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Foreword

In decision analysis, a typical aspect is the fact that quite often the decision problem combines hard and soft features. For instance, in many environmental policy problems, the costs and physical consequences of possible policies are relatively well-known whereas the health risks and the socio-economic consequences remain in the dark. As a consequence, we see that the hard features can be modelled and models can be evaluated by using model-based decision support systems. However, in the decision making process, the soft features should also be taken into account.

During her stay at IIASA as a participant in the 1994 Young Scientists Summer Program, Mari Pöyhönen has taylored and applied a new approach for treating imprecise information on two decision problems from the IIASA field of activities. These problems had in common that there existed already good models for the hard features. The new approach used by the author has been developed at her home university (Helsinki) by our colleagues Ahti Salo and Raimo Hämäläinen.

In the present Working Paper, the author explains the new approach, compares it with other approaches, and demonstrates it on two practical problems, namely "the improvement of water quality in a river basin" and "the planning of a power system expansion".

The work of Mari Pöhönen shows how fruitful the cooperation between substantive and methodological projects can be. I gratefully acknowledge the constructive contributions of Peter Dörfer and László Somlyódy.

Abstract

Multicriteria decision making techniques give a decision maker a way to thoroughly analyze complex problems and state his or her arguments for decisions. The techniques usually require precise numerical information of the decision maker's preferences and the parameters of a decision problem. However, it is most often difficult to get this information. There may be several decision makers which have different opinions and the parameters of the decision problem may be ambiguous. Preference Assessment by Imprecise Ratio Statements (PAIRS) is a hierarchical weighting technique which allows decision makers to give preference statements with intervals instead of single point estimates. Here this new technique is applied to two case studies where decision makers have to select the most suitable solution from a discrete set of alternatives for a problem which involves several conflicting environmental and economic factors.

Evaluation of Environmental Policy Strategies with Imprecise Preference Information

Mari Pöyhönen *

1. Introduction

A choice of the most suitable strategy for solving important economical and environmental problems requires thorough considerations as the problems are very complex and involve multiple conflicting criteria and several parties. Nowadays also legislation in many countries forces decision makers (DM) to state their arguments for decisions which may have environmental impacts. As Corner and Kirkwood (1991) state, analytical decision making tools could be of great help in this situation: "Decision analysis provides a systematic, quantitative approach to make better decisions."

The decision analysis literature proposes various techniques to support multicriteria decision making (MCDM). Recent surveys by Korhonen et al. (1992) and Stewart (1992) give descriptions of several MCDM techniques and their areas of applications. Most of these techniques require exact numerical information about the decision maker's preferences as well as of the alternatives. However, it is often very difficult to get this information. Several methods have been proposed to facilitate decision making so that the preference information could be given in an imprecise form.

One appealing way to give imprecise preference statements is to use words to indicate the strength of preferences during the decision making process. The Analytic Hierarchy Process (AHP) (Saaty 1980) allows decision makers to use words in the preference elicitation so that preference statements of relative importance are given with verbal expressions. These vague verbal expressions are, however, converted into exact numerical estimates. This conversion procedure has been criticized (Stewart 1992) and, indeed, there is empirical evidence that this conversion procedure does not correspond with a human interpretation of verbal expressions in the context of verbal ratio statements (Pöyhönen et al. 1994). The verbal expressions indicating the strength of preferences in decision making should rather be interpreted as intervals of numbers or, for example, in terms of fuzzy sets.

Other approaches to process imprecise information directly use information in a numerical form instead of words. Approaches based on fuzzy sets form a large group of methods which, however, are not discussed here. A reader may find information of fuzzy decision making methods in Zimmermann (1987). Granat and Wierzbicki (1994) give an example of the latest developments in this field.

Multiattribute utility theory (MAUT) (Keeney and Raiffa 1976) provides one normatively sound framework for decision making. The methods based on MAUT are also used extensively in real life applications. Corner and Kirkwood (1991) give a review of the applications which have been presented in the operations research literature during the last two decades. However, it is acknowledged that utility and value functions, probabilities and criterion weights are often imprecise. Salo and Hämäläinen (1992) give a review of approaches which let the decision maker give preference information in imprecise form with

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MAUT. One such approach, Preference Assessment by Imprecise Ratio Statements (PAIRS) (Salo and Hämäläinen 1992), is based on Value Tree Analysis (von Winterfeldt and Edwards 1986) and lets the decision maker give imprecisely defined values for alternatives as well as for criterion weights.

Validation of new decision analysis techniques requires applications with real decision problems and opinions from decision makers. In this paper we will use the term "decision maker" in a broader meaning so that it includes also the persons who are preparing the decisions but who are not necessarily making the final choices by themselves. Their duty is to analyze the problem throughoughly and then give their recommendations to the higher level decision makers who are, for example, politicians.

This study uses the PAIRS technique to evaluate environmental policy strategies in two cases. Both decision problems arose from the environmental research conducted at the International Institute for Applied Systems Analysis (IIASA). The first decision problem is to select a set of technologies to improve the water quality in a large river basin. The particular set of technology improvements is selected from alternatives which are generated by using a model based decision support system (Berkemer et al. 1993, Makowski et al. 1994). The second decision problem deals with the planning of power system expansion in Hungary. In this case, several alternatives for further analysis have been chosen by using the trade-off risk analysis which helps to analyze the risks of the decision problem (Crousillat et al. 1993, Dörfner 1991).

The decision analysis processes in these cases contain two phases. During the first phase, thorough multiple criteria optimization is conducted and several alternative scenarios are compared. This kind of scenario analysis gives the DM insight about the decision problem and helps in defining a set of alternatives. This phase is also motivated by the fact that the analysis of the DM's preferences is such a difficult task that: "Standard MCDM approaches could do with more caution before rushing into difficult and complex value judgment exercises erlier than is absolutely necessary." (Stewart, 1994). The initial analysis of the problem is not restricted to multiple criteria optimization and there exist various approaches for scenario analysis which help decision makers to learn more about their decision making problem (Bunn and Salo 1993).

During the second phase decision makers have to turn to their preferences and consider carefully which kind of reasoning leads to a certain solution from a discrete set of clearly defined alternatives. Hierarchical weighting methods make it possible to also take into account qualitative criteria which are often essential in making the final choice among alternatives. PAIRS now provides several advantages for the actual preference elicitation. PAIRS makes it easier for the DM to answer the difficult questions which usually require exact estimates of the strength of preferences. The preference elicitation process is no longer time consuming or difficult, and thus the DM may focus more on sensitivity analysis which is the essential part of the decision making process. The intervals of numbers describe the properties of alternatives and thus it is possible to include into the analysis the impreciseness of the measures of alternatives. PAIRS also provides a promising environment for group decision making as the intervals of preferences could contain the opinions of all group members.

This paper concentrates on the second phase of the decision making process. The analysis with both the water quality management and the energy planning is conducted with an expert who has been involved in the first phase of the process. The experts here are not real decision makers but the final decisions are made based on their work. The presented decision making sessions are examples of the decison analysis with PAIRS and show how the

DMs would be able to use the method as a remedy during their difficult task of decision making.

This paper is organized as follows. A short review of the hierarchical weighting techniques is given in Section 2, since the construction of the hierarchy and the basic ideas of hierarchical weighting are important and similar to PAIRS. Details of PAIRS are given in Section 3. Section 4 describes how the decision analysis process is conducted with PAIRS and how impreciseness in different situations is taken into account. Two case studies are presented in Sections 5 and 6. Conclusions of the case studies are presented in Section 7.

2. Hierarchical Weighting Techniques

Hierarchical weighting techniques are based on structuring criteria as a hierarchy. During the top-down design of a decision hierarchy a DM first defines general criteria for the decision problem. Each criterion may be further divided into subcriteria. These criteria may be either qualitative or quantitative to capture more profoundly the decision maker's view of the decision problem. At the lowest level of the hierarchy, each alternative is evaluated in terms of the lowest level criteria. These evaluations are combined through the hierarchy and a result is a score for each alternative indicating the rank order of alternatives. The decision hierarchy gives a decision maker a way to structure the problem as a whole and then concentrate on one part of the problem at a time. The weighting techniques define weights for each criterion and scores for alternatives through simple questions. Two most widely used techniques for weighting are the AHP (Saaty 1980, Vargas and Whittaker 1990) and the Value Tree Analysis (Keeney and Raiffa 1976, von Winterfeldt and Edwards 1986).

