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Endogenous Growth of Population and Income Depending on Resource and Knowledge

Prskawetz, A., Feichtinger, G., Luptacik, M., Milik, A., Wirl, F., Hof, F. and Lutz, W.

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ABSTRACT

We consider a three sector demoeconomic model and its interdependence with the accumulation of human capital and resources. The primary sector harvests a renewable resource (fish, corn or wood) which constitutes the input into industrial production, the secondary sector of our economy. Both sectors are always affected by the stock of knowledge. The tertiary sector (schooling, teaching, training, research) is responsible for the accumulation of this stock that represents a public good for all three sectors. Labor is divided up between the three sectors under the assumption of competitive labor markets. A crucial feature of this economy is the importance of public goods--stock of knowledge and the common--which requires collective actions. Absence of collective actions describes the limiting case of hunters and gatherers. The central focus of this study is whether and what kind of interactions between the economy, the population and the environment foster sustainability and, if possible, continuous growth.

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1. INTRODUCTION

History has told us that different countries may experience different economic growth rates during the same period of time. Even within a single country, growth rates can vary during different time periods, such that a country can enjoy rapid economic growth during one period and suffer from slow economic growth during other periods. To explain these differences in growth rates, or at least to partly give some possible explanations, is the challenge to the economist.

Out of this problem, the *endogenous growth theory* (Lucas 1988, 1993; Romer 1986, 1990) has been developed in the last ten years. Endogenous technological change (e.g. human capital and knowledge) has been identified as the engine of growth, and constitutes the heart of the theory. Compared to neoclassical growth models, where technological change was usually assumed to be exogenous to the economy, the new theory on economic growth defines technological change as endogenously evolving out of the economic environment. How does this technological change come about; which economic variables are influenced by this change; and how is technological change financed? These are the questions that constitute the difference between the various endogenous growth models developed in the last decade.

Regarding the question of the determination of technological change, two broad classes of theoretical models have been suggested. One focuses on technological change as a by-product of investment and production. This line of research dates back to Arrow's learning by doing model (Arrow 1962) and has recently been taken up by many authors (Lucas 1993). In the other models, resources are explicitly allocated to research (Romer 1990; Hartwick 1992). This idea is based on the work of Uzawa (1965) who constructed a planning model where knowledge is produced in a separate R&D sector. In line with this approach, models have been developed, where the total labor force is divided between two sectors: production and knowledge creation (Zhang 1990; Hartwick 1992). The second question, in which way is technological change introduced into the economy, has been dealt with quite similarly in the literature. Traditionally, combinations of labor and capital saving technologies have been used. Regarding the financing of technological change, the new literature on economic growth emphasizes that knowledge is a private property with substantial externalities. Private provisions therefore lead to market failure so that some kind of internalization seems necessary. One can carry this conclusion even to the extreme and stipulate (Hartwick 1992) that knowledge is entirely a public good in the notion of Samuelson (1964).

Drawing on these studies, we introduce a model that adds *demographic* and *environmental* inputs to the topic of endogenous growth theory. We choose a three sector demoeconomic model as the framework of our analysis. The standard two-sector model (production and knowledge creation) is extended by the primary sector, which harvests renewable resources (see Prskawetz et al. 1994 for this approach). While the labor force in the primary and secondary sector take part in production of the harvest and industrial goods, the labor force in the third sector is responsible for knowledge accumulation. We therefore model technological change as an increase in the knowledge stock produced by workers (scientists and engineers) specializing in knowledge production. In addition to the literature so far we assume that the labor force is free to migrate between the three sectors, i.e. at any instance of time, economic mechanisms endogenously determine the division of labor to the three sectors.[1] In formulating the endogenous technological change we rely on the extended Phelps model (Steinmann 1986, p. 102ff), where technological progress is determined by supply factors like the availability of teachers, per capita research equipment, etc. To also incorporate demand-induced technological progress in the line of Boserup[2] (1981), we add the dependence of technological progress on the population level. Therefore, in our model, technological progress not only fosters economic growth, but knowledge accumulation itself depends on the economic as well as demographic variables of the model. Similar to Hartwick (1992) and different to Romer (1990), we embody this new knowledge gratuitously and simultaneously in the workers of the primary and secondary sector (labor augmenting technological progress) and use it to shift up the production frontier of the secondary as well as of the tertiary sector itself. Technological change is purely a public good; no worker is

[1] In the literature, the division of labor to the production and knowledge creation sector is usually assumed to be exogenously given. "Accordingly, the intellectuals and physical workers are exogenously given to the system. In fact, we may consider the ratios as endogenous variables, though this will cause some analytical difficulties." (Zhang 1990, p. 1935)

[2] Boserup regards population growth as the cause, not the consequence, of the intensification of the means and methods of land cultivation, i.e. population levels themselves increase the pressure to create new knowledge.

excluded from receiving it. To finance the tertiary sector, its labor force and its factor inputs, all workers are taxed by an exogenously given amount (see also Hartwick 1992 for this approach).

There are two important extensions to the standard endogenous growth models in our study which will heavily influence and distinguish our results from the ones obtained in the literature on endogenous growth theory.

The first one deals with the shortcoming of modelling resources. Usually there are no references made to resources in standard growth models--indeed they are treated as free factors by excluding them from the production function and are assumed to provide constant services. On the contrary, in our model, resources (e.g. topsoil, fisheries, forests) are not fixed but can be depleted (by the harvest) as well as regenerate. The assumption of endogenous, renewable[3] resources is central to our study since it combines the literature on endogenous growth with the vast literature on *sustainability*.[4] As long as resources regenerate faster or at least at the same speed as the harvest grows, there will be no constraint on economic growth stemming from the availability of resources. If, however, the rate of utilization (harvest) exceeds the flow, a renewable resource will be depleted and in the extreme, it can be destroyed forever. In this case resources might well hinder, decrease or even terminate economic growth, i.e. the sustainability of the economy might be threatened. The critical point is exactly where the increasing annual harvest begins to exceed the reproductive capacity, so that a cumulative downturn in the productivity of the resource is initiated. If one would allow for a sufficiently high substitutability of labor and technology for resources, the depletable factor (resources) would eventually become inessential to the production process and economic growth would continue with the contribution of the reproducible factors only.[5] However our experience shows, that with simple and constant policies as used in our model, unconstrained and sustainable growth seems impossible. Furthermore to prevent overuse and degradation of the resources *(tragedy of the commons)*[6] we will restrict access to the

[3] Here we mainly stress renewable resources like water, soil, forests and fisheries, since these are the ones for which property rights are largely undefined, and which therefore largely fall outside the discipline of the market (Lee 1993). Also Keyfitz (1991b) regards renewable resources as the ones which come under increasing pressure and, contrary to what the words imply, these "renewable" resources are the ones of which we are more likely to run short.

[4] In traditional resource and environmental economics the sustainability criterion is simply that the current generation leaves to the next generation a natural endowment not inferior in value to the endowment which was received by the current generation.

[5] This is exactly what Dasgupta and Heal (1974) utilize in their analysis. They use a CES production function with an elasticity of substitution between capital and natural depletable resources greater or equal to one.

[6] With a given population a free-access resource is possibly subject to overuse and degradation, such that the existing population could be made better off on its own terms if use of the resource were limited. For example, free access, competitive fishery drives the rent from fishery to zero, which leads to too many fishermen, and thus overexploitation. For detail on this argument see Levhari and Mirman (1980), Clemhout and Wan (1985), and Lee (1990).

resources by taxing the output of the primary sector. In addition to the income tax mentioned above, this Pigouvian tax will be used to finance the input into the tertiary sector.

The demographic module constitutes the second important extension of the usual endogenous growth models. In particular we assume that population growth is influenced by economic and environmental factors. But these factors are in turn influenced by past population growth rates, so that population is endogenously determined as part of an interacting system. Essentially there are two conflicting opinions on the question about the dynamic relationship between population and economic growth (Gaburro and Poston 1991). The first one considers population growth as a negative phenomenon that creates impoverishment in the long run by generating a reduction in per capita output due to diminishing returns to scale to factors that can be accumulated. This is essentially the well-known Malthusian theory. In the second view, the incentive for economic development is human creativity, which in turn is stimulated by population growth. The diminishing returns to scale assumption is compensated by greater returns from human creativity and increasingly favorable conditions deriving from larger populations. Population therefore not only depends on economic growth but is in fact a variable that may explain growth. Two of the most famous proponents of this theory are Kuznets (1973) and Boserup (1981). In our model we will combine both considerations, since these two theories are complementary rather than contradictory as illustrated by Lee (1986). In particular we assume that the population growth rate increases with per capita income and knowledge up to a certain threshold level. To the right of this turning point, any further increase in per capita income and technology will have a negative effect on the rate of population growth. Additionally, population growth is linked to the environment[7] (via the economy) such that the above dynamics will be clouded by the question of sustainability, i.e. ever-increasing economic growth, accompanied by steadily growing population levels, will endanger the capacity of the resources.

Finally it should be mentioned that we abstract from the accumulation of capital in our model for two reasons: (i) physical capital formation is 'fast' within the long-run context of our model; physical capital stock typically lasts less than ten years, (ii) to retain a parsimonious and tractable framework.

Summing up, our model provides an analytical framework for the study of *economic development* and *sustainability* through integrating population growth, resource use and economic growth. Two recent conferences of the United Nations--on Environment and Development in Rio de Janeiro in 1992 and on Population and Development in Cairo in 1994--illustrate the importance of this topic.

[7] The population's adverse impact has most likely occurred wherever arable land and water are particularly scarce or costly to acquire, and wherever property rights to land and natural resources are poorly defined. In the opposite, the positive impact of population growth on the economy has been observed where natural resources are abundant, possibilities for scale economies are substantial, and where markets and other institutions (especially governments) allocate resources in a reasonably efficient way over time and space (Gaburro and Poston 1991).

Conservation of the ecosphere versus growth of population and the economy is a central issue of our time, but its relationships are quite controversial.[8] One group considers that growth comes at an intolerable expense to the environment, and others that the damage to the environment is slight and in any case so far in the future that it does not matter (or as Keynes put it so aptly, in the long run we are all dead). For example, technological optimists assume that new technologies will be developed to eliminate any resource constraints. As Keyfitz (1991a, p. 1) mentions,

> The economists are right in stressing the advantages of size and growth of population as well as of income. A larger population admits a finer division of labor. ...Surely it is finiteness that kills the chain letter, and finiteness is also the defect in the argument for indefinite growth of population and the economy. Real world growth of people and goods is entirely positive and beneficial, but only until the fisheries decline, the oil runs out, the planet warms up. And because of *nonlinearities* in man-environment systems the collapse of the environment comes on suddenly....

Hence, to achieve a sustainable pattern of resource use, population and economic growth, we have to understand and control the interactions of population and per capita resource consumption as mediated by technology and economic growth. This is exactly what we aim to capture within our model.

Central to our study are the endogenous *nonlinear* interactions between population growth, the economy and the environment. In particular, we totally abstract from any exogenous, stochastic forces in our model, which by no means will imply simple monotonic solution paths. Due to the highly nonlinear interactions incorporated in our model, different modes of behavior and stability can occur.

Due to the nonlinearity in our model, we have to rely on numerical methods in solving for the system dynamics. In particular, we will use the *theory of nonlinear dynamical systems*, which is based on the qualitative (geometric) theory of differential and difference equations. This theory renders it possible to obtain numerical results on the qualitative behavior of time paths for whole classes of parameterized functions. To analyze the effects of changes in the parameters, we use local bifurcation theory which replaces the comparative static analysis once the system is unstable (Zhang 1990).

The paper is organized as follows. In Section 2 we present a non-technical outline of the model to describe the considered economic, demographic and environmental interactions. The dynamic feedback rules, which we postulate to numerically illustrate the dynamics of the model, are given in Section 3. Using a cross-section study of 23 low-income countries, we give empirical evidence (Section 4) for the choice of the parameter set we use for the numerical simulation. In Section 5, we numerically analyze the model to quantify the resulting time paths of the variables involved. In particular, we show the dependence of the stability of the system dynamics on the parameters involved. We close with

[8] Keyfitz (1993) gives an excellent review on the debate of sustainability going on between economists and biologists. See also MacKellar and Horlacher (1993) for a nice summary on this topic.

some conclusions and suggestions for further research in Section 6. Mathematical calculations are given in the Appendix.

2. THE MODEL

We consider an entirely competitive, dynamic economy, where the labor force L is divided and migrates between three different kinds of employment:

1. the *primary sector* (L_1), which harvests natural renewable resources,
2. the *secondary* or *industrial sector* (L_2), and
3. the *tertiary sector* (L_3).

