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Application of the Generalized Reachable Sets Method to Water Resources Problems in the Southern Peel Region of the Netherlands

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APPLICATION OF THE GENERALIZED REACHABLE SETS METHOD TO WATER RESOURCES PROBLEMS IN THE SOUTHERN PEEL REGION OF THE NETHERLANDS

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PREFACE

This work is an interesting example of the application of the GRS method for multiobjective analysis to one of the case studies of the *Regional Water Policies Project* at IIASA. The simplification of models and computations for this example were conducted at the Computing Centre of the USSR Academy of Sciences in Moscow by the author of the GRS method, A.V. Lotov, and his colleague G.K. Kamenev. P.E.V. van Walsum, a member of the IIASA project was at this end of the collaboration, and saw to the qualitative significance of the results by comparing them with the results generated at IIASA using other methods.

S.A. Orlovski
Project Leader
Regional Water Policies Project

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APPLICATION OF THE GENERALIZED REACHABLE SETS METHOD TO WATER RESOURCES PROBLEMS IN THE SOUTHERN PEEL REGION OF THE NETHERLANDS

G.K. Kamenev, A.V. Lotov and P.E.V. van Walsum

1. INTRODUCTION

This research was carried out as a part of investigations undertaken by the International Institute of Applied Systems Analysis (IIASA) on the methods and procedures that can assist the design of policies aimed at providing for the rational use of water and related resources taking into account economic, environmental and institutional aspects. One of the case studies which was used for testing these methods is the problem of application of water in agricultural production in the Southern Peel region of the Netherlands. The problem was discussed in the paper by Orlovski and Van Walsum (1984). Here we apply to the investigation of water problems of the region under study a new approach – the generalized reachable sets (GRS) method. This approach makes it possible to present the information contained in a model implicitly in an aggregated explicit form. Within the framework of decision support systems the GRS approach can be applied for analysis of simplified models on screening stage of investigation. The method allows us to describe the set of all indicator values which are reachable (accessible) under feasible alternatives.

The decision makers (or experts) study two-dimensional cross-sections of the accessible set presented in dialogue regime on the screen of the computer, choose some "interesting" combinations of indicators and corresponding decisions. These decisions are checked afterwards in simulation experiments with more adequate models.

This approach gives to decision makers, experts and the interested public the understanding of potential possibilities of the system under study. It helps to formulate scenarios and decisions for simulation experiment and therefore helps to overcome the main disadvantage of simulation, consisting in the difficulty of choosing scenarios and decisions to be checked in simulation runs.

The generalized reachable sets method was applied at IIASA for investigation of water allocation in the Southwestern Skane region of Sweden (Bushenkov et al., 1982). This time the method is applied to water problems of the Southern Peel region in the Netherlands.

Two stages of the investigation are described herein. In the first stage a "modal subregion" of the region is studied. In the second stage the region is described as a combination of five economic clusters.

First of all we shall discuss the mathematical formulation of the GRS method.

2. THE GRS METHOD

The generalized reachable sets (GRS) method of investigation of controlled systems was developed for the analysis of the models with exogenous variables. The basic idea of the method consists of the following. The properties of the model under study are investigated using aggregated variables. The investigation is based on numerical construction of a set of all combinations of values of aggregated variables which are reachable (or accessible) using feasible combinations of original variables of the model. This set is called the GRS and for this reason the method is called the GRS approach. In multiple criteria decision making the GRS approach employs an explicit representation of the set of all accessible values of objectives or performance indicators.

The mathematical formulation of the method is as follows. Let the mathematical model of the system under study be

$$x \in G_x = \{x \in R^n : Ax \le b\} \quad ,$$

where R^n is n-dimensional linear space of variables x (controls), G_x is the set of feasible values of variables x, A is a given matrix, b is a given vector. Let $f \in R^m$ be the vector of aggregated variables (criteria). The vector f is connected with variables x by linear mapping described by the given matrix F, i.e.

$$f = Fx \quad .$$

The GRS for the model (1) with the mapping (2) is defined as

$$G_f = \{f \in R^m : f = Fx \ , \ x \in G_x\} \quad .$$

The GRS approach consists of the construction (or approximation) of the set G_f in the form

$$G_f = \{f \in R^m : Df \le d\}$$

and in the further analysis of the set G_f.

Note that the description of the set G_f has a form of an intersection of a finite number of hemispaces. This makes it possible to provide a decision maker with any two-dimensional cross-section of the set shortly after his request for it.

The presented mathematical formalization of the approach relates to finite dimensional models. Nevertheless it can easily be reformulated for linear functional spaces of general type (see Lotov 1981a, 1981b). In the latter case the feasible set and the mapping must be approximated by finite dimensional analogues.

The construction of the GRS is described in Section 5. The GRS approach was used for various purposes: for evaluation of potential possibilities of economic systems (Lotov 1981b, Bushenkov et al. 1982) for aggregation of economic models (Lotov 1982), for coordination of economic models (Lotov 1983). The approach can also be effectively applied to multiple criteria decision making (MCDM).

Application of the GRS approach applied to MCDM problems usually serves to provide the decision maker (DM) with the information on the effective set in objective space which is a part of the boundary of the GRS. In this case the GRS approach is related to so-called generating interactive methods (Cohon 1978) which inform the DM on the possibilities of the system under study while the process of choosing a compromise between competing criteria is left to the DM.

3. THE ORIGINAL MODEL

The model of the water resources of the Southern Peel Region was introduced and discussed in Orlovski and Van Walsum (1984). Here we use the aggregated versions of the same model to study possible development of the agricultural and water systems of the region during one year.

The model by Orlovsky and Van Walsum is a simplified model which links submodels of agricultural production, water quantity and quality processes and soil nitrogen processes. In the model a year is split into two parts: "summer" which starts on April 1 and "winter" which starts on October 1. The year is taken from the beginning of winter.

The Southern Peel region is divided into 31 subregions. This division is based on classes of groundwater conditions and soil physical units.

The agricultural production in the model is described by means of "technologies".

3.1. Agricultural Technologies

The term "technology" is used for a combination of agricultural activities involved in growing and processing of a certain crop and/or livestock. It is assumed that technologies differ from each other by their outputs and also by the inputs required to produce these outputs. For convenience, a distinction is made between agricultural technologies that use land and those that do not. The set of the former is denoted by JX, the set of the latter by JZ. It is also convenient to further subdivide the set JX into the subset JXL of land-use technologies involving livestock and the subset JXD of land-use technologies not involving livestock. A similar subdivision of the set JZ into the subsets JZL and JZD is made.

All technologies considered are explicitly characterized by the following types of inputs (resources): labour, capital, water. Land-use technologies of the set *JX* are additionally characterized by the input of nitrogen supplied by fertilization.

Each technology is also characterized by the output or production of the respective goods (crop yields, livestock products). Technologies that involve livestock are additionally characterized by outputs of animal slurries produced as byproducts.

The use of agricultural technologies is described in terms of their intensities. For land-use technologies intensities have the meaning of areas of land allocated to these technologies. For technologies that do not use land and that involve livestock (from set *JZL*) intensities have the meaning of a number of livestock-heads; for a technology from the set *JZD*, the intensity may have the meaning of for instance the amount of pig slurry transported to outside the region.

It is assumed that such inputs as labor and capital for every technology can be represented by corresponding quantities per unit of its intensity. (For example, amount of labor per unit area of land for a technology from set *JX*.) It is also assumed that the water inputs for technologies not using land can be quantified in the same normative way (amount per unit intensity).

But the situation is different with describing water inputs and the corresponding outputs for land-use technologies. One reason for this difference is that both the water availability and the output of land-use technologies depend on weather conditions. Another reason is that the availability of water is also influenced by activities in the region, especially pumping of groundwater. In order to take into account the respective possible variations in the performance of land-use technologies a finite number of options for each such technology are considered, which cover a suitable variety of typical water availability situations in each subregion. For the sake of brevity the term subtechnology is used to refer to such an option.

Each of subtechnologies k is characterized by the crop productivity cp_k, by the corresponding seasonal averages of the soil moisture vr_k and of actual evapotranspiration ea_k, as well as by the total nitrogen requirement nr_k (all amounts per unit area of land). The value vr_k is treated in the model as the "demand" for soil moisture, the satisfaction of which (together with the satisfaction of the requirement for nitrogen) guarantees obtaining the crop productivity not lower than cp_k.

The following notation is used for intensities of technologies and subtechnologies (r-subregions, j-technology, k-subtechnology):

$x(r,j)$ - area of land allocated to technology $j \in JX$,

$xw(r,j,k)$ - area of land allocated to subtechnology k of technology $j \in JX$,

$z(r,j)$ - intensity of technology $j \in JZ$.

In the model used in this report there were 10 land-use technologies divided into 3 subtechnologies and 5 technologies which do not use land. The following technologies are used in the model. For $j \in JXL$ we have

$j = 1$: glasshouse horticulture

$j = 2$: intensive field horticulture

$j = 3$: extensive field horticulture

$j = 4$: potatoes

$j = 5$: cereals

$j = 6$: maize with low nitrogen application

$j = 7$: maize with medium nitrogen application

$j = 8$: maize with high nitrogen application

$j = 9$: grassland with high cow density

$j = 10$: grassland with low cow density

For $j \in JZ$ we have:

$j = 1$: beef calves

$j = 2$: pigs for feeding

$j = 3$: pigs for breeding

$j = 4$: egg-laying chickens

j = 5: broilers

We obviously have

$$x(r,j) = \sum_{k=1}^{3} xw(r,j,k) \tag{1}$$

for all $r = 1, \ldots, 31$ and $j = 1, \ldots, 10$. The total area of agricultural land in subregion r is denoted by $xa(r)$. We have

$$\sum_{j=1}^{10} x(r,j) \leq xa(r) \quad . \tag{2}$$

We also have constraints on land allocated to certain groups of crops:

$$\sum_{j \in C_l} x(r,j) \leq x\,max\ (\mathrm{r},1) \quad , \tag{3}$$

where C_l, $l = 1, \ldots, 8$, are subsets of JX and $x\ max\ (r,l)$ are exogeneously fixed. We have $C_j = \{j\}$ for $j = 1, \ldots, 5$; $C_6 = \{6,7,8\}$, $C_7 = \{g\}$, $C_8 = \{10\}$.

3.2. Animal Slurry By-products

The technologies that involve livestock produce animal slurries as byproducts. These slurries are used as fertilizers for land-use technologies in the region. From the environmental viewpoint the slurries produced during the summer and the winter can best be temporarily stored in tanks till the next spring and only then applied to the land. The storage must not exceed storage capacities:

$$\sum_{m=1}^{5} \sum_{j=1}^{10} mx(j,m)x(r,j) + \sum_{m=1}^{5} \sum_{j=1}^{5} mz(j,m)z(r,j) -$$

$$- \sum_{m=1}^{5} \sum_{l=1}^{2} ma(r,l,m) \leq m\bar{\bar{c}}(r)\ ,\ r = 1, \ldots, 31 \quad ,$$

where $mxw(j,m)$ is the winter production of slurry m per unit technology $j \in JXL$, $mz(j,m)$ is the year production of slurry m per unit technology $j \in JZ$, $ma(r,l,m)$ is the autumn application of slurry m to the l-th direction of application, $mc(r)$ is the storage capacity. In the model five kinds of slurries are described:

$m = 1$: cattle slurry,

$m = 2$: beef calf slurry,

$m = 3$: pigs slurry

$m = 4$: chicken slurry,

$m = 5$: broiler manure.

Two directions of slurry application are described:

$l = 1$: on the arable land,

$l = 2$: on the grassland

We suppose that after spring application of slurries the tanks are empty:

$$\sum_{j=1}^{10} mx(j,m)x(r,j) + \sum_{j=1}^{5} mz(j,m)z(r,j) - \tag{5}$$

$$\sum_{l=1}^{2} ma(r,l,m) = \sum_{l=1}^{2} ms(r,l,m) \; , \; r = 1, \dots, 31 \quad ,$$

where $ms(r,l,m)$ is the spring application of slurry m in the l-th direction of application.

3.3. Labour Requirements

Labour requirements are described by the equality

$$\sum_{j=1}^{31} \sum_{j=1}^{10} lx(r,j)x(r,j) + \sum_{r=1}^{31} \sum_{j=1}^{5} lz(j)z(r,j) + lu - lh = lp \quad , \tag{6}$$

where lp is the amount of labour in the region, lu is the unemployment, lh is the amount of labour hired from outside of the region. There are the restrictions

$$lh \le lhmax \quad , \tag{7}$$

$$lu \le lumax \quad , \tag{8}$$

3.4. Income

The income y is calculated by the following equation

$$y = \sum_{r=1}^{31} [\sum_{j=1}^{10} \sum_{k=1}^{3} yxw(r,j,k)xw(r,j,k) + \sum_{j=1}^{5} yz(r,j)z(r,j) - rmc(r)mc(r)$$

$$- pf \sum_{l=1}^{2} fs(r,l) - peis(r)is(r) - peig(r) \cdot ig(r) - \tag{9}$$

$$rsc(r)sc(r) - rge(r) \cdot ge(r)] - plh \cdot lh \quad ,$$

where $fs(r,l)$ - is the amount of chemical fertilizer nitrogen applied to land type l, $l = 1,2$

$is(r)$ - is the amount of sprinkling from surface water

$ig(r)$ - is the amount of sprinkling from groundwater

$yx(r,j,k)$,

$yz(r,j)$, $rmc(r)$, pf, $peis(r)$, $peig(r)$, $rsc(r)$, $rgc(r)$, plh are the corresponding incomes (costs) per unit of the respective variables.

3.5. Water Quantity Processes

The water quantity processes are described in the following manner. Let $hs(r)$ be the groundwater level in the beginning of the summer, $hw(r)$ be the groundwater level at the end of the summer, where index r stands for subregion. Let $\overline{hs} = (hs(r),\ r = 1, \ldots, 31)$ and $\overline{hw} = (hw(r),$ $r = 1, \ldots, 31)$ be corresponding vectors. Let $\overline{gw} = (gw(r),$ $r = 1, \ldots, 31)$ be the vector of public water supply extractions during winter. Then in vector notation we have

$$\overline{hs} = \overline{hso} - A\overline{qw} \quad , \tag{10}$$

where $\overline{hso} = (hso(r),\ r = 1, \ldots, 31)$ is the vector of groundwater levels that would occur if there were no extractions, A is 31×31 matrix with non-negative elements describing influence of extractions.

