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MCBARG - A System Supporting Multicriteria Bargaining

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Bronisz, P., Krus, L. and Lopuch, B.

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WORKING PAPER

MCBARG - A SYSTEM SUPPORTING MULTICRITERA BARGAINING

P. Bronisz L. Krus B. Lopuch

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Foreword

This paper is one of the series of 11 Working Papers presenting the software for interactive decision support and software tools for developing decision support systems. These products constitute the outcome of the contracted study agreement between the System and Decision Sciences Program at IIASA and several Polish scientific institutions. The theoretical part of these results is presented in the IIASA Working Paper WP-88-071 entitled *Theory*, Software and Testing Examples in Decision Support Systems. This volume contains the theoretical and methodological bacgrounds of the software systems developed within the project.

This paper presents the MCBARG program. This program has been designed to support analysis of conflicting decision situations and mediations in multicriteria bargaining problem. The newly developed approach for supporting noncooperative gaming situations based on the aspiration-led paradigm constitutes the methodological background for implementation.

> Alexander B. Kurzhanski Chairman System and Decision Sciences Program

MCBARG A System Supporting Multicriteria Bargaining

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Contents

1	Extended summary	1
2	Theoretical foundations 2.1 Introduction 2.2 Problem formulation and definitions	2 2 3
	2.3 Interactive procedure	4
3	User guide	6
4	Example session	10
5	Technical information	22
6	References	22

Introduction

The purpose of this report is to provide user with theoretical foundations of the MCBARG system and with information necessary to use the system. Section 1 contains extended summary including general information about the bargaining problem and proposed solution approach. Section 2 contains mathematical formulation of the multicriteria bargaining problem and methodological background of solution concept and interactive algorithm. Section 3 explains using of the system, i.e. how to create model of bargaining game and support mediation. Section 4 presents an illustrative example of bargaining game played with the system support. Section 5 contains technical information.

1 Extended summary

Many aspects of economic, environmental, or technological activity are influenced directly by bargaining between and among individuals, firms, and nations ("players"). In the pure bargaining problem, considered in the MCBARG system, the bargaining conditions are determined entirely by the bounds of discussion, within which the final outcome is determined by the interaction of the players. Even in the case of one individual, firm or nation, there are many complex situations in which the decision maker needs help to learn about possible decision options and decision consequences. The MCBARG system enables learning process of the players, and supports reaching the final outcome in the multicriteria bargaining problem. It is based on the theoretical results presented in (Bronisz, Krus, Wierzbicki, 1988).

The multicriteria bargaining problem is a generalization of classical bargaining problem, under assumption that there are not given explicitly the utility functions of participants. This generalization follows from the fact that an aggregation of participants or player's objectives is often impossible because of various practical limitations of the utility theory. The problem is defined by an agreement set — the set of outcomes that can be reached under unanimous agreement of the players, and by a disagreement (status quo) point which is a result of the problem if there is no such an agreement.

The proposed interactive process consists in generation of sequence of the outcomes leading to the nondominatated solution. The process is based on limited confidence principle, taken from practical observation, which says that the players have limited confidence in their ability to predict consequences and possible outcomes, hence each player tries to prevent other players from receiving disproportionally large gains. The generated outcomes are consistent with preferences of the players. The process assure some fairness rules and is resistant to the various manipulations of the players.

The algorithm consists of a number of rounds. Each round starts at the current status quo point (the first round starts from the initial status quo point). At each round the player specifies his confidence coefficient (i.e. defines part of the maximal improvement of the outcomes the counter players can obtain in the round). Furthermore assuming some moves of the other players, he tests different improvement directions for his objectives. This phase of the work with MCBARG system consists in an interactive scanning of outcomes performed by each player who specifies reference points in the objective space. The reference points are composed of aspiration levels of each player for his objectives. The players get additional information about the range of possible outcomes for a given confidence coefficient and some assumed actions of the counter players. This information is useful for reasonable specification of the aspiration levels. The system generates also some initial values for the aspiration levels and calculates corresponding outcomes (called neutral outcomes). The scanning of the player outcomes is performed in the system through directional optimization and lexicographic improvement of the week Pareto outcomes. The system responds to the player with attainable, efficient (under the assumed confidence coefficient) outcomes that strictly correspond to the player-specified aspirations. The results obtained for a number of different reference points can be easily compared in both numerical and graphical form. To finish this phase the player is required to select, according to his preferences, his reference point indicating his preferable improvement direction. These points selected independently by all the players are basis for calculation of the result of the round. The result is calculated following the limited confidence principle (the minimal confidence coefficient is used for all players) trying to improve outcomes for all the players in the directions specified by their reference points. Thus, the system acts as a neutral mediator proposing a single-test provisional agreement improving the initial situation and forming a basic for next round of negotiations. The results are presented to the players in form of report, and the players can go to the next round assuming the obtained result as a new status quo point. The process terminates when the efficient, strict Pareto optimal solution in the agreement set is reached.

The system includes a generator and an editor of the model of the bargaining problem for which the interactive process is performed. The model describes the agreement set in form of a set of inequalities, and the status quo point. The generator and the editor enable introducing linear or nonlinear formulas defining the inequalities using standard operators and functions. An illustrative example has been prepared, which relates to the problem of cooperation of two farms. The problem consists in division of the products resulting from cooperation between the farm owners according to their preferences.

2 Theoretical foundations

2.1 Introduction

In most approaches (see Nash, 1950, Raiffa, 1953, Kalai and Smorodinsky, 1975, Roth, 1979), the bargaining problem has been considered in the case of unicriterial payoffs of players, i.e. when the preferences of particular players are expressed by utility functions. In many practical applications however, players trying to balance a number of objectives might have difficulties while constructing such utility functions. Moreover, the classic literature considers mostly axiomatic models of bargaining which yield one-shot solutions and do not result in procedures describing a process of reaching a binding agreement.

We consider n players, each with several objectives, so we deal with a multiobjective bargaining problem. In this problem, the players are faced with an agreement set of feasible outcomes. Any such outcome can be accepted as the result if it is specified by an unanimous agreement of all players. In the event that no unanimous agreement is reached, the status quo point is the result. If there are feasible outcomes which all participants prefer to the status quo point, then there is an incentive to reach an agreement. In most situations, players differ in their opinions which outcome is most preferable, hence there is a need for bargaining and negotiation.