The output $H = H(AL_1,R)$ of the *primary sector* is given by a standard production function in the inputs labor L_1 and the available resource stock R. In addition, technology, know-how, education, and social capital represented by the stock A, improves the labor productivity (e.g. by using tractors instead of horses).

The net growth of the renewable resource stock R is affected by two counteracting factors, indigenous, biological growth $g(R)$ and the harvest H; $\dot{R}=g(R)-H(AL_1,R)$.

Using the harvest H and labor L_2, the *secondary sector* produces the output $Y=Y(A,H,AL_2)$. In the following Y is taken as the numeraire good, such that its price π equals one. Again, technology A affects the labor productivity but it may also increase the production frontier over time.

Finally, the output of the *tertiary sector* $E=E(Z,L_3,P,A)$ adds new technologies and ideas to the stock of knowledge A.[9] The output E depends on two kinds of inputs, industrial products Z (ranging e.g. from buildings--schools, universities, courts--to desks, computers, research laboratories) and labor L_3 (teachers, researchers, professors, judges, etc.). As usual, this production function is itself affected by technological progress A as knowledge itself influences the creation of additional knowledge. In addition, the output E depends on the population or some measure of its density (Lee 1988), e.g. cities as centers of intellectual discourse. Put differently, schooling is insufficient for the population in order to enhance human knowledge because of the number of geniuses (Mozart, Newton, etc.) depending on the rise of the population. This argument dates back to Kuznets (1973, p. 3):

[9] Consideration of a scaler variable A requires taking a broad view of knowledge, including inter alia related public goods: education (from elementary schools to universities), research (basic and applied), establishment and maintenance of a legal system, including a police, etc. After all many institutions are the outcome of human knowledge plus spontaneous order, to use a phrase from Hayek.

> More population means more creators and producers, both of goods along established production methods and of new knowledge and inventions. Why shouldn't the large number achieve what the smaller numbers accomplished in the modern past--raise total output not only for the current population increase but also for a rapidly rising supply per capita.

Summing up, technological progress is easier when the population is both more numerous (P) and better educated (L_3 /P), and when industrial output (Z) and previous knowledge (A) have reached relatively high levels.

Human knowledge, as already mentioned, is a stock variable that also depreciates.[10] The net growth rate of knowledge can be described by the differential equation $\dot{A}=E-\delta A$, where δ denotes the depreciation factor of the stock A. Depreciation therefore constitutes the only countervailing force against the tendency of technology to increase.

The total ratio of the population entering the labor force, the labor participation ratio $l(w(1-t_2))$, depends positively on the real wage w net of lump sum taxes wt_2 (=T_2, percentage value of the wage collected as tax revenue) raised by the government on wage income.

As already noted in the introduction, the definition of the interaction between the demographic and the economic-environment sector via the endogenous population growth constitutes the core of our model.

While there are essentially two conflicting views regarding the dynamic relationship between population and economic growth (Malthusian versus Boserupian approach), things worsen if one starts to actually model the population growth function itself. While the literature indicates a clearly negative impact of increasing per capita income on mortality, the results are quite controversial and ambiguous regarding the impact of the economic status on fertility decisions.

Malthus (1798) was certainly one of the first to succeed in systemizing an economic theory of population growth. Basic to his theory were the following considerations. Any excess of production over requirements will give rise, via lower death rates and higher birth rates, to demographic expansion which will in turn drive the standard of living back down to its original level. This demographic trap fits the data from European countries in the pre-industrial period. However, it is contradicted by more recent evidence from industrialized countries during their period of rapid population growth, that demographic expansion can co-exist with improvements in living standards. But the wealth of the industrialized countries may distract attention from Malthusian forces that nevertheless are quite visible in many developing countries. This warning was recently expressed by a well-known political scientist

---

[10] Remember the surprise of the British romantics, notably Lord Byron, who expected the Greeks to be a people of philosophers but found by the nineteenth century a people of shepherds.

(Kennedy 1992). In fact Kennedy considers that Malthus will mean for the 21st century what Adam Smith means to the 20th century.

The inability of demographers to predict western birth rates accurately in the postwar period had a salutary influence on demographic research. In the 1960s the new economics of the family was developed, using the theory of the demand for consumer durables as a framework to analyze the demand for children. The basic message of these theories was the possible negative effect of increasing wealth on the number of children.

In a recent study, Winegarden and Wheeler (1992) present empirical findings that per capita income raised birth rates up to estimated turning points, and thereafter exerted negative effects. This curvilinear relationship of population growth was already outlined in Woods (1983) who identified three phases regarding the impact of economic growth on fertility. Where standard of living (as measured by per capita income) is high, fertility will not be significantly related to changes in per capita income; when it is at medium, fertility will decrease with increasing per capita income; and when the standard of living is low, fertility will increase with per capita income.

One can interpret these findings and considerations most properly using the transition theory (Blanchet 1991b). The transition theory essentially assumes that the relationship between the rate of population growth and the level of per capita income is positive in an initial phase, because better economic conditions imply lower mortality and often also higher fertility due to better food supply and health. Then after a critical point, a second phase emerges in which this relationship turns negative because gains in life expectancy become harder to attain and also have a lesser impact on the rate of population growth, and additionally, rising levels of income per capita lead to fertility decline.

The negative impact of increasing per capita income on population growth is strengthened by the inverse association between fertility rates and human capital (education, knowledge, etc.) as illustrated for example in Rosenzweig (1990). In particular, Rosenzweig emphasizes the striking evidence in aggregate cross-country data (whether examined cross-sectionally or over time) of the inverse associations between fertility rates and per capita incomes, and such indicators of human capital as schooling levels and survival rates. As a general rule, he concludes that high-income countries have been and are characterized by low fertility and high levels of human capital, while low-income countries are characterized by high fertility and low levels of human capital. Those countries that have experienced high rates of per capita income growth in the last 25 years have also experienced relatively rapid declines in fertility and increases in human capital levels.

A similar striking negative effect of education on the fertility rate is illustrated in the case of Mauritius (Lutz 1994), where during the fertility transition in the late 1960s and early 1970s, the total fertility rate declined from around six children per woman to under three children in less than 10 years. This was probably the world's most rapid fertility decline on a national level. The Mauritius fertility dropped on a strictly voluntary basis: the result of high levels of literacy and education for women, together with successful family planning programs were the main determinants.

The theoretical framework of the New Household Economics (Becker and Barro 1988; Becker et al. 1990) constitutes the microeconomic foundation of the nonlinear changes in fertility depending on economic growth (including human capital, etc.). It essentially applies the microeconomic theory of consumer demand to the childbearing decision, supplemented by a 'supply' side which takes such factors as child mortality and marriage into account. There is a trade-off, at a given level of income, between quantity (number of children) and quality (expenditures per child). At low levels of income, income gains will raise the demand for child quantity and quality; at some critical income level, the quantity-quality trade-off will depress the demand for numbers of children.

Following this discussion of the economic impact on the population growth rate, we define $n=n_1(y)-n_2(y,A,P)$. The first term captures the Malthusian effect, which might prevail for incomes below and incomes not too far above subsistence level. As long as output is below some exogenous given subsistence level y^{sub}, the population will decline, while a positive population growth rate will set in when output per capita exceeds y^{sub}. The second term relates to the second phase of the transition theory (increasing per capita income y leads to a decrease in population growth rate) as well as to the negative impact of increasing education A on fertility. The second term also includes population at whole, assuming that the size of the negative effects on population growth just described increase with increasing population pressure.

We know that in a world where population growth will affect and simultaneously will be affected by economic growth, we can observe neither causal relationship directly, i.e. what we observe is a mix, which is different from the two relationships taken separately. Additionally, the connection between population growth and economic growth is often the result of the interweaving of numerous factors (social, cultural and political factors) whose dynamics and cause-and-effect relationships with other factors are still both poorly understood and somewhat unstable.

Despite all these caveats, Figures 1.a and 1.b illustrate the suggested relationships quite well.[11] As Figure 1.a shows, the lower the per capita income, the higher the population growth rate will be in the average. In Figure 1.b we have used the third level (higher) gross enrollment ratio as a surrogate for the knowledge stock A. Again, the result is quite convincing. Higher levels of education are associated with lower population growth rates.

[11] Data have been taken from 219 countries.

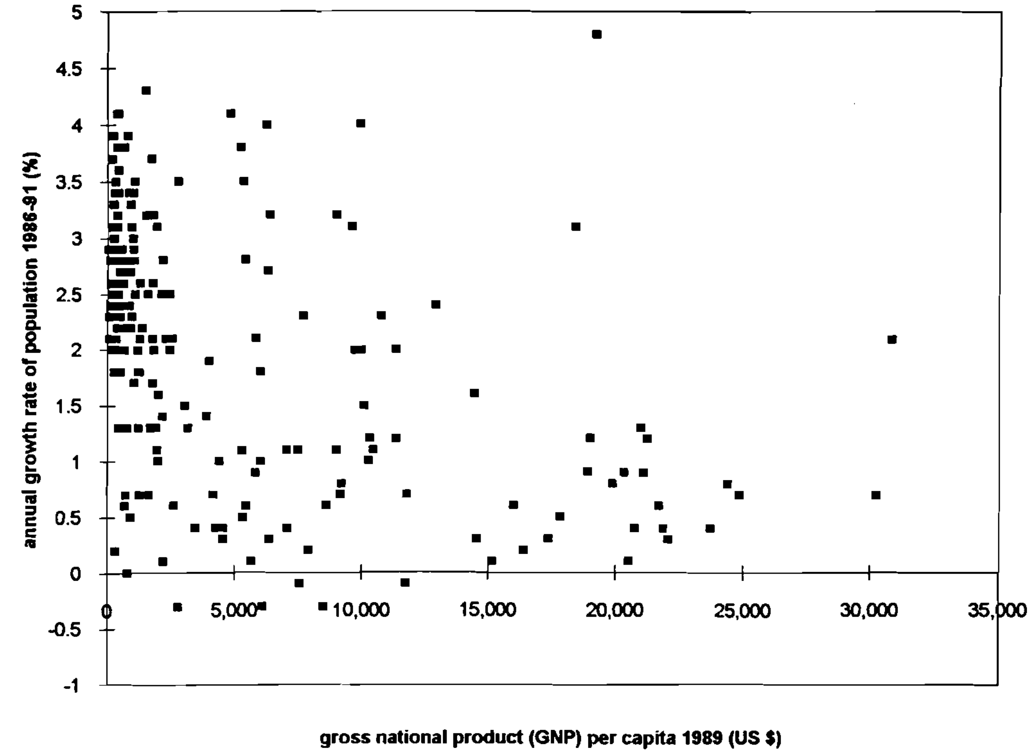


Figure 1.a. Annual growth rate of population versus GNP per capita. Source: Encyclopaedia Britannica 1992.

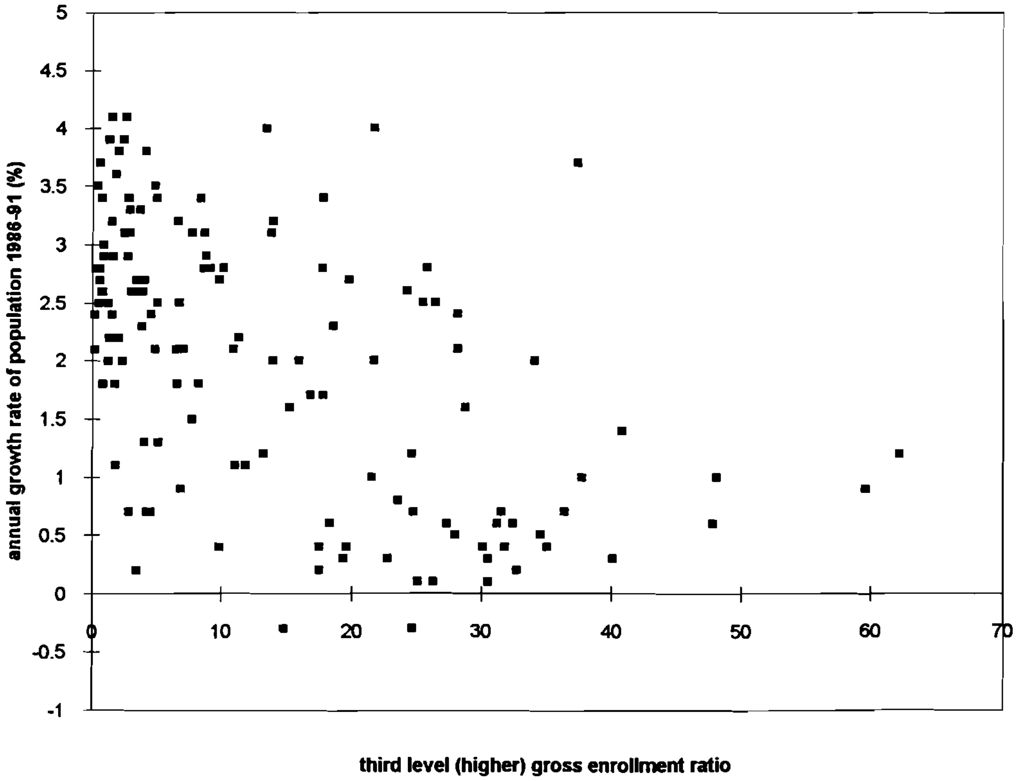


Figure 1.b. Annual growth rate of population versus third-level gross enrollment ratio. Source: Encyclopaedia Britannica 1992.

