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Foreword

This paper presents a prescriptive model for a decision maker's risk attitude toward financial outcomes that have important non-monetary effects, for example, effects on how the decision maker is judged by himself and by others. The model represents the risk attitude of a decision maker who is risk averse in the absence of such psychological effects, but who is risk prone in their presence for actions leading to net losses or the status quo. The model is examined for its adherence to normative principles. In particular, it is argued that the principle of dominance should be specified without any assumptions on preferences between conjunctions of lotteries; such assumptions are shown to imply the apparently stronger principle of risk neutrality.

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Introduction

Behavioral studies of people's risk attitudes have found that in a variety of contexts the majority of people are risk averse in their preferences among actions leading to gains or the status quo but are risk prone in their preferences among actions leading to losses or the status quo. Such preferences will be referred to in this paper as an *averse-prone risk attitude.*

During the early development of expected utility theory, Friedman and Savage (1948) and Markowitz (1952) discussed averse-prone risk attitudes (and even three- and four-piece risk attitudes). Relevant empirical work includes five studies, Barnes and Reinmuth (1976), Grayson (1960), Green (1963), Halter and Dean (1971), and Swalm (1966), that were examined by Fishburn and Kochenberger (1979), and were reexamined together with other empirical studies by Hershey, Kunreuther, and Schoemaker (1982). Other empirical work and analyses are in Dickson (1981), Fuchs (1976), Hershey and Schoemaker (1980a), Langhhunn, Payne, and Crum (1980), Schoemaker and Kunreuther (1979), Wehrung, Bassler, MacCrimmon, and Stanburg (1978), Wehrung, MacCrimmon, and Brothers (1984), and Williams (1966).

This paper is a modeling and a subsequent analysis of averse-prone risk attitudes. In the current milieu of research on preference models, it is important to emphasize that the models presented here are prescriptive rather than normative or descriptive. They are intended to help a decision maker whose actual preferences, i.e., the preferences lying behind his cognitive limitations, are such that he has an averse-prone risk attitude. These models may or may not conform to various assumptions of rationality (a matter that is examined in the second part of this paper). Moreover, they may or may not provide an accurate or predictive modeling of the behavior of most people (a matter that is commented on but it is not tested in this paper). Bell, Raiffa, and Tversky (1984) and Schoemaker (1982) contain recent, general discussions of the distinctions between the normative, descriptive, and prescriptive modeling of preferences.

Section 1 discusses the types of psychological responses to financial gains and losses that may induce an averse-prone risk attitude, and presents a rather general model of such risk attitudes. Section 2 then discusses a specialization of this model in which the psychological effects of gains and losses are priced-out as a "reward" that is independent of the magnitude of the gain and a "penalty" that is independent of the magnitude of the loss. In this model, anyone who is risk averse when the psychological effects are absent will have an averse-prone risk attitude over some range of monetary changes when the psychological effects are present.

Sections 3 and 4 examine which normative principles are or are not violated by an averse-prone risk attitude. The salient conclusion is not a list of yes/no answers but that certain conditions implicit in the common normative principles need to be made explicit and then examined in isolation. In particular, it is shown that certain conditions on preferences between conjunctions of lotteries that are often implicit in the dominance principle imply the apparently stronger principle of risk neutrality. It is argued that these conditions on conjunctions (and a condition implicit in the framing principle) should not be included as normative principles.

Section 5 then returns to a prescriptive viewpoint and discusses the potential uses and abuses of an averse-prone risk model for a decision analysis application in which risk aversion for gains but risk proneness for losses is an important preference issue.

1. Psychological-Effects Models

Several researchers have suggested that a decision maker's risk attitude may depend on various psychological effects of financial outcomes that might be included as part of the description of the decision maker's possible consequences. For example, Peter Fishburn, Ralph Keeney, and Richard Meyer (in the discussion following a paper presented by Tversky, 1977) suggest the use of an additional attribute to explain risk prone preferences for potential losses in financial choices. A similar idea is suggested in Keeney (1984) by his argument that the ethics of a deontological moralist need not violate the expected utility conditions provided that the social consequences of policy decisions are adequately defined (p. 122). Moreover, Raiffa (1984) discusses psychological effects by imagining that the decision maker has an external or internal kibitzer whose remarks could be used to provide a more sophisticated description of the consequences to the decision maker.

In this section, a preference model is developed that is consonant with the above ideas. Suppose that financial outcomes to the decision maker are measured by a variable x such that $x > 0$ represents net gains, $x < 0$ represents net losses, and $x = 0$ represents the status quo. An individual who is making a decision that appears to depend only on the financial amounts x may be influenced by the anticipation of his psychological responses to the possible consequences of a chosen action. To describe these effects, it will be helpful to distinguish between a decision maker who is acting on his own behalf and a decision maker who acting as an agent for an organization.

A person who is acting on his own behalf, e.g., a private entrepreneur or an individual investor, may feel embarrassment or a loss of self-esteem if a financial loss should occur. This person may also feel pride or increased self-esteem as a result of a financial gain. Such a type of psychological response is here distinguished from a change in financial position, that is, from the event that the person will have a certain lesser or greater amount to spend over his lifetime.

For a person who is acting as an agent for an organization, e.g., a business executive or a government administrator, there are external pressures in addition to the type of ego involvement described above. The person wishes to be favorably judged by those to whom he is accountable; for example, the person's primary concern may be to maintain or to enhance his reputation.

As a primary assumption in this paper, suppose that the psychological effects to be considered can be measured as components of the consequences to the decision maker. For modeling purposes, these types of psychological effects will be called *effects attributes*. Each effects attribute will be measured by a variable z_i, $i = 1,...,n$. Consequences will be described by both the variable x for the monetary attribute and the variables $z_1,...,z_n$ for the effects attributes; thus, each consequence will be denoted by a vector $(x, z_1,...,z_n)$.

By contrast, Bell (1982), (1983), (1985) has modeled psychological responses such as "regret" and "disappointment" that depend upon the decision maker's perception of all of the available lotteries or of the entire lottery that is chosen. As stated in Bell (1982) "regret is measured . . . as the difference in value between the assets actually received and the highest level of assets produced by *other* alternatives [italics Bell's]."

