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FOREWORD

This paper gives an overview of a project on stochastic control problems which is led by Academician V.S. Pugachev at the Academy of Sciences of the U.S.S.R. It reflects a presentation given by the author at IIASA in April 1989.

Alexander B. Kurzhanski
Chairman
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Stochastic Systems Research Program in the USSR Academy of Sciences

V.S. Pugachev

One of the most important scientific programs in the USSR is the State Program of wide-spread applications of mathematical modelling methods in all the branches of the national economy.

Any application of mathematical methods imply the design of a suitable mathematical model for the system or process under study, the necessary theoretical evaluations, sometimes very complex and sophisticated, based on this model, the problem algorithmization and producing the respective software. It demands the knowledge of the corresponding parts of mathematics, scientific skill and experience. But almost all the specialists working in various fields of human activity have neither the necessary mathematical knowledge nor the training in programming. And even in the future when the computerization of education provides all the fields of human activity with the specialists familiar with computers, these specialists in general will not have the necessary mathematical background. What then must these specialists do? Postpone all their daily work and study mathematics? I say no! People must not go to mathematics but mathematics must come to people! The mathematicians and scientists in the field of computer science must create such mathematical tools and such computer implementation and software for personal computers which give the possibility to solve various mathematical problems arising in any branch of science or national economy without any special knowledge of mathematics and computer science. The user should use a personal computer as he uses a radio-set or a TV-set.

Aiming at this goal, the Stochastic Systems Research Program was started in 1984 in the Institute for Informatic Problems of the USSR Academy of Sciences under my leadership in collaboration with Professor I.N. Sinitsyn. The choice of a stochastic system for study was motivated primarily by the fact that it serves as a suitable mathematical model for a lot of real-life phenomena. This Program includes the development of new mathematical methods in the stochastic systems theory, especially efficient approximate methods for studying the processes in stochastic systems, elaborating the respective algorithms and creating the corresponding software with some elements of artificial intelligence providing the user who has no special knowledge in mathematics and computer science, with tools for stochastic systems study using a personal computer in the interactive regime.

The first stage of the Program covers the problems of the statistical analysis of stochastic systems described by differential, integro-differential, difference, mixed differential and difference equations. The resulting methods, algorithms and software are designed for studying such systems whose mathematical model in the form of some of the above mentioned equations is completely known. This part of the Program will be finished in the early nineties.

The second stage of the Program covers the problems of real-time estimation of the state and unknown parameters and extrapolation of the state of a partially observed stochastic system of one of the above classes. The resulting methods, algorithms and software are designed for automatic construction of filters and extrapolators realizing the estimation processes in stochastic systems in the real time scale. This part of the Program wihich is supposed to be finished in the nineties, yields the tools for solving that part of the general identification problem which concerns the estimation of unknown parameters of a stochastic system described by known equations.

The third and the last stage of the Program covers the problems of automatic design of mathematical models from experimental and past statistical data. At this stage the theory of model fitting will be reviewed and developed including mathematical tools for the choice of an adequate model consistent with the available data and their uncertainty, estimation of its parameters, testing hypotheses and validation of the model chosen. Then the intelligent software and appropriate computer implementation will be elaborated to provide users with the tools for fully automatic design of mathematical models with possible user's guidance (depending on his desire and competence). Once the model is built, it may be used for studying the system functioning, to predict its behaviour under various managing decisions and thus supporting the decision making processes. This part of the Program, which is intended to be finished in the first decade of the coming century, will solve the most important and most difficult identification problem, i.e. the establishing of an adequate set of equations describing the system or process to be studied.