Up to now we have specified the demographic, environmental and economic interactions in our economy. But there is one more open question we have to handle: our economy set out exhibits two *externalities* that are not properly reflected in competitive markets.

First, free access to renewable resource harvesting may lead to the *tragedy of the commons* requiring restricted entry into the primary sector to sustain the resource basis. This restriction is most efficiently achieved by property rights that restrict entry or by taxing the output of the primary sector, i.e. a Pigouvian reasoning beyond the revenue motive to tax. In fact, agriculture was and is taxed as long as a dominant share of the population earns their living in the primary sector.[12]

The second externality arises due to the existence of a public stock *knowledge*, where we take a broad view and include education, technology, and knowledge in the broad sense including social capital, such as law, courts, police, defense, ethics, thrift and habits. This stock, which is purely public, shifts the production frontiers outward in all three considered sectors so that increasing returns to scale apply for the entire economy. Furthermore, this stock remains the same whether consumed by one or by a billion of people, although the build up of this stock may depend on the size of the population. Failures to organize collective actions properly, will hinder economic growth. Therefore total tax revenues $T=T_1H+T_2L$ are used to finance the factor inputs L_3 and Z for the production of the output E of the tertiary sector. Hence there exists a classical reason for creating governmental institutions, which should properly distribute the taxes. Of course, they may personally and directly affect the efficiency of this distribution, i.e. the amount of money they divert into their own and into the bureaucrat's pocket.[13] This implies that only part of the total tax revenue $(1-c)$, where c stands for corruption, is used to finance the factor inputs of the tertiary sector.

3. DYNAMIC FEEDBACK RULES

The evolution of the economy we are considering can be expressed by three nonlinear differential equations in the variables R (resource stock), A (technology, knowledge) and P (population)[14]:

---

[12] However, no regulation of access to resource harvesting may be optimal for low population densities, i.e., if harvest rates do not endanger sustainability and in particular, if enforcement is costly. Similarly, if industrialization due to technological progress would reduce the number of farmers too much, a subsidy might even be paid to keep farming above market clearing levels.

[13] Indeed, large sums of these revenues end up in bribery and rent seeking. According to *The Economist,* May 8th, 1993, p. 72, African leaders hold some $20 billion deposits in Swiss banks.

[14] Time arguments of the variables, e.g. *R(t), A(t), P(t)* are omitted in the following. Furthermore throughout this paper we will denote time derivatives by 'dots' on top of variables.

$$\dot{R}=dR/dt=g(R)-H(AL_1,R) \tag{1}$$

$$\dot{A}=dA/dt=E(Z,L_3,P,A)-\delta A \tag{2}$$

$$\dot{P}=dP/dt=n(y,A,P)P \tag{3}$$

All production functions are assumed to be of the Cobb-Douglas form commonly used in demoeconomic models (see, e.g. Blanchet 1991a):

$$H(AL_1,R)=R^{\alpha_1}(A^{\varepsilon_1}L_1)^{\alpha_2}, \qquad 0<\alpha_1,\alpha_2,\varepsilon_1\leq 1,\alpha_1+\alpha_2=1 \tag{4}$$

$$Y(A,H,L_2)=A^{\varepsilon_2}H^{\beta_1}(A^{\varepsilon_3}L_2)^{\beta_2}, \qquad 0<\beta_1,\beta_2,\varepsilon_2,\varepsilon_3,\leq 1,\beta_1+\beta_2=1 \tag{5}$$

$$E(Z,L_3,P,A)=\left[\left(\frac{Z}{P}\right)^{\gamma_1}\left(\frac{L_3}{P}\right)^{\gamma_2}A^{\gamma_3}\right]P^{\gamma_4} \qquad 0<\gamma_1,\gamma_2,\gamma_3,\gamma_4\leq 1,\ \gamma_1+\gamma_2=1 \tag{6}$$

In line with the new theory of economic growth, we assume that H, Y and E exhibit constant returns to scale with respect to the inputs provided by factor markets. Therefore competition is viable and the public good A provides for economy-wide increasing returns to scale. Additionally, all inputs are essential, in particular the availability of resources, i.e. $H \to 0$, thus $Y \to 0$ and $E \to 0$, for $R \to 0$.

Furthermore we use the standard model of renewable resource economics (Clark 1985). The net growth of the renewable stock R is affected by two counteracting factors, indigenous, biological growth $g(R)$ and the harvest H. The growth function g is assumed to be logistic

$$g(R)=\theta R(\overline{R}-R), \quad g(R)>0, g''(R)<0 \quad for \quad 0<R<\overline{R} \tag{7}$$

The coefficient $\overline{R}$ determines the saturation level (carrying capacity) of the resource stock, i.e. $\overline{R}$ is the stationary solution of R if the resource is not harvested (H=0), and the parameter θ determines the speed at which the resource regenerates, i.e. the intrinsic growth rate of the resource. (θ large corresponds to a fast-growing species and θ small to a slow-growing species, e.g. trees.)

For the endogenous population growth function $n=n_1(y)-n_2(y,A,P)$ we postulate the form

$$n=c_4\left(\frac{y-y^{sub}}{c_1+y-y^{sub}}-c_2\frac{AP(y-y^{sub})}{APy+c_3}\right) \tag{8}$$

where c_1, c_2, c_3 and c_4 represent constant scaling factors and y^{sub} the subsistence level.

The first term captures the Malthusian effect, while the second term describes the negative impact of technology A and increasing per capita output y on the population growth rate. But we restrict this negative relation to take place only when per capita income exceeds the subsistence level. This assumption is in line with the transition theory mentioned in Section 2. Initially (for low values of per capita output) any increase in the standard of living will lead to higher fertility and lower mortality. Only when a certain threshold of per capita output has been passed, fertility will decline while mortality stabilizes. Whenever per capita output is below the subsistence level, we assume that higher technology and increasing per capita income will counterbalance the negative Malthusian effect, i.e. foster population growth. Furthermore this second term is scaled by the total population level P assuming that higher population densities will strengthen the negative (positive for y less than y^{sub}) impact mentioned above.

As both terms in the endogenous population growth rate (8) are modelled to be concave functions in y and therefore bounded for any finite technology A and population P, the sum n will also be bounded. In particular n is restricted on the interval $[c_4(-y^{sub}/(c_1-y^{sub})+c_2y^{sub}AP/c_3),c_4(1-c_2)]$ for any finite level of technology A and population P.

Figure 2.a illustrates the form of the endogenous population growth rate as a function of per capita output y for differing values of the exogenous parameters c_1, c_2, c_3 and y^{sub} and fixed values of the variables c_4=1 and A=P=1. The population growth rate *na* constitutes the reference case, where the parameters are chosen as follows: $c_1=0.1,c_2=1,c_3=0.5,c_4=1,y^{sub}=0.01$. Increasing c_2, i.e. the negative impact of rising per capita income and technology, the rate of population growth will drastically decline once the subsistence level has been surpassed (*nb*). If instead the parameter c_1 decreases (*nc*) or the parameter c_3 increases (*nd*) the population growth rate will increase. In the first case the increase of the population growth rate has mainly occurred for values around the subsistence level, while in the second case, the rate of population growth shifts up over the whole range of y. Finally *ne* represents the population growth rate if we increase the subsistence level. The total effect on n is concentrated in the range of per capita output values below the subsistence level, where the higher subsistence level results in a strong decline of the population growth rate. In Figure 2.b the effect of increasing (*nf*: A=2) and respectively decreasing (*ng*: A=0.5), the technological level A is illustrated if the other parameters are fixed according to the population growth rate *na*. The rate of population growth decreases (increases) with increasing (decreasing) levels of technology.

Up to now we have specified the dynamic behavior of our economy as given by the three dimensional system of nonlinear differential equations (1)-(3). Next, we have to state the implicit constraints resulting from (a) the assumption of competitive markets, (b) the distribution of taxes to the tertiary sector, and finally (c) the endogenous labor supply function.

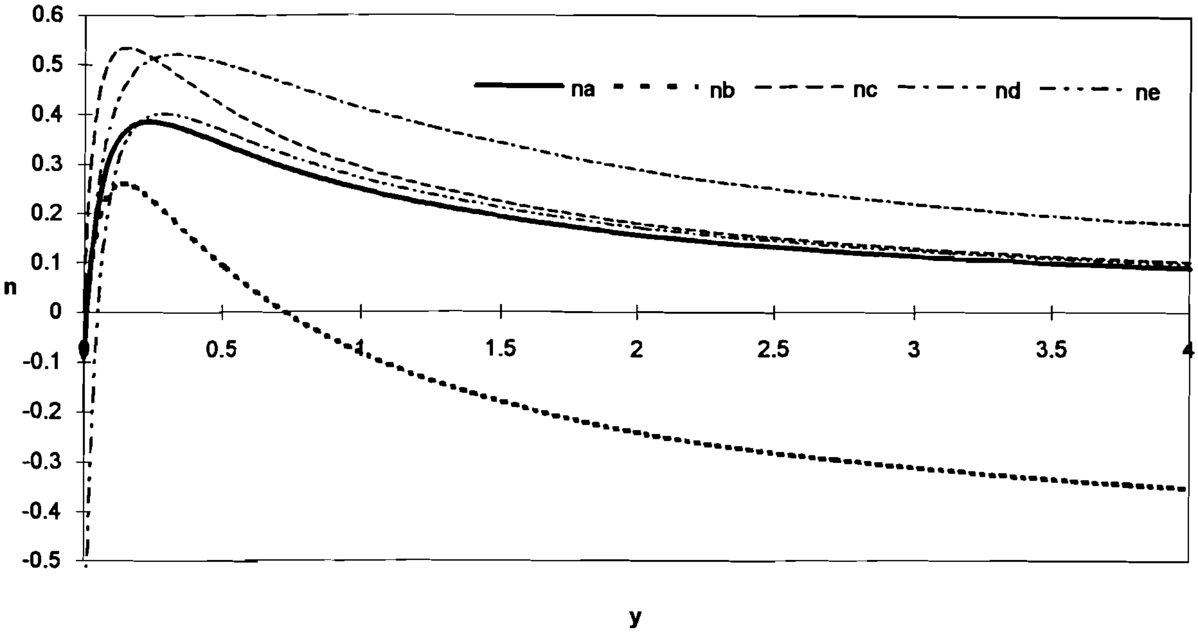


Figure 2.a. Endogenous population growth rate for different sets of parameters and fixed values of technology A and population P.

na: c_1=0.1,c_2=1,c_3=0.5,c_4=1,y^{sub}=0.01, A=1, P=1

nb: c_1=0.1,c_2=1.5,c_3=0.5,c_4=1,y^{sub}=0.01, A=1, P=1

nc: c_1=0.05,c_2=1,c_3=0.5,c_4=1,y^{sub}=0.01, A=1, P=1

nd: c_1=0.1,c_2=1,c_3=1,c_4=1,y^{sub}=0.01, A=1, P=1

ne: c_1=0.1,c_2=1,c_3=0.5,c_4=1,y^{sub}=0.05, A=1, P=1

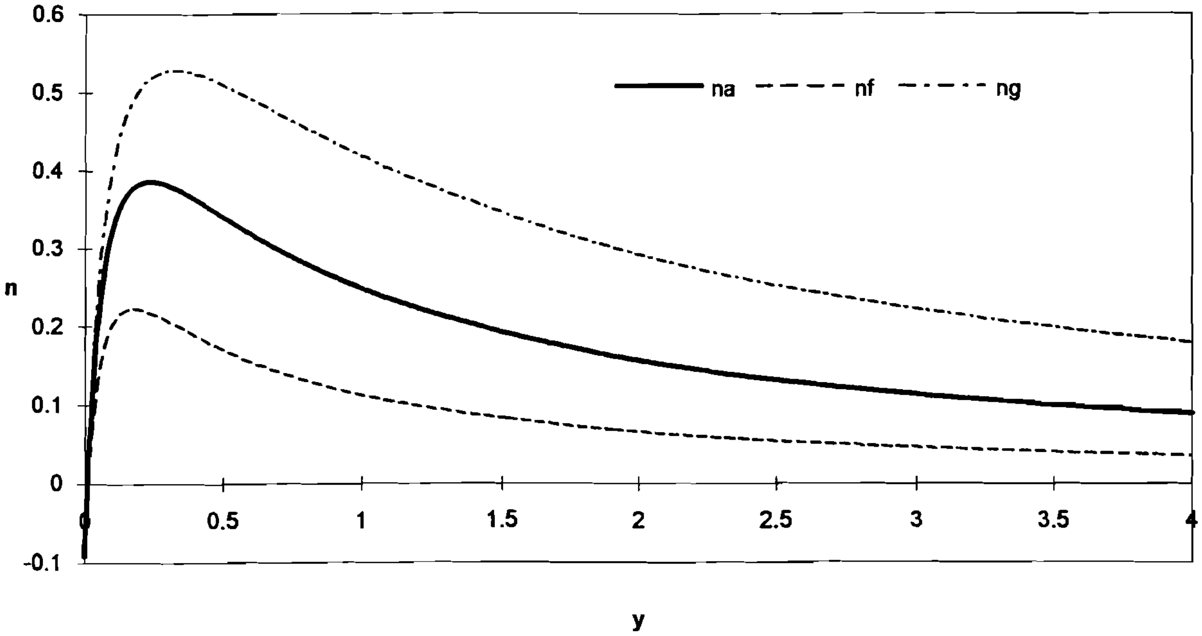


Figure 2.b. Endogenous population growth rate for different levels of technology A.

