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PLP - A Package for Parametric Programming

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Golebiowski, A.

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WORKING PAPER

PLP - A PACKAGE FOR PARAMETRIC PROGRAMMING

A. Golebiowski

December 1988 WP-88-118



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Foreword

This paper is one of the series of 11 Working Papers presenting the software for interactive decision support and software tools for developing decision support systems. These products constitute the outcome of the contracted study agreement between the System and Decision Sciences Program at IIASA and several Polish scientific institutions. The theoretical part of these results is presented in the IIASA Working Paper WP-88-071 entitled *Theory*, *Software and Testing Examples in Decision Support Systems* which contains the theoretical and methodological bacgrounds of the software systems developed within the project.

This paper presents the PLP package for parametric linear programming. This package constitutes the extension to MINOS, the well known linear and nonlinear programming code developed at Stanford University, and uses the MINOS as the solver of optimization problems. The PLP gives a complete parametric programming analysis for one, or more, of the following vectors: cost, rhs and bounds. In the same run several problems of this kind can be solved and for each, the starting point may be the original optimal solution obtained in the last problem. This property makes the PLP especially interesting for multiple criteria analysis using the reference point approach.

> Alexander B. Kurzhanski Chairman System and Decision Sciences Program

PLP - A PACKAGE FOR PARAMETRIC PROGRAMMING

Adam Golebiowski

Institute of Automatics, Academy of Mining and Metallurgy, Cracow, Poland.

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INTRODUCTION

PLP is a software package for parametric linear programming. It is an extension of MINOS, the well-known linear and nonlinear programming code developed by Saunders and Murtagh^{*}. PLP is initiated by adding some specifications to the original list of MINOS specifications.

The package PLP uses MINOS as the solver of optimization problems. It includes sections which create an iterative framework for parametric programming and perform ranging and housekeeping procedures.

The formulation of the linear problem analyzed by PLP is similar as for MINOS.

Optionally, PLP gives a complete parametric programming analysis for one, or more, of the following vectors: cost, rhs and bounds. Of course such analysis can also be performed for single elements of these vectors. In the same run, several problems of this kind can be solved and for each, the starting point may be the original optimal solution or the final solution obtained in the last problem.

The last current complete solution in MINOS format is printed or stored with frequency specified by the user. Additionally, the user specifies the frequency of printing of a short message about current changes of optimal basis.

^{*} B.A. Murtagh and M.A. Saunders. MINOS - A Large-Scale Linear and Nonlinear Programming System. User's Guide. Technical Report Sol 77-9, Systems Optimization Laboratory, Stanford University California, 1977.

A. THEORETICAL GUIDE

1. GENERAL INFORMATION.

As options of PLP can be expressed in terms of the internal formulation of the linear problem used by MINOS we shall begin with recalling this concept.

The external formulation (supplied by the user) of the linear problem to be solved by MINOS is: Minimize (or maximize) a linear cost function

$$F(\mathbf{x}) = \mathbf{a}_0 \mathbf{x} \tag{1}$$

subject to m row constraints:

$$d_i \leq a_i x \leq g_i , \quad i = 1, \dots, m \tag{2}$$

and n constraints on separate variables:

$$d_{m+i} \le x_i \le g_{m+i}, \quad i = 1, ..., n$$
 (3)

Here x is an n-dimensional column vector of decision variables, a_0 is an n-dimensional row vector of cost coefficients (also called the *objective row*), the a_i , i = 1,...,m, are ndimensional row vectors, the lower bounds d_i , i = 1,...,m+n, are real numbers or $-\infty$, and the upper bounds g_i , i = 1,...,m+n, are real numbers or $+\infty$. Of course, if the bounds take the values $+\infty$ or $-\infty$ the corresponding relation (2) or (3) must be replaced by a strict inequality. If $d_i = g_i$, then the variable x_i is said to be fixed. If $d_i = -\infty$ and $g_i = +\infty$ the variable x_i is said to be free. Analogous terms are used to describe the rows $a_i x$.

It should be recalled that in MINOS the two-sided inequality constraints (2) are not stated explicitly, but rather specified using ranges. More precisely, a one-sided inequality is introduced in the form $a_i x \leq g_i$ (type L) or $a_i x \geq d_i$ (type G), together with a real number r_i called the range. In the first case, the difference between the right-hand side g_i and this number yields the lower bound $(d_i = g_i - r_i)$; in the second case the sum of the right-hand side d_i and the real number r_i gives the upper bound $(g_i = d_i + r_i)$.

The linear programming problem is transformed by MINOS into the following internal form: Minimize (or maximize) the variable

$$-\tilde{x}_{n+1+\mathrm{obj}}$$
 (4)

subject to equality constraints:

$$\tilde{\mathbf{A}}\tilde{\mathbf{x}} = \mathbf{0} \tag{5}$$

and inequality constraints:

$$\tilde{l} \leq \tilde{x} \leq \tilde{u}$$
 . (6)

Here \tilde{A} is an $(m+1) \times (n+m+2)$ -matrix:

$$\tilde{A} = \begin{bmatrix} \tilde{a}_{1} & \tilde{b}_{1} \\ \cdot & \cdot \\ \cdot & \cdot \\ \tilde{a}_{m+1} & \tilde{b}_{m+1} \end{bmatrix} , \qquad (7)$$

where I denotes the $(m+1) \times (m+1)$ identity matrix and

$$\tilde{a}_i = a_i \quad i < \text{obj} , \quad \tilde{a}_{\text{obj}} = a_0 , \quad \tilde{a}_i = a_{i-1} \quad i > \text{obj} ,$$

$$\tilde{b}_i = b_i \quad i < \text{obj} , \quad \tilde{b}_{\text{obj}} = 0 , \quad \tilde{b}_i = b_{i-1} \quad i > \text{obj} ,$$

$$(8)$$

where

$$b_i = \begin{cases} 0 & \text{if } d_i = -\infty \text{ and } g_i = +\infty \\ d_i & \text{if } d_i \text{ is finite and } g_i = +\infty \\ g_i & \text{if } g_i \text{ is finite} \end{cases}$$

The first *n* components of the extended vector of decision variables $\tilde{x} \in \mathbb{R}^{n+m+2}$ form a subvector identical to *x*; these components are described as *structural*. Element \tilde{x}_{n+1} is called the *right-hand-side component*; it is fixed at -1. The remaining components of \tilde{x} are called *slack* or *logical components*. The objective variable $\tilde{x}_{n+1+obj}$ is free. The vector of lower bounds \tilde{l} and the vector of upper bounds \tilde{u} are defined as follows:

$$\tilde{l}_{i} = d_{m+i} \quad i = 1, ..., n , \quad \tilde{l}_{n+1} = -1, \quad \tilde{l}_{n+1+obj} = -\infty ,$$

$$\tilde{u}_{i} = g_{m+i} \quad i = 1, ..., n , \quad \tilde{u}_{n+1} = -1, \quad \tilde{u}_{n+1+obj} = +\infty .$$
(9)

Now let i = n + 1 + j, j = 1,...,m. Then

$$\tilde{l}_i = h_i$$
, $\tilde{u}_i = k_i$ for $j < obj$ and $\tilde{l}_i = h_{i-1}$, $\tilde{u}_i = k_{i-1}$ for $j > obj$, (10)

where

 $h_i = k_i = 0$ if the *j*-th row constraint is fixed (i.e., of type *E*) (11) $h_i = 0, k_i = +\infty$ if $d_j = -\infty$ and g_j is finite (one-sided constraint of type *L*) $h_i = -\infty, k_i = 0$ if d_j is finite and $g_j = +\infty$ (one-sided constraint of type *G*) $h_i = 0, k_i = g_j - d_j$ if d_j and g_j are finite $h_i = -\infty, k_i = +\infty$ if the *j*-th row constraint is free.

2. MATHEMATICAL THEORY

This section presents elements of ranging theory for the linear programming problem (4)-(6). Some nonconventional notation will be used in order to avoid discussion of many particular cases. The sign \leq will denote "less than or equal to" if the expressions on its both sides are finite and "less than" otherwise. Similarly, \geq will denote "greater than or equal to" or "greater than". The notation $[t_1, t_2]$ will be used for the closure of the open interval (t_1, t_2) ; that is, t_1 and/or t_2 do not belong to the interval if they are not finite. For the sake of simplicity we shall assume that obj = m+1, i.e., the objective row is the last row in matrix \tilde{A} . As the value of variable \tilde{x}_{n+1} is fixed at -1 we may remove it from the problem formulation, defining a new column vector of decision variables $y \in \mathbb{R}^{n+m}$, where $y_i = \tilde{x}_i$ i = n+1, ..., n+m. We also define an m x (n+m)-matrix

$$A = \begin{bmatrix} a_1 \\ \cdot \\ \cdot \\ a_m \end{bmatrix};$$

column vectors $b \in \mathbb{R}^m$ (see (8)), $l, u \in \mathbb{R}^{n+m}$, where $l_i = \tilde{l_i}$, $u_i = \tilde{u_i}$, i = 1,...,n and $l_i = h_{i+1}$, $u_i = k_{i+1}$, i = n+1,...,n+m; and a row vector $c \in \mathbb{R}_{n+m}$, where $c_i = a_0^i$

i = 1,...,n and $c^i = 0$ i = n+1,...,n+m.

The linear programming problem now takes the form: Minimize (or maximize) the linear cost function

$$F(\mathbf{y}) = c\mathbf{y} \tag{12}$$

subject to:

$$Ay = b \tag{13}$$

$$l \leq y \leq u \quad . \tag{14}$$

We denote the optimal solution of this problem by z and decompose it in the obvious way into the following subvectors:

 z_B - basic vector,

 z_l - vector of nonfixed, nonbasic variables which are at their lower bounds,

 z_u - vector of nonfixed, nonbasic variables which are at their upper bounds,

 z_i - vector of fixed variables (i.e., variables for which $u_i = l_i$).

Let I_u be the set of indices of all nonbasic variables at their upper bounds and let I_l be the set of indices of all nonbasic variables at their lower bounds. Fixed variables are not included in I_u or I_l . We shall let I_B denote the set of indices of all basic variables. This decomposition is also applied to the other vectors, yielding, for example, c_B , c_l , c_u ; l_B , l_l , l_u ; u_B , u_l , u_u . It is clear that $z_l = l_l$, $z_u = u_u$, $z_s = u_s$. Thus the constraint matrix is decomposed into the basic matrix B and matrices L, U, S such that

$$Bz_B + Lz_l + Uz_u + Sz_s = b$$

Hence we have

$$z_B = B^{-1}b - B^{-1}(Lz_l + Uz_u + Sz_s)$$
(15)

for the basic vector and

$$F(z) = c_B B^{-1} b + (c_l - c_B B^{-1} L) z_l + (c_u - c_B B^{-1} U) z_u + (c_s - c_B B^{-1} S) z_s \quad . \tag{16}$$

for the optimal cost.

Here and elsewhere we shall denote the *i*-th row of a matrix H by H_i and the *j*-th column by H^j . Define

$$D = B^{-1}$$
 . (17)

2.1. Parametric analysis of cost.

In every iteration of PLP COST the ranging problem has to be solved in the first place. Let Δc be a given nonzero row vector in R_{n+m} , where $\Delta c^i = 0$ for i = n+1,...,n+m and for fixed variables. We consider programming problems (12)-(13) with the cost vector c replaced by $\overline{c}(t)$, where

$$\overline{c}(t) = c + t\Delta c \tag{18}$$

and t is a real number, $t \in \mathbb{R}^1$. We wish to determine the largest range $[0, t_{\max}]$ in which the coefficient t may vary without affecting the optimal solution, i.e., the range of t values for which the optimal solution is equal to z.