The groundwater level at the end of the summer is described in a similar manner:

$$\overline{hw} = \overline{hwo} - B\overline{gs} - C\overline{ig} + D\overline{us} \quad , \tag{11}$$

where vector notation is used for the influence matrices B, C and D and the vector of amounts of subirrigation $\overline{us} = (us(r), r = 1, \ldots, 31)$. There are the restrictions

$$qs(r) \le qs_{max}(r) \tag{12}$$

$$qw(r) \le qw_{max}(r) \tag{13}$$

$$us(r) \le us_{max}(r) \tag{14}$$

$$gC(r) \le gc_{max}(r) \tag{15}$$

$$sc(r) \le sc_{max}(r) \tag{16}$$

describing hydrogeologic circumstances. The amount of sprinkling is restricted by sprinkling capacities

$$is(r) \le sc(r) \quad , \tag{17}$$

$$ig(r) \le gc(r) \quad . \tag{18}$$

Sprinkling from surface water is connected with subirrigation in the following manner

$$p \cdot is(r) \le us(r) \tag{19}$$

where p is a proportionality constant. The surface water supply capacity can be limited

$$us(r) + is(r) \le s_{max}(\mathrm{r}), \mathrm{r} = 1, \ldots, 31 \quad . \tag{20}$$

The supply of water for the whole region is restricted too:

$$\sum_{r=1}^{31} (us(r) + is(r)) \le st_{max} \quad . \tag{21}$$

The moisture content of the root zone in the middle of summer is described by the following equations

$$\sum_{j=2}^{10} \sum_{k=1}^{3} vr(r,j,k) xw(r,j,k) \le [icpvs(r) + lvs(r) \cdot hs(r)] + \beta \qquad (22)$$

$$[ps \cdot xa(r) + is(r) + ig(r) + vz_{max} \cdot xa(r) - \sum_{j=1}^{10} \sum_{k=1}^{3} ea(r,j,k) \cdot xw(r)] ,$$

$$\sum_{j=2}^{10} \sum_{k=1}^{3} vr(r,j,k) \cdot xw(r,j,k) \le [icpvs(r) + lvs(r) \cdot hs(r)] + \qquad (23)$$

$$\beta \{ ps \cdot xa(r) + is(r) + ig(r)$$

$$+ [\sum_{j=2}^{10} \sum_{k=1}^{3} vzo(r,j) \cdot xw(r,j,k) + lv(r)(hw(r) - hwo(r))] -$$

$$- \sum_{j=2}^{10} \sum_{k=1}^{3} ea(r,j,k) \cdot xw(r,j,k) \} \quad ,$$

where the total amount of moisture required for subtechnology $xw(r,j,k)$ is restricted by the moisture content in the beginning of summer (the first addendum, i.e. $lvs(r) \cdot hs(r)$) plus the change of soil moisture content in the middle of summer ($\beta = 1/2$). Here $icpvs(t)$ and $lvs(r)$ are the coefficient and proportionality constant describing the influence of the level of groundwater at the end of winter on the moisture content of the rootzone; $lvz(r)$ is the coefficient of a piecewise linear function describing the influence of a rise of the level of the groundwater at the end of summer on the capillary rise of moisture to the rootzone, and vz_{max} is the maximal amount of capillary rise of moisture to the rootzone in the case when the groundwater level rises above a certain critical level.

In our model meteorological parameters of the year 1976 were used. This is the year with low precipitation.

3.6. Fertilization, Mineralization of Organic-N

Each technology j that uses land has a specified level $nr(r,j)$ of the amount of nitrogen that is required for crop growth. This nitrogen can come from different sources – chemical fertilizer and various types of animal slurries. The simplified representation of the constraints prescribing the satisfaction of nitrogen requirements of technologies has the following form ($l = 1,2$)

$$\sum_{m=1}^{5} [ema(l,m)ma(r,l,m) + ems(l,n)ms(r,l,m)] + fs(r,l) = \sum_{j=1}^{10} nr(r,j)x(r,j) \quad (24)$$

where $ema(l,m)$ is nitrogen effectivity of slurry m applied in autumn in l-th direction;

$ems(l,m)$ is the same, but applied in summer.

The following restriction describes a minimum amount of chemical fertilizer nitrogen applied in spring ($l = 1,2$)

$$fs(r,l) \geq \sum_{j=1}^{10} rfs(l,j)x(r,j) \quad (25)$$

where $rfs(l,j)$ is the requirement of technology of l-th type (per unit area).

The leaching of nitrate to groundwater was not described in the first draft of the model, but it is included into the modified version of the model which is described later in Section 7.

3.7. Public Water Supply

If the demands of public water supply in winter qpw and in summer qps then the total of the extractions in the subregions must satisfy respectively for the winter and summer period

$$\sum_{r=1}^{31} qw(r) \geq qpw + \sum_{r=1}^{31}\sum_{j=1}^{10} wxw(j)x(r,j) + \sum_{r=1}^{31}\sum_{j=1}^{5} wzw(j)z(r,j) \quad (28)$$

$$\sum_{r=1}^{31} qs(r) \geq qps + \sum_{r=1}^{31}\sum_{j=1}^{10} wxs(j)x(r,j) + \sum_{r=1}^{31}\sum_{j=1}^{5} wzs(j)z(r,j) \quad ,(29)$$

where $wxw(j)$ is the water use per unit of $x(r,j)$ during winter,

$wxs(j)$ is the same during summer

$wzw(j)$ is the water use per unit of $z(r,j)$ during winter,

$wzs(j)$ is the same during summer.

3.8. Natural Ecosystems

The restrictions on groundwater levels in some subregions are given

$$hw(r) \geq hw_{\min}(r) \quad , \tag{30}$$

4. THE AGGREGATION

In order to make possible the application of the Generalized Reachable Sets Approach the model described in the previous section was aggregated. In the first stage of the investigation a "modal subregion" of the region was obtained. This "modal subregion" was used as a "characteristic representative" of the whole region in a preliminary calculation of potential possibilities of the region.

In the second stage of the investigation the region was divided into economic clusters, each of them consisting of a number of subregions. Let the total number of economic clusters be S. Let I_s be the set of subregions belonging to the s-th cluster. We suppose that any subregion belongs to one and only one cluster. The "modal subregion" can be treated as an economic cluster containing all subregions. Therefore aggregation in both stages of investigation can be described simultaneously (in the first stage $S = 1$ and on the second stage $S \neq 1$, in our case $S = 5$).

4.1. Aggregation of "Flow" Type Variables

The variables x, xw, y, qw, s, us, is, ig, ma, ms, mc have been aggregated in the following way:

$$\vartheta(s,j) = \sum_{r \in I_s} \vartheta(r,j) \quad .$$

For the capacities (i.e. upper limits) of these variables the same scheme of aggregation was used.

4.2. Aggregation of Groundwater Levels

The levels of groundwater have been aggregated according to the following scheme

$$hs(s) = \sum_{r \in I_s} hs(r) \frac{xa(r)}{\sum_{r \in I_s} xa(r)}$$

The initial groundwater levels hso, hwo in equations (10), (11) have been aggregated in the same manner.

4.3. Aggregation of Coefficients

The coefficients of the equations have been aggregated according to the type of aggregation of variables presented in these equations.

(a) If the equation is of the type

$$a_{\vartheta}(r)\vartheta(r) + a_h(r)hs(r) = B(r) \quad ,$$

for example, equations (9), (22), (23) where

$$\vartheta(s) = \sum_{r \in I_s} \vartheta(r) \, , \; hs(s) = \sum_{r \in I_s} hs(r) \frac{xa(r)}{\sum_{i \in Is} xa(r)} \quad ,$$

then the following method is suggested: To obtain coefficients $a_{\vartheta}(s)$, $a_h(s)$, $B(s)$ in

$$a_{\vartheta}(s)\vartheta(s) + a_h(s)hs(s) = B(s)$$

we use the formulae

$$G_{\vartheta}(s) = \sum_{r \in Is} \frac{\vartheta_{\max}(r)}{\sum_{r \in Is} \vartheta_{\max}(r)} \cdot a_{\vartheta}(r) \, , \; B(s) = \sum_{r \in I_s} B(r) \quad ,$$

$$a_h(s) = \sum_{r \in I_s} \frac{hso(r)}{hso(s)} a_h(r) \quad ,$$

where $\vartheta_{\max}(r)$ is the maximal value (capacity) of $\vartheta(r)$, and $hso(s) = \sum_{r \in Is} \frac{xa(r)}{\sum_{r \in Is} xa(r)} hso(r)$.

(b) If the equation is of the type

$$\sum_{r=1}^{31} a(t,r)\vartheta(r) = hs(t)\,,\ t = 1,\ldots,31 \quad ,$$

for example, equations (10), (11), then the following method is suggested: To obtain coefficient $a(s,q)$ in equation

$$\sum_{q=1}^{S} a(s,q)\vartheta(q) = hs(s)$$

we use the formula

$$a(s,q) = \sum_{t \in I_s} \sum_{r \in I_s} \frac{\vartheta_{\max}(r)}{\sum_{r \in I_q} \vartheta_{\max}(r)} \frac{xa(t)}{\sum_{t \in I_s} xa(t)} a(t,r) \quad .$$

This scheme has been used for aggregation of matrices A, B, C, D.

5. CONSTRUCTION OF THE GRS

In this Section, we briefly discuss numerical methods for the construction of the GRS. We suppose that the model of the system under study has the form

$$x \in G_x \equiv \{x \in R^n : Ax \leq b\} \quad , \tag{31}$$

where A is a given matrix, b is a given vector. The vector of criteria $f \in R^m$ is connected with variables by linear mapping

$$f = fX \quad , \tag{32}$$

where F is a given matrix. The GRS which is defined implicitly as

$$G_f = \{f \in R^m :\ f = Fx\ ,\ Ax \leq b\} \quad , \tag{33}$$

should be constructed in the form

$$G_f = \{f \in R^m :\ Df \leq d\} \quad . \tag{34}$$

To construct the GRS in the form (34), at the Computing Center of the USSR Academy of Sciences a group of numerical methods was developed. These methods were combined into the software system POTENTIAL for the computer BESM-6. The first version of the system POTENTIAL was published in (Bushenkov and Lotov, 1980), the second one was described in (Bushenkov and Lotov, 1982 and 1984).

The methods included into the system POTENTIAL are based on the construction of projections of finite dimensional polyhedral sets into subspaces. Suppose we have some polyhedral set M belonging to $(p+q)$-dimensional linear space R^{p+q}. Suppose this set is described in the form of solution of finite system of linear inequalities

$$Av + Bw \le c \tag{35}$$

where $v \in R^p$, $w \in R^q$, the matrices A and B as well as the vector c are given. The projection of the set M into the space R^q of variables w is defined as the set Mw of all points $w \in R^q$ for which there exists such a point $v \in R^p$ that $\{v,w\} \in R^{p+q}$ belongs to M. An example of the two dimensional set and its projection into one dimensional space is presented in Figure 1.

To construct the GRS for the system (31)-(32) lets consider the set

$$Z = \{\{x,f\} \in R^{n+m}: \ f = Fx \ , \ Ax \le b\} \quad .$$

The GRS is the projection of the set Z into the space R^m of the criteria f. The methods of the system POTENTIAL give the possibility to construct projections of polyhedral sets in the form of solution of a system of linear inequalities. Therefore, using the system POTENTIAL it is possible to construct the GRS in the form (34).

The first method of the construction of projections of polyhedral sets described as solutions of systems of linear inequalities was introduced by Fourier (1826). This method was based on exclusion of variables by combination of inequalities. The application of the method given by Fourier to the construction of the projection of the set given in Figure 1 is described in (Lotov 1981a).

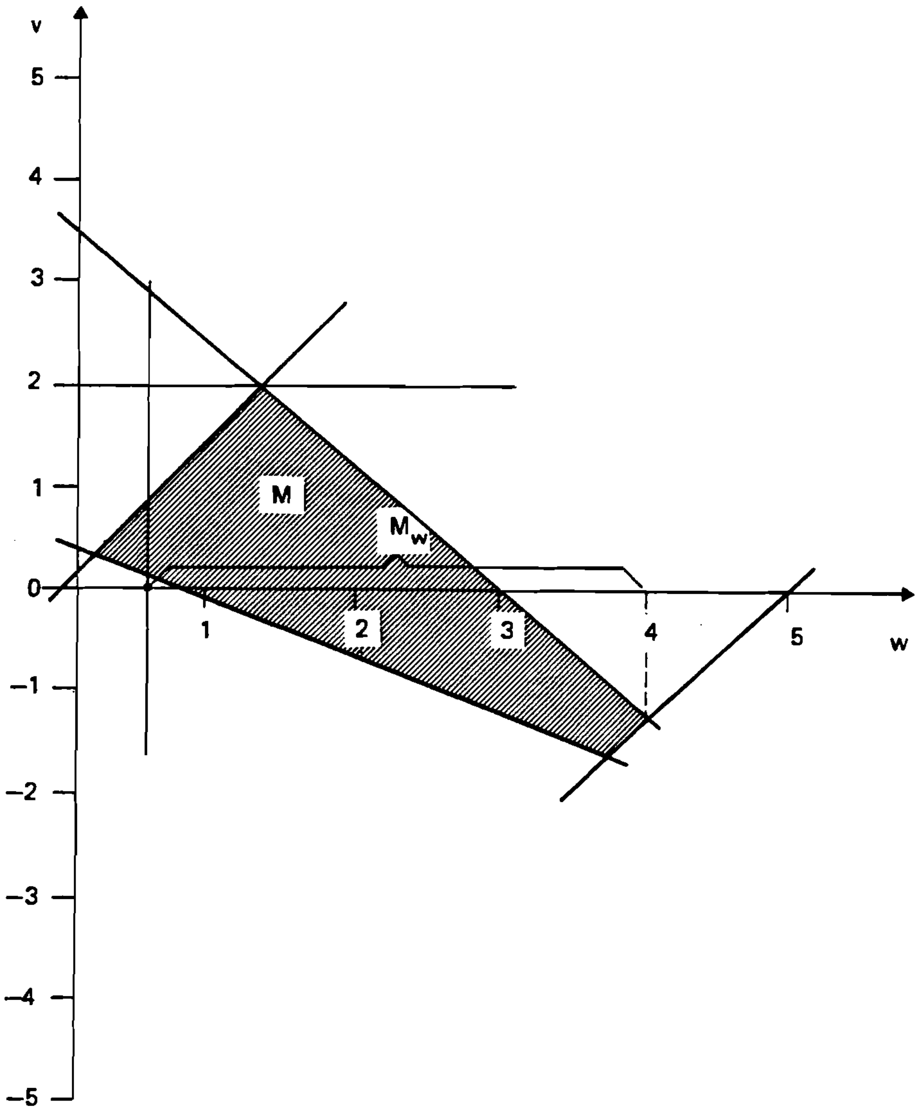


Figure 1

The idea suggested by Fourier was used in more effective methods of construction of projections of polyhedral sets (see Motzkin et al. 1953, as well as Chernikov 1965) based on exclusion of variables by combination of inequalitiess. The experimental study of these methods proved that the methods of this kind are effective for small systems only ($n \sim 10$–30). For mathematical models (31) with hundreds of variables new methods were suggested. The most effective of them at the moment is the method of improvement of the approximations of the projection (IAP). The IAP method gives the possibility to approximate the projection of the set Z for all the models (31) for which linear optimization problems can be solved. The IAP method is described in short below. For more detailed information see Bushenkov and Lotov (1982) as well as Bushenkov (1985).

The general idea of the IAP method consists of combining methods based on exclusion of variables with the optimization methods of construction of the GRS which are usually ineffective for $m > 2$. The IAP method consists of iterations having the following form.

