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Identifiable Age, Period, and Cohort Effects: An Exploratory Approach Applied to Italian Female Mortality

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This research was conducted in conjunction with a summer research seminar on heterogeneity dynamics, under the direction of James W. Vaupel and Anatoli I. Yashin, in the Population Program at IIASA led by Nathan Keyfitz.

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ACKNOWLEDGMENTS

This work was made possible, above all, by the interaction and spirit of cooperative effort which were present in IIASA's YSSP population group. All participants, whether program directors, young scientists, or visitors, provided valuable ideas and inspiration. Particularly useful suggestions and comments were made by Andrew Foster, Bill Hodges, and Mike Stoto. Jim Vaupel, both for his knowledge of the subject and his enthusiasm for the present research, deserves special recognition.

A special word of thanks also goes to Graziella Caselli, who knows better than anyone else how these data were collected, and who gave much help, both in data-related questions, and in more substantive interpretations.

FOREWORD

A group of eleven Ph.D. candidates from seven countries--Robin Cowan, Andrew Foster, Nedka Gateva, William Hodges, Arno Kitts, Eva Lelievre, Fernando Rajulton, Lucky Tedrow, Marc Tremblay, John Wilmoth, and Zeng Yi--worked together at IIASA from June 17 through September 6, 1985, in a seminar on population heterogeneity. The seminar was led by the two of us with the help of Nathan Keyfitz, leader of the Population Program, and Bradley Gambill, Dianne Goodwin, and Alan Bernstein, researchers in the Population Program, as well as the occasional participation of guest scholars at IIASA, including Michael Stoto, Sergei Scherbov, Joel Cohen, Frans Willekens, Vladimir Crechuha, and Geert Ridder. Susanne Stock, our secretary, and Margaret Traber, managed the seminar superbly.

Each of the elevent students in the seminar succeeded in writing a report on the research they had done. With only one exception, the students evaluated the seminar as "very productive"; the exception thought it was "productive". The two of us agree: the quality of the research produced exceeded our expectations and made the summer a thoroughly enjoyable experience. We were particularly pleased by the interest and sparkle displayed in our daily, hour-long colloquium, and by the spirit of cooperation all the participants, both students and more senior researchers, displayed in generously sharing ideas and otherwise helping each other.

One of the best of the research reports produced and probably the most original is the report by John Wilmoth that appears in this working paper. Building on some of the work on the mathematics of population surfaces, on the use of shaded contour maps for displaying population surfaces, and on Italian mortality that had been started at IIASA by the two of us and W. Brian Arthur, Bradley A. Gambill, and Graziella Caselli, Wilmoth developed an approach for analyzing age, period, and cohort effects that shows considerable promise for further development and application. This report thus exemplifies the productive role IIASA can play in bringing together diligent, creative scholars from different countries and disciplines.

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1. INTRODUCTION

It is of great interest to separate demographic trends into three components: age, period, and cohort effects. To what extent is mortality at age α and time t affected by the age of the individuals, the events of the present day, and the life history of the cohort now age α? Of special interest is the third component: Can cohorts carry their level of mortality with them? To what extent does high/low mortality early in life lead to high/low mortality later in life? If a cohort is disadvantage in childhood, can this early selection cause it to be an advantaged cohort late in life?

Several authors have discussed the importance of childhood mortality reductions in the historical improvements observed later in the lives of the affected cohorts. Kermack, McKendrick, and McKinlay (1934) studied death rates in nineteenth-century England and Wales and found clear cohort-specific mortality reductions. Coale and Kisker (1985) update this early work and corroborate the findings. They emphasize the importance of data quality, especially at older ages, in any such analysis. Preston and Van de Walle (1978) show similar results in French urban mortality in the nineteenth century. Horiuchi (1983) studies the mortality of war survivors in Japan, Germany, France, and other countries, and finds higher levels of mortality later in life for men who were in adolescence at the end of the wars. Caselli and Capocaccia (forthcoming), studying Italian data, link high mortality early in life with high levels in middle age. They show, however, that these effects tend to decline in importance after age 45.

Other authors have discussed the possibility of an inverse relationship between mortality in early childhood and mortality at advanced ages. Meindl (1982) finds such a relationship in studying nineteenth century rural New England communities. Vaupel, Manton, and Stallard (1979) and Vaupel and Yashin (1985) emphasize the theory of heterogeneity and selection as a means of explaining the phenomenon. McMillan and Nam (1985) give a comprehensive review of the literature on mortality crossovers and conclude that such crossovers are not mere artifactual phenomena.

In this work we attempt to define clearly what is meant by age, period, and cohort effects, and propose a graphical method of decomposition which may help to shed some light on these hotly-debated issues. The method is applied to Italian female mortality data for the period, 1869-1978, and very clear cohort and period variations in mortality are observed.

2. DEFINITIONS OF AGE, PERIOD, AND COHORT EFFECTS

We must first consider the factors which influence the level of mortality, $\mu(\alpha,t)$, for a given age α and time t. Many people have spoken of age-period-cohort models of mortality, but what exactly is meant by these three factors is often unclear. In this paper we are interested in the mortality of a national population, so a period is one calendar year of time, age refers to the time from birth for an individual or cohort, and a cohort is a group of people born in a single calendar year. The ultimate source of difficulty is that the three factors are related: a person who is age α at time t was obviously born at time t-α, if we think, for the moment, in continuous age and time. Knowing any two of these three factors, then, allows us unambiguously to know the third one as well.

This interrelationship between age, period, and cohort variables is the source of a well-known identification problem. If observed mortality, $\mu(\alpha,t)$, is decreasing with time, we cannot know whether this is because progress against mortality is being made now, at time t, or because the cohort born at time t-α is advantaged compared to the preceding cohort. If $\mu(\alpha,t)$ increases with age, we cannot know whether this is due to a natural tendency for mortality to increase with age, or again because the cohort from time t-α is advantaged compared to the preceding cohort (who is now slightly older). Finally, if $\mu(\alpha,t)$ tends to increase with respect to both age and time together (that is, in the cohort direction), we cannot determine how much of the increase is due to aging and how much is attributable to

period change.

In order to define cohort and period effects in a way which can be measured, it is best to consider not just simple age effects, which reflect some constant age-specific contribution to mortality, but to think in terms of the changing age structure of mortality. This would be a mortality surface which would reflect the age pattern of mortality as it is influenced by the smooth, long-term trends in mortality. This age structure of mortality would be a function of both the cohort and period trends in mortality.

For any one fixed point in time, t_0, the curve $\mu(\alpha,t_0)$ would take on the familiar bathtub shape. For any age, α_0, $\mu(\alpha_0,t)$ would show a steady (though not necessarily constant) rate of change. The changing shape of $\mu(\alpha,t)$ would reflect the changing, underlying age pattern of mortality. The essential requirement, though, is that this be a smooth surface in all directions, which, for our purposes, will mean that it must have smooth first derivatives.

We make no attempt, then, to explain why this pattern changes. There would be many factors to be considered. For example, health and medical progress would be important components, but progress at one age may mean higher mortality later in life for the affected cohorts due to heterogeneity and selection (Vaupel and Yashin, 1985). Again, though, we see that measuring the effects of selection versus long-term period progress is a non-identifiable problem. If real mortality progress is greater at younger than at older ages, is it due to heterogeneity and selection operating on the cohorts who are now at advanced ages, or is it simply that the health progress now being made favors the younger age groups?

It seems that answers to such questions will have to come from study of the biological mechanisms of aging in the presence of a changing health environment, which is beyond the scope of this paper. For this reason, we do not concern ourselves with such questions. Rather, we let all long-term mortality trends (whether they be period or cohort trends) be reflected in the changing age structure of mortality, which gives the expected mortality rate for a given age and time. We then consider deviations between the observed and expected rates.