It is clear from (16) that the optimal solution remains unchanged and equal to z for all values of t such that

$$\epsilon(\ \bar{c}_l(t) - \bar{c}_B(t)DL) \le 0 \tag{19}$$

and

$$\epsilon(\ \overline{c}_u(t) - \overline{c}_B(t)DU) \ge 0 \quad , \tag{20}$$

where

 $\epsilon = \begin{cases} +1 & \text{in the case of maximization} \\ -1 & \text{in the case of minimization} \end{cases}$

Hence

$$t\epsilon(\Delta c_l - \Delta c_B DL) \le \epsilon(c_B DL - c_l)$$

$$t\epsilon(\Delta c_u - \Delta c_B DU) \ge \epsilon(c_B DU - c_u) .$$
(21)

We shall use the following notation:

$$T_{j} = -c^{j} + c_{B}DA^{j}, \quad \Delta T_{j} = -\Delta c^{j} + \Delta c_{B}DA^{j}, \quad j \in I_{u} \cup I_{l} \quad .$$
⁽²²⁾

In the case of maximization we then have

$$t_{\max} = \min \left\{ -T_j / \Delta T_j \right\} \quad , \tag{23}$$

where the minimum is taken over all values of j from I_l such that $\Delta T_j < 0$ and all values of j from I_u such that $\Delta T_j > 0$.

In the case of minimization t_{\max} is determined from (23) but with the minimum taken over all values of j from I_i such that $\Delta T_j > 0$ and all values of j from I_u such that $\Delta T_j < 0$.

In all cases, if the set of indices over which the maximum (or minimum) is taken is empty, then $t_{max} = +\infty$.

If t_{\max} is finite, two situations are possible: either the optimal solution vanishes for all $t > t_{\max}$ or a new optimal solution exists for some $t > t_{\max}$. This change of the optimal solution is determined by MINOS in the following way.

A shifted value of the cost vector is determined

$$c(t') = c + (t_{\max} + \Delta')\Delta c$$
⁽²⁴⁾

 Δ' is an appropriately chosen increment (see below). For this cost vector, MINOS finds the corresponding optimal solution. Next, the value of the cost vector and optimal cost at $t = t_{max}$ are calculated

$$\bar{c}(t_{\max}) = \bar{c}(t') - \Delta' \Delta c \tag{25}$$

$$F(t_{\max}) = \bar{c}(t_{\max})z \tag{26}$$

where z is the right-hand limit of the optimal solution for $t = t_{max}$ and Δ' is computed from:

$$\Delta' = DELTA * z \tag{27}$$

where DELTA is given by the user in the keyword PLP INCREMENT and x is the greatest real for which the following inequality is satisfied

$$-\boldsymbol{x} \Delta T_{\boldsymbol{i}} \leq \boldsymbol{f}(\boldsymbol{x}) \tag{28}$$

where

$$f(\mathbf{z}) = \begin{cases} TOLD^{*} || (\overline{c}_{B}t_{\max} + \Delta \overline{c}_{B}\mathbf{z})B^{-1} ||, || (\overline{c}_{B}t_{\max} + \Delta \overline{c}_{B}\mathbf{z})B^{-} || > 1 \\ TOLD , \text{ otherwise} \end{cases}$$
(29)

This inequality is solved for all values of the subscript *i* which belong to the set I_{\sum} (see

2.2. Parametric analysis of rhs.

In every iteration of PLP RHS the ranging problem has to be solved in the first place. Let Δb be a given nonzero column vector in \mathbb{R}^m . We consider the family of linear programming problems (12)-(14) with the rhs vector b replaced by $\overline{b}(t)$, where

$$\overline{b}(t) = b + t\Delta b \tag{30}$$

and $t \in \mathbb{R}^1$. We wish to determine the largest range $[0, t_{\max}]$ in which the coefficient t may vary without affecting the optimal basis, i.e., the range of t values for which the optimal basis is equal to B.

Letting $\bar{z}_B(t)$ denote the vector of basic variables in the optimal solution corresponding to the rhs vector $\bar{b}(t)$, we have

$$\overline{z}_B(t) = z_B + t B^{-1} \Delta b \quad . \tag{31}$$

It is clear that the nonbasic variables do not change for values of $t \in [0, t_{\max}]$. The range $[0, t_{\max}]$ is determined by the feasibility constraint on the basic variables:

$$l_B \le \bar{z}_B(t) \le u_B \tag{32}$$

or

$$l_B - z_B \leq tD\Delta b \leq u_B - z_B \quad . \tag{33}$$

Define

$$t_{1} = \min_{j=1,...,m} \left\{ \frac{u_{Bj} - z_{Bj}}{D_{j}\Delta b} : D_{j}\Delta b > 0 \right\}$$

$$t_{2} = \min_{j=1,...,m} \left\{ \frac{l_{Bj} - z_{Bj}}{D_{j}\Delta b} : D_{j}\Delta b < 0 \right\}$$
(34)

We then have

$$t_{\max} = \min\left\{t_1, t_2\right\} \tag{34}$$

If $D_i \Delta b \leq 0$ for all i, i = 1,...,m, then we set $t_1 = +\infty$. Similarly, if $D_i \Delta b \geq 0$ for all i, i = 1,...,m, then we set $t_2 = -\infty$.

If t_{\max} is finite, two situations are possible: either the optimal solution vanishes for $t > t_{\max}$ or a new optimal solution exists for some $t > t_{\max}$. This change of optimal solution is determined by MINOS in the following way.

A shifted rhs vector is determined

$$\bar{b}(t') = b + (t_{\max} + \Delta')\Delta b \tag{35}$$

 Δ' is an appropriately chosen increment (see below). For this rhs vector, MINOS finds the corresponding optimal solution. Next, the value of the rhs vector, the basic vector and the optimal cost at $t = t_{\text{max}}$ are calculated

$$\bar{b}(t_{\max}) = \bar{b}(t') - \Delta' \Delta b \tag{36}$$

$$z_B = B^{-1}\bar{b}(t_{\max}) - B^{-1}(Lz_l + Uz_u + Sz_s)$$
(37)

$$F(z) = c_B B^{-1} \bar{b}(t_{\max}) + (c_l - c_B B^{-1} L) z_l + (c_u - c_B B^{-1} U) z_u$$

$$+ (c_s - c_B B^{-1} S) z_s$$
(38)

where; z_l , z_u , z_s and z_B are the decomposition of the right hand side limit of the optimal solution for $t = t_{max}$. The matrices B, L, U, S are the decomposition of constraint matrix A valid for the optimal solution for $t = t_{max}$.

 Δ ' is computed from

$$\Delta' = DELTA * x \tag{39}$$

where DELTA is given by the user in the keyword PLP INCREMENT and x is the greatest real for which the following inequality is satisfied

$$x \mid D_{i\max} \Delta b \mid \leq TOLX \tag{40}$$

where j_{max} is the subscript for which t_{max} is calculated in formula (34).

2.3. Ranging of bounds.

In every iteration of PLP BOUND the ranging problem has to be solved in the first place. Let col $(\Delta l, \Delta u)$ be a given column vector in $R^{2(n+m)}$, and be such that $\Delta l_i = \Delta u_i = 0$ if y_i is a fixed variable. We consider the family of linear programming problems (A.1) - (A.3) with the vectors of lower and upper bounds l and u replaced by $\overline{l}(t)$ and $\overline{u}(t)$, respectively, where

$$\bar{l}(t) = l + t\Delta l, \quad \bar{u}(t) = u + t\Delta u \tag{41}$$

and $t \in \mathbb{R}^1$. We wish to determine two ranges, $[0, t_{maxa}]$ and $[0, t_{maxb}]$. The first of these intervals is the largest range in which the coefficient t may vary without affecting the optimal solution (i.e., the range of t values for which the optimal solution remains equal to z); the second is the largest range in which t may vary without affecting the optimal basis (i.e., the range of t values for which the optimal basis remains equal to B).

The boundary t_{maxa} is easily determined from the following conditions: for every $t \in [0, t_{maxa}]$

$$t\Delta l_{i}=0 \text{ if } i \in I_{l}$$

$$t\Delta u_{i}=0 \text{ if } i \in I_{u}$$

$$l_{i}+t\Delta l_{i} \leq u_{i} \text{ if } i \in I_{u}$$

$$u_{i}+t\Delta u_{i} \geq l_{i} \text{ if } i \in I_{l}$$

$$l_{i}+t\Delta l_{i} \leq z_{i} \leq u_{i}+t\Delta u_{i} \text{ if } i \in I_{B}.$$

$$(42)$$

The first two conditions imply that $t_{\max} = 0$ if $\Delta l_i = 0$ for some $i \in I_l$ and/or $\Delta u_i = 0$ for some $i \in I_u$.

Let $\overline{z}(t)=z+t\Delta z$ denote the optimal solution corresponding to the vector of bounds col $(\overline{l}(t),\overline{u}(t))$. Then

$$\Delta z_l = \Delta l_l , \ \Delta z_u = \Delta u_u \tag{43}$$
$$\Delta z_R = -D(L\Delta l_l + U\Delta u_u)$$

The values of t_{maxb} may be calculated using the feasibility conditions

$$l_{l}+t\Delta l_{l} \leq u_{l}+t\Delta u_{l} , \ l_{u}+t\Delta l_{u} \leq u_{u}+t\Delta u_{u}$$

$$l_{B}+t\Delta l_{B} \leq z_{B}+t\Delta z_{B} \leq u_{B}+t\Delta u_{B}$$

$$(44)$$

or

$$t(\Delta l_l - \Delta u_l) \le u_l - l_l$$

$$t(\Delta l_u - \Delta u_u) \le u_u - l_u$$
(45)

$$t(\Delta l_B + DL\Delta l_l + DU\Delta u_U) \le z_B - l_B$$
$$t(\Delta u_B + DL\Delta l_l + DU\Delta u_u) \ge z_B - U_B$$

Define

$$t_{1} = \min_{j \in B} \left\{ \frac{u_{j} - l_{j}}{\Delta l_{j} - \Delta u_{j}} : \Delta l_{j} - \Delta u_{j} > 0 \right\}$$

$$t_{2} = \min_{j=1,...,m} \left\{ \frac{z_{B_{j}} - l_{B_{j}}}{\Delta u_{B_{j}} + D_{j}(L\Delta l_{l} + U\Delta u_{u})} : \text{denominator} < 0 \right\}$$

$$t_{3} = \min_{j=1,...,m} \left\{ \frac{z_{B_{j}} - l_{B_{j}}}{\Delta l_{B_{j}} + D_{j}(L\Delta l_{l} + U\Delta u_{u})} : \text{denominator} > 0 \right\}$$

$$(46)$$

Finally,

$$t_{\text{maxb}} = \min\{t_1, t_2, t_3\}$$
 (47)

If the set of indices j over which a minimum is taken is empty, we substitute $+\infty$ for t_1 , t_2 , or t_3 in (46). For instance, if $\Delta l_j - \Delta u_j \leq 0$ for all $j \in B$, we take $t_1 = +\infty$, and so on.

If t_{\max} is finite, two situations are possible: either the optimal solution vanishes for $t > t_{\max}$ or a new optimal solution exists for some $t > t_{\max}$. This change of optimal solution is determined by MINOS in the following way.

