \\ \frac{dx_2(t)}{dt} &= k_1\, x_1(t) - k_2\, x_2(t) \end{aligned} \qquad \text{(A3)}$$

which are indeed differential equations of the form (1). This means that the property discovered by the first investigator is a first level property (it allows one to uniquely identify the model of the system). In this special case, equations (A3) can be analytically solved for any initial condition. The solution in the case the second reservoir is initially empty ($x_2(0) = 0$) is

$$\begin{aligned} x_1(t) &= x_1(0)\, k_1\; exp(-k_1\, t) \\ x_2(t) &= x_1(0) \frac{k_1}{k_2 - k_1} \left[exp(-k_1\, t) - exp(-k_2\, t) \right] \end{aligned}$$

Noticing that the outflow rate $y_2(t)$ of the second reservoir is proportional to the storage $x_2(t)$ (see (A1)), one can conclude that $y_2(t)$ is maximum when $x_2(t)$ is such. But $x_2(t)$ is maximum when the function in square brackets is maximum, *i.e.*, for

$$t_{max} = \frac{ln\, k_1 - ln\, k_2}{k_1 - k_2}$$

This shows explicitly that the discovery of the second investigator (t_{max} independent upon $x_1(0)$) is a second level property, *i.e.*, a consequence of the discovery of the first investigator. On the contrary, it can be shown that equations (A1) are not a consequence of the independency of t_{max} upon $x_1(0)$. In conclusion, the first discovery is more informative than the second one.

Up to now, we have tacitly assumed that the two investigators have carried out their experiments on the same system. On the contrary, let us now assume that they have performed their experiments on two different pairs of reservoirs, randomly selected from a large set of reservoirs. Imagine that the first investigator has still discovered property (A1) while the second has found that t_{max} increases with the initial storage $x_1(0)$. Recalling the above discussion, one can immediately conclude that the property discovered by the first [second] investigator does not hold for the second [first] set of reservoirs. In other words, the properties are sample specific.

Appendix B

Proof of Property 1

For $x_1 = 0$ and $x_2 > 0$ eq. (2a) gives $dx_1/dt > 0$. Symmetrically, for $x_1 > 0$ and $x_2 = 0$ eq. (2b) gives $dx_2/dt > 0$. Moreover, for $x_1 = x_2 = 0$, eqs. (2) give $dx_1/dt > 0$ and $dx_2/dt > 0$. Hence, trajectories starting from the boundary of the positive quadrant enter into the positive quadrant and remain there forever.

Proof of Property 2

The isoclines $\dot{x}_1 = 0$ and $\dot{x}_2 = 0$ of the system are given by (see eq.(2))

$$x_1 = \frac{1}{\alpha_1}\left(R_1(x_2) + I_1(A_2)\right) \qquad\qquad x_2 = \frac{1}{\alpha_2}\left(R_2(x_1) + I_2(A_1)\right)$$

From properties (3) it follows that these isoclines intersect either at one point with positive coordinates (see Fig. B1a), or at three points, one with positive coordinates and two with negative coordinates (see Fig. B1b). These two cases correspond, by definition, to robust and fragile couples. On the first isocline, trajectories are vertical, while on the second they are horizontal. Studying the signs of dx_1/dt and dx_2/dt in the regions delimited by the isoclines, one can determine the direction of the trajectory of the system at any point (x_1,x_2). These directions, indicated with arrows in Fig. B1, allow one to conclude that the equilibrium points E^+ and E^- are stable nodes, while the equilibrium point S is a saddle. Moreover, if the couple is robust (Fig. B1a) the equilibrium E^+ is a global attractor, *i.e.*, all trajectories tend toward this point as time goes on. By contrast, if the couple is fragile (Fig. B1b), there are two attractors, namely points E^+ and E^-, and their basins of attraction are delimited by the stable separatrix of the saddle point S (not shown in the figure). Since point S has negative coordinates and the separatrix has a negative slope, it follows that the basin of attraction of the positive attractor (point E^+) contains the positive quadrant as indicated in Fig. 2.