The structure of the hierarchy needs thorough consideration. The hierarchy should be adequate and include all the relevant criteria of the problem, but on the other hand, there is no need to include criteria which do not help to distinguish alternatives from each other. The criteria at each level of the hierarchy should be independent from each other. This means that a DM should be able to consider an alternative with respect to only one criterion at a time independently from the attribute values which the alternative may have with respect to other criteria. Keeney and Raiffa (1976) and Von Winterfeldt and Edwards (1986) give more detailed instructions and examples for constructing decision hierarchies. The structure of the hierarchy should also be designed carefully in that sense that different structures of a hierarchy may cause some behavioral biases in the weight elicitation (Weber and Borcherding 1993).

The weight elicitation in the AHP is based on pairwise comparisons. All the way through the hierarchy the DM faces questions of the relative preference or importance between two alternatives or criteria. He or she is asked which one of the two criteria or alternative is more important or preferred and how much more. The DM gives answers with numbers or verbal expressions which are interpreted as weight ratios. Numbers are located into a reciprocal comparison matrix and the weights are calculated with an eigenvalue method. A reader may find details of the theory and a review of applications from Saaty (1980) and Vargas and Whittaker (1990).

Preference Programming (Salo and Hämäläinen 1994) allows the DM to give ratio statements of relative importance in AHP with intervals of numbers instead of exact point estimates. However, for this study we selected the method based on Value Tree Analysis since AHP has some serious drawbacks if the number of alternatives changes during the analysis. It may happen that an introduction of a new alternative or a deletion of some old alternatives causes changes in the rank order of the remaining alternatives. This rank reversal phenomenon was first observed by Belton and Gear (1982) and it motivates us to consider some other methods if the set of alternatives is likely to change during the analysis.

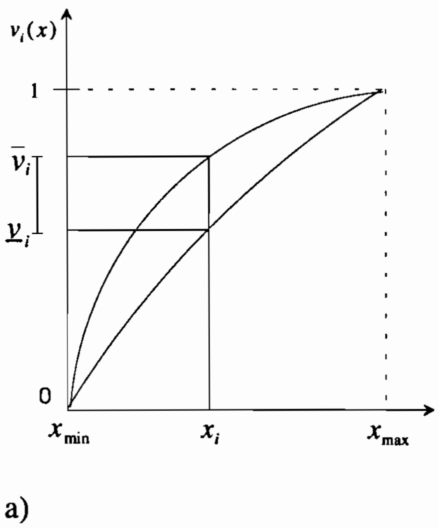
Value functions form the basis for the evaluation of alternatives in Value Tree Analysis. An alternative x is evaluated with respect to n criteria and x_i is the attribute value of an alternative x under criterion i. The value function $v_i(\cdot)$ is elicited for every criterion i separately. Usually $v_i(\cdot)$ is normalized to values from 0 to 1 so that the best value for an attribute has a value one and the least attractive attribute value has a value zero. The value of an alternative x is then $v_i(x_i)$ under criterion i. There are several methods for eliciting value functions (von Winterfeldt and Edwards 1986). For example the bisection method asks the DM to define attribute ranges which have equal value changes. The total value for an alternative x may be achieved with an additive value function $v(x) = \sum_{i=1}^{n} w_i v_i(x_i)$, where w_i is a weight for criterion i, only if the criteria are preferentially independent (Keeney and Raiffa 1976). This means that the DM's preferences between two attribute values should not depend on what the attribute values are for the other criteria.

The weights w_i for criterion i may be elicited in several ways. Frequently used techniques are the *TRADEOFF* method (Keeney and Raiffa 1976), *SWING* weighting (von Winterfeldt and Edwards 1986) and *SMART* (Simple Multiattribute Rating Technique) (Edwards 1977). SMART has some similarities to the pairwise comparison technique which is used in AHP and also forms the basis for the weight elicitation in PAIRS. The SMART procedure starts as the DM verifies the least important criterion and assigns a score 10 to this criterion. During the second step the DM evaluates the importance of every other criterion by comparing them pairwisely with the least important one and giving them scores up from 10. These scores are normalized so that the sum of the weights equals to one.

A normatively correct way to elicit criteria weights should take into account the ranges of attributes in the lowest level of the hierarchy (Keeney and Raiffa 1976). The weights indicate how much a change from the least attractive level of the attribute to the most attractive level of the attribute contributes to the final decision. SMART does not explicitly consider attribute ranges as is done for example with the TRADEOFF method. Thus it should be emphasized to the DM during the analysis that the criterion weights should reflect this decision situation and this set of alternatives. To avoid some of these difficulties it is recommended to work from the bottom upwards in the decision hierarchy, so that the DM clearly has in his or her mind the attribute ranges at the lowest level of the hierarchy. Weber and Borcherding (1993) give a review of other behavioral aspects which should be considered during the weight elicitation phase.

3. Preference Assessment by Imprecise Preference Statements - PAIRS

Here we shall consider only those aspects of PAIRS which a DM faces during the decision making process. A reader may find details of the theory in Salo and Hämäläinen (1992) and the way of computation of value intervals is presented in Appendix A. At the lowest level of the hierarchy the input for the decision model is a range of possible values for each alternative x with respect to each criterion i. The DM can define this range of values in several ways. The DM knows that the least and the most attractive attribute values are attached with values 0 and 1 respectively (See Figure 1). He or she can define directly a range $[\underline{v}_i, \overline{v}_i]$ of values to correspond with the attribute value x_i (Figure 1 a)). In this way we do not need to specify the shape of the value function. Another way is to give a range of possible attribute values $[\underline{x}_i, \overline{x}_i]$ for an alternative under criterion i . This range defines indirectly a range of the values through some value function. The value function could be known exactly or it can also be defined imprecisely as a set of appropriate value functions. The situation with a known value function and a range of possible outcomes for an alternative x under criterion i is shown in Figure 1 as a case b).



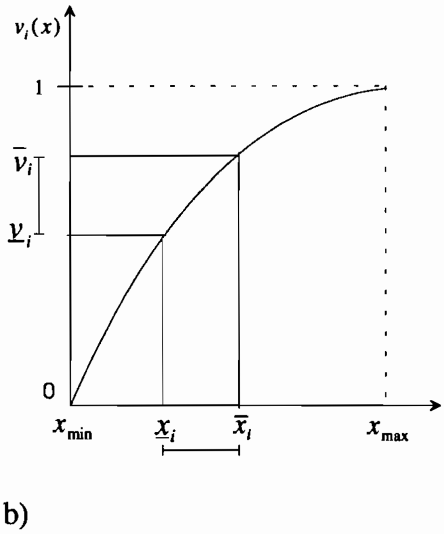


Figure 1: Evaluation of alternatives at the lowest level of the hierarchy
a) The DM gives directly a range of values $[\underline{v}_i, \overline{v}_i]$
b) The DM gives a range of attribute values of an alternative $[\underline{x}_i, \overline{x}_i]$ and this range defines indirectly a range of values

The weight elicitation in PAIRS is similar to SMART in that sense that the DM is asked to give estimates for weight ratios. However, PAIRS extends the SMART technique in two ways. First, the DM gives ranges of possible ratios instead of single estimates for a weight ratio. Second, comparison between criteria is not restricted to comparisons between the least important one and the DM gives ranges of weight ratios for selected pairs of criteria. New statements are required to be consistent with other statements. PAIRS uses so-called *consistency bounds* (Salo and Hämäläinen 1992) to help the DM in giving consistent comparisons. These bounds also help to reduce the number of pairwise comparisons because the DM is able to accept the intervals for weight ratios given by the consistency bounds.

The ranges of weight ratios are converted into inequalities which give constraints for the possible values of weights (Arbel 1989, Salo and Hämäläinen 1992). These inequalities, together with the ranges of values for alternatives under each criterion, impose constraints for a linear programming problem where the objective is to define the smallest and largest possible weights for alternatives in a value tree. The solutions to these problems define the weight range $V(x)=[\underline{v}(x),\bar{v}(x)]$ for each alternative *x*. This interval is recalculated after each new statement. The DM interactively gives more specific constraints as long as he or she finds it useful. The technique does not guarantee that the process converges to a solution. However, the goal is often to continue the working until a dominating alternative is found. Two dominance concepts are used to find out which one of the alternatives is preferred (Weber 1987, Salo and Hämäläinen 1992). An alternative *x* dominates an alternative *y* *absolutely* if the weight interval for an alternative *x* is totally above the weight interval for an alternative *y*. The so called *pairwise* dominance is possible even if the weight intervals for alternatives overlap. The reader may find details of the calculations of these dominances from Salo and Hämäläinen (1992).