The shifted vectors of lower and upper bounds are determined

$$\overline{l}(t') = l + (t_{\text{maxb}} + \Delta')\Delta l$$

$$\overline{u}(t') = u + (t_{\text{submaxb}} + \Delta')\Delta u$$
(48)

where Δ' is an appropriately chosen increment (see below). For these bound vectors, MINOS finds the corresponding optimal solution. Next, the values of the bound vectors, the basic vector and the optimal cost at t = tmaxb are calculated

$$l(t_{\text{maxb}}) = l(t') - \Delta' \Delta l \tag{49}$$
$$\overline{u}(t_{\text{maxb}}) = \overline{u}(t') - \Delta' \Delta u$$

$$u(\iota_{\max b}) = u(\iota) - \Delta \Delta u$$

$$= P^{-1}b P^{-1}(I_{\pi}(t)) + U_{\pi}(t) + S_{\pi})$$
(50)

$$z_{B} = B^{-1}b - B^{-1}(Lz_{l}(t_{maxb}) + Uz_{u}(t_{maxb}) + Sz_{s})$$

$$F(z) = c_{B}B^{-1}b + (c_{l} - c_{B}B^{-1}L)z_{l}(t_{maxb}) + (c_{u} - c_{B}B^{-1}U)z_{u}(t_{maxb})$$
(51)

$$\begin{aligned} f(z) &= c_B B^{-1} b + (c_l - c_B B^{-1} L) z_l(t_{\text{maxb}}) + (c_u - c_B B^{-1} U) z_u(t_{\text{maxb}}) \\ &+ (c_s - c_B B^{-1} S) z_s \end{aligned}$$
(51)

where z_l , z_u , z_s and z_B are the decomposition of the right-hand side limit of the optimal solution for $t = t_{maxb}$. The matrices B, L, U, S are the decomposition of the constraint matrix A valid for the optimal solution $t = t_{maxb}$ and Δ' is computed from:

$$\Delta' = DELTA * x \tag{52}$$

where DELTA is given by the user in the keyword PLP INCREMENT and

$$\boldsymbol{x} = \frac{TOLX}{|f|} \tag{53}$$

f is the denominator of that fraction in the two last definitions (46) which is equal to t_{maxb} .

3. THE METHODS.

3.1. The method of PLP COST.

The algorithm of PLP COST is as follows:

- 1. Set i := 0, $t_i := 0$.
- 2. MINOS finds the optimal solution for t_i with the basic matrix B and the basic vector z_B .
- 3. The boundary value of the parameter t_{i+1} is calculated, such that for all $a_0(t), t \in [t_i, t_{i+1})$ the optimal solution is constant. The set I_{∞} of nonbasic variables is determined, containing all nonbasic variables for which reduced costs:

$$a_0^{k}(t) - \tilde{a}_B(t)B^{-1}\tilde{A}^k \tag{54}$$

where \tilde{A}_k is the k-th column of the constant matrix \tilde{A} (see (7)), reach zero for some t in the interval $[t_i, t_i + 10^{-9})$. These variables are nonbasic in the decomposition valid for $t = t_i$.

4. Next, the value t' of the parameter is determined:

$$t' = t_{i+1} + \Delta', \, \Delta' = DELTA *\Delta, \, DELTA > 1$$
(55)

where Δ is the greatest increment of the parameter such that for $t = t_{i+1} + \Delta$ the nonbasic variable whose reduced cost reaches zero at t_{i+1} is still recognized by MINOS as optimal.

5. New cost vector is computed:

$$a_0(t') = a_0(t_i) + (t' - t_i)\Delta a_0$$
(56)

- 6. MINOS finds the new optimal solution for the new cost vector $a_0(t')$.
- 7. Set $t_i := t_{i+1}$ and shift the cost vector back to t_i

$$a_0(t_i) := a_0(t') - \Delta' \Delta a_0 \tag{57}$$

8. Set i := i+1 and go to 3.

3.2. The method of PLP RHS.

The algorithm of PLP RHS is as follows:

- 1. Set i := 0 and $t_i := 0$.
- 2. MINOS finds the optimal solution for t_i with the basic matrix B and basic vector z_B . At the same time it finds the optimal decomposition into basic and nonbasic variables.
- 3. The boundary value of the parameter t_{i+1} is calculated (see section 2.2), such that for all $\tilde{b}(t)$, $t \in [t_i, t_{i+1})$ the optimal basis (basic matrix) is constant and equal to B. The set I_{\sum} of the basic variables is determined containing all basic variables which reach their bounds for some value of t in the interval $[t_{i+1}, t_{i+1} + 10^{-9}]$. These variables are basic in the decomposition valid for $t = t_i$.
- 4. Next, the value t' of the parameter is determined

$$t' = t_{i+1} + \Delta', \ \Delta' = DELTA^*\Delta, DELTA > 1$$
(58)

where Δ is the greatest increment of the parameter such that for $t = t_{i+1} + \Delta$ the basic variable which reaches its bound at t_{i+1} is still recognized by MINOS as feasible.

5. New rhs vector is computed

$$\tilde{b}(t') = \tilde{b}(t_i) + (t' - t_i) \Delta \tilde{b}$$
⁽⁵⁹⁾

and the corresponding starting basic solution

$$z_B(t') = z_B(t_i) + (t' - t_i)B^{-1}\Delta \vec{b}$$
(60)

- 6. MINOS finds the optimal solution for the new rhs vector $\bar{b}(t')$, starting from the shifted basic solution (60) which is infeasible. The new optimal basis is denoted by B and the new basic vector by $z_B(t')$
- 7. Set $t_i := t_{i+1}$ and shift the solution back to t_i ,

$$z_B(t_i) = z_B(t') - (t' - t_i)B^{-1}\Delta \tilde{b}$$
(61)

also

$$\tilde{b}(t_i) = b(t') - (t' - t_i) \Delta \tilde{b}$$
(62)

8. Set i := i + 1 and go to (3).

3.3. The method of PLP BOUND

The algorithm of PLP BOUND is as follows:

- 1. Set i := 0 and $t_i := 0$
- 2. MINOS finds the optimal solution for t_i with the basic matrix B and the basic vector z_B . At the same time it finds the optimal decomposition into the basic and nonbasic variables.
- 3. The boundary value of the parameter t_{i+1} is calculated (see section 2.3), such that for all $\tilde{l}(t)$ and $\tilde{u}(t)$, $t \in [t_i, t_{i+1})$ the optimal basis (basic matrix) is constant and equal to *B*. The set I_{\sum} of basic variables is determined, containing all basic variables which reach their bounds for some value of *t* in the interval $[t_{i+1}, t_{i+1} + 10^{-9}]$. These variables are basic in the decomposition valid for $t = t_i$.
- 4. Next, the value t' of the parameter is determined

$$t' = t_{i+1} + \Delta'$$
, $\Delta' = DELTA^*\Delta$, $DELTA > 1$ (63)

where Δ is the greatest increment of the parameter such that for $t = t_{i+1} + \Delta$ the basic variable which reaches its bound at t_{i+1} is still recognized by MINOS as feasible.

5. New bound vectors are computed:

$$\tilde{l}(t') = \tilde{l}(t_i) + (t' - t_i)\Delta \tilde{l}$$

$$\tilde{u}(t') = \tilde{u}(t_i) + (t' - t_i)\Delta \tilde{u}$$
(64)

and the corresponding starting basic solution:

$$\mathbf{z}_{B}(t') = \mathbf{z}_{B}(t_{i}) + (t' - t_{i})B^{-1}(L\Delta \tilde{l} + U\Delta \tilde{u}).$$

$$\tag{65}$$

MINOS finds the optimal solution for the new bound vectors, starting from the shifted basic vector (65) (which is infeasible). The optimal basis is denoted by B and the basic vector by $z_B(t')$.

7. Set $t_i := t_{i+1}$ and shift the solution back to t_i

$$z_B(t_i) = z_B(t') - (t'-t)B^{-1}(L\Delta \tilde{l} + U\Delta \tilde{u}).$$
(66)

- 13 -
- 8. Set i := i+1 and go to 3.

B. USER MANUAL

1. BRIEF CHARACTERIZATION OF BASIC FUNCTIONS OF PLP.

1.1. Parametric analysis of cost (PLP COST).

The cost vector $a_0 = (a_0^1, a_0^2, \dots, a_0^n)$ (see (1)) is changed along a direction given by the user, $\Delta a_0 = (\Delta a_0^1, \Delta a_0^2, \dots, \Delta a_0^n)$ according to the formula:

$$a_0(t) = a_0(0) + t\Delta a_0, \ t \ge 0 \tag{67}$$

where $a_o(0)$ is the starting value of cost. If the structural variable, say \tilde{x}_i , is fixed then Δa_0^{i} is automatically set to zero, regardless of the value given in the data.

PLP determines a sequence of values of the parameter denoted by t_0, t_1, \ldots, t_k , such that $0 = t_0 < t_1 < t_2 < \cdots < t_k$ and in each of the intervals $[t_i, t_{i+1}), i = 0, \ldots, k-1$ the optimal solution is constant and in each case the optimal basis is different. The integer k: (1) may be defined by the user as the maximum number of iterations, (2) may be determined by the condition that the optimal solution is constant for every $t \ge t_k$ and different from that in $[t_{k-1}, t_k)$, (3) may be determined by the condition that there are no optimal solutions for every $t > t_k$.

1.2. Parametric analysis of rhs (PLP RHS).

The right-hand side $\tilde{b} = \operatorname{col}(\tilde{b_1}, \ldots, \tilde{b_{m+1}})$ $\tilde{b} = \operatorname{col}(\tilde{b_1}, \ldots, \tilde{b_{m\pm 1}})$ (see (7) and (8)) is changed along a direction given by the user, $\Delta \tilde{b} = \operatorname{col}(\Delta \tilde{b_1}, \ldots, \Delta \tilde{b_{m+1}})$, according to the formula:

$$\tilde{b}(t) = \tilde{b}(0) + t\Delta \tilde{b} , t \ge 0$$
(68)

where $\tilde{b}(0)$ is the starting value of rhs. The component of $\Delta \tilde{b}$ which corresponds to the objective row is automatically set to zero, $\Delta \tilde{b}_{obj} = 0$.

PLP determines a sequence of values of the parameter denoted by t_0, t_1, \ldots, t_k such that $0 = t_0 < t_1 < t_2 < \cdots < t_k$ and in each of the intervals $[t_i, t_{i+1}), i = 0, \ldots, k-1$ the optimal basis is constant and in each case different. The integer k : (1) may be defined by the user as the maximum number of iterations, (2) may be determined by the condition that the optimal basis is constant for every $t \ge t_k$ and different from that in $[t_{k-1}, t_k), (3)$ may be determined by the condition that there are no feasible solutions for every $t > t_k$.

1.3. Parametric analysis of bounds (PLP BOUND).

The vector of bounds $\operatorname{col}(\tilde{l}, \tilde{u}) \in R^{2(n+m+2)}$ (see (9)) is changed along a direction given by the user, $\operatorname{col}(\Delta \tilde{l}, \Delta \tilde{u})$, according to the formula:

$$\operatorname{col}(\tilde{l}(t), \, \tilde{u}(t)) = \operatorname{col}(\tilde{l}(0), \, \tilde{u}(0)) + t \, \operatorname{col}(\Delta \tilde{l}, \, \Delta \tilde{u}) \,, \, t \ge 0 \tag{69}$$

where $\operatorname{col}(\tilde{l}(0), \tilde{u}(t))$ is the starting value of bounds. The bound increments $\Delta \tilde{l}_i \Delta \tilde{u}_i$ which correspond to fixed variables are automatically set to zero regardless of the values given in the data.

If there is no lower and/or upper bound for the i-th variable \tilde{x}_i (see (6)) the corresponding increment $\Delta \tilde{l}_i$ and/or $\Delta \tilde{u}_i$, respectively, is also automatically set to zero.

