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IIASA Working Paper

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May 1987

Ovaskainen, V. (1987) Pollution-Induced Forest Damage, Optimal Harvest Age and Timber Supply: Some Theoretical Considerations. IIASA Working Paper. Copyright © 1987 by the author(s). http://pure.iiasa.ac.at/3015/

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PUBLICATION NUMBER 36 of the project:
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FOREWORD

This paper, by Ville Ovaskainen of the Finnish Forest Research Institute, examines the relationships of pollution-induced forest damage, optimal age of stand harvest, and timber supply from a theoretical perspective. Since future levels of forest damage will be important determinants of harvesting and silvicultural practices and thus of wood supply, they must be explicitly taken into account in any realistic analysis of alternative scenarios of future forest decline and appropriate policy responses.

The paper is an output of the IIASA Forest Study, which examines the consequences for the world's forests and the forest products sector of changes in forest patterns and growth due to airborne pollutants. The emphasis in the first phase of the study is on issues of major relevance to industrial and government policymakers in Europe. The research program includes an analysis of future wood supply in Europe under different assumptions about the rate and extent of forest decline. In addition, a number of papers are being produced to address various topics related to forest decline and the European forest sector in general.

The Forest Study is part of IIASA's Project on Ecologically Sustainable Development of the Biosphere, which seeks to clarify the policy implications of long-term, large-scale interactions between the world's economy and its environment. The Project conducts its work through a variety of basic research efforts and applied case studies. A list of the Project's publications appears at the end of this document.

R.E. Munn
Leader
Environment Program

ACKNOWLEDGEMENTS

I would like to thank Dr. H.A. Loikkanen for the stimulus to initiate this paper and for comments on the Finnish draft, as well as J. Kuuluvainen. Thanks are also due to Dr. P. Kauppi for discussions and the idea of collaboration with IIASA, and to Dr. S. Nilsson for his contribution to this collaboration.

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POLLUTION-INDUCED FOREST DAMAGE, OPTIMAL HARVEST AGE AND TIMBER SUPPLY: SOME THEORETICAL CONSIDERATIONS

Ville Ovaskainen

1. INTRODUCTION

Recently, a concern for the effects of changes in the atmosphere on forests has emerged. Forest damage due to air pollutants has been reported in Central European countries, accompanied by an anticipation of sanitation fellings and the subsequent adverse effects on the roundwood market. Less drastic changes in the productivity of forest ecosystems might even have long-term impacts through a decline in growth and sustainable harvest.

There is presently no reason to overestimate the scale and seriousness of the damage (e.g., there are no indications of increased sanitation fellings (Kauppi, 1987)). However, if some damage has been observed and changes may be anticipated in the future, the potential consequences are probably important enough to be worth a theoretical discussion, irrespective of the scale of the actual damage. For the moment, not too much is known about how pollution and its effects will proceed, or how pollutants actually affect tree vigor and growth. It is natural enough, then, that little analysis on the economic effects exists.

The present paper will not aim at forecasts. Instead, it inquires about the potential effects under alternative assumptions: given certain hypothetical damage levels in standing timber and changes in forest growth, what are the likely effects on harvesting decisions and the roundwood market equilibrium? A few hypothetical, intuitively plausible types of effects are considered to gain insight into the qualitative aspects of the problem. The technique employed is comparative statics, that is, we consider mainly the direction of changes.

First, the effect of changes in the growth curve on the optimal harvest age of forests is studied using the traditional Faustmann model. The drastic damage in older stands is by no means the only possibility, particularly when different time scales are recognized. The assumptions will then be described in more detail. Besides once-and-for-all changes assuming identical rotations, the harvest age for the standing timber will be analyzed by a modification that allows the future rotations and the current stands to possess different growth patterns.

The partial equilibrium analysis of the optimum rotation is limited by its restrictive assumptions, especially that of an exogenously fixed, constant price level. To recognize the price effects, a market equilibrium setting is introduced. Assuming that just a part of the exploitable forests is damaged, there will be responses other than the pulse of timber from sanitation fellings. The paper concludes with a discussion of the relevance and limitations of the results.

2. THE ASSUMPTIONS OF THE ANALYSIS

In the following analysis, the most traditional approach to forest utilization, the problem of optimum rotation, will be employed. The steady-state solution for the problem is well-known as the Faustmann rule (e.g., see Samuelson, 1976; Johansson and Loefgren, 1985). The model assumes perfect capital, timber and land markets. Prices, costs and interest rates are assumed to be known constants in the particular steady-state solution. Given the assumptions, the solution has been shown to be socially optimal (Johansson and Loefgren, 1985).

Implicitly, the model also assumes constant returns to scale in harvesting technology, so the optimal cutting decision can be made at the stand level (Johansson and Loefgren, 1985).

The time profile of fellings or the age-class composition of the entire forest is not constrained. Furthermore, the choice of optimal cutting time is assumed not to be restricted by the capacity of timber harvesting (cf. Heaps and Neher, 1979). The assumptions are strong, hence simplifying. In the present context, the assumption of a constant price level is especially restrictive, because non-marginal shifts in supply (e.g., due to sanitation fellings) imply price changes. In other words, the partial analysis ignores the price effects.

The standard Faustmann model aims at maximizing the present net value of an infinite chain of subsequent rotations, or land expectation value. This requires, besides constant economic variables, that the growth pattern is identical for all rotations. Thus, biotechnology (e.g., the quality of plants, seeds, and management intensity) as well as forest soil productivity are assumed to be unchanged. Then, all rotations will be identical, and the problem converges to the well-known compact formulation.

In a way, the simplification is overly restrictive. For example, the choice of tree species is not forever constant, since forest tree breeding changes the genetic properties of plants and seeds and the rate of tree growth. In analysing the effect of biotechnological improvements on optimum rotation, Johansson and Loefgren (1985) show how the assumptions of the model can be relaxed to allow for an explicit treatment of differences between characteristics of the standing timber and future rotations. Walter (1980) refers to the problem of maintaining forest soil productivity and how this influences the formulation of the problem. To the best of my knowledge, there has been little further study on this matter, reported in the optimum-rotation literature[1]. However, the discussion below on changes in the composition of the atmosphere, and soil acidification, and their effects on forests, suggests that the issue of soil productivity and the uncertainty of future growth conditions may become more relevant.

There is no straightforward answer to the question of how air pollutants affect forests. The reactions undoubtedly vary across a wide range of stages and degrees of pollution, as well as across various tree species and types of forest soil. Furthermore, in the atmospheric changes a number of different chemical compounds are involved (e.g., sulfur dioxide, nitrogen oxides, ozone and carbon dioxide).

Here it will suffice to refer to Kauppi's (1987) survey of study results thus far available. One conclusion is that old trees are particularly susceptible to damage. The sulfur content and the average age of foliage on needle-leaved trees are related. However, besides the hypothesis of immediate damage through toxic effects on the needles, the hypothesis of soil acidification as a link to delayed impacts on crown density and tree vigor has become more important. Thus, the damage may well be caused by accumulating soil acidification, the consequent changes in the balance of calcium, magnesium and aluminium, and their effect on plant roots along with heavy metal stress. On the other hand, increased carbon dioxide contents of the air may induce a so-called CO_2 fertilization effect.