At the first stage of the Program the following work has been completed:

1. The efficient approximate methods of stochastic differential systems theory developed for studying linear and non-linear systems described by Ito stochastic differential equations with any process with independent increments not necessarily a Wiener process (V.S. Pugachev, I.N. Sinitsyn. Stochastic Differential Systems. Analysis and Filtering. John Wiley & Sons, 1987) - this book is cited in the sequel as SDS - were extended to some class of systems described by stochastic integro-differential equations (I.N. Sinitsyn, 1984), to sampled data systems described by stochastic difference equations (V.S. Pugachev, 1984), and systems described by mixed sets of differential and difference stochastic equations (V.S. Pugachev, I.N. Sinitsyn, V.I. Shin, 1986).

2. For all these systems and methods, including stochastic differential systems (SDS) for which these methods were developed earlier, the complex of Fortran programs "Moment" was produced. This program complex is designed for studying multidimensional stochastic systems with polynomial and some nonpolynomial nonlinearities. The programs of the complex "Moment" automatically derive the equations for statistical characteristics of the state vector of a system from the initial stochastic equations and solve them. As we know the program complex "Moment" is the first

program product of such kind in the world.

3. The conditionally optimal filtering and extrapolation theory was extended to systems described by mixed sets of stochastic differential and difference equations, in particular, to problems of filtering and extrapolating the processes in SDS using discrete observations (V.S. Pugachev, I.N. Sinitsyn, V.I. Shin, 1986). This work is preparatory for the second stage of the Program.

4. The interactive package "StS-Analysis", Version 1.1, was produced for analyzing stochastic differential systems by users having neither knowledge of the stochastic systems theory nor any experience in programming. The first version of this package is based only on the normal approximation of the distribution of the instantaneous value of the state vector of a system. The subsequent versions which are in production will use more accurate methods for finding the distribution of the state vector.

Working with the package "StS-Analysis" the user only has to write on the display the relevant stochastic differential equations in the routine mathematical form with the aid of the keyboard, to input the initial data, to order the package which quantities must be calculated and in what form the results as time functions are to be presented on the display, either by graphics or by tables, and then to start the computations. When the computations with current visual representation of the results are finished the user may get the hard copies of the results of computations from the file.

The package "StS-Analysis" supports the following functions:

- receiving in the natural mathematical form, translating and storing the stochastic differential equations governing the system behavior,
- automatic derivation of the ordinary differential equations for the distribution parameters of the state vector of the system from the initial stochastic differential equations,

- solves these equations,
- gives the graphical visualization of the results of calculations both during the calculations and after their completion,
- outputs the results of calculations into the file for obtaining hard copies,
- performs the administration of equations introduced under the user's guidance (editing, copying, removal, revising, completing, etc.)

It is necessary to emphasize that the "StS-Analysis" package does not perform the *mathematical modelling* of a stochastic system. All the processes in stochastic systems are random by nature and each simulation of these processes gives nothing more than one realization of these processes which bears no information at all. The estimation of statistical characteristics of these processes demands many simulations, and the more is the dimension of a system the more simulations must be performed to obtain good estimates. The "StS-Analysis" package performs the *theoretical evaluation* of statistical characteristics of the processes in a system, and for calculating these characteristics a computer must be run only once. That is the main feature of the package "StS-Analysis" which is the first such package in the world.

Now I am going to present some mathematical details which will illustrate the "StS-Analysis" package.

Let

$$dZ = a(Z,t)dt + b(Z,t)dW\,. \qquad (1)$$

be the Itô stochastic differential equation representing the mathematical model of a system to be studied, where Z is the state vector of the system, $a(z,t)$ a vector-valued function, $b(z,t)$ a matrix-valued function, $W(t)$ any vector process with independent increments. The expectation $m = EZ$ and the covariance matrix $K = E(Z-m)(Z^T - m^T)$ of the state vector Z are determined by (SDS)

$$\dot{m} = Ea(Z,t)\,, \qquad (2)$$

$$\dot{K} = E\{a(Z,t)(Z^T - m^T) + (Z - m)a(Z,t)^T + b(Z,t)\nu(t)b(Z,t)^T\} \qquad (3)$$