PLP determines a sequence of values of the parameter denoted by t_o, t_1, \ldots, t_k such that $0 = t_0 < t_1 < t_2 < \cdots < t_k$ and in each of the intervals $[t_i, t_{i+1}), i = 0, \ldots, k-1$ the optimal basis is constant and in each case different. The integer k : (1) may be defined by the user as the maximum number of iterations, (2) may be determined by the condition that the optimal basis is constant for every $t \ge t_k$ and different from that in $[t_{k-1}, t_k)$, (3) may be determined by the condition that there are no feasible solutions for every $t > t_k$.

Each interval $[t_i, t_{i+1}]$ is optionally divided into two subintervals $[t_i, t_i^a], [t_i^a, t_{i+1}]$. The interval $[t_i, t_i^a]$ is the maximum interval where the optimal solution remains constant and not only the optimal basis. It often happens that $t_i = t_i^a$.

1.4. Dependent and independent work.

All three kinds of analysis can be performed in one run. The starting point for the next kind of analysis may be either the original starting optimal solution (The Independent Work) or the last optimal solution obtained in the preceding analysis (The Dependent Work). The continuation is impossible if the optimal solution vanishes.

1.5. Controlling output.

In each of the three kinds of analysis the following information is available. The user has to specify the frequency of printing the complete current optimal solution in MINOS format. This means that the complete printout is given for the values of parameters tequal to t_{0+} , t_{p+} , t_{2p+} ,..., and $t_{(k-1)+}$ or t_{k+} depending on whether the optimal solution exists for $t > t_k$. The notation t_{i+} means that we take the right-hand limit of the optimal solution at t_i . The user specifies frequency of printing the so called PLP ITERATION LOG. This is a short message containing most important information about current change of optimal solution (value of the parameter t, change of optimal basis, current value of objective function).

1.6. Tolerances.

The performance of PLP is strongly affected by the choice of tolerances. Especially important are two tolerances determined in MINOS : the tolerance of optimality (TOLD) and the tolerance of feasibility (TOLX). In the proper procedures of the PLP the following general rule is adopted. All quantities greater than or equal to 1.E+15 are taken as equal to infinity and all quantities whose absolute value is less than 1.E-9 are regarded as equal to zero.

2. INPUT

The input contains all necessary elements for MINOS with the conditions given below.

Key		Default	Meaning		
PLP ANALY	COST SIS n	off	This keyword activates the parametric analysis of cost. The integer n is the number of iterations to be performed. If no value or a zero value of n is given, all iterations will be performed (until the optimal solution becomes constant or the optimal solution vanishes).		
PLP ANALY	RHS SIS n	off	This keyword activates the parametric analysis of rhs. The integer n is the number of iterations to be performed. If no value or a zero value of n is given, all iterations will be performed (until the optimal basis becomes constant or the optimal solution van- ishes).		

2.1 .	New	key-words	in the	SPECS	file
--------------	-----	-----------	--------	-------	------

PLP BOUND ANALYSIS n	off	This keyword activates the parametric analysis of bounds. The absolute value of integer n is the number of iterations to be performed. If n is less than zero an additional output is printed in each iteration which gives the values of $t_i^a - t_i$ and $t_{i+1} - t_i$ and the corresponding boundary values of bounds.
PLP ORDER	off	This keyword activates the dependent work of PLP. If it does not occur, PLP performs each of the required kinds of analysis only once (keywords PLP COST, PLP RHS, PLP BOUND). In each of these, the starting point is the original optimal solu- tion. If PLP ORDER appears in the SPECS FILE, it must precede the sequence of keywords PLP COST, PLP RHS, PLP BOUND, which define the kinds of analysis to be performed in the same order. For each kind of analysis, the starting point is the last optimal solution obtained in the last analysis. If the optimal solution vanishes, the run stops. Each kind of analysis can be performed up to five times, in an arbitrary order (determined by the sequence of keywords PLP COST, PLP RHS, PLP BOUND). In each repetition of the same kind of analysis, the search direction and the max- imum number of iterations must be the same. The value of n given in the last keyword referring to a particular kind of analysis is valid for all its repeti- tions.
PLP SOLN n	n = 1	This keyword specifies the frequency of printing the current complete solution in the MINOS format. Full solution is printed after every n iterations. If this keyword is omitted or $n = 0$, the effect is the same as for $n = 1$.
PLP FREQUENCY n	n = 1	This command activates the frequency of printing the short message called PLP ITERATION LOG (see section 3 of USER MANUAL). A PLP ITERA- TION LOG is printed after every <i>n</i> iterations. If this keyword is omitted or $n = 0$, the effect is the same as for $n = 1$.
PLP SOLU- TION n	off	If this (optional) keyword is used with $n \ge 0$ complete outputs of optimal solution will be stored in file n with the frequency given in PLP SOLN m . If $n=0$ or this keyword does not occur, the complete outputs are stored in the printer file.
PLP FILE n	n = 5	The absolute value of n is the logical number of the data file for parametric programming. This file is read after processing other MINOS files has been completed. The parameter n also controls the output of the search directions. If n is less than zero, the search direction of each PLP analysis is printed. These directions are not printed for any other entry.

PRINT DATA PLP FILE	off	If this keyword is used, the whole DATA PLP FILE will be printed in the output. Otherwise, only the records with comments and the records NAME, SET and ENDATA are printed.		
PLP INCREMENT d	d = 1.1	This keyword specifies the value of factor Δ (see sections 3.1, 3.2 and 3.3 of THEORETICAL GUIDE). (4.2), (4.3)).		

2.2. DATA PLP file - input format.

The data for the PLP procedures are prepared in an MPS-like format and placed in the file specified by the key-word DATA PLP FILE n. The data sets for different PLP procedures may be given in any order. The beginning of the data set for each procedure is identified by the line NAME and its end by the line ENDATA. If it occurs, the line 'SET' must be given immediately after the line NAME in each data set; this line defines the default values of all the variables which are not explicitly defined. Every data set is identified by the name given in the line NAME.

The records in the DATA RANGING FILE should have the following (basic) form, which is analogous to MPS format:

Columns:	1-4,	5-12,	15-22,	25-36,	40-47,	5061
Fields:	f1,	f2,	f3,	f4,	f5,	f6

Below we give a detailed description of the data set for each parametric programming procedure.

Parametric analysis of cost (PLP COST)

	f1	f2	f3	f4	f5	f6
1. 2. 3. 4.	NAME 'SET' ENDATA	С	PLPC Comments Col. name	Value Value	Col. name	Value

Parametric analysis of rhs (PLP RHS)

	f1	f2	f3	f4	f5	f6
1.	NAME		PLPR			
2.	'SET'	0	Comments	Value		
3.			Row name	Value	Row name	Value
4.	ENDATA					

Parametric analysis of bounds (PLP BOUNDS)

	f1	f2	f3	f4	f5	f6
1.	NAME		PLPB			
2.	'SET'	Co	omments	Value		
3.		LOWER	Row/col. name	Value	Row/col. name	Value
4.		UPPER	Row/col. name	Value	Row/col. name	Value
5.	ENDATA					

Remarks:

- if field f2 in a given record is empty, this means that it is the same as in the previous record. Field f2 must not be empty in the first data record,
- the records with identifiers UPPER and LOWER may appear in any order,
- LOWER is used for increments of the lower bounds and UPPER for increments of the upper bounds.

The following general rules apply to all data sets:

- One of the fields f3, f5, (f4, f6) may be empty.
- If 'SET' appears, it must follow immediately after NAME. If 'SET' does not occur, the default for all variables whose values are not specified is zero. This has the same effect as:

'SET' 0.

- Comments may be entered in arbitrary positions in the data set. They are identified by an asterisk * in the first column.
- The values should be written as real numbers in a format accepted by FORTRAN.

2.3. Specification of zeros in the MPS file.

In two kinds of parametric analysis, PLP COST and PLP RHS the user has to specify explicitly some of the zero values of the objective row elements (vector a_0) and/or the rhs column elements (vector \tilde{b}), exactly in the same way as the nonzero values specified in the data (MPS file). This refers to those elements of the vector a_0 and/or \tilde{b} for which the corresponding elements of Δa_0 and/or $\Delta \tilde{b}$, respectively, are different from zero.

- Example $a_0 = (1.,0.,0.,3.,5.)$, $\Delta a_0 = (-1.,0.,0.1,1.,0.)$ In this case the element a_0^3 has to be explicitly specified in MPS
 - x3 obj 0.

where x_3 is the name of the third column (structural variable) and *obj* is the name of the objective row.

3. OUTPUT

The title of the output of PLP is:

P L P VERSION 1.0 JUNE 1986

In the case of dependent work of PLP the subtitle is printed:

DEPENDENT WORK OF PLP

Otherwise, this subtitle is omitted.

The output may be sent either to the printer file or to the file defined by the keyword PLP SOLUTION FILE. Only the output produced by the procedure SOLN of MINOS can be stored in the latter one.

Since the SOLN output is described in MINOS manuals we will confine ourselves to the output of PLP sent to the printer file, and so we will also skip the messages given by MINOS.

Each kind of parametric analysis procedures produces a printout containing the following information.

Title:

PLP COST	- for parametric analysis of cost
PLP RHS	- for parametric analysis of rhs
PLP BOUND	- for parametric analysis of bounds

Search direction (optionally):

For PLP COST it has the following format. For each structural variable \tilde{x}_i , i = 1, ..., n the following information is given:

- NUMBER Number of structural variable
- COLUMN Name of structural variable

DIRECTION - Increment component Δa_0^i

OBJ GRADIENT- Cost component a_0^i

M+J -m + 1 + i

In the case of PLP RHS the following information is given for each row (or each slack variable \tilde{x}_i , i = n+2, ..., n+m+2) except for the objective row (or slack variable \tilde{x}_{n+1} +obj):

NUMBER	- Number of slack variable
ROW	- Name of row
DIRECTION	- Component $\Delta \tilde{b_i}$ of increment vector
RHS	- Right-hand-side component $ ilde{b_i}$
I	- Row number

For PLP BOUND this printout is divided into two sections:

SECTION 1 - ROWS contains the following information for each slack variable \tilde{x}_i , $i = n+2, \ldots, n+m+2$ (or for each row), except for the slack variable \tilde{x}_{n+1+} obj which corresponds to the objective row:

NUMBER - Number of slack variable

ROW - Name of row

LL DIRECTION- Component $\Delta \tilde{l}_i$ of the lower bound inc. vector $\Delta \tilde{l}$

LOWER LIMIT- Lower bound l_i

UL DIRECTION- Component $\Delta \tilde{u}_i$ of the upper bound inc. vector $\Delta \tilde{u}$

UPPER LIMIT- Upper bound \tilde{u}_i

I - Row number

SECTION 2 - COLUMNS contains information analogous to that described above for each structural variable \tilde{x}_i , i = 1, ..., n with the following differences:

NUMBER - Number of structural variable

COLUMN - Name of structural variable

M+J -m+1+i

PLP iteration log printing:

Printing frequency is given in the keyword PLP FREQUENCY. It takes one of the following forms:

If only one variable in the optimal basis has been exchanged and none of the nonbasics has changed its state, the following message is printed:

PITN - Number of iteration of current parametric analysis

OBJ - Objective value

TMAX - Current boundary value of parameter t

VARIABLE "name" (number of the variable) FROM "bound" REPLACES BASIC VARI-ABLE "name" (number of the variable) WHICH PASSES TO "bound"

(LL is substituted for "lower bound" and UL for "upper bound")

In other cases the first three items are the same as above and the last row is replaced by the appropriate number of the following sentences:

VARIABLE"name" (number of the variable) FROM "bound" ENTERS THE BASIS

BASIC VARIABLE"name" (number of the variable) PASSES TO "bound"

VARIABLE"name" (number of the variable) FROM "bound" PASSES TO "bound"

If a variable which does not belong to I_{\sum} has changed its state, this row is preceded by the following message:

WITHIN THE GIVEN TOLERANCE ONLY THE FOLLOWING INFORMATION IS AVAILABLE

Special messages

1. If in the final iteration the situation arises in which the optimal basis is constant for every $t > t_{max}$, the following message appears in the printer file:

PITN - Number of iteration of current parametric analysis - FOR THE VALUE OF THE PARAMETER = Value of t_i INFINITE RANGE (TMAX.GE.1.E15)

where $\text{TMAX} = t_{i+1} - t_i$. In this case the last optimal solution is stored in the printer file or in the file defined by the keyword PLP SOLUTION FILE.