In the Faustmann rule of the optimum rotation for an even-aged forest (stand), the effect of changing growth conditions enters in terms of shifts or other changes in the growth curve expressing standing volume as a function of stand age. The ef-

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[1] In some versions of the Faustmann model (Hyde, 1980; Jackson, 1980; Chang, 1983), biotechnology is represented as a choice variable through endogenous silvicultural effort. The choice of a higher level of inputs results in a shift to a higher growth curve, and the optimal level, once determined, is maintained through the future. Here we are dealing with an exogenous change, or deterioration of the growth function due to pollution.

fect can not be unambiguously characterized, and it may also be different in different time scales. Therefore the present paper will consider three different possibilities. The cases, although hypothetical, seem to have some relevance in that a meaningful interpretation can be given to each of them from an intuitive point of view. They are squeezed into the following three assumptions. The effect of each type of once-and-for-all changes (i.e., assuming all rotations are identical) on the optimum rotation will be studied in turn by comparing the 'new' optimum with that of the reference conditions (the norm), or undamaged forest.

(1) "Forest damage". Young stands, being practically unaffected, grow normally, while growth is reduced later on in mature stands by various degrees of loss of vigor and increased tree mortality.

(2) "Poorer site". The trees do not drastically lose their vigor as in (1) above, but the productivity of forest soil declines. The standing volume is assumed to decline proportionally, i.e. by the same percentage at each age. The maximum potential volume per hectare decreases, as if the trees were effectively growing on a poorer site than the original one.

(3) "Lower input efficiency". The maximum potential volume per hectare is unaffected but stand growth is delayed, which implies a non-proportional decline in standing volume. The damaged stand resembles one growing under a lower management intensity, so one can say that the effectiveness of silvicultural inputs has decreased.

Next, the effect of observed and/or expected growth changes (damage) on the optimal harvest age of current standing timber is analyzed more explicitly. The growth curves of future rotations are assumed to be identical with each other but differ from the growth pattern of the standing timber. The standing timber is first assumed to be unaffected, and subsequently also damaged, while in both cases the future stands possess poorer growth patterns in comparison to the norm.

In the partial equilibrium setting of the optimum rotation, a constant parametric price for wood is assumed. However, the pulse of supply from damaged forests is not without implications to timber price whenever the demand for timber is not perfectly elastic. Furthermore, it is reasonable to assume that just a part of the forests in the relevant market area is damaged. A conventional market equilibrium setting will be employed to illustrate the point that, through the involved price changes, there will be a further indirect effect on supply from the undamaged forests.

3. THE EFFECT OF CHANGES IN THE GROWTH FUNCTION ON THE OPTIMAL HARVEST AGE

3.1. Once-and-for-all changes with identical rotations

In the traditional Faustmann model the problem is to choose the rotation length t so as to maximize the net present value V of receipts from an infinite chain of subsequent rotations (land expectation value):

$$(3.1) \qquad \underset{t}{Max}\, V = \frac{pf(t)e^{-rt} - c}{1-e^{-rt}} \quad ,$$

where: $p =$ stumpage price,

$f(t) =$ the growth function under reference conditions (the norm); $f'(t) > 0$, $f''(t) < 0$ over the relevant range,

$c =$ regeneration cost, and

$r =$ market rate of interest.

By differentiating (3.1) with respect to t and setting the derivative equal to zero, the first-order condition for a maximum is obtained:

$$(3.2) \qquad pf'(t) - rpf(t) - rV(t) = 0 \quad .$$

The condition is the Faustmann rule which says that a forest is profitably cut when its value increment is less than or equal to the opportunity cost (interest) on the capital tied up in the standing stock plus the value of forest land (for further analysis, see: Samuelson, 1976; Johansson and Loefgren, 1985). Ignoring regeneration cost (c = 0), the condition simplifies to:

$$(3.3) \qquad f'(t)/f(t) = r/(1-e^{rt}),$$

which says that at the optimum, the relative rate of growth of the stand must be equal to the market rate of interest corrected for the land rent element. Note that future rotations are recognized in the harvesting decision. To study the effect of changes in the growth function on the optimum rotation, we rewrite the function in the form

$$Q = f(t,\alpha)\ ;\ \alpha \in (0,1) \quad .$$

Thus, we have simply made explicit a parameter that changes the growth curve as compared to the norm without defining the form of the effect more specifically. The problem in (3.1), ignoring regeneration cost, now reads

$$(3.4) \qquad \underset{t}{Max}\ V = \frac{pf(t,\alpha)e^{-rt}}{1-e^{-rt}} \quad .$$

3.1.1. Damage in the mature forest

Old trees are particularly susceptible to forest damage due to pollutants (Kauppi, 1987). It may therefore be reasonable to assume that growth in the young stand is not disproportionately affected but declines in comparison to the norm in the mature forest. The degree of damage may range from minor growth reduction to serious damage through the loss of tree vigor. In the extreme case where the volume lost through mortality exceeds the increment of remaining live trees, implying a negative net volume increment, the term 'forest dieback' may be used and a sanitation felling may be expected.

One possible case of this type is shown in Figure 1. Regarding the formal analysis, there is no simple, special parametrization for this case, and the interpretation of the general form may become difficult. Therefore a simplifying regularity assumption will be made, thus resorting to a special yet still plausible case. As depicted in Figure 1, for all ages $t \in (t_0, t_{\max})$ the growth curve of the damaged stand is less steep than the norm. Thus the current growth or the derivative of the growth curve is less than that of the norm for all relevant ages. Also, the growth curve remains strictly concave:

$$f_t^{'}(t,\alpha) > 0,\ f_{tt}^{''}(t,\alpha) < 0 \ \text{all}\ \ t \in (t_0, t_{\max})\ ,\ \alpha \in (0,1) \quad .$$

This says simply that no irregularities in the sense of convex portions in the growth function are assumed to emerge. The first-order condition for an optimum now reads

$$(3.5) \qquad f_t'(t,\alpha)/f(t,\alpha) = r/(1-e^{-rt}) \quad .$$

Compared to the standard Faustmann rule (3.3), the left-hand side is changed. Under the regularity assumption we may conclude that the optimum rotation in the damaged forest will be shortened in comparison to the norm. This is clear from Figure 2, because

$$(3.6) \qquad f_0'(T)/f_0(T) = (A_0/B_0)/A_0 = 1/B_0 > 1/B_1 = f_1'(T)/f_1(T)$$

$$\text{all } T \in (t_0, t_{\max}) \quad ,$$

as $B_0 < B_1$ (f_0 refers to the norm and f_1 to the damaged forest). From the graphical interpretation of the derivative (the slope of a curve), it is seen that the relative rate of growth in the damaged stand for any relevant age T is lower than that of the norm, so the stand will be harvested at a lower age. The qualitative result holds true irrespective of the degree of damage whenever the regularity assumption holds.

3.1.2. "Poorer site" and "lower input efficiency" effects

Intuitively, the above case perhaps best represents the acute effects of air pollutants on the health of trees due to direct toxic effects through the foliage. However, regarding soil acidification, less extreme and qualitatively different long-term effects might also be possible (that is, the productivity of each type of soil or stand is reduced, yet without drastic 'dieback' effects). Two special parametrizations representing this possibility are considered here.

First, assume the effect is proportional. Thus, standing volume is reduced by the same percentage at each age, so the potential maximum volume also declines, and the trees are effectively growing on a poorer site. The growth curve is depicted in Figure 3, and the volume is given by the equation:

$$Q = \alpha f(t) \; ; \; \alpha \in (0,1).$$

Note that the case is formally identical with the time-neutral biotechnological improvement considered by Johansson and Loefgren (1985). The only difference is that the parameter α is interpreted as a 'pollution parameter', hence taking on values less than one.