Before the k-th iteration two polyhedral sets P_k and P^k should be given while it holds

$$P_k \subset G_f \subset P^k \quad .$$

The set P_k which is the internal approximation of the set G_f should be given in the two following forms simultaneously:

1) as the solution of a system of linear inequalities, i.e.

$$P_k = \{f \in R^m: \ (c_j, f) \leq d_j \ , \ j = 1, \ldots, s_k\} \quad ,$$

where c_j are the vectors and d_j are the numbers calculated on previous iterations,

$$(a, b) = \sum_{i=1}^{m} a_i b_i \quad ;$$

2) as the convex combination of points (vertices) $f_1, \ldots, f_{r_k}$:

$$P_k = \{f \in R^m: \ f = \sum_{l=1}^{r_k} \lambda_l f_l \ , \ \lambda_l \geq 0 \ , \ \sum_{l=1}^{r_k} 1_l = 1\} \quad .$$

The description of the polyhedral set using both forms is called double description (Motzkin et al. 1953). It is necessary to note that the conversion from one form to another is a very difficult task. Only if the number of inequalities s_k in the first form or the number of points r_k in the second one are rather small can this task be solved numerically. This is why on the zero iteration we construct a simple approximation P_1 for which the conversion between forms can be fulfilled easily. Subsequently, step by step the internal approximation is improved: from the set P_1 we obtain P_2 and so on. On k-th iteration we construct the set P_{k+1} for which

$$P_k \subset P_{k+1} \subset G_f \quad .$$

Each form of the presentation of the set P_{k+1} is calculated on the basis of the same form for the set P_k.

To obtain the set P_{k+1} on the basis of the set P_k we add to the set P_k a new vertex F_{r_k+1}. This vertex is chosen in the following manner. It is supposed that for any vector c_j in the first form of presentation the following optimiation problem was solved:

$$\left\{ (c_j, f) \longrightarrow \frac{\max}{f} = Fx \ , \ Ax \leq b \right. \quad . \tag{36}$$

Let $\{x_j^*, f_j^*\}$ be the optimal solution of this problem. Let

$$\Delta_j = ((c_j, f_j^*) - d_j) / \|c_j\| \quad ,$$

where $\|c_j\|$ is the norm of c_j. The values of Δ_j describe descrepancy between the internal approximation P_k and the external approximation P^k which is described as

$$P^k = \{f \in R^m: \ (c_j, f) \leq d_j + \Delta_j \cdot \|c_j\|, \ j = 1, \ldots, S_k\}$$

Since

$$P_k \subset G_f \subset P^k \quad ,$$

the value of $\max \{\Delta_j: \ j = 1, \ldots, S_k\}$ can be used as estimation of descrepancy between the sets P_k and G_f.

In Figure 2 for $m = 2$ we have the internal approximation P_k (its vertices are A, B, C and D), the external approximation P^k (its vertices are K, L, M and N) and the set G_f which is unknown for the researcher. The points E, F, G and H are the solutions of optimization problems (36) for inequalities describing the set P_k.

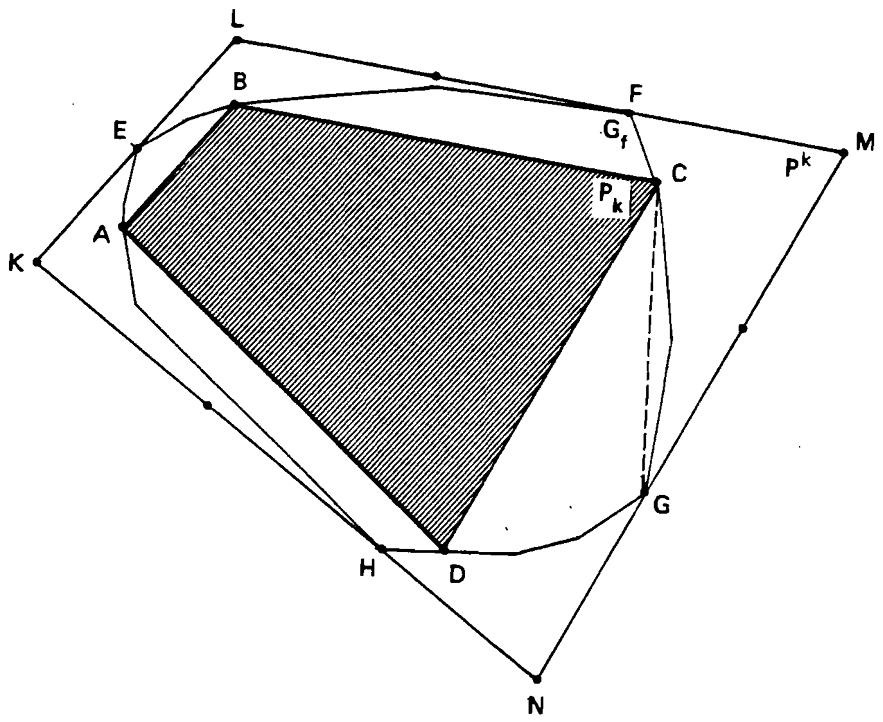


Figure 2

The set P_{k+1} is obtained by inclusion into the second form of description of the internal approximation the new vertex fr_{k+1} for which is chosen one of the points f_j^* obtained in optimization problems (36). It is rather effective to use the point with maximal value of Δ_j^k but different strategies can be applied as well. The description of the approximation of the set P_{k+1} in the second form is found. Note that to obtain fr_{k+1} the first form of the description of the set P_k was used. Now we will construct the first form of description of the set P_{k+1}.

In Figure 2 the point with maximal value of Δ_j is the point G. It is included into the description of the set P_{k+1} which therefore has vertices A, B, C, D, and G. To obtain the description of the P_{k+1} in the first form in two dimensional case presented in Figure 2 it is sufficient to exclude from the description the inequality corresponding to the line passing through the points D and C and to include in the description two new inequalities corresponding to the lines passing through points D and G as well as C and G. In general ($m > 2$) the problem of constructing the description of the set

P_{k+1} in the first form is not so simple and can be solved by using methods of excluding of variables in the system of linear inequalities.

First of all, we find the inequalities which are violated by the point $f_{\tau_{k+1}}$. These inequalities are excluded from the system. Let f_1, $f_2, \ldots, f_{N_k}$ be the vertices belonging to the excluded inequalities. Let us consider the cone with the vertex $f_{\tau_{k+1}}$ and the edges $f_1 - f_{\tau_{k+1}}$, $f_2 - g_{\tau_{k+1}}, \ldots, f_{N_k} - f_{\tau_{k+1}}$, i.e. the cone

$$K = \{f \in R^m : f = f_{\tau_{k+1}} + \sum_{l=1}^{N_k} \lambda_l (f_l - f_{\tau_{k+1}}) , \quad \lambda_l \geq 0\} \quad .$$

We shall present this cone in the form of solution of finite number of inequalities. For this reason we consider the set

$$H = \{\{f, \lambda\} \in R^{m+N_k} : f = f_{\tau_k+1} + \sum_{l=1}^{N_k} \lambda_l (f_l - f_{\tau_k}+1) , \quad \lambda_l \geq 0\} \qquad (20)$$

and construct its projection into the space R^m of variables f. This projection coincides with the cone K and can be constructed by means of methods of excluding of variables in systems of linear inequalities. Note that this problem has small dimensionality and can be solved easily. The obtained system of linear inequalities we include into the description of the approximation. The set P_{k+1} in the first form is constructed. Then we solve optimization problems (36) for new inequalities. The external approximation p^{k+1} is constructed as well. The k-th iteration is finished.

After a finite number of iterations the polyhedral set G_f could be constructed. Usually if the system (31) is large enough it is necessarily to fulfill millions of iterations to construct G_f precisely. Therefore in practical problems it is reasonable to find a good approximation of the set G_f. The good approximation is usually found after a small number of iterations. For example, for $m = 4$ the set G_f is approximated with 1% precision after 15–20 iterations. Note that we construct both internal and external approximations of the set G_f; so it is possible to decide after each iteration to stop or not to stop the process on the basis of graphical presentation of both approximations of the set.

It is necessary to note that the IAP method coincides in some details with non-inferior set estimation (NISE) method introduced by J. Cohon (1978). The main feature of the IAP method consists of using the double description of the polyhedral set and application of the methods of exclusion of variables from the systems of linear inequalities for the construction of the double description.

The IAP method was applied for the construction of the GRS for the model of Peel region.

6. THE RESULTS FOR THE "MODEL SUBREGION"

The GRS method was applied for investigation of the "model subregion" described earlier. Four indicators were chosen:

(1) income y; its maximal value equals to 2340 $\times 10^5$ fl;

(2) level of groundwater above the minimal level $hw - hw_{\min}$; its maximal value equals to 50.2 cm; $hw_{\min} = -200cm$;

(3) public water supply extraction during winter qw; its maximal value equals to 51 $\times 10^6 m^3$;

(4) public water supply extraction during summer qs; its maximal value equals to 51 $\times 10^6 m^3$.

The GRS in the space $\{y\ , hw - hw_{\min}, qw\ , qs\}$ was constructed. This set was studied in dialogue using presentation of two dimensional cross-sections (slices) of the set on the screen of the computer. Some of the cross-sections are presented in this paper. A number of interesting points belonging to the GRS were chosen. The values of the variables in this points are presented as well. The values of indicators corresponding to maximal income are pointed out by the star: $\{y^*\ , hw^* - hw_{\min}, qw^*\ , qs^*\}$.

Figure 3 shows the slices of the GRS in the space $\{hw - hw_{\min}, qs\}$. The value qw is fixed on the level corresponding to maximal income, i.e. $qw = qw^* = 11.75 \times 10^6 m^3$. The value of income y is changing from slice to slice. The slice with low value of $y = 1500 \times 10^5$fl contains points 1, 3, 3' and 1'. While the value of y is increasing the slice is getting smaller and smaller. Changes of slice caused by increment of the value of y are shown by the arrows. The related value of y is presented near the boundary of the slice. For maximal value of $y^* = 2340 \times 10^5$fl the slice consists of one point $hw^* - hw_{min} = 48.8$ cm, $qs = 11.8 \times 10^6 m^3$ (point 2).

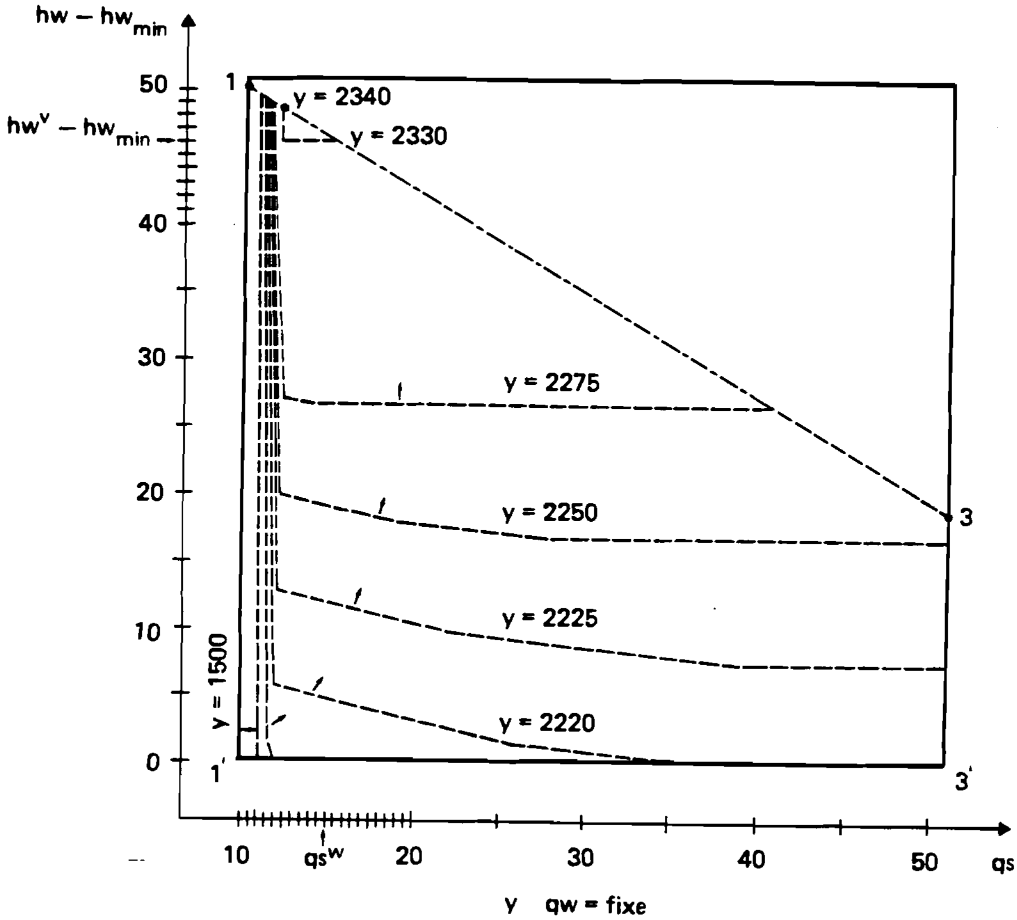


Figure 3

The analysis of slices presented in Figure 3 results in the following properties of the system under study. First of all point 1 with the maximal groundwater level can be achieved only if the income is rather small (in Figure 3 we have $y = 1500 \times 10^5$fl). If the value of y is increased the point 1 does not belong to the slice. But the fall of the maximum groundwater level belonging to the slice is rather small – only about 1 centimeter. On the smallest slice related to value $y^* = 2340 \times 10^5$fl we have $hw^* - hw_{min} =$

48.8 cm. It means that there exists a conflict between income and groundwater level but this conflict is limited to small changes in level of groundwater.

The second feature of the system under study which results from analysis of Figure 3 is the linear trade-off between hw and qs described by the line between points 1 and 3. Note that this boundary is the same for all slices and does not depend on the value of y. So we have evident conflict between groundwater level and summer extractions of water.

Figure 4 presents five slices of the GRS in the space $\{y\ ,\ qs\}$. These slices show the same properties of the system but from a different viewpoint. The value of qw is chosen to be technologically connected with the value of qs and to be optimal in this sense. The value of hw is changed from slice to slice. On the slice A the value of hw is minimal, i.e., $hw = hw_{min}$ = -200 cm. The possible values of y on this slice are not high, not greater than 2200 × 10^5fl. If the level of groundwater hw is increasing, the upper boundary of y increases as well (see slice B). This property is connected with the fact that the groundwater level hw and the income y have no conflict if the values of hw and y are rather small. On the slice C where the value of hw is higher ($hw - hw_{min}$ = 40cm) the conflict between hw and qs arises: to preserve this value of hw one has to extract less than 23 × $10^6 m^3$ of water during the summer.

On the slice D we have $hw = hw^*$ = -151.2 cm. In this case water extraction during the summer is limited by qs^* = 11.8 × $10^6 m^3$. The maximum value of income y^* = 2340 × 10^5fl can be achieved on this slice. On the slice E related to maximal value of hw, i.e., $hw = hw_{min}$ + 50.2 cm = -179.8 cm, we have only limited possibilities of obtaining income which is not higher than 1840 × 10^5fl. Note that the conflict between income and groundwater level arises if the value of hw is higher than hw^* = -151.2 cm.