- 2. If the optimal solution vanishes, one of the following MINOS messages is printed:
 - in the case of PLP COST:

EXIT - PROBLEM IS UNBOUNDED

this is followed by:

PITN - Number of iteration of current parametric analysis TMAX= Boundary value of the parameter t

- for PLP RHS and PLP BOUND:

EXIT - PROBLEM IS INFEASIBLE NO. AND SUM OF INFEASIBILITIES "number" and "value"

This is followed by:

PITN = Number of iteration of current kind of analysis TMAX = Boundary value of parameter

In both cases the SOLN output corresponding to the value t_{i+1} of parameter t is printed or stored in the file defined by the user in the keyword PLP SOLUTION FILE.

3. If MINOS cannot find the next optimal solution because of tolerances defined in MINOS, the following printout is displayed:

WITHIN THE GIVEN TOLERANCE NO NEW BASIS IS FOUND

This is a failure of the package. In order to continue the analysis, the user should decrease the appropriate tolerance (tolerances) in MINOS or to increase the factor DELTA in keyword PLP INCREMENT.

4. If the keyword PLP BOUND ANALYSIS n is less than zero an additional output is printed. It gives the values: $t_i^a - t_i = t_{maxa}$, $t_{i+1} - t_i = t_{maxb}$ and the corresponding boundary values of bounds:

PITN = Number of iteration of current kind of analysis TMAX = boundary value of parameter

This is followed by the information on t_{maxa} .

4. EXAMPLES

We shall now illustrate the performance of PLP using a simple example. The linear programming problem is as follows :

Maximize

 $F(x)=0.1x_1+x_2$

subject to:

 $x_1 + x_3 = 3.$ 0.7065 $(x_1 + x_2) + x_4 = 3.826$

$$\begin{aligned} x_2 + x_5 &= 3. \\ -0.7065(x_1 - x_2) + x_6 &= 1. \\ -x_1 + x_7 &= -1 \\ 0 &\le x_1 , x_2 &\le 5 , 0 &\le x_i &\le 2 , i &= 3, \dots, 7 \end{aligned}$$

Two runs of PLP are presented. The first shows the independent work of PLP. It contains all three kinds of parametric analysis: PLP COST, PLP RHS, PLP BOUND. In the second, we have the results of dependent work of PLP. The task for PLP was to perform one iteration of PLP RHS, then all iterations of PLP COST and then all iterations of PLP BOUND.

Below we give the MPS file common for both runs and then we give the MINOS and PLP specifications used to solve each of these problems.

Then we give the standard MINOS printout, followed by two outputs of PLP.

	test	
TOWE		
n ob		
e ri		
• r 2		
• r 3		
• r 4		
• r5		
columns		
x1	ob	0.1
x1	r1	1.
x1	r2	.7065
x1	r 4	7065
x1	гб	-1.
x2	ob	1.
x2	r2	.7065
x2	r3	1.
x2	r 4	.7065
x3	ri	1.
x4	r2	1.
хб	r3	1.
x6	r 4	1.
x 7	rб	1.
rhs		
rh	ri	3.
rh	r2	3.826
rh	r 3	3.
rh	r4	1.
rh	rб	-1.
bounds		
nd bo	x 2	
up bo	x 2	5.
up bo	ත්	2.
up bo	201	2.
up bo	жб	2.
up bo	305	2.
up bo	x7	2.
endata		

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specs file -----

begin	Second P L P Test
	meximize
	plp rhs analysis
	plp cost analysie
	plp bound analysis -10
	plp increment 1.5
	plp frequency 1
	data plp file -9
	print data plp file

end

problem name		ter	t	c	objective value 3		3.24164282164+00				
sta	tus		opt	imal	soln i	iteration 1		uperbasics	0		
obj	ectiv		ob		(max)						
rhe	1		rh								
гал	ges										
bou	nds		ъо								
500	tion	1 - 2	-045								
מח	mber	1		at	activity	slack activit	ty	lower limit.	upper limit.	.dual activity	i
	9	ob		bs	3.24154	-3.2415	54	none	none	1.00000	1
a	10	r 1		eq	3.00000	. 0000	20	3.00000	3.00000	. 00000	2
	11	r 2		eq	3.82600	.0000	00	3.82600	3.82600	14154	3
	12	r3		eq	3.00000	.0000	00	3.00000	3.00000	90000	4
a	13	r 4		eq	1.00000	.0000	00	1.00000	1.00000	. 00000	5
8	14	r 5		eq	-1.00000	.0000	00	-1.00000	-1.00000	.00000	6
500	tion	2 - 6	olumn	6							
טת	mber	.co]	uun.	at	activity	.obj gradient	t	lower limit.	upper limit.	reduced gradnt	m +j
	1	x 1		bs	2.41543	. 1000	00	.00000	none	.00000	7
	2	x2		bs	3.00000	1.0000	00	none	Б.00000	. 00000	8
	3	x3		bs	.58457	. 0000	00	.00000	2.00000	. 00000	9
	4	324		ц	.00000	.0000	00	. 00000	2.00000	14154	10
	5	хđ		11	.00000	. 0000	00	.00000	2.00000	90000	11
	6	хđ		bs	.58700	.0000	00	.00000	2.00000	. 00000	12

.00000

.00000

.00000

-1.00000

2.00000

-1.00000

.00000 13

-3.24154 14

PLP ~ - - varsion 1.0 june 1986, = = =

1.41543

-1.00000

data plp file -----

7 x7

8 rh

bs

eq

1	n at	me plpc		
2	18	et '	.00000003+00	
3		x1	1.000003+00	00+b00000.
4		x2	-1.00000d+00	.00000d+00
Б	*			
6	*	Note:		
7	*	Declaration of	dummy coefficients (=0	in MPS file) of the objective
8	*	is not necessar	ry because the above dir	ection is defined in the x1-x2
9	*	subspace of co	st vectors.	
10	*	-		

11 endata

plp	cost

		-		1 0000	1000	n 7			
	1 2	уд УД		-1 00000	1 0000	, / , e			
	3	x3		.00000	.0000	. J J 0			
	4	x4		.00000	.00000	5 10			
	Б	x6		.00000	.00000) 11			
	6	ක්		. 00000	.0000) <u>12</u>			
	7	x7		.00000	.00000) 13			
pita	n= 1	L 4	obj= 0.2	97848561+01	tamaar 2)	= 0.4	50004+00	variable x5	(5) from 1
rep			C VERIEDIO		3) which pa	see to	11		
broj	blem n		test		objective v	alue	2.97848556324+	00	
stai	tus		optimal so	ln	iteration	1	superbasics	0	
obje	ective	1	ob	(max)					
rhs			rh						
ranį bou	ges nds		Ъо						
8.0~ ⁻¹	tion 1		-						
	where		ow at	activity	alack act	ivity	lower limit	noner limit	dual activity
nu					aldun acu				
	9	olo	be	2.97849	-2.9	37819 	RORO	none	1.00000
	10	r1 	eq	3.0000		2000	3.00000	3.00000	- 77848
	12	r∡ 73	eq.	3 00000		00000	3 00000	3.00000	0 00000
a	13	r4	e0	1.00000		00000	1.00000	1.00000	.00000
a	14	гб	•q	-1.00000).	00000	-1.00000	-1.00000	.00000
sect	tion 2	t - c	oluma						
שות	mber	.col	umm. at	.activity	.obj grad	ient.	lower limit.	upper limit.	reduced gradmi
	1	x1	bs	3.00000) .1	55000	.00000	none	. 00000
	2	x2	bs	2.41543	i .i	55000	none	5.00000	. 00000
	3	хЗ	11	.00000). (00000	.00000	2.00000	0.00000
	4	x4	ц	.00000). (00000	. 00000	2.00000	77848
	Б	36	bs	58457		00000	.00000	2.00000	0.00000
,	6	365 	be	1.41300		00000	.00000	2.00000	.00000
d	7	४/ मो	be eq	2.0000) .	00000	.00000	2.00000	.00000
	-								
pit: rep:	n= 2 laces	basi	obj= 0.3 c variable	2999950d+01 x66 (tmax 6) which pa	= 0.1 sses to	0000d+01 ul	variable x4	(4) from]
pro	blem r		test		objective v	alue	3.299995009614	00	
stai	tus		optimal so	ln	iteration	2	superbasics	0	
oh i	ectiv		-	(****)	I		-		
rhs		-	rh	ر اللسب					
TEN	ges								
	• · · ·								

