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PREFACE

Many of today's most significant socioeconomic problems, such as slower economic growth, the decline of some established industries, and shifts in patterns of foreign trade, are inter- or transnational in nature. Intercountry comparative analyses of recent historical developments are necessary when we attempt to identify the underlying processes of economic structural change and formulate useful hypotheses concerning future developments. In this respect econometric models play an important role.

The paper relates changes of stocks of inventories and the supply surplus over demand. The assumptions of constant prices and restriction of production to a certain level reflect the background of the study, which relates to inventories in a socialized trade. The pattern of the relation is sketched on the basis of empirical evidence and pictured by means of beta functions. The paper ends with the statement of the author that an empricial verification is possible, but perhaps difficult.

A. Smyshlyaev

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INTENSITY OF DISEQUILIBRIUM AND CHANGES IN INVENTORIES

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INTRODUCTION

It is usually accepted in economic theory that one of the main reasons for keeping inventories is to balance supply and demand. However, the experiences of the recent years in Poland indicate that inventories play this role only in a limited way. The present paper is devoted to an interesting problem concerning the possibilities of using the changes of stocks of inventories as disequilibrium indicator for particular commodity markets. The analysis covers the changes of inventories in a socialized trade; the study, however, is of a more general character and the conclusions also apply to the inventories of finished products in enterprises, and to materials and semi-manufactured goods in the production process. The first part is an introduction into the problem of disequilibrium modeling. The changes of inventories in relation to the changes of market disequilibrium are analyzed in the second part. The third part contains a mathematical presentation of the above relationships and the proposal of building a model where the changes of inventories would serve as the disequilibrium indicator.

I. DISEQUILIBRIUM MODELING. GENERAL REMARKS

Generally speaking, a disequilibrium is opposite to the equilibrium state. This statement, however, does not release us from the necessity of defining the "equilibrium", because this expression is not unmistakably understood.

Equilibrium, in a static sense, is the equality of demand and supply in every period of time. Such a narrow understanding of the problem is inconsistent. Not even the most ideal market is equalized (in the above sense) if any of its elements changes. This is connected to the fact that real adjustment processes take time. This conclusion leads us to dynamic equilibrium. The system is in dynamic equilibrium[1)] if, starting with any value, all the variables tend towards their initial values (equilibrium) within their limits as time moves towards infinity. We can say that an equilibrium is stable if, after departure from the equilibrium caused by forces outside the system, the equilibrium is restored (initial position).

Static equilibrium is a degenerated case of dynamic equilibrium, where the equality of demand and supply is examined at every moment of time. Therefore we propose to speak of balance and imbalance rather than of equilibrium and disequilibrium, leaving these terms strictly to dynamic analysis[2)].

The reasons for the appareance of imbalance are not easy to find. To consider the problem more thoroughly, let us write down a model for any market in a system of three equations:

demand $D = f_D(x_D, \alpha_D) + \xi_D$ (1)

supply $S = f_S(x_S, \alpha_S) + \xi_S$ (2)

realized value $Q = \min \{D, S\}$, (3)

where

x_D, x_S - vectors of explanatory variables in equations of demand and supply of the size lxm and lxn,

α_D, α_S - vectors of parameters,

ξ_D, ξ_S - random components with a normal distribution.

$$\xi_D : N(0, \sigma_D^2) \quad , \quad \xi_S : N(0, \sigma_S^2) \quad .$$

The above system can be additionally extended by an identity defining the values of imbalance[3]:

$$SE = S - D \quad . \tag{4}$$

If SE > 0, then there is a supply surplus. Otherwise SE < 0, which means an excess demand.

Balancing the system (1)-(4), the equality is assumed:

$$D = S = O \quad , \tag{5}$$

which implies SE = 0.

The assumption that the system presented remains in equilibrium (in a dynamic sense) means accepting the hypothesis about the existence of certain mechanisms causing a return to the initial state. For markets with a freely negotiated price, the adaptation is realized by the changes of price, which can be written as:

$$\frac{dP}{dt} = f_P(S - D) \quad . \tag{6}$$

In fact, the above formula is of the notation of the commonly known L. Walras[4] hypotheses in the form of a dynamic continuous model.