ComPAIRS* is a software tool for PAIRS. It includes features to create decision hierarchies, to give ranges of attributes at the lowest level of the hierarchy and to make pairwise comparisons between criteria with ranges of possible weight ratio values. As a result from calculations, the DM is shown ranges of possible weights for alternatives and the dominance results for alternatives. The interface is totally mouse-driven and user friendly. The number of alternatives is limited to four.

4. Scenario Evaluation by PAIRS

There are several sources of imprecise information during the decision making process:

- Group of decision makers and experts:
  - different opinions about the criterion importance
  - different opinions about the valuation of alternatives
- Attribute outcomes to describe alternatives are not known precisely
- Decision makers themselves do not know their preferences

A single DM can use PAIRS straigthforwardly to give imprecise preference information as is described in section 3. He or she can give interval statements of the criteria importance and of the intervals of values for alternatives under each criterion. The studies of this paper concentrate on situations when PAIRS is used by a single DM. However, the situation is more complicated when there are several decision makers as the consensus about opinions is not guaranteed.

Computer supported group decision making has been studied extensively since early 1980's. A review of multiple criteria decision support techniques which have been developed for group decision making is given by Hwang and Lin (1987). Usually the results from these techniques are point estimates which represent the opinion of a group. However, the main

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benefit of the decision analysis tools with real applications has often been the presentation of conflicting opinions within the group instead of the exact numerical results (Islei and Lockett (1991).

PAIRS may be applied in group decision making so that the intervals of preferences include all the opinions of the group. The endpoints of the intervals may be defined to be the two most conflicting opinions. In this way intervals of the preferences may be wide at the beginning if the group strongly disagrees about their preferences. A group has to discuss and interactively change their preferences until they find a consensus solution.

The promising results from the group decision making studies with Preference Programming indicate that the PAIRS technique is greatly applicable for a group situation as the criteria weight elicitation is similar for these two methods. Hämäläinen et al. (1992) used Preference Programming to evaluate different energy options in Finland with student politicians. Nine groups discussed future traffic plans in the Helsinki metropolitan area in a study of Hämäläinen and Pöyhönen (1994). The main benefit from the interval presentation of preferences is that a group has clear guidance for their discussion as they are able to focus on those issues which have widest intervals of preferences and thus include more disagreement. The clear solutions are achieved only if the decision makers are ready to make concessions. This suggests that these methods are suitable only for a cooperative decision making situation when decision makers have to find a consensus solution.

The first phase of the analysis is the formulation of criteria as a decision hierarchy. This step is crucial as it defines all the relevant aspects of the problem which are taken into account during the subsequent steps. During this step the preferential independence assumption should be checked so that it is acceptable to use additive value function. In the beginning the alternatives should be defined as the analysis is tied with a certain discrete set of alternatives. Alternatives may be added or deleted during the analysis. However, the decision hierarchy should be changed if the new situation is different and there is, for example, a need to introduce a new criterion.

The number of suitable alternatives may be large. In that situation alternatives may be grouped so that similar alternatives according to one criterion form subgroups. The DM compares these subgroups. The ranges of values within one group form intervals for each subgroup at the lowest level of a hierarchy. Once the most attractive subgroup has been found, the decision maker can make a more detailed analysis with the alternatives within the subgroup.

Impreciseness in the measures of the alternatives is taken into account so that the attribute ranges in the lowest level of the hierarchy are ranges of possible values for an alternative. Uncertainty is not explicitly modelled with probabilities. However, the probabilistic information of the decision problem could be utilized for example by using the confidence intervals as the ranges of attributes for alternatives. Here it should be stressed that PAIRS assumes that all the values in the range are equally possible.

The weight elicitation is done in the hierarchy from the bottom upwards. First, all the alternatives are evaluated with the value functions. The next step is the evaluation of the criteria importance. This is done through pairwise comparisons and usually requires discussion if there are several decision makers. Conflicting and ambiguous opinions of decision makers are given as intervals of weight ratios.

The solution may be clear after these steps, but most often this is not the case. The PAIRS technique is supposed to be used interactively so that the DM gives more precise statements or tries totally different opinions and sees what kind of effect these have on results. The purpose is to seek answers to different what-if questions. This analysis is recommended and often the most beneficial part of the process.

5. Nitra River Basin Water Quality Management

This decision problem is formulated by Berkemer et al. (1993) as follows: "The scope of our problem is a river basin or a larger region composed of several basins where the water quality is extremely poor. We consider also a set of waste-water emission points, at which a waste-water treatment plant either exists or could be constructed or upgraded. At each emission point, one technology (to be selected out of the given set of possible technologies) can be implemented in order to improve the water quality in a region." The conflicting criteria of the problem are the improvement of the quality of the water and, on the other hand, the minimization of the costs. The actual decision is the choice of a set of waste water treatment technologies.

Berkemer et al. (1993) developed a model-based DSS which allows the DM iteratively analyze Pareto-optimal alternatives for technology improvements. The DSS is based on a mixed integer linear programming model and multiple criteria optimization. Makowski et al. (1994) improved the underlying simulation model so that the results are consistent with the more detailed simulation models (Somlyody et al. 1994). They used this decision support system to examine alternatives using an extension of the reference point method (Lewandowski and Wierzbicki 1989). A set of selected alternatives is further considered with PAIRS. The DM of this analysis is an expert of water quality management who has been working with this particular problem from the very beginning.

The decision model in Berkemer et al. (1993) and Makowski et al. (1994) includes six criteria, and the same set of criteria is also used here as a basis for the analysis. The water quality is measured with three attributes:

- DO, the minimum concentration of dissolved oxygen.
- CBOD, the maximum carbonaceous oxygen demand concentration.
- NH4, the maximum ammonia concentration.

The best water quality is achieved if DO is at the maximum level and the other two attributes are on the minimum level. The minimum of DO and the maximum of CBOD and NH4 are defined over the whole set of monitoring points along the river basin. Thus the measures give a description of the water quality at the worst monitoring points in the whole river basin.

Three criteria evaluate the costs. The total investment costs (INV) and the annual operating and maintenance costs (OM) were aggregated to total annual costs (TAC) according to the formula which enables the evaluation of all the costs on an annual level. TAC was used in the previous analysis as a separate criterion but now we are not able to do that since it violates the independence assumption. The DM would not be able to compare TAC with OM or INV if they depend explicitly on each other. In the aggregation annual INV and OM were treated as they would be equally important criteria. However, with PAIRS the DM is now able to trade-off investment and operating costs. Actually we know that investment costs for establishing new technologies are considered to be more important than the operating costs.For the PAIRS analysis TAC is taken as an upper level main criterion which is further divided into the INV and OM criteria.

The environmental water quality criteria, DO, CBOD and NH4, and economic cost criteria, INV and OM, are combined as a decision hierarchy which is presented in Figure 2.