In a specific decision context, the amounts $z_1,\ldots,z_n$ may be highly correlated to the financial amount x and may even be functionally dependent on x. However, it will be assumed that the decision maker is familiar with a sufficiently wide variety of contexts such that his preferences can be considered on a product set of potential consequences $(x,z_1,\ldots,z_n)$ where the variables $x,z_1,\ldots,z_n$ are defined on specific intervals.

(A) Suppose that the decision maker's tradeoffs satisfy the willingness-to-pay conditions (see, e.g., Keeney and Raiffa, 1976, pp. 125-127, and Harvey, 1985). Then, preferences among consequences in the product set can be represented by a value function of the form

$$V(x,z_1,\ldots,z_n) = x + g_1(z_1) + \ldots + g_n(z_n) \quad . \tag{1}$$

Here, the amounts $g_i(z_i), i = 1,\ldots,n$, can be assessed as ***pricing-out amounts*** for the psychological responses; that is, for some specified response $z_i{}^*$ of the i-th effects attribute, $g_i(z_i)$ is that amount such that the decision maker would just be willing to pay $g_i(z_i)$ in order to obtain z_i rather than $z_i{}^*$. The value V of a consequence $(x,z_1,\ldots,z_n)$ can be interpreted as the financial amount such that the consequence $(x,z_1,\ldots,z_n)$ is indifferent to the consequence $(V,z_1{}^*,\ldots,z_n{}^*)$.

(B) Suppose that the decision maker's risk attitude satisfies the conditions of expected utility. Then, preferences among lotteries can be represented by a utility function of the form

$$U(x,z_1,\ldots,z_n) = u\,(x + g_1(z_1) + \ldots + g_n(z_n)) \quad . \tag{2}$$

Here, the function u can be interpreted as a conditional utility function on financial amounts x given that the psychological effects are the specified amounts $z_1{}^*,\ldots,z_n{}^*$.

Consider next the causal relations between receiving a net gain or a net loss x is a specific context and the resulting psychological effects. Suppose that in any one decision context the psychological effects described by $z_1,\ldots,z_n$ are functionally dependent on the financial outcome x. Then, conditional on a given context: $z_i = r_i(x)$, $i = 1,\ldots,n$, for some functions r_i. The functions r_i will be referred to as *response functions*. Note that a decision context does not affect tradeoffs between the attributes but rather restricts the domain of potential consequences.

For prescriptive purposes, it will be useful to compare the following two types of decision contexts.

(i) *The effects context:* Here the decision maker anticipates that his psychological responses are important to him and should be included in describing the consequences of his actions.

(C) Suppose that the response functions $z_i = r_i(x)$, $i = 1,\ldots,n$, as discussed above denote the decision maker's psychological responses in the effects context. Suppose that larger monetary amounts x lead to responses $z_i = r_i(x)$, $i = 1,\ldots,n$, that are at least as preferred. The responses to maintaining the status quo, i.e., to $x = 0$, will be denoted by $z_i^* = r_i(0)$, $i = 1,\ldots,n$. The amounts z_i^*, $i = 1,\ldots,n$, will be called *standard effects*.

(ii) The *no-effects context:* Here the psychological responses in the effects context are either absent or are omitted from consideration.

(D) Suppose that in the no-effects context the psychological responses to any financial change x are the standard effects z_i^*, $i = 1,\ldots,n$. Thus, the effects of any change x in the no-effects context are the same as the effects of no change, $x = 0$, in the effects context.

Definition 1. A preference model as described in conditions (A)-(D) will be called a *psychological-effects model*. Any utility function $u(x)$ defined on monetary amounts x in the no-effects context will be called a *no-effects utility function*; any utility function $w(x)$ defined on monetary amounts x in the effects context will be called an *effects utility function*.

For a net gain or loss x, consider the associated *effects amount* $e(x)$ defined by

$$e(x) = \Sigma_{i=1}^{n} g_i(r_i(x)) \tag{3}$$

where $r_i(x)$, $i = 1,\ldots,n$, are the psychological effects of x in the effects context. The monetary amount $e(x)$ can be interpreted as that amount such that the consequence $(0, z_1,\ldots,z_n)$ is indifferent to the consequence $(e(x), z_1^*,\ldots,z_n^*)$, that is, $e(x)$ is the total pricing-out amount of the psychological effects $z_i = r_i(x)$, $i = 1,\ldots,n$.

Theorem 1. For a psychological-effects model, if $u(x)$ is any utility function for the decision maker's risk attitude in the no-effects context and $w(x)$ is any utility function for the decision maker's risk attitude in the effects context, then

$$w(x) = au(x + e(x)) + b \tag{4}$$

for some normalization constants $a > 0$ and b.

The psychological-effects model discussed above is far too restrictive to describe the heuristic biases that might be responsible for at least part of an observed averse-prone risk attitude. The model is intended as a possible formulation of that part of an averse-prone risk attitude which the decision maker regards as due to his underlying preferences. For such a prescriptive purpose, it is useful to further specialize the model; the following section presents one means of doing so.

2. The Reward-Penalty Model

This section discusses a special type of psychological-effects model for comparing a decision maker's preferences in the effects context and in the no-effects context. The model is intended to be sufficiently specific to be tractable for decision analysis applications.

As a strong causal assumption, suppose that in the effects context the psychological responses are constant in the sense that

$$r_i(x) = \begin{cases} z_i^+ & \text{for } x > 0 \\ z_i^* & \text{for } x = 0 \\ z_i^- & \text{for } x < 0 \end{cases} \tag{5}$$

for some constant amounts z_i^+ and z_i^-, $i = 1,\ldots,n$. Thus, the effects z_i in the effects context depend only on whether the financial change is a gain or a loss, and not on the magnitude of the gain or loss. This type of dependence may by appropriate as a modeling simplification for a variety of decision situations. For example, a decision maker acting as an agent may believe that he is being judged in part in a superficial manner by whether he succeeds ($x > 0$), maintains the status quo ($x = 0$), or fails ($x < 0$).

It follows directly from equations (3) and (5) that

$$e(x) = \begin{cases} \rho & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -\pi & \text{for } x < 0 \end{cases} \tag{6}$$

for some constant financial amounts $\rho \geq 0$ and $\pi \geq 0$. The tradeoffs amounts ρ and π can be regarded as the extra "reward" and "penalty" in the effects context of a gain and a loss respectively.

