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This research was conducted in conjunction with a summer research seminar on heterogeneity dynamics, under the direction of James W. Vaupel and Anatoli I. Yashin, in the Population Program at IIASA led by Nathan Keyfitz.

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
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Foreword

A group of eleven Ph.D. candidates from seven countries--Robin Cowan, Andrew Foster, Nedka Gateva, William Hodges, Arno Kitts, Eva Lelievre, Fernando Rajulton, Lucky Tedrow, Marc Tremblay, John Wilmoth, and Zeng Yi--worked together at IIASA from June 17 through September 6, 1985, in a seminar on population heterogeneity. The seminar was led by the two of us with the help of Nathan Keyfitz, leader of the Population Program, and Bradley Gambill, Dianne Goodwin, and Alan Bernstein, researchers in the Population Program, as well as the occasional participation of guest scholars at IIASA, including Michael Stoto, Sergei Scherbov, Joel Cohen, Frans Willekens, Vladimir Crechuha, and Geert Ridder. Susanne Stock, our secretary, and Margaret Traber managed the seminar superbly.

Each of the eleven students in the seminar succeeded in writing a report on the research they had done. With only one exception, the students evaluated the seminar as "very productive"; the exception thought it was "productive". The two of us agree: the quality of the research produced exceeded our expectations and made the summer a thoroughly enjoyable experience. We were particularly pleased by the interest and sparkle displayed in our daily, hour-long colloquium, and by the spirit of cooperation all the participants, both students and more senior researchers, displayed in generously sharing ideas and otherwise helping each other.

Robin Cowan succeeded in producing two papers over the course of the summer, the present paper on how cohort size affects total lifetime consumption being one of them. In it, Cowan develops a model, cleverly contrived to shed light on a darkly complex issue.

Anatoli I. Yashin
James W. Vaupel

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Notes on the Effects of Cohort Size on Intergenerational Transfer

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Introduction

As a member of the baby boom, I would like to be able to blame my poverty on the bad luck of having been born in a large cohort, rather than on some inability I suffer in the money-making department. This excuse is only viable, though, if there is a systematic relationship between cohort size and per capita lifetime earnings. Arthur (1984) describes a world in which such a relationship does exist. As my large cohort tries to move up through the job hierarchy pyramid, there will be a job market squeeze, and unless people before us retire early, many members of my cohort will find that their advancement is slower than otherwise might be expected. My concern in this paper is not with the job market, however. It is rather with the relationship between cohort size and intergenerational transfers. Arthur and McNicoll (1978) showed that in a regime of stable populations, under typical mortality and fertility schedules, the higher the growth rate, the lower the individual's welfare. Analysis of stable populations cannot capture the baby boom phenomenon though as a baby boom is very much an example of a non-stable population. If there is a systematic relationship between cohort size and per capita income, one effect of phenomena like baby booms is to create inequalities in living standards between generations. Keyfitz (1985) tries to devise ways in which to eliminate inequalities arising from intergenerational transfers. He discusses three transfer mechanisms which equilibrate the quasi-interest rates which cohorts see as the return on the money the pay into social security. His method involves projections of both birth and death rates. My interest is more general than this though. I wish to ask the question "What would be the effect on the net transfers of a cohort if it were of a size different than it actually is?" As one might expect, the answer depends on the nature of the transfer mechanism, and I will discuss a case in which the answer is, in fact, "Nothing." In this case, there is equity between the generations, but this equity is not the focus of the exercise.

In this paper I will describe a framework designed to answer certain questions about how cohort size affects total lifetime consumption. I will use an overlapping generations model in which intergenerational transfers are the mechanism through which changes in cohort size affect consumption levels. The model will seem rather contrived with respect to many economies, in that the state will intermediate all transfers. This assumption is not crucial to the results, as I will argue later, but it does facilitate discussion.