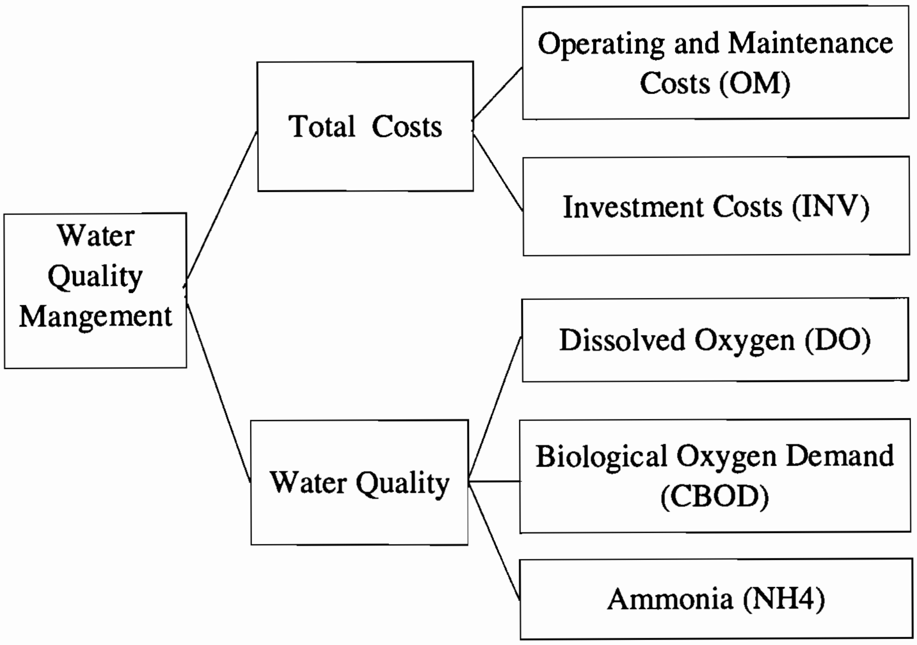

Figure 2: Decision hierarchy for the Nitra river basin water quality management

Makowski et al. (1994) generated 53 Pareto-optimal alternatives. The full list of the results from this multiple criteria optimization is in Appendix B. The purpose of the PAIRS analysis is to select the most attractive alternatives from this list. The analysis is conducted so that the alternatives are first divided into few groups so that in each group the alternatives are close to each other with respect to some criteria. The DM compares these groups of alternatives and tries to find if there is the most attractive group. The analysis continues after that with the comparison of few alternatives.

At the beginning of the reference point analysis in (Makowski et al. 1994), the so called *utopia* and *nadir* points were computed, which are the best and worst values for each attribute in the Pareto-set. These points give one possible range of outcomes for each attribute. With DO and INV we assumed some other attribute values to define best and worst outcomes. The investment costs are obviously very high if all the environmental criteria are on their best levels. However, the expert stated that the investment costs above 20 million US$ are not acceptable. The DO level may be very low and thus very bad. The DO level at this moment in the river is not the worst possible and the technology improvements at least do not make the situation worse. Thus we can assume that the worst attribute value for DO is the current situation in the river. For each criterion, the values one and zero are attached to the best and worst outcome of each attribute.

Alternatives which had investement costs above 20 million US$ were removed from the list of alternatives in the beginning of the analysis as they were considered to be too expensive. The first group (Group 1) includes those alternatives which have very low (below 10 million US$) investment costs. DO level between 4.6 and 5 define the second group (Group 2). All the alternatives in the third group have a very good DO level above 5 (Group

3). The technologies to reduce the amount of ammonia are very expensive and thus those alternatives which have best NH4 values are included into the fourth group (Group 4). For each group a range of attribute outcomes is found and given as a range of outcomes in PAIRS (See Figure 1, case b). Table 1 presents the ranges of attribute outcomes in each group.

	DO (mg/l)		BOD (mg/l)		NH4 (mg/l)		INV (10E6 US$)		OM (10E6 US$)	
best	5.38		9.81		1.70		0.00		1.55	
worst	3.60		25.80		4.71		20.00		8.69	
	$\underline{x}_1$	$\overline{x}_1$	$\underline{x}_2$	$\overline{x}_2$	$\underline{x}_3$	$\overline{x}_3$	$\underline{x}_4$	$\overline{x}_4$	$\underline{x}_5$	$\overline{x}_5$
Group 1	3.68	4.53	11.12	17.80	3.58	3.84	0.00	8.00	5.27	6.96
Group 2	4.67	4.98	10.20	10.66	3.09	3.29	10.00	16.00	6.21	6.95
Group 3	5.02	5.23	10.20	10.60	3.14	3.26	13.10	17.40	6.61	7.12
Group 4	4.94	5.24	10.10	10.40	2.86	3.00	13.50	17.60	6.47	7.50

Table 1: Attribute ranges within each group

The value functions are assumed to be linear for all the criteria. PAIRS does not state any restrictions for the shape of value functions and thus it is possible to implement nonlinearities in the valuation of attributes. The elicitation of the value functions is often difficult and thus the more practical approach is to use first linear value function and after that discuss with the DM about the value ranges. Actually this means that the expert gave value ranges directly, but the first suggestion for the value ranges is achieved with the assumption of the linear value function. The maximum value of DO is the best one and thus the value function for DO is increasing. All the other value functions are decreasing which means that lower attribute values are better.

An example of the analysis at the lowest level of the hierarchy is given in Figure 3 for DO. The value range for Group 2 is from 0.6 to 0.8 with respect to ammonia. These value ranges were defined for each alternative with respect to each criterion. The expert checked the resulting value ranges and some modifications were made so that the value ranges corresponded with his valuation.

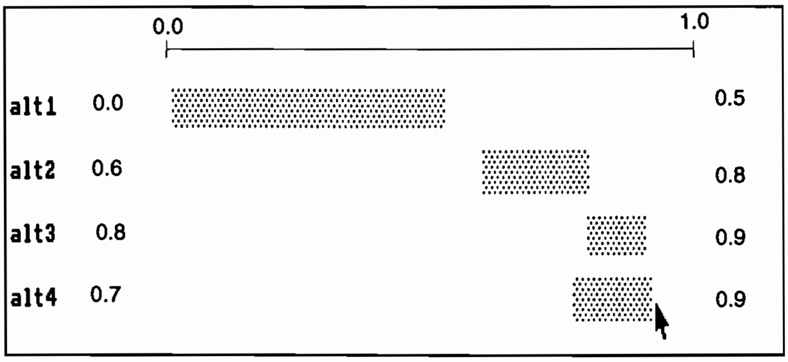

Figure 3: An example of the lowest level analysis for DO

The next step was the evaluation of different criteria. DO was considered to be the most important criteria of the water quality measures. Between NH4 and CBOD the expert was not able to give any statements, but he accepted the ratio ranges which were consistent with the other evaluations. At the cost branch the investment costs were more important than the operating costs. The changes in the water quality measures were considered to be more important than the changes in costs at the top-most level of the hierarchy.

Figure 4 shows the situation after these evaluations. The lower half of the screen presents the comparison of water quality measures. The middle line shows the point 1, where two criteria are evaluated to be equally important so that the weight ratio is one. The comparison between DO and CBOD in the screen shows that the expert evaluated DO to be at least 2, but no more than 5.5 times more important than CBOD. The exact numerical ranges are shown in the screen but in practice a DM is not focusing on giving exactly those endpoint of intervals as numbers. Some sensitivity analysis is usually done throughout the evaluation process as a DM moves intervals with the mouse and finds a range which satisfies him or her.

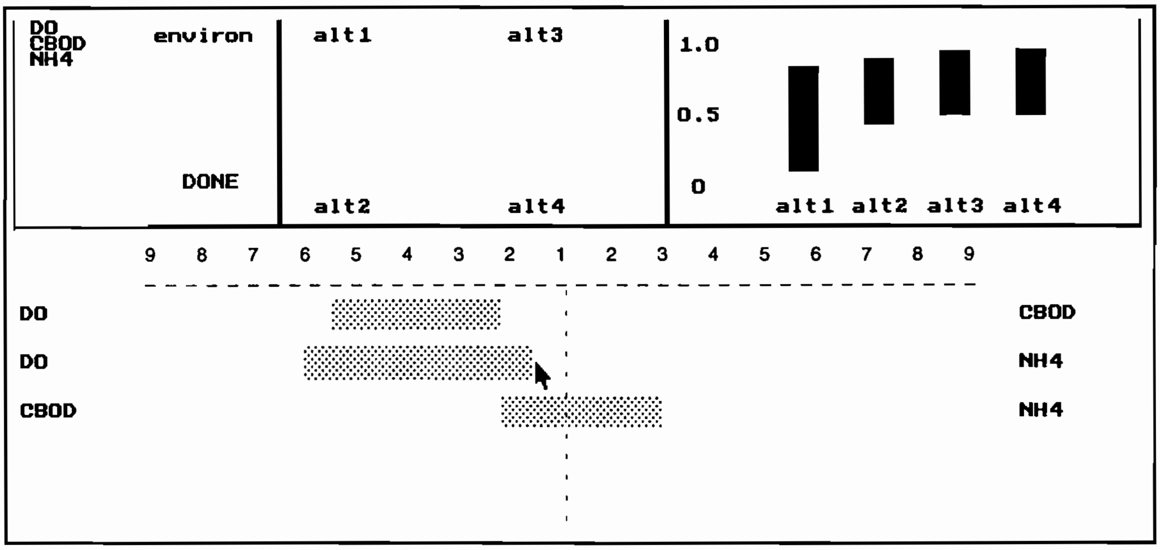

Figure 4: The analysis results after group analysis

The value range of an alternative is shown in the upper rightmost window as a vertical bar. For example, the value range for Group 2 is from 0.5 to 0.9 which means that Group 2 can get any values from that range. There is no dominance between groups. A DM would be able to detect absolute dominance between alternatives if some of the intervals are not overlapping. The pairwise and absolute dominance is indicated by arrows between alternatives in the middle window of the upper part of the screen.

