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SYSTEMS ANALYSIS IN FORESTRY AND FOREST INDUSTRIES

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editors

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SYSTEMS ANALYSIS IN FORESTRY
AND FOREST INDUSTRIES

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FOREWORD

The purpose of this book is to present a variety of articles revealing the state of the art of applications of systems analysis techniques to problems of the forest sector. Such applications cover a vast range of issues in forestry and the forest industry. They include the dynamics of the forest ecosystem, optimal forest management, the roundwood market, forest industrial strategy, regional and national forest sector policy as well as international trade in forest products. Forest industrial applications at mill level, such as optimal paper trimming, cutting, and production scheduling, are however, excluded.

This work was initiated by a suggestion from Robert E. Machol in 1980 to Markku Kallio, who invited Åke E. Andersson and Risto Seppälä to join the group of editors. Anne Morgan was later invited to join the editorial group, to be responsible particularly for coordinating the processes of reviewing and revising. Since 1983 the work has been carried out within the Forest Sector Project at IIASA (International Institute for Applied Systems Analysis), Laxenburg, Austria. The call for papers resulted in more than 60 proposed contributions, 30 of which finally have been included in this book.

The editors of this volume are grateful to Robert E. Machol, all the contributors and referees, members of the Forest Sector Project staff and Publication Department of IIASA, and members of the TIMS staff for all their efforts in producing this volume. We are grateful for the partial financial support of IIASA, Yrjö Jahnsson Foundation (Helsinki), Foundation for Economic Education (Helsinki), and of the Swedish Council for Planning and Coordination of Research.

Laxenburg, Austria
June 30, 1984

Markku Kallio
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SYSTEMS ANALYSIS FOR THE FOREST SECTOR

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This article is an overview of systems analysis in forestry and forest industries. The issues covered range from forest management and forest industrial strategy to international trade in forest products and structural change in the forest sector worldwide. The methodologies discussed include mathematical models of economics, statistics, and operations research.

1. Introduction

Systems analysis is often oriented toward improving *long-term* policy making. This implies an emphasis on strategic rather than tactical or operational issues. Frequently the work involves generating policies for major changes concerning production and marketing, and the use of capital, labor, and raw materials. This means that the analyst must ensure close cooperation with policy makers and planners – whether they are in industrial firms or in regional or national government.

Such long-term policy analysis involves some "well-behaved" uncertainties, as well as others that are less conveniently structured. The uncertainties are great, e.g., with respect to the prediction of future demand. It is also hard to make reasonable, quantitative predictions about the technologies and the politics that will predominate over a planning horizon of two to three decades. It is often necessary for this reason to use scenarios, sensitivity analysis, and other similar approaches to provide insight into probable consequences of the fundamental uncertainties involved in long-term policy making.

Modeling plays an important role in applied systems analysis, so much so that the two are sometimes assumed to be identical. Most systems-analytic studies are based on one or more explicit models. Often, however, models are used throughout the process, though their use is never made explicit to the final user.

When a policy decision is being made, it is extremely helpful for the decision maker to know what the consequences of this choice would be. Models used for predicting these consequences are often called systems-analytic models. Examples of basic modeling techniques are optimization, simulation, gaming, and game-theoretic models. A given model may employ more than one of these techniques.

The analysis often relates socioeconomic, ecological, and technological systems to each other in an essentially dynamic and spatial modeling effort. This implies that frequently a large number of variables have to be interconnected. It is necessary for the analyst to make a sensible trade-off between realism, simplicity, and possibilities of estimation.

In tactical and operational management analyses using models, the aim is often to generate quantitative recommendations or forecasts. For applied systems analysis oriented toward long-term policy problems, a more moderate goal would be to formulate qualitative policy recommendations or *policy options* as rules of thumb. This formulation involves a certain amount of judgemental information concerning possible substantial changes in the environment of the system modeled.

For very long-term perspectives even this may be too ambitious. In these cases applied systems analysis can only be used to create a better understanding of the long-term policy problems and their interdependence. To use models for such pedagogical purposes, it is very often necessary to generate projections for the future. In order to trace out the consequences of possible assumptions concerning uncertainties, this type of projection is defined as a series of scenarios.

The forest sector comprises two main components: forestry and the forest industry. The forest sector concept integrates all aspects connected with forests and their exploitation, i.e. activities from timber growth to the use of end products. Ecological, environmental, and socioeconomic factors are also included in this definition.

The forest sector has a number of specific features that influence its planning and policy making:

- Although forests are a renewable natural resource, the production time, i.e. the growth period of trees from seeds to logs, is usually very long, in temperate areas close to a hundred years. As a result of the long rotation time, structural changes in forests cannot take place very quickly. In addition, the soil and climatic conditions often restrict the options for timber growing dramatically.
- The growing stock of trees is at the same time both a product and a production machinery in which annual growth accumulates. This gives flexibility in choosing the exact time for the realization of production.

- Wood is one of the most versatile raw materials. Its use for fuel, for housing and other construction, for furniture, for printing, packaging and household paper, for rayon and cellulose derivatives and as a chemical feedstock gives the forest sector a very diverse potential within the economy.
- The forest industry is a processing industry, the bulk of which is very capital intensive. The normal life-span of machinery is several decades. (Some paper machines built before the first world war are still operating.) This is one reason why the forest industry is rather conservative and not very flexible.
- Production technology in the forest industry is to a large extent based on old and well-known principles. Therefore, the technology is international, and productivity is tightly connected to the age and size of the production plant.
- The life cycle of different forest products is long. Innovations have usually only meant improvements in existing products, and completely innovative goods have appeared on the market very rarely. One of the implications has been that price has become a pronounced factor in market competition. Therefore, world-wide profitability has become relatively low, and the traditional forest industry is often characterized as a mature industry.
- Consumption of forest products results almost entirely from input needs in other areas, such as the construction and information sectors.

In recent years there has been a pronounced interest in the introduction of systems-analytic approaches for studying the problems of the forest sector. The International Institute for Applied Systems Analysis (IIASA) has, for instance, launched a large-scale effort in this direction. In this project, scientists from more than twenty countries have been engaged in the development of systems-analytic tools to study development policies for the forest sector.

Although applied systems analysis is oriented toward interdependences between all levels of decision making, the combination of a large number of interrelated variables leads to a practical need for decomposition: different parts of the analysis are separated from each other. Ideally such a decomposition should be made so that interdependences between components are few but strong.

The following decomposition of the forest sector outlines the topics covered in the different articles in this volume:
- Global analysis
- National analysis
- Regional analysis

We shall discuss each topic in the following sections. [1]

[1] Throughout the article, an asterisk refers to other papers included in this book.

2. A global perspective

2.1. International issues

As the demand for forest products increases, world supply is constrained by the availability of wood raw materials, higher production and transport costs, environmental concerns, and competition between forestry and other use forms for land. For example, according to recent estimates of annual wood removals for the period 1970–2000, the Nordic countries, the traditional suppliers of Western European markets, are reaching their wood production limit and cannot very much increase their average production rate. Although there is some growth potential in the forest industries in other Western European countries, scattered ownership patterns and environmental issues limit this potential. Increasingly, Western European forest industries are becoming dependent on imports of roundwood and wood pulp. To remain competitive, they are concentrating on end products that do not have cost structures dominated by wood costs. In contrast, parts of North America still have forest resources with great potential for the future, as well as a developed industrial infrastructure that would allow industries in this region to become major suppliers of wood products on the world market.

Although most of the forest industry capacity is in the industrialized countries, more than half of the estimated forest land area of the world is located in the developing countries. Besides being raw material for forest product industries, these forests play an important role in providing fuel, and land for agriculture. However, temporary relief from food shortage has frequently occurred at the expense of the biological potential of the forests. Forest land once stripped cannot support agriculture for long due to erosion or low soil fertility. The devastation of forests is one of the most serious problems in developing countries. Each minute, some 20 hectares of tropical forest vanish and the relative pace is increasing.

The global forest sector system is an assembly of interacting national systems. The interaction takes place mainly via international trade. At present, such trade in forest products is small relative to total world production – about 85% of woodpulp, paper and paperboard is consumed in the country where it is produced. (There are, however, some countries such as Canada, Finland, and Sweden, which export most of their forest products.) Moreover, trade flows in quite circumscribed paths and so is of unequal significance to different countries and regions. However, the foreseeable changes, especially in the availability of wood raw material and the cost structures of products, may cause drastic changes in the patterns of trade, whose total is growing rapidly. The following factors have a fundamental impact on the development of world trade:

- Regional demand

- Regional forest resources
- Relative production costs
- Transportation costs
- Trade policies
- Exchange-rate policies

We shall now discuss each of these factors in detail.

2.1.1. Demand for forest products

Future demand for forest products is characterized by changing response patterns in different countries and areas. Over the long term, the impacts of a number of technological changes need to be evaluated. Examples such as advances in electronic information technologies, super absorbent materials, and packaging substitutes will affect the demand for forest products. This will be noticeable earlier and more strongly in some countries than in others. Also the impact of the changing energy scene on both forest products and on competitive products needs consideration.

The large variations between different countries cause considerable prediction problems. The per capita consumption of paper has, for instance, been more than twice as large in the USA as in Switzerland, over the recent time period when the two countries had approximately the same standard of living. It is thus evident that simple econometric analysis of price and income elasticities is not sufficient to permit us to understand and predict the level of demand. It has been argued by industrialists that the use of packaging paper, for instance, is related to the whole structure of production and consumption of commodities, and to the packaging requirements related to spatially dispersed producers and consumers. Modeling long-term demand and consumption development must therefore take into account not only income and price development but also developments in the location patterns of producers and consumers as well as life styles in different parts of the world.

2.1.2. Forest resources

Problems of future resources can be approached from several different viewpoints. First, the long-term resource potential may be studied. A steady-state analysis of forests demonstrating the ultimate potential could be a starting point (see for example, Kallio and Soismaa *). The advantage of this approach is that it avoids the mental traps of restricting the analysis to minor extensions of current practices.

Second, supply potentials under alternative social and agricultural land-use policies can be examined in conjunction with varying economic incentives for

* This volume.

timber production. Provisions for fuel, and agricultural uses of wood, taking into consideration the problems of erosion, may also be advisable.

2.1.3. Production and transportation costs

Another aspect of the supply of forest products is the restructuring process that forest industries are facing in many countries. A primary issue is the *comparative advantage* of the industry in the country concerned, i.e. how the relative abundance of wood raw material and the cost of inputs affect capital investments in the industry. Also, hypotheses from *product cycle theory* must be included in the analysis.

Key cost factors in the forest industry are wood, energy, labor, transportation, and capital costs. In the industrialized countries wages and capital costs are the most important items and for example, in the Nordic countries, their combined share is about half of the export price to Western Europe. On average, stumpage paid to forest owners, energy costs and transport costs each account for 10–15% of the export price in Finland, Norway, and Sweden.

In the long term, there should be no significant differences in capital, energy, and chemical costs between countries. In contrast, wages can vary considerably, but often differences in productivity bring the costs per product unit to the same level. The major differences in the production costs of the forest industry are thus found in wood and transportation costs.

Table 1 shows the drastic international disparity in pulpwood prices in 1982. It can be argued that the data indicate a disequilibrium in world trade in wood and wood products, rather than price differentials consistent with transportation and energy price differences.

Equilibrium in the world market would exist if the price difference between markets were, at most, the marginal cost of transportation for each commodity. The cost of transporting a Finnish forest product to Western Europe constitutes between 10 and 25% of the product's sales price. In deliveries to Western Europe, Sweden has a transport cost advantage of 1 to 4% of the

Table 1
Typical wood cost (softwood) at mill by region in November 1982 [a] (US\$/m^3) (Source: Jaakko Pöyry International)

	Brazil	Southern USA	British Columbia (coast)	British Columbia (interior)	Southern Sweden	Southern Finland
Sawlogs		42	45	20	36	52
Pulpwood	18	22			31	41
Chips			31	17		

[a]After devaluations in Sweden (total 27.6% in 1981–82) and in Finland (total 10% in 1982).

product's price compared to Finland, which itself has an advantage of 5 to 10% over U.S. and Canadian suppliers.

A general framework for analyzing the comparative advantage of forest industries is discussed by Kirjasniemi *. The economic comparative advantage of forest plantations in a number of tropical regions and implications for the traditional forest products supply regions are considered by Sedjo *.

2.1.4. Artificial trade barriers

Distance can be considered a natural barrier to trade. Tariffs, quota restrictions, subsidies, and trade agreements, on the other hand, are artificial resistance factors. Subsidies in particular have been used heavily in some major forest industry countries since the mid-1970s.

One factor of considerable importance for the 1980s and 90s is the growing tendency to return to protectionist or even mercantilist trade policies. This phenomenon has been observed with increasing frequency since the energy crises, and it may have serious impacts on a number of countries such as Canada, Finland, and Sweden, which are strongly oriented toward the export of forest products.

Noncompetitiveness is not the only reason for the use of subsidies. In some cases subsidies are used to create industries especially in developing regions. They have also been used in efforts to influence the production structure of the forest sector.

2.1.5. Exchange rate policies

The fluctuations in the values of various currencies, relative to one another and to the U.S. dollar, have caused disruptions of world trade patterns in forest products. During the last fifteen years, currency relations between some countries exporting forest products have developed as shown in table 2. We observe that the values of the Swedish and Finnish currencies were relatively high in the mid-seventies. Consequently, problems emerged for these countries

Table 2
Currency exchange rates 1968–1983 (US$/100 units of national currency) (Source: International Financial Statistics)

	1968	1972	1975	1978	1979	1980	1981	1982	1983
Canada	93	100	98	88	85	86	83	81	81
Finland	24	24	27	24	26	27	23	21	19
Sweden	19	21	24	22	23	24	20	16	13
Austria	3.8	4.3	5.7	6.9	7.5	7.7	6.3	5.9	5.9
Norway	14	15	19	19	20	20	17	15	14
Brazil	31.6	16.8	12.2	5.5	3.7	1.9	1.1	0.6	0.3
Indonesia	36	24	24	23	16	16	16	15	14

in both their international competitive position and their internal profitability. Within the last five years, however, the situation has reversed completely. In 1981–83 the position of these currencies changed markedly in relation to the U.S. and Canadian dollars. For this reason, the Nordic forest industry (especially in Sweden) witnessed a remarkable recapture of its international competitive power, with booming profits and greatly improved possibilities for future expansion.

For industries that strongly depend on international markets, uncertainty about the future values of currencies is a major concern. Such industries could benefit substantially from embarking upon portfolio-allocation strategies in terms of investments, marketing, and currency holdings.

2.2. Analysis of the global forest sector

According to Ohlin (1933) (see also Heckscher 1949), the theory of interregional and international trade is nothing other than a theory of interregional and international location of supplies. This follows from the basic assumption that the pattern of location and the pattern of trade are simultaneously determined. According to this view, countries tend to specialize in the production of commodities that contain a relatively large proportion of fairly immobile resources that are relatively abundant in the country concerned.

This implies that, although certain resources are not themselves directly mobile, the export of commodities containing a large amount of such immobile resources would be equivalent to a migration of the resources. From this follows the Heckscher–Ohlin factor-price equalization theorem: The indirect migration of factors would, in the long run, tend to even out the factor prices of different regions. Any absolute factor-price equalization is of course impossible as long as there are transportation and communication costs and other constraints on mobility.

It is evident that free trade is more beneficial to the world economy than an autarkic system. In an autarkic system, welfare in a given region is determined by the resources in that region. Any erasing of this constraint will improve the possibility of achieving a higher level of welfare. Free trade corresponds to a summation of regional resource constraints, and thereby to a relaxation of constraints for the individual regions.

The only constraints that cannot be removed are those associated with the immobility of factors, commodities, and information. The analysis must thus concentrate on the initial distribution of such factors (including resources and technological know-how) and on the communication and transportation systems.

Based on the static theory of comparative advantage, various propositions have been formulated that are relevant for an understanding of the future interregional division of labor in the world (see e.g., Ethier 1983). The first of these propositions is called the Rybczynski theorem. It states:

> At constant factor prices, an increase in the endowment of a factor used in at least two sectors, which leaves that factor fully employed, produces a more than proportional rise in the output of some good and a fall in the output of some other good.

As an application, consider the European and North American situation. Assume that wood resources are constant (over time) for Europe but growing in North America, and that the capital market in North America is tight (allowing no expansion of capacity in this illustration) whereas production capacity in Europe is growing. Then, applying the Rybczynski theorem twice, chemical wood processing in North America is shifting over time toward wood-intensive commodities like pulp, and in Europe toward capital-intensive (and wood-extensive) products like paper. This is in fact the conclusion formulated by Ryti *, a conclusion substantiated by the Rybczynski theorem.

Associated with the Rybczynski theorem, there is the Stolper–Samuelson theorem:

> An increase in the price of an initially produced good using an assortment of at least two factors necessarily causes some factor price to rise in even greater proportion and some other factor price to fall.

In a two-factor, two-commodity world, this theorem might be interpreted as follows. Consider an economy producing paper and pulp, and using machinery and land as factor inputs. Let k_1 and l_1, respectively, be the quantities of machinery and land necessary per unit output of paper, and k_2 and l_2 similar coefficients for pulp. We denote the price of paper by p_1, the price of pulp by p_2, the rent of machinery by q, and land rent by w. If sales prices are entirely imputed to factors of production, we have

$$k_1 q + l_1 w = p_1$$

$$k_2 q + l_2 w = p_2$$

When the commodity prices p_1 or p_2 change, the factor prices q and q must also change. If paper is more machinery intensive than pulp, i.e. if $k_1/l_1 > k_2/l_2$, then the rise in the prices of paper will cause a relative rise in the rent of machines that is greater than the relative price increase of paper, and it will also cause a fall in the land rent. For the same reason, a rise in the price of pulp, which uses relatively more land, will cause a relative rise in land rent that is greater than the relative price rise, and a fall in the rent of machines.

These ideas formulated by Ohlin, Rybczynski, Samuelson and Stolper can be extremely useful in understanding the simultaneous determination of location of supplies, demands, and trade. The paper by Buongiorno * is an example in this tradition of interregional location and trade analysis, applied to a study of the U.S. pulp and paper market.

For dynamic analysis of these issues, other procedures have been proposed. One prominent concept is the *product cycle theory* (Vernon 1966). According to this theory, every product tends to follow a *location cycle*. Each product cycle starts in one of the developed economies where research and development leads to the introduction of the product (or product quality). In the primary stage, profitability is high and thus other countries with large research and development capacity will quickly imitate the country that originally introduced the new product. In this stage, the knowledge about the new commodity is diffused among the most developed countries, which become the major world suppliers. Thereafter, the product technology becomes more widely known and ordinary comparative advantage determines the interregional pattern of production according to the availabilities of production factors and constraints on trade. Due to the maturing of the technology the final phase is then reached, during which a collapse of the production in the original countries of specialization may occur.

The essential difference between the theory of comparative advantage and the product cycle theory is the emphasis placed upon technological change as a driving force. In the traditional theory of comparative advantage, the technology is assumed to be given and in some versions of the theory even known to every participant in international exchange. By contrast, the theory of product cycles regards the research and development process as the essential driving force in the changing pattern of technological knowledge and in the international pattern of supply location.

Irrespective of what determines supplies in the different regions, there are a number of ways of determining trade flows in a model. Two basic approaches are as follows:
- Simultaneous determination of location of supplies, regional demands, and trade between the different regions (e.g., Leontief et al. 1977);
- Stepwise analysis in which supplies and demands are determined for each region separately, after which a procedure of linking is used to predict the trade flows (e.g., the LINK project, Klein 1976).

Both procedures have their own advantages and disadvantages in terms of analytical consistency and information needs. The simultaneous approach implies a gain in consistency but quite often a loss in he quality of the information on resources and technological conditions for each region. A linking system means that the heavy burden of regional information gathering is decentralized while the coordinating effort can be focused on balancing supplies and demands through trade flows. Furthermore, this procedure puts a greater responsibility for modeling on each of the participating regions.

If a linking procedure is chosen, there still remains the choice of a mechanism for determining the trade flows. When choosing such a mechanism, one is forced to ask whether the system actually behaves rationally (evolving according to some welfare criteria and possibly even in an almost deterministic way) or whether it behaves more or less erratically. If the first assumption is valid, there remains the problem of choosing a *performance criterion*. Among the criteria suggested, the following are prominent:

- Maximize economic surplus (equivalent to generating a constrained competitive equilibrium).
- Maximize the sum of profits (equivalent to a monopolistic equilibrium, with prices set by firms).
- Minimize total cost subject to quantitative constraints (corresponding to a specific oligopolistic equilibrium solution).
- Minimize transport costs (a special case of the preceding criterion).

If rational behavior cannot be assumed or if our knowledge of the behavioral relations is of limited quality, a suitable approach is to determine a *stochastic outcome*. This often amounts to maximizing the likelihood of the outcome subject to the economic and political constraints that can be formulated *a priori*.

3. National analysis

3.1. Economic issues

3.1.1. Supply of Capital

The investment of resources in the forest industries is an issue of primary importance in many of the forest-product supply countries. The allocation of capital between the forest industry and other sectors of the economy is of great concern in such countries as Canada, Finland, and Sweden. Particularly in the United States, Canada, and Sweden, there has been a long-term pattern of low savings and investment, as compared to countries like Japan, Finland, and Norway. The general lack of investment resources is especially problematic for the pulp and paper industry, which is extremely capital intensive. The ratio capital investment/annual sales for the pulp mills is normally larger than 2. Economies of scale in integrated pulp and paper plants only deepen the problem.

3.1.2. Regional employment

The supply of and demand for labor for the forest sector has in some countries become a question of a trade-off between regional employment goals and the goals of industrial growth and profitability. Therefore, the allocation of labor between different economic sectors cannot always be analyzed at the national level, but must sometimes be performed at the regional level.

Labor demand is often unnecessarily constrained by the wage costs associated with central negotiation schemes for the labor market. In such cases, it can even be nationally efficient to subsidize employment in the forestry regions. With a general employment subsidy, the forestry firms normally located in areas of high unemployment also have the option of hiring otherwise unemployed labor at lower wage costs. Such schemes have been successfully developed for some Scandinavian countries and are now in use.

3.1.3. Microeconomic questions

The most important decisions affecting the structure of an industry are decisions on investment. Four major interconnected questions must be answered: *What*, in terms of products? *How*, in terms of production technology and capacity? *For whom*, in terms of markets? *Where*, in terms of location of production sites?

The main factors affecting product choice in investment in the forest industry are:

- future demand for the products;
- market situation: competition, trade barriers;
- wood availability and price;
- availability and price of other production inputs: energy, chemicals, capital, human resources;
- production technology available.

If neither the markets nor the technology were to pose unavoidable barriers and if the production inputs were available in abundance and at a fixed price, the choice between different products could simply be based on a profitability criterion, i.e. return on investment. But when the procurement of wood is, in many cases, a bottleneck and the price of wood is determined on roundwood markets, ordinary profitability criteria are no longer so useful. Instead, the profit maximization is subject to resource and other constraints.

The manufacturing cost per product unit usually decreases with increasing mill size. However, with growing mill size the wood transportation costs increase. This, together with the finite amount of available capital, imposes a limit on how large a mill should be. The optimal mill size is likely to grow with technological development. In most tropical regions with fast growing trees, the lack of infrastructure and skilled labor reduces the advantages of economies of scale. In addition, the lack of capital may prevent the building of big mills, even though the raw material base would in principle provide good opportunities. In general, economies of scale in a sawmill are much more limited than in a pulp or paper mill.

In addition to the size of the mill, there are other mill-related factors that affect the profitability of production. These are the extents of vertical integration (i.e. using the product of one line as a raw material for another line in the same mill) and horizontal integration (i.e. grouping of products into a multi-product complex that uses different species and assortments of wood and manufactures several end products). The competitiveness of primary production can to some extent be improved by vertical integration. Most standard products of the Nordic paper industry are currently made in integrated mills.

The aim of horizontal integration is to utilize wood species and timber assortments in the same proportions as they occur in the forest. The manufacturing costs, capital charges, and wood harvest costs can be one-fifth less in a multiproduct complex than for separate mills making the same products.

The forest sector and especially the wood industries have traditionally paid little attention to R&D. The paper industry devotes 0.5 to 1% of the turnover to R&D. In the wood industry the share is typically only 0.1%. This low R&D intensity may have serious implications for the future of wood as an input in mechanical and chemical processing. Other feedstocks, especially oil and non-ferrous alloys, are the subject of much more intensive research and development which, in the long run, may tend to influence the competitiveness of those feedstocks in relation to wood.

3.2. Economic modeling

The problem of analyzing a set of industries like the forest industries is essentially a question of handling *interdependences* of a technological and economic nature. The standard method is *input–output analysis*, where each sector is characterized by a technological input–output coefficient a_{ij} indicating the amount of commodity i that is required to produce one unit of commodity j. The set of such input–output coefficients can be assembled into an input–output matrix $A = \{a_{ij}\} = \{x_{ij}/x_j\}$, where x_j is the gross production of commodity j and x_{ij} is the required input of commodity i. Forming the simplest possible static balance requirement for the economy we get

$$x = Ax + f,$$

where $x = (x_i)$ and $f = (f_i)$ is the final demand (defined to be exogeneously determined consumer, government, investment, and net export demand).

For example, increasing newsprint demand implies an increased demand for pulp, which in its turn implies an increased demand for chemicals. The increased demand for chemicals implies a further increase in the demand for oil, etc., in a diminishing series that finally involves the whole economy. Thus the influence of any change in final demand for a forest product – in terms of consumption, government demand, net exports, etc. – spreads both within the forest sector and onto other sectors, as well as returning eventually to the forest sector itself.

The prices are determined in this type of national model by the requirement that the price p_i of each commodity i ($i = 1, 2, \ldots, n$) covers the costs of all inputs:

$$p = pA + \omega,$$

where $p=(p_i)$ and $\omega=(\omega_i a_{li})$ is a vector indicating the cost of labor (and possibly other primary inputs) for each product, with a_{li} being the labor-output coefficient in sector i and ω_1 being the wage rate.

The influence of investment can be analyzed within an extended input–output framework. In this case investments in new machinery, building, etc., are distinguished in the demand. Let b_{ij} be an investment-output coefficient indicating the input of commodity i needed to achieve unit growth in the production capacity of sector j, and let t denote time. Then the dynamic input–output model is given by the balance requirement

$$x(t)=Ax(t)+B\dot{x}(t)+f(t),$$

where $B=(b_{ij})$ and $x(t)$ is the time derivative of production, i.e., the rate of increase in capacity. It can be shown that any decrease in an input–output or investment coefficient will increase the general rate of growth and lead to a structural change in the economy. It is also possible to show that under specific assumptions there exists a maximum growth rate with a corresponding balanced composition of industries. Such an equilibrium growth rate corresponds to the *minimal equilibrium rate of interest* associated with a balanced set of relative prices of all commodities.

One disadvantage of the class of models described above is the assumed inflexibility of technologies. A number of procedures have been proposed to overcome this weakness. One method proposed is as follows (see von Neumann 1937):

$Qx \geq Ax + f$	Quantitative equilibrium condition,
$pQ \leq pA + \omega$	Price equilibrium condition,
$pf = \omega x$	General equilibrium conditions,

where A is an input matrix, Q is an output matrix, f is the final demand and ω is the cost vector of primary inputs. The approach involves two basic assumptions:

- Each activity can produce *many* commodities, including capital goods (joint production).
- Each commodity can be produced by a number of activities (substitution).

This model is closely related to the theory of linear programming, although it dates back to the 1930s. In fact, it is equivalent to choosing technologies and production levels to satisfy final demand at minimum costs in terms of primary inputs.

A model based on input–output theory but allowing for partial substitution of inputs has been developed by Johansen (1972) and 1974). A variant of this model is described by Sohlberg *. In this approach, labor, capital, and energy are substitutable inputs, while all other inputs are regulated by fixed input–output coefficients.

Mathematical programming methods for handling the interdependences between a single sector and the rest of the economy have been proposed by Dantzig (1963). The idea here is to use an optimization model of the sector, in which an efficient choice of technologies is calculated assuming various supply functions for resources and demand functions for products. The prices are then determined together with the set of input–output coefficients optimally selected by the model. This approach has been adopted by Kallio, Propoi, and Seppälä * for studying the Finnish forest sector.

3.3. Analyzing economic structural change

The development of an industry is determined both by external technological progress and by consumer demand changes. An approach for analyzing these processes was introduced by Salter (1960) and further refined by Johansen (1972). In this approach, individual plants are assumed to be flexible regarding substitution at the investment stage only. After investment, each plant is almost rigid in terms of the energy, labor, and other input requirements per unit of output. At this later stage, substitution can only occur at the industry or corporate level by opening up new plants or closing down units constructed earlier. Another feature of this theory is the *asymmetry* of the closing down and investment criteria. The closure of a plant occurs when the product price decreases below the average variable cost. Investment in a new plant occurs when the expected price exceeds the average variable cost and *a proportion of* the fixed cost, properly discounted.

The analytical procedure can be illustrated by the empirical diagram in fig. 1, where the shaded area gives the gross profit of the paper industry in Sweden in 1978 and 1980. Similar productivity and cost curves are now produced for the Swedish and Norwegian industry sectors on an annual basis. The structure and potential uses of such a data base are discussed by Johansson *.

Against this background a model of economic structural change can be formulated based on the following assumptions:

- Each industry in each region consists a number of production units of given vintages (ages) with given output capacities.
- All production techniques are characterized by coefficients that vary across vintages and sectors only.
- Only labor is considered as primary input, though it is feasible to extend the model to cover other inputs such as energy.
- The investment possibility in each industry and region is given by a new vintage with a certain "*best practice technology*".
- Existing capacities are utilized and new capacities created according to an efficiency criterion of maximizing total profits for manufacturing industry

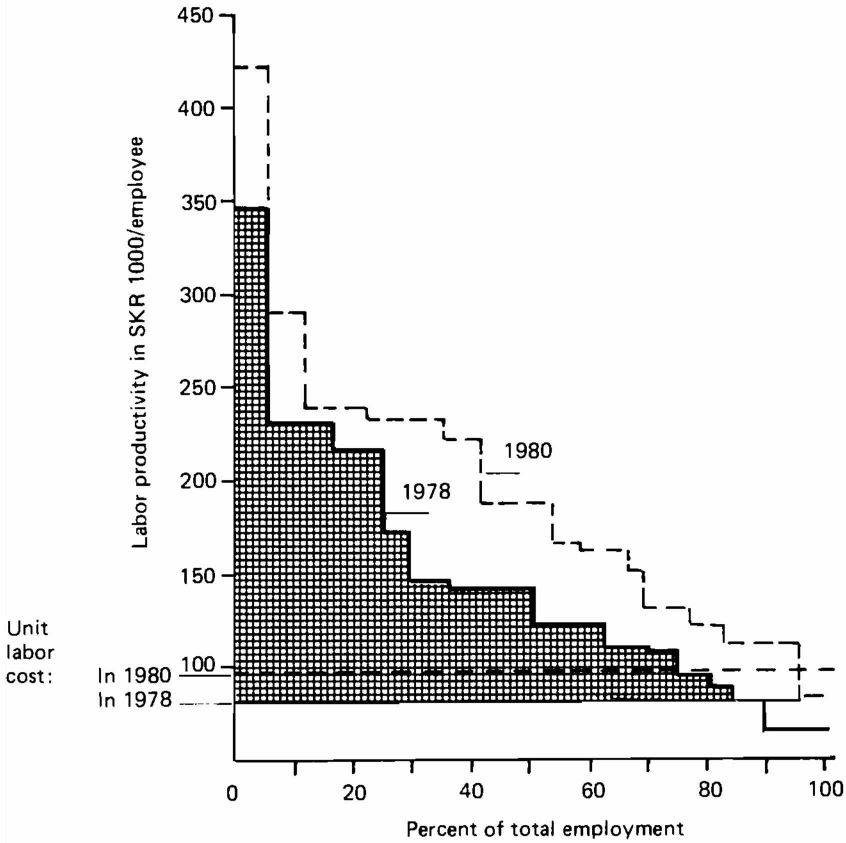

Fig. 1. Labor demand of the paper industry in 1978 and 1980 in Sweden.

as a whole. This is equivalent to the maximization of value added under constraints reflecting profitability.

Various industrial policy goals can be included either in the objective function or as constraints. The basic formulation employs maximization of profits. The other policy goals concerning employment and production, by industry and region, are taken into account by means of appropriate constraints. Additional constraints may specify availability of labor by region, availability of capital by industry, and employment in new vintages. The model framework is basically static, but the inclusion of the possibility of investment in new vintages allows some dynamic features to be simulated. The various sectoral and regional economic activities are only related to each other through competition for some resources such as labor and investment.

3.4. Forest sector modeling

In the preceding section it was implicitly assumed that a forest sector model exists that is compatible with modeling at higher levels of aggregation. This is not always the case. Forest sector models are often constructed without due consideration for requirements of consistency with macroeconomic models in terms of sectoral disaggregation, time periods to be covered, etc. This is an unfortunate situation in certain respects, but may also have advantages in other aspects: consistency in modeling simplifies cooperation whereas inconsistency can further the free development of new modeling ideas.

Forest sector models may be distinguished in a number of ways: disaggregation of inputs and outputs, treatment of time, treatment of regions, treatment of nonconvexities and nonlinearities of relations (such as economies of scale), and behavioral criteria such as optimization for single or multiple criteria. This section is limited to the consideration of three categories of models of particular significance. These are:
- Dynamic simulation
- Mathematical programming
- Spatial equilibrium models

3.4.1. Dynamic simulation

A dynamic simulation model is built on the fundamental assumption that there is a set of dynamically interacting decision bodies (constrained by technologies, resources, and other external factors) that determine a development trajectory. Generally a simulation model is a system of difference equations (for each time period t):

$$x_{t+1} = F(x_t, \dots, x_{t-T}, u_t, \dots, u_{t-T}, \varepsilon_t),$$

$$u_{t+1} = G(x_t, \dots, x_{t-T}, u_t, \dots, u_{t-T}, \xi_t),$$

where F and G are given functions, x_t is a vector of endogenous variables, u_t is a vector of decision variables (control variables), and ε_t and ξ_t are vectors of stochastic disturbances, for each time period t.

An example of such an approach is given by Lönnstedt *. His approach is rather generally applicable for studying the long-term development of the forest sector at the national level. An application is given for the specific case of Sweden.

It is well known from the theory of difference equations that a system of this type has to be restricted in terms of both functional forms and parameter values in order to obtain a well-behaved solution trajectory. As an example, it was shown many years ago by Slutzky (1937) that errors in the starting position of the system can be propagated through time and thus give rise to cycles, which would otherwise not occur. In many simulation models of the forest sector, formal stability analysis is made possible by certain linearity assumptions or by exclusion of stochastic elements. In such cases the model might take on the form

$$x_{t+1} = A(t)x(t) + B(t)u(t) + \varepsilon_t,$$

$$u_{t+1} = C(t)x(t) + H(t)u(t) + \xi_t,$$

where A, B, C, and H are given time-dependent matrices, and

$$x(t) = (x_t, x_{t-1}, x_{t-2}, \cdots) \text{ and } u(t) = (u_t, u_{t-1}, u_{t-2}, \cdots).$$

3.4.2. Mathematical programming

Dynamic linear programming (DLP) models of the forest sector employ deterministic versions of the linear system described above as an essential part of the constraint system. In addition to this constraint system, the DLP usually requires constraints on the initial and terminal states. Solutions of the model are obtained by formulating a goal function as follows:

$$\text{maximize} \sum_{t<T} [\omega_t x_t + \eta_t u_t] + \omega_T x_T,$$

where ω_t and η_t are given vectors, and T is the terminal time period. By superimposing a maximand, many of the equilibrium and stability problems of simulation models can be solved. Examples of such DLP approaches for forest sector analysis are given by Kallio, Propoi, and Seppälä * for Finland and by Hultkranz * for Sweden.

An essential problem with DLP models for the forest sector is to define a reasonable objective function to represent the goals of the industry and the multitude of other goals associated with the use of forests for recreation, as watersheds, etc. For this reason considerable efforts have been made in the field of *multiobjective programming*. Work in this area by Kallio, Lewandowski, and Orchard-Hays (1980) is an extension of the DLP model of the Finnish forest sector of Kallio, Propoi and Seppälä *. See also Rosenthal and Harrison * for an application of multiobjective optimization.

Another application of mathematical programming is reported by Hyman *. His study aims to assess alternative policies concerning fuelwood in the Philippines. The assessment considers economic, social, and environmental aspects of the problem.

3.4.3. Spatial equilibrium analysis

Regionalization of forest sector models is often required, because of geographical variation in ecological, institutional, or economic conditions. The primary problems associated with regionalization are a geometric increase in the number of variables and an arithmetic increase in the number of constraints; i.e., a considerable increase in model size.

Naturally, simulation and programming approaches apply to regional analysis as well. However, the spatial equilibrium approach is particularly suited for application to market economies as a means of determining an efficient allocation of production (over regions) to satisfy the demand in various regions (Lefeber 1958). Such allocations are assumed to be determined on the basis of cost competitiveness, subject to constraints on resources as well as constraints determined by national and regional policies.

The spatial equilibrium approach has been adopted by Dykstra and Kallio (1984) for studying long-term developments in the global forest sector. For another example of spatial equilibrium analysis, see Adams and Haynes *. Their model has been developed for the U.S. Forest Service for long-term planning purposes. This Timber Assessment Market Model (TAMM) involves regional production and consumption of roundwood and mechanical wood products in the U.S. A further application of a spatial equilibrium model is given by Buongiorno *. The long-term supply (and price) of timber has been studied by Kallio and Soismaa *, whose approach is also based on the economic equilibrium concept.

4. Regional analysis

4.1. Timber supply issues

In countries with a long tradition of forest industries, some of the most serious internal problems are connected with the availability and cost of wood raw material. The main constraints on the availability of wood for industrial use are the following:

- The physical and economic limits on wood production. The Nordic countries, for example, are approaching these constraints.
- The mixture of tree species in the tropical and subtropical regions.
- Institutional arrangements, in terms of ownership structures, principles of taxation, and regulation of cutting. The Nordic and Central European countries and some parts of the U.S. provide clear examples of this sort of constraint.
- Forests that are exploited for multiple use, such as tourism, outdoor recreation, hunting, and fishing, in addition to being the source of timber for the forest industry. The resulting multiple objective of forest management normally add to the constraints on the supply of timber. This is a major constraint in many Western European countries and in some regions of the U.S.

- Deficiencies in transportation capacity and high transport costs from wood-supplying to processing regions. The northern parts of the Soviet Union and Canada are examples here.

The article by Löfgren et al. * studies the effect of alternative taxation rules on the supply of roundwood. Sääksjärvi * reports a successful application of a game-theoretic approach for the fair division of costs between forest industries that cooperate in wood procurement in Finland. The estimation of forest resources is discussed in a paper by Ringo *.

4.2. Forest management

4.2.1. Classical theory

The problem of optimal rotation for forestry is a classic in the literature of forestry science. The basic assumptions of the analysis developed by Faustmann (1849), Pressler (1960), and Ohlin (1921) have been summarized by Löfgren and Johansson (1983):

- the capital market is perfect, i.e. allowing for immediate exchange on equal terms for all actors;
- the forest land market is perfect;
- future lumber prices are known without uncertainty;
- technical lumber-yield tables are available.

It is further assumed that the growth of timber is determined by a differentiable yield function $f(t)$ where t is the age of the forest stand. The time of harvesting $t = T$ is to be determined so as to maximize the present value of forest land. The present value is the sum of an infinite series of all future revenues:

$$\begin{aligned} V(0, T) &= pf(T)\mathrm{e}^{-rT}\left[1 + \mathrm{e}^{rT} + (\mathrm{e}^{-rT})^2 + \cdots\right] \\ &= \left[pf(T)\mathrm{e}^{-rT}\right]/(1 - \mathrm{e}^{-rT}), \end{aligned}$$

where p is the timber price reduced by variable production costs (such as labor), r is the interest rate, and T is the rotation period. This optimization criterion is known as the Faustmann formula. The determination of the optimal rotation period requires the maximization of $V(0, T)$ with respect to T. The optimality condition is the following

$$p\frac{\mathrm{d}f}{\mathrm{d}T} = [pf + V]r.$$

or expressed verbally:

> The forest stand should be harvested when the marginal rate of change of the value of the stand equals the interest on the stand value plus the forest land value.

A mathematical programming approach for determining the optimal rotation time is discussed by Dykstra *. A simulation approach is used by Lyons et al. * to study optimal rotation for energy wood plantations in Ireland.

4.2.2. Optimal control theory

The question of the optimal management of forest plantations is qualitatively different from the Faustmann problem in at least one fundamental respect: harvesting and planting levels are, in this context, the central decision variables and the rotation period is a secondary concern, determined as a by-product of optimal management. The problem can be formulated in terms of optimal control theory:

$$\text{maximize} \int_0^\infty [p(t, x)ux - c(u, x, s)]e^{-rt}\,dt,$$

$$\text{subject to } dx/dt = ax - bx^2 + s - ux,$$

where

x = the growing stock volume of the forest (frequently assumed to be determined by a logistic growth function), with growth parameters a and b,
p = price per unit volume of harvested forest,
u = the rate of harvesting,
s = the rate of seeding,
c = cost of harvesting, maintenance and seeding, and
r = rate of interest.

The papers by Hellman * and by Schmidt * are based upon optimal control theory oriented toward plantation management. The paper by Moiseev and Khmelevsky * addresses the complications caused by the dynamics of competition between different age groups of trees. The results from the qualitative analysis of forest management can be, and have been, used as a starting point for quantitative modeling. In this sort of exercise, the set of stands to be managed are normally modeled in a linear difference equation system of the Markov type. However, control variables associated with planting and harvesting activities are added to the standard Markov model. The development of the system is controlled by maximization of a performance index. An example of such a quantitative procedure based on dynamic linear programming is given by Kallio, Propoi and Seppälä *. In this model, forest management is treated as a submodel with a general forest sector framework. Other examples of quantitative forest management models are given by Weintraub and Navon *, by Phillips et al. *, and by Fowler and Nautiyal * (see also Dykstra 1984). Forest hazard management as an application of *decision analysis* is discussed by Barraer and Cohan *.

4.3. Multiple use of forests

The forests are, to industry and forestry managers, primarily a source of raw material for industrial processing into pulp and paper, building material, furniture, etc. To the general public, forests also have great value as pollution sinks (absorbing nitrogen compounds), as watersheds, for recreation, etc. Forests are consequently a *private* as well as a *public* good. Therefore, there is a probable need for public intervention in decisions on harvesting and other forest management issues. The resulting societal conditions of optimality are generally not consistent with the economic (forestry or forest industrial) conditions of optimality, which neglect the collective, nonindustrial uses of forests. For illustrations and applications of modeling for the simultaneous consideration of industrial and nonindustrial uses of forests, see e.g., Hyman *, Kilkki, Lappi and Siitonen *, and Navon and Weintraub *.

4.4. Forest ecosystem dynamics

Analysis of the forest sector is not sufficient if confined to the purely economic and social aspects of the matter. As has been shown by several authors (Baumol and Oates 1975, Clark 1976, Krutilla and Fisher 1975), the ecological issues have to be considered in any proper systems analysis of the forest sector. A few examples will clarify the need for the inclusion of ecological issues in any forest economic analysis:

- the forest industry is a polluting subsector, primarily influencing the rivers and lakes located near to the plants;
- forestry is a user of chemicals as insecticides, herbicides, and fertilizers;
- plantation management concentrates the numbers of species of trees and ground vegetation at levels that may be too low to be ecologically robust;
- animals that naturally belong to the forest ecosystems are sometimes excluded from their natural habitats by fencing, the use of chemicals, etc.;
- growing industrial output implies increasing pollution stocks of acid rain, which in the long term is likely to reduce the biomass growth potential. At present, large areas of the forests of central Europe are threatened by such environmental risks.

Analyses of dynamic interactions between forestry and ecology are reported by, for example, Isaev, Khlebopros, and Nedorezov *. Modeling of ecology–economy–technology interactions is also urgently needed. Holling, Dantzig and Winkler * give path-breaking examples of such modeling. By using a simple set of differential equations, the interactions between industry an forest ecology can be studied as an interdependent dynamic system evolving abruptly at times. These studies indicate that even smooth and continuous parameter changes can give rise to large structural changes in the forest ecosystem.

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Part I

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TRENDS AND LIKELY STRUCTURAL CHANGES IN THE FOREST INDUSTRY WORLDWIDE

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Jaakko Pöyry International Oy

Most forest products are of bulk type, so equivalent products from different sources are interchangeable and the main means of competition is price. Changes in the global structure of the forest industry are determined by the availability of wood and the cost structures in competing areas.

Technical development since the Second World War has broadened the range of wood species suitable for paper and board manufacture, and has made new forest areas potential sources of papermaking fibres. The most important of those new "low-wood-cost areas" are the U.S. West Coast, the U.S. Southeast, British Columbia, Brazil and Oceania. The consequences of the ensuing restructuring of the forest industry worldwide can be seen in Western Europe, where three different types of supplier compete in the market: the local forest industries, the Nordic forest industries, and the overseas suppliers. The different cost structures of these supplier groups are likely to lead them to different business strategies and to emphasis on different end products.

1. Introduction

The forest industry can be divided into a fibre-based industry producing pulp, paper and board, and a mechanical wood industry producing lumber, plywood, and wood-based panels. Many products are of bulk or semibulk type, products from different sources are interchangeable, and the main means of competition is price. Forest products have, as a rule, a long product life span; their markets have been characterized by continued growth over long periods.

The industry, especially the fibre-based industry, is capital intensive. This may explain its rather slow technical development. Considerable evolution has occurred over the years in sizes of production units and their technical details, but there have been very few revolutionary changes in technology.

Manufacturing technology is based on well known principles and readily available machinery. The success of a company can therefore only to a limited extent be based on its own technical innovation. This feature of the forest industry is not expected to change in the foreseeable future. Forecasts of changes in the global structure of the forest industry should therefore be based on the availability of suitable raw material in different areas and the cost patterns in these areas.

2. Some historical developments

The wood-fibre industries are based on inventions of a century ago. These inventions made it possible to use wood as a raw material for paper. In the western world, paper had until then been made from cotton and linen rags, which were in short supply. The shortage of paper for written and printed communication was a crippling constraint on cultural and economic development. These inventions removed the constraint, and the new wood-fibre industries enjoyed a fast-growing market. This growth continues today, although at a diminished rate.

The two most important of those inventions were the groundwood process [1] and the sulphite process [2]. The groundwood process is a mechanical process producing fibres of fairly light colour containing practically all the solid materials of the debarked wood. They are suitable for making printing papers with low permanency requirements, such as newsprint.

The sulphite process is a chemical process in which about half of the solid material in the wood is dissolved, setting the cellulose fibres free. These fibres are of rather light colour and easily bleached. They are suitable for manufacturing papers and boards with moderate strength and permanency requirements.

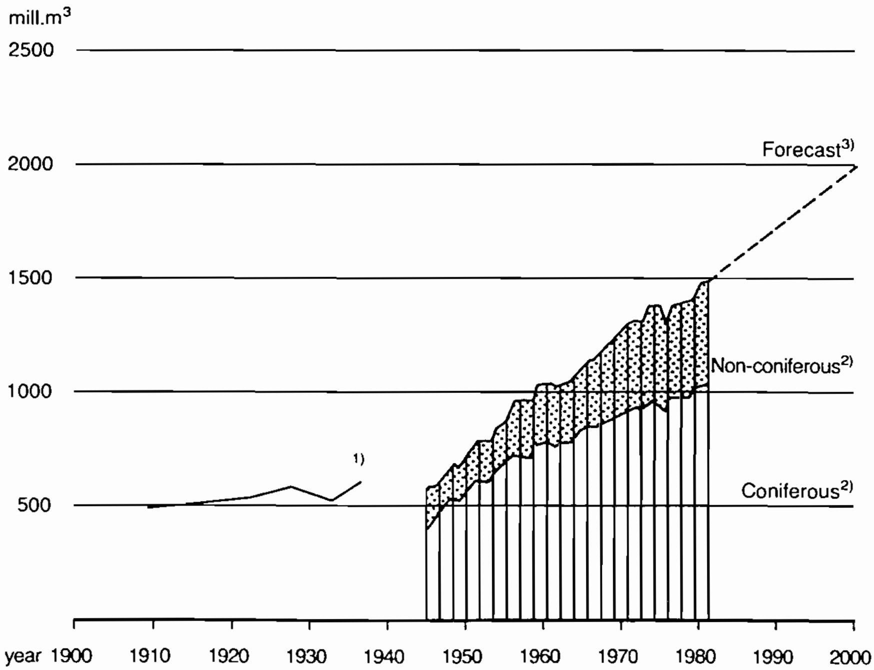
Groundwood and sulphite pulps cannot be made from all wood species. The best wood raw material for these pulping processes is spruce, so naturally the early wood fibre industries flourished in parts of the world where spruce was abundant: northern Europe and northeastern U.S.A.

The sulphate process [3], which is also a chemical process, is less dependent upon wood quality and species; practically all wood species can be pulped in this process. However, the resultant pulp is dark brown and difficult to bleach. In the early days of wood pulp manufacture, this pulp was therefore used only for manufacturing of packaging materials, such as wrapping paper.

This situation changed only in the early 1950s, when the modern process for bleaching sulphate pulp was developed in Scandinavia. This has had a profound effect on the world's forest industry. Now pine as well as spruce can be used for manufacturing bleached long-fibre chemical pulp; and a new type of pulp, short-fibre bleached hardwood pulp, has been introduced and found suitable for manufacturing many paper and board grades.

The original objective in developing a bleaching process for sulphate pulp was to expand the raw material base of the Nordic countries. This has certainly been achieved, but the economic effects have been still more dramatic in areas outside the Nordic countries. Forest areas with little or no spruce, which until the process was developed could be used only for lumber production, suddenly became potential sources of papermaking fibres.

The first areas to benefit were British Columbia, the U.S. West Coast and the U.S. Southeast, where mainly coniferous natural forests were taken into



1) *1910–1938, 5-year averages*
2) *1946–1980 (FAO "Yearbooks of Forest Products")*
3) *1981–2000, forecast (FAO "World Forest Products Demand and Supply 1990 and 2000", FAO Forestry paper 29, Rome, 1982)*

Fig. 1. World consumption of industrial wood [4].

use. The sulphate bleaching process has also made subtropical and tropical plantation forests a potential source of long-fibre and short-fibre chemical pulp. The results can be seen today in Brazil and Oceania.

The technology for bleaching sulphate pulp has been available for about 30 years. It is typical of the slow development of the forest industry that the industrial consequences of this development have been slow in coming. Only in the last few years has the worldwide competitiveness of the new, low-cost raw-material sources started to reshape the global structure of the forest industry.

3. Availability of wood raw material

The world demand for forest products is growing (fig. 1) [4]. To meet this growing demand the industry will have to expand. The most important constraint on this expansion is availability of wood raw material; subject to this constraint, expansion will be guided mainly by cost competitiveness.

Table 1
Wood removals and potential increase by region

Region	Removals [a]		Increase
	1970 mill.m^3/a	2000 mill.m^3/a	1970–2000 mill.m^3/a
Nordic countries	128	136	8
EEC	87	102	24
Western Europe outside EEC	74	91	17
US South	184	286	102
Canada, British Columbia (softwood)	68	83	15
Brazil (hardwood)	13	50–90	37–77
Indonesia (hardwood)	13	40–60	27–47
Western Africa (hardwood)	15	20–30	5–15

[a] Removals include merchantable wood, residues and non-commercial wood.

In most sectors of the forest industries raw material of a rather uniform nature is required. Natural forests in the temperate zone, and tropical and temperate plantation forests are technically suitable raw material sources for the forest industry but tropical rain forests have limited industrial value.

Table 1 shows the wood removal in 1970 and its potential increase up to 2000 in some important regions [5]. The figures are compiled from different sources using different calculating principles and assuming different levels of silvicultural measures. They are not therefore directly comparable, and should only be considered rough guidelines.

The Nordic forests are in virtually full use on a sustained yield basis. Only a rather limited expansion in the manufacture of base products can be expected there.

In Western Europe there still seems to be some growth potential. However, utilization of this potential is limited by institutional constraints, such as scattered ownership structure and environmental considerations. It therefore seems obvious that Europe as a whole will become more and more dependent on imports of forest products.

In North America there seems to be some additional growth potential. The U.S. South seems to be an area with great potential for expansion. As this is an area with a developed economy and industrial infrastructure, rapid expansion of the forest industry seems likely.

Some tropical countries, Brazil and Indonesia are examples, seem to have a considerable potential for expansion. This expansion would have to be based on plantation forests to a considerable extent. Whether this potential will really be used for industrial expansion depends greatly on political stability and government development policy in these countries.

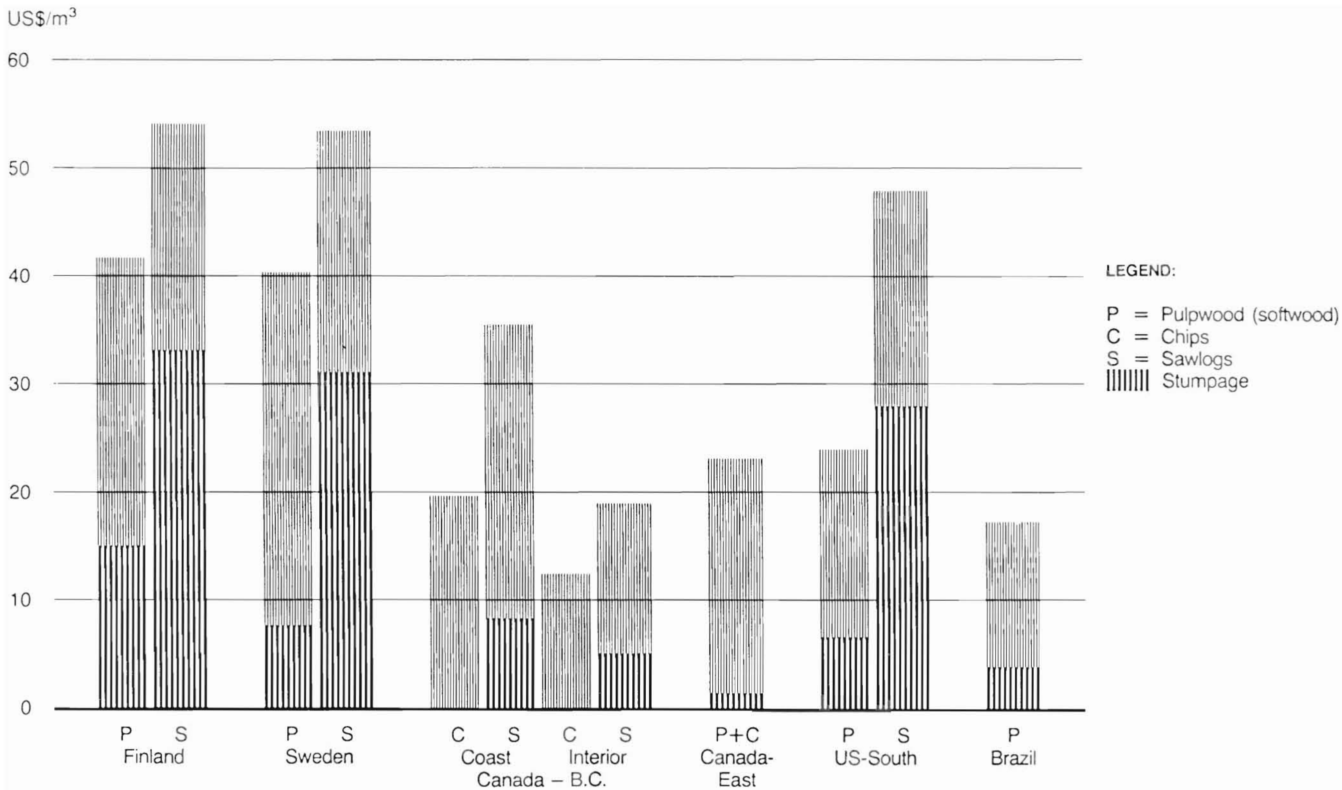


Fig. 2. Typical wood cost at mill by region in 1979 (Reference Jaakko Pöyry International Oy data files).

Fig. 2 shows some typical wood costs at the mill. It can be seen that the wood costs in areas where wood is a relatively scarce resource (here represented by Finland and Sweden) are much higher than in areas where it is more abundant (here represented by the U.S. South, British Columbia and Brazil). The reasons for these cost differences are complex. They include biological factors, ownership structure, taxation, government subsidies and wood supply and demand patterns.

4. The Nordic defence strategy

The Nordic countries are traditional suppliers of forest products to the rest of Western Europe and the forest industry is an extremely important sector of the national economy of these countries. A rapid decline of this industry would be extremely difficult to compensate through expansion of other industries. However, during the last two decades the Nordic countries have gradually lost their price leadership in the European market to overseas suppliers. This is mainly due to the larger difference in wood cost between the Nordic countries and their overseas competitors which is demonstrated in fig. 3, showing cost structures for hypothetical modern mills in different locations supplying their products CIF Rotterdam. The poor price competitiveness of the Nordic producers is rather pronounced in products with low yield for which the wood cost is a large part of the total cost structure. Chemical market pulp is typically

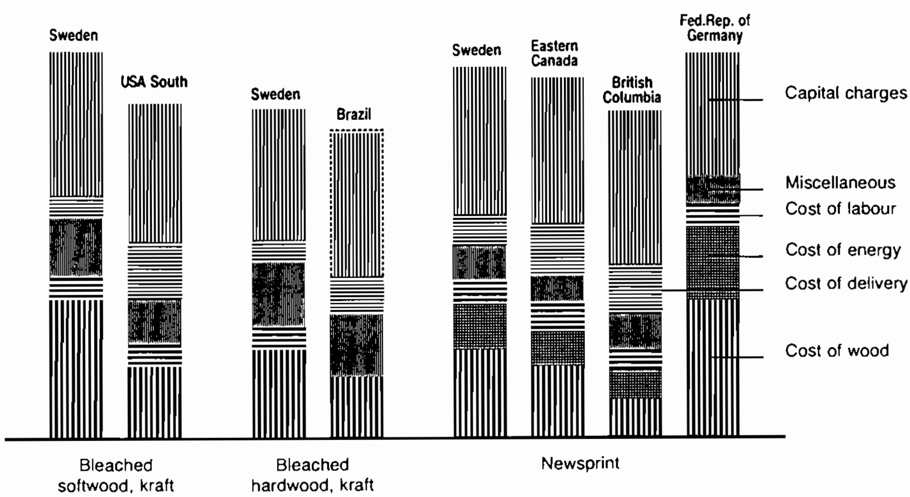


Fig. 3. Comparative cost structure of kraft pulps and newsprint delivered CIF Rotterdam. Capital cost in Brazil is uncertain due to high inflation and financing subsidies. Year 1980, arbitrary scale. (Reference Jaakko Pöyry International Oy data files).

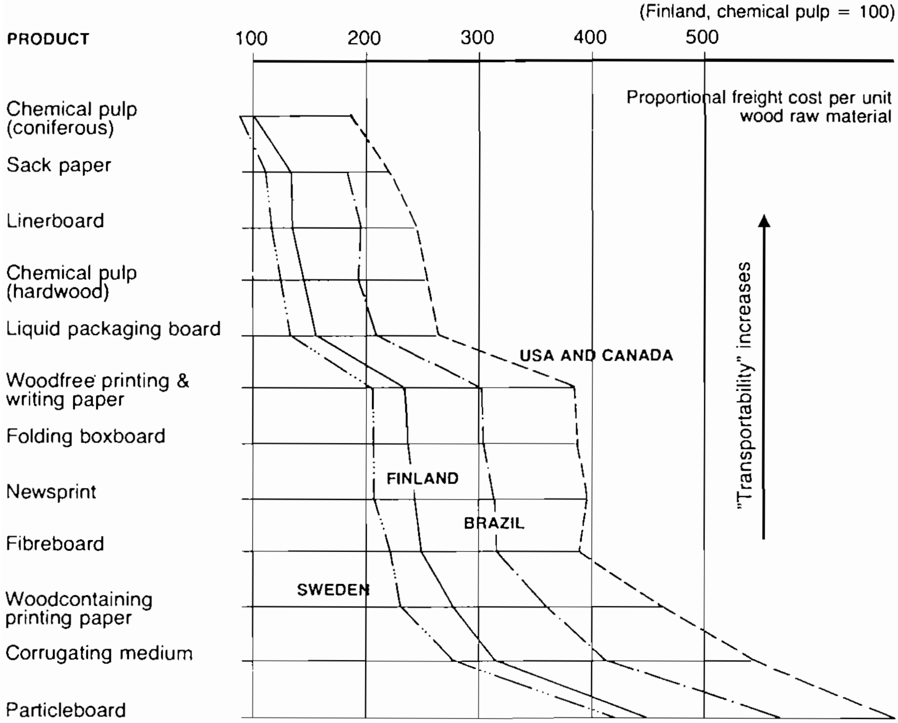


Fig. 4. Transportability of some forest products CIF Rotterdam from different producing areas [5].

such a product. Newsprint, on the other hand, is a high-yield product for which wood cost is a smaller part of the total manufacturing cost. A Nordic producer is therefore more cost-competitive in newsprint in the European market.

The cost-competitiveness of a Nordic producer also depends on freight costs, which are lower than for competing suppliers to the market. Fig. 4 shows the freight cost divided by the specific wood raw material consumption for different products and mill locations [5]. The "transportability" of the products increases when moving in the direction of the arrow, from particle board towards market pulp. For a Nordic producer, low transportability is advantageous when competing against an overseas supplier, whereas high transportability is favourable in competition with a Western European producer.

Obviously the cornerstone of the defence strategy of the Nordic producer must be the proper choice of end products. They should be high-yield products with a small wood cost relative to the total cost. They should also have a low transportability, if the competitors are overseas suppliers.

This clearly rules out chemical market pulp as a promising end product in the long run. The trend for the traditional market pulp producers in the Nordic countries will be to integrate into papermaking, either by building a paper mill on the same site as the pulp mill, thereby taking advantage of technical integration savings, or by financially integrating with an unintegrated paper producer on the continent, thereby benefiting from being close to the final customer.

There are some other difficult problems facing the Nordic forest industry in the long run.

How can the technical lead of the industry be maintained when limited raw materials do not allow the traditional path of combining modernization with expansion? How can requirements of economies of scale be satisfied, when technical developments tend to increase the "optimum" mill size, but scarcity of raw materials limits the expansion? How can the necessary growth of the forest industry be maintained when raw material limits the expansion of the production of base products? A thorough treatment of these items is not possible in this paper, so some comments will be given only about the last item – how to bring about necessary growth.

When raw material limits the expansion of base production, growth can still be brought about by increasing the refinement of the product. In practice, this means moving from market pulp into paper and board, and finally into converted products; or from unfinished lumber into planed board, and finally into joinery or prefabricated houses.

There are two conflicting aims in selecting the locations of the different operations in such a vertically integrated manufacturing system. Placing the operations at the same mill site usually brings some integration savings, such as the omission of pulp drying in an integrated pulp and paper mill, or efficient reuse of the converting broke in an integrated paper mill and converting plant. On the other hand, locating the final production step close to the market is often an advantage and sometimes a necessity.

When working toward greater refinement of the end product, and consequently more complex systems of vertical integration, some of the production steps must be placed in the market area. Where the production chain should be broken, and what combinations of integrated and unintegrated production systems should be used, are complicated problems which cannot be treated further in this context; but we can conclude that the defence strategy of the Nordic forest industry is likely to include internationalization of operations, possibly in cooperation with the industry in the rest of Western Europe.

5. Adaptation of Western European industry

The defence strategy of the Nordic countries – increased refinement of the end product, resulting in integration of pulp and paper making – is having a profound effect upon Western European industry. A considerable part of the paper industry in Western Europe is not integrated with pulp production, and it is now facing severe competition from integrated producers in the Nordic countries.

In the wood-free paper sector the situation is aggravated by a change in the structure of the market, which used to be a speciality market with small orders and individual quality requirements; consequently it was well served by small, versatile, unintegrated machines. In the last 10–15 years the market has standarized, and it can now be served by large, efficient machines. This development has had a dramatic effect on the real prices of these paper grades. The small, unintegrated mills are therefore facing considerable difficulties.

The paper industry in Western Europe is facing a period of adaptation in which it will have to make full use of its remaining advantages: good quality raw material (although available in limited quantities), availability of waste paper, and, above all, proximity to a large and growing market.

The available wood raw material will be used for chemical and mechanical pulping integrated with papermaking. Proximity to the market is likely to compensate for some cost disadvantages relative to Nordic or overseas competitors.

Manufacturing and transport cost structures, and savings from integration of converting and papermaking, justify converting imported pulp into some end products in the market area. Cleaning tissue is a good example.

Availability of waste paper is a benefit in the area, and packaging boards made from waste paper are natural products here. Waste paper is also making inroads into the furnishing of newsprint and printing papers.

Some unintegrated producers of wood-free papers might find a future in cooperation with Nordic integrated paper producers. The "satellite mill" in the market area would supplement the grade range of the Nordic mills by producing non-standard odd lots, and would carry out finishing operations for the Nordic mill in the market area.

6. North American offensive

Price leadership for many forest products in the Western European market has moved from the Nordic countries to North America. It is natural, then, that North American producers have increased their market share in the European market, especially in products such as chemical maker pulp, for which wood cost is a dominant factor in the total manufacturing cost structure (fig. 5). However, until now most North American producers, with the world's largest market on their own continent, have considered Europe as only a secondary market where marginal quantities can be sold at marginal prices when the home market is slack.

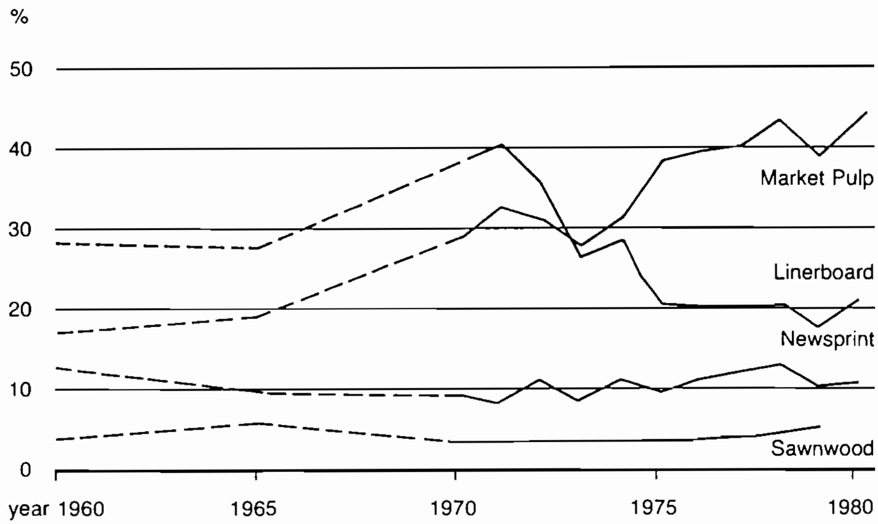


Fig. 5. North America's market share of some forest products in the Western European market (Reference Jaakko Pöyry International Oy data files).

Table 1 suggests that North America could become an increasingly important supplier of forest products for Western Europe. Whether this will really happen obviously depends on the judgment and decision of North American forest-industry companies. Important questions in this context include:

- Will the North American advantage in wood cost continue?
- Will changes in exchange rates not caused by differences in inflation rates influence North American competitiveness in overseas markets?
- Will trends in energy costs, which are a major cost factor for some forest products, be different in different producing areas, and thereby influence North American competitiveness in overseas markets?

These problems are not considered further here. If a North American offensive in the Western European market is explicitly decided upon by the industry, the next question is: What end products should be exported to this market? The principles in making such a choice and its compatibility with the "Nordic defense strategy" is illustrated in fig. 6. The idea behind this figure is as follows:

One of the basic criteria in choosing between alternative investment projects is Return on Capital. This assumes that capital is a scarce resource and its use must be optimized, whereas other resources, such as raw materials, are available at certain prices in sufficient amounts.

In North America this criterion is likely to be the most important one when choosing between alternative forest industry investment projects, but in the

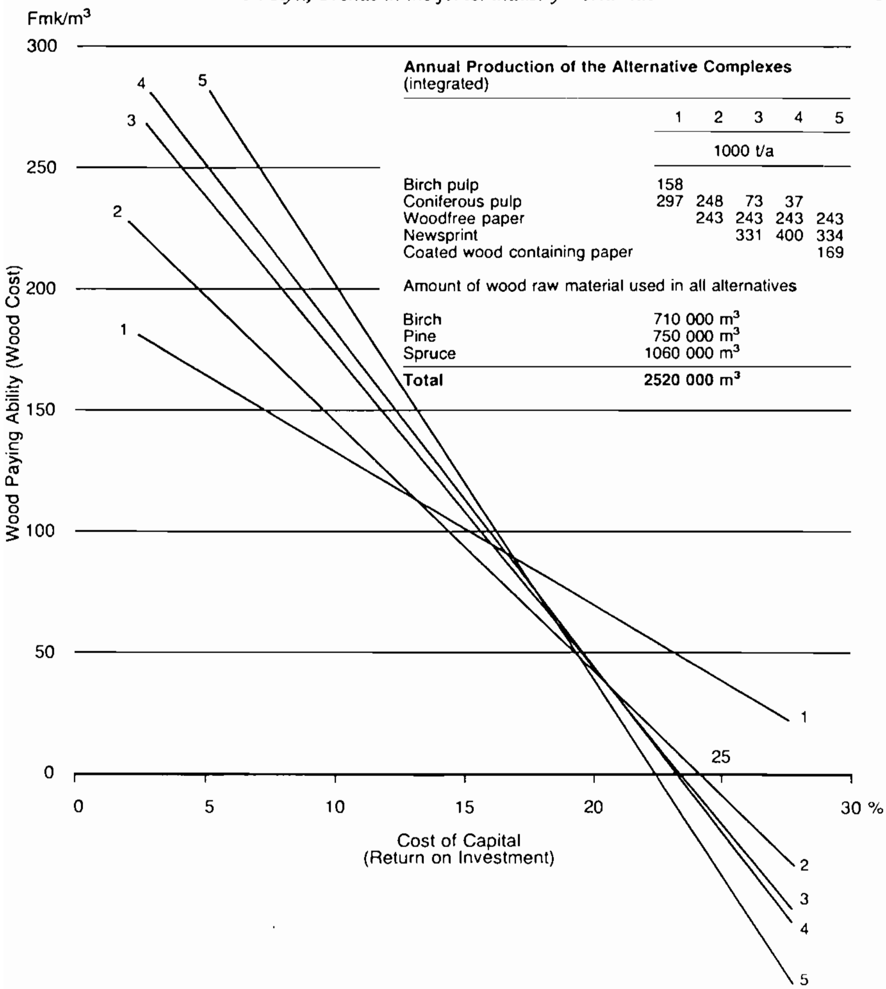


Fig. 6. Wood paying ability of some multiproduct forest industry complexes (Cost level first quarter 1981 in Finland) (Reference Jaakko Pöyry International Oy data files).

Nordic countries it is not fully valid. There the limiting factor for developing the forest industry is not capital, but wood raw material. A project increasing the demand for wood might have a profound effect on wood prices, and thus destroy its calculated profitability. It is therefore advisable in this case to use return on wood raw material, also called "wood paying ability", as the criterion when choosing among alternative investment projects. It is then assumed that wood is a scarce resource, the use of which has to be optimized, whereas other resources such as capital are available at certain prices in sufficient amounts.

In fig. 6 wood paying ability as a function of the cost of capital is shown for different end-product mixes, all based on a given raw material source. The figure suggests that wood paying ability increases when paper is substituted for market pulp as an end product. Papers including mechanical pulp in the furnish seem especially advantageous.

The figure can also be interpreted in a different way: "Wood cost" is substituted for "wood paying ability" and "return on investment" for "cost of capital" in the axes. The figure now suggests that in a low wood cost area the highest return on investment is achieved when producing market pulp only. Introducing more refined products into the product mix decreases the return on investment. [1] This exercise suggests that the choice of end products by a North American exporter, aiming for the Western European market could well be compatible with the defence strategy of the European producers. The end products that would seem most tempting for the American export-oriented producer are those which the European producer tries to avoid; and the products which the European producer tries to concentrate on will not give the American export mill the best possible rate of return.

7. Concluding remarks

There is a restructuring going on in the world's forest industries due to causes dating back many years. The development of the bleaching technology in the early 1950s expanded the potential raw material base of the wood fibre industry, and made new, low-cost forest areas available as raw material sources. On the other hand, in the 1960s the expansion of the forest industry in the Nordic countries, the traditional suppliers of the Western European market, reached the wood-producing limit of these countries. This has restricted the growth of the Nordic industry, and has contributed to a large increase in the cost of its wood raw material.

It seems clear that North America, with its still abundant wood resources and its developed industrial infrastructure, has a good short and medium term potential to expand its forest industry, not only to serve the North American market, but also to become an increasingly important supplier of forest products overseas.

The most interesting end products for an export-oriented forest industry in North America are products with a low yield, for which the cost of wood dominates the total cost structure. These products should also have a high transportability (i.e. a low freight cost per unit of wood raw material). Chemical market pulp and kraftliner board are such products.

[1] The calculations are based on hypothetical mill models and the Finnish cost level of the first quarter of 1981. The results should be used as a qualitative indication only, and should not be used for quantitative comparison of investment opportunities in different producing areas.

The European forest industry will not be able to increase its production of base products to meet the growing demand in the Western European market. To stay competitive, it will have to concentrate on end products which do not have a cost structure dominated by wood cost, and which have a low transportability. This implies a trend away from market pulp into paper and board making, increased emphasis on papers containing mechanical pulp, and increased use of waste paper.

The necessary growth of the forest industry enterprises will, to an increasing extent, have to come about through vertical integration, because lack of raw material prevents expansion of the base industry. It is therefore likely that we will see more integration, not only from pulp making to paper and board making, but also further into converting. This is likely to include internationalization of Nordic forest industry enterprises, and financial and technical integration between these and continental industries.

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A MODEL FOR INTERNATIONAL TRADE IN PULP AND PAPER *

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This paper outlines the mathematical structure of a world model of the pulp and paper industry. The model is designed to provide long-term forecasts of supply, demand, trade, and prices of raw materials, and of intermediate and finished products in the sector, based on the behavior of competitive markets. Supply and demand are distinguished by geographical regions. Raw-materials supply is represented by price-responsive supply functions. Manufacturing is modeled by a linear programming structure. Demand for various end products is a function of price, with income as a main shifter. All prices and quantities relating strictly to the pulp and paper sectors are endogenous. Econometric results regarding the magnitude of income and price elasticities of world demand for paper and paperboard are presented.

1. Introduction

In the United States, legislation requires the USDA Forest Service to make periodic assessments of long-term supply and demand of forest resources, including timber. Quantitative models have long been used in these planning exercises. However, only in the latest (1980) Renewable Resources Planning Act Timber Assessment Project, were consistent long-range projections made not only of timber consumption and production, but also of equilibrium prices. This was done thoroughly for the softwood timber sector, especially for lumber and plywood, by using the softwood Timber Assessment Market Model (TAMM) [1]. It was felt that some parts of the model, especially those dealing with the pulp and paper sector should be improved to prepare the 1985 Assessment.

The purpose of this paper is to outline this new model of the pulp and paper sector, as it is now being developed. Although the model is designed to assist the United States Forest Service in its periodic long-range planning activities, it is truly a world model. This is in agreement with a recent review of forestry economics research [15] which has shown the need for taking an international viewpoint in analyzing forest policies. This is true even when the analysis is

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focused on a single large country such as the United States [4]. To that effect, the form of the model is very general, it can accommodate a large number of geographical configurations, and it should be useful in a variety of national, regional or global analyses.

Most of the following exposition deals with the mathematical structure of the model since no run has yet been done with actual data. Nevertheless, some econometric estimates have already been obtained regarding the elasticity of the world demand for paper and paperboard with respect to price and income. These results will also be discussed.

2. General model structure

The model is a regional economic model in which both the quantities and the prices of pulp and paper products are endogenous. Although the model has an international scope, the United States and Canada will be more finely regionalized than the rest of the world, because of the requirement to use the model in the next United States' Assessment. Within the United States, eight geographic-supply regions and six final-product demand regions are recognized, identical with those used by the TAMM model. Preliminary plans are to disaggregate Canada into three sub-regions, to consider eastern Europe and Japan as two separate regions, and to have one single region for the rest of the world (table 1).

Table 1
Regions used in pulp and paper model

1. United States	
Supply	*Demand*
Pacific Northwest-West	Northwest
Pacific Northwest-East	Southwest
Pacific Southwest	North Central
Rocky Mountains	South
North Central	East
South Central	
Northeast	
Southeast	
2. Canada	
Western	
Central	
Eastern	
3. Western Europe	
4. Japan	
5. Rest of the World	

Table 2
Commodities used in pulp and paper model

1. Raw materials
Softwood
Pulpwood
Residues
Hardwood
Pulpwood
Residues
2. Intermediate products
Mechanical pulp
Chemical pulp
3. End products
Newsprint
Other printing and writing
Other paper and paperboard

The model uses nine commodity groups (table 2). These include four raw materials: softwood and hardwood pulpwood, and softwood and hardwood residues; two intermediate products: mechanical and chemical pulp; three final products: newsprint, other printing and writing paper, and other paper and paperboard; and three recycled commodities, one for each final product. This list may have to be expanded to describe transformation processes and final demand better. Nevertheless, to facilitate computations and interpretation of results, the number of commodities should be kept to a minimum consistent with the purposes of the model.

The model involves five general categories of activities: supply, manufactur-

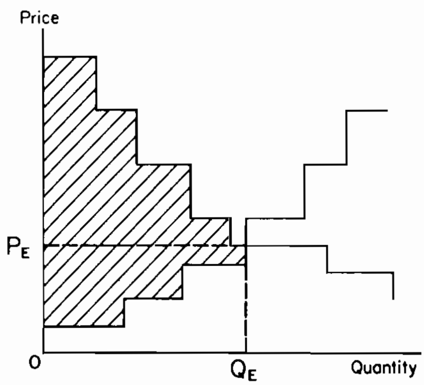

Fig. 1. Market mechanism of maximization for a single commodity and country. The shaded area is the net social pay-off, sum of consumers' surplus and producers' surplus. Maximizing net social pay-off is equivalent to determining equilibrium price, P_E, and quantity, Q_E.

ing, transportation, investment, and demand. Supply and demand activities are modeled as price-responsive functions. Manufacturing activities are modeled via input–output coefficients. All activities are integrated by a mathematical-programming model.

The optimization procedure simulates the market mechanism in which supplies and demands are balanced through equilibrium prices which equalize consumers' willingness to pay and producers' incremental costs. This is achieved by making the objective function being maximized the sum of producers' and consumers' surplus (fig. 1).

The model is long-term in nature, designed to produce forecasts for a 50-year period, divided into ten 5-year intervals. Investment activities link the submodels relating to each time period. Overall optimization is achieved by maximizing the discounted value of producers' plus consumers' surplus over the entire planning horizon.

3. Mathematical formulation

To simplify notations, this section will deal only with the static representation of the model at one specific point in time, namely at the end of the 5-year interval under consideration. The dynamic aspects of the model are discussed later. All activities are expressed per unit of time (year). The variables and parameters used by the model are defined in table 3.

The equations of the model are the following:

Price-responsive commodity supply:

$$\sum_{m=1}^{M} S_{ikm} - \sum_{j \in J_{ik}} T_{ijk} = 0 \quad \begin{matrix} k \in IC \\ i \in IS_k \end{matrix}, \quad 0 \leq S_{ikm} \leq S^*_{ikm}. \tag{1}$$

The quantity of a commodity supplied by a region equals the quantity shipped from that region. There is one constraint of this type for each commodity and region for which supply is specified explicitly as a function of price. This is the case of the supply of all primary materials: pulpwood and recycled paper. This representation of supply is also used for commodities and regions for which a more detailed representation of production is not possible or desirable.

Balance between material inputs and commodities produced:

$$\sum_{i \in I_{jk}} T_{ijk} - a_{jkl} Y_{jl} = 0 \quad \begin{matrix} l \in IC \\ j \in IR_l \end{matrix}. \tag{2}$$

The quantity of raw materials received by a region in which an intermediate or

Table 3
Definition of variables and parameters used in model (in order of appearance in the text)

Symbol	Definition
k, l	commodity subscripts
i, j	region subscripts
m	segment subscript in separable convex programming approximations
M	number of segments in separable convex programming approximations
S_{ikm}	mth segment of quantity of commodity k supplied from region i
S^*_{ikm}	upper bound on mth segment
T_{ijk}	quantity of commodity k transported from region i to j
t_{ijk}	cost of transportation per unit of commodity k from region i to j
J_{ik}	set of regions which can be supplied with commodity k from region i
	$j \in J_{ik}$ iff $t_{ijk} > 0$
IC	commodities set
IS_k	set of regions in which supply of commodity k is modeled as a function of price
I_{jk}	set of regions which can supply commodity k to region j
	$i \in I_{jk}$ iff $t_{ijk} > 0$
Y_{jl}	production of commodity l in region j
a_{jkl}	input of commodity k per unit of commodity l, in region j
IR_l	set of regions in which production of commodity l is modeled as an input–output process
	$i \in IR_l$ iff $K_{jk} > 0$
K_{jk}	manufacturing capacity for commodity k in region j at the beginning of the period under consideration
K^+_{jk}, K^-_{jk}	expansion, reduction of manufacturing capacity for commodity k in region j during period under consideration
D_{jkm}	mth segment of quantity of commodity k demanded in region j
D^*_{jkm}	upper bound on mth segment
ID_k	set of demand regions in which demand for commodity k is modeled as a function of price
r_{ik}	maximum possible recovery of commodity k by recycling, in region i
IC^r	set of recyclable commodities
	$k \in IC^r$ iff $r_{ik} > 0$ for some i
ID^r	set of regions in which waste paper recovery is considered
	$i \in ID^r$ iff $r_{ik} > 0$ for some k
Z	consumers' plus producers' surplus in entire sector
Z_D	area under demand curves
Z_S	area under supply curves
p_{ikm}	price at which the quantity $\Sigma^m_{n=1} D_{ikn}$ is demanded, or the quantity $\Sigma^m_{n=1} S_{ikn}$ is supplied, for commodity k in region i

continued overleaf

Table 3 (continued)

Symbol	Definition
Z_{R}	total manufacturing cost, excluding the cost of inputs of which supply and transportation are explicitly recognized in the model
c_{ik}	manufacturing cost per unit of output of commodity k in region i
Z_{I}	total cost of capacity expansion
e_{ik}^{+}	cost of capacity expansion in region i to manufacture commodity k
Z_{T}	total transportation cost
u, t	time period index
IT	planning horizon
K_{jk0}	capacity of production of commodity K in region j at the beginning of the production period

final commodity is manufactured is directly proportional to the quantity produced. There is one constraint of this type for each region and commodity for which production is modeled as an input–output process.

Changes in capacity of production:

$$Y_{jk} - K_{jk}^{+} + K_{jk}^{-} = K_{jk} \quad \begin{array}{l} k \in IC \\ j \in IR_k \end{array}. \tag{3}$$

The quantity of a commodity produced in a region at the end of a specific period must equal the capacity of production at the beginning of the period plus capacity expansion, minus capacity reduction in that period.

There is one constraint of this type for each region and commodity for which production is modeled as an input–output process.

Balance between production and shipments:

$$Y_{ik} - \sum_{j \in J_{ik}} T_{ijk} = 0 \quad \begin{array}{l} k \in IC \\ i \in IR_k \end{array}. \tag{4}$$

The quantity of a commodity produced within a region must equal the quantity shipped from that region. There is one constraint of this type for each commodity and region for which production is modeled as an input–output process.

Price-responsive commodity demand:

$$\sum_{i \in I_{jk}} T_{ijk} - \sum_{m=1}^{M} D_{jkm} = 0 \quad \begin{array}{l} k \in IC \\ j \in ID_k \end{array}, \quad 0 \le D_{jkm} \le D_{jkm}^{*}. \tag{5}$$

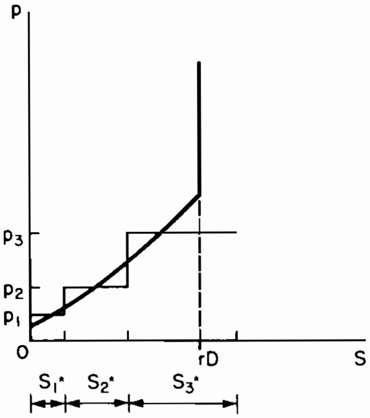

Fig. 2. The supply function for recycled paper is approximated by a step function where S_i^* represents the additional amount supplied at price p_i, D is the total amount of paper consumed, and r is the fraction of consumption that is recycled.

The quantity of a commodity transported to a region is equal to the quantity demanded in that region. There is one such constraint for each commodity and region for which demand is modeled as a function of commodity price. This is the case of the demand of all final products: paperboard and various types of paper. This representation is also used for commodities and regions for which a more detailed representation of demand is not possible or desirable. For example, it can be used for the demand for exports of raw materials or intermediate products from a country without elaborating the destination or end use of products.

Paper and paperboard recycling:

$$r_{ik} \sum_{m=1}^{M} D_{ikm} - \sum_{m=1}^{M} S_{ikm} \geq 0 \quad \begin{matrix} k \in IC^r \\ i \in ID^r \end{matrix} . \tag{6}$$

The quantity of a commodity which can be recycled cannot exceed a predetermined fraction of the quantity consumed. Within that limit the supply of recycled material is a direct function of price, as described by fig. 2. There is one constraint of this type for each recyclable commodity and for each region in which paper and paperboard recovery is considered.

Objective function:

$$\max Z = Z_{\mathrm{D}} - Z_{\mathrm{S}} - Z_{\mathrm{R}} - Z_{\mathrm{I}} - Z_{\mathrm{T}}, \tag{7}$$

where

$$Z_{\mathrm{D}} = \sum_{k \in IC} \sum_{i \in ID_k} \sum_{m=1}^{M} p_{ikm} D_{ikm},$$

is the area under the demand curve for commodities of which the demand is described as an explicit function of price;

$$Z_{\mathrm{S}} = \sum_{k \in IC} \sum_{i \in IS_k} \sum_{m=1}^{M} p_{ikm} S_{ikm}$$

is the area under the supply curve for commodities of which the supply is described as an explicit function of price;

$$Z_{\mathrm{R}} = \sum_{k \in IC} \sum_{i \in IR_k} c_{ik} Y_{ik}$$

is the total manufacturing cost, excluding the cost of inputs of which supply and transportation are explicitly represented in the model;

$$Z_{\mathrm{I}} = \sum_{k \in IC} \sum_{j \in IR_k} e_{jk}^{+} K_{jk}^{+}$$

is the cost of expanding manufacturing capacity; and

$$Z_{\mathrm{T}} = \sum_{k \in IC} \sum_{i \in I_{jk}} \sum_{j \in J_{ik}} t_{ijk} T_{ijk}$$

is the total transportation cost.

In summary, the quantity being maximized is the sum of producers' surplus plus consumers' surplus, i.e., the shaded area in fig. 1, which Samuelson [14] defined as net social pay-off. The optimization procedure is therefore similar to market mechanisms in which supplies and demands are balanced through an equilibrium price which equalizes consumers' willingness to pay and producers' incremental costs.

4. Treatment of nonlinearities by separable convex program

The first two terms of the objectives function (7) are integrands of the areas under the price-responsive demand and supply functions. Even if demand and supply equations are approximated by linear functions of prices, these two terms are quadratic functions of quantities demanded and supplied.

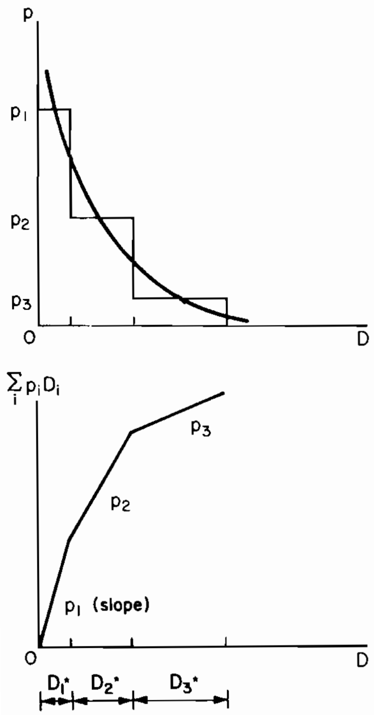

Fig. 3. Each demand curve is approximated by a step function where D_i^* is the additional amount demanded at price p_i. $\Sigma_i p_i D_i$ is the area under the demand curve.

Computer codes are available to solve mathematical programs with linear constraints and quadratic objective functions, and these have been used by several authors [8,10]. Nevertheless, quadratic programming algorithms are notably less powerful than the simplex method. Consequently, a piece-wise approximation is used.

The piece-wise approximation uses a simple method of separable convex programming [5]. For example, ignoring region and commodity subscripts, demand D during a specific time interval is described by a step function (fig. 3).

The area under the demand curve has the expression:

$$\sum_{m=1}^{M} p_m D_m, \tag{8}$$

where M segments of length D_m^* are used in the approximation, and D_m is such that:

$$0 \le D_m \le D_m^*. \tag{9}$$

There are very efficient algorithms available to solve linear programs with upper bound constraints on decision variables.

Although there is no explicit restriction on each D_m $(m > 1)$ that

$$D_m = 0 \text{ whenever } D_{m-1} < D^*_{m-1},$$

this restriction is automatically satisfied by any optimal solution of the linear programming problem because the slope of $\Sigma p_m D_m$ decreases as $\Sigma_m D_m$ increases (fig. 3).

5. Dynamic representation

The only equations of the model which tie together activities in different periods are those representing capacity expansion, because capacity added in one period is available for use in subsequent periods as well.

The model described above is made dynamic, as suggested by Bergendorff and Glenshaw [2] by (1) adding a time subscript to all variables, (2) discounting all costs and benefits so that the objective function expresses the present value of consumer plus producer surplus, (3) transforming the capacity expansion constraint into:

$$Y_{jkt} - \sum_{u=1}^{t} K^+_{jku} + \sum_{u=1}^{t} K^-_{jko} \quad k \in IC,\ j \in IR_k,\ t \in IT. \tag{10}$$

This equation states that production must be equal to initial capacity plus capacity expansion minus capacity reduction up to and including the current period.

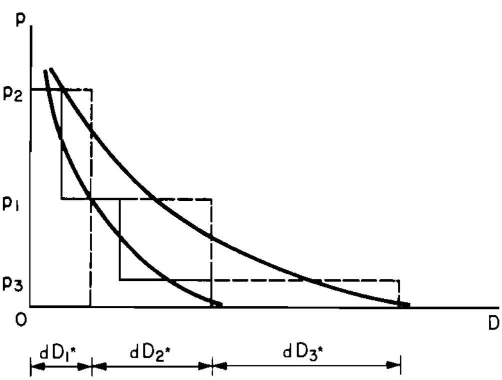

Fig. 4. A shift of a demand function approximated by a step function is simulated by multiplying each step D_i^* by the shifter d.

Shifts over time of the price-responsive demand and supply equations are conveniently handled by the piece-wise representation.

Assume for example that demand for a commodity has the following form:

$$D_t = aY_t^b P_t^c \tag{11}$$

where Y_t is income in the region of interest and P_t is commodity price. For a given price, D_t is related to demand in the base period D_0 by the equation

$$D_t = (Y_t/Y_0)^b D_0 = dD_0. \tag{12}$$

Therefore, the demand shift over the interval 0, t can be represented by multiplying each segment bound D_m^* by d. Such a shift is shown in fig. 4.

6. Paper and paperboard demand

An important part of the model described above is the set of price-responsive supply and demand equations. It is planned that most of the supply functions, at least those relating to pulpwood, will be developed by other researchers working on the 1985 U.S. Forest Service Assessment Model. The present discussion is restricted to the demand for final products: paper and paperboard. Since the results have been reported elsewhere [3], only a brief description of the work is given here.

The evidence on price-responsive demand for paper and paperboard has been obtained by pooling time-series data from different countries. This method alleviates the problems posed by short time series, multicollinearity between price and income variables within countries, and small variable fluctuations over the period of observation. Because the variation in income and price is much greater over countries than the variation in aggregates over time, the method permits the disentangling of income and price effects.

A static and a dynamic model of demand were used. The empirical results indicated specification error for the static model, and therefore only the dynamical model results are discussed here.

The demand model follows Nerlove and Addison's [11] specification:

$$C_{ikt}^* = e + fY_{it} + gP_{ikt} + hP'_{ikt} + \varepsilon_{ikt}, \tag{13}$$

where i, k, t refer to a particular country, commodity, and year, respectively. Y is income per capita, P is the price of the commodity of interest, and P' the price of the most direct substitute. C^* is the long-term equilibrium of consumption per capita which would be observed, provided that income and prices remained at a fixed level for a sufficiently long time.

All variables are expressed in logarithms; thus f, g and h are long-term income, own-price, and cross-price elasticities, respectively.

Since Y, P and P' change continuously, C^* is never observed. The hypothesis made here is that during one year, say from $t-1$ to t, consumption does not adjust totally to the level C_t^* described by eq. (13) but only to the observed level C_t such that:

$$C_t - C_{t-1} = \alpha(C_t^* - C_{t-1}), \tag{14}$$

where α is a fraction measuring the speed of adjustment. Using eq. (14), eq. (13) can be expressed in terms of observable quantities only as:

$$C_t = (1-\alpha)C_{t-1} + a[e + fY_t + gP_t + hP_t' + \varepsilon_t], \tag{15}$$

which can be estimated statistically, and from which the long-term elasticities of demand with respect to income, own-price, and cross-price elasticities can be obtained.

Eq. (15) was estimated with yearly data from 43 countries for the period 1963 to 1973. These countries consumed 85% of the world paper and paperboard production in 1973. Three groups of products were selected: newsprint, printing and writing paper other than newsprint, and other paper and paperboard. Consumption statistics were taken from the Food and Agriculture Organization [6]. Statistics on gross domestic product were provided by the FAO Commodities Division. Since consistent domestic price data were not generally available, unit value of imports and exports were used for net importers and exporters, respectively. Given competitive international markets, unit values should be good indicators of product price in each country. These statistics were obtained from the FAO [7].

The estimation of model (15) with pooled cross-section and time-series data requires some care because of the presence of a lagged endogenous variable (C_{t-1}) among explanatory variables. It has been shown [9,12,13] that ordinary least squares would in such a context lead to the coefficient of C_{t-1} being biased toward 1, and the other coefficients being biased towards 0. To resolve this difficulty, eq. (15) was estimated by a two-round generalized least squares procedure [9,12] which assumes a component of error structure in the residuals. Estimation was also done by analysis of covariance, since extensive Monte-Carlo experiments have given considerable support to that technique in estimating dynamic models with pooled cross-section and time-series data. Both techniques led to very similar results.

The main results appear in table 4. They indicate significant responses of demand to own-price changes, the responses appear especially high in low-income countries. In that table, P_t' refers to the price of other printing and writing paper in the newsprint equation and vice versa. These equations appear

Table 4
Paper and paperboard demand equations estimated from pooled cross-section and time-series data from 43 countries, 1963–1973 (C_t, Y_t, P_t and P_t' indicate the logarithms of consumption per capita lagged one year, income per capita, product price and price of direct substitute, respectively. R^2 and f are the coefficient of determination and number of degrees of freedom, respectively. Figures in parentheses are standard errors. * indicates a coefficient significantly different from zero at the 0.99 confidence level)

Product and countries group	Coefficients of independent variables				R^2	f
	C_{t-1}	Y_t	P_t	P_t'		
Newsprint						
High-income	0.16	0.70	−0.49	0.31	0.87	175
	(0.07) *	(0.19) *	(0.29) *	(0.18)		
Low-income	0.17	0.89	−0.63	0.02	0.90	157
	(0.07) *	(0.22) *)	(0.12) *	(0.11)		
Printing and writing						
High-income	0.35	1.01	−0.14	0.11	0.96	175
	(0.07) *	(0.15) *	(0.10)	(0.16)		
Low-income	0.41	0.71	−0.44	0.16	0.90	157
	(0.08) *	(0.22) *	(0.10) *	(0.10)		
Other paper and paperboard						
High-income	0.49	0.72	−0.15		0.98	177
	(0.06) *	(0.09) *	(0.04) *			
Low-income	0.56	0.73	−0.37		0.96	193
	(0.06) *	(018) *	(0.08) *			

Table 5
Long-term income and price elasticities of demand for paper and paperboard, derived from models in table 3 (Y_t, P_t and P_t' refer to gross domestic product per capita in current U.S. dollars, product price, and price of most direct substitute)

Product and countries group	Long-term elasticities		
	Y_t	P_t	P_t'
Newsprint			
High-income	0.84	−0.59	0.38
Low-income	1.07	−0.75	0.03
Printing and writing			
High-income	1.56	−0.21	0.17
Low-income	1.20	−0.74	0.27
Other paper and paperboard			
High-income	1.41	−0.29	
Low-income	1.65	−0.83	

to explain reasonably well the evolution of demand in each individual country [3].

The long-term elasticities computed from the data in table 4 appear in table 5. These are the elasticities used in the pulp and paper model to represent the slope of the final products-demand with respect to price, and to model the shift of the demand curve over time as income in each region increases.

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FOREST PLANTATIONS OF THE TROPICS AND SOUTHERN HEMISPHERE AND THEIR IMPLICATIONS FOR THE ECONOMICS OF TEMPERATE CLIMATE FORESTRY

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A broad conceptual framework for explaining many of the fundamental changes that are occurring in the nature of the world's forest resource is developed. Within this context, the role of the industrial forest plantation is analysed; and some results of an assessment of the comparative economics of plantations in twelve different world regions are presented. The implications of tropical plantations for the economics of the Nordic forests are examined and alternative strategies discussed.

1. Fundamental changes

Within the global context, some important recent trends in the nature of forest resources should be noted. First, the role of the traditional Northern Hemisphere temperate-climate producer regions has been declining over time for both industrial and nonindustrial wood. In the developing world the pressure of increasing population alone appears to guarantee greater utilization of wood for fuel and other nonindustrial purposes. The rising real prices of fossil fuels are certain to exacerbate this tendency. With regard to industrial wood, the developing regions of the world are putting increased pressure upon their own forest resources to feed the economic expansions that are occurring. In addition, where available, the forest resources of the developing regions are being increasingly utilized and exported to the major world markets.

These trends are suggestive of a growing worldwide demand for both industrial and nonindustrial roundwood and an increased role that the non-traditional producing regions are likely to play in meeting the increasing demand.

1.1. The forest transition

Forestry today is experiencing a transition similar to that which occurred in agriculture much earlier in human history. Just as mankind had progressed from gathering and hunting to cropping and livestock raising, we are today experiencing a transition from having our wood needs met from "old growth" naturally created forest inventories to a situation in which conscious decisions are made to plant, manage, and harvest a forest. Just as modern agriculture involves decisions as to location, crop type, technological inputs, and management mode, so too, forestry decisions increasingly involve questions of forest plantation location, species choice, technological inputs, and management regime.

In Europe, this transition is essentially complete. However, in many parts of the world the transition is still in process. Canadian production continues to rely heavily on "old growth" forest, while the United States finds some regions where man-made and second-growth forests dominate, e.g., the South, while other regions continue to draw heavily upon their inventories of "old growth" forests. In Southeast Asia the large inventories of "old growth" tropical hardwoods continue to provide wood products to much of the world.

Much of this transition phenomenon can probably be best explained in terms of a simple stock-adjustment model (Lyon 1981) that can be applied either to a particular forest (given qualifications) or to the global forest. In terms of this model, initially the actual stock of forest resources was in excess of the desired stock. Since the actual stock of existing timber, the old growth, exceeded the desired stock, the economically rational policy was simply to draw the old growth (actual stock) without any serious consideration being given to the establishment of new stands. In such a world, the initial price of stumpage would be at a very low level, approaching zero, and there would be no economic incentive to invest in tree growing. In fact, the stumpage price was often negative, since the timber resources had no economic value but were merely an obstacle to the use of the land in an alternative pursuit. Gradually, however, an adjustment was occurring. Increases in demand, together with a reduction in old growth stocks, brought desired and actual forest stocks into closer relationship. As this occurred, stumpage prices would be expected to exhibit real price increases. The historical occurrence of this phenomenon is well documented (Manthy 1978).

As long-term increases in the real price of timber resources occur and expectations develop that such price increases will continue, the economic incentives are created to induce investments in industrial forest plantations.

1.2. Industrial forest plantations

Reflective of increasing economic incentives, there is growing interest and increased activity throughout the world in the creation of man-made forest plantations. By 1975, over 3% of the world's forested area, or 90 million hectares, was man-made forest, although much of this would not be characterized as industrial forest (FAO 1978). Of this forested area, over 6.5 million hectares were in industrial plantations located in the tropics and the Southern Hemisphere. FAO projections forecast this figure to increase to over 21 million hectares by the year 2000 (Lanly and Clement 1979).

Major industrial forest plantation activities are under way in New Zealand, Australia, and South Africa. This trend is particularly powerful in South America. Many of the countries on that continent have some form of substantial industrial plantation development under way. In Brazil, some 3.5 million hectares have gone into industrial plantation since the early 1960s and an additional 300 000 to 500 000 hectares is planted annually. Chile currently has over 700 000 hectares of industrial plantation and is adding to these at about 70 000 per year. Venezuela is developing a vast forest plantation complex on previously unused interior lands. Similar activities are under way in other countries.

The implications of modern plantation forestry are potentially profound. First, forest resource production is becoming and will continue to become more like an agricultural crop. Second, and perhaps equally important, the location of industrial forests will gradually change. When relying on old growth for industrial wood, the critical concerns include the size of the inventory. For the future more and more industrial wood will be provided by plantations whose locations are the result of a conscious investment decision. Time is a critical component of any investment decision, and hence sites and species that permit rapid biological growth associated with tropical and semitropical regions will have an inherent economic advantage. Third, actual changes in forest-product trade flows are being observed (Francescon et al. 1983), and the potential of tropical and semitropical forest plantations to significantly affect major world markets and thus impact on traditional producers before the year 2000 appears substantial (Sedjo 1980).

2. The plantation study

This section summarizes the findings of a two and one-half year study of industrial forest plantations in twelve regions around the globe (Sedjo 1983). Nine of these regions are in the tropics and Southern Hemisphere: the Amazon, Central Brazil, Southern Brazil, Chile, West Africa, South Africa, Southeast Asia, Australia, and New Zealand. In addition, three regions in the temperate climate of the Northern Hemisphere are examined – the Pacific Northwest of North America, the U.S. Southeast, and the Nordic region in Europe.

2.1. The model

The methodology employed is described in detail elsewhere; therefore it will be described only very briefly here. The basic approach is economic and develops a simulation model which ties the output of the various plantations into one of the world's major market areas – Europe, the Eastern United States, and Japan. While the two final products examined are wood pulp and lumber, the model is designed to examine the economics of the forest resource rather than the economics of processing. Processing occurs in the same region as the plantation is located, with the processed product shipped to major markets. These major markets are integrated to the degree that price differentials greater than those attributable to transportation costs will result in a restructuring of forest-product flows from the low-price markets to the high-price markets. Market prices for the final product are determined exogenously. For each region a representative plantation is constructed – that is, a plantation of optimal size which embodies the biology and other features common in that region.

For each of the twelve regions, the following system of equations is used: Regional pulpwood stumpage price is given by

$$P_{\mathrm{sp}} = \alpha P_{\mathrm{p}} - \left(T_{\mathrm{p}} + M_{\mathrm{p}}\right) - H,$$

$$T_{\mathrm{p}} = \beta_0 + \beta_1(D),$$

where:

P_{sp} = price of pulpwood stumpage, \$/m^3
P_{p} = price of woodpulp in world market, \$/m.t.
T_{p} = international transport costs, \$/m.t.
M_{p} = woodpulp processing costs, \$/m.t.
H = harvest and local transport costs \$/m^3
α = conversion coefficient, m.t./m^3
D = distance from plantation region to market in thousand miles

$$H = \begin{cases} \overline{H}_{\mathrm{t}} & \text{if commercial thinning} \\ \overline{H}_{\mathrm{f}} & \text{if final harvest cost.} \end{cases}$$

Regional sawtimber stumpage prices are determined by a somewhat more complex set of equations (not presented here) that recognize that sawtimber stumpage prices reflect both the lumber and pulpwood values of the log as well as volume losses which occur in sawmilling.

The world stumpage market is represented by the following equations:

$$P_{\mathrm{spw}} = \alpha P_{\mathrm{pe}} - \alpha\left[\beta_0 +_1 (D_1) + M_p\right] - H,$$

$$P_{\mathrm{pu}} = \frac{1}{\alpha}\left[P_{\mathrm{spw}} + H\right] + \beta_0 + \beta_1(D_2) + M_p,$$

$$P_{\mathrm{pe}} = \frac{1}{\alpha}\left[P_{\mathrm{spw}} + H\right] + \beta_0 + \beta_1(D_3) + M_p,$$

where:
P_{pe} = woodpulp price in Europe, \$/m.t.
P_{spw} = pulpwood stumpage price, Pacific Northwest, North America, \$/m^3,
P_{pu} = woodpulp price, Eastern United States, \$/m.t.
P_{pj} = woodpulp price, Japan, \$/m.t.
D_1 = shipping distance between Pacific Northwest and Europe, thousand miles
D_2 = shipping distance between Pacific Northwest and Eastern United States, thousand miles
D_3 = shipping distance between Pacific Northwest and Japan, thousand miles

The world market price for lumber is determined in a similar manner.

Finally, the net present value per hectare (NPV) is the investment criterion used to evaluate the economic viability of prototype plantation

$$\mathrm{NPV} = \sum_{i=1}^{\infty} \frac{(R_i - C_i)}{(1+r)^i},$$

$$R_i = P_{\mathrm{sp}i} V_{\mathrm{p}i} + P_{\mathrm{ss}i} V_{\mathrm{s}i} = \text{gross receipts per hectare in period } i,$$

$$C_i = \overline{C}_i = \text{gross management costs per hectare in period } i.$$

where r = inflation-free discount rate.

Some of the preliminary results of the simulation study appear in table 1, which presents the NPV per hectare for the twelve regions under examination for what is called the "base case".

It will be noted that the prototype forest plantations of the Nordic region have relatively low NPVs, the lower among the regions examined. In the base case the negative sign indicates that, at a 5% real discount rate, the discounting costs exceed the discounted benefits.

Important determinants of economic returns in the simulation model are stumpage prices, biological growth and yields, rotation lengths, and silvicultural costs. To understand fully the implications of the model, we must comprehend how changes in these important determinants affect the estimates of NPV.

Stumpage prices are also important determinants of economic viability. In general, stumpage prices will decline as the distance between the resource and the final market increases, since the greater distance implies higher transportation costs.