nf: c_1=0.1,c_2=1,c_3=0.5,c_4=1,y^{sub}=0.01, A=2, P=1

nf: c_1=0.1,c_2=1,c_3=0.5,c_4=1,y^{sub}=0.01, A=0.5, P=1

Competitive allocation of factor inputs yields three constraints:

$$\tilde{p}=MC=\frac{w}{H_{L_1}} \quad (9)$$

$$Y_{L_2}=\frac{Y}{L_2}\beta_2=w \quad (10)$$

$$Y_H=\frac{Y}{H}\beta_1=p=\tilde{p}(1+t_1) \quad (11)$$

The first constraint (9) states that the primary sector maximizes the profit by setting marginal costs MC equal to the price $\tilde{p}$ of the harvest. Marginal costs are determined by the wage w paid for one additional worker divided by his (marginal) product, H_{L_1} (Varian 1984).[15] To take care of overharvesting we assume that the government introduces a Pigouvian tax[16] on the harvest. Now, the profit maximizing input allocation of the industry follows from the condition that the marginal products must equal the factor prices (10) and (11). Employing one more worker, the secondary sector has to pay the common market wage w, while the price p (=$\tilde{p}$ plus taxes T_1 (=t_1MC)) has to be paid for an additional unit of the harvest.

The distribution of the taxes to the tertiary sector yields two more constraints:

$$(1-c)s(T_1H+T_2L)=\pi Z \quad (12)$$

$$(1-c)(1-s)(T_1H+T_2L)=wL_3 \quad (13)$$

Total tax revenues are given by the sum of taxes levied on the harvest T_1H plus taxes levied on the workers T_2L. Admitting the possibility of corruption c (governmental institutions may divert part of the total tax revenue into their own pocket) only part of the collected taxes, i.e. $(1-c)(T_1H+T_2L)$, is used for financing the tertiary sector. The distribution of this amount is assumed to follow a constant pattern determined by a proportion s spent on the industrial products πZ and a proportion $(1-s)$ spent on the factor costs of the input labor wL_3. The endogenous labor supply function $l(w(1-t_2))$ yields the last constraint.

The dependence on w (net of lump sum taxes) is straightforward, increasing and concave.

$$L=l(w(1-t_2))P=\frac{w(1-t_2)}{w_0+w(1-t_2)}P \quad (14)$$

where w_0 is a constant.

---

[15] Subscripts denote partial derivatives of the function with respect to the corresponding argument.

[16] Instead of the Pigouvian taxes introduced in our model, one could also assume a tax on the labor force (a license for the harvesters).

Together constraints (9)-(14) yield six implicit equations in the variables w,p,Z,L,L_1,L_2, additionally to the state variables R,A,P. Carefully investigating these constraints reveals that all six variables can be written as functions of the state variables R,A,P and the additional variable L_1 (see Appendix).

Our model, therefore, boils down to a system of *algebraic-differential equations of index one* in the variables R,A,P and L_1 (see Griepentrog and März 1986 for an introduction to algebraic-differential equations, and the Appendix for the derivation of the system given below):

$$\dot{R}=f_1(R,A,P,L_1) \tag{15}$$

$$\dot{A}=f_2(R,A,P,L_1) \tag{16}$$

$$\dot{P}=f_3(R,A,P,L_1) \tag{17}$$

$$0=h_1(R,A,P,L_1) \tag{18}$$

given the functions

$$f_1(R,A,P,L_1)=g(R,A)-H(AL_1,R)$$

$$f_2(R,A,P,L_1)=E(Z,L_3,P,A)-\delta A$$

$$f_3(R,A,P,L_1)=n(y,A,P)P$$

$$h_1(R,A,P,L_1)=(1-c)(1-s)(T_1H+T_2L)-wL_3=(1-c)(1-s)(T_1H+T_2L)-\beta_2(Y/L_2)L_3$$

4. EMPIRICAL ILLUSTRATIONS

In the following, a short empirical assessment of the production functions stated in Section 3 is given (see also Hazledine and Moreland 1977).

Table 1 presents a cross-section of 23 low-income countries (per capita GNP $\leq$ US\$ 400 in 1989) in Africa. Data are taken from three sources: Encyclopaedia Britannica (1992), World Resources Institute (1990) and UNFPA (1993). Based on these data we estimate the elasticities of the production functions (4)-(6) by ordinary least squares.

As a measure of the labor force we use the percentage of population at working age (15-65 years). The percentage of population working in agriculture, industry and services yields the distribution of the labor force to each sector. The output H, Y and E of each sector is taken as the percentage of GDP out of the primary, secondary and tertiary sectors. Resources are equated with arable land. The percentage of GNP spent on education yields an estimate for Z. As there was no reliable estimate available for A, we excluded it from the list of independent variables.

Table 1. A sample of low-income countries in Africa. Data sources: Encyclopaedia Britannica (1992), World Resources Institute (1990) and UNFPA (1993).

	population: % annual growth rate 1986-91 (latest estimate)	population: total (latest census)	total labor force 1985	gross national product (GNP): nominal, 1989 ('000,000 U.S.$)	gross national product (GNP): per capita, 1989 (U.S.$)	origin of gross domestic product (GDP) by economic sector, 1988 (%): primary	secondary	tertiary	public expenditure on education (percent of GNP)	percentage of labor force (1980) in: agriculture	industry	services	agricultural population per hectare arable land, 1988
EASTERN AFRICA													
Burundi	2.6	5,356,266	2,520,000	1,149	220	53	13	25	3.1	93	2	5	3.5
Ethiopia	2.9	42,184,966	19,182,000	5,953	120	37	15	47	4.4	80	8	12	2.5
Kenya	4.1	15,327,061	8,389,000	8,785	380	26	16	42	7.1	81	7	12	7.2
Madagascar	3.3	7,603,790	4,510,000	2,543	230	43	16	41	2.5	81	6	13	2.8
Malawi	3.7	7,982,607	3,074,000	1,475	180	36	18	49	3.2	83	7	9	2.6
Mozambique	2.1	12,130,000	7,671,000	1,193	80	44	37	18	1.2	85	7	8	3.9
Rwanda	3.5	4,830,984	3,063,000	2,157	310	38	23	40	3.4	93	3	4	5.4
Somalia	3.1	4,089,203	1,999,000	1,035	170	76	6	18	0.4	76	8	16	4.9
Uganda	3	12,636,179	7,054,000	4,254	250	50	9	41	4.4	86	4	10	2.1
United Republic of Tanzania	2.8	23,174,336	10,913,000	3,079	120	53	7	40	3.7	86	5	10	3.9
Zambia	3.8	5,661,801	2,242,000	3,060	390	31	29	40	5.5	73	10	17	1
MIDDLE AFRICA													
Central African Republic	2.2	2,088,000	1,282,000	1,144	390	43	10	40	2.9	72	6	21	0.9
Chad	2.5	4,029,917	1,790,000	1,038	190	46	11	43	1.8	83	5	12	1.3
Zaire	2.4	29,671,407	11,666,000	8,841	260	55	7	35	1.4	72	13	16	2.8
WESTERN AFRICA													
Benin	3.1	3,331,210	1,964,000	1,753	380	41	11	49	5.1	70	7	23	1.2
Burkina Faso	2.8	7,964,705	3,765,000	2,716	310	46	17	35	2	87	4	9	
Ghana	3.2	12,296,081	4,963,000	5,503	380	51	15	34	3.3	56	18	26	1.5
Guinea-Bissau	2.1	767,739	427,000	173	180	46	14	36	2.8	82	4	14	2
Mali	1.8	7,696,000	2,598,000	2,109	260	57	15	29	3.3	86	2	13	2.5
Niger	3.4	7,250,383	3,203,000	2,195	290	52	9	42	3.1	91	2	7	2.2
Nigeria	3.3	55,670,055	36,568,000	28,314	250	54	11	35	1.5	68	12	20	3.4
Sierra Leone	2.6	3,517,530	1,352,000	813	200	48	8	44	2	70	14	16	1.8
Togo	2.9	2,719,567	1,244,000	1,364	39	47	14	39	5.2	73	10	17	2.1
data source	Britannica, 1992		World Ressources, 1990-91	Britannica, 1992		Britannica, 1992			Britannica, 1992	World Resources, 1990-91			Weltbevölkerungs-bericht, 1993

The results are shown in Table 2 (*t*-values are given between the brackets). In particular, the estimated coefficients are the marginal productivities of the corresponding factor inputs and contain information about the overall returns to scale.[17]

Table 2. Estimated production functions.

ln(H)		ln(Y)		ln(E)	
constant	5.998 (35.42)	constant	1.163 (0.65)	constant	4.375 (9.24)
$\ln(L_1)$	0.587 (2.91)	$\ln(L_2)$	0.199 (1.12)	ln(Z/P)	0.570 (7.72)
ln(R)	0.356 (1.63)	ln(H)	0.695 (3.09)	$\ln(L_3/P)$	0.298 (2.15)
				ln(P)	0.979 (14.70)
R^2	0.741	R^2	0.719	R^2	0.933

The estimated production functions reflect quite well the assumption of constant returns to scale with respect to the inputs provided by factor markets. Additionally, the *t*-statistics as well as the adjusted coefficients of determination (R^2) support the specified production functions.

5. NUMERICAL SIMULATION

As already mentioned in the introduction, the nonlinearity inherent in our model together with the implicit equation we have to solve at each time step forces one to rely on numerical tools to solve the system of algebraic-differential equations given by equations (15)-(18).

To numerically investigate the system outlined in Section 3, we use the interactive LOCal BIFurcation program LOCBIF (Khibnik et al. 1993). In particular, we shall illustrate the dependence of the levels and the stability of the stationary solutions on the model parameters.

We group the parameters into three categories:

1. *technological* parameters including
   the elasticities of the production functions H: $\alpha_1,\alpha_2,\varepsilon_1$
   Y: $\beta_1,\beta_2,\varepsilon_2,\varepsilon_3$
   E: $\gamma_1,\gamma_2,\gamma_3,\gamma_4$

   the saturation level of the resource stock $\overline{R}$
   the intrinsic growth rate of the resources θ
   the parameter w_0 in the labor supply function
   and the depreciation of technology δ

[17] The independent variables used in our model are likely to be correlated, such that the estimates might be biased upwards.

2. *policy* parameters including
   the tax rates t_1,t_2
   the corruption level c
   and the part of taxes s $(1-s)$ used for the input of $Z(L_3)$ into the tertiary sector
3. *demographic* parameters including
   the parameters c_1,c_2,c_3,c_4 and y^{sub}.

Table 3 gives the initial parameter settings, where the elasticities of the production functions have been chosen in correspondence to the estimates given in the previous section.

Table 3. Initial parameter setting.

technological parameters	*policy parameters*	*demographic parameters*
$\alpha_1 = 0.4, \alpha_2 = 0.6$	$t_1 = t_2 = 0.1$	$c_1 = 0.1, c_2 = 1.1$
$\beta_1 = 0.7, \beta_2 = 0.3$	c = 0.5	$c_3 = 0.5, c_4 = 0.1$
$\gamma_1 = 0.6, \gamma_2 = 0.4, \gamma_4 = 1$	$s = 0.5$	$y^{sub} = 0.01$
$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \gamma_3 = \varepsilon = 0.1$		
$\overline{R}=5$		
$\theta=1$		
$w_0=1$		
$\delta=0.01$		

To reduce the number of parameters, we assume that the elasticities of technology are equal in all sectors, i.e. $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \gamma_3 = \varepsilon$.