Quantity adjustment is in accordance with the Marshall law of price surplus:

$$\frac{dQ}{dt} = f_Q(Ps - Pd) \quad , \tag{7}$$

where

Ps, Pd - supply price, demand price.

The process of price adjustment may be disturbed by introducing administrative measures (i.e. decreed prices, maximum or minimum prices) or as a result of lags in price changes. In this case the quantity adjustments may take place either by increasing supply of the commodity under examination to the market or by diminishing the consumer demand. These adjustments will occur not as a result of direct market pressures, but as the consequence of the functioning of the other economic mechanisms (for example, a tendency to increase supply in order to follow the demand increase may arise from the motivation of socialized enterprises to maximize their global profits). They may also be the result of decisions of public administration. Let us consider these problems in greater detail.

The increase of supply for consumers may be a short-term increase, i.e. by means of decreaseing inventories (in the socialized trade) or--if there are free production capacities--by means of making use of them. This process will be manifested by the labor productivity increase (introducing the second and third shifts) or by the increase of the number of employees. Import is an additional possibility of increasing supply.

The decrease of consumer demand for the commodity under examination is possible to be achieved by constraining the personal incomes of the households. The constraints consist, for example, in imposing higher direct taxes or the so-called constant charges (rents, other charges connected with flats, etc.) by the government.

If the processes of quantitative adjustment are fast enough, then, in spite of fixed prices, imbalance will not appear at all or only in a very short period and will be invisible to the consumer.

It should be underlined that a mixed situation, consisting of partial price and quantity adjustments, may take place in practice.

Furthermore, we will be interested in such a case where adjustment processes--except for the changes of inventories--are partly blocked and lagged, which is followed by the appearance of imbalance (excess demand).

II. INVENTORY CHANGES IN CASE OF BARRIERS IN ADJUSTMENT PROCESSES

Let us assume that there is in the socialized trade an optimum, desired state of inventories of the commodity examined, which is, in the simplest presentation, proportional to the volume of sales, Sc[5]:

$$Ro = b + d\ S_c \quad , \tag{8}$$

where

$d \geq 0$.

Let us consider a situation where the increase of personal incomes--with other conditions unchanged--is followed by the increase of consumer demand. Additionally we accept the following assumptions: first, that prices are constant; second, that constraints in the production process, such as raw material, equipment or employment barriers, do not allow for an increase in the production[6]; and third, that the import of particular consumer goods or materials for their production is impossible.

The assumed demand increase causes a diminishing of inventories in the socialized trade below the optimum level, Ro. If unfavorable tendencies do not change, the process will continue, and the rate of inventory decrease will probably grow. This is shown on the A_1A_2 curve in Figure 1. The inventories ensure constant sales (that is: $S > D$) until reaching the critical state Rk, $Rk < Ro$. Having surpassed Rk, excess demand appears, $SE < 0$. It can be assumed that a tendency to restrict further inventory demand will take place, i.e. inventories will decrease at diminishing rates until their stock is completely exhausted, which is an equivalent of the minimum state of inventories Rn, $Rn < Rk$ (A_2A_3 curve). A further deepening of