Figures 5 and 6 present the same picture but the scale is changed. The new slice F is related to the value of hw which is less than hw_{max} but higher than hw^*. In Figure 7 three slices of the GRS in the space $\{y\ ,\ qw\}$ are presented. All slices are related to income-optimal value of groundwater level hw^*. The value of qs changes from slice to slice:

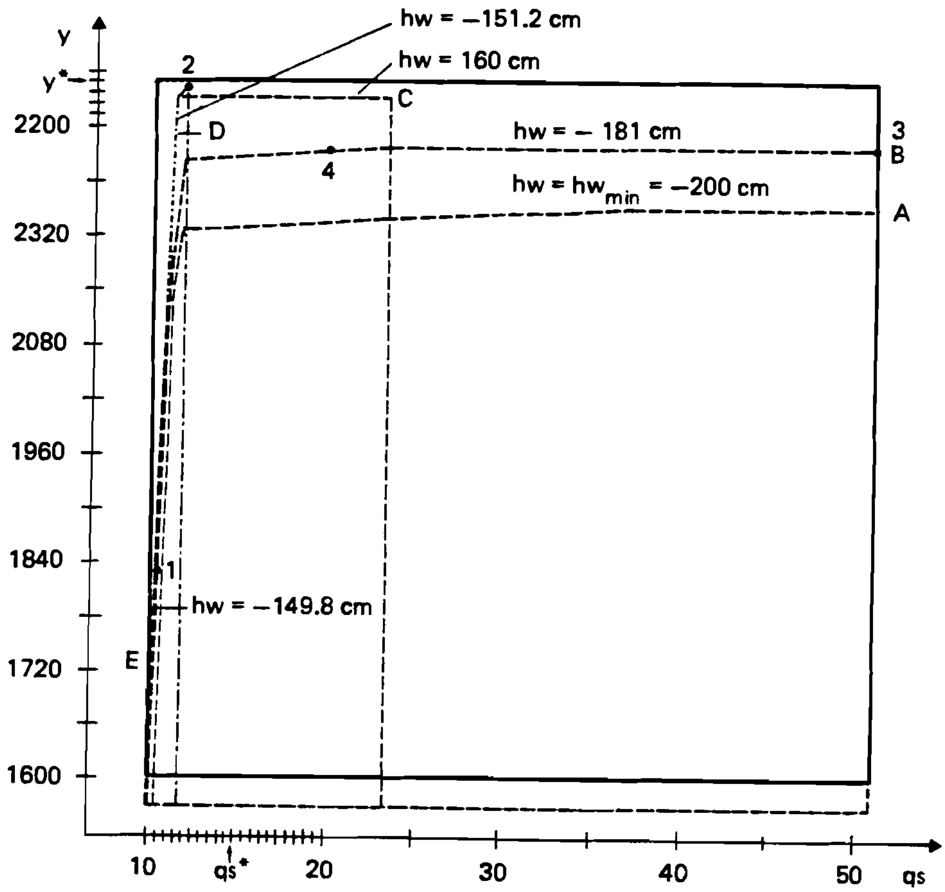


Figure 4

slice A is related to qs $= 10 \times 10^6 m^3$,

slice B is related to qs $= 10.3 \times 10^6 m^3$,

slice C is related to qs $= qs^{*}$ $11.82 \times 10^6 m^3$.

Figure 7 shows that if the value of qw is minimally (qw $= 10.10^6 m^3$)*, then the value of income can be not greater than 1700 × 10^5fl but a small additional value of qw makes the upper boundary of the income jump up. The boundary is kinked: after a sharp increase of possible income the boundary becomes horizontal. It means that the additional value of qw is used ineffectively. Note that the jump of the income near the minimal value

*Minimal values of qw and qs are connected with extractions for public water supply of $10^7 m^3$.

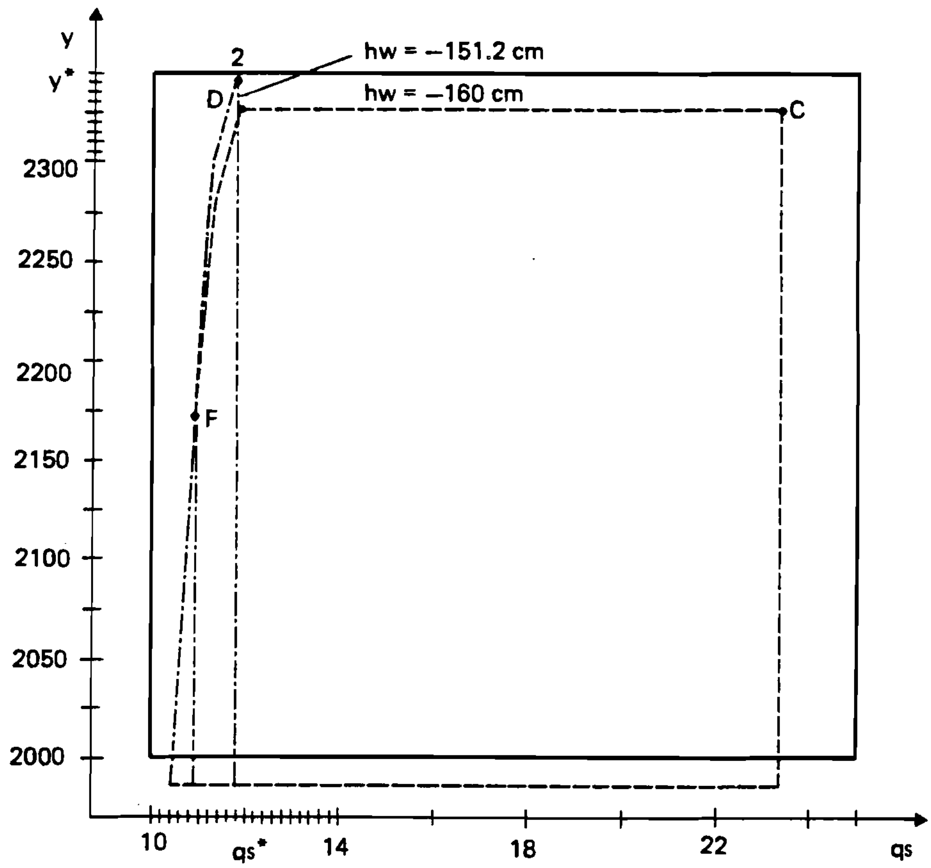


Figure 5

of qw depends on the value of qs. This means that qs and qw are "technologically" connected in the model.

Figure 8 presents the same effect. We have the same space $\{y, qw\}$ but five slices presented in Figure 8 correspond to different values of hw. The slice C is the same as for Figure 7. The correspondence between slices and the values of qs and hw is the following:

slice A: $qs = qs_{min} = 10^7 m^3$, $hw = hw_{max} = 149.8$ cm,

slice B: $qs = 10.27 \times 10^6 m^3$, $hw = -150$ cm,

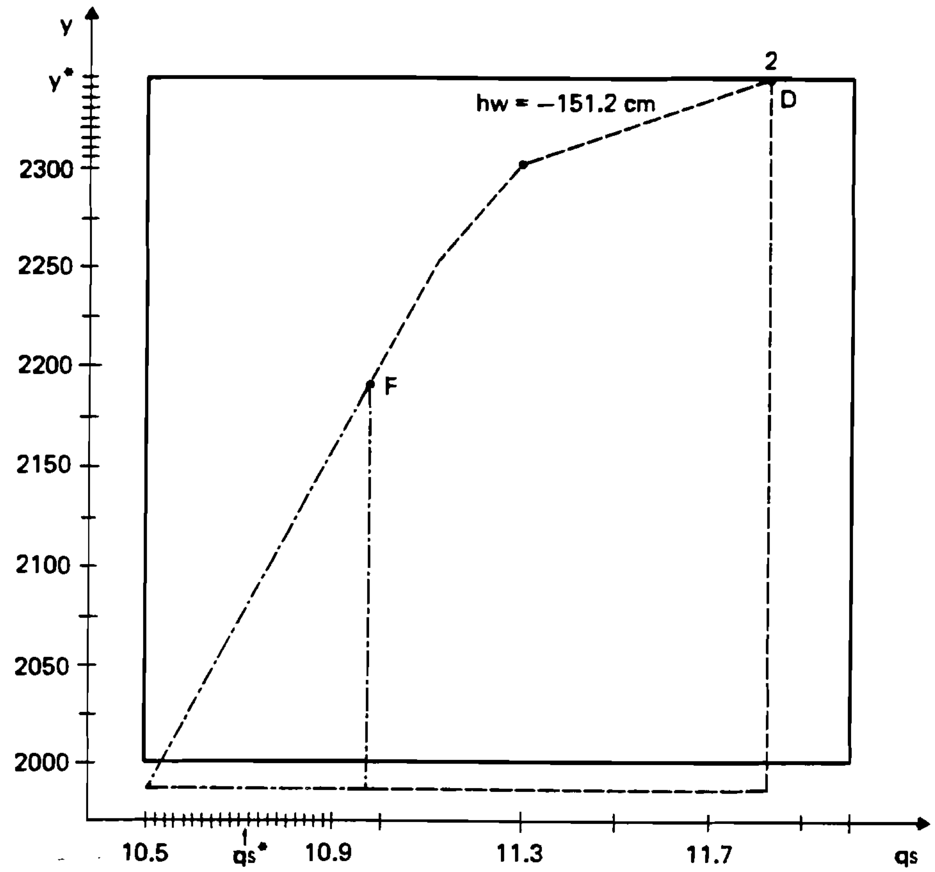


Figure 6

slice C: $qs = qs^{*} = 11.82 \times 10^{6}\ m^{3}$, $hw = hw^{*} = -151.2$ cm,

slice D: $qs = 35 \times 10^{6} m^{3}$, $hw = -170$ cm,

slice E: $qs = 51 \times 10^{6} m^{3}$, $hw = -192$ cm,

It is interesting to note that high extractions of groundwater during winter and summer ($qs = 35 \times 10^{6} m^{3}$, $qw = 15 \times 10^{6} m^{3}$) can result in sufficient high income ($y = 2300$). However, in this case the groundwater level in nature areas is very low ($hw = -170$ cm).

Figure 9 presents the results on cross-sections in the space $\{y\ ,\ hw - hw_{min}\}$. We have 9 slices here arranged in three groups. The group A corresponds to a minimal value of the summer extraction $qs = qs_{min} = 10^{7} m^{3}$, group B corresponds to $qs = qs^{*} = 11.82 \times 10^{6}$, group

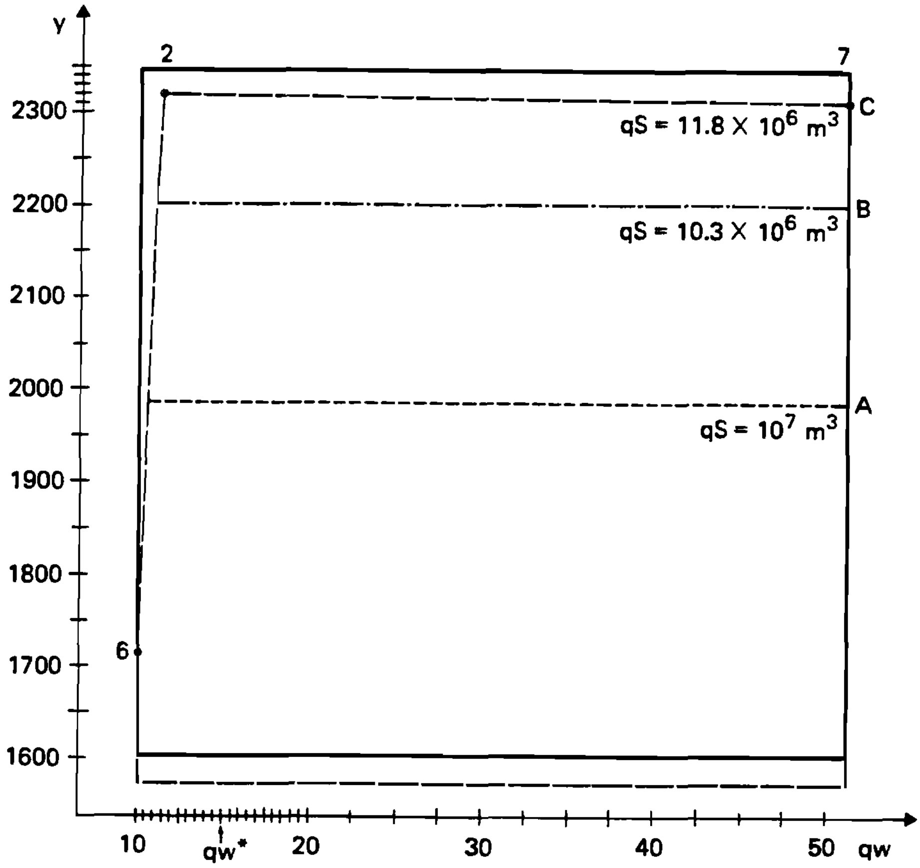


Figure 7

C corresponds to $qs = qs_{max} = 51 \times 10^6 m^3$. The slices in one group are distinguished by the value of qw. It is clear that the maximal value of hw which corresponds to group A is in sharp conflict with the income. The maximal value of income can be achieved by a small increase of the summer water extraction (group B). The further increasing of summer water extraction (group C) results in low groundwater levels (not higher than -180 cm) and reasonable drops in incomes.

Figure 10 shows slices in the same space $\{y \ , hw - hw_{min}\}$, but the scale is different.

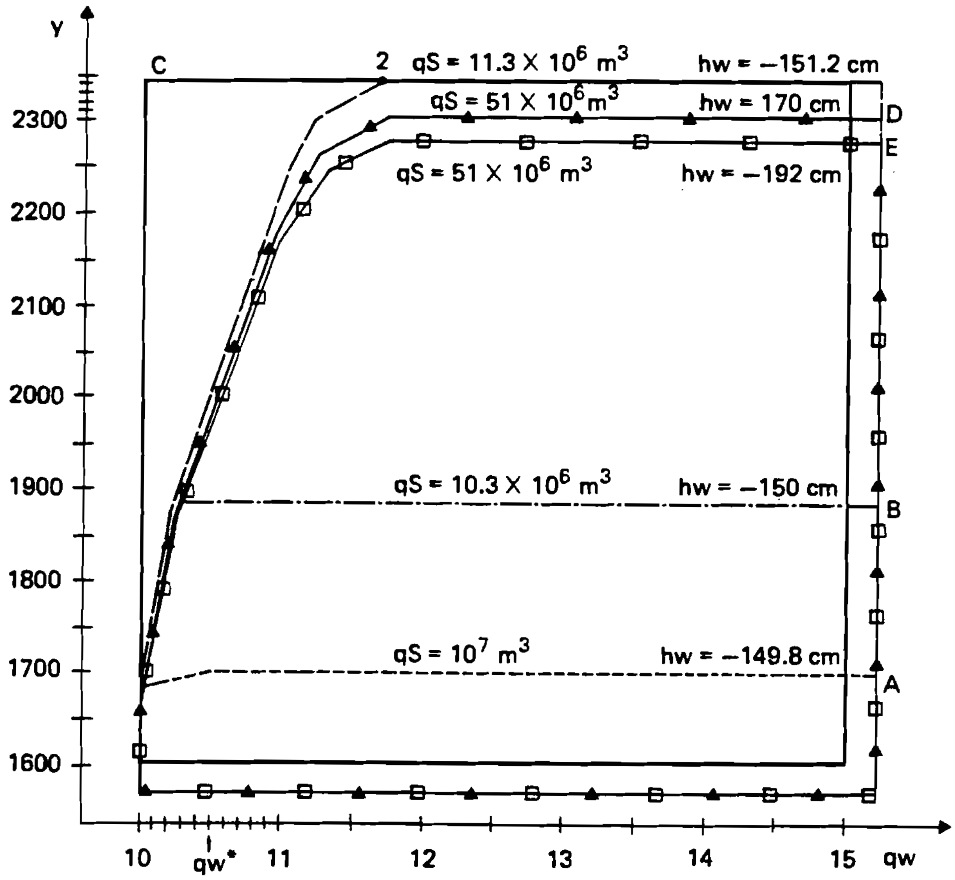


Figure 8

On the figures described above some points are presented (points 1-7). The controls resulting in these points are described in Appendix 1. The indicators related to points 1-7 are presented in Table 1.