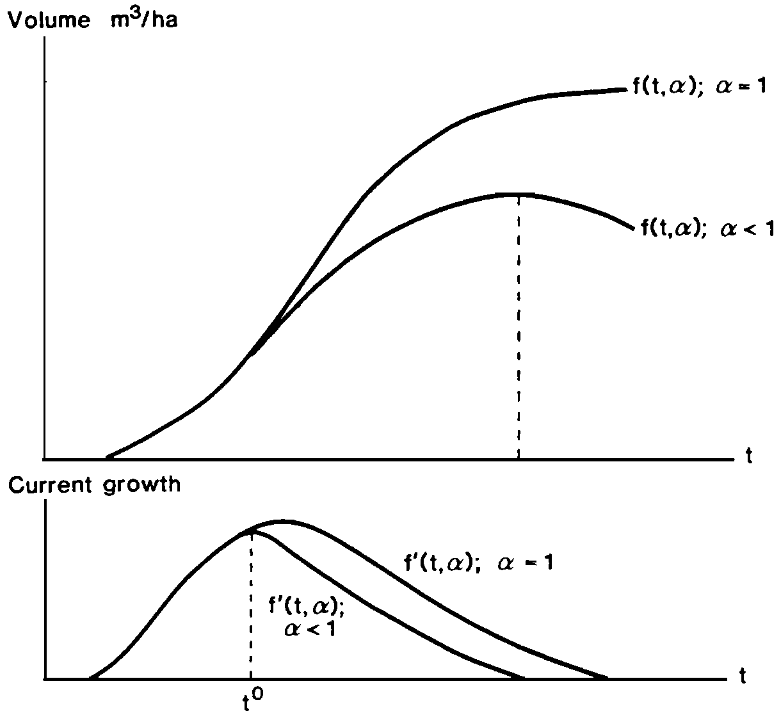

Figure 1. The change in the growth function with damage in the mature forest.

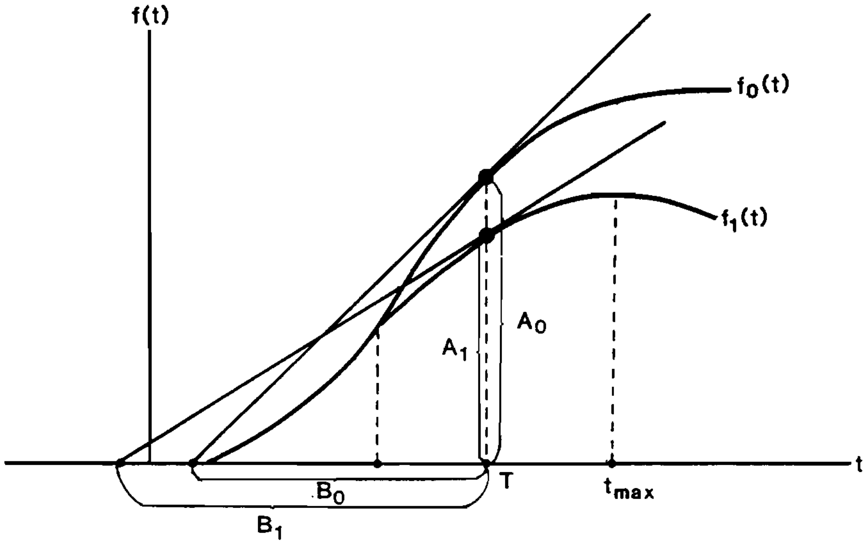

Figure 2. Diagrammatic presentation of the change in relative growth rate in Figure 1.

If regeneration cost is ignored for the moment, the problem can be written as

$$(3.7)\qquad \underset{t}{Max}\, V = \frac{p\alpha f(t)}{e^{rt}-1} \quad .$$

The first-order condition for an optimum, when arranged, is

$$(3.8)\qquad f'(t)/f(t) = r/(1-e^{-rt}) \quad .$$

Clearly, the standard Faustmann rule (cf. (3.3)) is obtained. Thus, a proportional change in the growth curve will not change the optimum rotation when regeneration cost is not taken into account ($c = 0$). The result is obvious because the optimum condition (3.7) does not depend on α. Intuitively, a proportional change leaves the relative growth rate unchanged.

It is also clear that the change in the growth curve will decrease the value of seeded (bare) land, because the rotation period remains unchanged but the volume of timber harvested declines.

By recognizing explicitly the regeneration cost ($c > 0$) the problem is rewritten as:

$$(3.9)\qquad \underset{t}{Max}\, V = \frac{p\alpha f(t)e^{-rt}-c}{1-e^{-rt}} \quad .$$

The first-order condition takes the form

$$(3.10)\quad p\alpha f'(t)/(p\alpha f(t)-c) = r/(1-e^{-rt}) \quad .$$

By rearranging to:

$$(3.11)\qquad pf'(t)/(pf(t) - c/\alpha) = r/(1-e^{-rt}) \;,$$

it becomes clear that a proportional downward shift in the growth curve ($\alpha\in(0,1)$) increases the relative rate of value growth in comparison to the norm ($\alpha=1$), and hence lengthens the optimum rotation. Because $\alpha < 1$, the effect of proportional deterioration of the growth function is effectively the same as a once-and-for-all relative increase of the regeneration cost with an unaltered growth curve (or a decline in price; cf. Johansson and Loefgren, 1985). For this it holds that $\partial t/\partial c > 0$, which establishes the claim.

Secondly, suppose the growth curve is given by the equation

$$Q = f(\alpha t)\ ;\ \alpha \in (0,1).$$

A curve of this type is presented in Figure 4. The change of the curve is clearly non-proportional.

The distinguishing feature of this case is that given enough time, the stand will potentially reach the same maximum volume as the norm. This property resembles the assumption employed by Chang (1983) where stands established at different densities on a given site eventually converge to the same volume (via a random pattern). In other words, we assume that the damaged soil can raise the same volume of trees but with a delay in comparison to the norm, as if the stand were growing under a lower management intensity. The response of timber yield to a given level of management intensity is lower than the norm. The other way round, then, the case is formally identical with a decreased efficiency of silvicultural inputs.

Assuming for simplicity that $c = 0$, the problem reads

$$(3.12) \qquad \underset{t}{Max}\ V = \frac{pf(\alpha t)}{e^{rt}-1} \quad .$$

The first-order condition for an optimum can be rearranged to

$$(3.13) \qquad \alpha f'(\alpha t)/f(\alpha t) = r/(1-e^{-rt}) \quad ,$$

or

$$(3.14) \qquad f'(\alpha t)/f(\alpha t) = \frac{1}{\alpha}[r/(1-e^{-rt})] \quad .$$

Taking into account that land value is maximized at the optimum t^*, $\partial V/\partial t = 0$, so the second-order condition is

$$(3.15) \qquad \alpha^2 f''(\alpha t) - r\alpha f'(\alpha t) < 0,$$

which holds by assumptions ($\alpha f'(\alpha t) > 0$, $\alpha^2 f''(\alpha t) < 0$ in the relevant range).