Cost obviously affects the economic viability of any investment. Three types of costs can be identified for plantations. The first type has been referred to as development costs, and may include the costs of land purchase as well as the

Table 1
Representative plantations: base case net present value: 5% discount rate. 1979 constant prices (1979 U.S. $/hectare)

Region/species	Regime	
	Pulpwood	Integrated, with sawtimber
North America		
U.S. South		
Pinus taeda, avg.-yield site	1748	2474
Pinus taeda, high-yield site	2830	3742
Pacific Northwest		
Pseudotsuga menziesii, avg.-yield site	616	902
Pseudotsuga menziesii high-yield site	1077	2248
South America		
Brazil, Amazonia		
Pinus caribaea	3087	4080
Gmelina spp.	2530	3184
Brazil, Central		
Eucalyptus spp.	3456	4027
Brazil, Southern		
Pinus taeda	3592	4715
Chile		
Pinus radiata	3649	4509
Oceania		
Australia		
Pinus radiata	2005	2141
New Zealand		
Pinus radiata	2903	4118
Africa		
South Africa		
Pinus patula	3051	3727
Gambia-Senegal		
Gmelina spp.	2289	2622
Eucalyptus spp.	1828	3262
Europe		
Nordic		
Picea abies	−100	154
Asia		
Borneo		
Pinus caribaea	1851	2364

costs associated with access, infrastructure development, and other costs necessary to allow for the establishment and operation of an industrial plantation. These costs may vary considerably, depending upon such factors as the location of the plantation and the types of infrastructure that are already in place. Of course, in a well developed market the excess of returns over costs will be capitalized in land prices. The second type of cost is referred to as establishment costs, incurred in the site preparation and planting associated with stand establishment in the first year of the rotation. The third set of costs is referred to as subsequent costs and is composed of those that occur subsequent to plantation establishment.

Establishment costs have a close relationship with estimates of the NPV obtained. Since the NPV represents the difference between discounted benefits (receipts) and discounted costs (expenditures) in the initial year of the investment, any increase in establishment costs will simply be added to the discounted costs (since this cost occurs in the initial year, no discounting is necessary) and thereby reduces the NPV accordingly. Since each dollar added to establishment costs will reduce the NPV by a corresponding dollar, a more complex sensitivity analysis is unnecessary.

The effect of subsequent costs is similar to that of establishment costs except that, since they are incurred in later years, the cost will be discounted back to the initial period. Therefore, a one-dollar increase in subsequent costs will reduce NPV by something less than one dollar, depending upon the point in time at which the cost is incurred and the discount rate that is used.

Table 2 summarizes the results for our twelve study regions with respect to rotations, yields, and harvest volumes. The rotations vary in length from seven years for the gmelina and eucalyptus pulpwood rotations in Brazil to eighty years for a sawtimber rotation in the Nordic region.

Generally, the short rotation plantations perform well under the economic criteria, while the very long rotations do relatively poorly. It should be noted, however, that some of the better performers have longer rotations. These include Chile, New Zealand, the U.S. South, and others. Thus, while rotation length is positively correlated with economic performance, other factors also influence the final economic performance.

Biological yield is also a factor to be considered when analyzing the elements that contribute to a strong performance on the investment criteria. Average annual yields range from 25 cubic meters per hectare per year in the Central Brazilian eucalyptus plantation and in the pine of New Zealand, to 5 cubic meters per hectare per year in the Nordic plantations. However, some plantations with lower yields perform better by the economic criteria than the higher yield plantations, e.g., Southern Brazil and Chile. In addition, while the Pacific Northwest high-site integrated plantation has a relatively high yield of 14.4 cubic meters per hectare per year, the economic performance of the Pacific Northwest plantations, as reflected in the investment criteria, is clearly inferior to several other prototype plantations with lower yields.

Table 3 summarizes the cost of establishment and subsequent management costs short of harvest. The lowest establishment costs are found in South Africa, followed by Chile and the gmelina plantations of Amazonia. For the one-cycle plantations, the high-cost regions are Australia, the Pacific Northwest of North America, and Southern Brazil. Costs subsequent to establish-

Table 2
Representative plantations: rotation, growth, and yields. (prices in 1979 dollars)

Region/species	Pulpwood regime			Integrated (sawtimber) regime		
	Rotation age (y)	Mean annual increment (m^3/ha/y)	Total commercial yield (m^3/ha)	Rotation age (y)	Mean annual increment (m^3/ha/y)	Total commercial yield (m^3/ha)
North American						
U.S. South						
Pinus taeda, avg.-yield site	30	11.9	356.7	35	12.4	433.9
Pinus taeda, high-yield site	39	17.9	538.0	35	17.7	620.2
Pacific Northwest						
Pseudotsuga menziesii, avg.-yield site	40	12.7	510.0	50	14.4	721.0
Pseudotsuga menziesii, high-yield site	30	13.6	408.0	40	20.0	801.0
South American						
Brazil, Amazonia						
Pinus caribaea	12	16.0	192.0	16	16.0	256.0
Gmelina spp.	7 [a], 13, 18	18.0	342.0	12	18.0	216.0
Brazil, Central						
Eucalyptus spp.	7 [a], 13, 18	25.0	475.0	19	25.0	475.0
Brazil, Southern						
Pinus taeda	12	20.0	240.0	20	20.0	400.0

Chile						
Pinus radiata	25	22.0	555.0	32	22.0	703.3
Oceania						
Australia						
Pinus radiata	29	20.7	600.0	35	17.4	609.0
New Zealand						
Pinus radiata	18	25.0	450.0	27	25.0	675.0
Africa						
South Africa						
Pinus patula	15	16.1	241.5	26	16.0	416.0
Gambia-Senegal						
Gmelina spp.	10 [a]	15.0	450.0	20, 40	15.0	600.0
Eucalyptus spp.	7 [a]	17.0	357.0	10, 20, 30	17.0	510.0
Europe						
Nordic						
Picea abies	50	5.0	250.0	80	5.0	400.0
Asia						
Borneo						
Pinus caribaea	15	14.0	210.0	20	13.5	270.0

[a] Involves multiple harvesting from a given root system based upon coppicing after the first planting.

Table 3
International plantations: management costs (1979 U.S. $/hectare)

Region/species	Pulpwood regime		Integrated regime	
	Initial costs	Subsequent costs	Initial costs	Subsequent costs
North America				
U.S. South/Loblolly pine	266	139	266	149
U.S. South/Loblolly pine/High Site	266	139	266	149
PNW/Doug.Fir/Short rotation/rvsd base	558	432	558	457
PNW/Doug.Fir/high site	558	420	558	445
South America				
Amazonia/pine	277	370	277	430
Amazonia/gmelina [a]	145	810	145	375
Brazil/central/eucalyptus [a]	523	547	523	647
Brazil/south/pine	523	382	523	502
Chile/radiata	176	92	176	114
Oceania				
Australia/radiata	651	173	651	192
New Zealand/radiata	456	445	456	567
Africa				
South Africa/P.patula	184	195	184	318
West Africa/gmelina [a]	350	250	450	265
West Africa/eucalyptus [a]	961	240	961	355
Europe				
Nordic/Norway Spruce	456	126	456	156
Asia				
Borneo/pine	277	415	277	490

[a] 3 harvest per rootstock.

ment also vary considerably, with the lowest (undiscounted) costs being experienced in the U.S. South, Australia, and Chile. High subsequent undiscounted costs are experienced in Brazil and the Pacific Northwest. The Nordic costs are toward the middle of the range examined in this study.

There is no clear, simple relationship between establishment and subsequent costs. High establishment costs are found among the strongest and weakest performers on the economic criteria. A similar lack of a clear relationship with NPV also applies to (undiscounted) subsequent costs.

2.2. Other considerations

In addition, there are considerations that should be introduced into the analysis that were not included in the formal model. First, the development costs required to begin to undertake a large-scale forest plantation may be massive and were not explicitly built into the analysis. It is clear that in the

Nordic region, as well as most of North America and some other regions, development costs are likely to be modest. By contrast, in some regions in the tropics the development costs are likely to be great.

An example of this problem is found in the famous Jari forest plantation in the Brazilian Amazon. The simulation study suggests that for this plantation, the economics of planting, growing, harvesting, processing, and shipping to European markets as woodpulp no longer appear favorable. However, this assessment does not consider the initial development costs, which were literally hundreds of millions of dollars for developing the infrastructure labor force, and so on, necessary for the plantation to function. These costs can radically alter the economic feasibility. The point here is simply that while the underlying biology and location of a region such as the Amazon may appear favorable for industrial forest plantations, development costs for primitive regions may be so great as to offset the economic advantages inherent in the region's biology and nearness to markets.

Second, the risks inherent in any long-term forest investment may vary systematically by region. The risk may be of at least three types – biological, economic, and political. Biological and economic risks are related to uncertainty associated with biological yields and the assurance of markets both after one rotation and into the future. Political risk relates to the question of stability in the political environment to allow the investor to see the investment through to maturity and realize the returns on the investment. It should be noted that the same political environment may present quite different risks to different investors; for example, the risk to a foreign private investor may be much greater than to a publicly owned domestic corporation.

To the extent that the risks vary by region, the prudent investor, whether private or public, will require an increased return premium to compensate for the risk. In an investment analysis, the premium required for risk could be explicitly incorporated by adding a risk component to the normal discount rate. Such a discount rate thereby explicitly recognizes the opportunity cost of capital and also any excessive risk that may be associated with an investment in the particular region. The higher discount rate requires that the project generate sufficient returns to compensate the investor for the higher risks associated with the project.

Third, the opportunity costs or alternative uses to which the potential forestlands could be put are often a relevant consideration. Some lands in the U.S. South, for example, have alternated between use as agricultural land and forestland. Should agricultural prices rise, economic theory would predict that some lands would shift out of forest into agricultural use.

2.3. The effects of tropic forest plantations upon the competitive position of temperate climate forests: The Nordic Case

Within the context of the plantation model as developed, the great strength of the Nordic forests lies in the high stumpage prices which are the result of favorable geographic location vis-à-vis the important world markets and the relatively risk-free environment, especially with respect to political and biological factors.

The Nordic countries enjoy the advantage of higher stumpage prices largely due to their proximity to the major European markets. To the extent that international transport costs are everywhere high, Nordic industry benefits from the protection against alternative, more distant supplies that these costs provide. Should international transport costs everywhere decline, this protection would erode. The relatively risk-free biological environment reflects the long-term experience of industrial forests in this region. Low political risk reflects the political stability of countries in this region.

The Nordic region's liabilities are found primarily in the very long rotation periods required and the low biological yields. Of course, improved silvicultural practices would be expected to increase yields and perhaps to reduce rotation lengths. (The economic effect is the same if the result of management is to improve quality and hence value and economic yield.) However, the costs also would be expected to rise. The economic question is whether the additional future returns are justified by the additional current costs.

Given these considerations, it appears that while the results of the study might somewhat overstate the competitive advantages of other regions relative to the Nordic region, nevertheless several of the regions examined have the economic *potential*, perhaps not yet fully realized, to compete very effectively with the Nordic producers of woodpulp and lumber in the European market.

3. Some policy options

There are a number of strategic options available to the temperate-climate forest industry to meet the competitive challenge of the tropics. In this section some of these are briefly explored and their implications examined.

3.1. Continuation of the status quo

The simulation study cited above suggests that the rates of return being generated to investments in forest plantations under the base case are above 4% in real terms in the Nordic countries and higher in other temperature forests. Although not high, such a rate of return may be acceptable, given the alternative uses available for its land capital and labor. Should real stumpage prices increase slowly, at a rate below that experienced for decades in North America, the effect would be to improve further the rate of return on forest investments. Therefore, a policy for the continuation of the present industry strategy and policies in the Nordic countries might be acceptable if real stumpage prices are expected to remain constant or to rise. Such price expectations are reasonable except if one anticipates massive inflows of tropical wood.

In addition, to the extent that some costs are systematically lower in temperate regions than in competing regions, the rationale for a continuation of policies similar to those of the recent past is enhanced.

3.2. Modest retrenchment

The options available to the Nordic forest industry are unlikely to be all-or-nothing. That is, under more favorable conditions one might expect some expansion of the industry, under less favorable conditions one might expect some industry contraction.

Thus, the adjustments in the industry are likely to be of an incremental nature and to occur on the economic margins. The Nordic countries are likely to continue at a high level of forestry activity indefinitely. The rationale is that the opportunity costs of most of the Nordic forestland are low and that forests are likely to grow on most of these lands even in the absence of intervention by man. Therefore, low-cost intensive management appears to be a viable economic alternative. Given the presence of an infrastructure and a hospitable terrain, the harvest and international transport costs should not become prohibitive.

In this context, some reduction of the level of the activity of the industry would be desirable if long-term rates of return became depressed as the result of rising competition from new supply regions. The forest industry could then concentrate its efforts upon the best growing regions with the best (low-cost) access and harvesting. Under this scenario, the better sites might continue to receive intensive management, while lower-class sites might even enhance their economic returns if they were managed less intensively. At lower stumpage prices it would be expected that the overall level of management intensity would decline. Gradually older and less efficient mills would be phased out and some perhaps not replaced, as the industry restructured to respond to its new competitors. Within this context the decision as to the relocation of capital within the Nordic economies would be determined by the economic returns. If the returns to capital were higher in other industries, capital would be free to be used there rather than in the forest industry. Labor would, of course, be allocated toward the expanding industries at the expense of the declining industries.

3.3. Reliance on alternative wood supply sources

It is generally agreed that the most efficient method of producing pulp over the long term is for the pulpmill to be located near the forest resource. The per-volume costs of transporting wood as woodchips are about twice those of transporting wood in the form of woodpulp (Eklund, 1977). Calculations using the simulation model confirm this and, for the base case, it will not generally be economically rational on a large scale for the Nordics to import woodchips from any of the regions under examination, since the opportunity cost of the wood is higher in local processing than it would be after shipping it as woodchips to Nordic ports. However, should the costs of woodpulp processing in the tropics and Southern Hemisphere be significantly higher than in the Nordic regions, thereby substantially changing relative stumpage prices, a system whereby a certain portion of the wood feedstock of the Nordic pulpmills could be provided by distant plantations may provide the economically most rational means for producing woodpulp.

This situation is similar to that of the projections of the Randers/Lönnstedt Scandinavian Forest Sector Model (1979) where domestic pressures cause stumpage prices to rise precipitously and thereby contribute to the instability of the forest industry system. While the Randers/Lönnstedt model explicitly precludes increased importation of foreign wood by assumption, the plantations simulation model examined the possibility of wood imports and estimated the stumpage price differentials at which the Nordic forest industry would find it profitable to begin to import woodchips from various foreign regions. For example, one scenario of the plantations model developed above suggests that if Brazilian pulpwood stumpage prices were 80% or less of Nordic stumpage prices, comparable quality woodchips imports from Brazil would become economically viable since international transport costs are about 20% of the Nordic stumpage price. Thus, under some conditions the efficient forest industry strategy would be one of increasing reliance upon inexpensive foreign wood to supplement domestic supplies.

4. Summary and conclusions

As the world's old-growth forests have gradually been utilized for their industrial wood resource, the world's future forest resource requirements are increasingly being met by man-made forest plantations. Rising real prices for the wood resource have provided economic incentives to undertake investments in man-made industrial plantations. The choice of location for new forest plantations is no longer confined to regions that have traditionally been important forest-product producers, but can now consider regions that have particularly favorable biological and/or locational features. A simulation study has indicated that the economics are quite favorable for plantation investments in many regions that have not been traditional forest product producers, including regions of the Southern Hemisphere. Recent experience with the establishment of plantations and Latin American exports tend to confirm these results.

Temperate forests have some important competitive advantages in the production of forest products, including a very favorable location vis-à-vis the important world markets. However, long rotations and low biological yields result in the Nordic region's relatively poor comparative performance, using common investment criteria. Other considerations, such as development costs and risk, while not formally introduced in the model, must also be considered.

Three alternative strategies for the Nordic forest sector were examined. These are (a) the continuation of the current strategy, (b) a modest retrenchment, and (c) the reliance on alternative wood supply sources. All of them may be viable.

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Part II

NATIONAL AND REGIONAL ANALYSIS

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A SPATIAL EQUILIBRIUM MODEL OF U.S. FOREST PRODUCTS MARKETS FOR LONG-RANGE PROJECTION AND POLICY ANALYSIS *

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The Timber Assessment Market Model (TAMM) was developed to facilitate long-range planning and policy analysis in the U.S. forest products sector. TAMM provides an integrated structure for considering the behavior of prices, consumption, and production in both stumpage and product markets. A competitive spatial equilibrium system is used to model markets for solid wood products (lumber and plywood). The paper describes the origins, structure, and simulation methods used in TAMM. Results of a baseline projection and a simulation of increased management intensity on private lands are presented to illustrate model applications.

0. Introduction

In both the public and private sectors, long-range projections of forest products markets and the condition of the resource base are essential tools for planning resource development. In the United States, organized efforts began a century ago to assess the state of the forest resource and to forecast its future use and condition. During the past three decades these "timber assessments" have become an integral part of U.S. Forest Service planning. Both the Forest and Rangeland Renewable Resource Planning Act (RPA) of 1974 and the National Forest Management Act (NFMA) of 1976 emphasize and strengthen the role of the Timber Assessment. The RPA, in particular, recognizes the problem of attempting to coordinate the short-term (annual) budgetary process of a public forestry agency and long-term goals and concerns in the forestry sector. Under the mandate of the RPA, the Forest Service develops decennial

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Timber Assessments which are used both to establish individual and agency perceptions about future markets and resource conditions (i.e., to identify potential policy issues) and as a means to evaluate the effectiveness of alternative policies on these emerging problems.

During the 1970s the projection methodologies employed in the Timber Assessment evolved from simple trend extrapolations of consumption, production, and price to econometric models that project supply and demand equilibria in both space and time. This paper discusses the modelling methodology employed in the most recent Timber Assessment for making long-term projections of future U.S. product and stumpage market activity and for evaluating the potential impacts of alternative private and public policy programs on resource use. A detailed discussion of model estimation, validation, and example results is given elsewhere [1].

1. Considerations in the development of a revised timber assessment model

Past effort to make long-range forecasts of timber production, consumption, and prices in the U.S. have relied on a process of independent projection of quantities "demanded" and "supplied" at prespecified future price levels (see, for example, ref. [8]). The intent in this approach was to approximate points on the actual demand and supply curves for timber as they shift through time. In real world markets, of course, prices will move so as to bring about equilibrium between quantities demanded and supplied. Thus, a projection of "demand" greater than "supply" for some future time period implies a price higher than that initially assumed in the analysis, while "supply" greater the "demand" implies a lower price. This approach has been termed "gap analysis," after the inevitable gaps between projected "demand" and "supply" levels.

While gap analysis played an important role in earlier national Timber Assessments, it has at least four serious faults. An evaluation of these problems was an important first step in developing the 1980 Timber Assessment model.

(1) The demand quantity projections in gap analysis are nothing more than trend extrapolations of past consumption levels, there being no explicit linkage between the quantity forecast and the assumed prices. Similarly on the supply side, quantities were projected with primary concern for basic forestry precepts such as cut/growth balance in the long-term rather than economic supply behavior. As a consequence the price inferences drawn from gap analysis are suspect. The relationships between prices and quantities consumed and produced must be clear and consistent in a long-range projection model. Models should be of a positive rather than normative nature, if projection results are to have broad credibility. Introduction of conditions, such as long-term growth/cut balance, should be treated as a policy analysis where such conditions are not already legally or socially mandated.

(2) Gap analysis focuses attention on quantity imbalances and establishes the initial price assumption as a standard. These restrictions limit the utility of projection results to policy analysts. Over the past decade, policy discussions have come to focus primarily on price as the measure of resource scarcity or abundance rather than quantity. As a consequence analysts need to know the specific levels and rates of price change resulting from a particular policy not simply that prices will be above or below some arbitrary datum. Further, any standard for acceptable price levels or rates of price growth will be established in the policy deliberation process and should not be introduced merely as an antifact of the projection process.

Application of gap analysis in past national Timber Assessment has also entailed a high degree of aggregation in the projections. This gives rise to two additional criticisms.

(3) In past national studies, quantities demanded and supplied have been defined in terms of total wood consumption (domestic use plus exports) and total timber availability (domestic harvest plus imports). In this way it has been impossible to judge whether prices of specific products (such as lumber, plywood, pulp, etc.) will be changing at the same or different rates. The substantial diversity in market structures and hence in market behavior for individual product categories is ignored in this aggregation. Since the analysis concentrates on end product demand and stumpage supply, the economic processes of *product* supply, i.e., conversion of stumpage to logs and ultimately to finished products, is also ignored. Additionally, no direct inferences can be made from the analysis about conditions or price movements at different market levels. Specifically, developments in resource markets (stumpage prices or log prices, for example) cannot be directly determined from the analysis. The future behavior of consumption, production, and prices of different products and of stumpage will affect widely different groups in society. Consequently, policy makers must have a less aggregative projection of future market behavior if they are to judge the important equity implications of alternative policies.

(4) A similar concern arises with regard to the high degree of geographic aggregation in past Assessments. The "gap" has generally been computed at the national level. Projections of regional developments (stumpage prices, product output, etc.) have not been available. This is a serious concern in light of the obvious significance of regional impacts in policy evaluation. Lumping all regions together also leads to a gross misrepresentation of market behavior. Production and consumption of forest products is not homogeneously distributed across the U.S. in a geographical sense. Production is concentrated in the South, Rocky Mountains, and Pacific Coast areas while consumption is concentrated in the North, South, and Pacific Southwest. Behavior of product and stumpage markets in producing regions is strongly influenced by the marketing advantages or disadvantages these regions face precisely because of

their location relative to consuming regions. A credible long-term projection model must recognize these spatial phenomena.

Finally, beyond the lessons learned from failures of the gap model, two other unique features of the U.S. forest products economy had a strong influence on the form and structure of the 1980 Timber Assessment model. In the U.S., production is oriented almost entirely toward filling the large domestic demand. Export markets play a relatively small role, unlike other major forest products producing countries. Imports, on the other hand, are significant in certain products, particularly from Canada. This suggests that the model should contain considerable detail on domestic trade and on certain import flows, with less emphasis on exports and other world markets. A further feature of significance is that forest management activities on private lands in the U.S. (harvesting, planting, and the whole array of intensive silvicultural practices) are only very loosely regulated by the public sector. Management on private forest lands is primarily a function of individual owner objectives. Prices of stumpage and costs of management have a significant influence on behavior of industrial owners, given their fairly consistent objectives of economic gain. Factors affecting behavior of non-industrial owners, however, are much less clear. As a consequence, a long-range projection model must contain a fairly elaborate structure to describe behavior at the stumpage level. The model must explain both current levels of timber harvest and rates of private investment in silvicultural practices.

2. Structure of the 1980 Timber Assessment Market Model

Guided by the foregoing discussion of deficiencies in the gap model and important structural considerations in U.S. markets, a revised projection model, termed the Timber Assessment Market Model or TAMM, was developed for the Forest Service's 1980 RPA Timber Assessment [9]. The broad objective was the construction of a model that could be used for making projections and evaluating a broad range of alternative policies over a 50 year projection period. The principal concern was with the identification of broad trends in market activity rather than short-term cycles. Thus, for the product markets where it was appropriate, a simple supply–demand framework was employed, utilizing annual time series data for estimation purposes. Spatial interactions in product markets was given explicit consideration.

An overview of the TAMM system is given in fig. 1. For lumber and plywood, demand equations were developed for each of seven demand regions; supply equations were developed for seven supply regions including Canada. Spatial equilibrium in these markets is determined by a process that explicitly considers transportation costs. Consumption of paper and paperboard products was projected using income–consumption relations for the entire U.S.

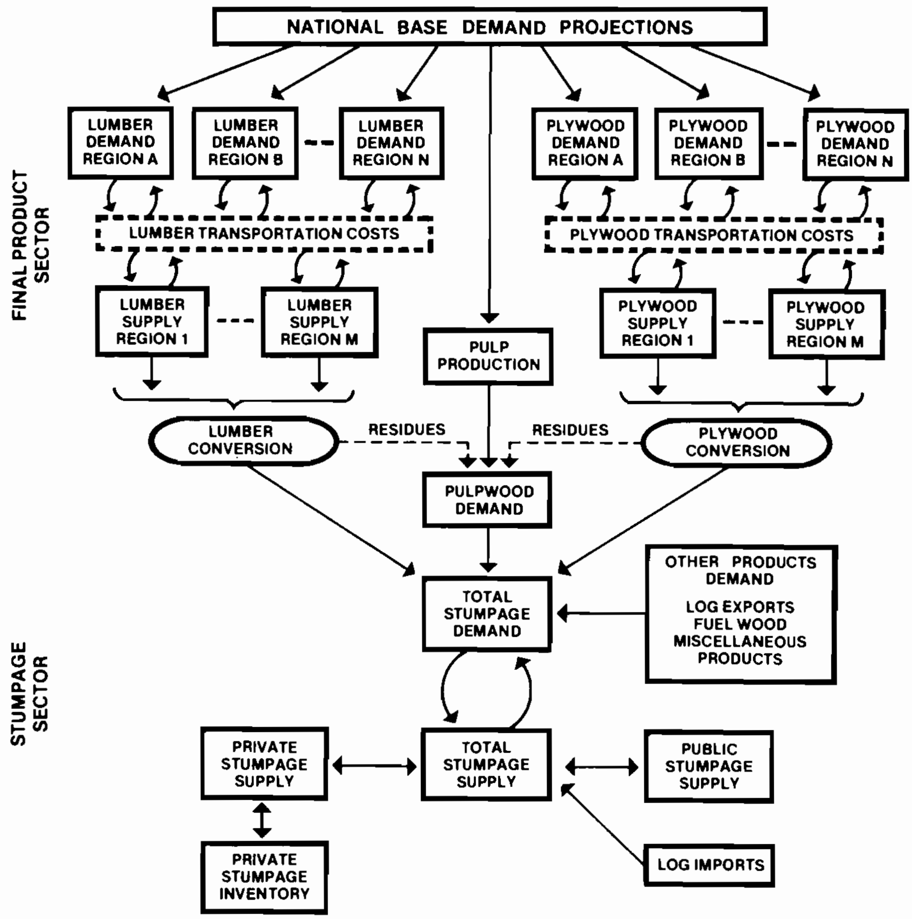

Fig. 1. Major interactions in the Timber Assessment Market Model. Curved arrows indicate locations of market price–quantity determination. Single or two-headed straight arrows indicate uni-directional or multi-directional causation. Detail in stumpage sector shown for only one region.

After adjustments for imports, projections of domestic pulp production were developed by supply region. Projected output of fuelwood, miscellaneous products, and log exports in each supply region were based on estimates of future consumption and trade in these products developed outside of the model. Total demand for stumpage in each supply region was derived from conversion of product supply volumes (for lumber, plywood, pulp, etc.) to roundwood equivalents. Supplies of stumpage consist of public harvests set by policies of federal and state agencies, and private harvests which are responsive to both stumpage price and available inventory volumes. Equilibrium in each regional stumpage market is determined by simple supply–demand intersection. Timber inventory volumes by ownership and region are projected over time using the TRAS model [4].

Boundaries of supply and demand regions in the model do not, in general, coincide. Demand regions were defined based on major geographical concentrations of forest products consumption. Lumber and plywood demand equations were estimated for these regions. Supply regions encompass principal concentrations of forest products output. Lumber and plywood supply equations, pulp production estimates, and all stumpage market activities are developed for each of the supply regions.

2.1. Specifying and solving the framework for lumber and plywood

Of the several major classes of forest products, lumber and plywood are the commodities whose markets in the U.S. are best represented by the classical competitive model. The numbers of buyers and sellers of these commodities are very large, while the market share of even the largest buyers and sellers is very small. Barriers to entry or exit are minimal and both consumption and production move freely in response to shifting prices or production costs. Supply and demand relationships were developed as appropriate for each supply and demand region, expressing price as a function of quantity and an array of predetermined shifters.

Solutions to these systems of supply and demand relations in competitive markets can be obtained in several ways. Samuelson [5] first showed that the process of finding a spatial equilibrium could be viewed as a maximization problem in which market forces act so as to maximize the sum of the areas between the demand and supply equations of all regions less the total transport costs for all shipments. The first order conditions for a maximum of this objective are twofold: (1) where trade does exist between two regions, the difference between unit costs in the supply region and unit price in the demand reion must equal the unit transportation cost between the two; and (2) where no trade takes place between two regions, the difference between demand region unit price and supply region unit cost must be less than unit transport costs. In practical applications, solutions may be found (using Samuelson's basic principle) by means of linear programming. In this case the supply functions may be represented by resource constraints or approximated together with the demand functions in a piecewise fashion by adding additional activities (see, for example, ref. [2]). Where demand and supply functions are linear, or approximately so, Takayama and Judge [6] have shown the applicability of quadratic programming in directly solving Samuelson's maximum problem. An alternative approach, and the one adopted in TAMM, has been developed by Tramel and Seale [7]. Termed "reactive programming," it employs an iteration approach to determine equilibrium. Assuming continuous demand and supply functions (of virtually any differentiable form), reactive programming involves the successive adjustment of quantities produced and their distribution to demand regions so as to maximize producer profits net of transport costs in each supply region. In what amounts to a *tattônement* process, each producing region is given an opportunity to adjust its production and shipments pattern until no further (profit increasing) moves are possible. The resulting solution is identical to solutions found by the several programming techniques.

In TAMM a spatial equilibrium in the lumber and plywood markets is found in each year of the projection period. It should be noted that these solutions do not represent, nor can the reactive programming procedure be readily used to find, intertemporal production or consumption strategies which are in some sense optimal. The production, consumption, and price time paths are only estimates of the outcomes of contemporaneous interactions in freely competitive markets.

2.2. The pulp sector

Owing to the substantial economies of scale in the production of pulp, paper, and board products, this industry has evolved a market structure not well approximated by the competitive model. In the short-term, rigidities in physical production processes and the high fixed costs of operation tend to keep production close to capacity and limit its responsiveness to price. Owing both to the limited availability of substitutes and the relative unimportance of the cost of paper and board products in total consumer expenditures and in total production costs of paper-consuming industries, the demand for paper and board tends to be highly inelastic when viewed in the aggregate. As a consequence, consumption of paper and board was projected by means of income–consumption or Engle's curves. Total domestic paper and board output was derived by deducting externally developed estimates of imports and adding estimates of exports. Total U.S. virgin pulp production was estimated by converting product output to pulp equivalents with due allowance for non-wood and recycled fiber inputs. Pulp production by supply region was projected as a function of roundwood and residue availability and relative regional labor costs.

Given estimates of regional pulp output, wood input requirements were generated using technical pulp yield relationships derived for the projected mix of pulping types in each supply region. Residue consumption in pulping was adjusted for consistency with total residue generation in lumber and plywood manufacture and residue consumption as exports, fuel, and as input to other residue-based industries such as particle board and other panel products. Roundwood demand generated by pulp producers was added to the total demand for stumpage in each region.

2.3. Product/factor market linkage

Most recent efforts to develop long-range projection models of the U.S. forest products economy have treated the relationship between product and stumpage markets in an *ad hoc* fashion. Specifically, stumpage prices have generally been projected by some link with product prices with no explicit model of stumpage supply and demand (see, for example, ref. [8]) or they have not been projected at all (e.g., ref. [3]). In TAMM these markets are explicitly linked and the solutions for market clearing prices in each period are obtained simultaneously.

The TAMM approach can be demonstrated as follows. The overall TAMM system may be represented by the equations:

Type of relation	*Function*					
Product						(1)
demands	$a_{11}D + a_{12}P^{D}$				$= g_{11}Z_1$,	(2)
Product						(3)
supplies		$a_{23}S + a_{24}P^{S}$		$+ b_{24}P^{H}$	$= g_{12}Z_2$	(4)
Stumpage						
demands		$c_{13}S$	$+ d_{11}H$		$= g_{21}Z_3$,	
Stumpage						
supplies			$d_{21}H$	$+ d_{22}P^{H}$	$= g_{22}Z_4$,	

where

D and P^{D} are 12×1 vectors of lumber and plywood demand region quantities and delivered prices, respectively;

S and P^{S} are 14×1 vectors of lumber and plywood supply region quantities and mill prices, respectively;

H and P^{H} are 8×1 vectors of supply region timber harvests and stumpage prices, respectively;

Z_1, Z_2, Z_3, and Z_4 are vectors of predetermined variables for the product demand, product supply, stumpage demand, and stumpage supply equations, respectively, and

a_{11}, a_{12}, a_{23}, a_{24}, b_{24}, c_{13}, d_{11}, d_{21}, and d_{22} are, respectively, 12×12, 12×12, 14×14, 14×14, 14×8, 8×14, 8×8, 8×8, and 8×8 arrays of coefficient estimates for the endogenous variables; and

g_{11}, g_{12}, g_{21}, and g_{22} are appropriately dimensioned arrays of coefficient estimates for the predetermined variables.

Eqs. (1) explain the quantities demanded (D) of lumber and plywood in each demand region in terms of delivered product prices (P^{D}) and an array of demand shifters (Z_1) including housing activity, manufacturing output, non-residential construction, and substitute prices. Eqs. (2) explain the quantities supplied (S) of lumber and plywood in each supply region in terms of product prices (P^S), stumpage prices (P^H), and predetermined variables (Z_2) including non-wood production costs. Product supply equations employ the familiar partial adjustment mechanism, so that Z_2 also includes once lagged supply quantities. Eqs. (3) give total stumpage demand (H) as the sum of the roundwood equivalents of lumber and plywood output plus exogenously determined production volumes of pulp, miscellaneous products, log exports and fuelwood (in Z_3). Finally, stumpage supply (equal to H in equilibrium) is explained by eqs. (4) as a function of stumpage price (P^H) and (in Z_4) growing stock inventory.

Stumpage market equilibrium involves simultaneous solution of eqs. (3) and (4). This leads to a partial reduced form expression for the endogenous variables in the stumpage sector, timber harvests (H) and stumpage prices (P^H), in terms of predetermined variables in these equations and the lumber and plywood supply quantities (S):

$$\begin{bmatrix} H \\ P^H \end{bmatrix} = \begin{bmatrix} d_{11} & 0 \\ d_{21} & d_{22} \end{bmatrix}^{-1} \begin{bmatrix} g_{21}Z_3 - c_{13}S \\ g_{22}Z_4 \end{bmatrix}$$

or

$$H = d_{11}^{-1}(g_{21}Z_3 - c_{13}S), \tag{5}$$

$$P^H = -d_{22}^{-1}d_{21}d_{11}^{-1}(g_{21}Z_3 - c_{13}S) + d_{22}^{-1}g_{22}Z_4. \tag{6}$$

Substituting eqs. (6) into (2) to eliminate stumpage market variables from the product supply equations we obtain (after simplification):

$$\begin{aligned}\left(a_{23} + b_{24}d_{22}^{-1}d_{21}d_{11}^{-1}c_{13}\right)S + a_{24}P^S = g_{12}Z_2 + b_{24}d_{22}^{-1}d_{21}d_{11}^{-1}g_{21}Z_3 \\ - b_{24}d_{22}^{-1}g_{22}Z_4.\end{aligned} \tag{7}$$

Eqs. (7) are the supply equations for lumber and plywood in each supply region expressed in a form which recognizes the interdependence of production costs of the two products. The supply equation for one product includes the supply volume of the other. This arises because changes in lumber supply, for example, influence stumpage prices which in turn influence plywood production costs. Thus, movements along the supply curve for one product lead to shifts in the supply curve of the other.

The reactive programming algorithm utilizes eqs. (1) and (7) to find a product market solution for the vectors D, P^D, S and P^S, given transport costs. Equilibrium in the stumpage market is then computed using eqs. (5) and (6). Thus the entire system is in simultaneous equilibrium.

2.4. Dynamic considerations

Introduction of key dynamic elements in product and stumpage supply adds two more model components. The first is a mechanism to expand and contract lumber and plywood processing capacity as economic activity changes and the second is a means for projecting resource inventory changes resulting from harvest and forest growth. Productive capacity of lumber and plywood is adjusted over time in response to changing profitability conditions. In a competitive market, large positive profits in a given region relative to other regions should serve as an inducement for capacity expansion. As capacity expands, production costs, particularly for stumpage, should rise (as derived demands for factors increase), eliminating the relative differences in regional profit.

The second dynamic element relates to private timber supply which is composed of an inventory simulator and stumpage supply relationships. For each region and private owner group there are two basic relations:

$$H_{o,t} = f_o(P_t, I_{o,t}), \tag{8}$$

$$I_{o,t+1} = I_{o,t} + G_{o,t} - H_{o,t}, \tag{9}$$

where

$H_{o,t}$ = timber harvest for private owner class o, during period t;
P_t = regional stumpage price in period t;
$I_{o,t}$ = growing stock inventory held by owner class o, at the start of period t;
f_o = a function relating current period price and start of period inventory to harvest for owner class o; and
$G_{o,t}$ = net growth of inventory, owner o, period t.

Eq. (8) is the private stumpage supply relation. In each region there are two private owner groups, industrial and non-industrial, and hence two sets of eqs. (8) and (9). Eq. (9) represents the basic growth model which employs a stand table projection method developed by Larson and Goforth [4]. The original Larson and Goforth model was modified so that net growth can be adjusted to account for changes in the intensity of forest management on private lands.

The model of private investment in intensified management assumes that private owners are economically rational in their evaluation of alternative investment opportunities, i.e., they base decisions on the present net worth of investments. In each year of the simulation period they examine potential investment alternatives to determine those with present net worths greater than zero. A fraction of the area available for intensive management is then enrolled in the various treatments. Net growth of the inventory is increased to reflect these investment decisions. The exact level and time path of changes in growth depend on the nature of the management practices adopted. For example, in the South most available practices involve regeneration of cutover softwood types or conversion of hardwood to softwood types through site preparation and planting. As a result, intensive management first impacts the growth of the smallest diameter classes, then gradually accelerates growth in larger classes as the plantations mature. As can be seen from eq. (8), intensive management influences quantities harvested only when inventory levels are changed. Because growth and inventory expand gradually over time in response to more intensive management, the resulting market impacts also increase in a gradual fashion.

3. Illustrative results

TAMM has been used both to make long-term projections and to conduct a number of policy simulations. In both cases, it has provided a number of insights into the structure of U.S. forest product markets and the effectiveness of various policies. Three such insights are discussed in this section: (1) long-term locational variability in the forest products industry, (2) the extent of substitution between sources of timber supplies (both interowner and interregional), and (3) the opportunities for, and the impacts of, increasing levels of intensive management on private lands.

The spatial framework of TAMM provides a useful tool for examining evolution of the location and mix of processing capacity through time. For example, projections of the market shares of U.S. lumber and plywood processors by region are shown in figs. 2 and 3. The driving force behind these projections is processing profitability. The shifts follow changes in regional profitability that stem from changes in production costs – most notably changes in stumpage prices. These latter costs are the most volatile components of total production costs. Stumpage price projections for representative regions

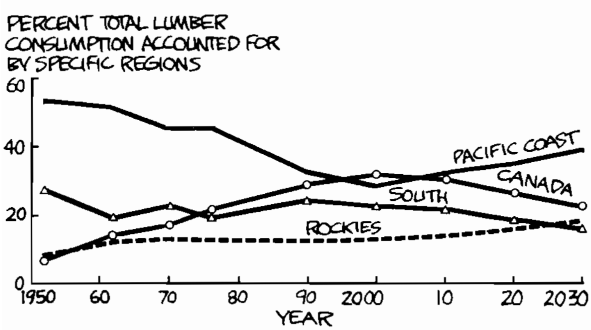

Fig. 2. Historical and projected regional shares of U.S. softwood lumber consumption.

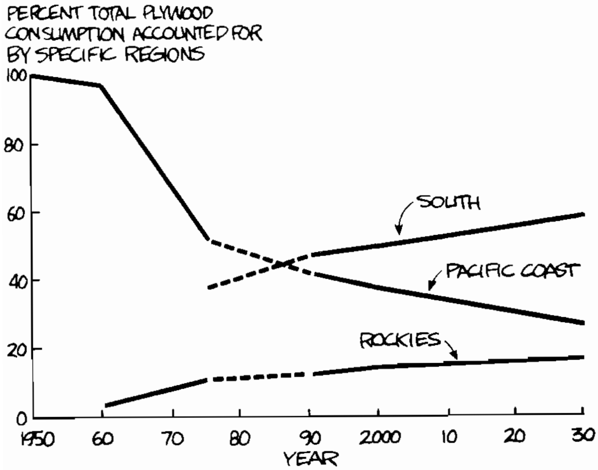

Fig. 3. Historical and projected regional shares of U.S. softwood plywood consumption.

are shown in fig. 4. Regional stumpage price differences have occurred in the past and remain an important feature of the future in TAMM projections. These differences stem from interregional variations in the resource situation, the mix and level of processing, capacity, and differences in location relative to emerging or declining final products markets.

The model framework has proven useful in simulating market responses

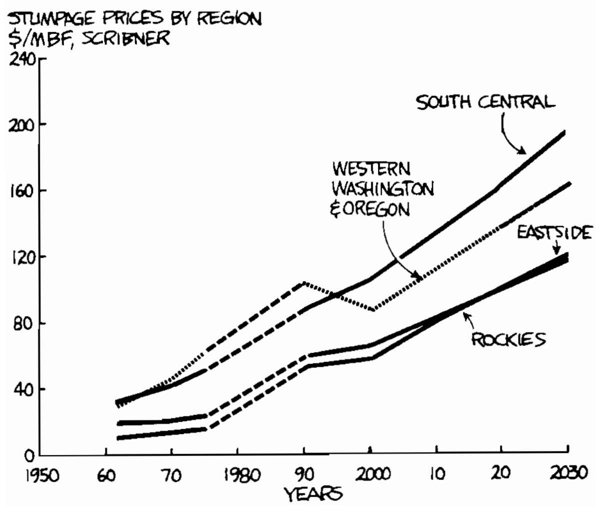

Fig. 4. Historical and projected regional stumpage prices.

associated with changes in harvest levels on publicly owned lands. It has illustrated that the impacts of various harvest policies on decision criteria such as prices, production levels, etc., depends on two key elements: (1) responsiveness of private timber owners to changes in price and inventory, and (2) the significance of the change in harvest vis-à-vis total national timber supply. The result has been a wider recognition that the impact of a change in harvest depends on the net change in U.S. supply rather than changes in harvest levels on public lands in isolation. This point can be illustrated using the results of a simulation of the withdrawal of all currently unroaded areas on the national Forests from the allowable cut base. These are unroaded areas of National Forests that may be withdrawn from harvest, depending on the eventual outcome of the second Roadless Area Review and Evaluation (RARE II). This withdrawal would be accompanied by a decline in allowable cut.

In the simulation, National Forest harvest declines in 1990 by 498 million cubic feet as a result of the withdrawals. Private owners and Canadian producers, facing higher stumpage and product prices, expand production and offset the decline by 381 million cubic feet. The net supply effect owing to this substitution is only 117 million cubic feet.

Another source of substitution is that among regions. In the above simulation most of the increases in private cut come in the South, while reductions in National Forest harvest come almost exclusively in the West. The result is a redistribution of production from the West to the South and Canada. This last point illustrates the usefulness of the spatial framework.

The third insight involves explication of the linkage between the level of private management intensity and long-term timber harvest. While this relationship is at present poorly understood, TAMM can be employed to experiment with various assumptions on future levels of management intensity, exposing thereby the possible range of harvest impacts that might be expected. An example of such experimentation is a simulation run assuming that owners adopt all available intensive practices yielding at least a four per cent real rate of return on invested capital. This is an extreme assumption and will lead to a very high level of investment, but it is useful as an illustration of impacts of widespread adoption of intensive management. Projection results for total inventories are shown in fig. 5 for the South and Pacific Coast regions. Intensive management has very large growth and harvest impacts in these two regions, which to some extent preempts opportunities in the other regions. Growth response is greatest in the South. The figure also illustrates the intertemporal impacts of intensive management, in that the effects are not realized until after 2000 in the South and 2020 in the Pacific Coast region. Results of this simulation suggest that intensification of management under this specific assumption will:

(1) accelerate concentration of lumber and plywood production in the South,

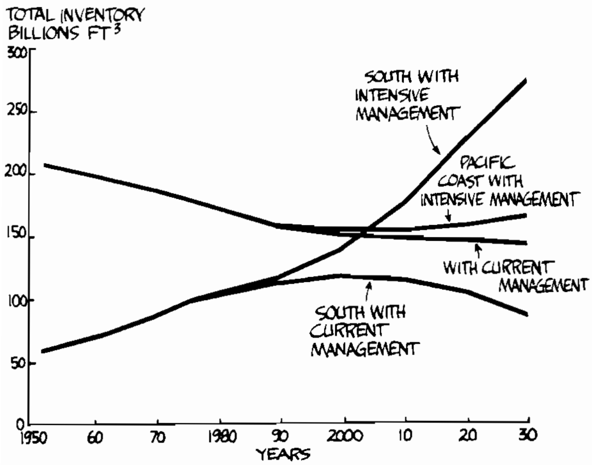

Fig. 5. Historical and projected private timber inventories by region under current and intensive management.

(2) lead to displacement of Canadian lumber imports and create opportunities for expanded exports, and
(3) stabilize lumber and plywood prices after 2000.

4. Concluding remarks

Long-range planning as now conducted by the U.S. Forest Service utilizes a systems approach with an explicit spatial structure. The approach has considerable potential for analyzing the effects of different forest policies and programs and for making conditional projections. The form of the TAMM model reflects market conditions specific to the U.S. in that competitive behavior is assumed (embodied) in the structures of the lumber, plywood, and stumpage markets but not in pulp and paper. The model is also a partial equilibrium analysis since conditions in other economic sectors are taken as exogenous. In other countries, where allocative decisions are based on other systems and considerations or where the forestry sector accounts for a large share of national economic activity, other structures will be required. Still, we believe the results to date with TAMM clearly illustrate the potential benefits, both to public policy and industrial decision makers, of large scale systems simulation in the forestry sector.

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A MODEL FOR THE FOREST SECTOR

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This paper describes a multicriteria dynamic linear programming model for studying long-range development alternatives of forestry and forest based industries at a national and regional level. The Finnish forest sector is used as an object of implementation and for numerical examples. Our model is comprised of two subsystems, the forestry and the industrial subsystem, which are linked to each other through timber supply. The forestry submodel describes the development of the volume and age distribution of different tree species within the nation or its subregions. In the industrial submodel we consider various production activities, such as the saw mill industry, panel industry, pulp and paper industry, as well as secondary processing of primary products.

1. Introduction

As is the case with several natural resources, many regions of the world are now at the transition period from ample to scarce wood resources. In addition, the forest sector plays an important role in the economy of many countries. Therefore, long-term policy analysis of the forest sector, i.e., forestry and forest industries, is becoming an important issue for such countries.

We may single out two basic approaches for analyzing the long-range development of the forest sector: simulation and optimization. Simulation techniques (e.g., system dynamics) allow us to understand and quantify basic relationships influencing the development of the forest sector (see Jeger et al. 1978, Randers 1976, Seppälä et al. 1980). Hence, using a simulation technique we can evaluate the consequences of a specific policy. However, using only simulation it is difficult to find a "proper" (or in some sense efficient) policy. The reason for this is that the forest sector is in fact a large-scale dynamic system and, on the basis of simulation alone, it is difficult to select an appropriate policy which should satisfy a large number of conditions and requirements. For this we need an optimization technique, or more generally, multiobjective optimization. Because of the complexity of the system in question, linear programming (Dantzig 1963) may be considered as the most appropriate technique for this case. It is worthwhile to note that the optimization technique itself should be used on some simulation basis; i.e., different numerical runs based on different assumptions and objective functions should be carried out to aid the selection of an appropriate policy. Specific applications of such an approach for planning an integrated system of forestry and forest industries have been presented, for instance, by Jackson (1974) and Barros and Weintraub (1979).

Already because of the nature of growth of the forests, the model should necessarily be dynamic. Therefore, in this paper we consider a dynamic linear programming (DLP) model for the forest sector. In this approach the planning horizon (e.g., a 50-year period) is partitioned into a (finite) number of time periods (e.g., 5-year periods) and for each of these shorter periods we consider a static linear programming model. A dynamic LP is then just a linear program comprised of such static models which are interlinked via various state variables (i.e., different types of "inventories," such as wood in the forests, production capacity, assets, liabilities, etc., at the end of a given period are equal to those at the beginning of the following period). In our forest sector model, each such static model comprises two basic submodels: a forestry submodel, and an industrial model of production, marketing and financing. The forestry submodel also describes ecological and land availability constraints for the forest, as well as labor and machinery constraints for harvesting and planting activities.

The industrial submodel is described by a small input–output model with both mechanical (e.g. sawmill and plywood) and chemical (e.g., pulp and paper) production activities. Also secondary processing of the primary products will be included in the model, in particular, because of the expected importance of such activities in the future.

The rate of production is restricted by wood supply (which is one of the major links between the submodels), by final demand for forest products, by labor force supply, by production capacity availability, and finally, by financial considerations.

The evaluation criterion in comparing alternative policies for the forest sector is highly multiobjective: while selecting a reasonable long-term policy, preferences of different interest groups (such as government, industry, labor, and forest owners) have to be taken simultaneously into account. It should also be noted that forestry and industry submodels have different transient times: a forest normally requires a growing period of at least 40 to 60 years whereas a major structural change in the industry may be carried out within a much shorter period. Because of the complexity of the system, it is sometimes desirable to consider the forestry and the industries on some independent basis, and to analyze an integrated model thereafter (see Kallio et al. 1979).

It is also important to take into account all the uncertainties which can happen with the forest and related sectors during such a long period. A stochastic model seems to be more appropriate for the case. However, high requirements to input data, difficulties of applying stochastic solution techniques, etc. make the suggested approach more realistic.

The paper is divided into two parts. In the first part (sections 2–4) we describe the methodological approach. In the second part (section 5) a specific implementation for the Finnish forest sector is described and illustrated with somewhat hypothetical numerical examples.

2. The forestry subsystem

Mathematical programming is a widely applied technique for operations management and planning in forestry (e.g., Navon 1971, Dantzig 1974, Kilkki et al. 1977, Newnham 1975, Näslund 1969, Wardle 1965, Ware and Clutter 1971, Weintraub and Navon 1976, Williams 1976). In this section we follow a traditional formulation of the forests' tree population into a dynamic linear programming system. We describe the forestry submodel, where the decision variables (control activities) are harvesting and planting activities, and where the state of the forests is represented by the volume of trees in different species and age groups. Because the model is formulated in the DLP framework, we single out the following: (i) state equations which describe the development of the system, (ii) constraints which restrict feasible trajectories of the forest development, and (iii) the planning horizon.

2.1. State equations

Each tree in the forest is assigned to a class of trees specifying the age and the species of the tree. A tree belongs to age group a $(a = 1, \ldots, N-1)$ if its age is at least $(a-1)\Delta$ but less than $a\Delta$, where Δ is a given time interval (for example, five years). In the highest age group $a = N$ all trees are included which have an age of at least $(N-1)\Delta$. (Instead of age groups, we might alternatively assign trees to size groups specified by the trees' diameter or volume.) We denote by $w_{sa}(t)$ the number of trees of species, s, $s = 1, 2, 3, \ldots,$ (e.g., pine, spruce, birch, etc.) in age group a at the beginning of time period t, $t = 0, 1, \ldots, T$. Index s may be extended to refer to region, soil class, and forest management alternative as well.

Let $\alpha^s_{aa'}(t)$ show the ratio of trees of species s and in age group a that will proceed to the age group a' during time period t. We shall consider a model formulation where the length of each time period is Δ. Therefore, we may assume that $\alpha^s_{aa'}(t)$ is independent of t and equal to zero unless a' is equal to $a+1$ (or a for the highest age group). We denote then $\alpha^s_{aa'}(t)=\alpha^s_a$ with $0 \le \alpha^s_a \le 1$. The ratio $1-\alpha^s_a$ may then be called the attrition rate corresponding to time interval Δ and tree species s in age group a. We introduce a subvector $w_s(t)=\{w_{sa}(t)\}$, specifying the age distribution of trees (number of trees) for each tree species s at the beginning of time period t. Assuming neither terminal harvesting nor planting, the age distribution of trees at the beginning of the next time period $t+1$ will then be given by $\alpha^s w_s(t)$ where α^s is the square $N \times N$ growth matrix, describing again, removals due to thinning and death of the trees resulting from natural causes. By our definition, it has the form

$$\alpha^p = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ \alpha^s_1 & 0 & \cdots & 0 & 0 \\ 0 & \alpha^s_2 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \alpha^s_{N-1} & \alpha^s_N \end{bmatrix}$$

Introducing a vector $w(t)=\{w_s(t)\}=\{w_{sa}(t)\}$, describing tree species and age distribution and a block-diagonal matrix α with submatrices α^s on its diagonal, the species and age distribution at the beginning of period $t+1$ will be given by $\alpha w(t)$.

We denote by $u^+(t)$ and $u^-(t)$ the vectors of planting and final harvesting activities at time period t. The state equation describing the development of the forest will then be

$$w(t+1)=\alpha w(t)+\eta u^+(t)-\omega u^-(t), \tag{1}$$

where matrices η and ω specify activities in such a way that $\eta u^+(t)$ and $-\omega u^-(t)$ are the incremental change in numbers of trees resulting from planting and harvesting activities, respectively.

It should be noted that the growth rate of the trees is dependent in fact on the number of trees growing in a unit area. Hence, instead of eq. (1) we should use a nonlinear equation. However it will complicate the problem considerably. Therefore, it would be more practical to use linear approximation (1) with some average figures in the matrix α. It is possible to check these figures after the solution has been obtained and update them if necessary.

A planting activity n may be specified to mean planting of one tree of species s which enters the first age group ($a=1$) during period t. Thus, matrix η has one unit column vector for each tree species s. The nonzero element of such a column is on the row of the first age group for tree species s in eq. (1).

A harvesting activity h is specified by variables $u^-_h(t)$ which determine the level of this activity. The coefficients ω^s_{ah} of matrix ω are defined so that

$\omega^{s}_{ah}u^{-}_{h}(t)$ is the number of trees of species s from age group a harvested when activity h is applied at level $u^{-}_{h}(t)$. Thus, these coefficients show the age and species distribution of trees harvested when activity h is applied.

2.2. Constraints

2.2.1. Land

Let $H(t)$ be the vector of total acreage of different soil types d of land available for forests at time period t. Let G^{s}_{ad} be the area of soil type d required by one tree of species s and age group a. We assume that each tree species uses only one type of soil d; i.e., only one of the elements G^{s}_{ad}, $d = 1, 2, \ldots,$ is nonzero. Thus, if we consider more than one land type, then the tree species s may also refer to the soil. Defining the matrix $G = (G^{s}_{ad})$, we have the land availability restriction

$$Gw(t) \leq H(t). \tag{2}$$

In this formulation we assume that the land area $H(t)$ is exogenously given. Alternatively, we may endogenize vector $H(t)$ by introducing activities and a state equation for changing the area of different types of land. Such a formulation is justified if changes in soil type over time are considered or if some other land intensive activities, such as agriculture, are included in the model.

Besides land availability constraints, requirements for allocating land for certain purposes (such as preserving the forest as a water shed or as a recreational area) may be stated in the form of inequality (2). In such a case (the negative of) a component of $H(t)$ would define a lower bound on such an allocation, while the left hand side would yield the (negative of) land allocated in a solution of the model.

Sometimes constraints on land availability may be given in the form of equalities which require that all land which is made available through harvesting at a time period should be used in the same time period for planting new trees of the type appropriate for the soil. Forest laws in many countries even require following this type of pattern.

2.2.2. Labor and other resources

Harvesting and planting activities require resources such as machinery and labor. Let $R^{+}_{gn}(t)$ and $R^{-}_{gh}(t)$ be the usage of resource g at the unit level of planting activity n and harvesting activity h, respectively. Defining the matrices $R^{+}(t) = \{R^{+}_{gn}(t)\}$ and $R^{-}(t) = \{R^{-}_{gh}(t)\}$, and vector $R(t) = \{R_{g}(t)\}$ of available resources during period t, we may write the resource availability constraint as follows:

$$R^{+}(t)u^{+}(t) + R^{-}(t)u^{-}(t) \leq R(t). \tag{3}$$

More adequately, resource availability may be given by supply functions for which piecewise linear approximations may be employed.

2.2.3. Wood supply

The requirements for wood supply from forestry to industries can be given in the form:

$$Vw(t) + S(t)u^-(t) = y(t), \tag{4}$$

where vector $y(t) = \{y_k(t)\}$ specifies the requirements for different timber assortments k (e.g., pine log, spruce pulpwood, etc.), matrix $S(t)$ transforms quantities of harvested trees of different species and age into the volume of different timber assortments, and $Vw(t)$ accounts for thinning activities. Note that the volume of any given tree being harvested is assigned in eq. (4) to log and pulpwood in proportion, depending on the species and age group of the tree. For each species, several dimension classes of log may be specified. The possibility of using any size of log as pulpwood may be included in the model. Thereby the dimension distinguishing log from pulpwood actually becomes endogenous.

2.3. Planning horizon

The forest as a system has a very long transient time: one rotation of the forest may in extreme conditions require more than one hundred years. Naturally, various uncertainties make it difficult to plan for such a long time horizon. On the other hand, if the planning horizon is too short we cannot take into account all the consequences of activities implemented at the beginning of the planning horizon. To set a terminal condition for the forests, the standard concept of sustainable yield shall be employed.

In order to analyze a stationary regime for the forests, we set $w(t+1) = w(t) = w$, for all t. The state equation (1) can then be restated as

$$w = \alpha w + \eta u^+ - \omega u^-. \tag{5}$$

Imposing constraints (2) through (4) on variables w, u^+, and u^-, we can solve a static linear programming problem to find an optimal stationary state w^* of the forest (and corresponding harvesting and planting activities). This approach has been used, for instance, by Rorres (1978) for finding the stationary maximum yield of a harvest. The solution of a dynamic linear program with terminal constraints

$$w(T) = w^* \tag{6}$$

yields the optimal transition to this sustainable yield state.

Another way of introducing sustainable yield is to consider an infinite period formulation and to impose constraints $w(t) = w(t+1)$, $u^-(t) = u^-(t+1)$ and $u^+ = u^+(t+1)$, for all $t \geq T$. If the model parameters for period t are assumed independent of time for all $t \geq T$, then the dynamic infinite horizon linear programming model may be formulated as a $T+1$ period problem where the last period represents a stationary solution for periods $t \geq T$, and the first T periods represent the transition from the initial state to the stationary solution.

There is a certain difference in these two approaches of handling the stationary state. In the first approach, when eq. (5) is applied, we first find the optimal stationary solution independently of the transition period, and thereafter we determine the optimal transition to this stationary state. In the latter approach we link the transition period with the period corresponding to the stationary solution. The linkage takes place in the stationary state variables which are determined in an optimal way taking into account both time periods simultaneously.

3. The industrial subsystem

We will now consider the industrial subsystem of the forest sector. Again the formulation is a dynamic linear programming model. We discuss first the section related to production and final demand of forest products, and the financial considerations thereafter.

3.1. Production and demand

Let $x(t)$ be the vector (levels of) of production activities for period t, for $t = 0, 1, \ldots, T-1$. Such an activity i may include production of sawnwood, panels, pulp, paper, converted products, etc. For each single product j, there may exist several alternative production activites i which are specified through alternative uses of raw material, technology, etc. Let U be the matrix of wood usage per unit of production activity so that the wood processed by industries during period t is given by vector $Ux(t)$. Note that matrix U has one row corresponding to each timber assortment k (corresponding to the components of the timber supply vector $y(t)$ in the forestry model). Some of the elements in U may be negative. For instance, saw milling consumes logs but produces raw material (industrial residuals) for pulp mills. This byproduct appears as a negative component in matrix U. We denote by $r(t) = \{r_k(t)\}$ the vector of wood raw material inventories at the beginning of period t (i.e., wood harvested but not processed by the industry). As above, let $y(t)$ be the amount of wood harvested in different timber assortments, and $z^+(t)$ and $z^-(t)$ the (vectors of) import and export of different assortments of wood, respectively during period t. Then we have the following state equation for the wood raw material inventory:

$$r(t+1) = r(t) + y(t) - Ux(t) + z^{+}(t) - z^{-}(t). \tag{7}$$

In other words, the wood inventory at the end of period t is the inventory at the beginning of that period plus wood harvested and imported less wood consumed and exported (during that period). Note that if there is no storage (change), and no import nor export of wood, then eq. (7) reduces to $y(t) = Ux(t)$; i.e., wood harvested equals the consumption of wood. For wood import and export we assume upper limits $Z^{+}(t)$ and $Z^{-}(t)$, respectively:

$$z^{+}(t) \leq Z^{+}(t) \text{ and } z^{-}(t) \leq Z^{-}(t). \tag{8}$$

The production process may be described by a simple input–output model with substitution. Let $A(t)$ be an input–output matrix having one row for each product j and one column for each production activity i so that $A(t)x(t)$ is the (vector of) net production when production activity levels are given by $x(t)$. Let $m(t) = \{m_j(t)\}$ and $e(t) = \{e_j(t)\}$ be the vectors of import from and export to the forest sector, respectively, for products j. Then, excluding from consideration a possible change in the product inventory, we have

$$A(t)x(t) + m(t) - e(t) = 0. \tag{9}$$

Domestic consumption is included in $e(t)$. Both for export and for import we assume externally given bounds $E(t)$ and $M(t)$, respectively:

$$e(t) \leq E(t), \tag{10}$$

$$m(t) \leq M(t). \tag{11}$$

Such constraints may be substituted by piecewise linear export and import functions in the model. The same applies to trade in timber.

Production activities are further restricted through labor and mill capacities. Let $L(t)$ be the vector of different types of labor available for the forest industries. Labor may be classified in different ways taking into account, for instance, type of production, and the type of responsibilities in the production process (e.g., work force, management, etc.). Let $\rho(t)$ be a coefficient matrix so that $\rho(t)x(t)$ is the (vector of) demand for different types of labor given production activity levels $x(t)$. Thus we have

$$\rho(t)x(t) \leq L(t). \tag{12}$$

Again, such a constraint may be substituted by piecewise linear supply function of labor.

We will consider the production (mill) capacity as an endogenous state variable. Let $q(t)$ be the vector of the amount of different types of such capacity at the beginning of period t. Such types may be distinguished by region (where the capacity is located), by type of product for which it is used and by different technologies to produce a given product. Let $Q(t)$ be a coefficient matrix so that $Q(t)x(t)$ is the demand (vector) for these types of capacity. Such a matrix has nonzero elements only when the region–product–technology combination of a production activity matches with that of the type of capacity. The production capacity restriction is then given as

$$Q(t)x(t) \leq q(t). \tag{13}$$

The development of the capacity is given by a state equation

$$q(t+1) = (I-\delta)q(t) + v(t), \tag{14}$$

where δ is a diagonal matrix accounting for (physical) depreciation and $v(t)$ is a vector of investments (in physical units). Capacity expansions are restricted through financial resources. We do not consider possible constraints of other sectors, such as machinery or the construction sector, whose capacity may be employed in investments of the forest sector.

3.2. Finance

We will now turn our discussion to the financial aspects. We partition the set of production activities i into financial units (so that each activity belongs uniquely to one financial unit). Furthermore, we assume that each production capacity is assigned to a financial unit so that each production activity employs only capacities assigned to the same financial unit as the activity itself.

Production capacity in eq. (14) is given in physical units. For financial calculations (such as determining taxation) we define a vector $\bar{q}(t)$ of fixed assets. Each component of this vector determines fixed assets (in monetary units) for a financial unit related to the capacity assigned to that unit. Thus, fixed assets are aggregated according to the grouping of production activities into financial units, for instance, by region, by industry, or by groups of industries.

Financial and physical depreciation may differ from each other; for instance, when the former is specified by law. We define a diagonal matrix $[I-\bar{\sigma}(t)]$ so that $[I-\bar{\sigma}(t)]\bar{q}(t)$ is the vector of fixed assets left at the end of period t when investments are not taken into account. Let $K(t)$ be a matrix where each component determines the increase in fixed assets (of a certain financial unit) per (physical) unit of an investment activity. Thus the components of vector $K(t)v(t)$ determine the increase in fixed assets (in monetary units) for the financial units when investment activities are applied (in physical units) at a level determined by vector $v(t)$. Then we have the following state equation for fixed assets:

$$\bar{q}(t+1) = [I - \bar{\sigma}(t)]\bar{q}(t) + K(t)v(t). \tag{15}$$

For each financial unit we consider external financing (long-term debt) as an endogenous state variable. Let $l(t)$ be the (vector of) beginning balance of external financing for different financial units in period t. In this notation, the state equation for long-term debt is as follows:

$$l(t+1) = l(t) + l^{+}(t) - l^{-}(t). \tag{16}$$

We will restrict the total amount for long-term debt through a measure which may be considered as a realization value of a financial unit. This measure is a given percentage of the total assets less short-term liabilities. Let $\mu(t)$ be a diagonal matrix of such percentages, let $b(t)$ be the (endogenous vector of) total stockholders equity (including cumulative profit and stock). Then the upper limit on loans is given as

$$[I - \mu(t)]\, l(t) \le \mu(t) b(t). \tag{17}$$

Alternatively, external financing may be limited, for instance, to a percentage of a theoretical annual revenue (based on available production capacity and on assumed prices of products). Note that no repayment schedule has been introduced in our formulation, because an increase in repayment can always be compensated by an increase of drawings in the state equation (16).

Next we will consider the profit (or loss) from period t. Let $p^{+}(t)$ and $p^{-}(t)$ be vectors whose components indicate profits and losses, respectively, for the financial units. By definition, both profit and loss cannot be simultaneously nonzero for any financial unit. For a solution of the model, this fact usually results from the choice of an objective function.

Let $P(t)$ be a matrix of prices for products (having one column for each product and one row for each financial unit) so that the vector of revenue (for different financial units) from sales $e(t)$ outside the forest industry is given by $P(t)e(t)$. Let $C(t)$ be a matrix of direct unit production costs, including, for instance, timber, energy, and direct labor costs. Each row of $C(t)$ refers to a financial unit and each column to a production activity. The (vector) of direct production costs for financial units is then given by $C(t)x(t)$.

The fixed production costs may be assumed proportional to the (physical) production capacity. We define a matrix $F(x)$ so that the vector $F(t)q(t)$ yields the fixed costs of period t for the financial units. According to our notation above, (financial) depreciation is given by the vector $\bar{\sigma}(t)\bar{q}(t)$. We assume that interest is paid on the beginning balance of debt. Thus, if $\epsilon(t)$ is the diagonal matrix of interest rates, then the vector of interest paid (by the financial units) is given by $\epsilon(t)l(t)$. Finally, let $D(t)$ be (a vector of) exogenously given cash expenditure covering all other costs. Then the profit before tax (loss) is given as follows:

$$p^+(t) - p^-(t) = P(t)e(t) - C(t)x(t) - F(t)q(t)$$

$$- \bar{\sigma}(t)\bar{q}(t) - \epsilon(t)l(t) - D(t). \tag{18}$$

The stockholder equity $b(t)$, which we already employed above, now satisfies the following state equation:

$$b(t+1) = b(t) + [I - \tau(t)]\, p^+(t) + B(t), \tag{19}$$

where $\tau(t)$ is a diagonal matrix for taxation and $B(t)$ is the (exogenously given) amount of stock issued during period t.

Finally, we consider cash (and receivables) for each financial unit. Let $c(t)$ be the vector of cash at the beginning of period t. The change of each during period t is due to the profit after tax (or loss), depreciation (i.e., noncash expenditure), drawing of debt, repayment, and investments. Thus we assume that the possible change in cash due to changes in accounts receivable, in inventories (wood, end products, etc.) and in accounts payable cancel each other (or that these quantities remain unchanged during the period). Alternatively, such changes could be taken into account assuming, for instance, that the accounts payable and receivable, and the inventories are proportional to annual sales of each financial unit.

Using our earlier notation, the state equation for cash is now

$$c(t+1) = c(t) + [I - \tau(t)]\, p^+(t) - p^-(t) + \bar{\sigma}(t)\bar{q}(t)$$

$$+ l^+(t) - l^-(t) - K(t)v(t) + B(t). \tag{20}$$

3.3. Initial state and terminal conditions

In our industrial model, we now have the following state vectors: wood raw material inventory $r(t)$, (physical) production capacity $q(t)$, fixed assets $\bar{q}(t)$, long-term debt $l(t)$, cash $c(t)$, and total stockholders equity $b(t)$. For all of them we have an initial value and possibly a limit on the terminal value. We shall refer to the initial and terminal values by superscripts 0 and *, respectively; i.e., we have the initial state given as

$$r(0)=r^0,\ q(0)=q^0,\ \bar{q}(0)=\bar{q}^0,$$

$$l(0)=l^0,\ c(0)=c^0,\ b(0)=b^0, \tag{21}$$

$$r(T)\geq r^*,\ q(T)\geq q^*,\ \bar{q}(T)\geq \bar{q}^*,$$

$$l(T)\leq l^*,\ c(T)\geq c^*. \tag{22}$$

The initial state is determined by the state of the forest industries at the beginning of the planning horizon. The terminal state may be determined as a stationary solution similarly as we described for the forestry model above.

4. The integrated system

We will now consider the integrated forestry–forest industries model. First we have a general discussion on possible formulations of various objective functions for such a model. Thereafter, we summarize the model in the canonical form of dynamic linear programming. A tableau representation of the structure of the integrated model will also be given.

4.1. Objectives

The forest sector may be viewed as a system controlled by several interest groups or parties. Any given party may have several objectives which are in conflict with each other. Obviously, the objectives of one party may be in conflict with those of another party. For instance, the following parties may be taken into account: representatives for industry, government, labor, and forest owners. Objectives for industry may be the development of profit of different financial units. Government may be interested in the increment of the forest sector to the gross national product, to the balance of payments, and to employment. The labor unions are interested in employment and total wages earned in forestry and different industries within the sector. Objectives for forest owners may be the income earned from selling and harvesting wood. Such objectives refer to different time periods t (of the planning horizon) and possibly also to different product lines. We will now give simple examples of formulating such objectives into linear objective functions. Nonlinear objectives may be employed as well.

4.1.1. Industrial profit

The vector of profits for the industrial financial units was defined above as $[I - \tau(t)]p^{+}(t) - p^{-}(t)$ for each period t. If one wants to distinguish between different financial units, then actually each component of such a vector may be considered as an objective function. However, often we aggregate such objectives for practical purposes, for instance, summing up discounted profits over all time periods, summing over financial units, or summing over both time periods and financial units.

4.1.2. Increment to gross national product

For the purpose of defining the increment of the forest sector to the GNP we consider the sector as a "profit center" where no wage is paid to the employees within the sector, where no price is paid for raw material originating from this sector, where no interest or taxes are paid. The increment to the GNP is then the profit for such a unit.

4.1.3. Increment to balance of payments

The increment of the forest sector to the balance of payments has a similar expression to the one above for the GNP. The changes to be made in this expression are, first, to multiply the components of the price vector by the share of exports in the total sales; second, to multiply the components of the cost vectors by the share of imported inputs in each cost term; third, to multiply by the share of foreign debts (among all long-term debts) of the financial unit; and finally, to replace the depreciation function by investment expenditures in imported goods.

4.1.4. Employment

Total employment (in man-year per period) for each time period t for different types of labor, in different activities and regions, has already been expressed in the left hand side expressions of inequalities (3) and (12).

4.1.5. Wage income

For each group of the work force, the wage income for period t is obtained by multiplying the expressions for employment above by the annual salary of each such group.

4.1.6. Stumpage earnings

Beside the wage income for forestry (which we already defined above), and an aggregate profit (as expressed in eq. (6)), one may account for the stumpage earnings; i.e., the income related to the wood price prior to harvesting the tree. Such income is readily obtained by the timber assortments if the components of the harvesting yield vector $y(t)$ are multiplied by the respective wood prices.

4.2. The integrated model

The integrated forestry–forest industry model may be illustrated by fig. 1. The interactions between the forestry and industrial subsystems takes place in round wood consumption. Both subsystems interact with the general economy (which in our model is exogenous). This latter subsystem determines the supply (i.e., prices and quantities) for capital, labor, energy, and land. Also domestic consumption and export demand (import supply) are determined by this subsystem.

We will now summarize the integrated forestry–industry model in the canonical form of dynamic linear programming (see e.g., Propoi and Krivonozhko 1978). Denote by $X(t)$ the vector of all state variables (defined above) at the beginning of period t. Its components include the trees in the forest, different types of production capacity in the industry, wood inventories, external financing, etc. Let $Y(t)$ be the nonnegative vector of all controls for period t, that is, the vector of all decision variables, such as levels of harvesting or production activities. An upper bound vector for $Y(t)$ is denoted by $\hat{Y}(t)$ (some of whose components may be infinite). We assume that the objective function to be maximized is a linear function of the state vectors $X(t)$ and the

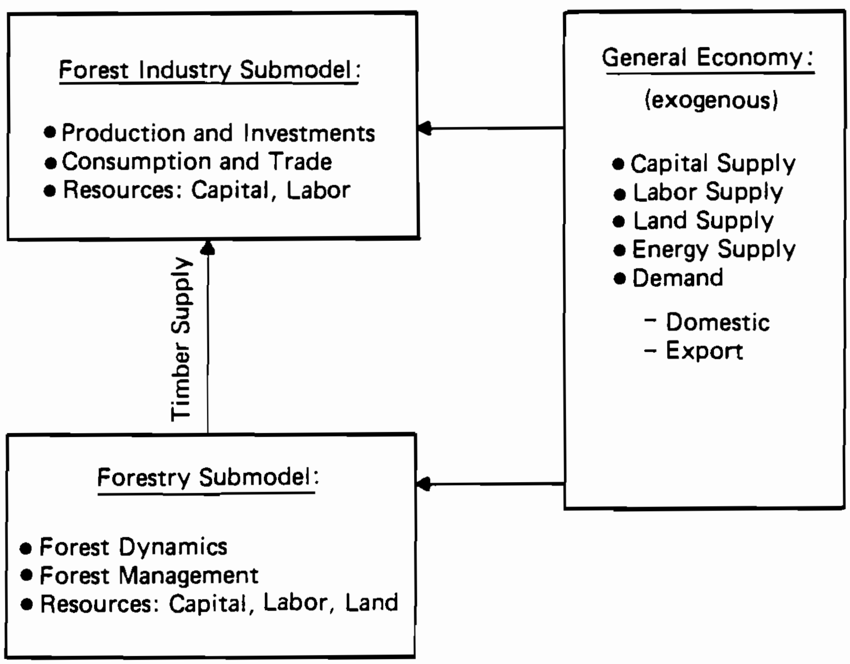

Fig. 1. Integration of forest sector models.

control vectors $Y(t)$, and we denote by $\gamma(t)$ and $\lambda(t)$ the coefficient vectors for $X(t)$ and $Y(t)$, respectively, for such an objective function. This function may be, for instance, a linear combination of the objectives defined above. For multicriteria analysis $\gamma(t)$ and $\lambda(t)$ are matrices. The initial state $X(0)$ is denoted by X^0, and the terminal requirement for $X(T)$ by X^*. Let $\Gamma(t)$ and $\Lambda(t)$ be the coefficient matrices for $X(t)$ and $Y(t)$, respectively, and let $\xi(t)$ be the exogenous right hand side vector in the state equation for $X(t)$. Let $\Phi(t)$, $\Omega(t)$, $\psi(t)$ be the corresponding matrices and the right hand side vector for the constraints. Then the integrated model can be stated in the canonical form of DLP as follows:

find $Y(t)$, for $0 \leq t \leq T-1$, and $X(t)$, for $1 \leq t \leq T$, to

$$\text{maximize} \sum_{t=0}^{T-1} [\gamma(t) X(t) + \gamma(t) Y(t)] + \gamma(T) X(T),$$

subject to

$$\begin{aligned} & X(t+1) = \Gamma(t) X(t) + \Lambda(t) Y(t) + \xi(t), && \text{for } 0 \leq t \leq T-1, \\ & \Phi(t) X(t) + \Omega(t) Y(t) \triangleq \psi(t), && \text{for } 0 \leq t \leq T-1, \\ & 0 \leq X(t),\ 0 \leq Y(t) \leq \hat{Y}(t), && \text{for all } t, \end{aligned}$$

with the initial state

$$X(0) = X^0,$$

and with the terminal requirement

$$X(T) \triangleq X^*.$$

The notation $\triangleq$ for the constraints and terminal requirement refers either to $=$, to $\leq$ or to $\geq$, separately for each constraint. The coefficient matrix (corresponding to variables $X(t)$, $Y(t)$, and $X(t+1)$) and the right hand side vector of the integrated forestry–industry submodel of period t are given as

$$\begin{bmatrix} -\Gamma(t) & -\Lambda(t) & I \\ \Phi(t) & \Omega(t) & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \xi(t) \\ \psi(t) \end{bmatrix},$$

respectively.

5. Application to the Finnish forest sector

5.1. Implementation

A version of the integrated model was implemented for an interactive mathematical programming system called SESAME (Orchard-Hays 1978). The model generator is written using SESAME's data management extension, called DATAMAT. An actual model is specified by the data tableaux of the generator programs. Our example has been designed for the Finnish forest sector. This model may have at most ten time periods each of which is a five year interval. The whole country is considered as a single region. Table 1 shows the dimensions of the model.

The seven product groups in consideration are sawnwood, panels, further processed (mechanical) wood products, mechanical pulp, chemical pulp, paper and board, and converted paper products. For each group we consider a separate type of production capacity and labor force. We have aggregated all production into one financial unit. Only one type of tree represents all tree species in the forests. The trees are classified into 21 age groups. Thus, the age increment being five years, the oldest group contains trees older than 100 years. Two harvesting activities were made available. The two timber assortments in consideration are log and pulpwood.

The data for the Finnish model were provided by the Finnish Forest Research Institute. They were partially based on the official forest statistics (Yearbook of Forest Statistics 1977/1978) published by the same institute.

Table 1
Characteristic dimensions of the Finnish forest sector model

Number of time periods [a]	10
Length of one period in years [a]	5
Number of regions	1
Number of tree species	1
Number of age groups for trees [a]	21
Harvesting activities [a]	2
Soil types	1
Harvesting and planting resources	1
Timber assortments	2
Production activities	7
Types of labor in the industry	7
Types of production capacity	7
Number of financial units	1
Number of rows in a ten period LP	520
Number of columns in a ten period LP	612

[a] The value may be specified by the model data. The numbers show the actual values being used.

Table 2
Assumed annual demand of forest products (mill. m^3, mill. ton)

Time period	Sawn-wood	Panels	Chemical pulp	Paper and board	Converted paper
1980–84	7.0	1.7	1.2	4.8	0.5
1985–89	7.5	2.0	1.1	5.8	0.7
1990–94	8.0	2.2	1.0	7.0	0.9
1995–99	8.8	2.5	0.9	8.3	1.2
2000–04	9.3	2.8	0.8	9.8	1.6
2005–09	9.7	3.2	0.7	11.6	2.1
2010–14	10.2	3.6	0.7	13.2	2.9
2015–19	10.7	4.1	0.6	15.1	3.8
2020–24	11.2	4.6	0.6	17.1	5.1
2025–29	11.6	5.2	0.6	19.2	6.9

Validation runs (which eventually resulted in our current formulation) were carried out by contrasting the model solutions with the experience gained in the preceding simulation study of the Finnish forest sector by Seppälä et al. (1980).

5.2. Scenario examples

For illustrative purposes we will now describe a few test runs. Most of the data being used in these experiments correspond approximately to the Finnish forest sector. This is the case, for instance, with the initial state; i.e., trees in the forests, different types of production capacity, etc. Somewhat hypothetical scenarios have been used for certain key quantities, such as final demand, and price and cost development. Thus, the results obtained do not necessarily reflect reality. They have been presented only to illustrate possible uses of the model.

For each test run a ten times five year period model was constructed. Labor constraints both for industry and for forestry were relaxed. At this state, one activity for converted paper products was considered. Both roundwood import and export were excluded. The assumed demand of wood products is given in table 2. Mechanical pulp is not assumed to be exported. At the end of the planning horizon, we require that in each age group there is at least 80% of the number of trees initially in those groups. For production capacity a similar terminal requirement was set to 50%. Initial production capacity is given in table 3 and the initial age distribution of trees in fig. 2.

5.2.1. Scenario A: base scenario

For the first run the discounted sum of industrial profits (after tax) was chosen as an objective function. Such an objective may reflect the industry's

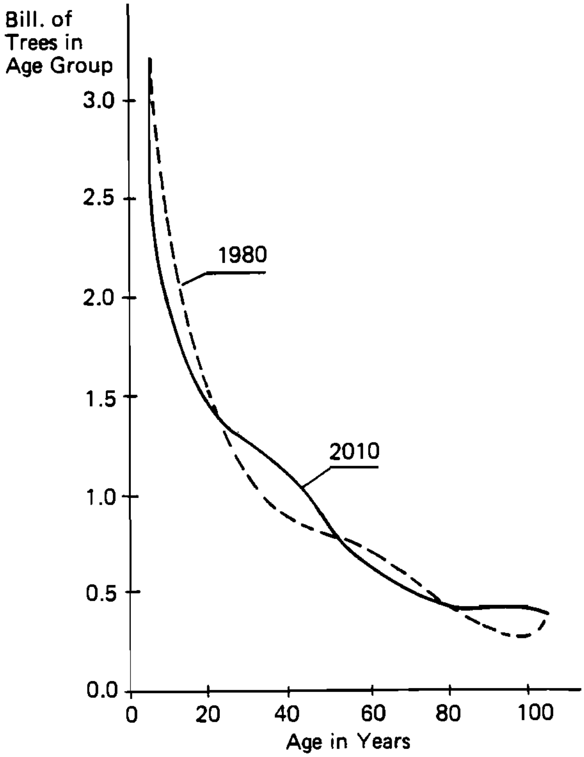

Fig. 2. Age distribution of trees in 1980 and in 2010 according to Scenario B.

behavior given the cost structure and price development. The resulting production is characterized as follows: the mechanical processing activities are limited almost exclusively by the assumed demand of sawnwood and panels. The same is true for converted paper products. However, the chemical pulp produced is

Table 3
Production capacity (mill. m^3/year, mill. ton/year) initially in 1980 and in 2010 according to Scenario A

Product	1980	2010
Sawnwood	7.0	10.2
Panels	1.7	3.6
Mechanical pulp	2.2	1.9
Chemical pulp	4.0	4.3
Paper and board	6.2	6.2
Converted paper products	0.5	2.9

almost entirely used in paper mills, and therefore, the potential demand for export has not been exploited. Neither have the possibilities for exporting paper been used fully. Paper export is declining sharply from the level of 5 million ton/y, approaching zero towards the end of the planning horizon. This is due to the strongly increasing production of converted paper products. The corresponding structural change of the production capacity of the forest industry over the 30 year period from 1980 to 2010 is given in table 3.

5.2.2. Scenario B: GNP potential

For the second run we have chosen the discounted sum of the increments of the forest sector to gross national product as an objective function. Compared with Scenario A, there is no significant difference in the production of sawnwood, panels and converted paper products for which export demand again limits the production. However, there is a significant difference in pulp and paper production. Pulp is now produced to satisfy fully the demand for export. Paper production is now steadily increasing from 5 million ton/y to nearly 9 million ton/y. Paper export is still declining again due to increasing use for the converting processes of paper products. Therefore, the export demand for paper is not fully exploited.

The bottleneck for paper production now is the biological capacity of the forests to supply wood. The annual use of roundwood increases from about 40 million m^3 to the level of 65 to 70 million m^3 (see fig. 3a). The increase in the yield of the forests may be explained by the change in the age structure of the forests during the planning horizon. Such change over the period 1980–2010 has been illustrated in fig. 2.

We notice a significant difference in the wood use between Scenarios A and B. In Scenario A (the profit maximization) the national wood resources are being used in an inefficient way; i.e., under the assumed price and cost structure the poor profitability of the forest industry results in an investment behavior which does not make full use of the forest resources.

5.2.3. Scenarios C and D: product price and demand variations

The prices of forest products in Scenario C represent average world market prices. The prices used in Scenario A are 10% above this level (accounting for possible quality differences). Investments are now unprofitable, and therefore, under profit maximization criterion, production is declining at the rate of capacity depreciation. As illustrated in fig. 3a, by the year 2020 the wood resource utilization is decreased by 50% compared to the level in 1980. The same is true for GNP as well.

In Scenario D demand for all forest products was doubled as compared with Scenario A. Because of poor profitability, only part of this demand potential will be exploited. The resulting wood consumption is given in fig. 3a.

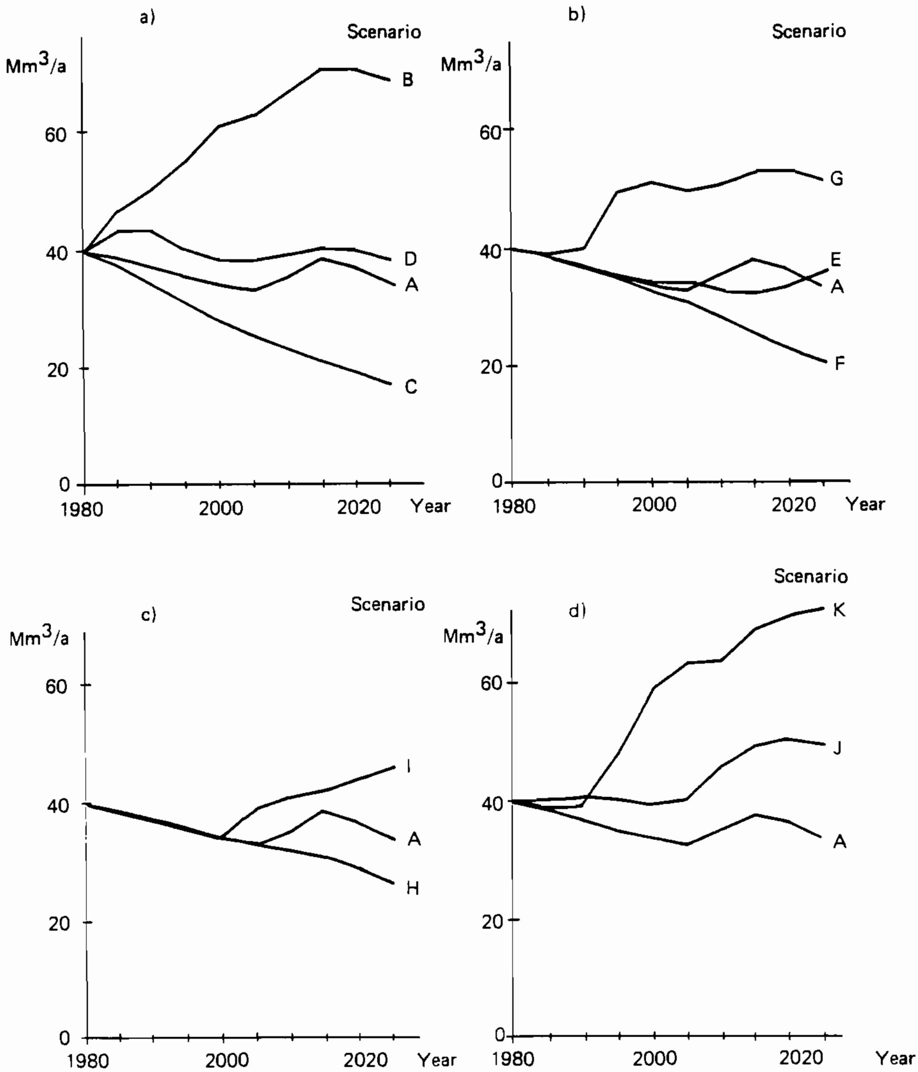

Fig. 3. Industrial consumption of roundwood (mill. m^3/y) for Scenarios A–K.

Compared with Scenario A, it is about 10% higher.

5.2.4. *Scenario E: inflation*

Next, we modified the base scenario by an eight per cent annual real inflation on all cost and price figures. This applies to production cost factors, such as wages, energy and wood costs, investment costs, to the rate of interest as well as to forest products prices. In this scenario, the inflation losses due to taxation and the gains due to a real reduction in loan repayments approximately outweighed each other so that profit maximization resulted is roughly the same production figures as in Scenario A (see fig. 3b).

5.2.5. Scenarios F and G: wage rates and productivity

For Scenario F, an annual real increase of 2% was assumed for all wage costs. The near future effect as compared with Scenario A is minor, whereas in the long term, such an increase results in a significant reduction in production (fig. 3b). On the other hand, a change in the opposite direction, i.e., a 2% annual increase in labor productivity influences production significantly earlier. As shown by Scenario G of fig. 3b, in 10 to 15 years wood consumption increases to the level of 50 million m^3 annually.

5.2.6. Scenarios H and I: energy cost increase

The energy costs of Scenario A were increased by 2.7% annually (i.e., 15% in 5 y). Scenario H in fig. 3c shows the resulting decrease in production which is significant only in the long term. In Scenario I we assume in addition, that the price of wood as a primary energy source would increase by the same rate of 2.8% annually. Around the year 2000 the energy use of wood becomes competitive with industrial wood processing and from then on the total consumption of wood starts increasing. The difference in Scenario I and H in fig. 3c represents the energy use of wood.

5.2.7. Scenarios J and K: timber cost variations

Finally, two experiments were carried out to study the sensitivity of wood price. In Scenario J a 20% decrease was imposed as compared to the wood costs in Scenario A. In Scenario K the corresponding decrease is 2% annually, The result in wood consumption are given in fig. 3d. The 20% decrease resulted in better profitability and thereby a steady immediate growth of the industry. The annual decrease of 2% results in a significant change only after 10 years. However, thereafter the forest industrial growth is fast and reaches the biological limits of forests (i.e., 60 to 70 million m^3 of annual wood consumption) around the year 2000.

6. Summary and possible further research

We have formulated a dynamic linear programming model of a forest sector. Such a model may be used for studying long-range development alternatives of forestry and forest based industries at a national and regional level. Our model comprises two subsystems, the forestry and industrial subsystem, which are linked to each other through the roundwood supply from forestry to the industries. We may also single out static temporal submodels of forestry and industries for each interval (e.g., for each five year period) considered for the planning horizon. The dynamic model then comprises these static submodels which are coupled with each other through an inventory-type of variables; i.e., through state variables.

The forestry submodel describes the development of the volume and the age distribution of different tree species within the nation or its subregions. Among others, we account for the land available for timber production and the labor available for harvesting and planting activities. Also ecological constraints, such as preserving land as a watershed may be taken into account.

In the industrial submodel we consider various production activities, such as saw milling, panel production, pulp and paper milling, as well as further processing of primary products. For a single product, alternative production activities employing, for instance, different technologies, may be included. Thus, the production process is described by a small Leontief model with substitution. For the end product demand an exogenously given upper limit is assumed. Some products, such as pulp, may also be imported into the forest sector for further processing. Besides biological supply of wood and demand for wood based products, production is restricted through labor availability, production capacity, and financial resources. Availability of different types of labor (by region) is assumed to be given. The development of different types of production capacity depends on the initial situation in the country and on the investments which are endogenous decisions in the model. The production activities are grouped into financial units to which the respective production capacities belong. The investments are made within the financial resources of such units. External financing is made available to each unit up to a limit which is determined by the realization value of that unit. Income tax is assumed to be proportional to the net income of each financial unit.

The structure of the integrated forestry–forest industry model is given in the canonical form of dynamic linear programs for which special solution techniques may be employed. (See, for instance, Kallio and Orchard-Hays 1979, Propoi and Krivonozhko 1978.) Objectives related to gross national product, employment and profit for industry as well as for forestry have been formulated. Terminal conditions (i.e., values for the state variables at the end of the planning horizon) have been proposed to be determined through an optimal solution of a stationary model for the forest sector.

The Finnish forest sector model has been implemented for the interactive mathematical programming system called SESAME (Orchard-Hays 1978). It is ten period models with each period of five years in length. The complete model amounts to 520 rows and 612 columns.

A number of numerical scenarios have been presented to illustrate possible uses of the model. Both the discounted industrial profit and the discounted increment to the GNP were used as objective functions. The base Scenario A illustrates a case where the internal wood price and wage structure results in a rather poor profitability for the forest industries. This in turn amounts to an investment behavior which provides insufficient capacity for making full use of the wood resources. However, because of somewhat hypothetical data used for some key parameters, no conclusions based on these runs should be made on the Finnish case.

Several scenario runs have been presented to illustrate the model use for studying the influence of demand and price assumptions concerning forest products, as well as for studying the impact of various production cost factors.

The purpose of this work has been the formulation, implementation and validation of the Finnish forest sector model. A natural continuation of this research is to use the model for studying some important aspects in the forest sector. For instance, the influence of alternative scenarios of the world market prices for wood products would be of interest. Furthermore, the studies could concentrate on employment and wage rate questions on labor availability restrictions and productivity, on new technology for harvesting and wood processing, on the influence of inflation, exchange rate changes, and alternative taxation schemes, on land use between forestry and agriculture, on site improvement, on ecological constraints, on the use of wood as a source of energy, etc. Given the required data, such studies could be carried out relatively easily.

Further research requiring a larger modeling effort may concentrate on regional economic aspects, on linking the forest sector model for consistency to the national economic model, and on studying the inherent group decision problem for controlling the development of the forest sector. The first of these three topics requires a complete revision of our model generating program and, of course, the regionized data. The second task may be carried out either by building in the model a simple input–output model for the entire economy where the non-forest sectors are aggregated in a few sectors. Alternatively, our current model may be linked for consistency to an existing national economic model. The group decision problem may be analyzed, for instance, using a multicriteria optimization which is based on the use of reference point optimization (Wierzbicki 1979).

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MACRO-ECONOMIC MODELS FOR LONG-TERM FOREST SECTOR ANALYSIS IN NORWAY

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This paper has two main objectives. First, it gives a brief account of two multi-sectoral growth economic models, called MSG-3 and MSG-R, which are of considerable interest with regard to the long term analysis of the development of the Norwegian forest sector. Second, the paper describes how these two models could be used for analysing major problems in the forest sector. It is argued that the advantage of using the MSG Multi-Sectoral Growth models as a basis for long term forest sector analysis is that the forest sector activities are consistent with the expected development of the other sectors of the economy. The particular advantage of the MSG-R model is that important regional differences regarding industry structure and forestry conditions as well as employment situation are taken care of.

0. Introduction

Forestry and forest industry activities are closely linked, and decisions regarding major changes in one of the sectors have to be considered in connection with changes in the other sector. It is, however, in many situations insufficient to look at these two sectors alone when analyzing their future. In particular, regarding long term analysis, it is in most cases necessary to include other sectors of the economy to prevent taking unwise decisions. Likewise, it is, in the Norwegian context, not sufficient to look at the national level only, when analyzing the forest sector. The regional effects are in many cases as important as the national aspects.

This paper has two main objectives. First, to give a brief account of two multi-sectoral economic growth models (MSG-3 and MSG-R), which are of considerable interest with regard to the long term analysis of the Norwegian forestry and forest industry – hereafter referred to as the forest sector. MSG-3 is a macroeconomic model, having MSG-R as a regionalized version. This part of the paper is largely based on refs. [6,8,10,11,12, and 14].

The second objective of the paper is to describe how these two models could be used for analyzing major issues in the forest sector.

1. Brief outline of MSG-3 and MSG-R

1.1. MSG-3

The macro-economic model of the Norwegian economy known as MSG (abbreviated from Multi-Sectoral-Growth) originates from Johansen [7]. This study attempted to construct a model covering important aspects of the process of economic growth with particular emphasize on explaining differences in growth rates between the various sectors of the economy. It was the explicit intention of the model's originator that the theoretical framework should be kept simple enough for the model to use existing statistics and to be solved by existing computational equipment. Since 1968 a revised version of the original model, MSG-2F, was used in the Ministry of Finance [13]. In 1974 this model was revised by the Central Bureau of Statistics, giving the MSG-3 model.

The model can be characterized as being a disaggregated neoclassical growth model. It is based on an input–output description of the economy, having for each production sector a fixed proportion between input and output commodities. A central feature of the model is that capital and labour are assumed to be homogeneous production factors freely moveable *between* sectors and substitutable *within* sectors according to a Cobb–Douglas production function for each sector of production. The function is of the form

$$X_t = A_i N_i^{\gamma_i} K_i^{\beta_i} \, e^{\epsilon_i t}, \tag{1}$$

where X_i, N_i and K_i indicate gross output in year t, employment and total capital stock in sector i. A_i, γ_i and β_i are constants. ϵ_i is a coefficient which expresses technological change and t is time. For all manufacturing industries and for the service sectors constant returns to scale are assumed, i.e. $\gamma_i + \beta_i = 1$. For Agriculture, Fishing, Mining and Electricity Supply diminishing returns to scale are allowed for.

The producers' decision is, apart from Agriculture and Fishing, in the model based on profit maximization with profits in sector i defined as:

$$\text{Profit} = P_i X_i - \sum_j P_i a_{ji} X_i - W_i N_i, \tag{2}$$

- value of inputs of noncompetitive imports
- depreciation
- indirect taxes
- interest on the value of the capital stock

where P_i is the price of output from sector i, W_i is the wage rate in sector i and a_{ji} are the input–output coefficients.

For the majority of sectors, where constant returns to scale is assumed, profits as defined by eq. (2) are zero in equilibrium, and output is unde-

Table 1
Exogenous and endogenous parameters of MSG-3

Exogenous	Endogenous
Capital stock	Allocation of capital
Employment	Allocation of labour
Population	Output by sector
Government demand	Output price by sector
Inventory changes	Return on capital
Exports less competitive imports	Private consumption by sector
Technological change	Non-competitive imports

termined, while equilibrium prices are determined by the expression in eq. (2) equalling zero. In the model as a whole this does not mean that outputs of the sectors with constant return to scale are completely determined by the demand side of the economy; the composition of total demand is influenced by relative prices and these are in turn influenced by conditions on the production side.

The growth of total labour force, total capital stock and technological improvements are exogenously given and determine the growth of production capacity of the economy. Most final demand components except private consumption are also exogenously given.

The level and composition of private consumption are endogenously determined in such a way that full utilization of capital and labour is ensured. For this, the "Complete Scheme"-price-elasticity system of Frisch [4] is used together with exogenously given elasticities of expenditures, the demand functions being dependent on per capita total consumption expenditures and all prices.

Thus, the model traces out paths of balanced growth with neither shortage nor surplus of commodities, labour and capital. The producers' and consumers' adaptations are affected by changes in relative prices and the equilibrium solution calls for a simultaneous determination of prices and quantities.

The main groups of exogenous and endogenous elements of the model are listed in table 1. The planning period, or horizon, of the model, has normally been 20 to 30 years. During a single run of the model, the non-linear equation system is solved at intervals of 1 to 5 years.

The model consists of 38 production sectors, 9 sectors for private consumption, 10 sectors for government consumption, 4 gross capital formation sectors, 1 sector for stock changes and 1 export sector.

1.2. MSG-R

An input–output model for regional analysis has been developed by the Norwegian Central Bureau of Statistics to improve the coordination between national economic forecasting and regional forecasting.

The model, MSG-R, was operational in 1979, comprising 38 production sectors and 20 regional units (19 counties and a dummy region). It is an interregional input–output model. Commodity flows are subdivided in intraregional flows (commodities produced and used in the same region), interregional flows (commodities produced in one region and used in another region), and international flows (exports and imports). Constant input coefficients are assumed for each type of commodity flow. The input coefficients vary between commodities, industries and regions. The production of intraregional commodities is determined by regional demand. The production of interregional commodities is determined by the assumption of constant regional shares for each commodity. The model does not specify interregional commodity flows between pairs of regions, but it is assumed that demand in one region is met by the same regional pattern of supply as demand in another region. Exports are exogenously given at the national level and distributed to regions according to estimated market shares.

Private consumption and investment are also specified by region and in different ways related to the production side of the regional economy. Private consumption is assumed to be determined by the total value added in the regions. For investment, national figures are exogenously given in each industry and subdivided to regions according to production shares calculated in the model. The consumption and investment in central and local government are exogenously given in each region. It is furthermore assumed that labour input in each regionally specified industry is determined by the calculated growth in production and exogenous growth in labour productivity.

As mentioned earlier the regional model is constructed to be used as a supplement to the MSG-3 model. The idea is that calculations with the MSG-3 model shall be a national starting point for calculations on the regional model. Most of the exogenous variables in the regional model are either exogenous or endogenous variables in the MSG-3 model. For all final demand categories except private consumption the regional calculations are based on the same assumptions as the MSG-3 calculations. To obtain a corresponding consistency for private consumption a mechanism is implemented which adjusts the calculated regional figures in accordance with given national figures. The main endogenous variables are production and employment specified by industry and region. For these variables the regional model will give other national results than MSG-3, but the differences will normally be of acceptable size. Table 2 lists the exogenous variables in MSG-R and how they relate to corresponding MSG-3 variables.

The national accounts by county for 1973 have served as a data base in implementing the regional model. The regional accounts are constructed by breakdown of national figures and give complete input–output tables by county containing about 180 industries and 300 commodities. Private consumption, investment and most of government final consumption are also

Table 2
Exogenous variables in MSG-R and their status in MSG-3. Source: Skoglund [14]

Exogenous in MSG-R	Number of variables	Status in MSG-3
Production	3	Exogenous
Gross capital formation	3	Exogenous or endogenous
Production inputs, government production activities specified by county	5×20	Exogenous
Gross capital formation, government production activities	5×20	Exogenous or endogenous
Export, whole country	30	Exogenous
Change in stock, whole country	30	Endogenous on Exogenous
Final use levels, whole country	16	Endogenous
Export of used fixed capital	2	Exogenous
Foreigners' consumption in Norway	1	Exogenous
Employment	3	Exogenously or endogenous
Gross capital formation for whole country	30	Endogenous
Production changes	30	Endogenous
Productivity changes	5	Exogenous or endogenous

classified by county. Regional figures for exports and imports and intraregional and interregional trade flows are not, however, estimated in the regional accounts. As these data were very important in model implementation, they were calculated directly in connection with the model building.

The sector specification of MSG-R corresponds to that of MSG-3. For further information about the model the reader is referred to Skoglund [14].

2. MSG-3 and MSG-R in forest sector analysis

Forestry is one sector in the MSG model, whereas the forest industry is represented by two sectors: wood industry and pulp and paper industry. A precondition for useful analysis of the forest sector is a more detailed specification of the forest industry activities. This can be done in at least two ways – either by initially expanding the number of activities, or by breaking up ex post the existing two forest industry sectors into more relevant activities assuming the development of the other sectors in the model is not altered significantly in doing so. The former method is correct from a theoretical point of view. However, in practice, the latter method, which is by far the easiest and less expensive, is likely to be favoured.

In general the forest sector optimization model can be formulated as an optimal control problem, where the choice of control variables depends upon what policy options are of interest to be analyzed. In Norway the following variables are important: for the forestry (by site classes and regions) the intensity of planting, tending, fertilizing, thinning and clearcutting, and for the forest industry (including wood for energy production, by regions) the changes in production capacity, as well as trade in timber and wood residuals. If effects of subsidies (regarding energy, transport, investments, etc.) are known with a reasonable degree of accuracy, such policy variables could also be included in the set of control variables.

The objective function will in most cases be discounted net of the present value of profits or of the contribution to net domestic product. Since choosing the objectives (including corresponding constraints on state and control variables) is a controversial matter reflecting the various interest groups involved, comparing the consequences of using different objectives should be an essential part of a forest sector analysis.

The forest sector model and the rest of the economy represented by the MSG-models, would be linked through the constraints regarding labour and investment capital available for the forest sector, and through the relative prices generated by the MSG-models. These are important input variables for the regionalized forest sector optimization model.

Ideally the production functions used in the macro-economic model should represent the ex ante technology which in some proportion of the existing production techniques generates a new average production technique in the subsequent time period. In practice, Cobb–Douglas production functions based on historical data are used in the MSG-models. By varying the coefficient ϵ_t in eq. (1) it is possible to make a sensitivity analysis of the robustness of the future structure of the forest sector regarding different probable technological development paths of the industry. Expected technology development can be included by using for instance the production function approach outlined in Førsund and Hjalmarsson [5]. Export and import prices of timber and forest industry products could be incorporated by applying an equilibrium world market model for forest industry products as proposed by IIASA [17]. Meanwhile, the relative price development of these products has to be exogenously given.

In my opinion a planning period of about 25 years would be appropriate in the forest sector analysis. Such a period seems appropriate in the forest sector analysis. Such a period seems realistic in the forest industry's long term planning, and forecasts of expected prices of forest industry products and technological changes are also possible without too much uncertainty involved. In the forestry sector a 20–30 year period is relevant for fertilizing and clearfelling of the present mature and maturing forests. For the investments in the forestry's primary production (planting, pruning, tending) a 60–120 year perspective is necessary in Scandinavia, and even long term macro-economic models like the MSG-models will here be of little use.

The Norwegian economy is already dominated by the oil sector, which in 1981 accounted for 33% of Norway's total export earnings and 16% of gross domestic product. Corresponding figures for the forest sector were respectively 3.3% and 2.5%. It is therefore unlikely that marginal changes in the forest sector will significantly influence the other sectors of the economy. An exception here are regions like Hedmark, Oppland and Østfold where the forest sector plays an important role. For these areas the regionalized multi-sectoral model should be adjusted according to the results of the forest sector optimization model regarding in particular the availability of capital and labour. With the existing models it is, in practice, not possible to incorporate such feedback mechanisms endogenously as a link between the multi-sectoral models and the forest sector model. The links have to be exogenously determined through iterations: information from the MSG-model is disaggregated in the MSG-R model and then fed into the forest sector model. The result here is then, for the above mentioned regions, used as input in the MSG-R model to get consistent solutions, and so on.

In this way one could arrive at a set of simulation and optimization sub-models exogenously linked to each other. A world market model and technology (production function) models give inputs to the MSG model (national and regionalized version), which then generate among other things labour and capital available for the forest sector as well as future relative prices. This information serves as a basis for a more detailed forest sector model.

The forest sector model has to be quite large to be realistic. In order to derive strategies for optimal control of a large scale system, it is often necessary to perform a model reduction giving a new model of reduced order and complexity based upon the more detailed and complete original model. As discussed by Balchen [1] an important part of control strategy is to find a *state estimator* which utilizes observations from part of the real system to compute estimates of the state variables in the whole model system. Equally important, when going for optimal solutions, is to develop a state reducer algorithm which brings the full model down to dimensions which are possible to handle. Various filtering techniques can be used for this. According to Balchen [1] the state vector dimension of the reduced model should not exceed 30, with existing computing power.

A forest sector model in a context as described here is of interest for several types of analysis. The allocation of timber between different demand categories – i.e. different kinds of forest industries, including wood for energy production, would be one interesting issue to analyze. It will be possible to arrive at estimates of optimal development paths of the forest sector seeing the cutting policy and some of the investments in forestry together with the development of the forest industries and of the other sectors of the economy. Most important, perhaps, will be the improved possibility for doing sensitivity analyses in order to find how flexible the various solutions are regarding changes of state variables, constraints and objectives. The shadow prices generated by the regionalized forest sector model give useful information regarding in which fields the forest sector's improvement potential is greatest.

Another field where the MSG model could be of considerable interest in connection with the forest sector, concerns the analysis of the domestic demand and supply of the various forest industry products, for instance as described in Statistisk Sentralbyrå [16] regarding energy.

The advantage of using the MSG models as a basis for long term forest sector analysis is that one is assured that the forest sector activities are consistent with the expected development of the other sectors of the economy. The particular advantage of the MSG-R model is that important regional differences regarding industry structure, forestry conditions (age classes, growth potential, species, etc.) and employment are taken care of in the analysis.

The forest sector discussed so far is deterministic. It may, however, in principle be expanded to take into account the stochastics regarding, e.g., timber stock levels, annual timber yield and future prices as described by Dixon and Howitt [3] using the Linear-Quadratic Gaussian approach in a forestry model. As mentioned by Kleindorfer [9] one major problem in such cases is to get sufficient computer power for the solution algorithms involved.

