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Foreword

For tracing the growth of populations over past time a useful indicator is cohort size. While a cohort moves through time, and therefore cannot be counted in the same way as the population of any given moment, yet its size can be measured as births less deaths up to some intermediate age. This may be estimated from a series of censuses, without reference to vital statistics or other data. The technique is applied to the onset of the world wide population expansion that followed World War II. In several Asian countries it took place in a single five-year period with a multiplication of earlier intercohort increases by as much as threefold. The jump occurred early in Burma, late in Indonesia, and suddenly in both of those countries; in India it was more gradual, so that the onset of the current population expansion is less sharply marked.

Calculation also shows a corresponding discontinuity in the rate of population change after World War I in a number of countries, but of lesser magnitude. Insofar as one may speak of a population explosion occurring in the world today the method of intercohort increase identifies its date of onset as immediately after World War II.

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Nathan Keyfitz
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THE PROFILE OF INTERCOHORT INCREASE

Nathan Keyfitz

1. Intercohort Increase Underlying Population Growth

The human population has increased during this century as a relatively smooth curve, concave upwards until very nearly the present day. But that smoothness results from the superposition of successive cohorts. For the world as a whole and for many individual countries cohorts changed by relatively small amounts until World War II, then within a single five year period their annual increase multiplied several fold. The result is a profile of intercohort increases that has the characteristic feature of a big step upwards around the middle of the century. Some of the sudden jump was due to a rise in births, most was the typical outcome of a fall in deaths, especially infant deaths. From the viewpoint of tracking population there is no need to distinguish between rising births and falling deaths; what counts is the increased number of living people. The paper develops a technique for identifying the cohorts that initiated what in the 1950s was designated the population explosion. A numerical example is given for Indonesia, but essentially the same picture appears for many other countries, and for the aggregate of the world. The intercohort increase can be estimated for each age using two successive censuses; the method here developed gives very nearly the same increase whatever age is used for the estimate.

This phenomenon—a smooth acceleration of total population underlying which is a sharp change in the intercohort increase—was noted early for Indonesia (Keyfitz, 1965); it was observed that the sudden increase of the intercohort growth rate about 1950 was by the 1960s about to result in large youth cohorts, just as a baby boom does. The approach accords with the perspective of Ryder (1964 and elsewhere) and Easterlin (1961) in which the cohort is the real population collectivity, the age distribution at a given moment of time is an abstraction. Study of the increase of given ages as shown by successive censuses has been pursued by Preston and Coale (1982). The political changes associated with large youth cohorts are discussed by Wriggins (1989).

Given censuses at five year intervals from 1950 onward, what can we infer about the joint effect of past fertility and mortality? In fact we do not have censuses at five year intervals for the postwar period for more than a very few countries, but there are some censuses nearly everywhere, and the United Nations has used these to establish a quasi-official set of estimates at five-year intervals, 1950, 1955, etc., that will here be treated as though they are actual counts. Errors in these estimates will appear as noise in the calculations to be made below, of the same character as irregular accurately recorded fluctuations in census age distributions, due for example to epidemics or migration. The theory will be for a population closed to migration, and the technique is such that variation arising from migration and other sources clearly reveals itself.

2. Formal Representation of the Intercohort Difference

Call $p_{a,t}$ the population aged a at last birthday at time t. This is the data, and the entity to be inferred is some combination of B_{t-a}, the births at time $t-a-1$ to $t-a$ and the survivorship $L_{a,t-a}$.

The intercohort increase is estimated as the first difference over time of the population at a particular age a:

$$\Delta_t p_{a,t} = p_{a,t+1} - p_{a,t}$$

The unit of time and age can be taken as one year, or with suitable definition of "last birthday" as five years.

The population aged a here means the people recorded in a census at time t from exact age a to exact age $a+1$. In symbols, with the continuous form of the functions indicated by parentheses,

$$p_{a,t} = \int_0^1 B(t-a-\delta)l(a+\delta)d\delta ,$$

if the life table survivorship $l(a+\delta)$ is unchanging with time through the relevant time interval. In the usual approximation the integral of the product is assumed equal to the product of the separate integrals, or

$$p_{a,t} = B_{t-a}L_a ,$$

L_a being the integral of $l(a+\delta)$ over the ages a to $a+1$, and B_{t-a} the births from time $t-a-1$ to $t-a$. With mortality changing over time as well as by age we need to indicate which life table is referred to, that will require a second subscript on L_a. The symbol

$L_{a,t-a}$ is survivorship of the cohort born at time $t-a-1$ to $t-a$.