Dealing with multiple payoffs, we do not assume that there exist explicitly given utility functions of the players. In this case the solution can be found in an interactive procedure. Such a procedure is considered here. The procedure starts from the status quo point and leads to a nondominated, individually rational solution belonging to the agreement set. During the interaction, players can express their preferences and can influence the course of the iterative process.

2.2 Problem formulation and definitions

Let $N = \{1, 2, ..., n\}$ be the finite set of players, each player having m_i objectives. A multiobjective bargaining problem is defined as a pair (S, d), where an agreement set S is a subset of $\sum_{i=1}^{n} m_i$ - dimensional Euclidean space, called R^{NM} , and a disagreement point (status quo point) d belonging to S.

The bargaining problem has the following interpretation: every point $x \in \mathbb{R}^{NM}$, $x = (x_1, x_2, \ldots, x_n)$, $x_i = (x_{i1}, x_{i2}, \ldots, x_{im_i})$, in the agreement set S represents payoffs for all the players that can be reached when they do cooperate with each other $(x_{ij}$ denotes the payoff of the *j*-th objective for the *i*-th player). If the players do not cooperate, the disagreement point is the result.

We assume that each player tries to maximize his every objective.

We employ a convention that for $x, y \in R^k$, $x \ge y$ implies $x_i \ge y_i$ for i = 1, ..., k, x > y implies $x \ge y$, $x \ne y$, $x \gg y$ implies $x_i > y_i$ for i = 1, ..., k. We say that $x \in R^k$ is a weak Pareto optimal point in X if $x \in X$ and there is no $y \in X$ such that $y \gg x$; $x \in X$ is a strict Pareto optimal point in X if there is no $y \in X$ such that y > x.

We confine our consideration to the class of all multicriteria bargaining games (S,d) satisfying the following conditions:

- (i) S is compact and there is $x \in S$ such that x > d,
- (ii) S is comprehensive, i.e. for $x \in S$ if $d \leq y \leq x$ then $y \in S$.
- (iii) For any $x \in S$, let $Q(S, x) = \{i : y \ge x, y_i > x_i \text{ for some } y \in S\}$. Then for any $x \in S$, there exists $y \in S$ such that $y \ge x$, $y_i > x_i$ for each $i \in Q(S, x)$.

Condition (i) states that the set S is closed, upper bounded and the problem is not degenerated. Condition (ii) says that objectives are disposable, i.e. that if the players can reach the outcome x then they can reach any outcome worse than x. Q(S, x) is the set of all coordinates in \mathbb{R}^{NM} , payoffs of whose members can be increased from x in S. Condition (iii) states that the set of Pareto optimal points in S contains no "holes". We do not assume convexity of S, however, any convex set satisfies Condition (iii).

The problem consists in supporting the players in reaching a nondominated solution, agreeable and close to their preferences.

Definitions:

A point $x^i \in S$ is defined as *i*-nondominated, $i \in N$, if there is no $y \in S$ such that $y_i > x_i^i$. A point $u \in \mathbb{R}^{NM}$ is defined as a *utopia point relative to aspirations* (RA utopia point) if for each player $i \in N$, there is an *i*-nondominated point $x^i \in S$ such that $u_i = x_i^i$.

The *i*-nondominated point is an outcome which could be achieved by a rational player *i* if he would have full control of the moves of the other players. Let us observe that if in the unicriterial set there is only one *i*-nondominated point, in the multicriteria case considered here there is a set of such points. Each player *i*, $i \in N$, is required then to investigate the set of *i*-nondominated points in S as m_i -dimensional multicriteria decision problem and then to select one *i*-nondominated point as his most preferable outcome.

The RA utopia point generated by the selected *i*-nondominated points, $i \in N$, carries information about the most preferable outcomes for all the players. The RA utopia point significantly differs from the ideal (utopia) point defined by the maximal values of all objectives in set S.

2.3 Interactive procedure

We are interested in a constructive procedure that is acceptable by all players, starts at the status quo point and leads to a strict Pareto optimal point in S. The procedure can be described as a sequence, $\{d^t\}_{t=0}^k$, of agreement points d^t such that $d^0 = d$, $d^t \in S$, $d^t \ge d^{t-1}$, for $t = 1, 2, \ldots$, d^k is a strict Pareto optimal point in S. (The assumption $d^t \ge d^{t-1}$ follows from the fact that no player will accept improvement of payoffs for other players at the cost of his concession.) At every round t, each player $i \in N$ specifies his preferable reference point $r_i^t \in R^{m_i}$, $r_i^t > d_i^t$ defining his improvement direction $\lambda_i^t \in R^{m_i}$, $\lambda_i^t = r_i^t - d_i^t$, and proposes his confidence coefficient $\alpha_i^t \in R$, $0 < \alpha_i^t \le 1$. The improvement direction λ_i^t indicates the *i*-th players preferences over his objectives at round t. The confidence coefficient α_i^t reflects his ability at round t to describe preferences and to predict precisely all consequences and possible outcomes in S. (For more detailed justification, see Fandel, Wierzbicki, 1985, and Bronisz, Krus, Wierzbicki, 1988).

We propose an interactive negotiation process defined by a procedure:

(*)
$$\{d^t\}_{t=0}^{\infty} \quad \text{such that} \quad d^0 = d, \\ d^t = d^{t-1} + \varepsilon^t * [u(S, d^{t-1}, \lambda^t) - d^{t-1}] \quad \text{for} \quad t = 1, 2, \dots,$$

where $\lambda^t \in \mathbb{R}^{NM}$, $\lambda^t = (\lambda_1^t, \lambda_2^t, \dots, \lambda_n^t)$, is the improvement direction specified jointly by all the players, $u(S, d^{t-1}, \lambda^t) \in \mathbb{R}^{NM}$ is the utopia point relative to the direction λ^t at round t defined by $u(S, d^{t-1}, \lambda^t) = (u_1(S, d^{t-1}, \lambda_1^t), u_2(S, d^{t-1}, \lambda_2^t), \dots, u_n(S, d^{t-1}, \lambda_n^t))$, $u_i(S, d^{t-1}, \lambda_i^t) = \max_{\geq} \{x_i \in \mathbb{R}^{m_i} : x \in S, x \geq d^{t-1}, x_i = d_i^{t-1} + a\lambda_i^t \text{ for some} a \in \mathbb{R}\}$, Moreover, $\varepsilon^t = \min(\alpha_1^t, \alpha_2^t, \dots, \alpha_n^t, \alpha_{\max}^t) \in \mathbb{R}$, where α_{\max}^t is the maximal number α such that $d^{t-1} + \alpha [u(S, d^{t-1}, \lambda^t) - d^{t-1}]$ belongs to S.