Table 1

indicators	1	2	3	4	5	6	7 N
$\tilde{y}$ (10^6 fl)	170.3	234.6	228.2	228.0	234.5	170.2	233.5
hw-hw sub min (cm)	50.2	48.8	19.8	19.9	48.8	48.8	48.8
$\tilde{q}^w$ ($10^6 m^3$)	10	11.75	51	11.77	11.75	10	51
$\tilde{q}^S$ ($10^6 m^3$)	10	11.82	51	20.61	11.82	11.82	11.82

7. THE FIVE CLUSTER MODEL

On the second stage of our investigation a more complicated model was studied. The model consists of five economic clusters (see Figure 11). It was obtained on the base of the model consisting of 31 subregions. The construction of five clusters model was based on aggregation described in Section 4, while $S = 5$. The original 31 subregional model described in Section 3 was slightly modified. The modifications are described for the cluster model.

B.1. INVESTMENT AND REORGANISATION OF THE ECONOMIC STRUCTURE

The modified model takes into account the possibilities of reorganisation of the economic structure of the region. For this purpose investment is used. The volume of investment needed is described in the following manner:

$$\text{inv} = \sum_{j=1}^{10} [pxi(j) \cdot x_i(j) - 0.3 \cdot pxi(j) \cdot x_d(j)] +$$

$$+ \sum_{j=1}^{5} pzi(j) \cdot z_i(j) - 0.2 \cdot pzi(j) z_d(j)] + \sum_{j=1}^{5} [psto \sum_{k=1}^{5} \sum_{l=1}^{2} ms(s,k,l)] +$$

$$+ sc_{\text{inv}} \cdot sc_i + gc_{\text{inv}} \cdot gc_i - 0.9 \cdot sc_{\text{inv}} \cdot sc_d - 0.5 gc\ \text{inv} \cdot gc_d \quad ,$$

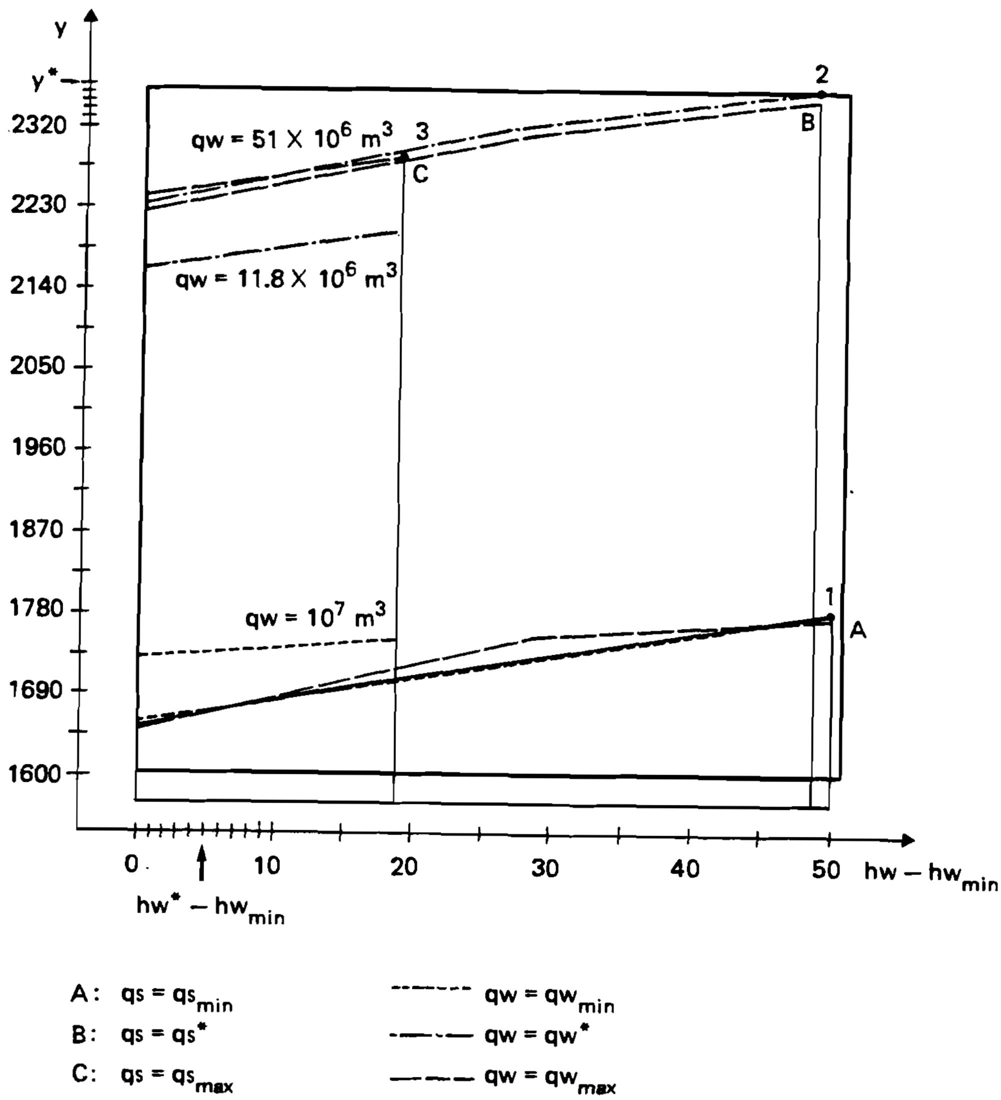
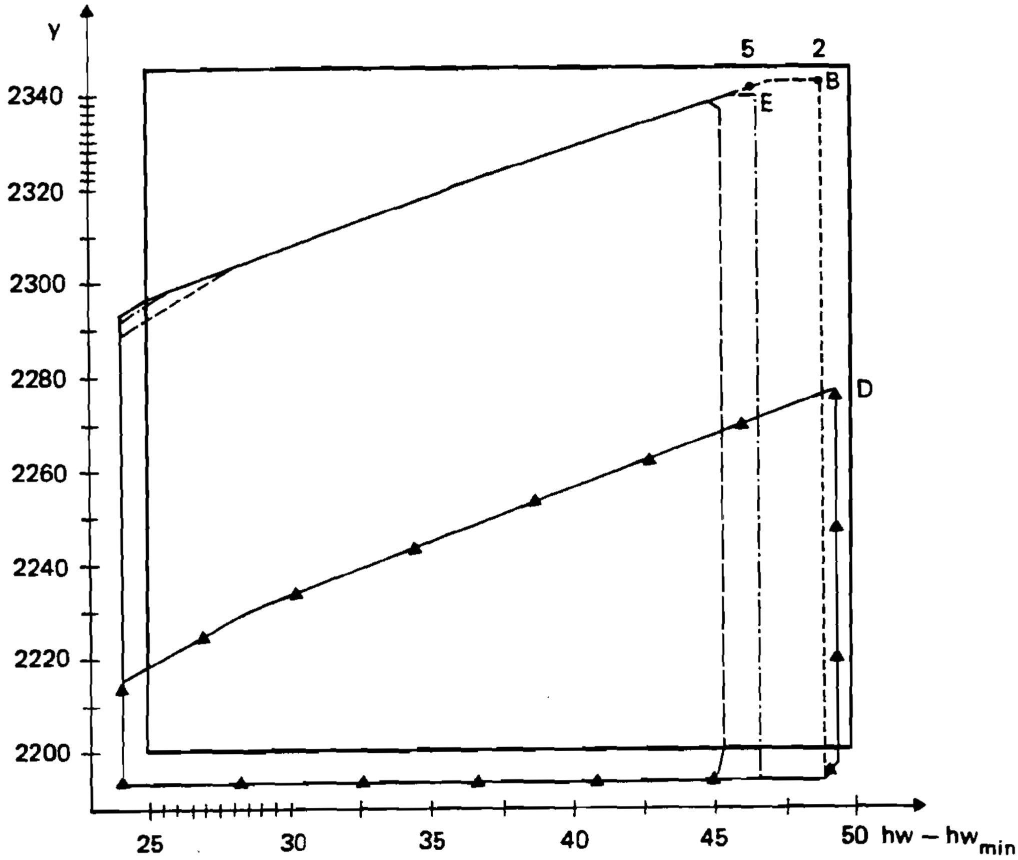


Figure 9

where inv is investment,

$x_i(j)$, $x_d(j)$ - increments and decrements of intensities of technologies $j \in JZ$,

$z_i(j)$, $z_d(j)$ - the same for $j \in JZ$,



D: qs < qs*   qw = qw*
B: qs = qs*
E: qs > qs*

Figure 10

sc_i , sc_d - increment and decrement of sprinkling capacities from surface water,

gc_i , gc_d - the same from groundwater

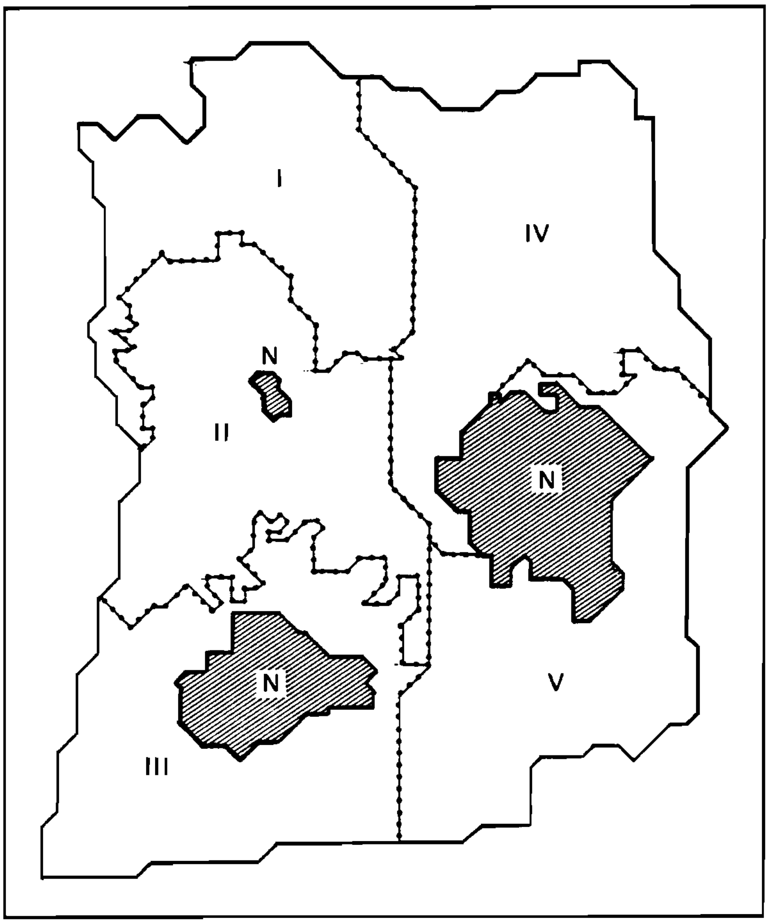
$pxi(j)$, $pzi(j)$, sc_{inv}, gc_{inv}, $psto$ - prices.

Increments and decrements are connected with initial capacities

$$x_i(j) - x_d(j) = \sum_{s=1}^{5} [x(s,j) - xo(s,j)] \, , \; j \in JX \quad ,$$

$$z_i(j) - z_d(j) = \sum_{s=1}^{5} [z(s,j) - zo(s,j)] \, , \; j \in JZ \quad ,$$

THE SOUTHERN PEEL AREA



I	Cluster:	1–6 Subregions		Natural zones:
II	Cluster:	7–14 Subregions (N)		10 Subregion
III	Cluster:	15–19 Subregions (N)		16 Subregion
IV	Cluster:	20–26 Subregions		27 Subregion
V	Cluster:	27–31 Subregions (N)		

Figure 11

$$sc_i - sc_d = \sum_{s=1}^{5} [sc(s) - sc_0(s)] , \; gc_i - gc_d = \sum_{s=1}^{5} [gc(s) - gc_0(s)]$$

The maximal possible value of investment is given by inv_{max}.

It is supposed that the total area of agricultural land is used

$$\sum_{j=1}^{10} x(s,j) = xa(s) , \; s = 1, \ldots, 5 \quad .$$

7.1. Animal Slurry By-products

It is supposed that in addition to spring application of slurry (see Section 3) autumn application is possible. In this case the autumn application of manure of k-th type in the s-th cluster should not exceed the half-year production. The possibility of manure transport between clusters is taken into account. We obtain

$$\frac{1}{2}\,[mt_e(s,k) - mt_i(s,k)] + \sum_{l=1}^{2} ma(s,l,k) \le \frac{1}{2} \sum_{j=1}^{5} mz(j,k) \cdot z(s,j) \quad ,$$

where $mt_e(s,k)$ - export of manure

$mt_i(s,k)$ - import of manure

The manure storage before spring application should not exceed storage capacities while in spring the total amount of manure should be applied to land:

$$\frac{1}{2}\,[\sum_{j=1}^{5} mz(j,k) \cdot z(s,j) + mt_i(s,k) - mt_e(s,k)] +$$

$$+ \sum_{j=1}^{10} mx(j,k) \times (s,j) - \sum_{l=1}^{2} ma(s,l,k) = \sum_{l=1}^{2} ms(s,l,k)$$

There is no export of manure outside the region

$$\sum_{j=1}^{5} mt_e(s,k) = \sum_{j=1}^{5} mt_i(s,k)\,,\; k = 1, \ldots, 5 \quad .$$

7.2. Income

The equation of income (9) should be modified in accordance with modification of the model. For this reason from income y, calculated by (9), the cost of manure transport $\sum_{j=1}^{5} pmt \cdot mt_e(s,k)$ is subtracted.

Further, the total income should exceed consumption

$$y \ge 0.3 \cdot lp$$

where lp is the amount of labour in the region.

7.3. Water Quantity Processes

In the modified model it was supposed that

$$qw(s) = 0.8 \cdot qs(s)\,,\ s = 1, \ldots, 5 \quad ,$$

Introducing the fall of groundwater level in cluster s from the level $hs0(s)$

$$\Delta hs(s) = hs(s) - hso(s)$$

we obtain

$$\Delta hs(s) = -\sum_{t=1}^{5} a(s,t) \cdot 0.8 \cdot qs(t) \quad .$$

For the fall of groundwater level in winter we obtain

$$\Delta hw(s) = -\sum_{t=1}^{5} b(s,t) \cdot qs(t) - \sum_{t=1}^{5} c(s,t) \cdot ig(t) - \sum_{t=2,3,5} d(s,t) \cdot u_s(s,t)$$

The description of moisture content of the rootzone is slightly changed while the process of deep percolation during the summer is taken into account. The amount of deep percolation during the summer in cluster s denoted by $ds(s)$ is described as follows:

$$ds(s) = \sum_{t=1}^{5} f(s,t) \cdot qs(t) + \sum_{t=1}^{5} g(s,t) \cdot ig(t) - 0.1 \cdot us(s) \quad .$$

The soil moisture requirements are modified as well:

$$\sum_{j=2}^{10} \sum_{k=1}^{2} vr(s,k) - xw(s,j,k) \leq \{icpvs(s) + lvs(s)\}$$

$$\cdot (hso(s) + \Delta hs(s)) + \beta\{ps \cdot [xa(s) - x(s,1)] + is(s) +$$

$$+ ig(s) - ds(s) + (\sum_{j=2}^{1o} \sum_{k=1}^{3} vz0(s,k)xw \cdot (s,j,k) +$$

$$+ lvz(s)(-\Delta hw(s))) - \sum_{j=2}^{10} \sum_{k=1}^{3} ea(s,k) \cdot xw(s,j,k)\} \quad .$$

7.4. Public Water Supply

It was supposed that values of qpw and qps equal zero. Since values of $qs(s)$ and $qw(s)$ are linked instead of two equations (28) and (29) we obtain

$$\sum_{s=1}^{5} qs(s) = \sum_{s=1}^{5} \sum_{j=1}^{5} wzs(j) \cdot zc(s,j) + \sum_{s=1}^{5} \sum_{j=1}^{10} wxs(j) \cdot x(s,j) \quad .$$

7.5. Water Quality

The modified version of the model contains a water quality block. The quality of water is described as an amount of nitrate in deep aquifers. All deep aquifers over the whole region are regarded in the model as one mixing cell. It is supposed as well that the volume of water does not change. Then the concentration of nitrate in deep aquifers denoted by cd is described as follows:

$$cd = \sum_{s=1}^{5} \{ \sum_{l=1}^{e} cfs(s,l) \cdot fs(s,l) + \sum_{k=1}^{5} \sum_{l=1}^{2} [cma(s,l,k) \cdot ma(s,l,k) + cms(s,l,k) \cdot ms(s,l,k)] \} \quad ,$$

where $fs(s,l)$ is the amount of nitrogen in chemical fertilizer applied on arable land ($l=1$) and grassland ($l=2$), $ma(s,l,k)$, $ms(s,l,k)$ are the autumn and spring applications of slurry in l-th direction, $cfs(s,l)$, $cma(s,l,k)$, $cms(s,l,k)$ are coefficients.