In terms of the entities of birth and death, then, the population increase at age a from time t to time $t+1$ is

$$\Delta_t p_{a,t} = B_{t+1-a} L_{a,t+1-a} - B_{t-a} L_{a,t-a} \,, \tag{1}$$

supposing a closed population. Now adding and subtracting the quantity $B_{t-a}L_{a,t+1-a}$, we obtain

$$\Delta_t p_{a,t} = (\Delta_t B_{t-a}) L_{a,t+1-a} + B_{t-a} \Delta_t L_{a,t-a} \,, \tag{2}$$

or alternatively, by adding and subtracting $B_{t-a+1}L_{a,t-a}$,

$$\Delta_t p_{a,t} = (\Delta_t B_{t-a}) L_{a,t-a} + B_{t-a+1} \Delta_t L_{a,t-a} \,. \tag{3}$$

Of the two terms in (2) or (3), the first is the survivors among the increase over a (1-year or 5-year period) of the absolute number of births that took place $t-a$ periods earlier, and the second is the improvement of survivorship over a time period multiplied by the births.

3. The Two Terms

For purposes of examining the relative sizes of the two terms of (2) divide by $B_{t-a}L_{a,t-a}$ to obtain the quantity

$$\frac{\Delta_t p_{a,t}}{p_{a,t}} = \left[\frac{\Delta_t B_{t-a}}{B_{t-a}}\right]\left[\frac{L_{a,t+1-a}}{L_{a,t-a}}\right] + \frac{\Delta_t L_{a,t-a}}{L_{a,t-a}} \,, \tag{4}$$

or expanding the second factor of the first term as a Taylor series, and neglecting the product of first differences, we have approximately

$$\frac{\Delta_t p_{a,t}}{p_{a,t}} \sim \frac{\Delta_t B_{t-a}}{B_{t-a}} + \frac{\Delta_t L_{a,t-a}}{L_{a,t-a}} \,. \tag{5}$$

The first term of (5) is the relative change of births from one interval to the next and the second the relative change of the probability of surviving. Thus the relative increase of population is equal to the relative increase of births plus the relative increase of survivorship. One can imagine circumstances where the first term is the larger, and other circumstances where the second is larger. In a Less Developed Country (LDC) with high and constant births and rapidly improving mortality the second term will dominate, as corresponds to early phases of the demographic transition.

Survivorship rises in nearly all the populations with which we are concerned, so the second term is positive; for More Developed Countries (MDCs) the first term can be positive or negative depending on whether births are in a rising or falling phase.

4. Constancy of Intercohort Differences

The usefulness of the method here proposed depends on the invariance among estimates of any given intercohort difference, which is to say on $\Delta_t p_{a,t}$ being approximately equal to $\Delta_t p_{a+1,t-1}$, etc., or in general $\Delta_t p_{a+u,t-u}$ being independent of u.

Indonesia is a large population for which this constancy may be examined. It took censuses in 1961, 1971 and 1980, on the basis of which the UN (1986, 1989) has calculated numbers at five-year intervals from 1950 to 2020. We cannot say that each five-year point is completely independent of the others, since all have been derived by interpolation from the same three censuses, and yet there is a measure of independence in the errors. They will be treated in this example as though they are separate counts.

Table 1 is an extract from the current estimate (UN 1989) provided by the United Nations for Indonesia along with 181 other populations.

TABLE 1. INDONESIA: EXTRACT FROM ORIGINAL UNITED NATIONS (1989) POPULATION NUMBERS IN FIVE-YEAR AGE INTERVALS FROM AGE 0 TO 29, 1950 TO 1985 (HUNDREDS OF THOUSANDS OF PERSONS)

Age	1950	1955	1960	1965	1970	1975	1980	1985
0-4	114	136	162	175	199	222	224	226
5-9	101	103	124	150	163	187	211	212
10-14	96	98	100	121	146	160	184	207
15-19	86	94	95	98	118	144	157	181
20-24	74	82	90	92	95	115	140	154
25-29	59	70	79	86	89	92	112	136

TABLE 2. INDONESIA: INTERCOHORT DIFFERENCES (EXCESS OF EACH COHORT OVER PREVIOUS COHORT) AS ESTIMATED FROM DATA SUCH AS THAT OF TABLE 1, AT DIFFERENT TIMES AND AGES (HUNDREDS OF THOUSANDS OF PERSONS)