Definition 2. A psychological effects model that satisfies the condition of constant psychological effects summarized in (5), (6) will be called a ***reward-penalty model***.

Figure 1 illustrates a reward-penalty model in which the utility function $w(x)$ for the effects context corresponds to a utility function $u(x)$ for the no-effects context that represents risk aversion. In the effects context, the penalty $\pi > 0$ leads to risk proneness among lotteries whose possible consequences are net losses and the status quo; the reward $\rho > 0$ leads to greater risk aversion among lotteries whose possible consequences are net gains and the status quo. These properties are illustrated by the dotted lines in Figure 1. Thus, $w(x)$ represents an averse-prone risk attitude over a range of amounts x including both net gains and net losses.

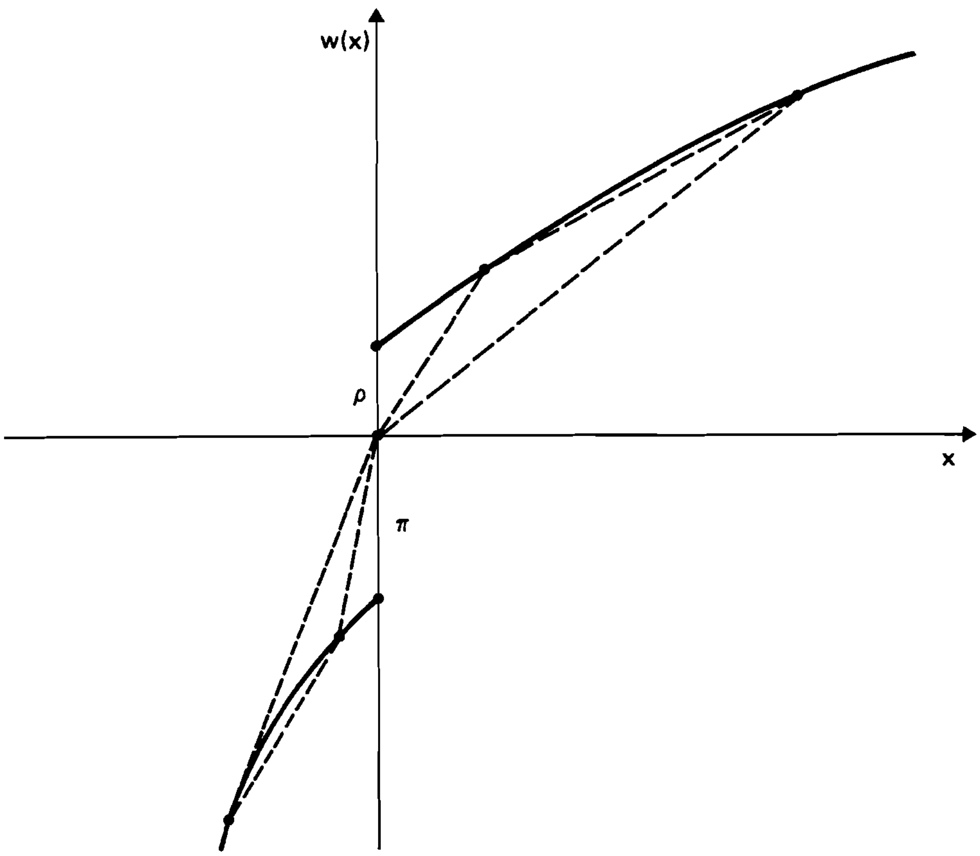

Figure 1. A Utility Function for the Effects Context

A numerical illustration of a reward-penalty model can be calculated by considering the following well-known choice problem presented in Tversky and Kahneman (1981). The percentages noted by the alternative actions are based on the responses of 150 students at Stanford University and the University of British Columbia.

Problem 1. Imagine that you face the following pair of concurrent decisions. First examine both decisions, then indicate the options you prefer.

Decision (i). Choose between:

A. a sure gain of \$240 [84 percent]

B. 25% chance to gain \$1000, and 75% chance to gain nothing [16 percent]

Decision (ii). Choose between:

C. a sure loss of \$750 [13 percent]

D. 75% chance to lose \$1000, and 25% chance to lose nothing [87 percent].

A majority [73 percent] of the respondents chose actions A and D. The expected monetary values of the four actions are: $E(A) = \$240 < E(B) = \$250$, and $E(C) = -\$750 = E(D)$. Thus, the students were risk averse in decision (i) but risk prone in decision (ii).

To model these preferences with a reward-penalty model, suppose that in the no-effects context a person has constant risk aversion (at least over the range of gains and losses considered) and assesses a certainty equivalent of \$245 for the lottery B. Then, in the no-effects context, the person is mildly risk averse; he prefers B to A and prefers C to D (the opposite of the preferences observed by Tversky and Kahneman).

Now, consider the person's preferences in the effects context. For a reward amount of $\rho = \$7$ or more, the person will become sufficiently risk averse among gains lotteries so that he prefers A to B. For a penalty amount of $\pi = \$21$ or more, the person will become sufficiently risk prone among losses lotteries so that he prefers D to C. Therefore, psychological responses to gains and losses that lead to only modest tradeoffs amounts ρ and π are sufficient to induce the choice behavior observed by Tversky and Kahneman.

The assumptions of a reward-penalty model imply in general a number of properties for the decision maker's preferences in the effects context. These properties are listed in the result below. Here, a lottery having a net gain or loss of x with probability p and a net gain or loss of x' with probability $q = 1 - p$ is denoted by $< p \ \$x, \ q \ \$x' >$.

Theorem 2. Suppose that a decision maker's preferences satisfy the conditions of a reward-penalty model.

Part I. If in the no-effects context the decision maker is risk averse, then in the effects context his preferences will have the following properties:

(a) If the penalty π is positive, then preferences are risk prone for moderate losses, that is, for any probability $0 < p < 1$, $< p \ \$x, \ q \ \$0 >$ is preferred to the amount $\$px$ for sufficiently small negative x.

(b) Preferences are risk averse for moderate or large gains, that is, for any probability $0 < p < 1$, the amount $\$px$ is preferred to $< p \ \$x, \ q \ \$0 >$ for any positive x.