Proof of Property 3

Assume that parameters and functions of the two individuals are such that the couple is fragile, *i.e.*, the isoclines are like in Fig. B1b. Then, increase gradually $I_1(A_2)$ and $I_2(A_1)$. Since an increase of $I_1(A_2)$ shifts the first isocline to the right, while an increase of $I_2(A_1)$ shifts the second isocline upward, after a while points S and E^- collide and disappear (through a fold bifurcation). Thus, sufficiently high values of $I_1(A_2)$ and $I_2(A_1)$ guarantee that the isoclines are like in Fig. B1a, *i.e.*, that the couple is robust.

Proof of Property 4

Refer to Fig. B1 and to the region delimited by the two isoclines in which the origin falls. Inside this region dx_1/dt and dx_2/dt are positive and trajectories cannot leave the region. This implies the property.

Proof of Property 5

Refer again to Fig. B1 and indicate with x_1^+ and x_2^+ the coordinates of the positive equilibrium E^+. Then, increase A_2 of a small quantity ΔA_2. This produces a rigid shift to the right of the first isocline while the second does not move. Thus, point E^+ shifts to the right along the second isocline. The consequence is that both components of the equilibrium increase, say of Δx_1^+ and Δx_2^+, respectively. Obviously, $\Delta x_2^+/\Delta x_1^+$ coincides with the slope of the second isocline and is therefore smaller than x_2^+/x_1^+ which is the slope of the straight line passing through the origin and point E^+. Hence, $\Delta x_2^+/x_2^+ < \Delta x_1^+/x_1^+$, which proves the property.

Proof of Property 6

Refer once more to Fig. B1 and note that an increase of the return function of the first individual from $R_1(x_2)$ to $R_1(x_2)+\delta R_1(x_2)$ with $\delta R_1 > 0$, moves the first isocline to the right of a quantity $\delta R_1(x_2)$. This implies that point E^+ moves to the right along the second isocline so that, using the same argument used in the proof of Property 5, one can conclude that $\Delta x_2^+/x_2^+ < \Delta x_1^+/x_1^+$.

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Figure captions

Fig. 1 The return function R_i of a secure individual *i vs.* the partner's love x_j.

Fig. 2 Evolution of the feelings in a robust couple (a) and in a fragile couple (b) for different initial conditions.

Fig. 3 Community structures in the appeal space: (a) two points corresponding to two couples (h,k) belonging to an unstable community (see (4)); (b) an example of a stable community (each dot represents a couple).

Fig. A1 Two cylindrical reservoirs in cascade. The variables $x_1(t)$ and $x_2(t)$ indicate storages at time t, while $y_1(t)$ and $y_2(t)$ indicate flow rates.

Fig. B1 The isoclines $dx_1/dt = 0$ and $dx_2/dt = 0$ of the system and the directions of trajectories in robust (a) and fragile (b) couples. The equilibrium points E^+ and E^- are stable nodes, while S is a saddle.