	Excess of cohort born in 5 years starting with the year									
Age	1925	1930	1935	1940	1945	1950	1955	1960	1965	1970
0						22	26	13	24	23
5					2	21	26	13	24	24
10				2	2	21	25	13	24	22
15			8	2	2	21	25	13	24	22
20		9	8	2	3	20	25	14	24	22
25	11	9	8	2	3	20	25	14	24	22
30	10	9	8	3	3	20	24	14	24	23
35	10	9	8	3	4	20	24	14	24	23
40	10	8	8	3	4	20	24	14	24	23
45	10	8	8	4	4	19	24	14	24	23
50	9	9	8	4	4	19	23	14	23	22
55	9	8	8	4	4	18	22	14	23	
60	9	8	8	4	4	17	21	13		
65	8	7	7	4	4	16	20			
70	7	7	6	4	4	14				

From Table 1, in units of 100,000, the survivors to 1950 of the births of 1945-50 are 114, and the survivors to 1955 of the 1950-55 births are 136, a difference of 22. This last, the increase at age 0 to 4 from 1950 to 1955, is the figure shown at the top of the column under 1950 in Table 2, that gives the difference over time of given age groups according to (1) and subsequent formulas. Similarly, the number at the top of the column headed 1970, 23 expressed in hundred thousands, is the difference between the children 0-4 in 1970 (199) and the number 0-4 in 1975 (222).

One can well be surprised at the near constancy in each column of the inferred intercohort differences of Table 2. One would not expect absolute constancy, since mortality enters, there is some migration and the several estimates of the cohorts are based on censuses that suffer differently from errors of enumeration and tabulation. Some of the columns of Table 2 gently increase, some gently decrease but the changes within columns are small compared with the differences between columns.

The first term of (2) or (3) includes a factor of survivorship that multiplied by the (for any cohort) essentially constant ΔB_{t-a} causes a decrease in the counts as one estimates the same increase from older and older cohorts, while the second is a function of improving survivorship that causes a rise in the overall value of (2) or (3) and as we see from Table 2 can offset or more than offset the change in the first term.

5. Reconstruction of the Age-Time Table

The closeness to constancy of the differences within columns of Table 2 means redundancy in Table 1, the original age-time data. Hence we should be able to average numbers in each column of Table 2, and reconstruct Table 1 without appreciable loss of information. The result of such averaging is the profile of intercohort differences over historical time until about 1970. These summarize the age-time distribution.

We hesitate to go beyond 1970 because most of the numbers subsequent to that are projections, and for them our present method merely reads out the assumptions made in calculating the projections. And though in principle the method provides information back as far as the ages of the oldest group of people living in 1950, yet one is disinclined to use the information for any group in which the number of persons alive is not large, and for which enumeration is notoriously inaccurate. Hence the method's usefulness is confined to inferences about the first half or two thirds of the 20th century.

For Indonesia the averages of the intercohort differences of Table 2 are given by Table 3. Thus the average of the column headed 1950 in Table 2 is in the row for 1950 in Table 3.

TABLE 3. AVERAGE OF THE COLUMNS OF TABLE 2 SHOWING THE AMOUNT BY WHICH EACH COHORT IS LARGER THAN THE PRECEDING (HUNDREDS OF THOUSANDS OF PERSONS)

t	Cases (1)	Total (2)	Average D_t = (2)/(1)
1895	4	1	0
1900	5	15	3
1905	6	25	4
1910	7	21	3
1915	8	11	1
1920	9	46	5
1925	10	93	9
1930	11	90	8
1935	12	92	8
1940	13	40	3
1945	14	47	3
1950	15	289	19
1955	14	334	24
1960	13	177	14
1965	12	286	24
1970	11	250	23

Fig. 1 shows the numbers of Table 3 as the middle of the three bars for each year. The left-hand bar for each year is the lowest of the values in each column of Table 2; the right-hand bar is the highest. The profile of intercohort increase is much the same for the low, average, and high values.

Table 3 and Fig. 1 (expressed in hundreds of thousands of persons) show the cohort of 1915-20 larger than that of 1910-15 by only 0.1 million, while the cohort of 1920-25 is larger than that of 1915-20 by 0.5 million, and this is followed by an increase of 0.9 million; this latter jump is much more than any preceding five-year period shown. Similarly the 1950-55 cohort is larger than the 1945-50 by 1.9 million persons, a further step over the preceding history.