A Welfare State

Each individual born in this mythical economy has three distinct stages of life. From birth to age $x-1$ he leads the life of a child, producing nothing and being supported by his parents who receive baby-bonus cheques from the government which exactly offset all child-rearing expenses. For purposes of exposition, I will speak as if the child receives these cheques directly. From age x to age $y-1$ he works and receives nothing from the state except tax bills. Over this time his production can be described by an age-earnings profile which will be called $f(a)$ where a is his age. During this period of life he also pays taxes which go solely to support those who are not in the working stage of life. At age y he retires and produces nothing. The government pays him an old age pension on which he lives until his dying day. The government has only one role, namely to collect transfer payments from those working and disperse transfer receipts to those not working. This it does costlessly, and operates only under the constraint that its budget must be balanced at all times.

I now introduce a cohort in whose welfare we will be interested. It is born at time zero, and is of size L. The cohort born just before it was born at time -1 and is of size L_{-1} the cohort born just after it will be born at time 1 and will be of size L_1. No one saves in this economy, so in each period (i) of its life a cohort consumes either its transfer receipts, R_i or its after-tax production, that is, production less transfer payments, $Q_i - P_i$. We should note that the production of our cohort in period i is $Q_i = L \cdot f(i)$.

The total lifetime consumption of our cohort can be expressed as a sum:

$$C = \sum_{i=0}^{\omega} [Q_i + R_i - P_i]$$

where ω is the age at death (which equals time of death for our cohort). Since the time when the cohort is receiving is distinct from the time when it is producing and paying, we can divide its consumption into distinct parts:

$$C = \sum_{i=0}^{x-1} R_i + \sum_{i=x}^{y-1} (Q_i - P_i) + \sum_{i=y}^{\omega} R_i \quad .$$

For simplicity, I will refer to the time when it is receiving as A, and the time when it is producing as A^c. So

$$C = \sum_{i \in A} R_i + \sum_{i \in A^c} (Q_i - P_i) \quad .$$

The government plays the central role in determining the magnitude of total transfers each period. Every period, it considers the total number of people who are receiving transfers that period, L_i^T, and the aggregate output of the period, TQ_i, to determine the total quantity that should be transferred, TT_i. Clearly, the number of recipients must be considered, since if it changes, the value of transfers necessary to support them at a given level also changes, and in the same direction. There is nothing to say that the level of individual support must be fixed, but I will assume it bounded; below by subsistence, and above by some level of luxury. This is enough to make L_i^T an argument. I will assume as well that it is a "nice" argument. That is to say that $\frac{\partial TT_i}{\partial L_i^T}$ is finite and continuous everywhere. Total output is considered by the government since it is the tax base. The larger the tax base, the greater the quantity (though not necessarily the proportion) that can be extracted. Each period then, total transfers can be written as $TT_i = TT(TQ_i, L_i^T)$.

Having determined total transfers, the government must then decide the share of total receipts (payments) that each receiving (paying) cohort is to be assigned. In making this assignment, it does not differentiate between people by age, and ignores other potential differentiating characteristics. As a result, receipts are distributed to a cohort according to the ratio of its size to the total number of recipients. So when our cohort is receiving (during time A) in each period i it receives $\frac{L}{L_i^T} \times TT_i$. On the other hand, individuals paying can be distinguished by the quantities they produce. The government uses this information, and when our cohort is paying (on A^c) it pays $P_i = \frac{Q_i}{TQ_i} \times TT_i$ each period, where TQ_i is the aggregate production of the economy at time i. Now the total consumption of our cohort can be written as

$$C = \sum_{i \in A} \frac{L}{L^{T_i}} \times TT_i + \sum_{i \in A^c} Q_i - \frac{Q_i}{TQ_i} \times TT_i \quad ,$$

or more explicitly,

$$C = \sum_{i \in A} \frac{L}{L_i^T(L)} \times TT(L_i^T(L),\, TQ_i) + \sum_{i \in A^c} L \times f(i) - \frac{L\, f(i)}{TQ_i(L)} \times TT(L_i^T, TQ_i(L)) \quad .$$

Dividing through by the size of our cohort L, we get lifetime consumption per capita,

$$\frac{C}{L} = \sum_{i \in A} \frac{1}{L_i^T(L)} \times TT(L_i^T(L), TQ_i) + \sum_{i \in A^c} f(i) - \frac{f(i)}{TQ_i(L)} \times TT(L_i^T, TQ_i(L)) \quad .$$

To find the effect of cohort size on total lifetime consumption, simply differentiate with respect to L.