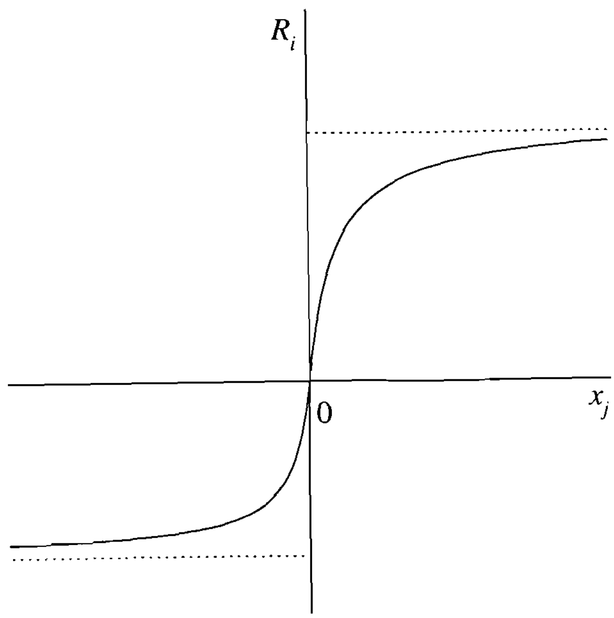
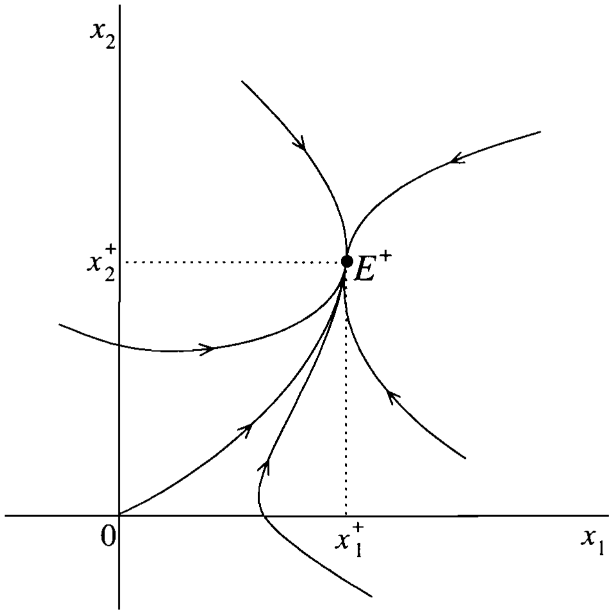
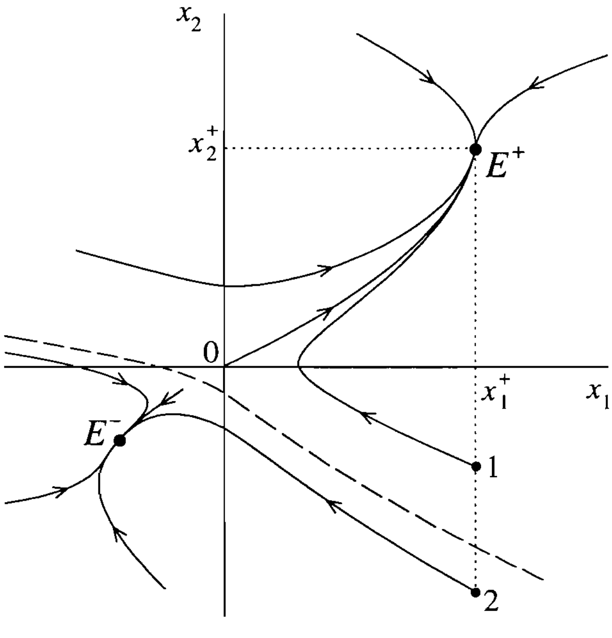


Figure 1

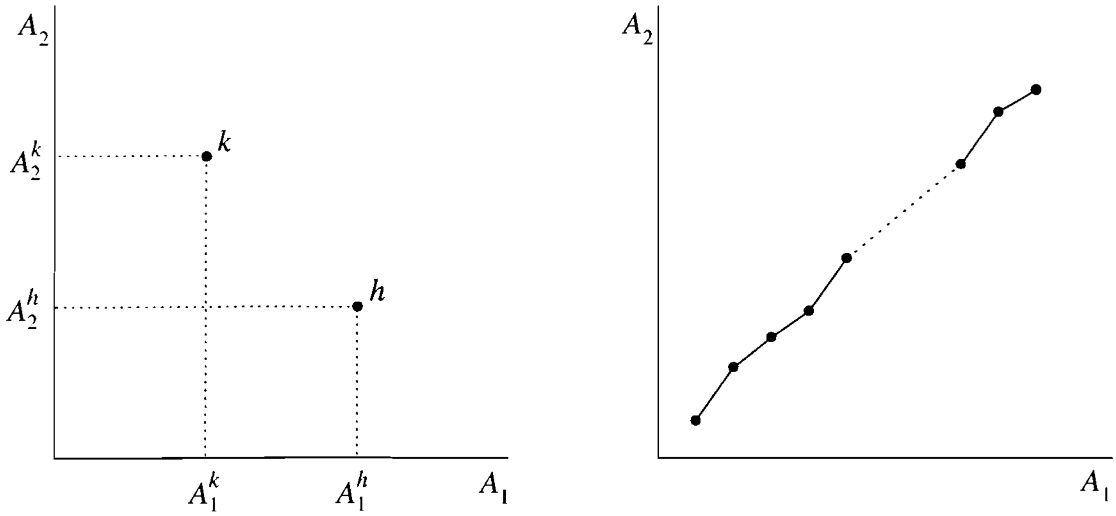


(a)