(c) If π is greater than ρ, then preferences are risk averse for symmetric lotteries, that is, $\$0$ is preferred to $< \frac{1}{2} \$x, \ \frac{1}{2} \$(-x) >$ for any non-zero amount x.

Part II. If in the no-effects context the decision maker has decreasing risk aversion and has unbounded utility from below (i.e., for any amounts $x < x'$ there is an amount $x'' < x$ such that x is preferred to $< \frac{1}{2} \$x'', \ \frac{1}{2} \$x' >$), then in the effects context his preferences have the following additional properties:

(d) Preferences are risk averse for large losses, that is, for any probability $0 < p < 1$, $\$px$ is preferred to $< p \ \$x, \ q \ \$0 >$ for sufficiently large negative x.

(e) If $\pi > 0$, then preferences are more risk averse in the effects context then in the no-effects context for strict losses, that is, for any two strict losses $x, x' < 0$, the certainty equivalent of $< p \ \$x, \ q \ \$x' >$ is less in the effects context than in the no-effects context.

(f) If $\rho > 0$, then preferences are less risk averse in the effects context than in the no-effects context for strict gains, that is, for any two strict gains $x, x' > 0$, the certainty equivalent of $< p \ \$x, \ q \ \$x' >$ is more in the effects context than in the no-effects context.

The purpose of Theorem 2 is not to provide a means of testing the descriptive accuracy of a reward-penalty model. The intent is rather to make explicit to a decision maker some of the implications of adopting such a prescriptive model.

3. Conformity with Normative Principles

This section examines whether a risk attitude in a reward-penalty model, or a averse-prone risk attitude in general, confirms to certain conditions on preferences that have been regarded from a normative viewpoint as principals for rational decision making. The violation of these principles by people's choice behavior in non-transparent decision problems has been well documented, for example, in the work of D. Kahneman and A. Tversky (1979), (1981), and (1986).

3.1. *Transitivity, Dominance, and Independence..* For any reward-penalty model in which there is risk aversion in the no-effects context and a non-zero penalty π, there will be in the effects context an averse-prone risk attitude as described in parts (a), (b) of Theorem 2. Such preferences are represented by a utility function $w(x)$ as in (4), and hence are consistent with the normative principles of expected utility. It follows that, in particular, these averse-prone risk attitudes satisfy the principles of transitivity, stochastic dominance, and independence.

3.2. *Framing Consistency*. Consider any averse-prone risk attitude with respect to a variable x. In this subsection, the risk attitude may or may not confirm to the principles of expected utility. Let $l = < p_i, x_i >$ denote a lottery having possible consequences x_i with probabilities p_i where $i = 1,\ldots,m$.

The variable x is intended to measure changes in monetary position, i.e., either changes in the decision maker's personal finances or changes in the finances of an organization for which the decision maker is acting as an agent. The consequences to the decision maker can also be described as final asset positions measured by a variable y. Assume that: (1) a current asset position c can be defined (but not necessarily evaluated), (2) the amounts x are net monetary gains and losses, e.g., net present values or net after-tax profits, and (3) the final asset position y implied by c and x is specified by the formula $y = c + x$.

For a situation in which x measures rates of return or net pre-tax profits, the formula relating y to x for a fixed current asset position c will differ from $y = c + x$, but there will still be a one-to-one relationship $y = f(x \mid c)$ between financial changes x and final asset positions y. The results in this subsection easily generalize to such situations.

The preference relation concerning financial changes x conditional on a current asset position c will be denoted by $\succsim_{x|c}$. An associated preference relation $\succsim_y$ concerning final asset positions y is defined by

$$< p_i, y_i > \succsim_y < p_j', y_j' > \; iff \; < p_i, x_i > \succsim_{x|c} < p_j', x_j' > \tag{7}$$

where $y_i = c + x_i$, $y_j' = c + x_j'$ for $i = 1,\ldots,m$ and $j = 1,\ldots,m'$. The equivalence (7) between preferences when outcomes are described by net gains or losses and preferences when outcomes are described by asset positions will be called a *framing transformation.*

Definition 3. A decision maker's preferences such that the framing transformation (7) is satisfied for every current asset position c will be called *framing consistent.*

The condition of framing consistency can also be regarded as a consistency condition on preferences concerning financial changes for different current asset positions c and c'. More precisely, framing consistency implies the condition that

$$< p_i, x_i > \succsim_{x|c} < p_j', x_j' > \; iff \; < p_i, x_i + d > \succsim_{x|c'} < p_j', x_j' + d > \tag{8}$$

where c, c' are any two current asset positions and $d = c - c'$. Conversely, the condition (8) implies that if a preference relation $\succsim_y$ on final asset positions is defined by (7) with a specific amount c, then (7) is also satisfied for any other amount c'.

Kahneman and Tversky (1979, p. 273) have demonstrated that the manifest behavior of students who are presented with hypothetical choice problems is not in accord with the condition of framing consistency; often there are reversals in preference depending on whether the problem is framed in terms of a current asset position c or in terms of another current asset position c'.

Theorem 3. The condition that the preference relations $\succsim_{x|c}$ are averse-prone risk attitudes for more than one current asset position c is inconsistent with the principle of framing consistency.

The result identifies a conflict between a preference condition having a strong normative appeal and the choice behavior reported in many articles, e.g., Fuchs (1976), Green (1963), Grether and Plott (1979), Halter and Dean (1971), Hershey and Schoemaker (1980a), Hershey and Schoemaker (1980b), Kahneman and Tversky (1979), Langhhunn, Payne, and Crum (1980), Payne, Langhhunn, and Crum (1980), Payne, Langhhunn, and Crum (1981), Slovic, Fischhoff, Lichtenstein, Corrigan, and Combs (1977), Swalm (1966), and Williams (1966). Theorem 3 and the remarks in subsection 3.1 imply that the conflict often alluded to in the literature between averse-prone choice behavior and the rules of rationality is not a conflict with the expected utility conditions, e.g., independence, but is a conflict with a consistency requirement. It is possible, for example, for a person's preferences $\succsim_{x|c}$ to be an averse-prone risk attitude and to satisfy expected utility conditions for each current asset position c and yet not to be framing consistent.