6. Redundancy

From the averages of Table 3 (the middle bars of Fig. 1) we can reconstruct the original Table 1 using relatively little data. Aside from what is contained in Table 3 for the average intercohort difference, we need only the age distribution of one cohort as a starting point. If the reconstitution comes close to the original we can say that the original age-period table contains a corresponding degree of redundancy.

FIG. 1. Intercohort increase 1885-90 to 1950-55, Indonesia.

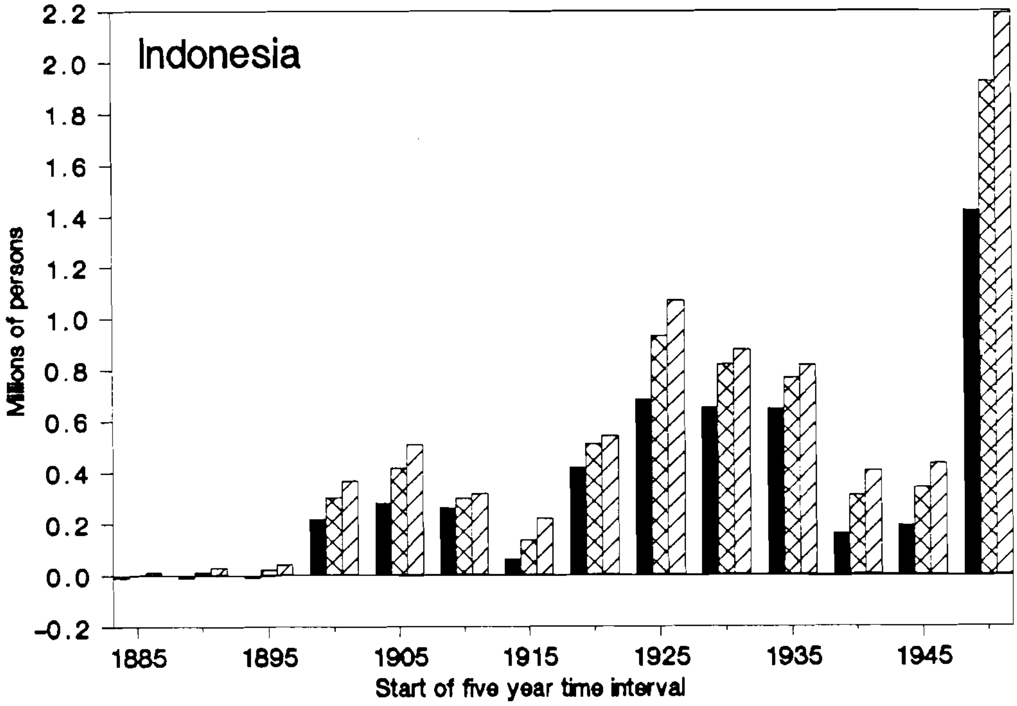
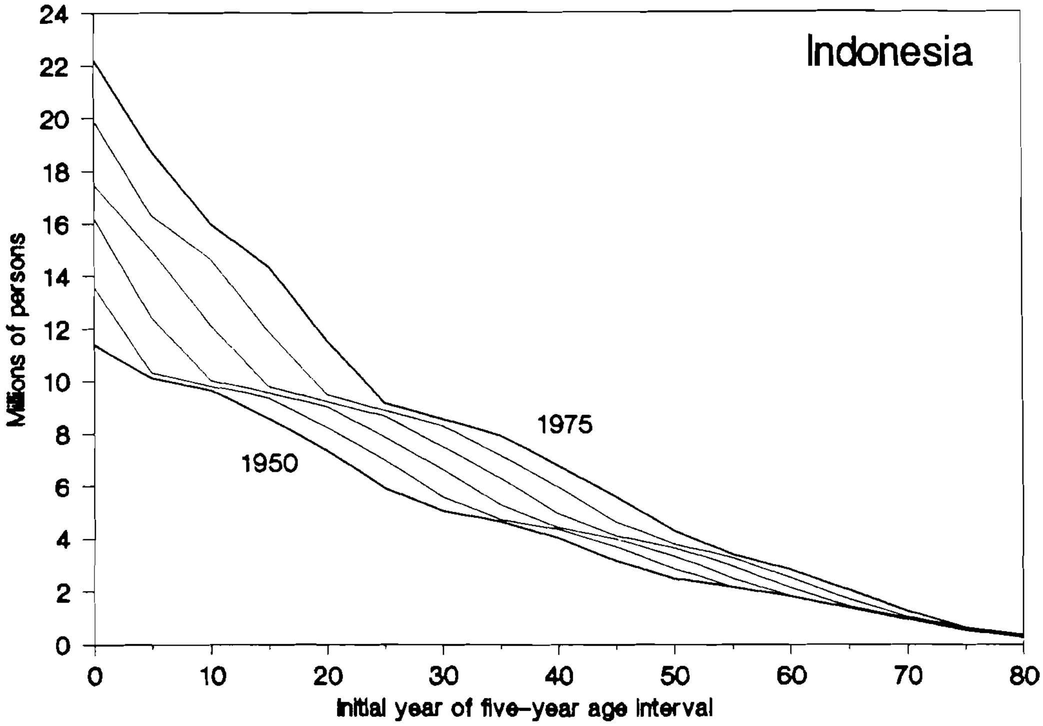