(b)

Figure 2



(a)

(b)

Figure 3

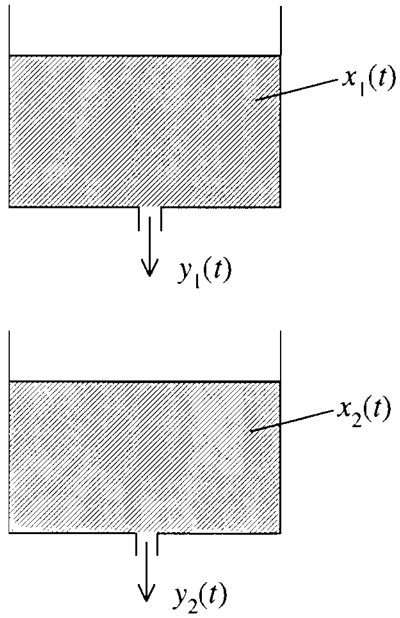
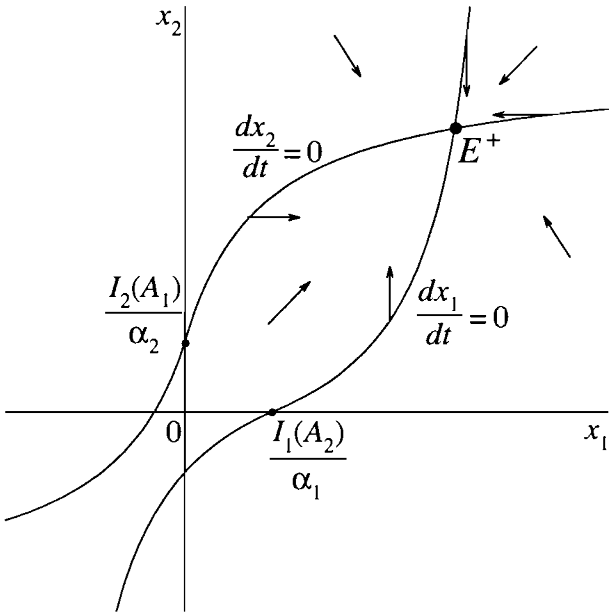
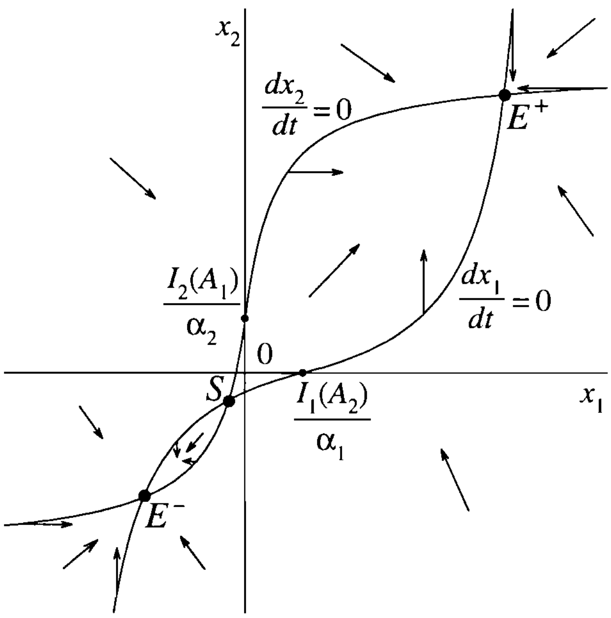


Figure A1



x_2
$\frac{dx_2}{dt} = 0$
E^+
$\frac{I_2(A_1)}{\alpha_2}$
$\frac{dx_1}{dt} = 0$
0
$\frac{I_1(A_2)}{\alpha_1}$
x_1

(a)



x_2
$\frac{dx_2}{dt} = 0$
E^+
$\frac{I_2(A_1)}{\alpha_2}$
$\frac{dx_1}{dt} = 0$
0
S
$\frac{I_1(A_2)}{\alpha_1}$
x_1
E^-

(b)

Figure B1