3. Some final remarks

The current MSG-3 model uses constant coefficients for the input–output relationships in the economy. However, the need to take into account systematic changes in some of the input–output coefficients is recognized, and it is hoped to introduce such changes explicitly into the model.

For the specification of the production structure of the MSG-model the Cobb–Douglas functions might be replaced by CES-functions. However, as mentioned by Johansen [8] the estimation of elasticities of substitutions remains a very uncertain undertaking and the evidence against the Cobb–Douglas functions is not yet conclusive. Similarly, introducing putty–clay assumptions leads to almost insurmountable data problems when many sectors are involved.

Regarding analysis of regional employment effects the present MSG-R model has some deficiencies. It is most likely unrealistic to assume equal productivity development in all counties for each sector. Also, the employment is decided by demand only, and labour supply relationships are not taken into account. The Socio-Demographic Research Group at the Central Bureau of Statistics is, however, working on improving these aspects of the model.

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A DYNAMIC SIMULATION MODEL FOR THE FOREST SECTOR WITH AN ILLUSTRATION FOR SWEDEN

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This paper describes a dynamic simulation model for long-term development of the forest sector. The model is developed for analyzing the impact of cost competitiveness and wood availability on the structural change of the forest sector in a particular country over a 20–30 y period. It is intended to aid decision makers on strategic issues about possible futures of the forest sector.

The model consists of two symmetric competing forest sectors: one for the national forest sector under investigation and another aggregating competing forest sectors of other countries. Both submodels cover all activities ranging from timber growing to the consumption of forest industrial products such as paper (pulp), sawnwood and panels. They are represented in the model through eight modules: demand of forest products, product market, forest industry, roundwood market, forest management, inventory of standing volume, construction sector and regulation of the forest sector. At the end a Swedish example is discussed.

1. Introduction

This paper gives a formal description of a prototype model of long-term development of the forest sector (Adams et al. 1982, Lönnstedt 1983a, b). The model is developed for analysis over a 20–30 y perspective, the impact of cost competitiveness and wood availability on the structural change of the forest sector under investigation (Grossmann and Lönnstedt 1983). The model is able to produce scenarios for the forest sector under different assumptions about the economic environment of the forest sector when applying different strategies. The model is intended for interactive use in strategic discussions in government, forest industrial companies, and other organizations concerned with the future of the forest sector.

Under given assumptions, the model produces scenarios for the sector development. The long-term price will follow the long-term development of processing costs. However, the market share and price will fluctuate around these trends due to, for example, information delays, planning time, and building time, which will cause excess demand or excess supply. Unfavorable cost development, limited wood resources or opportunities for better investment outside the sector will alter this scenario in the sense that the market share for the sector under investigation will decrease. If this share is already small and the sector under investigation is not the price leader, the price will remain unchanged. Actions taken by the sector will, however, change the scenario.

Existing forest sector models (Kuuluvainen and Seppälä 1982, Kallio et al. 1982, Kalgraf and Lönnstedt 1981, Adams and Haynes 1980, Lönnstedt and Randers 1979) as well as the general economic theory lie behind the construction of this model (e.g., Douglas 1979, Layard and Walters 1978, Gold 1971, Cohen and Cyert 1965). The demand and supply curves are indirectly represented. One main assumption is that the system continually tries to equalize potential demand and supply, but never succeeds due to delays. A simulation approach will be used where the solution is found recursively over time.

The model consists of two symmetric competing forest sectors – one for the national forest sector under investigation; and the other represents competing forest sectors of other countries (see fig. 1). Each forest sector covers all activities ranging from timber growth to the consumption of forest industrial products such as paper, sawnwood and panels.

In the text below, the general structure of the model system is described starting from an aggregated level, i.e., the linkage between demand and supply will be described. Thereafter, the description of the supply modules follows.

In principle, each sector is thus defined by domestic long-term *demand* and *supply* of forest industry products and by a product market. *Trade* is introduced by linking the two sectors. This means that imports of forest industrial products and roundwood are introduced on the domestic markets. At the same time, the forest sector under investigation has the possibility to export its products. Exogenous variables are gross domestic product (GDP), size of population, price of substitutes, exchange rate, price of input factors and technological change.

The price of the bulk products studied, which is essential for the feed-back mechanisms that exist in the model, is defined by the aggregated long-term demand and long-term supply for the two sectors. For example the increasing relative price of forest products will, after some time, decrease the demand for forest industry products which in its turn will affect the price. Price also affects supply via profits and investments. Decreasing price means, everything else unchanged, reduced profit per product unit, and less money for investments. Fewer investments in new capacity will after some years reduce the long-term supply of forest industry products, which in turn will increase the price.

Supply of forest industry products from each forest sector is made up in the model of five submodels or modules representing: (1) forest industry; (2) roundwood market, (3) forest management, (4) inventory of standing volume, and (5) regulation. The linkage of these modules to the rest of the system is carried out through the forest industry module. Potential supply and processing costs are given to the product market module, from which product price, actual demand and market imbalance are received.

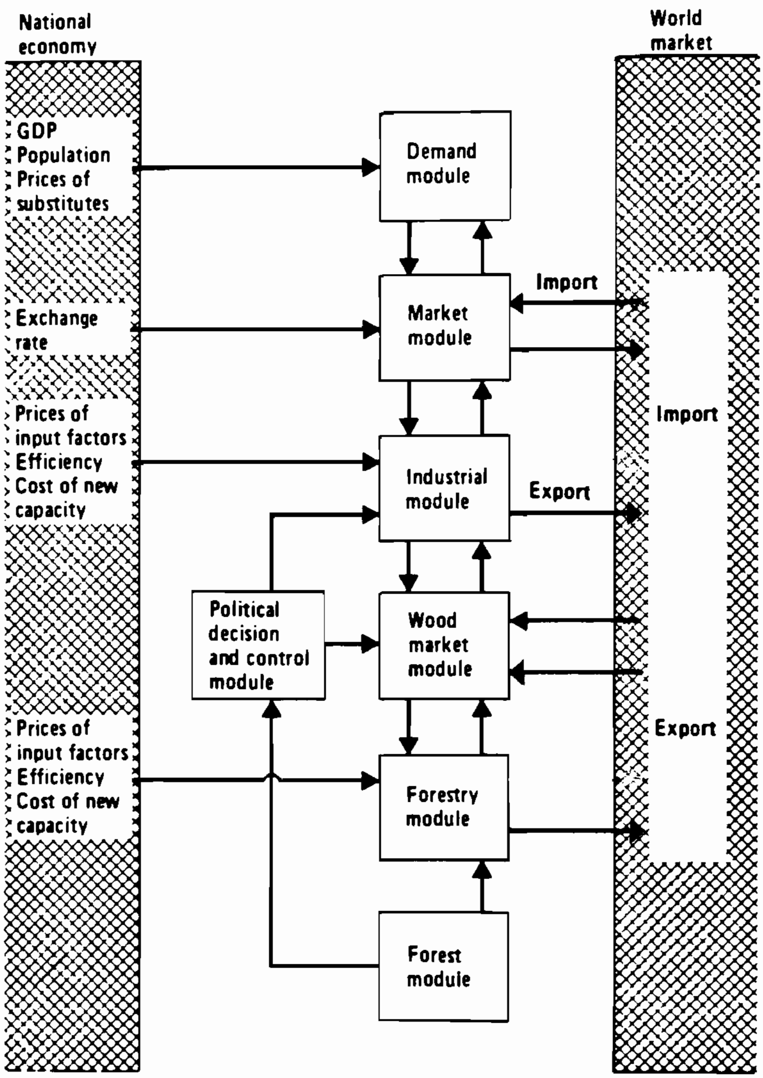

Fig. 1. Outline of the prototype model used on the forest sector under investigation.

The forest industry module defines production capacity and processing costs. The wood raw material base for the industry are the domestic forest resources and import of wood. Gross felling, import of wood, and delivery price are defined in the roundwood market module. Potential supply based on existing forest resources and harvesting capacity, as well as the harvesting costs are calculated in the forest management module. The regulation module allows the regulation of investments in new industry capacity and gross felling.

Each of the eight modules is described in the following text. Each section contains a description of how each module is constructed.[1] Possible future extensions of the model are also pointed out.

2. A prototype model

2.1. Demand of forest industrial products

Demand per capita for paper, sawnwood, and panels, may be defined by a logistic function of GDP per capita, and product price relative to price of substitutes[2] (fig. 2). Total demand is calculated by multiplying demand per capita with size of population. Demand for fuelwood relative to total demand of wood, may be calculated along the same principles. However, in this model version it is defined as a constant share of demand for wood raw material.

Demand for pulp and paper, sawnwood, and panels will, in this version of the model, be aggregated in the market module into just one demand curve. The output of the forest industry is thus only one product. The industrial demand for wood together with the demand for fuelwood meet the supply of wood in the roundwood market. The principles outlined in this paper can easily be followed in future versions of the model when the number of products is increased. Another important aim in future model versions is to relate the demand functions to the end use of wood based products.

2.2. Product market

Sawnwood, panels, and pulp can be considered as bulk products the prices of which are determined by the condition of an international market – in this case defined by the two forest sectors under study. Thus, after getting total potential demand from the demand module and total potential supply from the industry module, the market module determines:

(a) the product price (in U.S.$) which is the same for the two sectors under investigation,

(b) the ratio between aggregated potential demand and aggregated potential supply, which is called market imbalance.

[1] For a detailed mathematical formulation, see Lönnstedt 1983b.

[2] The chosen function is just one of several candidates here as well as later on the paper. The function is an *example* for giving structure to the model. The final choice must be made after looking at the data for the forest sector under investigation.

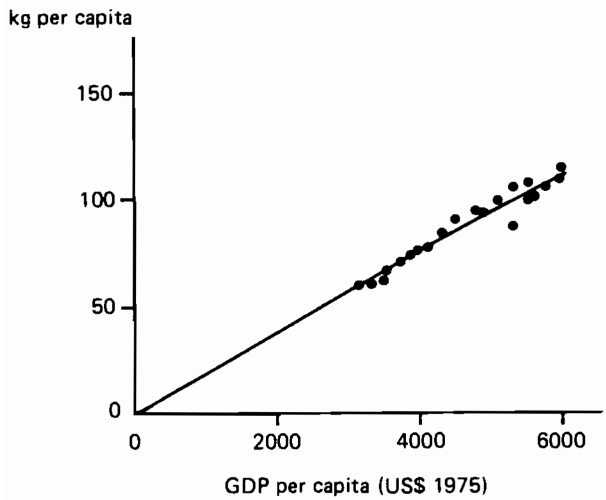

Fig. 2. Example of a demand function for paper and paperboard in Western Europe. (Sources: UN Statistical Yearbook, Yearbook of National Accounts Statistics and FAO Yearbook of Forest Products.)

2.2.1. Market price

The principle for formulating the price in the product market is to calculate what long-term price is needed in order to cover both variable and fixed costs. The price is calculated as the long-term average variable cost for those producers supplying the market plus a mark-up for required long term return on capital. As a weight when calculating the average variable cost the quantity produced by each producer has been used. Mark-up can for example be defined by an exponential function of the market imbalance (fig. 3). Excess supply implies a lower mark-up than a market with excess demand. Market imbalance information is somewhat delayed in the model. Therefore, it will take some time for the price to change when the market situation changes.

2.2.2. Import and export

Import is determined as a delayed share of potential demand (Nagy 1983). The share can for example be defined by a logistic function of relative processing cost, taking transportation cost into consideration (Considine et al. 1983, see also Andersson and Persson 1982). Export is calculated in the same way but using the conditions in the competing forest sector for which potential demand is calculated following the same principles as for the forest sector under investigation. Depending on the actual case it can be necessary to introduce trade with one more market.

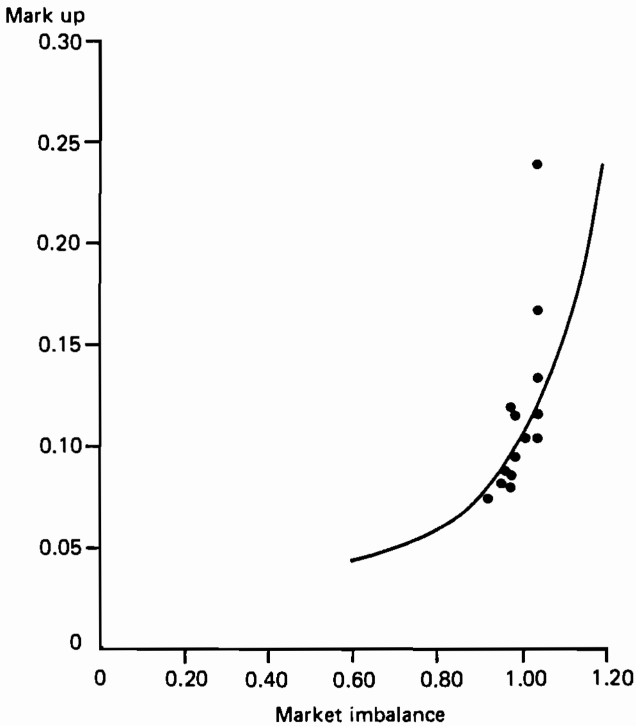

Fig. 3. Example of mark up defined as an exponential function of market imbalance. Plotted in the figure is gross profit for the Swedish forest industry related to a calculated value of capacity utilization. The capacity utilization when production is peaking is assumed to be 0.95 of the capacity. The values thus calculated have been related to the average capacity utilization. (Sources: Yearbook of Forest Product, Yearbook of Swedish Forest Statistics and SOS INdustry.)

2.2.3. Actual demand and supply

The price that is fixed for a specific time period generates potential demand and supply. In the long run demand and forest industry capacity will follow each other. This means that inventory fluctuations can be neglected. There will of course be periods when potential demand exceeds potential supply, triggering price increases that increase production capacity and decrease the potential demand – and vice versa if there is an excess supply. At the same time demand and supply change due to income changes and changes in technology. If everything remains unchanged or develops in the same way the market shares of the two forest sectors in the model will also be unchanged. If the competitive conditions are getting worse for one of the sectors or if it faces limited wood resources the market share of this sector will decrease (if no actions are taken for offsetting those disadvantages).

Actual demand and supply is calculated in the module as the minimum value of potential demand and potential supply taking export and import into consideration.

2.3. Forest industry

In the forest industry module the long-term potential supply is determined taking an eventual shortage of wood deliveries into consideration. The module consists of three submodels: (a) gross profit module, (b) cash flow module, and (c) production capacity module.

2.3.1. Gross profit

Gross profit is defined by subtracting total variable costs from total income. Gross margin is gross profit as a share of total income. Gross profit for the forest sector under investigation is calculated in the model through adding profit for domestically sold products to profit for exported products. The difference in profit between these two types of sales is explained by differences in transportation costs. The market price, expressed in U.S. dollars, is transferred into the national currency.

The variable costs are calculated in this version of the model from two groups of input factors: wood and other input factors. The formulation assumes fixed coefficients for labor and capital usage. This means that the relative cost of labor and capital is constant. The transportation cost from mill to market is calculated as a share of total variable processing cost.

The cost of wood is calculated by adding the transportation cost to the delivery price, which is defined in the roundwood market (see section 2.4). Transportation cost is assumed to be a constant share of the roundwood price. For calculating the wood raw material cost per product unit, cost of wood at mill is multiplied by wood volume needed per product unit; this transformation quotient in the model is a constant parameter.

The principle for calculating the production cost related to input factors other than wood is the same. The price of the input factors is devided by the quantity of forest industry products produced per unit input factor (shortly called average efficiency). If we take labor as an example the calculation is as follows: the wage cost per hour is divided by the number of units produced per hour – a number that changes with the technological development, investments, and shutdowns – and the result is wage cost per produced unit. Compared with the calculation of the cost of wood, the prices of the other input factors than wood are treated as exogenous variables and the efficiency in utilization of the factors as a variable.

When developing the model it is worthwhile to remember that (a) a production function with fixed input coefficient for labor and capital is used, (b) cost with different dynamic behaviors is combined into just one cost factor,

(c) transportation costs and efficiency in wood utilization is treated as constant, and (d) the same fuelwood share is used for import as for domestic harvest.

2.3.2. Cash flow

In this version of the model we are operating with two inflows and one outflow of cash. One inflow is earnings before depreciation after subtracting a constant share representing interest payments, dividends, and taxes. Another financial source is net inflow of external money such as new loans after repayment of old ones. The external financing is expressed as a share of the first mentioned inflow – a share that can for example be a linear function of the gross margin for new investments relative to a gross margin reference value for alternative investment opportunities. The financial resources that these two cash inflows make up are all, with the exception of a certain share that is withheld as working capital, used for investments. One part of this financial outflow is for investments in new forest industry capacity and the other part is for investments outside the sector (see next section).

The amount of borrowed money and repayments follow, in this version of the model, given a profitability of the forest sector comparable with alternative investment opportunities, the development of the gross profit and financial resources. The model will become more realistic by introducing an accounting system that keeps track on balance of payment and allows the industry to manipulate its profit and taxes by changing the depreciations. This will probably not have any major impact on the dynamic behavior of the model.

2.3.3. Production capacity

As mentioned in the previous section when describing the cash flow two types of investments exist. One type is investment in new forest industry capacity and the other type is investment outside the sector. The share of financial resources available for forest industry investments can for example be a linear function of expected changes in the market situation as measured by a moving average of the imbalance between potential demand and supply. Changes in the market imbalance indirectly reflect the changes that can be expected in the product price and profitability. If no regulation exists (cf. section 2.8) the "planned" change in capacity, in the model defined by dividing the outflow from the financial resources intended for new industrial capacity with unit investment costs, will be realized. If regulation exists the change of the industrial capacity will be affected by authorized change and regulation power. Financial resources not used due to regulation are assumed to be used for investments outside the sector.

Between the investment decision and the start of production several years will elapse because it will take some years to build new capacity. The model operates with a shutdown rate that for example can be expressed as an exponential function of profit margin (fig. 4).

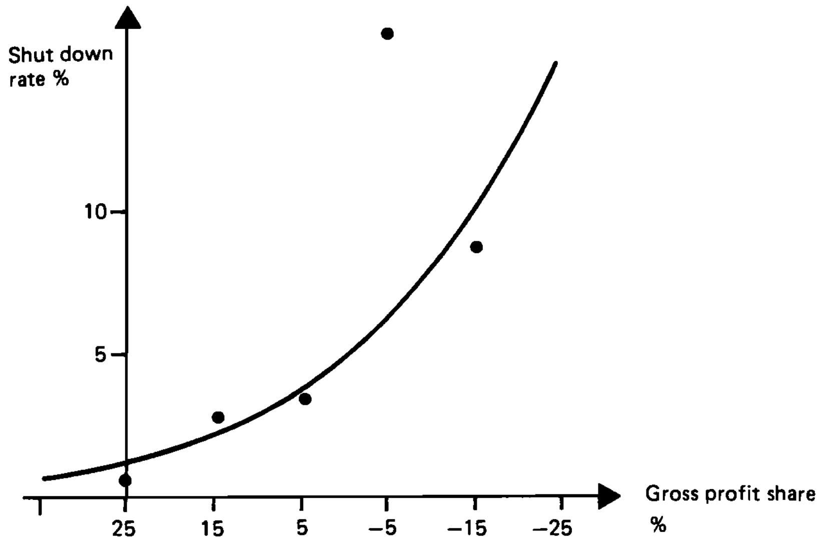

Fig. 4. Example of a shutdown function derived for the Swedish pulp and paper industry (Source: taken from SIND 1983: 1.)

As pointed out above, it is important to keep track of the production efficiency of the existing capacity as this will affect the variable costs. In the model we handle this by saying that each new capacity unit allows the input factors to be used with a certain efficiency. The average efficiency of the operating capacity is thus affected by the rate of introduction of new capacity and the shutdown of old capacity.

For calculating average efficiency a technical term, called *efficiency memory*, has been introduced. The efficiency memory is defined as existing capacity times average efficiency for existing capacity. Changes in the efficiency memory depend on new capacity times production efficiency for new capacity with respect to the shutdown of old capacity and the production efficiency for this old capacity. Average capacity is calculated through dividing the efficiency memory by the capacity.

In this version of the model we have not distinguished between investments in existing or new capacity. Furthermore, investment costs are not affected by the type and size of investment. Neither have we incorporated the effect maintenance investment will have on the lifetime of capacity.

Potential forest industry production is, taking the conversion quotient into consideration, defined from gross felling, export and import of wood and use of fuelwood. Potential domestic demand of wood is calculated through multiplying the production capacity with the conversion quotient and adding fuelwood.

2.4. Roundwood market

Forest industry and forest owners are looked upon in this module as two independent parties. The roundwood market is characterized by negotiations between the parties about the actual stumpage price.

2.4.1. Price of roundwood

The actual delivery price is decided upon after negotiations and depends on the demand and supply of wood. In the model all roundwood will be sold as delivered timber. The delivery price is calculated from the sum of the average variable cost for logging and the stumpage price. The stumpage price depends on the negotiation power set somewhere between the maximum stumpage price the buyers are willing to pay and the minimum stumpage price the sellers are willing to accept for doing any cutting. The negotiation power can for example be defined as a linear equation of timber balance. Two extreme possibilities exist. If negotiation power equals one the realized stumpage share equals maximum possible stumpage share that the forest industry can pay. On the other hand if the forest owners have no negotiation power at all the realized stumpage price equals the minimum acceptable stumpage share for forest owners. If negotiation power equals one half the stumpage share will be between those two extreme values. What parameter values to use is partly a policy question when experimenting with the model.

Depending on the type of market in different countries the structure of this module will be changed. If, instead of negotiations, there is just a fixed price for standing timber that has to be paid to the forest owner (for example the national forest service) this "price" can just be added to the average variable cost instead of the negotiation structure now outlined. If the owner structure of the forest sector is other than the one outlined, for example closer economic links between forestry and industry, the net profit of the forest sector management will go into the financial resources of the forest industry. The outflow of financial resources will in its turn be divided between investment in the industry and in forestry.

2.4.2. Import and export

Import and export of wood is calculated following the same principles as for import and export of forest industry products (compare section 2.2). Import of wood is thus calculated as an s-shape share of potential demand of wood – a share that increases if domestic wood at the mill becomes relatively more expensive than imported wood. Export is calculated in the same way but using the conditions in the competing forest sectors.

2.4.3. Gross felling

Gross felling is calculated as the minimum value of demand for wood taking trade into consideration and potential supply of wood. If cutting is regulated the potential supply will be a compromise between the forest owners' willingness to cut and the allowable cut calculated by the regulators. Where between those two extremes the compromise will be depends on the regulation power (see section 2.8).

2.5. Forest management

The long term goal of the forest managers is assumed to be to adjust the harvesting capacity to the industrial capacity, considering economic and biological restrictions. The effects on the long-term supply due to changed owner structure, inflation, taxes, lack of investment opportunities, recreational and environmental considerations (Grossmann 1983), and lack of forest labor (Kauluvainen and Seppälä 1982) will not be taken into consideration in this version of the model.

The principles behind the structure of the forestry module are the same as for the industrial part – the same mechanisms are functioning. Therefore, in order not to try the patience of the readers the equations of this model will not be described.

2.6. Inventory of standing volume

The structure of the forest industry module is formulated without detail. This is reflected in the modeling of the inventory module. The aim is to give an estimate of the wood raw material base for the forest industry. In this model version we do not distinguish between different species, site quality classes, or age classes. Neither do we distinguish between different types of felling. We have not taken into consideration the future possibility of decreasing residues and losses or of using branches, roots, leaves, and needles. The effects of different silvicultural activities have not been handled.

The standing volume of the forest in the model is increased by increment and decreased by total drain and mortality. The increment is calculated by multiplying an increment percentage by the standing volume. The increment percentage can for example be defined as an exponential function of density. Mortality is calculated in the same way. These relationships are assumed to be a mirror image of the relationship between increment and age. The forest land area has been made a constant. The total drain is calculated as the sum of gross felling and mortality. The latter is a constant factor of standing volume.

In a future version of the model it may be possible to incorporate the effects of silvicultural activities (Lönnstedt and Randers 1979). Other possibilities are to introduce different age classes, qualities, and species.

2.7. Construction sector

The forest industry and management modules get investment cost and efficiency, used when calculating new capacity and efficiency memory, respectively, from the construction sector module. Investment cost is calculated from the cost of input factor divided by the efficiency in using the input factors when producing new machineries. Both factor costs and efficiency are exogenous.

2.8. Regulation of the forest sector

In some countries such as the Nordic countries, there is a long established tradition to utilize the forest in accordance with long-term allowable cut. This was, for a long time, only done through regulations on the cutting. Now, even the investments in new sawmilling, panel, and pulp capacity are regulated.

The allowable cut can for example be defined as a linearly adjusted value of the annual increment. If the inventory is above a target value, this represents in the model, a forest with an uneven distribution towards mature stands and the allowable cut calculated from increment will be somewhat increased and vice versa.

The regulation power can for example be defined as a linear function of the timber balance. A tight timber balance – total drain exceeds allowable cut – implies a high regulation power, and vice versa. By setting the regulation power to zero the regulating authorities will be put out of play (compare with section 2.4).

3. The Swedish case

3.1. Technological development

In this section the presented model will be used for illustrating the impacts of technological development on the dynamics of the Swedish forest sector. When parameterizing the model for the Swedish forest sector, the primary manufacturing industries have been treated under one coverage. The assumptions for the reference scenario are that the historical trends (GDP, population increase, factor costs and technological development) will continue. In the two following scenarios the assumption is more rapid and slower technological development, respectively, than in the reference run. The two following scenarios show the effect of (1) stimulating investments in new capacities through different economical and financial means and (2) the same as (1) plus shorter life time for capacity. The impact of technological development on the production process is defined as the change in the relationship between output and input factors, for example, amount of pulp per working hour, per cubic meter of wood or per energy unit. Technological development in the sense of the improvement of existing processes or introduction of a new process is thus measured as the increase in output at a given input.

3.2. A reference scenario

The gross domestic production per capita for Sweden has, as an average until the end of this century, been assumed to increase by 1% in real terms compared with 2% for Western Europe. (The corresponding percentages for 1960–1980 are 2.8 and 3.5, respectively.) Also the population is assumed to increase slower in Sweden than in Western Europe, 0.3% compared with 0.7% (0.5 and 0.7, respectively, for the period 1960–1980). The result will be a slower increase in demand for wood products in Sweden than in the rest of Europe. Another reason for this is that consumption of wood products in Sweden seems to be in a more mature phase than in the rest of Europe. This means a scenario where the Swedish consumption of wood products increase from 5.6 million tons in 1980 to 7.0 million tons in 2000. The corresponding figures for Western Europe are 106 and 199 million tons, respectively.

Other assumptions made are that the factor costs (excluding wood raw material) in nominal terms as an average for the decades to come until the end of this century will increase by 10% both in Sweden and in competing countries. Technological development, defined as change of output–input ratio for new equipment, is assumed to increase by 4% for this period. New equipment is available for every company that wants to buy. The value of the Swedish currency relative to the European ones is assumed to be unchanged for the years to come.

One main difference between the Swedish forest sector and its competitors in North America is wood availability and cost of wood. The Swedish forest industry capacity is close to the sustainable yield compared with, for example, competitors in the southern U.S.A. This situation will have an impact on (1) marginal costs for wood, for example, due to expensive import, (2) transport distances and harvesting costs for part of the domestic supply and (3) negotiation power of the forest owners. Investment in new primary manufacturing capacity has been regulated in Sweden for almost one decade. The silvicultural law that regulates the cutting has an history of more than 100 years. Those "institutional conditions" will quite certainly affect the scenarios for Sweden.

The reference run shows increasing forest industry capacity for the competitors of the Swedish forest sector for the whole period (fig. 5). Some decades are characterized by a quicker capacity increase than others depending on the market situation and applied investment policy. The capacity of the Swedish forest sector levels off in the scenario due to limited domestic supply of wood, increased costs for wood, and regulation of industrial investments. The Swedish export of wood products follows the capacity development. However, there is a tendency, due to high production costs relative to competitors, to lose market shares on the Western European market.

Increasing cost for wood means lower profit, difficulties in borrowing money and an outflow of money from the sector to better investment oppor-

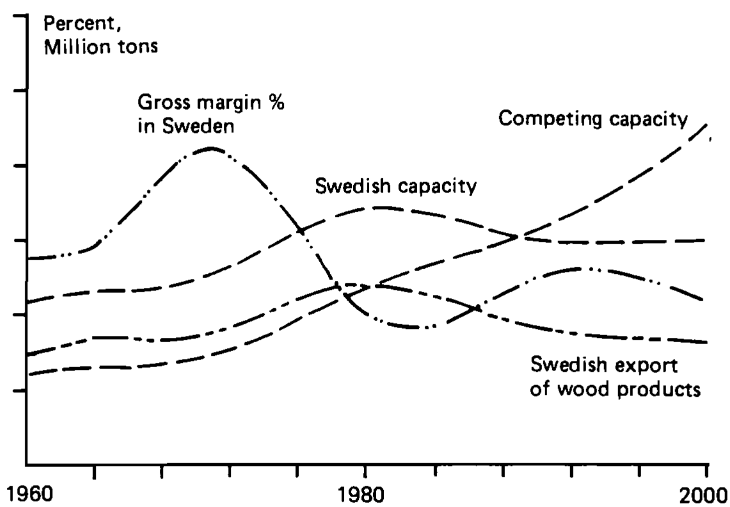

Fig. 5. A reference scenario for Swedish and competing forest industry capacity (different scales). Export from Sweden and gross margin for the industry is illustrated.

tunities. This decrease in the cash flow means less money for investments. The result of the model is that the output–input ratio develops unfavorably for the Swedish forest sector compared with its competitors. The situation could be better or worse depending on the market situation. A favorable market situation for the producers, defined as excess demand, which in the model develops at the end of the 1980s and the beginning of the 1990s after a bad end of the 1970s and the beginning of the 1980s, means higher profit and increased selling. Another explanation for the increasing profit is reduced costs of wood as a consequence of reduced industrial capacity. Increased profit means more money for investment.

3.3. Changed technological development

The two model runs (in this section) show the impact of the technological development on the dynamics of the Swedish forest sector. In the reference run the output–input ratio for new equipment was assumed on average to increase by 4% per year for the rest of this century. This assumption is in the two model runs increased to 6% and decreased to 2%, respectively. Everything else is unchanged.

Those changes have quite a substantial impact on the result. The assumption about a quicker technological development means a slower long-term increase of the wood product price than in the reference run – and the other way around. Furthermore, the development of the Swedish forest industry capacity, wood product export, profit level, and harvesting level is changed – just to mention some examples.

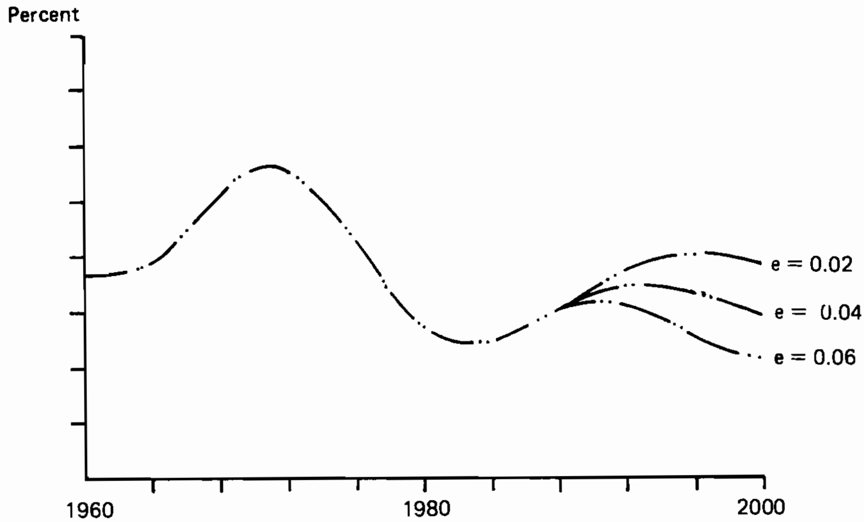

Fig. 6. Scenarios of the gross margin for of the Swedish forest industry for three different assumptions about the technological development of new industrial equipment (e = average annual increase of output–input ratio for new equipment).

The assumption about a quicker technological development than in the reference scenario makes the situation worse for the Swedish forest sector (see fig. 6). The explanation is that an unfavorable profitability development due to increasing wood costs relative to the competitors leaves, as was mentioned above, less money for investment in new equipment. A decreased investment rate as compared with competing forest sectors means a slower introduction of new equipment and thus a slower increase of the average output–input ratio than for the competitors. The disadvantage with limited investment possibilities will be increasingly clear the quicker the technological development. This is one of the reasons for a widening gap between developed and developing countries. Another factor to remember is the effect of the technological development on the long-term price development.

3.4. Consequences of different investment strategies

One problem in the scenarios for the Swedish forest sector as a whole, but not necessarily for an individual company, is the increase of the forest industrial capacity combined with expensive wood raw material. This means less money for investments with the drawbacks discussed above. One strategy for offsetting this drawback with limited and expensive wood resources is to make more money available for investments or to replace old capacity quicker. A decrease of the prime rates will help because the pulp and paper industry is

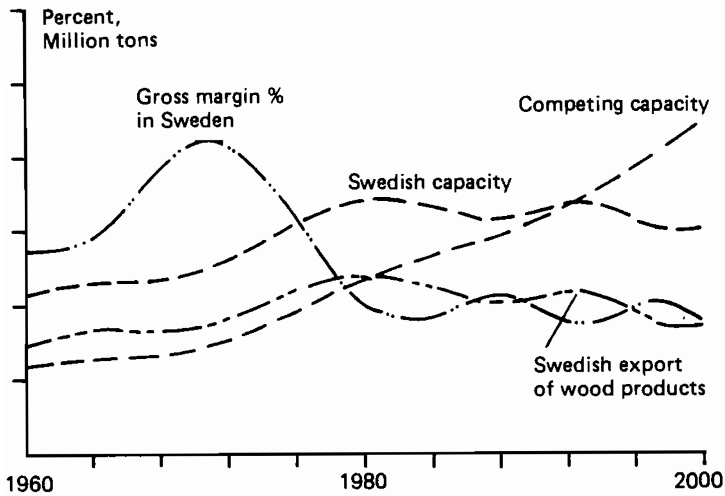

Fig. 7. A scenario for the Swedish forest sector when the financial resources for investments are doubled as compared with the reference run during the middle of the 1980s. (Competing and Swedish capacities are on different scales.)

very capital intensive. Reduction of taxes is another way. The banks could be asked to lend more money to the forest sector – for example special and favorable forest industrial bonds could be issued with the help of the government. The sector could be subsidized and so on.

Fig. 7 shows a scenario where the assumption is that the financial resources available for investments in new industrial capacity are almost doubled in the middle of 1980s as compared with the reference scenario. (This dramatic increase is used to make the result more clear.) It takes some years, due to different delays in the system as for example building time, before the result is seen. Compared with the reference scenario the industrial capacity after the beginning of the 1980s does not decrease as much as in the reference run. But what is more important is that the output–input ratio is increased, as a consequence of increased investments. The result is decreased processing costs.

However this strategy means increasing cost of wood and interest rates for borrowed money. These cost increases can counter-balance the decrease in processing costs. This is what happens in the model run.

A strategy with increased investments could be combined with a shortened lifetime. The goal could be to allow an even more rapid introduction of new technology than in the previous scenario through a shortened lifetime of capacity. The result will, in the model, be a quicker increase than in the previous scenario of the output–input ratio. This will have a favorable effect on the profit development.

4. Concluding remarks

Forest management and investments in new plants involve long-term decisions. Consequently, strategic planning is vital for the companies involved and has for a long time been one of management's most important functions. This is even more underlined by the fact that forest companies are facing slow growth of their markets, cyclical fluctuations of demand, high investments requirements for new capacity and high interest rates (Kalgraf et al. 1983).

The first and most important step in strategic planning is understanding the environment in which a company operates, i.e., the market, the competitors, the supply of raw material, and the society at large. The presented forest sector model is a tool for achieving this. The model is basically a formulation of the logical relationships between the forest products market, the forest sector under investigation and competing forest industrial production, and wood supply.

The prototype model presented is to be looked at as a suggestion or example of how a national forest sector model could be built. Some of the modules will be of more interest than others depending on the conditions for the forest sector under study. For example in some countries the forest resource will be of more importance than in others. For some cases, it will be found that modules are missing, for example, representing land use and pollution (Grossmann 1983). The structure of some modules will probably also need some changes. For example, in countries where the forest area is mainly owned by the state the stumpage price could be decided upon in another way. More detail could be wanted. This is true when it comes to the number of products.

Technological development of production processes, defined as the output–input ratio, is of importance for the competition between companies and between industrial sectors. A company that introduces technological changes quicker than its competitors will have a more favorable development of the output–input ratio and thus also of the processing cost. This means higher profitability than for the competitors. The profit could be used for new investments which will still further improve the advantage of this company. The profit could also be used for improving the services given to the customers or for given price discounts. The result will probably be an increased market share and selling. The profit will increase even more.

The technological development affects, via the processing costs, the long term price development. This means that a product characterized by a slow technological development, as compared with other products, will be substituted. This explains the structural changes taking place between different industrial sectors.

One disadvantage for the Swedish forest sector during the decades to come is that the industrial capacity can be expected to increase only slowly due to limited supply of wood. It will be easier for the quickly expanding competitors, for example in the southern U.S.A., to introduce technological novelties. Therefore it seems extremely important for the Swedish forest sector quickly to discover and introduce technological progress. This is a strategy that the industry has successfully followed for several decades. However it is important for the future to invest even more money in R&D activities. This will also open up the possibility for discovering new products and processes where the industry can find a competitive advantage. Society can help to achieve this development through creating good financial conditions for the forest sector.

The use of the model has been educational and has given the user possibilities of answering "what if questions" (Lönnstedt and Schwarzbauer 1984, Lönnstedt 1983c). The inexpensive and simple interaction between man and computer increases the understanding of the way the environment of an individual company behaves.

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A STRUCTURAL CHANGE MODEL FOR REGIONAL ALLOCATION OF INVESTMENTS *

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The industrial establishments of the forest industry are often concentrated in distinct regions, in which they employ a significant part of the labor force. This paper presents a model which provides a means to analyze and evaluate investment patterns and programs in such regions. The model contains two integrated parts: one describes the obsolescence and renewal processes in the industry sectors of a region. This part of the model is formulated within the framework of a regionally specified multisectoral model. The other part is an optimization model which generates investment and production for regions, given national and regional constraints on production and employment levels.

1. Introduction

In Sweden the forest industry constitutes a sector which is extremely oriented towards the world market. At the same time this industry is characterized by a set of basic rigidities. First, the long run supply of inputs is determined by the slow growth process of forests in regionally concentrated areas. Second, the paper and pulp sector, here called "paper and paper products industry", has a high average capital coefficient and the capital equipment is characterized by extraordinarily long durability. Third, different sectors of the forest industry are highly concentrated in a distinct set of separated regions.

In small regions in which the forest industry plays a dominant role, structural change may have severe effects on the employment situation. In this paper we suggest an approach to formulate investment policies for the industry in a region such that the structural change of the industry satisfies both certain profitability conditions and regional employment targets. Using a vintage type production model, we derive structural change requirements from international and national medium term scenarios. These scenarios are combined with regionally specified employment and production targets. We also suggest how this analysis may be extended to an interregional framework. To illustrate the

* This revised version has been influenced by recent joint efforts between the author and Håkan Petersson.

method developed, we present a single-region application which focuses on the forest industry.

This paper belongs to a framework employing three levels of models: international, national and regional level models. The top level consists of world trade models. For a country in which a large share of the forest industry output is exported, world trade scenarios form a necessary basis for medium and long term analyses of the domestic development of the forest industry. A minimum requirement here is that a world trade scenario should specify the development of world market prices.

At the national level, a multisectoral model is designed to capture structural change between and within sectors of the economy. World market prices are obtained from the top level in the system, while domestic prices are determined endogenously by the national model. For each sector, this model [10] utilizes a specification of how production techniques are distributed over the production capacity of the sector. Structural change emerges as a change of the capacity associated with each production technique including new techniques. Capacity increase is obtained by investment in new capital equipment.

At the regional level, the structural change process will have to satisfy certain employment and production targets which may affect the set of feasible changes on the national level. The national and regional models of the system [4,5,10] are formulated so as to reflect rigidities of the type described in models adhering to the vintage tradition, with early contributions by Houthakker [1], Johansen [2,3] and Salter [11].

We should finally emphasize that the high capital coefficients in the forest industry sectors means that structural change in these sectors may use up large proportions of the total "investment budget" in nations with a large forest industry sector. This fact makes it meaningful – perhaps necessary – to treat the investment process in a multisectoral framework.

2. A vintage model with regional and intersectoral specification

The basic ideas in this section rely on the early contributions to vintage analysis by Johansen [2] and Salter [11]. We shall identify industrial establishments as production units. The design of an establishment is defined as the choice of specific capital equipment with a given production capacity and an associated fixed relation between labor and other inputs. This means that ex post, i.e., after investment in an establishment, its input coefficients and the production capacity remain unchanged. However, since an establishment generally consists of separate subsystems, we assume that as a result of renewal investments (i) new capacity may be added to the already existing capacity in the establishment, and (ii) parts of the old capacity may be removed simultaneously as new capacity is added.

2.1. Distribution of vintages over the input space

Let $x_{jr}^{\tau}(t)$ denote the total output in period t from the set of establishments, in sector j and region r, which are of the vintage class (production technique) τ, where $\tau = 1, 2, \ldots$. For this set of establishments we shall introduce the following notations

$$\begin{aligned} \bar{x}_{jr}^{\tau}(t) &= \text{production capacity in period } t, \\ a_{ij}^{\tau} &= \text{input from sector } i \text{ per unit of output in sector } j \text{, given technique } \tau \\ l_j^{\tau} &= \text{labor input per unit of output in sector } j \text{, given technique } \tau \end{aligned} \tag{1}$$

We assume that $\bar{x}_{jr}^{\tau} \geq 0$, $l_j^{\tau} > 0$, $a_{ij}^{\tau} \geq 0$. The production technique specified in eq. (1) represents the average technique belonging to the vintage (technique) class τ. By α_j^{τ} we denote the average input vector $\alpha_j^{\tau} = \{a_{ij}^{\tau}; l_j^{\tau}\}$ of this class.

For a single establishment, with capacity $\bar{x}$ and input coefficient vector α_j, we shall introduce the following assumption

$$\begin{aligned} &\bar{x}(t+1) = \bar{x}(t) \text{ if no capacity-increasing investments in the establishment occur between } t \text{ and } t+1. \\ &\alpha_j(t+1) = \alpha_j(t) \text{ if no investment in the establishment occurs between } t \text{ and } t+1. \end{aligned} \tag{2}$$

Given the assumption in eq. (2) it is natural to assume that the production capacity in a production unit can be increased, and that the vintage classification τ may be changed as a result of investment. From this assumption we may conclude that $\bar{x}_{jr}^{\tau}(t)$ and $\bar{x}_{jr}^{\tau}(t+1)$ may differ for three reasons: capacity increasing investments in existing establishments (indexed j, r, τ), exit from, and entry to the set of establishments with index j, r, τ. Exit may occur as the result of either shut down or investments changing the production techniques from τ to τ^* for one or several existing establishments.

Entry occurs analogously: either completely new units enter the economic sector or existing units change technique from τ' to τ.

Let $\bar{x}_j(t)$ and $\bar{x}_{jr}(t)$ denote the total production capacity in sector j in the whole economy and in region r, respectively. Then we have

$$\begin{aligned} \bar{x}_{jr}(t) &= \sum_{\tau} \bar{x}_{jr}^{\tau}(t), \\ \bar{x}_j(t) &= \sum_{r} \bar{x}_{jr}(t). \end{aligned} \tag{3}$$

Studies of industrial change should avoid relying on aggregates of the type described in eq. (3). The usefulness of such studies increases if they focus on the distributions of techniques and capacities forming the aggregates. For example, one may consider sequences like $(\bar{x}^1_{jr}, 1/l^1_j), \ldots, (\bar{x}^\tau_{jr}, 1/l^\tau_j), \ldots,$ where $1/l^\tau_j$ represents the labor productivity of units belonging to class τ. In the very short run these distributions can change only if production units in a class are closed down. In a medium term perspective new establishments may enter and existing establishments may renew their technique through renewal investments. This study focuses on how distributions, of the type described above, change as the compound result of simultaneous shut down and investment.

2.2. *Profits, exit and entry*

In the type of framework which is presented here, it is tempting to introduce regionally specified prices and wages. Resisting this temptation, we shall assume that the price of product j (output from sector j), p_j, and the wage level in sector j, w_j, are economy-wide, i.e., equal in all regions. Using the notation in eq. (1) we may then define the value added of a specific production unit as follows

$$\left(p_j(t) - \sum_i p_i(t) a^\tau_{ij} \right) x^\tau_{jr}(t). \tag{4}$$

The wage share of value added, θ^τ_j, is then determined as

$$\theta^\tau_j(t) = w_j(t) l^\tau_j / \left(p_j(t) - \sum p_i(t) a^\tau_{ij} \right). \tag{5}$$

Consider now the sequence of techniques $\tau = 1, 2, \ldots,$ each with its average input vector. Assume that techniques are arranged so that [1]

$$\theta^1_j(t) < \theta^2_j(t) < \ldots. \tag{6}$$

Let A^0_j be an input set which becomes available at time t. Suppose next that the average input vector $\alpha^0_j \in A^0_j$ satisfies

$$\theta^0_j(t) < \theta^1_j(t) < \ldots. \tag{7}$$

Suppose also that $1 - \theta^0_j(t)$ is greater than the capital cost per value added which obtains when the new technique is installed. Then establishments entering into this new class, $\tau = 0$, may through competition force the price in

[1] Observe that if the wage, w_j, is equal for all establishments, and if $\Sigma a^1_{ij} \leq \Sigma a^2_{ij}, \ldots,$ then the ranking according to productivity, $1/l^1_j$, $1/l^2_j, \ldots,$ is necessarily equivalent to the ranking according to gross profit share, $(1 - \theta^1_j)$, $(1 - \theta^2_j), \ldots.$

sector j to be lowered relative to $w_j(t)$ and other prices. According to eq. (5), this implies that $\theta_j^\tau(t)$ has to increase for $\tau = 1, 2, \ldots$. This process, characterized by growing values of θ_j, reflects economic ageing of the existing establishments.

The ultimate effect of economic ageing is that θ_j approaches unity or becomes even greater. Suppose that $\theta_j^\tau(t) \geq 1$. If the objective is to retain the production capacity of an establishment in class τ, with non-positive gross profit, there are two options: the establishment can be shut down and replaced by a completely new production unit with an input vector $\alpha_j^0 \in A_j^0$. The second option is partial renewal. In that case new equipment is added to the old, or parts of the old production equipment are replaced by new equipment. Of course, this also includes reorganisation. Partial renewal should result in a change from technique τ to γ such that $\theta_j^\gamma(t) < 1 \leqslant \theta_j^\tau(t)$.

2.3. International, national and regional interfaces

In small countries many industry sectors are highly oriented towards the world market – both with regard to selling its output and buying its inputs. In Sweden, an extremely high proportion of the products from the forest industry is exported. Inputs are primarily domestic.

In the previous section economic ageing was related to the introduction of new and competitive techniques. For industries interacting with the world market we may add changes in world market prices as an important cause of economic ageing. In this case, we may argue as follows: if the world market price of the output from sector j is falling relative to other prices on the world market, the domestic price of sector j will also be reduced in relative terms. [2] From eq. (5) it follows that this change in relative prices will cause economic ageing, i.e., the wage share will increase in the sector experiencing a reduced relative price. In this situation establishments in sector j will have to search for new techniques and renew their old techniques, if possible.

We may now introduce a domestic multisectoral model of the following kind

$$x(t) = A(t)x(t) + B(t)c(t) + f(t), \tag{8}$$

where all variables refer to the last year of period t (e.g. five years), and where $x(t) = \{x_i(t)\}$ is a vector of sector outputs, $A(t) = \{a_{ij}(t)\}$ is a matrix of average input–output coefficients, $B(t) = \{b_{ij}(t)\}$ is a matrix in which an element b_{ij} denotes investment deliveries from sector i per unit of new capacity (equipment) in sector j, $c(t) = \{c_i(t)\}$ is a vector of annual capacity

[2] If this is not the case, there is no regular interaction between the domestic and the world market.

increments in sector j during the period, and where $f(t) = \{f_i(t)\}$ is a vector summarizing export, import and net final demand.

Different ways of solving a system of the kind described in eq. (8) are presented in refs. [7,9,10]. In order to establish a link between the domestic and the world economy, $f(t)$ may be specified as a vector function of domestic income and prices, and of the relation between domestic and world market prices.

In addition, we may embed a structural change model in the multisectoral framework. With the features specified in the previous section, such a model will react to changes in prices, wages and demand. Those reactions include removal of existing capacities and construction of new capacities embodying new techniques. As described in ref. [10], the reactions in such a structural change model will also affect the formation of domestic prices. In this way each sector may simultaneously satisfy total demand and counteract economic ageing.

A solution to the system in eq. (8), together with the interacting model components, consists of a set of equilibrium prices and wages together with an associated balanced structure of outputs, investments, etc.

The system in eq. (8) could be regionalized in several ways. For the moment we just note that according to eq. (2) the aggregates of the system in eq. (8) consist of regionally specified quantities x_{jr}^{τ}. A similar observation can be made with regard to the average input–output coefficients of the matrix A. Such an element is determined as follows

$$a_{ij} = \sum_{\tau} \sum_{r} a_{ij}^{\tau} \bar{x}_{jr}^{\tau} / \bar{x}_j. \tag{9}$$

Entry of new production units applying technique $\tau = 0$ and renewal of existing units will change the distribution of capacities $\bar{x}_{jr}^{\tau}$ between time $t-1$ and t. The same is of course true for regionally specified (average) coefficients a_{ijr} such that

$$a_{ijr} = \sum_{\tau} a_{ij}^{\tau} \bar{x}_{jr}^{\tau} / \sum_{\tau} \bar{x}_{jr}^{\tau}. \tag{10}$$

3. Structural change models

This section presents different ways of modeling structural change within sectors on the national level. In a first step structural change is analyzed without considering renewal of existing establishments. Thereafter renewal or restoration is considered explicitly. It is shown how these two approaches may be integrated. The structural change model which includes renewal is utilized in the regional models of sections 4 and 5.

3.1. Structural change without renewal

Consider a medium term period. Let the prices and wages during this period be given. By using formula (5) we may then calculate the wage share θ_j^τ for each class τ. Let ϵ_j be a function describing the frequency of capacity removed during the period for each value of θ_j^τ. The total capacity removed, e_j, will then be

$$e_j = \sum_\tau \epsilon_j(\theta_j^\tau)\bar{x}_j^\tau. \tag{11}$$

The function ϵ_j will reflect the obsolescence policy of sector j. For medium term periods empirical observations show (see refs. [4,10]) that in general, the obsolescence policy is delayed so that the following three conditions are satisfied for each τ: (i) $\epsilon_j(\theta_j^\tau) \geq 0$, (ii) $\partial\epsilon_j/\partial\theta_j^\tau > 0$, and (iii) $\epsilon_j(\theta_j^\tau) < 1$ for $\theta_j^\tau > 1$. In a strict version of vintage theory [3,11] we should have: (i) $\epsilon_j(\theta_j^\tau) = 0$ if and only if $\theta_j^\tau < 1$, and (ii) $\epsilon_j(\theta_j^\tau) = 1$ if and only if $\theta_j^\tau \geq 1$.

Having established the form of ϵ_j, we can calculate the value of e_j as shown in eq. (11). One may then determine the new capacity, $c_j(t)$, which must be created in order to reach the capacity level $\bar{x}_j(t)$ at the end of the period

$$c_j(t) = \bar{x}_j(t) + e_j(t) - \bar{x}_j(t-1). \tag{12}$$

For every vector of given prices, the investment costs per unit capacity in sector j are determined as $\sum_i p_i b_{ij}$. We shall compare these costs with the associated profits. By $\{a_{ij}^0, l_j^0\}$ we denote the input coefficients which obtain from the new technique embodied in the new capacity. The associated profits are

$$\begin{aligned} \pi_j^0 &= p_j - \sum_i p_{ij} a_{ij}^0 - w_j l_j^0, \\ \pi_j &= \pi_j^0 c_j(t) + \sum_\tau \pi_j^\tau \left[\bar{x}_j^\tau(t-1) - \epsilon_j(\theta_j^\tau) \right], \end{aligned} \tag{13}$$

where π_j^0 denote profits per unit in the new technique, and π_j total sector profits after removal of old capacities. For every wage level the first function is exclusively determined by the price structure. With each of the functions one may associate an investment criterion such that the investment process is reflected in the determination of equilibrium prices of the multisector model. In Persson and Johansson [10] an average return criterion is used such that

$\sum p_i b_{ij} c_j \leq r_j \pi_j$, where r_j is an estimated coefficient. With a standard rate of return coefficient β_j the requirement $\beta_j \geq \bar{\beta}_j$ yields

$$\bar{\beta}_j \leq \pi_j^0 / \sum_i p_i b_{ij}, \quad j = 1, \ldots, n. \tag{14}$$

Since $\theta_j^0 = w_j l_j^0 / [\pi_j^0 + w_j l_j^0]$ and $1 - \theta_j^0 = \pi_j^0 / [\pi_j^0 + w_j l_j^0]$a similar criterion may be expressed in terms of θ_j^0 or $1 - \theta_j^0$.

Remark 1. For given wages the n prices $p_1, \ldots, p_n$ are directly determined by the n equations in eq. (14). This remark may be related to the empirical observation that for each sector the medium term average of θ_j^0 is approximately constant over time. When θ_j^0 is rising as time goes by we observe contracting sectors. [3]

In this subsection we have described structural change without renewal. It is possible to embed this change process in the national model related to eq. (8). In a second step one may apply to the regional level a change process which includes renewal. By requiring mutual consistency between the two levels, the two approaches may be integrated. The renewal or restoration type of process is presented in the subsequent section.

3.2. Structural change with restoration

In this section we will introduce a way of distributing the new capacity, $c_j(t)$, over the set of old and entirely new establishments, where units belonging to the same class are treated as a group. We shall do this by focusing on the following property of observed behavior [4]. Over time a high proportion of production units are renewing their techniques as if their objective was to keep the gross profit share, $1 - \theta_j$, approximately constant. We shall call this type of renewal a restoration policy.

Consider establishments in technique class τ, and imagine a change of prices and wages such that $\theta_j^\tau(t-1) < \theta_j^\tau(t)$. Let γ be a technique such that $\theta_j^\gamma(t) = \theta_j^\tau(t-1)$ and let $\mu_j^{\tau\gamma}$ be defined by

$$\mu_j^{\tau\gamma} = \theta_j^\gamma(t) / \theta_j^\tau(t). \tag{15}$$

We may now define restoration as an investment policy by which production units shift from technique τ to γ so as to satisfy the condition $\theta_j^\gamma(t) = \theta_j^\tau(t-1)$.

[3] This may for example be caused by external competition from countries which are increasing their production and which have lower wages and other input costs (raw materials, etc.).

Let $\bar{x}_j^{\tau\gamma}$ be the capacity in class γ which obtains as a result of a restoration shift from τ to γ. Assuming that restored units retain their initial labor force we may add the constraint

$$\bar{x}_j^{\tau\gamma}(t) \leq S_j^{\tau\gamma}(t-1)/l^{\gamma}, \tag{16}$$

where $S_j^{\tau\gamma}(t-1)$ denotes the number of persons employed during period $t-1$ in units shifting from τ to γ, and l_j^{γ} is the labor input coefficient of class γ. The restoration is called universal if either $S_j^{\tau\gamma}(t) = S_j^{\tau}(t-1)$ or $\epsilon_j[\theta_j^{\tau}(t)] = 1$. The new capacity (in the form of new equipment) in class γ, $c_j^{\tau\gamma}$, is approximated by

$$c_j^{\tau\gamma} = \left(1 - \mu_j^{\tau\gamma}\right)\bar{x}_j^{\tau\gamma}. \tag{17}$$

Remark 2. The exact value of $c_j^{\tau\gamma}$ is $[1 - \mu_j^{\tau\gamma}(F_j^{\gamma}/F_j^{\tau})]\bar{x}_j^{\tau}$, where $F_j^{\delta} = [p_j - \Sigma p_i a_{ij}^{\delta}]$. Formula (17) obtains for $F_j^{\gamma} \approx F_j^{\tau}$.

The remark follows from eq. (15) where $\mu_j^{\tau\gamma} w_j l_j^{\tau}/F_j^{\tau} = w_j l_j^{\gamma}/F_j^{\gamma}$. For $F_j^{\tau} = F_j^{\gamma}$ this yields $\mu_j^{\tau\gamma} = l_j^{\gamma}/l_j^{\tau} = \bar{y}_j^{\tau}/\bar{x}_j^{\tau\gamma}$, where $\bar{y}_j^{\tau} = S_j^{\tau\gamma}/l_j^{\tau}$ and $\bar{x}^{\tau\gamma} = S_j^{\tau\gamma}/l_j^{\gamma}$. This implies that $(1 - \mu_j^{\tau\gamma})\bar{x}_j^{\tau\gamma} = \bar{x}_j^{\tau\gamma} - \bar{y}_j^{\tau} = c_j^{\tau\gamma}$.

By setting $\tau = \gamma = 0$ and $\mu_j^{00} = 0$ for completely new units, these are included in eq. (17). Consistency between the multisectoral model and universal restoration requires that the following additivity condition is satisfied

$$\sum_{\tau} c_j^{\tau\gamma} = c_j(t), \text{ and } \sum_{\tau} x_j^{\tau\gamma} = \bar{x}_j(t). \tag{18}$$

Similar conditions may be formulated for structural change with partial or non-universal restoration.

The investment costs corresponding to eq. (17) may be expressed as $I_j^{\tau\gamma} = \Sigma_i p_i b_{ij} c_j^{\tau\gamma}$. From this one may determine an investment coefficient, $k_j^{\tau\gamma}$, which relates to value added. This yields

$$k_j^{\tau\gamma} = \sum_i p_i b_{ij} \Big/ \left(p_j - \sum_i p_i a_{ij}^{\gamma} \right)$$

and we may write

$$I_j^{\tau\gamma} = k_j^{\tau\gamma}\left(1 - \mu_j^{\tau\gamma}\right)\bar{x}_j^{\tau\gamma}\left(p_j - \sum_i p_i a_{ij}^{\gamma} \right) \tag{19}$$

4. Regionally specified structural change

In this section we start by specifying regional and interregional constraints which may be derived from a national multisectoral model providing production and employment targets and an economic ageing scenario for a region or a system of regions.

With this as a background we focus on a single region and present two optimization models which generate structural change solutions for the region. If the approach presented is applied to the complete set of regions simultaneously, it provides a means by which to examine the feasibility of the national multisector model when regional rigidities and employment policies are taken into account. Formulating an interactive scheme between the national and multiregional structural change model, removal and investment in the national model may be adjusted so as to reflect the rigidities on the regional level.

4.1. Interregional interdependencies and constraints

A structural change model may either focus on one single region or it may, for example, be formulated as a multiregional programming model utilizing an interregional input–output system as in Lundqvist [8]. In the latter case the national and regional levels of structural change merge into one multiregional level. In both cases we have to consider balance constraints such that regional solutions are consistent on the national level and in an interregional perspective. One constraint of this type concerns the breakdown of $\bar{x}_j(t)$ to a regionally specified vector $\{\bar{x}_{jr}(t)\}$ such that

$$\bar{x}_j(t)=\sum_r \bar{x}_{jr}(t). \tag{20}$$

In order to capture all regional interdependencies we should need a complete multiregional input–output system. Information from such a system may be added as constraints to the more simple scheme presented here which basically utilizes eq. (10) together with employment constraints as restrictions. In this context we shall sketch an optimization model which is able to generate consistent scenarios of multiregional structural change.

First, assume that we can determine an employment target, $S_r(t)$, denoting the desired number of persons employed in the industry sectors in region r. Observe then the following relationships based on eq. (1)

$$S_{jr}^{\gamma}=l_j^{\gamma}\bar{x}_{jr}^{\gamma},$$

$$S_{jr}=\sum_{\tau} S_r^{\gamma}, \tag{21}$$

$$S_r=\sum_j S_{jr}.$$

In the sequel we assume universal restoration so that there are unique pairs (τ, γ) for which $S_{jr}^{\tau\gamma}(t-1) = S_{jr}^{\gamma}(t)$, and S_{jr}^{γ} denotes $S_{jr}^{\gamma}(t)$.

We shall formulate a restoration policy, as specified in section 3.2, for region r with the following objective function which implies universal restoration

$$L_0^r = \sum_{\gamma} \sum_{j} \rho_j h_{jr}^{\tau\gamma} S_{jr}^{\gamma} - \sum_{\gamma} \sum_{j} \delta_j^{\tau\gamma} S_{jr}^{\gamma}, \tag{22}$$

where ρ_j denotes an interest rate (discount factor), where $\delta_j^{\tau\gamma}$ denotes the gross profit per person employed in all establishments which are changing technique from τ in period $t-1$ to γ in period t so that [4]

$$\delta_j^{\tau\gamma} = \left(1 - \theta_j^{\gamma}\right)\left(p_j - \textstyle\sum_i p_i a_{ij}^{\gamma}\right) \bar{x}_{jr}^{\gamma} / S_{jr}^{\gamma} \tag{23}$$

and where $h_j^{\tau\gamma}$ denotes the investment cost per person employed in establishments changing technique from τ to γ during period t. From eq. (19) we have that [4]

$$h_j^{\tau\gamma} = k_j^{\tau\gamma}(1 - \mu^{\tau\gamma})\left(p_j - \sum_i p_i a_{ij}^{\gamma}\right) \bar{x}_{jr}^{\gamma} / S_{jr}^{\gamma}. \tag{24}$$

Using eq. (22) we introduce the following multiregional objective

$$\mathrm{Min} \sum_r L_0^r,$$

subject to

$$\begin{aligned}
&\text{(i)} && S_{jr}^{0}(t) \leq \bar{S}_{jr}^{0} \text{ for all } j, \\
& && S_{jr}^{\gamma}(t) \leq S_{jr}^{\tau}(t-1) \text{ for all } r,\ j, \text{ and } (\tau, \gamma), \\
&\text{(ii)} && \sum_{\gamma} \sum_{j} S_r^{\gamma}(t) = \bar{S}_r \text{ for all } r, \\
&\text{(iii)} && \sum_{\gamma} \sum_{r} S_{jr}^{\gamma}(t) / l_j^{\gamma} = \bar{x}_j(t) \text{ for all } j,
\end{aligned} \tag{25}$$

where (i) restrains the introduction of the new technique and states that transition from τ to γ does not increase the labor force in the establishments, (ii) states that the regional employment target for the industry must be satisfied, and (iii) states that the national production target as specified in eq. (20) must be satisfied. Observe that (i) is an indirect capacity constraint and

[4] Observe that for new establishment, $\gamma = 0$, we write δ_j^{00} and h_j^{00}. These coefficients are calculated separately, according to the specification of technique $\gamma = 0$.

that (iii) is the solution to the national multisectoral model. Observe finally that the interregional objective in eq. (25) is equivalent to a maximization of the difference between annual gross profits and annual costs associated with investment in the new technique $\tau = 0$ and with the restoration investments. Ultimately the gross profit determined by the solution to eq. (25) must be consistent with the income formation in the national multisectoral model. In particular, one may introduce a consistency criterion for regional profits, formulated in a similar way as the additivity condition in eq. (18).

From eq. (25) we may specify the following Lagrange function for distinct (τ, γ)-pairs

$$L = \sum_r L_0^r - \sum_j \lambda_j \left(\bar{x}_j(t) - \sum_\gamma \sum_r S_{jr}^\gamma(t)/l_j^\gamma \right)$$

$$- \sum_r \lambda_r \left(\hat{S}_r - \sum_\gamma \sum_j S_{jr}^\gamma(t) \right) - \sum_\gamma \sum_j \lambda_{jr}^\gamma \left[S_{jr}^\tau(t-1) - S_{jr}^\gamma(t) \right]. \qquad (26)\ [5]$$

The optimum conditions may be described as follows

$$S_{jr}^\gamma \left[\partial L / S_{jr} \right] = 0 \Rightarrow \begin{cases} S_{jr}^\gamma = 0 \text{ or} \\ \delta_{jr}^\gamma = \rho_j h_j^{\tau\gamma} + \lambda_r + \lambda_j / l_j^\gamma + \lambda_{jr}^\gamma. \end{cases} \qquad (27)$$

For this solution we may interpret λ_r as a regional policy parameter and λ_j / l_j^γ a sectoral policy parameter specified with respect to technique class γ, where policy parameter means "subsidy" of "tax". Finally, λ_{jr}^γ represents the additional profit that obtains if the capacity constraint is binding.

4.2. Structural change in one region

In section 5 the structural change analysis is illustrated by an application to a single region. There we utilize two different "mini-versions" of the optimization model in eq. (25). In both versions we minimize the objective function L_0^r in eq. (22). The first version utilizes constraints (i) and (ii) in eq. (25), which means that it allows free allocation between sectors up to the limits set by the constraints. This version should only be applied to a small region, since it has no mechanism which relates it to the sector balance on the national level in eq. (20). The Lagrangian becomes for (τ, γ)-pairs

$$L_\mathrm{I} = L_0^r - \lambda_r \left(\hat{S}_r - \sum_j \sum_\gamma S_{jr}^\gamma \right) - \sum_j \sum_\gamma \lambda_{jr}^\gamma \left[S_{jr}^\tau(t-1) - S_{jr}^\gamma(t) \right]. \qquad (28)$$

The second version differs from version I in only one respect. The target $\hat{S}_r$

[5] In this formulation $\bar{S}_{jr}^0$ is denoted by $S_{jr}^0(t-1)$, in order to simplify notation. Observe that technique $\tau = 0$ did not exist in period $t-1$, by definition.

is taken away and replaced by sector-specific targets, $\hat{S}_{jr}$. Then the following Lagrangian applies for distinct (τ, γ)-pairs

$$L_{\mathrm{II}} = L_0^r - \sum_j \lambda_{jr}\left(\hat{S}_{jr} - \sum_\gamma S_{jr}^\gamma(t)\right) - \sum_j \sum_\gamma \lambda_{jr}^\gamma\left[S_{jr}^\tau(t-1) - S_{jr}^\gamma(t)\right]. \tag{29}$$

Observe finally that the price–wage scenario affects L_0^r through $h_j^{\tau\gamma}$ and $\delta_j^{\tau\gamma}$ in eq. (22). For each such scenario, eqs. (28) and (29) generate associated structural change scenarios. In the following section we provide a comparison between scenarios generated by eqs. (28) and (29).

5. Illustration of structural change scenarios with single-region allocation

In this section we shall illustrate an application of the models associated with eqs. (28) and (29). We call them Programs I and II, respectively. The programs have been applied to a Swedish region, Värmland, for the period 1978–1985. In table 1, the initial employment structure (1978) is described. The techniques are aggregated to 5 technique classes, of which $\tau = 0$ represents the new technique. The restoration profile for the period 1978–1985 is illustrated by fig. 1, which shows that the period has been divided into 2 + 5 years.

Table 1
Initial employment structure and capacity constraints. Wärmland (1978) (Source: ref. [5])

	Paper products	Wood products	Summation over remaining 16 industry sectors
Employment target	6250	2911	24850
Capacity constraints in terms of persons employed in class:			
$\tau = 0$	620	300	2520
1	2750	730	4580
2	2560	1490	11040
3	750	440	6410
4	190	260	2850
Total capacity constraint	6870	3220	27400

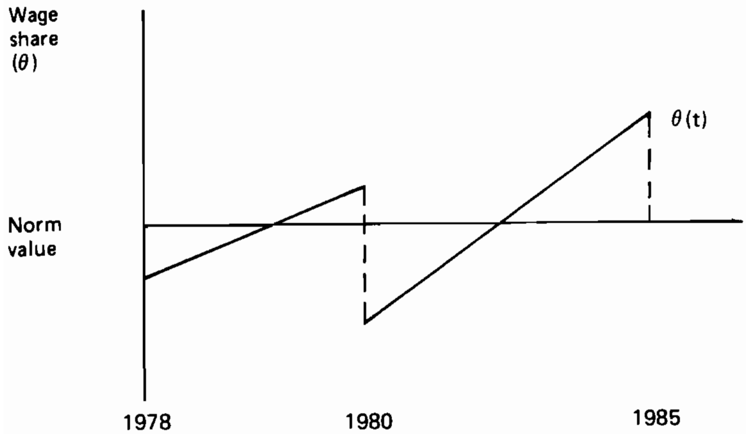

Fig. 1. The change of the wage share in a specific sector and profit share class with a restoration program.

5.1. Two structural change scenarios

Table 2 summarizes two structural change scenarios with associated investment allocations. The two scenarios are based on the same price–wage scenario. The scenarios utilize Programs I and II, respectively.

In table 2 we have not described the distribution of techniques. This is illustrated in table 3. Program I generates the same total employment as

Table 2

Structural change scenarios with Programs 1 and II. Net surplus = Gross profits minus costs associated with the investment (1.000 Swkr). Annual growth of economic age refers to the difference $\alpha - \beta$, where α, in per cent, is the annual growth of the wage level and β is the annual growth of the value added price index, in per cent

	Annual net surplus per person (Swkr)		Wood products	Paper products	All 18 sectors
		Annual growth of economic age	1,0	2,3	2,5
Base alternative	35400	Employment 1978	2911	6250	34011
		Investment 1972–78 per year (1.000 Swkr)	48500	197200	604100
Program I	50554	Employment (%)	102	107	99
		Investment (%)	45	100	104
Program II	49155	Employment (%)	100	100	99
		Investment (%)	45	53	85

Table 3
Illustration of Programs I and II for the forest industry

	Index class τ	Capacity utilization in per cent	Gross profit $\delta_i^{\tau\gamma}$	Inv. costs $\rho_i h_i^{\tau\gamma}$	Regional shadow prices λ_i^{γ}	λ_i	Wage share, per cent
Program	0	100	106.1	59.2	46.9	–	39
I	1	100	106.1	10.4	95.7	–	39
Wood	2	100	47.1	2.8	44.3	–	59
products	2	100	16.9	0.5	16.4	–	80
	4	0	−10.3	–	–	–	118
Program	0	100	106.1	59.2	30.5	16.4	39
II	1	100	106.1	10.4	79.3	16.4	39
Wood	2	100	47.1	2.8	27.9	16.4	59
products	3	89	16.9	0.5	0	16.4	80
	4	0	−10.3	–	–	–	118
Program	0	100	172.1	172.1	0	–	30
I	1	100	149.8	0	149.8	–	33
Paper	2	100	51.3	17.9	33.4	–	59
products	3	100	18.4	6.0	12.4	–	80
	4	0	−21.1	–	–	–	140
Program	0	31	172.1	172.1	0	0	30
II	1	100	149.8	0	149.8	0	33
Paper	2	100	51.3	17.9	33.4	0	59
products	3	100	18.4	6.0	12.4	0	80
	4	0	−21.1	–	–	–	140

Program II. It does so by requiring a greater total investment effort. At the same time it generates a higher net surplus which is the difference between the annual gross profits and the annual costs associated with the investment allocation. One should then observe that Program I, which refers to eq. (28), allows free allocation between sectors, while Program II operates with one employment target per sector. The outcome of the two programs is specified as a percentage of a base alternative which is also presented in table 2.

In table 3 we illustrate Programs I and II in more detail for the two subsectors of the forest industry. The similarity between the two outcomes makes it easy to grasp the structure of the solutions. One should observe that every class τ includes at least three establishments, since this is a confidentiality criterion in the industrial statistics of Sweden. If we regard Program II and focus on the wood products sector, the parameter value $\lambda_i = 16.4$ indicates the taxation per employee on a marginal unit in class $\gamma = 3$ that would force the reduction of this class to 89% of full capacity utilization. For further details, we may refer to Johansson and Strömquist [4] and Johansson [5].

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THE USE OF ECONOMIC VALUE OF WOOD IN COMPARING LONG-TERM STRATEGIES FOR FOREST INDUSTRIES

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This article describes the long-term planning problems of the forest industry. The mixed integer mathematical model developed optimizes the structure of the forest industry under the limited forest resources available. The model is a single-period model concentrating on the most important structural factors in the industry: economies of scale and economies of integration. The strengths and weaknesses of the major competitors, possible outcomes of future changes in markets, the economic environment, and technical developments in the forest industry and in competing industries are also assessed. The results of the optimization are intended to give a solid basis for drawing conclusions about long-term development strategies, rather than to give solutions ready for implementation.

1. Objectives of the long-term planning

The forest industry is highly capital intensive. Typical of this industry is the relatively long economic life of production units, with consequent rather modest profitability prospects for most new mill projects.

These basic characteristics of the forest industry call for comprehensive in-depth long-term planning, with more emphasis on seeking consistent operating conditions than on occasional business opportunities. In addition, the long payback times of forest industry projects and intensifying international competition mean that the structure of the industry (i.e. mill sizes, degree of integration, productivity etc.) has a major effect on profitability.

2. Planning criteria

2.1. Principle

From the standpoint of the national economy, the forest industry is the instrument for obtaining economic benefit from a nation's forest resources. This benefit can be measured in many different ways. The traditional measure of performance is profitability, calculated by dividing net return by capital investment. According to this assessment, the alternative with the highest return on capital investment is the most beneficial. This thinking is based on the assumption that capital is the limiting factor for which each alternative must compete; it is assumed that other factors such as raw materials and labour are available at a known price and in abundance. But for the forest industry these conditions no longer apply in most major production regions.

In many instances the procurement of wood raw material is the bottleneck which limits the development prospects in the long term. In these circumstances evaluation based on the traditional profitability calculations is not sensible. It is more realistic to evaluate each alternative in terms of its net return per unit of the factor which most limits the scale of the operation, and that is the availability of wood raw material. The most favourable alternative is then that with the highest net return per unit of raw material used. This thinking is based on the assumption that other factors, such as capital, are available at a known price and in quantities sufficiently large for them not to impose critical limitations on the scale of operation.

In accordance with this principle, the best measure of the value of a forest-based industrial enterprise from the national economy viewpoint is the *economic value of wood*, which is defined as the maximum theoretical raw material price which a company could pay, given a known sales price for its product, and still cover all manufacturing costs, including capital charges. The following formula is used for calculating the economic value of wood denoted by v_i for a certain product i:

$$v_i = p_i / r_i, \tag{1}$$

where p_i is the sales price less non-wood manufacturing costs, and r_i is the consumption of wood raw material.

For a forest industry business unit, the economic value of wood is a very useful measure of long-term profitability prospects. When forest resources are limited and there is competition for wood between different companies, wood prices tend to increase. The company which has the highest economic value of wood has the highest wood-paying capacity and the best growth prospects; competing companies may have to withdraw.

2.2. Maximization of the economic value of wood

The principle of the economic value of wood is applied to determine which products and production methods should be chosen to process a known, limited annual harvest of wood to give the highest possible financial result. The principle of the calculation can be expressed in the formula (2) for the economic value of the total quantity of wood harvestable, denoted by V.

$$V = P - C, \tag{2}$$

where P is the total sales revenue from forest products and C is the total manufacturing cost excluding wood but including capital charges. Subject to the limitations set by the wood supply, the desired production programme is that which gives the highest economic value for the total amount of wood raw material available. This is the optimum programme for a forest-based industry with a limited supply of wood raw material. The problem can be solved by well-known linear programming methods (e.g. Wolfe and Bates 1968) or mixed integer programming methods (e.g. Bender et al. 1981).

Goal programming methods, in which both the economic value of wood and return on investment are simultaneously taken into account, are not applied to the problem. Instead it is useful to see how the optimum programme is sensitive to the changing cost of capital.

3. Mill models

3.1. Basic principles

Mill models have been developed to represent the modern process and mill design practices of the forest industry. Certain specific mill site conditions which affect mill designs and costs are not taken into account. The mill models are thus hypothetical aimed at representing average conditions of the region.

In the forest industries, economic efficiency depends on two main factors: *economies of scale* and *economies of integration*. Mill models have been developed so that these factors are properly taken into account.

3.2. Economies of scale

For bulk products such as newsprint, linerboard and market pulp, labour costs, other fixed costs and investment requirements per ton markedly decrease with increasing mill size. If there were no limitations on wood consumption, the optimum mill size for bulk products would be the maximum mill size technically feasible. For products with more specialized and heterogeneous markets, the optimum mill size also depends on specific market conditions.

The investment requirements I as a function of mill size T (expressed by annual production) can be estimated with the following equation:

$$I/I_0 = (T/T_0)^d, \tag{3}$$

where I_0 and T_0 are given fixed values for investment required and annual production, respectively, and d is the degression factor.

In general cases the degression factor varies between 0.4 and 0.9. The lowest degression factors are for continuous kraft cooking plants in one-line operation; groundwood mills have high degression factors.

However, this generalized formula is not very suitable for mathematical programming, and the number of parallel production lines required at different capacities brings stepwise increases in the investment required. Mixed integer programming methods are therefore used. The investments required are then approximated as follows:

$$I = aT + b, \tag{4}$$

where a and b are constants specific to each type of production unit. The constants a and b are chosen so that the estimates of the investments required are most accurate at most realistic mill sizes from the techno-economic and marketing points of view. Similarly, labour costs, maintenance costs and other fixed costs are estimated as functions of mill size.

In a few cases the feasible mill size range must be divided into 2–4 smaller ranges, and calculate separately the above-mentioned approximations for each smaller range. This is required in cases where the process design, number of production lines, labour input etc. depend strongly on mill size.

3.3. Economies of integration

The economies in technically integrating several production lines vertically or horizontally together in a multiproduct forest industry complex decrease production costs for many reasons. First, certain auxiliary functions such as steam and power generation may in a one-product mill be small, and economies of scale can be achieved only by integrated operations. Second, certain intermediate operations can be eliminated in integrated mills. For example, chemical pulp for papermaking is not dried, baled, stored, transported and repulped in an integrated pulp and paper mill complex.

Additionally, certain wood species and grades are normally best suited for certain end products, and their use for second-best end products is normally economically feasible only if economies of scale are achieved. In most regions forests contain several different wood species with a wide age distribution of trees. One-product mills in these circumstances have difficulties in achieving rationalized forestry operations or the optimum wood mix for the process. In an integrated forest industry complex, the forestry operations and the production processes can be balanced so that wood costs at mill are minimized and the benefits from the specific quality characteristics of different wood species maximized.

Alternative ways of using wood and integrating forest industry operations are illustrated in fig. 1. The integration alternatives can be analysed and the optimum forest industry complex found by dividing the production processes into different mill departments or blocks. When these blocks are logically put together, all required process functions and auxiliary departments are taken

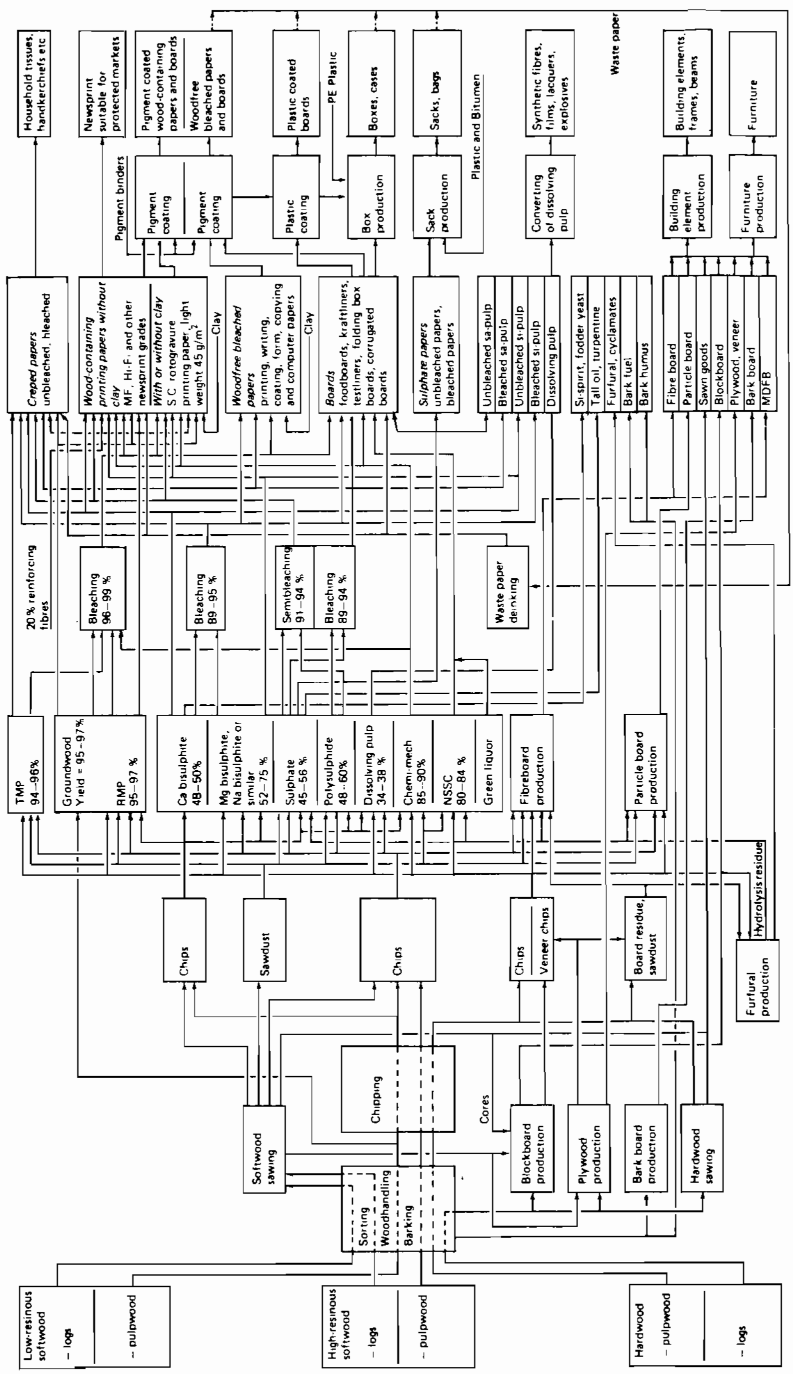

Fig. 1. How to use wood: strategic alternatives.

into account at the required sizes. Block definition is to a large extent based on physical mill departments, with allowance for connections between process blocks and certain indirect and service functions. In the mill model the auxiliary departments, or common blocks, which may be common to several production blocks, are:

- woodhandling at mill site,
- regeneration of pulping chemicals,
- process steam generation,
- back-pressure power generation,
- power distribution,
- raw water intake and mechanical treatment,
- chemical treatment of process raw water,
- effluent treatment.

The process design of these blocks depends to some extent on the configuration and sizes of the production departments. The mill models represent normal design principles for large, modern operations. For instance, hog fuel from wood barking is burnt in high-pressure steam boilers, and effluent treatment includes both primary and secondary treatment. The costs of common functions such as the central maintenance shop and head office are not expresssed in the model as separate blocks, but are included in the cost estimates of other blocks.

Table 1
Mill model: structure of block data. The blocks are grouped in four categories for calculating maintenance and general overhead costs, and determining differences in productivity between major production regions

Data entities	Remarks
Block number	1–110
Block indicator	1–4
Number of block sizes	1–4
Consumption of fibre raw materials	wood, purchased pulp, waste paper
Net consumption of fuel	
Consumption of power	
Costs of other raw materials	chemicals, additives etc.
Costs of operating materials	
Costs of packaging materials	
Loading of other production blocks	steam and power generation,
Loading of common blocks	chemical recovery, water handling, etc.
Block data as a function of block size:	
Fixed capital required	
Number of operating personnel	
Number of maintenance personnel	
Number of administration personnel	

The production lines are broken down into different blocks so that alternatives for vertical integration can be analysed. The connections between the blocks are determined by defining the fibre furnishes in papermaking, the bleaching losses, the pulping yields, the steam and power consumptions, the water usages and other similar process parameters. For each of the production or common blocks, the operating costs and capital expenditure required are estimated as functions of mill size, and put into the mixed integer programming model. The data contents of each block are indicated in table 1.

The mill models have been used in various corporate planning and regional studies by Jaakko Pöyry Companies since late 1960s (Eklund and Kirjasniemi 1969).

4. Optimum industrial structure

4.1. Optimization model

The mill model described is the essential part of the optimization model illustrating the structural characteristics of the forest industry. When we additionally include in the model the marketing possibilities and sales prices, fibrous raw material and other resource limitations, as well as the costs of other raw materials, energy, labour and capital, then all necessary aspects for strategic considerations are included in the model (see fig. 2).

This model can describe fairly satisfactorily only the industry and its resources, present economic and market situation, and technological state. It is a single-period model, in which future changes are studied in a sensitivity analysis.

4.1.1. Objective function

The objective function is the economic value of wood as defined in section 2. The economic value of wood for an individual product is composed of:
- the direct charges of the production blocks,
- the indirect charges of auxiliary blocks,
- the direct compensation from sales.

In the model, the economic value of wood is, however, not calculated for each product but in terms of a total value. The economic values of wood corresponding to a given block size is the sales revenue less the following: cost of fuel, cost of other raw material costs, packaging material costs, operating supply costs, personnel costs, maintenance material costs, general overheads, interest on capital, and investment annuity. For the production and auxiliary blocks the sales prices are equal to zero. For the sales variables, the sales prices are positive but cost terms are zero.

For a block size within given limits, the economic value of wood can be calculated by linear approximation in the following way. Let the block size

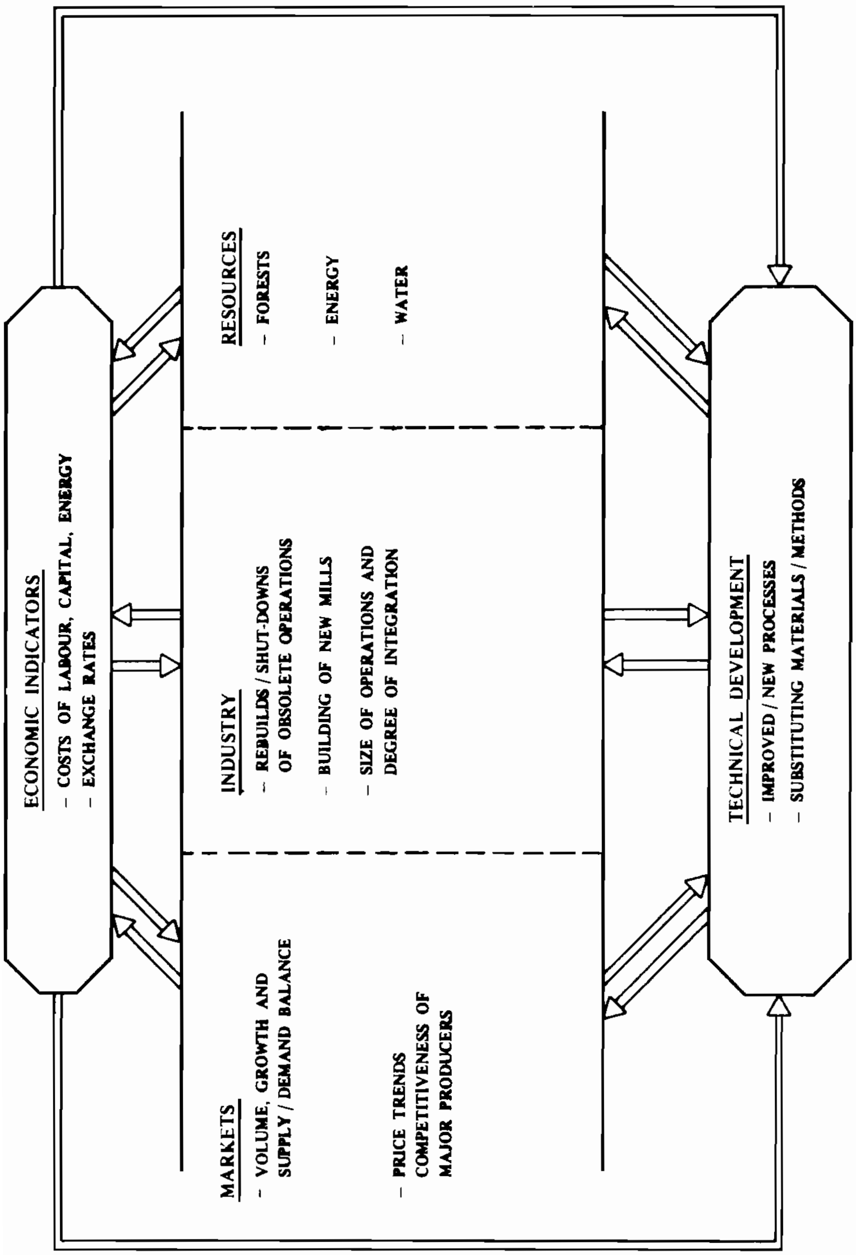

Fig. 2. Optimization model: areas of analysis.

alternatives be T_1, T_2 and T_3, and let the corresponding economic value of wood per product unit be C_1, C_2 and C_3. Then the total economic values of wood at the points mentioned are C_1T_1, C_2T_2 and C_3T_3. If the desired block size is for example in the range (T_1, T_2), the corresponding total economic value of wood Y is

$$Y = xC_1T_1 + (1-x)C_2T_2 = ax + b, \tag{5}$$

where $x \geq 0$. From this equation, the coefficients of the objective function are taken for each variable.

4.1.2. Furnish balances

The furnish constraints or raw material balances are divided into two groups: balances for individual raw materials and compound balances for several raw materials. The latter are used to determine furnishes, i.e. the ratios between different raw materials to be fed into the production process. The general form of furnish constraints is

$$\sum_{i \in I} a_i x_i - \sum_{j \in J} b_j x_j \geqslant 0, \tag{6}$$

where I is the set of indexes of raw material producing activities, J is the set of indexes of raw material consuming activities, x_i is the activity level and coefficients a_i and b_j the corresponding furnish coefficients, which are non-negative.

4.1.3. Production balances

Production constraints and material balances are divided into three types: (i) quantity constraints, (ii) linkage of the existence and (production) quantity variables of a certain block size, and (iii) linkage of existence variables of different block sizes.

The first type restricts the output at least to the level of consumption. The second type allows the quantity variable to be nonzero only if at the same time the corresponding existence variable equals one. The third type ties the various block sizes so that at most one size gets chosen.

The general form of the first type for a given product is

$$\sum_{i=1}^{k} x_i - \sum_{j \in J} b_j x_j - x \geqslant 0, \tag{7}$$

where k is the number of block sizes, x_i the corresponding quantity variables, J an index set of the block sizes using the product as raw material, x_j the corresponding quantity variables, b_j the input coefficient and x the amount of the product to be sold.

The general form of the second type for a given block size is

$$aI - y \leqslant x - yI \leqslant 0, \tag{8}$$

where a is the lower limit of the block size, y is the upper limit, x is a production variable (between a and y), and I the corresponding existence variable equal to zero or one.

The general form of the third type for a given mill block is

$$\sum_{i=1}^{k} I_i \leqslant 1, \tag{9}$$

where k is the number of mill block sizes and I_i is the ith existence variable.

4.1.4. Operation balances of auxiliary blocks

The operating balances of the auxiliary blocks are, in general, formed in the same way as production balances. They may also be divided into three types. The only exception is the primary boiler for which the material balance has the form

$$\sum_{i=1}^{k} x_i + \sum_{h \in H} a_h x_h - \sum_{j \in J} b_j x_j \geqslant 0, \tag{10}$$

where k, x_i, b_j, J have the same meanings as above, and H is the index set of block sizes corresponding to the block that increases the available primary boiler capacity.

4.2. Markets

4.2.1. Marketing potential

Product prospects (i.e. the volumes, growths, and regional supply and demand balances) provide very important basic information for considering the long-term development of a forest industry company or sector. However, it is difficult to provide the market data to the optimization model.

The present market positions of the domestic industry and competing foreign suppliers in each market are the result of a long historic development. In many cases, a comparison of the present market share and competitiveness of each producer suggest that the market shares will considerably change in favour of the most powerful producers.

On the other hand, there are regions like Finland and Sweden where the forest industry is an essential part of the entire national economy. The economic policy in these countries must remain such that the forest industry as a whole can keep its market position in its major markets. Naturally, the product mix of the industry could be changed in these countries towards more competitive products.

An evaluation of marketing potential must therefore be based on an analysis of the strengths and weaknesses of potential competitors, as well as product prospects. With regard to bulk products, this analysis must be based on cost competitiveness. For more specialized products, more emphasis should be placed on marketing capabilities, distribution efficiency and similar factors.

Marketing potential, expressed as upper bounds or right-hand-side limitations, is therefore not a very sensible starting point in the optimization model. Instead, it is found more useful to run the model first without any market limitations; if the results are clearly unrealistic from the marketing point of view, the limitations can be added one by one. The gradual increase of market limitations makes it easier to illustrate their influence on the optimization results as well.

4.2.2. Sales prices

The straightforward method of using present market prices as such in the long-term considerations, which was still a general practice in late 1960s and early 1970s, is often now considered misleading.

As in many other sectors, market disruptions in the mid 1970s broke down the earlier relatively steady price trends and showed how the prices of different forest products react very differently to any severe economic recession. For instance, a cheaper substitute to advertising on TV was found to be newspapers and magazines, and consequently the demand and price of certain printing paper grades were firm during the recession. On the other hand, packaging papers and paperboards, various office papers and products with demand directly related to industrial production and general economic activity, were hard hit in the mid 1970s.

Present prices may not be suitable for long-term considerations, as different forest products may be in the different phases of market cycles, there may be some medium-term imbalances in supply and demand, etc. These short-term or medium-term fluctuations should be eliminated, and long-term trends in the price relations between the different forest products found. One way of doing this is to calculate longer-term average prices. This would give good results if all forest products had market cycles of about the same frequency, but this is not necessarily true. Another problem is that prices and price relations before the first oil crisis in 1974 probably have rather limited relevance for estimating future prices, and the years after 1974 are quite few for calculating averages.

Another perhaps more relevant approach is to assume that the long-term trends in future prices and price relations are increasingly being determined by the market or price leaders. This is the expected result of the future keener competition between producers as the growth in demand in major markets becomes slower than in the past. Exposure to international competition will also be greater in the future (Leone and Meyer 1980). The total manufacturing costs, capital charges and distribution costs – in other words the required sales prices – of the major producers or production regions are estimated as part of the analysis of competitiveness. The lowest-cost producers or production regions are thus found, and their potential for increasing market shares assessed.

In bulk products this approach should result in a good basis for estimating long-term prices, whereas in estimating prices for semi-bulk or specialty grades certain other factors should also be taken into consideration. A comparison of actual market prices and the required sales prices of the identified price leaders could give indications of the pricing policies in the more specific market sectors.

The mill models previously described are also used in analysing cost competitiveness. Economic indicators are estimated for each competitor or competing region, the representative industrial structure, labour productivity, etc. defined and the total manufacturing and distribution costs (i.e. sales prices required) calculated using the mill models.

4.3. Industry

The optimization problem may concern either a single company or a whole forest industry sector in a certain region. In both cases there may already be an extensive industry with relatively little expansion potential left. Optimization could begin with mill models representing new mills with modern technology. The results may indicate vast differences between the present situation and the optimum, if the industry could be built from scratch. In these cases the needed shutdowns and rebuilds must be studied in more detail.

In the next phase the short-term viability of the present industry is analysed by calculating the economic value of wood by excluding capital charges on fixed investments at the industry's present capacity. If the industry is then different from the optimum calculated results, some old capacity should be shut down and replaced by more profitable units.

It may be necessary to study in more detail the units of the present industry that, according to the optimization results, seem viable in the short term, but could not bear the full capital charges considered necessary for long-term viability. The actual operating costs of these mills or production lines, and the need for major rebuilds in the near future are then analysed, and the corresponding actual total costs, plus capital charges covering future major rebuilds, are put into the optimization model. The optimization results should then give more reliable indications of the viability of these plants.

The optimum number of mill sites within a certain region of given forest resources can normally be determined reliably enough by simple case studies.

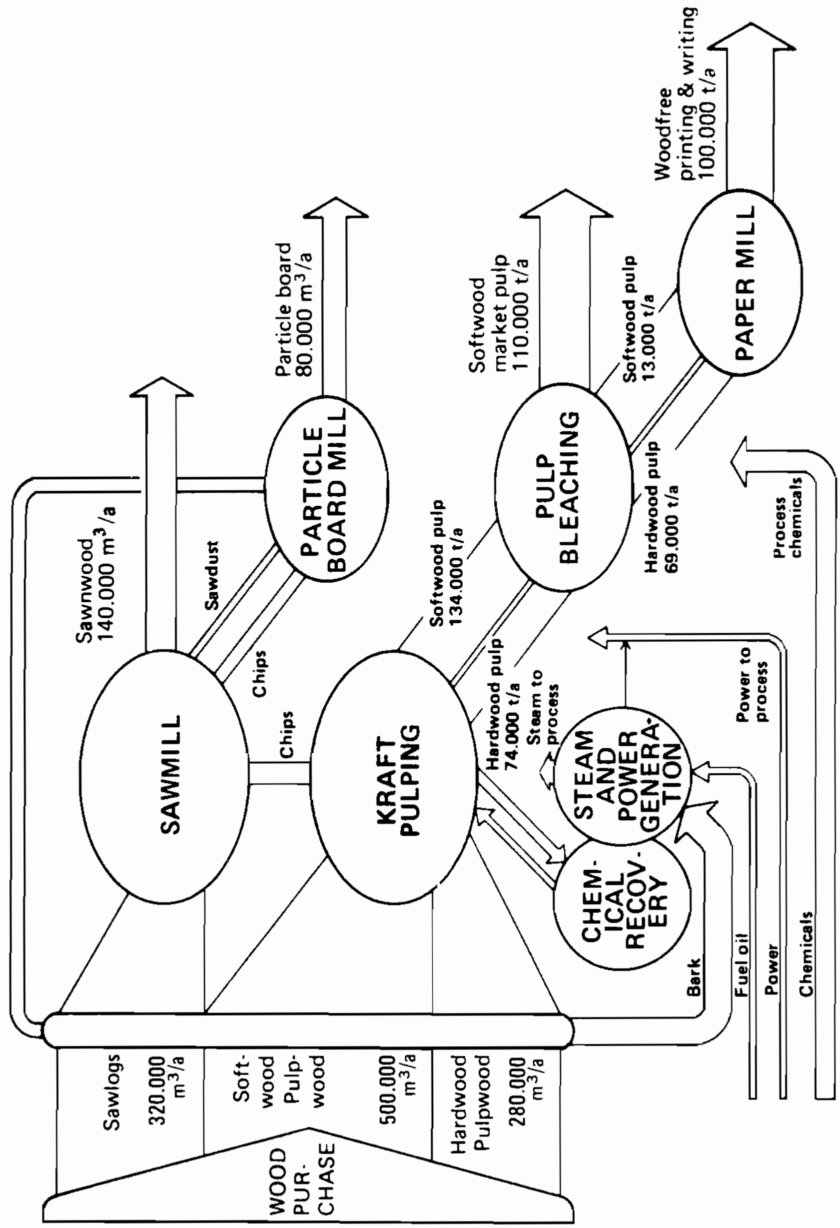

Fig. 3. Optimization results at low level of wood procurement (1 million m^3/a).

The inclusion of this problem in the optimization model would dramatically increase the size of the model, and might make the costs of running the model unacceptable.

For example, the optimum size of a full-scale integrated forest industry complex in southern Finland using all wood species in proportion to their allowable cuts can be calculated simply so that the production line for which economies of scale are greatest, the chemical pulp line, is given its optimum size, about 300 000 t/a. When the most suitable wood species (pine and birch), together with the corresponding wood residues from the mechanical wood mills, are used for chemical pulp manufacture in southern Finnish conditions, the result is that the optimum annual roundwood consumption of one forest industry mill site is about 3 million m^3, solid underbark measure.

Optimization at considerably lower wood consumption levels may result in rather weak production units. As shown in fig. 3, optimization at 1 million m^3/a wood consumption suggests that in order to achieve an economic enough

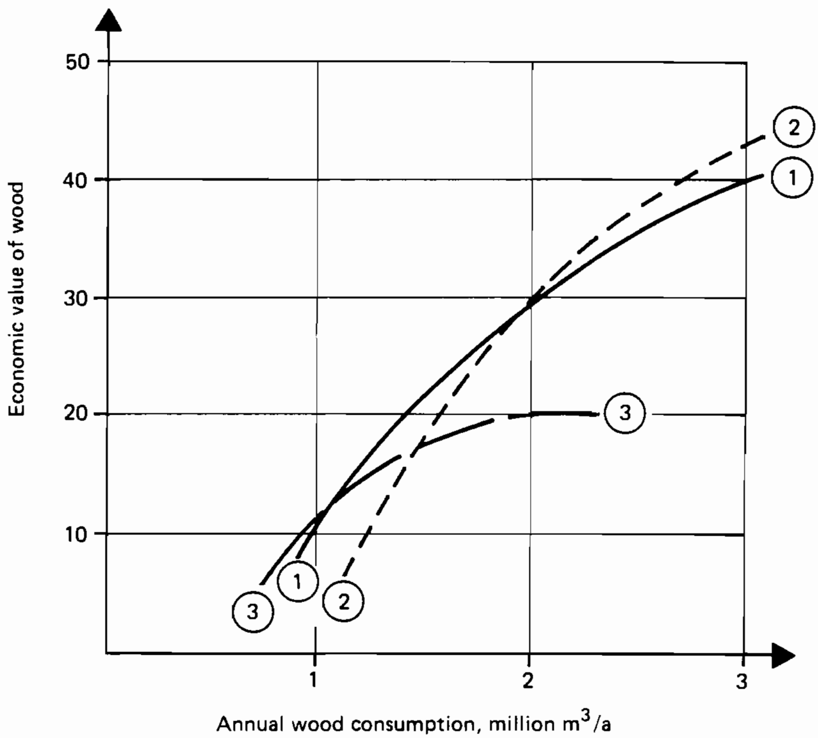

Fig. 4. Economic value of wood as a function of product mix and size of operations. Product Mix 1: as described in fig. 3. Product Mix 2: as in fig. 3 plus mechanical printing paper less market pulp. Product Mix 3: unintegrated market pulp mill.

big kraft pulp mill, all softwood pulpwood species and lower-value sawlogs would have to be kraft pulped. This results in an imbalance between the softwood and hardwood pulp production and the paper mill requirements of softwood/hardwood pulp, so some of the softwood pulp must be sold as a relative low-value-added product. On the other hand, some of the softwood pulpwood species may be more suitable for mechanical than kraft pulping, but at this low level of wood procurement mechanical pulping and production of mechanical printing papers or paperboards would not be economically feasible.

In fig. 4 some examples of the effect of the size of operations and product mix are given. When the wood consumption exceeds 2 million m^3/a, the production mix with mechanical pulps becomes better than the mix shown in fig. 3.

Aside from these full-scale integrated complexes with chemical pulp manufacture, it is found that certain small-scale sawmills, converting plants etc., where the economies of scale and economies of integration may be less important than flexibility of operations, can also be viable in separate small units.

In principle basic bulk products should be produced near raw material sources, and converted products near markets. The best way of dividing the production chain between these two alternative locations depends on the product. This problem is therefore normally analysed separately through case studies, and not included in the optimization models.

4.4. Resources

4.4.1. Forests

Forest resources, divided between species, log diameter classes and grades, and into wood residues or roundwood are expressed in the model as right-hand-side limitations.

As a basic principle, the annual allowable cut equals the annual forest growth rate. By improving less productive forest areas, fertilizing, reforesting etc., intensified forestry will increase the allowable cut. By comparing the increase in the economic value of wood and the costs of the intensified forestry it can be seen how far it would be practical to intensify the forestry operations. The optimum results could also give indications of the relative values of different wood species and log diameter classes for different strategies of intensified forestry.

Normally it is not necessary to include wood transport in the optimization model. The consumption of different wood species and grades in the same proportions as the forests produce them has proved to be so clearly optimum that this principle needs to be reconsidered only in very unusual forestry conditions; and increases in wood transport costs as the total consumption of wood increases are normally quite marginal compared with the economies of scale and economies of integration of the industry.

In areas where the forest industry is based on fast-growing plantations, long-term development problems are essentially simpler in terms of the number of different feasible products, fibre furnish alternatives and different forest resource categories. In these cases it is not useful to use the comprehensive optimization models; conventional profitability analyses are better.

4.4.2. Energy

Products based on chemical pulp have high wood consumptions, but their production is almost energy self-sufficient; products based on mechanical pulp have low wood consumption, but most of the energy for their production is purchased.

If the price of purchased energy is high, burning wood for energy generation may be feasible, especially for forest harvesting residues, bark and some lower-value wood residues from the mechanical wood industry. The optimization model should therefore include as one alternative the burning of different wood grades for energy generation.

Optimization at different energy prices indicates sensitivity of the product mix, especially the relation between chemical and mechanical pulping, to changes in real energy prices, and to what extent wood should be used for energy generation.

4.5. Economic indicators

The optimization model has to be build so that the sensitivity of the results to the most critical economic indicators can be analysed. Such indicators include: costs of labour, capital (interest rates) and energy, as well as exchange rates. Manpower and capital requirements, and energy balances are thus included in the model as separate lines.

Certain future changes in the economic indicators alter costs for all competitors. If, for instance, for a certain product one company has an energy cost structure similar to the market/price leader's, international energy price increases can be expected to be passed on as product price increases, so the competitive situation should not change.

The economic indicators are interdependent. For instance, as wood can be used as fuel, the price of wood cannot for long be less than its value as fuel. One aspect of the sensitivity analysis can be optimization assuming that energy prices are so high that all the required energy is generated from wood. The sales prices should be adjusted accordingly to correspond to the prices required by the market leaders in this situation.

Changes, which would critically affect competitiveness, such as exchange rates, should be identified and their probable outcomes in the long term assessed. Changes in exchange rates directly affect export sales prices, imported fuel oil prices etc., whereas their effects on mill capital requirements depend on the international markets for the mill equipment, and have to be analysed in more detail when analysing the whole effect of the exchange rates on competitiveness.

4.6. Technological changes

Technological developments have widened the range of raw materials suitable for chemical pulping, increased the sizes of production lines and cheapened fibre furnishes. Real manufacturing costs and real sales prices have thus decreased in the long term. It seems probable that future technical developments will affect the cost structure of the forest industry similarly, but at an unknown rate.

Some technical innovations and developments would affect all producers in about the same way, and thus be less critical from the planning point of view. More interesting are developments which would essentially change the competitive situation. For instance, recent and likely developments in mechanical and chemi-mechanical pulping will improve the competitiveness of high-wood-cost regions. The economic potential of these technical developments can be assessed through sensitivity analyses in the optimization process.

Technical development of competing materials and systems, such as plastic packaging and electronic media, will affect both the demand for forest products and the quality required of these products. In assessing future risks and potential one should be aware of the probable outcomes of these likely long-term development trends.

5. Concluding remarks

Although the optimization model describes structural factors of the forest industry fairly accurately, it is relatively simple and compact in relation to the rather complex and comprehensive long-term problems to be analysed. The simple model, however, enables us to make a large variety of different sensitivity analyses to illustrate the effects of probable or potential changes on the optimum allocation of limited resources. On the other hand, the results of the simple models are not directly applicable; they are merely intended to form a more solid basis for drawing conclusions about the most favourable long-term development strategies for a forest industry company or sector (Naylor and Schauland 1976).