$$\frac{\partial(\frac{C}{L})}{\partial L} = \sum_{i \in A} \frac{1}{L_i^{T2}} \times TT(L_i^T, TQ_i) + \frac{1}{L_i^T}\frac{\partial TT_i}{\partial L_i^T} +$$

$$\sum_{i \in A^c} f(i)[f(i) \times \frac{TT(L_i^T, TQ_i)}{TQ_i^2} - \frac{f(i)}{TQ_i}\frac{\partial TT_i}{\partial TQ_i}] \quad .$$

After simplifying (and noting that $TT_i = TT(L_i^T, TQ_i)$, we get

$$\frac{\partial(\frac{C}{L})}{\partial L} = \sum_{i \in A} \frac{1}{L_i^T} \times [\frac{\partial TT_i}{\partial L_i^T} - \frac{TT_i}{L_i^T}] + \sum_{i \in A^c} \frac{f(i)}{TQ_i}[f(i) \times \frac{TT_i}{TQ_i} - f(i) \times \frac{\partial TT_i}{\partial TQ_i}] \quad . (1)$$

As expected, cohort size affects consumption in both stages of life, and in both cases there are positive and negative effects. The first term represents the effect of cohort size on transfer receipts. When our cohort is receiving, the positive effect may be seen as a marginal effect: Adding a person raises the total quantity of transfers. The negative effect is an average effect: That there is one more person means that whatever transfer receipts are available are spread among more people. Clearly, the net effect on transfer receipts depends on how the quantity of total transfers responds to changes in the number of recipients. If $\frac{\partial TT}{\partial L_i^T}$ is a decreasing function of L_i^T, as seems likely to be the case, then the effect on per capita transfer receipts, of increasing the cohort, size is negative. The second sum in $\frac{\partial(\frac{C}{L})}{\partial L}$ represents the effect of cohort size on transfer payments. Here, effects of population are transmitted through their effects on production. The positive effect is the average effect: Individual payments are smaller as there are more people contributing to the transfer fund. The negative effect can be seen as marginal: A larger cohort means that its production, and so total production, will be larger. This will have a positive effect on the size of total transfers. Here, whether the net effect is positive or negative depends on how total transfers responds to total output.

The net result of changing the size of a cohort is, in general, indeterminate. There are some special cases, however, where the transfer mechanism is of a type for which determinate results can be obtained.

In the first case, suppose that in calculating the quantity of total transfers, the government maintains a fixed tax rate and pays no attention to the total number of recipients. Here transfers are always a fixed proportion, α, of total output, and so our cohort, in each period when it is paying, pays $100 \cdot \alpha$ percent of its production into the transfer fund. This is the only qualification to the general scheme described above. Here, the receipts in period i (on A) for our cohort are $R_i = \frac{L}{L_i^T}(\alpha \cdot TQ_i)$. On A^c production is unchanged, and the payments our cohort makes are

$$P_i = \frac{Q_i}{TQ_i}(\alpha TQ_i) = \alpha Q_i \quad .$$

So for our cohort, total lifetime consumption per capita becomes

$$\frac{C}{L} = \sum_{i \in A} \alpha TQ_i \frac{1}{L_i^T} + \sum_{i \in A^c} f(i)(1-\alpha) \quad .$$

And, differentiating with respect to the size of the cohort

$$\frac{\partial(\frac{C}{L})}{\partial L} = \sum_{i \in A} -\frac{1}{L_i^{T2}} \times \alpha TQ_i \quad .$$

which is unambiguously negative. This result corresponds with our intuitions: If each individual contributes a fixed proportion of his income to the transfer fund, then a large cohort, when receiving, cannot affect the size of the fund, so the same quantity is divided among more people. On the other hand, after tax production is constant, so per capita consumption on A^c is unchanged. Notice also that under this scheme those receiving when the larger cohort is receiving are hurt by its size, but those receiving when the large cohort is producing gain by its size. (For the cohorts very near in birthdate to our cohort the net effect will also be negative though less so than for our cohort.)