TABLE 4. RECONSTRUCTION OF TABLE 1: POPULATION IN FIVE-YEAR AGE GROUPS (HUNDREDS OF THOUSANDS OF PERSONS)

Age	1950	1955	1960	1965	1970	1975	1980	1985
0-4	114	133	157	171	194	217	220	225
5-9	100	103	122	146	160	184	206	209
10-14	94	97	100	119	143	157	181	203
15-19	84	91	94	98	117	141	154	178
20-24	72	81	88	91	95	114	138	151
25-29	60	69	78	85	88	92	111	135

The sense in which the usual age-period table is redundant is seen by comparing the extract shown as Table 1 with the reconstruction presented as Table 4. The initial cohort of Table 4 is copied from Table 1: 114, 103, 100, 98, etc. The part of Table 4 for the more recent cohorts is made by adding the intercohort differences of Table 3 to the initial cohort. Thus the 133 at the top of the 1955 column is equal to 114 plus the 19 given opposite 1950 in Table 3. The part of Table 4 for cohorts preceding that born 1945-50 is made by subtracting the intercohort differences from the 1945-50 cohort.

FIG. 2. Age distributions, 1950-1975.



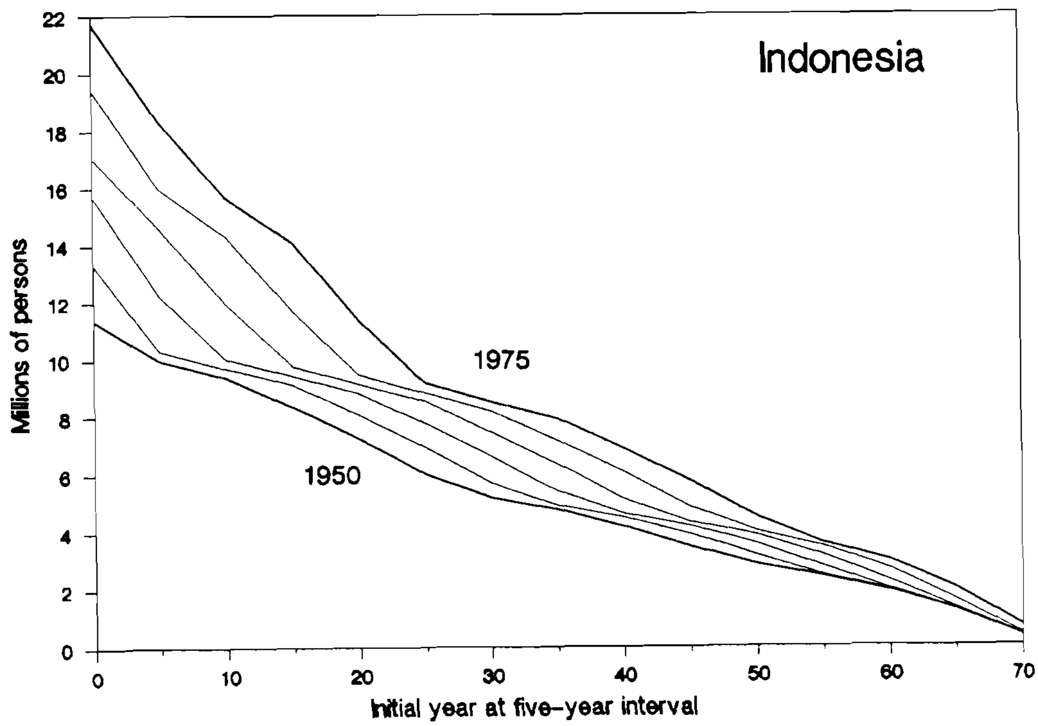
Shown as Tables 1 to 4 are windows on the full tables to save space in printing. The full Table 1 may be said to contain $t \times a$ items, where t is the number of its columns (points of time) and a the number of its rows (age groups). Table 4 has been made by the $t + a$ items of Table 3 along with the a items of the first main diagonal of Table 1. The degree of redundancy in Table 1 is thus the difference between $t \times a$ and $t + 2a$.