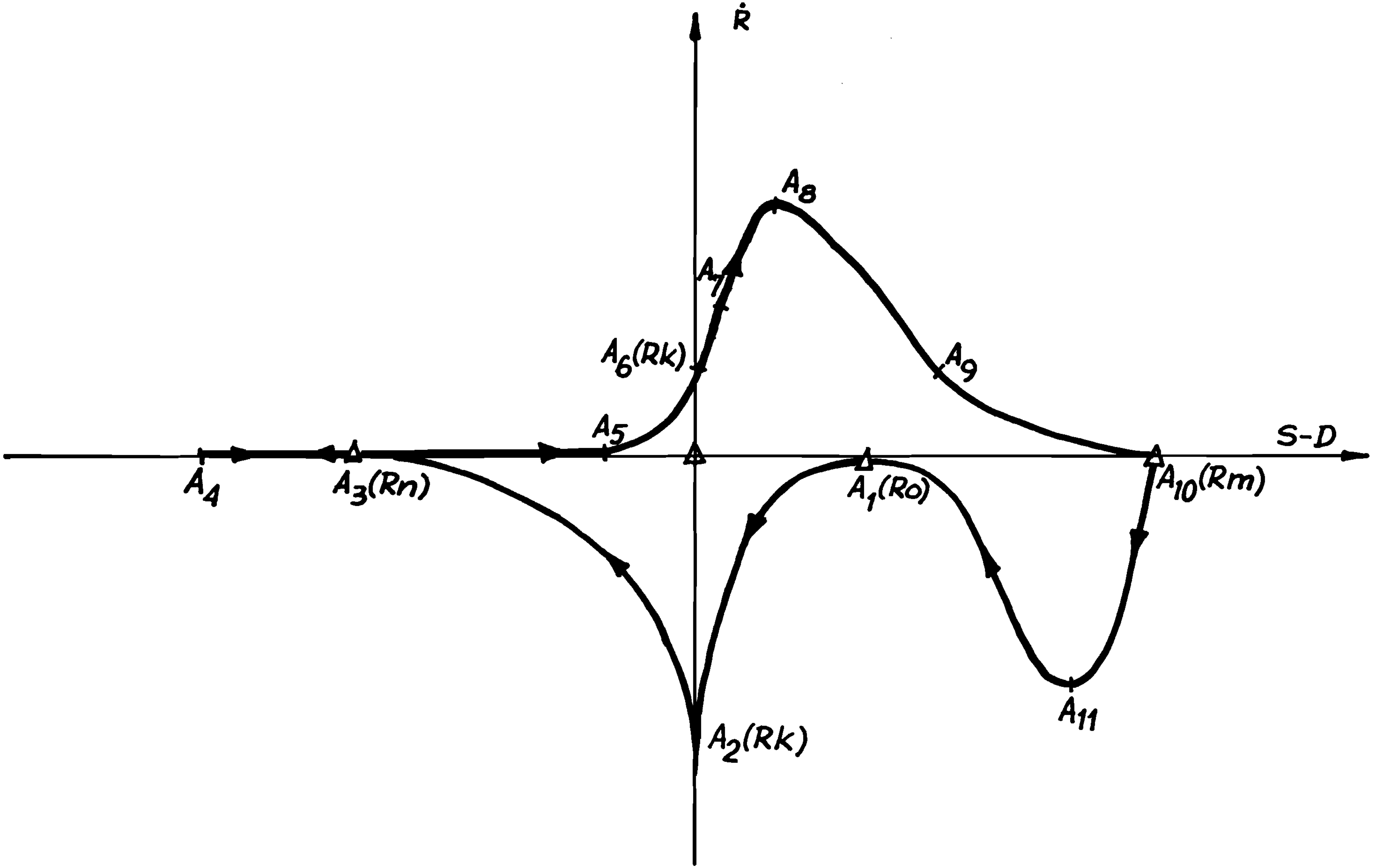

Figure 1. Inventory changes and the intensity of disequilibrium.

imbalance will not result in changes of inventories, $\dot{R} = 0$, where $\dot{R} = \frac{dR}{dt}$ (section A_3A_4). In case of the increase of supply of the commodity examined in the market--given the consumer demand--all the supplied commodities flowing into the socialized trade channels will be bought by the households. This results in diminishing demand, but the inventories do not grow (section A_3A_5). In case of a further increase of supply, restoring of stocks of inventories begins; it is fast at the beginning (A_5A_7 curve), then moves more slowly (curve A_7A_8) while approaching the optimum state by the inventories Ro. It should be underlined that the initial increase of inventories $\dot{R} > 0$ (A_5A_6 curve) cannot be interpreted as accumulation of supply surpluses till the moment of surpassing the critical level Rk by the inventories.