This identification of a specific normative principle that is in conflict with averse-prone choice behavior suggests a question for future empirical work. For which persons and in which contexts is a person's averse-prone choice behavior a result of a preference issue that is of genuine importance to the person rather than a result of his information processing limitations and of his limited experience in risk taking. Dickson (1981) found that a risk manager, i.e., a person "confronted continually with problems holding out the chance of loss" is likely to be considerably more risk averse for potential loss-producing choices than is a non-risk manager, i.e., a person who "will rarely be concerned with decisions where only a loss or break even point is in prospect". A survey by Freifelder and Smith (1984) has similar results. Hershey and Schoemaker (1980a) and Schoemaker and Kunreuther (1979) found that risk aversion is far greater when decisions are presented in an insurance context than when the same decisions are presented as standard gambles. Experimental studies in Hershey and Schoemaker (1980b) that examine the reflection hypothesis of Kahneman and Tversky (1979) at both the across-subject and within-subject levels "seriously question the generality of prospect theory's reflection hypothesis."

One approach to the question posed above would be to investigate the extent to which the same person in different contexts and different persons in the same context would modify their averse-prone choice behavior after being informed as to its conflict with the normative principle of framing consistency. Slovic and Tversky (1974) examine similar questions concerning a person's modification of his preferences when informed as to its conflict with Savage's independence principle for expected utility.

4. Combinations of Lotteries

Tversky and Kahneman (1981), (1986) discuss a type of dominance principle that is concerned with the sum of two lotteries. In this section, a general definition of this principle is specified, and the principle is shown to be violated by any averse-prone risk attitude. Dominance concerning sums and multiples of lotteries appears to be appropriate as a normative principle; however, it is shown to have implications that render it it far too restrictive. Indeed, in an expected utility model, this type of dominance is shown to imply that the decision maker is risk neutral.

Consider the specific choice problem due to Tversky and Kahneman (1981) that is described in Section 2. While most of the students in the sample (73 percent) preferred A to B and also preferred D to C, none of these students preferred the sum of A and D to the sum of B and C:

Problem 2. Choose between:

A and D. 25% chance to win \$240, and
75% chance to lose \$760. [0 percent]

B and C. 25% chance to win \$250, and
75% chance to lose \$750. [100 percent]

Tversky and Kahneman argue that the students' decisions are "violations of the rules of rational choice" and, more specifically, "violations of dominance" for those students who preferred A to B and preferred D to C even though the sum of B and C stochastically dominates the sum of A and D.

It will be useful to define precisely the principle that is being violated by the students' decisions in this example. Definition 4 below is intended to do so; there are other definitions that have a greater or lesser apparent generality but are in fact equivalent.

Definition 4. A preference relation on lotteries will be called *sum-dominance consistent* provided that for any probability $0 < p < 1$, both of the preferences

$$x_1 \succ \langle p \,\$x_2,\ q \,\$x_3 \rangle,\quad \langle p \,\$x_2',\ q \,\$x_3' \rangle \succ x_1' \tag{9}$$

do not occur whenever the lottery $\langle p \,\$(x_1 + x_2'),\ q \,\$(x_1 + x_3') \rangle$ is dominated by the lottery $\langle p \,\$(x_2 + x_1'),\ q \,\$(x_3 + x_1') \rangle$ in that

$$x_1 + x_2' < x_2 + x_1' \quad \text{and} \quad x_1 + x_3' < x_3 + x_1' \ . \tag{10}$$

In the above Problems 1 and 2, $p = .25$ and $x_1 = \$240$, $x_2 = \$1000$, $x_3 = \$0$, $x_1' = -\$750$, $x_2' = \$0$, and $x_3' = -\$1000$.

Before discussing the implications of sum-dominance consistency, we will describe a second, analogous type of dominance. To focus the discussion, consider a person who wishes to invest a certain amount of money, which will be referred to as the person's *current fund*. It is meaningful to measure the consequences of the available investment lotteries not only by net gains and losses but also by percent increases and decreases in the person's current fund. Consider, for example, the following choice problem.

Problem 3. Imagine that you face the following pair of concurrent decisions. First examine both decisions, then indicate the options you prefer.

Decision (i). Choose between:

A. a sure gain of 6%

B. a one-fourth chance to gain 25%, and a three-fourths chance to gain nothing

Decision (ii). Choose between:

C. a sure loss of 15%

D. a three-fourths chance to lose 20%, and a one-fourth chance to lose nothing.

This problem has not been empirically tested. However, it is similar to the Problem 1 tested by Tversky and Kahneman in that mild risk aversion for gains leads to a preference of A over B and any degree of risk proneness for losses leads to a preference of D over C.

Now, consider the sequential occurrence of the investment lotteries in the decisions (i) and (ii). (The resulting percent changes are the same regardless of whether decision (i) or decision (ii) occurs first.)

Problem 4. Choose between:

A and D. a one-fourth chance to gain 6%
a three-fourths chance to lose 15.2%

B and C. a one-fourth chance to gain 6.25%
a three-fourths chance to lose 15%

Note that the combination of the lotteries B and C dominates the combination of the lotteries A and D in this problem just as the combination of B and C dominates the combination of A and D in Problem 2.

In general, let z denote a percent increase ($z > 0\%$), a percent decrease ($-100\% < z < 0\%$), or no change ($z = 0\%$) in a person's current fund. Let $< p(z\%), q(z'\%) >$ denote a lottery having a percent gain or loss of z with probability p and a percent gain or loss of z' with probability $q = 1 - p$. Note that if first a percent change z_1 occurs and second a percent change z_2 occurs, then the overall percent change is $z_1 + z_2 + (z_1 z_2/100)$. Let $z_1 \circ z_2$ denote this resulting percent change.

Definition 5. A preference relation on lotteries will be called ***percent-dominance consistent*** provided that for any probability $0 < p < 1$, both of the preferences

$$z_1 \succ < p(z_2\%), q(z_3\%) >, \quad < p(z_2'\%), q(z_3'\%) > \succ z_1' \tag{11}$$

do not occur whenever the lottery $<p(z_1 \circ z_2'\%), \ q(z_1 \circ z_3'\%)>$ is dominated by the lottery $<p\ (z_2 \circ z_1'\%), \ q(z_3 \circ z_1'\%)>$ in that

$$z_1 \circ z_2' < z_2 \circ z_1' \quad \text{and} \quad z_1 \circ z_3' < z_3 \circ z_1' \quad . \tag{12}$$

In the above Problems 3 and 4, $p = .25$ and $z_1 = 6\%$, $z_2 = 25\%$, $z_3 = 0\%$, $z_1' = -15\%$, $z_2' = 0\%$, and $z_3' = -20\%$.