Compare Fig. 2, which shows the original numbers provided by the United Nations for 1950-75, and Fig. 3, the reconstruction using intercohort differences. In all essentials the two sets of curves are the same. Both show the same two bends, one after each of the World Wars, and have otherwise similar configurations. This way of compacting the age-time table is useful for comparing populations in respect of their acceleration after World War II and other features.

7. Other Populations

Fig. 4 is made up of pairs of charts corresponding to Fig. 1 and Fig. 2 for Malaysia, Burma and India. The three sets of profiles of intercohort increase are placed alongside charts of the data on which they are based; the age distributions turned left to right help

FIG. 3. Reconstruction of age-time distribution.



to understand the respective profiles. Visual comparison shows how the profiles of inter-cohort differences correspond to age distributions. Corresponding to each sharp bend on the left there is a sharp rise in the bars on the right.

Looking at the age distribution for Malaysia on the upper left of Fig. 4, we can see by the vertical distance between the bottom curves that there is a large increase between 1950 and 1955 in the 0-4 age group, as well as in the 5-9, but the 10-14 group increases little if at all. That is to say that the births less deaths of 1945-50 must have been much greater than those of 1940-45, while the births less deaths of 1940-45 must have been about the same as those of 1935-40.

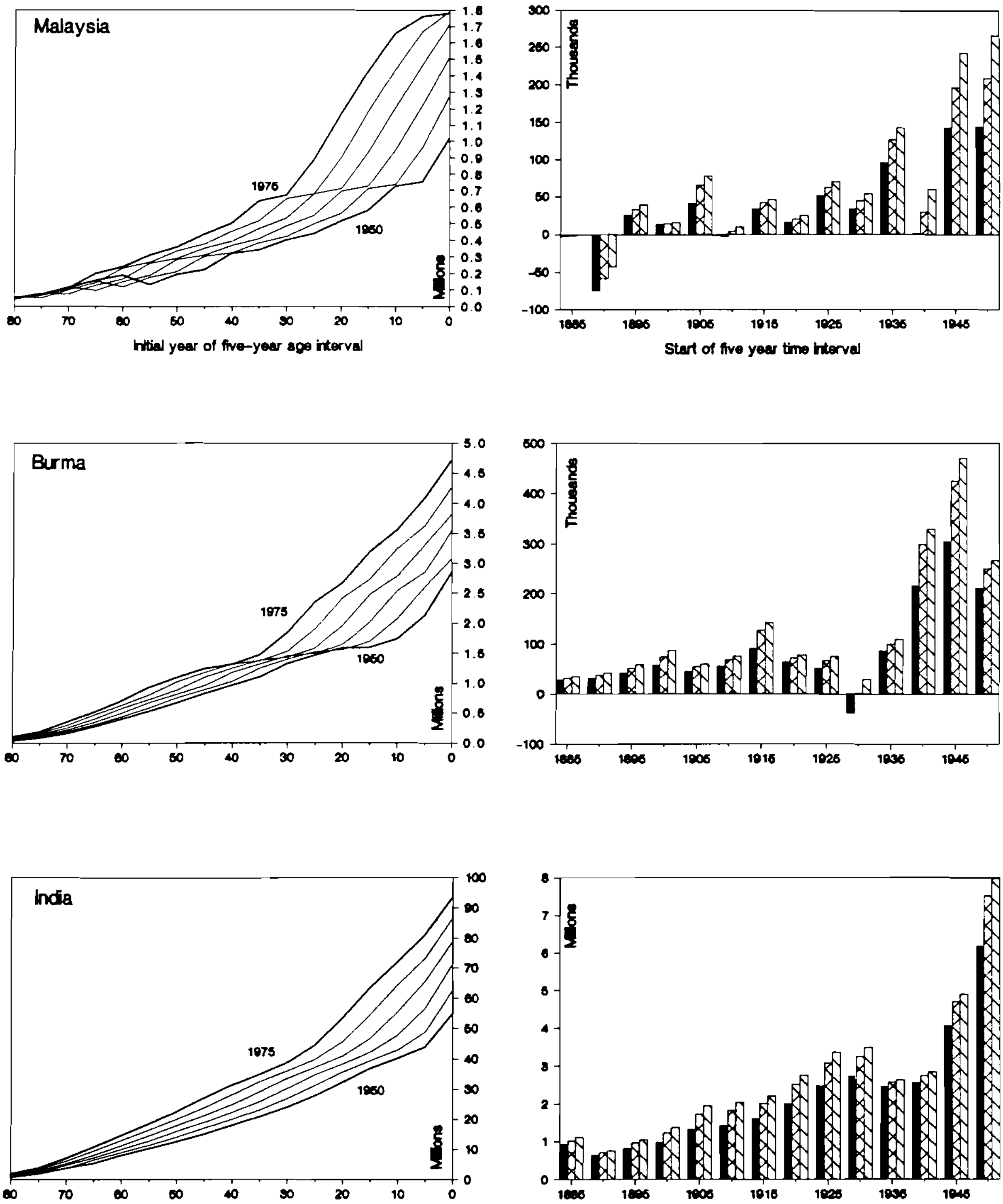
Corresponding to all this, still for Malaysia, we have on the right hand side low bars for the year 1940, which means a low increase for the cohort born 1940-45, i.e. the cohort 1940-45 was not much greater than that of 1935-40; but on the other hand there was a large increase up to the cohort born 1945-50, i.e. it was much greater than the 1940-45 cohort.