Market fluctuations and the consequent severe financial problems of many forest industry companies in the 1970s have made it more important to achieve a production structure less sensitive to market disruptions. Analysis of the strengths and weaknesses of the major competitors has therefore become a more and more important part of long-term planning studies, and the results of the competitiveness analysis is an essential part in considering the optimum allocation of limited forest resources.

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A NOTE ON THE OPTIMAL ALLOCATION OF LABOR IN THE FOREST INDUSTRY

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In this paper the problem of the optimal allocation of labor resources in the forest industry among works on current production output and the construction of new forest industrial complexes is investigated. Subject to several assumptions this problem is reduced to a mathematical problem of optimal control with nonfixed time, and is solved completely for the case when the production function is of the Cobb–Douglass type. The solution is obtained in the form of simple formulas permitting natural economic interpretation.

1. Introduction

One of the most important problems in forest industry management, subject to the condition of full employment, is the problem of the optimal allocation of labor resources among existing enterprises and the construction of new forest industrial complexes. The latter require massive investments but promise to give substantial gains in the form of rapid increases in output. It is assumed here that there is no influx of labor from other sectors of the economy, and that all technical problems related to complex construction have been solved, since the relevant planning bodies have all the necessary resources at their disposal. The forest stocks in the given simplified version are assumed to be unlimited.

The time at which such forest industrial complexes could commence production is apparently determined by the supply of labor that can be used for construction. Slow construction work will thus lead to small reductions in the current output, but they defer the moment of completion of construction and the related increase in general production. There then arises the question as to which labor use strategy is in some sense the best and most feasible?

The purpose of this paper is not to generate optimal policies for the solutions of practical problems of this kind but is more modest, namely, to study the simple but useful mathematical model of the allocation of labor in the forest sector at the initial stage of so-called pre-institutional analysis. This simple model, retaining the major causes of the system's behavior, helps to

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crystallize our understanding of the important features of the process under control. The solution is obtained in closed analytical form permitting an almost self-evident economic interpretation for some range of the parameters involved.

Our perception determines the methods we use and the solution we see. Originally this model was elaborated for the study of allocation of labor in the forest industry but the results proved to be more general and can be applied in those sectors of the economy where our assumptions hold. We hope that these results can be used as the starting point for further research in the field. Previous studies of economic models of renewable resources which are structurally similar to a number of two-sector models of economic growth and in which detailed analysis was done when the production function was of the Cobb–Douglas type and the ecological equation was of the "logistic" form have been published by different authors (see, for example, Beddington et al. 1975 and Clark et al. 1979). The allocation of labor in these studies was ignored. The joint consideration of the problem of the long-term allocation of labor in the forest industry and of the appropriate model of forest growth seems to be a natural and interesting step for further research in the not so distant future.

In order to make this consideration possible many things must be clarified, since from the mathematical point of view, the problem of forest modeling for any suitable long-term goals is an extremely difficult one. Some useful approaches to this problem were discussed by Holling (1978). More advanced mathematical models for forest growth and management were recently proposed and investigated by Hellman (1979, 1980).

2. Statement of the mathematical problem

Let the construction of a forest industrial complex be started at time $t = 0$ and finished at $t = T$. Let also the current output in the forest industry be described by the production function

$$Y = F(K_0, L_c) \equiv \varphi(L_c), \quad L_c \leq L_0$$

where Y is the output per unit of time, constant K_0 is the capital of the forest industry up to the end of construction, L_c is the labor used in production, and L_0 is the total amount of labor in forestry. We assume that the function is concave of L_c.

After completion of the forest industrial complex the capital will increase by K_1. The production function $F(K_0, L_c)$ is repaced at $t = T$ by some other function G, while the following relation holds:

$$G(K_0 + K_1, L_0) = kF(K_0, L_0),$$

where $k > 1$. In other words, the forest industry output after construction of the complex will increase k times. Note that in this case a knowledge of G is not necessary; one needs only to know the parameter k. It is further proposed that after construction all labor resources will be used only for production purposes.

It is natural to consider the following function as a criterion of control:

$$I = \int_0^\infty u(Y)\mathrm{e}^{-\rho t}\,\mathrm{d}t = \int_0^T u(Y)\mathrm{e}^{-\rho t}\,\mathrm{d}t + \{u[Y^*(T)]/\rho\}\mathrm{e}^{-\rho T},$$

where ρ is the discount factor, $Y^*(T)$ is the output after the end of construction, and $u(Y)$ is a social preference function. We will limit ourselves here to a consideration of one particular case when $u(Y) \equiv Y$, since it is the most interesting from a practical point of view. The following inequality holds

$$L_c(t) + L_p(t) \le L_0,$$

where $L_p(t)$ is the labor employed in construction in the interval $[0, T]$. Let us consider L_c and L_p as control variables, and denote by L_1 an amount of labor (for example, measured in man-hours) that is needed for the entire construction process. It is evident that

$$L_1 = \int_0^T L_p(t)\,\mathrm{d}t.$$

Let us introduce a new variable $x(t)$ by the relations

$$\dot{x}(t) = L_p(t),\ x(0) = 0,\ x(T) = L_1.$$

Now we are able to formulate the following problem of optimal control:

$$\max\left(\int_0^T \varphi(L_c)\mathrm{e}^{-\rho t}\,\mathrm{d}t + (k/\rho)\varphi(L_0)\mathrm{e}^{-\rho T},\ \varphi(L_0)/\rho\right) \tag{1}$$

subject to

$$\dot{x} = L_p(t), \tag{2}$$

$$x(0) = 0,\ x(T) = L_1, \tag{3}$$

$$L_p + L_c \le L_0,\ L_p,\ L_c \ge 0, \tag{4}$$

$$Y = \varphi(L_c). \tag{5}$$

The second term in expression (1) $\varphi(L_0)/\rho$, corresponds to the case when there is no construction work at all, and all the labor is used only for production purposes. This case should not be excluded since, subject to some relations among parameters, this case could be optimal. Note also that T is unknown and remains to be found.

3. Solution to the problem

The problem formulated above is one of optimal control with non-fixed time and fixed initial and terminal conditions(3). In its solution it is convenient to apply Pontryagin's Maximum Principle (see Pontryagin et al. 1969, Moiseev 1975, Schmidt, 1979). In this case the Hamilton function $H(x, p, L_c, L_p)$ and the Lagrange function $L(x, p, q, L_c, L_p)$ have the following forms:

$$H(x, p, L_c, L^p) = \varphi(L_c)e^{-\rho t} + pL_p,$$

$$L(x, p, q, L_c, L_p) = H(x, p, L_c, L_p) + q(L_0 - L_p - L_c),$$

where p is a dual variable and q is a Lagrange factor such that

$$q(L_0 - L_p - L_c) = 0, \ q \geq 0.$$

For p we have

$$\dot{p} = -(\partial L/\partial x) = 0 \text{ i.e. } p = p_0 \geq 0. \tag{6}$$

The values of p is unknown and must be found from the solution of the corresponding boundary problem.

The transversality condition is as follows (see Moiseev 1975):

$$H[x(T), p(T), L_c(T), L_p(T)] = k\varphi(L_0)e^{-\rho T}. \tag{7}$$

The necessary conditions of the maximum Hamilton function, subject to conditions (4) will bring us to the following relations

$$0 < \varphi'(L_c)e^{-\rho t} = q = p, \tag{8}$$

$$L_p = L_0 - L_c. \tag{9}$$

Let us denote the solution of eq. (8) by $L_c(p_0, t)$. We obtain

$$\dot{x} = L_0 - L_c(p_0, t). \tag{10}$$

Let the solution of eq. (10) be $x = \psi(A, p_0, t)$, where A is an arbitrary constant that has to be chosen in such a way as to satisfy the initial condition $x(0)=0$. The condition on the right hand size is as follows

$$\psi(A, p_0, T)=L_1. \tag{11}$$

Using eqs. (7) and (9) we obtain

$$\varphi[L_c(p_0, T)]e^{-\rho T}+p_0[L_0-L_c(p_0, T)]=k\varphi(L_0)e^{-\rho T}. \tag{12}$$

Thus the original problem in eqs. (1)–(5) is reduced to the solution of two transcendental equations (11) and (12) for the determination of p_0 and T.

Let us consider now a case when the production function is a Cobb–Douglas function:

$$F(K_0, L_c)=CK_0^{1-\alpha}L_c^{\alpha}, 0<\alpha<1.$$

Then $\varphi(L_c)=\gamma L_c^{\alpha}$, where $\gamma=CK_0^{1-\alpha}$. It is easy to show that the optimal control L_c is as follows:

$$L_c(p_0, t)=\left(\frac{\alpha\gamma}{p_0}e^{-\rho t}\right)^{(1/1-\alpha)}.$$

Thus $L_c(p_0, t)$ is a continuous, monotonically decreasing function of t. Further, we have

$$x(t)=L_0t+B(e^{-\rho t/(1-\alpha)}-1),$$

where

$$B=\frac{1-\alpha}{\rho}\left(\frac{\alpha\gamma}{\rho}\right)^{1/(1-\alpha)}.$$

We are able now to write equations for the determination of p_0 and T:

$$L_0T+B(e^{-\rho T/(1-\alpha)}-1)=L_1, \tag{13}$$

$$p_0\left[e^{-\rho T/(1-\alpha)}\left(\frac{\alpha\gamma}{p_0}\right)^{1/(1-\alpha)}\left(\frac{1}{\alpha\gamma}-1\right)+L_0\right]=k\gamma L_0^{\alpha}e^{-\rho T}. \tag{14}$$

It should be noted that only the positive roots of eqs. (13) and (14) that are feasible do not violate the condition

$$0\le\frac{L_c(t)}{L_0}\le 1.$$

Let $p_0 = ze^{-\rho T}$. Then from eq. (14) we obtain for z:

$$z\left(L_0 + \frac{b}{z^\beta}\right) = k\gamma L_0^\alpha, \tag{15}$$

where

$$b = \left(\frac{1}{\alpha\gamma} - 1\right)(\alpha\gamma)^{1/(1-\alpha)}, \ \beta = \frac{1}{1-\alpha} > 1.$$

Let us now consider two different cases:

(1) $\frac{1}{2} < \alpha\gamma < 1$. Then $b > 0$ and, due to the value of k, eq. (15) could have not more than two roots. When k is sufficiently large both roots exist and it is easy to obtain the asymptotic expressions:

$$z_1 = \left(\frac{b}{k\gamma L_0^\alpha}\right)^{1/(\beta-1)} + o\left(k^{-1/(\beta-1)}\right),$$

$$z_2 = k\gamma L_0^{\alpha-1} + 0(1).$$

The first root is not suitable because

$$\frac{L_c(t)}{L_0} = \left(\frac{k}{\alpha}\right)^{1/\alpha} e^{\rho(T-t)(1-\alpha)}\left(\frac{1}{\alpha\gamma} - 1\right)^{1/\alpha} > 1.$$

For the second root we have

$$\frac{L_c(t)}{L_0} = \left(\frac{\alpha}{k}\right)^{1/(1-\alpha)e\rho(T-t)/(1-\alpha)} \leq 1 \quad \text{if } k \geq \alpha e^{\rho(T-t)}.$$

Thus the second root z_2 is suitable.

(2) $\alpha\gamma \geq 1$. In this case we have $b < 0$ and eq. (15) has only one root which, in the first approximation, coincides with z_2. So in both cases we have

$$p_0 = k\gamma L_0^{\alpha-1} e^{-\rho T} = \frac{Y^*(T)}{L_0} e^{-\rho T}.$$

It is not difficult to note that p_0 has a clear economic interpretation: this is the productivity of labor at time T, which is reduced to an initial time by the discount factor. In other words, this is the estimate of possible wages at the initial time. On the importance of discounting see also Koopmans (1974). (It should be mentioned here that the case of a relatively underdeveloped economy (where $0 < \alpha\gamma \leq \frac{1}{2}$) is of special interest, but is not considered in this paper.)

Substituting p_0 in eq. (13) we obtain for the determination of T:

$$T + \frac{1-\alpha}{\rho}\left(\frac{\alpha}{k}\right)^{1/(1-\alpha)}\left(1 - \mathrm{e}^{\rho T/(1-\alpha)}\right) = \frac{L_1}{L_0}. \tag{16}$$

This equation can have no more than two roots. It is not difficult to show that for eq. (16), with at least one root, the following inequality must hold:

$$\frac{1}{\rho}\ln\left(\frac{k}{\alpha}\right) \leq \frac{L_1}{L_0} + \frac{1-\alpha}{\rho}\left[1 - \left(\frac{\alpha}{k}\right)^{1/(1-\alpha)}\right]. \tag{17}$$

In the case when eq. (17) is an equality, eq. (16) has only one root, and when the sign in eq. (17) is opposite, eq. (16) has no roots at all: this means that the optimal solution consists of the utilization of all labor resources in current production.

Let eq. (16) have two different roots. Taking into consideration

$$L_c^* = \left(\frac{\alpha}{k}\right)^{1/(1-\alpha)} \mathrm{e}^{(T-t)/(1-\alpha)} L_0.$$

we obtain the following expression for $I^*(T)$:

$$I^*(T) = \frac{k\gamma L_0^\alpha}{\rho}\mathrm{e}^{-\rho T} + \gamma L_0^\alpha\left(\frac{\alpha}{k}\right)^{1/(1-\alpha)} \mathrm{e}^{a\rho T/(1-\alpha)} \int_0^T \mathrm{e}^{-\rho T/(1-\alpha)}\,\mathrm{d}t.$$

Using this formula and eq. (16) we obtain finally for $I^*(T)$:

$$I^*(T) = \frac{\gamma L_0^\alpha}{\rho}\left[k\mathrm{e}^{-\rho T} + \rho\left(\frac{k}{\alpha}\right)\mathrm{e}^{-\rho T}\left(T - \frac{L_1}{L_0}\right)\right]. \tag{18}$$

Only in the case when the expression in square brackets is greater than unity, will the construction of a forest industrial complex provide a real economic gain. This will not occur for all parameter combinations. For example, when eq. (16) has two equal or almost coinciding roots, it is easy to show that the expression in square brackets is less than unity; the construction of a forest industrial complex is not profitable in this case.

It is interesting to note that $I^*(T)$ (see eq. (18)) has a maximum in the point

$$T_{\max} = \frac{L_1}{L_0} + \frac{1-\alpha}{\rho},$$

which is situated between the two roots of eq. (16). Thus we need an additional check on for which of the two points T_1 or T_2 the value of $I^*(T)$ is the greater. However, for parameters with economic sense, in most cases $I^*(T_1) > I^*(T_2)$. T_1 can be calculated approximately using an iterative method. In the first approximation we have

$$T_1 = \frac{L_1}{L_0} + \frac{1-\alpha}{\rho}\left(\frac{\alpha}{k}\right)^{1/(1-\alpha)}\left(e^{\rho(L_1/L_0)/(1-\alpha)} - 1\right).$$

which shows that the optimal time for the construction of a forest industrial complex in the sense of the accepted criteria is only a little longer than the minimal time for construction, which is equal to L_1/L_0. Thus our asymptotic solution is obtained for sufficiently large k, which is the most interesting case. In other cases one should apply numerical methods.

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Part III

TIMBER MARKET

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STEADY STATE ANALYSIS OF THE FINNISH FOREST SECTOR

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The purpose of this study is to investigate long term equilibrium prices for wood (and thereby total round wood costs) under various conditions of the world market for wood products for the Finnish forest sector. First a (discrete time) dynamic linear model for the forest sector is discussed. The steady state version is then analyzed in more detail. An application of the steady state forestry model is carried out and alternative sustained yield solutions for Finnish forests are obtained. Then, a steady state sectorial model is adopted and elaborated to carry out a Stackelberg equilibrium analysis for the round wood market.

1. Introduction

During the last few years a growing research effort has been directed towards (renewable) natural resources. Since the prosperity of many nations is dependent on a sensible exploitation of these resources the significance of studies dealing with such problems becomes great. For the Finnish economy forests represent the most important national resource. Not only as such but because an entire line of production ranging from pulp and sawn goods to paper, furniture and prefabricated houses, is based on wood, not to mention the associated industry producing machinery for forestry and wood processing. This also emphasizes the importance of the forest sector, which includes both forestry and forest based industries, for employment and foreign trade.

In the past the total cost of round wood for the Finnish forest industry has been of the order of US$1.5 billion annually. The share of stumpage price represents roughly one half whereas harvesting, transportation, etc. account for the rest. The purpose of this study is to investigate long range equilibrium prices for wood (and thereby total round wood cost) under various conditions for world market of wood products.

In section 2 we will present a dynamic linear model for the entire forest sector. In section 3 we take forestry separately and determine the optimal harvesting policies in a steady state. Section 4 deals with a steady state model for the forest industries. In section 5 we combine these two parts and formulate a steady state model for the forest sector. In section 6 we supplement the model of section 5 to make it applicable for solving the long range equilibrium wood prices as a solution to a Stackelberg game. In section 7 we present numerical results from experiments with Finnish data. Finally, in section 8 we present a summary and conclusions.

2. A dynamic linear model for the forestry and wood processing industry

We shall consider first the integrated and dynamic system of wood supply and wood processing; i.e. forestry and forest industry. The model has been adopted from Kallio et al. (1980). The discussion begins with the forestry part describing the growth of the forest given harvesting and planting activities, as well as land availability over time. The wood processing part consists of an input–output model describing the production process as well as production capacity and financial resource considerations. Each part is a discrete time linear model describing its object over a chosen time interval.

2.1. Forestry

Let $w(t)$ be a vector determining the number of various tree species (say pine, spruce and birch) in different age categories at the beginning of time period t. We define a square transition (growth) matrix Q so that $Qw(t)$ is the number of trees at the beginning of period $t+1$ given that nothing is harvested or planted. Thus, matrix Q describes the aging and natural death of the trees. Let $p(t)$ and $h(t)$ be vectors for levels of different kinds of planting and harvesting activities, respectively (e.g. planting of different species, terminal harvesting, thinning, etc), and let the matrices P and H be defined so that $Pp(t)$ and $-Hh(t)$ are the incremental increase in the tree quantity caused by the planting and harvesting activities. Then, for the state vector $w(t)$ of the number of trees in different age categories we have the following equation:

$$w(t+1) = Qw(t) + Pp(t) - Hh(t). \tag{1}$$

Planting is restricted through land availability. We may formulate the land constraint so that the total stem volume of trees in forest cannot exceed a given volume $L(t)$ during t. Thus, if W is a vector of stem volume per tree for different species in various age groups, then the land availability restriction may be stated as

$$Ww(t) \leq L(t). \tag{2}$$

Given the level of harvesting activity $h(t)$, there is a minimum requirement for the planting activity $p(t)$ (required by the law, for instance) as follows:

$$p(t) \geq Nh(t), \tag{3}$$

where N is the matrix transforming the level of harvesting activities to planting requirements.

In this simple formulation we shall leave out other restrictions such as harvesting labor and capacity. Finally, the wood supply $y(t)$, given the level of harvesting activities $h(t)$, is given for period t as

$$y(t) = SHh(t). \tag{4}$$

Here the matrix $S = (S_{ij})$ transforms a tree of a certain species and age combination j into a volume of type i of timber assortment (e.g. pine log, spruce pulpwood, etc).

2.2. *Wood processing industry*

For the industrial side, let $x(t)$ be the vector of production activities for period t (such as the production of sawn goods, panels, pulp, paper, and converted wood products), and let U' be the matrix of wood usage per unit of production activity. The wood demand for period t is then given by $U'x(t)$. It cannot exceed wood supply $y(t)$:

$$y(t) \geq U'x(t). \tag{5}$$

Note that the matrix U' may also have negative elements. For instance, sawmill activity consumes logs but produces pulpwood as a residual.

Let A be an input–output matrix so that $(I-A)x(t)$ is the vector of net production. If $D(t)$ is the corresponding (maximum) external demand, we require

$$(I-A)x(t) \leq D(t). \tag{6}$$

Production is restricted by the capacity $c(t)$ available:

$$x(t) \leq c(t). \tag{7}$$

The vector $c(t)$ in turn has to satisfy the state equation

$$c(t+1) = (I - g)c(t) + v(t), \tag{8}$$

where g is a diagonal matrix accounting for (physical) depreciation and $v(t)$ is the increment from investments during period t. The vector $v(t)$ of investment activities is restricted through financial considerations. To specify this, let $m(t)$ be the state variable for cash at the beginning of period t. Let $G(t)$ be the vector of sales revenue less direct production costs per unit of production including, for instance, wood, energy, and direct labor costs. Let $F(t)$ be the vector of monetary fixed costs per unit of capacity, let $l(t)$ be the amount of external financing employed by the industry at the beginning of period t, let s be the interest rate for external financing per period, let $l^+(t)$ be new loans taken during period t, let $l^-(t)$ be loan repayments during t, and let $E(t)$ be the vector of cash expenditure per unit of increase in the production capacity. Then, the state equation for cash may be written as

$$\begin{aligned} m(t+1) = m(t) + G(t)x(t) - F(t)c(t) \\ - sl(t) - l^-(t) + l^+(t) - E(t)v(t). \end{aligned} \tag{9}$$

Finally, for the industrial model, we may write the state equation for external financing as follows:

$$l(t+1) = l(t) - l^-(t) + l^+(t). \tag{10}$$

3. Sustained yield in forestry

In the previous section we presented a dynamic linear programming model encompassing both forestry and forest based industries. In this section we focus our attention solely on forestry. We present forestry in a steady state by assuming that one period follows another without changes. We shall investigate alternative sustained and efficient timber yields in various timber assortments. We also present an application to Finnish forestry.

3.1. The steady state formulation

In section 2.1 we presented a general dynamic formulation for forestry. In this section we deal with a steady state case of this model.

We consider a forest land with a single tree species and with uniform soil, climate, etc. conditions. We assign the trees to age groups a, for $a = 1$,

$2, \ldots, N$. Let d be a time interval; e.g. 5 years. A tree belongs to age group a if its age is in the interval $[(a-1)d,\ ad]$ for all $a < N$. Trees which have an age of at least $(N-1)d$ belong to age group N. We consider a discrete time steady state formulation of the forest, where each time period is also an interval of d. Let p be the number of trees entering the first age group during each period (e.g. through planting or natural regeneration), and let $w(a)$ be the number of trees in age group a at the beginning of each time period, for $1 \leq a \leq N$ (cf. eq. (1)). Let $h(a)$ be the number of trees harvested during each period from age group a. In this case, we assume that the harvesting activities equal the number of trees harvested from each age group during each period. We denote by Q_a the ratio of trees proceeding from age group a to group $a+1$ in one period given that no harvesting occurs. Without loss of generality we assume $0 \leq Q_a \leq 1$, for all a. Factors $(1-Q_a)$ account for the natural death of trees, forest fires, etc., as well as for thinning of forests in age group a. The state equations for forestry in a steady state may then be written as follows (cf. eq. (1)):

$$w(1) = p, \tag{11}$$

$$w(a+1) = Q_a w(a) - h(a), \quad 1 \leq a \leq N, \tag{12}$$

where we define $w(N+1) = 0$.

The land constraint prevents excessive planting (cf. eq. (2)). Let W_a be the amount of land consumed by each tree in age group a, $1 \leq a \leq N$, and let L be the total amount of land available in the forests. Alternatively, the space limitations may be taken into account denoting by W_a the volume of wood per tree in age group a and L the total possible volume of wood in the forests. In either case the land constraint is given as

$$\sum_{a=1}^{N} W_a w(a) \leq L. \tag{13}$$

As a performance index for forestry we consider the physical wood supply. (Experience shows that when we maximize the physical wood supply we usually get a policy which also meets other important requirements, such as preserving the watershed.) The timber assortments vary in value (e.g. log, pulpwood, fuel wood). Let j $(=1, 2, \ldots)$ refer to different timber assortments. Accordingly, let e_{aj} be the yield (in m^3/tree) of timber assortment j when a tree in age group a is harvested, and let g_{aj} be the yield per tree in age group a resulting from thinning activities. As stated earlier, our objective is to find an efficient timber yield using the yields of timber assortments as criteria. Let e_a and g_a be convex combinations (weighted sums) of the coefficients e_{aj} and g_{aj}, respectively. The objective is to maximize the weighted sum of the yields of various timber assortments and it is given as

$$\sum_{a=1}^{N} \left[e_a h(a) + g_a w(a) \right]. \tag{14}$$

The weights to be used may be proportional to the market prices of the timber assortments. Also other weights may be considered for studying efficient yields (see section 3.2 below). The forestry problem, denoted by (F), is to find nonnegative scalars $h(a)$ and $w(a)$, for each a, which maximize eq. (14) subject to eqs. (11)–(13). The following result is used to derive an optimal solution to this linear program.

Proposition. For an optimal solution of the forestry problem (F) there is an age group A such that $h(a) = 0$, for all $a \neq A$, and $w(a) = 0$, for all $a > A$.

Thus in the optimal harvesting schedule, all trees are harvested, clearcut (besides thinning activities) if and only if they reach age group A. Therefore, there are no trees in age groups higher than A. Problem (F) may then be solved, for instance, by checking all alternative harvesting policies of this type. For a proof of the Proposition, see Appendix 1.

We consider now a particular policy $a = A$ where trees are harvested in an age group A. Then, according to eq. (12),

$$w(a) = \begin{cases} p \prod_{i<a} Q_i & \text{for } a \leq A \\ 0 & \text{for } a > A. \end{cases} \tag{15}$$

For the corresponding level of planting p_A the land constraint (13) yields:

$$p_A = L \Big/ \left(\sum_{a=1}^{A} W_a \prod_{i<a} Q_i \right). \tag{16}$$

The number of trees harvested, when policy A is applied, is given as

$$h(A) = Q_A w(A). \tag{17}$$

The efficient yield of timber assortment j from clearcutting when policy A is applied is given as

$$e_{Aj} h(A). \tag{18}$$

As for cutting and thinning, the efficient yield of timber assortment j under policy A is

$$\sum_{a=1}^{A} g_{aj} w(a). \tag{19}$$

3.2. Application to Finnish forestry

We will now apply this approach to forestry in Finland. Let the age group interval d be 5 years and $N = 21$ (so that the oldest group includes trees at least 100 years old). We consider two timber assortments: pulpwood ($j = 1$) and log ($j = 2$).

Table 1 gives estimates for the transition probabilities Q_a, the average volume of pulpwood and log per tree in age group a, e_{a1} and e_{a2}, respectively, as well as the total volume W_a. We assume that all losses indicated by the Q_a coefficients for age groups less than 20 are due to thinning. Based on this, the yield coefficients g_{aj} can be given as

$$g_{aj} = (1 - Q_a)e_{aj}, \tag{20}$$

for $4 \leq a < 20$. We assume $g_{aj} = 0$ for each j, for $a \geqslant 20$. The land constraint (13) requires that the total volume of log and pulpwood cannot exceed an amount $L = 1700$ million m^3, which is around 10% above the actual current level in Finland. According to the transition coefficients, 5.6 trees have to be

Table 1
Transition coefficients Q_a, volume W_a, pulpwood yield e_{a1} and g_{a1}, and log yield e_{a2}. Yield g_{a2} equals 0.0 for all age groups except for $a = 13$ for which $g_{a2} = 0.003$ (Volumes in m^3/tree)

a	Q_a	W_a	e_{a1}	e_{a2}	g_{a1}
1	0.68	0.0	0.0	0.0	0.0
2	0.93	0.0	0.0	0.0	0.0
3	0.90	0.001	0.001	0.0	0.0
4	0.80	0.006	0.006	0.0	0.001
5	0.80	0.014	0.014	0.0	0.003
6	0.82	0.026	0.026	0.0	0.005
7	0.90	0.041	0.041	0.0	0.004
8	0.93	0.061	0.061	0.0	0.004
9	0.93	0.085	0.085	0.0	0.006
10	0.93	0.114	0.114	0.0	0.008
11	0.97	0.146	0.146	0.0	0.004
12	0.97	0.182	0.182	0.0	0.005
13	0.97	0.222	0.138	0.084	0.004
14	1.0	0.263	0.113	0.150	0.0
15	1.0	0.308	0.102	0.206	0.0
16	1.0	0.353	0.081	0.272	0.0
17	1.0	0.399	0.076	0.323	0.0
18	1.0	0.446	0.071	0.375	0.0
19	1.0	0.494	0.064	0.430	0.0
20	0.99	0.543	0.060	0.483	0.0
21	0.95	0.600	0.060	0.540	0.0

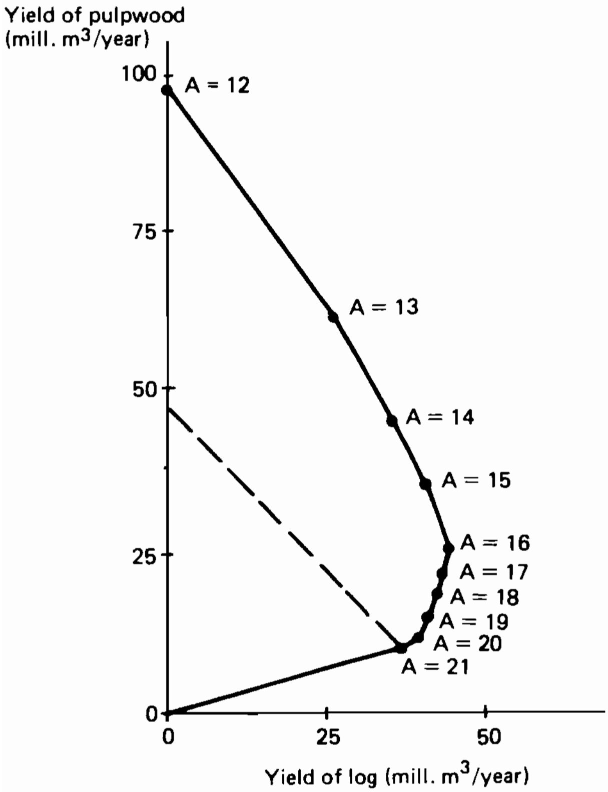


Fig. 1. Alternative yield of log and pulpwood.

Table 2
Yield by timber assortments, trees harvested and trees planted for harvesting policies $A = 12, \ldots, 16$

Harvesting policy A	12	13	14	15	16
Log yield, mill. m^3/a					
Harvesting	0	24.1	35.0	39.8	43.9
Thinning	0	3.7	2.7	2.3	2.0
Total	0	27.8	37.7	42.1	45.9
Pulpwood yield, mill. m^3/a					
Harvesting	76.2	39.5	26.5	19.7	13.0
Thinning	22.7	24.0	19.7	16.3	13.6
Total	98.9	63.5	46.2	36.0	26.0
Total yield, mill. m^3/a	98.9	91.3	83.9	78.1	72.5
Harvesting, mill. trees/a	420	290	230	190	160
Planting, mill. trees/a	2060	1620	1320	1090	910

planted for each grown tree harvested when policies $A = 14, 15, \ldots, 21$ are applied. This number is roughly what is enforced by the Finnish law today.

Fig. 1 shows the annual yield of log and pulpwood when harvesting policies $A = 12, 13, \ldots, 21$ are applied. We may note that alternatives $A = 17, 18, \ldots, 21$ are dominated by other alternatives; i.e. there is another alternative whose yield is better for both of the two timber assortments. The optimal alternative depends on the weighting of log and pulpwood. If the weight for log is at least 150% larger than that for pulpwood, then $A = 16$ is optimal; i.e. a tree gets harvested when it grows 75 to 80 years old. If this percentage is 100 (which roughly corresponds to the current price levels of log and pulpwood in Finland) then the alternatives $A = 14$ and $A = 15$ are about equally good; i.e. trees in the age interval 65 to 75 should be harvested. When the weight for log drops to only 75% above that for pulpwood, the harvesting age falls to 60 to 65 years.

The yield along the line segment between the corner points in fig. 1 may be obtained when two policies are combined. Logs may also be used as pulpwood.

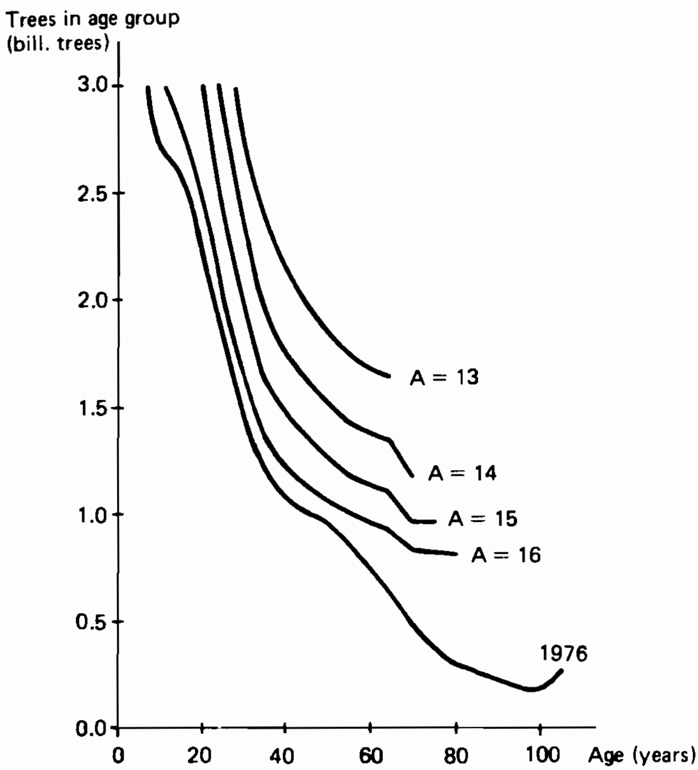


Fig. 2. Age distribution of trees for policies $A = 13, \ldots,$ and comparison with the situation in 1976.

This has been illustrated for $A = 21$ by points along the broken line of fig. 1. Note that each such point is inferior to the efficient frontier, and the same is true for any other policy alternative A. Thus, in the stationary state logs should not be used as pulpwood regardless of the price ratio of log and pulpwood. (In a transition period this of course may not hold.)

Table 2 summarizes the alternatives $A = 12, 13, \ldots, 16$. It shows, for each policy alternative A, the yields of log and pulpwood separately from the harvesting and the thinning activities when the total volume L of forests is assumed to be 1700 mill. m^3. Also the number of trees to be harvested and planted annually is shown in table 2.

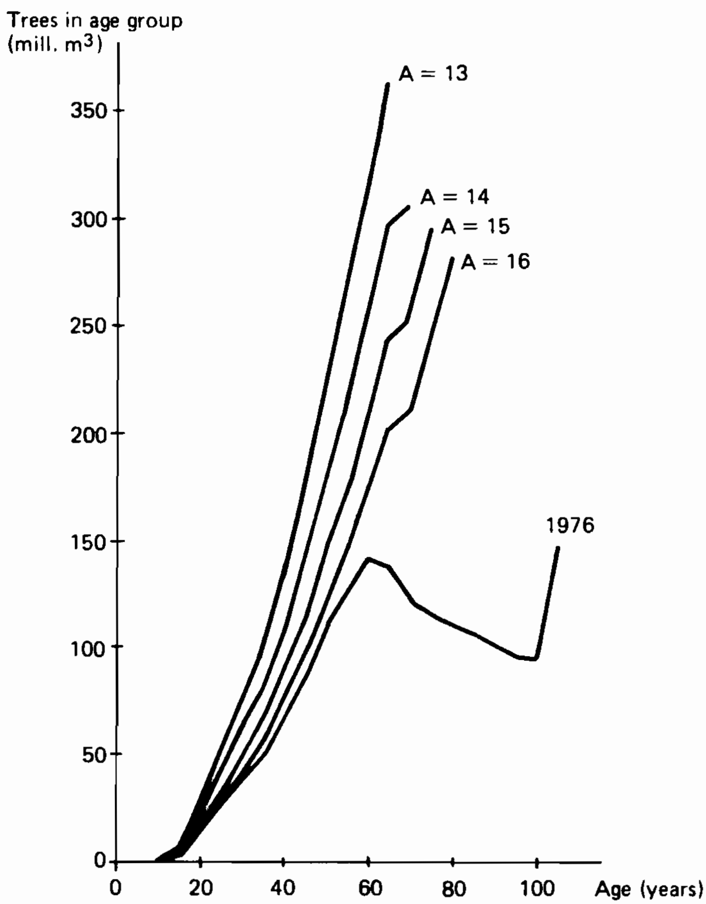


Fig. 3. The volume distribution of trees in age groups for policies $A = 13$, 14, 15 and 16 as compared to the situation of 1976.

The age distribution of trees resulting from policy alternatives $A = 13,\dots,16$ has been illustrated in fig. 2. For comparison, the estimated age distribution in 1976 adjusted to the same total volume of forests has been shown in fig. 2. In fig. 3 we have presented the distribution of the volume of trees in different age groups for policies $A = 13$, 14, 15 and 16. The estimated distribution of the volume for the year 1976 has also been presented.

4. A steady state model of the forest industries

In this section we shall consider the wood processing part of the model of section 2 in a steady state.

Suppressing the time index t in the industrial part of the model, eq. (5) yields

$$U'x \leq y, \tag{21}$$

i.e. industrial usage of wood $U'x$ cannot exceed wood supply y.

Eq. (6) requires that net production $(I-A)x$ supplied to the external market cannot exceed demand D:

$$(I-A)x \leq D. \tag{22}$$

As in the dynamic version (7), gross production is limited by capacity c

$$x \leq c. \tag{23}$$

The state equation (8) for capacity yields

$$gc = v, \tag{24}$$

i.e. investments v equal (physical) depreciation gc. The state equation (10) for external financing is rewritten as

$$l^- = l^+, \tag{25}$$

in other words, the level of external financing remains constant in the steady state formulation.

Taking into account eqs. (24) and (25) the modification of eq. (9) results in the following formulation

$$Gx - (F + Eg)c - sl = 0. \tag{26}$$

Eq. (26) states that the net income from sales equals the expenditures caused by capacity (fixed costs and depreciation) plus external financing (interest payments). Alternatively we may replace the equality in eq. (26) by an inequality. The slack can then be interpreted as a constant flow out from the forest sector.

It is obvious that in the optimal solution eq. (23) holds as an equality:

$$x = c. \tag{27}$$

We define a vector d for the external demand which equals net production. Solving x from this, yields

$$U'(I - A)^{-1}d \leq y. \tag{28}$$

In summary, for the steady state solution we have to find d which satisfies eq. (28),

$$(G - F - Eg)(I - A)^{-1}d \geq 0 \tag{29}$$

and

$$0 \leq d \leq D. \tag{30}$$

5. A steady state model of the forest sector

Above we have presented two steady state models, one for forestry and another for wood processing industries. In this section we shall merge these two parts to obtain a steady state model for the entire forest sector.

Efficient yields of pulpwood and log are shown in fig. 1, in which the feasible region of yields can be defined by a set of linear inequalities:

$$Ry \leq s,$$

where y is a vector of m components signifying the different timber assortments, R is a matrix and s a vector. For the two-dimensional case of fig. 1, the components of R and s can be obtained immediately.

Thus, for $U'x$, the industrial use of wood, we require

$$RU'x \leq s,$$

or

$$RU'(I - A)^{-1}d \leq s. \tag{31}$$

The steady state solution d for the entire sector is then one which satisfies eqs. (29), (30) and (31).

So far we have not included into the model any objective functions. For forestry we might choose to maximize stumpage earnings; i.e. the income from selling wood to industry less the production costs for that wood (e.g. harvesting and transportation costs). As for industry, industrial profit, the left-hand side of eq. (29) offers one possible objective function to be maximized. The sum of these two could constitute a joint objective function (the joint profit) for the entire forest sector. We shall discuss this subject further in section 6.1.

6. A Stackelberg game

In this section we shall discuss a specification of the steady state model to be applied for timber market analysis. The model will be augmented with objective functions both for forestry and the wood processing industry. Furthermore, the demand for final products is represented by a demand function of constant price elasticity. The round wood market is viewed as a Stackelberg game.

It is apparent that the game situation in the forest sector involves two parties: forestry and forest industry. So far this bipartition has been revealed by separate models for each party. These models are interconnected through the amount of round wood supplied by forestry to the industry and through the prices of round wood.

The market mechanism which determines (round) wood prices may be described as follows: given the prices and the availability of different timber assortments (at these prices) the industry chooses the quantity it will buy by maximizing its profit; the problem for forestry is to choose prices to maximize its profit (given the resulting wood demand for that price).

The decision process described above was first described in Stackelberg (1952) and it is, therefore, called a Stackelberg game. The party making the first decision (on prices) is called the leader of the game and the other party the follower. In our application, forestry acts as the leader and the industry as the follower. We assume that both the leader and the follower are *profit maximizers* and that they both have *perfect information* about the game (e.g. on profit functions, supply and demand).

The complexity of the game arises from the fact that the profits of both parties depend on raw wood prices. The revenues of forestry are determined by the price of wood and the quantity sold. In addition, the production cost for wood (e.g. harvesting and transportation costs) influence the profit of forestry. For the industry, the price of wood influences the cost of production. The sales price of an industrial product influences its demand.

At the solution of the game, i.e. at the Stackelberg equilibrium, prices for timber assortments are at a level which maximizes forestry's profit taking into account the effect of this price level on wood demand.

6.1. The profit functions

In order to solve the (Stackelberg) equilibrium prices we shall append to the steady state model of section 5 profit functions for both parties.

Let $p = (p_i)$ be the vector of unit prices for industrial products i on the international market, let vector $c = (c_i)$ stand for the costs of one unit of production including labor, energy and fixed costs, depreciation, and real interest on total invested capital but excluding wood cost. Let z be the vector of wood prices for the different timber assortments. Denote

$$U = U'(I - A)^{-1} \tag{32}$$

as the vector of timber assortments required for one unit of (industrial) production. *Industrial profit*, denoted by P_I, is given by

$$P_I = (p - c - zU)d, \tag{33}$$

where vector d stands for the volume of export.

As for forestry, denote by e the unit production cost for wood. *Forestry profit*, denoted by P_F, is given by

$$P_F = (z - e)y, \tag{34}$$

where y is the quantity of wood sold to the industry.

6.2. Demand functions and optimal prices for wood products

In section 5, we assumed that the external demand for wood products is limited by an (exogenous) upper bound. However, for the Stackelberg analysis it is convenient to use a demand function with constant price elasticity

$$d_i = k_i p_i^{-b_i}, \tag{35}$$

(for each wood product i) where p_i is the price, k_i is a constant, and $-b_i$ is the price elasticity of demand. We may assume that b_i is greater than 1.

Denote by $\bar{p}_i$ the world market price which results in the (reference level) of demand $\bar{d}_i$. For example, if $\bar{d}_i$ is the current (external) demand, then $\bar{p}_i$ shall refer to the current price. Using $\bar{p}_i$ and $\bar{d}_i$ we solve for k_i. Substituting into eq. (35) yields

$$d_i/\bar{d}_i = (p/\bar{p}_i)^{-b_i}. \tag{36}$$

Inserting $d = (d_i)$ from eq. (36) into eq. (33), we can solve the (profit maximizing) optimal price p_i^* for wood products. As a result we have

$$p_i^* = [b_i/(b_i - 1)](c_i + zU). \tag{37}$$

6.3. The profit maximization problem for forestry

In eq. (37) we obtain the optimal price p_i^* as a function of wood price z; in other words, $p_i^* = p_i^*(z)$. Thus, external (optimal) demand d_i is actually a function of wood price z. We shall denote the vector of optimal demand quantities as a function of z by $d(z)$. The wood usage $y = y(z)$ corresponding to the optimal wood product prices is then given as a function of wood price:

$$y(z) = U'(I - A)^{-1}d(z). \tag{38}$$

According to eq. (31), the wood availability from forests restricts wood consumption as follows:

$$RU'(I - A)^{-1}d(z) \leq s. \tag{39}$$

We have combined the two models, one for forestry and another one for industry, to yield the following optimization problem for forestry:

$$\max_z P_F(z) = (z - e)y(z) \tag{40}$$

subject to

$$Ry(z) \leq s. \tag{41}$$

The forestry profit maximizing wood price vector, denoted by z^*, is the (Stackelberg) equilibrium price.

7. Equilibrium solutions for Finland

In this section we shall present numerical results for the Stackelberg game with Finnish data. We will carry out the numerical tests using a model dealing with two timber assortments (log and pulpwood) and with seven wood products: sawn goods, panels, other mechanical wood products, mechanical pulp, chemical pulp, paper, and converted paper products.

For the forestry sector we employ the alternative sustained yield solutions derived in section 3. The set of sustained yield solutions of fig. 1 is used to define the constraints (41) defining the convex polyhedral set of feasible round wood yield.

For the industrial model, we assume demand functions with price elasticity coefficients $b_i = b$ being equal for each product. According to the representatives of the Finnish forest industry, a reasonable assumption concerning the value of b is the range between 10 and 30. However, sensitivity analysis shall be presented for the whole range of $1 < b \leq \infty$.

Another highly sensitive and uncertain figure in the analysis is the reference level p_i of the world market price. For sensitivity analysis, three price scenarios based on the study of Jaakko Poyry (1979) were constructed for each forest product. Scenario 1: an *optimistic* world market price is defined as total production cost in Finland (including wood cost at present prices and a 10% real interest on total invested capital). Scenario 3: a *pessimistic* price is defined reflecting such production costs for the major future suppliers (such as North American and Latin American producers) in the world market. Scenario 2: a more *likely* scenario, is the average of the two above. According to our data, the price in Scenario 1 is higher than in Scenario 3 for each wood product separately.

7.1. The single product–single timber assortment case

For qualitative analysis of the model we shall first study the case of a single timber assortment and a single product. In this case, the equilibrium can actually be solved analytically.

Depending on the value of b the results shall be studied in two cases. We consider first the case when b is small and when forest land is not fully exploited. To solve the equilibrium wood price z^* we maximize forestry profit as defined in section 6.3. Taking into account eqs. (36) and (37) and omitting constants we have

$$P_{\mathrm{F}} = (z-e)\bar{d}\bar{p}^{b}(b/(b-1))^{-b}(c+z)^{-b},$$

$$= (z-e)(c+z)^{-b}. \tag{42}$$

The equilibrium wood price z^* from eq. (42) is

$$z^* = (c+be)/(b-1). \tag{43}$$

Notice that z^* is independent of the world market reference price $\bar{p}$. It is a decreasing function of b, which asymptotically approaches wood production cost e (harvesting, transportation, etc.) as b increases.

Inserting z^* into eq. (42) the maximum forestry profit is

$$P_F = [(c+e)/(b-1)]\{[b/(b-1)]^2[(c+e)/\bar{p}]\}^{-b}\bar{d}. \tag{44}$$

As for industrial profit given by eq. (33), the following formula results

$$P_I = (p-c-z^*)d = [b/(b-1)]P_F. \tag{45}$$

Along with b, forest utilization increases until the total forest land area is exploited. In this second case, when forest land is fully exploited, we solve the equilibrium wood price z^* assuming that the demand for round wood equals the maximum supply. The maximum production is denoted by d^*. From eqs. (36) and (37) we get

$$d^* = \bar{d}[b/(b-1)]^{-b}[(c+z)/\bar{p}]^{-b}. \tag{46}$$

Solving the equilibrium wood price z^* from eq. (46) results in

$$z^* = (\bar{d}/d^*)^{1/b}[(b-1)/b]\,\bar{p} - c. \tag{47}$$

In this case, the equilibrium price z^* is a concave function of b which asymptotically approaches $(\bar{p}-c)$ (the unit profit when wood cost is omitted) as b increases.

Inserting eq. (47) into eq. (37) yields the optimal product price

$$p^* = \bar{p}(\bar{d}/d^*)^{1/b} \tag{48}$$

which asymptotically approaches $\bar{p}$ (the world market price) as b approaches infinity.

Using eqs. (45), (47) and (48) the industrial profit is defined as

$$P_I = (1/b)(\bar{d}/d^*)^{1/b}\bar{p}d^*. \tag{49}$$

As b increases the equilibrium price z^* asymptotically approaches a level absorbing all profit of the forest sector into wood price.

As for forestry profit, eq. (34) gives us

$$P_F = (z^* - e)d^*, \tag{50}$$

which asymptotically approaches $(\bar{p}-c-e)d^*$ (the maximum profit of the entire forest sector) as b approaches infinity.

In fig. 4a we present the equilibrium price z^* of raw wood as a function of

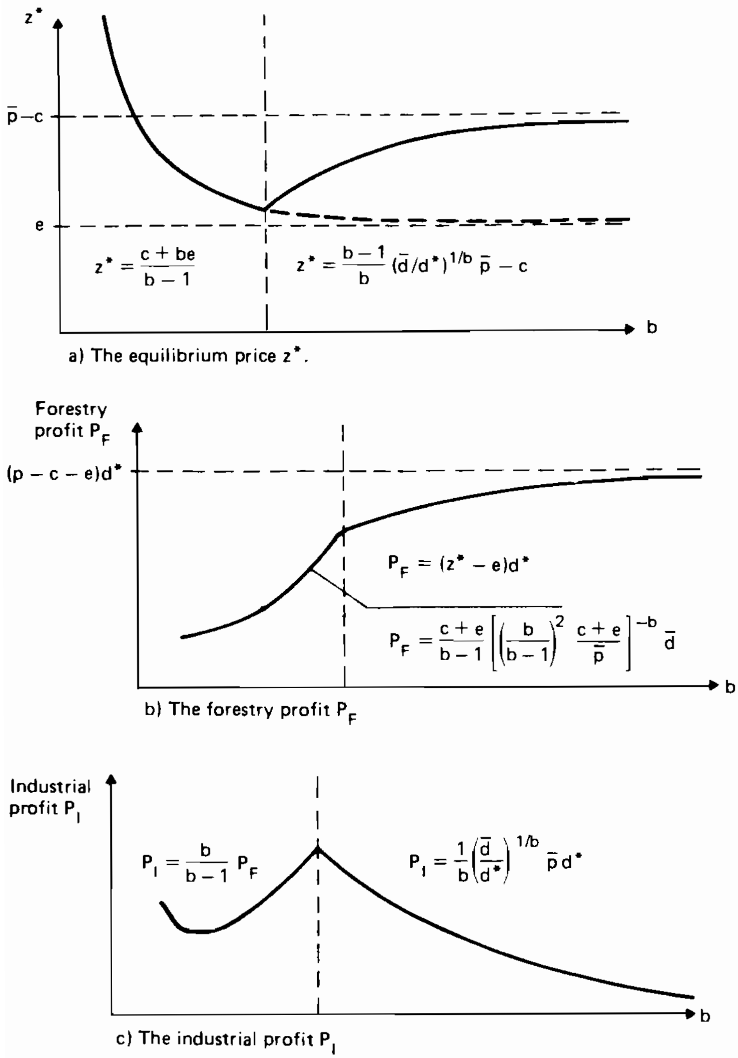


Fig. 4. Equilibrium prices and profits as function of the price elasticity coefficient b for the single product–single timber assortment case.

b. Figs. 4b and 4c show the behavior of forestry profit P_F and industrial profit P_I as functions of b, respectively.

7.2. *The seven products–two timber assortments case*

For each wood price vector z, the profit maximizing solution for industry, and thereby wood demand $y(z)$, can be expressed analytically. Thus the problem of determining the equilibrium price z^* can be stated as an explicit nonlinear programming problem, eqs. (40)–(41), with nonlinearities both in the objective and in the constraints.

We shall redefine the variables so that the resulting problem has nonlinearities only in the objective. Let the inverse function of $y(z)$ be defined as

$$z = g(y). \tag{51}$$

Substituting this into eqs. (40) and (41) yields the following problem with linear constraints

$$\max_{y} \bar{P}_{\mathrm{F}}(y) = (g(y) - e)y \tag{52}$$

subject to

$$Ry \leq s. \tag{53}$$

For moderate values of b we can solve this problem using standard nonlinear programming codes. The MINOS code discussed in Murtagh and Saunders (1978) was employed in this study.

Since we only know $g(y)$ through its inverse function, the following procedure was implemented for evaluating the gradient: (i) Employing iterative methods, solve for the price vector z corresponding to the current value for y; (ii) determine the Jacobian matrix $E(z) = (\partial y_i(z)/\partial z_j)$ for current y and z, and finally, (iii) calculate the gradient as $\nabla_y P_{\mathrm{F}}(y) = \nabla_z P_{\mathrm{F}}(z) E^{-1}(z)$.

For large values of b, the problem is ill-behaved and thereby nonsolvable. However, for $b = \infty$ we obtain the equilibrium price z^* from the dual solution of the following linear program maximizing joint profit for industry and forestry as follows:

$$\max_{d,\, y} (p - c - eU)d. \tag{54}$$

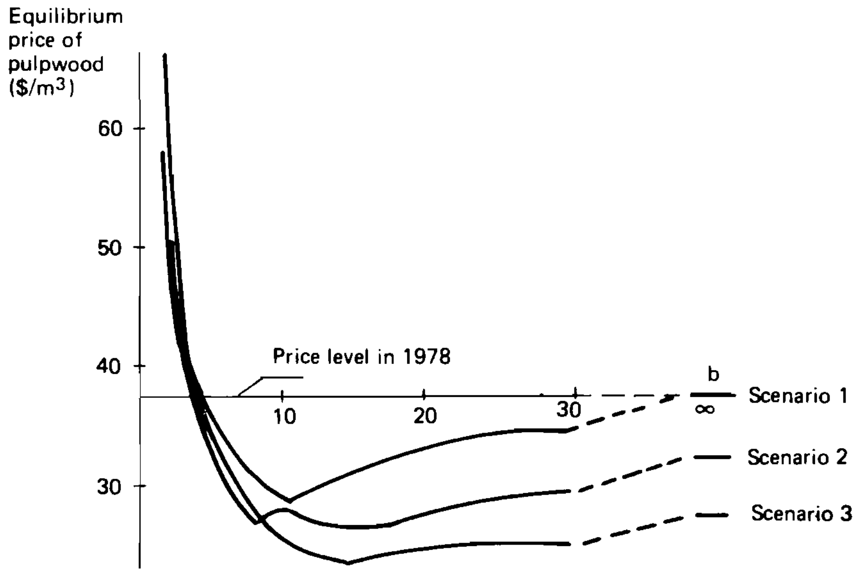
subject to

$$Ud - y = 0, \tag{55}$$

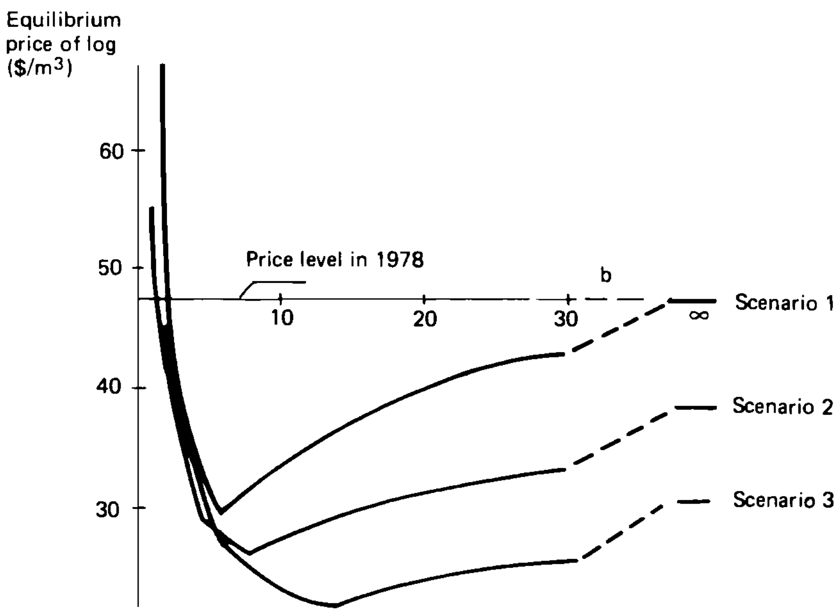
$$Ry \leq s, \tag{56}$$

$$d,\ y \geq 0. \tag{57}$$

Proposition. Assume the problem, eqs. (54)–(57), to be nondegenerate. If μ^* is the dual optimal solution corresponding to constraint (55), then $z^* = e + \mu^*$ is the Stackelberg equilibrium price for $b = \infty$.



a) The equilibrium price for pulpwood



b) The equilibrium price for log

Fig. 5. Equilibrium round wood prices as functions of the price elasticity coefficient *b* for world market price Scenarios 1–3.

When forestry sets the stumpage price at μ^* and $y \leq y^*$ (the optimal wood consumption) it will maximize its earnings, which, in this case, are equal to the total profit for the entire forest sector. For a proof of the Proposition, see Appendix 2.

In figs. 5a and 5b we have the equilibrium wood prices as functions of the elasticity parameter b for the three world market price scenarios.

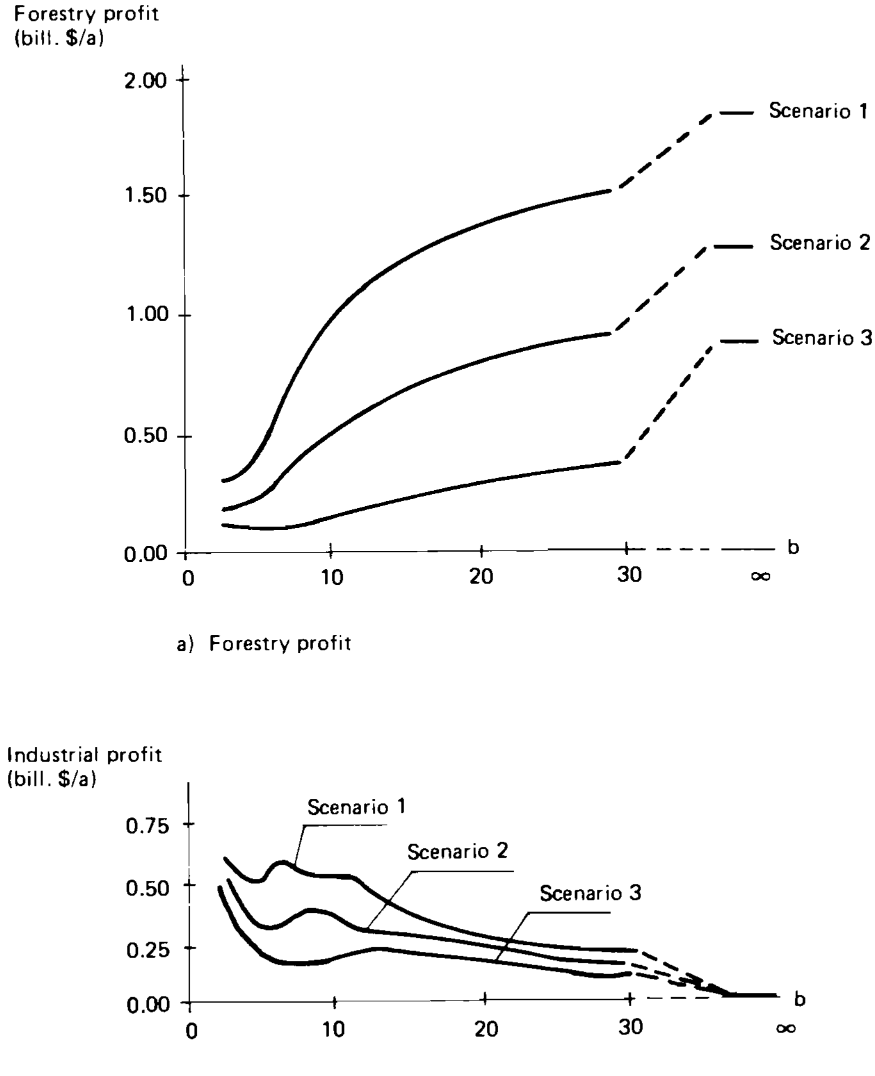


Fig. 6. Equilibrium profit for the forestry and the industry as a function of the price elasticity coefficient b for world market price Scenarios 1–3.

Table 3
Equilibrium prices for pulpwood and log as compared to current prices when b equals 10, 20, 30 and ∞

(a) The case of $b = 10$

$b = 10$		Current price ($/m^3)	Equilibrium price ($/m^3)		
			Sce 1	Sce 2	Sce 3
Wood	log	48	34	28	23
price	pulpwood	38	28	28	26
Stumpage	log	30	17	11	5
price	pulpwood	15	6	6	3

(b) The case of $b = 20$

$b = 20$		Current price ($/m^3)	Equilibrium price ($/m^3)		
			Sce 1	Sce 2	Sce 3
Wood	log	48	41	32	24
price	pulpwood	38	33	28	25
Stumpage	log	30	23	14	7
price	pulpwood	15	11	5	2

(c) The case of $b = 30$

$b = 30$		Current price ($/m^3)	Equilibrium price ($/m^3)		
			Sce 1	Sce 2	Sce 3
Wood	log	48	43	34	25
price	pulpwood	38	34	29	24
Stumpage	log	30	26	16	8
price	pulpwood	15	12	7	2

(d) The case of $b = \infty$

$b = \infty$		Current price ($/m^3)	Equilibrium price ($/m^3)		
			Sce 1	Sce 2	Sce 3
Wood	log	48	48	39	31
price	pulpwood	38	38	33	28
Stumpage	log	30	30	21	14
price	pulpwood	15	15	10	5

Figs. 6a and 6b show the profits for forestry and for industry at equilibrium. For large values of b (i.e. $b=\infty$), forestry, absorbs the total profit of the sector. (Note that the necessary return on capital has been taken into account as a cost factor for the forest industry. Zero profit for industry means, therefore, that return on capital equals this minimum.)

For $b = 10$, 20, 30 and ∞, the numerical results have been given in table 3.

Generally, we conclude that the current price levels for pulpwood and log are much higher than the equilibrium prices resulting from our analysis. On the other hand, there are substantial differences between prices resulting from the different price scenarios.

8. Summary and conclusions

In the first part of this paper a (discrete time) dynamic linear model for the forest sector was discussed. The steady state version of it was analyzed in more detail. An application of the steady state forestry model was carried out for the Finnish forests. As a result, alternative sustained yield solutions for the Finnish forestry were obtained.

In the second part of the paper, a steady state sectorial model was adopted to carry out a Stackelberg equilibrium analysis for the round wood market of Finland. Further elaboration was needed for the steady state model until it became suitable for this game theoretic analysis. This elaboration involved definitions of objective functions for the forestry and for the industry.

For the industrial model, a demand function with a constant price elasticity coefficient b was chosen for each product. A reasonable assumption concerning the value of b is in the range between 10 and 30. If b is greater than 30 we price ourselves out of the market with a 10% increase in price. On the other hand, when b is under 10 the demand is very rigid; in other words, changes in price do not affect demand, which does not correspond to the present market situation. However, sensitivity analysis was carried out on the whole range of $1 < b \leqslant \infty$. The other highly uncertain and sensitive figure in the analysis is the world market price (defined as sales price when b approaches infinity). For sensitivity analysis, three price scenarios were constructed for each forest product as follows: (1) An optimistic world market price is defined as total production cost in Finland (including wood cost at present prices and a ten per cent real interest on total invested capital), (2) a pessimistic world market price is defined as being roughly equal to the production cost of our major future competitors in the world market, and (3) a likely scenario which is the average of the two above.

As the numerical results presented in section 7 show the current price levels for pulpwood and log are much higher than the equilibrium prices resulting from our analysis. On the other hand, there are substantial differences between prices resulting from the three price scenarios for the world market prices of wood products.

Appendix 1

Proposition. For an optimal solution of the forestry problem (F) defined in section 3.1 there is an age group A such that $h(a)=0$, for all $a \neq A$, and $w(a)=0$, for all $a>A$.

Proof. Clearly, for an optimal solution $w(1)=p>0$. Let $a=A$ be the smallest age group for which $w(A+1)=0$. Then $h(A)>0$ and $w(a)=h(a)=0$ for all $a>A$. To show that $h(a)=0$ for $a<A$, we consider the optimal basis B for (F) partitioned as follows:

$$B=\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix},$$

where B_{11} and B_{22} are square matrices and $B_{21}=0$, and where the columns of B_{11} correspond to basic variables p, $w(1),\ldots,w(\mathrm{A})$ and $h(A)$, and the rows of B_{11} correspond to constraints (11)–(13) for all $a \neq A$. Because $B_{21}=0$, B_{22} is nonsingular and therefore, $h(a)$ is nonbasic for $a<A$. ||

Appendix 2

Proposition. Assume problem, eqs. (54)–(57), to be nondegenerate. If μ^* is the dual optimal solution corresponding to constraint (55), the $z^*=e+\mu^*$ is the Stackelberg equilibrium price for $b=\infty$.

Proof. Consider the problem (G) of maximizing the profit for the entire forest sector:

$$\max_{(d,\,y)\geq 0} (p-c-eU)d \tag{G.1}$$

subject to

$$Ud-y=0, \tag{G.2}$$

$$Ky \leq s. \tag{G.3}$$

Let $(d^*,\ y^*)$ be the optimal primal solution for (G), and $(\mu^*,\ \lambda^*)$ the optimal dual multipliers (for constraints (G.2) and (G.3), respectively). Let $\epsilon>0$ and define a wood price vector $z(\epsilon) = e + (1-\epsilon)\mu^*$. For this wood price the profit maximization problem (I) of industry is the following:

$$\max_{(d,\, y) \geq 0} \left[p - c - z(\epsilon) U \right] d \tag{I.1}$$

subject to

$$Ud - y = 0, \tag{I.2}$$

$$Ry \leq s. \tag{I.3}$$

Optimal primal and dual solutions for (I) are denoted by (d', y') and (μ', λ'), respectively.

To prove the proposition, we shall show that an optimal solution (d', y') for (I) is optimal for (G) as well, and that the profit thereby obtained by forestry can be made arbitrarily close to the optimal profit for (G), the profit for the entire sector. The latter is achieved when ϵ approaches zero corresponding to the limiting wood price $z(0) = \mathrm{e} + \mu^*$.

One can readily check the optimality conditions for (I) and observe that the primal and dual solutions (d^*, y^*) and $(\epsilon\mu^*, \epsilon\lambda^*)$, respectively, are optimal for (I). Because of the primal nondegeneracy assumption for (G), and thereby for (I), the dual optimal solution for (I) is unique. Therefore $(\mu', \lambda') = (\epsilon\mu^*, \epsilon\lambda^*)$. This together with the optimality condition for (I) applied to the primal solution (d', y') and the dual solution (μ', λ'), imply the optimality conditions of (G) for (d', y') and (μ^*, λ^*), i.e. an optimal solution (d', y') for (I) is optimal for (G) as well. From the optimal profit $(p - c - eU)d^*$ of (G), an amount of $\epsilon\lambda^* y^*$ belongs to the industry, and this share approaches zero with ϵ. □

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THE OPTIMAL EQUILIBRIUM OF THE SWEDISH WOOD MARKETS ON THE SHORT AND MEDIUM RUN

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Evaluation of different harvesting programs, short-run supply curves and subsidies to single industry-plants are made with one and three-period, as well as short and medium-run linear programming models.

1. Introduction

The supply and demand of wood and timber in Sweden is regulated by the government in many ways. Certain obligations and constraints are imposed by law upon the use of forest land. Besides the general economic and tax policies, which have a strong impact on forestry, special taxes and subsidies are directed towards this sector. The state also plays an important role as an active agent of the markets for wood and timber. One fourth of the forest land, as well as some companies in the forest-based industry, are publicly owned.

The government's exercised and potential power on the wood market is in some respects stronger than its knowledge of how to direct the power. The costs and benefits of different measures are often not very well identified. There is therefore a need for empirical models which will enable economic welfare theory to be applied to wood market problems.

In this article, I will present a collection of LP-models which are devoted to this purpose. The models have been used to analyze problems discussed in the political debate. Some results from these applications will also be reported.

Although the forest sector produces a significant share of Sweden's exports, the Swedish industry is a rather small agent on the world market for forest products. Hence, world market prices can be regarded as the ultimate determining factor for the level of activity in forestry. This relationship works in two ways. The price level determines to what extent the resources are economically available. High prices make it profitable to harvest stands with high exploitation costs and to utilize trees and components of trees with low quality or thin dimension. Secondly, the expected development of future prices as compared to the current level affects the decision concerning the time to harvest, now or later.

The long-run planning problem of forestry (for an introduction to this problem, see Löfgren and Johansson 1982) is quite complex. One obvious reason is the high degree of uncertainty involved due to the long production period. Another reason is the complicated mathematics necessary when the planning level extends from the micro level of a single tree or even-aged stand to the macro level of the forests of a nation.

However, the world will not stop spinning just because the theoreticians are stumbling on their way. Historically, the Swedish forest policy has relied on some rules of thumb such as the requirement of a certain balance between the volume of fellings and forest growth of the same period. The current policy is based on a principle of a sustained constant harvest volume from now till kingdom come.

During the seventies refined model techniques have been developed to make long-term predictions of the effects on the future state of the forests using different programmes for harvesting, aforestation, etc. Combined with methods to assess the differences in economic value between such long run management plans, this certainly boosts the possibility to increase the economic rationality of the forest policy.

The models presented in this study calculate the value of the near future forest production. The economic availability of the forest resources in the short run can be illustrated by a supply and demand schedule. The maximal harvestable volume is set by the long-run production plan. Given the intertemporal programme, the short-run supply curve shows the marginal costs of felling and transporting the accessible stands. The demand curve shows the industry's ability to pay for the raw materials. This reflects current world market prices and the distribution of the capital world market prices and the distribution of the capital stock over plants with different output and efficiency levels.

The intersection A between the two curves in fig. 1 is the market equilibrium. If the demand and supply curves correctly take account of all relevant costs and benefits, this is the optimal short-run solution.

Note that, since the main capital stock in forestry cannot be changed as fast as the capital stock in the industry, the expression "short-run" has different meanings to the demand and supply curve. A period of five to ten years is short-run in forestry but medium-run in industry.

In the medium-run, when some new investments can be made, the demand curve shifts to, e.g., the dotted line in the figure, reflecting the need for maintenance investments and the possibilities to invest in new equipment. The equilibrium consequently moves to point B.

The net value of the optimal market solution is defined in the supply/demand schedule by the area between the two curves to the left of the intersection point. The part of this area above the equilibrium price is the contribution profit of the industry, i.e. the quasi-rents to industrial capacity, while the area below represents the quasi-rents to forest land.

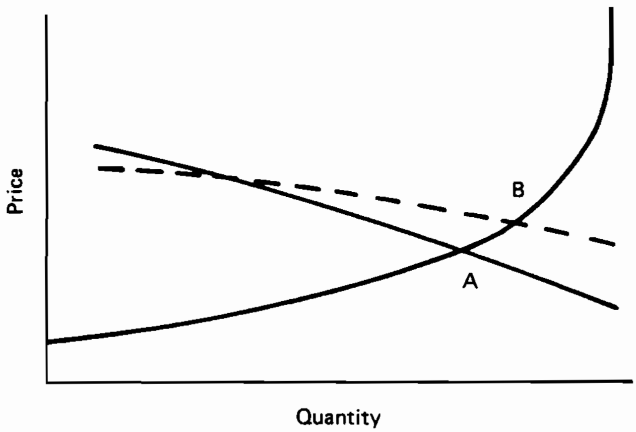

Fig. 1. Timber supply and demand curves.

Fig. 2 shows a case with two short-run supply curves, S and S + , corresponding to two alternative long-run forest production programmes. The short-run (industry short or medium-run) difference in net value between the programmes is expressed by the area 0AB.

There is much room for market, as well as planning, failures in the wood market. The dotted lines in fig. 2 shows an actual short-run supply curve above the social marginal cost curve. The cost to society of the corresponding inoptimal equilibrium C is equal to the area DAC (if the unharvested stands remain unused; otherwise, if the momentary loss is compensated later – before

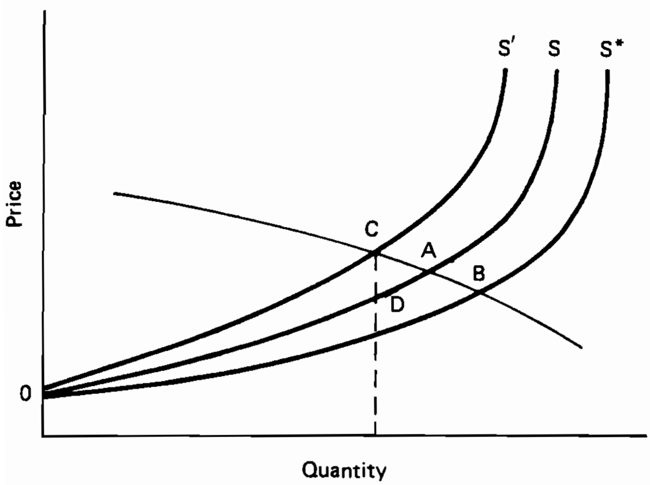

Fig. 2. Effect of alternative forest production programmes.

it becomes irreversible – by an increase in harvests, the cost will be reduced to a loss of interest income).

2. The models

The optimal wood market solutions have been calculated with linear programming. The models used are designed to be both detailed in certain respects and small, making it possible to simulate many alternative cases. Therefore, three separate models with different richness of detail have been constructed: a genuine short run model, called the "1979-model"; an industry medium run static model, the "1984-model"; and an industry medium run three-period model, called the "1979–1993-model".

The models maximize the quasi-rents to industrial capacity (gross profits net of the capital costs of new investments) and quasi-rents to forest-owners (price of stumpage), subject to the constraints set by industrial capacity, investment possibilities and available volumes of wood in different cost classes. World market prices on the forest-industrial products and domestic prices of various inputs (except the wood resources) are exogenous. A model solution is, in principle, equivalent to the (partial equilibrium) outcome of a perfect competition market for wood resources.

The general structure of these models is described below.

List of variables

Indices

Periods (in the three-period model)	$t = 1, 2, 3$
Production activities	$j = 1, 2, \ldots, J$
Provision activities	
(a) wood supply activities	$r = 1, 2, \ldots, R_1$
(b) other	$r = R_1 + 1;\ R_1 + 2, \ldots, R$
Commodities	$i = 1, 2, \ldots, I$
Production capacities	$s = 1, 2, \ldots, S$

Endogenous variables

$X_j(t)$ = activity level of production activity j period t
$M_r(t)$ = activity level of provision activity r period t
$Y_s(t)$ = activity level of provision activity s period t

Exogenous variables

K_s = initial production capacity s
$E_r(t)$ = capacity of wood supply activity r period t

$\overline{Y}_s(t)$ = feasible expansion of capacity s period t

$P_j(t)$ = unit value of the finished commodity produced by production activity j in period t

$W_r(t)$ = cost in period t when provision activity r is used at the unity level

C_s = annuity cost when investment activity s is used at the unit level

$U_s(t)$ = investment cost when investment activity s in period t is used at the unity level

$a_{ij}(t)$ = production requirement of commodity i period t when production activity j is used at the unity level (positive elements represent outputs, negative elements inputs)

$m_{ir}(t)$ = provision of commodity i when provision activity r in period t is used at the unity level

$g_{sj}(t)$ = utilization of capacity s when production activity j in period t is used at the unity level

ρ = rate of interest

θ = expected life length of investments

One period model

Objective function

$$\max \pi = \sum_{j=1}^{J} P_j X_j - \sum_{r=1}^{R} W_r M_r - \sum_{s=1}^{S} C_s Y_s,$$

where

$$C_s = U_s\left(\frac{\rho}{1-(1+\rho)^{-\theta}}\right).$$

The maximand is a profit function. Prices on finished goods are specific for each production activity which makes it possible to take into account price differences of similar products. The costs include cutting and transporting costs for wood materials, other costs of operations and capital (annuity) costs for new investments. Thus, the value of the profit function is equal to the sum of the quasi-rents to forest land and to industrial capacity.

Commodity balances

$$-\sum_{j=1}^{J} a_{ij} X_j - \sum_{r=1}^{R} m_{ir} M_r \leq 0 \text{ for all } i.$$

The consumption of a commodity (negative a_{ij}) is required not to exceed the production (positive a_{ij}) and/or external purchases.

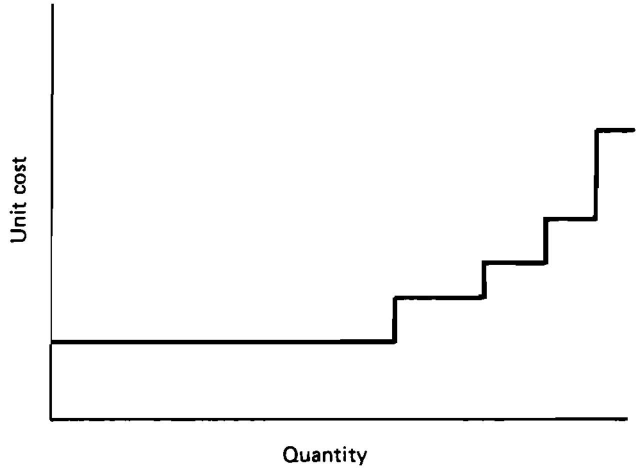

Fig. 3. Piecewise constant approximation of the marginal cost function of timber supply.

Capacity and investment constraints

$$\sum_{j=1}^{J} g_{sj} X_j - K_s - Y_s \leq 0 \quad \text{for all } s,$$

$$Y_s \leq \overline{Y}_s \quad \text{for all } s.$$

The production activities are constrained by the initial capacity endowments and/or the feasible expansion volume.

Wood-supply constraints

$$M_r \leq E_r \quad \text{for } r = 1, 2, \ldots, R_1.$$

Unlike demand on finished products and the supply of other raw materials, wood supply is not assumed to be perfectly elastic. The wood-supply curve is built up with a number of capacity constrained delivery activities as illustrated by fig. 3.

Non-negativity constraints

$$X_j \geq 0 \quad \text{for all } j,$$

$$M_r \geq 0 \quad \text{for all } r,$$

$$Y_s \geq 0 \quad \text{for all } s,$$

Three-period model

Objective function

$$\text{Max } \pi(3) = \sum_{t=1}^{3} \phi(t)\left(\sum_{j=1}^{J} P_j(t) X_j(t) - \sum_{r=1}^{R} W_r(t) M_r(t)\right)$$

$$- \sum_{t=2}^{3} \psi(t) \sum_{s=1}^{S} U_s(t) Y_s(t),$$

where

$$\phi(t) = (1+\rho)^{-5(t-1)} \sum_{\tau=1}^{5} (1+\phi)^{-(\tau-1)},$$

$$\psi(2) = (1+\rho)^{-5},$$

$$\psi(3) = (1+\rho)^{-10} \frac{(1+\rho)^{-10}}{(1+\rho)^{-10} + (1+\rho)^{-15}}.$$

The maximand is now a discounted profit function. Each of the three periods is five years long. Investments are assumed to be made in the beginning of the second and third period and to have a maximal life length of ten years. The discount factor of period 3 is adjusted to compensate for the shorter life length of the investments in this period.

Commodity balances

$$- \sum_{j=1}^{J} a_{ij}(t) X_j(t) - \sum_{r=1}^{R} m_{ir}(t) M_r(t) \le 0 \quad \text{for all } i \text{ and } t.$$

Capacity and investment constraints

$$\sum_{j=1}^{J} g_{sj}(t) X_j(t) - \sum_{j=1}^{J} g_{sj}(t-1) X_j(t-1) - Y_s(t) \le 0 \quad \text{for all } s \text{ and } t,$$

where

$$\sum_{j=1}^{J} g_{sj}(0) X_j(0) = K_s,$$

$$Y_s(1) = 0 \qquad \text{for all } s,$$

$$Y_s(t) \le \overline{Y}_s(t) \qquad \text{for } t = 2, 3 \text{ and for all } s.$$

Wood supply constraints

$$M_r(t) \leq E_r(t) \quad \text{for } r = 1, 2, \ldots, R_1 \text{ and for all } t.$$

Non-negativity constraints

$$X_j(t) \geq 0 \quad \text{for all } j \text{ and } t,$$
$$M_r(t) \geq 0 \quad \text{for all } r \text{ and } t,$$
$$Y_s(t) \geq 0 \quad \text{for all } s \text{ and } t.$$

The "1979-model" has been applied separately to the three forest regions into which Sweden can be divided (see the map, fig. 4), whereas the medium run models have been run for the Northern region only.

A major part of the data material has been collected from the companies' returns to the official Swedish statistics for the manufacturing industry 1979. Input–output data have been calculated for separate production lines within the plants. For example, in a plant which produces both pulp and paper, there are different activities for the production of semi-finished pulp, market pulp and paper etc. In most cases there are several options to the composition of different kinds of pulp in a specific paper product.

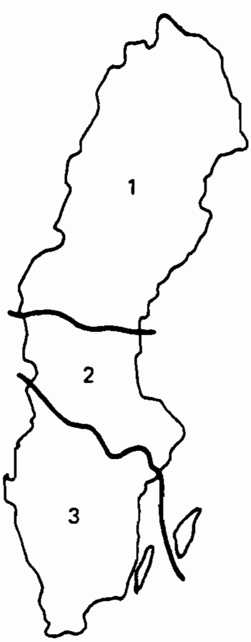

Fig. 4. The three forest regions under study in Sweden.

The models comprehend the pulp, paper and board industry and wood-based large-scale heating stations (blue-print technology). The sawmill industry's net demand for wood material (timer, less waste-products used by the pulp or board industry) is exogenous to the model. [1]

Wood (conifer wood and leafwood) is supplied by five (one large intramarginal and four marginal or near-marginal) capacity-constrained supply activities; in this way, the short-run exploitation-cost distribution of the allowable cut [2] is represented in the models. The main data source is found in the cost accounts for the government owned forests.

A possibility of a minor "import" of softwood from the southern part of Sweden is also given.

The first model, the "1984-model", includes the investment alternatives that were actually considered in the autumn of 1980 by the companies in the region. The data on the projects have been supplied by the corporations. The wood market some years ahead (for instance 1984) is described as a relevant to the investment-decision maker of 1980. Thus, capital costs (annuities) are only regarded for investments.

The "1984-model", is a static model, although investment activities are included. The second model, the "1979–1993-model", is three-periodical. The first five-year period is constrained by the 1979 capacities. The second period capacities can be expanded. More investments can be made for utilization in the last period. [3] The costs of the latter are modified to compensate the shorter life-time of these. A solution is chosen which maximizes the sum of the discounted quasi-rents of the three periods.

In the three-periodical model, unlike the case in the "1984-model", there is only one production line corresponding to each final or semi-finished product, and the whole industry is described as if this consisted of only one multiproduct plant. The size of the model is kept down at the expense of the scope for substitution.

[1] To make the allocation of timber to sawmills endogenous, it is necessary to introduce into the model the quality and dimension distribution of the wood/timber supply as well as the substitution possibilities in the sawmills between different kinds of timber. This would greatly increase the size of the model and the requirements of detailed data. Moreover, since one half of the timber input becomes chips and sawdust used as input to the pulp and board industry, the sawmill's net demand for wood material is fairly inelastic with regard to the price of pulpwood. Thus, the solution of the wood market can be reasonably well calculated given a solution of the timber market.

[2] The forest owners are supposed to have only a static problem of whether to cut or not cut a permissible volume of trees. The dynamic problem of when to harvest the growing resources is solved by government regulation.

[3] The shadow prices of the volume constraints on investment possibilities can be interpreted as quasi-rents imputed to the scarce business organizational, ground, and other resources which are necessary for an investment. The interpretation is valid for the second period, but more ambiguous in the third period, since the constraints for this period are more arbitrary.

The simulations have been made for a full and a 90% degree of maximal capacity utilization. Labour and investments costs have been considered as fixed.

3. Some results

The supply of wood and timber was sufficient for a full utilization of the industry's capacity during the 1979–80 boom years. The opinions concerning the nature of this problem are divergent. Some have claimed that supply is too small while others have stated that industry's capacity is too great. The policy implications of these opinions are equally divergent. The results of the "1979-models" give some light on this controversy.

The "1979-models" calculate the optimal allocation of wood resources to the capacity existing in 1979. Table 1 shows the total volume of wood and timber in the solutions of three cases, together with some actual data.

An average rate of 90% throughout the business-cycle can be regarded as a rather high capacity utilization level. Wood and timber requirements for this average rate, given the current capacity of industry, is 57 mill. m^3, which is approximately equal to the maximum harvestable volume (A), according to the long run management plan which governs the forest policy.

The optimal volume (A), as calculated by the "1979-model", is however, 3–4 mill. m^3 less. The actual supplied volume is still less, viz. 51 mill. m^3. The actual market equilibrium corresponds to a supply curve (the "actual supply curve") approximately 30 Sw. Cr./m^3 above social marginal costs.

Thus, the results so far suggest a compromise judgement. Supply is too small and industry is too big. This is, however, not the end of the story.

The age distribution of Swedish forests is uneven, with a high proportion of old slow-growing trees. This suggests that a long-run programme which maximizes the present value of forest income will allow an increase of harvests in

Table 1
Supply of and demand for wood and timber at a 90% capacity utilization rate

	Mill. m^3 (net volume)
Wood and timber consumption capacity 1980	57
Harvestable volume (A)	56
Harvestable volume (B)	61
Optimal volume (A)	53
Optimal volume (B)	56
Actual supply 1976–1980	51
Optimal volume (A) with actual supply curve	51

the near-distant future, followed by a reduction later on. In another study, using a linear age-class forest model, I have demonstrated that this is likely to be true even if the rate of interest is low and there are some adjustment costs (proportional to increases or decreases in harvested volume from one period to another).

Regardless of his, the ruling long-run harvest-programme is in any case technically inefficient. Model simulations made by the Swedish National Board of Forestry have shown the existence of a long-run programme which is indisputably better (although not in conformity with the principle of a constant harvesting level through time) than the one on which the forest policy is based. Harvests can be increased during the 1980s without incurring, in comparison with the even-flow programme, smaller harvests later. This is due to the increase in average growth which follows when old trees are replaced by seedling. (John Keane has given a similar example from the U.S. See Hyde (1980), p. 28).

The short and medium-run effects of such an increase in the allowable cut (the "potential supply curve") have been evaluated with the industry models. The higher harvestable volume (B) and the corresponding optimal volume (B), according to the "1979-model", are also shown in table 1. The new solution, as is seen when compared with the capacity figure on the first line of the table, omits only a small fraction of the industrial capacity.

This result is further modified slightly when the investment opportunities in the "1984-model" are introduced. The simulations for the Northern region show that the optimal harvested volume is increased even more. These investments also cause a close down of some plants and production lines, which moderates the effect on total wood consumption.

The calculated shadow prices on wood are higher than actual market prices 1979. For example, the equilibrium price on conifer wood corresponding to the "actual" supply curve is estimated by the "1984-model" to be 25% above the 1979 price. With the "potential" supply curve the price difference is 6%. This result could have been expected, since there obviously was an excess demand on the wood market 1979.

If the "actual" supply curve could be replaced by the "potential" supply curve, the annual net income to the Swedish forest sector is estimated to increase by 214 mill. Sw. Cr. The relatively low figures is due to the assumption of an optimal allocation of resources within the sector. The difference can be greater, if a too low supply is accompanied by frictions in the structural adjustments, so that the most efficient plants of the industry are forced to reduce their production. This difference is the net result of an increase in industry's profits and a reduction of forest-owner's rent income. The size of the redistribution is estimated to be about 1.400 mill. Sw. Cr. Thus, one would not expect forest owners to be the first to demand government measures to increase wood supply!

The difference in employment in the forest sector is estimated to be 5500 persons, of which 3300 are in industry and the rest in forestry.

One mill excluded from the solutions is the state-owned sulphate mill Köpmanholmen (which was shut down 1982). If some reconstruction investments had been subsidised, it would have been possible to continue the operation of the plant. This would, the model shows, have had two effects. The equilibrium volume of wood would have been increased and some other plants would have been forced out of the solution. The net in employment is less than the number employed in the subsidized factory. The cost (loss of net income) per net extra employed is estimated to be 9.000 Sw. Cr. per year (which may be regarded as a moderate cost in comparison with other regional policy measures). If two additional plants outside the optimal solution also had been subsidized the cost increases to 21.000 Sw. Cr. per year and net employed.

The three-period model has been applied to mainly two different scenarios for the relative prices of finished goods. The first one is based on "informed persons' guesses" and the other on historical average price relations. The first one favours, in comparison with the latter, various types of bleached paper but disfavours, "brown" paper, leaf-wood pulps and sulphite pulp. In both cases the model expands the capacity of printing paper, but the investments in new capacity for newsprint paper in the "informated guesses" scenario are substituted by a more favourable development of unbleached sulphate paper production in the "average" scenario. The net effect on wood consumption, through various adjustments of the pulp production, is however nearly the same. Wood consumption is successively raised, which indicates that the long run (from industry's point of view) equilibrium lies above the level estimated by the "1979"- and "1984-models".

The difference in discounted net value in 1979–1993 between a solution with the actual supply curve and the potential one is, according to this model, 330 mill. Sw. Cr. (at a 10% rate of discount). The difference in employment in the industry of the region is 200 persons in the first period, 600 in the second period and 1100 in the last period.

The sensitivity of the results to various input prices and other parameters has been studied. For instance, a 25% reduction or increase in investment costs turns out to have a significant impact on investment activities in the three-period model, but does not considerably change total wood consumption. In the last period wood consumption is 4% lower when investment costs are increased and less than 1% higher when costs are decreased.) Also, the sensitivity to changes in sawmills' net demand for wood material has been studied. The sum of timber and wood consumption is shown to be unaffected, except in the short run and if the more favourable supply curve is used. A special study on energy price elasticities and the impact of the oil price on wood fuel use and on total wood consumption is reported elsewhere (Hultkrantz 1983).

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THE SUPPLY OF ROUNDWOOD AND TAXATION *

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Two ways of modeling the supply of roundwood are introduced: the present value maximizing forestry firm and the self-active forest farmer. These models are investigated with respect to the properties of the supply function. Results with particular interest for the effects of changed tax rates are derived. The theoretical results are used to discuss the effects of a changed system of forest taxation in Sweden.

1. Introduction

The purpose of this paper is to introduce two ways of modeling the supply of roundwood, the present value maximizing forestry firm (PVFF-model) and the self-active forestry farmer (SAFF-model), and analyse how the supply of roundwood is affected by parameter changes such as changes in prices, changes in tax rates, and a switch from one system of taxation to another. The theoretical results are e.g. relevant for the recent discussion in Sweden concerning the effects of a switch from a proportional or progressive tax on the income from forestry to a lump sum taxation, where the magnitude of the lump sum is determined by the site qualification of the forest land. [1]

The paper is structured as follows: in section 2 we introduce the PVFF-model, and we derive some relevant properties of the supply function. These properties are then used to analyse how changed tax rates, and a switch from one system of taxation to another affects the supply of roundwood. A similar strategy is followed in section 3, where the SAFF-model is analysed. The results are summed up and commented on in section 4.

* The authors acknowledge comments on an earlier version of this paper from two anonymous referees.

[1] The supply of roundwood in Sweden has for several of the recent years been "too low", and it has been claimed that a switch from the present system of taxation to a system of lump sum taxation would increase the supply of roundwood.

2. The present value maximizing forestry firm

In this section, we will introduce a simple model of the supply of roundwood from a particular forestry firm – the PVFF-model. To begin with no explicit taxes are introduced.

We will make a couple of simplifying assumptions, which will generate the linear programming problem of maximizing the present value of the income stream from cutting subject to the constraints of a linear technology. [2]

The first simplifying assumption that we make is to assume that the planning horizon is finite. This assumption is, however, not quite satisfactory. At any given future point of time the age distribution of the forest will have implications for the future. If one stops the analysis after a finite number of periods, say T, it would in principle be necessary to include some "scrap value" of the forest stand. The only logical way of doing so is to determine the maximum present value attainable in the further future, starting with any given age distribution of the forest.

On the other hand, if one assumes that prices are bounded, it is easy to show that the loss in land value from stopping after a finite time can be made arbitrarily small by making T large enough.

It is also appropriate to say a few words about our target function. The maximization of the present value can be shown to be an objective investment criterion – in the sense that an investment which has a positive present value can be recommended independently of the preferences of the investor – provided that the capital market is perfect. If the capital market is imperfect a variation of the present value method can under certain conditions be used as an objective investment criterion, but there are unfortunately no general methods which can deal with investment decision in an imperfect capital market. [3]

The technology of the forestry firm can be introduced by defining x_{ti} = the number of acres of land occupied at time t by trees of age i. As an initial condition one has

$$\overline{M} = \bar{x}_{00} + \bar{x}_{01}, \dots, + \bar{x}_{0n}. \tag{1}$$

i.e. trees of different age classes and seeded land ($\bar{x}_{00}$) cover the initial amount

[2] This formulation has been borrowed from Berck [1]. A similar formulation of the finite planning horizon, discrete time case is due to Karl G. Jungenfelt, and can be found in ref. [4]. Many of the results which are presented in this paper can also be derived if the problem is cast in the usual capital theoretic terms due to Faustmann [3], but the supply function would in such an approach be a "long run supply curve" as Faustmann's formula presupposes the same price in every period.

[3] Compare Puu [8,9].

of land ($\overline{M}$). Clearly it holds for all t that

$$\sum_{i=0}^{n} x_{ti} = \overline{M} \quad \text{all } t. \tag{2}$$

It is also convenient to define the harvest as a joint product consisting of timber and seeded land and to assume that there is a highest age class (n), and that the lowest age class is seeded bare land (0). Hence, bare land at the end of the period 1 is

$$x_{10} = c_{11} + c_{12} + c_{13} \dots + c_{1n}, \tag{3}$$

where c_{1i} = the number of acres with trees of age i, which is cut in period one. i.e. equal to the sum of the cut in all age classes during period 1. The area of land occupied by trees of age $i < n$ at time $t \leqq T$ will be equal to

$$x_{ti} = -c_{ti} + x_{t-1, i-1} \quad \text{all } t \quad i = 1, \dots, n-1. \tag{4}$$

i.e. equal to what was left from the previous period minus what was cut during the period. For $i = n$, we will assume, to end the recursion, that what was left from the previous period of trees belonging to age class n will remain in this age class for ever. In other words

$$x_{tn} = -c_{tn} + x_{t-1, n-1} + x_{t-1, n}. \tag{5}$$

Combined with the growth function we will apply below, this can be interpreted as if the growth of the biomass of an even-aged stand ended after n periods at which time any extra growth is balanced by mortality. For all practical purposes the assumption behind eq. (5) is no large violation of reality. It can be viewed upon as a pragmatic method of keeping the dimensions of the problem finite.

The growth function is specified as a vector $G' = (0, g_1, \dots, g_n)$, and the net price vector is defined as $p_t = (p_1, \dots, p_T)$, where $p_t > 0$ is the present value of the constant profit/m³ in period t. $g_i > 0$ is the number of m³/acres in a stand of age i. The maximization problem can now be specified as

$$\operatorname*{Max}_{c} \sum_{t=1}^{T} p(t) c'(t) G \tag{6}$$

subject to the restriction specified by eqs. (1)–(5) where

$$c'(t)G = c_{t1} g_1 + \dots, + c_{tn} g_n = c_t.$$

If we define $c = (c_1, \dots, c_T)$ the "profit function" or the present value "function" is defined as

$$\pi(p, \bar{x}) = \max_{c} \{ pc/c, \ x \in C \} = pc(p, \bar{x}), \tag{7}$$

where $\bar{x} = (\bar{x}_{00}, \dots, \bar{x}_{0n})$ are the initial endowments of forest land occupied by trees of different ages, and p is the net price vector. C is the feasible choice set determined from the convex technology defined by eqs. (1)–(5). Further restrictions could however, be added without any substantial changes in the qualitative properties of the model (e.g. a nonlinear cost function).

2.1. The properties of the supply function

The present value function in eq. (7) embodies all relevant properties of the technology. We will now introduce a few properties of the solution of the forest management problem, some of which are relevant for the effect of taxation. We start with the following claim. [4]

Claim 1. Convexity (C)

$$\pi(p, \bar{x}) = \max_{c} \{ pc/p, \ x \in C \} = pc(p, \bar{x})$$

is a convex function in p, i.e.
(i)

$$\pi(p^{\lambda}) \leqq (1 - \lambda)\pi(p') + \lambda\pi(p''),$$

where $p^{\lambda} = (1 - \lambda)p' + \lambda p''$, $0 \leqq \lambda \leqq 1$.

For the proof of this claim the reader is referred to a modern textbook in microeconomic theory, e.g. ref. [10]. The present value function is formulated in such a manner that it shares its properties with the profit function of the profit maximizing firm in neoclassical microeconomic theory.

As we will show below, convexity and differentiability of the present value function with respect to p imply some neat results with respect to the properties of the supply function. We will, however, start off with a few results, which hold irrespectively of whether the function is differentiable or not.

Let us assume that the felling program c^* solves the maximization problem (7), when the net price system is p^*; by definition it then holds that

$$p^* c^* \geqq p^* c \quad \text{all } c \in C, \tag{8}$$

[4] Note, that convexity implies continuity at any interior point. Compare, e.g. Nikaido [7] p. 45.

where C denotes the feasible choice set, i.e. the compact set [5] defined by the restrictions (1)–(5). Eq. (8) simply says that c^* gives a present value at least as large as any other feasible felling program. Now, multiply the net price system by a scalar $\lambda > 0$. Since, the restrictions are not changed by the change of the price system and since eq. (8) holds, it must also hold that

$$\lambda p^* c^* \geqq \lambda p^* c \quad \text{all } c \in C \tag{9}$$

and we have shown.

Claim 2. Homogeneity of degree zero (H). If c^* is "profit" maximizing for the net price system p^*, then c^* is "profit" maximizing for the net price system λp^*, $\lambda > 0$.

It obviously also holds that we can add a constant cost (T_x), independent of the felling program, to the problem without any change in the present value maximizing cutting program c^*, i.e.

$$p^* c^* - T_x \geqq p^* c - T_x \quad \text{all } c \in C \tag{10}$$

and hence

Claim 3. Independence of addition of constant (I). If c^* is present value maximizing for the price system p^*, it will be present value maximizing if a constant is added to the "profit function".

Let c' be the present value maximizing choice when the net price system is p', and let c'' be the corresponding optimal cutting program when the net price system is p''. By using a simple revealed preference argument it is now easy to show

Claim 4. Supply is neoclassical (N) $(p'' - p')(c'' - c') \geqq 0$.

Let us now introduce the following definition

Definition 1. A feasible supply program c is efficient if and only if there is no other feasible supply program c' such that $c'_t \geqq c_t$ all t and $c'_t > c_t$ for at least one t.

The definition says simply that a cutting program is efficient if it is impossible to increase the supply of roundwood in one period without at the same time

[5] A set is compact if it is closed and bounded.

decreasing it in another period. It should, however, be pointed out that the definition presupposes a given (biological) production technology and not necessarily the best technology. It could, however, be argued that it is in the interest of the present value maximizing forest owner to use the best known "production function", as a more efficient technology would increase net revenues from forestry.

It is easy to show that a present value maximizing program according to eq. (7) is efficient provided that $p^* \gg 0$.

If c^* is present value maximizing, but not efficient, then there exists a c^0 such that $c_t^0 \geq c_t^*$ for all t and $c_t^0 > c_t^*$ for at least one t. From the assumption that c^0 is efficient and c^* is inefficient it follows that $p^*c^0 - p^*c^* > 0$. This statement contradicts the assumption that c^* is profit maximizing relative to p^*, which means that $p^*c^* - p^*c^0 \geqq 0$. Thus, if c^* is profit maximizing relative to p^*, it is also efficient.

Claim 5. Efficiency (E). The present value maximizing cutting program c^* is efficient, provided that $p_t > 0$, all t.

For the two period cast ($T = 2$) this could be illustrated by the following diagram. The feasible cutting possibilities are given by the shaded area in fig. 1. The efficient programs are on the boundary AB. All interior points, such as c^0, are inefficient.

Assume now that the present value function is differentiable. [6] We can then prove the following well-known result often referred to as Hotelling's lemma. [7]

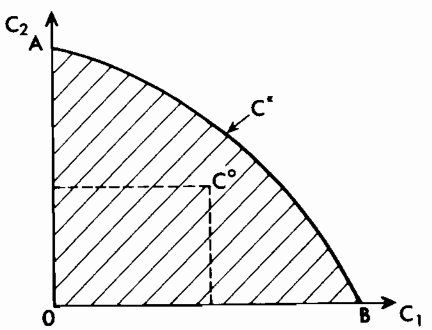

Fig. 1.The feasible cutting possibilities and the efficient points.

[6] The fact that the profit function is convex means that it is continuously differentiable for $p \gg 0$ except possibly for a set of measure zero. Compare Katzner [5] p. 199.

[7] For a proof compare e.g. Varian [10] p. 31.

Claim 6. Hotelling's lemma (HL).

$$\frac{\partial \pi(p, \bar{x})}{\partial p} = \pi_p(p, \bar{x}) = c(p, \bar{x}) \text{ if } c_t > 0 \text{ all } t.$$

The result means that the present value maximizing cutting program is obtained by differentiation of the present value function with respect to p. In other words, the knowledge of the first order derivative of the present value function embodies all the knowledge of the supply of roundwood over the planning horizon. Note also that the differentiability of the present value function with respect to the net price system vector means that the supply functions are single valued.

If we assume that the present value function is twice differentiable with respect to p it follows from the convexity of the function in p and (HL) that the first order derivatives of the supply functions are related in the following way

Claim 7. Positive semidefinite quadratic form (PD)

$$\begin{vmatrix} \frac{\partial c_1}{\partial p_1} & \cdots & \frac{\partial c_1}{\partial p_i} \\ \vdots & & \vdots \\ \frac{\partial c_i}{\partial p_1} & \cdots & \frac{\partial c_i}{\partial p_i} \end{vmatrix} \geqq 0 \quad i = 1, \ldots, T.$$

The proof is omitted.[8] The result means that the supply function is a non-decreasing function of the net price – supply is not backward bending. This has, however, nothing to do with differentiability, as it follows directly from Claim 4. By assuming that the only net prices that differ between p' and p'' are the prices in period i, (N) reduces to

$$(p_i'' - p_i')(c_i'' - c_i') \geqq 0, \tag{11}$$

i.e. an increased net price in a period will never mean a lower supply of roundwood.

The remaining implications of the proposition relate the partial derivatives of the supply function to each other in an empirical meaningful, but fairly complicated manner.

[8] The result is well known from calculus. Convexity implies a positive semidefinite quadratic form and this is in this case equivalent to Claim 7.

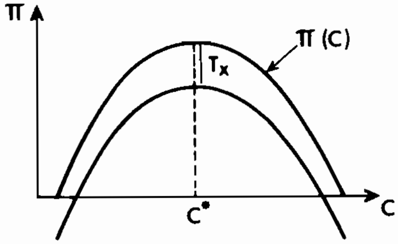

Fig 2.The effects of a lump sum tax.

As the effects of $\bar{x}$ on the present value function have a limited relevance for tax consequences we will not examine this problem here. [9]

2.2. The effects of taxes

It is now time to use the claims established in section 2.1 above in order to derive the theoretical effects of taxation on the supply of roundwood.

A proportional tax on the present value of the harvest in each period is equivalent to a scalar multiplication of the price system p^* by the scalar $\lambda = 1 - t > 0$, where $0 < t < 1$ is the proportional tax rate. Hence, the optimal cutting program c^* is not changed by the imposition of a proportional tax. This follows directly from the (H) homogeneity property of c^* (Claim 2).

A lump sum tax, the magnitude of which is determined by the site quality classification of the forest land, means that the firm has to pay the same amount each period independently of the cutting program. Over the whole planning period a constant sum independent of the cutting has to be paid (T_x), and Claim 3 shows that the optimal cutting program is independent of the size of the lump sum.

The principles which are involved can be illuminated by the following diagrams. Fig. 2 above shows that the lump sum tax does not change the cutting program that gives maximum present value, it just shifts the present value function $\pi(c)$ downwards. The proportional profit tax does not affect the determination of the optimal cutting program, since it does not influence the relative sizes of costs and revenues. This is displayed in fig. 3.

It should also be obvious that a switch from a proportional tax to a lump sum tax does not change anything, and that both tax systems induce efficient (in the sense of definition 1) cutting programs.

There is, however, an important difference between a proportional tax system and a lump sum tax system. Consider a case where the tax rate and the lump sum are allowed to vary between periods. As

[9] One can probably prove that $\pi(p, \bar{x})$ is concave in $\bar{x}$.

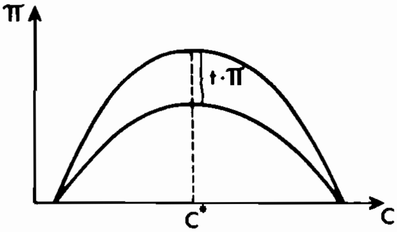

Fig. 3. The effects of a proportional profit tax.

$$T_x = \sum_{i=1}^{T} T_i,$$

it does not matter if the size of the lump sum varies between periods. Another way to express this is to say that the optimal cutting program cannot be affected by differentiating the level of the lump sum between different periods.

If the proportional tax rate is raised in one period, the only thing that differs between the net price vectors before and after the change is the net price in that particular period. From Claim 4 it then follows that the supply of roundwood will remain unchanged or decrease. Compare eq. (11) above. It is, hence, in principle possible to affect the supply of roundwood by differentiating the proportional tax rate between periods.[10]

Although the Swedish corporate tax is a proportional tax, we will discuss what happens if profits are taxed by a progressive tax rate. When the rate of taxation is variable and dependent on net revenues the maximization problem becomes

$$\underset{c}{\text{Max}}\, \pi(p, \bar{x}, c) = \sum_{t=1}^{T} \left[1 - t(p_t^* c_t)\right] p_t^* c_t$$

subject to restrictions (1)–(5) where

$$t_x(p_t^* c_t) = t_x(y_t) \quad \partial t_x / \partial y_t \geqq 0$$

is the progressive tax rate. When we consider the whole planning period $t = 1, \ldots, T$, the tax rate has different values in different periods depending on the total income during each particular period. As the tax rate is variable, it is not a scalar multiplication of the net price vector this time, and the profit maximizing felling program (c') will in general differ from c^*. If, however,

[10] If the profit function is twice differentiable, $\partial c / \partial p_t = \pi_{p_t p_t} > 0$, except possibly at single points.

$p_t^* > 0$ all t and $0 < t(y_t) < 1$ all y_t, it can be proved that c' is an efficient solution relative to the net price system

$$p' = [p_1^*(1 - t_x(y_t'))], \ldots, p_T^*[1 - t_x(y_t')].$$

The solution will in the two period case, be situated on the border of the feasible region OAB in fig. 1.

3. The self-active forest farmer

In this section we will turn to an analysis of the self-active forest farmer, who has the choice between work on his forest farm, work on his agricultural farm, or work in the industry.

To deal with his management problem we will assume that he possesses the utility function

$$U = U(y, \bar{L} - l_1 - l_2 - l_3), \tag{12}$$

which is assumed to be quasi-concave, twice continuously differentiable, and increasing in each argument ($\partial U/\partial y > 0$, $-\partial U/\partial l_i > 0$). Moreover,
$\bar{L}$ = the total available number of hours,
l_1 = the total amount of labor supplied for work in forestry,
l_2 = the total amount of labor supplied for work in industry,
l_3 = the total amount of labor supplied for work in agriculture,
y = the consumption of consumer goods,
$\bar{L} - l_1 - l_2 - l_3$ = leisure time.

The fact that $\bar{L} - l_1 - l_2 - l_3$ is included as an argument in the utility function means that the disutility of one hours work is independent of where the self-active farmer works. In other words, he has no preference for any particular working place. This might seem a bit unrealistic, but is no large violation of reality, and it simplifies the analysis considerably.