The cohort effects which interest us, then, are deviations which occur in a cohort-specific pattern. To the extent that we find a deviation at the point, (α,t), which is similar to those at $(\alpha+1,t+1)$, $(\alpha+2,t+2)$, etc., we may conclude that there are real, identifiable cohort effects in operation. If the cohort variations are such that the observed mortality is lower than the expected, we say that a

cohort is *advantaged*. Likewise, a cohort is *disadvantaged* if observed mortality is higher than expected.

Similarly, period variations are deviations which follow a period-specific pattern. A deviation at (a,t) which resembles the deviations at $(a+1,t)$, $(a+2,t)$, etc., is a period effect. A positive deviation (observed greater than expected) will be call an *unfavorable* period, while a negative deviation (observed less than expected) will be called a *favorable* period.

In both cases, these period and cohort variations are relative effects, in that they are relative to the overall trend of mortality, which is contained in what we have already defined as the changing age structure of mortality. The crux of the age-period-cohort identification problem is the question of how much of this trend should be attributed to periods and how much to cohorts. Our approach in this paper is that it is the relative effects (i.e., the variations around the trend) which are of interest, and we make no attempt to separate the overall trend of mortality into what may be its period and cohort components. (Caselli and Capocaccia, in a forthcoming paper, take a similar approach in considering a range of reasonable cohort and period trends.)

3. MATHEMATICAL FORMULATION

In order to analyze the mortality surface $\mu(a,t)$, it is often more useful to consider, not the level of mortality at a given point, but rather the rate of change in three possible directions. We thus define three quantities which will form the basis for the mathematical model to be developed. These will measure the relative change in three directions at the point $\mu(a,t)$. Thus, let

$$k_p(a,t) = \frac{\frac{\partial}{\partial a}\mu(a,t)}{\mu(a,t)}$$

$$= \frac{\partial}{\partial a}\log\mu(a,t)\,, \tag{1}$$

$$\rho(a,t) = \frac{-\frac{\partial}{\partial t}\mu(a,t)}{\mu(a,t)}$$

$$= -\frac{\partial}{\partial t}\log\mu(a,t)\,, \tag{2}$$

$$\text{and } k_c(a,t) = \left.\frac{\frac{\partial}{\partial x}\mu(a+x,t+x)}{\mu(a,t)}\right|_{x=0}$$

$$= \left.\frac{\partial}{\partial x}\log\mu(a+x,t+x)\right|_{x=0}. \qquad (3)$$

These are, respectively, the relative rates of change in $\mu(a,t)$ with respect to age alone, time alone, and age and time together.

Obviously, these are equivalent to the derivatives in the same directions of $\log\mu(a,t)$. It seems reasonable to consider the effects of cohort and period variations on the relative rates of change in $\mu(a,t)$, but, for ease of expression, we will henceforth refer only to the absolute change in $\log\mu(a,t)$. It should be remembered, though, that the justification for the use of $\log\mu(a,t)$ is that its derivative gives the relative rate of change in $\mu(a,t)$.

This notation was not chosen just by hazard. The rate of increase in $\log\mu$ for a real cohort is given by k_c, while the similar rate of increase within a synthetic cohort (period life table) is denoted by k_p. The subscripts c and p thus serve to remind us that we are considering the derivatives of $\log\mu$ for a cohort and for a period, respectively. The rate of decrease in $\log\mu$ over time is given by ρ. This is a different kind of measure from k_c and k_p and thus deserves a different notation. It is a measure of progress against mortality at the point (a,t), and tends to be of a much smaller magnitude than k_c and k_p. Furthermore, the plots of k_p and k_c are of a similar nature, since the major component in both is the biological tendency for mortality to change with age. The plot of ρ is very different. (The choice of k is due to Horiuchi and Coale (1983) and Horiuchi (1983), who define $k(x)$ for a period life table in the same way as $k_p(a,t)$ is here defined.)

These three derivatives, $k_p(a,t)$, $\rho(a,t)$, and $k_c(a,t)$, are exactly analogous (after a change of sign) to the quantities $\nu(a,t)$, $r(a,t)$, and $\mu(a,t)$ from the generalized theory of stable populations (Preston and Coale, 1982; Arthur and Vaupel, 1984). Just as we may write

$$\mu(a,t) = \nu(a,t) - r(a,t), \qquad (4)$$

we have

$$k_c(a,t) = k_p(a,t) - \rho(a,t). \qquad (5)$$

The mathematical equivalence of this identity can be established using the same

methods given in the sources cited, so the proof will not be repeated here. Instead, an intuitive, demographic explanation will be attempted.

The derivative, $k_p(\alpha,t)$, represents the tendency for mortality increase (or decrease) within a synthetic cohort (i.e., period). Similarly, $k_c(\alpha,t)$ expresses the rate of mortality increase (or decrease) within a real cohort. Finally, progress against mortality is contained in $\rho(\alpha,t)$. If mortality is constant, then $\rho(\alpha,t)=0$, and $k_c(\alpha,t)=k_p(\alpha,t)$. This means that a person who is age α at time t is expected to experience the same mortality increase as expressed in the appropriate period life table for that time. If $\rho(\alpha,t)$ is non-zero, though, progress against mortality is being made (though it may actually be negative "progress"), and the actual mortality experience of the individual at (α,t) will reflect the period pattern of mortality, k_p, less any progress being made, ρ. That is, the equation,

$$k_c = k_p - \rho\,, \tag{5}$$

is equivalent to the expression,

cohort mortality increase = period mortality increase - progress against mortality.

When dealing with discrete data, it is necessary to find a discrete analog to equations (1), (2), and (3). The derivatives could be approximated by moving spline functions, but a simple and equally effective approach is to calculate the following first differences:

$$\begin{aligned} k_p(\alpha,t) &= \Delta_\alpha \log\mu(\alpha,t) \\ &= \log\mu(\alpha+1,t) - \log\mu(\alpha,t)\,, \end{aligned} \tag{1a}$$

$$\begin{aligned} \rho(\alpha,t) &= -\nabla_t \log\mu(\alpha,t) \\ &= \log\mu(\alpha,t-1) - \log\mu(\alpha,t)\,, \end{aligned} \tag{2a}$$

$$\begin{aligned} k_c(\alpha,t) &= \nabla_{\alpha,t} \log\mu(\alpha,t) \\ &= \log\mu(\alpha,t) - \log\mu(\alpha-1,t-1)\,. \end{aligned} \tag{3a}$$

The symbols, Δ and ∇, are used to indicate forward and backward differences, respectively. The choice to define k_p by a forward difference, while defining k_c and ρ by backward differences, was made for reasons which may not be immediately clear for the reader. Essentially, this is done so that a cohort is always compared to the preceding one (in the case of k_p), and so that a period is likewise always

compared to the one just before (in the case of ρ and k_c).

In the case where k_p, ρ, and k_c are calculated from discrete data, we must alter the fundamental identity, equation (5), which expresses the relationship between these three quantities, so that we have

$$k_c(\alpha+1,t+1) = k_p(\alpha,t) - \rho(\alpha+1,t+1) . \qquad (5a)$$

On the discrete mortality surface, $\log\mu(\alpha,t)$, these are quantities calculated from three values of $\log\mu$ which form a triangle. The identity is thus a trivial arithmetic equation.

Now that we are talking about real data, it is necessary to make a notational distinction between what we have called the observed and underlying rates of mortality. For the observed rates, we will use a small superscript 0 after the quantity: μ^0, $\log\mu^0$, k_p^0, ρ^0, and k_c^0. The underlying rates will be written μ, $\log\mu$, k_p, ρ, and k_c. It will be necessary to estimate these underlying rates, and the estimated quantities will be denoted by a small circumflex: $\hat{\mu}$, $\log\hat{\mu}$, $\hat{k_p}$, $\hat{\rho}$, and $\hat{k_c}$.