The second case with an unambiguous result is the situation in which the government guarantees a certain standard of living, call it β, for those receiving. In this case the size of our cohort is positively related to its welfare. Here, over time A, when it is receiving, our cohort receives $R_i = \beta L$. On A^c production is unchanged, but payments are $P_i = \frac{Q_i}{TQ_i} \times \beta L_i^T$. Total lifetime consumption per capita can be written as

$$\frac{C}{L} = \sum_{i \in A} \beta + \sum_{i \in A^c} f(i) - \frac{f(i)}{TQ_i} \times \beta L_i^T \quad .$$

Again, differentiating with respect to L yields

$$\frac{\partial(\frac{C}{L})}{\partial L} = \sum_{i \in A^c} TT_i \times \frac{f(i)^2}{TQ_i^2} \quad ,$$

which is positive. A larger cohort means that total production rises. The number of recipients does not change, however, so each producer is required to pay less in order to provide support at the prescribed level to those receiving.

The final unambiguous case which I will describe is one in which cohort size has no effect on lifetime consumption. Again, this case can be described simply by specifying the total transfers function. If we specify that total transfers are calculated by $TT_i = K \times L_i^T \times TQ_i$, where K is a constant, then

$$\frac{\partial TT_i}{\partial L_i^T} = K \times TQ_i = \frac{TT_i}{L_i^T} \quad .$$

Likewise,

$$\frac{\partial TT_i}{\partial TQ_i} = K \times L_i^T = \frac{TT_i}{TQ_i} \quad .$$

From equation (1) (page 4), we see that under this condition, cohort size has no effect on lifetime consumption. The major drawback of such a scheme, were it to be implemented in our world, is that the pattern of consumption over an individual's lifetime is determined (apart from his age-earnings profile) by the sizes of the cohorts which are alive when he is. This means that, at least in principle, it is possible for someone to spend part of his life in the lap of luxury, and part of it below subsistence. (That's all right I suppose, as long as they come in that order.)

Discussions and Extensions

I stated earlier that the large role which the state plays in this economy is not crucial to the results of the model. That the state play some role is important, but not that it play the all-encompassing role described in the model. I claim that the transfer mechanism described here is not grotesquely different in effect from what we might observe in an economy with less government intervention. This is clearly so for transfers to the young, which are generally parent to child transfers. If a couple has more children, they are surely going to spend more in total on their children. If parents find themselves with more income, again it seems very likely that they will spend more on their children. One word of caution: The model describes a world in which the production of the economy as a whole affects the size of transfers. It is possible that the economy could get richer without parents of receiving children getting richer. I would argue as follows: Most economies provide many services out of general tax revenues: public education; defense; transportation system subsidies and so on. If these services are used by the non-producing part of the population, then they are a transfer. The level of these services increases as the production of the economy increases. With regard to transfers to the retired, for any economy with an old age security program, the model needs no additional explanation. But even so, it seems very likely that any intra-family transfers to the retired will be a function both of the number of recipients (both parents or only one) and the income of those paying.

There are two obvious ways in which this model can be extended. The first has to do with the nature of production in this economy. Until now we have assumed that production is linear in labour. That is, changing the size of a cohort does not change its average product, $f(a)$. However Finis Welch (1979) has shown that the age-earnings profile of the baby-boom generation is lower than one would have expected had that generation not been so large, and this result can be easily incorporated into the model. The second extension is to the results rather than to the model. The model can be used to answer the question "What would be the effect on lifetime consumption of our cohort if some other cohort were of a size other than it actually is?"

I will first address the case of non-linear production. The model changes only in that cohort size is now included as an argument in the age-earnings profile. In order to make the problem tractable, I will assume that the size of a cohort has no effect on the age-earnings profiles of other cohorts. Now $f(a)$ becomes $f(a,L)$. It seems reasonable to expect that $\frac{\partial f}{\partial L}$ is negative in sign. If we accept diminishing marginal product, then as cohort size increases, average product, which is one way that $f(.)$ can be viewed, must fall. Per capita lifetime consumption does not change from the simple model, but the first derivative has added terms:

$$\frac{\partial \frac{C}{L}}{\partial L} = \sum_{i \in A} \frac{1}{L_i^T}\left(\frac{\partial TT_i}{\partial L_i^T} - \frac{TT_i}{L_i^T}\right) + \tag{2}$$

$$\sum_{i \in A^c} \frac{\partial f}{\partial L}\left(1 - \frac{TT_i}{TQ_i}\right) - \frac{f(i,L)}{TQ_i}\left[(f(i,L) + L\frac{\partial f}{\partial L}\right) \times \left(\frac{\partial TT_i}{\partial TQ_i} - \frac{TT_i}{TQ_i}\right)] \ .$$

As in the simple model, the sign is in general indeterminate, depending here not only on the transfer mechanism, but also on the responsiveness of average product to changes in cohort size. Again, however, there are special cases in which the results are determinate.