While approaching the optimum level Ro, the increase of inventories is declining in the following (A_8A_9 curve). Difficulties with stopping the process of increasing inventories cause further growth of the inventories until a maximum state Rm, Rm > Ro is reached. The latter may be understood as inventories provoking no more orders for the commodity under examination from the trading sector. The costs the trade pays for maintaining inventories larger than optimum cause a tendency towards diminishing such inventories. The rate of inventory decrease is declining while approaching point Ro ($A_{10}A_{11}$ curve).

The results of the above analysis[7)] are quite general. First, inventory decrease cannot be uniquely interpreted as the manifestation of the increase of disequilibrium intensity[8)]. On the other hand, in the period of recovery from the economic crisis, the inventory increase does not mean meeting consumer demand until the moment of surpassing the critical inventory level. The third conclusion and the most important one is that the inventories "behave" according to the prevailing type of the imbalance intensity changes (whether it is growing or decreasing) and the type of imbalance itself (whether it is characterized by excess demand (SE < 0) or supply (SE > 0)).

Figure 1 gives the hypothetical shape of the curve representing changes of inventories in relation to the type and intensity of disequilibrium. It is worth underlining that, firstly, the position of Rn, Rk, Ro, and Rm was chosen in a way allowing for graphic presentation of separate process phases. We think however, that in many cases maximum inventories (point Rm) will be close to optimum inventories (point Ro).

Secondly, some phases of the "cycle" of inventory changes presented may not appear at all, if adjustment processes are efficient. This concerns mainly the period when a deepening disequilibrium is not accompanied by changes in inventories (section A_3A_4). An earlier appearance of adjustment processes does not allow for complete exhaustion of inventories.

Thirdly, every "crossing" of the cycle through the inventories may differ from another, as is shown in Figure 2. Especially the values of minimum inventories (points Rn, Rn', Rn") and maximum inventories (points Rm, Rm', Rm") do not have to be the same. According to formula (8) the place of point Ro may change in a separate cycle (not shown in Figure 2).

It is worth mentioning that Figure 2 illustrates the dynamic process in one plane. However, we have to consider it in three-dimensional space where "time" is the third coordinate.

Then, we are mainly interested in the deflection of inventories from steady equilibrium--point Ro--leading to the decrease of inventories. In reality it may happen that supply increase takes place (under a given demand) despite the fact that the inventories were initially at the level Ro, as for example, due to an increase of production capacities (new plant coming into operation), or changes in import-export relations (export constraints, abolishment of tax barriers). As shown in Figure 3, initial, quick increment of inventories (B_1B_2 curve) will stop after some time (B_2B_3 curve). After reaching a maximum level, inventories will drop to the optimal level (point B_3).