8. THE RESULTS FOR CLUSTER MODEL

The cluster model described above contained 460 variables and 672 linear restrictions. Four performances indicators have been chosen:

1) Investment (inv). The maximal admissible value of the indicator was 250 $\times 10^6$ fl;

2) Additional income (difference between income and given consumption: $\Delta y = y$ - consumption). In our case consumption equals 154 $\times 10^6$ fl and maximal accessible value of Δy was 163 $\times 10^6$ fl;

3) Concentration of nitrate in deep aquifers (cd). The maximal admissible value of this indicator was 88.6 mg/l;

4) The fall of groundwater level under initial level (maximal for economic clusters containing natural zones: $\Delta h\hat{w} = \min_{s=2,3,5} \Delta hw(s)$; where $\Delta hw(s) = hw(s) - hwo(s)$). The values of $hwo(s)$ for $s = 2,3,5$ were the following ones: $hwo(2)$ = - 180.4 cm, $hwo(3)$ = -177.4 cm, $hwo(5)$ = -143.8 cm. Since minimal admissible level of groundwater was chosen to be -200 cm, the minimal value of $\Delta h\hat{w}$ is equal to -19.6 cm.

The GRS for these four indicators was constructed. Most important two-dimensional cross-sections (slices) of the GRS are presented on Figures 12–28. On these two-dimensional cross-sections some points were chosen. The indicator values for these points are presented in Table 2 while corresponding values of variables are presented in Appendix 2.

Table 2

N indicators	1	2	3	4	5	6	7		
inv (10^6 fl)	0	0	0	0	0	250	0	250	0
Δy (10^6 fl)	0	46.5	60.7	64.5	0	144	24	163.6	46
cd (mg/e)	21	20.7	18.7	20.9	8.3	24.4	51.6	25.9	15.6
$-\Delta h\hat{w}$ (cm)	0.5	5	10	20	5	5	5	20	5

8.1. The Cross-Sections Containing Income

The cross-sections containing income are presented in Figures 12-16. In Figure 12 slices of the GRS containing investment (inv) and additional income (Δy) are presented while values of cd and $\Delta h\hat{w}$ are fixed on a series of levels. The slices are grouped into three groups each of them corresponding to definite value of cd:

for group A we have cd = 8 mg/l;

for group B we have cd = 21 mg/l;

for group C we have cd = 80 mg/l.

Every group contains three slices corresponding to definite values of $\Delta\hat{hw}$. The boundary of the slice corresponding to $\Delta\hat{hw}$ = -5 cm is presented with the broken line, to $\Delta\hat{hw}$ = -10 cm is presented with dot and dash line, to $\Delta\hat{hw}$ = -20 cm is presented with solid line.

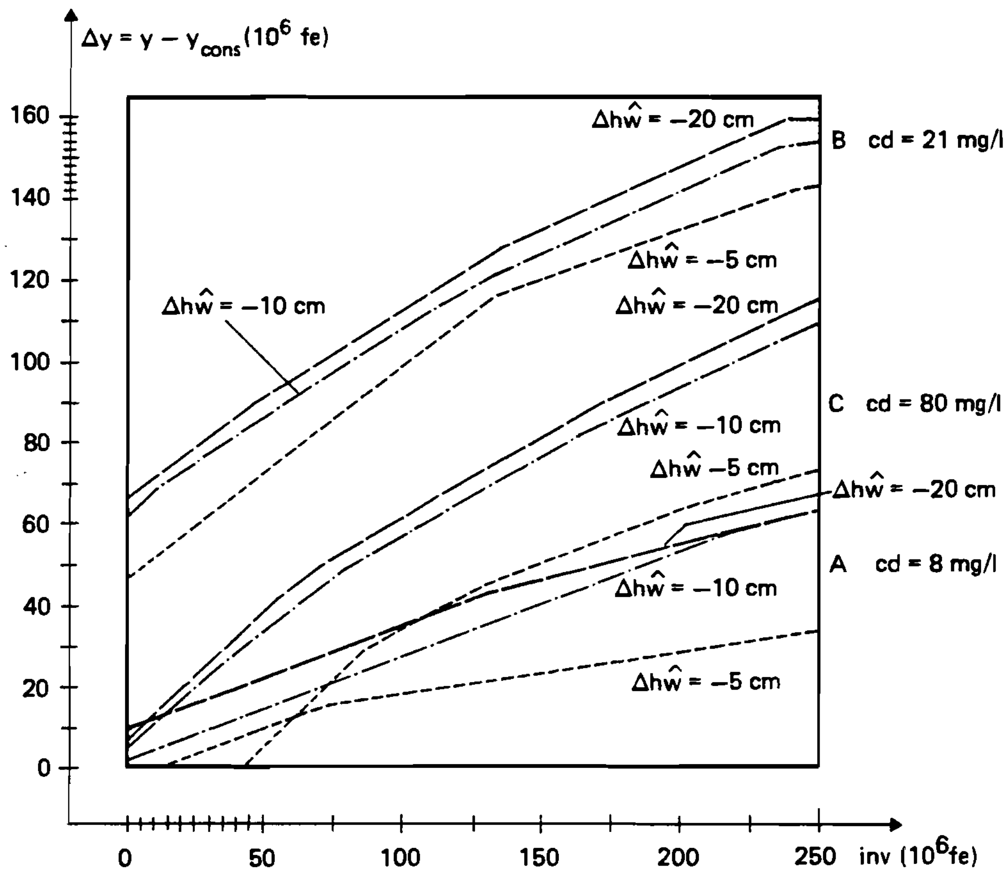


Figure 12

First, in all slices the increment of investment results in increment of possible value of income, but the effectiveness is slightly going down. It is important to mention that the dependence of maximal accessible income on pollution level cd is nonmonotonous. If the values of inv and $\Delta\hat{hw}$ are fixed and the values of cd are small the increment of cd results in the increment of maximal accessible value of Δy (compare slices of groups A and B). But if the values of *cd* are great than the increment of *cd* results in the decrement of maximal accessible value of Δy (compare slices of groups B and C). So, these values of *cd* are inefficient. The dependence of maximal accessible income Δy on the fall of groundwater level is monotone.

The dependence of the income on the groundwater level is more clearly presented in Figure 13 where slices in the space $\{\Delta y \ , \ -\Delta h\hat{w}\}$ are given while the values of *cd* and *inv* are fixed. There are two groups of slices on this figure: on the slices of group A the value of *cd* equals to 8 mg/l, and on the slices of group B the value of *cd*. The values of inv are presented on the boundaries of the slices. This figure shows that after sharp rise of income due to fall of the groundwater level further fall of the level of the groundwater is quite inefficient. It is interesting to mention that the analogous effect was obtained on the "modal subregion" model (see Figures 9 and 10). The only difference between Figures 9, 10 and Figure 13 consists in the fact that in Figures 9 and 10, the income begins to fall down but in Figure 13 the income stops to rise.

It is interesting to note that in the case of zero investment the level $\Delta y = 0$ can be achieved in the case of nonzero fall of groundwater level only (see point 1).

Five points were chosen in Figure 13. Four of these belong to the boundary of the slice with $cd = 21$ mg/l and $inv = 0$. The corresponding values of variables are presented in Appendix 2. The study of the values of variables shows that in the case of small values of $\Delta h\hat{w}$ (points 1 and 2) the agricultural production (especially livestock production) is limited by water resources. The unemployment in this case is quite great and the pollution level is high due to extensive application of chemical fertilizers. If the fall of the groundwater level is getting greater, about 10 cm, then livestock production and income are increasing and unemployment is going down (point 3). The additional fall of the groundwater level (point 4) results in practically constant income and in changing the structure of land-using production. The production of technologies that do not use land is decreasing.

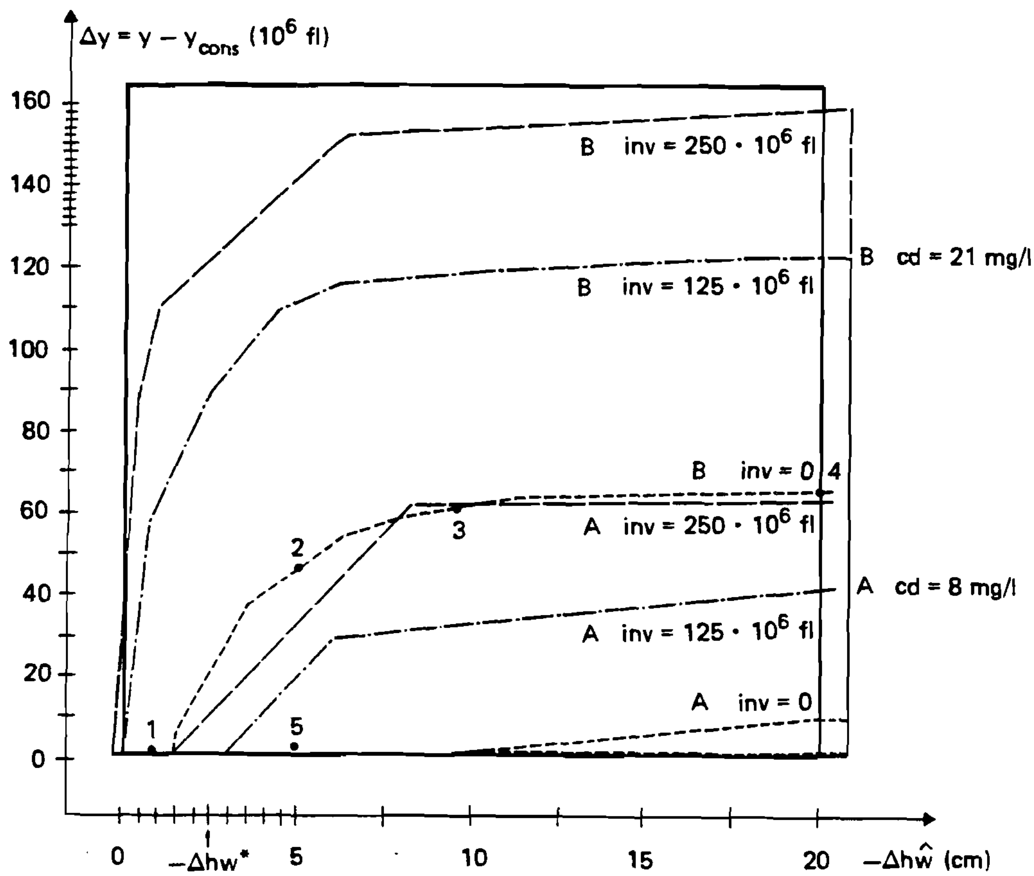


Figure 13

The monotonous dependence of the income on the pollution is clearly presented in Figure 14 where two groups of cross-sections in the space $\{\Delta y\ , cd\}$ are presented. For group A we have $inv = 0$, and for group B we have $inv = 250 \times 10^6$ fl. The values of $\Delta h\hat{w}$ are presented near the boundaries of the slices. All boundaries of the slices which are in this case related to maximal accessible income have clear maximum for $cd \sim$ 15-30 mg/l. It is interesting that the values of cd less than 7 mg/l are not accessible. The increment of cd above the minimal value results in sharp increments of Δy. After the sharp increment the increase of the income is getting smaller and smaller and after some value of the pollution the value of the income begins to drop.