Definitions 4 and 5 formalize certain requirements on preferences as proposed normative principles. The implications of these proposed principles are as follows.

Theorem 4. Consider a decision problem in which the possible consequences are financial changes measured either as absolute gains and losses x or as percent gains and losses z. Assume that the decision maker's preference relation on lotteries satisfies (i) the conditions of expected utility and (ii) the condition that larger amounts are preferred. Then:

(a) The preference relation is sum-dominance consistent if and only if the decision maker has a constant risk attitude for possible net gains and losses.

(b) The preference relation is percent-dominance consistent if and only if the decision maker has a constant proportional risk attitude for possible amounts of his current fund.

(c) Therefore, the preference relation is both sum-dominance consistent and percent-dominance consistent if and only if the decision maker is risk neutral.

Sum-dominance consistency and percent-dominance consistency appear to be reasonable as normative principles. They exclude the type of choice behavior that was observed by Tversky and Kahneman (1981) for Problems 1, 2, and that is conjectured to occur for Problems 3, 4. As Theorem 4 demonstrates, however, these principles exclude not only an averse-prone risk attitude but any attitude toward risk other than risk neutrality.

The assumptions (i), (ii) in Theorem 4 require that there exists a strictly increasing utility function for the preference relation. They do not require, however, that the utility function has derivatives of any order or even that it is continuous. Thus, Theorem 4 applies in particular to the preference relation for the effects context in a reward-penalty model.

5. Uses of the Reward-Penalty Model

This section discusses first the possible testing of the reward-penalty model as a descriptive model, and second its possible usefulness as a prescriptive model in a decision analysis study.

The reward-penalty model is sufficiently restrictive to be testable as a descriptive model of preferences in choice behavior. In this sense, it is not subject to the criticism of Tversky (1977) that introducing additional variables as descriptors of psychological consequences is ad hoc. However, any empirical study of the model that compares a person's preferences in two different contexts (what we have called the effects context and the no-effects context) will need to be carefully designed either to convey to the persons being interviewed a clear understanding of the effects context and of the no-effects context or to elicit by indirect questioning their preferences in the no-effects context. The design problems will be simpler for an empirical study that considers only the effects context and tests the implications (a)-(f) in Theorem 2 of a reward-penalty model. Such a study could offer indirect evidence as to the descriptive accuracy of a reward-penalty model for different types of persons and under different circumstances.

The evidence is strong that we humans have systematic and stubborn weaknesses in our ability to process information on our preferences and our probibalistic beliefs (see, for example, Kahneman, Slovic, and Tversky, 1982 and the review by Schoemaker, 1982). For this very reason, the distinction between descriptive models and prescriptive models can be useful; descriptive models have the purpose of aiding our understanding as scientists of how people make decisions and prescriptive models have the purpose of aiding our ability as decision makers to process information for making decisions (see, for example, Bell, Raiffa, and Tversky, 1984, Howard, 1980, Keeney, 1982, and Raiffa, 1961.) Leaving aside the operational difficulties of assessing simple preferences, this argument depends on the premise that a prescriptive model can capture the major preference concerns of the decision maker. Thus, it is also important to distinguish between those behavioral deviations from a prescriptive model that are due to suboptimal information processing, i.e., heuristic biases, on the part of the decision maker and those deviations that are due to preference concerns of the decision maker that are not included in the model.

The reward-penalty model is prescriptive rather than descriptive in that it is not intended to model a person's heuristic biases. Rather, it is intended to identify those psychological effects that are regarded by the decision maker as an important aspect of his consequences (and that cause his preferences to deviate from the preference condition of framing consistency). The recognition of these effects in the model is to be regarded as the inclusion of a preference issue of importance to the decision maker rather than as the inclusion of systematic imperfections in the reasoning of the decision maker. Thus, the purpose of the reward-penalty model is more restricted than the purposes of such general models as prospect theory (Kahneman and Tversky, 1979), weighted utility theory (Chew and MacCrimmon, 1984), and SSB theory (Fishburn, 1982).

In addition to the question of whether averse-prone choice behavior may accurately reflect a decision maker's preferences, and in that sense is rational, there is also the question of whether such choice behavior is in the best interest of an organization or of society, and in that sense is ethical. The psychological qualities that are hypothesized in Section 1 to be associated with averse-prone choice behavior are not flattering. In the case of an individual who is acting on his own behalf, these qualities can be negatively described as ego-centric and inflexible; the person would rather risk losing a larger amount than admit even to himself that he has suffered a loss. In the case of an individual who is acting on behalf of an organization, there is the additional quality that his concern for his own reputation may at times be at variance with concerns for the effectiveness of the organization; in common parlance, the person may be primarily concerned with "covering his ass."

Thus, for decision analysis applications in which an averse-prone risk attitude is an important feature of the decision maker's preferences, there may be cogent reasons for formulating two versions of the decision analysis model, one that includes π and ρ as additional parameters, and one that does not (and thus assumes that preferences are framing consistent). Both the effects context and the no-effects context would thereby by considered.

This caution as regards the modeling of a decision maker's complete preferences is similar to the point made in Bell (1985) "that what is currently omitted from expected utility analysis *deserves* to be omitted and that a formal analysis may be exactly what is needed to prevent a decision maker's intuition from forcing economically inefficient decisions [italics Bell's]." This point is made also in Raiffa (1984). In a decision analysis application for an organization, the most important preferences to be examined may be those of an exemplifying individual representing the entire organization rather than those of any single individual within the organization.