4. WHY CONSIDER DERIVATIVES?

A natural question is why we would want to consider the first derivatives on the mortality surface $\log\mu$. The essential reason is that the derivatives, k_p and k_c, are very stable measures of mortality. Although the level of mortality as expressed by μ or $\log\mu$ can change very quickly, the shape of the mortality curve for successive periods or cohorts changes much more slowly, so that k_p and k_c have a nearly constant shape for close periods or cohorts. The result is that it is relatively simple to estimate the underlying values, $\hat{k_p}$ and $\hat{k_c}$, even by considering just simple averages.

Another reason to calculate the derivatives over the surface, $\log\mu$, is that, within a period or cohort, it is easier graphically to look at the entire age range using derivatives than using the original data. Above age 30, for example, it is well known that mortality follows approximately a Gompertz curve, so that the derivative of $\log\mu$ is nearly constant. This constancy of the underlying curve makes deviations all the more apparent.

The derivatives may also be used effectively to discover inconsistencies in the data. Outliers become immediately apparent (in the national Italian mortality data, several gross errors were discovered which had previously passed unnoticed); and

other peculiar tendencies become apparent which are hidden in the raw data (the Italian data again provides an example which will be discussed later).

Finally, taking the derivative of $\log\mu$ in one of the three directions becomes a means of approximately removing one of the three effects (i.e., age, period, or cohort). This is true only in a neighborhood around the age, period, or cohort involved, and remains valid only so long as the change in the age structure of mortality is slow in that neighborhood. It can be shown that, if $\rho(a,t)=\rho(t)$ is constant for all ages at any time t, then $k_p(a,t) = k_p(a)$ does not change with time, and thus $k_c(a,t) = k_p(a)-\rho(t)$. Since this assumption should hold approximately in a neighborhood around (a,t), and since $\rho(a,t)$ is generally quite small, k_p should change little for adjacent periods, and k_c should change little for adjacent cohorts.

Thus, in taking the derivative with respect to age and considering k_p^0 for adjacent periods, the deviations found in the various periods must be stochastic or result from cohort variations in mortality (Horiuchi, 1983). Likewise, in taking the derivative with respect to age and time and considering k_c^0 for adjacent cohorts, the deviations found in the various cohorts must be random or result from period variations in mortality. The derivative with respect to time alone, ρ^0, will show, above all, the confounding of cohort and period variations in mortality. In all three cases, though, the derivatives will be affected by the overall trend of mortality, $\rho(a,t)$, but this will be a relatively small effect, and, in a neighborhood around (a,t), an almost negligible one.

We may say, then, that k_p^0 approximately removes period variations from the observed data. Likewise, k_c^0 approximately removes cohort variations, and ρ^0 nearly eliminates age effects. This is the final motivation for considering derivatives on the mortality surface $\log\mu(a,t)$. They provide the first step in a filtering process which attempts to effectively separate long-term mortality trends from period and cohort variations.

5. APPLICATION TO ITALIAN FEMALE MORTALITY DATA

In order to illustrate the technique of age-period-cohort decomposition just described, we will examine historical mortality data for Italian females. The data are for the years 1869 to 1978 and give observed, single-year, conditional probabilities of death, q_x^0, for ages 0-79. We estimate the observed force of mortality, $\mu^0(x)$, by the simple equation,

$$\mu^0(x+\frac{1}{2}) \approx -log(1-q_x^0) . \qquad (6)$$

In the discussion that follows, the extra one-half year in $\mu^0(x+\frac{1}{2})$ will not be written but will remain understood.

All data are age-cohort data; that is, all events are grouped by age and year of birth. It is thus impossible to know the exact period of a death. If someone born in 1900 dies at age 38, for example, this can happen in either 1938 or 1939. Thus, our classification of cohorts is exact; but, when we speak of period x, that could mean that death occurred either in year x or $x+1$.

Further examination of the methods employed in collecting and manipulating the original data is probably necessary in order to check whether any of the results are mere artifacts of the data collection techniques. (For example, if a cohort were systematically undercounted in the national censuses, and yet deaths were correctly counted, the cohort would show exaggerated levels of mortality throughout life and would thus be incorrectly classified as disadvantaged.) Some preliminary investigation in this direction has failed to turn up any such systematic errors in the data which would alter the validity of the technique; but data quality must always remain suspect, and the absence of any systematic errors must repeatedly be verified.

As examples of the curves, k_p^0, k_c^0, and ρ^0, we will consider the periods of 1950-55, the cohorts of 1870-75, and the age groups, 0, 1-4, 5-14, 15-24,..., 65-74. These three graphs, then, are shown in Figures 1-3. Figure 3 shows the ups and downs of mortality progress over time. The values for each age group are a simple average of the derivatives for the individual ages. Our ultimate goal will be to decompose this confused pattern of progress into three parts: period variations, cohort variations, and the overall trend of mortality.

5.1. Analyzing Cohort Effects

Figure 1 shows the tendency for mortality change within the synthetic cohorts from 1950 to 55. Two things are immediately apparent: first, the curves are by no means smooth; second, deviations in one period may resemble those within another, but usually with a shift of one year. For example, a strong positive deviation occurs at age 30 in 1950, at age 31 in 1951, etc. Other similar patterns (positive or negative deviations) are noticeable starting at ages 4, 9, 10, 38, 39, 40, 41, 42, 46,