Starting from the initial conditions $(R,A,P,L_1)=(1,1,1,0.1814817)$,[18] the system converges towards the equilibrium point $(R^*,A^*,P^*,L_1^*)=(4.8,2.7,1.2,0.3)$ as indicated in Figure 3 (where the first integration steps have been skipped).

The phase portrait of the system exhibits three different demoeconomic regimes. At moderate levels of technology A (region I) the Malthusian effect dominates, i.e. any increase in the size of population will lower per capita income. But as the stock of technology builds up, the population growth rate slows down and the system passes region 2, where population P and per capita output y simultaneously increase, which in turn fosters the accumulation of technology. Once the negative effect of technology

[18] The labor force L_1 has been computed (using the program MATHEMATICA; Wolfram 1988) such as to fulfill the implicit constraint given by equation (18).

on population growth dominates, population will start to decline (region III). Due to our assumption that technology depends on population, indefinite growth of technology together with a steadily decreasing population is not feasible. Once population is too small to increase or even to keep up technology, the trajectories will bend backwards, i.e. decreasing population levels and increasing per capita income levels are accompanied by decreasing technology. Finally, the system converges to the equilibrium point (R^*, A^*, P^*, L_1^*) characterized by a stationary population ($n=0$) and zero growth rates of technology and resources. It is interesting to note that the influence of resources on the system dynamics is almost negligible. In a first conjecture this would suggest regarding the environment as the slowest moving variable compared to the demographic (P) and economic (y) variables in our model. Later on it will become apparent that this is true as long as we are in a pre-catastrophe time, i.e. as long as the sustainability of the environment is not threatened.

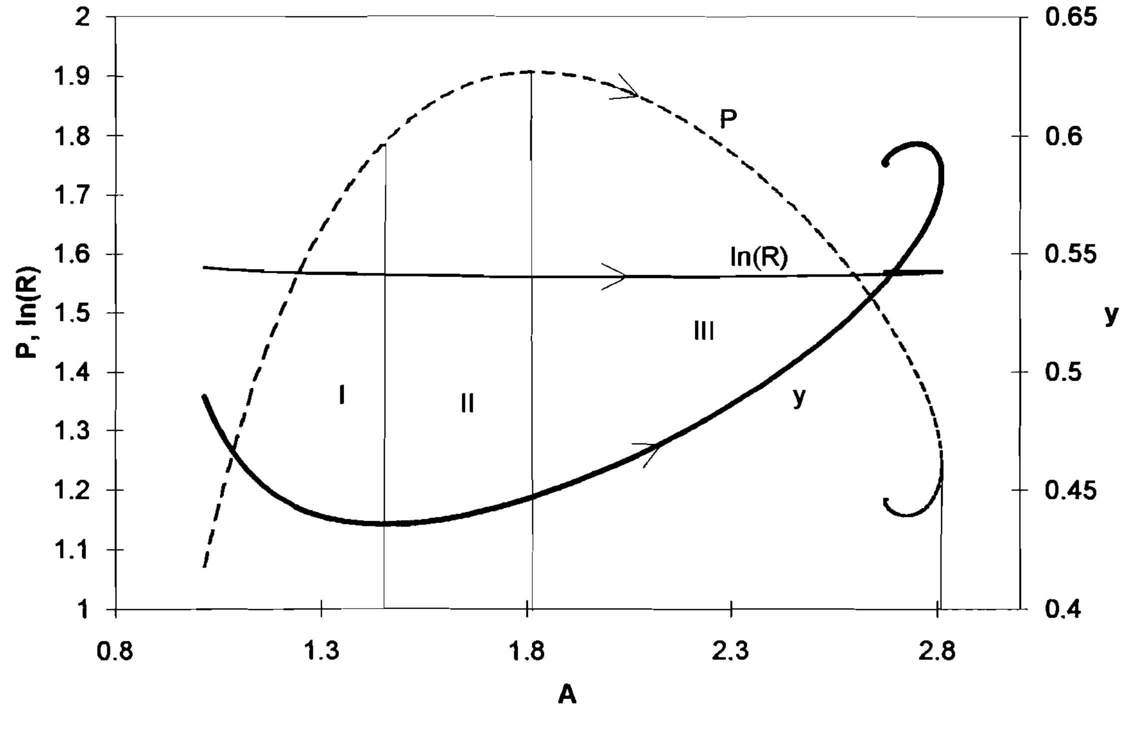
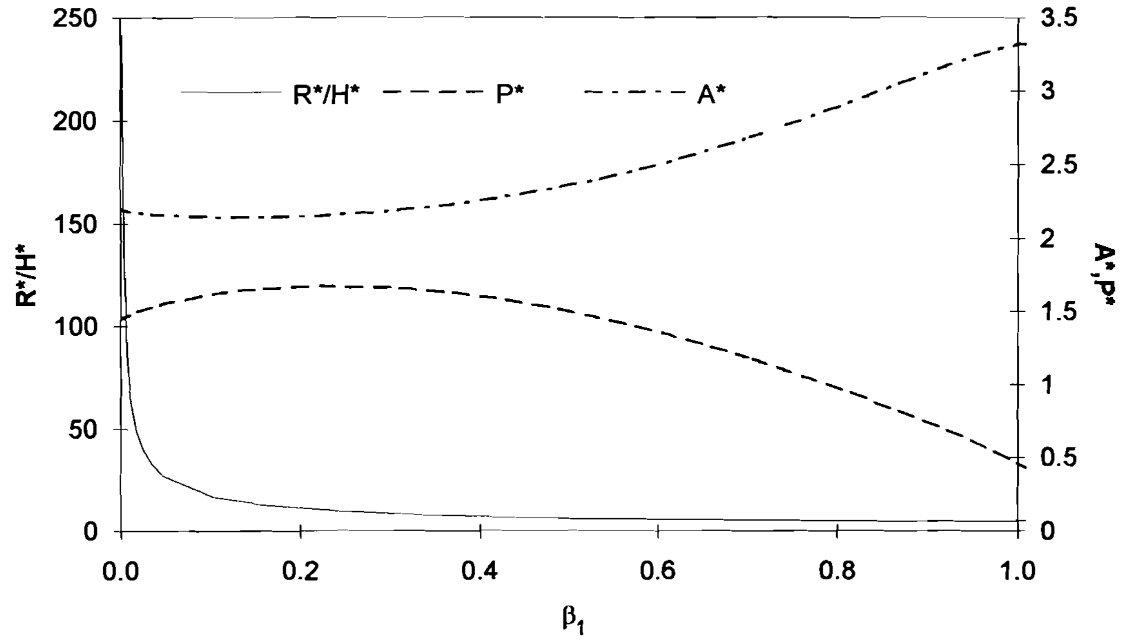


Figure 3. Characteristic trajectories in phase space for the parameter values given in Table 3.

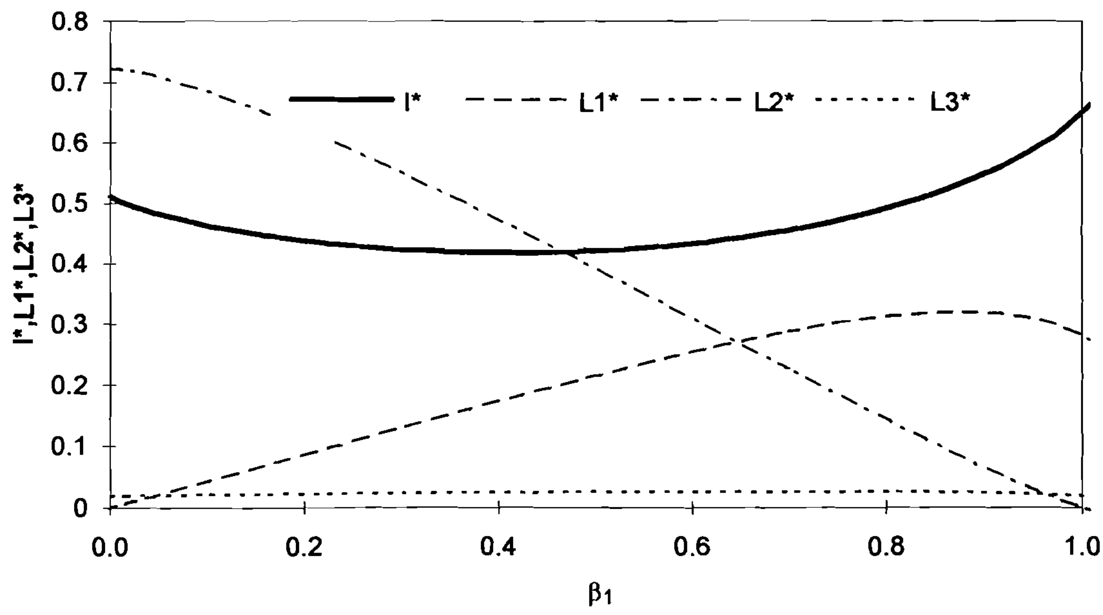
The stable stationary point to which the system converges in Figure 3 can be used as a starting point to compute *equilibrium curves*. These curves are defined in the phase-parameter space of our model by $\dot{R}=0, \dot{A}=0, \dot{P}=0$ and $h_1=0$, where all but one parameter are fixed.

Figures 4.a-4.c. Equilibrium curves for selected system variables given the parameter setting in Table 3 and choosing β_1 to be the bifurcation parameter.

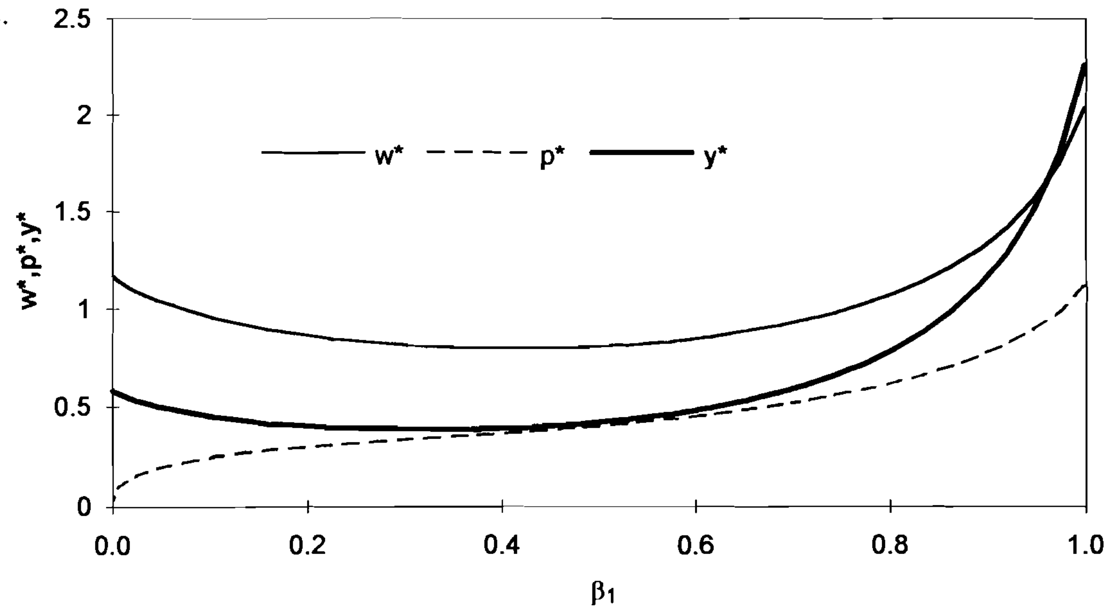
4a.



4b.



4c.



In Figures 4.a to 4.c, equilibrium curves for various variables of the model have been computed if the *production elasticity* β_1 of the industrial output Y with respect to the resource stock R is varied between 0 and 1. (The 'star' on top of the variables denotes stationary values.) The assumption of Constant Returns to Scale production functions implies that by varying β_1 we implicitly also vary β_2 between 1 and 0.