Turning to the second set, for Burma, we can see that the corresponding initiation of rapid growth took place earlier, and the bend where it started is more gentle, meaning that it accelerated somewhat more gradually but over a longer period. Thus on the left

FIGURE 4.

Age distributions, 1950–1975

Intercohort increase 1885–90 to 1950–55



the age 20-24 shows little growth from 1950 to 1955, while the 15-19 shows medium growth and and the 10-14 rapid growth. That means that the births of 1930-35 were just about the same as 1925-30, that 1935-40 increased somewhat over 1930-35, and that 1940-45 increased greatly over 1935-40. On the right hand side there is virtually zero increase for 1930-35 over the preceding five years, a small increase for 1935-40, and a large increase for 1940-45.

India's acceleration is more gradual than either of these, as one can see both on the left and the right figures. If there is a speedup in the increases anywhere it is in the same place as Malaysia's, i.e. the cohorts under 10 years in 1950 being much greater than their predecessors, reflect acceleration in 1945-50.

If we go back to Indonesia we see that in Fig. 2 the 5-9 increase little from 1950 to 1955, the 0-4 increase greatly. That means births less deaths must have risen between 1945-50 and 1950-55. And in Fig. 2 it is the 1950-55 column that shows a large increase, i.e. the 1950-55 cohort was much larger than its predecessor.

Summing this up in respect of the discontinuity of change in surviving births, we can say that the turning point for Burma was about 1940 or earlier, for Malaysia and India about 1945, for Indonesia about 1950, later than either. These facts constitute a dating in the respective countries of the onset of the demographic expansion of the mid-twentieth century.

8. A Three Dimensional Portrayal

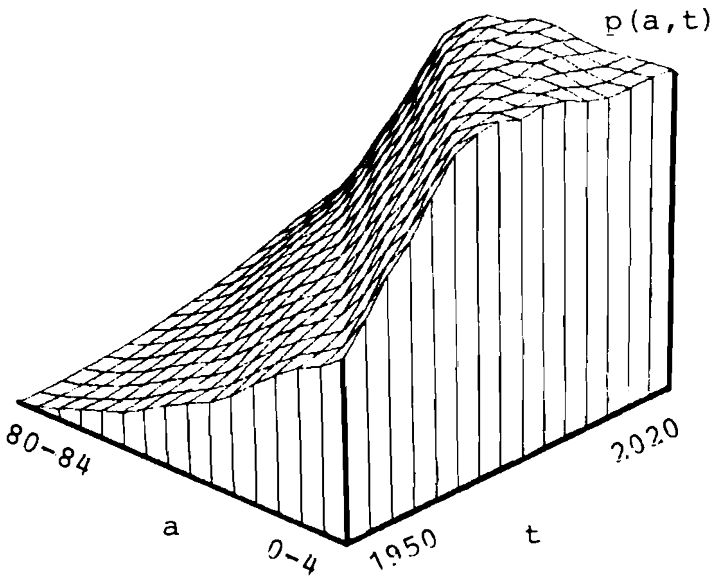
A three dimensional portrayal makes clearer what the algebra and the numbers given above tell us, and in particular how it can be that the age-time table contains so much redundancy. In Fig. 5 we think of years as the t-axis, ages as the a-axis, and the number of individuals as the $p(a,t)$ axis. This is a graphical representation in space of the familiar age-time distribution of Table 1.