There were no clear dominances between the groups of alternatives, even with very extreme statements about criteria importance. The expert thought that the evaluation of the groups of alternatives is not very useful as the ranges of attributes are too wide and each group contains alternatives which are very attractive. The evaluation of the groups may lead to an exclusion of very good alternatives. The analysis with groups of alternatives is useful only if we have very similar alternatives so that a DM is not able to distinguish them.

Next we decided to focus the analysis on few alternatives. The expert chose three alternatives. The attribute values for these alternatives are shown in Table 2. The attribute values were converted first to values with the linear value functions. These values were not used directly and not precisely, but rather as an aid for the expert to define value ranges directly for each alternative. For example, with the CBOD level there are differences in the attribute outcomes of the alternatives but the expert said that these differences are not enough to make a distinction between alternatives and all the alternatives have the same value range from 0.8 to 0.9 with respect to CBOD.

Alternative	DO (mg/l)	CBOD (mg/l)	NH4 (mg/l)	INV (10^6\$)	OM (10^6\$)
1	4.45	11.13	3.61	6.70	6.92
2	4.67	10.61	3.29	10.00	6.54
3	5.05	10.19	3.00	13.50	6.95

Table 2: Attribute outcomes for three alternatives

The criteria evaluations between the water quality measures and the economic factors were kept at first unchanged. The first analysis question was that are we able to detect dominances between alternatives if we change the trade-off between costs and water quality measures. Figure 5a) shows the point when the alternative 1 starts to dominate alternative 3 if more weight is given to cost factors. However, even if the water quality measures are considered to be extremely more important, it is not possible to achieve dominance relations between alternatives (Figure 5b)). This indicates that the cheapest alternative 1 is a very robust one. It is not the worst in any case and at least is better than the alternative 3 if the costs are stressed.

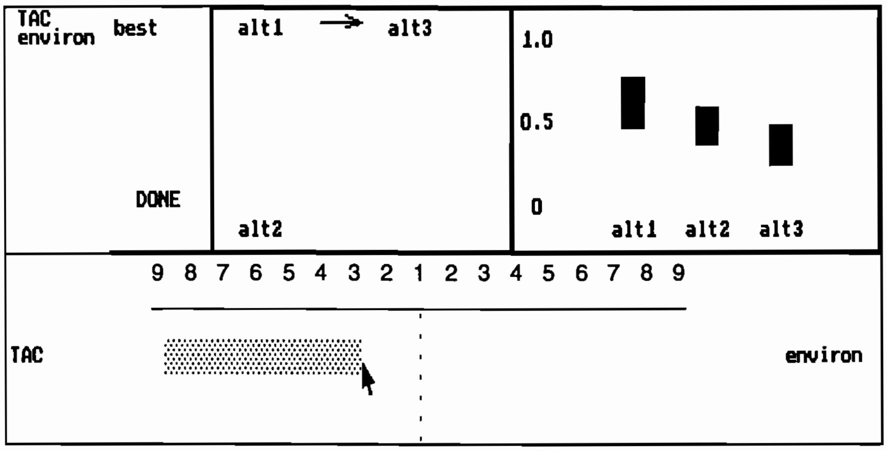

Figure 5a): The situation when the alternative 1 starts to dominate the alternative 3 when the costs are more important than the environmental factors

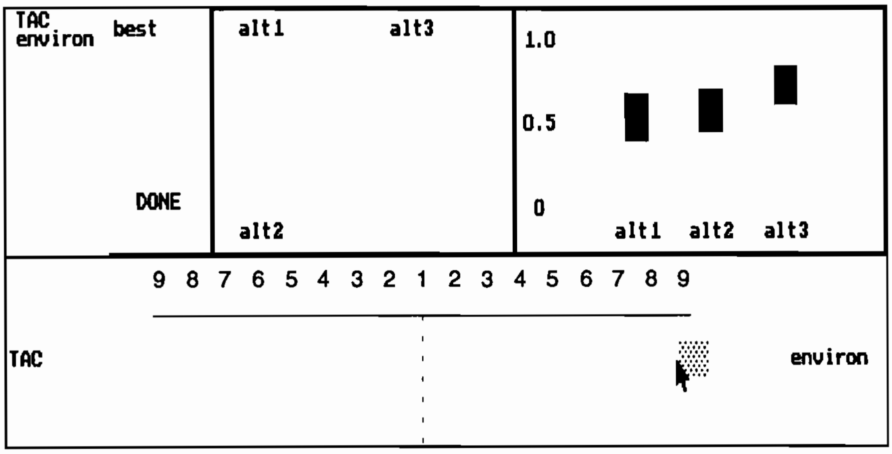

Figure 5b): There are no dominance between alternatives even if the environmental factors are evaluated to be much more important

The next interesting question was to find out if some other alternatives would be better. The expert chose one very cheap alternative that was included in the analysis. Value ranges were defined for that alternative at the lowest level of the hierarchy. All the other statements were kept unchanged so that at the topmost level the situation was as in Figure 5 a) and the costs are more important. The very cheap new alternative dominated both alternatives 2 and 3, but not alternative 1. This means that again alternative 1 is acceptable even if we introduce some new alternatives.

The last sensitivity analysis was conducted with the ammonia values. The alternatives have different values of ammonia and they are clearly different if the water is supposed to be used for drinking. This situation was assumed during the analysis. However, the ammonia levels of alternatives do not actually make any distinction between alternatives if water is used only for other purposes. So the question was: Does the situation change if we change the value ranges of alternatives with respect to ammonia so that the three alternatives are similar? This was done, but it did not have any significant impact on results. Actually the dominance of the alternative 1 became stronger since this alternative has the worst value with respect to ammonia.

The conclusions from the analysis were that the relatively cheap alternative 1 is a very good alternative. Cheaper alternatives or alternatives which lead to better water quality do not dominate this alternative. The expert thought that the PAIRS analysis was useful for seeking answers to different what-if questions. The careful data analysis with simulation and optimization is essential as the DM gets information of the problem. The analysis of preferences helps to compress this information.

6. Power System Expansion in Hungary

Hungary is now in the situation when they have to decide how to fulfill the growing demand of energy in the difficult economic situation. The power expansion planning generally is a continuous problem for the power systems. However, during the next transitional period the task is more complex than it used to be before the year 1990. Following characteristics of the Hungarian power system describe the situation (Dörfner 1991). The share of hydrocarbon fired units is roughly 40% of the installed capacity. Thus the cost of electricity depends highly on the world market oil price. Beside power generation, the Hungarian power system provides significant amount of district heating. The old district heating cogeneration power plants require reconstruction. The main pollution sources of the Hungarian power system are the old, inefficient coal fired power plants. Their contribution to the reliable power supply is significant, but they have to be retired because of their age and environmental regulations.

There are four main options for the energy planning (Dörfner 1991). The actual decision problem is to select a suitable combination of these options.

- The introduction of a ripple control system for the residential electric storage heating devices can decrease the peak load and does not require high investments.
- The establishment of combined cycle units can improve the system efficiency, decrease the environmental impacts, and does not require high investments.
- New base load plants would reduce the dependency on imported hydrocarbons. However, the current financial situation forces to consider carefully new big power projects.
- The open cycle gasturbines have an important role in fulfilling the peak demand.

The planning problem can not be considered as a simple least cost problem because of significant uncertanties. Main uncertanties in this planning problem are the load growth, the world market oil price and the domestic coal price. The decision problem also involves several conflicting criteria which are not measurable in monetary terms.