Looking at (3.14), the right-hand side of the equation increases as compared to the norm because $\alpha < 1$. But the left-hand side also changes in a way that is not readily clear. It can be shown mathematically that a non-proportional change in the growth curve of the form $Q = f(\alpha t)$; $\alpha \in (0,1)$ will lengthen the optimum rotation in comparison to the norm. The formal proof is technical, and is presented in Appendix 1. It is obvious again that the deterioration of the growth curve de-

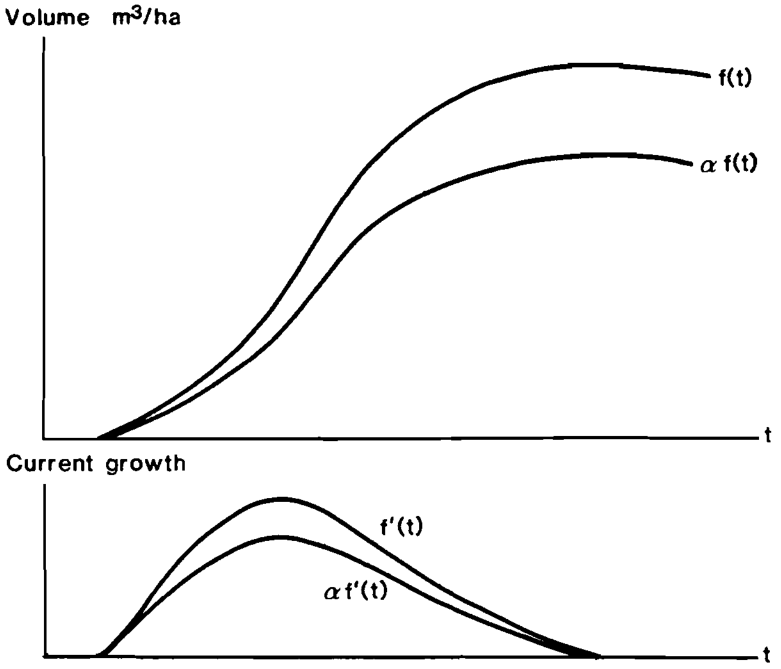

Figure 3. A proportional change in the growth curve (the "poorer site" effect).

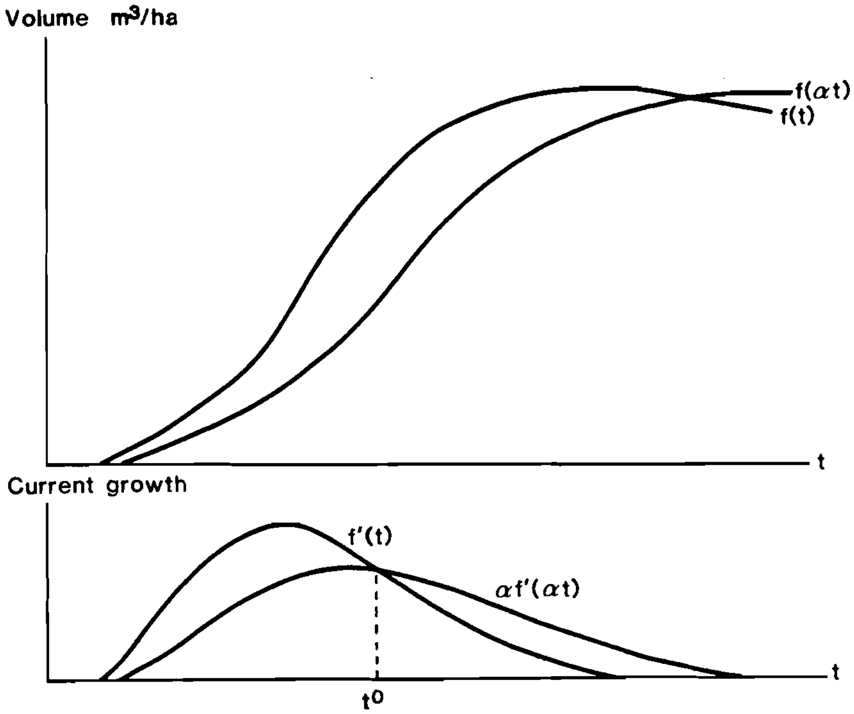

Figure 4. A non-proportional change in the growth curve (the case of "lower input efficiency").

creases the land value, because the rotation age is lengthened and the volume of harvest is lower than the norm.

3.2. The optimal harvest age for standing timber under expected changes in future stands

3.2.1. Standing timber unaffected

Next, assume that current mature stands are not yet seriously affected by air pollutants, but there is reason to believe that future rotations will show a reduced growth. The assumption intuitively refers to areas like the Nordic countries where no acute forest damage has actually been observed, but still a concern for the future growing conditions exists. The problem is, then, whether standing timber should be harvested more or less rapidly than the "planned" rotations (for reference conditions) imply.

In what follows we explicitly recognize the standing timber, aged b years, as the starting point instead of bare land. Regarding realism, the initial age b should refer to mature stands at an early stage of maturity. The standing timber is assumed to follow the norm (original forecast growth curve for the site) while the future rotations, being identical to each other, follow a lower growth pattern. The problem is to maximize the present value of the standing timber plus the seeded forest land (the present value of the series of rotations starting on the newly seeded land) (cf. Johansson and Loefgren, 1985):

$$\text{(3.16)} \qquad \underset{T,t}{Max}\, PV = pf(T)e^{-r(T-b)} + e^{-r(T-b)}\, V(t,\alpha) \quad ,$$

where: b = the age of the standing timber;
T = optimal harvest age for the standing timber; and
t = optimum rotation for the future stands.

By differentiating (3.16) first with respect to t, the first-order condition for the optimum rotation in the future stands is obtained:

$$\text{(3.17)} \qquad \frac{\partial}{\partial t} PV = e^{-r(T-b)}\, \frac{\partial}{\partial t}\, [V(t,\alpha)] = 0 \quad ,$$

which holds if and only if

$$(3.18)\qquad \frac{\partial}{\partial t}[V(t,\alpha)] = 0.$$

Thus, the optimum rotation t for the future stands is independent of the harvest age T of the standing timber. Thus the problem can be solved by first solving independently for the optimum t^* and substituting the corresponding optimal land value into equation (3.16) (cf. Johansson and Loefgren, 1985). For the optimal harvest age of the standing timber, the following condition is obtained:

$$(3.19)\qquad \frac{\partial}{\partial T} PV = pf'(T) - rpf(T) - r[V(t^*,\alpha)] = 0 \quad ,$$

which can be presented in the form

$$(3.20)\qquad f'(T)/f(T) = r + rV(t^*,\alpha)/pf(T) \quad .$$

From (3.20) it is easy to see that the expected change of the growth function (decline in soil productivity) in the future rotations affects the harvest age of the standing timber through the value of seeded forest land, i.e. through a change in land rent or shadow cost of land use. From (3.19) or (3.20) it is obvious that the expected deterioration in the growth of future stands will lengthen the optimal harvest age for the standing timber. Because all kinds of growth reduction decrease the value of bare land, the interest on forest land also declines, and the right hand side in (3.20) declines. The equilibrium is re-established by letting the trees grow older.

The result is intuitive in that a reduced growth of future stands naturally reduces the implicit loss owing to the delay of receipts from future harvests or land rent attributed to growing the current stand further. As long as no damage is observed in the standing timber, it is also natural to utilize the growth potential of existing stands longer, *ceteris paribus*.