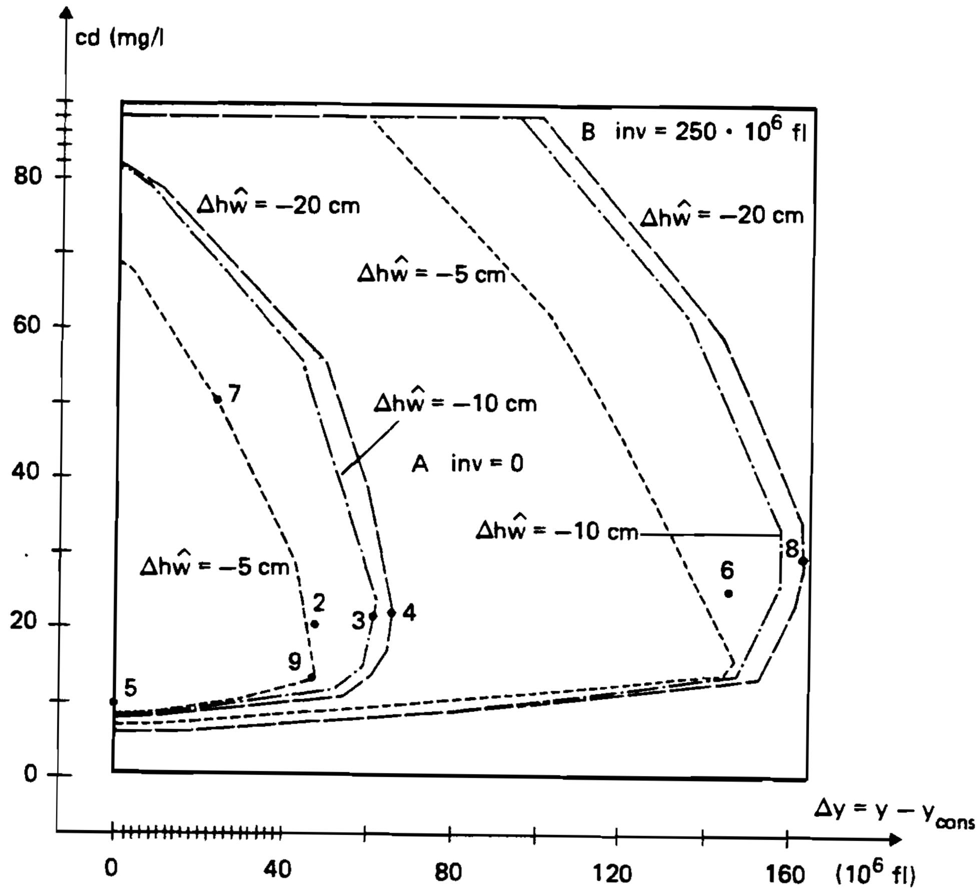


Figure 14

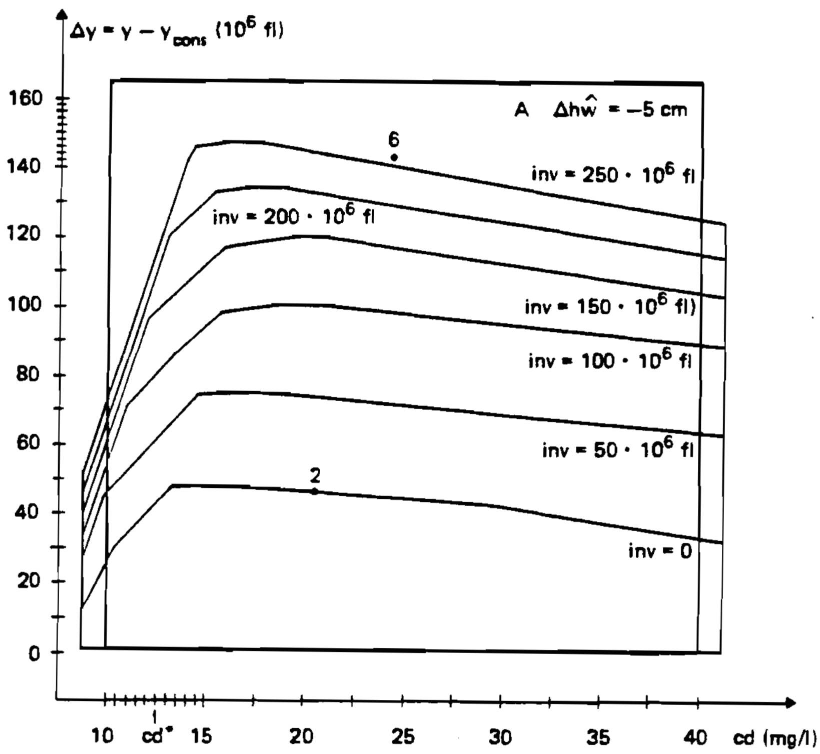


Figure 15

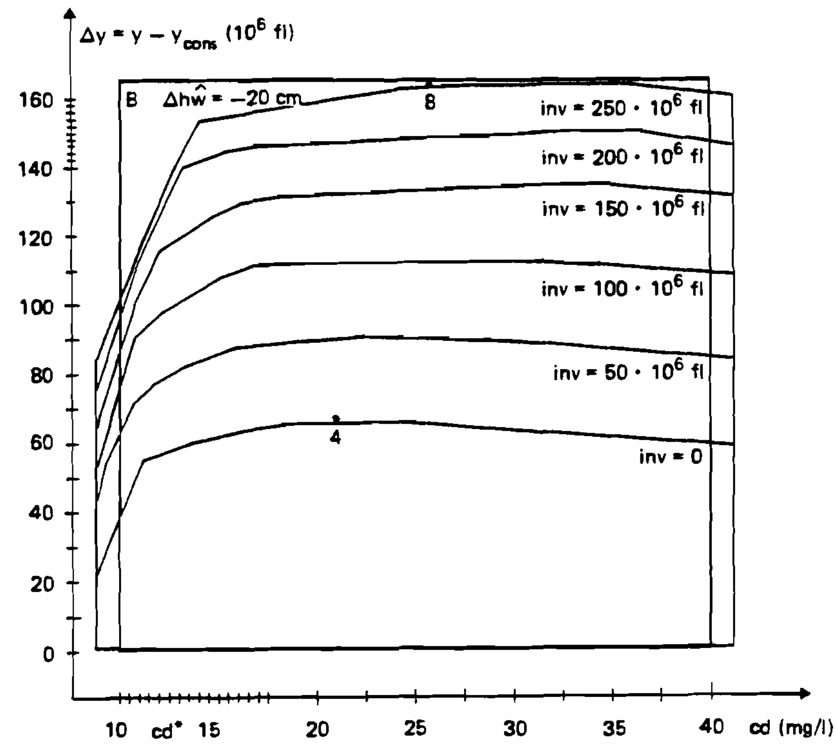

Δy = y − y_cons (10⁶ fl)
160
140
120
100
80
60
40
20
0
B Δhŵ = −20 cm
8
4
inv = 250 · 10⁶ fl
inv = 200 · 10⁶ fl
inv = 150 · 10⁶ fl
inv = 100 · 10⁶ fl
inv = 50 · 10⁶ fl
inv = 0
10 cd* 15 20 25 30 35 40 cd (mg/l)

Figure 16

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APPENDIX 1

Points	1	2	3	4	5	6	7
Income (10^6 fl): Δy −16.3	y −170.3 Δy −80.6	y −234.6 Δy −74.2	y −228.2 Δy −74	y −228 Δy −80.5	y −234.5 Δy −16.2	y −170.2 Δy −79.5	y −233.5
Labour (hrs)	lh -2100 ln-0	lh -2100 ln-0	lh -2100 ln-0	lh -2100 ln-0	lh -2100 ln-0	lh -2100 ln-0	lh -2100 ln-0
Areas (ha) of technol. that us land $(x(j)$ (k -subtechnology $(j,k)-x(j)$	(1.1)-380 (3.3)-3873 (4.3)-5955 (6.2)-5955	(4.2)-5955 (8.2)-4695 (9.2)-5955	(4.2)-5955 (8.2)-5798 (9.2)-3172	(4.2)-5955 (8.1)-4694 (9.2)-4319	(4.2)-5955 (8.2)-4690 (9.2)-5955	(1.1)-384 (3.3)-3718 (4.3)-5955 (6.2)-5955	(4.2)-5955 (8.2)-4018 (9.2)-5955
Intensities of techn. that do not use land $(z(j))$	(1)-25195 (2)-77154	(1)-31000 (2)87848	(1)-3100 (2)-74557	(1)-25214 (2)-77083		(1)-27690 (2)-67892	
Chemical Fertilizer $(FS(c))(tons)$	(1)-152	(1)-235 (2)-2382	(1)-290 (2)-1269	(1)-235 (2)-1728	(1)-235 (2)-2382	(1)-153	(1)-201 (2)-2382
Animal slurry application (10^3 tons) $ma(k,l)$ $ms(k,l)$ l =1-arable land l =2-grassland mc-storage capacity	(3.2)-135.3	ma: (1.1)-297.8 (2.1)-755.8 (3.1)-930 (3.2)-721 ms: (3.1)-169.2 mc-169.2	ma: (1.1)-158.6 (2.1)-930 (3.1)-930 (3.2)-98.2 ms: (3.1)-403.5 mc-403.5	ma: (1.1)-216 (2.1)-930 (3.1)-930 (3.2)-135.3 ms: (3.1)-164.6 mc-164.6	ma: (1.1)-297.8 (2.1)-756.4 (3.1)-930 ms: (3.1)-168 mc-168	(3.2)-135.3	ma: (1.1)-297.7 (2.1)-830.7 (3.1)-930 ms: (3.1)-21 mc-21
Water supply (10^6 tons) $qw,qs;\Delta qw=qw-qpw$ $\Delta qs=qs-qps;is,ig,us$	qw-10 qs-10 Δqw-0 Δqs-0 us-15	qw-11.75 qs-11.82 Δqw-1.75 Δqs-1.82 us-15	qw-51 qs-51 Δqw-41 Δqs-41 us-15	qw-11.77 qs-20.61 Δqw-1.77 Δqs-10.61 ig-2.49	qw-11.75 qs-11.82 Δqw-1.75 Δqs-1.82 us-14.97	qw-10 qs-11.82 Δqw-0 Δqs-1.82 us-15	qw-51 qs-11.82 Δqw-41 Δqs-1.82 us-15
Groundwater levels ($cm-gl$): Fall of GWL (cm): $\delta hw=hw-hw_{min}$ $\Delta hw=hw-hw\Delta$ hw_{min}= 200cm $hw0=-$ 162.9 cm	hs:-97.0 hw:-149.8 δhw: 50.2 Δhw: 13.1	hs:-97.6 hw:-151.2 δhw: 48.8 Δhw: 11.7	hs:-97.6 hw:181.2 δhw: 19.8 Δhw: -18.3	hs:-97.6 hw:-181.1 δhw: 19.9 Δhw: -18.2	hs:97.58 hw:-151.2 δhw: 48.8 Δhw: 11.7	hs:-97.0 hw:-151.2 δhw: 48.8 Δhw: 11.7	hs:-111.7 hw:-151.2 δhw: 48.8 Δhw: 11.7

APPENDIX 2

Point 1

		Increments:	Decrements:	
Investments:	inv=0	x_i:(4)=9710	x_d(9)=11024	z_d(1)=1640
Addit. income:	Δy=0	x_i:(6)=2308	x_d(10)=2839	z_d(2)=36890
Income:	y=154.10^6 fl	x_i(8)=1864		z_d(3)=8300
N-Conc.In.D.A.:	cd=21 mg/l		sc_d=0.91	z_d(4)=3276
Lowering GWL:	$\Delta h\hat{w}$=-0.5 cm		gc_d=14.23	z_d(5)=3596
In employment:	Lu=2051 hrs			

Clusters	1	2(N)	3(N)	4	5(N)
Areas (ha) of Technol. that use land ($x(j)$) (k-subtechnology) (j,k)	(3.1)=630 (4.1)=461 (8.1)=1865	(1.1)=78 (4.1)=4356	(1.1)=21 (2.1)=473 (4.1)=5190	(2.3)=808 (6.1)=4876	(2.2)=40 (4.1)=966 (5.1)=334 (6.1)=3722
Intensities of technol. that do not use land ($z(j)$)					
chemical fertilizer ($FS(e)$) (tons)	(1)=1841	(1)=902.3	(1)=1165	(1)=1470	(1)=1204
Animal slurry Application (103 tons) ($ma(k,l), ms(k,l)$)					
Water Supply ($10^6 m^3$)	is=0.27 us=0.05	is=0.31 us=0.06	is=0.1 us=0.02	is=2.21 us=0.44	is=0.08 us=0.02
Groundwater levels (cm): hs, hw Fall of GWL Δhs, Δhw (cm)	hs=-89.7 Δhs=0 hw=-163.2 Δhw=0	hs=-110.3 Δhs=0 hw=-163.7 Δhw=0.5	hs=-87.7 Δhs=0 hw=-180.9 Δhw=-0.5	hs=-90.6 Δhs=0 hw=-177.4 Δhw=0	hs=-90.4 Δhs=0 hw=-144.3 Δhw=-0.5

Point 2

		Increments:	Decrements:	
Investments:	inv=0			
Addit. income:	Δy=46.49 10^6 fl	x_i:(4)=7074	x_d(9)=4215	z_d(1)=1640
Income:	y=200.49 10^6 fl		x_d=2839	z_d(2)=36890
N-Conc.In.D.A.:	cd=20.71 mg/l	sc_i=12.33		z_d(3)=8300
Lowering GWL:	$\Delta\hat{hw}$=-5.0 cm		gc_d=8.7	z_d(5)=3596
Unemployment:	Lu=865 hrs			

Clusters	1	2(N)	3(N)	4	5(N)
Areas (ha) of Technol. that use land ($x(j)$) (k-subtechnology) (j,k)	(4.2)=546 (6.1)=364 (9.1)=2049	(6.1)=1190 (9.1)=3244	(1.1)=99 (3.1)=630 (4.1)=4888	(2.2)=1321 (4.3)=2426	(4.2)=478 (5.1)=334 (6.1)=1937 (9.1)=1449
Intensities of technol. that do not use land ($z(j)$)			(4)=229	(4)=2376	(4)=671
Chemical fertilizer ($FS(e)$) (tons)	(1)=45 (2)=820	(1)=60 (2)=1298	(1)=1051 (2)=27	(1)=416	(1)=478 (2)=580
Animal slurry Application (103 tons) ($ma(k,l)$,$ms(k,l)$) l = 1 arable land l = 2 grassland	ms (1.1)=47.8 (1.2)=34.2	ms (1.1)=75.7 (1.2)=54.1	ma (4.1)=7.9 (4.2)=1.2 ms (4.1)-9.2	ma (4.1)=95 ms (4.1)=95	ma (4.2)=26.8 ms (1.1)=58 (4.1)=26.8
Water Supply ($10^6 m^3$)	is=4.3 us=0.86 qs=0.37	is=2.8 us=0.56	is=0.95 us=0.19 qs=0.18	is=7.23 us=1.45 ig=5.54	is=0.99 us=0.19
Groundwater levels (cm-gl)=hs,hw Fall of GWL (cm): Δhs,Δhw	hs=-90.3 Δhs=0.6 hw=-165.3 Δhw=-2.1	hs=-110.6 Δhs=0.3 hw=-185.4 Δhw=-5	hs=-87.8 Δhs=0.1 hw=-182.4 Δhw=-5	hs=-90.6 Δhs=0 hw=-174.1 Δhw=22.5	hs=-90.4 Δhs=0 hw=-148.8 Δhw=-5

Point 3

		Increments:	Decrements:	
Investments:	inv=0			
Addit. income:	Δy=60.7 10^6 fl	$x_i(4)$=367	$x_d(3)$=630	$z_d(1)$=1640
Income:	y=214.7 10^6 fl	$x_i(8)$=490	$x_d(9)$=677	$z_d(2)$=36890
N-Conc.In.D.A.:	cd=18.74 mg/L	sc_i=12.33	$x_d(10)$=2839	$z_d(3)$=4046
Lowering GWL:	$\Delta h\hat{w}$=-10 cm	gc_i=0.05		$z_d(5)$=3596
Unemployment:	Lu=0 hrs			

Clusters	1	2(N)	3(N)	4	5(N)
Areas (ha) of Technol. that use land ($x(j)$) (k-subtechnology) (j,k)	(4.2)=1189 (9.1)=1767	(4.2)=721 (5.1)=334 (6.1)=413 (9.1)=4838	(1.1)=99 (4.1)=192	(2.3)=1321 (4.3)=1812 (6.1)=199	(4.2)=1026 (6.1)=3902 (6.1)=2801
Intensities of technol. that do not use land ($z(j)$)	(4)=107			(3)=4253	(4)=2851
Chemical fertilizer ($FS(e)$) (tons)	(1)=60 (2)=707	(1)=57 (2) 1186	(1)=67 (2)=1935	(1)=378 (2)=310	(1)=253
Animal slurry Application ($ma(k,l)$,$ms(k,l)$) $l=1$ arable land $l=2$ grassland	ma (4.2)=4.3 ms (1.1)=54.1 (1.2)=16.6 (4.2)=4.3	ms (1.1)=69.2 (1.2)=49.4	ms (1.1)=112.9 (1.2)=80.6	ma (3.1)=91.5 (3.2)=20 ms (3.1)=93.6 (4.1)=12.7	ma (4.1)=114 (4.2)=114
Water Supply (million m^3)	is=4.3 us=0.86	is=7.28 us=1.96 ig=0.017 gs=0.76	is=1.88 us=0.38 gs=0.02	is=0.64 us=0.13 ig=14.27	is=209 us=0.42
Groundwater levels (cm-gl):hs,hw Fall of GWL Δhs, Δhw-cm	hs=-90 Δhs=-0.3 hw=-166 Δhw=-3	hs=-111 Δhs=-0.7 hw=-190.4 Δhw=-10	hs=-88 Δhs=-0.3 hw=-187.4 Δhw=-10	hs=-90.6 Δhs=0 hw=-209.5 Δhw=-57.9	hs=-90.4 Δhs=0 hw=-153.8 Δhw=-10