The utopia point $u(S, d^{t-1}, \lambda^t)$ relative to the aspirations of the players (and relative to direction λ^t) reflects the preferences of the particular players when the improvement direction λ^t is specified at round t. The individual outcome $u_i(S, d^{t-1}, \lambda_i^t)$ is the maximal payoff in S for the *i*-th player from d^{t-1} according to the improvement direction λ_i^t , while ε^t is the minimal confidence coefficient of the players at round t (we assume that no player can agree on a coefficient greater than his) such that a new calculated agreement point belongs to S. The preferable direction λ_i^t at round t is specified on the basis of interactive scanning of a number of solutions generated for assumed by players different reference points. The proposed approach is very closed to the achievement function concept (Wierzbicki, 1982) from the point of view of the user. Analogously, a special way of the parametric scalarization of the multiobjective problem is utilized to influence on the selection of solutions by changing reference points. To solve the problem, directional maximization is applied, using a bisection method. (see Bronisz and Krus, 1988). The scanning (called in the system the improvement directions testing) is performed independently by each of players. Given the information about the current status quo and ideal point the player proposes a number of reference points and confidence coefficient. For each reference point $r_i^t > d_i^t$ at the round t and confidence coefficient α_i^t given by the player, the system calculates:

RA-utopia:

$$u_i(S, d^{t-1}, \lambda_i^t) = \max_{\geq} \Big\{ x_i \in \mathbb{R}^{m_i} : x \in S, \ x \geq d^{t-1}, \ x_i = d_i^{t-1} + a\lambda_i^t \quad \text{for some} \quad a \in \mathbb{R} \Big\},$$

one shot solution:

$$x^t = \max_{\geq} \left\{ x \in S : x = d^{t-1} + a * [u(S, d^{t-1}, \lambda^t) - d^{t-1}] \text{ for some } a \in R
ight\},$$

tentative solution:

$$y^t = d^{t-1} + \varepsilon^t * [u(S, d^{t-1}, \lambda^t) - d^{t-1}]$$

maximal confidence coefficient:

$$\alpha_{\max}^t = \max_{\geq} \Big\{ a \in R : d^{t-1} + a * [u(S, d^{t-1}, \lambda^t) - d^{t-1}] \in S \quad \text{for some} \quad a \in R \Big\},$$

where $\lambda^t \in \mathbb{R}^{NM}$, $\lambda^t = (\lambda_1^t, \lambda_2^t, \dots, \lambda_n^t)$, $\lambda_j^t = \lambda_j^{t-1}$ for $j \neq i$, $\lambda_i^t = r_i^t - d_i^t$, $u(S, d^{t-1}, \lambda^t) \in \mathbb{R}^{NM}$ is the utopia point relative to the direction λ^t .

Having the above information for a number of reference points, the players select his preferable one. It defines the improvement direction of this player. Defined in this way improvement directions of all the players λ_i^t are used for calculation of the result d^t of the negotiation round.

The procedure is based on the following theoretical result (see: Bronisz, Krus, Lopuch, 1987, and Bronisz, Krus, Wierzbicki, 1988):

Theorem 1. For any multicriteria bargaining game (S, d) satisfying conditions (i), (ii) and (iii) and for any confidence coefficients α_i^t such that $0 < \epsilon \leq \alpha_i^t \leq 1, t = 1, 2, ..., T$ there is a unique process d^t , t = 0, 1, ..., T, $T \leq \infty$, described by (*) satisfying the following postulates:

- P1. $d^0 = d, d^t \in S \text{ for } t = 1, 2, ..., T,$
- P2. $d^t \ge d^{t-1}$ for t = 1, 2, ..., T,

P3. $d^T (= \lim_{t\to\infty} d^t$ if $T = \infty$) is a strict Pareto optimal point in S.

P4. Principle of α -limited confidence. Let $0 < \alpha_i^t \le 1$ be a given confidence coefficient of the *i*-th player at round *t*. Then acceptable demands are limited by:

$$d^{t} - d^{t-1} < \alpha_{\min}^{t} [u(d^{t-1}) - d^{t-1}]$$

for t = 1, ..., T, where α_{\min}^t is a joint confidence coefficient at round t, $\alpha_{\min}^t = \min \{\alpha_1^t, ..., \alpha_n^t\}, u(d^{t-1})$ is the RA utopia point of the set $\{x \in S : x \ge d^{t-1}\}$ reflecting the preferences of the players.

- P5. Principle of recursive rationality. Given d^t , at each round t, there is no such outcome $x \in S$, $x > d^t$, that x satisfies P4.
- P6. Principle of proportional gains. For each round t, t = 1, ..., T, there is a number $\beta > 0$ such that

$$d^{t} - d^{t-1} = \beta[u(d^{t-1}) - d^{t-1}].$$

The presented approach has been examined in a case of one-round process with confidence coefficients of the players equal one. Corresponding one shot solution has been characterized axiomatically (Bronisz and Krus, 1987a). It is easy to notice that in the unicriterial case, each game (S, d) has a unique utopia point which coincides with the ideal point and the one shot solution coincides with the Raiffa solution (see Raiffa, 1953, Roth, 1979).