Dörfner (1991) analyzed over 100 alternatives (conbinations of energy production options) for 54 different futures with the so called trade-off risk analysis. The goal of the trade-off risk analysis is to determine some robust plans which represent a good compromise under all futures. A future here is one realization of uncertainties. For example, one future is that the load growth is 3% per year and the price of oil increases 1%. The analysis also includes different criteria. The selected robust alternatives are not best for all futures and with respect to all criteria, but they offer a good compromise. The trade-off risk analysis of Dörfner (1991) used five criteria to evaluate alternatives:

- Total discounted cost which includes investment and operation costs (TC)
- Capital requirement (CC)
- Quality of supply which is measured with the loss of load probability
- Oil and natural gas consumption (OC)
- SO_2 emissions (SE)

All the possible attribute values of alternatives were derived. Experts evaluated the probabilities for each future. For example, the probability that the load growth is 1% is 0.2. Experts also evaluated attribute values for some combinations of energy options and these evaluations formed a basis for the derivation of attribute values for all alternatives. The trade-off risk analysis was then used to find out robust alternatives. The solution from this analysis was a list of robust alternatives.

The PAIRS analysis was conducted with an expert who has been involved in the initial analysis and works at the Hungarian Power Companies ltd. Three potential alternatives were selected from the list of robust alternatives for the PAIRS analysis. The goal of the PAIRS analysis is to give the DM a more detailed picture of the alternatives. The main task is to determine the desirable proportions of combined cycle units and new base load plants. The combinations of options for energy production for these three alternatives are shown in Table 3. The numbers in Table 3 indicate how much the energy supply increases yearly as a result of that action (MW/year). The combined cycle units are the basis for Alternative 1 and Alternative 2 includes new base load plants. Alternative 3 is between these Alternatives 1 and 2 as it includes some amounts of both.

Alternative	Combined Cycle	Ripple Control	Gas Turbines	New Base Load Plants
1	1120	300	140	-
2	280	450	280	600
3	700	300	560	300

Table 3: Alternatives for energy production in Hungary (MW/year)

The public opinion of different ways to produce energy is very important and should be a part of the analysis. From the initial list of criteria we left out the loss of load probability since all the alternatives were very similar with respect to this criterion and it would not have an effect on decision. The criteria TC, CC and OC formed together a group of economic factors. The oil and natural gas consumption is an economic criterion as they are not domestic energy sources. The price variation of oil and natural gas forms a big risk for energy production and economically it would be better to decrease the dependence on imported energy. The decision hierarchy was formed and discussed with the expert so that the attributes fulfill the preferential independence assumptions. Figure 6 presents the decision hierarchy for the energy planning problem.

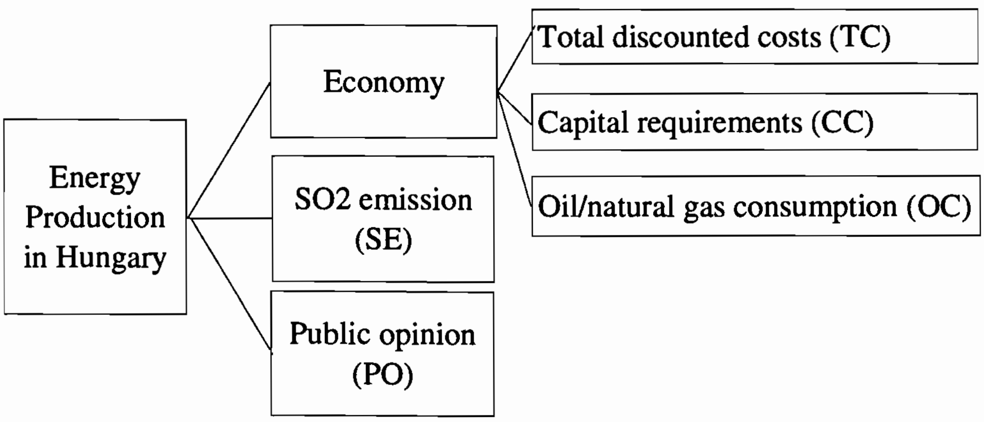

Figure 6: Decision hierarchy for energy planning in Hungary

During the trade-off risk analysis the uncertainty of the decision problem was already included into the analysis since a wide range of different possible realizations were analyzed. Intervals of values for PAIRS represent now the uncertainty of the decision problem in a different way by focusing on the impreciseness of preferences. The data from the previous analysis were available and so we had all the possible attribute values to help the valuation process. One way to use this information is to form intervals of attribute outcomes for each alternative so that the smallest and largest possible values of each attribute would be the endpoints of the intervals. The maximum and minimum values of each attribute for each alternative and the averages over all 54 futures are presented in Table 4.

Alternative	Total discounted costs Gft*10			Capital requirement Gft*10			Oil consumption kt			SO2 emissions kt		
	min	max	average	min	max	average	min	max	average	min	max	average
1	4316	6592	5388	330	338	335	210	255	230	332	360	348
2	4385	6524	5493	394	403	400	197	244	218	346	367	356
3	4365	6611	5499	371	383	379	206	251	226	348	364	354

Table 4: Ranges and averages of attribute values over all futures

The information from Table 4 is not enough to give a full picture of this decision problem. The average does not give information about the risks and uncertanties and the ranges of attribute outcomes are too large to distinguish alternatives. However, the situation under different futures is the main motivation to study this problem deeper and thus we need also some other measures to describe alternatives. In this phase of the analysis we decided that the definition of explicit value functions would be unnecessary and with the additional information the DM is able to define value ranges for each alternative directly with respect to each criterion (According to the situation in Figure 1 a)).

During the trade-off risk analysis experts defined differences in attribute values which are big enough to distinguish alternatives (Dörfner 1991). For example, the alternatives are equivalent if the difference in oil consumption between them is less than 100 kt and if the difference is 200 kt, then the better alternative is accepted to be much better with respect to this criterion. The verbal expression "much better" does not have any special meaning in this case. The estimates for differences were opinions of the real DMs and are considered to be very reliable. For each possible future, we calculated the differences between three alternatives. This information gave us a possibility to look more deeply at the DMs' opinions of different alternatives with respect to each criterion. Table 5 presents percentages of each of the 54 possible futures when there is an equivalence between two alternatives or when one of the two alternatives is considered to be much better. For example, with respect to total costs Alternative 1 is equivalent to Alternative 3 in 81% of futures and much better than alternative 3 in 13% of possible events. There is one possible future (2% of all the futures) when Alternative 3 is much better than Alternative 1 and this happens if the oil and coal price are on their highest levels.

TC

Equivalence

Alt	1	2	3
1	-	59%	81%
2	93%	-	93%
3	98%	100%	-

CC

Equivalence

Alt	1	2	3
1	-	0%	89%
2	100%	-	100%
3	100%	100%	-

OC

Equivalence

Alt	1	2	3
1	-	100%	100%
2	24%	-	83%
3	96%	100%	-

SE

Equivalence

Alt	1	2	3
1	-	2%	22%
2	100%	-	96%
3	100%	13%	-

Much better

Alt	1	2	3
1	-	13%	13%
2	7%	-	7%
3	2%	0%	-

Much Better

Alt	1	2	3
1	-	0%	0%
2	0%	-	0%
3	0%	0%	-

Much Better

Alt	1	2	3
1	-	0%	0%
2	0%	-	0%
3	0%	0%	-

Much Better

Alt	1	2	3
1	-	87%	52%
2	0%	-	0%
3	0%	44%	-

Table 5: The percentage of dominances between alternatives for each attribute and under each future

Tables 4 and 5 and the information of all the possible futures are the basis for the valuation of alternatives with respect to each criterion. With respect to total costs, Alternative 1 is better than Alternatives 2 and 3 in 13% of futures. Alternatives 2 and 3 are better in those cases when the prices of oil and coal are high. Alternative 2 is also better than Alternative 3 under those futures. The average of total costs is more for Alternative 3 than for Alternative 2, but a more detailed examination shows that actually Alternative 3 has lower total costs in 93% of futures. However, the differences in total costs are not significant. This robustness of Alternative 3 indicates that the value of Alternative 3 is more than for Alternative 2, but the value ranges are overlapping as the high prices of imported energy may come true. From the attribute values of capital requirements, it is possible to see that Alternative 1 is always better than Alternatives 2 and 3 and also Alternative 3 is better than Alternative 2, but the differences are not enough to distinguish alternatives clearly. With respect to oil consumption Alternative 2 is better than Alternative 1 in 76% of futures but none of the alternatives is ever much better than the other. Alternative 1 has smallest SO_2 emissions and Alternatives 2 and 3 are never better than 1. However, Alternative 1 is not significantly better than these two for every future. Between Alternatives 2 and 3 it is seen that Alternative 3 is most often (87% of the futures) better than Alternative 2 with respect to SO_2.