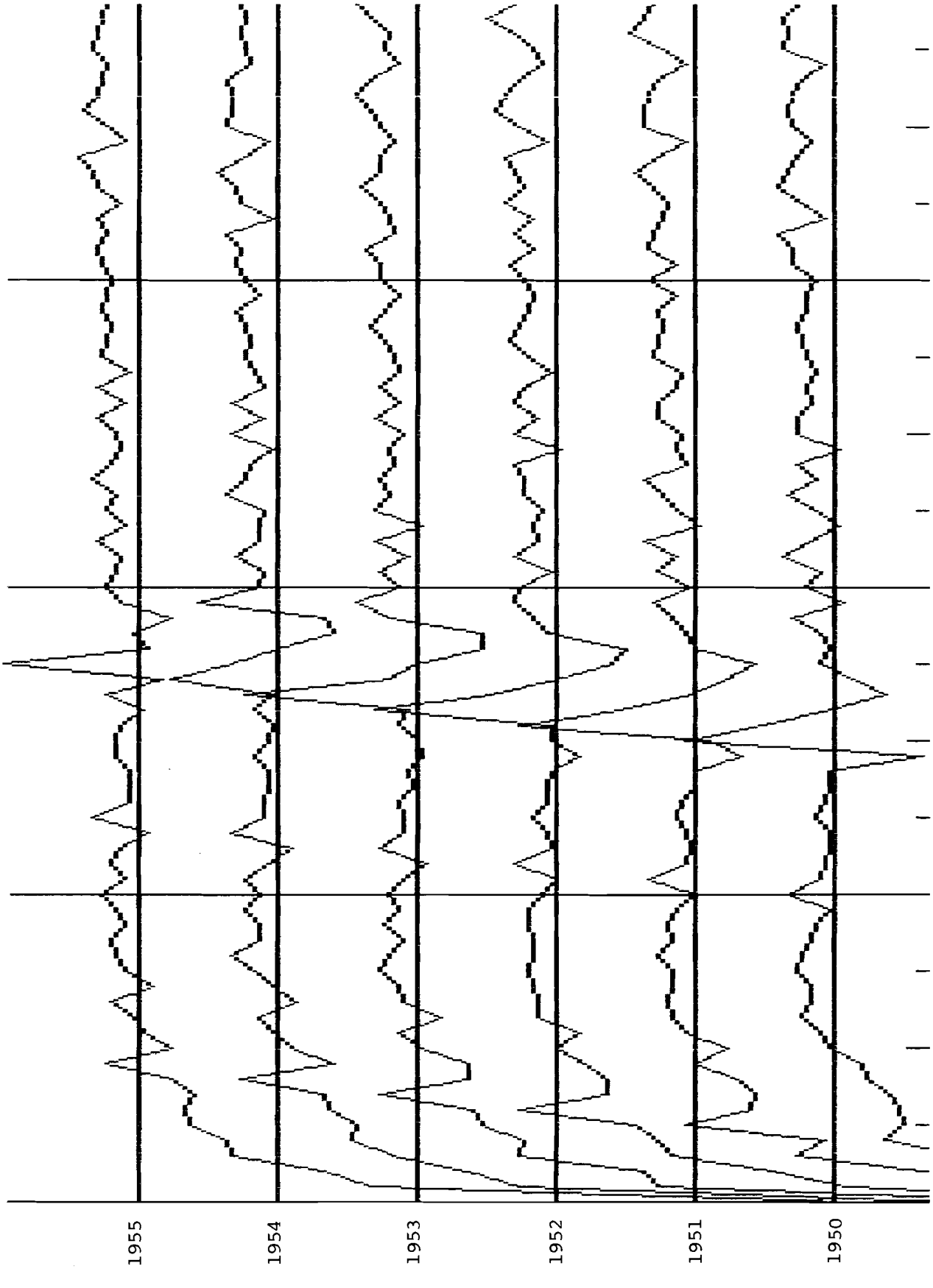

Figure 1: $k_p^o(a,t)$ by periods: 1950-55 (horizontal lines mark zero level for each period).

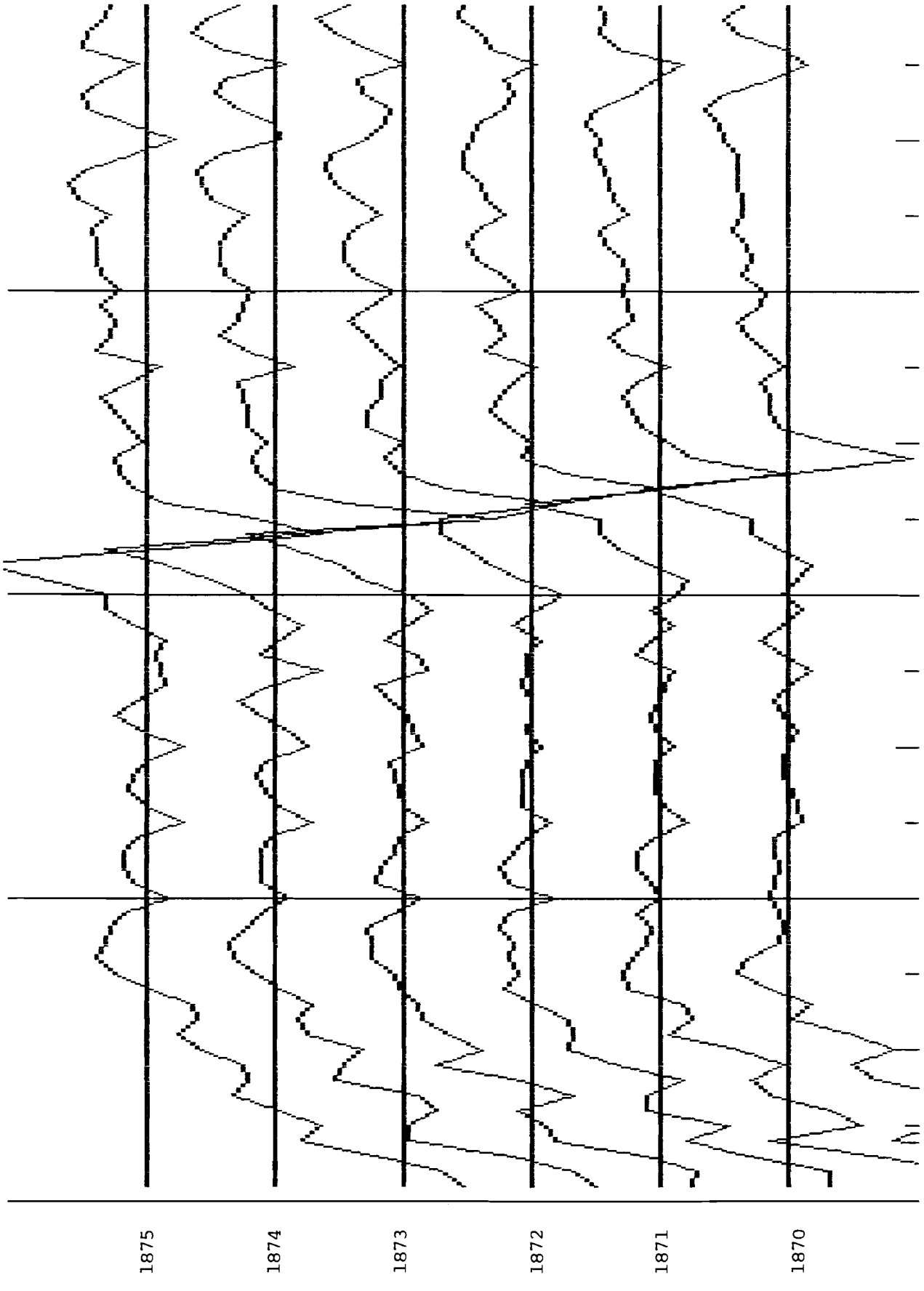

Figure 2: $k_c^o(a,t)$ by cohorts: 1870-75 (horizontal lines mark zero level for each cohort).