The testing of improvement directions by particular players is really important element of the procedure. It is assumed that after the testing each player selects his preferable direction. However, it may happend that a player has not sufficiently tested his set of nondominated points and selects a weak Pareto outcome as his preferable result. In such a case, even if all the players assume the confidence coefficients greater than the values of maximal confidence coefficients, the procedure should proceed in several iterations more, till it will reach strict Pareto solution in S. In the system this inconvenience is removed by application of an option of lexicographical improvement of weak Pareto solution to strict Pareto one without interaction of the players. The option is used only in the case that all the players assume their confidence coefficients greater then the values of maximal confidence. In such a case it is assumed that they are going to finish the interactive process. The lexicographical improvement proceeds in the following way:

Let us assume that in round t the obtained agreement point $d^t = d^{t-1} + \varepsilon * [u(S, d^{t-1}, \lambda^t) - d^{t-1}]$ is weak Pareto optimal. For a finite subsets of integer numbers I, J, let $e(I, J) = (e_1(I, J), \ldots, e_n(I, J)) \in \mathbb{R}^{NM}$ be such that $e_{ij}(I, J) = \lambda^t$ for $i \in I$ and $j \in J$, otherwise $e_{ij}(I, J) = 0$.

Given $y \in S$ with $Q(S, y) \neq \emptyset$, define $x(S, y) \in S$ by

$$x(S,y) = \max_{\geq} \left\{ x \in S : x = y + a * e(Q(S,y)) \text{ for some } a \in R \right\}.$$

Intuitively, the vector e(Q(S, y)) includes all the coordinates of vector λ^t , along with the solution can be improved. Otherwise, corresponding coordinate of the vector e(Q(S, y)) is equal to 0. Then the lexicographical improvement can be defined by the sequence $\{x^j\}_{j=0}^{\infty}$ such that $x^0 = d^t$, and $x^j = x(S, x^{j-1})$ for $j = 1, 2, \ldots$. It is shown (Bronisz and Krus, 1988) that there is exactly one such sequence, moreover this sequence is finite.

The presented lexicographical improvement has been examined in a case of one-round process, i.e. when weak Pareto optimal solution is reached in the first round. It is shown (Bronisz and Krus, 1988) that in such a case the solution of the bargaining process can be described with the Rawlsian lexmin principle (Rawls, 1971). Moreover in the unicriterial case, the solution coincides with the Imai solution (Imai, 1983).

Remark. In the presented approach we assume that each player tries to maximize his every objective. In a case that some of objectives should be minimized, we can use a following linear transformation of objectives T such that

$$Tx = (T_1x_1, \ldots, T_nx_n) \in \mathbb{R}^{NM},$$

$$T_ix_i = (T_{i1}x_{i1}, \ldots, T_{im_i}x_{im_i}),$$

$$T_{ij}x_{ij} = a_{ij} - \gamma_{ij}x_{ij},$$

where $a_{ij} \in R$, $\gamma_{ij} = 1$ if the *j*-th objective of the *i*-th player is maximized, $\gamma_{ij} = -1$ if the *j*-th objective of the *i*-th player is minimized. The selection of a_{ij} is arbitrary, it can not influence on the solution if we use the same transformation on the reference points (see the third objectives in presented example, section 4).

3 User guide

The MCBARG system is a decision support system designed to help in analysis of decision situation and mediation in multicriteria bargaining problem in which a mathematical model of the problem can be formulated by a status-quo point and a system of inequalities describing agreement set in objective space of the players.

The program is recorded on one diskette that should be installed on an IBM-PC-XT, AT or compatible computer with Hercules Graphics Card, Color Graphic Adapter (CGA) or Enhanced Graphics Adapter (EGA). A diskette contains compiled code of the program together with some data files for a demonstrative example of the bargaining problem.

The system supports the following general functions:

- 1. The definition and edition of a model of bargaining problem.
- 2. Interactive mediation.
- 3. Report of successive agreement outcomes.

The interactive mediation proceeds in a number of rounds and in each round the system supports:

- Initial multiobjective analysis of the bargaining problem for each player, resulting in estimation of bounds on efficient outcomes and learning about the extreme and neutral outcomes.
- Unilateral, interactive analysis of the problem with the stress on learning, organized through system response to user specified confidence coefficients and aspiration levels for objective outcomes. The systems responds with efficient (under the assumed confidence coefficient) objective outcomes.
- Calculation of the multilateral, cooperative solution of the round. Reporting the results of the already performed rounds.

The system is self-explaining, it includes a set of information facilitating working with the system. User is provided with a set of menu which allows him to select a needed option easily.

In the following sections more detailed information about particular options of the system menus are presented.

INFO option

The INFO option presents general information about the system.

SET UP MODEL option

The SET UP MODEL option enables definition and edition of a model of bargaining problem. The bargaining problem is described by a set of players having several criteria, by an agreement set and by a status quo point.

The following options of SET UP MODEL menu allow:

- LOAD loading an existing model or starting creation of a new one.
- EDIT editing an existing model or creating a new one.
- **SAVE** saving a currently edited model.

COMPILE — compiling the loaded model and performing initial calculation.

The LOAD option enables selection of one of displayed file names or input of a new file name (default extension of file names is .MOD).

The EDIT option enables edition or creation of a model. The built-in editor is specially designed for this purpose. To create a new model the following values must be introduced:

- number of players (PLAYERS option).
 Default: 2 players. Maximum: 5 players.
- number of criteria of each player (CRITERIA option). Default: 2 criteria. Maximum: 9 criteria.
- names of criteria (CRITERIA option).
- status quo value for each criteria (CRITERIA option). Default: 0.0.
- set of formulae describing the agreement set (FORMULAE option).

The players are recognized by the successive numbers. To facilitate description of criteria names in formulae, the criterion of the given player has the following alphanumeric code: "1n", where 1 means letter of the given player (a or A — first player, b or B — second player, and so on), n denotes number of the criterion. (For example, "c2" means variable for the second criterion of the third player).

The set of formulae describing an agreement set should be in the form:

```
formula_name_1 = expression_1 ;
formula_name_2 = expression_2 ;
....
formula_name_N = expression_N ;
```

what gives the following set of inequalities:

expression_I >= 0 for $I = 1, 2, \ldots, N$.