The utility function is maximized subject to a budget constraint.

$$\pi_f(l_1) + wl_2 + \pi_a(l_3) - py \geqq 0, \tag{13}$$

where
$\pi_f(l_1)$ = the imputed income [11] from l_1 hours of work in forestry,
w = the wage rate (wl_2 = the wage income),
$\pi_a(l_3)$ = the imputed income from l_3 hours of work in agriculture,
p = the price of consumer goods.
$\pi_f(l_1)$ and $\pi_a(l_3)$ could be apprehended as the yearly interest on the capital stocks of agricultural and forest farm respectively. We will assume that

$$\pi_f(0) \geqq 0,$$

$$\pi_a(0) \geqq 0,$$

and

$$\frac{\partial \pi_f}{\partial l_1} > 0, \quad \frac{\partial \pi_a}{\partial l_3} > 0, \quad \frac{\partial^2 \pi_f}{\partial l_1^2} < 0 \text{ and } \frac{\partial^2 \pi_a}{\partial l_3^2} < 0. \tag{14a}$$

This means that the incomes from forestry and agriculture are non-negative and that they are differentiable, and increasing functions of the input of labor

$$\partial \pi_f / \partial l_1 = \pi_f'(l_1) > 0, \; \partial \pi_a / \partial l_3 = \pi_a'(l_3) > 0. \tag{14b}$$

Moreover, there are diminshing returns to the inputs of labor.

$$\partial^2 \pi_f / \partial l_1^2 = \pi_f''(l_1) < 0, \text{ and } \partial^2 \pi_a / \partial l_3^2 = \pi_a''(l_3) < 0. \tag{14c}$$

A possible shape of the $\pi_f(l_1)$ function is depicted in fig. 4.

The budget restriction simply says that the value of consumption cannot exceed the "income" of the self-active forest farmer.

Let us now introduce taxation. We assume that the income from forestry and agriculture are taxed by the proportional tax rate $1 - \alpha$, $0 < \alpha \leqq 1$, while wage income is taxed by the proportional tax rate $1 - \beta$, $0 < \beta \leqq 1$. (Nothing essential is changed if the tax rates are assumed to be progressive.) Moreover, we introduce a lump-sum tax T_x. The budget constraint can now be written as [12]

$$\alpha\left[\pi_f(l_1) + \pi_a(l_3)\right] + \beta w l_2 - py - T_x = 0. \tag{15}$$

The self-active forest farmer's choice theoretical problem is now to maxi-

[11] From the Faustmann–Ohlin theorem, we know that the land value of a forest stand (the value of the capital) is given by

$$\underset{T}{\text{Max}} = \frac{\bar{p} f(T) e^{-rT}}{1 - e^{-rT}} = \frac{\bar{p} f(T)}{e^{rT} - 1} = K,$$

T = time of harvest,
$\bar{p}$ = the price of roundwood,
r = the interest rate,
K = the value of the capital.

If T^* solves the problem, we have from the fact that income equals interest on the capital, that imputed income from forestry equals

$$\pi_f = rK^* = \frac{r\bar{p} f(T^*)}{e^{rT^*} - 1}.$$

[12] As the utility function is increasing in each argument the budget constraint will hold with equality.

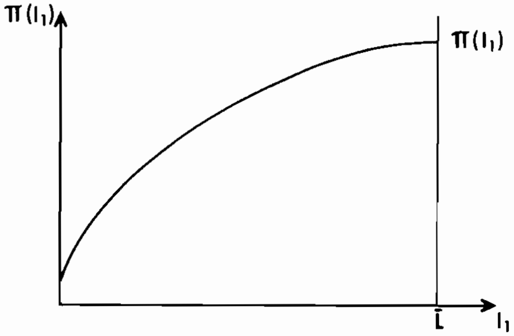

Fig. 4. The capital income from forestry.

mize the utility function (12) subject to the budget constraint (13). The first order conditions for an interior solution can be written.

$$
\begin{aligned}
&\text{(a)} \quad U_y - \lambda p = 0,\\
&\text{(b)} \quad -U_{\bar{L}} + \lambda\alpha\pi_f'(l_1) = 0,\\
&\text{(c)} \quad -U_{\bar{L}} + \lambda\beta w = 0,\\
&\text{(d)} \quad -U_{\bar{L}} + \lambda\alpha\pi_a'(l_3) = 0,\\
&\text{(e)} \quad \alpha[\pi_f(l_1) + \pi_a(l_3)] + \beta w l_2 - py - T_x = 0,
\end{aligned}
\tag{16}
$$

where

$\partial U/\partial y = U_y$ = the marginal utility of consumer goods,

$-\partial U/\partial l = U_{\bar{L}}$ = the marginal utility of leisure,

$\pi_a'(l_3)$ = $\left\{ \begin{array}{l} \text{the increase in income of the last hour spent in} \\ \text{agriculture and forestry respectively,} \end{array} \right.$

λ = a Lagrange multiplier.

These are five equations in five variables (y, l_1, l_2, l_3 and λ), and it is particularly easy to solve for the degree of self-activity in forestry, and agriculture (l_1 and l_3). By combining eqs. (16b) and (16c) one obtains

$$\alpha\pi'(l_1) = \beta w, \tag{17}$$

which can be interpreted as an equality between the net return from the last hours of work in industry and forestry respectively. As eq. (17) only contains one unknown, one can solve to obtain

$$l_1 = \pi_f'^{-1}(\beta w/\alpha). \tag{17a}$$

Two things should be noted. The number of hours worked as a self-active forest farmer is independent of the lump-sum tax, and if income from work in

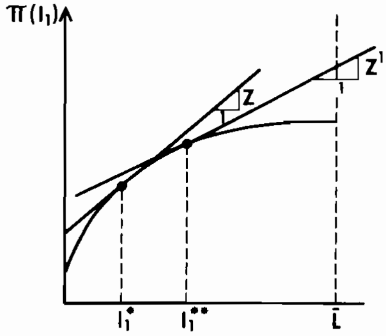

Fig. 5. The optimal self-activity in forestry.

forestry and income from work in industry are taxed in the same manner ($\beta = \alpha$), then the degree of self-activity will depend on the wage rate (w).

If we put $\beta w/\alpha = Z$ we can rewrite eq. (17) as

$$\pi_f'(l_1) = Z \tag{17b}$$

and the solution is found diagrammatically, where the slope of the income function coincides with the number Z. Compare fig. 5 above.

The optimal self-activity in forestry given Z is equal to l_1^*. If we eliminate the tax on income from forestry ($\alpha \to 1$), e.g. by introducing a lump-sum tax on the possession of the forest land, Z decreases to Z' and self-activity in forestry increases.

Claim 8. (i) The degree of self-activity in forestry is independent of the magnitude of the lump sum. (ii) The degree of self-activity in forestry is an increasing function of the proportional tax levied on the income from work in industry, and a decreasing function of the proportional tax levied on the income from forestry. (iii) A switch from a proportional (or progressive) taxation of income from forestry to a lump sum taxation increases the number of hours worked in forestry.

If one assumes that an increased number of hours worked in forestry also increases the supply of roundwood Claim 8 could be rewritten in an obvious way. This assumption does not seem unreasonable as a changed taxation does not favour any particular activity; it does not e.g. make thinning more attractive than final cutting.

3.1. The self-active forest farmer with income constraints

It is very often argued that the self-active farmer, for one reason or another, has an income target. [13] A logical way to model this is to allow income from all sources to add up to the income target. We will now carry out an analysis analogous to the one above. The only difference is that we now introduce a binding income constraint on the sum of the net incomes from forestry, and work in the industry i.e.

$$\alpha\pi_{\mathrm{f}}(l_1) + \beta w l_2 - T_x = \bar{\pi}, \tag{18}$$

where $\bar{\pi}$ is the net income target.

When the self-active forest farmer maximizes the utility function (12) subject to the budget constraint (13) and the income constraint (18), the first order conditions for an interior solution can be written

$$\begin{aligned}
&(\mathrm{a}) && U_y - \lambda p = 0,\\
&(\mathrm{b}) && -U_{\mathrm{L}}' + (\lambda + \mu)\alpha\pi_{\mathrm{f}}' = 0,\\
&(\mathrm{c}) && -U_{\mathrm{L}}' + (\lambda + \mu)\beta w = 0,\\
&(\mathrm{d}) && -U_{\mathrm{L}}' + \lambda\pi_{\mathrm{a}}' = 0,\\
&(\mathrm{e}) && \alpha[\pi_{\mathrm{f}}(l_1) + \pi_{\mathrm{a}}(l_3)] + \beta w l_2 - py - T_x = 0,\\
&(\mathrm{f}) && \alpha\pi_{\mathrm{f}}(l_1) + \beta w l_2 - T_x = \bar{\pi}.
\end{aligned} \tag{19}$$

The tax rates on incomes from forestry is $(1-\alpha)$ and on incomes from industry $(1-\beta)$, and μ is the Lagrange multiplier corresponding to the constraint introduced in eq. (18).

By combining eqs. (19b) and (19c) one obtains

$$\alpha\pi_{\mathrm{f}}'(l_1) = \beta w \tag{20}$$

which implies that

$$l_1 = \pi'^{-1}_{\mathrm{f}}(\beta w/\alpha). \tag{20a}$$

In other words, all the statements of Claim 8 are valid for this "augmented" problem. The intuitive reason is simple. The utility maximizing self-active forest farmer will strive to satisfy the income target with minimum loss of leisure time, and this implies the equality between the marginal net income from forestry and work in the industry. If the marginal net income from forestry increases, he will, under restriction (18), increase his working time in forestry and decrease his working time in the industry in order to re-establish the mentioned equality.

This is quite contrary to the common contention that an income target results in a backward bending supply curve of roundwood. In this context a

[13] He might e.g. want to buy a new Volvo.

backward bending supply curve would be equivalent to the statement that an increased taxation of income from forestry would result in an increased number of hours worked in forestry, and indirectly in an increased supply of roundwood.

To derive a backward bending supply curve in the present context one has to assume that it is impossible to vary the work intensity in the industry, i.e.

$$\alpha\pi_{\mathrm{f}}(l_1) + \beta w\bar{l}_2 - T_x = \bar{\pi}, \tag{21}$$

where $\bar{l}_2 =$ the constant supply of labor directed towards industry.

Eq. (21) can now be solved directly for l_1. We have that

$$\pi_{\mathrm{f}}(l_1) = \alpha^{-1}(\bar{\pi} + T_x - \beta w\bar{l}_2) \tag{22}$$

and

$$l_1 = \pi_{\mathrm{f}}^{-1}\left(\frac{1}{\alpha}(\bar{\pi} + T_x - \beta w\bar{l}_2)\right) = \pi_{\mathrm{f}}^{-1}(\underset{+}{\theta}), \tag{23}$$

where the sign under the argument denotes the sign of the partial derivate.

4. Summary and concluding comments

The optimal cutting programs within the PVFF-model, which solve the maximization problems when the firm faces a lump sum tax, a proportional tax rate constant over periods, or no taxation at all are identical. The optimal cutting program can, however, very easily be induced to deviate from the program, which is optimal when no taxation is presented by e.g. using different proportional tax rates in different periods. Another method would be to use a progressive tax system.

The optimal cutting program can, provided that certain differentiability conditions are fulfilled, be obtained as the first derivative of the present value function. The supply of roundwood is a nondecreasing function of the net discounted price of roundwood.

Self-activity in forestry and the supply of roundwood in the SAFF-model is independent of a lump sum tax. It is a decreasing function of the wage rate in the industry, an increasing function of the tax rate on income from work in the industry, and a decreasing function of the tax rate on income from forestry.

This is also true if the self-active forest farmer has an income target amounting to a sum of income from different income generating activities. On the other hand, if forestry is the sole income source from which income easily can be varied, this results, together with an income target, in a backward bending supply curve of roundwood.

Translated into Swedish conditions the results indicate that the cutting programs of present value maximizing forest firms, like the Swedish Domain Group, will be unaffected by a switch from the present tax system to a lump sum tax system similar to the system which dominates the taxation of forestry in Finland, where the site quality classification determines the magnitude of the lump sum. [14]

If we consider the behaviour of the self-active forest farmer under the above mentioned switch from one tax system to another the number of hours spent in forestry should increase. It would be strange if this did not result in an increased supply of roundwood. As a considerable share of the Swedish forest land is managed by self-active forest farmers this might mean an important contribution to supply in the Swedish roundwood market.

We must, however, remember that the models analyzed in this paper are two of many, and that the differences in cutting policy between the tax systems might turn out to be considerable, if e.g. forests and forest land are analyzed as a part of an investor's portfolio problem.

Let us for a moment leave the world of models, and ask us a little tentatively what will happen to the cutting activities among those who have small forest areas, which they manage "inoptimally", due to the small losses involved, [15] and perhaps also due to a speculation on future price increases. The willingness to avoid the inoptimality losses are not to any considerably extent affected by a proportional tax or a progressive tax on the income from forestry. A lump sum tax would, however, make it more expensive to manage a forest inoptimally, and make it most expensive on the land with the highest site qualifications. A switch to a lump sum tax may therefore in the long run improve on the distribution between active and inactive forest owner for two reasons. Former inactive forest owners may turn active, or choose to sell their land to active forest farmers.

Finally, it should be mentioned that we have in this paper analyzed very limited aspects of the tax system. In a perfect market economy all kind of taxes, except lump sum taxes, can deteriorate the over all efficiency of the market economy.

[14] It should also be emphasized that the Finnish tax system has some complicated provisions about the improvements of the quality or growth potential of the forest land, so that taking it to be a pure lump sum tax is a simplification. These provisions make the Finnish system non-neutral in a subtle manner.

[15] The high marginal tax rates in Sweden can make net revenue from forestry very small, if gross revenue is added to a normal income, even if there are certain provisions available to spread the net revenues from forestry over many periods.

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COST ALLOCATION IN COOPERATIVE WOOD PROCUREMENT: A GAME THEORETIC APPROACH *

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The problem of allocating the joint costs of timber procurement among the cooperating companies is formulated as a cooperative game. Some known methods are compared, and an application is presented to share the costs among a group of major Finnish pulp and paper companies.

1. Introduction

Timber is a central natural resource in Finland: nearly a half of the annual exports is based on wood products. As the demand for timber intensifies, the cost of material also rises rapidly, due partly to higher purchasing prices, and partly to the extra costs incurred by the overlapping purchasing areas of the companies. The overlap tends to increase due to the hardening competition for timber caused by the trend of the woodworking industries to guarantee their supplies by widening their geographical areas. Thus, through cooperative purchasing and by optimal transportation methods it is possible to achieve remarkable savings.

The current trend has been towards the formulation of large coalitions of companies to minimize their total costs using large-scale optimization models. But, in order to obtain acceptance all companies must be assured of a fair cost. The companies must pay compensation to each other because optimal solutions are often unfair. Depending on the factory locations, some companies may face even higher costs than they would if they operated individually. We need methods with which to determine a fair cost allocation.

This paper is based on an actual case in which a coalition of woodworking industries is considered. Cost allocation is formulated as a cooperative game in which the factories are the players. We compare the numerical results of some

* This research was supported by Puulaaki Oy, a major organization supplying timber to several forest industrial companies in Finland. The numerical examples and the case material are derived from this organization. The author would like to thank Matti Ylinen and Arja Winter for inspiring cooperation.

known methods and discuss the practical difficulties in applying these methods.

2. The general problem of procurement costs

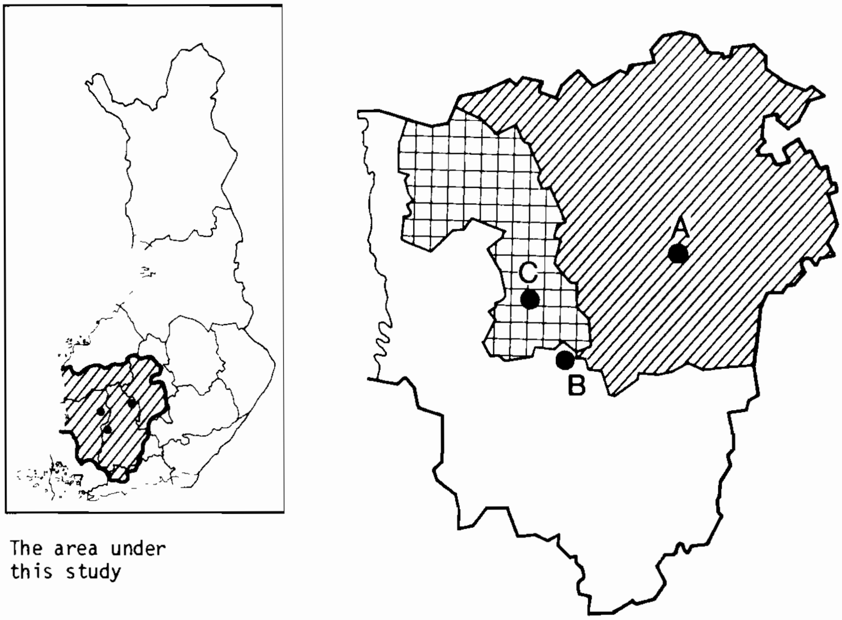
The problem of cost sharing in cooperative wood procurement can be illustrated as follows: there are N companies purchasing timber and trying to minimize their procurement costs. Since the locations of the factories often lead to an overlapping of optimal purchasing areas, there will be competition for timber, especially close to the factories. Uncoordinated, competitive purchasing will result in high cost timber, due to the many cross transports. Through cooperation, however, both fixed organizational costs and the variable transportation costs can be greatly reduced. But the problem is how to get the acceptance of all partners, since some factories in good locations tend to lose when optimizing total costs.

This problem can be solved using a cooperative game theory in which the players may form coalitions and gain by doing so; some coalitions may gain more than others. The question is thus how to divide the benefits among the players. Depending on the criteria that we want the solution to satisfy, there are different cost allocations, which will result in different compensations (side payments).

3. Joint cost functions

Let us consider three factories that compete for timber supplies. In fig. 1 we can see the purchasing area of the companies, the volumes of wood transported, and the costs of each factory. Apparently, there are considerable savings if the proposed optimal division of the area will be acceptable. If factories cooperate they may achieve savings by minimizing costs (by sharing the wood supplies so that cross transports are eliminated). Fig. 1 shows the optimal division of the area and the costs of each factory as a result of a linear programming (LP) model. There will be a total saving of 0.945 million FIM as compared with the competitive solution. But the savings in the optimal solution seem to be unfair: for example factory C reduces its costs by 2.3% while B's savings are only 1.4%.

More generally, let $\{1, 2, \ldots, N\} = N$ represent a group of cooperating factories. The costs of each subgroup S, denoted by $c(S)$, are defined by solving the least-cost procurement program for all factories of S. To determine the least-cost alternative, the sharing of wood must be known at each source in a fixed geographical area. Thus, we must know the amount of wood shared



	TIMBER COST (mill.FIM)		TIMBER VOLUME
	CONFLICT	OPTIMAL	(m^3)
Factory A	30.970	30.483	279 000
Factory B	21.211	20.914	184 000
Factory C	6.887	6.726	63 000
TOTAL	59.068	58.123	526 000

Fig. 1. Profiles of companies: locations, wood procurement regions, and annual timber costs and volumes.

between S and $N-S$. The joint cost function $c(S)$ has property

$$c(S)+c(T) \geq c(S \cup T) \tag{1}$$

for any two separate groups S and $T \subset N$, because there will be no more cross transportation for S and T together if they minimize their total costs by LP.

Typically the optimal costs of a large group are about 1–2% lower than in the alternative case:

$$\sum_N c(i) > c(N). \tag{2}$$

The costs of group S depend not only on the number of members of S, but also on their locations and their total timber demand. The justification of a fair allocation of costs is not only a matter of solving the minimal cost of the main coalition; the contribution of each of the subgroups of factories should also be taken into account. Minimum requirements must at least be satisfied to obtain the acceptance of all partners.

4. Some basic conditions and the core

A fundamental minimum requirement of a fair cost allocation is that no factory should pay more in a cooperative venture than it would pay on its own. This is the *individual rationality* principle of game theory. In our case (fig. 1) the costs allocated to factories A, B, and C must satisfy the conditions

$$\begin{aligned} x_{\mathrm{A}} &\leq 30\,970, \\ x_{\mathrm{B}} &\leq 21\,211, \\ x_{\mathrm{C}} &\leq 6887. \end{aligned} \tag{3}$$

where x_i is the cost allocated to factory i. Similar conditions can be applied to all relevant subgroups of factories. We must consider, for example, all factories belonging to the same company as one subgroup. The condition that no group of factories should pay more than its alternative cost is the *group rationality* principle.

In general, there are n factories, and the individual and group rationality conditions can be expressed in the form of the well known core of the game:

$$\begin{aligned} &\sum_{i \in S} x_i \leq c(S), \quad S \subset N, \\ &\sum_{i \in N} x_i = c(N). \end{aligned} \tag{4}$$

The function $c(S)$ represents the *characteristic function* of the cooperative game (c, N).

In our case (fig. 1) the core of the game is

$$
\begin{aligned}
&x_A + x_B + x_C = 58\,123;\\
&x_A + x_B \le 51\,814,\\
&x_A + x_C \le 37\,633,\\
&x_B + x_C \le 27\,958,\\
&x_A \le 30\,970,\\
&x_B \le 21\,211,\\
&x_C \le 6887.
\end{aligned}
\tag{5}
$$

It is easy to see that in this case the optimal solution is a point of the core – but not a very central one.

This approach, as clear as it is, may involve practical difficulties. The core may be nonexistent; for convex games such as our case, the core always exists, but, it may be very wide. To determine a fair cost allocation we need a unique point of the core. Thus, we look for methods narrowing down the core of the game.

5. Least core

The idea behind the least core (Shapley 1971) is to narrow the core down to a single point by determining the largest decrement ϵ for all subcoalitions so that the core still exists. This can be solved by LP as follows:

max ϵ

$$
\begin{aligned}
&\sum_{i \in S} x_i \le c(S) - \epsilon, \quad S \subset N,\\
&\sum_{i \in N} x_i = c(N).
\end{aligned}
\tag{6}
$$

In our case the least core will be formulated as follows:

max ϵ

$$
\begin{aligned}
&x_A + x_B + x_C = 58\,123,\\
&x_A + x_B \le 51\,814 - \epsilon,\\
&x_A + x_C \le 37\,633 - \epsilon,\\
&x_B + x_C \le 27\,958 - \epsilon,\\
&x_A \le 30\,970 - \epsilon,\\
&x_B \le 21\,211 - \epsilon,\\
&x_C \le 6887 - \epsilon.
\end{aligned}
\tag{7}
$$

The solution to this problem is $\epsilon = 289$, with $x_A = 30\,681$, $x_B = 20\,844$, and $x_C = 6598$. If we accept this as the fair allocation, the side payments (compensation) will be as follows (in million FIM):

$$\begin{aligned} \delta_A &= -0.198 \text{ (pays more)}, \\ \delta_B &= 0.070 \text{ (receives from A)}, \\ \delta_C &= 0.128 \text{ (receives from A)}, \end{aligned} \tag{8}$$

where δ_i is the compensation received by factory i from other factories in order to make the cost allocation a fair one.

The percentage savings – and also the savings in the factory unit cost of timber – are highly nonhomogeneous. It seems that the absolute decrement ϵ does not offer a good way to determine a single point of the core in our case, since the smallest factories will benefit at the expense of others.

6. Proportional least core

The above method of narrowing the core presents a maximum principle: we determine an allocation that maximizes the smallest reduction in the total cost of coalition S. But, instead of uniformly reducing the costs of coalitions we must control the unit cost of timber. Thus, the maximum subsidy should be combined with the total volume or cost of timber of each coalition. For this purpose, we apply the proportional least core (Young et al. 1979) as follows:

$$\max\ p$$

$$\begin{aligned} &\sum_{i \in S} x_i \leq (1-p)c(S), \quad S \subset N, \\ &\sum_{i \in N} x_i = c(N). \end{aligned} \tag{9}$$

This solution maximizes the smallest relative decrease in the coalition costs when compared with the alternative cost. This seems to be an attractive method that is easy to justify, and was originally proposed by Young et al. (1979) who also developed an application. They also presented the idea of applying the proportional relaxation of the constraints to the cost-saving game (not to the cost function itself) in order to avoid difficulties in the case of a nonexistent core. Our case always has a core (Sääksjärvi 1976), and we can avoid the risk of nonrational solutions.

The proportional least core can be found by solving the following program:

$$
\begin{aligned}
&\max\ p\\
&x_A + x_B + x_C = 58\,123,\\
&x_A + x_B \le (1-p)\ 51\,814,\\
&x_A + x_C \le (1-p)\ 37\,633,\\
&x_B + x_C \le (1-p)\ 27\,958,\\
&x_A \le (1-p)\ 30\,970,\\
&x_B \le (1-p)\ 21\,211,\\
&x_C \le (1-p)\ 6887.
\end{aligned}
\tag{10}
$$

The solution to this problem is $p = 0.00985$ with $x_A = 30\,443$, $x_B = 20\,861$, and $x_C = 6819$. This allocation is much smoother than the allocation of the least core (eq. (7)) and leads to the following compensation:

$$
\begin{aligned}
\delta_A &= 0.040,\\
\delta_B &= 0.053,\\
\delta_C &= -0.093.
\end{aligned}
\tag{11}
$$

7. Unique relative savings

If we do not allow any coalitions other than individuals the proportional least core will be

$$
\begin{aligned}
&\max\ p\\
&x_A + x_B + x_C = 58\,123,\\
&x_A \le (1-p)\ 30\,970,\\
&x_B \le (1-p)\ 21\,211,\\
&x_C \le (1-p)\ 6887.
\end{aligned}
\tag{12}
$$

This gives about the same percentage reduction in unit costs to all factories. The solution, which can also be solved without LP, gives $p = 0.0160$ with

$x_A = 30\,474$, $x_B = 20\,872$, and $x_C = 6777$. In this case the compensations needed are

$$\begin{aligned} \delta_A &= -0.009, \\ \delta_B &= -0.042, \\ \delta_C &= 0.051. \end{aligned} \tag{13}$$

8. Comparison of results

The various cost allocations of the previous methods are given in fig. 2, where the resultant unit factory cost of timber is shown for each factory. Note that the methods show quite large differences in unit costs. It is evident that only the proportional cost allocations can be accepted in our case.

It is of great interest to look at the compensation that results from fair cost allocation when compared with optimal costs. Remember that the optimal solution was accepted as the basis for the operation of the main coalition. Remember also that the decision makers of the cooperating companies have strong subjective beliefs in levels of fair compensation. For that reason, we collected a summary of the compensations and compared their signs with the

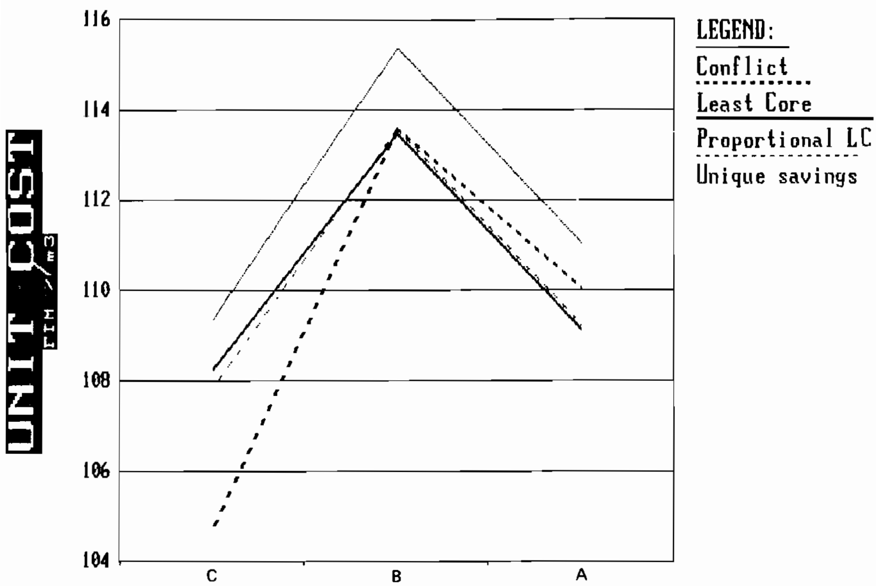


Fig. 2. Comparison of unit costs for timber for companies A, B, and C when alternative cost allocation schemes are applied.

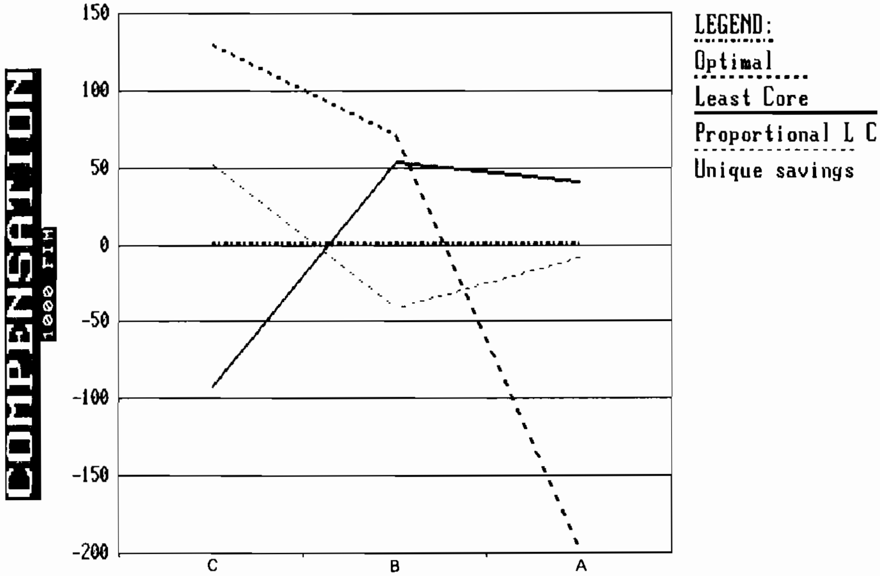


Fig. 3. Comparison of compensation received by companies A, B, and C when alternative cost allocation schemes are applied.

subjective estimates of a selected specialist group. Fig. 3 shows that no two methods give identical sign profiles. But, after looking at the results, the specialists were willing to accept the proportional least core.

The difference between the equal share and the proportional least core can be explained by the varying savings of pairs of factories. As factory C seems to have fewer cross transports with A and B, it will benefit less from cooperation. The proportional least core seems to be quite sensitive to conflict costs.

9. A key question: conflict costs

The above examples show that the proportional methods of narrowing down the core of the game seem to give acceptable cost allocations. It seems also that at least in symmetric cases, where the savings of the subcoalitions of factories are of about the same magnitude, the proportional core gives a cost allocation that is easy to accept. But, since fair division depends very strongly on conflict costs it is important to justify them properly. How can one estimate the sharing of timber if factories do compete? The key question is not only the rule of fairness, but also the premises upon which it will be based on. In order to justify and accept cost allocations the method of defining the alternative costs is urgently needed.

We estimated the conflict costs by simulating wood sharing between the

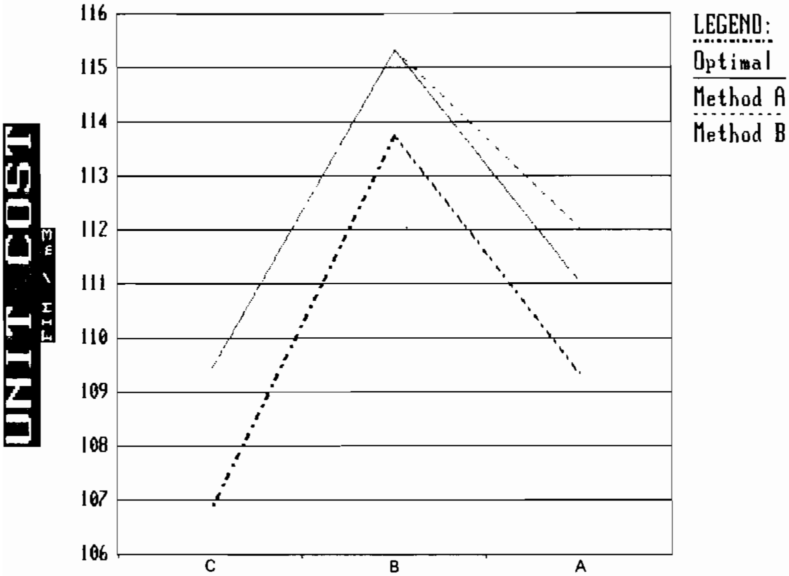


Fig. 4. Unit costs of timber resulting from two simulations (Methods A and B) of the conflict situation. Optimal timber cost is plotted for comparison. *Method A*: Factory j's share of source i is $s_{ij} = p_{ij}(a_i/\Sigma_j p_{ij})$ where a_i is the total amount of timber in i and where $p_{ij} = C_j^2/(C_j^2 + r_{ij}^2)$, with r_{ij} being the distance between factory j and the source i, and C_j is constant. *Method B*: The same as Method A with $p_{ij} = \exp(-C_j^2/r_{ij}^2)$.

factories by several methods, again evaluated by a group of specialists. Two methods of sharing each source of wood between the competing factories are illustrated in fig. 4.

The timber from each source was shared between competing factories in relation to their "purchasing force", calculated as a function of the distance of the source from the factory (see fig. 4). The costs of factories and coalitions of factories were determined by LPs, minimizing transportation costs. If the competitive areas of factories overlapped, the coalition could benefit by cooperation. In our case there was no significant variance between different methods of estimating the conflict costs of factories. Despite small differences in the distribution of wood the overall costs of each factory were not sensitive to these differences.

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Part IV

LONG-TERM FORESTRY PLANNING

TIMS Studies in the Management Sciences 21 (1986) 257–267

A MODEL FOR LAND USE PLANNING

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This paper describes a reformulation of a mixed integer programming land use planning model which maximizes net social welfare and incorporates area, price, cost, and product distribution constraints. An example developed for about 2850 square kilometers in Ontario is described. This example was solved in about 1/200th of the previous time. Potential uses of the model are considered.

0. Introduction

Nautiyal et al. (1975) described a pilot study of a "land use model for planning (LUMP)" developed for the Ontario Ministry of Natural Resources. The model used mixed integer programming to allocate 635 land units, each of about 450 hectares, to agricultural, timber, mining, urban, or recreational uses for an area including and surrounding Sault St. Marie and some of the shoreline of Lake Superior. This article describes a restructured version of the problem which eliminated many of the integer variables and substantially reduced its size. The consequence of these revisions was that a solution can now be obtained in fewer minutes than the number of hours previously required. This reduced solution time makes such a model a feasible planning tool for almost any organization. Alternatively, those with the resources to use the previous model will now have the ability to work with much larger problems or a base for developing even more sophisticated models by incorporating additional parameters and constraints.

1. Model formulation

The planning model maximizes the net benefit for the five land uses considered. Each use is assumed to yield a variety of products in constant proportion to the area devoted to that use. To understand the formulation, consider one use and assume it produces a single uniform product. Fig. 1 illustrates the economics of the problem. In the figure the socially optimal quantity, q_1, occurs at the level of output indicated by the intersection of the demand, D, and marginal cost, MC, curves. At this level, the total revenue

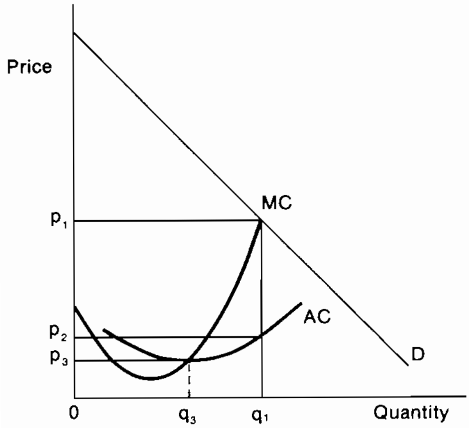

Fig. 1. Demand and cost curves which determine net benefit.

would be the rectangle p_1q_1. The lower third of this rectangle, p_2q_1, defined by the intersection of the optimal quantity and the average cost curve, is the total cost of producing q_1 units. The upper two-thirds represents the net revenue. The area above p_1 and below the demand curve is the consumers' surplus. The net benefit is the net revenue plus consumers' surplus, or the area above p_2, below D, and between 0 and q_1.

The use of consumers' surplus as a benefit measure has been criticized. But as Willig (1976) pointed out, both the compensating income and equivalent income variations (which are the correct theoretical measures of the welfare impact of changes in price and income for an individual) can be well approximated by consumers' surplus. We further assume that the extension of his approach aggregating individual consumer's surpluses into consumers' surplus produces a reliable approximation to society's welfare.

In anticipation of our formulation below, it is useful to consider the geometry of fig. 1 somewhat further before we proceed. We note first that the total cost of producing q_1 units may be represented not only by the rectangle p_2q_1, but also by the area under the marginal cost curve – the derivative of the total cost curve – to q_1 units. Finally, observe that for any quantity q greater than that to the right of the intersection of the marginal cost and average cost the cost may be represented as the area p_3q_3 plus the area under the marginal cost curve beyond q_3.

In summary, the objective function of the planning model is designed to maximize net social welfare in the economic sense: i.e., the direct annual benefit (revenue plus consumers' surplus), from using each particular land unit in its assigned way, less costs of producing the indicated outputs and transporting them to market.

Two of the constraints in the model are very easily incorporated. One allows each land unit to be allocated once; i.e., it can be half used for timber and half for agriculture, but not fully for agriculture and fully for timber. The other ensures that a fixed proportion of each type of output can flow to the market centers typical for that product. (The pilot study assumed a total of five market centers.) The revenue and cost constraints of the model are separately discussed because they are more complex and because their reformulation led to the increased efficiency of the new model.

2. Revenue

The unit revenue for each product should be represented as a decreasing function of quantity. This is achieved by piecewise constant approximation of the usual demand curve of economics into a step function and constraining the quantity that can be sold at each price. Revenue constraints of the following from might be used:

$$\sum_{in} q_{ij} X_{ijmn} \leq R_{jm}. \tag{1}$$

In this inequality X represents the unknown area of land which should be devoted to a particular use (denoted by j), and q is the output per hectare land unit i when it is devoted to use j. The summation over all land units and destinations (represented by n) to which products are shipped, yields the total output for a given land use j and price condition m. The constraint is thus, of course, actually a set: one for each use j and condition m. For a particular use $j = J$, say timber production, with demand schedule D shown in fig. 2 (and its approximated) form D'), the first constraint with $m = 1$ says that at most R_{J1} units of timber may be produced at the high sale value implied by pricing assumption 1. The second constraint says that at most R_{J2} units of timber may

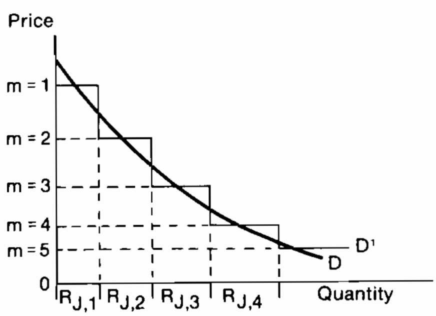

Fig. 2. Demand schedule for use J.

be produced at the next highest sale value indicated by pricing assumption 2. Since the problem objective is to maximize net benefit, the model elects high sale values to the maximum extent possible. Thus, these constraints ensure that some set of X_{iJ1n} variables will have positive values sufficient to produce a timber output of R_{J1} before any X_{iJmn}, $m > 1$, assumes a positive value. It is the priority for high prices of the objective function and the monotonic nature of the demand curve that makes this formulation valid.

An alternative formulation, using X_{ijn}, reduces the dimensionality of the grid assignment activities by shifting the price condition to the aggregate output variables. We replace eq. (1) by eqs. (2) and (3)

$$\sum_{in} q_{ij} X_{ijn} - \sum_{m} Y_{jm} = 0, \tag{2}$$

$$Y_{jm} \leq R_{jm}. \tag{3}$$

Here Y_{jm} is the output of land use j that can be sold at each price level m. Set (3) restricts this amount to some upper bound. The set (2) ensures that the total output of use j that is to be sold must have been produced and shipped to the market centers.

If I, J, M and N represent the maximum limits on the corresponding subscripts, this alternative formulation adds one constraint for each land use but significantly reduces the land allocation activities from $JIMN$ to $J(IN + M)$.

3. Costs

Unlike revenues, our cost functions are not monotonic throughout the range of interest. The treatment of costs is thus more complex. Several approaches are possible. One is a series of integer variables to select appropriate cost levels. Another is to utilize an iterative solution procedure where costs are adjusted from iteration to iteration. We have combined these strategies to provide a heuristic procedure that appears very satisfactory for this sort of problem.

Before describing this procedure, the "cost zones" should be explained. For each land use one would expect the cost curve appropriate to a grid unit to depend on locational factors such as soil type, microclimate, availability of labor, etc. On the cost side, the land uses are thus sub-divided into zones. In the Ontario model, the five land uses led to 17 cost zones – four for agriculture, two for timber, etc. Each grid unit capable of producing timber had associated with it an indicator of the zone which would dictate the cost level for using the area for timber production. The actual cost assigned in the

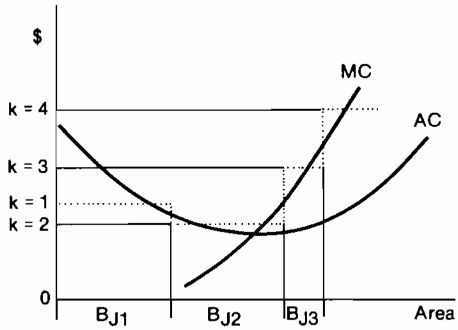

Fig. 3. Development of the linearized cost curves for the planning problem.

zone was a function of the total area devoted to that use within the zone. The total cost for timber production would be obtained by summing the costs for the two timber zones.

For each zone a piecewise constant cost curve like that in fig. 3 is obtained. The curve is based on the marginal cost to the right of the minimum average cost and on a temporary linearization of the average cost curve to the left. If the area allocated in this zone, J, is greater than B_{J1}, we use the monotonically increasing portion of the cost function to determine the total cost in exactly the way of marginal revenue curve was used above.

Next, an integer variable is introduced to force use of the cost level $k = 1$ if the area allocated is less than B_{J1}. This formulation results in one integer variable for each cost zone. More could be introduced to better represent the average cost region. However, given sufficient land, one would expect the optimal level of use to be either to the right of the minimum average cost, or zero. Further, the earlier model makes clear the solution cost effects of any extra integer variables. Hence, an iterative procedure was adopted using the single integer for each zone.

First, the continuous problem is solved and examined for cases where the area allocated in a zone is so much less than B_{J1} that the cost at level $k = 1$ is inappropriate. Using the solution area, a new cost for level $k = 1$ is calculated and B_{J1} reduced. After these revisions, the optimal basis from the previous solutions is inserted and a new continuous solution obtained. (Some of the integer variables may be tentatively set during this process.) In our problem, one would expect this procedure to converge rapidly because, *ceteris paribus*, higher costs lead to the allocation of less land, and most of the rest of the problem remains constant. Once the costs for the $k = 1$ levels have been set for all zones, the integer solution is sought. By specifying the integer values associated with the total area allocated in the continuous solution, one should find a very good initial integer solution and thus eliminate many branches of the MIP problem. In many cases the integer portion of the problem will simply become a solution confirmation process.

Certainly this iterative process is less satisfactory than the use of several integer variables for the declining average cost region from the theoretical perspective. While we believe it was useful in the Ontario case, we recognize that the simplicity of the single iteration approach with more integers may be preferable for others. The remainder of our approach is independent of the number of integer variables used in each zone.

4. The model

The model may be mathematically expressed as

$$\text{Max} - \sum_{ijsn} t_{ijn} X_{ijsn} + \sum_{jm} p_{jm} Y_{jm} - \sum_{jsk} X_{jsk} U_{jsk}$$

subject to:

$$\text{(i)} \quad \sum_{jsn} X_{ijn} \leq A_i,$$

$$\text{(ii)} \quad \sum_{i} q_{ij} X_{ijsn} - d_{jn} \sum_{m} Y_{jm} = 0,$$

$$\text{(iii)} \quad \sum_{in} q_{ij} X_{ijsn} - \sum_{m} Y_{jm} = 0,$$

$$\text{(iv)} \quad Y_{jm} \leq R_{jm},$$

$$\text{(v)} \quad \sum_{in} X_{ijsn} - \sum_{k} U_{jsk} = 0,$$

$$\text{(vi)} \quad U_{jsk>1} \leq B_{jsk},$$

$$\text{(vii)} \quad - \sum_{k} U_{jsk} + B_{js1} Z_{js} \leq 0,$$

$$\text{(viii)} \quad \sum_{k} U_{jsk} - M Z_{js} \leq B_{js1},$$

$$\text{(ix)} \quad U_{js1} + B_{js1} Z_{js} \leq B_{js1},$$

$$\text{(x)} \quad \sum_{k>1} U_{jsk} - M Z_{js} \leq 0,$$

where the decision variables U, X and Y must be greater than or equal to zero, and Z equal to zero or one. Each variable is defined as follows:

A_i is the land area of grid unit i.

B_{jsk} is the maximum area in cost zone s that can be devoted to use j at cost level k (before another cost at level $k+1$ must be incurred).

c_{jsk} is the cost/hectare at level k in cost zone s for use j. (For $k = 1$, these values are adjusted for each iteration as described above.)

d_{jn} is the proportion of output j which must be delivered to destination n.

M is a large number such that its use as an upper bound does not restrict the problem.

p_{jm} is the price/unit of output at level m for use j.

q_{ij} is the physical output/hectare resulting if grid i is devoted to land use j.

R_{jm} is the maximum number of units of output j which may be sold at price level m (before a lower price level $m+1$ must be used).

t_{ijn} is the transportation cost/hectare for shipping the output for use j from grid unit i to destination n plus the cost/hectare of converting grid unit i to that use if necessary.

U_{jsk} is the total area utilized for use j in cost zone s for which the cost/hectare is established at level k on the linearized cost curve.

X_{ijsn} is the area of grid unit i devoted to use j in cost zone s, having its output shipped to destination n.

Y_{jm} is the total output from use j which is sold at price level m on the linearized demand curve.

Z_{js} is a zero-one integer variable having the value one if the total area in a zone s for use j is sufficient for the minimum average cost to apply. Otherwise (i.e., when the area is less than B_{js1}), it has the value zero.

The terms of the objective function represent the gross social benefits from producing certain quantities of each product less the cost of producing the product and transporting it to its market destination. The constraints ensure that:

(i) each hectare is used only once;
(ii) appropriate proportions of the output of each land use are delivered to appropriate market centers;
(iii) the output to be sold has been produced;
(iv) only a limited output can be sold at higher prices before the demand curve forces a lower price to be realized;
(v) costs are paid for each acre managed;
(vi) higher costs are incurred when larger areas used imply greater marginal costs; i.e., that $k > 1$. When the area used implies that $k = 1$, constraint (ix) makes higher costs apply.
(vii) if the area in a zone devoted to a use makes the first cost level appropriate, the integer variable for the use and zone will be forced to zero;
(viii) if the area in a zone devoted to a use is too large for the first cost level to be appropriate, the integer variable is forced to one,
(ix) if the integer variable for a use and zone is one, the first cost level cannot be used:
(x) if the integer variable for a use and zone is zero, only the first cost level can be used.

Perhaps we should point out that the t_{ijn} coefficients may be used to incorporate more than production costs – (see Found 1971). In the present study these numbers were generated from the distance from each grid unit to all market destinations, the shipping cost per kilometre per unit of output for each product, and the outputs per hectare for each use. The product of these represents the transportation cost per hectare for a particular product. The raw data also included the area currently in use j and the cost per hectare of converting from any use to another. Thus a land conversion cost was added to the transportation cost. Where grid units did not originally support a single land use, this approach somewhat distorts actual conversion costs. Appropriately defining grid units to ensure homogeneity of existing of use would eliminate this difficulty. Homogeneity of product output within a grid unit is also, of course, an implicit assumption of linear programming. Consequently, care should be taken when defining grid units.

5. Results

The initial LUMP model used 71 integer variables to restrict the ranges of land inputs and product outputs to which particular prices applied. It had about 72 000 variables and 819 constraints. Using UMPIRE and a UNIVAC 1108, about 30 min of CPU time was required to generate and input the problem. The continuous solution was obtained 17 min later. The optimal integer solution then required an additional 10.5 hours.

Two types of changes have been made to the earlier model. First, the dimensionality of the grid assignment activities was reduced by shifting the price and cost conditions so that they apply to aggregate levels of output and input. This change was described above in relation to the revenue constraints. Extending the same approach to the cost constraints resulted in a total reduction of about 66 000 variables.

The second change was to eliminate most of the integer variables. The representation of the problem in marginal rather than average terms accounts for much of this reduction. That is, instead of ranges of input or output each having an appropriate price and made effective by an integer variable, we have a certain benefit (or cost) for the first range of units, a second added benefit increment for the next range of units, etc. Authors of the original model recognized that their revenue constraints could have been structured in marginal terms, but probably underestimated the significance of multiple integer variables. Because of the investment in their input generating programs they decided to continue with their original formulation. Adopting an iterative solution strategy made it possible to represent the portion of the cost curve where average costs are falling with even fewer cost zones and corresponding integer variables.

The reformulated LUMP model was run using the MPSX MIP code on an IBM 3033 using CMS. The MPSX input was generated from the original data using a FORTRAN program. This program generated the t_{ijn}'s, defined the appropriate X's, positioned the coefficients, etc., and also generated an initial basis for the LP by assigning each grid unit to the use currently predominating on it. The input generating program required 0.24 min of CPU time. The revised model had 764 rows and 5122 variables (including 17 integers) with a

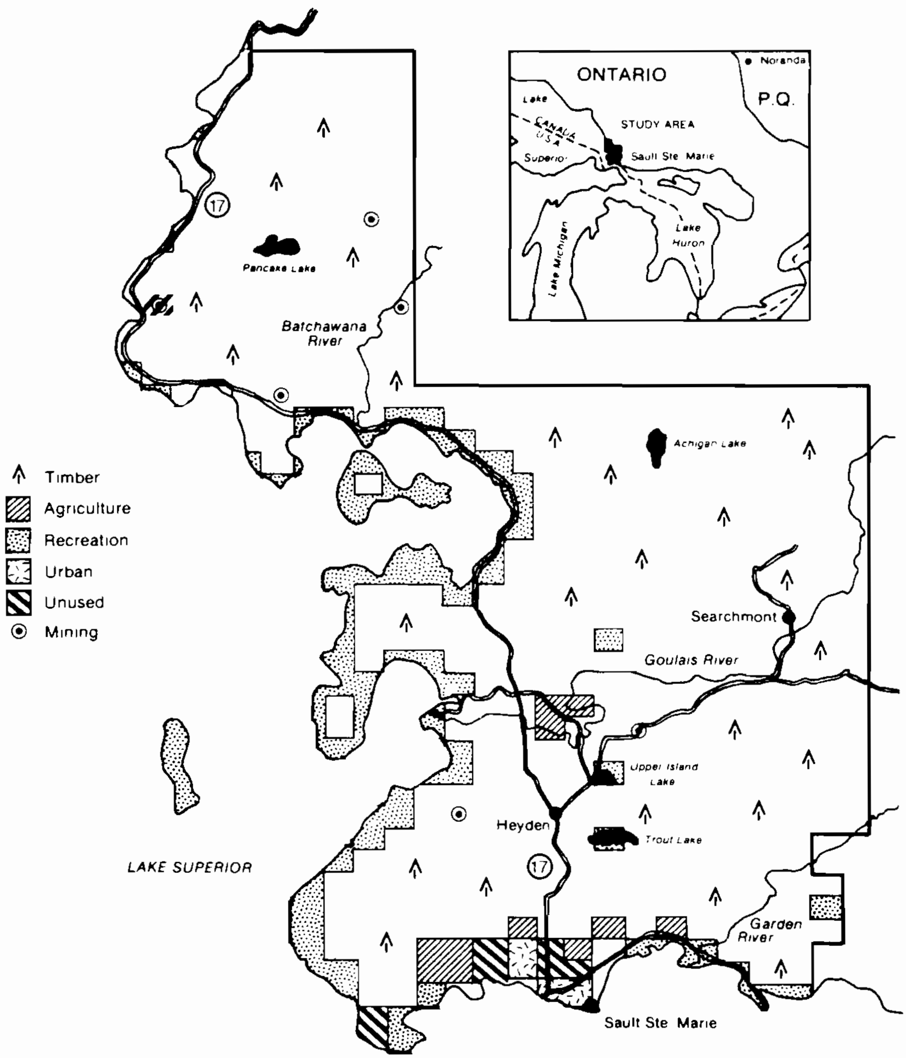

Fig. 4. Map of the optimal solution.

matrix density of 0.52. Approximately 0.14 min was required to load the input and prepare for the optimization. Eleven hundred iterations and 0.73 min were required to obtain the initial solution. For three of the seventeen cost zones the area allocated was positive yet significantly below the quantity corresponding to the $k = 1$ input costs. These parameters were revised three times in 0.18 min. The zero/one levels for the integer variables were then specified and the first corresponding integer solution was obtained in 39 iterations (0.15 min). A second better integer solution was then obtained in 20 more iterations (0.11 min). This change again made the cost level for the area allocated in one of the earlier revised zones inconsistent with the average cost curve. The cost and maximal areas for the $k = 1$ level of this zone were again revised. Forty-three iterations (0.16 min) were required to obtain a new first integer solution. After 0.16 min and another 38 iterations it was confirmed that no better solution existed.

An attempt was made to solve the problem before some data input errors were discovered and before it was decided that current land use should constitute a reasonable initial basis. This first solution required 1676 iterations. We thus conclude that in such problems current land use practice may provide a substantially better initial basis than one mechanistically obtained by the LP algorithm.

The land use pattern indicated by the reformulated LUMP is shown in fig. 4. Results for the earlier model are quite similar but not identical to those shown in the figure.

6. Future use of the model

Certainly no computerized model of planning will replace planners. There are simply too many relevant factors to include all of them in any model. Models should abstract some features and thus vastly reduce the complexity of the real problem. LUMP allocates land to various uses by considering the costs of conversion to that use, transportation of its products to a market, and economies of scale in production through better servicing, etc., as more or less area is assigned in any particular category. It further recognizes that the demand for any product is a function of price. The interaction of these economic factors may be extremely complex. Freed from these details, a planner may view a variety of other considerations which may suggest or require alternatives to portions of this planning base. At the very least, LUMP provides a means of estimating the opportunity costs of these alternatives.

The dual variables output by linear programming may be one of the most useful features to this approach to land use planning. These variables represent the marginal value of each particular grid unit and cost of deviating from the solution. Thus the analyst who wishes to estimate the opportunity cost of perhaps withdrawing some area from a use has a basis for doing so. Alternatively, if one wished to increase the output of some product he would scan the dual variables to determine where the land use might be changed so as to minimally affect economic objectives.

The revised LUMP is efficient enough that it may be re-solved to incorporate updated data or to indicate the sensitivity to changing economic assumptions about product demands or costs. As with other quantified models, LUMP will make more explicit many of the assumptions upon which a plan is ultimately based. By providing an economically efficient base, and focusing attention on the economic assumptions and the effects of other criteria as the solution plan is varied, LUMP should lead to a more efficient use of planners' time and consequently to better land use decisions.

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A MULTIOBJECTIVE OPTIMIZATION APPROACH FOR SCHEDULING TIMBER HARVESTS ON NONINDUSTRIAL PRIVATE FOREST LANDS

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The United States is expected to experience a shortage of timber early in the 21st century. In response to this problem, the Tennessee Valley Authority (TVA) developed a forest management program specifically for small landowners, who comprise what is known as the NIPF (nonindustrial private forest) sector. These landowners were chosen as target users because they collectively own the majority of forest lands in the United States but have made only limited use of modern forestry practices. TVA's forest management program, called WRAP, is a multiobjective optimization program. It considers eight landowner objectives: (i) the net present value of timber income, (ii) a measure of recreation, and (iii–viii) the populations of six wildlife species (deer, grouse, squirrel, quail, turkey and rabbit). These are handled with a nonlinear utility function that is tailored individually to each landowner. The program has been used by more than 1300 landowners in the southeastern United States to date (April, 1983) with encouraging indications of success.

1. Introduction: the need to increase timber production in nonindustrial private forests

According to estimates of the United States Forest Service [21], the U.S. will experience timber shortages and rapid increases in the real price of timber early in the 21st century. It takes at least 20 years and often much longer to grow a crop of merchantable trees, so this predicted shortage can be viewed as a current problem. [1] Unfortunately, the United States cannot rely solely on normal competitive economic forces to supply the necessary stimulation to wood production because timber producers do not constitute a profit-maximizing market system. The majority of United States forest lands are owned in

[1] Forest economists do not unanimously agree that projected timber shortages are sufficient justification for attempting to increase timber productivity [15,38]. However, they do agree that increased productivity is a basic, defendable goal of forestry.

small lots by individuals who do not regard profits from timber harvests as their only objective in owning land. As surveys have shown [25,29,41], these small landowners tend to have a variety of objectives, including recreational benefits, aesthetic appreciation, and long-term insurance, in addition to timber sales.

These small landowners are known in the forestry literature as nonindustrial private forest (NIPF) landowners. In the southern United States these individuals own 72% of all commercial timberland (as compared with 19% owned by corporations and 9% held by governments) [43]. Their landholdings are mostly under 100 acres each [41], and they typically have professions unrelated to forestry. As a group, they have made only limited application of modern forestry practices that could substantially increase their timber yields.

Recognizing that intensification of forest management in the NIPF sector is in the national interest, the Tennessee Valley Authority (TVA) developed a program specifically for NIPF landowners. The program, called WRAP (Woodland Resources Analysis Program), has been used by more than 1300 landowners in the southeastern United States to date (April 1983). There have been earlier efforts to induce NIPF landowners to practice more intensive forest management, but these generally met with limited success [51]. An important reason why some of the past programs failed to stimulate increased forest management was that they concentrated solely on maximizing financial return.

WRAP was designed from the point of view that a forest management tool for the NIPF sector must accommodate the multiple objectives of the landowner and it should do so on an individual basis. WRAP's methodology is fairly unsophisticated, but it is a multiobjective optimization program that has the desired flexibility and meets other design criteria necessitated by the special nature of the NIPF user. As reported in the evaluation (section 7), WRAP has had success in improving NIPF acceptance of modern forest management and in potentially stimulating production.

Numerous forestry applications of optimization, including linear [18,30,40,47,48], dynamic [9,14], and nonlinear [28,46] programming, have been reported for industrial [46,48] and governmental [18,30,36,40,49] timberlands. But the literature is apparently void of approaches designed around the unique requirements of NIPF landowners. This is surprising considering the controlling interest this group holds over commercial forests in the United States.

2. Design considerations

WRAP is administered to a NIPF user in the following manner. Upon request of the service, a consulting, industrial, State, or Federal forester visits the NIPF site. The forester partitions the property into timber stands, performs a timber inventory [23,24,42], makes technical recommendations, and conducts an interview with the landowner concerning his or her landowning objectives. The forester's data are mailed to the TVA Division of Land and Forest Resources, where they are coded and input to WRAP. TVA personnel check the results, call for clarifications or resubmittal if necessary, and return the output to the forester, typically in 2 weeks' time. The forester then makes a return visit to the landowner to carefully explain WRAP's output, which contains substantial amounts of textual information about forest management in addition to the recommended harvest schedule.

Several aspects of this procedure affect the design of the multiobjective optimization program. The most important aspects are: first, the landowner's contribution to WRAP's input is limited to a single interview, second, this interview is conducted by a forester, not a management scientist and third, no computing equipment is present at the site of the interaction with the landowner. None of these factors could be changed while WRAP was under development. The number of forester/landowner contacts is limited by the sheer number of NIPF users. The forestry expertise of the interviewer is absolutely essential for providing the technical inputs to the program. Furthermore, an important side benefit of WRAP is provision of a vehicle to introduce the NIPF landowners to current research results and forest management practices.

The disadvantage of not having a management scientist conduct the interview is that the landowner's preferences for the multiple objectives may not be assessed with as much sophistication as the system designers might like. Only a very small portion of the forester's time on the NIPF site was allowed for assessment of the objectives. Computer terminals could not be present at the interview for reasons of cost as well as for fear they might inhibit landowner acceptance. [2]

The restriction to a single, brief, computer-free session for collecting information on decisionmaker preferences ruled out the use of interactive optimization approaches such as those reported in [5,10,18,19,26,27,36,52]. These approaches have demonstrated their effectiveness on a range of interesting problems and will be considered carefully for potential applicability in the present context when TVA develops an interactive multiobjective optimization procedure. [3]

In summary, the design criteria of WRAP are that: (i) it is a multiobjective optimization program with sufficient flexibility for tailoring to the individual landowner; (ii) it is very easy to use and understand in a setting where neither the decisionmaker nor the administrator of the program has a quantitative methods background; (iii) it is computationally feasible for use with a large

[2] Added in proof: A successor program to WRAP, named FORMAN, does not have this limitation.

[3] Added in proof: The interactive approach taken is described in refs. [53] and [54].

number of individual landowners; and (iv) the recommended solutions adequately reflect the capabilities of the land and the desires of the landowner in the views of all parties involved, i.e., the foresters, the management scientists, and, most importantly, the NIPF users themselves.

3. Modeling and methodology of WRAP

The harvest schedules recommended by WRAP all have the following form. The forester subdividies the NIPF property into tracts, drawing the boundaries so as to obtain approximate homogeneity in tree speciation, geography, and land use within each tract. WRAP handles the tracts independently. For each one, the forester decides how many harvests to schedule on both the existing (first) stand and the future (second) stand. The program determines the timings of these harvests. It is assumed that the same pattern of harvests selected for the future stand will be repeated indefinitely on a third, fourth, fifth, etc., stand. Consequently, the only decision variables in the WRAP optimization model are the timings of harvests on the first and second stands. The number of harvests per stand is never more than four, so the number of decision variables for each tract is at most eight.

The constraints on timings are relatively simple: (i) the harvests must not occur more frequently than the forester deems advisable and (ii) the first harvest must fall within a time range dictated by technical concerns of the forester or personal concerns of the landowner.

Because of the very small number of decision variables and the small number of values they can take on, it is evident that choosing the optimization methodology was not one of the critical issues in designing WRAP. The methodology is, in fact, simply a cyclic coordinate search in which one decision variable is modified, with all the other variables fixed, until no further improvement can be achieved; then another variable is modified in this way, then another, etc. [32]. The critical issue in designing WRAP was to define a function for measuring the overall quality of a harvest schedule, i.e., the function to which this optimization methodology should be applied.

WRAP considers eight landowner objectives: (i) the net present value of timber income, (ii) a measure of recreation (nonhunting), and (iii–viii) the populations of six wildlife species (deer, grouse, squirrel, quail, turkey and rabbit). This set was chosen as representative of the most important timber and nontimber benefits of the NIPF landowner. The eight objective functions are denoted $f_i(x)$, $i = 1, \dots, 8$, where x is the vector of harvest decision variables. (We also employ the vector function notation $F(x)$ where $F(x) = [f_1(x), \dots, f_8(x)]$.) Given a proposed value of x, the $f_i(x)$ are evaluated with a growth and yield simulation program in the case of timber, and with tabular functions in the cases of recreation and wildlife. (These evaluation methods were synthesized from the forestry literature and are described in greater detail in section 6.) What WRAP seeks to maximize in the cyclic coordinate search is $U(F(x))$, where U: $R^8 \to R$ is a utility function discussed in the next two sections. The motivation for using a utility approach is given in section 4; the specific form of utility function employed in WRAP and the method of its assessment are presented in section 5.

4. Theoretical considerations in the use of a utility function

The forest management problem addressed by WRAP belongs to a class of important but ill-defined problems known as multiobjective optimization problems. There is no precise mathematical statement for this type of problem, although a notation like "maximize $F(x)$ over all x belonging to the feasible set X" has been used to describe it in the past. Unfortunately, the phrase "maximize $F(x)$" lacks meaning, because the set $\{F(x): x \in X\}$ lacks a natural ordering whenever F is vector-valued. This means that, given two feasible alternatives y and z, there may be no definite answer as to whether $F(y)$ is "greater than," "less than," or "equal to" $F(z)$. The inability to specify which of two alternatives is greater implies the inability to specify which of several is greatest.

A meaningful definition of multiobjective optimization requires the use of some subjectivity, regardless of how rigorously $F(x)$ and the feasible region X may be defined. A reasonable statement of the problem is: find a feasible x so that the most preferred vector of objective function values $F(x)$ is attained. Here the subjectivity is captured in the term "preferred," which has no universally applicable definition. The reason for this brief philosophical digression is that some of the multiobjective optimization literature treats the problem as if it were amenable to cut-and-dried solutions, which it is not. There is in fact no absolute meaning of "best decision" in the multiobjective context.

A theoretically ideal approach to multiobjective optimization is to construct a scalar-valued utility function U with the property that $U(F(y)) > U(F(z))$ if and only if the vector of objective attainments $F(y)$ is preferred to $F(z)$. Given this construction, the multiobjective problem reduces to a well-defined, single-objective optimization problem. Unfortunately, the ideal is not generally attainable in practice because of the difficulty of constructing a U with the desired property. It may be seductive, therefore, to dismiss utility theory as unworthy of practical consideration, but it turns out that many of the techniques in current practice are founded on unwittingly specified utility functions. For example, the common procedure of assigning "weights" w_i and maximizing $\Sigma_i w_i f_i(x)$ is nothing more than constructing a linear utility function. (Another example is goal programming, whose underlying utility function is a very rigidly defined piece-wise linear function [16,35].)

There are some promising multiobjective techniques such as those mentioned in section 2 and others that do not require the specification of a utility function, but in all cases these methods require a much larger information burden on the decisionmaker than is feasible within WRAP.

In summary, WRAP is designed as a system for maximizing an explicitly defined utility function for the following reason. The use of a utility function of some form is unavoidably necessary in treating the subjective aspects of the multiobjective decision problem. We should, therefore, choose our utility function consciously and attempt to have it reflect the landowner's preferences very closely, even if it cannot reflect them perfectly.

5. Form and assessment of the utility function

In an earlier version of WRAP [22], the utility function was implicitly assumed linear. A set of weights w_i, $i = 1, \ldots, 8$, was determined by the Churchman–Ackoff procedure [11], and the function $\Sigma_i w_i f_i(x)$ was maximized. The current version of WRAP departs from this design for two reasons. First, the foresters who administered the Churchman–Ackoff procedure to the landowners complained about excessive time and paperwork requirements. (The procedure is a highly structured sequence of questions involving comparisons between various combinations of benefits. The interviewer cannot preset the questions; they must be formulated in a precise manner depending on previous answers.)

The second reason for discarding the original WRAP objective function design is more fundamental than the first. It is our disagreement with linear utility as a reasonable model of a person's preferences. Linear utility implies that the marginal value of any benefit is constant, i.e., the utility gained from a change of $f_i = a$ to $f_i = a + 1$ is independent of a. This conflicts with many generations of economic thought (including Bernoulli's 1730 St. Petersburg Paradox) which established that these marginal values should not be constant [2]. These values ordinarily vary as perceived "needs" vary. For instance. one more unit of f_i may seem more valuable to a landowner when f_i is scarce than when it is plentiful. An assumption of linear utility (whether implicit or explicit) denies the possibility of this normal human perception.

A further implication of linear utility is that the rate at which a person would be willing to trade off attainment of one benefit for another (i.e., the marginal rate of substitution, $[\partial U/\partial f_i(x)]/[\partial U/\partial f_j(x)]$) never varies. Again this seems unreasonable, because the things a person has in short supply are probably more difficult to trade off than the things that are in abundance.

For these reasons WRAP was redesigned with a nonlinear utility function. The form of the utility function assumed is $U(F(x)) = \Sigma_i u_i(f_i(x))$, where the eight functions $u_i(f_i)$ are derived by means of a 2-part questionnaire, reproduced in the Appendix. The first part of the questionnaire asks for comparisons *among* benefits, resulting in a set of weights, like the Churchman–Ackoff procedure, but with much less work. These weights are obtained by simply having the interviewee distribute 100 points among the eight benefits. [4]

The second part of the questionnaire allows for the nonlinearity. It asks for comparisons *within* benefits, i.e., between different levels of the same benefit. The landowner is asked to rate, on a 1-to-9 scale of value going from very unsatisfactory to very satisfactory, each of four levels of benefit (none, low, medium, high). A simple chart requiring one checkmark per column is the device for obtaining these responses. The entire questionnaire is reproduced in the appendix.

The results of Part II of the questionnaire yield a shape to the single-attribute utility functions $u_i(f_i)$, and the results of Part I yield a scale parameter. By this process, the utility function $U(F(x))$ is well defined, nonlinear, and easily obtained within the strict limitations on landowner and forester participation described in section 2.

There is presently one unavoidable drawback to this process. WRAP's modeling of landowner preferences would be more accurate if it were permissible to use a nonseparable utility function. The advantage of nonseparability (also known as nonadditivity) is that it allows for situations where the marginal rate of substitution between two benefits, say i and j, depends on the amount on hand of some other benefit, say k. For instance, a landowner may be more willing to trade off deer (f_i) for increased timber income (f_j) when quail (f_k) are plentiful than when quail are scarce. Unfortunately, incorporating nonseparability in WRAP at present would add far too much complexity to the utility assessment process. The limitation of nonseparability is shared by many other approaches to multiobjective problems (sometimes unwittingly [16]). The test suggested by Dyer [16] for assessing the validity of separability is not implementable in WRAP, but past performance indicates the effect of this drawback is not significant.

6. Form and assessment of the benefit functions

The set of benefits that WRAP examines can be partitioned into two categories: timber and nontimber. Timber benefit is measured as the present value of monetary returns from timber sales. These returns depend on the species, volume, and timing of the timber harvested. The nontimber benefits, recreation and the six wildlife populations, are not translated into financial measures.

[4] Added in proof: This method was deemed the most appropriate among several alternative weighting methods in a study conducted by Schoemaker and Waid [55].

Table 1
Sources for WRAP timber growth and yield simulators

Species/type	Publication(s) cited
Loblolly pine plantation	Smalley and Bailey [39]
	Pienaar [33]
	Schumacher and Coile [37]
	Williams and Hopkins [50]
Longleaf pine plantation	Lohrey and Bailey [31]
	Farrar [17]
	Williams and Hopkins [50]
Slash pine plantation	Bennett [7]
	Clutter and Jones [12]
	Schumacher and Coile [37]
	Williams and Hopkins [50]
Natural loblolly pine	Brender and Clutter [8]
Natural slash pine	Bennett [6]
Natural shortleaf pine	Schumacher and Coile [37]
Natural longleaf pine	Farrar [17]
	Anon. [1]
	Williams and Hopkins [50]
Yellow-poplar	Beck and Della-Bianca [3,4]
Upland oak types	Dale [13]

Harvest volumes are computed through timber growth and yield simulation. There are nine separate simulation models. They address the species and species groups listed in table 1. These models were synthesized from the forestry literature [34,42] as indicated in table 1. Timber growth and yield is simulated as a function of tree species, stand age, and site quality. These models were derived by: (i) establishing plots of trees that included a variety of forest conditions, (ii) obtaining detailed measurements of these trees over time, and (iii) fitting a series of regression equations to these data to determine estimates of the major components of growth and yield – diameter growth, mortality, and height growth. Since stand age is one of the independent variables, these models are designed only for even-aged stands.

The wildlife benefit simulations use functional relationships that Giles et al. [20] obtained through numerous interviews of wildlife biologists. The biologists were asked to predict the forest's potential carrying capacity for each species as a function of timber stand age. These predictions (called "opinion curves" due to their subjective derivation) were aggregated over all biologists to form functions that relate habitat potential to stand age. The recreation benefit function was similarly derived through aggregation of expert opinion.

7. Evaluation of WRAP

It will take decades to determine how well WRAP stimulates increased timber production and helps avert future shortages. Most of WRAP's recommendations are for timber harvests scheduled years into the future. There is, of course, no certainty that all these recommendations will be followed. Nor is there a guarantee of the accuracy in WRAP's projection of the timber volume these harvests would yield. However, several substantive indications that WRAP has achieved a promising degree of success can be reported.

The most important result is a definite change in attitude among the NIPF users. This was documented by the U.S. Forest Service [45] in a 1978 survey of WRAP users in Alabama. At the time of the survey, WRAP was still undergoing development so that some of its present features were not in place and some long turnaround times were experienced. Nevertheless, the user group was quite receptive to the program: 90% found the program sufficiently understandable, 76% found it sufficiently personalized, and 97% felt that WRAP was applicable to their situation. Two-thirds of the sample were landowners who had never before made an effort to formulate a management plan for their timberland. Of this group, 46% reported they had changed their management technique or outlook as a result of WRAP. This figure should not be taken lightly; it has been observed that only 20% of NIPF landowners own timberland for the sale of forest products [41]. Among the 80% who hold timberland for other reasons are people who refuse ever to cut a living tree, regardless of what they are told about the increase in overall growth that can result from thinning a stand. Consequently, the 46% rate of attitude change in the Alabama pilot study is very encouraging. Moreover, the rate has probably increased since 1978, because the program is past the developmental stage.

In terms of timber volumes, some promising results can be reported. These results represent only order of magnitude estimates due to the absence of control over landowners to ensure they perform WRAP's recommendations. Also, the stochastic nature of the timber growth and yield system (which only shows *potential* growth and does not include the effect of catastrophic losses due to fire, insects, etc.) adds a large degree of uncertainty. However, if all recommendations are followed by the first 993 WRAP users, a predicted total of 4.4 billion cubic feet of timber will be harvested in the next 50 years. (This consists of 10.2 billion board feet of sawtimber and 23.6 million cords of pulpwood.) A representative sample of this group found an average annual growth rate of 115 cubic feet per acre per year. As a comparison, TVA [41] estimated in 1969 that the long-term yield on NIPF lands in the Tennessee Valley would be 30 cubic feet per acre per year. (This figure *does* include the effect of catastrophic loss.) Improvement from 30 to 115 cubic feet per acre per year is undoubtedly overoptimistic, but these figures certainly demonstrate the potential for increased timber yields in the NIPF sector of the southeastern

United States and that programs such as WRAP can help to achieve these gains.

8. Summary and conclusion

TVA developed WRAP to stimulate production of forest benefits on NIPF lands. Owners of these small properties collectively hold a controlling interest in United States timberlands but have made only limited use of modern forestry practices. Consequently, they experience smaller timber yields than possible. The historical root of this problem is that NIPF landowners have multiple landowning objectives, with timber sales possibly less important than recreation, aesthetics, or other objectives.

To attract this particular group to more intensive forest management, WRAP was designed as a multiobjective optimization program that considers timber, recreation, and wildlife objectives. The program is administered onsite by a forester, who has only a limited amount of time and resources available for ascertaining the landowner's individual preferences for these objectives. The instrument the forester uses for assessing these preferences is a simple questionnaire that is later used as the source of an individually tailored nonlinear utility function. The utility function is maximized by a coordinate ascent method, and the forester reports the recommended harvest schedule to the landowner. The WRAP output also contains textual information to help educate the landowner on the benefits and techniques of modern forest management.

Although there have been indications of WRAP's success, some aspects of the current system require improvement. First, the age-dependent nature of the growth and yield simulators restricts WRAP to use in even-aged stands. A more general method for predicting growth and yield that is species and age independent is desirable. Second, the restriction to a single, computer-free interview is an obvious drawback in the assessment of the landowner's objectives. Preferably, an interactive, multiobjective optimization procedure would remove the necessity of specifying an explicit utility function a priori. This would permit the landowner to clarify his or her perception of management objectives in a learning situation. The third potential area of improvement is in the nontimber benefit functions, which provide only rough estimates because they are difficult to quantify. It is hoped that present and future developments will yield a new system that improves upon WRAP in all three of these areas. [5] Until then, WRAP is probably better suited for making management guidelines rather than rigid harvest prescriptions.

[5] Added in proof: These and other improvements are incorporated in the FORMAN system described in refs. [53] and [54]).

At present, the most important result of WRAP is that it has brought about a definite change in attitude within the NIPF sector. It has been used by more than 1300 landowners to date (April 1983) in the southeastern United States, and many of these users are implementing intensive forest management practices for the first time.

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Resource Objective Ranking Questionnaire for WRAP

Explanation

One purpose of WRAP is to schedule timber harvests to maximize woodland benefits for the landowner. In all, WRAP considers eight different resource objectives, which are named below.

We need to ask you some questions about your feelings toward these objectives. What we want to determine from your answers are the <u>relative strengths of your satisfaction with each objective</u>.

There are two reasons why WRAP needs this information:

1) Different landowners probably have different ideas about the relative importances of the eight objectives. WRAP does not presume to know what you want most from your land.
2) The eight objectives compete with one another. Taking a high level of one might force you to accept a low level of another. For example, if you direct WRAP to find the harvest schedule that attains the largest possible timber yield, then you may have to accept low levels of squirrel population.

The questionnaire has two parts. Part I asks you to think about the objectives together. In Part II you consider them individually.

Part I

Objectives	
Timber Income (periodic)	Turkey
Recreation (undeveloped)	Grouse
Squirrel	Quail
Deer	Rabbit

1. List the objectives on the lines at the left below, with the most important objective on the first line, the second most important on the second line, etc. Any objectives of benefits which are of no importance to you should be given 0 points and listed at the end.

Objective Benefit	Points
____________________	________
____________________	________
____________________	________
____________________	________
____________________	________
____________________	________
____________________	________
____________________	________
	Total = 100

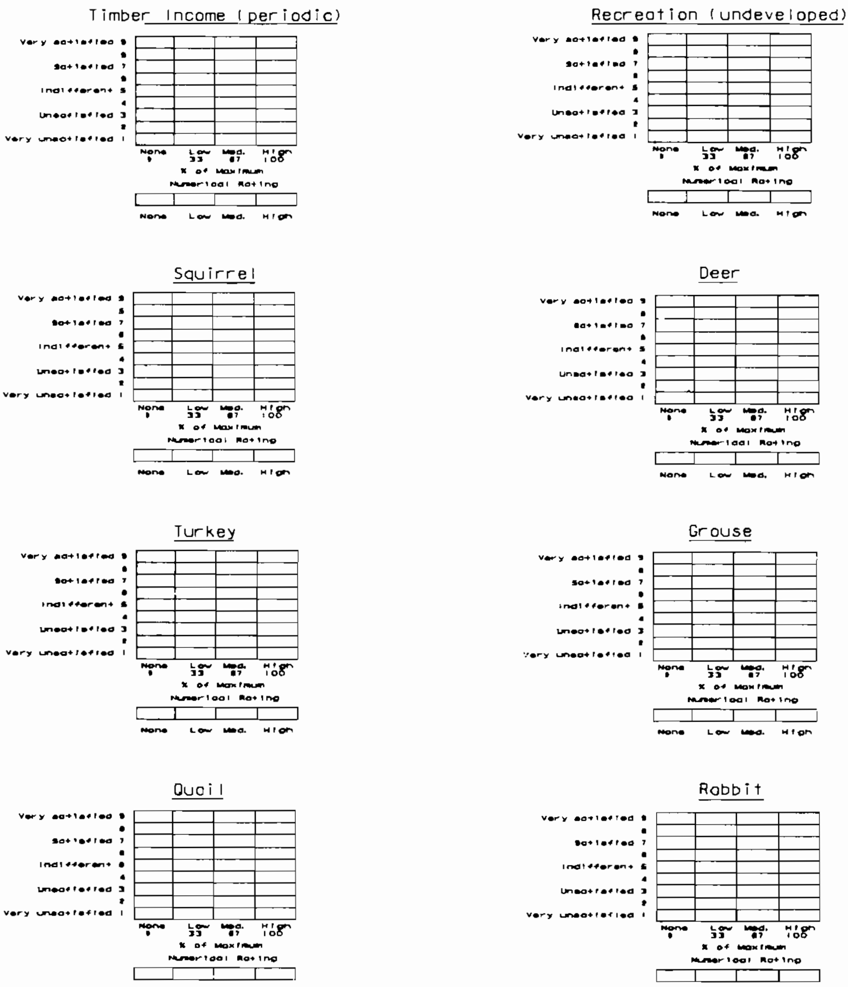
2. Give point scores to each of the benefits on the lines at the right above. The point scores should represent the <u>strength of your desire to attain the objective</u>. The numbers you put down should be positive and they must sum to 100. Take your time. Feel free to cross out and change answers as you go along.

Part II

In Part II you must consider your interest in each objective that was not crossed out in Part I.

1. For each objective, there are four levels of attainment across the bottom and nine levels of satisfaction along the side. For each level of attainment (None, Low, Medium, or High) determine your degree of satisfaction and place an X in the appropriate square.
2. After determining the placement of your four X's for each objective, determine the numerical rating by transferring the number beside the degree of satisfaction to the numerical rating squares.

Resource Objectives Rating Charts



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LONG-TERM TIMBER PRODUCTION PLANNING VIA UTILITY MAXIMISATION

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An approach to long-term timber production planning is presented. The forest and its growth are described by individual trees which grow in homogeneous units, forest stands. The forest stands are grouped into calculation units for which a number of management schedules is simulated. The set of these management schedules constitutes the production function which describes the timber production process.

The value of the timber production policy is determined by the utility it provides for the decision maker. Utility is expressed as a function of the products and production factors of the timber production process. A multiplicative, linearly homogeneous utility function is suggested and the use of direct search methods in its maximisation is demonstrated.

1. Introduction

The task of timber production planning is to provide the decision maker with a relevant set of feasible production policies. Each production policy should yield the criteria needed in the decision making. Furthermore, there has to be an operational and computationally feasible procedure in finding out the timber production policy which is optimal with regard to these criteria.

The performance of the timber production policy has usually been measured either by such general economic indicators as net present value of the future income (e.g. ref. [7]) or by some criteria characteristic to forestry. Because of the reproductive capacity of the trees such a requirement as sustainable yield, unknown to many other industries, has frequently been set to forestry. A commonly used criterion for the sustainable yield is the allowable cut which indicates the level of the annual or periodical timber drains from the forestry unit over the planning horizon (see e.g. ref. [8]).

Neither of these indicators satisfactorily describes the goals of all decision makers. The maximisation of the net present value may lead to great variation in timber drains over time. On the other hand, timber unlike food, for example, can be substituted by other raw materials. Thus, the maximisation of the allowable cut cannot be accepted as the policy objective without analysing its consequences.

It is evident that several decision criteria have to be taken into account in forestry decision making. The long planning horizon in timber production still increases the number of decision variables since physically similar production factors and products have to be treated as different variables when they occur at different points of time.

The employment of mathematical programming methods has made it possible to evaluate several decision variables at the same time. In the usual LP (and NLP) formulation of the timber production optimisation problem (see e.g. ref. [9]) one output (input) variable has been taken as the value of the objective function and the other relevant variables have been employed as constraints.

This formulation is efficient only if the objective function and goals are not substitutable or if the goals of the decision maker are expressed by the objective function, i.e. the constraints are imposed by the environment. If the constraints are set by the decision maker they clearly constitute part of his goals. If the objective function and constraints are substitutable the previous formulation of the problem may lead to a laborious search process before optimal or acceptable values for the constraints are attained.

One way to tackle the problem of the multiple objective decision making is to condense all goals into one variable which describes the utility of the decision maker. A Dictionary of the Social Sciences (ref. [2] p. 740) states: "Utility is now generally understood to mean the direct satisfaction that goods and services yield to their possessors.". Consequently, it is usually assumed that decision makers want to maximise their utility: "The ultimate goal of forest management is the maximization of the utility of the forest to the owner" [9].We have suggested utility as a means to evaluate the constraints of a LP timber production model [5]. If the objective function and the constraints of the LP model constitute the determinants of a strictly monotone utility function, the marginal rates of substitution derived from the utility function are in the optimal solution equal to those derived from the shadow prices of the LP model. However, we did not present any operational utility model.

Even though utility looks like an ideal decision criterion, strong doubts about the applicability of the unidimensional utility models have been presented (see, for example, [3, p. 433]): "In particular, they fail to express desirable minimum and maximum limits in either of the goal elements involved, and/or they fail to propose a method by which to assign weights to the goals, which people in the specific decision situation agree are appropriate.".

The difficulties in the derivation of utility models must not be underestimated. On the other hand optimisation may be markedly simplified if the objective function and with it substitutable constraints of the original LP (or NLP) problem are combined into a unidimensional utility function.

The purpose of this paper is fourfold: first, to present a system which yields for a forestry unit both nondiscrete and integer production possibility sets (production functions) with potentially interesting long-term input and output variables of the timber production process.

Second, to suggest an operational unidimensional utility model where the utility is a function of the input and output variables of the production process.

Third, to discuss possible solution techniques for the utility maximisation problem.

Fourth, to demonstrate some sample problems.

2. Timber production process

Our approach is to describe the forestry unit and its production process by the forest stands and their production models. The number of forest stands has to be adjusted to the computational capacity. For a large forest area a representative sample may suffice. Each stand is described by its area, site variables, and by the number of sample trees. The number of sample trees may be as small as one. As a maximum, the whole tree population of the stand is employed.

Each sample tree is described by a number of variables such as frequency, tree species, breast height diameter, height, age, volume, timber assortment volumes, and stem value. The respective growing stock variables are calculated as sums or other simple functions of the tree variables.

To facilitate the organisation of the stand information, the stands are grouped into calculation units. The stands belonging to one calculation unit should be similar with regard to the stand characteristics and future management of the stand.

Feasible production processes of each forest stand are described by a number of simulated management schedules. The calculation units are furnished with instructions of the possible measures in various decision situations. The measures comprise thinnings, clear cuttings, regeneration, tending of young stands, fertilisation, etc.

The growth simulation is done individually for each forest stand in accordance with the instructions of the calculation unit. The development of the growing stock is simulated by models describing the ingrowth, growth, and death of the trees. Even though the growth of the trees close to the stand border may be influenced by the neighbour stand, the forest stands of a certain forest area may usually be assumed without any great error to be independent of each other as far as the growth process is concerned. It is also assumed that the logging and transportation costs are independent of the spatial distribution of the stands.

The planning period is divided into subperiods. The calculation unit sums of all interesting forest stand variables are calculated within each management schedule for these subperiods. These variables include volumes by tree species and timber assortments in the beginning of each subperiod, and volume increment, drain, income, cost, etc. during each subperiod.

The following formalism in the description of the timber production process is used:

m = number of calculation units,

n_i = number of management schedules for unit i,

w_{ij} = the proportion of the calculation unit i managed according to management schedule j,

p = number of input and output variables,

x^{ij} = $(x_1^{ij}, x_2^{ij}, \ldots, x_p^{ij})$ is the vector of quantities produced or consumed by unit i if schedule j is applied. (The components of x^{ij} refer to specific time periods as well.)

x^i = $(x_1^i, x_2^i, \ldots, x_p^i) = \Sigma_{j=1}^{n_i} w_{ij} x^{ij}$ is the vector of quantities produced or consumed by unit i when schedules j are applied in proportions w_{ij},

$\overline{X}^i$ = $\{x^i \mid w_{ij} \geqslant 0$ for all i and j, and $\Sigma_{j=1}^{n_i} w_{ij} = 1$ for all $i\}$ is the production possibility set for unit i if any combination of schedules j is feasible (nondiscrete formulation),

$\overline{X}$ = $\{x \mid x = \Sigma_{i=1}^{m} x^i,\ x^i \in \overline{X}^i\}$ is the production possibility set for the whole forestry unit i (nondiscrete formulation),

X^i = $\{x^i \mid w_{ij} = 1$ or 0 for all i and j, and $\Sigma_{j=1}^{n_i} w_{ij} = 1$ for all $i\}$ is the production possibility set for unit i (integer formulation),

X = $\{x \mid x = \Sigma_{i=1}^{m} x^i, x^i \in X^i\}$ is the production possibility set for the whole forestry unit (integer formulation).

3. Utility model

When trying to develop an operational utility model to our problem we have the following goals in mind:

(i) Net present value and allowable cut criteria should be special cases of the utility model.

(ii) Parameters should have obvious interpretations that decision maker can utilise in determining their values.

(iii) Quantitative comparisons between utility values should be meaningful.

Multiplicative functions have been commonly suggested as a basis for the utility models (see e.g. ref. [4]). We propose the use of the following variant of the multiplicative function as an instrument for long-term timber production planning

$$u = u(x) = \prod_{k=1}^{p} y_k^{c_k},$$

where $y_k = x_k + b_k$ and b_k and c_k are parameters given by the decision maker.

To make the function linearly homogeneous with respect to y_k variables it is further required that

$$\sum_{k=1}^{p} c_k = 1,$$

so that

$$u = \sum_{k=1}^{p} \frac{\partial u}{\partial y_k} y_k .$$

The economic reasoning behind the utility model is as follows:

(i) If $x_k > 0$ then x_k is a produced commodity.

- If $b_k > 0$ then decision maker has in his possession the amount b_k of commodity k from other sources than the forestry unit under surveillance.
- If $b_k < 0$ then b_k expresses the minimum amount of commodity k needed by the decision maker.

(ii) If $x_k < 0$ then x_k is a commodity consumed by the production process. Then $b_k > -x_k$ for all k.

(iii) The marginal rate of substitution between variables x_k and x_l is

$$-\delta_{lk} = -\frac{\mathrm{d}x_k}{\mathrm{d}x_l} = (c_l/c_k)(x_k + b_k)/(x_l + b_l).$$

The formula of the marginal rate of substitution helps the decision maker to choose the values of the c_k parameters. First, he decides which ratio y_k/y_l represents a favourable, balanced situation. Then c_k and c_l are chosen in such a way that in the balanced situation the marginal rate of substitution is acceptable. For instance, if y_k and y_l are net incomes in different periods, c_k and c_l can be chosen so that in the even income situation ($y_k = y_l$) the decision maker is marginally maximising the net present value with respect to some rate of interest.

(iv) If $b_k = b$ for all k and $b \to \infty$ the surplus utility is

$$\prod_{k=1}^{p} (x_k + b)^{c_k} - \prod_{k=1}^{p} b^{c_k} \to \sum_{k=1}^{p} c_k x_k .$$

If y_k variables are net incomes in different periods and c_k parameters are discounting factors the decision maker wants in this case to maximise the net present value of his future income from the forestry unit.

(v) If $x_k > 0$, $b > 0$, and $b_k = -b$ for all k, a point $x \in X$ is feasible ($u(x)$ is defined) only if $x_k \geqslant b$ for all k. If b increases, the number of feasible points decreases and finally there is only one feasible point which represents the maximal even output (or allowable cut) situation. In this case c_k parameters do not have any influence. Thus if x_k variables refer to net incomes in periods k the decision maker can by means of b_k parameters weight continuously the net present value and even flow of income criteria which represent two extremes of our utility model.

(vi) Because the function is linearly homogeneous it measures the utility in ratio scale [1]. We think that quantitative comparisons between utilities are meaningful and easily explicable in forestry context, at least in the case when x_k variables represent products and $b_k = 0$ for all k. The utility can also be interpreted as the sum of the products of the available commodities and their marginal utilities.

4. Optimisation methods

After the production possibility sets are given and the utility model is defined the problem is to find

$$\max_{x \in \bar{X}} u(x) \text{ (nondiscrete formulation)},$$

or

$$\max_{x \in X} u(x) \text{ (integer formulation)}.$$

To solve the problem in its nondiscrete formulation we can employ any efficient nonlinear optimisation method (see e.g. ref. [6]).

Our multiplicative utility model can be made separable and concave by taking logarithms

$$\log(u) = \sum_{k=1}^{p} c_k \log(x_k + b_k).$$

Consequently, separable LP can be applied.

If the integer formulation is more appropriate the following direct search method, for example, can be employed:

step 1. Select randomly a starting point $x \in X$.

step 2. $r = 1$

$\quad$ opt2 $= u(x)$

step 3. opt $= \max_j u(x^{rj} + \sum_{i \neq r}^{m} x^i)$

$\quad$ replace x^r by the optimal schedule x^{rj}

step 4. $r \to r + 1$

if $r \leqslant m$ go to step 3

step 5. If opt > opt2 then go to step 2, otherwise stop and deliver x as a local optimum.

By one application of this algorithm we get a local optimum. The usefulness of the algorithm is dependent on the production possibility set and on the utility model. If the utility model is strictly monotone, the calculation units and the management schedules are clearly different, and the calculation units are approximately of the same size, the algorithm may directly lead to the global optimum.

Even if the global optimum is not easily attainable, the algorithm seems reasonable in this kind of planning context. Because the production processes in the forest stands (and in the calculation units) were assumed to be independent of each other (see section 2) it is unlikely that there exists one or few superior solutions.

5. Sample problems

Our sample problems refer to a forest area in Southern Finland. The forest data are based upon the national forest inventory in 1978. A total of 2935 sample trees on 395 sample plots were employed. Each sample plot represents a forest stand. The sample plots were grouped into 43 calculation units. For all calculation units a total number of 1326 management schedules over a 50-year planning period was simulated. This number of schedules was due to the assumption that for every stand in each calculation unit two levels of thinning and a clear cutting were the potential measures in the middle of each 10-year subperiod.

Case 1. In the first case the utility model was

$$u = x_1^{0.278} x_2^{0.206} x_3^{0.154} x_4^{0.114} x_5^{0.085} x_6^{0.163},$$

where

x_k = volume drain during the kth 10-year period, $k \leqslant 5$,

x_6 = volume of the growing stock at the end of the 50-year period.

In this example b_k parameters were set to zero. The values of the c_k parameters were based upon the assumption that in even timber flow situation the marginal rates of substitution equal the discounting factors calculated according to 3% rate of interest. Thus when $k > l$ and $x_k = x_l$

$$-\delta_{lk} = -\frac{\mathrm{d}x_k}{\mathrm{d}x_l} = (c_l x_k)/(c_k x_l) = c_l/c_k = 1.03^{10(k-l)}.$$

In the determination of parameter c_6 it was further assumed that after the 50-year period the annual volume drain is 4.5% of the total volume. Thus the even flow situation exists when

$$x_5/10 = 0.045 x_6.$$

In the even flow situation

$$-\delta_{56} = -\mathrm{d}x_6/\mathrm{d}x_5 = (c_5 x_6)/(c_6 x_5) = (c_5 x_6)/(0.45 c_6 x_6)$$

$$= c_5/(0.45 c_6) = 1.03^5,$$

or $c_6 = 1.92 c_5$.

Furthermore, the c_k parameters were scaled to yield a linearly homogeneous function.

The problem was solved 3000 times by the direct search algorithm given in section 4. A total of 718 different local optima were found. The best local optimum which supposedly equals the global optimum within the production possibility set (see section 4) was attained 5 times. The values of the decision variables in the best local optimum were

$$x_1 = 7070,\ x_2 = 5753,\ x_3 = 4802,\ x_4 = 6202,\ x_5 = 6394, \text{ and } x_6 = 15\,684.$$

The utility of this solution was 7100.9. The utility of the worst local optimum was 7042.5 (99.2% of the best local optimum) and the mean utility of all local optima was 7082.7 (99.7%). Of all local optima 270 were closer than 0.1% from the best local optimum. The derivation of one local optimum took on an average 3...4 rounds through the calculation units and 11.7 s of VAX-11/780 CPU-time.

The matrix of the marginal rates of substitution $-(\delta_{lk})$ for the best local optimum is

$$-(\delta_{lk}) = \begin{pmatrix} 1.00 & 1.09 & 1.22 & 2.13 & 2.95 & 3.77 \\ 0.91 & 1.00 & 1.12 & 1.95 & 2.69 & 3.45 \\ 0.82 & 0.90 & 1.00 & 1.74 & 2.41 & 3.09 \\ 0.47 & 0.51 & 0.57 & 1.00 & 1.38 & 1.77 \\ 0.34 & 0.37 & 0.41 & 0.72 & 1.00 & 1.28 \\ 0.27 & 0.29 & 0.32 & 0.57 & 0.78 & 1.00 \end{pmatrix}.$$

A few modifications of the direct search algorithm were tested.

First, the starting point probability of the management schedule was increased whenever it occurred in a local optimum. The performance of the algorithm was markedly improved. The major part of the learning process occurred during the search for the first 10 local optima, but slight improvements were discernible up to 500 local optima.

Similarly to the original direct search algorithm, the best result was found 5 times out of the 3000 local optima derived by the modified algorithm. This local optimum equalled the one found by the original algorithm. The worst result was 7054.7 (99.4%) and the average utility 7089.7 (99.8%). Of the local optima 816 were closer than 0.1% from the best local optimum. The algorithm found 573 separate local optima, 183 being different from those found by the original algorithm. The derivation of one local optimum took on an average 8.2 s of CPU-time.

In the second modification the calculation units were divided into 1...10 (on an average 2.6) parts in proportion to their volume drain potential. The best solution was

$$x_1 = 7049,\ x_2 = 5620,\ x_3 = 4867,\ x_4 = 6387,\ x_5 = 6448, \text{ and } x_6 = 15\,638.$$

The utility of the best local optimum was 7101.0, the utility of the worst local optimum 7088.7 (99.8%), and the average utility of 300 local optima was 7098.7 (100.0%). The number of separate local optima was 64. The average CPU-time per one local optimum was 110.8 s.

Case 2. In the second case it was assumed that the decision maker wants to maximise the minimal level of the drain during the 10-year periods, i.e. the allowable cut. Assuming 4.5% annual volume increment the final volume of the growing stock x_6 is multiplied by 0.45 to make it comparable with other variables. The problem can now be stated

$$\begin{aligned} &\max\ b \\ &\text{s.t.}\, x_k \geq b \ \text{ for all } k, \end{aligned}$$

Three alternative methods were tested. In every case the direct search algorithm was set to maximise the following objective function

$$u(x) = \sum_{k=1}^{6} \min(x_k - b, 0).$$

In the first method b is given different trial values and for every value of b the direct search algorithm is used to calculate local optima.

In the second method the direct search algorithm first seeks a local optimum x for $b = 0$. Then b is replaced by $\min_k(x_k)$, and the direct search is started from the old local optimum. Iteration is continued as long as new optima are attained.

In the third method the direct search algorithm first seeks a local optimum x for $b = 0$, as in the previous case, but then b is replaced by $0.5\,[b + \min_k(x_k)]$. Iteration is continued until the change of b is smaller than a chosen limit Δb.

The third method gave clearly best results. In our example with $\Delta b = 10$ the algorithm stopped after 21 changes of b and 92 s of CPU-time. The solution was

$$x_1 = 6058,\ x_2 = 6052,\ x_3 = 6033,\ x_4 = 6129,\ x_5 = 6037, \text{ and } x_6 = 6058.$$

The value of x_6 is 13 462 in its original units.

Case 2 demonstrates one extreme limit of our utility model. In this case the production variables are not substitutable by each other. If the decision maker wants to maximise the level of the even yields he is evidently inclined to pay any price in other input and output variables of the production process.

The other extreme limit of our utility model is attained when the decision maker wants to maximise the net present value of the future income (or yield). Then, the marginal rates of substitution are constants for all values of x_k. In this case the solution is trivial. The management schedule which maximises the net present value of the future income is chosen for each calculation unit.

The ending inventory is not satisfactorily taken into account in our utility models. Construction of the utility models would be easier if planning periods were longer than the 50 years employed in our sample problems. The ending inventory would then be a less important utility determinant.

6. Discussion and conclusions

It was assumed that the forest resources (forestry unit described by calculation units) and the production technology (management schedules for each calculation unit) are the only constraints which determine both nondiscrete and integer production possibility sets.

It was further assumed that the utility of the decision maker is a function of the input and output variables of the timber production process. Other utility determinants were assumed to be constants. The utility was described by a linearly homogeneous function. The optimal solution is the feasible production policy which maximises the utility function.

Due to this formalism efficient algorithms can be employed in the solution of the optimisation problem. In this paper we have demonstrated the use of simple direct search algorithms which are appropriate for the integer formulation of the problem.

In long-term, large area timber management planning both integer and nondiscrete formulations of the problem yield practically equal results. This was indicated by Case 1 where the division of the calculation units increased the utility only by 0.0014%. Accordingly, the choice of either of the two formulations is mainly dependent on the efficiency of the solution algorithms.

The utility model may assume two different tasks: (1) it may estimate the true utility of the decision maker, or (2) it may only be an instrument to derive efficient production policies. In the first task the production policy which maximises the utility is the solution we have been looking for. In the second task the values of the utility model parameters are varied until a satisfactory production policy is attained. To facilitate the second type use an interactive system should be developed. The operator would indicate the values of the c parameters as interest rates, for example. The computer would then scale these values into appropriate discounting factors for the linearly homogeneous utility function.

The methods described in this paper constitute a part of the timber production planning system developed for the national forest inventory in Finland. The inventory is repeated regularly. The database comprises 60000 relascope sample plots with 300000 measured trees and covers an area of 22 mill. forest hectares.

Further research and development are required to fully utilise the utility maximisation method. Even in its present stage the method has proven to be a practical tool when compared to the earlier trial and error approach to find acceptable constraints for large scale LP problems.

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THE INFLUENCE OF LOGGING COSTS ON OPTIMAL ROTATION AGE FOR SITE I *Pinus patula* IN TANZANIA *

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The determination of optimal rotation age for a pine plantation in Tanzania is cast as an unconstrained, integer nonlinear programming problem in one variable. Continuous models are fitted to growth and yield data for the plantation under study. Data are presented suggesting that logging costs are functionally related to tree size and thus to stand age. Under such conditions, the global optimum may be found by a simple direct search procedure. For the plantation under study, the optimal rotation age when logging costs are considered is found to be 28 years, or 12% longer than the presently specified rotation of 25 years. The soil expectation value is some 2.3% greater for the optimal rotation than for the presently specified rotation. By comparison with the rotation of maximum mean annual increment, the optimal financial rotation is 40% longer and provides an increase in soil expectation value of 66%. If logging costs are presumed to be unaffected by tree size, then the optimal rotation age is reduced to 25 years.

1. Introduction

Tanzania is a tropical country on the east coast of Africa (fig. 1) with a total land area of about 890000 km^2, of which about 0.5% comprise softwood plantations, established mainly as afforestation projects. Some fourteen such plantations are currently under management, primarily utilizing various species of tropical pines and cypress. Though small in total area, these plantations constitute virtually the only source of softwood logs in the country, and thus help offset a rather large net import bill for industrial wood products [13]. The productivity of the plantations is relatively high; the largest plantation, at Sao Hill (fig. 1), produces an average of more than 25 m^3 per hectare annually, averaged over all site classes, at culmination of mean annual increment [1]. This compares to an average of less than 2 m^3/ha y for natural savanna and open woodland areas [7].

The plantation at Sao Hill currently extends over nearly 23000 hectares and an additional 3700 hectares are being planted annually with financial assistance from the World Bank. About 90% of the current planting is in *Pinus patula* with the remainder being primarily *Pinus elliottii* and a small amount of

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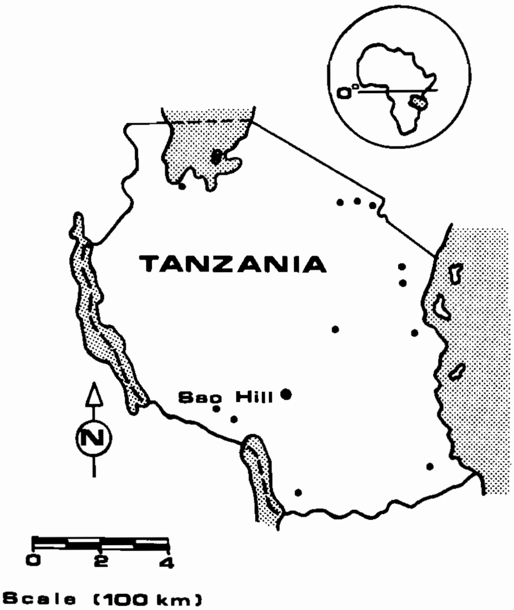

Fig. 1. Map of Tanzania showing the location of the Sao Hill Forest Project (large black dot) and the locations of the other 13 industrial forestry plantations in the country (small black dots). The inset shows the location of Tanzania on the continent of Africa.

eucalyptus. A sawmill, built under an agreement with the government of Norway, began producing in 1976 and by 1981 was consuming logs at the rate of nearly 50 000 m^3 annually. A pulp mill, presently under construction near Sao Hill, is scheduled to begin production in the mid-1980s and will require an annual log input of about 300 000 m^3.

2. Management of Site I *Pinus patula*

Approximately 25% of the older stands at the Sao Hill plantation are classified as Site I. This means that the dominant trees reach an average height of 30 m at 20 years of age. The Site I lands are thus referred to as having a *site index* (S.I.) of 30. These fast-growing stands are of particular interest because their soil expectation value is high and thus there is considerable urgency about turning over the standing volume frequently through short rotations. The management of plantations in Tanzania is regulated by a standing technical order issued by the Forest Division [9]. Silvicultural operations stipulated under this order, together with current estimated costs for Sao Hill, are summarized in table 1. This schedule applies only to Site I *Pinus patula*;

Table 1
Schedule of silvicultural activities required for Site I *Pinus patula* in Tanzania, with estimated costs for Sao Hill in 1980 shillings

Age	Operation	Cost (shs/ha)
0	Acquisition of nursery seedlings	488
0	Planting	115
2	Beating up (replanting)	30
2	Weeding and cutting climbers	145
3	Weeding and cutting climbers	145
4	Access pruning	150
7	Full-scale pruning	240
9.5	Precommercial thinning	300
13.5	First commercial thinning	[a)]
17.5	Second commercial thinning	[a)]
21.5	Third commercial thinning	[a)]
25	Clearfelling	[a)]

[a)] Net returns (see table 4). Sources: the schedule of operations is from ref. [9]; estimated costs are from ref. [14].

similar schedules have been prepared for other site classes and other plantation species.

For this analysis, the silvicultural schedule in table 1 is assumed to be fixed, with the exception of final felling. In fact, the treatments specified in the schedule constitute a legally enforceable ordinance as all resources in Tanzania are owned by the government. For purposes of policy analysis, however, a reasonable question to ask is "what silvicultural treatments should be applied in order to maximize soil expectation value?" Procedures for answering such questions have been worked out using dynamic programming [2,4] and nonlinear programming [11]. A major impediment to the use of these methods for the

Table 2
Official royalty price schedule for *Pinus patula* in Tanzania as a function of average stand diameter at breast height (Source: [18])

Dbh range (cm)	Royalty fee (shs/m^3)
$0 < \text{Dbh} < 32$	20
$32 \leq \text{Dbh} < 40$	65
$40 \leq \text{Dbh} < 48$	100
$48 \leq \text{Dbh} < 56$	120
$56 \leq \text{Dbh}$	125

present study is the fact that data on the response of *Pinus patula* to various pruning and thinning treatments are only sketchy at present, and an analysis of this type would require considerable extrapolation which could not be statistically justified.

In Tanzania, forest projects and the associated manufacturing plants are managed as separate profit centers, although both are government parastatal organisations. Ownership of logs is transferred from the forest project to the manufacturing plant upon payment of a royalty based on the diameter of the standing tree at breast height [18]. The schedule of royalties is given in table 2. Logging is the responsibility of the manufacturing plant.

3. Methodology for determining optimal rotation age

The current rotation age as prescribed for Site I *Pinus patula* in Tanzania is 25 years [9]. It is not clear how this was determined, but it appears likely that it was perceived as being near the culmination of mean annual increment [1]. The purpose of the present analysis is to compute the age of the optimal financial rotation, assuming the schedule of silvicultural activities in table 1, by using the soil expectation criterion [5]. This may be done by solving the following nonlinear programming problem:

$$\text{MAXIMIZE } S_e = \left(\frac{Y_r - P(1+i)^r + \sum_j T_j(1+i)^{r-j} - \sum_k S_k(1+i)^{r-k}}{(1+i)^r - 1} \right) - \frac{A}{i}, \tag{1}$$

where S_e = soil expectation value, in Tanzanian shillings per hectare (the 1982 exchange rate is US\$1.00 = shs 8.32); Y_r = value yield at rotation age, net of logging costs, in shs/ha; P = planting cost, in shs/ha; i = discount rate expressed as a fraction (the Tanzania Investment Bank uses 8%, or $i = 0.08$, for forestry projects); r = rotation age, in years; T_j = revenues derived from commercial thinnings at age j, net of logging costs, in shs/ha; S_k = cost of silvicultural treatments in year k (including precommercial thinnings), in shs/ha; and A = annual cost of forest administration and protection, in shs/ha (estimated at shs 10/ha y for Sao Hill).