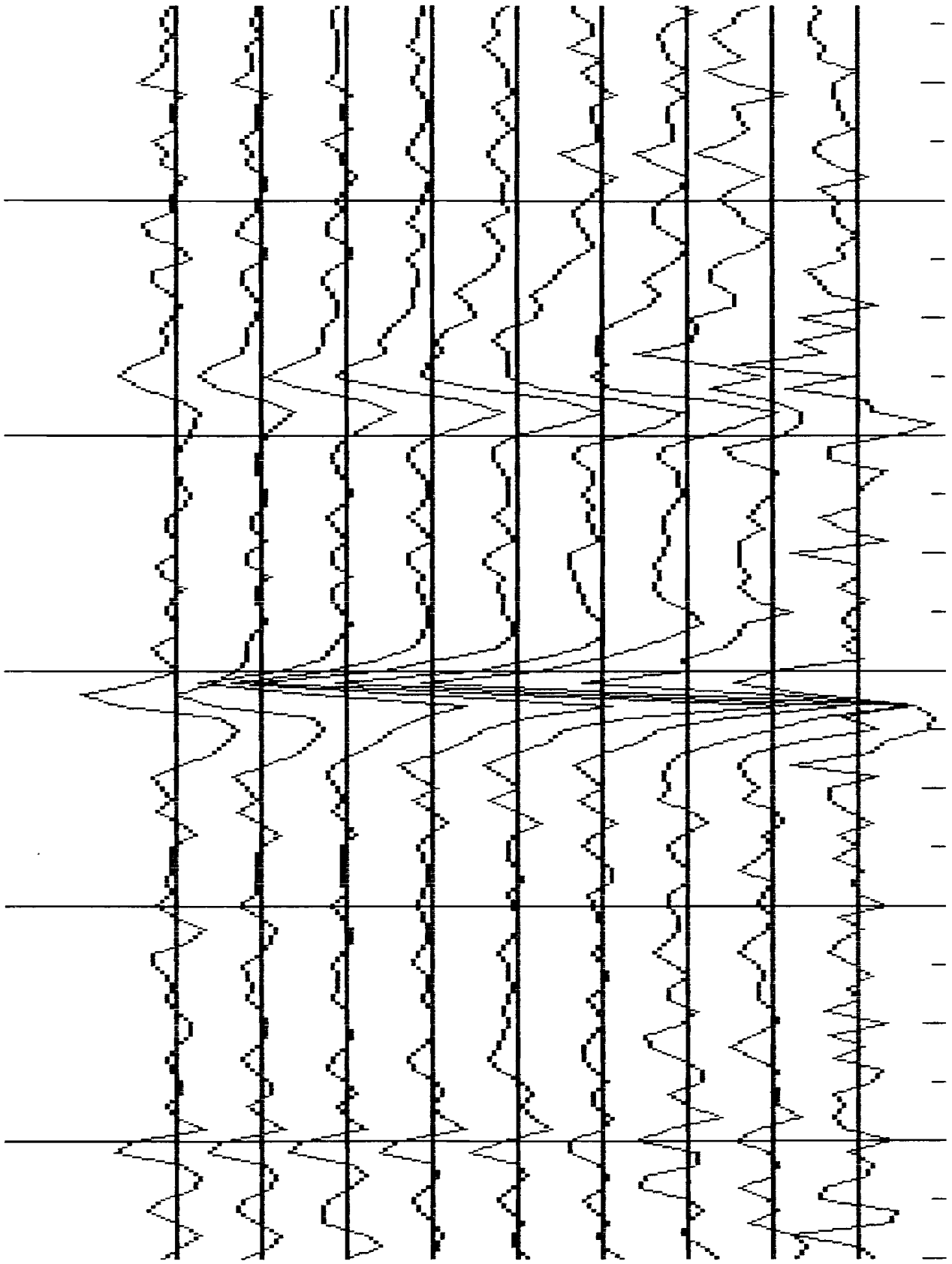

Figure 3: $p^{o}(a,t)$ for various age groups.

47, and so forth, in 1950. The fact that the deviations follow a consistent pattern leads us to believe that they cannot be stochastic in nature (although a proper statistical test of significance still needs to be developed). If they are not an artifact of the data, then they are clearly cohort variations in mortality.

It is natural to ask what these deviations mean in terms of the affected cohorts. The spike observed in 1950 at age 30 means that $k_p^o(30,1950)$ is much larger than would normally be expected. Since

$$k_p^o(30,1950) = \log\mu^o(31,1950) - \log\mu^o(30,1950) ,$$

this must mean that the observed rate of mortality at age 31 is significantly larger than at age 30. Of course, we consider it perfectly normal for mortality to increase as we go from age 30 to age 31, but the spike in k_p^o indicates that the increase is more than expected. Thus, we say that the cohort now age 30 (born in 1920) is advantaged compared with the cohort now age 31 (born in 1919), over the period, 1950-55.

Our comparisons, for the moment, are always of this nature: we compare each cohort to its elder neighbor and attempt to determine if the more recent cohort is advantaged or disadvantaged in comparison with the preceding one. We will be especially interested to observe how the advantaged-disadvantaged relationship changes over time. For instance, does the magnitude of the observed deviations change over the life of the cohorts? If so, at what age does this change begin? Also, can two cohorts change the direction of their advantaged-disadvantaged relationship at some point in life?

In order to answer these questions, it is necessary to measure the magnitude of the observed deviations. To do this, we must estimate the underlying levels of k_p for the relevant cohorts. It turns out that this is rather easily accomplished. As noted before, k_p changes very little for close periods of time, so a reasonable means of smoothing the observed curve, k_p^o, is through simple-moving-averages. After a little experimentation, it was decided that a 7-year moving-weighted-average (using binomial weights) was slightly superior to a number of other possibilities which were tried. We thus let

$$\hat{k}_p(a,t) = \frac{1}{64}\sum_{n=-3}^{3}\binom{6}{3-|n|}k_p^o(a,t+n) . \qquad (7)$$

There are surely still better smoothing techniques to be found, but, for the moment, we can be content to employ this one. The effect of averaging is that the de-

viations, which tend to be both positive and negative, work to cancel each other out, producing a nearly smooth curve, $\hat{\kappa}_p$.

Since we applied this to the entire matrix of data, containing periods 1869-1978, it was necessary to make some sort of adjustment at the endpoints (i.e., for the periods, 1869-71 and 1976-78). For simplicity, we let $\hat{\kappa}_p$ for these periods equal $\hat{\kappa}_p$ from the final periods for which the moving-weighted-average was possible (i.e., 1872 and 1975). Figure 4 shows the estimated values, $\hat{\kappa}_p$, for the periods, 1950-55.

In examining the estimated values, $\hat{\kappa}_p$, one is immediately struck by the 5-year cyclical pattern of mortality present in this data. This is the data peculiarity, referred to earlier, which had not been previously noticed in the Italian data. It is present for both males and females up until around 1957. Curiously enough, the year 1957 does not correspond to any particular change in data collection techniques, or even to a national census (there were censuses in 1951 and 1961). The cause of this pattern is thus still not completely clear, but for our purposes it has no great effect: since we are interested in removing the underlying age-pattern of mortality within a period, we should remove the pattern which exists *in the data itself*. In this way we remove, not only the actual pattern of age-specific mortality, but also any consistent (with respect to age) data peculiarities.

Another noticeable characteristic of the $\hat{\kappa}_p$ curves, during the periods, 1950-55, is that the cohort effects between ages 30 and 40 have not been completely removed. This is a criticism of the smoothing technique used and must eventually be improved upon. The important thing for now, though, is that we may examine the residual quantities, $\kappa_p^0 - \hat{\kappa}_p$, as in Figure 5. These show, even more clearly than before, the cohort pattern of deviations in the observed values, κ_p^0. Especially at the younger ages, the deviations are clearer. Also at the more advanced aged, deviations which were once obscured by the 5-year cyclical pattern are now more apparent.

Another result which is now easier to observe, is the relative magnitude of the cohort variations. Obviously, the greatest difference between adjacent cohorts occurs for the cohorts of 1919 and 1920. Also, the cohort of 1946 is strongly advantaged compared with 1945; 1930 is advantaged compared with 1929; 1916 is disadvantaged compared with 1915; 1917 is disadvantaged compared with 1916; and 1918 is disadvantaged compared with 1917. By cumulating the effects, we can say that all the cohorts from 1916 to 1919 are strongly disadvantaged. Even for the cohorts now at advanced ages, though, consistent patterns can be observed in the