A formula_name_I, I = 1, ..., N, should begin with a letter. It can be treated as a comment to an expression_I. An expression_I, I = 1, ..., N, is Pascal-language-like expression, which can include:

- constants in normal or in exponential form;
- variables for criteria in presented above alphanumeric code "1n";
- operators divided into 4 categories, listed by their order of precedence:
 - 1) Unary minus: -,
 - 2) Exponentiation: ^,
 - 3) Multiplying operators: *, /,
 - 4) Adding operators: +, -;
- parenthesizes: (,);
- arithmetic functions: sin, cos, exp, log. The argument of each function should be closed in parenthesizes.

sin - returns the sine of the argument (the argument is expressed in radians), cos - returns the cosine of the argument (the argument is expresses in radians),

- exp returns the exponential of the argument,
- log returns the natural logarithm of the argument.

EXAMPLE of formula for two players, each one having two criteria:

formula = $1000 - a1 - exp(a2+b1) - (b2 - 1.05e-2)^2$;

An edited model is stored in memory, if you want to use the same model in future you should write it on the disk with SAVE option.

To utilize the model in the next rounds (TEST DIRECTION option, CALCULATE NEW AGREE MENT option) you should perform COMPILE option. This option produces as an output a binary code as well as performs some initial calculations including, among others, checking correctness of the model of bargaining problem.

TEST DIRECTION option

The TEST DIRECTION option enables testing different improvements direction starting from current status quo. It allows interactive scanning of outcomes variants. The variants are generated for the player specified reference points. The points correspond to the improvement directions. The resulting outcomes are calculated for player assumed confidence coefficient and assumed improvement directions of other players. By the scanning the player can test his preferences and select his preferable outcome and corresponding reference point.

The following options allow :

CONF_COEF — specification of confidence coefficient, it should be in the range (0, 1],

- NEW_REF specification of reference point and creation of a new variant, it should be greater or equal to the current status quo point,
- SELECT selection of the preferable variant,
- GRAPH presentation of solution in graphical form,
- SCROLL_CRIT scrolling criteria,

SCROLL_VAR — scrolling variants,

EXIT — exit to the main menu.

Using this options the player can create and compare a number of outcomes variants, specifying confidence coefficient and different reference points. The system presents current status quo and ideal point. They define range of possible outcomes and reasonable reference points. Created variants are stored on the disk. The first variant is created by the system, under assumption of the reference point equal to ideal point, which serves the player introductory information. For each variant the system calculates and presents:

- solution outcomes (Solution),
- maximal confidence coefficient (Max conf coeff.),
- one shot solution (One shot solution),
- utopia point relative to aspiration of the player (RA-utopia point).

One shot solution denotes outcomes the player could obtained under assumption of the full confidence (i.e. confidence coefficient equal to one). The RA-utopia point denotes outcomes the player could obtained under assumption that other players have outcomes on the level of status quo. The scrolling and graph options facilitate comparison of any variants on the screen.

Testing improvement direction phase should be performed by all the players, and finished with the selection of the preferable variant by each of them.

CALCULATE NEW AGREEMENT and REPORT options

The CALCULATE NEW AGREEMENT option enables calculation of a new agreement outcome according to the preferable reference points and minimal confidence coefficient assumed by the players in the TEST DIRECTION phase. The result can be presented independently to the players using the REPORT option. The calculated agreement solution is assumed as the current status quo in the next iteration. If the current agreement outcome is strictly Pareto optimal in the agreement set, the bargaining process is finished.

4 Example session

After starting from the distributed disk using command line: MCBARG, our system will be loaded and you will see on the first screen full name of the program, names of the authors and institutions, version number and date of release (Figure 1).

If you want to work with the system just press any key. The program will display the first menu (Figure 2).

It will be called as the MAIN menu and will appear in the left upper corner of the screen. On the bottom of the screen you can see bright blue bar (help bar) with brief explanation of all active keys at the moment. The help bars will appear during all phases of the work with the system.

Notice that one of the commands (options) of menu is highlighted. This is the command where the cursor is resting. You can activate this command pressing the Enter key or you can move to another command using the arrow keys. There is, however, one thing you should remember of: you can not use the commands in any order. First you have to establish the active model (the SET UP MODEL option) and then you can start playing with it (the TEST DIRECTION, CALCULATE NEW AGREEMENT, and REPORT options). If it is not obvious what to do next, just press F10 key (HELP) or select the INFO option for more information.

MCBARG
SYSTEM SUPPORTING MULTICRITERIA BARGAINING
by P. Bronisz, L. Krus, B. Lopuch
Version 2.2 September 1988
in Systems Research Institute, Polish Academy of Sciences, Warsaw, Poland
for System and Decision Sciences Program, International Institute for Applied Systems Analysis, Laxenburg, Austria (Copyright)

Press any key ...

Figure 1

BARGAIN

MAIN MENU INFORMATION SET UP MODEL TEST DIRECTION CALCULATE NEW AGREEMENT REFORT EXIT

♣ Home End - move bar ← select option F10 - help

Figure 2

Let us continue our session. If the cursor remains on the SET UP MODEL option, just press the Enter key. In other case first move the cursor to it. The system will transfer control to the **SET** UP MODEL menu displayed below:

SET UP MODEL

SET UP MO	DDEL MENU	
INFORMATION		
LDAD		
EDIT		
SAVE		
COMPILE		
EXIT		

🛉 Home End - move bar 🛛 - select option 🛛 F10 - help

Figure 3

Once again, you are asked to select an option from the above menu. At first, you have to select the LOAD option which allows you to establish the active model by creating a new one or selecting an existing one. Other options are inactive until an active model name is available. So move the cursor to the LOAD option (if it does not remain in it) and press Enter key. Your screen will look like in the figure 4.