Point 4

		Increments:	Decrements:	
Investments:	inv=0			
Addit. income:	Δy =64.51 10^6 fl	$x_i(4)$=4438	$x_d(1)$=53	$z_d(1)$=1640
Income:	y =218.51 10^6 fl	$x_i(8)$=598	$x_d(3)$=630	$z_d(2)$=36890
N-Conc.In.D.A.:	cd =20.85 mg/L		$x_d(9)$=1493	$z_d(3)$=2102
Lowering GWL:	$\Delta h\hat{w}$ =-20 cm	sc_i =11.93	$x_d(10)$=2839	$z_d(5)$=3596
Unemployment:	Lu =0 hrs	gc_i =1.95		

Clusters	1	2(N)	3(N)	4	5(N)
Areas (ha) of Technol. that use land ($x(j)$) (k-subtechnology)	(4.2)=530 (8.1)=120	(4.2)=2419 (9.1)=2015 (9.1)=2306	(1.1)=46 (8.1)=479	(2.3)=1321 (4.3)=1622 (9.1)=5159 (9.1)=50	(4.3)=1130 (5.1)=334 (6.1)=2691
Intensities of technol. that do not use land ($z(j)$)				(3)=6198 (4)=204	(4)=2522
Chemical fertilizer ($FS(e)$) (tons)	(1)=325 (2)=922	(1)=121 (2)=806	(1)=38 (2)=2064	(1)=414 (2)=20	(1)=236
Animal slurry Application ($ma(k,l)$,$ms(k,l)$)(10^3 tons) l = 1 arable land l = 2 grassland	ms (1.1)=53.8 (1.2)=38.4	ma (4.1)=22 ms (1.1)=80.6 (4.1)=6 (4.2)=16	ms (1.1)=1103.8 (1.2)=86	ma (3.1)=136.3 (4.1)=8.2 ms (1.1)=2 (3.1)=136.3 (4.1)=7.8 (4.2)=0.4	ma (4.1)=100.9 ms (4.1)=100.9
Water Supply ($10^6 m^3$)	is =233 us =0.47	is =7.28 us =1.45	is =3.74 us =0.75	ig =14.27	is =2.44 us =0.98
Groundwater levels (cm): hs, hw Fall of GWL Δhs, Δhw (cm)	hs =89.8 Δhs =-0.1 hw =-166.8 Δhw =-3.4	hs =110.7 Δhs =-0.4 hw =-195.8 Δhw =-15.4	hs -88.2 Δhs =-0.5 hw =-197.4 Δhw =-20	hs =-90.6 Δhs =0 hw =-209.7 Δhw =-58.1	hs =-90.4 Δhs =0 hw =-163.8 Δhw =-20

Point 5

		Increments:	Decrements:	
Investments:	inv=0			
Addit. income:	Δy =0	x_i(4)=5034	x_d(3)=630	z_d(2)=36890
Income:	y =154 10^6 mg/l	x_i(5)=6422	x_d(6)=6289	z_d(3)=8300
N-Conc.In.D.A.:	cd =8.29 mg/L	sc_i =12.22		z_d(5)=3596
Lowering GWL:	$\Delta h\hat{w}$ =-5 cm	gc_d =14.23	z_d(4)=3096	
Unemployment:	Lu =1362 hrs			

Clusters	1	2(N)	3(N)	4	5(N)
Areas (ha) of Technol. that use land ($x(j)$) (k-subtechnology)	(5.2)=1682 (9.1)=1274	(5.1)=1763 (10.1)=2671	(4.1)=2661 (9.1)=3023	(2.3)=710 (4.3)=1494 (10.1)=168	(1.1)=99 (2.3)=611 (9.1)=2210
Intensities of technol. that do not use land ($z(j)$)				(1)=1640	(4)=180
Chemical fertilizer ($FS(e)$) (tons)	(2)=573	(2)=668	(1)=133 (2)=1361	(1)=713 (2)=42	(1)=229 (2)=994
Animal slurry Application (10^3 tons): ($ma(k,l), ms(k,l)$)(10^3 tons) $l=1$ arable land $l=2$ grassland	ms (1.1)=51	ms (1.1)=53.4	ms (1.1)=120.9	ms (1.1)=3.4 (2.1)=49.2	ms (1.1)=88.4 (4.1)=14.4
Water Supply ($10^6 m^3$)	is =4.3 us =0.86 qs =0.37	is =3.37 us =0.67 qs =0.1	is =0.95 us =0.19 qs =0.13	is =6.25 us =1.25	is =1.21 us =0.24
Groundwater levels (cm): hs, hw Fall of GWL Δhs, Δhw (cm)	hs =-90.3 hs =-164.5 Δhs-0.6 Δhw =-1.3	hs =-110.7 hw =-185.4 Δhs =-0.4 Δhw =-5	hs =87.8 hw =-182.4 Δhs =-0.1 Δhw =-5	hs =-90.6 hw =-151.7 Δhw =-0.02 Δhw =0.1	hs =-90.4 hw =-148.8 Δhs :-0 Δhw =-5

Point 6

		Increments:	Decrements:	
Investments:	inv$=250.10^6$			
Addit. income:	$\Delta y=143.95.10^6$ fl	$x_i(u)$:21806	$x_d(2)=670$	$z_d(1)=1640$
Income:	$J=297.95.10^6$ fl	$z_i(4)=1628$	$x_d(3)=630$	$z_d(2)=35869$
N-Conc.In.D.A.:	$cd=24.43$ mg/l		$x_d(5)=334$	$z_d(3)=3596$
Lowering GWL:	$\Delta h\hat{w}=-5$ cm	$sc_i=12.33$	$x_d(6)=6289$	
			$x_d(9)=11027$	
Unemployment:	$Lu=0$	$gc_d=1.84$	$x_d(10)=2839$	

Clusters	1	2(N)	3(N)	4	5(N)
Areas (ha) of Technol. that use land ($x(j)$) (k-subtechnology)	(4,1)=2596 2	(4.1)=4434	(1.1)=99 (4.1)=5585	(2.3)=651 (4,1)=5033	(4,1)=5062
Intensities of technol. that do not use land	(4)=1167	(4)=514	(2)=1031 (3)=2383 (4)=1225	(3)=5917	(4)=1998
Chemical fertilizer ($FS(e)$) (tons)	(1)=148	(1)=691	(1)=309	(1)=349	(1)=253
Animal slurry Application ($ma(k,l), ms(k,l)$)(10^3 tons) $l=1$ arable land $l=2$ grassland	ma (4.1)=46.7 . ms (4.1)=46.7	ma (4.1)=20.6 ms (4.1)=20.6 (3.1)=60.7	ma (3.1)=60.7 (4.1)=49 (3.1)=130 (4.1)=49	ma (3.1)=130 ms (4.1)=79.9	ma (4.1)=79.9 ms
Water Supply ($10^6 m^3$)	$is=4.3$ $us=0.86$ $qs=0.24$	$is=2.07$ $us=0.41$	$is=0.95$ $us=0.19$ $qs=0.18$	$is=8.3$ $us=1.66$ $ig=12.39$	$is=0.58$ $us=0.12$
Groundwater levels (cm):hs, hw Fall of GWL (cm) $\Delta hs, \Delta hw$	$hs=-90.1$ $hw=-166$ $\Delta hs=-0.4$ $\Delta hw=-2.8$	$hs=-110.5$ $hw=-185.4$ $\Delta hs=-0.2$ $\Delta hw=-5$	$hs=87.9$ $hw=-182.4$ $\Delta hs=-0.2$ $\Delta hw=-5$	$hs=-90.6$ $hw=-201.8$ $\Delta hs=-0$ $\Delta hw=-50.2$	$hs=-90.4$ $hw=-148.8$ $\Delta hs=0$ $\Delta hw=-5$

Point 7

		Increments:	Decrements:	
Investments:	inv=0			
Addit. income:	ΔJ=23.99.10^6 fl	x_i(4):4221	x_d(1)=99	z_d(1)=1640
Income:	y=177.99.10^6 fl	x_i(8)=7390	x_d(9)=8652	z_d(2)=28021
N-Conc.In.D.A.:	cd=51.57 mg/l		x_d(10)=2839	z_d(5)=3596
Lowering GWL:	$\Delta h\hat{w}$=-5 cm	sc_i=12.33		
Unemployment:	Lu=669 hrs	gc_d=14.23		

Clusters	1	2(N)	3(N)	4	5(N)
Areas (ha) of Technol. that use land ($x(j)$) (k-subtechnology) (j,k)	(8,1)=2956	(8.1)=4434	(3.1)=630 (4.1)=3720 (6.1)=1334	(2.3)=1321 (4,3)=1029 (6.1)=333 (9.1)=2372	(4,2)=735 (5.1)=334 (6.1)=1621
Intensities of technol. that do not use land		(2)=8869 (3)=8300 (4)=680		(4)=2596	
Chemical fertilizer ($FS(e)$) (tons)	(1)=2571	(1)=2002	(1)=1217	(1)=416	(1)=419 (2)=949
Animal slurry Application (10^3 tons) ($ma(k,l), ms(k,l)$) l = 1 arable land l = 2 grassland		ma (3.1)=253.5 (4.1)=27.2 ms (3.1)=253.5 (4.1)=27.2		ma (4.1)=103.9 ms: (4.1)=103.9	ms (1.1)=55.3 (1.2)=39.5
Water Supply ($10^6 m^3$)	is=3.98 us=0.8 qs=0.49	is=3.4 us=0.68	is=0.95 us=0.19 qs=0.14	is=6.67 us=1.33	is=1.19 us=0.24
Groundwater levels (cm-gl): hs, hw Fall of GWL (cm) $\Delta hs, \Delta hw$	hs=-90.4 hw=-164.8 Δhs=-0.7 Δhw=-1.6	hs=-110.7 hw=-185.4 Δhs=-0.36 Δhw=-5	hs=87.8 hw=-182.4 Δhs=-0.1 Δhw=-5	hs=-90.6 hw=-151.7 Δhs=0 Δhw=-0.1	h=-90.4 hw=-148.8 Δhs=0 Δhw=-5

Point 8

		Increments:	Decrements:	
Investments:	$inv=250.10^6$ fl			
Addit. income:	$\Delta J=163.6.10^6$ fl	$x_i(1)=2.6$	$x_d(2)=931.4$	$z_d(2)=4.68$
Income:	$J=317.6.10^6$ fl	$x_i(4)=22066$	$x_d(3)=630$	$z_d(5)=3596$
N-Conc.In.D.A.:	$cd=25.93$ mg/l		$sc_i=12.33$	$x_d(5)=334$
Lowering GWL:	$\Delta h\hat{w}=-20$ cm	$gc_i=4.7$	$x_d(9)=11024$	
Unemployment:	$Lu=0$ hrs			

Clusters	1	2(N)	3(N)	4	5(N)
Areas (ha) of Technol. that use land ($x(j)$) (k-subtechnology) (j,k)	(4,1)=2956	(1.1)=98 (4.1)=9336	(2.3)=4.3 (4.1)=5680	(2.3)=390 (4,1)=5294	(4,1)=735
Intensities of technol. that do not use land	(4)=1167	(1)=1640 (3)=2289 (2)=4649 (4)=111	(2)=16912	(3)=6011	(4)=1998
Chemical fertilizer ($FS(e)$) (tons)	(1)=1481	(1)=246	(1)=285	(1)=323	(1)=253
Animal slurry Application (10^3 tons) ($ma(k,l)$, $ms(k,l)$) $l=1$ arable land $l=2$ grassland	ma: (4.1)=46.7 ms: (4.1)=46.7	ma: (2.1)=24.6 (3.1)=87.5 (4.1)=3.9 ms: (2.1)=24.6 (3.1)=87.5 (4.1)=5	ma: (3.1)=135.3 ms: (3.1)=135.3	ma: (3.1)=132.2 ms: (3.1)=132.2	ma: (4.1)=79.9 ms: (4.1)=79.9
Water Supply ($10^6 m^3$)	$is=4.3$ $us=0.86$	$is=3.35$ $us=0.67$ $ig=4.65$ $gs=0.62$	$is=3.66$ $us=0.73$	$ig=14.3$	$is=4.89$ $us=0.98$
Groundwater levels (cm): hs, hw Fall of GWL (cm) Δhs, Δhw	$hs=-89.7$ $hw=-167.7$ Δhs:0 $\Delta hw=-4.5$	$hs=-110.4$ $hw=-200.4$ $\Delta hs=0.1$ $\Delta hw=-20$	$hs=-88.2$ $hw=-197.4$ $\Delta hs=-0.5$ $\Delta hw=-20$	$hs=-90.6$ $hw=-210$ $\Delta hs=0$ $\Delta hw=-58.4$	$h=-90.4$ $hw=-163.8$ $\Delta hs=0$ $\Delta hw=-20$

Point 9

		Increments:	Decrements:	
Investments:	inv=0			
Addit. income:	Δy =	$x_i(4)$:7022	$x_d(9)$=4163	$z_d(1)$=1640
Income:	y=200.10^6 fl		$x_d(10)$=2839	$z_d(2)$=36890
N-Conc.In.D.A.:	cd=15.631mg/l	sc_i=12.33		$z_d(3)$=8300
Lowering GWL:	$\Delta h\hat{w}$=-5 cm		gc_d=8.96	$z_d(5)$=3590
Unemployment:	Lu=861 hrs			

Clusters	1	2(N)	3(N)	4	5(N)
Areas (ha) of Technol. that use land ($x(j)$) (k-subtechnology) (j,k)	(4.2)=532 (6.1)=372 (9.1)=2052	(6.1)=1190 (9.1)=3244	(1.1)=99 (3.1)=630 (4.1)=4907 (9.1)=47.3	(2.3)=1321 (4.3)=2357 (6.1)=1421 (9.1)=586	(4.2)=489 (5.1)=334 (6.1)=3307 (9.1)=933
Intensities of technol. that do not use land ($z(j)$)			(4)=1378		(4)=1898
Chemical fertilizer ($FS(e)$) (tons)	(1)=45 (2)=221	(1)=60 (2)=1298	(1)=375 (2)=19	(1)=1094 (2)=264	(1)=190 (2)=373
Animal slurry Application (10^3 tons) ($ma(k,l), ms(k,l)$) $l=1$ arable land $l=2$ grassland	ms (1.1)=47.9 (1.2)=34.2	ms (1.1)75.7 (1.2)=54.1	ma (4.2)=0.9 ms: (1.1)=1.9 (4.1)=109.4	ms (1.1)=23.4	ma (4.1)=58.6 (4.2)=17.3 ms (1.1)=37.3 (4.1)=75.9
Water Supply ($10^6 m^3$)	is=4.3 us=0.86 qs=0.38	is=2.82 us=0.56	is=0.95 us=0.19 qs=0.18	is=7.19 us=1.44 ig=5.27	is=0.94 us=0.19
Groundwater levels (cm):hs, hw Fall of GWL (cm) $\Delta hs, \Delta hw$	hs=-90.3 hw=-165.3 Δhs=0.6 Δhw=-2.1	hs=-110.6 hw=-185.4 Δhs=-0.3 Δhw=-5	hs=-87.9 hw=-182.4 Δhs=-0.2 Δhw=-5	hs=-90.6 hw=-173 Δhs=0.0 Δhw=-21.4	hs=-90.4 hw=-148.8 Δhs=0.0 Δhw=-5