While values for P and S_k are given in table 1, Y_r and T_j are dependent upon the size of the timber harvested because of the nature of the royalty schedule (table 2) and the fact that logging costs in general are size-dependent (see, for example, ref. [6]).

Special note should be made of the fact that a discount rate of 8% is used in this analysis. Although higher than recommended by many economists for forest analyses, this is the official rate used by the Tanzania Investment Bank. As forestry decisions in Tanzania are based on this rate, its use is therefore essential for a realistic analysis of the type considered in this paper.

The explicit consideration of logging costs through eq. (1) is a key issue of considerable importance. The point of view adopted here is that, although ownership of the trees is transferred from one government agency to another at the stump, the value of the trees (or logs) remains vested in the public until the logs have actually been processed and the lumber or other products sold. Therefore the optimal rotation age should reflect all costs which are dependent upon log size (and thus upon stand age). A complete analysis would in fact include log hauling and milling costs as well. However, no quantitative data presently exist which relate hauling and milling costs to log size at Sao Hill, and according to logging and sawmill managers, any such relationship is likely to be small for the limited range of diameters processed at Sao Hill [10,17]. It should be noted, however, that this conclusion is largely conjecture, and further studies should include such data if they can be obtained.

4. Growth and yield relationships

Adegbehin and Philip [1] have prepared site index curves for Sao Hill *Pinus patula*. The following equation describes the height curve for S.I. 30:

$$H_{\mathrm{D}} = 41 - 53.5 \exp(-0.079A), \tag{2}$$

where H_{D} = average height of dominant trees, in m; A = stand age, in years.

Similarly, the yield equation for S.I. 30 based on data summarized in ref. [1] is as follows;

$$V = 1400 - 1840.5 \exp(-0.0477A), \tag{3}$$

where V = expected total volume, in m^3/ha, at age A.

The analysis is complicated somewhat by the intermediate thinnings undertaken according to the schedule in table 1. Conceptually, eq. (3) should measure stand volume over time if the stand is unthinned. However, the data of Chamshama and Philip [3] suggest that the thinnings are scheduled so that, at the time of each commercial thinning, total stand volume will have recovered from the previous thinning to approximately the level predicted by eq. (3), as shown in fig. 2. After a thinning, the residual trees grow at an increased rate. Virtually all of this incremental growth is added to tree diameter; there appears to be no significant difference in H_{D} between thinned and unthinned stands at the end of the thinning recovery periods (points b–e in fig. 2). Although a yield function based on average tree diameter would be more

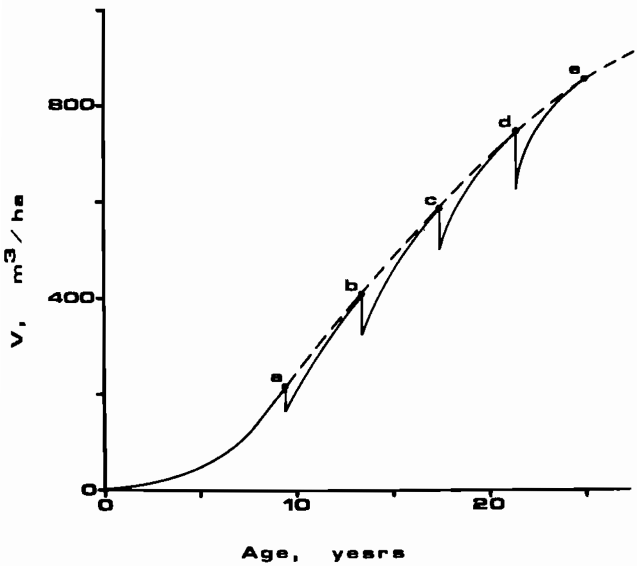

Fig. 2. Yield curve for Site I *Pinus patula* (site index 30). Point a marks a precommercial thinning; points b–d are commercial thinnings; and point e is the scheduled rotation age.

convenient for considering the effect of thinnings (and would be essential for the determination of optimal thinning regimes), eq. (3) is adequate for the present analysis where thinning treatments are considered fixed.

5. Logging costs

In order to derive equations relating logging costs to tree size, the expected volume per tree at any age must be known. Given eq. (3), this requires an estimate of the number of trees per hectare. For this purpose, data reported by Chamshama and Philip [3] on stocking levels over time were fitted to an exponential function via simple linear regression:

$$N = 1999\ \exp(-0.0648A) \quad (r^2 = 0.899), \tag{4}$$

wher N = expected stocking level, in trees per hectare, at age A. Then the average volume per tree, V_T, is given by V/N.

Philip et al. [15] have derived the following volume function for *Pinus patula* at Sao Hill:

$$V_T = 0.000055 D_B^{1.749} H^{1.137}, \tag{5}$$

where V_T = volume per tree, in m^3; D_B = diameter at breast height, in cm; and H = total tree height, in m.

Rearranging this equation and substituting H_D for H, the following expression is obtained for average stand diameter at breast height:

$$D_B = 272.6 V_T^{0.572} H^{-0.65}. \tag{6}$$

It should be noted that the rearrangement of eq. (5) into eq. (6) will provide a biased estimate of D_B unless eq. (5) reveals the exact relationship among the variables. The magnitude of this potential bias is negligible for the present study because of the large increment in D_B corresponding to changes in the royalty price (table 2). An analysis of potential changes in the royalty price structure, however, would require an independent model for estimating D_B as would an investigation of optimal silvicultural regimes.

Both logging costs and value yield are related to V_M, merchantable tree volume, rather than V_T. For the utilization standard used at Sao Hill, Philip et al. [15] have estimated the following relationship:

$$V_M = 1.015 V_T - 0.071, \tag{7}$$

where V_M = merchantable volume per tree to a 15 cm top diameter, in m^3.

To estimate logging costs the individual log volume, rather than tree volume, must be known as all logging at Sao Hill is done by the shortwood system [17]. Using data on the relationship between merchantable height and total height for *Pinus patula* in Tanzania [8] and adjusting from a 10 cm top diameter to a 15 cm top, the following regression equation was obtained:

$$H_M = (0.44 \log_e D_B) H_D - 0.96 H_D \quad (R^2 = 0.96), \tag{8}$$

where H_M = merchantable height to a 15 cm top diameter, in m. The difference between stump height and ground level is usually less than 10 cm and is ignored in this analysis as being negligible by comparison with height measurement errors.

At Sao Hill, trees are bucked into log lengths varying from 3 m to 6 m. The logs average 4.6 m in length including trim [17]. Thus, the average number of logs per tree, L, is given by

$$L = H_M / 4.6 \tag{9}$$

and volume per log in m^3, V_L, is

$$V_L = V_M / L. \tag{10}$$

5.1. Felling and bucking cost

We are now in a position to estimate the cost of individual logging activities. Felling and bucking crews at Sao Hill are paid on a piecework basis at the rate of shs 1.00 per log [17]. Thus, felling and bucking cost in shs/m^3, C_F, is given by

$$C_F = 1.00/V_L. \tag{11}$$

5.2. Skidding cost

Although various types of equipment are used for skidding at Sao Hill, most skidding is done by Ford 6600 farm tractors equipped with Igland two-drum winches [17]. Each tractor carries six chokers and averages eight logs per load. Truck road spacing is such that the average skidding distance is about 70 m. Round-trip skidding time may be estimated as follows:

$$T = T_H + T_U + 2(ASD)/v, \tag{12}$$

where T = average round-trip skidding time, in min; T_H = average hooking time, in min; T_U = average unhooking and decking time, in min; ASD = average skidding distance, in m; and v = average skidding and return speed, including an allowance for delays, in m/min.

Estimating T_H at 6 min, T_U at 2 min, and v at 60 m/min, we compute $T = 10.33$ minutes per load. This estimate must be considered preliminary; it is based on three days of observations at Sao Hill and interviews with the logging manager, skidding foreman, and tractor crews. More reliable estimates will have to await a full-scale logging study.

The estimate of skidding time may be combined with the figure for hourly skidding cost from table 3 to obtain unit skidding cost as a function of log volume:

$$C_S = (C_H T)/(60 L_T V_L), \tag{13}$$

where C_S = skidding cost, in shs/m^3; C_H = hourly skidding cost, in shs/h (table 3); T = skidding time per round-trip from eq. (12); L_T = logs skidded per round-trip; and V_L = volume per log from eq. (10). Thus,

$$C_S = 1.41/V_L. \tag{14}$$

Table 3
Calculation of hourly skidding cost for the logging operations at Sao Hill

Item	shs/h
Equipment depreciation	38.32
(Based on a purchase price of shs 170000 and salvage value of shs 17000 [17], a depreciable life of 5 y, capital recovery at 8% interest, and annual use of 1000 h).	
Operating costs	15.50
(Extracted from records on a Ford 6600 tractor used at the University of Dar es Salaam Training Forest).	
Labour cost	11.87
(Two chokersetters and one tractor driver, each paid the minimum wage of shs 18.45 per 7 h day. Cost includes fringe benefits – noon meal, transportation, housing, medical care, insurance, and pension payments – amounting to 50% of wages)	
Total cost	65.69

Finally, combining eqs. (11) and (14), total logging cost, C_L, is:

$$C_L = C_F + C_S = 2.41/V_L. \tag{15}$$

6. Return from commercial thinnings

The model in eqs. (2)–(15) can be used to estimate the value yield of the commercial thinnings required by the schedule in table 1. However, the study by Adegbehin and Philip [1] indicates that the average total height of trees removed in thinnings is less than the average height of the dominant trees. The relationship in their report is given as follows:

$$H_T = 0.77 + 0.84 H_D, \tag{16}$$

where H_T = average total height of trees removed in a thinning, in m.

To estimate the value yield of commercial thinnings, eq. (16) was used to substitute H_T for H_D in eqs. (6) and (8). It was assumed that the volume of the average tree removed is 85% of the average volume per tree prior to thinning, based on an analysis of thinning data summarized in ref. [1]. Furthermore, it was assumed that the number of trees removed in each thinning follow the schedule in ref. [9]. Finally, following a suggestion by Salehe [17], skidding cost

Table 4
Value yield, net of logging costs, from commercial thinnings (in current shillings)

Age (y)	N_T trees/ha removed	D_B of trees removed (cm)	C_L (shs/m^3)	R [a)] (shs/m^3)	V_T (m^3)	Value yield $(N_T)(R-C_L)(V_T)$ (shs/ha)
13.5	370	20.9	15.94	20	0.33	498
17.5	250	28.0	10.95	20	0.69	1562
21.5	120	36.9	7.83	65	1.28	8806

[a)] R = royalty paid, in shs/m^3, based on D_B and the schedule in table 2.

for thinnings was estimated to be 15% higher than that computed by eq. (14).

The estimated value yields from commercial thinning, net of logging costs, are listed in table 4.

7. Optimal rotation age

Eqs. (2)–(15) plus the information in tables 1–4 provide the information necessary for computing a solution to the nonlinear programming problem in eq. (1). While eqs. (2)–(15) all give rise to a convex function in S_e, the royalty schedule in table 2 is nonconvex, and this introduces the difficulty that the Kuhn–Tucker theorem [12] cannot be relied upon to define necessary and sufficient conditions for a global optimum. However, the data on which the model is based are sufficiently imprecise that it would be unreasonable to define the optimal rotation age more closely than the nearest integer year. A global optimum to the *integer* nonlinear programming problem in eq. (1) can then be guaranteed by executing the following direct-search procedure:

(1) Initialize rotation age, r, at some sufficiently low level, say 15 y, which will guarantee that r is on the "low" side of the optimal rotation age, r^*.

(2) Using eqs. (2)–(15), substitute r for A to compute N, D_B, V_M, and C_L; select the royalty price, R, associated with D_B. Compute the current value yield at rotation age r:

$$Y_r = (R - C_L)V_M N. \tag{17}$$

Using Y_r and data from tables 1 and 4, compute $S_e | r$ from eq. (1). Save this value.

(3) Increment r by one year.

(4) Repeat steps (2) and (3) until $S_e | r$ begins to decline when $D_B \geq 56$. This value of D_B corresponds to $R = 125$ (table 2); for all larger values of D_B

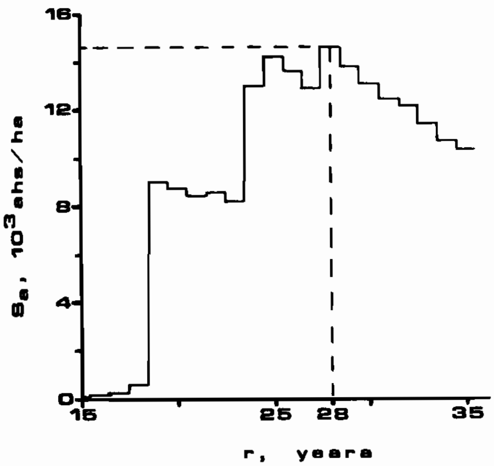

Fig. 3. Soil expectation value as a function of rotation age for Site I *Pinus patula* at Sao Hill. The global (integer) optimum occurs at a rotation age of 28 y.

(and hence all larger values of r), R is constant and thus convex. When S_e begins to decline beyond this point it will never again increase.

(5) Choose the optimal rotation age, r^*, corresponding to the maximum value of S_e saved in step (2).

The simplicity of this procedure is a considerable advantage for doing an analysis of this type in a developing country where access to even microcomputers is rare. For the present analysis the procedure was done entirely on a Hewlett-Packard Model 97 programmable calculator. Such calculators are within the budget means of any of the forest projects in Tanzania.

The fact that the schedule of commercial thinnings in tables 1 and 4 is fixed by law introduces a complication which must be accommodated. Consider $r = 20$. This rotation age falls at a time when the stand will not yet have recovered from the thinning at age 17.5 (see fig. 2). However, it is unlikely that a thinning would be undertaken at 17.5 y if the stand were to be clearfelled at 20 y. thus, in this analysis it has been assumed that the thinning scheduled for year t will not be undertaken unless $(r-t) \geq 4$ for the first and second commercial thinnings and $(r-t) \geq 3.5$ for the third commercial thinning. These intervals reflect the recovery periods shown in fig. 2.

The response surface relating soil expectation value to rotation age for this analysis is shown in fig. 3. As indicated, the optimal rotation corresponds to a stand age of 28 y. This represents a 12% increase beyond the established rotation of 25 y and a corresponding increase in soil expectation value of 2.3%. Comparisons with the culmination of mean annual increment (MAI) rotation show even greater differences. Using eq. (3),

$$MAI = [1400 - 1840.5 \exp(-0.0477r)]/r.$$

Culmination of MAI occurs at the point where $\mathrm{d}(MAI)/\mathrm{d}r = 0$:

$$\mathrm{d}(MAI)/\mathrm{d}r = (87.79r + 1840.5)\exp(-0.0477r) - 1400 = 0. \tag{18}$$

Solving eq. (18) numerically, $r^*_{MAI} = 19.54$, or approximately 20 y. Thus, r^* from fig. 3 represents a 40% increase in rotation age and a 66% increase in soil expectation value by comparison with the optimal MAI rotation.

8. Effect of logging costs

Since this analysis assumes that logging costs influence the determination of optimal rotation age, it is of interest to test the hypothesis explicitly. This was done by solving eq. (1) with $C_L = 0$ (any fixed value for C_L would yield an identical value for the optimal rotation age). With logging costs thus removed, the optimal rotation age was reduced to 25 y, a change of 12%. Clearly, then, logging costs significantly influence the determination of optimal rotation age because of the relationship between logging costs and tree size as expressed by eq. (15). The arguments for including logging costs in the analysis were advanced in section 3. Whether they will be incorporated in future determinations is, of course, a matter which must be decided by persons responsible for forest policy decisions.

9. Discussion

The irregular shape of the response surface (fig. 3) is due entirely to the influence of the royalty price schedule (table 2). If the royalty schedule were to be replaced by a convex function (such as a linear equation), then the response surface would become unimodal. Abrupt increases in the magnitude of soil expectation value at rotation ages of 19, 24, and 28 y are due to the fact that in each case the trees reach an average diameter during that year which corresponds to one of the thresholds in table 3. The final royalty price increase occurs at $r = 32$ y, but at this point the marginal increase is so small relative to the influence of the discount rate that the net effect registers as a small decline in soil expectation value. This suggests that both the royalty schedule and the discount rate should be carefully reexamined in the context of their influence on forest policy in Tanzania. Such an examination has not been undertaken in the present study because, so far as the Forest Project Manager at Sao Hill is concerned, both are fixed by law. Forest management decisions at Sao Hill must therefore be taken in the context of both the existing royalty schedule and the specified discount rate.

10. Concluding remarks

The results of this analysis must be considered preliminary, and in any case are limited to a single site class of *Pinus patula*. Perhaps the most important conclusion to be drawn from the exercise is the fact that the treatment of logging costs as a function of stand age tends to increase optimal rotation length. In short-rotation tropical plantations, an increase of only a few years can be substantial.

It should be noted that the soil expectation value for the 28 y rotation is only slightly more than that for the 25 y rotation. Other factors not quantified here, such as the risk of fire, might thus tend to encourage the selection of the shorter rotation. On the other hand, if log hauling and milling costs are shown to be functionally related to log size, then the optimal rotation age might be extended even further. A recent analysis by Ringo and Klem [16] suggests that wood density in *Pinus patula* is related to stand age, and that harvesting of stands which are too young produces wood with low strength properties. If this effect could be quantified in economic terms, it would tend to support longer rotations.

Finally, this analysis has treated the current schedule of thinnings as fixed. This is appropriate for an analysis done from the point of view of the Sao Hill Forest Project Manager, since the thinning schedule has been specified in a legally enforceable forest ordinance [9]. However, a high priority should be given to the acquisition of data on the response of *Pinus patula* to various thinning treatments. This would enable the simultaneous determination of optimal thinning regime and optimal rotation age. Experience with other species (e.g. refs. [2,4]) suggests that such an analysis might significantly increase both soil expectation value and optimal rotation length in tropical softwood plantations.

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ECONOMIC ANALYSIS OF SHORT-ROTATION FORESTS FOR ENERGY IN IRELAND *

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Short-rotation coppice forests can provide biomass fuels on a sustained basis to replace declining peat supplies for process heating and electricity generation in Ireland. Economic analysis of energy plantation establishment, production and harvesting indicate that cost per metric ton of chips produced and delivered would be most sensitive to yields, cutting cycle and management costs. Total costs range from IR£10.92 to IR£34.87 per green metric ton for a 5 y cutting cycle, the most cost effective harvesting frequency. Real returns depend on fuel chip prices and range from 6.9 to 20.7% when prices are based on the energy equivalence of oil. This study shows that production of biomass for energy can have attractive returns, offset increasing imports of foreign energy supplies, and provide a suitable land use for harvested peat bogs.

1. Introduction

Increasing costs of fossil fuels have directed much attention towards renewable energy sources. Ireland, a country poor in indigenous energy supplies, but with a low population density, may find biomass a particularly promising alternative to imported fuels. Short-rotation coppice forests may have potential for production of process heat and electricity in existing utilities (Szego and Kemp 1973, Howlett and Gamache 1977, Neenan and Lyons 1980a). This paper investigates the economics of short-rotation forestry energy plantations on low-grade land in Ireland. A financial modelling approach is applied to examine production economics and identify optimum plantation decisions.

Ireland has extensive areas of mined-out peatland known as "cutaway". This shallow bogland is of low agricultural potential, but may be suitable for energy plantations which would maintain local fuel production enterprises. This concept might be suitably adapted to other low-grade soil types, such as

* This work is part of a study being conducted under contract from the Science, Research and Development directorate (DGXII) of the Commission of the European Communities.

glacial soils, occurring in the north-western counties. However, it may have more immediate potential on peatland cutaway in the midland plain. Here, existing fuel production facilities, including transportation systems and peat-fueled power stations, would allow rapid development of energy plantations.

About 250 thousand hectares of peatland cutaway occur in the midlands. A further 70 thousand hectares of bogland, currently under development by Bord na Mona (the Irish Peat Development Board), will become available as cutaway between 1985 and 2030 (Healy 1979).

Production of short-rotation coppice forests could eventually replace the finite peat resource for fuel, should the economics prove favourable. At present, 13.7% of total primary energy demand and up to 16% of national electricity output is generated from indigenous peat fuel (Electricity Supply Board 1978–1981). Peat-based power stations are mainly on the midland bogs and range in generating capacity from 40 MW to 90 MW. These stations are designed to burn peat in either sod or milled form, but with some modifications could accommodate wood chips as the main or supplementary fuel.

Space and process heating in the commercial, domestic and industrial energy demand sectors may also become a large market for the wood biomass; fuels directly consumed by these sectors currently account for 45% of the nation's total primary energy demand (Department of Energy 1981).

2. Analysis

Our main objective was to examine the economics of short-rotation biomass plantations by detailed financial systems analysis. Performance of plantations is presented for comparison with other opportunities for development of the study regions. Decisions relating to plantation management, such as planting density, cutting cycle, rotation and fuel market price, are enabled by the analysis.

The FORTRAN IV computer model, SALIX, that was developed for the study, considers biomass yields, production costs and fuel revenues within a framework of scheduled management activities. These activities may range from site preparation to harvesting, transportation and fuel sales. SALIX provides a series of economic indicators, including biomass production cost per metric ton, rate of return, net present value and benefit-cost ratio, for the plantation lifetime. These may be evaluated for both single and perpetual rotations. Biomass yields are read (or interpolated) from tables provided by the user. Estimates of optimum cutting cycle and planting density may be derived from several model runs. SALIX also shows the effects of economic variables such as discount rate, costs, revenues, and yields on financial performance. The SALIX model is described in the Appendix.

Four steps are required in developing a SALIX analysis: (1) identification of plantation management options, (2) preparation of production and harvesting cost estimates, (3) establishment of biomass yield expectations, and (4) derivation of appropriate fuel prices. Provided with this data, the model conducts a complete investment appraisal of any proposed scenario.

2.1. Management options

A separate economic analysis was made for each of 7200 production options. Each option consisted of a different combination of planting density, cutting cycle and rotation age (i.e. age of rootstock replacement), production cost, discount rate, fuel chip price, and estimated yield. Planting densities used were 6666 stems per hectare (300 × 50 cm spacing), 10 000 stems (200 × 50 cm), 13 333 stems (150 × 50 cm), and 20 000 stems (100 × 50 cm). Cutting cycle ranged from 2 to 5 y with rootstocks replaced after 24 y (25 y for a 5 y cutting cycle). This resulted in 12 cuts for a 2 y cycle, 8 cuts for a 3 y cycle, 6 cuts for a 4 y cycle, and 5 cuts from a 5 y cycle. Discount rates of 6, 8 and 10% in real terms (i.e. above inflation) were used. Production costs included a high, medium and low cost scenario. These are described in further detail below. Fuel chip prices considered were 9.95, 16.00, 22.40, 29.80 and 48.33 pounds Irish per green metric ton delivered. Their derivation is discussed below.

The investment alternatives were evaluated for a single initial rotation and for perpetual rotations. Fixed costs for land preparation are included only in initial rotation costs as they do not recur for later plantings. Also, yields were varied to reflect 75, 90, 100, 110 and 125% of expected yields.

2.2. Production and harvesting costs

Biomass production includes 12 different activities which are classed as (1) land preparation, (2) plantation establishment and management, and (3) utilisation. Low, medium and high costs were assigned for each activity (table 1); these costs represent current commercial rates for similar activities performed in agriculture and conventional forestry. Land costs reflect recent market sale values for low-grade land in the cutaway and glacial soil categories. All production costs were specified in real terms.

Land preparation includes purchase, roading, drainage, fencing and site preparation. Plantation establishment was by rooted cuttings and it was assumed that 10% of initial planting material would not survive the first growing season[1]. Dead plants would be replaced in the first year. Site management involves the periodic application of fertiliser and regular maintenance operations to roads, drains and fencing, as well as weed control. Site

[1] Survival rate of cuttings planted in short-rotation forestry experimental plots, in Ireland, has been in excess of 90% (Neenan and Lyons 1980b). A 10% failure rate is assumed here.

Table 1
Cost assumptions used in the analysis (IR £ per unit of activity)

Activities	Units	Management cost £(Irish)		
		High	Medium	Low
Land preparation				
Land acquisition	ha	2,500	1,000	750
Site preparation	ha	350	100	40
Drainage	ha	200	100	50
Fencing	ha	200	150	100
Access	ha	350	150	50
Plantation establishment and management				
Planting	plant	0.125	0.085	0.040
Plant replacement	plant	0.125	0.085	0.040
Fertilisation	ha	94.5	81.0	00.0
Maintenance	ha	120.0	87.5	66.5
Regeneration	ha	200.0	100.0	50.0
Utilisation				
Harvesting	ton	5.00	3.00	1.80
Transportation	ton-km	0.060	0.036	0.036

regeneration by ploughing and stump removal takes place at the end of the useful lifetime of initial planting stock, which is assumed to be about 24 y.

Complete harvesting systems for short rotation coppice forests are not as yet commercially developed. The system envisioned in this analysis involves felling with a feller-buncher, skidding to a central location, and chipping in the field with a mobile chipper. The chipped material would be blown into 20 metric ton wagons for delivery. Costs were calculated for three production rates, 12, 20 and 37.5 metric tons per hour, for the high, medium and low cost scenarios, respectively. An average transportation distance of 20 km was assumed in determining delivery costs. Harvesting and transportation costs include capital, interest, maintenance, fuel and labour charges. These cost estimates are given in table 1, and were assumed to increase at 1.5% per year, in real terms.

2.3. Biomass yields

Complete yield data for species suitable for planting on low-grade soils are unavailable at present. Trials with varieties of willow, including *Salix viminalis* and *Salix aquatica gigantea*, have shown significant potential in preliminary yield studies conducted by the Irish Agricultural Research Institute and by the Northern Ireland Ministry of Agriculture (Neenan and Lyons 1980b). These early results have proven to be similar to yields reported for American Sycamore (*Platanus occidentalis*) by Dutrow (1971) and Dutrow and Saucier (1976).

Table 2
Cumulative yields per fully stocked hectare given in green metric tons of standing biomass for initial planted stands and subsequent coppice stands (Source: Based on Dutrow (1971), and Dutrow and Saucier (1976))

Density (stems/hectare)	Stand age (y) 2	3	4	5
	Initial planted stand yields			
6666	9.0	34.7	55.9	78.2
10000	12.3	39.1	61.5	85.4
13333	15.6	43.6	67.1	92.3
20000	21.2	47.0	70.0	92.8
	Subsequent coppice stand yields			
6666	17.9	50.4	80.6	114.8
10000	22.9	56.5	89.0	124.3
13333	27.9	62.6	97.3	133.9
20000	31.9	67.7	101.4	136.6

Although actual yields of willow or other species may exceed or fall short of reported sycamore yields, the latter formed the basis for yield estimates. Yield tables for both initial and coppice cutting cycles, representing 100% of sycamore yields, were input to the SALIX model (table 2). These tables provide a more accurate simulation of short-rotation coppice growth than a single assumed mean annual increment figure. To account for possible variation in actual willow yields, a range from 75% to 125% of reported sycamore yields was included. This range should adequately represent yield expectations of cutaway and other selected sites with proper management.

Dutrow and Saucier (1976) suggest that soil compaction by equipment and stem damage during harvest may cause deterioration of rootstocks. Consequently, a decline in yields of 0.35% per year was built into the economic analysis. This decline amounts to between 8 and 9% over a 24 y rotation (Dutrow and Saucier 1976).

2.4. Fuel prices

The technical feasibility of using wood chips as a primary combustion fuel for process heat and electricity generation is well developed (Karchesy and Koch 1979). Initial tests have shown that chips can be burned efficiently at a moisture content up to 35% of wet weight on travelling grate furnaces in Ireland and the U.S. (O'Connell and Gallagher 1980, Morford 1978). Such furnaces are installed in Irish sod-peat burning power stations.

In the economic analysis presented, wood chips were assigned prices equivalent to the fuel prices of peat, coal and oil. In estimating fuel equivalents, we considered conversion efficiency as well as gross heat content of each fuel. Economic comparisons of fuel value are based on the actual amount of primary heat derived from fuel combustion in large scale boilers. This takes into account the heat value and moisture content of the fuel as fired and the efficiency of the boiler. The gross calorific value of oven-dry wood chips is about 18.6 megajoules per kilogramme (8000 Btu per pound) (Rose and Cooper 1977). After adjustments for moisture content and losses due to boiler efficiency, the useful fuel energy of green chips is reduced to 6.13 MJ/kg (2640 Btu per pound).

Availability of an energy market for wood chips is fundamental to the system economics. The expected fuel chip price is sensitive to projected market prices for the cheapest competing fuel, such as sod and milled peat. On the other hand, the existence of alternative markets for wood chips might establish a premium price. At present, extensive supplies of fuelwood are not available in Ireland from any domestic sources. Further, no domestic markets are developed for utilisation of small diameter wood or chips.

Five values were assigned to delivered green wood chips; each represents the current (or projected) price of an equivalent amount of an alternative fuel. In each case, the chip price was set at the amount which could be paid to purchase an equivalent amount of useful fuel energy in the form of chips. For example, one of the values assigned to a metric ton of green wood chips is £29.80 which is the cost of one-third metric ton of bituminous coal – enough to yield an equivalent amount of energy. Fuel values were assumed to be increasing at an average annual rate of 3.5% above inflation. The five fuel values used for the analysis are shown in table 3, together with the conventional fuel prices for comparison. Prices ranged from a low value of £9.95 per green metric ton delivered when compared with milled peat to a high value of £29.80 per metric ton of chips to replace bituminous coal. In addition, a price of £48.33 was used in the analysis as an upper limit on prices. This chip price corresponds to a projected 1985 fuel oil price, extrapolated from the oil price inflation trend since 1974.

Table 3

Comparison of equivalent fuel values (Sources: Bord na Mona (Irish Peat Board): Annual Reports 1979/80–1980/81. Department of Energy, Dublin. Electricity Supply board: Annual Reports 1980–1981.)

Chip value, green (IR£/ton)	Alternative fuel	Final price (IR£/ton)	Year
9.95	Milled peat	5.67	1979/80
16.00	Sod peat	11.50	1979/80
22.40	Fuel oil	120.75	1980/81
29.80	Bituminous coal	89.40	1980/81
48.33	Fuel oil	261.00	1985 (Projected)

3. Results

Production of short-rotation forestry wood fuel returned at least the investment cost for most levels of management and chip prices tested. Only for a chip price of £9.95 per metric ton and high management costs was the rate of return negative.

In general, the cutting cycle which maximises rate of return decreases as fuel prices increase. Further, as management costs decrease the cutting cycle which maximises rate of return also decreases. Hence, if maximum rate of return is the chosen decision criterion, the cutting cycle can be selected on the basis of density and price.

3.1. Cost per ton

Cost per metric ton delivered to the conversion facility as green chips was considered as one of the most important criteria. Results were calculated for discount rates of 6, 8, and 10% in real terms, but only values discounted at 8% are shown in this paper.

Average production costs for delivered fuel chips ranged from £10.92 to £24.02 per metric ton for the low cost scenario. Medium costs ranged from £18.06 to £39.63 per metric ton and high costs were £31.83 to £70.87 per metric ton. These costs assumed an 8% discount rate in real terms for perpetual rotations. The lowest cost per ton was for plantations harvested every 5 y for all cost scenarios. Plantations with densities ranging from 6666 to 13 333 produced fuel at nearly the same cost. Table 4 shows the relationship between cutting cycle, planting density, and cost per ton for the high, medium and low management cost levels.

Average cost per ton was relatively insensitive to the range of planting densities used in this study. For a 5 y cutting cycle, the average cost per ton varied less than £0.50 for densities from 6666 to 13 333 stems per hectare for all cost scenarios. Densities of 10 000 or 13 333 stems per hectare may be preferable to 6666 stems even though production costs may be greater. Higher planting densities can exclude competition from undesirable weed species and achieve crown closure more quickly. Needs for cultivation or weed control may thereby be reduced. Also, somewhat higher average yields per hectare result from higher planting densities; this reduces the need for additional land to meet production requirements. However, planting densities may ultimately be determined by width specifications of cultivation and harvesting equipment.

With cutting every 2 y, cost per metric ton of chips varied by less than £0.50 for densities of 13 333 to 20 000 stems per hectare. However, at the same cutting cycle, a density of 6666 was 25 to 30% more costly than 13 333 stems per hectare. This difference, amounting to almost £16.50 per ton for the high management cost scenario, occurs at low densities because the stand does not fully occupy the site before harvest.

Table 4
Cost per green metric ton (£Irish) by density, cutting cycle, and management cost [a]

Cutting cycle	Planting density (plants/hectare)			
	6666	10000	13333	20000
	High management cost			
2	70.87	60.90	54.47	53.99
3	40.77	39.55	38.58	40.34
4	35.32	34.74	34.28	36.92
5	31.96	31.89	31.83	34.87
	Medium management cost			
2	39.63	34.61	31.37	31.73
3	22.94	22.58	22.29	23.74
4	19.93	19.87	19.84	21.75
5	18.06	18.26	18.43	20.55
	Low management cost			
2	24.02	20.66	18.50	18.29
3	13.99	13.56	13.21	13.75
4	12.14	11.93	11.76	12.59
5	10.99	10.95	10.92	11.89

[a] Discount rate for cost calculations is 8%. Costs shown are per green metric ton of fuel chips delivered to the conversion facility and calculated for perpetual rotations.

Calculations on a perpetual basis rather than for the first rotation reduces costs by as much as 9%. Minimum costs per metric ton for a single rotation were £11.49, £19.06 and £33.90 at an 8% discount rate. The initial rotation is loaded with early fixed costs for land purchase, drainage, roading and fencing that do not recur in subsequent rotations.

3.2. Rate of return

The internal rate of return was sensitive to yield, cutting cycle, and fuel chip price for each set of cost assumptions. Returns ranged from negative for high costs to 9.6% for low costs at a chip price of £9.95 per metric ton. At £48.33 per ton, returns were as high as 37.4% for low costs. Rates of return on perpetual series are shown in table 5. All returns are shown in real terms.

For a high cost scenario, return was maximised by a 5 y cutting cycle, regardless of chip prices. At medium costs, a 5 y cycle generated the highest return for all chip prices except £48.33, for which a 4 y cycle was preferable. At low prices, a 5 y cycle had highest returns at prices up to £16.00, 4 y cuts were highest for a price of £22.40, and 3 y cuts had highest returns with chip prices above £29.80.

Fig. 1 shows the effect of fuel chip prices on rate of return. The band shown

Table 5
Rate of return (%) by cutting cycle, management cost level and price of fuel chips. Values are real rates of return calculated on perpetual investments. N stands for negative rate of return. The highest rate of return among all densities is shown. The density is indicated in parenthesis as (A) – 6666 stems per hectare, (B) – 10000, (C) – 13333, (D) – 20000

Cutting cycle	Current fuel chip price (IR£/ton, green)				
	9.95	16.00	22.40	29.80	48.33
	High management cost				
2	N	N	2.5 (D)	5.1 (D)	10.3 (D)
3	N	2.3 (C)	5.3 (C)	8.2 (C)	14.2 (C)
4	N	3.1 (C)	6.2 (C)	9.2 (C)	15.0 (C)
5	N	3.8 (C)	6.9 (C)	9.7 (C)	15.2 (C)
	Medium management cost				
2	N	3.7 (D)	7.5 (D)	11.2 (C)	18.8 (C)
3	2.1 (C)	7.4 (C)	11.7 (C)	15.9 (C)	24.5 (C)
4	3.2 (C)	8.4 (C)	12.7 (B)	16.8 (A)	25.0 (A)
5	4.0 (C)	9.3 (A)	13.4 (A)	17.2 (A)	24.7 (A)
	Low management cost				
2	4.5 (D)	10.2 (D)	15.2 (D)	20.3 (D)	31.2 (D)
3	7.9 (C)	14.5 (C)	20.1 (C)	25.7 (C)	37.4 (C)
4	9.0 (C)	15.4 (C)	20.7 (C)	25.8 (C)	36.1 (C)
5	9.6 (C)	15.6 (C)	20.6 (A)	25.3 (A)	34.4 (A)

for each cost level represents the range of returns for 2 y to 5 y cutting cycles at the most profitable density for each cycle. Results for fully stocked yields are shown.

3.3. Yield effects

Results reported so far in this study have assumed yields expected for sycamore plantation trials in the Southern United States. Although early willow yields in Ireland are similar, full rotation yields may fall short or exceed these expectations. Tree growth, management practices, site conditions, or other factors may cause yields to vary by as much as 25%. The effect of this yield variation on production costs and returns was tested.

In general, cost per ton would increase and rate of return would decrease if expected yields were not achieved. If yields exceeded the expectations, cost per ton would drop and return would increase, but this relationship is not linear. Effects are shown in table 6 for one selected plantation option – 13333 stems per hectare and a 5 y cutting cycle. This option is a least-cost combination of density and cutting cycle.

At 75% of expected yields, cost per ton increased by 27% for high and medium management costs, and 26% for low costs. A 25% increase in expected

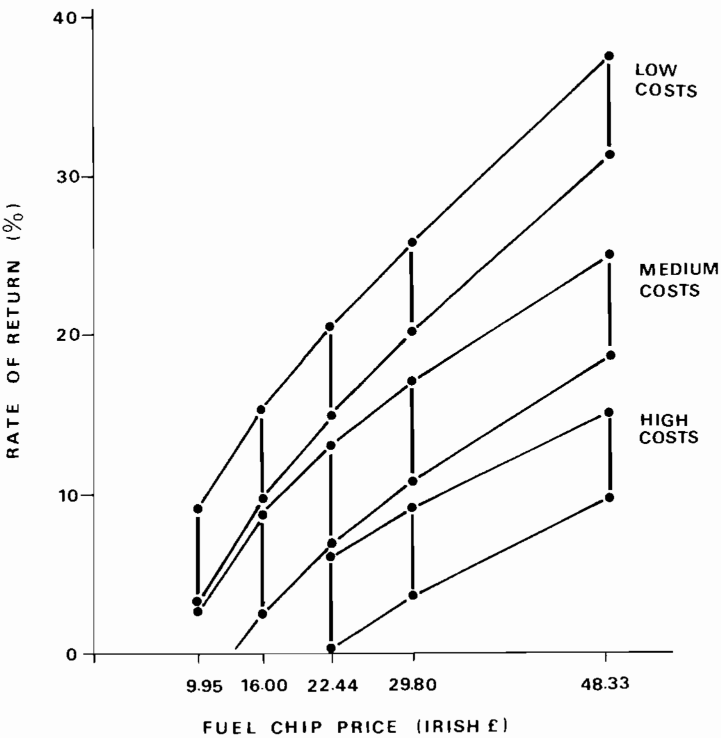

Fig. 1. Rate of return by fuel chip price for high medium and low cost scenarios. Range of returns for each cost-price combination is for 2 y to 5 y cutting cycles with the high-return density for each.

yields would reduce cost per ton by about 16% for all management costs.

4. Summary

Short-rotation energy plantations on low-grade land in Ireland can produce fuel chips at costs competitive with conventional fuels. Biomass produced on this land can replace peat in existing electrical generating facilities and provide space and process heating for industry. Plantations of 13 333 stems per hectare with harvests every 5 y yield more than 20 green metric tons per hectare annually for the least cost.

The effects of cutting cycle, planting density, management cost level and fuel chip price were tested on single and perpetual rotations at three discount rates. Deviations from expected yields were also considered. Average annual yields ranged from 8 to 24 green metric tons per hectare for cutting cycles of 2

Table 6
Effect of yield variation on cost per metric ton and rate of return for planting density 13 333 and 5 y cutting cycle. Discount rate for cost calculations is 8%. Rate of return and costs are given for perpetual rotations. N stands for negative rate of return

Per cent of expected yields	Cost per ton (£Ir)	Rate of return (%)				
		Current fuel chip price (£Ir)				
		9.95	16.00	22.40	29.80	48.33
	High management cost					
75	40.41	N	2.1	4.8	7.3	12.2
90	34.69	N	3.2	6.1	8.8	14.1
100	31.83	N	3.8	6.9	9.7	15.2
110	29.48	0.4	4.4	7.6	10.6	16.3
125	26.67	0.9	5.2	8.6	11.8	17.8
	Medium management cost					
75	23.36	2.0	6.5	10.1	13.4	19.8
90	20.07	3.2	8.1	11.9	15.4	22.3
100	18.83	4.0	9.0	13.0	16.7	23.9
110	17.08	4.7	9.9	14.0	17.9	25.4
125	15.47	5.7	11.2	15.5	19.6	27.5
	Low management cost					
75	13.73	7.1	12.4	16.7	20.8	28.8
90	11.86	8.7	14.4	19.0	23.4	32.1
100	10.92	9.6	15.6	20.5	25.1	34.1
110	10.15	10.5	16.8	21.8	26.6	36.0
125	9.23	11.8	18.4	23.7	28.8	38.6

y to 5 years at planting densities of 6666 to 20 000 stems per hectare. Costs per green metric ton delivered were 31.83, 18.06 and 10.92 pounds Irish for high, medium and low management cost levels when discounted at 8%. Rates of return depended on fuel chip price but were as high as 6.9, 13.4 and 20.6% in real terms for high, medium and low management costs and a price of £22.40, the energy equivalent price for fuel oil.

The factors most critical to the economic feasibility of short-rotation forestry energy plantations are biomass yield, management costs and fuel price. A species that can sustain an average annual green weight yield of 20 metric tons per hectare could result in a biomass production cost as low as £18.06 per metric ton, with medium range costs assumptions. This, in fuel energy terms, is equal to £97.52 per ton of oil equivalent.

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Appendix. Description of SALIX model

SALIX is a financial analysis model designed for economic assessments of biomass energy plantations, such as short-rotation forests. In an investment analysis approach, it addresses the effects of biomass yield, production costs, fuel prices, management options and economic factors on financial returns. It thus enables the comparison of alternative production strategies (species, cultivations, fertilisation rates, etc.), harvesting/transportation systems, and market opportunities (for stumpage, wood fuel chips, or secondary fuels), as well as plantation design factors (cutting cycle, planting density, and overall rotation).

The main feature of SALIX is a discounted cash flow procedure which provides a full range of economic indicators, including: present net worth, present value of costs and revenues, annualised equivalent net worth, annualised equivalent costs and revenues, and a benefit/cost ratio. The secant method for solution of non-linear equations is employed to determine the internal rate of return; and the production cost per tonne of fuel is derived from yield tables or equations supplied by the user. For each run, these indicators are evaluated for cash flows resulting from combinations of 5 planting densities, 5 yield levels and 5 separate sets of activity costs and product prices. These are discounted by up to 3 discount rates, for both single and perpetual rotations. Each specified cost or price can be assigned a unique inflation rate; a general inflation rate may also be applied.

Inputs to the model consist of (1) activity profiles (activity descriptor, value, type, inflation rate, etc.), (2) yield tables (seedling and coppice cycles at different planting densities and stand ages), (3) activity schedule (timing of activities), and (4) run control data (discount rates, selected planting densities, run I.D. etc.).

In the analysis presented here, (1) activity profiles were based on the operations and costs presented in table 1; (2) yield tables were those presented in table 2; (3) activity schedules depended on selected cutting cycle within the range 2 to 5 y; and (4) run control data enabled analysis of 3 discount rates, 6, 8 and 10%, and 4 planting densities (as shown in table 2).

During each run, the model develops a year to year cash flow table from the inputed activity profile and schedule; a growth routine reads or interpolates the required yield information from the data provided, and also applies a yield decline multiplier, if specified. The secant and discounted cash flow routines evaluate the economic indicators described above. And finally, these indicators, a cash flow table, and a graph of present net worth against discount rate, are provided as output.

SALIX is written in ANSI standard FORTRAN IV and has about 1200 statements grouped in 9 subroutines and a main control program. It currently operates in batch mode and input is by punched cards, magnetic disk or tape. A typical input deck consists of about 70 cards. The program is currently installed at the USDA Forest Service, Southeastern Forest Experiment Station, on an IBM 370; and at the Agricultural Institute, Dublin, on a DEC VAX 11/750. The program and further documentation (SALIX – User's Guide, 1981) is available from the authors.

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A LINEAR PROGRAMMING MODEL FOR ANALYZING ECONOMIC, SOCIAL AND ENVIRONMENTAL IMPACTS OF WOODFUEL POLICY ALTERNATIVES IN THE PHILIPPINES *

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This paper represents an application of the FUELPRO model to analyze the policy options for dealing with woodfuel shortages. The first stage of the model combines quantitative and qualitative information from an extensive benefit–cost analysis. In the second stage, a linear programming model is employed for optimal selection of the policy options. The data in this application are largely derived from the author's studies in the Philippines.

1. Introduction

The problems of woodfuel (wood and charcoal) shortages and associated deforestation are receiving increasing attention in developing countries; e.g., in the Philippines (Hyman 1983b). In this article, a linear programming model, called FUELPRO, is employed to allocate resources among nine policy options (table 1) in order to deal efficiently with these problems. This model is designed to aid decision makers in selecting an appropriate mix of such policy options.

The approach consists of two stages. In the first stage, scaled goals-achievement scores are calculated for each policy option k ($k = 1, 2, \dots, 9$) in terms of five objectives $i (i = 1, 2, \dots, 5)$. The benefits and costs that can be expressed in monetary terms are first escalated to a common base year (1981) according to the consumer price index (Central Bank of the Philippines 1981) and then discounted. Shadow prices are employed to reflect the social cost of using scarce resources of the national economy. Some scores are qualitative. All objective scores will be placed on a common scale ranging between -1 and $+1$. Scaled scores, denoted by S_{ik}, are obtained by dividing each score by the highest absolute value of the scores for the same objective among all policy options (Miller 1980).

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Table 1
Policy options employed in the FUELPRO model

k	Policy option	Definition	Unit
1.	Reforestation	Are of public lands reforested and maintained as protection forests	Hectares
2.	Treefarming loans	Area of private smallholder tree-farms at the village-level, financed through government loans	Hectares
3.	Resettlement	Area of deforested public lands leased to small farmers for a mixture of forestry and agri-cultural purposes	Hectares
4.	Bamboo substitution	Substitution of other biomass fuels such as bamboo and coconut parts for fuelwood	Number of house-holds switching from fuelwood to other biomass fuels
5.	Woodstoves adoption	Adoption of more fuel-efficient woodstoves	Number of house-holds adopting improved stoves
6.	Charcoal kiln adoption	Adoption of more efficient charcoal-conversion techniques	Number of open charcoal pits replaced by metal drum kilns
7.	LPG substitution	Substitution of liquefied petroleum gas for fuelwood in cooking	Number of house-holds switching from fuelwood to LPG
8.	Electricity substitution	Substitution of electricity for fuelwood in cooking	Number of house-holds switching from fuelwood to electricity
9.	Kerosene substitution	Substitution of kerosene for fuelwood in cooking	Number of house-holds switching from fuelwood to kerosene

In the second stage, the scaled objective scores S_{ik} are employed in the objective function of a linear programming model. By adopting a set of relative social welfare weights w_i for the objectives, a scalar objective is computed representing the utility of each policy mix. This linear programming model also contains constraints to ensure that certain energy resource and environmental quality restrictions are met.

Formally, the model structure may be summarized as follows:

$$\underset{x_k}{\text{maximize}} \sum_{k=1}^{9} x_k \sum_{i=1}^{5} w_i S_{i_k}$$

subject to constraints on energy supply and environmental quality, where x_k is the level of policy option k to be chosen in a policy mix.

The linearity assumption in the model is justified by the nature of the policy options considered: smallholder forestry, household substitution of other fuels, and adoption of more efficient wood use technologies are unlikely to be subject to economies of scale. The static characteristic of a linear programming model does not pose a problem because the optimal rotation length for production of Giant Ipil-Ipil fuelwood in the country has already been determined by forestry experts.

Such linear programming models are commonly used in forestry applications, particularly in the determination of forest rotations (see e.g., Schreuder 1971, Ware and Clutter 1971, Navon 1976, and Johnson and Scheurman 1977). In order to account for multiple goals, goal programming and other multiobjective optimization techniques may be used (Bell et al. 1977, Starr and Zeleny 1977). Scaling and weighting approaches, in particular, have been proved useful in urban and regional planning, water resources management, and in forestry. For applications in forestry, see for example Hartman 1976, Calish et al. 1978, Nguyen 1979.

2. Data

The study region is the province of Ilocos Norte in the Philippines. This province of 373 000 hectares (ha) is located in the upper northwest part of Luzon. Excluding the coastal fringe, it is primarily mountainous. A Landsat analysis estimated that degraded second-growth forest of noncommercial species occupies 54 500 ha in Ilocos Norte. Since the province has only 24 000 ha of old-growth forest, most of which is inaccessible, there is no logging for domestic or export production. There are 270 000 ha of noxious Imperata grasslands, open lands, and developed areas. Nevertheless, forest cutting rates are high due to fuelwood, polewood, and construction wood demands, and in some areas, slash-and-burn agriculture. The population of Ilocos Norte in 1980 was 393 000, comprising of approximately 17 000 urban and 58 000 rural households.

Much of the data comes from a survey of the demand for energy in Ilocos Norte (Hyman 1984a, b). Additional data sources were necessary for information on the costs and benefits of treefarming (Hyman 1983a, d, and 1984c, Philippine Bureau of Forest Development (BFD)); wood yields (Bonita 1981); costs and benefits of agroforestry crops (Philippine National Economic and Development Authority (NEDA) 1975, Deomampo 1976, Coronel 1979); woodstove efficiencies and costs (Gould and Joseph 1979, Earthman Society 1980, Volunteers in Technical Assistance (VITA) 1980); charcoal kiln efficiencies and costs (Earl 1974, Medrano et al. 1977); and population and incomes

(Philippine National Census and Statistics Office (NSCO) 1975 and 1980, U.S. Agency for International Development (AID) 1980).

3. The objectives

This application of the FUELPRO model is based on a set of objectives and associated weights developed by NEDA in 1976 (table 2). It uses the shadow prices of capital and foreign exchange and the two discount rates selected by NEDA: the 9% social rate of return, and the 15% opportunity cost of capital.

FUELPRO expresses the social development goals in terms of energy supply, community participation, institution building and education. Energy supply benefits are in terms of the value of the fuelwood produced or saved by adoption of the various policy options. Since reforestation is intended to maintain protection forests, it does not yield any energy supply benefits (except for a terminal period value). Fuelwood is valued in two ways: by its market price or by a shadow price for collection time of non-marketed fuelwood.

The model handles equity by applying distributional weights to net economic growth benefits accruing to each of four socioeconomic classes j. This distributional weight, d_j, can be expressed as

$$d_j = Y^*/Y_j,$$

where

Y^* is the poverty level of income (Abrera 1976),

Y_i is the average income of the socioeconomic class i.

The economic growth score is penalized by the social costs of capital, as well as by the operating, maintenance, and replacement required for each policy option. The economic growth score is contributed by the value of forest products, crops, and terminal period values.

The employment objective counts the wages paid to previously unemployed

Table 2
Policy objectives and relative social welfare weights in the FUELPRO model

	Objective	Relative social welfare weight (%)
1.	Social development	9
2.	Equity in the distribution of income	26
3.	Rate of economic growth	38
4.	Level of employment	19
5.	Environmental quality	8

labor as a benefit. The unemployment rate selected is the one used by NEDA in project evaluation.

Goals for the environmental quality account include water quality, air quality, noise, protection of endangered and threatened species, habitat stability and diversity as well as for aesthetics.

4. Constraints and scenario specifications

In order to test the sensitivity of the analysis to changes in key assumptions, the weighted sums of scaled scores have been calculated for sixteen scenarios (table 3). The scenarios differ in terms of the discount rate, the price of fuelwood, whether treefarmers followed recommended silvicultural practices or the typical traditional practices, and the opportunity cost of land. Each of the sixteen scenarios is produced by the linear programming model twice: once with additional constraints on the rate of implementation of each policy option and once without these constraints. The input data are available from the author.

Some constraints have been imposed on the maximization. First, the maximum amount of wood produced or saved through implementation of the policy options must be less than the total demand for fuelwood in the province by households and tobacco-curers. Second, a given amount of wood must be produced or saved through the options.

Wood is produced by the two treefarming options. The output depends on the product of the number of hectares developed, expected yield of a hectare of trees, and survival rate of seedlings to maturity. The amount of fuelwood saved when households adopt more efficient woodstoves is the product of the minimum total amount used in a year by a household, the number of households switching to improved woodstoves, and the ratio of the efficiency of the improved stove to the efficiency of the simple stove.

Table 3
Notation of scenarios. A specific scenario is denoted by a *scenario number* (ranging between 1 and 16) appended by a letter A or B. The *scenario number* is defined as $1+\Sigma_m 2^{n(m-1)}$, where n and m are given below. The letters A or B apply depending on whether the rate of policy implementation is unconstrained or constrained

m	Scenario parameter	Alternative n	
		0	1
1	Cost of land	included	not included
2	Fuelwood price (P/ton)	90	170
3	Discount rate (%)	9	15
4	Treefarming practices	ideal	actual

Third, the forestry options cannot take up more than the amount of land available for planting trees after accounting for other land uses such as residential settlements, agriculture, and public reserves and parks. A fourth constraint ensures that a minimum forest cover is produced through reforestation. Since trees on reforested lands may not be legally cut, the forest cover would be maintained, at least in theory. The purposes of this forest cover could include watershed management, erosion control, or plant and animal habitat preservation.

The fifth constraint sets the minimum amount of charcoal that must be produced. The sixth constraint limits total charcoal production below total demand. In Ilocos Norte, few households use charcoal for cooking. Charcoal is used only by industries such as restaurants, bakeries, blacksmiths, and a proposed large pig iron plant. Seventh, the sum of the number of households that adopt improved woodstoves or switch to other biomass fuels, kerosene, LPG, or electricity must not exceed the total number of households. Finally, there are nine additional constraints in half of the scenarios that restrict the optimal solutions to a practical rate of implementation for each of the policy options.

5. The results

5.1. Scaled scores for policy options

Table 4 contains a sample of the multiple-objective evaluation scores for each policy option under the various scenarios. A high degree of consistency appeared evident in the results across the different scenarios. In nearly all

Table 4
Scaled and weighted scores in Scenarios 1A and 1B

Policy option	Scaled scores					Weighted score
	Social development	Income distribution	Economic growth	Employment	Environmental quality	
Reforestation	0.00	0.00	0.00	0.20	1.00	11.80
Treefarming	0.67	0.04	0.13	0.55	0.59	27.18
Resettlement	1.00	1.00	1.00	1.00	0.59	96.72
Bamboo	0.00	0.07	−0.08	0.36	0.02	5.78
Woodstoves	0.56	0.05	0.04	0.01	0.77	14.21
Charcoal kilns	0.33	0.14	0.22	0.01	0.70	20.57
LPG	0.00	−0.18	−0.17	0.00	0.70	−5.54
Electricity	0.00	−0.16	−0.15	0.00	0.18	−8.42
Kerosene	0.00	−0.08	−0.07	0.00	0.50	−0.74

cases, the policies ranked from best to worst are as follows: land for treefarming, loans for treefarming, charcoal kilns, woodstoves, reforestation, biomass substitution, kerosene, LPG, and electricity. As an exception, in scenarios 5, 13, and 15 (which combine a high discount rate with inclusion of the opportunity cost of land) adoption of improved charcoal kilns turned out superior to loans for treefarming. In scenario 13, adoption of improved woodstoves also won over loans for treefarming. In six of the scenarios (1, 2, 5, 6, 13 and 14), kerosene substitution had a negative score as did LPG substitution in fifteen scenarios (1, 2, 3, 5–10, and 13–16). The substitution by electricity turned out to be undesirable in every case contrary to the major efforts by the Philippine government at promoting rural electrification.

Let us define a "sensitive" assumption as one that changes the weighted score by 25% or more and consider changes in one assumption at a time. The reforestation option only proved sensitive to changing the costs and mortality rates from the ideal to the actual estimates. The option of treefarming loans was sensitive only to the price of fuelwood which increased net benefits and to the inclusion of the opportunity costs of land which decreased them.

In all of the scenarios, the agroforestry resettlement option turned out to be the top choice, although its net benefits are reduced in absolute terms when the opportunity costs of land are included or the discount rates increase. The main reason for the greater economic benefits of agroforestry is the high value of the upland rice and mango crops combined with fuelwood production.

Bamboo substitution was most sensitive to the price of fuelwood, with higher prices favoring this option. An increase in the discount rate improved the relative desirability of bamboo substitution because the economic costs are spread out evenly over the entire period of analysis.

The weighted score for adoption of improved woodstoves did not vary as much as 25% when any of the assumptions was changed one at a time. Yet, the adoption of improved charcoal kilns was favored relative to the other options by increases in the price of fuelwood as well as an increase in the discount rate from 9% to 15%.

Substitution by LPG, electricity, or kerosene became relatively more desirable (or less undesirable) as the price of fuelwood rose. On the other hand, increases in the discount rate hurt these options.

5.2. Optimal policy mixes

The linear program modifies the evaluation results by imposing limits on the achievement of the objectives. It also provides additional information by recommending at what level each policy option should be implemented (table 5).

Due to the effect of the constraints, the results show more homogeneity across the scenarios. In scenarios 1A to 8A, resources would be devoted to

Table 5
Optimal mix of policies in scenario alternatives

Options	Scenarios			
	1A to 8A	9A to 16A	1B to 8B	9B to 16B
Reforestation (ha)	99000	79000	20000	20000
Treefarming (ha)	–	–	–	290
Resettlement (ha)	4300	26000	11000	20000
Bamboo substitution [a]	–	–	–	10000
Woodstoves [a]	76000	–	10000	10000
Charcoal kilns [a]	20	20	20	100
LPG substitution [a]	–	–	–	–
Electricity substitution [a]	–	–	–	–
Kerosene substitution [a]	–	–	–	–

[a] The unit is the number of households switching to new technology.

reforestation, land for treefarming, woodstoves, and charcoal kilns. Note that a large amount of reforestation is called for. The program of agroforestry resettlement dominated the treefarming loan option because of the greater profitability of agroforestry crops compared to pure treefarming and the higher tree mortality rates experienced by loan recipients. Improved woodstoves could be so beneficial that the entire household population of the province is urged to change stove types. Despite the high weighted scores for improved charcoal kilns, the low consumption of charcoal by households in the province limits the total potential wood saving from this source.

The assumption of *actual* rather than *ideal* treefarming practices multiplied the optimal number of hectares of agroforestry resettlement by sixfold as a result of the larger area devoted to mangos and upland rice relative to fuelwood. The increased resource allocation for agroforestry was at the expense of the woodstoves program and reforestation.

The B scenarios are more practical than the A scenarios because they recognize that there are practical limits on how fast any particular program can be implemented and that a greater diversity of approaches reduces the risk of failures. In scenarios 1B to 8B, the reforestation and woodstoves programs were reduced to more feasible levels, the charcoal kiln program was unchanged, and the provision of land for treefarming increased.

Scenarios 9B and 16B, combining *actual* treefarming practices with the constraints on program implementation, increased the diversity of the mix of policies. Reforestation, land for treefarming, woodstoves and now, biomass substitution were selected to the maximum extent allowed. The option of improved charcoal kilns was expanded and treefarming loans showed up for the first time in the optimal set of options. Compared to scenarios 9A to 16A, the increases in loans for treefarming, biomass substitution, woodstoves, and charcoal kilns occurred at the expense of the reforestation and land for treefarming programs.

In summary, the critical assumptions in the linear programming analysis turned out to be the *ideal versus actual* treefarming practices and the constraints on the rates of specific program implementation. The final results were insensitive to an increase in the discount in the discount rate from 9% to 15%, a change in the price of fuelwood from P90 to P170 per ton, or inclusion of the opportunity costs of land. Despite the large number of scenarios, the results fell into four groups which would tend to confirm the robustness of the analysis, if the right assumptions have been tested. The policy significance of the analytical findings should be tempered by an examination of the actual experience with these options elsewhere. For example, improved woodstove projects have proven quite difficult to implement in many developing countries.

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Part V

MEDIUM AND SHORT-TERM FORESTRY PLANNING

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MATHEMATICAL PROGRAMMING IN LARGE SCALE FORESTRY MODELING AND APPLICATIONS

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Mathematical programming (MP) techniques used to model various aspects of the forest management problem are reviewed and evaluated. The mathematical structure of two linear programming models widely used for timber harvest scheduling by public enterprises are compared, and their operational implications discussed. The character of other MP models used for public forest land allocation are considered next, followed by alternative MP techniques used to model forest transportation activities. Modifications in these models required for planning the management of private forest holdings are pointed out, and the modeling implications for strategic and tactical planning are identified. Attempts to deal with the inherent variability of the parameters of the forest management problem are reviewed. In the concluding section, the contribution of MP to modeling and planning forest management are assessed.

1. Introduction

Mathematical programming (MP) is the most widely used operations research (O.R.) technique in planning the management of forest land. The prevalent use of mathematical programming is attributable to its adaptability to the wide range of problems encountered in forest management. Forest management problems vary greatly with the type of enterprise (public or private), the ecological setting (plantation or natural), and the focus of management concerns (strategic or tactical as defined in [12]). The type of mathematical programming technique selected, and the specification of the mathematical program, vary accordingly.

In this paper we review modeling techniques widely used in forest management, and we reference some of the seminal models published in the O.R. and forestry literature, as well as models which although unpublished are widely used and for which some documentation is available. A comprehensive but somewhat outdated survey of applications of O.R. techniques in forestry is given in ref. [2], and an up-to-date international bibliography of MP applications to forestry is available from the authors.

The MP models used by public forest enterprises are reviewed first, followed by the models used by private enterprises. The exposition is roughly chronological. It reflects the rapid increase in modeling sophistication which has been achieved as algorithms and computing facilities have improved.

2. Public forestry models

The management of public forest lands poses special problems. The area managed by public forest enterprises are often very large: hundreds of thousands of acres, and in the case of the U.S. Forest Service, over 200 million acres. The character of the land on any management unit is typically heterogenous, ranging from barren rock to range, brush fields, and dense forest. The objectives of management are numerous, diffuse, and often conflicting: produce timber and forage, provide recreation opportunities, regulate the flow and control the quality of water runoff, maintain the productivity and the natural character of the land, manage efficiently. Private forest enterprises also share these objectives but their imperative to prosper requires that they give strong priority to financial objectives which are more easily quantified. While the objectives of the public enterprise are broader, its decision space is more severely restricted by institutional constraints: conservation laws, administrative regulations and traditions, and political considerations [6].

A central and critical aspect of the problem faced by the public enterprise is the ordering – and, when they conflict, the reconciling – of these objectives. In order to model these multiple objectives, techniques such as goal programming have been proposed, for example in ref. [31], but have not actually been implemented. While in principle very simple to use, there are several shortcomings to goal programming; most notably goal programming requires assuming linear utility functions. An interactive multiple objective programming approach which is more complex and attempts to overcome many limitations of goal programming has been proposed in ref. [35], but its effectiveness has not been demonstrated.

Until very recently, public enterprises have used mathematical programming mostly to plan timber management and to allocate forest land among timber, range, recreation, and other "multiple uses". A discussion of MP models for these two types of applications is followed by a review of models which address transportation issues and the special problems posed by the very large areas typically managed by public enterprises.

2.1. Timber management scheduling

The focus of the analysis is to evaluate timber management options. The problem posed is to find an optimal schedule of silviculture treatments (planting, thinning, cultivating, spraying insecticides or herbicides, fertilizing, harvesting) which is consistent with each management option. Optimality is usually specified in terms of maximum volume harvested, present net worth, or minimum cost.

Timber Management options can be roughly defined by three parameters: rotation age, spatial and temporal cutting regime, and the time path of the aggregate harvest level. The rotation age specifies the age at which timber stands will be regenerated. Species, productivity of the site, intended use of the logs, and concerns for wildlife and scenic beauty determine the rotation age and the cutting regime. The latter is also influenced by climatic and economic factors. For example, the incidence of high winds may rule out cutting in strips which would be blown down. Costly access or hauling may rule out partial harvesting. Partial or intermediate cutting is practiced either to maintain perpetual "uneven-aged" stands, or to increase the productivity of stands which eventually will be "clear cut" and will be regenerated as even-aged stands. The determination of the time path of harvest can in principle be left to the invisible hand of the market when present net worth is maximized. In practice, at least for public enterprises, the time path of aggregate harvest is often manipulated to promote rapid economic development, provide a stable supply of raw material, or maximize the total timber harvested over time.

The degree of professional discretion allowed to the forest planners by the objectives of the public enterprise and by the institutional setting will determine both the number of management options which will be evaluated, and the specification of rotation ages, cutting regime and time path of the aggregate timber harvest. For example, federal law places narrow bounds on these specifications for National Forests in the USA: rotation age is restricted to the age at which average annual growth is highest (culmination of mean annual increment), only cutting regimes consistent with multiple use are allowed, and the time path of the aggregate timber harvest must be "non-declining" on each National Forest – that is the volume harvested in a decade must be at least 95% of the volume harvested in the preceding decade. Exceptions are allowed but require approval by the Chief of the Forest Service.

Even when rotation age, cuts and harvest levels are narrowly restricted the size of the problem is very large. The wide diversity of forest conditions can seldom be characterized by fewer than 100 "resource classes". For each resource class several combinations of rotation ages and patterns of cuts can be specified, and each of these "silvicultural alternatives" can be scheduled differently.

Dynamic Programming [1], Markov Chains [14,21], Dantzig–Wolfe Decomposition [22], and optimal control [23] have been proposed to determine optimal management schedules, but they have not been actually used in applications, due to the excessive simplification of the real life situation which they require, or because of the cost of setting up and solving. Forest planners have turned to linear programming (LP) both because of the availability of computerized algorithms for solving very large problems and because of the adequacy they provide in representing many planning situations.

The LP models proposed in the literature fall into two categories referred to as Models I and II [19]. In Model I [26], timber classes – that is acreages of homogeneous forest land – preserve their physical identity until the planning horizon is reached. In Model II [19], once an acre has been completely harvested, it is combined with all the acres harvested in the same period which have similar potential for timber production.

In Model I, every management activity consists of a sequence of silvicultural treatments expressed as a vector of inputs and outputs per acre for a sequence of time periods stretching from the present up to the planning horizon T.

Total timber production for period t is expressed as:

$$H_t = \sum_{ij} a_{ijt} Y_{ij}; \quad t = 1, 2, \ldots, T,$$

where Y_{ij} is the number of acres in resource class i managed with the jth sequence of silvicultural treatment from the present to the planning horizon T, and a_{ijt} corresponds to the volume timber per acre harvested with activity Y_{ij}, in period t.

The desired time path for periodic harvest levels can be modelled by constraining the H_t variables. For example, $kH_t \leq H_{t+1}$ will ensure non-declining periodic yields when $k = 1$, and no worse than moderately declining yields when k is slightly less than one. Control over the acreage managed is achieved by the set of constraints: $\Sigma_j Y_{ij} = A_i$, where A_i is the acreage of the ith resource class. The objective function is simply:

$$Z = \sum_{ij} c_{ij} Y_{ij},$$

where c_{ij} is the total cost, net worth, or volume harvested corresponding to activity Y_{ij}.

In Model II, the activities are also scheduled sequences of silvicultural treatments. However, since timber classes are combined after harvest cuts, the vectors of inputs and outputs span only the interval between harvest cuts.

Total timber production for period H_t must now be expressed as:

$$H_t = \sum_{ij} a_{ijt} Y^*_{ij} + \sum_{rs} b_{rst} X_{rs} \quad t = 1, 2, \ldots, T,; \ r < t,$$

where Y^*_{ij} is the number of acres in resource class i managed with the jth sequence of treatments from the present to *the first harvest cut* – which typically will occur well before the planning horizon T. X_{rs} is the acres regenerated in period r and clear cut in period s, and b_{rst} is the volume per acre harvested with activity X_{rs} in period t. Note that $r < t \leq s$, and that for clarity it is assumed that all acres clear cut in period s have similar silvicultural characteristics and can be aggregated in the same resource class. The extension required when silvicultural characteristics call for more than one resource class is straightforward.

Control over the acreage managed now requires two sets of constraints. One set is for the acreage of timber standing at the beginning of the planning period: $\Sigma_j Y_{ij}^* = A_i$, where A_i is defined as for Model I; and the other set is for the acreage of the regenerated timber classes which will be created in every planning period:

$$\sum_s X_{rs} = \sum_{ij\,Jr} Y_{ij}^* + \sum_q X_{qr} \quad r, 1 = 1, 2, \ldots, T,$$

where Jr is the set of activities Y_{ij} which are harvest cut in period r.

The objective function is now expressed as:

$$Z^* = \sum_{ij} c_{ij}^* Y_{ij}^* + \sum_{rs} c_{rs} X_{rs} \quad r = 1, 2, \ldots, T,$$

where c_{ij}^* and c_{rs} are the total cost, net worth, or volume harvested corresponding to activities Y_{ij}^* and X_{rs} respectively. Note that Y_{ij}^* and X_{rs} activities must be specified to account for the acres not harvested by the end of the planning period. Constraints on the inventory remaining in period T can be specified with constraints of the type:

$$L_{T-r} \leqq \sum_{s|s\rangle T} X_{rs} \leqq U_{T-r}; \quad r = 1, 2, \ldots, T,$$

where U_{T-r} and L_{T-r} are the upper and lower bounds on the acreage of timber of age $T-r$ which is desired at time T. Note that similar constraints may have to be placed on the Y_{ij}^* variables if acreage of the original stands is desired in period T. Similar constraints for controlling the final inventory can be specified by Model I.

The Model I structure has the advantage of preserving the location of the timber classes. This greatly facilitates translating the LP solution into operational plans and eventually into management decisions. The Model I structure also permits a more realistic specification of the management alternatives. On the other hand, the specification of complete sets of alternatives through the planning horizon can require an exceedingly large number of variables – particularly when harvest cutting can be specified for many different rotation ages, and permutations of rotation ages are specified in a single activity [16]. A recursive technique which exploits the inherent decomposition characteristics of the timber management problem has been proposed in ref. [4] for solving problems of practically unlimited size. Applications of this technique have not yet been made.

The Model II structure requires a smaller number of variables, but may require a much larger number of constraints than the equivalent Model I LP. This is likely to occur when the potential silvicultural characteristics of clear cut land vary greatly, and a large number of regenerated timber classes are required.

The Model I and Model II formulations have been combined to plan the management of the National Forest in the U.S.A. (MUSYC). Model I was used to specify site specific treatments up to the first harvest cut, and Model II for specifying a large number of alternative rotations for the regenerated stands. In ref. [11], tests for comparing this hybrid Model with Model I were made on hypothetical pine plantations with six timber classes for LP models having approximately the same dimensions. A six to ten per cent improvement in the optimal value of the objective function over the Model I formulation was noted with the hybrid model. Considerably more research is needed before the respective advantages of the Model I, Model II, and hybrid models can be fully identified and exploited.

2.2. *Land allocation*

Timber is only one of the multiple outputs of forest land. Recreation, wilderness, forage, wildlife habitats, and water have great – and often equal – importance with timber in the management of public lands. In the land-allocation problem the management options acquire another dimension. In addition to rotation age, regime of cut, and time path of timber harvest, the balance among multiple uses must be specified. The problem posed is to find efficient allocations of the land resource among multiple uses which are consistent with predetermined multiple output targets, usually specified by an acceptable range for each output.

Initially, the land-allocation problem was modeled without consideration for output variations over time. In these models [25] the activities represent the average periodic inputs and outputs after a resource class had been brought to a stable state by a sequence of silvicultural treatments. Several stable states are specified for each resource class, each state corresponding to an alternative "allocation" of the resource class to a ranked mix of uses, e.g., timber–recreation–wildlife–water, water–wildlife–recreation, etc. As computing capabilities increased, the models were expanded to permit an analysis of alternative paths to stable states [18].

The difficulties of modeling the management of complex forest ecosystems over long periods of time have not been entirely resolved. The reliability of the information generated with linear models of intrinsically nonlinear phenomena remains to be established, and the level of resolution at which the allocation problem should be specified remains to be determined. These issues are discussed further below.

2.3. Modeling transportation

The construction, maintenance, upgrading, and closing of roads, and the determination of efficient traffic patterns are critical issues in forest planning. On plantations, road construction or upgrading is usually of secondary importance. The main decisions require mapping and scheduling timber hauling. On natural forests, access is an important issue, and a large fraction of total management costs is spend on road construction or upgrading. Adequate access must be provided before management activities can be scheduled. Several approaches have been used to handle this problem. Timber and other forest activities can be analyzed independently of road construction. Thus, in ref. [26] access provided by road construction in different periods is considered in the model as exogenous information, and the optimization process assumes that a specific transportation scheme will be carried out. Conversely, in refs. [36] and [37] the traffic flow associated with a given resources management plan is taken as a given parameter and the model finds the optimal combination of construction and hauling activities for directing this traffic from its origin to market points.

In their simpler forms, these models consider only hauling decisions, determining the best (cheapest) routes for the traffic generated at each node in the network. In more complete versions, road construction decisions are also included. This leads to mixed integer LP programs, where the number of 0–1 variables is determined by the number of possible road projects that can be carried out in different periods. A simpler, heuristic scheme is proposed for this problem in ref. [38] but it has severe limitations in that it can handle only one period and it can unpredictably lead to substantial sub-optimality.

As proposed in ref. [36], the timber and the transportation models can be interfaced, the solution to one model serving as an input for the other, and an iterative procedure followed. This approach has severe limitations, in that it can be very costly and it can lead to sub-optimal solutions. A small prototype case presented in ref. [41] showed a 6% loss in objective function value. Greater losses could occur in larger problems.

When road construction plays an important role, a more accurate way of dealing with this problem is to represent explicitly in the same model both transportation and timber resources management activities. In ref. [41], a linear program for managing timber is integrated with a mixed integer program for planning the development and use of a transportation network. In this model, timber management activities specified for a resource class can be carried out only if access is provided to one of the nodes in the transportation network associated with the resource class. In ref. [20] however, a broad range of resource management activities can be defined, including construction of camp sites and erosion control structures, range management, etc. For each node in the network, one or more "resource" projects are specified. Each project can require several types of resource management activities and can span one or more time periods. The amount of traffic per period generated by each resource project must be specified. "Transportation" projects can be specified for every link in the network. Both continuous "traffic" variables and integer construction variables can be specified for any link. The model is used to find combinations of resource management and transportation projects which optimize some index of performance subject to constraints on accessibility, traffic flow, and selected input and output levels. Limited experience acquired with these models suggests that this project approach is better suited to detailed short-range planning, while the resource class approach used in ref. [41] is more effective for long-range planning on very large acreages. However, the potential usefulness of the project approach for strategic planning is being investigated by the authors.

Two general strategies are typically used for modeling access for forest management. In refs. [36] and [41], access is specified by a limited number of routes from each traffic generating node to a specific market node. That is, the acreage of a resource class is considered accessible only if all the links of the route can carry traffic. The route strategy has the disadvantage that specific routes have to be defined in order to be considered by the model. This not only requires preliminary work in order to determine what routes should be included in the model, but also there is no guarantee that routes needed for an optimal solution will not be left out. The criteria for pre-selecting routes are log hauling and construction costs. An important advantage of this route strategy is that cost associated with hauling logs over long distances (e.g. per diem, etc.) can be more easily modeled. In ref. [20], access is specified in terms of flow along the links. The optimization process, therefore, considers every possible route in the network and can find the true optimum. In both approaches one integer variable is required for each link defined. This can lead to a large number of 0–1 variables in particular as links can be constructed or upgraded at several standards (one or two lanes, types of materials, etc.) in any of several time periods. In such cases, the number of resource management activities which can be specified must be restricted, and even then available mixed integer algorithms may fail to produce optimal, or even feasible, solutions.

The costs of solving these problems can quickly become prohibitive as the size of the problem increases. Approximate solution strategies for solving such very large mixed integer problems as defined in ref. [20] are being tested by the U.S. Forest Service.

2.4. Large forest enterprises

When forest holdings cover very large areas, effective management requires their partition into semi-autonomous units. The linear program required to plan the management for such enterprises is likely to be extremely large and expensive to solve. Very significant savings can be realized by using special algorithms based on the Dantzig–Wolf decomposition to solve such LPs [27]. Each management unit is represented by one of the sub-matrices forming the block diagonal of the decomposed LP (DLP) tableau. Global constraints on cash flow, and selected inputs and outputs, appear in the "master" program linking the management unit matrices.

Public enterprises must consider explicitly trade-offs among inputs and outputs which are not linearly related. One approach combining decomposition and integer programming has been proposed in ref. [27]. A global optimal solution is obtained for the linear related inputs and outputs with a decomposed LP algorithm. The DLP solution is translated into a range of acceptable output levels for each management unit. Alternative plans for both linearly and non-linearly related inputs and outputs are prepared for each unit. Integer programming is then used to select one plan for each unit. This integer program optimizes an objective such as present net benefits or costs, subject to global constraints on selected inputs and outputs. Each integer variable is essentially a complete plan for a management unit.

The cost of preparing each plan is so high that only very few can be specified for a management unit. Therefore, only a few of the myriads of management options available can be represented in the integer program. The use of DLP in the initial stage of planning increases the probability that the few unit plans will capture some of the more efficient options for managing the linearly related inputs and outputs.

3. Private enterprises

Enterprises in the private forestry sector can be divided into land holdings where timber management and sales are the only concerns, and vertically integrated forest industries, which in addition to land ownership, have production facilities to process the timber, such as sawmills, pulp plants, etc. Timber holdings in the private sector may be natural forests but more often are plantations. Timber lands are divided into classes, or areas, which are considered homogeneous with respect to silvicultural management. This classification is far more difficult for natural forests since usually several species co-exist and the definition of homogeneity is necessarily only approximate. For plantations, age, site quality, and location characteristics usually define homogeneity adequately. There is no basic difference in the modeling of timber management for natural forests and for plantations. The differences reside in the definition of the resource classes which is much more straightforward for plantations. For short rotation species (20–25 y), such as pine plantations, initial periods of one year are normally defined, and subsequent periods are extended to 2, 3, 5, or more years. For longer rotation species (70–80 y), one or two 5 y periods are followed by several 10, 20, and 40 y periods.

For private enterprises which only manage forest land, the timber harvesting models are very suitable. Modeling in this case will reflect basically timber output and financial considerations. There may be constraints imposed to reflect other concerns, such as conservation, but these are much easier to take into account in private models, as they will usually be explicitly enforced on the firm.

Some of the first LP models proposed for forest planning were designed for private industry [24], but several models currently in use by private enterprises were derived from models such as ref. [26] developed for public forestry. All these models – with the exception of ref. [5] – use the Model I formulation.

For vertically integrated industries, production is supplied in part or entirely from timber lands owned or leased by the enterprise. Typically, such enterprises own timber lands, mills, possibly a pulp plant and other installations. In the models developed for pine plantations in Australia [7,8] and Chile [3], the industrial production units are modeled as "black boxes", for which capacities, costs, and timber requirements needed to produce given amounts of final products have been specified. The objective is to find an optimal management plan for the timber lands corresponding to a mix of finished or semi-finished products to be sold in specified markets (logs, raw timber, pulp, etc.). There is typically an interaction among the production units (e.g., sawdust can be used to generate energy for running the mills, and chips can supplement timber lands as a source of supply for pulp plants).

In private-enterprise models, financial aspects including taxes and market considerations, are far more relevant and must often be modeled in greater detail. A more detailed level in management planning is also often required, because decisions which in public lands are left to contractors, such as harvesting in different seasons, must be considered. In some cases access to certain areas in the rainy season may be difficult or completely closed off and annual planning must be divided into winter and summer periods.

Strategic planning is essential in private enterprise, where the size, assets, and financial state of the firm are a vital part of the decision making process. Practically no modeling of these types of decisions for private enterprises has been reported in the literature.

Typical tactical models for cutting and planting do not consider trade-offs involving large investments in industrial plants or acquisition of forests, etc. Such decisions can, however, be analyzed with tactical models, by "parameterizing" the investment decision and using a "what if" modeling strategy.

This approach can also be used to analyze the development of a forest transportation system. Some attempts at strategic planning have been made by linking a short range with a long-range planning process [33,28]. This approach presents serious interfacing difficulties.

Strategic planning is particularly relevant for vertically integrated industries [42]. Strategic decisions considered include: investing in new mills, expanding an existing plant, constructing other industrial installations, acquiring land, revising financial policies. In strategic planning, timber cutting and planting can be considered at a much more aggregate level than is generally used for tactical planning. Timber classes are grouped into "macro" classes. For example, land holdings stratified into several hundred classes for tactical planning are re-stratified into 15–20 macro classes, and only a few representative management alternatives are defined for each macro class. The aggregation of timber strata is based on their proximity, their approximate age and site "index" or potential productivity. The resulting problem leads to a mixed integer program, with as many integer variables as the number of discrete strategic decisions to be evaluated in each period. From 30 to 100 integer variables are typically required. To achieve computational efficiency, it is essential to reduce to a minimum the number of continuous variables and constraints. Therefore, the aggregation of timber classes – and the consideration of only a few management options – is absolutely essential.

The consequences on the quality of the planning information attributable to aggregation of classes and the "specification" of only a few management alternatives as well as the interface between tactical and strategic models have not yet been adequately studied.

4. The problem of uncertainty

There exist several elements of uncertainty in forest planning. One area of uncertainty is that of future market behavior, which is reflected through prices. This is normally introduced in the objective function, and is relatively simple to handle, if probability functions exist for the "uncertain" parameters [32]. Some work has been published in this area, though no actual applications have been done.

Random variations in natural phenomena introduce another source of uncertainty. The modeling of this source is more complex because it translates into variability of the coefficients of the constraint matrix. This problem has not yet been tackled in actual decision making. Some work has been done to determine the character of the variability in projected yields. The corresponding yield coefficients in the LP appear to behave approximately as random normal variables [34]. This work is currently being expanded to obtain reliable estimates of the mean and standard deviation for timber strata in the Western

U.S.A. This problem can be modeled as a chance-constrained LP where the equivalent deterministic formulation of the probabilistic constraints is non-linear. A heuristic procedure for solving this problem is proposed in ref. [39]. Feasible solutions to the original problem, and bounds on the optimal solution are obtained by solving a series of LP problems. For typical MP Forest Planning Models, solutions which are very close to the bound have been obtained in a CPU time not much greater than running the original LP [40].

Another approach [10] proposes using the Linear-Quadratic Gaussian Control Model. This method is used in two stages. In the first stage, a deterministic version of the optimization problem is solved yielding a time series of target levels for the system variables. For the second stage, the LQG solution is used to adjust the control actions as the system is actually evolving through time, allowing the system variables to be kept close to their predetermined target levels. This approach presents an approximate method for stochastic optimization.

5. Conclusions

The use of MP has resulted in substantial improvements in both the modeling and the planning of forest management. Until the late 1960s, when the first applications of MP to forestry occurred, modeling techniques used in forest management consisted mostly of arithmetical formulas and simple simulation [13]. With these techniques, only an extremely crude representation of silvicultural characteristics of the forest could be achieved. A typical assumption required by those arithmetical formulas was that the entire forest acreage be homogeneous with respect to all ecological and economic parameters. The information provided by these models consisted only of the volume or area to be cut in the next planning interval, usually a decade [9].

Simulation models [30] allowed the representation of age variations in the timber but required that the oldest timber always be cut first, a highly questionable rule given the productivity differentials typically encountered on forest holdings. Formulae and simulation techniques addressed only the silvicultural problem of transforming the helter-skelter collection of stands characterizing virgin and wild forests into managed or "regulated" properties capable of yielding the same timber harvest in perpetuity. No economic considerations were entertained, except indirectly by the "cut the oldest first" rule used in simulations. The economics of forest management were modeled with the calculus of capital theory, but only for individual homogeneous stands [29]. Thus, the optimality claimed for the results applied only in the "small" and was largely irrelevant to the problem of managing even small forest holdings. This reflects the natural limitations of simulation procedures in obtaining optimal, or even feasible, solutions to problems when management on several timber classes interact.

MP made possible a much more realistic representation of the silvicultural characteristics of the forest, and the integration of economic and silvicultural considerations. The production opportunities elaborated with MP reflect more accurately the silvicultural capabilities of the forest, and achieve at least a measure of efficiency, if not the optimum.

Mathematical Programming has also had a great impact on the actual planning process. Large forest enterprises in both the public and private sector are systematically exploring their production space with MP techniques to evaluate proposed policies and to elaborate management plans.

Some challenging modeling and application problems remain unsolved. In modeling, the assumptions of divisibility and independence of the activities, and of non-random variability in the parameters cast long shadows. Mixed integer techniques have been proposed to deal with the non-divisible activities, and with spatial interdependence. Computational difficulties have impeded their application. Promising heuristic methods are still only in the development stage. Economic models of forest management are still primitive, particularly for competitive markets. Price/quantity relationships have been modeled [15], but only under highly restrictive assumptions: a single commodity is produced, or the price of a commodity is independent of the harvest of other commodities. For very large forest enterprises, such assumptions may seriously bias the selection of activities.

The large errors in estimating the technical coefficients of production, and the great uncertainty associated with natural phenomena and with distant objectives, raise serious questions about the adequacy of the deterministic MP techniques used to date. Heuristic methods which allow for the consideration of stochastic variation in the technical coefficients have been proposed, but these remain to be applied.

Forest management planning encompasses production processes which are only partially understood and can only be approximately quantified, and objectives which are ill defined and often conflicting. The modeling of the forest management problem as a mathematical program can never capture more than a few of the critical aspects of this problem. Professional judgement must continue to play a central role in forest management planning. But the information developed with MP models can play the vital role of guiding and validating what must remain a predominantly intuitive process.

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OPERATIONAL MODEL OF SUPPLY FOR WILDLAND ENTERPRISES

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Conceptual and practical considerations restrict enterprises which manage wildlands to a very limited exploration of their production possibilities. A heuristic procedure is presented which facilitates the elaboration of supply alternatives or *plans* that are ecologically sound, administratively acceptable, and reasonably efficient. Mathematical programming and professional expertise are combined to provide information for strategic planning. The moderate cost of the procedure makes practical the identification of the options foreclosed by current and proposed policies, an essential step in allocating wildland resources wisely. The structure and specification of the mathematical programs, and computational results are presented. An application of the model to a hypothetical case study based on U.S. Forest Service data is discussed.

1. Introduction

Private and public enterprises conduct only a limited search among their production possibilities to guide the "strategic" decisions which will have large and long-term impacts on their operations. The exhaustive search of the production space required by profit or benefit maximization would be too costly, and would frequently lead to politically and socially unacceptable results. For wildland enterprises which manage the timber, forage, water, wildlife and the recreation opportunities of uncultivated and sparsely populated lands, the search is further limited by the dearth and unreliability of the information on their production processes. The response of trees to various cultural treatments is well known only for some plantations, and this knowledge can only occasionally be transfered to wildland environments. And silviculture is the best known of the wildland production processes.

The use of linear programming (LP) has long been indicated for strategic planning when the enterprise has only limited information (Boulding and Spivey 1960). However, the economy of information achieved with LP exacts a price. It requires the assumption of linear or proportional relationships among all the relevant inputs and outputs. On wildlands, the application of LP is problematic since some of the production processes are ill defined, are scarcely known or are unmeasurable, and therefore cannot be represented by mathematical relations. Furthermore, a subset of the remaining processes cannot be represented in a linear program because the relationship among inputs and outputs is non-linear. Typically a change in input does not result in a proportional change in output (runoff does not vary in proportion to the number of acres harvested or grazed) or there are only a few discrete choices for relating inputs and outputs (a bridge or a recreation complex must either be constructed at one of only a few specified standards, or not constructed at all). An allocation of wildland resources based only on the solution to a linear program would be unacceptably biased, since it could not account properly for the inputs and outputs not represented in the LP. Consequently, at best, LP can be used to prepare partial production alternatives or "plans" which allocate only the resources corresponding to the linearly related inputs and outputs. To prepare complete plans, the remaining resources must be allocated without benefit of the LP, on the basis of professional judgement aided by whatever available analytical techniques are appropriate.

In this paper we present a heuristic procedure for elaborating supply alternatives or plans for a few "neighborhoods" in the decision or policy space of the wildland enterprise. Each of the plans is designed to satisfy present and expected demands with a modicum of efficiency, in a manner consistent with current and proposed policies which define the neighborhoods to be explored. The procedure permits the exploitation of the powerful optimizing capabilities of mathematical programming up to the limits posed by the peculiarities of wildland management. The evaluation and the final selection of a management plan requires the use of benefit–cost analysis and lies outside the scope of this paper.

2. The heuristic procedure

The procedure can be more easily described by considering first a wildland enterprise which is sufficiently small to be managed as a single unit. Knowledge of the conditions and potential of the land is essential for allocating the non-linearly related resources, and for determining whether a plan is ecologically sound and politically acceptable for such an enterprise. Let this knowledge reside in a "manager" who has elaborated a linear programming model of the enterprise. In this setting, the procedure requires the manager to define neighborhoods in the enterprise's decision space which are to be explored, and to specify for each neighborhood:

(1) a linear objective function (e.g. minimize management costs, maximize timber harvest or discounted net revenues over the next 50 years, etc.),

(2) levels, or ranges of acceptable levels, for all quantifiable inputs and outputs for specified time intervals (e.g. browse, forage, water, sediment, camping capacity).
(3) statements of policy for dealing with intangible or non-measurable goods (e.g. protection of game and endangered species, conservation of scenic vistas) and "bads" (e.g. stream sedimentation).

For each of the neighborhoods, the manager must elaborate one or several plans. Each plan requires two tasks:

Task 1. Formulate and solve the linear program (LP) optimizing the objective function subject to constraints corresponding to the desired levels – or ranges – for the linearly related inputs and outputs.

The activities of the LP must be consistent with the constraints specified for the non-linearly related inputs and outputs and with the statements of policy related to the intangible goods and bads. Thus typically, only a subset of the activities initially specified by the manager in the linear programming model will be included in each LP. The solution to each LP represents only a partial plan since only linearly related inputs and outputs are explicitly represented in the LP.

Task 2. Complete the partial plan by allocating the non-linearly related resources, and by estimating the corresponding costs and outputs with whatever procedures are available (e.g. non linear programming, simulation, survey or delphi methods). This allocation must be consistent with the LP solution and with the policies for intangible goods and bads.

Typically several complete plans will be prepared for each LP solution corresponding either to different combinations of non-linearly related inputs and outputs, or to different interpretations of the policies on intangibles. If no acceptable allocation of non-linearly related inputs and outputs can be found, the constraints of the LP must be adjusted, a new LP solution calculated, and the process repeated.

The use of LP assures a measure of local efficiency, since each plan is the combination of linearly-related inputs and outputs which optimizes some index of efficiency defined by the objective function. The use of LP also permits a systematic analysis in each neighborhood of tradeoffs among these inputs and outputs. LP also permits an evaluation of changes in the policies – to the extent that these policies can be translated into incremental changes in the constraint levels of the LP.

Whether the solution of the LP should precede or follow the allocation of all the non-linearly related inputs and outputs can only be decided on a case by case basis. For example, resources could be first allocated to wilderness recreation use. Then the most promising opportunities for managing timber and forage could be identified with LP. And finally, resources for controlling water runoff, sediment yield, and for producing the remaining nonlinearly related outputs, could be allocated. The elaboration of a single plan could require iterating this procedure – and therefore solving several LP's – to achieve an ecologically and politically acceptable balance in allocating linearly and nonlinearly related inputs and outputs.

A firsthand knowledge of the condition and capability of the resources is essential to manage wildlands. Large wildland ownerships are therefore typically partitioned into semi autonous management units. The procedure sketched above must be modified to allow the manager of each unit to exploit his personal knowledge of the land in allocating wildland resources while allowing for the pursuit of the goals of the enterprise. This is accomplished by letting units managers elaborate complete plans for their respective unit. Supply alternatives or plans for the enterprise are then elaborated by selecting one complete plan for each unit which optimize an appropriate objective function. Fractional combinations of unit plans cannot be selected – even if the fractions add up to one – because of the non-linearity of the relationships among some of the inputs and outputs of the complete unit plans.

The supply function for the "large" wildland enterprise cannot be represented by the familiar continuous graph. This supply function consists of the set of combinations of unit plans obtained by selecting one plan for each unit. Integer programming (IP) can be used to search this set systematically for combinations of unit plans which optimizes an index of performance subject to constraints on any of the measurable inputs or outputs. Note that in the IP, non-linearly related inputs and outputs can be represented as long as they are additive over the management units. The index of performance can be a single input or output, or any function of the measurable inputs and outputs. The IP therefore can be used to evaluate policy alternatives which can be translated into incremental changes in any of the measurable inputs and outputs. Supply alternatives for the enterprise can be calculated by solving an IP including all the plans prepared by the unit managers, or only subsets of these plans corresponding to one or more of the decision space neighborhoods defined. In either case, additional supply alternatives for the enterprise can be calculated by specifying incremental changes in the IP constraints, or by optimizing a different index of performance.

A measure of local efficiency is achieved by solving the LP for each unit, and a measure of global efficiency is achieved by optimizing the index of performance for the enterprise as a whole in the IP. The level of efficiency achievable in the IP is clearly limited by the LP solutions calculated independently by each unit manager. Efficiency can be substantially increased by solving a "global" enterprise-wide LP. The solution to this global LP will identify promising allocations of linearly-related resources among the units. The LPs solved subsequently to generate partial plans for each unit can then be restricted to a range around these resource levels. This may serve to insure that resource allocation among the units is not unduly restricted by the parochial tendencies of unit managers.

The global LP combines the LPs specified by the managers for their respective unit with a "master program" representing enterprise-wide activities and constraints. The resource levels specified by managers for their respective unit LPs will have to be relaxed to allow an adequate exploration of tradeoffs among units. For the short run, these tradeoffs may be severely restricted by current commitments. In the longer run the global LP can be used to identify opportunities for significant increases in efficiency requiring a substantial reallocation of outputs and budgets among the units.

The solution to the global LP has embedded in it a solution for each of the unit LPs. These "unit" solutions can in principle be used directly as partial plans by the unit managers to elaborate complete plans for their respective units. In practice, these unit LP solutions will more likely be used to specify target levels or ranges for each unit. Each unit manager must then respecify and solve the unit LP to ensure consistency with these targets. Unit managers are free to explore tradeoffs among linearly related resources, subject of course to satisfying the targets derived from the global LP solution. Adjustments in the targets can be negotiated either during the process of solving the unit LPs or during the process of completing the partial plans.

Solving the global LP will clearly promote a more efficient allocation of linearly related resources. To compensate for possible bias against non-linearly related inputs and outputs, several global LP solutions can be calculated corresponding to a different balance between linearly and non-linearly related inputs and outputs. Unit targets can then be specified for each global LP solution, the corresponding partial and complete unit plans prepared, and IP used to generate supply alternatives for the enterprise.

Only unit managers have the direct and continuous access to the resource information needed to identify and to resolve – or to forestall – ecological problems. By making them responsible for the specification and solution of the unit LPs, and for the allocation of non-linear resources, this procedure promotes the elaboration of ecologically sound supply alternatives for very large areas of wildlands.

Administrative acceptance is also promoted – if not assured – by the procedure. A delicate balance between authority and responsibility is essential to the decentralized processes characteristic in wildland management. Such a balance can be maintained through the negotiation of unit targets based either on the solution of a global LP or one some less formal evaluation of comparative productivity among the units. Mathematical programming provides managers with information needed to determine whether allocated targets are feasible and do represent a reasonable use of wildland resources. In the case of public enterprises, this information can in addition be useful in arbitrating conflicts with – and among – interested publics.

In the next section, the structure and specification of the global LP and of the IP are presented, and solution strategies are sketched. In the following section computational experience with currently available algorithms is presented. In the concluding section we turn to a consideration of prices and of the nature and level of efficiency which can be legitimately claimed for the supply alternatives elaborated with the model.

3. Mathematical programs

When the wildland enterprise is managed as two or more units, a global linear program is specified, consisting of unit LPs, and of a coordinating or master program. This "master" consists of activities and constraints on enterprise-wide aggregations of inputs and outputs (e.g., timber harvested, forage grazed, recreation opportunities provided, dollars spent) and of enterprise-wide activities required to coordinate management among the units.

A typical global linear program will have the following form:

$$\text{optimize} \quad z = \sum_{i=0}^{N} C_i X_i, \tag{1}$$

subject to:

$$\text{Enterprise constraints} \quad \sum_{i=0}^{N} R_i X_i = d_0, \tag{2}$$

$$\text{Unit constraints} \quad D_i X_i = d_i \quad i = 1, \ldots, N, \tag{3}$$

$$X_i \geqq 0, \tag{4}$$

where

X_i $(i = 1, 2, \ldots N)$ is the vector of all the management activities specified for the ith unit, and X_0 is a vector of enterprise-wide "support" activities. (X_i is typically measured in acres, miles or some other unit of management activity.)

C_j $(i = 1, 2, \ldots, N)$ is the vector of discounted total costs, or of some other index of performance such as present net revenue or value, for the management activities of the ith unit, and C_0 is the vector of costs for the enterprise support activities.

R_i $(i = 1, 2, \ldots, N)$ is the matrix of coefficients for the inputs and outputs of the ith unit which will be subjected to the enterprise constraints, and R_0

is the matrix of coefficients for the inputs and outputs specifying enterprise support activities. d_0 is the RHS for the enterprise support activities.

D_i $(i = 1, 2, \dots, N)$ is the matrix of coefficients for the inputs and outputs of the ith unit which will be subjected to unit constraints.

d_i $(i = 1, 2, \dots, N)$ is the RHS for the ith unit.

The objective function Z, eq. (1), captures the costs and values associated with the management and support of the units. Relations (2) are referred to as the "master program" and represent constraints at the enterprise level, while relations (3) represent constraints on each unit. Equality is achieved in each constraint by the addition of slack variables where necessary.

Each column in D_i, the matrix of coefficients for the ith unit, is the activity vector of input and output coefficients corresponding to the treatment of one acre of a resource class. All the acres in a unit which can be considered homogeneous for purposes of resource allocation are collected in a resource class. The level of the activity vector in the LP solution is therefore the number of acres to be managed.

For some resource classes such as stream-side zones, the unit of treatment may be a mile instead of an acre. In any case, for every resource class, material balance relations must be specified to ensure that the sum of all the activity levels selected by the LP is less than or equal to the area – or in the case of stream-side zones, the length – available for treatment.

The inputs and outputs in each activity are expressed as total amounts per acre during a period of time, usually several years: e.g. the total volume of timber harvested, forage grazed, and the associated management costs and revenues over a 5 or 10 year period.

In order to insure the continuing productivity of the land, inputs and outputs per acre are estimated – and constrained – for a sequence of time periods stretching from the present to the point in the future when the treatment will have brought the wildland resources to a stable or quasi stable state. Activities can also be specified for shorter periods if constraints are added to preserve the production potential of the resources.

The constraint vectors d_i in eq. (3) expresses local priorities and policies for the ith unit, while d_0 in eq. (2) expresses enterprise priorities and policies. For example, d_i may place some restriction on maximum expenditures, and on the minimum level of forage and timber volume to be produced on a unit. Whereas d_0 may place similar restrictions on output levels and expenditures for the entire enterprise or for two or more units.

A global LP solution is an efficient point with respect to linearly related resources in one of the neighborhoods of the decision space of the enterprise. Different points can be found by changing the specifications of the unit LPs (3), of the master (2), or of the objective function (1). Unit managers must be allowed to specify the part of the global LP corresponding to their own unit, to insure that the solution will be ecologically sound and politically acceptable at the local level. But each unit LP must be made consistent with the overall goals and objectives of the enterprise.

The solution to the global LP provides a partial plan for every unit. To capture the range of supply opportunities available in this neighborhood of the decision space, additional partial plans can be obtained by resolving the unit LPs individually. To control the corresponding loss in global efficiency, constraints must be imposed to restrict the solutions to a narrow range around the global LP solution. For example, to elaborate a partial plan requiring higher timber production for a particular management unit, the constraint coefficients (d_i) corresponding to periodic timber harvests would be specified as a range around the level calculated by the global LP. Increases in the constraint coefficients corresponding to periodic budgets may also be required to maintain feasibility or acceptable levels of other outputs.

Nonlinear activities, such as supplying developed recreation opportunities, can be taken into consideration in the unit LP only implicitly by excluding some activities or by reducing the acreage of resource classes. Such changes must be carefully reviewed to control the loss in global efficiency. To elaborate a "high recreation" supply alternative the acreage available for timber would be reduced, some clear cutting activities excluded and the budget and timber harvest constraints adjusted. To elaborate an ecology oriented alternative, intensive timber management activities for certain areas could be excluded from the LP. Some bias will exist towards the linear activities, but by the implicit consideration of nonlinear activities this bias can be considerably reduced.

It is the responsibility of the enterprise to specify targets for the units that are consistent with a neighborhood in its decision space. The enterprise must use its knowledge about the capabilities of the units to exploit the comparative advantages of each unit in the production of various goods and bads. This is a process with feedback; the enterprise knows the rough limits of feasible inputs and outputs for each unit and can reallocate budgets and requirements among the units in accordance with information acquired from solving the global LP. The shifting of these inputs and outputs among units, as well as varying enterprise inputs and outputs, will typically require specifying and solving several global LPs.

The enterprise Integer Program has the following form:

$$\text{optimize} \quad W = \sum_{ij} w_{ij} y_{ij}, \tag{5}$$

subject to:

$$\text{Support activities} \quad \sum_{ij} r_{ij} y_{ij} = r_0, \tag{6}$$

Linearly related inputs and outputs $\sum_{ij} \bar{d}_{ij} y_{ij} = \bar{d}_0$, (7)

Non linearly related inputs and outputs $\sum_{ij} \bar{\bar{d}}_{ij} y_{ij} = \bar{\bar{d}}_0$, (8)

The number of plans selected for each unit $\sum_{j} y_{ij} = 1$, (9)

$y_{ij} = 0, 1$, (10)

where

y_{ij} represents the jth complete plan for the ith unit. y_{ij} takes the value 1 if and only if plan j is chosen for unit i.

w_{ij} is the contribution of the plan y_{ij} to the index of performance to be optimized in eq. (5) (e.g. discounted expenditures, or gross or net revenues over a specified number of years).

r_{ij} is the vector of inputs and outputs unit which must be provided by the enterprise if the activity y_{ij} is selected.

$\bar{d}_{ij}$, $\bar{\bar{d}}_{ij}$ are respectively the linearly and nonlinearly related input–output vectors corresponding to activity y_{ij}.

r_0 is the vector of levels for the set of constraints on the level of enterprise support inputs and outputs (6).

$\bar{d}_0$, $\bar{\bar{d}}_0$ are, respectively, the enterprise-wide constraint levels on the linearly (7) and nonlinearly related (8) inputs and outputs.

Slack variables must be specified for relations (6), (7), (8) which are not equalities.

The mathematical structure of the enterprise IP is transparently clear. The specification of IPs to elaborate supply alternatives for the enterprise is not quite as straightforward. Supply alternatives for the enterprise can be elaborated by including in the IP only a subset of the unit plans y_{ij}. For example, to evaluate the opportunity cost of a policy shift to higher environmental standards, all unit plans which violate these standards would be excluded from at least one of the IP runs, and if necessary, the minimum output levels for timber and forage would be lowered to maintain feasibility.

4. Computer algorithms

The practicality of the approach described above was tested on representative data. Global LPs were solved with a decomposed LP algorithm specifically designed for this class of problems (Ingram 1974). This algorithm is based on the decomposition principle (Dantzig 1961). Integer programs were solved with a modified version of an implicit enumeration algorithm (Geoffrion 1969).

The reason for using a decomposition approach to solve the global LP was twofold. First, it kept down the costs of computation. For problems with the characteristics corresponding to wildland management the proposed decomposition approach was more efficient than the use of straight LP. For example, a problem for 15 units. with 10 master constraints and with overall dimensions of 1000 rows by 3000 columns took 240 s on a CDC 7600, when solved as an LP with the ALPHAC algorithm (Ingram 1973). The same problem took only 50 s when solved on the same computer with the special decomposition algorithm.

Second, using a decomposition approach preserves the institutional characteristics of the allocation problem for the wildland enterprise. The global LP has a block diagonal form, each block corresponding to a management unit. This has advantages. For example, when additional runs require introducing changes in one unit LP, it is much easier to handle the modifications in a decomposed format. It can be particularly advantageous in terms of CPU time when several enterprise LP runs are made, and the final solution of one run is used as a starting solution for the next (Weintraub 1981).

The basic integer algorithm written by Geoffrion was modified so that attractive suboptimal solutions could be obtained (Geoffrion 1969). For example, if W_0 is the net return of the optimal solution, the program can determine all solutions with a value better than $W_0 - e$, where e is a parameter predetermined by the decision maker. This allows the decision maker to obtain a set of solutions which are all very close to the optimal.

It is well known that as the number of 0–1 variables increases, the processing time needed for obtaining an optimal solution to an integer program can increase dramatically. In our case, problems represented 17 hypothetical wildland units in California. On a CDC 7600, computer times for solving 10 problems of 68 integer variables oscillated in response to changes in constraint levels between 2.5 and 40 s. Obtaining suboptimal solutions was more costly because of the need to perform several passes through the basic algorithm. As an example, a run in which e was taken to be 0.1% of W_0 led to 27 solutions in 70.5 s, compared to 18.3 s needed for obtaining the optimal solution. More recently, integer programs with over 700 variables and 24 constraints were solved in less than 10 min on an IBM 370 with a code written by R. McBride of USC.

These results strongly suggest that supply alternatives for very large wildland enterprises can be elaborated at tolerable computing costs by solving very large decomposed LPs and IPs.

5. Conclusion

Periodically, wildland enterprises must re-evaluate policies, and analyze major investment–or disinvestment–opportunities. This process of strategic planning, although restricted in the near term to incremental changes (Wildavsky 1971) often requires a broad exploration of the decision space of the enterprise. The heuristic procedure presented above uses mathematical programming to structure the strategic planning process, and to provide a measure of efficiency in the elaboration of supply alternatives.

The specification of the linear and integer programs brings into sharp focus information needs. The setting of constraint levels defines the lines of responsibility for strategic planning between unit managers and enterprise decision makers, and the solutions to the mathematical programs provide an instrument for negotiation between levels of the enterprise. Infeasibilities, or unacceptable objective function levels, are used to renegotiate targets and budgets, and to reconsider policies.

The measure of efficiency which can be claimed for the supply alternatives is admittedly modest. The mathematical programming techniques require assumptions which cannot do full justice to the complexity and unpredictability of nature. The linearity assumption is essentially respected by including only a subset of the measurable inputs and outputs in the linear programs, and by stratifying the land into large resource classes, each in excess of several thousand acres. This greatly reduces the variance of the input and output coefficients but at a substantial price. Ecological characteristics often vary greatly over small areas of wildlands; therefore large resource classes must typically consist of scattered parcels. Consequently, unit LP solutions require expert – and somewhat free – interpretation before they can be expanded into complete plans. The principal difficulties are the spatial allocation of the management prescriptions corresponding to the unit LP solutions, and the determination of transportation-related activities and of their cost. The spatial allocation problem can be greatly reduced by partitioning the forest into contiguous areas or compartments (e.g. watersheds, transportation areas) and restricting resource classes to a single compartment. The consequent increase in the size of the LP can be somewhat controlled by combining similar resource strata, but only at the cost of increasing the variance of the input–output coefficients and thus decreasing the reliability of the LP solution. The use of stochastic programming techniques is not yet computationally feasible. A heuristic technique is available to solve the unit LPs individually, given desired levels of confidence for the RHS and the objective function (Weintraub 1979). Mixed integer models are available to represent spatial dependence among production activities (Weintraub 1976, Navon 1975, Kirby 1981) but computational limitations also restrict their use to solving small individual unit programs.