In the bottom window you will see file names of the models created during the previous sessions. Each of displayed file names has an extension . MOD, it is the default extension. Lets choose the AGRICO.MOD. Just press the Enter key. This will be our illustrative example to show you what it is the MCBARG system and how you can work with it. The AGRICO model shows us an bargaining problem related to cooperation of two farms. The farm owners try to maximize production of the agricultural products and minimize inputs for the production under constraints related to the conditions of the production and applied technologies. The following two situations can be considered. In the first one, each owner acts independently, that is, he does not plan his production with the other. In this case, having substantial model of the farm production, to formulate production plan according to his preferences, he can use, as a supporting tool, one of the multiobjective optimization packages from the DIDAS family (Rogowski, Sobczyk, Wierzbicki, 1987). In the second situation, the owners decide to cooperate. Typically, in such a case, some surplus is obtained in comparison to the first case. It is a result of scale effects and better utilization of production inputs. Assuming cooperation, the farm owners have the problem how to allocate the surplus between them. They would like to take into account their preferences on the objectives which can be in general case different, and also they would like to make the division having a feeling of fairness. A solution obtained by the owners in the first case, called further as a status quo point, is a starting point for negotiation on the surplus division. This is an example of the

bargaining problem you can support with the MCBARG system. In the AGRICO model each of two players (farm owners) has three objectives, namely: the milk production, the rye production, and savings of tractor power. Denoting the objectives: a1, a2, a3 for the first



Figure 4

player, and b1, b2, b3 for the second one, respectively, the agreement set is described by the set of the following constraints:

$$\begin{array}{l} a1, a2, a3, b1, b2, b3 \geq 0;\\ 14-(a1+b1)^{1/2}-a2-b2 \geq 0;\\ 17-(a1+b1)^{1/2}-a3-b3 \geq 0;\\ 6-a2 \geq 0;\\ 8-a3 \geq 0;\\ 8-b2 \geq 0;\\ 9-b3 \geq 0; \end{array}$$

To see how the model is introduced into the system, select the EDIT option and then select in a sequence the options of the EDIT MODEL menu: PLAYERS, CRITERIA, FORMULAE exiting each of them by pressing F9 key. You will obtain respectively the screens presented in the figures 5, 6 and 7.











In the figure 5 you have defined number of players (2). In the figure 6 the names of criteria and the status quo point are given. On the left hand side you see a notation assigned by the system to the particular criterion. This notation is used in description of the formulae defining the agreement set, as you can see on the figure 7. The left side of each equation are treated by the system as a comment.

We suggest to leave the model unchanged and to exit the EDIT menu option. You can omit the SAVE option because the model was not changed. Select the COMPILE option of the



🚽 Ctrl-I - insert line Ctrl-Y - delete line Ins - change mode F9 - enter

Figure 7	7
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TEST IMPROVEMENT DIRECTIONS Player no 1 Iteration no 1 INFO CONF_COEF NEW_REF SELECT GRAPH SCROLL_CRIT SCROLL_VAR EXIT Your confidence coefficient : 1.0000

Var	Status quo	Reference point	Solution	Max conf coeff.	One shot solution	RA-utopia point	Ideal point
1	MILK 20.40	PRODUCTION 113.00	42.10	0.50	h liters] 42.10	63.79	113.00
1	RYE 0.60	FRODUCTION 5.43	1.73	0.50	h tonnes] 1.73	2.86	5,43
1	SAVIN(0.00	B OF TRACTO B.00	DR POWER 1.87	[th tract 0.50	pr hours] 1.87	3.75	8.00



Figure 8

SET UP MODEL menu. Wait please a moment. The system compiles the model and makes initial calculations. Now, you can return to the MAIN menu and start the mediation process.

Select the TEST DIRECTION option. The system goes to the first iteration of interactive mediation procedure, which starts from unilateral analysis of the problem. The analysis is performed independently for each player. So the system displays the players numbers and you should select one of them. Select first the first player. On the screen presented in the figure 8 you can see first variant of tentative solution.

For each objective the following values are given: number of variant, status quo, reference point, solution, maximal confidence coefficient, one-shot solution, RA-utopia point and ideal point. The status quo and ideal points are constant within the unilateral analysis, while the other values depend on the confidence coefficient and reference point. Using the CONF_COEF option you can modify the confidence coefficient. Lower value of the coefficient means you have more limited confidence on the future outcomes of the game and it results in greater number of iterations in the mediation procedure. Let assume the value to be 0.3000. The NEW REF option allows you by specification of reference points to generate the different variants of tentative solutions dependently on the currently obtained results and on your preferences. The status quo and ideal points define reasonable bounds for the reference points. The presented first solution, called as neutral solution, has been obtained for the reference point equal to the ideal point. Let us assume that as the owner of the first farm (the first player), you prefer to obtain first of all more milk and to save more tractor power. So select the NEW_REF option and type (for the second variant) ideal point values for milk and savings of tractor power (113.00 and 8.00, respectively), and lower value for the rye production (4.00). In third variant, try to decrease the milk production (100.00) and the rye production (3.00). The screen will look like this:

	TEST IMPROV	EMENT DIRE	CTIONS	Flaye	erno 1 It	teration no 1	
INFO	CONF_COEF	NEW_REF	SELECT	GRAPH	SCROLL_CRIT	SCROLL_VAR	EXIT
Your	confidence c	oefficient	: 0.30	000			

Var	Status quo	Reference point	Solution	Max conf coeff.	One shot solution	RA-utopia point	Ideal point
	MILK	PRODUCTION			h liters]		
1	20.40	113.00	<u>دى.42</u>	0.50	42.10	63.79	113.00
2	20.40	113.00	35.73	0.50	45.94	71.49	113.00
3	20.40	100.00	37.00	0.50	48.04	75.74	113.00
	RYE	PRODUCTION			h tonnes]		
1	0.60	5.43	1.28	0.50	1.73	2.86	5.43
2	0.60	4.00	1.16	0.50	1.54	2.48	5.43
3	0.60	3.00	1.10	0.50	1.43	2.27	5.43
	SAVIN	GS OF TRACTO	R POWER	[th traci	or hours]	<u>├</u>	
1	0.00	8.00	1.12	0.50	1.87	3.75	8.00
2	0.00	8.00	1.32	0.50	2.21	4.41	8 00
3	0.00	8.00	1.67	0.50	2.78	5.56	8.00
	lome End-mo	<u>ب</u> لو vebar	-select op	tion F1	L	<u> </u>	-help

-			~
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n 1	U	 •	<u>ч</u>
	5.	 <u> </u>	~

To compare the values of solutions in the graphical form, press the GRAPH option. The screen will look like in the figure 10.

In the session, the decrease of the rye production in the third variant seems to be too high for the player, so he assumes the second variant as the best. As you see, one of variant numbers is highlighted. This variant is called as active. Using the F1 key you can change the active variant. Set please the second variant as the active one and press the SELECT option.