3.2.2. Observed damage in the standing timber

Empirically, the most relevant case may be the observed acute damage (deterioration of the health of trees) in mature stands with the consequent reduced rate of growth (this is what is said to happen in Central Europe). In terms of our setting the phenomenon is most appropriately presented as a growth change in the standing timber; on the other hand, it is natural in this case to expect that future rotations also differ from the norm (the predicted growth pattern of the relevant site under reference conditions).

To illustrate the situation we write a slight modification of the formulation in (3.16). It is assumed that the standing timber is subject to a change of the "forest dieback" type (Section 3.1.1 above; Figure 1, satisfying the regularity assumption), while the specific type of deterioration in the future stands does not matter:

$$\underset{T,t}{Max}\, PV = pf(T,\alpha)e^{-r(T-b)} + e^{-r(T-b)}V(t,\beta) \quad . \tag{3.21}$$

Again, the optimum rotation in the future stands t^* is solved independently. Then it holds for the optimal harvest age T^* of the standing timber that

$$\frac{f'(T,\alpha)}{f(T,\alpha)} = r + \frac{rV(t^*,\beta)}{pf(T,\alpha)} \quad . \tag{3.22}$$

In comparison to the norm or the standard Faustmann rule, the equations have two differences that affect the outcome in opposite directions. First, the expected lower growth curve of the future stands decreases the value of land, hence land rent, and *ceteris paribus* tends to lengthen the optimal harvest age. On the other hand, the observed damage in the standing timber affects the left-hand side of the equation. According to discussions above, the relevant change implies a decreasing relative rate of growth, and hence a shorter optimal harvest age in comparison to the norm. Also, the second term on the right-hand side of (3.22) is affected; the decline in the standing volume $f(T)$ raises, loosely speaking, the "effective guiding rate of interest" and *ceteris paribus* shortens the optimal harvest age.

Owing to these changes of opposite direction, the net effect is ambiguous, depending on quantitative relations of different effects (e.g., the relative strength of pollution effects in the standing timber *vs.* in the future stands). However, intuitively we may expect that the effect through the standing timber dominates. The effect through future rotations, even if unrealistically considered as known with certainty, tends to be small, not to say discounted out of existence, in "European" forestry with long rotation periods. Hence we may conclude that observed damage in the standing timber will typically shorten its optimal harvest age as compared to the planned rotation even under expectations of reduced growth in the future rotations.

3.3. Concluding remarks

The results concerning the effect of pollution-induced changes in the growth function on the optimal harvest age are summarized below. The potential implications to timber supply will be briefly discussed.

First, the standard Faustmann model was employed assuming that all rotations will be identical. Regarding the direction of the effect on optimum rotation, the specific type of growth change, i.e. the relative effects at various tree ages, turned out to be decisive:

(1) Damage observed first in mature stands (that "locally" reduces the relative rate of growth) will shorten the optimum rotation as compared to the norm, while

(2) both proportional and non-proportional changes affecting entire rotations (causing a "poorer site" or a delay of growth) will lengthen it.

Second, the effects on the optimal harvest age for existing mature stands were examined with allowance for the growth pattern of future rotations to differ from the standing timber. Then,

(3) under expected growth reductions in the future rotations, the optimal harvest age for unaffected standing timber will be lengthened.

This case might have some relevance in, for example, the Nordic countries, where no considerable damage has been observed for the moment (there may even have been signs of increased forest growth). On the other hand,

(4) acute damage observed in standing timber will typically shorten its optimal harvest age.

By "typically" I refer to the fact that forest damage may range from minor growth reduction to serious damage (labelled "forest dieback") that implies immediate sanitation fellings. In the usual case we assume that the unexpected rapid decline in the relative rate of stand growth is marked enough to outweigh the counter-affecting change in land rent. Intuitively, the case might resemble the situation in Central Europe with reported decline in crown densities and increased mortality of trees.

If the shift parameter of the growth curve would, instead, be given values above unity, the analysis could be easily reinterpreted as the effect of the so-called "CO_2 fertilization" effect (see Kauppi, 1987). Naturally, the impacts of an exogenous improvement in forest growth would be reversed in comparison with the above analysis (see Johansson and Loefgren, 1985).

Sometimes, the maximization of sustained yield could be taken as the objective of forest management. If the average output over the rotation period were to be maximized, the qualitative results would be identical to those presented here. This, in fact, is no surprise because it is well-known that the Faustmann rotation period approaches the one maximizing sustained yield in the case where the rate of interest tends to zero.

Turning to the implications of the changes in forest rotations to roundwood supply, the limitations of the model should be recognized. The relevance of the results is partly hampered by the underlying *ceteris paribus* assumption concerning the constant level of timber price. Ignoring this for the moment, we may reasonably summarize the effects as "potential" supply implications. These may be considered at two levels. First, the effect on short-term supply refers to the response during the adjustment period, i.e. when the changes have been observed and the adjustment to the new optimum rotation starts by cutting stands that have turned overmature or by letting trees grow older (cf. Clark, 1976). The long-term effect, on the other hand, refers to "supply" in the sense of maximum sustained harvest levels (the quantity $f(t\ )/t\ $), i.e. the supply potential as a flow concept.

The qualitative supply implications, again in comparison to the norm or "forecast" without forest damage, are summarized in Table 1 for the cases with identical rotations. Table 1 shows the intuitive expectation of increased short-term supply due to forest damage, and the decline in long-term supplies in all cases, while the last two short-term reactions are hard to grasp intuitively. Note that besides the rotation period, the growth curve or the volume harvested at rotation age also changes. So, for example, long-term supply in the proportional case would even be reduced without regeneration cost (i.e. if the rotation period remains unchanged).

The cases with non-uniform growth curves are more interesting. The short-term response, again, refers to adjustment effects due to rotation changes, but the long-term response may be examined at two time spans. In Table 2, the quantity $f(T)/T$ refers to the present stands (standing timber), and $f(t)/t$ to the flow from future stands.

In areas with no damage in standing timber, the expectation of growth reduction in future stands would imply a decline in short-term supply through a lengthening harvest age. On the other hand, observed damage will increase short-term supply whenever the shortening of the rotation period dominates over the change in land rent.

Table 1. Potential supply responses from once-and-for-all changes in the forest growth curve ("+" means positive effect; "–" means negative effect).

Case	Effect on Rotation Period	Supply Response	
		Short-term	Long-term
Damage in the Mature Forest	-	+	-
"Poorer Site" (proportional)	+	-	-
"Lower Intensity" (non-proportional)	+	-	-

Table 2. Potential supply responses when standing timber and future stands differ

Case	Effects on harvest age of standing timber	Supply responses		
		From Standing Timber		From Future Stands
		Short-Term	$f(T)/T$	$f(t)/t$
Expected Growth Reduction in Future Stands Only	+	-	+	-
Observed Damage in Standing Timber	-	+	-	-

In the above partial equilibrium analysis of the optimal harvest age, a parametric price was assumed. However, owing to shifts in timber supply both in the short term (changes in the timing of fellings of standing timber) and the long term (changing flows of wood), the equilibrium position of the roundwood market, and hence timber price, will be affected whenever the demand for wood is not perfectly elastic. Timber price, on the other hand, matters for the choice of optimum rotation (e.g. Johansson and Loefgren, 1985)[2]. Therefore, if all the exploitable

---

[2] The formulation in Section 3.2 allows the treatment of price as a function of time (age), i.e. exogenous price variation (cf. Lohmander, 1984; Comolli, 1984). This will not do here because there is a two-way causation between price and harvest age.

forests are not equally subject to damage, there will be a two-fold effect on aggregate supply. In what follows, a conventional supply-demand framework will be introduced to make this point more clearly.