What are the effects on the level of the stationary values of the variables if we vary β_1? The higher the production elasticity β_1 becomes, the more the secondary sector will substitute the inputs into production away from labor towards the input of harvest. Consequently, the primary sector increases its output H and thereby intensifies the pressure on the resource stock R. But as indicated in Figure 4.a, the increase in the harvest outweighs the decrease in the resource stock, i.e. the resource intensity R/H decreases with increasing β_1. Additionally, varying the elasticity β_1 leads to a redistribution of labor from the secondary to the primary sector. This follows directly from the endogenous competitive allocation of labor as stated by equation (A.1) given in the Appendix. In particular, both labor forces will be equal if $\beta_1 = 0.65$. As the industrial labor force L_2--which essentially determines the wage rate w--decreases, the marginal productivity of labor, i.e. w, goes up. This in turn leads to a growing labor force participation rate though the population at whole declines (due to the rising technological level). Finally, the more expensive labor force and the smaller amount of resources forces the primary sector to go up with the price of the harvest $\tilde{p}$ (Figure 4.c).[19]

Now the question arises how to judge whether the economy is better or worse off by increasing the production elasticity β_1. This question is not easy to answer in a purely descriptive model like the one

[19] In Figure 4.c, p has been plotted instead of $\tilde{p}$, but since the tax rate t_1 is constant, $\tilde{p}$ is just proportional to p.

presented here. But we can approach this problem by using the stationary value of per capita income $y^*=Y^*/P^*$ as a measure of 'social welfare'. With the exception of small values of β_1 the steady state value of per capita income increases as the productivity of the harvest, β_1, becomes higher. In the sense defined above, the economy will therefore enjoy higher welfare. Of course, this 'welfare measure' could be enhanced by incorporating some function of the resources, i.e. people might as well derive some utility from forests, etc., which might considerably decrease the 'optimal' value (the argument which maximizes social welfare) of the productivity of the harvest.

One of the most interesting parameters in our model, which provides for economy-wide increasing returns to scale (IRS) is the production elasticity of technology ε. While IRS build the core of endogenous growth, this is no longer true once we endogenize the resource stock.

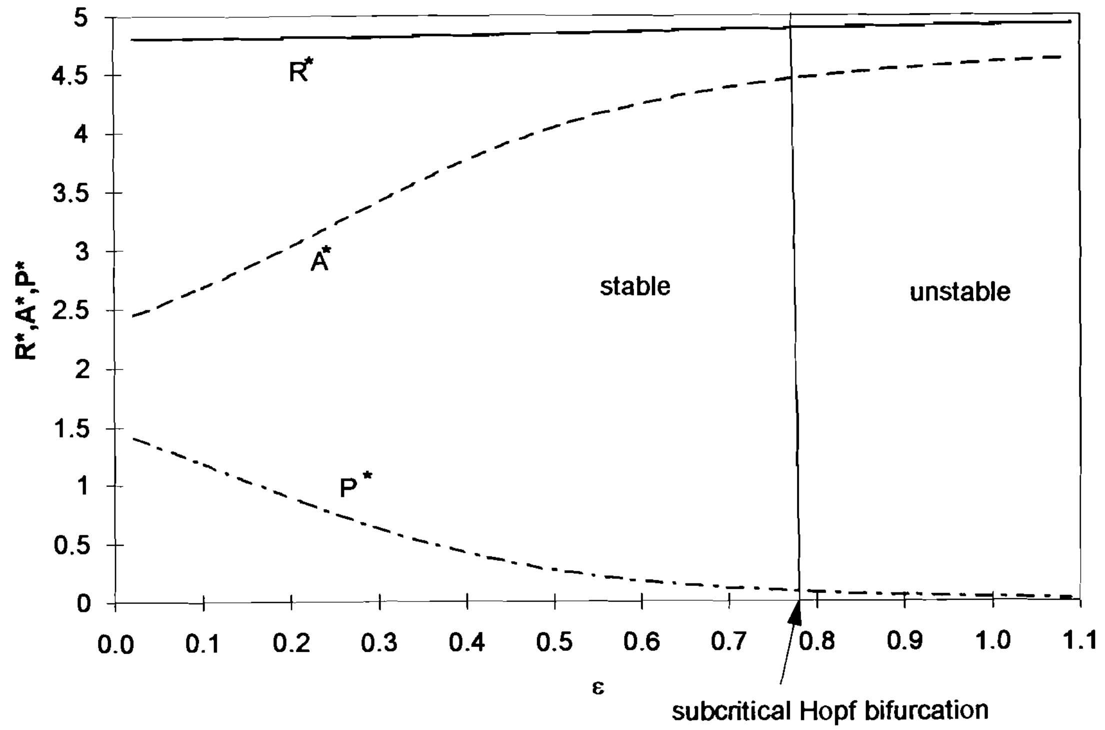


Figure 5.a. Equilibrium curves for selected system variables given the parameter setting in Table 3 and choosing ε to be the bifurcation parameter.

In Figure 5.a the equilibrium curves for the stationary values of resources (R^*), population (P^*) and technology (A^*) have been computed if the *production elasticity* ε varies. As expected, the more productive technology as well as labor become, the higher the stationary values of technology will be. This in turn decreases the population growth rate, such that population will decline with increasing values of ε. Once the production elasticity becomes too high, the stationary solution loses its stability via a subcritical Hopf bifurcation, i.e. the stable solution becomes unstable and a coexisting unstable periodic orbit disappears. This can be termed a 'dangerous bifurcation' because the equilibrium becomes unstable and no other stable solution is created. Indeed the unstable region to the right of the

Hopf bifurcation point in Figure 5.a is characterized by resource catastrophes, i.e. the resource stock tends towards zero because technology increases much faster than population declines, which implies increasing pressure on the resource stock. While the resource stock is the slowest-moving variable in the pre-catastrophe period, its speed suddenly accelerates during the transition towards the catastrophe.

Hence the incorporation of depletable resources bounds the system's growth, i.e. in equilibrium (to the left of the Hopf bifurcation point) all system variables will cease to grow. In the extreme (to the right of the Hopf bifurcation point) resources might even tend towards zero, such that the economy will end in a 'natural catastrophe', i.e. all variables will finally become extinct.[20]

To give some intuition on these numerical, results let us consider the conditions which have to be satisfied for the growth rates to be greater than 0.

Let $\gamma_R, \gamma_A, \gamma_P$ be the growth rates of the resource stock R, the technology A and the population P. Using (1)-(3) it follows that:

$$\frac{\dot{R}}{R} = \gamma_R = \theta(\overline{R}-R) - H/R = g_R + \theta R - H_R/\alpha_1 \tag{19}$$

$$\frac{\dot{A}}{A} = \gamma_A = E/A - \delta \tag{20}$$

$$\frac{\dot{P}}{P} = \gamma_P = n \tag{21}$$

Obviously the conditions for positive endogenous growth will heavily rely on the growth rate of the resources as indicated by equation (19). In particular to keep γ_R greater than zero requires a high growth rate of the resources g_R together with high levels of the resource stock R compared to the marginal productivity of the resources H_R. But these conditions are difficult to fulfill since the growth rate g_R will be positive only for $R<\overline{R}/2$. But the lower the resource stock, the higher will be the marginal productivity H_R.

In the case of Figure 5.a the higher production elasticity of technology ε even further increases the marginal productivity of the resources H_R such that the growth rate of resources will decline, cease or even become negative.

[20] Of course the system (1)-(3) guarantees that all variables will remain in the positive orthant. But once the state variables are too small, numerical integration techniques will run into difficulties. This problem is even strengthened by the fact that our model equations are very stiff.

The sustainability of the environment might not only be threatened for high values of the production elasticity ε, but might also crucially depend on the demoeconomic environment where the system starts (*sensitive dependence on initial conditions*). This demonstrates that the potential positive effects of a thinly populated world, i.e. reduction of population growth, can be overdone.

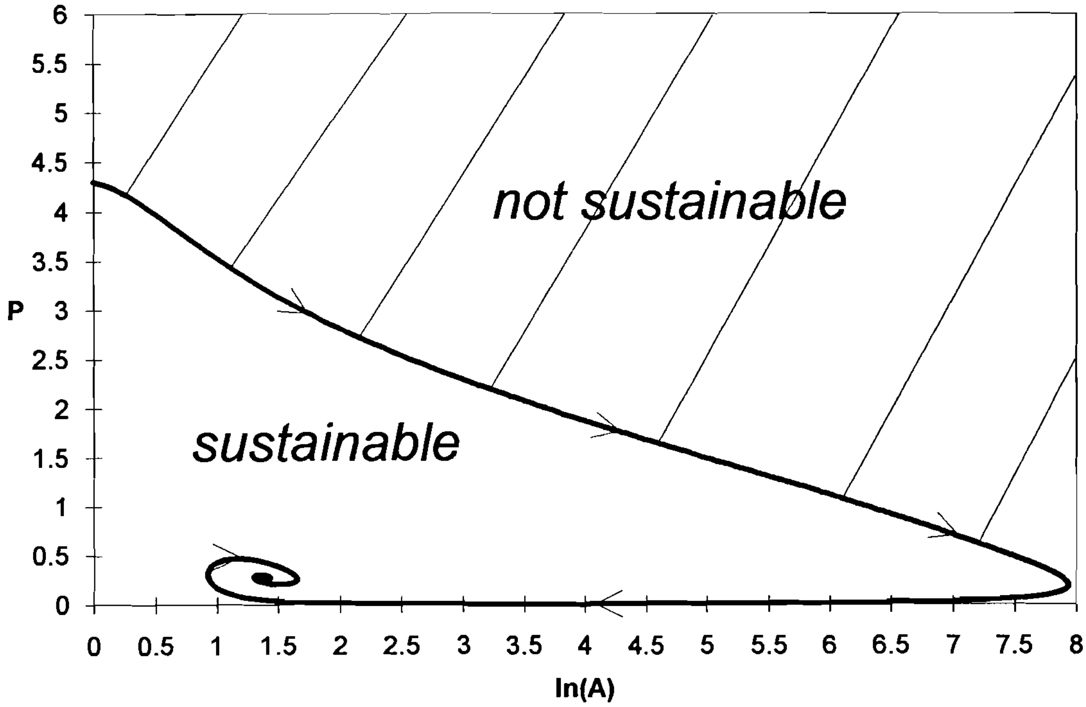


Figure 5.b. Sustainable region in the *(ln(A),P)*-phase diagram (with the initial value of the resources *R* fixed at a value of 5) given the parameter setting in Table 3 and fixing ε at the value 0.5.

Figure 5.b illustrates the dependence of the system dynamics on the initial values of the population level and technology, while the initial values of the resources are fixed at a value of 5 and ε is set equal to 0.5. The solid trajectory in the *(ln(A),P)*-phase diagram delimits the region of sustainability from the region, where the system ends up in a natural catastrophe, i.e. where the resources are totally exploited. As Figure 5.b indicates, the higher the initial technological level, the lower the initial population level has to be to sustain the environment. This tradeoff between technology and population essentially implies that the pressure of technology and population on resources (via the harvest *H*) operates in the same direction. Of course, if the growth of resources would be positively influenced by technology as well, i.e. $\theta(A),\theta_A>0$ or $\overline{R}(A),\overline{R}_A>0$, the overall effect of technology on the resource stock might be positive. On the other hand, even though we exclude the positive influence of technology on the parameter θ, our model is still quite optimistic since we limit our discussion only to renewable resources (compared to exhaustible resources).

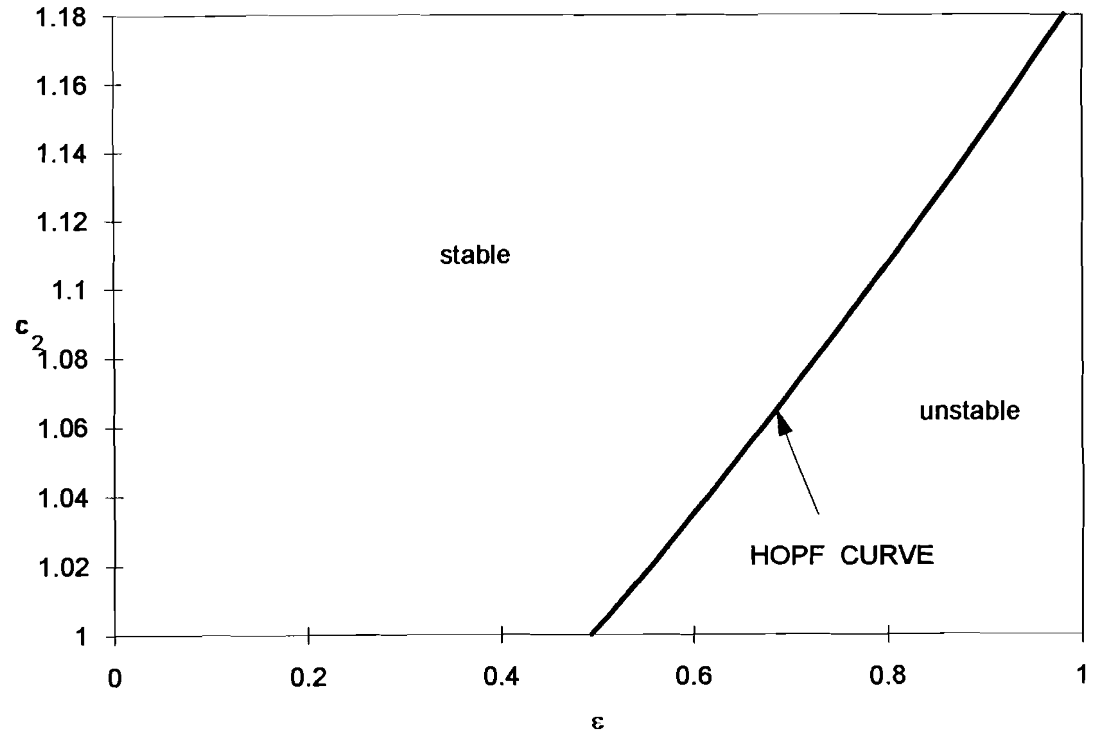


Figure 5.c. Hopf curve for the parameters given in Figure 5.a and choosing c_2 and ε to be the bifurcation parameters.