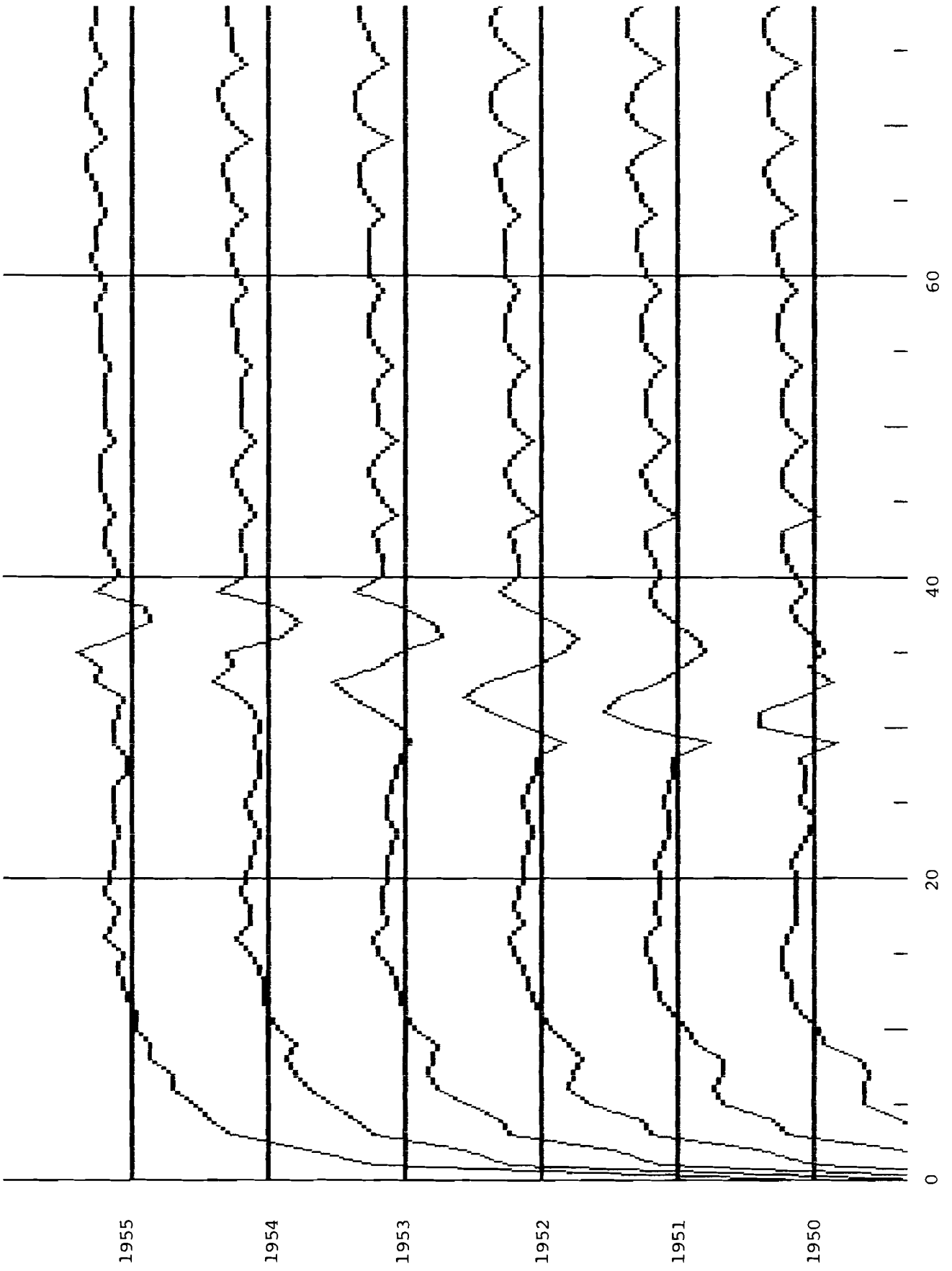

Figure 4: $\hat{k}_p(a,t)$ for periods 1950-55.

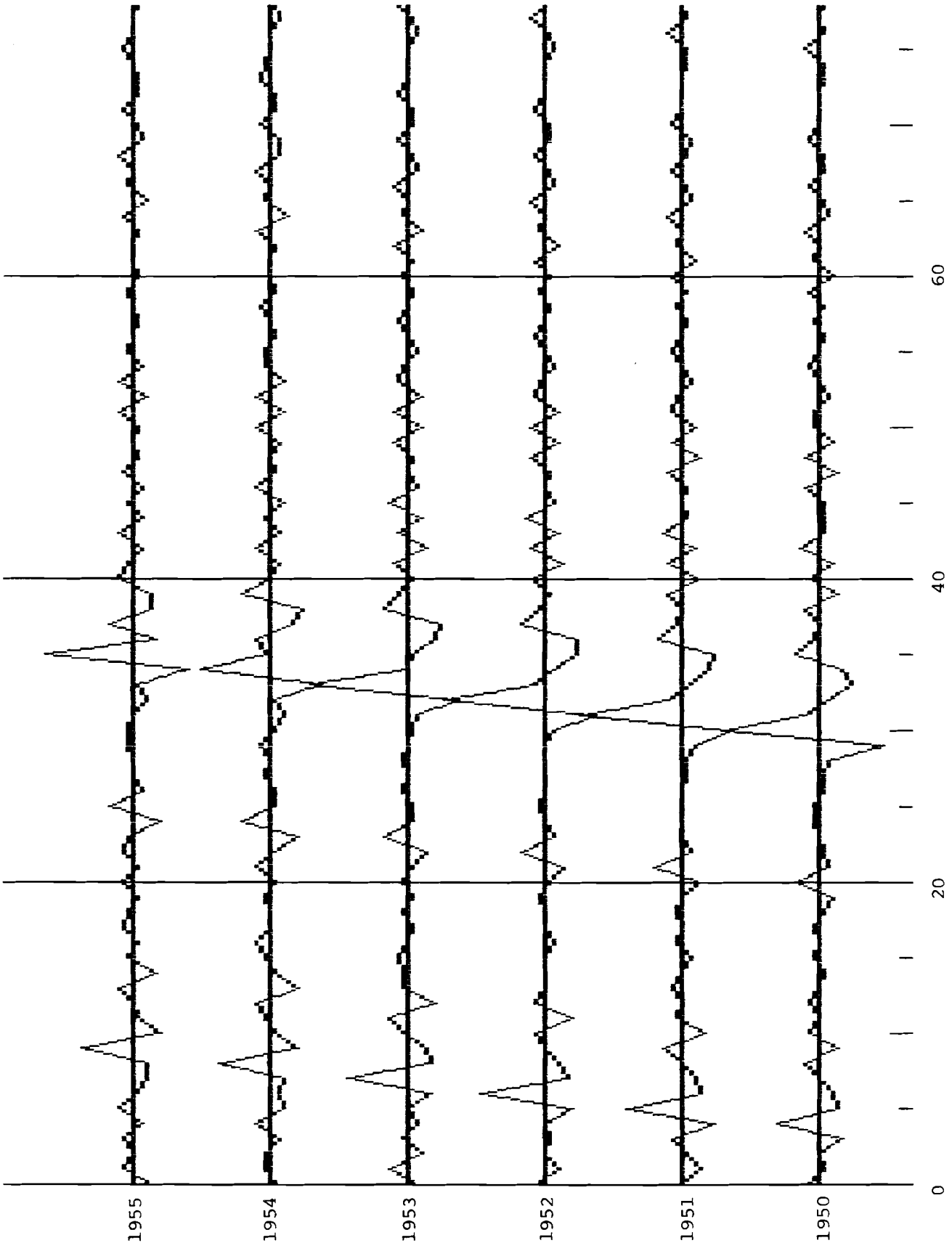

Figure 5: $k_p^o - \hat{k}_p$, for periods 1950-55.

residuals, although the differences are clearly of a smaller magnitude.

To continue the analysis of cohort effects, we can make a graph of all the k_p-residuals, $k_p^0 - \hat{k}_p$, using a program, called "Lexis", developed by Bradley Gambell and James Vaupel at IIASA. This permits us to see the overall pattern of the residuals. For simplicity, we examine only the sign of the residuals in Figure 6.

The diagonal pattern of the residuals is most striking; furthermore, it is interesting to note that the width of the diagonal lines often encompasses only one or two cohorts. This would indicate that there may be cohort variations in mortality through a large portion of life which may be observed even between adjacent cohorts.

Two inconsistencies in the diagonal pattern are immediately noticeable. First, the triangle in the upper, left-hand corner, corresponding to the cohorts before 1862, shows a less regular diagonal pattern, and the bands seem to be of greater width than elsewhere. This corresponds to a different technique of calculating the q_x^0 probabilities for the early cohorts, where only five-year data was available. For our purposes, then, we consider only the cohorts from 1862 onward.

Second, there is clearly a change in the pattern which begins in 1957, where the diagonals lines are still evident, but nevertheless not as clear as before. Either this corresponds to some data-related artifact, not yet discovered; or it indicates a change in the mortality regime which tends to obscure, in a very real sense, cohort variations in mortality. This second theory is supported by the fact that the change in the pattern actually seems to start for the cohort of 1925. This whole period, beginning in 1925, was a time of very rapid change in Italian mortality, and it may very well be that the type (and regularity) of cohort effects observed in a country may be a function of the current age structure of mortality.

Aside from possible data irregularities, though, something should be said about what the model can tell us that we did not already know. Of course, no one would be surprised to learn that the cohorts born during the First World War tended to be successively more and more disadvantaged; nor is it surprising that the 1920 cohort is advantaged compared with the final war cohort. It is more astonishing, though, that cohorts, even those born in times of peace, may show consistently higher or lower levels of mortality than the adjacent cohorts (after adjusting for changes in the age structure of mortality, of course).