4. THE EFFECT ON EQUILIBRIUM IN THE ROUNDWOOD MARKET

The standard forecast concerning the economic consequences of forest damage is a considerable pulse of high timber supply in the short term, based on anticipations of sanitation fellings. Increased damage would result in a lower price for timber (e.g. Kroth and Bartelheimer, 1984). On the other hand, the reduced productivity of forest ecosystems would mean a decline in the sustainable harvest (potential supply) in the long term. Intuitive as the above views are, they consider sanitation fellings and the decline in price as the one and only response. This, in turn, seems to require an implicit underlying assumption that all forests in the economy under consideration will be damaged, so the question of indirect effects does not arise.

Because the effects of pollution vary widely with respect to soil types, tree species, etc., it might be reasonable to analyze the problem theoretically, assuming that just one part of the economy's forest area is damaged while the other remains unaffected. Under this assumption timber supply consists of two subsectors that interact through the market. An exogenous shift of supply from the damaged forests will now have further effects on the undamaged forests (cf. Dykstra and Kallio, 1987).

4.1. Short-term effects

The market for roundwood is formulated by making some very general and conventional assumptions concerning the supply and demand functions. Actually, we only know that the short-term supply of timber depends positively on its price. Such an assumption may be theoretically justified by various models (e.g., see: Johansson and Loefgren, 1985; Binkley, 1987), including the response of the above Faustmann model in the adjustment period, but only under the particular assumption of static, unitary elastic price expectations. No matter what the underlying theoretical model is, empirical results quite consistently support the normal, positively sloped supply curve.

The model is written as follows (subscripts are used throughout to denote partial or ordinary derivatives of functions):

$$
\begin{array}{llll}
(4.1) & Q_S^D = & s^D(p,\beta) & s_p^D \geq 0,\ s_\beta^D > 0 \\
 & Q_S^U = & s^U(p) & s_p^U > 0 \\
 & Q_D = & d(p) & d_p < 0 \\
 & Q = & Q_S^D + Q_S^U & \text{(equilibrium condition).}
\end{array}
$$

The first equation defines supply from damaged forests as a non-decreasing function of the current price for timber and increasing in β, a parameter describing the level of pollution. Increasing β means increased pollution and forest damage, hence increasing supply in the short term (cf. Sections 3.1.1 and 3.2.2 above). Second, the supply of wood from undamaged forests is an increasing function of price only, while the demand for timber (third equation) is decreasing in p. The equilibrium is given as an equality between demand and aggregate supply, the latter being a horizontal summation of supplies from both sub-sectors.

In an initial equilibrium the following conditions hold:

$$
\begin{array}{ll}
(4.2) & d(p) - Q^U - Q^D = 0 \\
 & s^D(p,\beta) - Q^D = 0 \\
 & s^U(p) - Q^U = 0
\end{array}
$$

To point out the qualitative effects of increased pollution on the equilibrium price and quantities in the market, suppose a change in β. Then the following comparative static results hold true (for proof, see Appendix 2):

$$
\begin{array}{ll}
(4.3) & dp/d\beta = \dfrac{-s_\beta^D}{|J|} < 0 \quad , \\[2ex]
 & dQ^D/d\beta = \dfrac{s_\beta^D(-d_p + s_p^U)}{|J|} > 0 \quad , \quad \text{and} \\[2ex]
 & dQ^U/d\beta = \dfrac{-s_p^U s_\beta^D}{|J|} < 0 \quad ,
\end{array}
$$

where $|J| = s_p^D - d_p + s_p^U > 0$.

The results in (4.3) unambiguously tell that increased pollution results in increased sales (fellings) from the damaged forests and in a lower price for wood. What is interesting is the fact that the sales from undamaged forests at the same time decline, which is due to the indirect effect of a change in price. The total quantity traded being the sum of the two sub-sectors, its change is the net effect (sum) of the two changes:

$$(4.4) \qquad dQ/d\beta = \frac{-d_p s_\beta^D}{s_p^D - d_p + s_p^U} > 0 \quad .$$

The net effect of the opposite changes on the quantity traded is unambiguously positive. However, note that the net effect is less than the mere increase of supply from the damaged forests. Thus, the effect of forest damage on the equilibrium is partly cancelled by the simultaneous response of the undamaged forests to the price change. The magnitude of changes in price and quantity depends on the slopes of supply and demand curves.

We next modify the model slightly by introducing timber price expectations into the supply function of undamaged forests. In contrast with the above myopic supply behavior, reacting only to current price changes, it is assumed that timber suppliers make forecasts of future short-term price fluctuations. The expectations are adjusted (albeit imperfectly) on the basis of observed price changes. Expectations are assumed not to enter the supply function of the forests subject to damage. The intuitive reasoning is that along with observed damage, the health aspects of the forest tend to dominate, and "there is no space for price speculation".

The model remains the same as in (4.1) except for the supply equation for the undamaged forests, which now reads:

$$(4.5) \qquad Q_S^U = s^U(p, p^e) \; ; \; s_p^U > 0 \, , \; s_{p^e}^U < 0$$

$$\text{and } p^e = p^e(p) = p^e(p(\beta));$$

$$dp^e/dp \in (0,1), \; dp^e/d\beta < 0.$$

Thus, supply from unaffected forests is a decreasing function of price expectations p^e, which in turn are an increasing function of the current price. Because the equilibrium price depends negatively on the pollution parameter β, the price expectation p^e also becomes a decreasing function of the level of forest damage.

The result is that the effects are not unambiguous, because the signs of the comparative static derivatives are not unambiguously known without reference to the relative magnitudes of the different price effects involved:

$$(4.6) \qquad dp / d\beta = \frac{-s^D_\beta}{|J|} \gtrless 0 \quad ,$$

$$dQ^U / d\beta = \frac{-(s^U_p + s^U_{p^e}(\frac{dp^e}{dp}))s^D_\beta}{|J|} \gtrless 0 \quad ,$$

$$dQ^D / d\beta = \frac{s^D_\beta(-d_p + s^U_p + s^U_{p^e}(\frac{dp^e}{dp}))}{|J|} \gtrless 0 \quad , \quad \text{and}$$

$$dQ / d\beta = \frac{-s^D_\beta d_p}{|J|} \gtrless 0 \quad ,$$

where $|J| = s^D_p - d_p + s^U_p + s^U_{p^e}(dp^e / dp)$.

The ambiguity results from the change in price expectations and their effect on timber supply from undamaged forests. However, note that by assumption, the elasticity of expectations is between zero (expectations independent of current price changes) and unity (perfect adjustment of expectations), i.e. $0 < dp^e / dp < 1$. Then it is sufficient (but not necessary) for $|J| > 0$ and the results to be unambiguous that the effect of the expected price on supply from undamaged forests, $s^U_{p^e}$, is not greater in absolute value than that of the current price, s^U_p. In the long-term, price is at the long-run equilibrium level (path), and expectations tend to be correct, so the expected price is equal to the current (actual) price. Then p^e and p collapse to the same variable, and the effect of the price variable is the sum of the two opposite effects.