The diagonal vertical section from the nearest corner, at the middle of the diagram, identified by age 0-4 and year 1950, is what has above been called the central cohort of Table 1. Other cohorts are planes parallel to this one.

On the other hand the diagonal plane at right angles to this main one, connecting the extreme left and the extreme right of the diagram, i.e. connecting age 70-74 in 1950 with 0-4 in 2020, is the profile shown in Table 3 and Fig. 1 and represented algebraically in (3). The shape of the surface is such that we can reconstruct the whole of it once we know these two diagonals. Any of the sections parallel to the main diagonal, along with any of the sections at right angles to this main diagonal, in principle allow at least a part of the top surface to be reconstructed.

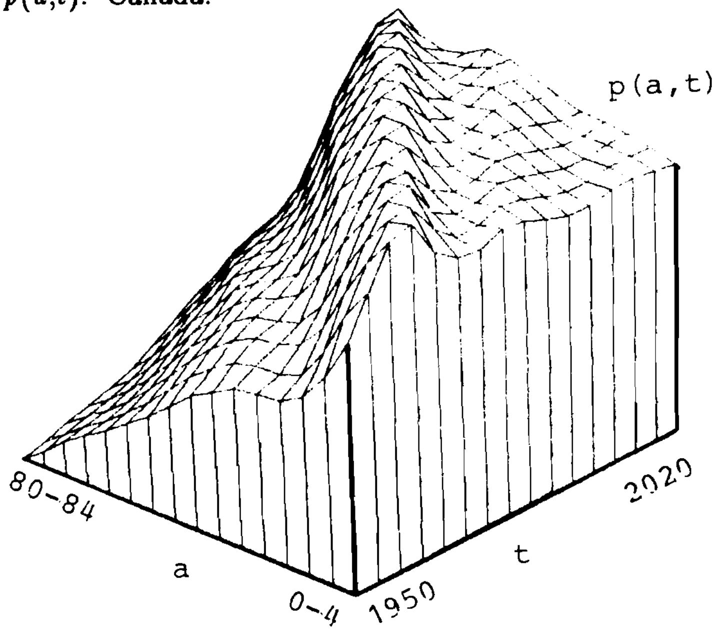
For a country in which the fluctuations of births have been more irregular, the point comes out even more strikingly. Consider the three dimensional diagram for Canada shown as Fig. 6. Once again we can construct the whole solid figure knowing only a pair of diagonals at right angles to one another, say as before (1) a vertical plane drawn through the points in the base age 0-4 in 1950 and age 70-74 in 2020, and (2) the plane at right angles to this one through the point in the base 70-74 in 1950 and 0-4 in 2020. The baby boom after each of the two world wars stands out conspicuously, as does the echo of the peak in the 1960s that is now starting to appear.

FIG. 5. Age-time diagram, showing three dimensions of age (a), time (t), and number of persons $p(a,t)$: Indonesia.



As throughout this paper the source of the numerical data used is the estimates of population by age at five year intervals provided by the United Nations in 1986 on the basis of data up to 1984. It is left to another place to comment on the quality of that data, and especially on the projections included in it.

FIG. 6. Age-time diagram, showing three dimensions of age (a), time (t), and number of persons $p(a,t)$: Canada.



9. Identifying Mortality and Births Separately

What has been obtained so far is a combination of survivorship and births expressed by (2) to (5), where validity is confirmed by the ability to reconstruct the original age-time table from intercohort increase and the age distribution for one cohort. Separating out the births from survivorship is more hazardous. We could choose a set of life tables, guess which members apply to which cohorts, deflate the age-time table by survivorships, then see whether the discrepancies between the reconstructed and the original age-time tables was less, and so iterate. The present paper has not ventured on this terrain, that corresponds to the quite different problem of reconstructing past births and deaths from age distributions, as Andrei Rogers (1989) has done. Put in the fewest possible words, the object of this paper is to locate the start of what used to be called the population explosion.

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