where in addition to previous notations $\nu(t)$ is the intensity of the process $W(t)$, i.e., the time derivative of its instantaneous covariance matrix (certainly this derivative is supposed to exist). Equations (2), (3) do not represent a closed set of equations for m and K as the expectations in the right-hand sides depend on the unknown one-dimensional distribution of the process $Z(t)$. If this distribution is approximated by a normal one (2), (3) become a closed set of approximate ordinary differential equations determining the expectations, variances and covariances of all the components of the state vector of the system. This approximation underlies the Version 1.1 of the "StS-Analysis" package. Another essential assumption in Version 1.1 is that the components of the vector-function $a(z,t)$ in (1) are polynomials in the components of vector z of not higher than the third degree, and the elements of the matrix-valued function $b(z,t)$ are linear in the components of vector z. Both these assumptions will be removed in the following versions of the "StS-Analysis" package.

To obtain better approximation to the true distribution of the state vector Z the approximation of the probability density $f(z;t)$ of the vector Z by a segment of some of its orthogonal expension may be used,

$$f(z;t) \approx w(z)\{1 + \sum_{|\nu|=1}^{N} c_\nu p_\nu(z)\} \qquad (4)$$

where $w(z)$ is some "standard" density, $\{p_\nu(z)\}$ a system of polynomials in z which may be either orthonormal with the weight $w(z)$ or biorthonormal with another system of polynomials $\{q_\nu(z)\}$, ν being the vector subscript of the same dimension as the vector z and $|\nu|$ represents the sum of all the components of the vector ν. In the special case where $w(z)$ has the same expectation m and covariance matrix K as the state vector Z of the system then $c_\nu = 0$ for all ν for which $|\nu| = 1$ or 2 (SDS). The evaluation of the expectations in (2), (3) using the approximation (4) of the distribution of the vector Z, makes the right-hand sides of (2), (3) functions of m,K and $c_\nu(|\nu| = 3,\cdots,N)$. To get a closed set of equations it is necessary then to derive ordinary differential equations for the coefficients

$c_\nu(|\nu| = 3,\ldots,N)$ or for the moments of the state vector Z of orders up to Nth in terms of which the coefficients c_ν may be represented (SDS). The approximation (4) of the distribution of the state vector Z of the system is used in the subsequent versions of the "StS-Analysis" package. As a "standard" density $w(z)$ the normal density is generally used with the same m,K as the state vector Z. In this case $p_\nu(z)$, $q_\nu(z)$ are known Hermite polynomials.

To study system (1) in a routine way it is necessary first of all to derive specific equations (2), (3) and perhaps the respective equations for the coefficients $c_\nu(|\nu| = 3,\ldots,N)$ as well. This is a very difficult and time consuming job, especially for multi-dimensional systems. The larger the dimension p of the state vector Z the more equations for the distribution parameters are needed. So for instance equations (2), (3) contain $p(p+3)/2$ scalar equations if (1) consists of p scalar equations. Much more equations are necessary if the approximation (4) of the distribution is used. That is the main reason why the problem of automatic derivation by a computer of equations for m,K and other distribution parameters arises. Without solving this problem, a wide-spread application of stochastic systems theory is quite impossible. And therefore all our software for stochastic systems study contain the automatic derivation of equations for distribution parameters as a necessary part. In particular, the "StS-Analysis" package, Version 1.1 automatically derives $p(p+3)/2$ scalar equations (2), (3) from the original p scalar equations (1), written on the display in a usual mathematical form, and solves them.

In conclusion, I would like to recall the names of the main contributors to the interactive package "StS-Analysis" and the preceding experimental package "Signal". In addition to the leaders V.S. Pugachev and I.N. Sinitsyn, these are A.A. Cherednichenko, V.I. Shin, I.M. Zatsman, E.V. Andreeva, A.N. Karpenko, A.P. Khatuntsev, T.D. Konashenkova, E.Yu. Maisheva, I.V. Novikova.