Based on this kind of information, the expert gave all the ranges of values and also some ranges of values for the public opinion of different alternatives. The value intervals for PAIRS were left quite wide in every case to reflect the ambiguous situation. Figure 7 shows the total value intervals for each alternative after the lowest level valuation of alternatives.

Figure 7: The weight intervals of alternatives after the lowest level valuation

The evaluation of three economic criteria was done for two different situations. In the first case, the expert considers the relatively short time period goals when the role of the capital requirements is more important because of the recent weak financial situation of the country. The second case examines a longer time period when it would be better to emphasize those alternatives which decrease the dependence of foreign energy sources and thus decrease total costs. After this discussion and before the actual evaluation of two situations, the expert evaluated the criteria importance at the topmost level. Public opinion is very important at this moment in Hungary and the opinion of the citizens can even change the decisions which are already made. Thus change in public opinion was considered to be more important than the changes of costs or SO_2 emissions. Figure 8 shows the criteria evaluations at the topmost level of the hierarchy and the weight intervals for alternatives after these statements.

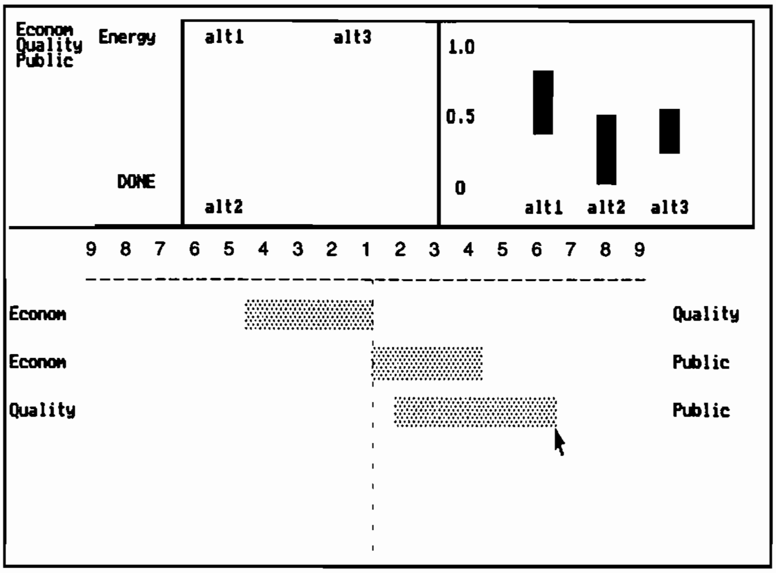

Figure 8: The evaluation of main criteria

The capital resources in Hungary are scarce at this moment. At this situation, the capital requirement is more important than discounted costs or oil and natural gas consumption. The discounted costs are also more important than oil and natural gas consumption. Figure 9 shows the value ranges of alternatives after these statements of relative importance. Alternative 1, which is based on an increased combined cycle program and which does not require big investments, dominates Alternative 2. However, there is no dominance between Alternatives 1 and 3, while Alternative 3 is a compromise alternative which includes some amount of all the options.

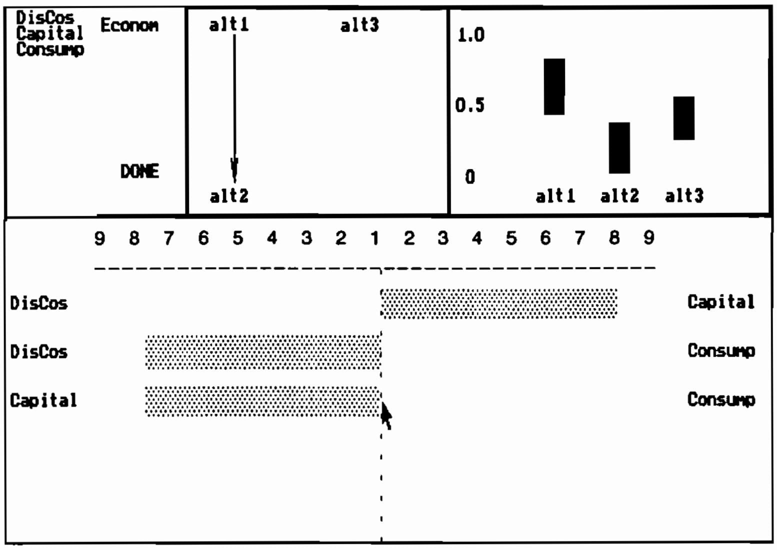

Figure 9: The situation if the expert consideres a short time period of 10 years

The new base load capacity would have advantages if we think of a longer time period. Would the decision change if the oil and natural gas consumption is given more weight and the other statements remain unchanged? The statements for the second situation for economic factors and the results after these statements are shown in Figure 10. There are no dominances but it can be seen that the weight interval for Alternative 1 is almost above the weight interval of Alternative 2. However, the most useful information is that even if we now give less weight for the capital requirements, Alternative 1 is anyway among the best solutions.

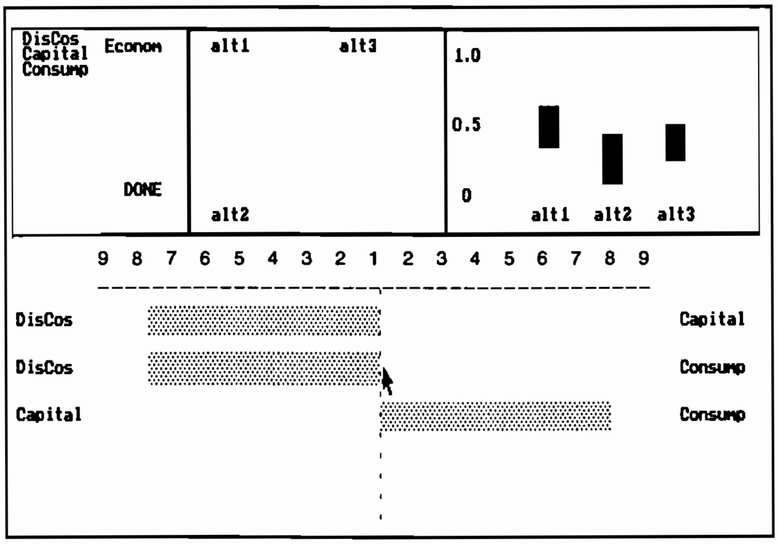

Figure 10: The situation if the capital requirements gets less weight than the other economic factors

The PAIRS analysis showed that the decision based on a combined cycle program is a very robust one and is a very good alternative if DMs also think about the forthcoming situations. For the DM it is quite clear that the country can start the combined cycle program and can install to some extent ripple control devices.

The energy planning is a very dynamic process and the data for the analysis change as the times go by. The following citation describes the situation: "Plans are nothing, planning is everything". PAIRS fulfills the requirements of the changing situation and could be updated and used continuously during the next decision steps. The main purpose with PAIRS is not to find the best alternatives, but rather conduct the planning process and help to compress the information which is achieved during the trade-off risk analysis.

7. Conclusions

PAIRS is a new decision analysis method which allows imprecise preference statements. This study uses PAIRS for two cases and gives examples how the method is used in different situations during the decision making process. The PAIRS analysis focuses on decision makers' preferences. Those preferences evolve during the decision making process and the information of the problem guides this evolution. Valuable information is achieved with other decision support tools as well. The important lesson of the case studies in this paper is that the combination of different techniques leads to best results if we consider the practical use of the decision support systems.

The underlying theory of PAIRS is based on multiattribute utility theory and the assumptions of preferential independence should be filled when we use additive value functions. In our cases, preferential independence was examined by discussing with the DMs but it may be that this is not enough. The elicitation of criteria weights in PAIRS is similar with the SMART technique. The questions which are asked from the DM are not directly bound with ranges of attributes. During the analysis sessions it was explained to the DM that the ratios of weights are actually ratios of changes when we move attribute values from the worst level to the best level. However, the analysis with PAIRS is maybe even too easy and quick so that it may be difficult to the DM to keep this in mind all the time. It is also acknowledged that in the decision hierarchy it is very difficult to ask meaningful questions of the criteria weights as we move upwards and compare clusters of criteria. These drawbacks of the traditional value tree analysis are not as serious with PAIRS as all the statements are imprecise. The imprecise model of preferences includes also the ambiguity of the methodology.