For the global DLP, spatial considerations can only be represented implicitly. For example, limitations on the size of contiguous clearcut acreage can be represented by LP activities which prorate clear-cutting over several decades. When road construction is required to provide access for harvesting or recreation, additional constraints can be used to restrict the acreage accessible in any period. The determination of these acreages requires drawing a road construction schedule and mapping the areas made accessible as the transportation network evolves. This can be done at the unit level with the mixed integer techniques referenced above when the units are small, or on the basis of professional judgement. Although such procedures must lead to suboptimal results, the loss of efficiency is likely to be acceptable for strategic planning.

Wildland enterprises are often large enough to influence the market price of the commodities they supply. If a price–quantity function can be reliably estimated, it can be approximated by a piece-wise linear function and represented in the unit or global program (Hrubes 1976). The specification and estimation of reliable price–quantity functions for individual units or enterprises pose problems which yet remain to be solved. Note also that the enterprise integer program will be transformed into a mixed integer program, and that price–quantity relationships can only be represented in the unit or in the global programs, but not in both.

The heuristic procedure proposed relies heavily on the expertise and experience of managers to identify the neighborhoods in the decision space of the enterprise which will be explored, and to complete the partial plans calculated with the global or unit LPs. Therefore at best only a modicum of efficiency can be claimed for the plans. Nevertheless, this procedure allows the full exploitation of the knowledge and information available for planning while respecting the inherent limitations of the mathematical programming techniques used.

Unless special care is taken in specifying the mathematical programs and in completing partial plans, the procedure is likely to result in overvaluing the outputs represented in the LPs. The balance between linearly and non-linearly related inputs and outputs can be improved by conducting a sensitivity analysis on the unit LPs before solving the global LP for the enterprise in order to identify appropriate bounds on the linearly related inputs and outputs. As the state of the art in wildland management advances, the set of inputs and outputs which can be represented in the LPs may be increased, thus raising the level of efficiency of the plans and further balancing the analysis between linearly and non linearly related inputs and outputs.

To solve the probabilistic problems of resource allocation on wildlands with deterministic methods such as LP and IP a large number of plans must be generated. By limiting the analysis to specific neighborhoods in the decision space, the procedure allows the elaboration of a sufficient number of plans to assess the consequences of current and proposed policies for alternative scenarios of future conditions. The procedure is particularly useful for generating supply alternatives representing a selective application of a policy such as maximizing present net value. Even for a private enterprise, maintaining timber production above the current level over the very long run may be desirable for one more unit. For these units, the only complete plans which would be specified in the IP would be those based on unit LPs incorporating constraints which assure a non declining timber harvest over time.

The cost of elaborating alternative plans can be controlled by selecting the level of sophistication and of resolution appropriate for the enterprise. A spartan planner would solve either no global LP or at most one or two, would define only one or two neighborhoods in the decision space, and would settle for a very crude level of resolution by aggregating resource strata, reducing the number of alternative management prescriptions, or spanning the planning period with fewer – and therefore longer – time intervals. Even when spartan standards of analytical sophistication and resolution are adopted, the use of mathematical programming required by the procedure will promote efficiency in each supply alternative. The analytical structure provided by the procedure will also facilitate the identification of conflicts among uses of wildlands, and the arbitration of conflicting claims on the limited resources of the enterprise by the unit managers. For public enterprises, the specification of the mathematical programs will reveal many of the assumptions underlying the decision process. This may bring a measure of reason to the discussion of volatile wildland management issues which are of growing importance to the general welfare in every society.

Appendix: A hypothetical case study

The elaboration of supply alternatives with the heuristic procedure is illustrated for a hypothetical enterprise consisting of six wildland management units. The data for these management unit are roughly representative of conditions encountered on the 17 National Forests located in California.

In this case study, only the inputs and outputs associated with timber and forage production are assumed to be linearly related. Inputs and outputs associated with recreation and environmental quality are assumed to be measurable but to be non-linearly related. Timber is measured in MBF (thousand board feet), forage in millions of pounds available for grazing, recreation potential is measured in visitor days. Environmental quality is measured by an index ranging from 1 to 5, where 5 reflects the highest level of environmental quality consistent with current or contemplated policies and with probable budget levels. A crude measure of the contribution of a supply alternative to environmental quality was obtained by multiplying this index by the total area of each unit.

Five steps are required to generate supply alternatives for the enterprise when there are two or more management units, and a global LP is solved.

Step 1. Elaboration of partial unit plans

Each manager specifies a unit LP for timber and forage production on his unit. The constraints of the unit LP must reflect the policies and objectives corresponding to a neighborhood in the decision space of the enterprise, as well as the unit manager's assessment of local priorities and of the capabilities of the land.

On public wildlands in the United States, decennial timber harvests are

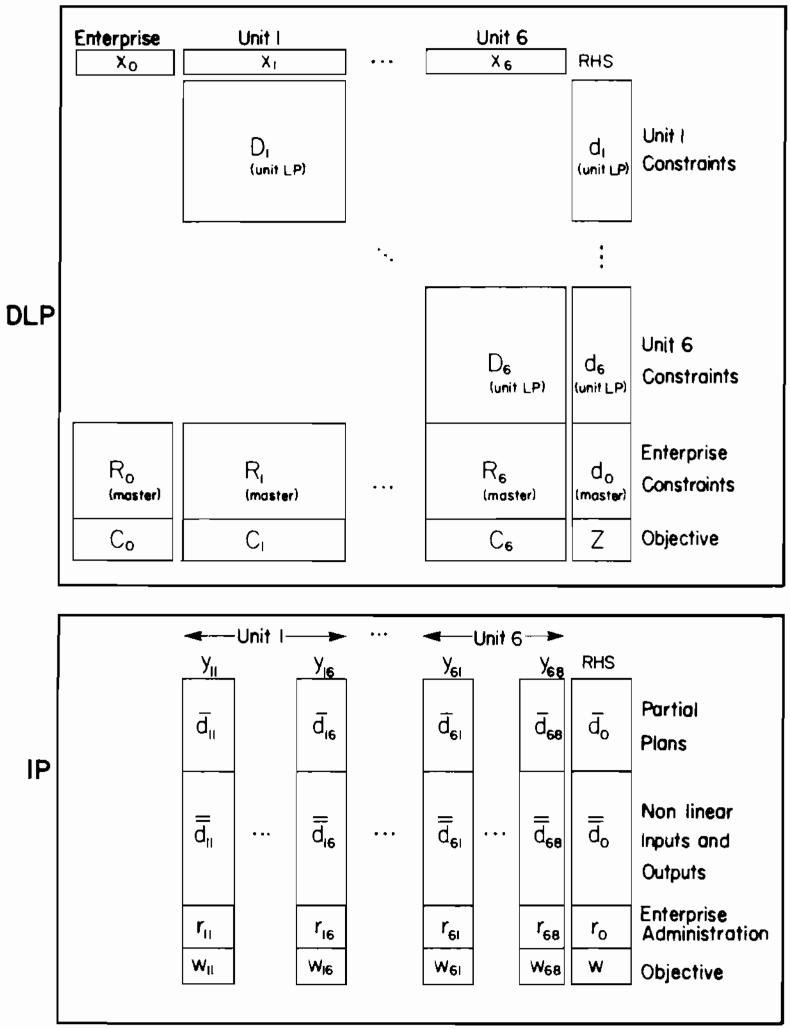

Fig. 1. Schematic tableaux of the enterprise linear and integer programs.

often constrained to produce a non-declining yield over time. In this illustration, to permit a significant reallocation among the units of the level of timber management and harvesting, the enterprise requires each unit manager to allow an increase or decrease in decennial timber harvest of up to 10% from one decade to the next. Note that each manager is free to select the type of timber and range management practices to be used on his unit. Thus managers can draw on their understanding of local needs and conditions in specifying the unit LPs for forage and timber. Before submitting their LP to the enterprise, each unit manager will test it to ensure that it is in fact feasible, and consistent with the capabilities of the land, administrative requirements, expected budget levels, and with the goals and policies of the enterprise.

Step 2. Specification of the global LP and translation of the global solution into partial plans for each unit

The enterprise combines the unit LP for each of the six units with "master" constraints to specify the enterprise LP. This global LP has the block diagonal

Table 1
Partial plans for the six management units

Management unit	1	2	3	4	5	6	
	Partial plans						
	$\bar{d}_{11}$	$\bar{d}_{21}$	$\bar{d}_{31}$	$\bar{d}_{41}$	$\bar{d}_{51}$	$\bar{d}_{61}$	
	Timber harvest (MMBF/decade)						
Decade							
1st	371	2074	894	115	1542	630	5626
2nd	334	2074	894	115	1542	630	5589
3rd	391	2074	894	115	1588	630	5692
4th	352	2074	894	115	1588	630	5653
5th	317	2074	894	115	1591	630	5621
6th	285	2074	894	115	1591	630	5589
7th	478	1205	1003	117	1602	630	5035
8th	430	1279	1032	117	1602	630	5090
9th	387	1279	1032	130	1602	630	5060
10th	348	1120	1314	130	1602	630	5144
	First decade timber management budget						
	6.770	13.708	14.769	1.616	16.651	8.028	61.542
Forage by decade	Available forage (MM lbs/decade)						
1st	1.4	7.6	4.6	1.2	5.3	1.6	21.7
2nd	2.7	9.2	4.3	1.4	4.8	1.4	23.8
3rd	4.5	9.8	5.8	1.9	5.9	1.3	29.2
4th	3.2	10.5	5.5	2.3	6.5	2.0	30.0
	Total timber and range management present costs (MM$)						
	14.674	28.504	3.620	40.015	17.127	19.863	123.803

structure typical of decomposable LPs (fig. 1). In this case study it is assumed that the "master" or enterprise-wide constraints restrict the total enterprise decennial harvest to the current level plus or minus 5% over the next 10 decades. These master constraints reflect a policy decision to even out the flow of timber supplied by the enterprise over time, while allowing greater fluctuations in the harvest levels of individual units. We shall further assume that the index of performance to be optimized in the objective function is to minimize the present cost of management over the next four decades. The global LP is solved, and the solution disaggregated into one partial plan for each of the six units. The partial plan for each unit consists, in this simplified situation, of the

Table 2
Complete plans for Unit 1. Plans d_{11}^1 and d_{11}^2 are based on partial plan $\bar{d}_{11}$, and Plans d_{12}^1 d_{12}^2 are based on small variations from this partial plan

Complete plan	d_{11}^1	d_{11}^2	d_{12}^1	d_{12}^2
Decade	Timber harvest (MMBF/decade)			
1st	371	371	365	365
2nd	334	334	330	330
3rd	391	391	380	380
4th	352	352	345	345
5th	317	317	315	315
6th	285	285	290	290
7th	478	478	450	450
8th	430	430	420	420
9th	387	387	400	400
10th	348	348	360	360
	Available forage (MM lbs/decade)			
1st	1.4	1.4	1.2	1.2
2nd	2.7	2.7	2.1	2.1
3rd	4.5	4.5	3.9	3.9
4th	3.2	3.2	3.0	3.0
	Recreation capacity (M visitor days/decade)			
1st	100	110	141	136
2nd	110	112	196	193
3rd	122	124	214	206
4th	134	138	307	325
	Environmental quality index			
	3	4	2	3
	Unit budget (MM\$/1st decade)			
	9.0	9.05	8.5	8.9
	Total unit management present costs (MM\$)			
	26.4	29.0	26.0	28.5

decennial harvest volumes for 10 decades, decennial forage volume over 4 decades, and of the investment, discounted to the present, that is required over the first 40 years (table 1). For unit 1 the decennial harvest volumes are successively 371, 334, 391, etc. million board feet, and the present value of the investment required in the first decade is 14.674 million dollars (see column $\bar{d}_{11}$ in table 1).

Step 3. Completion of unit plans

Each unit manager prepares several complete plans based on the partial plan extracted from the solution to the enterprise DLP. The levels of the nonlinearly related inputs and outputs must be consistent with the level of the inputs and outputs in the partial plan. Complete plans for Unit 1 are given in table 2.

The unit manager uses his discretion in elaborating these complete plans. He must produce several complete plans reflecting different mixes of non-linearly related inputs and outputs (plans d_{11}^1, d_{11}^2 table 2). In addition, he may elaborate complete plans corresponding to small deviations in the partial plan extracted from the global DLP solution (plans d_{12}^1, d_{12}^2 table 2).

Step 4. Steps 2 and 3 will be repeated for every neighborhood of the decision space which the enterprise wants to explore and for which a DLP has been solved

Step 5. Enterprise integer program

Once the complete plans have been elaborated, integer programming is used to calculate combinations consisting of one plan from each unit which satisfies specified input and output levels for sets of management units, and which optimizes the objective function (fig. 1). Several integer programs are solved corresponding to different subsets of complete unit plans and RHS. Each integer program defines a neighborhood in the decision space of the enterprise which may correspond to the union of neighborhoods used for the global DLP.

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SIMULTANEOUS PLANNING OF WILDLAND MANAGEMENT AND TRANSPORTATION ALTERNATIVES

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Our mathematical programming model deals with the interactions between natural resource investments and transportation network investments as a means of generating alternative land management plans. Spatial and network relations are formulated as disjunctive or conditional (contingency) constraints. The mixed-integer programming formulation employs continuous or 0–1 variables for natural resource investments, continuous variables for traffic flow, and 0–1 variables for road investments. The model is a composite of two models for multiple commodities and time periods – a transhipment model with fixed-charge arcs and a land allocation model. It has been implemented at National Forests in Alaska, California, Colorado, Idaho, Montana, Oregon, and Washington.

1. Introduction

The many conflicting priorities facing wildland planners today require careful weighing of the various land-use options available to each planning area. The growing number of constraints being placed on planners by legislation defy the continued use of manual planning methods. Computer-based technology offers a means to solve the multiproduct output decisions on wildlands. One product of such technology is a mathematical programming model, the Integrated Resource Planning Model (IRPM) [25].

Introduction of the model signals a major departure from existing planning practices. At present, most plans are formulated through a separate determination of land-use allocations and the road network required to support these allocations. With IRPM, these two facets of the plan are determined simultaneously. Input data and model results of IRPM are "site specific".

Although simultaneous planning is new and unconventional, it offers several advantages over separate planning. It allows freedom of interaction between alternative road networks and alternative wildland activities, hence the solutions are optimal. For example, a least cost objective function would include not only the costs of wildland activities but the costs of travel, road building and maintenance as well. Since the transportation costs comprise the bulk of the total cost, transportation activities must be represented explicitly in the model formulations in the construct of a network. IRPM also permits spatial constraints on activity locations. For example, most of the non-transportation investments in wildland activities are for vegetative treatments such as tree planting and timber harvesting, or for physical facilities such as streamside stabilization and fuel break clearings which require road access; but some uses, like wilderness and wildlife, prohibit most kinds of road access. Geographical dispersion of intensive timber harvest activities is another form of spatial constraints permitted in the model. Thus, input data and solution results of IRPM are "site specific". And, each plan then is a combination of explicit activities that are defined site by site, throughout the planning area. The IRPM model is operational and officially approved for use by the U.S. Forest Service. Two companion user guides [25,26] provide the background on mathematical formulation and the job control language statements to execute the programs. The model has been implemented in national forests in Alaska, California, Colorado, Idaho, Oregon, and Washington. [1]

This paper describes the model formulation, solution strategies for large-scale applications, and two Forest Service applications.

2. Background of current methods of solution

American foresters have long recognized the importance of integrating transportation with timber management planning. Chapman [11], in discussing management plans stated, "The three factors upon which a self-contained working circle [2] depends are market, transportation, and production." He concluded that economies of scale for plant size are dominated by the cost of transporting logs to the mill.

How mathematical programming applies to urban land-use planning is addressed by Bammi and Bammi [4,5], Charnes et al. [12], Bagby and Thesen [3], and Patterson [32]. Johnson [17], Johnson et al. [18], Navon [30], and Nazareth [32] present models for allocation of forest land primarily on the basis of silvicultural [3] treatments. Benninghoff and Ohlander [8] present a model that integrates silvicultural treatment considerations with environmental quality. A model for allocation of forest land for multiple uses is discussed by Nautiyal et al. [29].

[1] Applications have been completed or are underway (*) in 13 National Forests: Plumas, Sierra and Klamath (by contract) in California; Gifford Pinchot and * Colvile in Washington; * Malheur, Mt. Hood, * Willamette, * Umpqua, * Umatilla, Wallawa-Whitman, * Fremont, in Oregon; Payette in Idaho; and Tongass in Alaska. The cost savings mentioned subsequently in this paper pertain to the Sierra National Forest application.

[2] Working circle is an area capable of permanently producing wood fiber.

[3] The art and science of growing trees for timber production.

The advantages of analyzing a group of wildland resource management projects and their required road network jointly was first demonstrated by Kirby [22]. This paper offered the stimulus in the U.S. Forest Service to depart from the long standing piece-meal network planning for each project separately to that of a global network analysis for all projects with shared access requirements. A mixed integer programming (MIP) model was formulated to maximize benefit from projects less road construction cost subject to management constraints, relations between each project and its immediate access road, and relations between adjacent roads. Vehicle operating cost was not accounted for in this model since traffic variables were not introduced.

To account for both construction cost and vehicle operating cost, Barnes and Sullivan [6] and Sullivan [35] developed a MIP model for forest network planning to maximize net revenue (timber receipts less road building, and vehicle operating cost). The timber location, timber volume available, and timing of timber cuts are predetermined as part of the input. In order to keep the MIP problem to a practical size, Sullivan [35] used an ingenious approach of generating K-shortest paths between each timber sale and the timber demand points. Users may delete and add paths. Optimization is restricted to these paths, which comprise a small select sample of all possible paths.

Weintraub and Navon [39] and Navon [31] combined the network analysis scheme of Sullivan [35] and the timber harvest scheduling capability of Navon [30]. Due to the large number of silvicultural treatment variables, the network is necessarily simplified. This is achieved by approximating the network by a hypothetical network consisting of only major corridors. Optimization is still restricted to a small number of paths generated for this network, the same as for the Sullivan model [35].

It was found in practice that for large and complex networks, optimization based on paths consumes much analyst time in manually adding paths and may lead to a poor solution as well. This is caused by the fact that road construction costs are not used to generate the K-shortest paths. There is no simple way of incorporating road construction cost for each path since a portion of this cost may be shared by many other paths. Hence, desirable paths with respect to both operating cost and a share of construction cost are often overlooked. The model of Kirby et al. [24] overcame this difficulty by considering all possible paths using the classical transhipment formulation. With this approach, all possible paths are considered. Timber harvest location, volume and harvest are user specified in the same way as in Sullivan [35]. Buongiorno and Svanqvist [10] use a separable goal-programming model for determining the geographic pattern of forest exploitation, industrial processing, and transportation that minimizes total costs. The transportation network is grossly approximated by arcs connecting each permissible pair of points representing forests, timber processing plants, ports, and demand locations.

The formulations of general land management planning problems require additional capabilities that were not formulated by any of the preceding models. Briefly these capabilities are:

- to include recreation traffic as well as timber traffic on the same road network;
- to represent sediment and/or heat transfer in a stream system as flow in a network;
- to represent the construction of facilities such as streamside rehabilitation and campgrounds by means of 0, 1 variables;
- that certain land uses and facility investments are treated as mutually exclusive options because of their spatial locations. For example, clear cuts adjacent to wilderness areas or road building through wildlife sanctuaries are mutually exclusive options;
- to include wildlife habitat, visual degradation measures, recreation visitation, and mineral uses as part of the constraint system and/or as alternative objective functions in addition to the usual inclusion of cost and revenue.

3. Description of IRPM structure

The model is formulated as an MIP with embedded networks for roads and sediment transport. The road network is a multicommodity, multiperiod, fixed charge, capacitated network with mutually exclusive road capacities. The sediment networks are single commodity, multiperiod capacitated networks. The non-network parts of the formulation are described as follows.

For each contiguous land parcel [11,27], several land management alternatives (vegetative treatments or facility construction) are postulated as column vectors. The vector components pertain to joint multiple requirements and responses by period. Requirements include funding and land area. Responses include water run-off, recreation usage, wildlife usage, erosion, timber yield, visual degradation, employment and revenue. The jointness is a fundamental characteristic of virtually all wildland investments. Each column vector is multiplied by a variable, with value bounded between zero and one, and the resultant vector sum represents the total of the requirements and responses. By varying the multipliers, alternative mixes of total requirements and responses are obtained. In practice, this is done by means of alternative objective functions and/or right hand side constraints or by means of goal programming. This part of the formulation resembles the well known product mix problem [36].

But since spatial aspects are ignored by this simple formulation, it is incomplete. Some of the responses require a more elaborate treatment. Erosion becomes sedimentation which is transported by a network of streams. Recreation usage and timber yields correspond to traffic which must flow on a road network. Hence imbedded networks are inherently part of the problem and must be part of the formulation. The sediment transport on the stream system is formulated as an inbedded multiperiod capacitated network. The transport of timber and recreationists on the road system is formulated as an imbedded multiperiod, multicommodity capacitated network but with mutually exclusive capacities that correspond to different road standards. Since all networks take the form of transhipment problems, the routes are implicit rather than explicit, so the optimal choices are made from all possible routes rather than only from a selected subset of routes as was done in Sullivan [35] and Weintraub and Navon [39].

The periodic yields from timber harvesting are governed by restrictions on logging traffic flow through the demand nodes of the road network. This feature permits harvest scheduling to be a time–location function. Besides timber, other typical investments include watershed stabilization, mining, reforestation, vegetative type conversion, clearings for fire prevention and treatments of the fuel residue of logging activities.

Still other departures are made from a strict product mix problem. The best wildlife habitat, in many places, requires a given age distribution of trees because older stands provide cover and newly cutover stands provide forage. And, the ratios between the age classes should be maintained for each time period.

Some departures from the strict transhipment network formulation are also made. Projects such as thinning, planting, clearings for fire prevention, mining and streamside rehabilitation require access to roads but generate only trivial amounts of traffic. Hence conditional constraints are employed that directly connect such projects with the road network by the methods developed in 1972 by Kirby [22]. Conversely, these same methods of conditional constraints are used to prohibit access. Some road construction variables are made mutually exclusive with certain land uses such as wilderness and wildlife sanctuary (eqs. (12) and (13) of section 4).

Conditional constraints are also used to formulate connections between adjacent arcs of the road network so that the construction of some arcs is contingent upon the construction of other arcs (eq. (10)). Such connections are useful because they force many of the solution values of the road option variables in the continuous LP solution to be closer to the subsequent MIP solution values obtained via the branch and bound algorithm. (This phenomenon is more frequently observed for tree-like than for grid-like networks).

The IRPM model software [25] consists of a package of computer programs written in standard FORTRAN and COBOL languages. The model components are the following: Data Base, Network Program, Matrix Generator, Optimization System, and Report Writer. The data base contains technological coefficients for all variables defined for the problem. Data can be added, deleted, and modified readily in the data base. The network program automatically generates all linear relations associated with the road or other networks

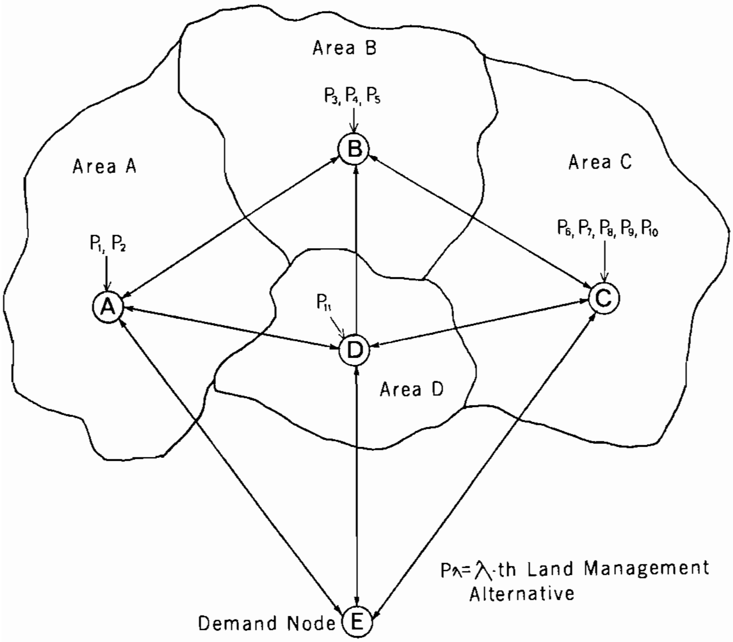

Fig. 1. Management area, land management alternatives and network.

(see eqs. (1)–(4), (10)–(13) of section 4) using bare minimum input regarding the network configuration and traffic generation locations and amounts. The matrix generator constructs the LP matrix for a particular computer run using information from the data base, linear relations generated by network program, any additional linear relations between any variables in the data base, and specifications of objective function or goal values and constraints. The state-of-the-art UNIVAC's Functional Mathematical Programming System [28] constitutes the optimization system. It solves both LP and MIP problems. The report writer can produce more than twenty reports of a LP solution based on user specified scheme for aggregating and disaggregating information. The large number of reports facilitates the transfer of solution information to the decision makers.

The mathematical formulation is presented in section 4. Some of the notations that follow are used in land management planning and are illustrated in fig. 1. For brevity the formulation omits constraints pertaining to sediment and heat transfer networks. Since any linear relationship between variables can be specified in IRPM, the model formulation is presented in the most general form and illustrated by specific examples.

4. Mathematical formulation of the integrated resource planning model

Subscripts

c	traffic class
i, j, k, l	node numbers
λ, λ'	land management alternatives
r	index for rth resource constraints
s	index for road options [4]
t, t'	time period
v	row index for vth linear relationships between land management alternatives, road options, or both

Sets

E = (i, j): An arc between nodes i and j is defined, where nodes i and j are lexically ordered
F = (i, j): Traffic flow from node i to node j is permitted
L_j = λ: land management alternative λ is associated with node j
T_j = i: $(i, j) \in F$
T'_j = k: $(j, k) \in F$

Variables

Δ_{ijs} = 1 if road option s is selected for the arc between nodes i and j, 0 otherwise
D_{jtc} = amount of traffic of class c terminating at demand node j, during period t
$\epsilon_r^- (\epsilon_r^+)$ = under (over) achievement variable, rth resource goal
P_λ = the fraction of the λth land management alternative selected (defined as either a 0–1 integer variable or a continuous variable upper bounded by one, depending on whether a fractional value is meaningful in the problem context)
X_{ijstc} = amount of traffic of class c traveling from node i to node j, during period t, using road option s

Constants

$a_{\lambda v}$ coefficient for variable P_λ in the vth linear relation between land management alternatives, road options, or both

[4] The triplet ijs denotes an arc that connects nodes i and j, with option s.

a'_{ijsv}	coefficient for variable Δ_{ijs} in the vth linear relation between land management alternatives, road options, or both
β	a positive constant
b_r	constraint amount for the rth resource constraint (linear program), or target value for the rth resource goal (goal program)
d_v	constraint amount for the vth linear relation between land management alternatives, road options, or both
$f_{\lambda tc}$	amount of traffic of class c generated by the λth land management alternative during period t
g_{jtc} (g'_{jtc})	minimum (maximum) percentage requirement on the amount of traffic of class c, terminating at node j, during time period t
$l_{jtc}(u_{jtc})$	lower (upper) bound on the amount of traffic of class c that is allowed to terminate at demand node j, during period t
m_{ijstcr}	amount of contribution (input or output) to the rth resource constraint per unit traffic of type X_{ijstc} amount of contribution (input or output) to the rth resource constraint by the λth land management alternative
q_{ijst}	traffic capacity of the arc between nodes i and j and option s, during period t
ρ_{ijsr}	amount of contribution (input or output) to the rth resource constraint, if option s is selected for arc between nodes i and j
w_r^- (w_r^+)	penalty weight for each unit of under (over) achievement from the rth resource goal

Constraints

Multually exclusive relations for alternative road options defined on arcs between nodes i and j:

$$\sum_s \Delta_{ijs}\{\leq, =\}1 \quad \text{for all } (i, j) \in E. \tag{1}$$

Traffic flow-capacity relations:

$$\sum_c \left(X_{ijstc} + X_{jistc}\right) \leq q_{ijst}\Delta_{ijs} \quad \text{for all } (i, j) \in E,\ s, \text{ and } t. \tag{2}$$

One of the terms on the left-hand side of the relation is deleted if the direction of flow is restricted to one-way flow on the arc between nodes i and j.

Conservation of flows:

$$\sum_{i \in T_j} \sum_s X_{ijstc} + \sum_{\lambda \in L_j} f_{\lambda tc} P_\lambda - \sum_{k \in T'_j} \sum_s X_{jkstc} - D_{jtc} = 0 \quad \text{for all } j, t, c. \tag{3}$$

The second term is omitted when there is no traffic generated by any land management alternative defined at node j. Similarly, the term D_{jtc} is dropped if node j is not a demand node.

Demand node capacities:

$$l_{jtc} \leq D_{jtc} \leq u_{jtc} \quad \text{for all demand nodes } j, t, c. \tag{4}$$

Resource constraints:

$$\sum_{(i,j)\in F} \sum_{s,t,c} m_{ijstcr} X_{ijstc} + \sum_{(i,j)\in E} \sum_{s} \rho_{ijsr} \Delta_{ijs} + \sum_{\lambda} \theta_{\lambda r} P_{\lambda} \{\leq, =, \geq\} b_r \quad \text{for } r = 1, 2, 3 \ldots . \tag{5}$$

Linear relationships between land management alternatives, road options, or both:

$$\sum_{\lambda} a_{\lambda v} P_{\lambda} + \sum_{(i,j)\in E} \sum_{s} a'_{ijsv} \Delta_{ijs} \{\leq, = \geq\} d_v \quad \text{for } v = 1, 2, 3, \ldots . \tag{6}$$

(A) Typical relations between land management alternatives
Mutually exclusive relations:

$$\sum_{\lambda \in L_j} P_{\lambda} \{\leq, =\} 1 \quad \text{for some } j. \tag{7}$$

Companion relations:

$$P_{\lambda} - P_{\lambda'} = 0 \quad \text{for some } \lambda \text{ and } \lambda'. \tag{8}$$

Contingent relations:

$$P_{\lambda} - P_{\lambda'} \leq 0 \quad \text{for some } \lambda \text{ and } \lambda'. \tag{9}$$

(B) Typical relations between road options
Road-to-road triggering relations:

$$\Delta_{ijs} - \sum_{s} \Delta_{jks} \leq 0 \quad \text{for some } i, j, k \text{ and } s. \tag{10}$$

Mutually exclusive relations:

$$\sum_{s} \Delta_{kls} + \sum_{s} \Delta_{ijs} \{\leq, =\} 1 \quad \text{for some } i, j, k \text{ and } l. \tag{11}$$

(C) Typical access relations between land management alternatives and road options
Access requirements:

$$P_\lambda - \sum_s \Delta_{ijs} \le 0 \quad \text{for some } (i, j) \in E \text{ and } \lambda. \tag{12}$$

Access prohibition (mutually exclusive):

$$P_\lambda + \sum_s \Delta_{ijs} \le 1 \quad \text{for some } (i, j) \in E \text{ and } \lambda. \tag{13}$$

Relationships that govern distribution of traffic between demand nodes:
(A) Set the ratio for the traffic at two demand nodes to a specified constant:

$$D_{jtc} = \beta D_{itc} \quad \text{for some } i, j, t \text{ and } c. \tag{14}$$

(B) Set upper and lower limits on the ratio for the traffic at a demand node and at all demand nodes:

$$\sum_k g_{jtc} D_{ktc} \le D_{jtc} \le \sum_k g'_{jtc} D_{ktc} \quad \text{for some } j, t \text{ and } c. \tag{15}$$

Goal programming objective. Some of the resource constraints (5) are used as objective functions. Examples of such possible resource constraints are as follows:
- budget constraint (by period)
- present net worth of all costs and benefits for all periods
- timber volume production (by period)
- sediment yield by period
- fuel consumption for all periods.

Adding slack and excess use variables e_r^- and e_r^+, respectively, to constraints (5), yields the objective function for the goal programming formulation as follows:

$$\min \sum_r w_r^- \epsilon_r^- + \sum_r w_r^- \epsilon_r^+ . \tag{16}$$

5. Solution strategies

Usually the land management alternative variables are defined as continuous variables that are upper bounded at 1.0, since fractional values can easily be interpreted. The road construction options, however, must be defined as 0–1 integer variables.

When the size of the mixed-integer program is relatively small (the combined number of rows and columns is less than 500, and number of integer variables is less than 50), any state-of-the-art software package can be used to solve the problem. A judicious choice of branch cut-off value, appropriate priority ordering of the integer variables, periodic analysis of the branching history to direct its progression, and integer fixings are some of the techniques that may be used to hasten the solution process [28,33].

When the problem is so large as to make the cost of the mixed-integer computer runs excessive, a heuristic procedure requiring a sequence of LP runs is recommended. This procedure [37,38] for a cost minimization problem is outlined as follows:

First solve an LP relaxation of the problem; i.e., drop the integrality requirements. From this solution, round to 1.0 all fractional values for fixed-charge arcs. In instances where multiple road options are defined for an arc, however, set to 1.0 the option with the lowest cost and with sufficient capacity for the traffic assigned to it for all time periods. Try to improve the solution. On the basis of some simple cost calculations or by inspection eliminate selected roads, or substitute a road not in the solution for one that is in the solution. Candidates for elimination are roads with high cost and little traffic, or those where less expensive alternate routes exist. A simultaneous relaxation of the resource constraints may be necessary to maintain feasibility. The last step may have to be repeated; however, the computer time required is relatively little because each new run can start from an old basis. For several problems ranging in size from 1200 to 3200 rows, 2000 to 4600 columns, 66 to 420 binary variables, the heuristic procedure obtained solutions within 0.3 to 12% of optimum, with only 2 to 6 LP runs for each problem.

A major component of IRPM is a multicommodity, multitime, capacitated fixed-charge transhipment problem for which extremely efficient network codes have been recently developed [1,2,7,9,13,14,15,19,20,21]. Some of the constraints of the model that do not conform to or cannot be transformed into a standard network formulation must be deleted when a network algorithm is used. The significant reduction in cost of computer runs when using network codes, however, would allow the analyst to make numerous network runs and evaluate the desirability of the solutions with respect to the omitted constraints. A special network algorithm is being developed for handling as many of the constraints as possible, without sacrificing too much efficiency in the network code. At the time of this writing we are testing these ideas in cooperation with Kennington and Helgason [21]. they are attempting to develop an efficient algorithm for the fixed-charge multi-commodity network flow with side constraints problem.

6. A case study

In 1976, IRPM was implemented on the French Creek Basin of the Plumas National Forest in northern California. The study area has high productive capability for a variety of resources including minerals, timber, hydroelectric generation, wildlife, and fisheries.

Table 1
French Creek Basin application results for minimizing total erosion under four alternative harvest levels

	Alternative harvest levels			
	1	2	3	4
Activity levels				
Harvest area (acres)	3000	6000	10000	16000
Planting area (acres)	200	1000	2000	3000
Type conversion (acres)	–	2900	5600	6500
Hardwood understory (acres)	300	1300	2400	3500
Thinning (acres)	150	480	500	260
Fuel break clearing (acres)	–	170	170	190
Fuel removed (tons)	20	57	89	109
Road building and maintenance (miles)	65	90	108	129
Economic effects				
Stumpage (mm \$) [c]	8	15	23	30
Harvest cost (mm \$)	2	4	6	8
Road construction (mm \$) and maintenance	1.5	2.2	2.7	3.3
Haul cost (m \$)	20	50	80	332
Energy used (mm gals)	5.1	3.9	4.0	4.1
Employment (local) (m hours)	74	115	150	185
Resource responses				
Harvest level [a)e)] (mm bf)	40	80	120	160
Erosion – harvest (acre ft)	8	59	137	260
Erosion – transport (acre ft)	8	0 [b)]	5	11
Fire protection (m acre)	4	8	12	16
Recreation [d)] (PAOT)	400	800	1200	1600
Forest base [a)] (m acre)	0.3	2	3	4
Growth [f)] (m unit)	22	50	75	99
water yield (m acre ft)	7	17	29	43
Visual index (m points)	6	8	10	11

a) Constrained.
b) Approximately zero.
c) $m = 1000$.
d) PAOT = persons at one time.
e) bf = board foot measure.
f) Cunit = volumetric wood measure.

The purpose of the application was to determine the feasibility of harvesting timber without causing excessively adverse effects on fisheries, wildlife, visual quality, and hydroelectric generation. The model was used to simulate combinations of various land management alternatives. These alternatives are the decision variables in the model formulation.

The formulation contained 1200 variables divided, about equally, between land management alternatives, traffic variables and road construction options. Associated with each decision variable were the financial and human resources needed to carry out a unit level of the activities associated with the variable, and the social and biological consequences. The formulation contained erosion control projects with negative coefficients for erosion that came into the

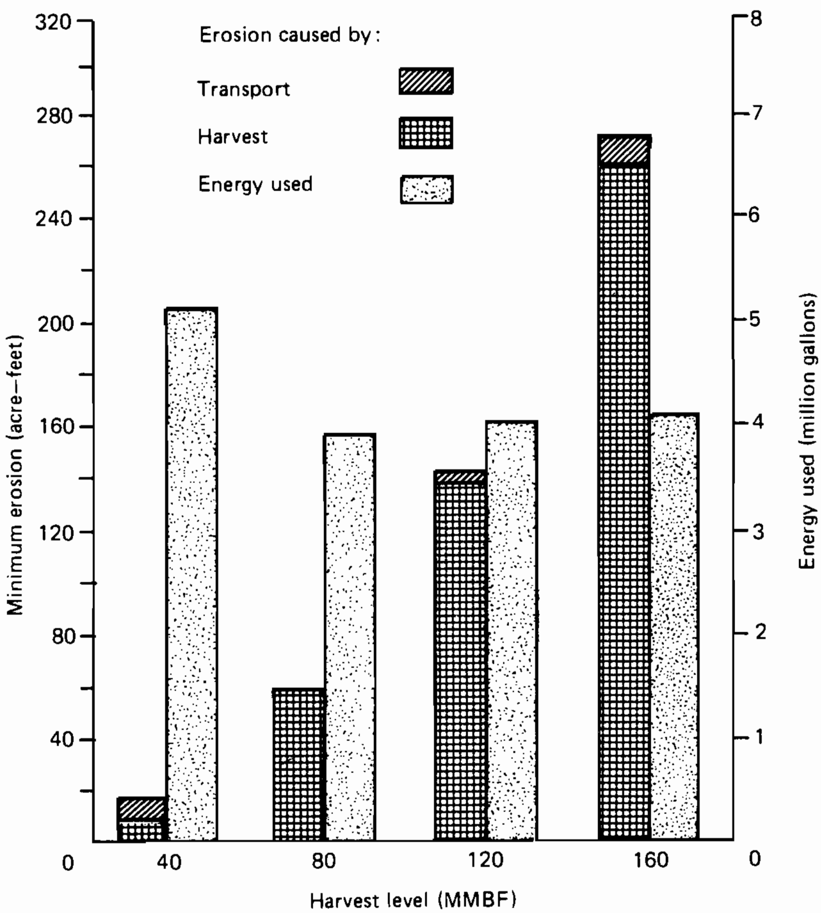

Fig. 2. Erosion responses.

solution only for higher levels of timber harvest (80 million and more). The result can be seen in the unusual erosion response for transport shown in table 1 and in fig. 2.

At the beginning we assumed that transportation activities, rather than timber harvesting, was the dominant cause of erosion. The converse proved to be true, however. The erosion control projects reduced erosion and, although erosion rates from road construction were higher than those for timber harvesting, the harvest areas are much larger than the road areas (fig. 2). The "flat" energy response to increased harvest activity resulted from decreased haul distances and shifts in traffic directions. Timber harvest contracts are based on assumed mill locations, and the best mill locations depend upon the best road network which, in turn, depends upon the total land use in the Basin. The preparation of timber sale contracts without regard for the ultimate road system, in this situation, could have been misleading.

An idea of the kind of measurements that were used as input to this model can be obtained by examining the summary data of table 1. For each of the four runs shown, the harvest level was constrained and total erosion was minimized.

7. Practical and theoretical efficiencies of integrated planning

The cost savings of an integrated approach to land allocation and transportation planning are demonstrated by a strategic level application at the Sierra National Forest in California. Table 2 documents the results of two planning approaches that used the same data and produced virtually identical outputs except for total costs. Under the sequential (traditional) approach, a least cost land allocation was determined first, then a least cost transportation system was found for this allocation. This is contrasted with the IRPM integrated approach which simultaneously solved for the least combined cost for both the transportation system and the land allocation. (The data represent the capital investments plus the average annual operating costs over a 10 year period).

The magnitude of such cost savings as this example may not be typical; but since our comparison is based on a forest-wide example and since it also

Table 2
Cost comparison of sequential and integrated approaches to land management planning

Approach	Estimated costs ($millions)		
	Resource activities	Transportation	Total
Sequential	32	25	57
Integrated	33	12	45

confirms many smaller examples, we use it to indicate the potential savings. Indeed, in light of current expenditures by the Forest Service and its timber purchasers, estimated at \$1.5 to \$2 billion annually for construction, maintenance, and haul on its 200000 mile road system [4] the potential for cost savings is very significant.

8. Summary

We may assert that the cost of implementing a plan that is generated by means of an integrated formulation will always be either smaller or the same as a plan that is generated by means of a sequential formulation. In other words, sequential formulations cannot yield optimal plans except by coincidence. The reason is simply that integrated formulations are less constrained than sequential formulations. They have more degrees of freedom. No single class of variables, such as vegetative treatments, are dominant as is the case in sequential formulations. Instead, all the investments are included as variables in a single model and the interplay among variables is restricted by overall constraints rather than constraints on each class of variables. This means, for example, that a slight increase in timber harvesting cost due to a shift in harvest locations can be more than offset in reduced transportation system costs. But such shifts could never be seen in sequential planning because a separate determination of harvest locations would precede the determination of a transportation system. And since transportation costs tend to be much larger than all other resource investment costs combined, it is easy to see why the implementation cost of integrated formulations is less than for sequential formulations. So the results exemplified in table 2 are not unusual.

The user's guide [25] explains the mathematical ideas and shows how to devise suitable mathematical formulations for a wide range of planning situations.

The computer software, designed for IRPM, has been used on 14 National Forests to explore the consequences of proposed actions over various time horizons. In all of these applications the mathematical formulations maintain the integrity of geographic and temporal space. Reviews of application results have been very favorable, and the model is expected to be used even more widely in the future.

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A DATA-BASE SYSTEM FOR WOOD HARVEST AND TRANSPORTATION PLANNING *

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This paper describes an automated technique for selection for timber salvage operation sites and determination of wood flow to associated market locations. Salvage volume allocation strategies are based upon the cost minimization of new road construction and of wood transportation. Program output establishes the minimum total and unit costs of wood delivery and the optimal wood volumes to be transported from each salvage source to known markets. Computer mapping, network analyses, and mathematical programming techniques are combined into a systems approach. The effectiveness of the system and its uses in timber distribution strategies is discussed.

1. Introduction

The spruce budworm is an insect pest which has coexisted with the spruce–fir forests of the state of Maine (U.S.A.) for centuries. In the past it has followed cycles of epidemic outbreak during which vast acreages of spruce–fir timber were destroyed followed by long intervals during which budworm population levels were barely discernible and forest regrowth occurred [5,17]. Currently this natural cycle is in conflict with man's use of the forest and can have a severe impact on the economic well-being of the region [18].

The State of Maine is approximately ninety percent forested (7.5 million hectares), of which over 96% is in private ownership [11]. Individual ownerships ranging from four thousand to one million hectares occupy the majority of this forested area. Forest products-related industries are the major contributors to Maine's economy. Forty percent of the state's total value of manufactured products can be attributed to the lumber and to the pulp and paper industries [13]. Among the states in the USA, Maine has been the leading producer of paper products.

Timber harvesting decisions of firms involved in wood utilization are generally based on long-term strategies. Typical goals reflect the trade-off

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between sufficient timber harvesting necessary to meet current consumption requirement and the necessity to maintain forest inventories for future requirements [10]. This posture is further confounded by a rationale toward expansion in the production of wood products in anticipation of increasing demand.

Harvest strategies can be grossly upset when large areas of timberlands are destroyed or threatened with destruction by natural agents such as insect or disease. When this occurs, attempts are made both to protect as much of the resource as economically feasible and to recover as much of the value of the timber being destroyed as economically practical. During these periods, harvest location decisions can be utilized to decrease both the loss in timber value and the cost of protection. It is generally desirable to meet this potential as cost effectively as possible.

In addition to private industry, the public sector has an interest in maintaining a wood supply as the resource base for employment and for the continued economic viability of a region. During periods of high timber loss, it could be beneficial for the public sector to financially, legally or philosophically motivate cooperation between independent private firms to allow as much salvageable timber as possible to be recovered while conserving areas with little or no damage for future wood supply.

Concerted and frequently joint efforts to control budworm have been made within both the private and public sectors of responsibility for Maine's forests [18]. The primary weapon has been the aerial application of insecticide which has met with limited success. Although massive destruction has thus far been averted, tree growth has been reduced and expensive spray programs have been repeatedly required. Environmental and cost factors have prompted questioning of the continued use of spray as a means of control. Cessation of control measures could result in widespread devastation of the spruce–fir resource and require extensive salvage operations. Budworm-damage wood not harvested within roughly 2–3 y will deteriorate below minimum quality standards and be lost forever to productive use. The potential wood supply which could be generated under these conditions would, in all likelihood, be far in excess of the state's current processing and harvesting capabilities or market needs. This excess may be on the order of twelve times the potential annual industrial consumption [7], and necessitates the development of effective wood allocation schemes.

Wood movement strategies have been explored in regard to general models for regional applications [1,16,24] and to the budgetary planning of a single firm [4,20,26,3]. Mapping by computer has found favor in recent years [15,21] especially for visual descriptions of use patterns, vegetative cover, and urban planning schemes. However, for the most part those were subjective in nature and have not taken advantage of the mathematical potential inherent in computer-based systems [19,12]. Maine's forest area is relatively vast by U.S. standards [2]. Since it is primarily in large private ownerships, public roads are minimal in number and span. Maine, therefore, represents an ideal locale for development of procedures to optimally plan wood flow strategies using a computer-based system.

2. Program development

As an aid to planning for spruce budworm control within the State of Maine, a computer program was developed to tabulate and display information collected on spruce budworm levels and stand conditions [22]. The display is based on grid coordinates which divide the state into rectangular units (cells) of approximately 240 hectares. With each cell is associated a single spruce budworm hazard rating (the highest of which indicates an immediate salvage need) determined from sample data.

During development of this earlier program, it was realized that information about transportation networks, wood markets, and wood volumes could be associated with grid cells along with the information on spruce budworm hazard rating [23]. As an extension of this earlier program, accomodations were made to allow each grid cell to hold information indicating which, if any, of the 8 adjacent locations (2 horizontal, 2 vertical, and 4 diagonal) have a transportation network element connecting them to the cell and specifying the road class of this interconnection. Each class can be assigned a different transportation cost per unit volume of wood which is utilized in computing transportation route costs.

Fixed grid size and fixed relationships between cells allows mathematical manipulation of the transportation network information within the data base, which once established can be saved and utilized for any number of processings. In its current form, the program elements consist of a data base which contains the information associated with each cell (spruce budworm hazard rating and transportation element), a routine which creates lineprinter maps displaying any element of the data base information, and the routines to be discussed below: new road construction, determination of the costs associated with each source/market combination, and linear programming.

3. Program elements

The process by which timber sources and market destinations are associated through the transportation network is multiphased and composed of (1) the definition of the base area, (2) the simulated creation of new roads required to reach potential timber sources, (3) the determination of minimum transportation cost per unit volume or weight of wood from each market to all potential timber source locations, and (4) the selection and association of timber sources and market locations as an optimal strategy.

3.1. Defining the base area

The first phase is the definition of the geographical boundary which describes the area to be processed, which may represent only a portion of the total area available to the program (in this case, the State of Maine). The boundary and transportation networks (i.e. existing roads) are specified by giving end point locations of straight line segments which comprise them. Within the current program these locations are indicated by the grid designation. However, any designation, such as latitude and longitude, from which a grid index could be calculated can be easily implemented. A grid corresponding to one minute by one minute or approximately 240 hectares in area was utilized (fig. 1). A map grid of 4 hectares has also been produced for portions of Maine, but budworm data is not available at this scale. Any rectangular grid system can be utilized provided its size is constant over the area. Map printout will be distorted for grid sizes whose vertical to horizontal ratio differs greatly from the ratio of lineprinter character height to character width. This distortion of printout in no way effects computed values, optimal or otherwise, and can be reduced or eliminated in a map printout by using multiple print positions for each grid cell. The obvious limitations of lineprinter maps (e.g. the representation of roads in fig. 1, where adjacency does not necessarily imply interconnection) clearly demonstrate the need for computer graphics capability. This potential is currently being developed for the system described herein.

The transportation cost is composed of two elements – new road construction to access timber salvage areas and the unit cost of transporting the material over the network to market(s). Since new roads are frequently required and are expensive to build and maintain, priority is given to minimizing new road construction. The cost of wood flow over the network, including hypothetical new roads, is then minimized. No attempt was made to bring new construction costs directly into the transportation cost minimization criterion. This was done for two reasons. First, the roads thus constructed can be considered an ongoing asset even after a particular unit has been harvested. Secondly, current algorithms for solving the integer programming model necessary to effectively include these costs are computationally inefficient. However, the costs of new road construction can be incorporated by using higher unit transportation costs for the newly constructed roads if these costs are adequately determined. This would have the potential to bias against new road construction.

3.2. Creation of new roads

The second phase begins with the specification of the location and timber volume of each area that is to be considered as a source for harvest. The

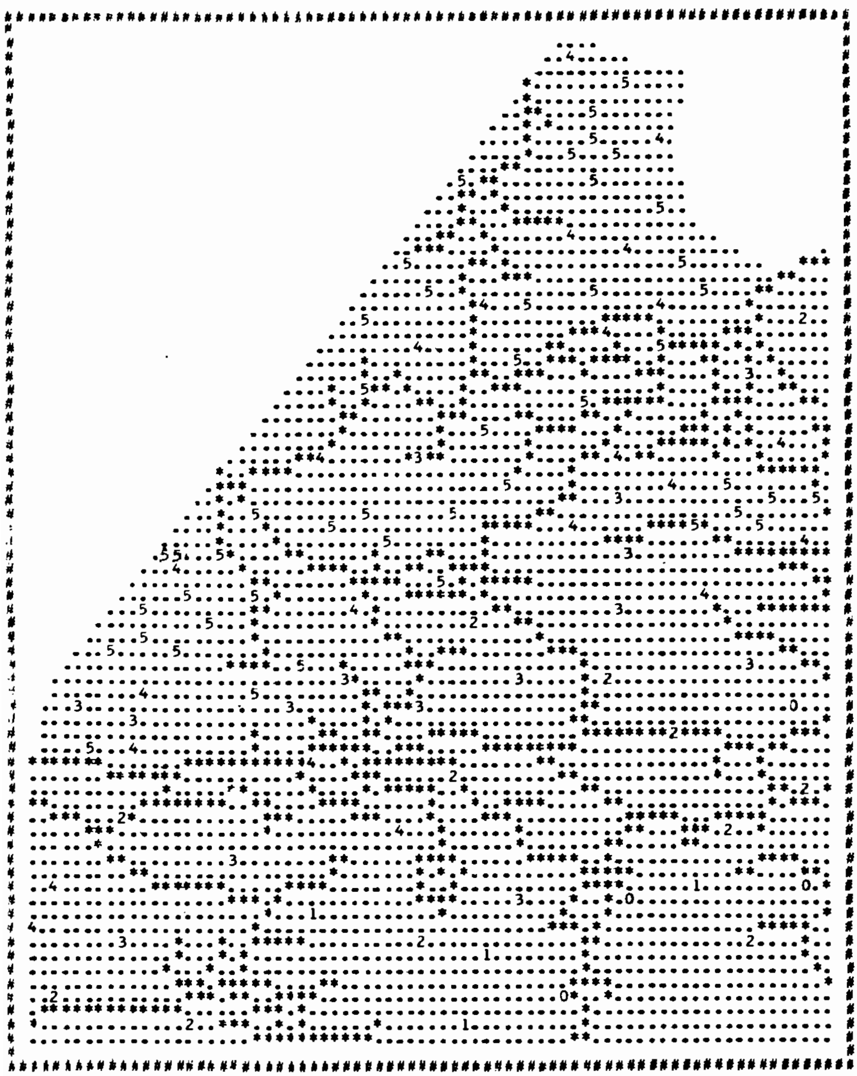

Fig. 1. Lineprinter map showing existing road network (*) and potential source locations by hazard ratings (0–5).

volume associated with a grid location can represent either the volume of wood available in that grid area or the volume available in a larger contiguous area of which that grid represents the centroid. As would be expected, some executional efficiency is achieved with the later representation. However, the time gain in program execution is not appreciable within real problem limits.

From each specified timber source location not adjacent to an existing road,

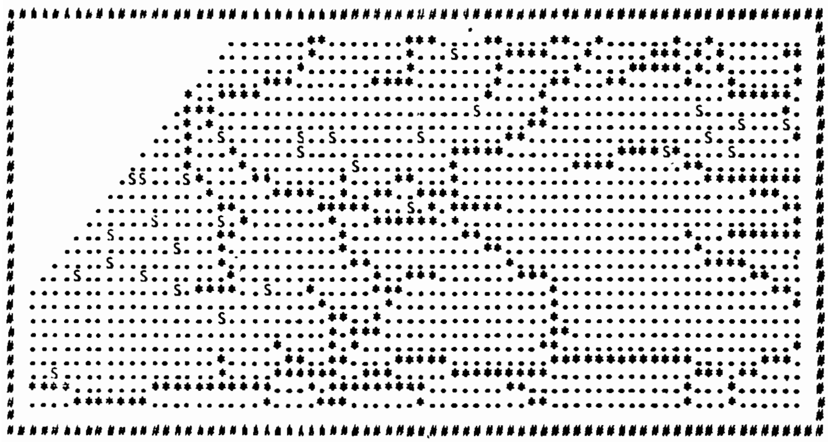

Fig. 2. Map showing existing road network (*) and potential salvage sites (S). Site selection on basis of spruce budworm hazard rating (5).

a new road must be computed and connected to the transportation network (figs. 2 and 3). In order to allow newly constructed roads to be shared by multiple timber sources while attempting to minimize new road construction, two passes are made over the source location data. During the first pass, the shortest distance to an existing road is calculated for each source location, and the source locations are sorted by these distances (shortest to longest).

During the second pass the roads are hypothetically constructed by the following procedure. Beginning at the top of the sorted list resulting from the

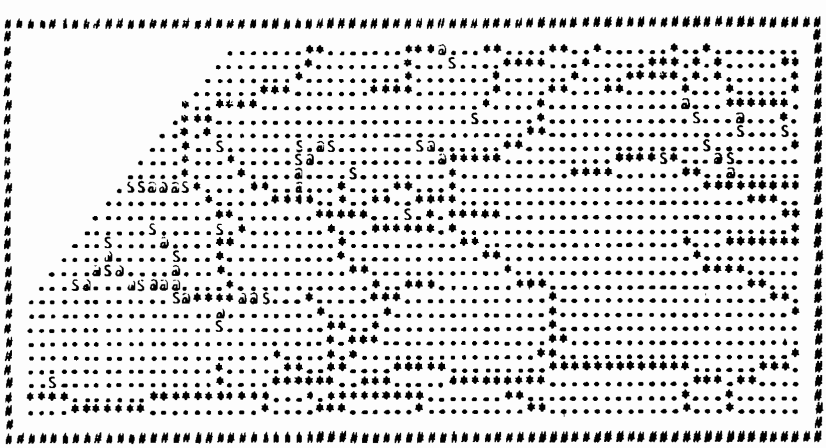

Fig. 3. Map showing existing road network (*), potential salvage sites (S), and program-determined locations for new roads (@), based on distance minimization.

first pass (i.e., the source associated with the shortest distance) each source location (k) is processed in a similar manner: for a given source location k_i (where locations k to k_{i-1} have already been processed), the shortest distance to the road network (existing roads and roads hypothetically constructed up to and including k_{i-1}) is determined and this road is "built". When k_{i+1} is processed, it has available to it all existing roads as well as all roads "built" up to and including the road "built" during the processing of source location k_i.

Although this method does not necessarily yield a minimum of new roads, in most situations it is the minimum or closely approximates the minimum. An algorithm which produces a minimum under virtually all situations is also available in the program. The manner in which this is accomplished is to repeat the first pass following the addition of each new road. This involves repeated recalculation of distances and relinking of locations into the distance-sorted list. It is much less efficient in terms of execution time and is usually unnecessary.

Currently new road location is determined by the minimum straight line distance to an element of the transportation network and the road is hypothetically constructed along the line. Additional information about each grid cell would permit different cells to be designated as impassable or assigned a higher transportation cost level. A more complex new road location algorithm could then be employed. Since the program was developed as a macro-policy model for timber flow decision making, it differs in one sense from a classical transshipment problem [6] in that a retained knowledge of the actual transportation routes to be used is not of concern.

3.3. Determination of minimum transportation cost

Once all source locations have been connected to the transportation network, the minimum transportation cost per unit volume from each source to each destination is calculated by a shortest path network analysis technique [9]. The process begins at the destination location with a specified initial cost value and proceeds outward along the network paths. The initial cost value may be zero or may reflect some fixed cost representing further transportation or handling charges or even an adjustment to standardize market price for delivered wood. The cost of transporting one unit of volume between adjacent cells is the product of the road class cost per unit distance and the distance between grid cell centers as calculated from the grid dimensions and the direction of travel along the grid. For any cell whose transportation cost has been determined, the transportation cost of reaching any connected adjacent cell is computed by adding the "between cell" cost to the transportation cost of the "costed" cell. Minimum cost routing is obtained by sequentially extending the "costed" portion of the network from the current least cost grid cell. This process continues until all source locations are "costed" or all elements of the transportation network connected to the destination have been processed. An over-simplified example, where the total system is comprised of just two sources and two destinations, is given in fig. 4.

3.4. Selection and association of harvest sites

After the transportation costs per unit volume for each source–destination pair have been calculated, this information along with source supply potentials and destination requirements are formed into the well-known linear programming transportation model [25].

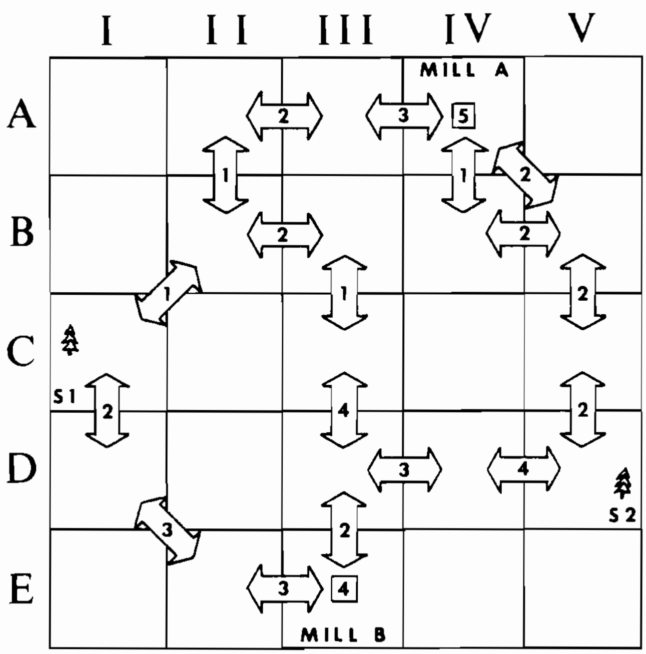

Fig. 4. A graphical representation of minimum transportation cost determination.

Symbols

⇔– Between cell connection and associated cost.

□– Assigned value of "costed" cell.

– Source location (harvest site).

○– Least cost value to source.

Beginning at mill A (initial cost value of 5) costing extends into all adjacent interconnected cells (AIII, BIV, BV).

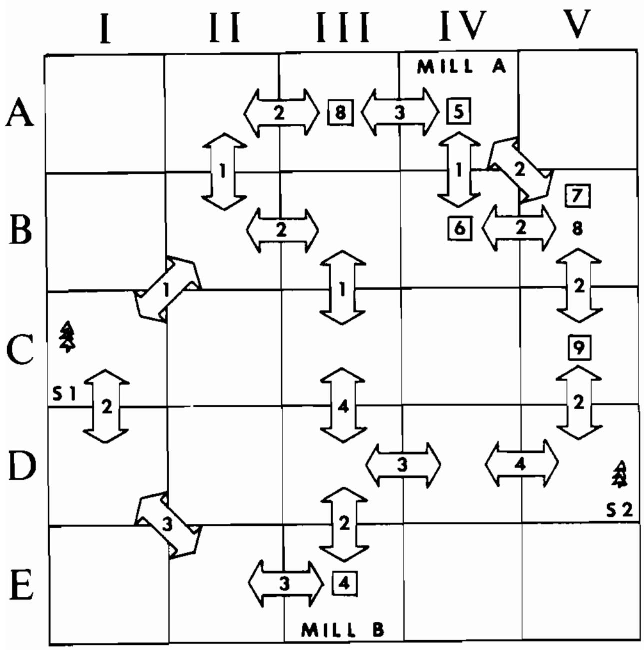

Fig. 4 continued.
BIV is the least cost cell, so its adjacent interconnected cells (AICs) are costed. Since minimum cost routing to cell BV is along the diagonal, the path from mill A through BIV is discarded. The AIC of the current least cost cell (BV) are costed and routing is extended to cell CV.

$$\text{Minimize} \quad Z = \sum_{i=1}^{n} \sum_{j=1}^{m} C_{ij} X_{ij},$$

$$\text{subject to:} \quad \sum_{j=1}^{m} X_{ij} \le S_i \quad \text{for all } i,$$

$$\sum_{i=1}^{n} X_{ij} \ge D_j \quad \text{for all } j,$$

$$X_{ij} \ge 0 \quad \text{for all } i \text{ and } j,$$

where
Z = total cost of wood hauled over existing and new roads,
C_{ij} = computed value of the minimum transportation cost from source i to destination j,
X_{ij} = volume of wood to be shipped from source i to destination j,
S_i = volume of wood available at source i,
D_j = volume of wood required at destination j,
n = number of source locations, and
m = number of destination locations.

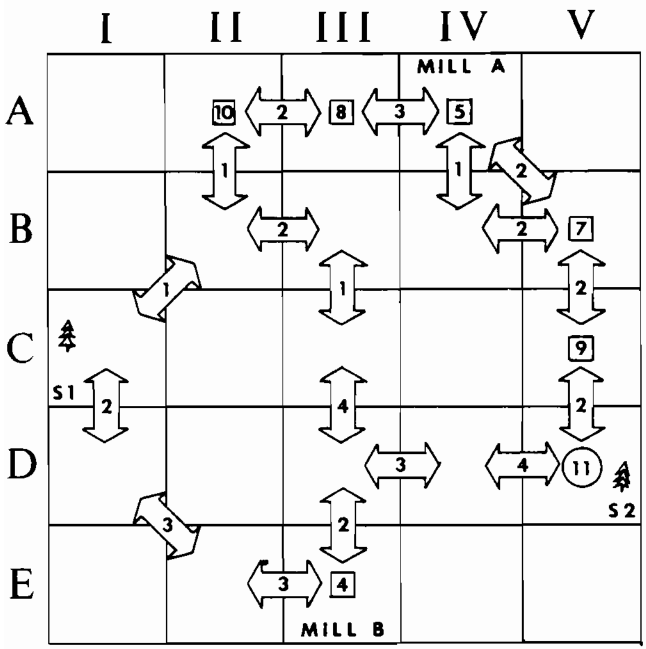

Fig. 4 continued.
At this point, cell AIII becomes the current least cost cell. The AICs of AIII are costed and routing is extended to cell AII. The current least cost cell is now CV and costing of its AICs (DV) completes the minimum cost routing from mill A to source S2.

The minimum cost solution is computed using the Vogel advanced start method and algorithmic techniques [14]. A dummy node is used to absorb the excess supply or provide additional demand as required. The resultant output indicates those source locations which are to allocate their supply, in whole or in part, to specified markets. Selection is based on a total transportation cost minimization criteria. Table 1 illustrates one such output. Obviously some potential sources will be excluded if the requirements of all markets are below the total supply of all defined sources. The converse is true if market requirements are in excess of total source supply. In this case specific market destinations will be identified as being required to obtain wood resources by means exterior to the defined set of sources.

Ideally, additional criteria such as dynamic changes in the hazard rating, stand age, stand composition, operability, and silvicultural prescription options would be included in the salvage site selection decision-making process.

4. Additional considerations

Once a data base for an area has been created, execution time is a function of the number of source and destination locations and of the transportation

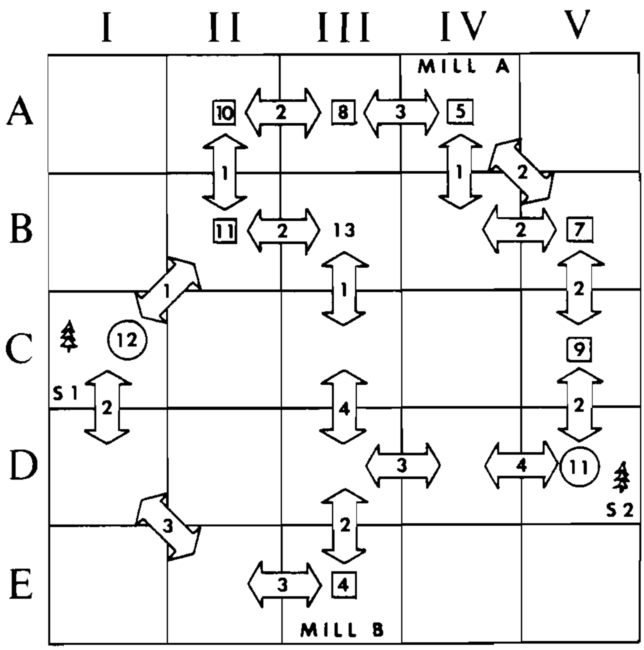

Fig. 4 continued.
The process continues to source S1 by extending from AII to cell BII and the costing of its AICs. Cell CI becomes the current least cost cell, which completes the minimum cost routing from mill A to source S1.
Now the routine processes mill B and computes minimum cost routing to sources S1 and S2, in a similar manner.

network length and organization. For conditions existing in Maine, execution time has been found to be surprisingly modest. For example, a problem on the order of 100 sources and 10 destinations will execute in less than two minutes on the University of Maine's central computer facility IBM 370 (168 series) computer. Even with significant increases in the transportation network, computing time remains very reasonable. Except for the linear programming transportation algorithm, the execution time can be expressed as a second order polynomial of the network length and number of source and destination locations. In regard to execution time, an increase in the number of destinations is more significant than an increase in the number of sources. The latter has a minimal impact on the execution time unless extensive amounts of new roads are added to the transportation network.

Extensions of the system that employ digitizing, on-line plotting, microcomputing, and data storage equipment have progressed to the point of implementation. This equipment, having been acquired in the spring of 1980, is currently maintained in-house by the authors' research unit and is completely interactive with the University main-frame computer. Further development utilizing the

Table 1
Sample program output [a)]

DESTINATION	SOURCE	QUANTITY		UNIT COST	TOTAL COST	
1: D21610						
	25: B19621	510		56.67	28900.93	
	35: C20415	970		46.09	44707.87	
	34: C20408	630		50.77	31985.48	
	6: C19614	1390		47.19	65593.13	
	TOTAL		3500			171187.31
2: E19723						
	5: C18806	890		60.96	54258.50	
	46: D16204	920		98.05	[illegible]	
	41: C16913	130		[illegible]	[illegible]	
	30: C18322	530		[illegible]	[illegible]	
	18: C1E507	500		[illegible]	[illegible]	
	49: B19405	970		[illegible]	[illegible]	
	47: B19719	560		[illegible]	69648.38	
	TOTAL	[illegible]				416704.81
3: C21615						
	[illegible]	[illegible]		30.31	20608.92	
	[illegible]	[illegible]		32.39	13280.11	
	25: [illegible]	190		39.11	30894.69	
	34: [illegible]	1020		37.35	38094.16	
	[illegible]	960		41.94	40266.20	
	[illegible]17213	970		41.99	40728.34	
	2: D17124	370		41.56	15375.43	
	TOTAL		5200			199247.69
TOTAL UTILIZED			35040			1964855.00
SURPLUS						
	10: B18609	740		0.00	0.00	
	9: B18118	960		0.00	0.00	
	20: A19415	1050		0.00	0.00	
	23: A19802	970		0.00	0.00	
	21: A19503	960		0.00	0.00	
	46: D16204	220		0.00	0.00	
	43: C16819	520		0.00	0.00	
	44: D16206	840		0.00	0.00	
	40: A19521	920		0.00	0.00	
	16: A19922	1250		0.00	0.00	
	TOTAL		8430			0.00
GRAND TOTAL			43470			1964855.00

a) Presented for each destination are source locations, wood volumes and computed transportation costs as determined from the cost minimization criteria. Node numbers and Maine grid designations are used to identify sources and destinations. Source locations which are not optimal for harvest are listed as surplus.

full potential of the equipment continues to be a challenge and will be the subject of future discourse.

5. Conclusions

The association of timber source locations in need of salvage operation with market locations based on a cost minimization as provided by the procedures presented herein can be utilized to determine where harvesting efforts should be concentrated when the areas in need of harvesting represent a total timber supply far in excess of market requirements. For the public sector it can be useful both in locating areas for cooperative effort and in allocating costs between cooperators. In addition, areas which need expensive insect control measures would be analyzed and those which could be harvested most economically could be removed from the control operation. This would reduce the overall control costs.

The techniques applied in this system should be readily adaptable to areas in other regions or nations, particularly those where the forest resource is quite extensive, roads are limited, and the timber sources and delivery locations are usefully differentiable. Whether forest land ownership is private or public is not all critical to a successful application.

Under a U.S. government grant, the program is currently in use as a research tool in developing harvest scheduling procedures [8]. While this study was motivated by a forest insect problem, a more generalized use has high potential because of the inherent flexibility allowed for during program development. Improvements to the system such as the incorporation of computer graphics will enhance system performance. Digitizing the data base information will increase the speed of input and the use of electromechanical plotters will improve map quality.

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VALUE OF INFORMATION IN FOREST FIRE MANAGEMENT DECISIONS

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Throughout the United States National Forest system there are numerous opportunities for improving forest-fire management through the gathering or development of better information about fuel conditions (the quantity and characteristics of flammable forest materials) and fire behavior. However, there are costs associated with collecting data and carrying out research. The purpose of the analysis described in this paper is to determine the appropriate level of expenditures on improving fuel and fire behavior information.

A methodology for determining the economic value of improved information on fuels and forest fire behavior is described. The methodology uses a decision-analytic approach to evaluate the expected value of perfect information, using a quantitative model of fire-management costs and losses due to forest fires. Insights gained through an application to a national forest in Oregon are discussed.

1. Introduction

Information plays a key role in fire management planning. The cost of forest fires can be reduced by learning more about where fires will start and how they will behave. Standby forces can be reduced if it is known, for example, that an area of forest is virtually fire-proof under certain weather conditions. Similarly, the effectiveness of fire suppression efforts is improved if planners can accurately predict the behavior of a fire, often measured by its intensity and rate of spread. Such predictions depend on knowing what the fuel conditions are at the site of the fire and in its likely path, as well as weather conditions. Fuel conditions depend on the quantity and characteristics of living or dead vegetative materials. A predictive capability also requires information about how each combination of forest fuels will burn once it is ignited.

The accuracy of information about forest fuel conditions varies widely among different areas. Relatively little is known about fuel conditions over vast areas of the National Forest System because these areas are remote or inaccessible and conditions are constantly changing as vegetation grows, dies, and decays. In many areas records of past fires and logging operations (both of which affect fuel conditions) are incomplete or unavailable. Detailed fuel inventories have been carried out in only a few places.

Scientists have made considerable progress in developing methods for predicting fire behavior in many different kinds of fuel and weather conditions. The methods are based on the results of experiments with forest fires and with mathematical models that simulate the physical processes involved in combustion. Even under the best of conditions, however, current methods are only accurate within small, homogeneous collections of fuel. Fuel conditions in an actual forest usually vary widely, even within a relatively small area. For this reason, predictions of fire behavior can be uncertain even with detailed knowledge of local fuel conditions. Planners would like to know more about the mechanisms of fire spread and about how different fuels interact in order to improve the predictive capability of fire behavior models.

Improvements in knowledge about fuel conditions and fire behavior are possible and, in many cases, economically beneficial. However, there are also costs associated with collecting and analyzing local data and doing research on methods of cataloging fuels and on predicting how forest fires will act under various conditions. The primary purpose of this analysis is to identify the appropriate level of expenditures on improving fuel and fire behavior information, given our current level of knowledge and available information improvement options.

The economic value of information depends on how it is used in making decisions [6,7]. New information has value if it reduces uncertainty and enables forest managers to make better decisions and thereby reduce management costs and fire losses. In this paper, the decision on which we concentrate is the choice of an annual forest-wide fire management budget.

Ideally, each year's fire management budget is set at a level that minimizes management costs plus net fire losses (potential losses include damage to timber, property, wildlife, watershed, and recreation areas). Fire losses are a function of the fire management budget level; spending more to prevent and detect fires and to keep more personnel and equipment ready to fight fires should result in smaller losses due to fire. The objective is to minimize the sum of these costs plus losses.

Choosing the optimal budget would be straightforward if the relationship between losses and management expenditures were known with certainty. The budget decision is complicated, however, by the fact that the budget/loss relationship is highly uncertain. The fire losses that will result after any given level of expenditure are difficult to estimate. The major contributors to this uncertainty are the number and location of fires in the coming year, fuel conditions, weather, fire behavior, fire effects, and the effectiveness of management expenditures.

The analysis is based on a case study carried out on the Mt. Hood National Forest, which covers an area of about 1 000 000 acres on the western slope of the Cascade Mountain Range near Portland, Oregon. Much of the area is heavily forested with commercially valuable Douglas fir.

With the Mt. Hood situation as a starting point, a variety of management conditions were examined to provide insights applicable to other forests, private as well as public. Full documentation of the analysis is contained in the case study report [2]. The report includes an analysis of the value of information in site-specific fuel treatment decisions in addition to the forest-wide budget decision described here.

2. Representation of fuels and fire behavior

Forest fuels include standing timber, brush, down and dead material resulting from natural processes, and debris left after logging operations. The quantity and characteristics of fuels play a key role in determining at what intensity a forest fire burns, how fast it spreads, and how difficult it is to suppress.

Two general techniques are used to represent local fuel conditions. The first method is to represent each area of forest by a "stylized fuel model". The appropriate stylized fuel model is selected from a set of such models, each of which characterizes a broad class of vegetation. Two sets of stylized fuel models are in use, the National Fire-Danger Rating System models [5] and the Northern Forest Fire Laboratory (NFFL) models [1]. Stylized fuel models are often an appropriate method to represent fuel conditions for decisions involving large forest areas, such as the annual forest-wide fire management budget decision. The NFFL fuel models, currently used on the Mt. Hood National Forest, represent areas such as Chaparrel (NFFL model 4), timber with litter and understory (NFFL model 10), and light, medium, and heavy logging slash (NFFL models 11, 12, and 13). A set of photographs and a description of each fuel model are typically used as guides in the selection of a fuel model to represent a particular area of forest. Use of stylized fuel models implies that two kinds of approximations are being made: (1) it is often not clear which fuel model is most appropriate, and (2) the chosen model itself is an approximation over a range of fuel conditions.

The second general technique for representing fuel conditions is to directly estimate the quantity of fuels (usually in several diameter classes) using sampling techniques [4]. Sampling procedures are well developed but tend to be labor-intensive and thus costly to carry out. Representing fuel conditions by specific quantitative measures based on sampling is most useful for decisions involving relatively small forest areas, such as a particular logging site. An example for the Mt. Hood Forest is discussed in ref. [2].

Characteristics of fire behavior that are of interest include flame length, fire intensity, rate of spread, and mechanism of spread. In this analysis we concentrate on fire intensity, expressed as the rate of heat release per unit length of the burning front of a fire (Btu/ft/s), and fire size in total acres burned. Fire intensity is a good indication of the resistance of a fire to suppression efforts. Hand crews can work right next to a fire having an intensity of less than 100 Btu/ft/s, whereas fires with intensities greater than 700 can only be fought by indirect efforts such as setting back-fires from natural fuel breaks.

3. Fire management budget decision analysis

The specific decision analyzed was that of setting the annual budget level for prevention, detection, and presuppression activities at the national forest level. Data used were representative of the Clackamas and Estacada Districts of the Mt. Hood National Forest. Alternatives evaluated involved incrementally increasing or decreasing the budget from a nominal base budget. The budget level for fire management operations in the two districts during 1979 was about 266000 U.S. dollars.

3.1. Cost-plus-loss model

A simple mathematical model was developed to evaluate fire losses and management costs associated with alternative budget levels and a range of settings of uncertain parameters. The primary purpose of the model, outlined in fig. 1, is to estimate the annual number of forest fires, broken down by size, location, and intensity, and to calculate the costs associated with these fires. The model can be thought of as an accounting framework in which to integrate information on fuel characteristics, ignitions (events due to man or nature that result in the start of a fire), fire intensity, fire size, timber losses, and forest rehabilitation costs. The model is run for each path through the decision tree.

Fuel characteristics were represented in the fuels submodel (fig. 4) by specifying the number of acres of Mt. Hood National Forest best represented by each NFFL stylized fuel model. Mt. Hood fuels and fire management staff were uncertain as to what proportions of the total area should be assigned to each fuel model. The impact of this uncertainty is analyzed below. The nominal distribution of forest area by stylized fuel model was 280000 acres in Model 10 (timber litter and understory), 80000 acres in Model 8 (closed timber litter), and 20000 acres in each of Models 12 and 13 (medium and heavy logging slash). The fuel conditions in a given area, and thus the number of acres best represented by a given fuel model, can change due to fuel treatment and logging activities.

The fire location submodel distributes the ignitions among the fuel types, with the assumption that all industrial ignitions occur in slash fuels (slash results from logging operations, often involving the use of vehicles and other equipment that can cause fires). The expected annual number of fire ignitions

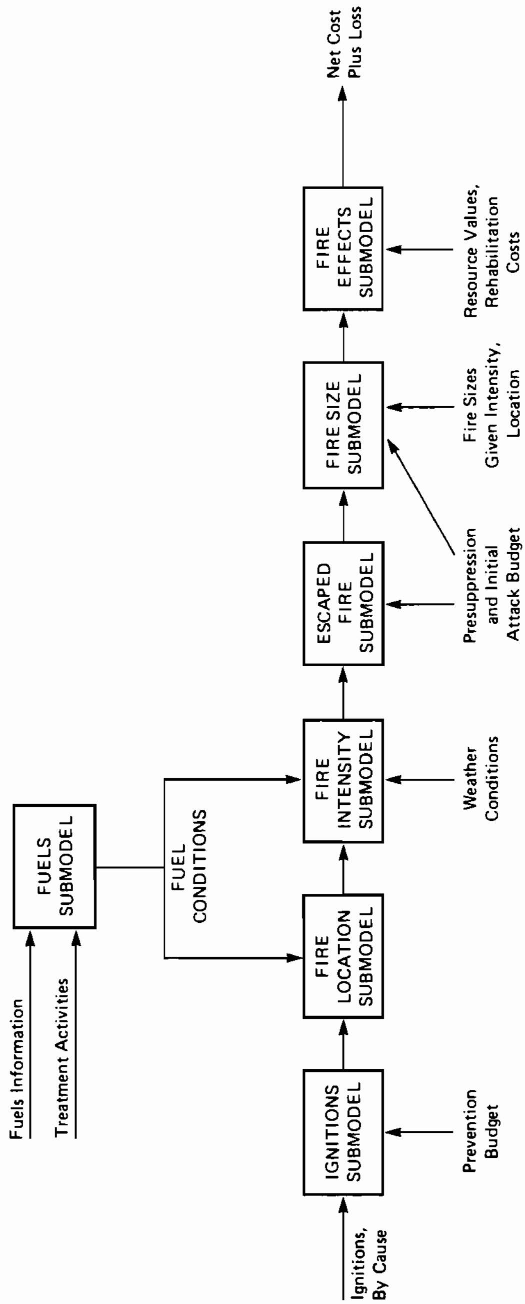

Fig. 1. Cost-plus-loss model block diagram.

Table 1
Fire intensity probabilities given type of fuel. Fire intensity is measured in Btu/ft/s

Type of fuel	Fire intensity class		
	Low (0–100)	Moderate (100–700)	High (> 700)
Timber litter	1.00	0.00	0.00
Litter + understory	0.78	0.19	0.03
Medium slash	0.03	0.61	0.36
Heavy slash	0.00	0.28	0.72

on the Mt. Hood National Forest totaled 34; namely 8 lightning fires, 1 industrial fire, and 25 other people-caused fires.

In forest areas represented by each possible stylized fuel model, conditional probability distributions for fire intensity given an ignition were calculated using the Rothermel fire behavior model [8], incorporating uncertainty in weather conditions (wind speed and fuel moisture). These distributions are summarized in table 1. The fire intensity submodel uses this information to calculate the number of fires in each of three intensity classes, given the number and location of ignitions and the number of acres represented by each stylized fuel model.

Given the fuel type and fire intensity, the fires are divided by the escaped fire submodel into those that are controlled at a small size by initial attack forces and those that escape suppression by initial attack forces. The fraction of fires that escape initial attack depends on the fuel type, fire intensity, and the fire management budget level.

The fire size submodel uses escape status, intensity, and location (fuel type) to predict ultimate fire size, divided into four size classes. The cumulative result of the submodels discussed to this point is to provide a detailed estimate of the number of fires in a particular year, broken down by fuel type (represented by the stylized fuel model), fire intensity, fire size, and whether or not the fire escaped initial attack.

Resource losses due to fire, represented in the fire effects submodel, are assessed on a per-acre basis as a function of size and intensity. Loss figures were on the order of $1000 per acre burned. The largest component of fire losses on the Mt. Hood National Forest is damage to timber. Resource loss figures used were based on estimates provided by Mt. Hood National Forest personnel. Total losses are calculated by multiplying the expected acres burned in each fire size and intensity combination by the corresponding per acre resource loss, and summing over all fire categories. The cost of suppressing escaped fires are calculated in a similar manner and added to fire losses and management costs to give the total annual cost-plus-loss estimate.

Table 2
Base case results. Expected fire damages by fuel type and intensity class ($1000/y)

Fuel type of fire start	Low (0–100)	Moderate (100–700)	High (> 700)	Total
Heavy slash	0.00	6.31	64.84	71.95
Medium slash	0.01	13.55	32.42	45.98
Timber litter	0.79	0.00	0.00	0.79
Litter + understory	2.14	74.67	52.13	128.94
Total	2.94	94.53	149.39	246.86

3.2. Results using nominal data and nominal budget level

Using the current budget and nominal values for fuel types, fire intensities, and other parameters, the model estimates that an average of about 200 acres per year will be burned, resulting in about $250 000 in resource losses and about $100 000 in suppression costs. A summary of the base-case expected fire damages (net resource losses) is provided in table 2. Fires burning 10 acres or more occur at an average rate of about five per year. One might expect a fire in the 100-acre class once every one to two years, and a very large fire (of 1000 acres or more) about once every 15 years. These results are independent from year to year; the occurrence of a large fire in one year does not imply that several years must pass before another is possible.

An important observation from table 2 is that almost half the losses result from fires starting in slash fuels, even though these represent only 10% of the total area. This is due to the high intensity of fires starting in slash, and the likelihood of such fires escaping initial attack. The majority of resource losses are due to escape fires.

3.3. Sensitivity analysis

A wide range of sensitivity tests was carried out to examine the implications of model assumptions and important uncertainties. Several of the sensitivity cases are listed in table 3. Fire loss estimates were most sensitive to variations in the proportion of slash fuels, and to changes in fire intensity. The great sensitivity of the net cost plus loss to changes in the representation of fuel conditions and predicted fire intensities suggests that the fire management budget decision would be sensitive to such uncertainties. The value of information that would reduce uncertainty in fuel conditions and fire intensity is analyzed in the following section.

The primary purpose of sensitivity analysis is to identify critical uncertainties that must be analyzed in greater detail, as discussed above. Sensitivity analysis can also be used directly to provide insights that may be useful in

Table 3
Sensitivity analysis (in thousands of dollars per year)

Sensitivity case	Expected fire losses	Expected suppression costs	Losses + suppression costs: difference from base case
Base case	247	100	–
Ignitions			
Subtract one industrial ignition	220	88	−39
Subtract one other human-caused ignition	240	98	−9
Slash area			
50% less slash area	202	80	−65
100% more slash area	369	164	+186
Fire intensity			
2 × fire intensities	560	191	+404
$\frac{1}{2}$x fire intensities	200	84	−63
Number of escaped fires			
10% more escaped fires	268	109	+30
25% more escaped fires	301	121	+75
10% fewer escaped fires	225	92	−30
25% fewer escaped fires	193	80	−74

refining fire management expenditures. For example, table 4 shows that the annual expected value of eliminating one industrial ignition is about $39 000; this can be compared with the cost of actions that can be taken to prevent such fires.

Table 4
Uncertainties in area distribution of fuel types and in predicted fire intensities

Fuel distribution	Probability [a]	Thousands of acres assigned to each stylized fuel model			
		8	10	12	13
Nominal	0.4	80	280	20	20
Low slash	0.3	100	280	10	10
High slash	0.3	80	200	80	40
Intensity case	**Probability [a]**	**Fire intensity distribution**			
Nominal	0.4	As predicted [b]			
Low	0.3	Half of predicted intensities			
High	0.3	Twice predicted intensities			

[a] Reduced uncertainty scenarios were also examined, using probabilities of 0.8 for the nominal case and 0.1 for low or high case.

[b] By Rothermel fire behavior model.

4. Value of information analysis

New information has value if the corresponding reduction in uncertainty leads to better decisions and improved outcomes (in an expected-value sense). The expected value of perfect information (EVPI) is defined as the expected value (which may be negative) of the optimal strategy given perfect information less the expected value of the best strategy given the current state of information. For the budget decision, the available strategies are to raise the fire management budget, maintain it at the present level, or lower the budget. Several different amounts of potential budget increase or decrease were investigated.

4.1. Value of fuels and fire intensity information

The sensitivity analysis identified two uncertainties as being critical to the budget decision: (1) the assignment of stylized fuel models (in particular, slash models) to the total forest area and (2) the fire intensity given the fuel type. The value of information about these two items was analyzed first.

A difficult question was how to model the change in fire management effectiveness (expressed in fire suppression costs and net resource losses), given changes in budget. Since the purpose of this analysis was to get a good quantitative estimate of the value of new information, rather than to determine the optimal budget level for a specific year, a parametric approach was used. This provided an internally consistent framework within which a set of budget decision cases could be evaluated. After discussions with Mt. Hood National Forest personnel, we assumed that the dominant effect of a change in the annual budget would be to change the probability of controlling a fire with initial attack forces. This could occur through changes in presuppression, fuel treatment, or initial attack activities. Changing the probability of achieving control during initial attack implies a change in the number of escaped fires – the dominant category in fire losses.

We first assumed that a 15% change in the budget would change the number of escaped fires by 10%. The complete decision tree used for the analysis of the value of information under this assumption is shown in fig. 2. The expected value of cost plus loss is computed using the probabilities of stylized fuel model types and of fire intensities listed in table 4. The best choice given this decision is to increase the budget by 15%, with a net expected cost plus loss of 742 000 dollars. The expected value of perfect information on the fuel model area distribution was about $0.01 per acre per year; information on fuels could change the budget decision and lead to improved outcomes. The EVPI for fire behavior was about three cents per acre per year, or about 30 thousand dollars for the entire Mt. Hood National Forest.

A range of cases was then evaluated. For each case we determined (1) the best alternative (having the minimum expected value) given the initial uncer-

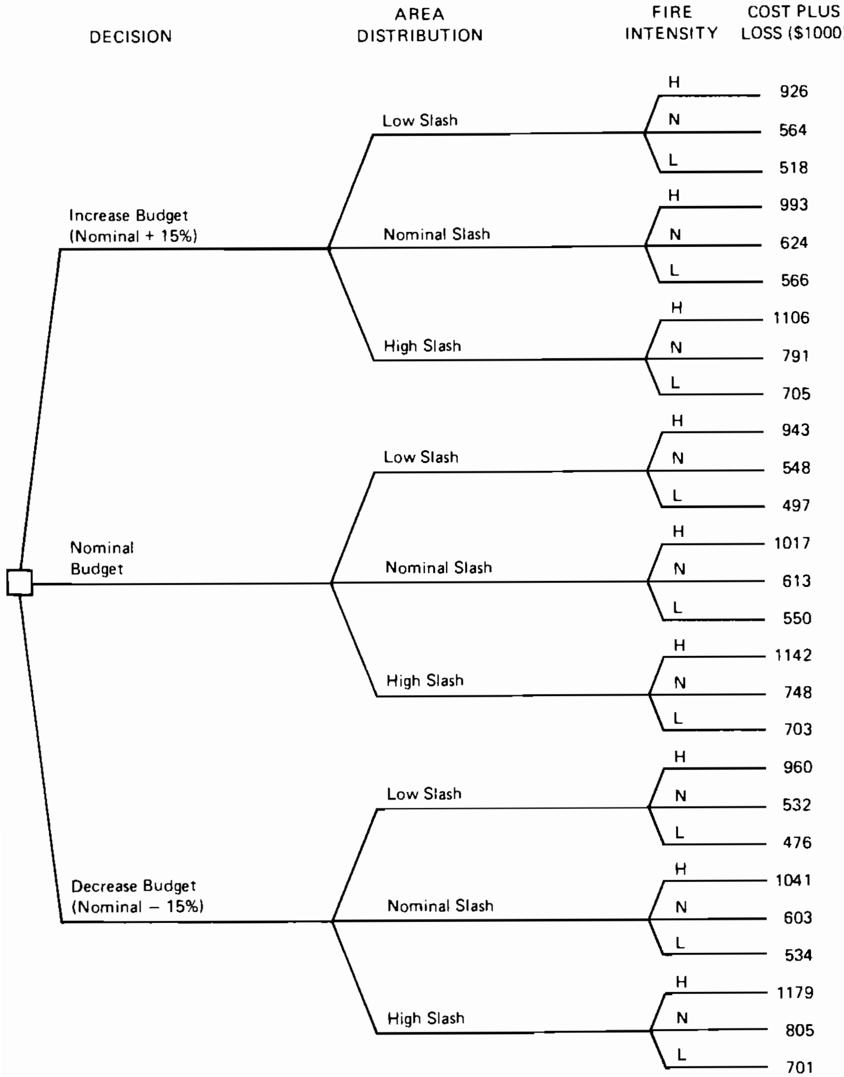

Fig. 2. Decision tree for value of information analysis.

tainty, (2) the expected value of perfect information on the fuel model area distribution, and (3) the expected value of perfect information on fire intensity. Budget changes of up to 75% and impacts (changes in escape fires) of up to 50% were analyzed. Several of the interesting cases are summarized in table 5. Case 1 corresponds to the assumptions of the preceding paragraph. Asymmetric as well as symmetric sets of alternatives were tried; for example, Case 5, where the budget increase is 10% and the budget decrease is 20%. For numerous other cases the settings of the budget changes result in one of the alternatives being dominant in all situations, implying no value for new information. Thus, the cases shown illustrate the magnitude of an upper bound on the expected value of perfect information in the context of the fire management budget decision.

For decisions involving relatively small changes in the budget level (on the order of 10–20%), the value of obtaining perfect information on the distribution of area (or fuel) types was very small, typically less than $0.01 per acre per year. Most annual decisions regarding budget level will involve alternatives of this magnitude, which corresponds to a fuel information maximum value of roughly $10 000 per year for the entire Mt. Hood National Forest. Recall that this is the value of perfect information; most information gathering alternatives will still leave one short of certainty. This suggests that no forest-wide fuels inventory effort would be economically justified on the basis of fire management decisions. Funds invested in the development of an improved system of records-keeping (applicable to many national forests) to better utilize information already available (e.g., from timber stand surveys, fuel treatment activities, and other resource management activities) may be of greater net value.

Radical changes in the fire management budget, on the order of a 50% increase or decrease, could result in situations where the value of information on fuel conditions is considerably higher, up to about $0.05 per acre. This is intuitively reasonable – the value of new information is likely to be greater when more resources are involved in the decision. Such major decisions merit detailed analysis on a case-by-case basis.

The expected value of perfect information on fire intensity was typically in the range of $0.01 to $0.05 per acre per year. While this is probably insufficient to merit any forest-specific research or detailed data gathering, it supports continued research in the development of improved models of fire behavior that would be of use on a number of forests.

We also investigated the value of obtaining perfect information simultaneously on fuel conditions and fire behavior, to see whether information synergism exists. The results showed no clear pattern. In some cases the value of joint information was greater than the sum of the individual values, while in others it was less. The joint value was never very different from the sum. For example, in Case 1 the expected value of perfect information on both fuel conditions and fire behavior was just over $0.03 per acre per year, while the sum of the individual EVPI values is $0.04 per acre per year.

4.2. *Value of increasing the resolution of the stylized fuel models*

A simple decision tree analysis was also used to investigate the value of adding additional stylized fuel models to the existing NFFL set; that is, of

Table 5
Summary of value of information cases

Case	Case descriptions				Results			
	Decision alternatives [a]		Change in number of escaped fires (%) [b]	Fuel and fire intensity uncertainty	Best alternative under uncertainty	Expected cost + loss ($1000/y)	Expected value of perfect information ($/acre/y)	
	Increase budget (%)	Decrease budget (%)					On fuel model distribution	On fire intensity
1.	15	15	10	nominal	increase budget	742	0.01	0.03
2.	10	10	10	nominal	increase budget	729	0.00	0.00
3.	15	15	10	low initial uncertainty on fire intensity and fuel types	decrease budget	652	0.005	0.01
4.	15	10	10	nominal	increase budget	742	0.008	0.02
5.	10	20	10	nominal	increase budget	729	0.01	0.03
6.	60	60	50	nominal	increase budget	698	0.00	0.04
7.	75	50	50	nominal	increase budget	738	0.04	0.08

[a] A third alternative, no budget change, is always considered.
[b] Change resulting from change in fire management budget.

providing a set of fuel models with greater resolution. This is a compromise between the existing limited number of fuel models and the more expensive direct measurement and sampling techniques for assessing fuel conditions.

In order to investigate this issue, we defined a budget decision problem in which it was assumed as an extreme case that all uncertainty in the assignment of stylized fuel models to the Mt. Hood National Forest was due to an insufficiently detailed set of fuel models, and that all such uncertainty could be removed by the addition of one more correctly chosen model. In other words, we set up a hypothetical decision that would be most likely to benefit from an additional model. This should provide an upper bound on the value of adding an additional fuel model to the existing NFFL set (see table 1) in the context of fire management budget decisions.

An additional model was defined with characteristics falling between the existing medium and heavy slash models (NFFL 12 and 13). The budget decision was analyzed both with the new model (and no uncertainty) and without the new model (and thus with uncertainty in the fuel area distribution). The results of the analysis were clear: adding an additional fuel model (and thus removing a source of uncertainty) generally did not result in a change in the preferred decision alternative. Since the additional model failed to affect the decision, its availability has no value. A few decision scenarios were found in which the use of the new model could result in a changed decision, but in these cases the difference in net cost plus loss among the alternatives was insignificant. These results were insensitive to wide ranges of settings for other input parameters. This finding suggests that the sets of stylized fuel models now available provide the appropriate amount of detail for decisions at the level of determining the annual fire management budget.

5. Summary and conclusions

Simple decision analyses and value-of-information calculations have proven useful in providing insights into forest fire and fuel management strategies. The primary purpose of this analysis was to identify the appropriate level of expenditures on improving information on fuel conditions and fire behavior, in the context of forest-wide fire management decisions. These values were found to be relatively small – of the order of \$0.01 per acre of forest per year for fuels information, and \$0.03 to \$0.05 per acre annually for fire behavior information. These small numbers imply that efforts at collecting information (e.g. on fuel inventory) would not be justified. While numerous modeling assumptions and approximations were necessary, a wide range of sensitivity analyses – and the relative insensitivity of the decisions to variations in the assumptions – suggest that the results of the analyses are quite robust.

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All views expressed in this paper are those of the authors and do not necessarily represent the views of the individuals cited or of the U.S. Forest Service.

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Part VI

FOREST ECOSYSTEM DYNAMICS

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MODELS OF POPULATION DYNAMICS WHICH TAKE INTO ACCOUNT AGE GROUP COMPETITION

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A class of mathematical models assuming a wide interpretation is considered. Under certain conditions these can be used to provide a mathematical description of the forest cenosis. Several proposals on the mathematical nature of problems are formulated, on the basis of which a research program is proposed.

1. Introduction

Problems of mathematical modeling in ecological and economic ecological studies differ considerably from similar problems in physics where facilities and experiments can give good numerical results by means of mathematical models. The complexity of biological processes, and the difficulty of experiments do not lend themselves to the formulation of models for forecasting characteristics of the processes involved with the same accuracy as those in physics.

Ecological models play different roles: they describe conservation laws of mass and energy, they reflect experimenter's view of the structure of corresponding flows and of the character of interconnections between system elements. Sometimes the use of mathematical models is the only method of experimentation with which to study a system.

Despite the fact the values of coefficients are rarely exact, mathematical models are more useful in complicated studies. The structure of interconnections in biological systems is so intricate that it is often impossible to trace development trends in a particular process, not to mention the estimation of critical parameter values on the basis of logical analysis of experimental material only, without use of mathematical models.

In this paper a class of mathematical models that assumes a wide interpretation that under certain conditions can be used to describe mathematically the forest cenosis is considered. Several proposals on the mathematical nature of such problems are formulated, on the basis of which a research program is proposed.

2. Basic equations

In any community two independent variables play essential roles: natural time t and specific time (e.g., the age of individual organisms) τ. Correspondingly, we denote by $x(t, \tau)$ vector components $x_i(t, \tau)$ which describe the biomass density of species of age τ in a population of number i at time t. Since variables t and τ change simultaneously, the equation of mass conservation has the form

$$\frac{\partial x}{\partial t} + \frac{\partial x}{\partial \tau} = F(x, t), \quad t, \tau \geq 0, \tag{1}$$

where F is the evolutional operator. Under certain external conditions this denotes the correspondence between the vector function x and the magnitude of mass flow. For example, in the case of a population the operator can be written as

$$F = \begin{cases} \left(a(t, \tau) - \int_0^T b(t, \tau, s) x(t, s)\,\mathrm{d}s\right) x(t, \tau) & \text{if } x \geq 0 \\ 0 \quad \text{if } x < 0, \end{cases} \tag{2}$$

where $a(t, \tau)$ is the specific biomass growth under optimal conditions (e.g., the absence of competition). The function $b(t, \tau, s)$ describes the decrease in biomass growth of trees of age τ due to competition from trees of age s. In the case when a community comprises n populations, we have

$$F_i = \begin{cases} \left(a_i(t, \tau) - \sum_{j=1}^{n} \int_0^T b_{ij}(t, \tau, s) x_j(t, s)\,\mathrm{d}s\right) x_i(t, \tau) \\ \text{if } x_i \geq 0,\ i = 1, 2, \ldots, n, \text{ and} \\ 0 \text{ otherwise.} \end{cases} \tag{3}$$

We should add the equation of nativity to eq. (1):

$$x_i(t, 0) = \int_0^T \alpha_i(t, s) x_i(t, s)\,\mathrm{d}s, \quad t \geq 0. \tag{4}$$

We call model (1)–(4), which contains the only nonlinear term describing competition, a quasi-linear model. This determines one of the most important peculiarities of forest cenosis dynamics, i.e., the phenomenon of self-thinning. This is a highly simplified macro model of a forest system where integration is made over space variables. In its simplest form it gives aggregated information that is needed to solve various problems that arise during forest exploitation. The model for the case when $n - 1$ has been the subject of many studies (see, for instance, Stronina 1980).

In his works Hellman uses models similar to eqs. (1), (3) and (4) to study problems of optimal control of the growth of a forest (see, for example, Hellman 1985) and advocates the applicability of this type of models to the forest sector. The basic difficulties of practical use of models (1)–(4) seem to be associated with computation of competition coefficients b_{ij}.

In this paper we consider only quasi-linear models and list results obtained by the authors during 1979–1981, on the basis of which we formulate a research program.

3. Theorems of existence and uniqueness

Assuming that all the functions in (1)–(4) are sufficiently smooth we formulate a Cauchy problem: to determine a non-negative continuous vector function $x(t, \tau)$ to satisfy eq. (1), boundary condition (4), and initial condition

$$x_i(0, \tau) = \varphi_i(\tau), \quad i = 1, 2, \ldots, n \tag{5}$$

in the domain

$$\Lambda_\infty\colon 0 \le \tau \le T;\ 0 \le t < \infty,$$

$$\forall \tau \varphi_i(\tau) \ge 0.$$

Without strict constraints on the matrix elements b_{ij}, there is no global solvability of the Cauchy problem because functions $x_i(t, \tau)$ may have a vertical asymptote. This case must not be excluded from consideration since one could observe such a situation in communities when a biomass of one species starts to grow quickly, leading to a complete reorganization of the cenosis. This process cannot be described within the framework of a quasi-linear model.

Let us denote by Λ_ϵ the domain $0 \le \tau \le T$, $0 \le t \le \epsilon$, where ϵ is a fixed positive number.

Theorem 1. (*Local theorem of existence*). Let the functions $a_i(t, \tau)$ and $b_{ij}(t, \tau, s)\alpha_i(t, \tau)$, $\varphi_i(\tau)$ be continuous in corresponding domains. Then there exist such $\epsilon > 0$ that the Cauchy problem is solvable in the domain Λ_ϵ.

Theorem 2. (*Theorem of uniqueness*). Subject to the constraints of Theorem 1, the Cauchy problem has no more than one solution in any domain $\Lambda_\epsilon (0 < \epsilon < \infty)$. Following Volterra we call a matrix $\|b_{ij}\|$ dissipated if, for any $t \geq 0$, the quadratic form

$$W(t) = \int_0^T \int_0^T \sum_{i,j=1}^{n} b_{ij}(t, \tau, s) x_i(t, \tau) x_j(t, s) \, \mathrm{d}\tau \, \mathrm{d}s$$

is non-negative.

Theorem 3. (*Gobal theorem of existence*). If matrix $\|b_{ij}\|$ is dissipated, then the Cauchy problem has a unique solution in the domain Λ_∞. We can say that communities are purely competitive if all $b_{ij} > 0$.

Theorem 4. (*Theorem of boundedness*). If all species are strongly limited (see Stronina 1980), the community is purely competitive, and the derivatives $\partial\alpha_i/\partial t$ are bounded in the domain Λ_∞, then all the functions $x_i(t, \tau)$ are bounded in the domain Λ_∞. (*Note*. The local theorem of existence and the theorem of uniqueness are also valid for eq. (1), subject to quite general assumptions on the evolutional operator F and the nativity operator.)

4. Steady states

The steady state of a biocenosis is determined as nontrivial, non-negative solution of the following:

$$\frac{\mathrm{d}x_i}{\mathrm{d}\tau} = \left(a_i(\tau) - \int_0^T \sum_j b_{ij}(\tau, s) x(s) \, \mathrm{d}s \right) x_i(\tau), \tag{6}$$

$$x_i(0) = \int_0^T \alpha_i(\tau) x_i(\tau) \, \mathrm{d}\tau, \quad i = 1, 2, \ldots, n, \tag{7}$$

where the coefficients a, b, α do not depend on time t. The problem, eqs. (6) and (7), always allows a trivial solution $x_i \equiv 0$. In order to exclude from consideration all trivial solutions, it is convenient to replace eqs. (6) and (7) with a system of integral equations.

Theorem 5. The finding of nontrivial solutions to the problem, eqs. (6) and (7), is equivalent to finding the functions $Z_i(\tau)$ and non-negative numbers λ_i which satisfy the system of integral equations

$$\int_0^T \alpha_i(s) \exp\left(\int_0^s a_i(s_1) \, \mathrm{d}s_1 \right) \exp[-Z_i(s)] \, \mathrm{d}s = 1, \tag{8}$$

$$Z_i(\tau) = \sum_j \lambda_j \int_0^\tau \int_0^T b_{ij}(s_1, s) \exp\left(\int_0^s a_j(s_2) \, \mathrm{d}s_2 \right) \exp\left[-Z_j(s)\right] \, \mathrm{d}s \, \mathrm{d}s_1, \tag{9}$$

where $\lambda_i = x_i(0)$, and function $x_i(\tau)$ is determined from:

$$x_i(t) = \lambda_i \exp\left(\int_0^T a_i(s_1)\,\mathrm{d}s_1\right) \exp[-Z_i(\tau)].$$

In the general case the system eqs. (8) and (9) is extremely difficult; no general results exist for the case when no restrictions on b_{ij} are imposed.

5. The fining of steady states

Let us consider the case when

$$b_{ij}(\tau, s) = q_i(\tau) p_{ij}(s). \tag{10}$$

This expression gives satisfactory approximation to real competition. The biological sense of eq. (10) is evident: it is assumed that the resistance to competition in relation to "external influence" depends only on the tree age and type. If we introduce the notations

$$K_i(\tau) = \int_0^T q_i(s)\,\mathrm{d}s, \quad c_i = \sum_j \int_0^T p_{ij}(s) x_j(s)\,\mathrm{d}s$$

then the system of eqs. (8) and (9) can be reduced to

$$\int_0^T \alpha_i(s) \exp\left(\int_0^s a_i(s_1)\,\mathrm{d}s_1\right) \exp[-c_i K_i(s)]\,\mathrm{d}s = 1, \tag{11}$$

$$c_i = \sum_j \int_0^T \lambda_j p_{ij}(s) \exp\left(\int_0^s a_j(s_1)\,\mathrm{d}s_1\right) \exp\left[-c_j K_j(s)\right]\,\mathrm{d}s. \tag{12}$$

The left-hand side of eq. (11) is the convex function of c_i, which is why each equation in this system has no more than two solutions. Thus the system of eqs. (11) and (12) has no more than 2^n solutions.

Theorem 6. Let interactions b_{ij} between species satisfy expression (10). It is then possible to construct an algorithm that finds all the steady states of biocenosis in a finite number of steps. This algorithm uses the following operations, besides four arithmetic operations: (1) computation of exponents; (2) integration; (3) solution of inequalities, i.e., the recognition of the correctness of the formula $\exists x[\varphi(x) > 0]$, where φ is given; and (4) the search of the root of the equation $f(x) = 0$, where $f(x)$ is a continuous monotonous function.