משנית	ber	го	r a t	activity	<pre>slack activity</pre>	lower limit.	upper limit.	dual activity	i
	9	ob	ba	3.30000	-3.30000	none	Rone	1.00000	1
	10	r 1		3 00000	00000	3 00000	3 00000	-1 09999	2
	11			3.82600	.00000	3 82600	3.82600	00001	3
a	12	r 3		3.00000	.00000	3.00000	3.00000	.00000	4
-	13	r 4	~1	1 00000	.00000	1 00000	1 00000	0,00000	5
	14		~	-1.00000	.00000	-1 00000	-1.00000	0,00000	
•	14	10	•4	-1.0000		-1.0000	-1.0000		0
sect	ion	2 - col	umns						
num	ber	.colum	m. at	activity	.obj gradient.	lower limit.	upper limit.	reduced gradnt	≞+j
	1	x 1	bs	3.00000	1.10000	.00000	none	.00000	7
	2	x2	bs	1.58457	0.00000	none	5.00000	.00000	8
	3	хЗ	11	.00000	.00000	.00000	2.00000	-1.09999	9
	4	304	bs	.58700	.00000	.00000	2.00000	00001	10
	Б	x65	bs	1.41543	.00000	.00000	2.00000	.00000	11
	6			2.00000	.00000	.00000	2 00000	0.00000	12
a	7		ha	2 00000	00000		2 00000	00000	13
•	8	rh	•9	-1.00000	.00000	-1.00000	-1.00000	-3,30000	14
							<i>.</i>		
pitn	=	3 10	xr the v	alue of the pare	meter = 1.	.00000 infinite	range (tmax.ge.1		
data	חות	file							
	12	name		plpr					
	13	'set'		.00	000001+00				
	14			r3 1.0	00003+00	.00000d+0	o		
	15	*							
	16	* N	lote the	it all components	of the vector ar	e defined in MP	S file.		
	17	*		•					
	18	endat	a						
plp	rhs								
num	ber	ro	1	direction	rhsi	i			
	10	 1		00000					
	44				3 00000 5	,			
	11	r a		~~~~	3.00000 2	2			
	14	-9		.00000	3.00000 2 3.82600 3	2 3			
		r3		.00000	3.00000 2 3.82600 3 3.00000 4	2 3 1			
	13	r3 r4		.00000 1.00000 .00000	3.00000 2 3.82600 3 3.00000 4 1.00000 6	2 3 1 3			
	13 14	г3 г4 г5		.00000 1.00000 .00000 .00000	3.00000 2 3.82600 3 3.00000 4 1.00000 6 -1.00000 6	2 3 1 5 3			
pitn	13 14 = 1	r3 r4 r5	obi=	.00000 1.00000 .00000 .00000	3.00000 2 3.82600 3 3.00000 4 1.00000 6 -1.00000 6 tamax= 0.4	2 3 1 5 5 115434+00	variable 15	(5) from 11	
pitn repl	13 14 = 1 aces	r3 r4 r5 basic	obj= variab]	.00000 1.00000 .00000 .00000 0.36154282d+01 a x6 (6	3.00000 2 3.82600 3 3.00000 4 1.00000 6 -1.00000 6 tmmx= 0.4	2 3 4 5 5 115434+00 11	variable x5	(5) from 11	
pitn repl prob	13 14 = 1 aces 1en	r3 r4 r5 basic	obj= variabl test	.00000 1.00000 .00000 .00000 0.36154282d+01 (a x6 (6	3.00000 2 3.82600 3 3.00000 4 1.00000 6 -1.00000 6 tmax= 0.4	2 3 4 5 5 11543d+00 5 11 3.6154281824d+	variable x5 ©	(5) from 11	
pitn repl prob	13 14 = 1 aces 1em us	r3 r4 r6 basic	obj= variab] test optima]	.00000 1.00000 .00000 0.36154282d+01 1 x6 (6 . soln i	3.00000 2 3.82600 3 3.00000 4 1.00000 6 -1.00000 6 tmax= 0.4) which passes to bjective value teration 1	2 3 4 5 5 11543d+00 > 11 3.6154281824d+ superbasics	variable x5 00	(5) from 11	
pitn repl prob stat	13 14 = 1 aces 1em us	r3 r4 r5 basic	obj= variab] test optima]	.00000 1.00000 .00000 0.36154282d+01 1a x6 (6 .soln i	3.00000 2 3.82600 3 3.00000 4 1.00000 6 -1.00000 6 tmax= 0.4) which passes to bjective value teration 1	2 3 4 5 5 11543d+00 5 11 3.6154281824d+ superbasics	variable x5 00 0	(5) from 11	
pitn repl prob stat	13 14 = 1 aces 1em us ctiv	r3 r4 r5 basic name	obj= variabl test optimal ob	.00000 1.00000 .00000 0.36154282d+01 0 x6 (f soln i (max)	3.00000 2 3.82600 3 3.00000 4 1.00000 6 -1.00000 6 tmmx= 0.4) which passes to bjective value teration 1	2 3 4 5 5 3 11543d+00 5 11 3.6154281824d+ superbasics	variable x5 ©	(5) from 11	
pitn repl prob stat obje rhs	13 14 = 1 aces len us	r3 r4 r5 basic	obj= variabl test optimal ob rh	.00000 1.00000 .00000 0.36154282d+01 0 x6 (6 0 soln i (max)	3.00000 2 3.82600 3 3.00000 4 1.00000 6 -1.00000 6 tmmx= 0.4) which passes to objective value teration 1	2 3 4 5 5 3 11543d+00 5 11 3.6154281824d+ superbasics	variable x5 ©	(5) from 11	
pitn repl prob stat obje rhs rang	13 14 = 1 acces lem us ctiv	r3 r4 r5 basic	obj= variabl test optimal ob rh	.00000 1.00000 .00000 0.36154282d+01 0 x6 (6 1 soln i (max)	3.00000 2 3.82600 3 3.00000 4 1.00000 6 -1.00000 6 tmmx= 0.4) which passes to objective value .teration 1	2 3 4 5 5 3 4 5 5 3 4 15 4 3 4 5 5 3 4 15 4 3 4 4 4 5 5 5 3 4 5 5 5 5 5 4 15 4 3 4 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	variable x5 00	(5) from 11	
pitn repl prob stat obje rhs rang boun	13 14 = 1 acces lem us ctiv es ds	r3 r4 r5 basic name	obj= variabl test optimal ob rh bo	.00000 1.00000 .00000 0.36154282d+01 ie x6 (6 i soln i (max)	3.00000 2 3.82600 3 3.00000 4 1.00000 6 -1.00000 6 tmax= 0.4) which passes to bjective value teration 1	2 3 4 5 5 11543d+00 5 11 3.6154281824d+ superbasics	variable x5 00	(5) from 11	
pitn repl prob stat obje rhs rang boun sect	13 14 = 1 acces lem us ctiv es ds ion	r3 r4 r5 basic name	obj= variabl test optimal ob rh bo	.00000 1.00000 .00000 0.36154282d+01 0 x6 (f 0 soln i (max)	3.00000 2 3.82600 3 3.00000 4 1.00000 6 -1.00000 (tmax= 0.4) which passes to bjective value teration 1	2 3 4 5 8 11543d+00 • 11 3.6154281824d+ superbasics	variable x5 ©	(5) from 11	
pitn repl stat obje rhs rang boun sect	13 14 = 1 aces len us ectiv ves ds ion ber	r3 r4 r5 basic name 1 - row	obj= variabl test optimal ob rh bo me	.00000 1.00000 .00000 0.36154282d+01 0 x6 (6 0 soln i (max)	3.00000 2 3.82600 3 3.00000 4 1.00000 6 -1.00000 (tmax= 0.4) which passes to bjective value teration 1 slack activity	2 3 4 5 6 11543d+00 5 11 3.6154281824d+ superbasics lower limit.	<pre>variable x5 00 0upper limit.</pre>	(5) from 11 .dual activity	i
pitn repl grob stat cbje rhs rang boun sect num	13 14 = 1 acces lem us ctiv ves ds ion ber 9	r3 r4 r5 basic name 1 - row row	obj= variabl test optimal ob rh bo r. at bs	.00000 1.00000 .00000 0.36154282d+01 Le x6 (f L soln i (max) activity	3.00000 2 3.82600 3 3.00000 4 1.00000 6 -1.00000 6 tmax= 0.4 bjective value teration 1 slack activity -3.61543	2 3 4 5 8 11543d+00 > 11 3.61542B1824d+ superbasics lower limit.	<pre>variable x5 0 0upper limit. none</pre>	(5) from 11 .dual activity 1.00000	1

	11	r 2	eq 3.82	600 . 00 0	00 3.82	500 3.826	0077849	3
a	12	r3	eq 3.41	544 .000	00 3.41	544 3.415	44 .00000	4
	13	r 4	eq 1.00	000. 000	00 1.000	000 1.000	00 - 63694	Б
a	14	r 5	eg -1.00	000. 000	00 -1.000	-1.000	000000. 00	6
			•					
sect	ion :	2 - column						
nun	ber	.column.	atactivity	obj gradien	tlower lim	itupper limi	t. reduced gradnt	ma+j
	1	x1	bs 2.00	.100	00.00	000 non	.00000	7
	2	x2	bs 3.41	543 1.000	00 лог	ne 5.000	00000.00	8
	3	x3	bs 1.00	000.000	00.00	000 2.000	00 .00000	9
	4	x4	11 .00	000 .000	00.00	000 2.000	0077849	10
d	5	x5	bs 0.00	.000 .000	00.00	000 2.000	00 .00000	11
	6	x65	11 .00	.000	00.00	000 2.000	0063694	12
	7	x 7	bs 1.00	.000 .000	00.00	000 2.000	.00000	13
	8	rh	•q -1.00	000 .000	00 -1.00	000 -1.000	-3.61543	14
and t		nuchlam ia	infoncible					
10		ann of inf	initialities	1 2 5731388	01-05			
100.				1 2.0/01000				
nita	= 2	tan	ax= 0 24154d+0	1				
P =	. –	-		-				
data	plp	file						
	19	name	plpb					
	20	'set'		00+5000000 ·				
	21	uppe	r x2	-1.000001+00	. 0000	0d+00		
	22	endata						
plp	bou	nd						
sect	ion	1 - rows						
				. .				
nun	ber	Tow	.11 direction.	.lower limit	.ul direction.	.upper limit	1	
	10	_1		~~~~			•	
	10	ri _0	.0000	.00000	.00000	.0000	2	
	47	-7	.0000	.00000	.0000	.0000	3	
	12	-1		.00000	.0000	.0000	4 E	
	14				.0000	.0000	0 6	
	14	10					0	
sect	ion	2 ~ column						
			-					
num	ber	.column.	.11 direction.	.lower limit.	ul direction.	.upper limit.	mo+i	
						· • • • • • • • • • • • • • • • • • • •	- ,	
	1	x1	. 00000	.00000	.00000	none	7	
	2	x2	.00000	none	-1.00000	5.00000	8	
				.00000	.00000	2.00000	9	
	3	хЗ	. 00000			2 00000	10	
	3 4	x3 x4	.00000. 00000.	.00000	. 00000		10	
	3 4 5	х3 х4 х5	.00000 .00000 .00000	.00000	.00000 .00000	2.00000	10	
	3 4 5 6	х3 х4 х5 х6	00000. 00000. 00000. 00000.	.00000 .00000 .00000	.00000 00000 00000	2.00000	10 11 12	
	3 4 5 6 7	x3 x4 x5 x8 x7	00000. 00000. 00000. 00000. 00000.	00000. 00000. 00000. 00000.	00000. 00000. 00000. 00000.	2.00000 2.00000 2.00000	10 11 12 13	
	3 4 5 6 7	xය x4 x5 x6 x7	.00000 .00000 .00000 .00000 .00000	.00000 .00000 .00000 .00000	.00000 .00000 .00000 .00000	2.00000 2.00000 2.00000	10 11 12 13	
a . n	3 4 5 6 7	x3 x4 x5 x6 x7 x7	.00000 .00000 .00000 .00000 .00000 .00000	.00000 .00000 .00000 .00000	. 00000 . 00000 . 00000 . 00000	2.00000 2.00000 2.00000	10 11 12 13	
a. n f	3 4 5 6 7 0 ch	x3 x4 x5 x6 x7 ange in th e range (t	.00000 .00000 .00000 .00000 .00000 e optimal soluti manca= 0.20000d	.00000 .00000 .00000 .00000 .00000	. 00000 . 00000 . 00000 . 00000	2.00000 2.00000 2.00000	10 11 12 13	
a. n f	3 4 5 6 7 in ch	x3 x4 x5 x7 x7 ange in th e range (t	.00000 .00000 .00000 .00000 .00000 .00000 e optimal soluti mmoza= 0.20000d	.00000 .00000 .00000 .00000 .00000	. 00000 . 00000 . 00000 . 00000	2.00000 2.00000 2.00000	10 11 12 13	
a. n f b. n	3 4 5 6 7 0 ch init	x3 x4 x5 x6 x7 ange in th e range (t ange in th	.00000 .00000 .00000 .00000 .00000 e optimal soluti mmoca= 0.20000d	.00000 .00000 .00000 .00000 .00000	. 00000 . 00000 . 00000 . 00000	2.00000 2.00000 2.00000	10 11 12 13	
a.n f b.n	3 4 5 6 7 init init	x3 x4 x5 x7 ange in th e range (t ange in th e range (t	.00000 .00000 .00000 .00000 .00000 • optimal soluti maca= 0.20000d • optimal basis macb= 0.20000d	.00000 .00000 .00000 .00000 *01)	. 00000 . 00000 . 00000 . 00000	2.00000 2.00000 2.00000	10 11 12 13	