Instead of adding another functional relationship ($\theta(A), \theta_A > 0$) to the model, Figure 5.c[21] illustrates how the stability domain (and therefore implicitly also the sustainability region) can be enhanced by simply increasing the negative effect of technology on the population growth rate, i.e. c_2, in correspondence to increasing values of ε.

The opposite happens, i.e. the stability region shrinks, if one would use the parameters c_1 instead of c_2 as the second bifurcation parameter, as illustrated in Figure 5.d, because increasing c_2 reinforces population decline once technology and per capita income are high, while increasing the parameter c_1 already depresses population growth around the subsistence minimum y^{sub}. Only for low values of c_1 will the slope of the Hopf curve be positive, i.e. there exists an optimal value of c_1 which maximizes the size of the stability region.

Additionally for low values of c_1, the Hopf curve changes from a subcritical (above the zero Lyapunov point) to a supercritical Hopf-bifurcation (below the zero Lyapunov point). Contrary to the subcritical Hopf-bifurcation, the supercritical Hopf-bifurcation is 'safe', because although the equilibrium becomes unstable, a new attractor is created, namely a limit cycle.

[21] Figure 5.c was constructed by using the bifurcation point in Figure 5.a as the initial point for a Hopf curve, where all but two parameters are fixed.

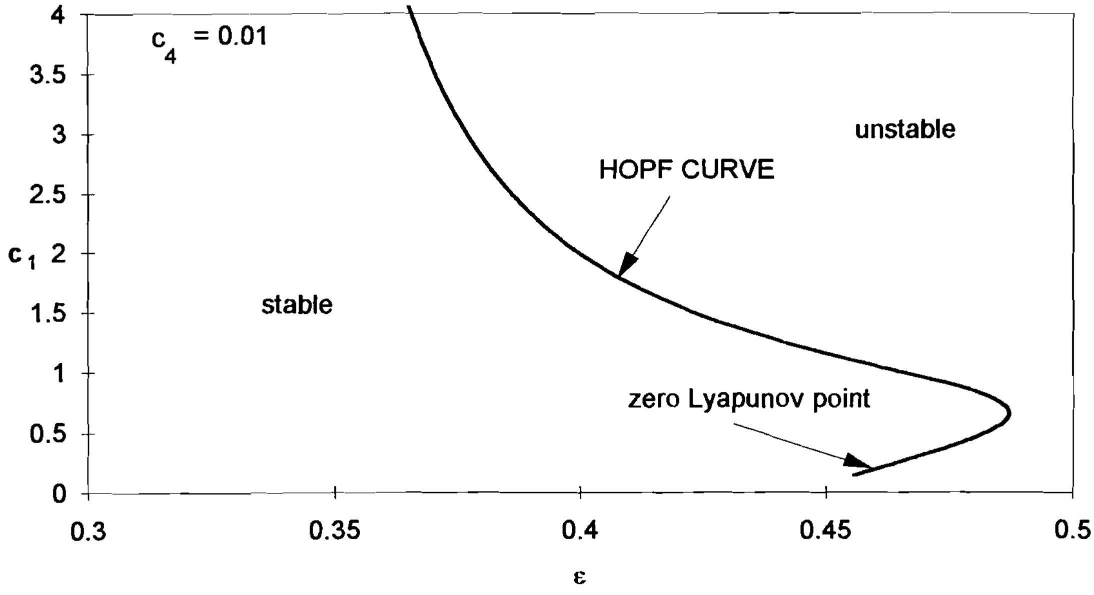


Figure 5.d. Hopf curve for the same parameter setting as in Figure 5.c except $c_4 = 0.01$, and choosing c_1 and ε to be the bifurcation parameters.

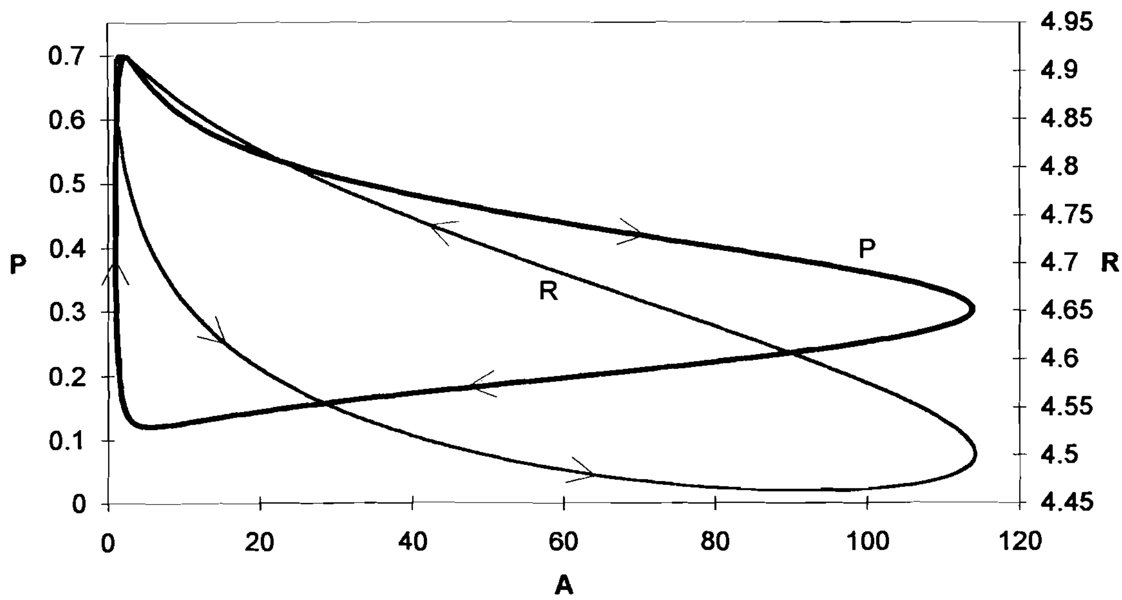


Figure 5.e. Stable limit cycle for the parameter values given in Figure 5.d and fixing ε at 0.46 and c_1 at 0.15.

Figure 5.e illustrates the stable limit cycle for $c_1 = 0.15$ and $\varepsilon = 0.46$. Essentially, the behavior along the limit cycle can be divided into three regions. In the first region technology *A* increases, whereas population *P* and resources *R* decrease. Once the population is too sparse, technology declines (second region). While still being accompanied by decreasing population levels, the resources recover again. For low values of technology, not only resources but also population increases, while technology almost remains constant.

Up to now we have concentrated our investigations on the effects of changing production elasticities together with varying demographic parameters. Next we will consider changes in the regenerative capacity of the resources on the system dynamics.

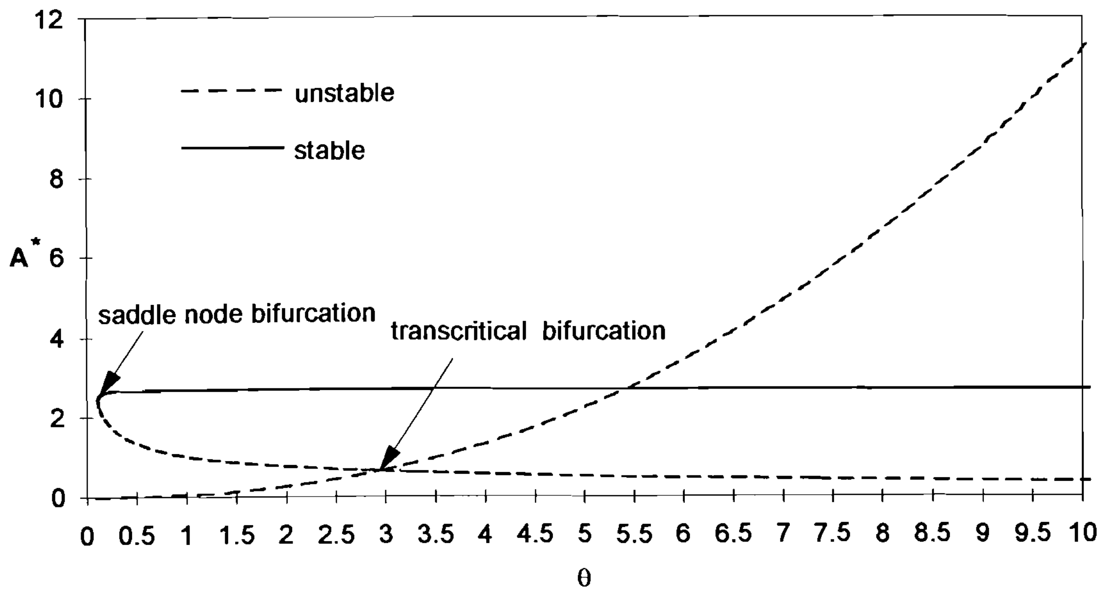


Figure 6.a. Equilibrium curves for the stationary value of technology A^* given the parameter setting in Table 3 and choosing θ to be the bifurcation parameter.

In Figure 6 the equilibrium curves are plotted for the stationary values of technology A^* if the *speed at which resources regenerate* θ is varied. In this case three branches of equilibria (1 stable and 2 unstable) coexist, which emerge through a saddle node bifurcation and a transcritical bifurcation, respectively. If θ is too small, no stable equilibrium exists at all. Different to the equilibrium curves up to now, the level of the stable equilibrium is almost unaffected by changes in θ. Once more this suggests regarding resources as the slowest-moving variable if the system is near a stable equilibrium. But as Figure 6.a illustrates, this no longer holds in the case of unstable equilibria, where the levels of the variables change. Similar to Figure 5.b we have examined the basin of attractions (sustainability domain) of the stable equilibrium. In particular the basin boundaries, i.e. the border lines between sustainable and not sustainable regions of initial conditions, are shifted outward by increasing the parameter θ.

Regarding the branch of equilibria separated by the saddle node bifurcation, there is a nice interpretation if we consider the change in the resource stock and the population as well (Figure 6.b). In particular the stable branch is characterized by low values of the population and high values of technology and the resource stock, i.e. characteristics inherent to more developed countries according to Rosenzweig (1990). The opposite holds in the case of the unstable equilibrium branch.

Finally we present the changes in the system dynamics, if one of the strategic parameters is altered.

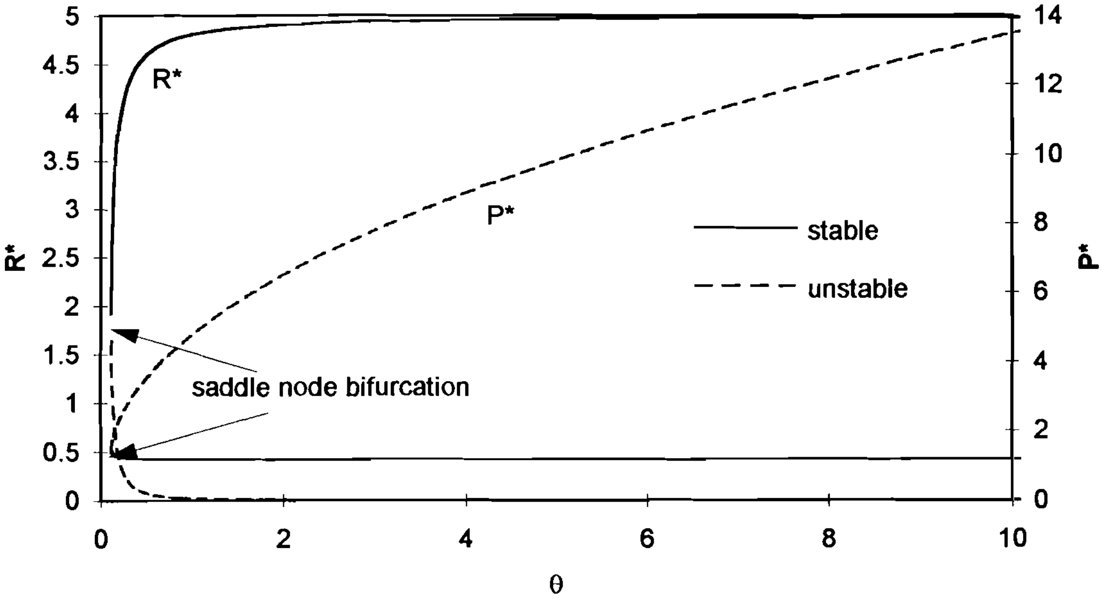


Figure 6.b. Equilibrium curves for the stationary value of resources $R*$ and population $P*$ given the parameter setting in Table 3 and choosing θ to be the bifurcation parameter.