Hence, an interpretation for the required non-negativity of the sum $s^U_p + s^U_{p^e}$ is that the long-term supply curve of the undamaged forests is not backward bending.[3] If this is true then the signs in (4.6) are the same as in (4.3), and the changes in price and the total quantity traded are larger than without decreasing price expectations. Otherwise the situation is ambiguous depending on relative magnitudes, and unfortunately, the possibility of a backward bending long-term supply curve

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[3] Suppose the market is in an equilibrium where timber price p is unchanging over a considerable time. With all previous values of p constant ($p = \bar{p}$), adaptive expectations imply that eventually the expectation of price in period $t + 1$, held in period t, equals the actual price or $p^e_t,t+1 = \bar{p} = p_t$. With $p^e = p$, the current and expected price effectively collapse to one variable, and the net effect is the sum of the two opposite effects.

may exist (see next section).

4.2. Long-run effects

Conceptually, short-term supply means a supply from a given stock resulting from the adjustment of the standing volume to a new equilibrium. On the other hand, we may be interested in the impact on long-run supply. This can be taken as a flow concept, the sustainable harvest level, which requires a time span long enough for the growing stock to adjust to the new equilibrium level. In what follows, the above model is straightforwardly used by making some modifications to the sign assumptions.

In the long run, pollution implies a lower sustainable harvest due to the reduced productivity of (a part of) forest soils. Hence, the pollution parameter β now affects the supply from damaged forests contrary to the above case, and we have $s_\beta^D < 0$.

Regarding the effect of price on long-term supply, two possibilities will be considered. Thinking about long-run supply in terms of the Faustmann model, it is well-known that the rule implies a backward-bending supply curve if rotation age is the only decision variable (Hyde, 1980; Johansson and Loefgren, 1985; Binkley, 1985). An economic interpretation of the situation is that silvicultural effort and land use fail to react to the (expected) price, so the only effect of an increased price level is a shorter rotation, and subsequently lower sustainable harvest $f(t^*)/t^*$. The supply equations under these assumptions are as follows:

$$(4.7) \qquad Q_s^D = s^D(p,\beta)\,; \qquad s_p^D < 0\,,\ s_\beta^D < 0$$

$$Q_s^U = s^U(p)\,; \qquad s_p^U < 0$$

The negative slopes of supply curves make the situation a bit complicated. Namely, there is a possibility of market instability (an unstable price adjustment process) that emerges when the aggregate supply curve is flatter than the demand curve (e.g., see Johansson and Loefgren, 1983). Because there have been no signs of such a phenomenon in the real world, we restrict the analysis to the normal case with a stable adjustment. Hence, the following results are derived by assuming that aggregate supply is steeper than the demand curve; i.e., $d_p < s_p^D + s_p^U$. Then the Jacobian determinant is positive, and the following results are obtained:

$$(4.8)\quad \begin{aligned} & dp/d\beta > 0 \quad , \\ & dQ^U/d\beta < 0 \quad , \\ & dQ^D/d\beta < 0 \quad . \end{aligned}$$

The derivations are not repeated because the comparative static derivatives given in (4.3) and Appendix 2 hold except for changed signs of the derivatives of the relevant functions.

As could be expected, decreasing sustainable harvest due to reduced soil productivity in damaged areas will induce a higher price level. But again, there is a simultaneous reaction from undamaged areas. Because the price level rises, stands will be cut at a younger age, implying lower sustainable harvests also in undamaged areas. This strengthens the decline in aggregate supply.

Declining supply from undamaged forests, however, depends critically on the assumption of negative price elasticities. Suppose next that long-run supply from both sub-sectors reacts positively to timber price. Then we have supply equations

$$(4.9)\quad \begin{aligned} & Q_s^D = s^D(p,\beta)\ ; \quad S_p^D > 0\ , \quad s_\beta^D < 0 \\ & Q_s^U = s^D(p)\ ; \quad s_p^U > 0 \quad . \end{aligned}$$

The more conventional effect of price on long-run supply is obtained from the Faustmann model by assuming an endogenous silvicultural effort (e.g., Hyde, 1980). The intuitive reasoning behind the positive effect is that, in undamaged areas, a higher price attracts more advanced silvicultural practices (more efficient regeneration, fertilization, etc), and land use might also react. On the other hand, in damaged or susceptible areas a higher price might justify investments in the protection of forest soils against the impacts of acidification (improving the Ca and Mg balances, perhaps) or more appropriate management practices (felling or regeneration methods, for example). Under these assumptions, the results are unambiguously as follows:

$$(4.10)\quad \begin{aligned} & dp/d\beta > 0 \quad , \\ & dQ^D/d\beta < 0 \quad , \\ & dQ^U/d\beta > 0, \text{ and} \\ & dQ/d\beta < 0 \quad . \end{aligned}$$

The results indicate that timber price would rise, yet much less than under negatively sloped supply curves. The total quantity traded would also decline as before but less than above ($|J|$ in the denominator is greater). A qualitative difference is that supply from undamaged forest areas would increase in the long run due to a positive change in price level. Hence, the price effect would offset part of the decline in supplies from damaged areas, that is, the neutralizing effect is the mirror image of that in the short term.

5. DISCUSSION

The effects of forest damage attributed to air pollutants on optimum forest rotations and the equilibrium position of the roundwood market were considered above in theoretical terms. The main results and conclusions can be summarized as follows.

(1) Once-and-for-all changes of the growth function may either shorten or lengthen the optimum rotation, depending on the specific type of change.

(2) Concerning the effect of acute forest damage observed in current mature stands, the phenomenon is to be viewed as an unexpected decline of the relative growth rate of the forest. This will imply a shortened optimal harvest age for the standing timber, hence increased supply in the short term. Theoretically, however, expected growth changes in future rotations are also important here according to the Faustmann rule.

(3) In the short term, considerable forest damage would result in a pulse of increased supply of wood to the market, hence higher quantity traded and a lower price for timber.[4] However, the indirect effects through the price change on supply from the undamaged forests should be recognized. Thus, the effect of damage on aggregate supply may be partly neutralized by a price-induced reallocation of fellings. This result can be taken as a simple formal justification for the consideration of the neutralizing effect in quantitative scenarios made with the IIASA forest sector model (Dykstra and Kallio, 1987). The situation is complicated by short-term price expectations held by timber suppliers.

(4) In the long run, declining productivity of (a part of) the forest soils implies a reduced sustainable harvest, i.e. decreased supply and a higher price for timber. However, the magnitude of the quantitative effect depends on whether

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[4] Kroth and Bartelheimer (1984) suggested that along with increased damage, timber supply becomes more price inelastic. If this is the case, a lower price level is not the only change. Assuming variations in the demand for timber (shifting demand schedule), a steeper supply curve will require larger price changes for a given increase in quantity traded. Thus, given shifting demand, price variations tend to be strengthened.

the level of silvicultural effort responds to the price changes, and whether the long-run supply from undamaged forests is positively or negatively price elastic.

The relevance of the results, as a theoretical basis for quantitative scenarios, for example, depends on the empirical relevance of both growth and behavioral assumptions. In the analysis of optimum rotations, the assumption of present-value-maximizing behavior is made. However, qualitatively identical results will be obtained if timber management is volume-oriented (sustainable yield-maximizing). Regarding acute damage in current stands, more rapid fellings and increased short-term supply are clear from purely biological considerations. It should be noted here that the optimal harvest age refers to the final cut in even-aged management (strictly speaking, clear-cutting), being oversimplified in the sense that special management practices, e.g. selective harvesting of damaged trees, may become more important in practical terms as forest damage progresses.[5]

On the other hand, the assumptions behind the short-term changes in the roundwood market equilibrium are very general and non-restrictive (the usual slopes of supply and demand curves). The price-induced neutralizing effect due to the response from undamaged forests is also likely to have its counterpart in forestry under a non-price-oriented or price-insensitive management. A reallocation of fellings may be expected to take place under the institutional practices of public forests, or in planned economies as well (Dykstra and Kallio, 1987). The price sensitivity of long-run timber supply, on the other hand, is an open issue.