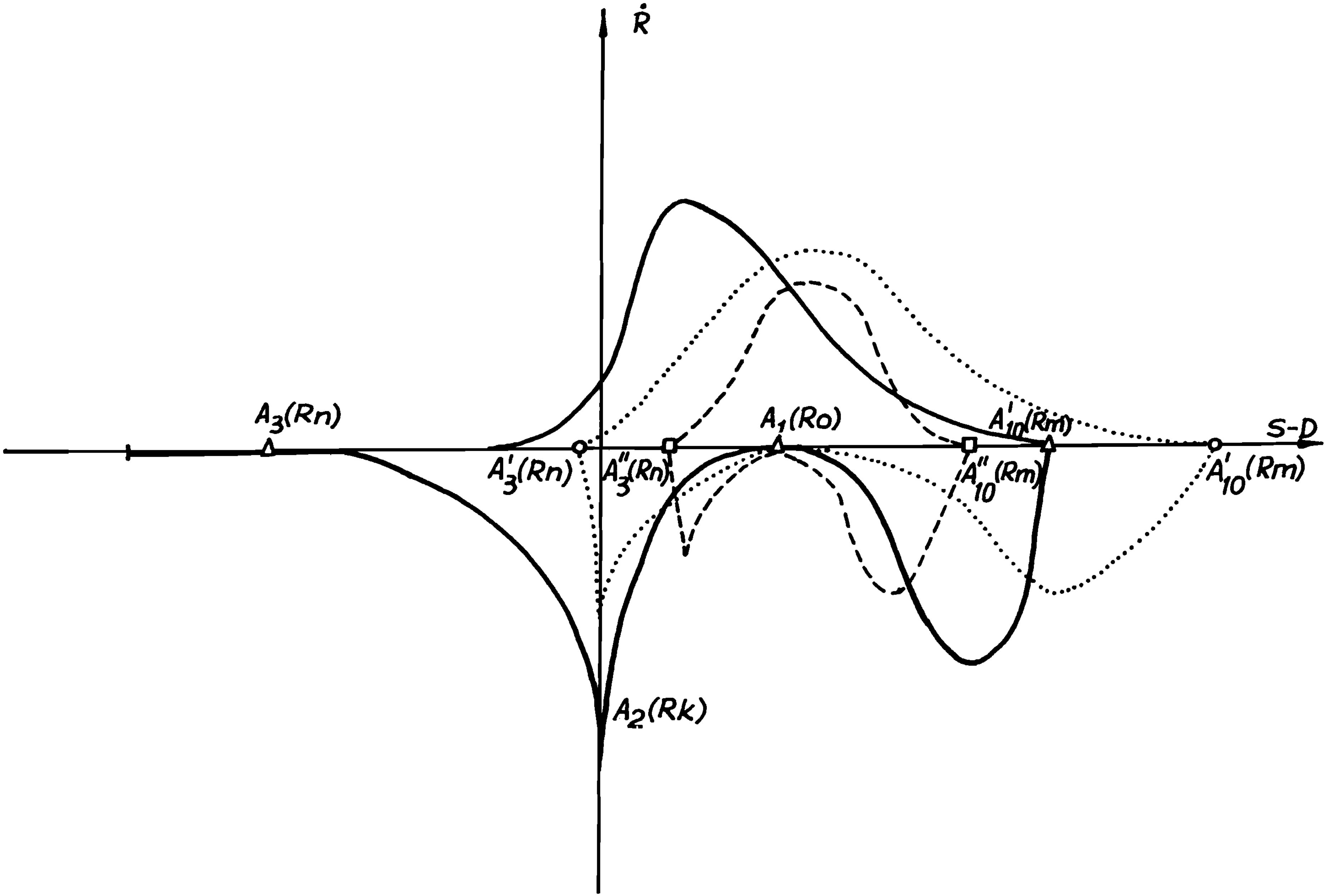

Figure 2. Cycles of inventory changes.