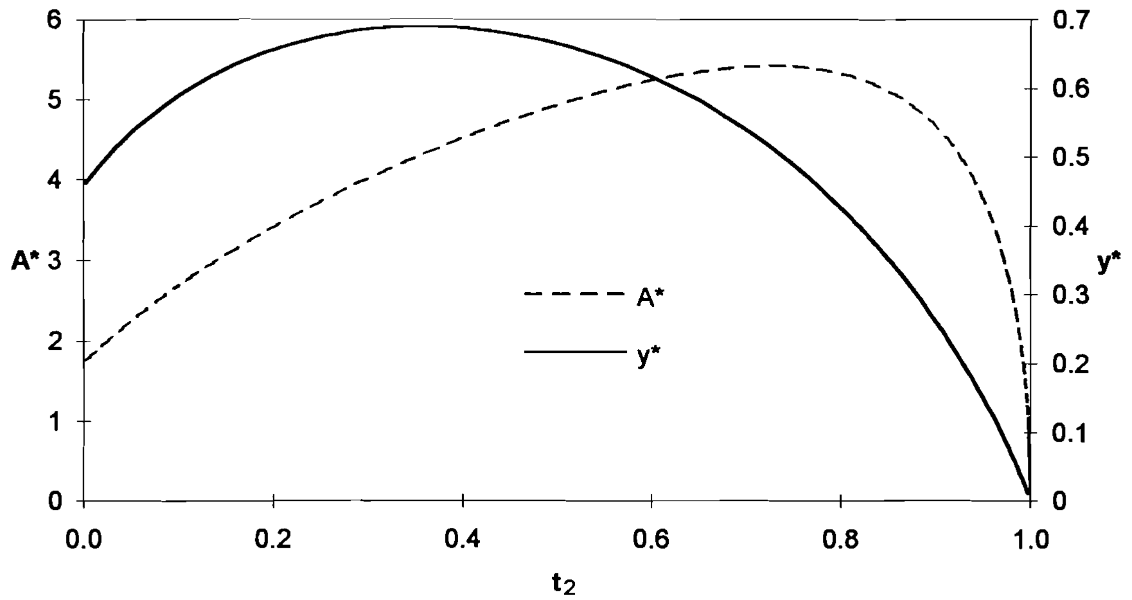


Figure 7. Equilibrium curves for selected system variables given the parameter setting in Table 3 and choosing t_2 as the bifurcation parameter.

Figure 7 illustrates the effect of varying *the tax rate* t_2 on the stable equilibrium of the system variables. The rate at which people's income is taxed determines the labor supply, but also the money available to finance the inputs into the tertiary sector. Therefore the higher the tax rate t_2, the more the labor force L_3* and the following technical progress $A*$ will increase. But at the same time, the higher tax rate on wages decreases the labor participation rate such that the labor force in the primary and secondary sector decline. Consequently also the harvest and industrial production will decline. Hence, too high values of the tax rate t_2 may even be harmful, since the supply of inputs into the tertiary sector decreases such that the level of technology might even decline for high values of the tax rate as indicated in Figure 7.

In determining which tax rate t_2 is optimal, we again might use per capita income as a surrogate for the economy's welfare. A tax rate between 30 and 40 percent of the wage would be optimal in terms of maximizing per capita income (given the parameters of the model as stated above). Note that maximizing technology would imply much higher tax rates. If, instead, we vary the tax rate t_1, per capita income y will increase over the whole range of possible tax rates $t_1 \in [0,1]$.

6. CONCLUSIONS

Based on the literature on endogenous growth we have built up a three-sector, demoeconomic model. The primary sector harvests a renewable resource which constitutes an input into industrial production, the secondary sector of our economy. Both sectors are always affected by the stock of knowledge (a public good) which is produced by the tertiary sector and financed through taxes levied on the wage and on the harvest. The homogeneous labor force is endogenously divided between the three sectors under the rules of competitive labor markets. In addition to the endogenous growth literature, not only technology but also population and resources are assumed to endogenously evolve over time.

These two extensions mainly influence our results, which differ considerably compared to the predictions of the endogenous growth literature. In particular indefinite growth of population and the economy, i.e. constant growth rates, is not sustainable under the assumption of endogenous depletable resources, which need time to regenerate.

Though stable equilibria are characterized by zero growth rates in our model, the economy can change to higher stationary levels of e.g. technology or per capita outcome by changing specific parameter values, i.e. the stable equilibria might be of temporary nature only. For example, it has been illustrated that per capita income rises with increasing production elasticity β_1 or that there exists an optimal tax rate on wages such as to maximize per capita income. Changing two parameters at a time even renders it possible to investigate the tradeoff between different demographic-economic, environmental-economic, etc., policies.

But not only the level of the stationary solutions will change if the technological, demographic or strategic outset of the economy is altered; also the stability of the equilibria might be altered. In particular, the coexistence of stable and unstable equilibria in our model means that history and luck are critical determinants of a country's growth experience. That means that depending on the initial outset, the system might end in a natural catastrophe (resources are totally depleted) or in a sustainable region of population, technology and resource utilization. Basin boundaries, i.e. the border between sustainable and not sustainable regions, might not only depend on the initial conditions but also on the parameters chosen. In addition to stable and unstable equilibria, the equilibrium solution may be of a higher dimension than zero, e.g. a limit cycle (that is an equilibrium of dimension one) rather than a point in state space (that is an equilibrium of dimension zero). Most important is that all these system dynamics, ranging from stable to unstable equilibria, sustainable and catastrophic scenarios, etc., emerge endogenously from the model without resorting to irregularities or stochastic elements in the system itself. The dynamics are simply the outcome of the nonlinear interactions of the demographic, economic and environmental modules, together with the lags implicitly involved.

One might ask why the system does not even exhibit more complex dynamics. The following hypothesis should give some hints for further work on this question. We found in our numerical studies that during the first timesteps, the resource moves very fast and the other two system variables A and P seem to be nearly constant, i.e. for the dynamic of the resource A and P are parameters rather than co-moving system variables. During the period of fast movement, the resource comes to an equilibrium $R^*(A,P)$ which depends on the 'parameters' A and P. This forms a two-dimensional manifold in the complete phase space of the system. After the resource has settled down on this equilibrium manifold we are left with slow dynamics of A and P on the manifold. And since it is two-dimensional, nothing more complex than a limit cycle can emerge.

The model we propose is fairly general and may, in principle, cover as limiting case a society of hunters and gatherers (see Prskawetz et al. 1994) that lacks collective action.[22] In the other extreme, it may characterize highly-civilized and organized societies observed in industrialized countries. However, and despite this attempt of generality, one has to take a humble view of such a modelling exercise. That is, it remains a 'toy model' in the words of Romer (1993), and it is necessarily so, because the growth of human knowledge by definition escapes the description by simple formulas.

Given the results, what conclusions can be drawn?

First of all, if we continue to ignore natural ecosystems, we may drive the entire system down while we think we are building it up. That is, propositions that high rates of population growth stimulate economic development through inducing technological change often ignore the danger of environmental depletion implicit in unchecked economic growth. In understanding the population-development paths we cannot rely on fixed causal relationships, since the system dynamics may drastically change if only one of the system parameters or the initial conditions are slightly altered. This is of particular importance because the environment may be the slowest moving variable, which degradation escapes casual observation here. Another important question concerns optimal tax rates, which again have to be set accordingly to the economic, demographic and environmental conditions under consideration. As to the role of education in the population-resource question, we know it plays an ambiguous role.

[22] By setting the tax rates t_1,t_2 equal to zero such that the accumulation function of technological progress E is 0, and eliminating the second differential equation, one directly gets the two-dimensional system studied in Prskawetz et al. (1994), which gives the limiting case of hunters and gatherers.

On the one hand, literacy and higher levels of education are needed to enable individuals and societies to better understand and manage the technological and administrative processes of the modern global economy and the process of rationalizing environmental transformation. On the other hand, higher levels of education also encourage higher levels of resource consumption among the higher income groups. In fact, education and the resulting (excess) reduction of population growth may be self-defeating, at least for particular societies.

Finally it should be mentioned, though the model is purely descriptive up to now, that not all time paths are equally desirable. In particular, high per capita income rates, high education levels, high resource stocks, etc., are goals of socio-economic and ecological development paths. The model could therefore be enhanced by including a social welfare function such that the character of the model changes from descriptive to normative. Further extensions of our model would include incorporating the dependence of the growth rate of natural resources on technology or adding the stock of knowledge A to determine the labor force participation rate covering the tradeoff between work and leisure time depending on A. Another extension would be to model the subsistence level y^{sub} as a function of the past history or per-capita income, thus making y^{sub} higher in rich countries than in poor countries.

Last but not least, it should be said that we do not mean to suggest that the actual development paths for any country will exactly exhibit the stylized patterns shown. But computer simulations can help to understand part of the interdependence between ecological, demographic and economic systems. To summarize, we conclude similar to Zhang (1990) with a phrase from Kac (1969, p. 699):

> Models are, for the most part, caricatures of reality, but if they are good, then like good caricatures they portray, though perhaps in a distorted manner, some of the features of the real world. The main role of models is not so much to explain and to predict--though ultimately these are the main functions of science--as to popularize thinking and to pose sharp questions.

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APPENDIX

The system we start from consists of three nonlinear *differential equations* (1)-(3)

$\dot{R}=dR/dt=g(R)-H(AL_1,R)$ (1)

$\dot{A}=dA/dt=E(Z,L_3,P,A)-\delta A$ (2)

$\dot{P}=dP/dt=n(y,A,P)P$ (3)

and six *implicit equations* (9)-(14).

$p=\tilde{p}+t_1=MC(1+t_1)=(w/H_{L_1})(1+t_1)$ (9)

$Y_{L_2}=(Y/L_2)\beta_2=w$ (10)

$Y_H=(Y/H)\beta_1=p$ (11)

$(1-c)s(T_1H+T_2L)=\pi Z$ (12)

$(1-c)(1-s)(T_1H+T_2L)=wL_3$ (13)

$L=l(w(1-t_2))P=[w(1-t_2)/(w_0+w(1-t_2))]P$ (14)

From (9) we can directly express L_2 as a function of L_1:

$$L_2=L_1\frac{\beta_2(1+t_1)}{\beta_1\alpha_2} \qquad \text{(A.1)}$$

Furthermore we can use (10) and (11) to express the wage w and the price p as functions of the variables R,A,P and L_1.

To express Z and L_3 as functions of the same variables (R,A,P and L_1) only, we can use constraints (12) and (14):

Constraint (14) yields:

$$L_3=L-L_1-L_2=\frac{w(1-t_2)}{w(1-t_2)+w_0}P-L_1(1+\frac{\beta_2(1+t_1)}{\beta_1\alpha_2})$$

If we plug L into equation (12) we explicitly get Z as a function of R,A,P and L_1.

We are left with a system of three differential equations in 4 variables R,A,P and L_1.

To solve this system we use the remaining constraint (13) to get one implicit condition in the variables R,A,P and L_1 only.

$$h_1(R,A,P,L_1)=(1-c)(1-s)(T_1H+T_2L)-wL_3=(1-c)(1-s)(T_1H+T_2L)-\beta_2(Y/L_2)L_3$$

This exactly yields the system of *algebraic-differential* equations (three differential equations and one implicit equation):

$$\dot{R}=f_1(R,A,P,L_1) \tag{15}$$

$$\dot{A}=f_2(R,A,P,L_1) \tag{16}$$

$$\dot{P}=f_3(R,A,P,L_1) \tag{17}$$

$$0=h_1(R,A,P,L_1) \tag{18}$$

given the functions stated in Section 3.

Let us combine the variables R,A,P into one common vector denoted by x and L_1 respectively into a vector denoted by y. Then it can be easily verified that our system is of the special structure:

$$\dot{x}=f(x,y)$$

$$0=h(x,y)$$

where the components of f and h are just given by f_1, f_2, f_3 and h_1 respectively. This special form of *algebraic-differential equations* is termed to be *of index one* in the literature.