In the general case, the problem is qualitatively more complicated; moreover, the main difficulties are seen in the case when $n = 2$, when one cannot construct a finite algorithm to find all solutions, or even to recognize whether the system has a solution through the operations indicated in *Theorem 6.*

6. On the optimal control of the growth of a forest

The problems of forest exploitation can be formulated in terms of optimal control theory so far as they are reduced to the rational choice of control actions: the whole list of actions can be attributed to them. First of all, this is the felling of trees. We denote by $u_i(t, \tau)$ the fraction of the biomass of trees of type i and age τ, which are cut down at time t. Other measures, such as ameliorative works, fertilizer usage, planting, etc., can also be employed, all such measures lead to changes in the character of biomass growth and nativity parameters. In order to see peculiarities of the optimization problems arising here we confine ourselves only to consideration of the case when the forest cenosis is controlled by felling. The introduction of the function u_i leads to the fellowing growth equation

$$\frac{\partial x_i}{\partial t} + \frac{\partial x_i}{\partial \tau} = \left(a_i - u_i - \sum_j \int_0^T b_{ij} x_j \, \mathrm{d}s \right) x_i. \tag{13}$$

The choice of the function $u_i(t, \tau) \in [0, 1]$ is governed by the need to maximize the objective function that characterizes the end product (biomass) obtained:

$$y(u) = \int_{t_1}^{t_2} \sum_i \int_0^T \mu_i(\tau) u_i(t, \tau) x_i(t, \tau) \, \mathrm{d}\tau \, \mathrm{d}t, \tag{14}$$

where $\mu_i(\tau)$ is a weight coefficient for the biomass of trees of type i and age τ.

In the problem of forest control a special problem is its continuous exploitation: to find that amount of forest that can be felled without violating the steady state of forest massif. In other words, it is necessary to determine a regime of felling under which the quantity of wood obtained annually is at a maximum, while the steady state is maintained.

This problem can be formulated as follows: to determine piecewise continuous functions $u_i(\tau) \in [0, 1]$ and $x_i(0)$ supplying a maximum value to the function

$$I(u) = \int_0^T \sum_i \mu_i(\tau) u_i(\tau) x_i(\tau) \, \mathrm{d}\tau \tag{15}$$

subject to the constraints

$$\frac{dx_i}{d\tau} = \left(a_i - u_i - \int_0^T \sum_j b_{ij} x_j(s)\, ds\right) x_i(\tau),$$

$$x_i(0) = \int_0^T \alpha_i(s) x_i(s)\, ds. \tag{16}$$

The solution to the problem of eqs. (15) and (16) determines an optimal exploitation regime under steady state conditions, which we denote as x^*, u^*. The transition to the steady state gives rise to a number of optimization problems with functions of type (14).

For all similar problems it is not difficult to formulate necessary conditions of optimality of maximum principle type. However, these prove to be non-constructive since conjugate variables also satisfy some system of integral–differential equations. For a solution of optimization problem it is necessary to develop special methods of finite-dimensional approximation.

7. Stability

Problems of the type, eqs. (15) and (16), seem to be the most interesting from the practical point of view, but in order to use rules of felling and not to resort to feedback operator construction, it is necessary to be sure of the stability of the solutions obtained. Essentially this problem is equivalent to that of the stability of steady state solutions of the problem of eqs. (1)–(4).

We confine ourselves only to consideration of a stationary medium (environment), which means that the functions a, b and α are assumed to be independent of time. Let us denote by $\hat{x}(\tau)$ steady solutions and suppose that at time $t = t_0$ the environment is perturbed and passed to the state $\hat{x}(\tau) + \epsilon(\tau)$. Let us denote also by $x(t, \tau)$ the solution of the Cauchy problem with the initial condition

$$x(t_0, \tau) = \varphi(\tau).$$

We call the state $\hat{x}(\tau)$ at solutely stable if for any $\epsilon(\tau)$

$$\lim_{t \to \infty} x(t, \tau) = \hat{x}(\tau). \tag{17}$$

If the condition (17) occurs only for sufficiently small ϵ then we call the solution $\hat{x}(\tau)$ a stable one. Assuming

$$y_i(t, \tau) = x_i(t, \tau) - \hat{x}_i(\tau)$$

we reduce relations (1), (4), and (5) to the form

$$\frac{\partial y_i}{\partial t} + \frac{\partial y_i}{\partial \tau} = m_i y_i - \hat{x}_i(\tau) \int_0^T \sum_j b_{ij}(\tau, s) y_j(\tau, s) \, \mathrm{d}s - W_i, \tag{18}$$

$$y_i(t, 0) = \int_0^T \alpha_i(s) y_i(t, s) \, \mathrm{d}s, \quad y_i(0, \tau) = \epsilon_i(\tau), \tag{19}$$

where

$$m_i = a_i - \int_0^T \sum_j b_{ij} \hat{x}_j \, \mathrm{d}s, \quad W_i = y_i \int_0^T \sum_j b_{ij} y_j \, \mathrm{d}s.$$

We call eqs. (18) and (19) the deviation problem. If we reject the magnitude W_i in eq. (18) then we proceed to the problem that we call a linear one. In relation to the solvability of linear problems the following theorem holds.

Theorem 7. Let conditions of *Theorem 1* be fulfilled. Then, for a linear problem, the global theorem of existence, theorem of uniqueness, and the following estimate of the solution hold:

$$\| x(t, \tau) \| \leq C\mathrm{e}^{qt},$$

where

$$\| x(t, \tau) \| = \max_{\tau, i} | x_i(t, \tau) |,$$

and C, q are positive constants.

For practical control problems it is sufficient to study the stability of a steady solution, that is, to study properties of the Cauchy problem for a linear deviation problem. A numerical solution of the Cauchy problem, eqs. (18) and (19), at $W = 0$ can be obtained without difficulty, but no general method of studying the behavior of these solutions with any ϵ is known so far. One exception is the case $n = 1$, which is studied in some detail in Stronina (1980).

8. Research program

We have given an account of the main results of a study of models describing the functioning of a forest cenosis, which enables us to outline a research program that could serve as a basis for a mathematical theory for the exploitation of similar communities. From our point of view the following problems should be studied first:

(a) The creation of finite-dimensional approximations of the continuum models models described in this paper, and the subsequent development of effective numerical methods. The development of applications of the theory should be connected first of all with the improvement of these methods.
(b) Optimization problem statements within a framework of finite-dimensional approximations and the development of corresponding numerical methods of optimal exploitation control on the basis of these approximations.
(c) Expansion of the model, taking into account the spatial distribution of vegetation.
(d) A study of the set of bifurcational characteristics of a forest cenosis and changes in the structure of equilibrium states depending on the peculiarities of exploitation. Let us note that if we are to consider the evolutional operator as a function of loading (i.e., control), then a corresponding optimality condition will be analogous to the vanishing of the Gateau derivative for the evolutional operator. Hence bifurcational values of forest loadings must be situated in the neighborhood of optimal values of control actions. In its turn, this means that it will be possible to radically reorganize forest communities.
(e) A study of model sensitivity in relation to its parameters. This will enable experts to determine the most important directions of experimental studies (from a practical point of view) in order to make the model more precise, and hence to increase the efficiency of exploitation.

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ON THE OPTIMAL CONTROL OF A FOREST WHICH CONSISTS OF SEVERAL SPECIES OF TREES

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A mathematical model is suggested for a forest which consists of several species of trees. Trees of the forest produce seeds which will grow into trees. The shading effect of other trees upon a smaller one is taken into account in a special way. Further, the seeding is also carried out by the forest management which also does thinning operations in the forest. The purpose of the model is to be a part of a larger mathematical model of an economical system which is using wood as raw material.

1. Introduction

The laws governing the growth of a real forest are extremely complicated. A mathematical model which would be reasonably well based on those laws would very likely be most complicated, in such a way, that it would be difficult to use it as a part of a mathematical model of an economical system; one which is by nature large and complicated. Therefore, a mathematical model is needed which is of such a structure that it fits into an economical model. On the other hand, the model should reproduce the basic processes of a forest well enough. If the number of species of trees in the forest is n, our model will have $n(n+4)$ parameter functions available for the identification of the model. This would be done by using well-known methods of the theory of identification from a real forest. If it turns out that the identification has been a successful one, the model is valid, whatever laws are governing the growth of a forest.

2. Basic equations

We shall assume that a forest, which will consist of n species of trees, is planted at time $t=0$. The trees are assumed to be growing according to given laws

$$\mathrm{d}f/\mathrm{d}t = l_k(f, f_{mk}) \quad k = 1, 2, \ldots, n, \tag{1}$$

where $l_k(f, f_{mk})$ are given functions with f a quantity which characterizes the size of a tree of species k, where $k = 1, 2, \dots, n$. Here $f_{mk} = (f_{mk1}, \dots, f_{mkr})$ is a vector consisting of r parameters which are assumed to govern the growth of a tree of species k.

In what follows we shall write instead of, say, $x^{(k)}$, $k = 1, 2, \dots, n$, simply $x^{(k)}$. We shall denote the solution of eq. (1) with condition that $f(0) = 0$ by $f_k(t)$. If $l_k(f, f_{mk}) = 1$, then $f_k(t) = t$, i.e., $f_k(t)$ is now simply the age of the tree.

A basic quantity of the following theory will be function $x^{(k)}(t, f)$, such that $x^{(k)}(t, f)\mathrm{d}f$ is, at time t, the number of trees of species k, whose size is in the range of $(f, f + \mathrm{d}f)$.

A portion $\mu^{(k)}(f)\mathrm{d}t$ of the trees of size f of the species k are assumed to die during time $(t, t + \mathrm{d}t)$, due to natural mortality. Furthermore, it is assumed that trees of species l larger than f will increase the mortality rate of trees of species k and of size f by an amount

$$\int_f^{f_k(t)} n_{kl}(\bar{f}) x^{(l)}(t, \bar{f}) \,\mathrm{d}\bar{f}. \tag{2}$$

Therefore, if the portion $u^{(k)}(t, f)\mathrm{d}t$ of tree species k of size f are felled during time $(t, t + \mathrm{d}t)$, the portion

$$\left(\mu^{(k)}(f) + u^{(k)}(t, f) + \sum_{l=1}^{n} \int_f^{f_k(t)} n_{kl}(\bar{f}) x^{(l)}(t, \bar{f}) \,\mathrm{d}\bar{f} \right) \mathrm{d}t \tag{3}$$

of tree species k of size f will die and is felled during time $(t, t + \mathrm{d}t)$. The term

$$\sum_{l=1}^{n} \int_f^{f_k(t)} n_{kl}(\bar{f}) x^{(l)}(t, \bar{f}) \,\mathrm{d}f$$

will represent the shading effect of the forest on a tree species k whose size is f.

A tree species k of size f is assumed to produce, during the time interval $(t, t + \mathrm{d}t)$, $\lambda^{(k)}(f)\mathrm{d}t$ seeds which begin to grow. There will be, therefore, during time $(t, t + \mathrm{d}t)$,

$$\left(\int_0^{f_k(t)} \lambda^{(k)}(f) x^{(k)}(t, f) \,\mathrm{d}f \right) \mathrm{d}t \tag{4}$$

trees of size $f = 0$, i.e., trees of age 0. If the rate of seeding of tree species k at time t is $\nu^{(k)}(t)$, we obtain the relationship

$$l_k(0, f_{mk}) x^{(k)}(t, 0) = \int_0^{f_k(t)} \lambda^{(k)}(f) x^{(k)}(t, f) \,\mathrm{d}f + \nu^{(k)}(t). \tag{5}$$

Now

$$x^{(k)}(t+\Delta t,\, f+\Delta f) = x^{(k)}(t,\, f) - \left(\mu^{(k)}(f) + u^{(k)}(t,\, f) + \sum_{l=1}^{n} \int_{f}^{f_k(t)} n_{kl}(f) x^{(l)}(t,\, f)\,\mathrm{d}f\right) x^{(k)}(t,\, f), \tag{6}$$

which yields, after a straightforward calculation, the following equations

$$(\partial/\partial t) x^{(k)}(t,\, f) + l_k(f,\, f_{mk})(\partial/\partial f) x^{(k)}(t,\, f) = -\left(\mu^{(k)}(f) + u^{(k)}(t,\, f) + \sum_{j=1}^{n} \int_{f}^{f_k(t)} n_{kj}(\bar{f}) x^{(j)}(t,\, \bar{f})\,\mathrm{d}\bar{f}\right) x^{(k)}(t,\, f),$$

$$k = 1, 2, \ldots, n, \tag{7}$$

valid for $0 < t$ and $0 < f < f(t)$.

The aim of the theory is to get the maximum value for the expression

$$J(u,\, \nu) = \sum_{j=1}^{n} \int_{0}^{f_k(T)} l_j(f) x^{(j)}(T,\, f)\,\mathrm{d}f - \sum_{j=1}^{n} \int_{0}^{T} K_{ij}(t) \nu^{(j)}(t)\,\mathrm{d}t - \sum_{j=1}^{n} \int_{0}^{T} \mathrm{d}t \int_{0}^{f_k(t)} L_j(t,\, f) u^{(j)}(t,\, f) x^{(j)}(t,\, f)\,\mathrm{d}f. \tag{8}$$

Here

$$\int_{0}^{f_k(T)} l_j(f) x^{(j)}(T,\, f)\,\mathrm{d}f$$

is the amount which the forest returns, when it is felled at time T, from trees of species j, while the functions $K_j(t)$ and $L_j(t,\, f)$ represent the expenses due to seeding and felling, respectively.

We shall assume that there are for the functions $\nu^{(l)}(t)$ and $\mu^{(l)}(t,\, f)$ – the control functions of the theory – the following constraints:

$$0 \le \nu^{(k)}(t) \le \tilde{\nu}^{(k)}(t), \quad k = 1, 2, \ldots, n \tag{9}$$

and

$$0 \le u^{(k)}(t,\, f) \le \tilde{u}^{(k)}(t,\, f), \quad k = 1, 2, \ldots, n, \tag{10}$$

where $\tilde{\nu}^{(k)}(t)$ and $\tilde{u}^{(k)}(t,\, f)$ are given functions.

3. The optimal control problem

By substituting

$$x^{(k)}(t,f)=y^{(k)}(t,f)\exp\left(-\int_0^f \mu_k(\bar f)/l_k(\bar f,f_{mk})\,\mathrm{d}\bar f\right), \tag{11}$$

$$z^{(k)}(t,f)=\sum_{j=1}^{n}\int_f^{f_k(t)} n_{kj}(f)x^{(j)}(t,\bar f)\,d\bar f \tag{12}$$

and

$$w^{(k)}(t,f)=\int_f^{f_k(t)}\lambda_k(f)x^{(k)}(t,\bar f)\,\mathrm{d}\bar f, \tag{13}$$

we now obtain from eqs. (5), (7), (8), (11), (12) and (13) the following optimal control problems:

$$y_t^{(k)}(t,f)+l_k(f,f_{mk})y_f(t,f)$$
$$+\left[u^{(k)}(t,f)+z^{(k)}(t,f)\right]xy^{(k)}(t,f)=0, \tag{14}$$

$$z_f^{(k)}(t,f)+\sum_{j=1}^{n}\hat n_{kj}(f)y^{(j)}(t,f)=0, \tag{15}$$

$$w_f^{(k)}(t,f)+\hat\lambda_k(f)y^{(k)}(t,f)=0, \tag{16}$$

$$l_k(0,f_{mk})y^{(k)}(t,0)=w^{(k)}(t,0)+\nu^{(k)}(t), \tag{17}$$

$$z^{(k)}(t,f(t))=w^{(k)}(t,f(t))=0, \tag{18}$$

$$J(u,\nu)=\sum_{j=1}^{n}\int_0^{f_k(T)}\hat l_j(f)y^{(j)}(T,f)\,\mathrm{d}f-\sum_{j=1}^{n}\int_0^T K_j(t)\nu^{(j)}(t)\,\mathrm{d}t$$
$$-\sum_{j=1}^{n}\int_0^T\mathrm{d}t\int_0^{f_k(t)}\hat L_j(t,f)u^{(j)}(t,f)\,\mathrm{d}f. \tag{19}$$

let a_k denote any of the functions λ_k, η_{ik}, l_k, L_k. Then the notation a_k used above is defined as follows:

$$\hat a_k=a_k\exp\left(-\int_0^f\mu_k(f)/l_k(f,f_{mk})\,\mathrm{d}f\right).$$

We also denote partial differentiation simply by subindices, for instance, $x_t^{(k)}(t, f)\}$ *instead of* $(\partial/\partial t)x^{(k)}(t, f)$.

4. Conditions for optimality

As indicated by Hellman (1981), we shall now apply the method of Egorov (1966) for obtaining the conditions of optimality. Let $\Psi_k(t, f)$, $\xi_k(t, f)$, and $\theta_k(t, f)$, where $k = 1, 2, \ldots, n$, be continuous arbitrary functions for $0 < t$ and $0 < f$. Then we have the identity

$$\begin{aligned}
&\int_0^T \mathrm{d}t \int_0^{f_k(t)} \sum_{k=1}^{n} \Big[\Psi_k(t, f)\big[y_t^{(k)}(t, f) + l_k(f, f_{mk})\, y_f^{(k)}(t, f) \\
&\quad + \big(+ u^{(k)}(t, f) + z^{(k)}(t, f)\big) y^{(k)}(t, f)\big] \\
&\quad + \xi_k(t, f)\left(z_f^{(k)}(t, f) + \sum_{j=1}^{n} \hat{n}_{kj}(t, f)\, y^{(j)}(t, f)\right) \\
&\quad + \theta_k(t, f)\big[w_f^{(k)}(t, f) + \hat{\lambda}_k(t, f)\, y^{(k)}(t, f)\big]\Big] \,\mathrm{d}f = 0. \qquad (20)
\end{aligned}$$

Now

$$\begin{aligned}
\int_0^T \mathrm{d}t \int_0^{f_k(t)} \Psi_k y_t^{(k)} \,\mathrm{d}f &= -\int_0^T \mathrm{d}t \int_0^{f_k(t)} \Psi_{k_t} y^{(k)} \,\mathrm{d}f + \int_0^T \mathrm{d}t \frac{\mathrm{d}}{\mathrm{d}t} \int_0^{f_k(t)} \Psi_k y^{(k)} \,\mathrm{d}f \\
&\quad + \int_0^T \mathrm{d}t\, l_k(f(t), f_{mk})\, \Psi_k(t, f(t)\, y^{(k)}(t, f(t)) \\
&= -\int_0^T \mathrm{d}t \int_0^{f_k(t)} \Psi_{k_t} y^{(k)} \,\mathrm{d}f \\
&\quad + \int_0^{f_k(T)} \Psi_k(T, f)\, y^{(k)}(T, f) \,\mathrm{d}t \\
&\quad - \int_0^{f_k(0)} \Psi_k(0, f)\, y^{(k)}(0, f) \,\mathrm{d}f \\
&\quad - \int_0^T l_k(f(t), f_{mk})\, \Psi_k(t, f(t))\, y^{(k)}(t, f(t)) \,\mathrm{d}t. \qquad (21)
\end{aligned}$$

$$\begin{aligned}
\int_0^T \mathrm{d}t \int_0^{f_k(t)} \Psi_k l_k y_f^{(k)} \,\mathrm{d}f &= -\int_0^T \mathrm{d}t \int_0^{f_k(t)} (\Psi_k l_k)\, Y_f^{(k)} \,\mathrm{d}f \\
&\quad + \int_0^T \mathrm{d}t \big\{ \Psi_k(t, f(t))\, l_k(f(t), f_{mk})\, y^{(k)}(t, f)) \\
&\quad - \Psi_k(t, 0)\, l_k(0, f_{mk})\, y^{(k)}(t, 0)\big\}, \qquad (22)
\end{aligned}$$

$$\int_0^T \mathrm{d}t \int_0^{f_k(t)} \xi_k z_f^{(k)}\, \mathrm{d}f = -\int_0^T \mathrm{d}t \int_0^{f_k(t)} \xi_{k_f} z^{(k)}\, \mathrm{d}f + \int_0^T \mathrm{d}t \left\{ \xi_k(t, f(t)) z^{(k)}(t, f(t)) - \xi_k(t, 0) z^{(k)}(t, 0) \right\}. \tag{23}$$

and

$$\int_0^T \mathrm{d}t \int_0^{f_k(t)} \theta_k w^{(k)}\, \mathrm{d}f = -\int_0^T \mathrm{d}t \int_0^{f_k(t)} \theta_{k_f} w^{(k)}\, \mathrm{d}f + \int_0^T \mathrm{d}t \left\{ \theta_k(t, f(t) w)^{(k)}(t, f(t)) - \theta_k(t, 0) w^{(k)}(t, 0) \right\}. \tag{24}$$

Requiring that in

$$\Psi_k(T, f) = \hat{l}_k(t), \tag{25}$$

$$\Psi_k(t, f(t)) = 0, \tag{26}$$

$$\Psi_k(t, 0) + \theta_k(t, 0) = 0. \tag{27}$$

$$\xi_k(t, 0) = 0, \tag{28}$$

and by using eq. (18), the identity (20) becomes

$$\sum_{l=1}^{n} \int_0^{f_k(t)} \hat{l}_k(f) y^{(k)}(T, f)\, \mathrm{d}f - \sum_{k=1}^{n} \int_0^T \Psi_k(t, 0) \nu^{(k)}(t)\, \mathrm{d}t + \int_0^T \mathrm{d}t \int_0^{f_k(t)} \mathrm{d}f \sum_{k=1}^{n} \left[-\xi_{k_f} z^{(k)} - \theta_{k_f} w^{(k)} + \left(-\Psi_{kt} - (l_k \Psi_k)_f + (+u^{(k)} + z^{(k)}) \Psi_k + \sum_{j=1}^{n} \hat{n}_{jk} \xi_j + \theta_k \hat{\lambda}_k \right) y^{(k)} \right] = 0. \tag{29}$$

Let us now write

$$\Delta\nu^{(k)} = \nu_{\text{opt}}^{(k)} - \nu^{(k)},\ \Delta u^{(k)} - u_{\text{opt}}^{(k)} - u^{(k)}, \text{ etc.} \tag{30}$$

It now follows from eqs. (19) and (29) that

$$\Delta J(u, \nu)=\sum_{k=1}^{n}\int_0^T[\Psi_k(t,0)-K_k(t)]\,\Delta\nu^{(k)}(t)\,\mathrm{d}t$$

$$+\int_0^T\mathrm{d}t\int_0^{f_k(t)}\mathrm{d}f\sum_{k=1}^{n}\left[-\Psi_k(t,f)-\hat{L}_k(t,f)\right]y_{\mathrm{opt}}^{(k)}(t,f)\Delta u^{(k)}(t,f)$$

$$+\int_0^T\mathrm{d}t\int_0^{f_k(t)}\mathrm{d}f\sum_{k=1}^{n}\Bigg[\Delta z^{(k)}\Big(+\xi_{k_f}-y_{\mathrm{opt}}^{(k)}\Psi_k\Big)+\Delta w^{(k)}\theta_{k_f}$$

$$-\Delta y^{(k)}\Bigg(-\Psi_{k_t}-\big(l_k\Psi_{k_f}\big)+\big(u_{\mathrm{opt}}^{(k)}+z_{\mathrm{opt}}^{(k)}\big)\Psi_k+\sum_{j=1}^{n}\xi_j\hat{n}_{jk}$$

$$+\theta_k\hat{\lambda}_k+\hat{L}_k(t,f)u_{\mathrm{opt}}^{(k)}\Bigg)+(\Delta u^{(k)}+\Delta z^{(k)})\Delta y^{(k)}\Psi_k$$

$$+\hat{L}_k\Delta u^{(k)}\Delta y^{(k)}\Bigg].\tag{31}$$

Let us now require that functions $\Psi_k(t,f)$, $\xi_k(t,f)$, and $\theta_k(t,f)$ satisfy equations

$$\Psi_{k_t}(t,f)+\big(l_k(f,f_{mk})\Psi_k(t,f)\big)_f-\big(u_{\mathrm{opt}}^{(k)}(t,f)$$

$$+z_{\mathrm{opt}}^{(k)}(t,f)\big)\Psi_k(t,f)-\sum_{j=1}^{n}\xi_j\hat{n}_{jk}-\theta_k\hat{\lambda}_k-L_k(t,f)u_{\mathrm{opt}}^{(k)}=0,$$

$$\xi_{k_f}+y_{\mathrm{opt}}^{(k)}\Psi_k=0.\tag{33}$$

and

$$\theta_{k_f}=0.\tag{34}$$

Then eq. (31) becomes

$$\Delta J(u,\nu)=\sum_{k=1}^{n}\int_0^T[\Psi_k(t,0)-K_k(t)]\,\Delta\nu^{(k)}(t)\,\mathrm{d}t$$

$$+\sum_{k=1}^{n}\int_0^T\mathrm{d}t_0^{f_k(t)}\,\mathrm{d}f\left[-\Psi_k(t,f)-\hat{L}_k(t,f)\right]y_{\mathrm{opt}}^{(k)}(t,f)\Delta u^{(k)}(t,f)$$

$$+\sum_{k=1}^{n}\int_0^T \mathrm{d}t\int_0^{f_k(t)\,\mathrm{d}f}\{(\Delta u^{(k)}(t,f)+\Delta z^{(k)}(t,f))\Delta y^{(k)}(t,f)$$

$$\times\Psi_k(t,f)$$

$$+\hat{L}_k\Delta u^{(k)}\Delta y^{(k)}\} \tag{35}$$

Since $\Delta J(u, v)\geq 0$ and since the first two sums determine the sign of $\Delta J(u, v)$, we arrive at the following optimal controls:

$$\nu^{(k)}(t)=\begin{cases}\tilde{\nu}^{(k)}(t), & \text{for } l_k(0)\Psi_k(t,0)>K_k(t)\\ 0, & \text{for } l_k(0)\Psi_k(t,0)<K_k(t)\end{cases} \tag{36}$$

and

$$u^{(k)}(t,f)=\begin{cases}\tilde{u}^{(k)}(t,f), & \text{for } \Psi_k(t,f)>\hat{L}_k(t,f)\\ 0, & \text{for } \Psi_k(t,f)<\hat{L}_k(t,f).\end{cases} \tag{37}$$

where the conditions (9) and (10) were used. It turned out that the function $\Psi_k(t, f)$ determines the switching points of both $\nu^{(k)}(t)$ and $u^{(k)}(t, f)$. The functions $\Psi_k(t, f)$ are obtained by using conditions (36) and (37) in the familiar way, by solving the following boundary value problem:

$$\Psi_{k_t}(t,f)+l_k(f,f_{mk})\Psi_k(t,f)_f-(u^{(k)}(t,f)+z^{(k)}(t,f))\Psi_k(t,f)$$

$$-\sum_{j=1}^{n}\xi_j(t,f)\hat{n}_{jk}(f)-\theta_k(t,f)\hat{\lambda}_k(f)-\hat{L}_k(t,f)\mu^{(k)}(t,f)=0,$$

$$y_t^{(k)}(t,f)+l_k(f,f_{mk})y_f^{(k)}(t,f)$$

$$+(u^{(k)}(t,f)+z^{(k)}(t,f))y^{(k)}(t,f)=0.$$

$$z_f^{(k)}(t,f)+\sum_{j=1}^{n}\hat{n}_{kj}(t)y^{(j)}(t,f)=0,$$

$$w_f^{(k)}(t,f)+\hat{\lambda}_k(f)y^{(k)}(t,f)=0,$$

$$\xi_{f_k}(t,f)+y^{(k)}(t,f)\Psi_k(t,f)=0$$

and

$$\theta_{k_f}(t,f)=0,$$

under the conditions

$$l_k(0, f_{mk})y^{(k)}(t, 0) = w^{(k)}(t, 0) + \nu^{(k)}(t),$$

$$w^{(k)}(t, f(t)) = 0,$$

$$z^{(k)}(t, f(t)) = 0,$$

$$\Psi_k(t, f(t)) = 0,$$

$$\xi_k(t, f(t)) = 0,$$

$$\xi_k(t, 0) = 0,$$

$$\Psi_k(t, 0) + \theta_k(t, 0) = 0,$$

$$\Psi_k(T, f) = \hat{l}_k(f).$$

5. Comments

An obvious drawback of the model of the present paper is its deterministic nature, since the growth of a forest is obviously stochastic in several ways. The quantities of our theory are to be interpreted as average values. Now, a forest model which would be stochastic in the sense that there are equations for the average values as well as for the variances, could already be fairly useful in practice. One way to achieve such a model would be to introduce the probability density $P_i(n, f, t)$ such that $P_i(n, f, t)\Delta f$ is the probability that there are, at time t, n trees of kind i whose size is in the range $(f, f + \Delta f)$. After reformulating (2), the law of the shading effect, in a suitable way, it is possible to derive difference-differential equations for the probability densities $P_i(n, f, t)$. The average values and the variances become known as soon as the functions $\sum_{n=1}^{\infty} nP_i(n, f, t)$ and $\sum_{n=1}^{\infty} n^2 P_i(n, f, t)$ are known. It is possible to obtain equations for these functions from the above mentioned difference-differential equations.

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A NOTE ON FORECASTING THE FOREST STATE WITH A FIXED NUMBER OF MEASUREMENTS

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The problem of forecasting forest state in the absence of complete information on the characteristic phenomena is considered. The number of observed measurements is fixed. The author has constructed an approximation of the forecasting vector using the ideal of the Tchebyshev center. A certain regularization method of computation is proposed.

1. Introduction

The problem of forecasting the state of a forest cenosis is considered. By the term state we mean a set of various characteristics such as number of trees of a certain age, size, foliage, etc., in one-species communities, or the number of trees of certain age of all species in many-species communities, etc. A forest cenosis is subject to systematic external perturbations such as anthropogenic stress, tree harvesting, solar activity, weather, etc.; these effects are beyond exact quantitative analysis but they are limited and their effects are known to the observer. Some characteristics are almost inaccessible to direct measurement. In addition, the collection of some data on the forest state are labor-consuming and expensive so that they are made at fixed points in time and usually only some of them and their linear combinations are measured. The question thus arises: What will be the state of the forest cenosis after some period of time? We thus come to the problem of forecasting in the absence of complete information. Obviously it is generally impossible to obtain true (exact) values so that estimates, in a certain sense, may be considered optimal. A simplified model in which changes in forest characteristics in the neighborhood of some state has therefore been built using a system of linear differential equations with additive perturbations. The perturbations may change over time. It should be noted that the errors inherent in linear modeling may be considered as one of the perturbations.

2. Problem formulation and solution

The state equation of the system considered is as follows:

$$\dot{x} = A(t)x + B(t)u, \tag{1}$$

where $x \in R^n$, $u \in R^p$, $0 \leq t \leq T$, $x(0) = x_0$, $A(t)$ and $B(t)$ are continuous matrix value functions on $[0, T]$; and u is a measurable vector function satisfying a constraint $u(t) \epsilon P$, where P is a convex compact set in R^p. The observer knows only the values

$$y(t_i) = G(t_i) x(t_i),$$

where $0 \leq t_1 < t_2 < \ldots < t_N \leq \Delta$, $N \geqslant 1$, $0 < \Delta < T$, $y \in R^m$, and values of $G(t_i)$ matrices. The observer's problem is to forecast the vector $x(T)$ according to

$$y = \operatorname{col}[y(t_1), y(t_2), \ldots, y(t_N)], \tag{3}$$

where the matrices $A(t)$, $B(t)$, $0 \leq t \leq T$ and $G(t_i)(i = 1, \ldots, N)$ are known, and the function $u(t) \in P$, $0 \leq t \leq T$ is unknown. The initial state x_0 is also unknown, and the observer knows only that $x_0 \in X_0$ is a non-empty convex compact set in R^n.

The forecasting problem has different ecological interpretations, but we fix our attention on the following:

(a) State equation (1) describes the dynamics of the increase in biomass in a one-species community of a forest farm, or in a many-species community with systematic tree harvesting, such as thinning or with permanent anthropogenous stress $u(t)$. N measurements of some state characteristics or their linear combinations can be made on the interval $[0, \Delta]$. The observer also knows the set of possible values of the function $u(t)$. We suggest that the set P is convex and compact. The problem is to forecast the vector $x(T)$. The ecological process considered is described by linear differential equations. The possibility of constructing linear models of similar ecological processes is discussed in Patten (1978). Note that in order to obtain the matrices $A(t)$ and $B(t)$, additional considerations are required.

(b) State equation (1) describes the nutrient cycling process in a forest ecosystem (see Harwell et al. 1981), where x_i is the quantity of nutrients in compartment i (there are n compartments: foliage, trunks, roots, leaf fall, herbivores, etc.) and $u_i(t)$, $j \epsilon \overline{1, p}$ are perturbations with a given set of possible values.

Generally speaking, it is impossible to obtain a true value of $x(T)$, so that an estimate $\hat{x}(T)$ of vector $x(T)$ is required. We shall construct one nonlinear estimate, using the concept of the Tchebyshev center. According to the Cauchy formula we have

$$y(t_i) = G(t_i) \varphi(t_i) x_0 + G(t_i) \varphi(t_i) \int_0^{t_i} \varphi^{-1}(s) B(s) u(s) \, \mathrm{d}s, \tag{4}$$

where $\varphi(t)$, $0 \le t \le T$, is fundamental matrix of the uniform equation

$$\dot{x} = A(t)x,$$

and $\varphi(0) = E$ is a unit $n \times n$ matrix. Let

$$\nu_1 = \int_0^{t_1} \varphi^{-1}(s)B(s)u(s)\,\mathrm{d}s,$$

$$\nu_2 = \int_{t_1}^{t_2} \varphi^{-1}(s)B(s)u(s)\,\mathrm{d}s, \ldots,$$

$$\nu_N = \int_{t_{N-1}}^{t_N} \varphi^{-1}(s)B(s)u(s)\,\mathrm{d}s,$$

where $t_0 = 0$ if $N = 1$. Using the notation ν_i we obtain from eq. (4)

$$y(t_i) = G(t_i)\varphi(t_i)x_0 + G(t_i)\varphi(t_i)(\nu_1 + \cdots + \nu_N). \tag{5}$$

Let

$$Z = \mathrm{col}(x_0, \nu_1, \ldots, \nu_N); \tag{6}$$

It then follows from eqs. (3) and (5) that

$$y = AZ, \tag{7}$$

where A is a $mN \times (N+1)n$ matrix and may be obtained without difficulty. Note that

$$\nu_i \in V_i = \int_{t_{i-1}}^{t_i} \varphi^{-1}(s)B(s)P\,\mathrm{d}s$$

is an integral from a many-valued mapping $\varphi^{-1}(s)B(s)P$. It follows from the theory of many-valued mapping that V_i is convex and compact. We then have $Z \in B^* = X_0 \times V_1 \cdots \times V_N$, where B^* is convex compact.

Under the information postulated, any given vector y satisfying eq. (7) and belonging to set B^* is possible, and these vectors are indistinguishable. All of the vectors Z of interest form a non-empty set

$$C_y = (A^+y + L) \cap B^*, \tag{8}$$

where A is a pseudo-reciprocal matrix, and L is the linear subspace of A. According to the Cauchy formula,

$$x(T) = \varphi(T)x_0 + \varphi(T)\int_0^T \varphi^{-1}(s)B(s)u(s)\,\mathrm{d}s$$

$$= \varphi(T)x_0 + \varphi(T)(\nu_1 + \cdots + \nu_N) + \varphi(T)\int_{t_N}^T \varphi^{-1}(s)B(s)u(s)\,\mathrm{d}s$$

$$\epsilon DC_y + \varphi(T)\int_{t_N}^T \varphi^{-1}(s)B(s)P\,\mathrm{d}s, \tag{9}$$

where the block matrix D consists of $(N+1)$ matrices $\varphi(T)$ written in rows, and $+$ denotes the algebraic composition of sets. It is easily seen that any element of the set

$$F_y = DC_y + \varphi(T)\int_{t_N}^T \varphi^{-1}(s)B(s)P\,\mathrm{d}s \tag{10}$$

is equally permissible for our conditions. It is also easily seen that F_y is convex and compact at an arbitrary permissible vector y.

We shall take as the estimate for $x(T)$ the vector $\xi_y \in F_y$ such that

$$\gamma_y = \min_{\eta\in F_y}\max_{\zeta\in F_y}|\eta - \zeta| = \max_{\zeta\in F_y}|\xi_y - \zeta|. \tag{11}$$

It follows from the convexity and compactness assumption that the vector ξ_y is defined uniquely. The problem of a practical definition of ξ_y is incorrect. A certain regularizing method of calculation, that enables one to avoid this kind of instability, is proposed. From the assumption on the boundedness of the set B, it follows that there is a large constant $R > 0$, so that we may change L to $M = L \cap S_R$, where S_R is an $(N+1)n$-dimensional sphere of radius R with the center placed at zero. Thus

$$C_y = (M + A^+y) \cap B^*. \tag{12}$$

Let calculations of M, A^+y, B^* be accurate to $\epsilon > 0$, so that we know the convex and compact set M_ϵ, vector x_ϵ, and convex and compact set B^*_ϵ such that

$$H(M_\epsilon, M) \le \epsilon, \tag{13}$$

$$H(x_\epsilon, A^+y) \le \epsilon, \tag{14}$$

$$H(B^*_\epsilon, B^*) \le \epsilon \tag{15}$$

where $H(.,.)$ denotes the Hausdorff distance. From eqs. (13)–(15) it follows that

$$M + A^{+}y \subset M_{\epsilon} + x_{\epsilon} + S_{2\epsilon}, \tag{16}$$

$$B^{*} \subset B^{*}_{\epsilon} + S_{\epsilon}. \tag{17}$$

From eqs. (12)–(17) it follows that

$$\begin{aligned} C_y + S_{\epsilon} \subset C_y^{\epsilon} &= (M_{\epsilon} + x_{\epsilon} + S_{3\epsilon}) \cap (B^{*}_{\epsilon} + S_{2\epsilon}) \subset B_y^{\epsilon} \\ &= (M + A^{+}y + S_{5\epsilon}) \cap (B^{*} + S_{3\epsilon}). \end{aligned} \tag{18}$$

Let the non-empty convex compact A_y^{ϵ} instead of C_y^{ϵ} be known, and let A_y^{ϵ} satisfy the condition

$$H(A_y^{\epsilon}, C_y^{\epsilon}) \leq \epsilon. \tag{19}$$

It is not difficult to obtain following:

Lemma 1. Under a given vector y the convex and compact sets C_y^{ϵ} converges to a set C_y when ϵ tends to zero from the above.

Lemma 2. Under a given vector y and $\epsilon \to +0$, we have $H(A_y^{\epsilon}, C_y) \to 0$. As an approximation of γ_y (see eq. (11)), we shall take the value

$$\gamma_y^{\epsilon} = \min_{\eta \in \tilde{F}_y^{\epsilon}} \max_{\zeta \in \tilde{F}_y^{\epsilon}} |\eta - \zeta|,$$

where $\tilde{F}_y^{\epsilon}$ is a convex and compact set which approximates the compact set

$$F_y^{\epsilon} = DA_y^{\epsilon} + \varphi(T) \int_{t_N}^{T} \varphi^{-1}(s) B(s) P \, ds,$$

and satisfies the condition

$$H(\tilde{F}_y^{\epsilon}, F_y) \leq \epsilon.$$

In view of *Lemma 2* it is not difficult to prove that with a fixed vector y, $\tilde{\gamma}_y^{\epsilon} \to \gamma_y$ when $\epsilon \to +0$. If $|\gamma_y^{\epsilon} - \tilde{\gamma}_y^{\epsilon}| \leq \epsilon$, then taking into account the above considerations we have $\gamma_y^{\epsilon} \to \gamma_y$ when $\epsilon \to +0$. Thus the regularization method considered is recommended when making approximate calculations with an accuracy $\epsilon > 0$ using the set

$$C_y^{\epsilon} = (M_{\epsilon} + x_{\epsilon} + S_{3\epsilon}) \cap (B^{*}_{\epsilon} + S_{2\epsilon}),$$

which contains interior points.

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THE DYNAMICS OF THE PHYTOPHAGE–ENTOMOPHAGE SYSTEMS

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This paper presents an analysis of a model of the phytophage–entomophage systems. Based on this analysis the types of mass propagation of forest insects are defined and their classification given.

1. Introduction

Forest biogeocenoses are among the most complicated ecological systems. It is difficult to investigate them, however, because of both their complex nature and their long-term development. Information on the main stages of forest formation has been accumulated for many years, so that it is difficult and sometimes even impossible to carry out direct experiments.

In order to understand forest biogeocenosis dynamics it is first necessary to study the behavior of components and their interactions. This can be achieved by constructing phase portraits of the population dynamics based on the principle of stability of dynamic ecological systems (Isaev and Khlebopros 1973, 1977). The essence of this principle is that a biogeocenosis has a domain of stability, and analyzing the phase portrait it becomes possible to establish typical points and curves, their crossing qualitative changes in the biogeocenosis, to differentiate precisely between the effect of modifying and regulating factors, to establish the rate of system regulation, and to build specific models of system behavior.

2. Qualitative analysis of the "predator–prey" model

The numbers in two interacting populations can be described by the following system of differential equations:

$$\frac{dx}{dt} = xF_1(x, z), \quad \frac{dz}{dt} = zF_2(x, z),$$

where x, z are the densities of the phytophagous and entomophagous populations, respectively, and F_1, F_2 are the relative rates of density change. The

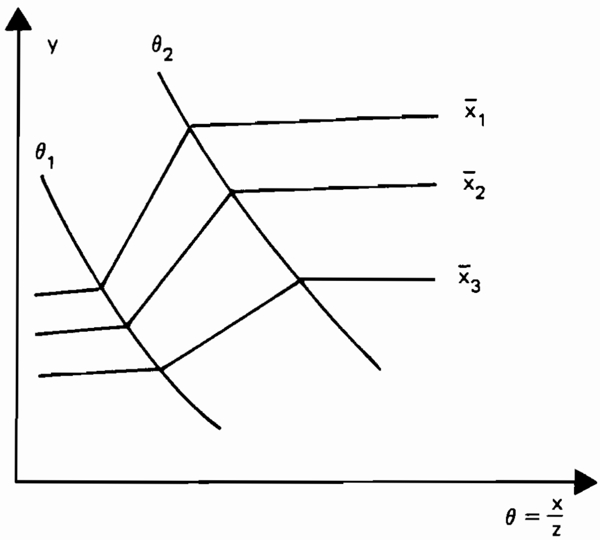

Fig. 1. Phytophage reproductive rates as a function of food supply for their natural enemies θ. Each curve corresponds to x = constant, $\bar{x}_1 < \bar{x}_2 < \bar{x}_3$. θ_1, θ_2 are threshold curves.

following inequalities are necessary and sufficient to describe the model of the predator–prey type:

$$\partial F_1/\partial z < 0, \partial F_2/\partial x > 0, \quad \partial F_2/\partial z < 0.$$

It has been proven experimentally (Yanovsky and Kiselev 1975) that the entomophage –phytophage interaction is characterized by the presence of threshold values of the density relationship $v = x/z$, where v is the entomophage food supply. At threshold point v_1 and v_2 the nature of the regulation changes. Nondimensional rates can be presented as

$$F_1(x, z) = F(x) - f(x, z), \quad F_2(x, z) = -G(z) + g(x, z),$$

where the functions F and G describe self-regulation, and f and g characterize the result of the phytophage–entomophage interaction. As self-regulation in a population is realized according to the principle of negative feedback, F and G meet the following conditions (Vorontsov 1978):

$$F(0) > 0 > F(+\infty), \quad \mathrm{d}F/\mathrm{d}x < 0, \quad G(0) > 0, \quad \mathrm{d}G/\mathrm{d}z > 0.$$

The behavior of the curves in the (v, y) plane (fig. 1) shows that the following conditions are met for the functions f and g:

$$\partial f/\partial x < 0, \quad \partial f/\partial z > 0, \quad \partial g/\partial x > 0, \quad \partial g/\partial z < 0.$$

Between v_1 and v_2 the behavior of the isocline $F_1 = 0$ is to a considerable extent dependent on the degree of inequality. If the derivative $\mathrm{d}y/\mathrm{d}v$ is large enough, the isocline increases monotonically. The isocline $F_2 = 0$ increases monotonically throughout the phase portrait. Non-monotonic behavior of the first isocline may result in intersections that characterize one or three points of the system in the phase portrait.

2.1. A fixed outbreak

Let us consider in detail phase portraits with three stationary points (fig. 2): (x_1, z_1) is the stable density of thinned out populations; (x_2, z_2) is the stable

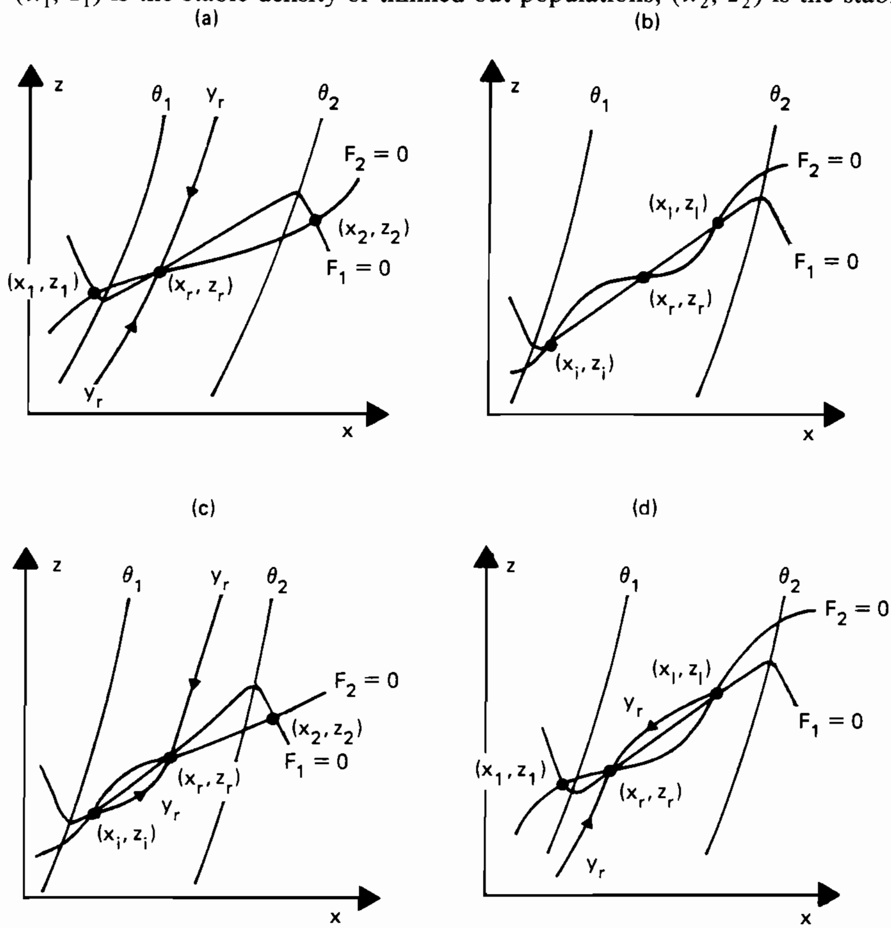

Fig. 2. Behavior of main isoclines in the phytophage–entomophage model with three stabilization points: (a) fixed outbreaks, (b) permanent outbreaks, (c) reverse outbreaks, (d) phytophage mass propagation outbreaks proper. (x_1, z_1) is the stable state of the system, (x_r, z_r) is the escape point, (x_2, z_2) is the second stable state, (x_i, z_i) is the minimum density at the phase of population stabilization at the initial level.

density of high-density populations,; and (x_r, z_r) is the escape point. The threshold curve y_r divides the phase portrait into two regions (fig. 2a).

When the fluctuations of modifying factors are slight, the predator–prey system may be near one or another stable point for any period of time. Only when the density fluctuations are great does the system cross the threshold curve y_r and the nature of the phytophage–phytocenosis interaction changes qualitatively as high-density populations are thinned, and vice versa. A similar situation, with an alternative interpretation was first described by Takahashi (1964).

Evolving natural systems usually have a tendency to eliminate such essentially unstable biogeocenoses. At present such a situation is especially typical of artificial biogeocenoses such as fruit gardens, forest plantations, as well as of plantations subject to intensive anthropogenic effects.

2.2. *A permanent outbreak*

Population stabilization is not necessary for ecosystem stability. In principle, forest biogeocenosis stability may be preserved even if the amplitudes of fluctuations in the phytophagous insect population are large (permanent outbreaks). In the (x, z) phase plane the intersection of the main isoclines within region II reflects this situation (fig. 2b). All three points are in this case unstable and are situated within the stable-limit cycle.

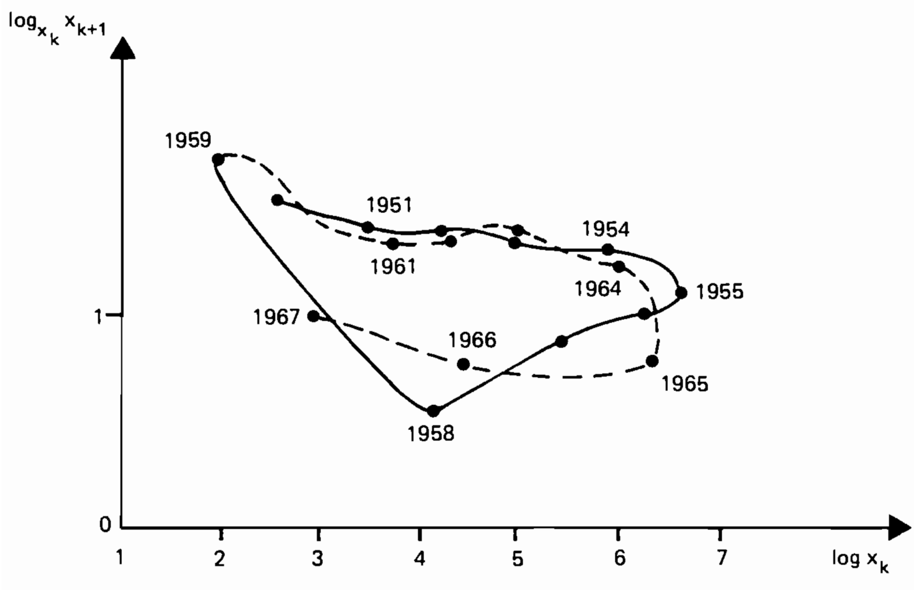

Fig. 3. *Zeiraphera diniana Cin* number dynamics. Experimental data (Baltensweiler 1970).

In a natural situation, in order for a permanent insect outbreak to occur, the effect on the forest biogeocenosis has to be rather weak in order to preserve the stability of the forest ecosystem. The peculiarities of mass propagation of *Zeiraphera diniana Cin* in Swiss Alpine forests may be used as an example of a permanent outbreak. This species does not cause the destruction of stands even when the larch crown is considerably injured during the period of the needle unfurling because the needles recover quickly. But as the tree grows, it decreases greatly (Baltensweiler 1970). The experimental data are presented in fig. 3.

2.3. A reverse outbreak

A peculiar regulation of insect number dynamics is possible in the case when the equilibrium points (x_i, z_i) and (x_r, z_r) lie in region II and the stable points (x_2, z_2) lie in region III (fig. 2c). Incidentally, the stable point (x_2, z_2) in the system performs the role of the stable point (x_1, z_1).

2.4. An outbreak proper

Outbreaks resulting in the disturbance of forest ecosystem stability are most characteristic in mass forest insect propagation. According to the phase

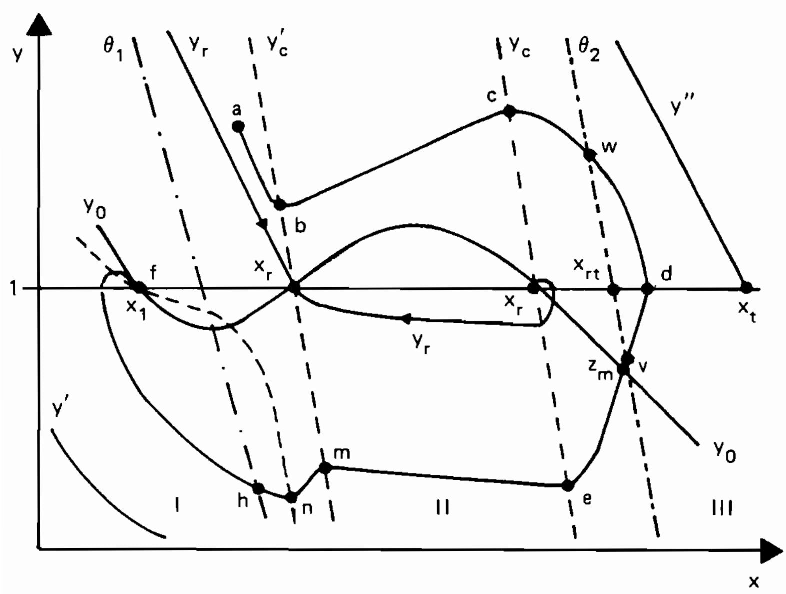

Fig. 4. The phase portrait of a phytophage population. y_r is the threshold curve, y', y'' are the boundaries of the phase portrait, y_c, y_c', y_c'' of reproductive rate, cd is the peak phase, de is the phase of population decrease, eh is the phase of depression, hf is the phase of population stabilization at the initial level, ac is the phase of population growth, *ab* is the lag period.

portrait of the population dynamics (x, y), for an outbreak to develop it is necessary and sufficient for the stabilization point (x_1, z_1) to lie in region I. The equilibrium points $((x_r, z_r)$ and (x_l, z_l) lie in region II (fig. 2d). In this case the third intersection of the main isoclines [point (x_l, z_l)] becomes unstable.

In fig. 4 is shown a phase portrait of phytophage dynamics in the (x, y) coordinate system. Let us consider in detail the structure of the phase plane where the following typical points of the population dynamics are exhibited: x_1 is the stable density of a thinned-out population usually showing that a species exists in a stable biogeocenosis; x_r is the escape point, above which (at $y = 1$) a mass outbreak will occur; x_l is an unstable equilibrium characterizing the density at its minimum limit when an outbreak passes through its maximum phase. The point x_{rt} at $y = 1$ delimits regions II and III, and the point x_i is the phytophagous insect density at its limit. The static curve y_0 at x_r and x_1, divides the phase portrait into the domains of stability and outbreak. The curves y_r and v_1 bound a specific "layer" of the phase space characterizing the part where a population is as near as possible to the domain of a mass outbreak. The volume of this layer, located in the region II, essentially depends on the degree of the numerical response changes; the layer may spread throughout the region II, thus making an outbreak impossible.

The locations of the maximum and minimum reproductive rate y (i.e., the maximum and minimum rates of population change along a trajectory) is an important characteristic of the phase trajectories. The extreme points of all trajectories inside the phase portrait form three curves y_c, y_c', y_c'' crossing x_l, x_r, x_1, respectively. With these curves it is possible to estimate qualitatively the different parts of an outbreak phase trajectory.

As shown, mass outbreaks of insects pass through five qualitatively different stages: population growth, peak phase, population stabilization at the initial level, etc. Experimental data on *Dendrolimus sibiricus superans Ts*, number dynamics are presented in fig. 5. As one can see, all effects of the phase trajectory may provide an explanation of the boundaries of our phenomenological theory.

3. A classification of number dynamics

An analysis of phase portraits of forest insect populations indicates that there are different regimes of the number dynamics that determine the role of one or other of the species in a forest biogeocenosis. The structure of the phase portraits may therefore be used to classify the types of mass propagation according to the dimensionality (the relative width) of the phase space, Δx, and to the static curve y_0 behavior.

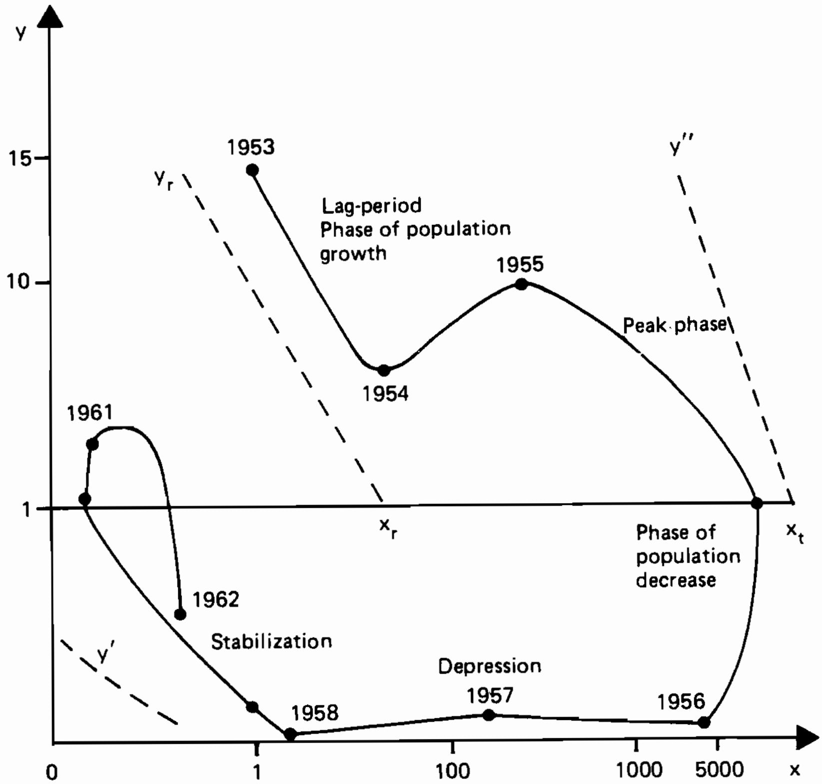

Fig. 5. *Dendrolimus sibiricus superans Ts.* number dynamics. Experimental data (Kondakov 1974).

3.1. Number dynamics of species with a narrow phase space

In a narrow phase portrait the difference between boundary values of densities Δx is much less than the values of a stable density x_1 and x_2. A narrow phase portrait is characteristic of forest insect species whose numbers are stable under normal conditions.

Three groups of insects with narrow phase portraits and monotonically decreasing static curves y_0 are distinguished. The first group consists of indifferent species whose numbers undergo practically no change, regardless of weather and food conditions. A substantial number of phytophagous insects that form entomocomplexes of forest biogeocenoses belong to this group.

The second group includes species whose numbers may vary over a wide range, but which are limited mainly by weather conditions. As soon as the weather becomes favorable these species are able to increase their numbers rapidly, and have a perceivable economic effect on a forest biogeocenosis. When weather conditions become less favorable these populations immediately return to their initial levels. Among these species there are insects that live outside trees, are not limited by food resources, and have either a multispecific set of parasites and predators or well organized intraspecific self-regulation.

The third group of insects consists of those whose numbers are limited mainly by available food resources. Most of the species that live inside trees belong to this group, e.g., xylophagous insects whose propagation is functionally dependent on the appearance of favorable foods. There are species belonging to the third group whose phase portraits are characterized by non-monotonic behavior of the static curve. In certain ecological situations these insects change the nature of their connection with the forage plant and are thus able to stabilize their numbers at a higher level (Vorontsov 1978).

3.2. Number dynamics of species with wide phase portraits

A wide phase portrait is typical of insect species whose number regulation is associated with lag effects. All thresholds in a wide portrait are displayed when the system intersects the typical curves, while a narrow phase portrait is characterized by the presence of the critical points only.

Insects with wide phase portraits can be divided into two groups: those unable to escape the regulating effects of entomophage complexes under any ecological conditions, and those that can escape their natural enemies at a certain alignment with the number of entomophagous insects. In the phase portrait this corresponds to the nonmonotonicity of the static curve (fig. 2) which, if it creates three equilibrium points, results in mass outbreaks.

As mentioned above, four types of outbreaks may be distinguished: fixed, permanent, reverse, and an outbreak proper. This suggested classification gives an objective assessment of the function and the character of the interaction of insects in a forest biogeocenosis. The approach considered above is thus important as a basis for integrated forest control and the choice of optimum strategies in forest management.

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DETERMINING OPTIMAL POLICIES FOR ECOSYSTEMS

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1. Introduction

The problem of finding the optimal policy for controlling the spruce budworm – an insect whose outbreaks from time to time do great damage to the fir forests of New Brunswick, Canada – represents a rare opportunity to develop and to successfully apply the methodology of optimization to a prototypical ecosystem problem.

The boreal forests of North America have, for centuries, experienced periodic outbreaks of a defoliating insect called the spruce budworm. In any one outbreak cycle a major proportion of the mature softwood forest in affected areas can die, with major consequences to the economy and employment of regions like New Brunswick, which are highly dependent on the forest industry. An extensive insecticide spraying programme initiated in New Brunswick in 1951 has succeeded in reducing tree mortality, but at the price of maintaining incipient outbreak conditions over an area considerably more extensive than in the past. The present management approach is, therefore, particularly sensitive to unexpected shifts in economic, social and regulatory constraints, and to unanticipated behaviour of the forest ecosystem.

Most major environmental problems in the world today are characterized by similar basic ingredients: high variability in space and time; large scale, complex interactions between many variables; and a troubled management history. Because of their enormous complexity there has been little concerted effort to apply systems analytic techniques to the coordinated development of effective descriptions of, and prescriptions for, such problems. The budworm–forest system seemed to present an admirable focus for a case study with two objectives. The first, of course, was to attempt to develop alternative policies appropriate for the specific problem. But the more significant purpose was to see just how far we could stretch the state of the art capabilities in ecology, modeling, optimization, policy design and evaluation to apply them to complex ecosystem management problems. This paper describes the critical first steps in applying optimization methodology to a typical ecosystem problem.

Every resource environmental problem presents three principal challenges to existing techniques. The first is complexity. The resources that provide the food, fibre and recreational opportunities for society are integral parts of ecosystems characterized by the complex interrelationships of many species among each other and with the land, water and climate in which they live. The interactions of these systems are highly nonlinear and have a significant spatial component. Events in any one point in space, just as at any moment in time, can affect events at other points in space. The resulting high order of dimensionality becomes all the more significant as these ecological systems couple with complex social and economic ones.

The second challenge derives from the incomplete knowledge of the variables and relationships governing the systems. A large body of theoretical and experimental analysis and data has led to an identification of the general form and kind of functional relations existing between organisms. But only occasionally is there a rich body of data specific to any one situation. To develop an analysis which implicitly or explicitly presumes sufficient knowledge is therefore to guarantee management policies that become more the source of the problem than the source of the solution. In a particularly challenging way, present ecological management situations require concepts and techniques which cope creatively with the uncertainties and unknowns that in fact pervade most of our major social, economic and environmental problems.

The third and final challenge reflects the previous two: how can we design policies that achieve specific social objectives and yet are still "robust"? Policies which, once set in play, produce intelligently linked ecological, social and economic systems that can absorb the unexpected events and unknowns that will inevitably appear. These "unexpecteds" might be the one-in-a-thousand-year drought that perversely occurs this year; the appearance or disappearance of key species; the emergence of new economic and regulatory constraints; or the shift of societal objectives. We must learn to design in a way which de-emphasizes minimizing the probability of failure and instead emphasizes minimizing the cost of those failures which will inevitably occur.

The analysis of this forest management problem, presented below under the headings Descriptive and Prescriptive Analysis, was a cooperative effort between the International Institute for Applied Systems Analysis, Vienna; the Institute of Resource Ecology, University of British Columbia; and the Maritimes Forest Research Centre, Environment Canada, New Brunswick. Its development relied on the dedicated energies of a team – Gordon Baskerville, David Bell, William Clark, Michael Fiering, Dixon Jones, Charles Miller, Zafar Rashid and Carlos Winkler. We particularly note the contributions of Carlos Winkleer, who developed the details of the optimization procedure which was compared with existing methods for determining forest management policy. The authors, Holling and Dantzig, guided this research.

2. The descriptive analysis

The descriptive analysis of the budworm/forest system aimed to produce a well validated simulation model that could be used as a laboratory world to aid in the design and evaluation of alternative policies. The key requirement of that laboratory world is to capture the essential qualitative behaviour of the budworm forest ecosystem in both space and time. Extensive data concerning forest–pest and economic interrelations had been collected over the past 30 years by Environment Canada as one of the earliest interdisciplinary efforts in the field of renewable resource management (Morris 1963). There are many missing elements, but this is an inevitability rather than a drawback. If systems analysis is to be applied successfully to the management of ecological systems, it must be able to cope with unknowns.

The essential qualitative behaviour in time has been identified through an analysis of tree ring studies from eight regions of eastern North America (Blais 1968) and covering a period from 1704 to the present. During the inter-outbreak periods the budworm is present in barely detectable densities which, when appropriate conditions occur, can increase explosively over three orders of magnitude during a three to four year period (fig 1). The period between outbreaks is typically 30 to 44 years with occasional periods of 60 to 100 years. The outbreaks persist in a region for 7 to 16 years and finally collapse with attendant high mortality of trees.

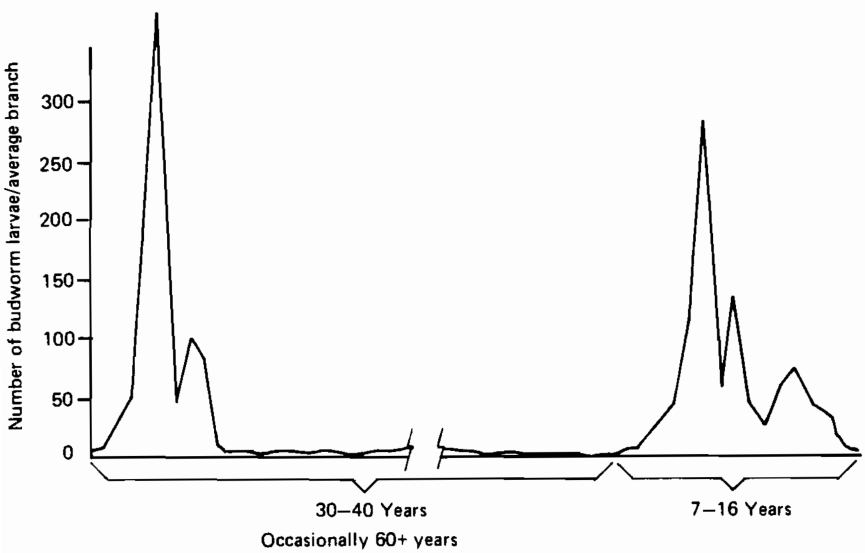

Fig. 1. The pattern in time. Representative historical pattern of spruce budworm outbreak. There have been four major outbreaks since 1770.

The distinctive pattern in time is paralleled by one in space. The historical outbreaks typically start in one to three or four local areas of eastern Canada and from those centres spread to and contaminate progressively larger areas. Collapse of the outbreaks occurs in the original centres of infestation in conjunction with mortality of the trees and similarly spreads to the areas infested at later times. The resulting high degree of spatial heterogeneity in the forest age and species composition is closely coupled to the "contamination" feature caused by the high dispersal properties of this insect.

The first step we took in order to simulate this system was to discretize the prime variables: space and time. From the very start of the analysis, the grid sizes had to be carefully selected since all other steps in the analysis depend on these decisions and they profoundly influence the final form and relevance of the policies. The principle followed with regard to the space, time, and later for other variables, was to select the coarsest grid possible which still retained, in our opinion, the essential properties of behaviour and needs for management.

2.1. Time grid

Because of the pattern of outbreaks shown in fig. 1, the minimum time horizon required is that which can contain two outbreaks – that is 150 to 200 years. In order to capture the dynamics of this system it is essential to have a time resolution of one year with seasonal events implicitly represented.

2.2. Space grid

As in many pest species, the budworm disperses over long distances. The modal distance of dispersal is about 50 miles from one site, but dispersal distances of several hundred miles have also been recorded. It was thought essential to have a minimum total area based on at least twice this modal distance, leading to a minimum modeled region of 14 000 to 15 000 square miles. The area chosen in this study was a 17 500 square mile area containing much of the Province of New Brunswick (fig. 2). But even events in this size of area are profoundly affected by contagion from outside. It was therefore necessary to add a buffer zone around the area in order to compensate for edge effects. The behaviour of this system is as highly heterogeneous in space as it is in time, and because of the contagion problem spatial disaggregation is essential. There is high variation in the spatial distribution of the primary tree species, of harvesting activities and of recreational potential, in part as a consequence of the historical interplay between the forest and the budworm. The 50 mile modal dispersal distance also suggests a minimum resolution of about one-fifth to about one-tenth of that distance. Hence the overall area is divided into 265 distinct subregions, called sites, each approximately 64 square miles (fig. 3).

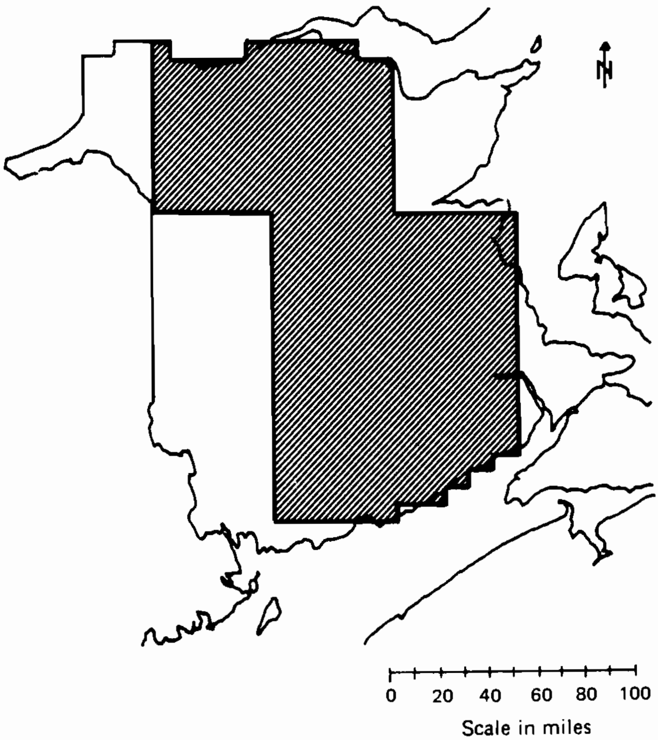

Fig. 2. Study area within the Province of New Brunswick used in the current study. The hatched area includes the primary forested regions of New Brunswick.

2.3. Limiting the number of essential variables

An ecosystem of this extent has many thousands of species and potential variables. Our understanding of the dominant budworm–forest dynamics is sufficiently detailed, however, that the system's relevant behaviour can be captured through the interrelations among five species, each of which represents a key role in determining the major dynamics of the forest ecosystem and its resulting diversity. The links between key variables are depicted in fig. 4.

The principal tree species are birch, spruce and balsam. In the absence of budworm and its associated natural enemies, balsam tends to out-compete spruce and birch and so would tend to produce a monoculture of balsam. Budworm, however, shifts that competitive edge since balsam is most susceptible to damage, spruce less so, and birch not at all. Thus there is a dynamic rhythm, with balsam having the advantage between outbreaks and spruce and birch during outbreaks. The result is a diverse species mix.

As noted earlier, between outbreaks the budworm is rare but not extinct. Its

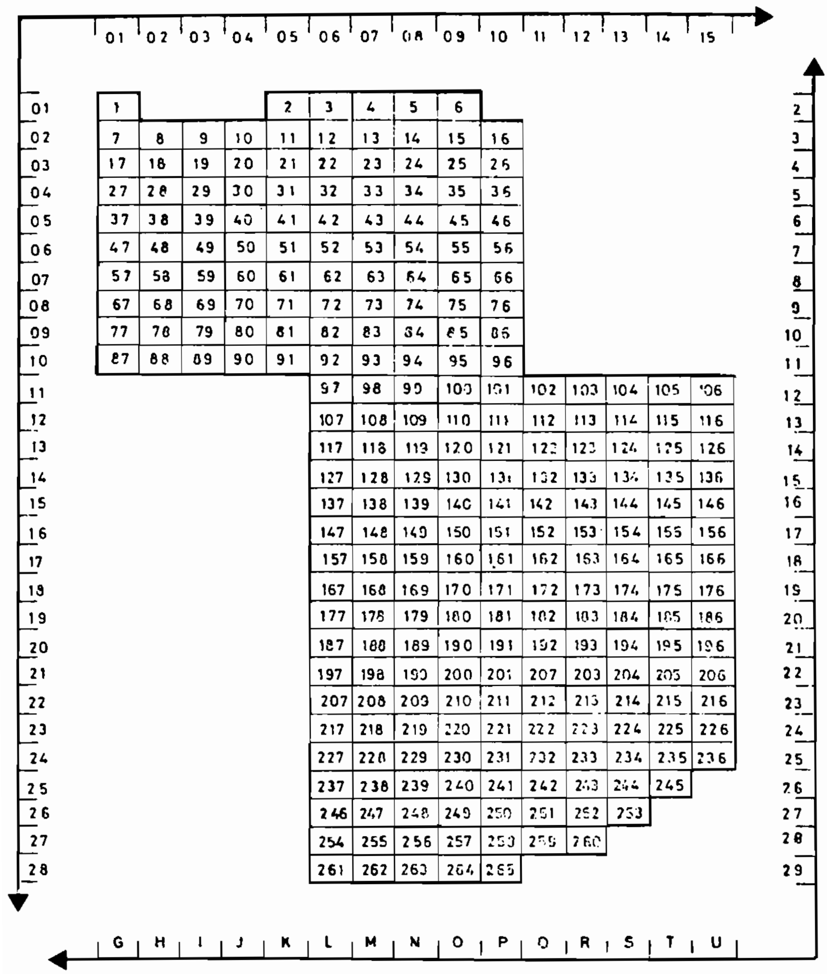

Fig. 3. This figure shows the numbering and indexing system for the 265 subregions, or "sites", in the study area.

numbers are then controlled by natural enemies such as insectivorous birds and parasites. But a key feature of this control is that there exists an upper threshold of budworm numbers which, once exceeded, allows the budworm to "escape" predation and multiply unchecked. There is, in other words, a distinct *but limited* stability region at low budworm densities.

In addition to tree species and natural enemies there is a key stochastic driving variable, weather, which affects survival of the budworm and can flip the system out of the low density stability region if forest conditions are appropriate. Outbreaks cannot occur unless the forest has sufficiently recovered from the previous outbreak to provide adequate food. Even with the

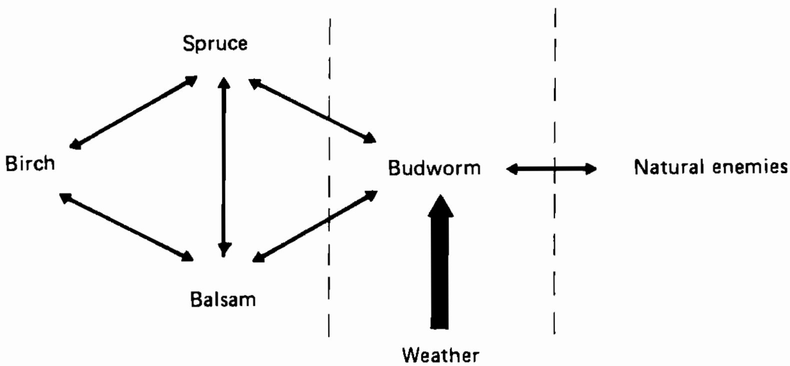

Fig. 4. The key roles or variables and their interrelations in the natural ecosystem. The principal tree species (birch, spruce, and balsam fir) have a dynamic interaction of their own. This interaction is altered by the presence of budworm which consumes some spruce but primarily balsam. The budworm is in turn affected by a complex system of natural enemies and a stochastic weather variable. Only budworm, balsam, and weather are treated as explicit dynamic variables.

food conditions met, however, the budworm can remain at low densities under control by natural enemies until there occurs several warm dry summers in a row. Such conditions allow larvae to develop so rapidly that densities above the escape threshold are achieved. An outbreak in the years that follow is then inevitable, irrespective of their weather patterns. To describe this behaviour we need, at the very least, the distribution by age for spruce and balsam, budworm count, something about natural enemies, weather, and the condition of the trees. Even with a highly simplified representation, the number of state variables generated is enormous.

Table 1 summarizes what we estimate to be the minimum number of state variables necessary to represent the essential behaviour of the system in space and time. In any one subregion, there are 112 state variables; for all 265 sites (subregions), a total of 112×265 or 29680 state variables are required. It would, of course, be quite possible to develop a simulation model with this number of state variables. This would be expensive and time consuming to run and debug, but it would be possible. Our key goal, however, is to provide a useable and well tested model for exploring behaviour and policy alternatives. With such a high dimensionality the simulation results would become nearly as incomprehensible to follow (unless summarized) as the real world and the opportunities for systematic exploration would be greatly reduced.

As a consequence, a systematic series of further compressions and tests were made to determine whether the number of state variables could be significantly reduced. This led to four prime variables: tree density, two variables expressing foliage condition, and budworm density. The tree population had to be represented by 75 state variables representing the number of balsam trees of various ages (although techniques were developed to collapse these into one for

Table 1
The number of state variables by type emerging after problem simplification

In one subregion:	
Birch	1
Spruce	30 age groups
Balsam	75 age groups
Budworm	1
Natural enemies	1
Weather	1
Tree stress	1
Foliage new	1
Fliage old	1
Dimensionality of the state	
for a subregion	112
for all 265 subregions	$112 \times 265 = 29680$

descriptive purposes). The effects of all the other variables can be incorporated implicitly so that this ultimate compression requires essentially 78 state variables per site and 78×265 or 20,670 for the entire forest.

2.4. The model and its behaviour

A detailed description of the simulation model is presented in Jones (1976) and of its behaviour and validation, in Clark, Jones and Holling (1979). For the present purpose of describing the optimization procedure, we will only briefly describe the structure of the model and its response. The goal of developing this simulation model was simply to provide a laboratory world with sufficient credibility in its structure and behaviour that it could be used to provide a realistic comparison of the optimization policies (derived from even more simplified models) with any other proposed policies.

The basic form of the simulation model structure is shown in fig. 5. At the beginning of a year, changes in the condition of each of the 265 subregions are computed as independent regions since contagion does not occur at this time of the year. Budworm grow, and a certain proportion survive depending on the weather (a stochastic variable) and on the state of the forest and of the budworm at the beginning of the year. Similarly, trees grow and die from natural causes as a function of the starting age structure, and are stressed or killed depending upon the density and quality of the budworm populations. Finally, the budworm that survive to adults deposit some of their eggs on site, the number depending upon their nutritional state as larvae. At this time of the year contagion can occur. The adults with the remaining eggs are dispersed by wind to other sites throughout the forest where they deposit their eggs. These eggs entering a site are accumulated with those deposited locally by resident

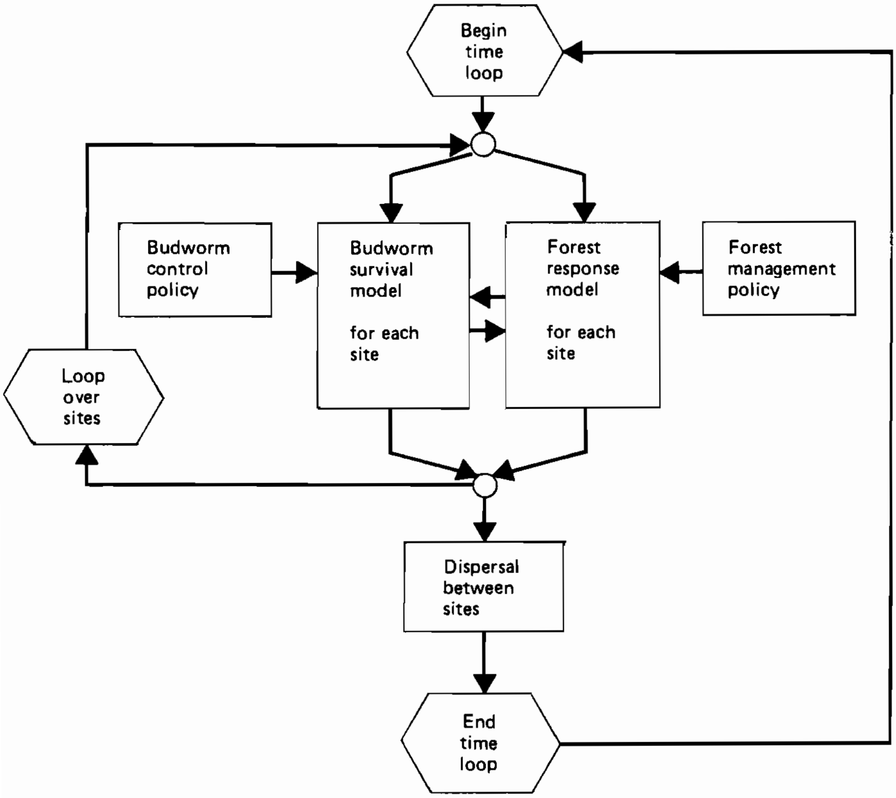

Fig. 5. The basic model structure for the budworm/forest simulation model.

moths to set the starting egg densities for the next year. At the same time the surviving trees are aged one year.

A dynamic descriptive model of this kind is useful for studying the affects of management intervention based in policy decisions. There are two main classes of policy action possible – one relating to control of the budworm and the other to management of the forest. These are structured in broad terms allowing for the exploration of not only insecticide control of budworm but for biological and other methods of control as well. Similarly the forest management policy can include specific actions of cutting by age in different regions of the forest and also, at least implicitly, a variety of silvicultural and tree breeding actions. Although the model is structured to accommodate a wide range of possible actions, to demonstrate the optimization method in this paper, attention will be limited to budworm control using insecticides or bacterial agents and forest management using different techniques of scheduling cutting in space, time, and by tree age and condition.

The credibility of an ecological simulation model depends in part on the information that defines its causative structure and in part on how closely its behaviour matches the real world. The causative structure was based, in part, on the information in the extensive monograph of Morris (1963) that analyzed the processes contributing to variance in budworm populations in New Brunswick. The form of the functions describing these processes of growth, mortality, and reproduction was drawn from the accumulated experimental and modeling analyses of key ecological processes such as predation, parasitism, reproduction, and competition (e.g., Holling and Buckingham 1976). The latter have led to simplified and well-tested modules that have broad descriptive generality.

But however credible the parts of an ecological model are, validation in terms of overall forest behaviour is always a difficult problem. In this example a statistically rigorous validation would require detailed historical information on all the state variables over a large spatial area and covering a very long period of time – at least 70 to 150 years. Only in that way could the full dynamic interplay of the system over time and space be adequately tested. The budworm case is rare in that such data are in fact available for a 30 year period, but that is scarcely long enough to instill a profound confidence in the model. Our present validation approach has therefore been to combine a quantitative comparison of state variable values over the period for which detailed historical data are available, with a qualitative comparison of gross

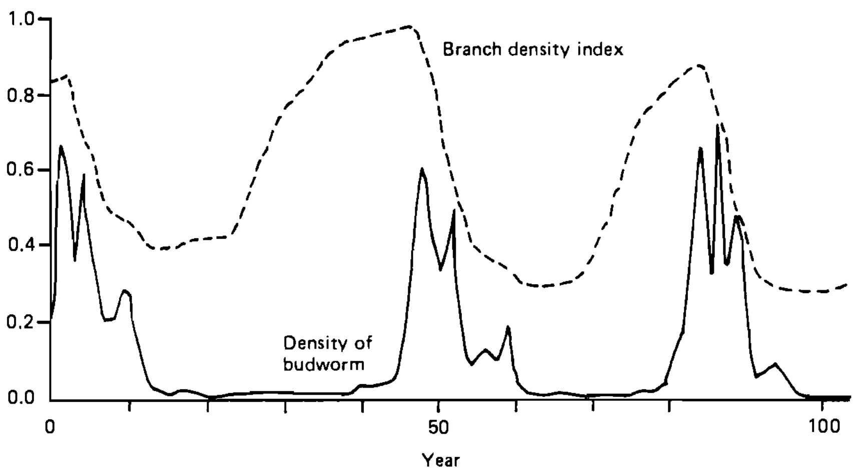

Fig. 6. Typical outbreak pattern generated by model with no management or harvesting imposed. This represents mean conditions in 265 subregions of the simulated province, starting with initial conditions known to exist in 1953. Budworm densities are in 1000 eggs/ten square feet of branch area. The Branch Density Index is a relative scale which closely parallels average forest age and forest volume.

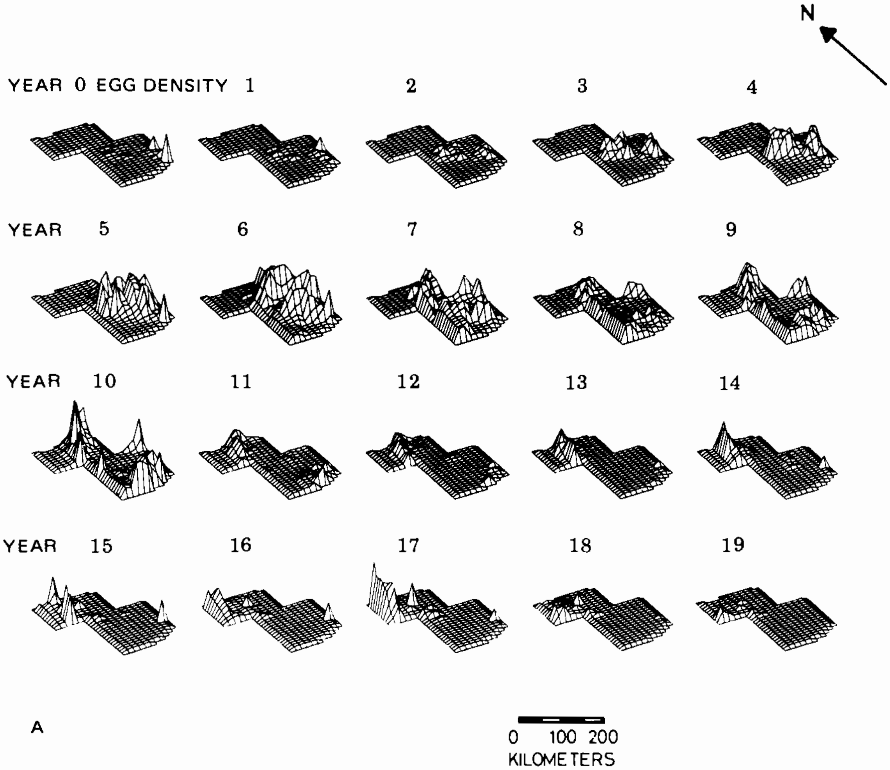

Fig. 7. Annual egg density patterns for a typical outbreak under no management.

behavioural properties (mean outbreak densities and variances, length of inter- and intra-outbreak periods, and so on) for the longer term. Some sense of this latter exercise can be drawn from figs. 6 and 7. Fig. 6 shows changes in the budworm and trees averaged over the 265 sites, under conditions of no management. Fig. 7 shows, for the same condition, the behaviour of budworm in space as well as time during a typical outbreak sequence. The principal point is that these simulations, together with ones in which historical management rules are applied, generate patterns in space and time, for a variety of conditions, that are similar to independent historical data from New Brunswick and elsewhere in eastern Canada.

Hence, both the causative structure of the model and its behaviour give reasonable grounds for at least a skeptical suspension of disbelief. And achieving that degree of belief is a critical requirement in the procedure we describe, for the existence of a credible simulator provides a kind of "court of last resort" for evaluation of policies developed by any technique ranging from intuition and informed judgment to formal, though necessarily simplified, optimization models. By cycling such policies through the full simulator, the question of the reality of the assumptions of the optimizer can be replaced by the only important question: will the policy manage the forest better than any other proposed policy?

3. The presciptive analysis

The goal of the prescriptive analysis is to provide a management tool which can aid in policy design and evaluation. There are three parts to the analysis as we implemented it. The first was the definition of a number of different objectives, the second the application of optimization techniques to develop policy rules for each objective, and the third the development of a framework to broadly evaluate the consequences of each policy in terms of a wide range of potential management goals.

3.1. Objectives and utilities

One place where the formulation of models runs into difficulty is in the identification and quantification of objectives. This is certainly true of the forest management problem where a variety of general objectives can be stated, such as: (1) obtaining high yields of lumber, (2) minimizing cost and environmental impact of insecticides, (3) preserving the forest as a recreational area, and (4) making the forest resilient to unexpected events. Sometimes objectives are enunciated only after a solution of a model is presented and its bad characteristics noted. Sometimes objectives are stated in the form of requirements.

In general there appears to be no completely satisfactory way to state what is to be optimized when a model has multiple objectives. Probably the best approach to an understanding of objectives is through *dialogue*. Any particular run of the model becomes a dialogue at the time its solution is reviewed. New runs can then be re-initiated and the dialogue process continued until a satisfactory compromise solution is obtained. But to make the dialogue useful a variety of indicators should be generated that cover all reasonable interests.

Hence the state and control variables of the model were transformed into a set of social, economic, resource, and environmental indicators such as profit, unemployment, diversity, budworm damage, logging effects, insecticide burden, and recreational quality. The time streams of selected indicators can be evaluated independently or they can be combined, using techniques from decision analysis, to yield a single utility function (Bell 1975). The latter can either be used to rank policies or can itself be used as an objective function.

A number of different objective functions have been used to develop policies, but in order to demonstrate the optimization procedure, we will only give examples of one typically used. This is simply to maximize discounted value of all timber harvested in the future. Policies generated by such a simple objective function, at an early stage, can be revealing. If it turns out that the policy that yields the maximum discounted value does not cause the forest to be in an undesirable state at some future time, then this purely economic criterion is likely to become the one accepted for comparison of policies. Should it result, however, in a solution in which, say, the lumber industry cuts down a large number of trees one year and a few the next, then such a solution may receive a lower rating than some other that yields lower profits but has a more even employment pattern. It may also be given a low rating if it does not result in a good mix of stands of trees of various ages, since a mixed forest is desirable because it can support a greater variety of wildlife and because it is more attractive as a recreational area.

3.2. Finding optimal policies

A policy for the forest model is a statement of whether or not to (1) cut down a tree and replant, (2) leave it alone until the following year, or (3) spray (if the latter, then the amount of spray to be used on the insect and when must also be stated). As noted, the simulator is very efficient for the comparison of one policy with another. It can also be programmed so that one (or more) of the parameters defining a policy can be varied to see if some change in them (e.g., the amount or insecticide) will lead to improvement of some stated objectives.

For complex systems this type of search is, however, hopelessly inefficient unless it can be combined with a more analytic approach. A natural question to ask is this: is there any hope that a practical analytic method can be devised for finding the optimal policy? Let us note some difficulties: the New Brunswick forest consists of 265 sub-forests, each with its own distribution of trees and different ages from 0 to 75 years (or more) in various states of stress (health) and degree of infestation by the budworm. Let us note the complex mathematical equations and stochastic weather factors which govern the change of the forest from one year to the next. One is naturally discouraged by all this complexity from trying the analytic approach unless there is a way to simplify the model. If the resulting policy based on a simplified model turns out to be better (when compared by using the simulator), than one obtained using, say, intuitive rules of thumb, then this would justify the use of more analytic tools.

The approach we have taken is to regard the simulator as a means of bringing the real world into the laboratory. The various policies (whether obtained by common sense, or by common practice, or through the use of an "optimizer") can always be compared by making a sufficient number of runs on the simulator. An analyst weak in analytic skills, poorly trained in the formulation of models, poorly informed about algorithms for solving classes of models, or unfamiliar with software availability may well opt to run many cases on the simulator to see if local improvements in a proposed policy are possible. Most simulation efforts unfortunately end up this way. Unfortunate because the high cost of using simulators to test many cases usually exhausts the patience and funds of sponsors to support development of an optimizer. If these funds had been used instead to develop a simplified model, then the process of determining an optimal policy for the simplified model could serve as a "brain" for the simulator and would have resulted, we believe, in significantly better policies being found.

Generally speaking there are two types of analytic models that have had many successful applications: (1) "linear programming," and (2) "dynamic programming" models.

The linear programming model is characterized mathematically by a system of linear inequalities. Many kinds of non-linear relations can be practically approximated by such systems which can be both dynamic and stochastic. Software is available for solving such systems at reasonable costs even when they involve thousands of inequalities and variables.

The dynamic programming model is characterized by a dynamic system that moves from any given state in time to the next without being affected by the past history of how it arrived at its given state. Many practical models can be cast in this form. In practice, however, applications are narrowly limited to those whose "state space" may be approximated by a low number of cases. In our research, however, we have pursued an alternative possibility – one which allows the state space to be multi-dimensional and continuous in certain components. We were able to do this by finding a practical way to approximate the "payoff" for each state if henceforth one follows an optimal policy.

3.3. The dynamic-programming optimization model

For the Budworm Optimizer we used a mathematical model closely related to the dynamic program – the so-called *Markov Process* (Winkler 1975). At each point in time t, the system is in some state A, B, C,.... If in state A it will move to state A or B or C,..., at time $t+1$ with probabilities $P(\mathrm{A}\,|\,\mathrm{A})$, $P(\mathrm{B}\,|\,\mathrm{A})$, $P(\mathrm{C}\,|\,\mathrm{A}),\ldots$; similarly if in state B it will move to A or B or C at time $t+1$ with probabilities $P(\mathrm{A}\,|\,\mathrm{B})$, $P(\mathrm{B}\,|\,\mathrm{B})$, $P(\mathrm{C}\,|\,\mathrm{B}),\ldots$, etc. see table 2. In our application these probabilities can be changed at a price by engaging in certain alternative actions. The problem is to find the best choice of these alternative actions. This is easy to do if we know the value $V(\mathrm{A}\,|\,t+1)$, $V(\mathrm{B}\,|\,\mathrm{t}+1),\ldots$ of being in various states at time $t+1$. Thus the *expected value* $V(\mathrm{A}\,|\,t)$ is given by

$$V(\mathrm{A}\,|\,t)=P(\mathrm{A}\,|\,\mathrm{A})\left[V(\mathrm{A}\,|\,t+1)-C_{\mathrm{AA}}\right]+P(\mathrm{B}\,|\,\mathrm{A})\left[V(\mathrm{B}\,|\,t+1)-C_{\mathrm{AB}}\right]$$
$$+P(\mathrm{C}\,|\,\mathrm{A})\left[V(\mathrm{C}\,|\,t+1)-C_{\mathrm{AC}}\right]\ldots,$$

Table 2

Time t			Time $t+1$	
Value	State		State	Value
		$P(\mathrm{A} \mid \mathrm{A})$		
$V(\mathrm{A} \mid t)$	A		A	$V(\mathrm{A} \mid t+1)$
		$P(\mathrm{B} \mid \mathrm{A})$		
$V(\mathrm{B} \mid t)$	B		B	$V(\mathrm{B} \mid t+1)$
		$P(\mathrm{C} \mid \mathrm{A})$		
$V(\mathrm{C} \mid t)$	C		C	$V(\mathrm{C} \mid t+1)$

where C_{AB}, for example, is the cost (revenue if negative) of transitioning from A to B in time period t. If there are alternative actions in period t which can affect these probabilities and costs, then the action that yields the maximum value of $V(\mathrm{A} \mid t)$ is chosen. The procedure is thus a backward induction to time $t = 0$ but requires (in order to get it started) the knowledge of $V(\mathrm{A}, t)$, $V(\mathrm{B}, t)$, $V(\mathrm{C}, t)$ for some time $t = T$ in the future.

As noted, a Markov-type model was the one used for the budworm study. The key idea used to develop this analytic model was to view the *single* tree as an entity which changes state from year to year – its state being defined by its age, stress, and the number of budworms it hosts. The tree, depending on the weather and whether or not it is sprayed or cut will (with certain probabilities) become one year older with certain stresses and egg densities *or* reverts to age zero and is replanted. This model has the merit that *all* relations can be used with little or no simplification or change except for the spread from one timber stand to another of budworm eggs by the adult moth. This leaves open the question of how to take into account contagion effects. Several approaches have been considered some of which will be discussed in the monograph in preparation. But acute simplification is the rule in dynamic programming applications, and we shall show how we have presently elected to live with this one after outlining the existing model and its solution.

For the simplified model we wish to find the optimal policy for every state (tree age, stress, and egg density). One way to determine optimal policy is to begin with a guess V_0 as to the entire discounted future value of a tree starting at age zero including the value of all its future harvesting and replanting (to time infinity) when we always carry out an optimal policy in the future with regard to the tree and its replantings. A tree planted a year from now has present value of $0.95V_0$ for its time stream from 1 year to infinity where 5%, say, is the discounted factor (without inflation). If for the moment we accept our guess V_0, we are in a position to evaluate the present value of *all* other states. One begins by noting that as far as the value of a tree as lumber is concerned it has the same value if 60 years does not pay to allow a tree to become older than 60 years. The present value of a 60 year old tree (or older) is

$V_{60} = V_0 + L_{60}$ where L_{60} is the value of the 60 year old tree as lumber less the cost for replanting it. The present value of a 59 year old tree (which is in some state of stress and egg density) depends on how it is treated; if it is cut, then $V_{59} = V_0 + L_{59}$; if it is left alone, then $V_{59} = 0.95\{ pV_{60} + (1-p)V_0 \}$ where p is the probability of a tree living; and if it is sprayed, then $V_{59} = -S + 0.95\{ \bar{p}V_{60} + (1-\bar{p})V_0 \}$ where S is the cost of spraying and $\bar{p}$ is the probability of the tree living after it is sprayed. The policy which yields the highest value is selected as optimal. Note that the effect of random weather factors is part of the calculations (i.e., weather affects the probabilities of dying or the probabilities of moving from one state to another) so that values (and optimal policies) of various states can be determined backwards from the highest age 60 down to age 0. If it turns out that our guess V_0 checks with the value V_0 obtained by the backward calculations, we accept it – if not, then we revise our guess up or down until it does check. (This evaluation could also be done by setting down a series of inequalities and then maximizing V_0.)

This procedure defines therefore an optimal way to apply the variety of management acts for a specific objective in terms of the values of the key state variables. These policy rules may be represented in the form of policy tables such as those shown in fig. 8. For any age of tree, foliage condition and density of insects the manager can either do nothing, spray (and the spray can be at different intensities and concentrations), or harvest. The advantage of such policy tables is that they are clear, unambiguous, and can be easily applied by a forest manager attempting to manage a stand in isolation from the rest of the regional forest system.

But to achieve these "optimal" rules required gross simplifications due to the limitations of available optimization techniques. Two major simplifying assumptions were necessary. The first concerned a simplified expression of the objective function, and the second assumed that resulting policies would be so effective that dispersal between spatial areas was unimportant and could be ignored. (If this turned out not to be so, then some sort of corrective procedure would have to be invented later.) It was only in this way that the high dimensionality of the problem could be simplified to the point where dynamic programming could be successfully applied. Similar gross simplifications will be required in most problems involving dynamic management of resource and environmental systems.

Dynamic programming is a particularly powerful and valuable tool for use in ecosystem management studies. But unless really substantial advances are made in its ability to handle certain classes of high dimensionality, it will properly remain a special-use "sub = optimizer" methodology only.

Sub-optimal or partial optimal solutions have a useful role to play, however. The key to their constructive utilization is an ability to cycle such simplified policies through the full simulation model with all its complexity. By using a variety of indicators, each of these policies can be assessed in terms of a

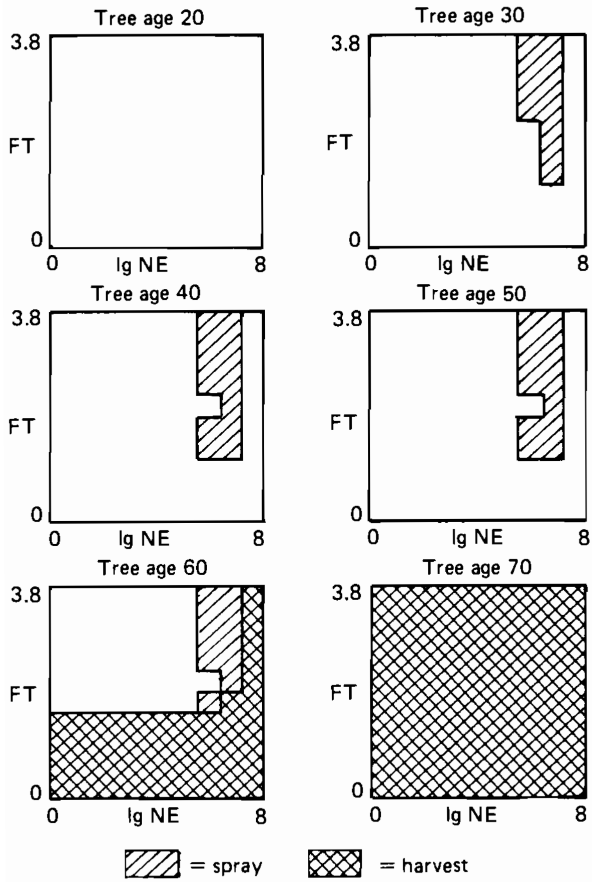

Fig. 8. Representative policy tables generated by the Winkler optimization. A separate table is provided for each age of tree (or, in practice, age of stand). The table tells what management act should optimally be applied to the tree as a function of the tree's present complement of foliage (FT) and its resident budworm egg density (here plotted as $\lg E_D$). Available management options are to do nothing, to spray, and to harvest or log the tree.

possible drift of solution from some broader societal and environmental goal. When this is detected, then *ad hoc*, heuristic modifications of the policies can be employed to produce more desirable behaviour.

This process, again, should be in the form of dialogue with both managers and interest groups. As we said earlier, the optimization model was designed to provide a "brain" for the simulator. But that "brain" is a childish thing and for its proper functioning it requires the guidance that can only be provided by those who make policy and those who have to endure its implementation.

As an example of the approach described, fig. 9 shows the results of applying the optimal rules described earlier and summarized in fig. 8, in the

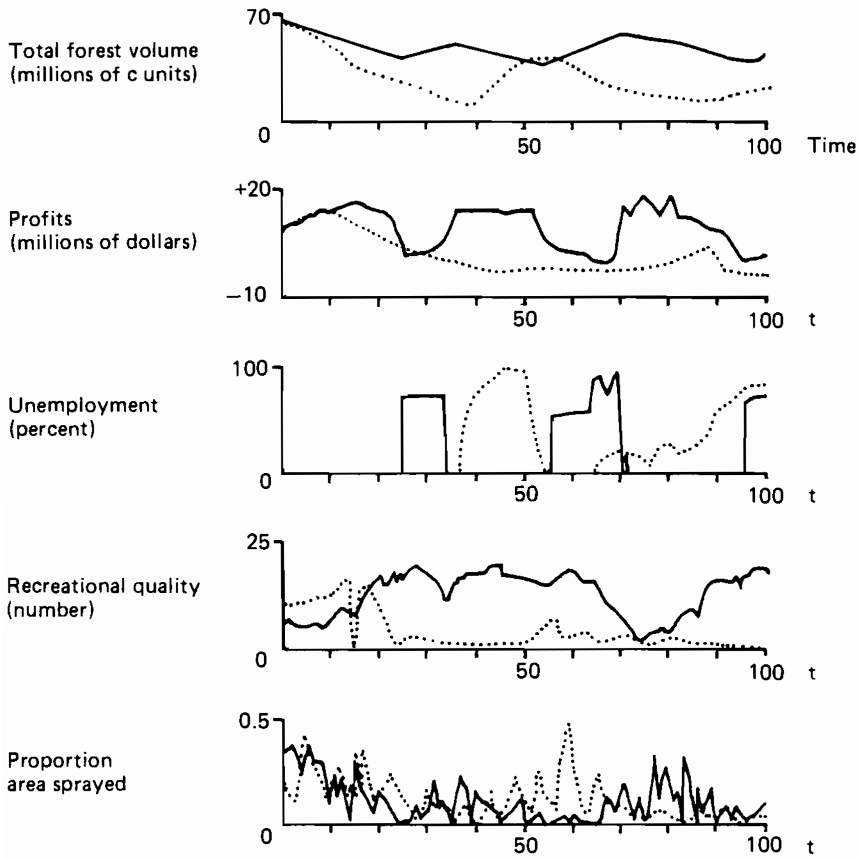

Fig. 9. A comparison between evaluation indicators under "constrained" Winkler management ——— and historical budworm management - - - - - -.

full simulation model, starting with conditions in 1953. The five indicators happen to be the five that were of interest to one particular policy maker. The results can then be compared to the policy that has been applied since 1953 in New Brunswick. That policy basically aimed to maintain employment by protecting the forest in the face of an impending budworm outbreak. That was achieved by only spraying insecticide on those stands of trees that were stressed by past and present defoliation. The results of applying this historical policy in the simulation model are compared also in fig. 9.

It is evident that for all five criteria it is superior and for three of the criteria the optimizer's policy is markedly superior to that developed by experts using common sense, common practice and mature judgment. See table 3 where a cummulative comparison is made. It is interesting to note that, in spite of the fact that the objective used by the optimizer was economic value only, the

Table 3
Comparison of objectives using optimizer instead of historic policies

	Cumulative total forest volume [a]		Cumulative profits [a]		Cumulative % unemployment		Cumulative recreational quality [a]		Cumulative % sites sprayed	
	Optimizer rules	Historic rules	Optimizer rules	Historic rules	Optimizer rules	Historic rules	Optimizer rules	Historic rules	Optimizer rules	Historic rules
by year 10	590	614	130	138	–	–	54	74	266	212
20	1109	1074	271	244	–	–	172	171	400	438
30	1572	1390	333	297	370	–	349	190	466	556
40	2081	1623	432	291	660	126	508	191	634	719
50	2560	1885	574	288	660	977	690	191	634	943
60	2997	2244	646	302	910	1156	857	211	667	943
70	3492	2538	670	289	1714	1682	986	234	755	1084
80	4047	2781	838	275	1734	3454	1026	244	923	1129
90	4555	3003	984	230	1734	5519	1140	252	1076	1154
by year 94	4740	3102	1019	227	1734	8278	1210	252	1112	1167

[a] Forest volume in millions of cunits, profits in millions of \$, recreational quality = number of sites suitable for recreation.

derived policy markedly improved this attribute and two others forest volume and recreational quality; less people were unemployed. The proportion of area sprayed remained about the same. Hence the optimizer policy is robust to other objectives as well.

But, however attractive such optimal policies are, they are highly simplified probes that only identify interesting starting points in policy space. There are, in particular, operational constraints in the real world that were not considered in the simplified dynamic program. For example, there are limits set by mill capacity to the amount of wood cut in any year; there are standards that limit the dosages of insecticide used; and only large blocks of trees can be sprayed practically, rather than single trees. So many of these constraints exist that our approach has been to add them heuristically as they are suggested by those who know the reality of that world. In fact, the three constraints mentioned above were included in generating the results shown in fig. 9.

Quite apart from existing constraints, from such starting points we can also identify other actions and rules that could, for example, eliminate insecticide spraying altogether. We have thus found that applying a biological control agent that produces modest mortality (e.g., a host-specific virus) when the forest is in an endemic "threat" state can dramatically reduce the need for insecticide. Such agents exist but have never been given priority for economical development because it has not been obvious how they might be used. The optimizer, by producing an interesting starting policy for the simulator, thus suggests special simulation runs as a way to modify and make practical an improved management of the forest ecosystem.

The conclusions are inescapable: (1) that well-tested simulators should be used to bring the real world into the laboratory; (2) that policies independently developed by experts should be compared on the simulator with those developed by optimization of simplified models and (3) that a simulator guided by an optimizer can manage the forest significantly better. This suggests that this approach, appropriately refined to cover certain exceptional circumstances, should be used in place of existing management procedures.

Discussions with regard to implementation are in process between members of the U.B.C. Institute of Resource Ecology and branches of the Canadian Government responsible for developing forest management policy. The discussions are facilitated by the prior cooperation between these groups and member of the Ecology and Methodology Projects of IIASA that took place throughout the development of this study.

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