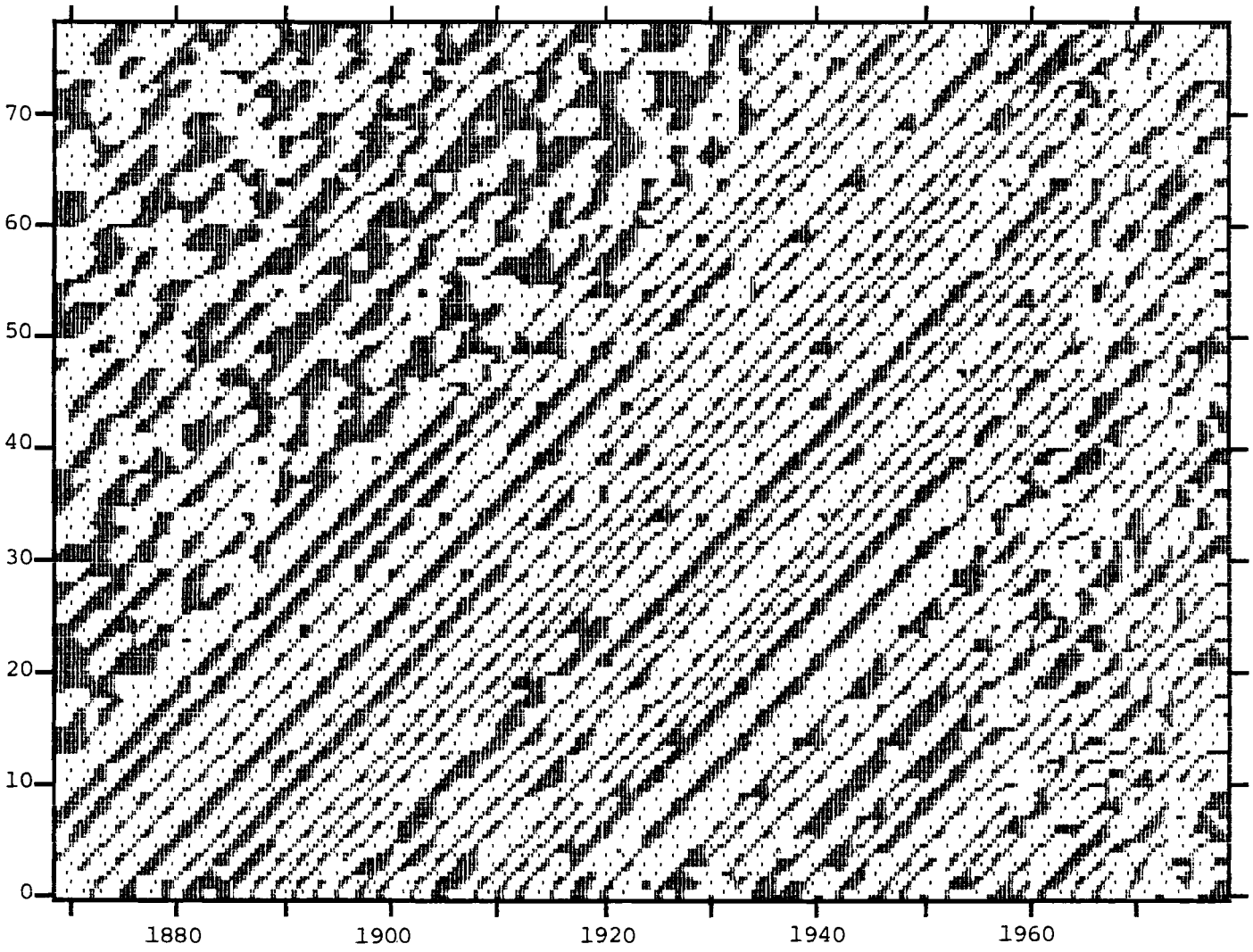

Figure 6: "Lexis" map of k_p-residuals (advantaged cohorts are dark, disadvantaged ones are light).

It might be instructive to examine only the larger residuals in order to see which cohorts experience significantly different mortality from the preceding ones. Figure 7 shows only the residuals greater than 0.1 or less than -0.1. In terms of Gompertz mortality ($\mu(x)=Be^{\alpha x}$), we expect the parameter, α, should be approximately in the range

$$0.08 < \alpha < 0.12 .$$

Since these residuals may be interpreted as deviations from the Gompertz α parameter (at least at the older ages), we are now examining only the residuals which are approximately larger (in absolute value) than the parameter itself.

It is also possible to view the residuals for a particular cohort. Again, we choose the generations, 1870-75, in order to examine completed cohorts. The residuals, $k_p^0-\hat{k}_p$, by year of birth are shown in Figure 8. The simple interpretation is that, in comparison with the cohort born just one year earlier, the generations from the years, 1870, 1871, 1873, and 1874, were disadvantaged, while those from 1872 and 1875 were advantaged. The relationship between adjacent cohorts usually remains stable through a large part of life; it is apparent, however, that the advantaged-disadvantaged relationship often tends to change direction late in life, at least for the cohorts born in 1871, 1872, and 1873.

5.2. Analyzing Period Effects

We may make a similar analysis of period effects by examining the residuals, $k_c^0-\hat{k}_c$. Since period effects tend to last longer than cohort effects, it was thought that a wider moving-average was appropriate. Again, this part of the technique still needs some improvement, but, for now, we employ an 11-year simple-moving-average:

$$\hat{k}_c(\alpha,t) = \frac{1}{11}\sum_{n=-5}^{5} k_c(\alpha,t+n) . \qquad (8)$$

This formula is used wherever possible, and at the endpoints, like before, the final values are extended to the years for which an 11-year moving-average is not possible.

The estimated values of $\hat{k}_c$ show a 5-year cyclical pattern, as with $\hat{k}_p$, but we remove this tendency and examine only the residuals, $k_c^0-\hat{k}_c$, in Figure 9. We observe a large positive spike with its peak at age 47 for the 1870 cohort. A similar