In this example you do not use the SCROLL_CRIT option and SCROLL_VAR option. The first one allows scrolling criteria in a case when the number of criteria is greater than three. In such a way, any combination of three criteria from criteria set is possible to display. The second one allows scrolling variants. In general case the player can introduce relatively large TEST IMPROVEMENT DIRECTIONS

Flaver no 1 Iteration no 1

INFO CONF_COEF NEW_REF SELECT GRAPH SCROLL_CRIT SCROLL_VAR EXIT Your confidence coefficient : 0.3000

Var	Status que	Reference point	Solution	Graphical representation of solutions
1 N 19	MILK 20.40 20.40 20.40	PRODUCTION 113.00 113.00 100.00	33.42 35.73 37.00	[th liters]
1 2 3	RYE 0.60 0.60 0.60	PRODUCTION 5.43 4.00 3.00	1.28 1.16 1.10	[th tonnes]
1 2 3	SAVIN 0.00 0.00 0.00	55 OF TRACTO 8.00 8.00 8.00 8.00	DR POWER 1.12 1.32 1.67	[th tractor hours]

Press any key ...

Figure 10

TEST IMPROVEMENT DIRECTIONS Flayer no 2 Iteration no 1 INFO CONF_COEF NEW_REF SELECT GRAPH SCROLL_CRIT SCROLL_VAR EXIT Your confidence coefficient : 0.2000

Var	Status quo	Reference point	Solution	Max conf coeff.	One shot solution	RA-utopia pcint	' Ideal point
1 2 4	MILK 31.00 31.00 31.00	PRODUCTION 123.60 70.00 60.00	39.68 36.18 35.19	0.50 0.50 0.50 0.50	h liters] 52.70 43.92 41.44	74.39 56.90 51.97	123.60 123.60 123.60
1 2 4	RYE 1.40 1.40 1.40	PRODUCTION 6.23 6.23 6.23	1.85 2.04 2.10	0.50 0.50 0.50 0.50	h tonnes] 2.53 3.00 3.14	3.66 4.61 4.89	6.23 6.23 6.23
1 2 4	SAVINO 0.00 0.00 0.00	5 DF TRACTO 9.00 9.00 8.00	R POWER 0.84 1.20 1.16	[th tract 0.50 0.50 0.50	or hours] 2.11 2.98 2.88	4.22 5.98 5.79	7. 00 7. 00 7. 00

Figure 11

number of variants. By properly setting the active place and scrolling it, you can compare any combination of three variants from all the introduced ones.

Exit please this menu. Systems will ask you for the number of the next player making the interactive analysis. Select the second. The interactive analysis is performed in the same way as for the first player, however the values of particular quantities and the player preferences are in general case different. Let the second player has more limited confidence than the first one and assume the confidence coefficient equal to 0.200. The player tries to increase

first of all the rye production and savings of tractor power at a cost of decreasing the milk production. Then select the fourth variant. The screen will look like in the figure 11.

Exit this phase using the EXIT option and go back to the main menu pressing F9 key. Having the preferable variants for the both players selected, the cooperative solution can be calculated for the first iteration. Select the CALCULATE NEW AGREEMENT option and wait a moment for the system calculations. The system reports end of the calculations. Then press any key to go back to the MAIN menu. You can see the obtained values for particular objectives using the REPORT option.

Select the TEST DIRECTION option to start interactive analysis in the second iteration. The interactive analysis is done in the same way as in the first iteration. Screens of the analysis for the first and the second players are presented in the figures 12 and 13, respectively.

	TEST IMP	ROVEMENT DIR	ELTIONS	Flayer	- no 1	ICERACION	
INFC Your	CONF_CO	EF NEW_REF e coefficien	SELECT	GRAFH	SCROLL_CRI	T SCROLL_V	VAR EXIT
Var	Status Quo	Reference point	Solution	Max conf coeff.	One shot solution	RA-utopia point	Id eal point
1 2 3	MILK 30.62 30.62 30.62	FRODUCTION 84.19 84.19 80.00	38.36 39.63 39.32	0.50 0.50 0.50 0.50	th liters] 43.48 45.58 45.06	56.42 60.66 59.62	84.19 84.19 84.19
1 2 3	RYE 0.98 0.98 0.98	PRODUCTION 3.79 3.00 3.00	1.38 1.32 1.33	• 0.50 0.50 0.50	th tonnes] 1.65 1.54 1.57	2.33 2.11 2.16	3.79 3.79 3.79 3.79
1 2 3	SAVIN 0.88 0.68 0.88	S DF TRACTO 7.73 7.73 6.00	DR POWER 1.87 2.03 1.78	(th trac 0.50 0.50 0.50	or hours] 2.53 2.79 2.38	4.18 4.72 3.89	7.73 7.73 7.73

Figure 12

The cooperative (agreement) solution obtained at the first iteration is assumed as the status quo at the current, second one. The solution dominates strictly the status quo of the game and is calculated along the improvement direction defined by the preferable reference point selected in the first iteration. Observe decreasing of the set of agreement outcomes dominating the new status quo for both the players (i.e. the values of the ideal point are smaller than those in the previous iteration). In the second iteration both the players assume the confidence coefficient equal to 0.3000. The first one tries, as previously, to increase the milk production and savings of tractor power in the second variant. In the third variant tries slightly increase the rye production, however the increase was very small for relatively significant decrease of savings of tractor power. So he decides to select the second variant as the best. The second player (Figure 13) tries to increase the rye production. From the introduced variants he selects the fourth one as the best. Repeat please the players actions and calculate the new agreement solution.