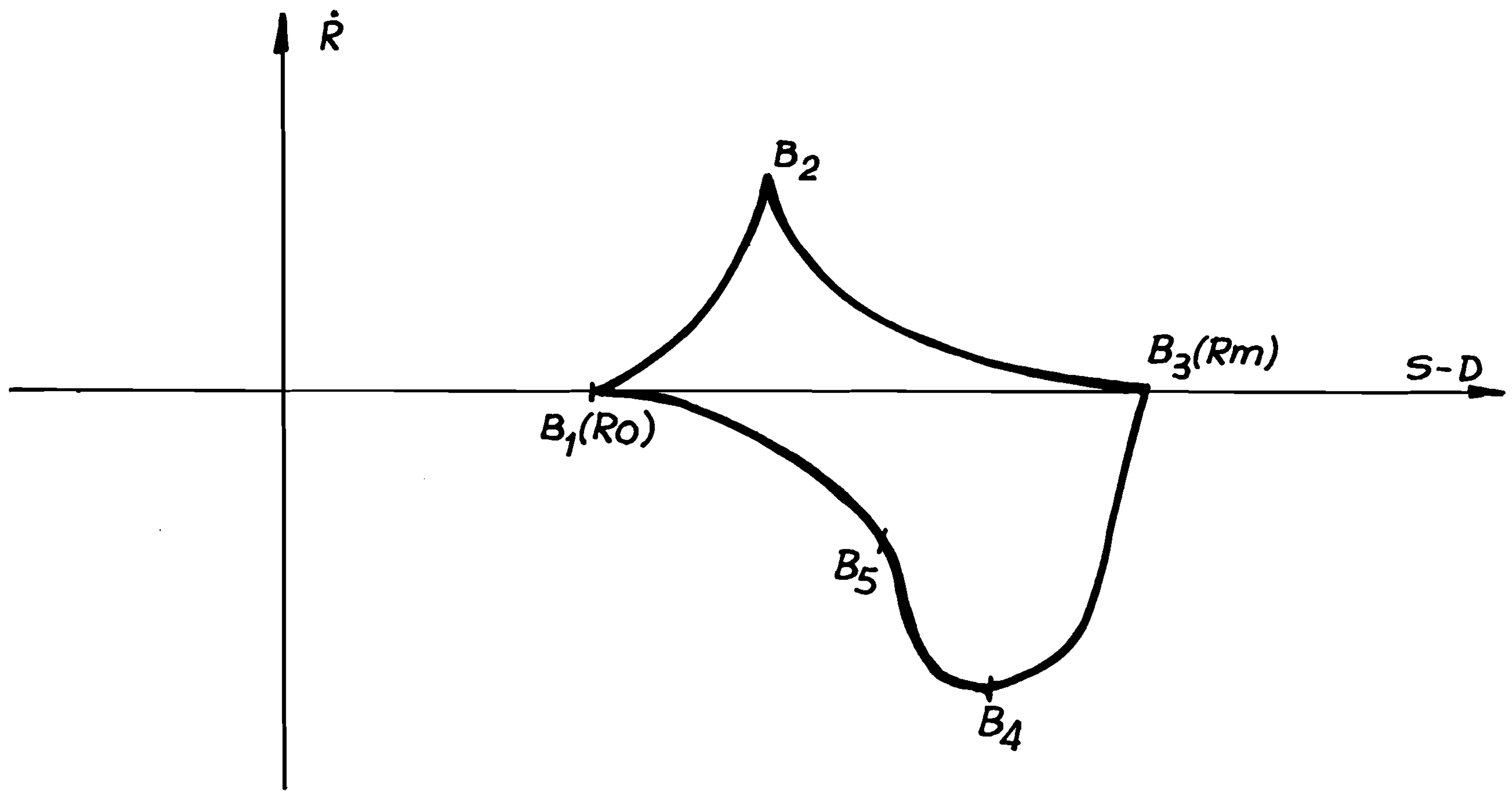

Figure 3. Inventory changes and the intensity of disequilibrium (second type).

III. SPECIFICATION OF THE FUNCTIONS DESCRIBING INVENTORY CHANGES

The curve shown in Figure 1 may be presented by means of the following implicit function:

$$F(\dot{R},\ SE,\ t) = 0 \tag{9}$$

Therefore the complete disequilibrium model for commodity j consists of 5 equations:

$$D^j = f_D(x_D^j,\ \alpha_D^j) + \xi_D^j \tag{10a}$$

$$S^j = f_S(x_S^j,\ \alpha_S^j) + \xi_S^j \tag{10b}$$

$$Q^j = \min\ \{D^j,\ S^j\} \tag{10c}$$

$$F(\dot{R}^j,\ SE^j,\ t) = 0 \tag{10d}$$

$$SE^j = S^j - D^j \tag{10e}$$

We have no clear reasons for choosing a specific class of functions to describe respective segments of the curve given by equation (9). It seems, however, that they should meet two basic conditions: firstly, they should be continuous and twice differentiable; secondly, they ought to contain few parameters to be estimated. We suggest the following forms of the functions in corresponding variability intervals:

I	$\dot{R} = -(A_1 - SE)^{V_1}$	for $SE \in (0,\ A_1\rangle\ ,\ \dot{R} < 0$
II	$\dot{R} = -(SE - A_3)^{V_2}$	for $SE \in \langle A_3,\ 0)\ ,\ \dot{R} < 0$
III	$\dot{R} = 0$	for $SE \in \langle A_4, A_3)$
IV	$\dot{R} = 0$	for $SE \in (A_4, A_5)$

$$\text{V} \qquad \dot{R} = C_1(SE-A_5)^{V_3}(A_{10}-SE)^{V_4} \quad \text{for } SE \in (A_5, A_{10}>, \ \dot{R} > 0$$

$$\text{VI} \qquad \dot{R} = -C_2(SE-A_1)^{V_5}(A_{10}-SE)^{V_6} \quad \text{for } SE \in (A_{10}, A_1), \ R < 0.$$

Formulae V and VI represent the so-called beta function exact to the fixed value. Functions I and II may be considered as a special case of the beta function.

Empirical verification of model (10), taking into account I-VI, is possible, although this would be very difficult. Overcoming all the numerical problems consists partly in accepting some new assumptions concerning the process of inventory changes and, consistently, the position of points A_1, A_3, A_4, A_5, and A_{10}. The application of the proposed disequilibrium model with explicitly introduced inventory changes will be subject of a separate paper.

NOTES

1. P.A. Samuelson calls it the dynamic-quilibrium of the first type. See: P.A. Samuelson, Zasady analizy ekonomicznej (Foundations of Economic Analysis), PWN, Warszawa 1959, pp. 250-259.

2. We can speak of imbalanced but equalized systems, as indicated by J. Kornai. He introduced the term "normal" disequilibrium to express the state of imbalance in the market to which the system tends if there are no additional forces. This state cannot be understood as an optimum or desirable situation. See: J. Kornai, Growth, Shortage and Efficiency. A Macroeconomic Model of the Socialist Economy, Basil Backwell, Oxford, 1982.

3. The imbalance is often defined in an opposite sense as excess demand; then the identity has the following form: DE = D - S. See: W. Charemza, Ekonometryczne modele nierownowagi. Problemy specyfikacji i estymacji (Econometric Disequilibrium Models. Problems of Specification and Estimation), Gdansk, 1981, pp. 30-32; A. Welfe, Analiza popytu w warunkach nierownowagi (Demand Analysis in Conditions of Disequilibrium), "Ekonomista", No. 5, 1984, pp. 1045-1064.

   For reasons which will be explained later, it will be more convenient for us to use formula (4), i.e.: SE = - DE.

4. Passing to discontinuous time, equation (6) has the form: $\Delta P = f_p(S_t - D_t)$, where Δ is the operator of the first difference. See: R.G.D. Allen, Ekonomia matematyczna (Mathematical Economics), PWN, Warszawa 1961, pp. 25-38.

5. See: W. Juszczak, Zasada elastycznego akceleratora oraz wykorzystanie informacji ex ante w modelach zapasow (Principle of Elastic Accelerator and Utilization of ex-ante Information in Inventory Models), "Studia Prawno-Ekonomiczne", t. XVII, 1981, pp. 129-146.

6. Barriers in the production process are discussed more broadly in: W. Welfe, Models of the Socialist Economy, in: L.R. Klein, Lectures in Econometrics, North Holland Pc., Amsterdam, 1983.

7. See also: W. Welfe, Popyt i podaz (Demand and Supply), PWE, Warszawa, 1962, pp. 206-207, and A. Welfe. Analiza popytu w warunkach nierownowagi (Demand Analysis in Conditions of Disequilibrium), "Ekonomista", No. 5, 1984, pp. 1045-1064.

8. This interpretation can be only accepted after introducing many constraining assumptions. See: A. Welfe, Popyt konsumpcyjny w warunkach niedostatecznej podazy (Consumer Demand in Conditions of Insufficient Supply), "Studia Prawno-Ekonomiczne", t. XXXIII, 1984, pp. 187-202. Non-fulfillment of these assumptions may lead to false conclusions as for the state of the market examined.