section 1 - rows

number	r 0	w11	direction.	.11 boundary a	.11	boundary b	.ul	direction.	. 11	boundary a	.ul	boundary b	i
10	-1		00000	00000		00000		00000		00000		0000	2
11	-2		00000	00000				00000		.00000		.00000	3
12	r3		.00000	.00000		.00000		.00000		.00000		.00000	4
13	r4		,00000	.00000		.000000		.00000		.00000		.00000	Б
14	r6		.00000	.00000		.00000		.00000		.00000		. 00000	6
section	2 - co	lums											
number	.colu	m11	direction.	.11 boundary a	. 11	boundary b	.ul	direction.	.ul	boundary a	.ul	boundary b	ma+j
1	x1		.00000	.00000		.00000		.00000		none		none	7
2	×2		.00000	none		none		-1.00000		3.00000		3.00000	8
3	x3		.00000	.00000		. 00000		.00000		2.00000		2.00000	9
4	X4		.00000	.00000		.00000		.00000		2.00000		2.00000	10
6	305 e		.00000	.0000		.00000		.00000		2.0000		2.0000	11
0 7	320 - 77		.0000	.0000		.00000		.0000		2.0000		2.0000	12
	x			.0000				.0000		2.0000		2.0000	5
pitn= :	L	obj=	0.324152784+0	1 tamax=	0.3	20000d+01		variable x5		(5) fra	n 11		
replaces	s basic	variab]	le x2 (2) which pass	es t	o ul							
problem	name	test		objective val	ue	3.241527803	21+	00					
status		optimal	i soln	iteration	1	superbasics		0					
objectiv	78	ob	(max	x)									
rhs		rh											
ranges													
bounds		ьо											
section	1 - ro	6/18		- 9 10				.	• •				
number	r c	W 80	activity.	Black activ	159	lower limi	τ.	upper 11m	17.	. dual activ	100	1	
- 10	-1	Dis	3.241	b3 −3.24 m m	153	non 3 000	•	na 2 00		1.00	~~~	1	
a 10	-71	ब्यू ब्यू	3.000	00 .00 00 00	~~~~	3.000	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	3.00	800	- 14	154	4	
a 12	-3	~ ~	3 000	00 00	000	3 000	00	3.00	200		000	4	
a 13	74		1.000	00 .00	000	1.000	00	1.00	200	.00	000	5	
a 14	тб	•q	-1.000	.00	000	-1.000	00	-1.00	000	.00	000	6	
		-											
section	2 - co	lums											
number	.colu	man.at	activity.	obj gradie	nt.	lower limi	t.	upper lim	it.	reduced gra	dnt	m+j	
1	x1	bs	2.415	43.10	000	.000	00	ла	ne -	.00	000	7	
2	x2	บไ	3.000	00 1.00	000	non	8	3.00	000	.90	000	8	
3	x3	bs	. 584	Б7 .00	000	.000	00	2.00	000	.00	000	9	
4	x4	ц	.000	00.00	000	.000	00	2.00	000	14	154	10	
d B	х£	bs	0.000	00.00	000	.000	00	2.00	000	.00	000	11	
6	x6	bs	.587	00.00	000	.000	00	2.00	000	.00	000	12	
7	x7	bs	1.415	43 .00	000	.000	00	2.00	000	.00	000	13	
8	rh	eq	-1.000	00.00	000	-1.000	00	-1.00	000	54	154	14	
a. no cl finit	uange i ta rang	n the op e (tmecc	ptimal solution = .00000d+	n 00)									
b. no cl finit	unge i te rang	n the op re (tamed	ptimal basis p= 0.58457d+	00)									
section	1 - ra	AMB											
number	r	w11	l direction.	.11 boundary a	. 11	boundary b	.ul	direction.	.11	boundary a	.ul	boundary b	i

				~~~~~			~~~~	~
10 F1		.00000	.00000	.0000	.00000	.00000	.0000	
11 12		.00000	.0000	.0000	.0000	.0000		3 1
14 13		.00000	.0000	.0000	.00000	.00000		
13 74		.00000	.00000	.0000	.00000	.00000	.0000	
14 10		.00000	.0000	.0000	.0000	.0000	.0000	0
section 2 - co	lums							
number .colu	znll dire	ction11	boundary a .11	boundary b .ul	direction 1	boundary a .ul	boundary b	m+j
1 - 1		00000	00000	00000	00000	B07.6	7056	7
2 22		.00000	none	3050	-1.00000	3.00000	2.41543	8
3 x3		.00000	.00000	.00000	.00000	2.00000	2.00000	9
4 x4		.00000	.00000	.00000	.00000	2.00000	2.00000	10
5 x5		.00000	.00000	.00000	.00000	2.00000	2.00000	11
6 x6		.00000	.00000	.00000	.00000	2.00000	2.00000	12
7 x7		. 00000	.00000	. 00000	.00000	2.00000	2.00000	13
						<i>.</i>		
pitn= 2	obj= 0.2/1	54131 <b>d</b> +01	tomext 0.	25846d+01	variable 324	(4) from 11		
Tehraces mesic		· · ·	A MILCI PERSON C	0 11				
problem name	test	0	bjective value	2.71541314576	-00			
status	optimal soln	i	teration 2	superbasics	0			
objective rhs	ob rh	(max)						
ranges								
bounds	bo							
section 1 - ro	W5							
numberro	w., ata	ctivity	slack activity	lower limit.	upper limit.	.dual activity	<b>i</b>	
9 ob	bs	2.71541	-2.71541	none	ncne	1.00000	1	
a 10 r1	eq	3.00000	.00000	3.00000	3.00000	.00000	2	
a 11 r2	eq	3.82600	.00000	3.82600	3.82600	. 00000	3	
a 12 r3	eq	3.00000	.00000	3.00000	3.00000	.00000	4	
a 13 r4	eq	1.00000	.00000	1.00000	1.00000	. 00000	Б	
14 <del>r</del> 5	eq	-1.00000	.00000	-1.00000	-1.00000	. 10000	6	
section 2 - co	lums							
number .colu	mn. ata	ctivity	.obj gradient.	lower limit.	upper limit.	reduced gradnt	m+j	
1 121	be	3.00000	. 10000	.00000	none	.00000	7	
2 x2	ul	2.41543	1.00000	none	2.41543	1.00000	8	
d 3 x3	bs	.00000	.00000	.00000	2.00000	.00000	9	
d 4 x4	bs	0.00000	.00000	.00000	2.00000	.00000	10	
5 x5	bs	.58457	.00000	.00000	2.00000	.00000	11	
6 336	bs	1.41300	.00000	.00000	2.00000	.00000	12	
7 x7	ul	2.00000	.00000	. 00000	2.00000	. 10000	13	
8 rh	eq	-1.00000	.00000	-1.00000	-1.00000	10000	14	
a. no change in finite rang	n the optimal e (tmacca=	solution .00000d+00)						
b. no change i	n the optimal	basis						
TIUTO LEUG	e (um <u>etro</u> = 0	. 0.30000+00)						
section 1 - ro	1415							
numberro	wll dire	ction11	boundary a .11	boundary b .ul	l directionul	boundary a .ul	, boundary b	<b>i</b>
10 r1		.00000	.00000	. 00000	.00000	.00000	.00000	2

		_		~~~~		~~~~~	~~~~	00000	~~~~	2
	11	74		.0000	.00000	.00000	.00000	.0000	.00000	3
	12	<b>T</b> 3		.0000	.00000	.0000	.00000	.00000	.00000	4
	13	<b>r</b> 4		.00000	.00000	.00000	.00000	.00000	.00000	ь
	14	тб		.00000	.00000	.00000	.00000	.00000	.00000	6
sect	ion	2 - cc	lumns							
num	ber	.colu	aan	direction1	L boundary a	ll boundary b	.ul direction.	.ul boundary a .	ul boundary b	m+j
	1	x1		.00000	.00000	.00000	.00000	none	none	7
	2	x2		.00000	none	none	-1.00000	2.41543	1.58457	8
	3	x3		.00000	.00000	. 00000	.00000	2.00000	2.00000	9
	4	201		.00000	.00000	. 00000	.00000	2.00000	2.00000	10
	5	x6		.00000	.00000	.00000	.00000	2.00000	2.00000	11
	6	хđ		.00000	.00000	. 00000	.00000	2.00000	2.00000	12
	7	<b>x</b> 7		.00000	.00000	.00000	.00000	2.00000	2.00000	13
nita	= 3		obi= (	0 188155061+01	tmere (	0 341544+01	variable v7	( 7) from	מ	
repl	- 0 AC06	basic	variable	a x66 ((	5) which passes	to ul	Ver India XI			
prob	len	name	test		bjective value	1.884550626	61+00			
stat	116		optimal	soln	iteration 3	superbesics	0			
			·•••				-			
obje	ctiv	•	ob	(max)						
rhs			rh							
rang	96									
boun	ds		ъо							
sect	ion	1 - ra	we							
num	ber	<b>r</b>	w at	activity	slack activit;	ylower limi	t upper limi	tdual activit	yi	
	9	ob	bs	1.88455	-1.8845	5 nom	e non-	e 1.0000	00 1	
8	10	r1	eq	3.00000	. 0000	0 3.000	00 3.000	0000.0000	0 2	
a	11	r2	eq	3.82600	. 0000	0 3.826	00 3.826	0000.000	0 3	
a	12	r3	eq	3.00000	. 0000	0 3.000	00 3.000	0000.000	0 4	
	13	<b>r</b> 4	eq	1.00000	.0000	0 1.000	00 1.000	00 . 1415	45	
a	14	r6	eq	-1.00000	.0000	0 -1.000	-1.000	0000.0000	0 6	
sect	ion	2 - co	luma							
nun	ber	.colu	mon. at	activity	.obj gradient	lower limi	tupper limit	t. reduced gradm	ıt ma+j	
	1	x1	be	3.00000	1000	0 000	00 1.00		0 7	
	2		 "]	1 58457	1 0000	 	e 1 5241	57 1 1000	- , 10 8	
A	-	~		0.0000	0000		00 2.000	· · · · · · · · · · · · · · · · · · ·	n 0	
4	3		106 1	U.0000		0.000	⊷ ∡.000 ∩∩ າ.∩∽	~~	~ ¥	
	-			4 44549		0.000	₩ <u>2.000</u>	·····	N 10	
	5	жо е	DB	1.41543	.000	000.	0 2.000		N 11	
	6	200	<u>о</u> т ,	2.0000	.0000	000.	00 2.000	.1418	4 12	
d	7	<b>x</b> /	be	2.00000	.0000	U .000	00 2.000	0000.UU	0 13	
	8	rh	eq	-1.00000	.0000	0 -1.000	-1.000	00 . 1418	4 14	

minos --- version 4.0 mar 1981 =====

# specs file

begin Second P L P Test maximize plp order plp rhs analysis 1

plp cost analysis plp bound analysis -3 plp increment 1000. plp frequency 1 data plp file -9 print data plp file and PLP --- version 1.0 june 1986, = = = DEPENDENT WORK of PLP ------ - data plp file 12 name plpr 13 'set' .000000d+00 1.000001+00 .0000004+00 14 т3 15 * 16 * Note that all components of rhs vector are defined in MPS file. 17 * 18 endata plp rhs _____ 10 r1 .00000 3.00000 2 11 r2 .00000 3.82600 3 12 r3 1.00000 3.00000 4 1.00000 13 r4 .00000 Б 14 гБ .00000 -1.00000 6 pitn= 1 obj= 0.36154282d+01 tamex= 0.41543d+00 variable x5 (5) from 11 replaces basic variable x6 (6) which passes to 11 objective value 3.6154281824d+00 problem name test status optimal soln iteration 1 superbasics 0 (max) objective оЪ rhs rh TANZOS bounds Ъо section 1 - rows number ...row.. at ...activity... slack activity ..lower limit. ..upper limit. .dual activity ..i 9 ob 3.61543 -3.61543 1.00000 bs none none 1 10 **r**1 3.00000 .00000 3.00000 .00000 2 3.00000 eq 11 12 3.82600 .00000 3.82600 3.82600 -.77849 3 eq 3.42251 3.42251 .00000 3.42251 .00000 12 r3 4 eq i -.63694 5 .00000 6 13 r4 1.00000 .00000 1.00000 1.00000 eq 14 гБ eq -1.00000 .00000 -1.00000 -1.00000 section 2 - columns number .column. at ...activity... .obj gradient. ..lower limit. .upper limit. reduced gradnt m+j .00000 1 x1 2.00000 . 10000 .00000 none 7 bs 2 32 bs. 3.41543 1.00000 none 5.00000 .00000 8