Appendix: Proofs of results

Proof of Theorem 1. If $u(x)$ is any utility function for the no-effects context, then $u(x) = U(x, z_1^*, \ldots, z_n^*)$ for some utility function U as described in (2). If $w(x)$ is any utility function for the effects context, then $w(x) = \tilde{U}(x, r_1(x), \ldots, r_n(x))$ for some (possibly different) utility function $\tilde{U}$ as in (2). Moreover, $\tilde{U} = aU + b$ for some constants $a > 0$ and b. Therefore,

$$w(x) = \tilde{U}(x, r_1(x), \ldots, r_n(x)) = aU(x, r_1(x), \ldots, r_n(x)) + b =$$

$$au(x + g_1(r_1(x)) + \ldots + g_n(r_n(x))) + b = au(x + e(x)) + b.$$

Proof of Theorem 2. Assume that the utility function u for the no-effects context is normalized so that $u(0) = 0$. Let l denote the lottery to be compared.

To prove (a), observe that for $x < 0$, $w(px) = u(px - \pi) = u(-\pi) + o(1)$ while $w(l) = pw(x) + qw(0) = pu(x - \pi) = pu(-\pi) + o(1)$. Thus $pu(-\pi) > u(-\pi)$ implies $w(l) > w(px)$ for $x < 0$ sufficiently near to 0.

To prove (b), observe that for $x > 0$, $w(px) = u(px + \rho)$ while $w(l) = pu(x + \rho) + qu(0)$. Since u is strictly concave and $\rho \geq 0$, $pu(x + \rho) + qu(0) < u(p(x + \rho)) \leq u(px + \rho)$, and thus $w(l) < w(\rho x)$.

To prove (c), it suffices to consider $x > 0$. Then,

$$w(l) = \tfrac{1}{2}u(x+\rho) + \tfrac{1}{2}u(-x-\pi) < \tfrac{1}{2}u(x+\pi) + \tfrac{1}{2}u(-x-\pi) < u(0) = w(0)$$

since $\rho < \pi$ and u is strictly increasing.

To prove (d), we must show that $w(l) = pu(x - \pi)$ is less than $w(px) = u(px - \pi)$ for sufficiently large negative x. As a first observation, the condition of unbounded utility from below is well-known to imply that the strictly increasing function $u(x)$ is unbounded from below (hence the name). There are two cases to consider: first, that in which the amounts x have a finite lower bound a (with $a < -\pi$ so that $w(x)$ is defined for some $x < 0$) and $\lim_{x \to a^+} u(x) = -\infty$, and second, that in which the amounts x have no lower bound and $\lim_{x \to -\infty} u(x) = -\infty$.

In the first case, $\lim_{x \to (a+\pi)^+} pu(x - \pi) = -\infty$ while $\lim_{x \to (a+\pi)^+} u(px - \pi) = u(a + (p-1)(a+\pi))$ is finite since $(p-1)(a+\pi) > 0$. Therefore, $pu(x - \pi) < u(px - \pi)$ for all x less than some amount x_p between $a + \pi$ and 0.

In the second case, the condition of decreasing risk aversion implies that $\lim_{x \to -\infty} u'(x) = +\infty$. To show this, note that $u''(x) \leq 0$ for all x implies that $u'(x)$ is decreasing for all x. Thus, $\lim_{x \to -\infty} u'(x)$ is a finite number b or is $+\infty$. If $\lim_{x \to -\infty} u'(x) = b$, then $\lim_{x \to -\infty} \sup u''(x) = 0$ and hence $\lim_{x \to -\infty} \inf -u''(x)/u'(x) = 0$. However, decreasing risk aversion implies that the local risk aversion function $-u''(x)/u'(x)$ is positive and decreasing (Pratt, 1964), and this property contradicts the previous statement.

Now, assume that $0 < p < 1$ is fixed. If $\pi = 0$, then for any $x < 0$ the line from the point $(x, u(x))$ to the point $(0,0)$ lies below the graph of u, and therefore $pu(x) + qu(0) = pu(x) < u(px + q \cdot 0) = u(px)$.

If $\pi > 0$, then there exists a number $x_0 < -\pi$ such that the line from $(x_0, u(x_0))$ to $(-\pi, 0)$ lies above the graph of u. Since $\lim_{x \to -\infty} u'(x) = \infty$ and $u(x)$ is unbounded from below, there exists a unique $x_1 < x_0$ such that this line intersects the graph of u also at $(x_1, u(x_1))$. Moreover, for any $x_p < x_1$, the line from $(x_p, u(x_p))$ to $(-\pi, 0)$ will also intersect the graph of u at a unique point $(x_p', u(x_p'))$ with $x_0 < x_p' < -\pi$. Choosing x_p so that $px_p - \pi < x_p'$, it follows that for any amount $x < x_p$, $pu(x - \pi) + q \cdot 0 < u(p(x - \pi) + q(-\pi))$ and hence $pu(x - \pi) < u(px - \pi)$ as was to be shown.

To prove (e), note that the certainty equivalent of l is $x_0 = u^{-1}(pu(x) + qu(x'))$ in the no-effects context and $x_1 = w^{-1}(pw(x) + qw(x'))$ where $w(t) = u(t - \pi)$ is the effects context. Since u represents an attitude of decreasing risk aversion, it follows by a result in Pratt (1964, Theorem 2) that $x_1 < x_0$.

Finally, the proof of (f) is similar to that of (e) above except that $w(t) = u(t + \rho)$, $\rho > 0$, and hence $x_1 > x_0$.

Proof of Theorem 3. Suppose that for two different current asset positions $c < c'$, the preference relations $\succsim_{x|c}$ and $\succsim_{x|c'}$ satisfy the condition of an averse-prone risk attitude. Then, framing consistency implies that the preference relation $\succsim_y$ is risk averse on the range $y > c$ and risk prone on the range $y < c'$. For the intersection range, $c < y < c'$, these risk attitudes are contradictory.

Proof of Theorem 4. To show part (a), we will relate the condition of sum-dominance consistency to the condition of a constant risk attitude. Let I denote the interval of possible monetary changes x. One definition Harvey (1981, 1986) of a constant risk attitude is that for any amounts $h_1 < h_2 < h_3$ and any probability $0 < p < 1$, if

$$x + h_1 \sim \langle p \,\$(x + h_2),\ q \,\$(x + h_3) \rangle \tag{A1}$$

for some x such that $x + h_1$, $x + h_2$, and $x + h_3$ are in I, then

$$x' + h_1 \sim \langle p \,\$(x' + h_2),\ q \,\$(x' + h_3) \rangle \tag{A2}$$

for any x' such that $x' + h_1$, $x' + h_2$, and $x' + h_3$ are in I.