The software implementation of PAIRS is based mainly on visual information. The DM gives all information by moving horizontal bars with a mouse and also the results are vertical bars. It seems to be so that usually DMs do not consider the numbers which are shown in the screen, but rather accept choices which look good. This includes the requirement that the visual information should be interpreted in a right way so that we can rely on the results. The future research topic is to study if there are some biases when the DMs give imprecise preference information visually and what would be the most efficient way to utilize human ability to quickly understand and interpret visual information.

The new decision support methods can be validated only by applying them to true decision problems. Although this study does not fulfill all the requirements of a real application (real DMs, real problem, real data, real decision and an implementation of that decision) it provides examples how PAIRS is used from the decision maker's point of view by using real data and opinions from specialists. According to the experts of this study, the main benefit from the PAIRS analysis was the learning during the process as it was very easy to seek answers to different what-if questions. The questions which were dealt with during the analysis were mainly of two types: what happens if the DM changes preferences (evaluation of criteria weights) or what happens if the description of the decision problem changes (attribute outcomes change or the valuation of attribute outcomes changes). The PAIRS does not give ready answer to the question: "What is the final decision?", but instead it gives a tool for seeking answer to the question: "Why this decision should be made?".

Acknowledgment

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Appendix A

The computation of value intervals in a decision hierarchy (Salo and Hämäläinen 1992)

The set of all criteria is A={a_0, a_1, ... }. The set D(a_i) ⊂ A includes those criteria which are structured immediately under a_i in a decision hierarchy. T is a set of lowest level criteria. The topmost criterion a_0 is on level 0 of the decision hierarchy and there are *m* levels in the hierarchy. The set of attributes can be partitioned into sets A_0 ... A_m so that the criteria at the *i*th level of the hierarchy are in A_i. The feasible region for the local weight of criterion a_i is S_i. These feasible regions consists of the local weights which satisfy inequality constraints which the DM gives as intervals of weight ratios. For example the statement: "the investment costs s_i are more important than the operationg costs s_o" implies that the weights should satisfy an inequality $s_i \geq s_o$.The weight of the topmost criterion a_0 is one.

For a fixed set of local weights the value of the alternative *x* is $v(x) = \sum_{a_i \in T} w_i v_i(x_i)$. The DM gives value ranges [$\underline{v}_i(x_i)$, $\bar{v}_i(x_i)$] for each lowest level criterion. The maximum value of the alternative x, $\bar{v}(x)$, is achieved if each $v_i(x_i)$ is in its upper bound $\bar{v}_i(x)$. Same applies to the lower bounds and thus the value range for alternative x is:

$$\bar{v}(x) = \max \sum_{a_i \in T} w_i \bar{v}_i(x) \qquad (1)$$

$$\underline{v}(x) = \min \sum_{a_i \in T} w_i \underline{v}_i(x) \qquad (2)$$

Optimizations are done with respect to the local weights at the branches of the decision hierarchy.

Formulas (1) and (2) are not efficient in computing value intervals for alternatives. The following theorem suggests a more useful algorithm for the derivation of value intervals. The criterion weights are obtained from the local weights by the equality $w_j = w_i s^i_j$, $a_j \in D(a_i)$, $s^i \in S_i$. The weight of a criterion a_j depends thus only on the local weights of those criteria which are in the same branch and above a_j in the decision hierarchy. This property enables to decompose (1) and (2) into a series of linear programming problems over the feasible regions. Each solution is a optimal way to allocate weight from a_j to the already solved parts of the hierarchy under a_j. This is formulated as a theorem and the proof of this theorem can be found in Salo and Hämäläinen (1992).

Theorem 1: Let i takes values from m-1 to 0 and for each $aj \in A_i \neg T$ define the absolute bounds

$$\underline{v}_j(x) = \min_{s \in S_j} \sum_{a_k \in D(a_j)} \underline{v}_k(x) s_k \qquad (3)$$

$$\bar{v}_j(x) = \max_{s \in S_j} \sum_{a_k \in D(a_j)} \bar{v}_k(x) s_k \qquad (4)$$

Then $V(x) = [\underline{v}_0(x), \bar{v}_0(x)]$.

Algorithm for computing the value intervals of alternatives starts then as (3) and (4) are solved for criteria at level *m-1*. The value ranges are given by the DM. Then the absolute bounds on level *m-2* are computed by using the bounds of level *m-1* as coefficients in (3) and (4). Iteration continues towards the topmost criterion and at the top of the hierarchy bounds $[\underline{v}_0(x), \bar{v}_0(x)]$ give the value interval for alternative x.

Appendix B

The optimization results from Makowski et al. (1994) for the Nitra river water quality management problem.

Alternative	TAC	INV	OM	DO	BOD	NH4
0	1.55	0.00	1.55	0.14	25.75	4.71
1	6.65	13.50	5.06	2.80	17.52	3.17
2	3.95	2.80	3.62	2.85	21.46	4.38
3	3.65	1.50	3.47	2.89	21.86	4.40
4	3.98	2.50	3.69	3.03	21.81	4.38
5	4.25	1.00	4.13	3.06	19.53	4.02
6	7.50	21.50	4.97	3.45	16.78	3.14
7	6.06	0.00	6.06	3.60	11.62	3.84
8	5.46	1.20	5.32	3.73	17.71	3.78
9	5.45	1.50	5.27	3.87	17.76	3.69
10	5.56	2.00	5.32	3.96	17.76	3.69
11	5.85	2.60	5.54	4.02	17.68	3.68
12	6.19	1.00	6.07	4.05	11.52	3.68
13	6.36	1.00	6.24	4.05	11.52	3.68
14	6.64	2.30	6.37	4.24	11.16	3.68
15	7.71	6.70	6.92	4.45	11.13	3.61
16	7.77	7.70	6.87	4.51	11.13	3.59
17	7.90	8.00	6.96	4.53	11.12	3.58
18	7.72	10.00	6.54	4.67	10.61	3.29
19	7.91	12.00	6.50	4.75	10.39	3.23
20	8.68	28.50	5.33	4.77	17.25	3.00
21	7.52	17.00	5.52	4.82	20.25	3.85
22	7.87	10.50	6.64	4.83	10.61	3.29
23	7.94	10.60	6.69	4.84	10.61	3.29
24	8.08	16.00	6.20	4.91	10.66	3.29
25	8.14	12.50	6.67	4.92	10.20	3.09
26	8.15	11.30	6.82	4.94	10.60	3.29
27	8.23	15.00	6.47	4.94	10.20	2.89
28	8.15	13.60	6.55	4.95	10.60	3.29
29	8.40	15.10	6.63	4.95	10.20	2.88
30	8.44	15.10	6.67	4.95	10.20	2.88
31	8.77	14.60	7.05	4.95	10.20	2.88
32	8.65	22.50	6.01	4.97	16.48	2.95
33	8.94	15.00	7.18	4.97	10.37	2.87
34	8.43	12.60	6.95	4.98	10.60	3.26
35	8.44	13.10	6.90	5.02	10.19	3.14
36	8.31	14.50	6.61	5.05	10.60	3.26
37	8.32	14.00	6.68	5.05	10.60	3.26
38	8.54	13.50	6.95	5.05	10.19	3.00
39	8.76	16.00	6.88	5.05	10.19	2.86
40	10.04	23.10	7.33	5.06	10.18	2.38
41	8.44	14.80	6.70	5.08	10.60	3.26
42	8.45	14.30	6.77	5.08	10.60	3.26
43	8.70	13.90	7.07	5.09	10.16	3.00
44	8.83	15.00	7.07	5.13	10.15	2.99
45	10.95	26.30	7.86	5.15	10.13	2.15
46	10.30	28.00	7.01	5.20	10.12	1.82
47	9.16	17.40	7.12	5.23	10.58	3.22
48	9.46	17.40	7.42	5.24	10.10	2.91
49	9.57	17.60	7.50	5.24	10.10	2.90
50	10.76	29.90	7.25	5.31	10.06	1.70
51	11.83	32.90	7.97	5.34	9.89	1.70
52	14.06	50.30	8.15	5.38	9.81	1.70
53	14.36	50.30	8.45	5.38	9.81	1.70