In the first special case discussed in the simple model, total transfers were not affected by the number of recipients, but were a linear function of aggregate production. In this case,

$$\frac{\partial TT_i}{\partial L_i^T} = 0 \quad , \quad TT_i = \alpha TQ_i \ .$$

Under this transfer mechanism, the derivative of lifetime consumption with respect to cohort size has an added term in the extended model:

$$\frac{\partial \frac{C}{L}}{\partial L} = \sum_{i \in A} - \frac{\alpha TQ_i}{L_i^{T^2}} + \sum_{i \in A^c} \frac{\partial f}{\partial L}(1 - \alpha) \ .$$

This result, as in the simple model, is unambiguously negative. If our cohort is large, the fixed quantity of available transfers is divided among more people, so the cohort is hurt. As well, during the time when our cohort is producing, their average product, of which they each consume the proportion $(1 - \alpha)$, is driven down. Under this transfer mechanism then, the negative effect of being born in a large cohort is more severe than it is in the simple model.

The second special case specified that total transfers were not affected by the level of aggregate production, but were a linear function of the number of recipients:

$$\frac{\partial TT_i}{\partial TQ_i} = 0 \quad , \quad TT_i = \beta L_i^\tau \quad .$$

Under this transfer scheme, cohort size has no effect on per capita transfer receipts, either in the simple model or in the extended model. Cohort size does affect both per capita production and transfer payments in the extended model however. Here the first derivative is

$$\frac{\partial \frac{C}{L}}{\partial L} = \sum_{i \in A^c} \frac{\partial f}{\partial L}\left(1 - \frac{TT_i}{TQ_i}\right) + \frac{f(i,L)}{TQ_i}\frac{TT_i}{TQ_i} \times \left(f(i,L) + L\frac{\partial f}{\partial L}\right) \quad .$$

Noting that $(f(i,L) + L\frac{\partial f}{\partial L})$ is equal to the derivative of aggregate output with respect to cohort size, $\frac{\partial TT_i}{\partial L}$, we see that the sign of $\frac{\partial \frac{C}{L}}{\partial L}$ is ambiguous. (It is safe to assume, I think, that aggregate output rises if the size of a producing cohort increases.) As in the simple model, we have a positive effect--the second term describes the gain from having more producers among whom to divide a fixed quantity of transfer payments. This effect is smaller than in the simple model, though, since average product is driven down by the extra cohort members. The first term represents the per capita loss of after tax production due to the growth of the cohort. Under these conditions, the higher is the ratio of total transfers to aggregate production, the larger the positive effect relative to the negative effect.

In the original model there was a very simple transfer mechanism which would guarantee that no cohort gained or lost solely due to its size, *viz.* setting total transfers to $TT_i = K \times L_i^\tau \times TQ_i$. Under this mechanism, at each stage of a cohort's life, whether receiving or paying, the net gain from changes in cohort size is zero. Indeed, this is true at every period in its life. In the world where average product is a function of cohort size, this can still be arranged without difficulty for the periods where the cohort is receiving ($i \in A$). Simply devise a transfer mechanism that is linear in L_i^T. Unfortunately it is more difficult for the periods when the cohort is producing and paying ($i \in A^c$). The same approach would involve setting the summand of the second term of equation (2) equal to zero:

$$0 = \frac{\partial f}{\partial L}(1 - \frac{TT_i}{TQ_i}) - \frac{f(i,L)}{TQ_i}(f(i,L) + L\frac{\partial f}{\partial L}) \times \frac{\partial TT_i}{\partial TQ_i} - \frac{TT_i}{TQ_i}) \quad .$$

If we assume that $\frac{\partial TT_i}{\partial TQ_i}$ falls as TQ rises, and this seems perfectly reasonable, then in order for this equation to have a solution, the nature of production must be such that $f(i,L) + L\frac{\partial f}{\partial L} > 0$. But we have observed that $f(i,L) + L\frac{\partial f}{\partial L}$ is equal to the derivative of aggregate output with respect to cohort size. If we can assume that the marginal product of labour never goes to zero, then this quantity is always positive, and the equation is, in principle, solvable, and so we can find a transfer scheme that is neutral to changes in cohort size.