For a preference relation $\succsim$ that satisfies the conditions of expected utility and such that larger amounts x are preferred, this condition holds if and only if $\succsim$ can be represented by a utility function of the form

$$u(x) = \begin{cases} e^{rx}, & r > 0 \\ x, & r = 0 \\ -e^{rx}, & r < 0 \end{cases}$$

for some parameter value r (Harvey, 1986). Note that no assumptions of differentiability or even of continuity of the utility function $u(x)$ are required.

Assume that there is a constant risk attitude and, in particular, that $\gtrsim$ is represented by a utility function $u(x) = \exp(rx)$ for some $r > 0$. Consider amounts x_i, x_i' $i = 1,2,3$, and a probability $0 < p < 1$ as in Definition 4. Then,

$$u(<p^{\#}(x_1 + x_2'), q^{\#}(x_1 + x_3')>) = u(x_1)u(<p^{\#}x_2', q^{\#}x_3'>)$$

$$u(<p^{\#}(x_1' + x_2), q^{\#}(x_1' + x_3)>) = u(<p^{\#}x_2, q^{\#}x_3>)u(x_1') .$$

Therefore, the preferences (9) imply that

$$u(<p^{\#}(x_1 + x_2'), q^{\#}(x_1 + x_3')>) > u(<p^{\#}(x_1' + x_2), q^{\#}(x_1' + x_3)>)$$

which implies that (10) is false. Thus, $\gtrsim$ is sum-dominance consistent. Similar arguments can be given for the cases $u(x) = x$ and $u(x) = -\exp(rx)$, $r < 0$.

To show the converse implication, first assume that there is not a constant risk attitude but that the utility function $u(x)$ is continuous. Then, there exists amounts $x + h_i$, $x' + h_i$, $i = 1,2,3$, in I and a probability $0 < p < 1$ such that (A1) is true but (A2) is false. By a slight change in p, it follows that the indifferences in (A1), (A2) can be replaced by opposite preference. Without loss of generality, assume that there is the preferences $\succ$ in (A1) and the preference $\prec$ in (A2). Then, by the continuity of $u(x)$, these preferences will remain true when $x + h_1$ in (A1) is changed to a slightly smaller amount $x + h_1 - d$, $d > 0$. It follows that for $x_1 = x + h_1 - d$, $x_2 = x + h_2$, $x_3 = x + h_3$ and $x_1' = x' + h_1$, $x_2' = x' + h_2$, $x_3' = x' + h_3$ the conditions (9) and (10) are satisfied, i.e., there is a violation of sum-dominance consistency.

Second, assume that the utility function $u(x)$ is not a continuous function on the interval I. Since $u(x)$ is strictly increasing, it has only a finite or countably infinite number of points of discontinuity. Thus, we may choose a point x_1 at which $u(x)$ is discontinuous and a point x_1' at which $u(x)$ is continuous. Suppose, for example, that $sup\ \{u(x) : x < x_1\} < u(x_1)$. Choose $h > h' > 0$ such that $x_2 = x_1 + h$ and $x_2' = x_1' + h'$ are in I. There exists a probability $0 < p < 1$ such that

$$x_1 \succ <p^{\#}(x_1 + h), q^{\#}x_3>$$

for all $x_3 < x_1$. Since $u(x)$ is continuous at x_1', we can choose an $x_3' < x_1'$ sufficiently near to x_1' so that

$$< p^{*}(x_1' + h'), q^{*}x_3' > \succ x_1' \ .$$

Now, choose x_3 sufficiently near to x_1 so that $x_1 - x_3$ is less than $x_1' - x_3'$. Then, $x_1 + (x_1' + h') < (x_1 + h) + x_1'$ and $x_1 + x_3' < x_3 + x_1'$. Hence, there is a violation of sum-dominance consistency. A similar argument can be given for the case $inf\{u(x) : x > x_1\} > u(x_1)$.

We will derive part (b) of Theorem 4 from part (a) by means of a change of variable argument. Suppose that the decision maker's current fund is denoted by a variable $y = c + x$ where c is his initial fund and x is the subsequent net change in the fund. We assume that $y > 0$ for all net gains or losses x in the interval I. Consider a new variable, $w = \log y$. The preference relation associated with y, which we will denote here by $\succsim_y$, induces a preference relation associated with w, which we will denote by $\succsim_w$.

It is well-known that the preference relation $\succsim_y$ satisfies the condition of a constant proportional risk attitude if and only if the preference relation $\succsim_w$ satisfies the condition of a constant risk attitude (see, e.g., Harvey, 1986 for a detailed discussion). Thus, to prove part (b), it suffices to show that $\succsim_y$ is percent-dominance consistent if and only if $\succsim_w$ is sum-dominance consistent.

A lottery $< p(z\%), q(z'\%) >$ expressed in terms of percent changes z, z' is equivalent to the lottery $< p \$ky, q \$k'y >$ with $k = 1 + (z/100)$, $k' = 1 + (z'/100)$ expressed in terms of asset positions. Therefore, Definition 5 can be restated as the condition that for any probability $0 < p < 1$ both of the preferences

$$k_1 y \succ < p^{*}k_2 y, q^{*}k_3 y >, < p^{*}k_2' y, q^{*}k_3' y > \succ k_1' y \tag{A3}$$

do not occur whenever

$$k_1 k_2' < k_2 k_1' \text{ and } k_1 k_3' < k_3 k_1' \ . \tag{A4}$$

However, a lottery $< p^{*}ky, q^{*}k'y >$ for the preference relation $\succsim_y$ corresponds to a lottery $< p(\log k + \log y), q(\log k' + \log y) >$ for the preference relation $\succsim_w$. Thus, there exist amounts k_i, k_i', $i = 1,2,3$, such that (A3), (A4) are satisfied if and only if there exist amounts x_i, x_i', $i = 1,2,3$, such that (9), (10) are satisfied.

Part (c) follows immediately from parts (a) and (b) since a utility function $u(y)$ is both linear-exponential and logarithmic-power if and only if it is linear.

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