TEST IMPROVEMENT DIRECTIONS Player no 2 Iteration no 2 INFO CONF_COEF NEW_REF SELECT GRAPH SCROLL_CRIT SCROLL_VAR EXIT Your confidence coefficient : 0.3000

Var	Status quo	Reference point	Solution	Max conf coeff.	One shot solution	RA-utopia point	Ideal point	
1 2 4	MILK 35.19 35.19 35.19	PRODUCTION B8.76 70.00 60.00	42.93 41.18 40.01	0.50 0.50 0.50 0.50	th liters] 48.09 45.16 43.20	60.99 55.16 51.25	88. 76 88.76 88.76	
1 2 4	RYE 2.10 2.10 2.10	FRODUCTION 4.91 5.00 5.00	2.51 2.60 2.66	0.50 0.50 0.50	th tonn es] 2.78 2.93 3.03	3.45 3.76 3.98	4.91 4.91 4.91	
1 2 4	SAVIN 1.16 1.16 1.16	55 DF TRACTO 8.00 8.00 6.00	R POWER 2.15 2.33 2.10	[th tract 0.50 0.50 0.50	tor hours] 2.81 3.12 2.72	4.45 5.08 4.29	8.00 8.00 8.00	
◆◆ Home End-move bar ◆ -select option F1-active variant F10-help								

Figure 13

Start the next iteration selecting the TEST DIRECTION option from the main menu. The interactive analysis performed by the players is presented in the figures 14 and 15.

TEST IMPROVEMENT DIRECTIONS Flaver no 1 Iteration no 3 INFO CONF_COEF NEW_REF SELECT GRAPH SCROLL_CRIT SCROLL_VAR EXIT Your confidence coefficient : 1.0000

Var	Status quo	Reference point	Solution	Max conf coeff.	One shot solution	RA-utopia point	Ideal point
	MILK	PRODUCTION	44.75	[t	h liters]	40.00	
	37.03	60.43	44.73	0.50	44./5	47.88	60.43 40.43
3	37.63	60.00	47.96	0.52	47.96	55.52	60.43
	RYE	PRODUCTION		j	th tonnes]	<u></u> }	
1	1.32	2.41	1.59	0.50	1.59	1.86	2.41
2	1.32	2.00	1.53	0.50	1.53	1.74	2.41
3	1.32	1.50	1.39	0.52	1.39	1.46	2.41
	SAVIN	S OF TRACTO	R POWER	l Lith tract	or hours]		
1	2.03	5.98	3.01	0.50	3.01	3.98	5.98
2	2.03	6.00	3.25	0.50	3.25	4.48	5.98
3	2.03	6.00	3.66	0.52	3.66	5.13	5.98
★+ Home End-move bar ↓-select option F1-active variant F10-help							

Figure 14

TEST IMPROVEMENT DIRECTIONS Player no 2 Iteration no 3 INFO CONF_COEF NEW_REF SELECT GRAPH SCROLL_CRIT SCROLL_VAR EXIT Your confidence coefficient : 1.0000

Var	Status quo	Reference point	Solution	Max conf coeff.	One shot solution	RA-utopia point	Ideal point
1 2 3	MILK 40.01 40.01 40.01	PRODUCTION 60.81 50.00 45.00	45.14 42.91 41.70	0.50 0.50 0.50 0.50	th liters] 45.14 42.91 41.70	50.26 45.82 43.40	60.81 60.81 60.81
1 2 3	RYE 2.66 2.66 2.66	PRODUCTION 3.76 4.00 4.00	2.93 3.05 3.12	0.50 0.50 0.50 0.50	th tonnes] 2.93 3.05 3.12	3.20 3.44 3.57	3.76 3.76 3.76
1 1 1 13	SAVIN 2.10 2.10 2.10	55 DF TRACTO 6.04 6.00 5.00	R POWER 3.07 3.23 3.08	[th tract 0.50 0.50 0.50	or hours] 3.07 3.23 3.08	4.04 4.37 4.07	6.04 6.04 6.04

★◆ Home End-move bar →-select option F1-active variant F10-help

Figure 15

Follow their actions. Both the players assume the confidence coefficients as equal to 1.0000. That means they decide to have relatively high confidence on the future outcomes and to not prolong the procedure for the next iteration. The first player tries consequently increase the milk production but not at the cost of too high decreasing of the rye production. So he decides to select the second variant as the best. The second player increasing the rye production selects the third variant as the preferable one. Calculate the new agreement solution. After the calculation the systems reports you that the final, efficient solution is found.

Type any key and select the REPORT option. Select then the first player. You will obtain the first player report as in the figure 16.

You can see values of the agreement points in the successive iterations. The values are presented also in the graphical form compared to the initial status quo point. The iteration numbers in the bars indicate the values. The values obtained at the last, third iteration are equal to the one-shot solution. This means the final, efficient solution (on the Pareto frontier) in the agreement set has been reached.

The SCROLL_CRIT option allows you to compare any combination of three criteria in a case of greater number of the criteria.

Exit the report and select the second player to obtain his final result as presented in the figure 17.

Exit the report, go back to the MAIN menu, and leave the system.

REPORT Player no 1 Iteration no 3

INFO SCROLL_CRITERIA EXIT

Joint confidence coefficient : 1.0000

) 5	MIL) Status quo 20.40	< PRODUCT: Agreement 1 30.62	IDN points in 2 39.63	successive 3 45.88	[th liters] iterations	2	One shot solution 43,88
5	RYE Status quo 0.60	PRODUCT: Agreement 1 0.98	ION points in 2 1.32	successive 3 1,53	[th tonnes] iterations	2	One shot solution 1.53
	SAV	INGS OF TRA	ACTOR POWER	lth tra	actor hours]		
9	Status	Agreement	points in	successive	iterations		One shot
	quo	1	2	3			solution
	0.00	0.88	2.03	3.98			3.98
		1		2			2

++ Home End - move bar - select option

Figure 16

REPORT Player no 2 Iteration no 3

INFO SCROLL_CRITERIA EXIT

Joint confidence coefficient : 1.0000



★◆ Home End - move bar → - select option

Figure 17

5 Technical information

The program is recorded on attached diskette that should be installed on an IBM PC family. This diskette contains the following files:

- MCBARG.EXE binary code of the program,
- INFOSYS.DAT file of records containing internal information presented in the program by INFO option,
- HELPSYS.DAT file of records containing additional information presented in the program by HELP option,
- AGRICO.MOD --- file containing the illustrative example of bargaining problem.

The program is activated by the command: MCBARG. It automatically recognizes one of the following graphic cards: HERCULES, CGA, EGA.

The program enables a user to create new models of bargaining problems and to save the models on the active disk. Default extension of the files containing the models is .MOD. During every session with the system the following auxiliary files are generated on the disk: TRY.DAT, REP.DAT. They can be erased after a session.

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