I turn now to the question of other cohort effects. The model can be used to examine the effect on our cohort of changes in size of other cohorts. For this exercise I will retreat to the simple model. Incorporating the effect of cohort size on average product is not a problem, but it makes the presentation considerably less transparent. The time of birth of the other cohort will effectively divide the life of our cohort into several stages. These stages can be characterized by the activities of the two cohorts. In each stage the effect of the size of the other cohort on our cohort will take on a particular nature. I give detailed examples of two different times of birth for the other cohort. There are, of course, many types of birth times, but for each type, the general procedure of determining $\frac{\partial \frac{C}{L}}{\partial L}$ is the same.

Suppose we are interested in the effect on the transfers of our cohort of a change in size of the cohort born s periods after ours. These people are s periods younger, and their cohort is of size L_s. The life of our cohort can be divided into six stages:

1) when our cohort is alive but cohort C_s is not--from period zero to period $s-1$;

2) when both cohorts are alive and both are receiving--periods s to $x-1$;

3) when our cohort is producing and C_s is receiving--periods x to $x+s-1$;

4) when both cohorts are producing--periods $x+s$ to $y-1$;

5) when our cohort is receiving and C_s is producing--periods y to $y+s-1$;

6) when both cohorts are again receiving--periods $y+s$ to ω.

Consequently, per capita lifetime consumption can be divided into these six stages, as can the effects of size of cohort C_s. If we take this derivative we get:

$$\frac{\partial \frac{C}{L}}{\partial L_s} = \sum_{i=0}^{s-1} 0 + \sum_{i=s}^{x-1} \frac{1}{L_i^T}\left(\frac{\partial TT_i}{\partial L_i^T} - \frac{TT_i}{L_i^T}\right) +$$

$$+ \sum_{i=x}^{x+s-1} -f(i)\left(\frac{1}{TQ_i}\frac{\partial TT_i}{\partial L_i^T}\right) +$$

$$+ \sum_{i=x+s}^{y-1} f(i)\left[\frac{f(i-s)}{TQ_i}\frac{TT_i}{TQ_i} - \frac{f(i-s)}{TQ_i}\frac{\partial TT_i}{\partial TQ_i}\right] +$$

$$+ \sum_{i=y}^{y+s-1} \frac{f(i-s)}{L_i^T}\frac{\partial TT_i}{\partial TQ_i} +$$

$$+ \sum_{i=y+s}^{\omega} -\frac{1}{L_i^T}\left(\frac{\partial TT_i}{\partial L_i^T} - \frac{TT_i}{L_i^T}\right) .$$

These stages only apply when $s < x$. Clearly, if $s = 0$, stages 3) and 5) disappear, and we have the original results. Also if s is small, the cross effects will be similar to the own-cohort effect, particularly if the age-earnings profile is relatively flat.

As opposed to examining these general results in detail, it may be more instructive to look at the first two special cases. Recall that in the first case total transfers were a linear function of aggregate output $TT_i = \alpha TQ_i$. Under this transfer mechanism $\frac{\partial TT_i}{\partial L_i^T} = 0$, and $\frac{\partial TT_i}{\partial TQ_i} = \frac{TT_i}{TQ_i}$, and so many terms in $\frac{\partial \frac{C}{L}}{\partial L_s}$ disappear. In stages 1, 3 and 4 of the life of our cohort, the size of cohort C_s has no effect on consumption. So we can write

$$\frac{\partial \frac{C}{L}}{\partial L_s} = \sum_{i=s}^{x-1} -\frac{TT_i}{L_i^{T^2}} + \sum_{i=y}^{y+s-1} \frac{\alpha f(i-s)}{L_i^T} + \sum_{i=y+s}^{\omega} -\frac{TT_i}{L_i^{T^2}} .$$

The first and third sums are over periods when both cohorts are receiving. The middle sum is over periods when our cohort is receiving but the other is producing. Here, an added member of C_s increases total transfers by $\alpha f(i - s)$ each period, of which each member of our cohort receives $\frac{1}{L_i^T}$. This is a positive effect; the others are negative effects. Clearly, exactly when C_s is born (i.e. the size of s) is important in determining the relative sizes of the positive and negative effects. As s approaches x, the first sum disappears, the second sum gets large, and the third sum gets small, very possibly disappearing (if $x > \omega - y$).