Although the intuitive reasoning behind the analysis, hence the qualitative implications, seem fairly realistic, nothing can be concluded about the quantitative significance of the economic consequences. Other limitations also exist, so the paper is at best a first approximation of the problem. Some of these limitations are pointed out below.

The analysis ignores the inherent uncertainty involved in the problem. Rather than as changes represented deterministically, the effect of pollution could be represented as changes in the stochastic properties of the growth process (for some results on optimum cutting rules with random elements, see Johansson and

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[5] In mountainous regions where the most serious damage has been reported, no large-scale clear-cutting is possible. However, in the limit the arguments here apply to individual trees; so the "forest stand" may refer to very small plots of forest land.

Loefgren (1985)). The expected optimal rotation under stochastic growth can probably not be straightforwardly compared with that in the deterministic case. Regarding supply behavior in a two-period setting, the expected future effects of pollution (forest damage) could be considered as increasing uncertainty of the revenue from cutting tomorrow. Modifying the results of Johansson and Loefgren (1985), we could expect that both a spread-preserving decrease in the random revenue from tomorrow's cut, and a mean-preserving increase in its variability, will increase today's cut, ***ceteris paribus***, if risk aversion is assumed.

By definition, the comparative static illustration of the roundwood market only considered the directions of change by comparing the equilibrium positions before and after the change. Dynamic adjustments, especially the dynamics of the standing timber volumes, as a determinant of market equilibrium can be important. Also the development of timber price over time, and expectations of it, may be essential because the time path of price has impacts on optimal cutting decisions apart from those of price levels.

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APPENDIX 1. The Proof to (3.14)

By differentiating (3.12) with respect to t and dividing through by p the following condition is obtained, to be denoted by F:

(A.1) $$F(t,\alpha,r) = \alpha f'(\alpha t) - rf(\alpha t) - \frac{r}{e^{rt}-1} f(\alpha t) = 0 \quad .$$

By the implicit function rule,

$$\partial t / \partial \alpha = -F_\alpha / F_t \quad .$$

Because $F_t = \partial^2 V / \partial t^2 < 0$ by virtue of the second-order condition, the sign of the comparative static derivative is the same as that of F_α, and

(A.2) $$F_\alpha = f'(\alpha t) + f''(\alpha t)t - f'(\alpha t)\,[\frac{r}{1-e^{rt}}]$$

$$= f'(\alpha t)\,[1 - \frac{r^t}{1-e^{-rt}}] + \alpha t f''(\alpha t) \quad .$$

Because $f'(\alpha t) > 0$ and $f''(\alpha t) < 0$, it is sufficient for $F_\alpha < 0$ that

(A.3) $$[1 - \frac{rt}{1-e^{-rt}}] < 0$$

$$\Longleftrightarrow 1 - e^{-rt} - rt < 0 \quad \Longleftrightarrow e^{-rt} - 1 > -rt \quad .$$

Denote $e^{-rt} - 1 = h(t)$ and $-rt = g(t)$. Then $h(0) = 0$, $h(t) < 0$ and $g(0) = 0$, $g(t) < 0$ *all* $t, r > 0$. Thus, $h(0) = g(0) = 0$ and $g'(t) = -r < -re^{-rt} = h'(t)$ *all* $r,t > 0$ as $e^{-rt} < 1$ *all* $r,t > 0$.

Hence the functions are always negative, starting at zero when $t = 0$, and $g(t)$ declines faster for all $t > 0$, so $h(t) > g(t)$ *all* $t, r > 0$:

Thus, $[\cdot] < 0$ *all* $r,t > 0$, hence F_α is negative (for figure see p. 28), and

(A.4) $\partial t / \partial \alpha < 0$.

APPENDIX 2. The proof to (4.3), (4.4), and (4.6).

The effect on market equilibrium price and quantities of a disequilibrating change in β is obtained by taking the total derivative of each identity in (4.2) with respect to β and rearranging to get the matrix equation

$$\text{(A.5)} \qquad \begin{bmatrix} d_p & -1 & -1 \\ s_p^D & 0 & -1 \\ s_p^U & -1 & 0 \end{bmatrix} \begin{bmatrix} dp/d\beta \\ dQ^U/d\beta \\ dQ^D/d\beta \end{bmatrix} = \begin{bmatrix} 0 \\ -s_\beta^D \\ 0 \end{bmatrix} .$$

Using Cramer's rule, the comparative static partial derivatives are obtained. The Jacobian determinant of the system is, to be stated first,

$$|J| = s_p^D - d_p + s_p^U > 0 \quad .$$

Then the following results hold true:

$$\text{(A.6)} \qquad dp/d\beta = \frac{\begin{vmatrix} 0 & -1 & -1 \\ -s_\beta^D & 0 & -1 \\ 0 & -1 & 0 \end{vmatrix}}{|J|} = \frac{-s_\beta^D}{|J|} < 0 \quad ,$$

$$dQ^D/d\beta = \frac{\begin{vmatrix} d_p & -1 & 0 \\ s_p^D & 0 & -s_\beta^D \\ s_p^U & -1 & 0 \end{vmatrix}}{|J|} = \frac{s_\beta^D(-d_p + s_p^U)}{|J|} > 0 \quad ,$$

$$dQ^U/d\beta = \frac{\begin{vmatrix} d_p & 0 & -1 \\ s_p^D & -s_\beta^D & -1 \\ s_p^U & 0 & 0 \end{vmatrix}}{|J|} = \frac{-s_p^U s_\beta^D}{|J|} < 0 \quad .$$

By writing the equilibrium identities corresponding to (4.2) and taking the total derivatives with respect to β, we obtain a matrix equation with one element of the coefficient matrix changed as compared to (4.3):

$$\text{(A.7)} \qquad \begin{bmatrix} d_p & -1 & -1 \\ s_p^D & 0 & -1 \\ s_p^U + s_{p^e}^u \left(\frac{dp^e}{dp}\right) & -1 & 0 \end{bmatrix} \begin{bmatrix} dp/d\beta \\ dQ^U/d\beta \\ dQ^D/d\beta \end{bmatrix} = \begin{bmatrix} 0 \\ -s_\beta^D \\ 0 \end{bmatrix}$$

The relevant derivatives in (4.6) are solved by using Cramer's rule. Clearly, it is sufficient for $|J| > 0$ that $s_p^U \geq -s_{p^e}^U$ because $0 < dp^e / dp < 1$, otherwise the sign is indeterminate *a priori*. The same applies to the numerators in $dQ^U / d\beta$ and $dQ^D / d\beta$.

PUBLICATIONS FROM THE PROJECT ON ECOLOGICALLY SUSTAINABLE DEVELOPMENT OF THE BIOSPHERE

The following publications appear with their serial numbers from the compilation "Publications of IIASA's Project on Ecologically Sustainable Development of the Biosphere". Serial number and date of final publication do not always coincide because papers are assigned a serial number when first circulated as preprints. Copies of as yet unpublished documents are available from the Project.

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