		~	<b>L</b> -	4 00000	00000	00000	0.00000	00000	
	3 X	3	D6	1.0000	.00000	.00000	2.0000	.00000	
	4 x	4	ц	.00000	. 00000	.00000	2.00000	77849	10
d	5 x	5	bs	0.00000	.00000	.00000	2.00000	.00000	11
	6 x	£	11	.00000	.00000	.00000	2.00000	63694	12
	7 x	7	bs	1.00000	.00000	. 00000	2.00000	. 00000	13
	8 r	ħ	eq	-1.00000	.00000	-1.00000	-1.00000	-3.61543	14

# data plp file

1	nan	me plpc			
2	'se	rt'	.0000000d+00		
3		<b>x1</b>	1.000004+00	.000000d+00	
4		x2	-1.000004+00	.000000d+00	
Б	*				
6	*	Note:			
7	*	Declaration of	dummy coefficients	(=0 in MPS file) of the	objective

- 8 * is not necessary because the above direction is defined in the x1-x2
- 9 * subspace of cost vectors.
- 10 *
- 11 endata

#### plp cost

number .column. ...direction.. .obj gradient. m+j 1.00000 .10000 7 1 x1 1.00000 2 22 -1.00000 8 .00000 .00000 3 x3 9 .00000 .00000 10 4 x4 5 x5 .00000 .00000 11 .00000 6 x6 .00000 12 7 x7 .00000 .00000 13 pitn= 1 obj= 0.29784856d+01 tamex= 0.45000d+00 variable x6 (6) from 11 replaces basic variable x3 (3) which passes to 11 objective value 2.9784855632d+00 problem name test iteration 1 superbasics O status optimal soln objective (max) ob rhs rh ranges bounds ъо section 1 - rows number ...row.. at ...activity... slack activity ..lower limit. ..upper limit. .dual activity ..i 9 ob 2.97849 -2.97849 1.00000 1 bs none none 10 r1 3.00000 .00000 3.00000 3.00000 -.00580 2 eq 3.82600 11 r2 .00000 3.82600 3.82600 -.77849 eq 3 12 r3 3.42251 .00000 3.42251 3.42251 .00000 4 . eq 13 r4 .00000 1.00000 1.00000 1.00000 .00000 8 eq Б -1.00000 .00000 -1.00000 -1.00000 14 тб .00000 6 a eq section 2 - columns number .column. at ...activity... .obj gradient. ..lower limit. ..upper limit. reduced gradnt m+j 3.00000 .55000 - .00580 7 i x1 bs .00000 none 2 22 bs 2.41543 .55000 5.00000 .00000 8 none

.00000 . 00000 .00000 2.00000 -.00580 9 3 x3 ц - . 77849 4 xx4 п .00000 .00000 .00000 2.00000 10 2.00000 . 00000 1.00708 .00000 .00000 11 5 x5 bs 6 x6 1.41300 .00000 .00000 2.00000 .00000 12 bs 2.00000 đ 7 x7 bs 2.00000 .00000 .00000 .00000 13 -1.00000 .00000 -1.00000 -1.00000 -2.99590 14 8 rh •9 pitn= 2 obj= 0.33000001d+01 tamax= 0.10000d+01 variable x4 ( 4) from 11 replaces basic variable x6 ( 6) which passes to ul problem name test objective value 3.300000715d+00 status optimal soln iteration 2 superbasics 0 objective (max) ob rhs rh ranges bounds ь section 1 - rows number ...row.. at ...activity... slack activity ..lower limit. ..upper limit. .dual activity ..i 9 ob 3.30000 -3.30000 none 1.00000 1 bs none 3.00000 .00000 10 r1 3.00000 3.00000 -1.09717 2 eq 11 r2 3.82600 .00000 3.82600 3.82600 -.00200 3 eq 12 r3 3.42251 .00000 3.42251 3.42251 .00000 4 a eq 13 r4 1.00000 .00000 1.00000 1.00000 .00200 5 eq. -1.00000 .00000 -1.00000 -1.00000 .00000 a 14 r5 eq 8 section 2 - columns number .column. at ...activity... .obj gradient. ..lower limit. .upper limit. reduced gradnt m+j 1 x1 3.00000 1.10000 .00000 none 0.00000 7 bs 0.00000 1.58457 5.00000 . 00000 2 12 bs none 8 3 x3 ц .00000 .00000 .00000 2.00000 -1.09717 9 .58700 .00000 2.00000 -.00200 4 x4 .00000 bs 10 5 x65 1.83793 .00000 .00000 2.00000 .00000 bs 11 2.00000 .00000 .00000 2.00000 . 00200 6 x6 ul 12 2.00000 .00000 .00000 .00000 đ 7 🛫 7 bs 2.00000 13 8 rh eq -1.00000 .00000 -1.00000 -1.00000 -3.29717 14 pitn= 3 for the value of the parameter= 1.00000 infinite range (tmex.ge.1.e15)

data plp file

19	name	plpb		
20	'set'		.00000001+00	
21	upper	x2	-1.00000d+00	.0000001+00
22	endata			

plp bound

section 1 - rows

number:	<b>row</b>	.11 direction.	.lower limit	.ul direction.	.upper limit	<b>i</b>
10	ri	.00000	.00000	.00000	.00000	2
11	<b>r</b> 2	. 00000	.00000	.00000	.00000	3
12	r3	.00000	.00000	.00000	.00000	4
13	<b>r</b> 4	.00000	.00000	.00000	.00000	Б

- 32 -

14	r6	. 00000	.00000	.00000	. 00000	6		
section (	2 - co]	unns						
number	.colum	m11 direction.	.lower limit	.ul direction.	.upper limit	m+j		
1	x1	. 00000	.00000	.00000	none	7		
2	x2	.00000	none	-1.00000	5.00000	8		
3	хЗ	.00000	.00000	.00000	2.00000	9		
4	x4	.00000	.00000	. 00000	2.00000	10		
Б	жБ	.00000	.00000	.00000	2.00000	11		
6	жő	.00000	.00000	.00000	2.00000	12		
7	x7	. 00000	.00000	.00000	2.00000	13		
a. no ch	ange in	the optimal solut:	ion					
finit	e range	(tame) (t	1+01)					
b. no ch	ange in	the optimal basis						
finit	e range	(tamedo= 0.34154d	1+01)					
section	1 - row	•						
number			.11 boundary a	.11 boundary b	.ul direction.	.ul boundary a .	ul boundary b	<b>i</b>
10	ri	.00000	.00000	.00000	.00000	.00000	.00000	2
11	72	.00000	.00000	.00000	.00000	.00000	.00000	3
12	r3	.00000	.00000	.00000	. 00000	.00000	.00000	4
13	<b>r</b> 4	.00000	.00000	.00000	. 00000	.00000	.00000	5
14	rб	.00000	.00000	.00000	. 00000	.00000	.00000	6
section	2 - co]	.111121.5						
number	.colum	nll direction.	.11 boundary a	.11 boundary b	.ul direction.	.ul boundary a .	ul boundary b	m+j
1	<b>x1</b>	.00000	. 00000	.00000	.00000	none	none	7
2	x2	.00000	none	none	-1.00000	1.58457	1.58457	8
3	хЗ	.00000	.00000	. 00000	.00000	2.00000	2.00000	9
4	x4	. 00000	.00000	.00000	. 00000	2.00000	2.00000	10
5	x5	.00000	.00000	.00000	. 00000	2.00000	2.00000	11
6	305	.00000	.00000	. 00000	.00000	2.00000	2.00000	12
7	<b>x</b> 7	.00000	.00000	.00000	. 00000	2.00000	2.00000	13
pitn= 1 replaces	basic	obj= 0.33000001d variable x2	01 tmmax= (2) which pass	0.34154d+01 es to ul	variable x3	( 3) from	11	
problem	nane	test	objective val	ue 3.30000010	13d+00			
status		optimal soln	iteration	1 superbasic	<b>B</b> 0			
objectiv	•	ob (1	max)					
rhs		rh						
ranges								
bounds		Ъо						
section	1 - row	<b>a</b>						
number	<b>ro</b> v	atactivity	slack activ	itylower lim	itupper limi	itdual activit	yi	
9	ob	bs 3.30	0000 -3.30	000 na	ne nor	e 1.0000	10 1	
<b>a</b> 10	<b>r</b> 1	eq 3.00	00. 000	000 3.00	000 3.000	000.000	0 2	
a 11	r2	eq 3.82	<b>1600</b> .00	000 3.82	800 3.826	.0000	0 3	
a 12	r3	eq 3.42	2251 .00	000 3.42	251 3.422	151 .0000	0 4	
13	<b>r</b> 4	eq 1.00	00. 000	000 1.00	000 1.000	00 1.5569	75	
<b>a</b> 14	r5	eq -1.00	00. 000	000 -1.00	000 -1.000	0000.0000	06	

section	2	- columns
---------	---	-----------

num	ber	.column.	at	activity	.obj gradient.	lower limit.	upper limit.	reduced gradnt	<b>m</b> +j
	1	<b>x</b> 1	bs	3.00000	1.10000	.00000	none	.00000	7
	2	x2	ul	1.58457	0.00000	none	1.58457	1.10000	8
d	3	хЗ	bs	0.00000	. 00000	.00000	2.00000	.00000	9
	4	x4	bs	.58700	.00000	.00000	2.00000	. 00000	10
	5	x5	bs	1.83793	.00000	.00000	2.00000	. 00000	11
	6	хđ	nl	2.00000	. 00000	.00000	2.00000	1.55697	12
d	7	x7	bs	2.00000	. 00000	.00000	2.00000	. 00000	13
	8	rh	eq	-1.00000	. 00000	-1.00000	-1.00000	1.55697	14

a. no change in the optimal solution finite range (tmsca= .00000d+00)

b. no change in the optimal basis finite range (tmath= 0.16207d+00)

section 1 - rows

nunber	<b>row</b>	.11 direction.	.11 boundary a	.ll boundary b	.ul direction.	.ul boundary a	.ul boundary b	<b>i</b>
10	<b>r1</b>	.00000	.00000	. 00000	.00000	.00000	. 00000	2
11	r2	.00000	.00000	.00000	.00000	.00000	.00000	3
12	r3	.00000	.00000	.00000	.00000	.00000	.00000	4
13	<b>r</b> 4	. 00000	.00000	.00000	.00000	.00000	.00000	5
14	r6	.00000	.00000	. 00000	.00000	.00000	. 00000	6

section 2 - columns

number	.column.	.11 direction.	.11 boundary a	.11 boundary b	.ul direction.	.ul boundary a	.ul boundary b	<b>m</b> +j
1	<b>x</b> 1	.00000	.00000	.00000	.00000	none	none	7
2	22	.00000	none	none	-1.00000	1.58457	1.42251	8
3	x3	.00000	.00000	. 00000	.00000	2.00000	2.00000	9
4	xx4	. 00000	.00000	. 00000	. 00000	2.00000	2.00000	10
5	хб	.00000	.00000	.00000	.00000	2.00000	2.00000	11
6	<b>36</b>	. 00000	.00000	.00000	.00000	2.00000	2.00000	12
7	<b>x</b> 7	. 00000	.00000	.00000	.00000	2.00000	2.00000	13

excit -- problem is infeasible.

no. and sum of infeasibilities 1 9.99999046d-03

pitn= 2 tmax= 0.35775d+01