Let us now look at the other transfer mechanism, where $TT_i = \beta L_i^T$. In this case, the size of cohort C_s has no effect on the consumption of our cohort in stages one, two, five and six. Per capita receipts are guaranteed, so effects on our cohort can only occur when it is producing. We can write

$$\frac{\partial \frac{C}{L}}{\partial L_s} = \sum_{i=s}^{x-1} -\beta \times \frac{f(i)}{TQ_i} + \sum_{i=x+s}^{y-1} f(i) \times \frac{f(i-s)}{TQ_i} \times \frac{TT_i}{TQ_i} \quad .$$

Here, when our cohort is producing and the other is receiving, an extra member will raise total transfers by β, of which each member of our cohort must pay $\frac{f(i)}{TQ_i}$. When both cohorts are producing, an extra member of the other cohort will raise aggregate output and so lower the proportion of the total which our cohort produces, thus reducing its payments. Again, the value of s is crucial in determining the relative sizes of the positive and negative effects.

All of the above analysis has assumed that the other cohort was born before ours started producing, i.e., $s < x$. This need not be the case. Suppose, for example, that we are interested in the relation between the number of children our cohort has and its lifetime consumption. In this case the other cohort will almost certainly be born after ours starts working. Suppose for simplicity that in each family all children are born in one, possibly multiple, birth.

Under these conditions the stages of life of our cohort have different characteristics. I will number them 1' to 5'.

1') Our cohort is receiving and the other is not yet alive--periods 0 to $x-1$.

2') Our cohort is producing and the other is not yet alive--periods x to $s-1$.

3') Our cohort is producing and the other is receiving--periods s to $x+s-1$.

4') Both cohorts are producing--periods $x+s$ to $y-1$.

5') Our cohort is receiving, and the other is producing--periods y to ω.

In constructing these stages of life, I am thinking of the following 'typical' life plan. An individual begins to work at about age 20. Childbearing does not begin until after the individual begins work and is completed by age 40. The individual retires from the labour force at age 65, after which he lives another 15-20 years. This, I think, is not an unreasonable approximation for the life plans of the average North American, say. Under these circumstances, parents and children will spend some of the same years in the labour force together, but will not both be retired at the same time. Consequently, only during three stages of its life does the number of children affect our cohort:

$$\frac{\partial \frac{C}{L}}{\partial L_s} = \sum_{i=s}^{x+s-1} \frac{f(i)}{TQ_i} \frac{\partial TT_i}{\partial L_i^T} +$$

$$+ \sum_{i=x+s}^{y-1} f(i)\left[\frac{f(i-s)}{TQ_i} \frac{TT_i}{TQ_i} - \frac{f(i-s)}{TQ_i} \frac{\partial TT_i}{\partial TQ_i}\right] +$$

$$+ \sum_{i=y}^{\omega} \frac{f(i-s)}{L_i^T} \frac{\partial TT_i}{\partial TQ_i} .$$

With qualifications to cover particular transfer mechanisms, having more children will hurt our cohort, as there are more people for it to support. On the other hand, when our cohort is retired, there are more people to support it, which is clearly a benefit. When both parents and children are working, the net effect will be positive if total transfers increase with total product, but at a decreasing rate. Once again, in general the result is ambiguous, but specific transfer mechanisms will generate unambiguous results.

Conclusions

The main conclusion is that the size of one's cohort can affect the level of one's total lifetime consumption through intergenerational transfers. Whether this effect is positively or negatively related to cohort size depends on the specific transfer mechanism employed however. And, indeed, there is at least one simple mechanism in which positive and negative effects cancel each other, and so consumption is unrelated to cohort size.

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