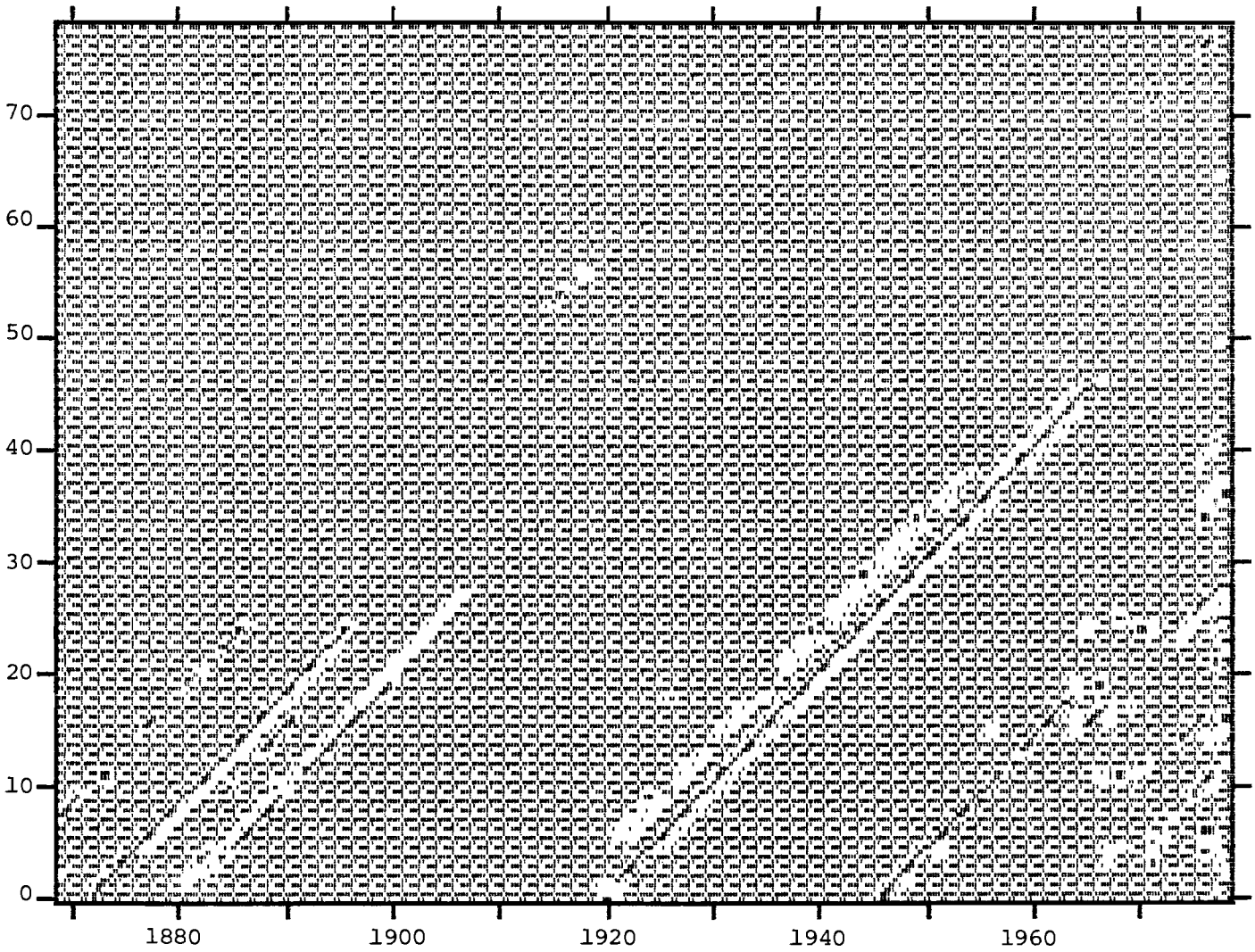

Figure 7: "Lexis" map of large k_p-residuals (> 0.1 or < -0.1).

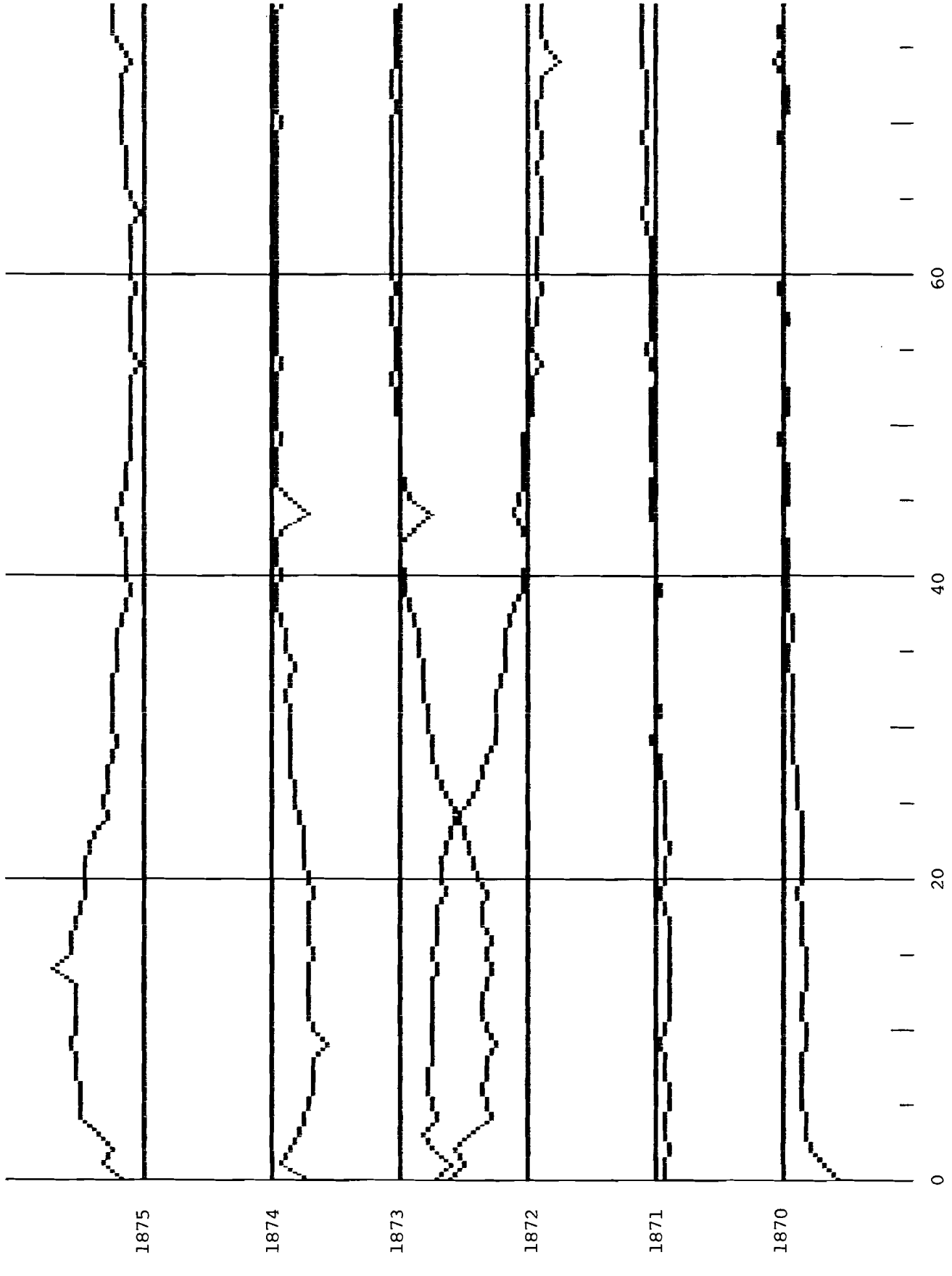

Figure 8: k_p-residuals by cohorts: 1870-75.

spike occurs one year earlier in the lives of the successive cohorts. This indicates that the period, 1917-18, was very unfavorable compared with the previous one. The width of the spike indicates that conditions deteriorated starting in 1914-15 and lasting until 1917-18. The period, 1918-19, saw little change, but strong improvement came in the period, 1919-20. This seems consistent with our observations earlier concerning cohorts: the generations from 1916-1919 tend to be strongly disadvantaged in comparison, both with earlier cohorts, and with the cohort of 1920. The period, 1915-19, corresponds exactly to the time when these cohorts were being conceived and first exposed to the world.

As with the k_p-residuals, it is possible to examine all the residuals, $k_c^o - \hat{k}_c$, using the program, "Lexis". The period variations in mortality become quite clear in Figure 10, where we can see the positive and negative k_c-residuals. Figure 11 eliminates the weak residuals (those between -0.1 and 0.1) and thus shows the periods where strong variations were present. These correspond, not surprisingly, to the First and Second World Wars. If a strong positive variation occurs during the war, then a large negative one must occur after the war in order to move back in the direction of the overall mortality trend.

5.3. Final Decomposition

Since we know that

$$k_c = k_p - \rho, \tag{5}$$

we may estimate $\hat{\rho}$ according to

$$\hat{\rho} = \hat{k}_p - \hat{k}_c. \tag{9}$$

This should reflect the long-term trend in mortality and should thus be a small, positive value for all ages and periods over 1869-1978. At this point in the development of the technique, however, all errors in estimating $\hat{k}_p$ and $\hat{k}_c$ (i.e., all the inadequacies of the averaging methods) show up in $\hat{\rho}$. Nevertheless, the general pattern of positive progress can be seen in Figure 13, with negative progress in times of war, or for the more severely disadvantaged cohorts. In further work, it is hoped that these difficulties can be overcome.

The eventual goal would be to complete the decomposition of the observed, ρ^o, into three components: long-term health progress, cohort effects, and period effects. This can be expressed mathematically as

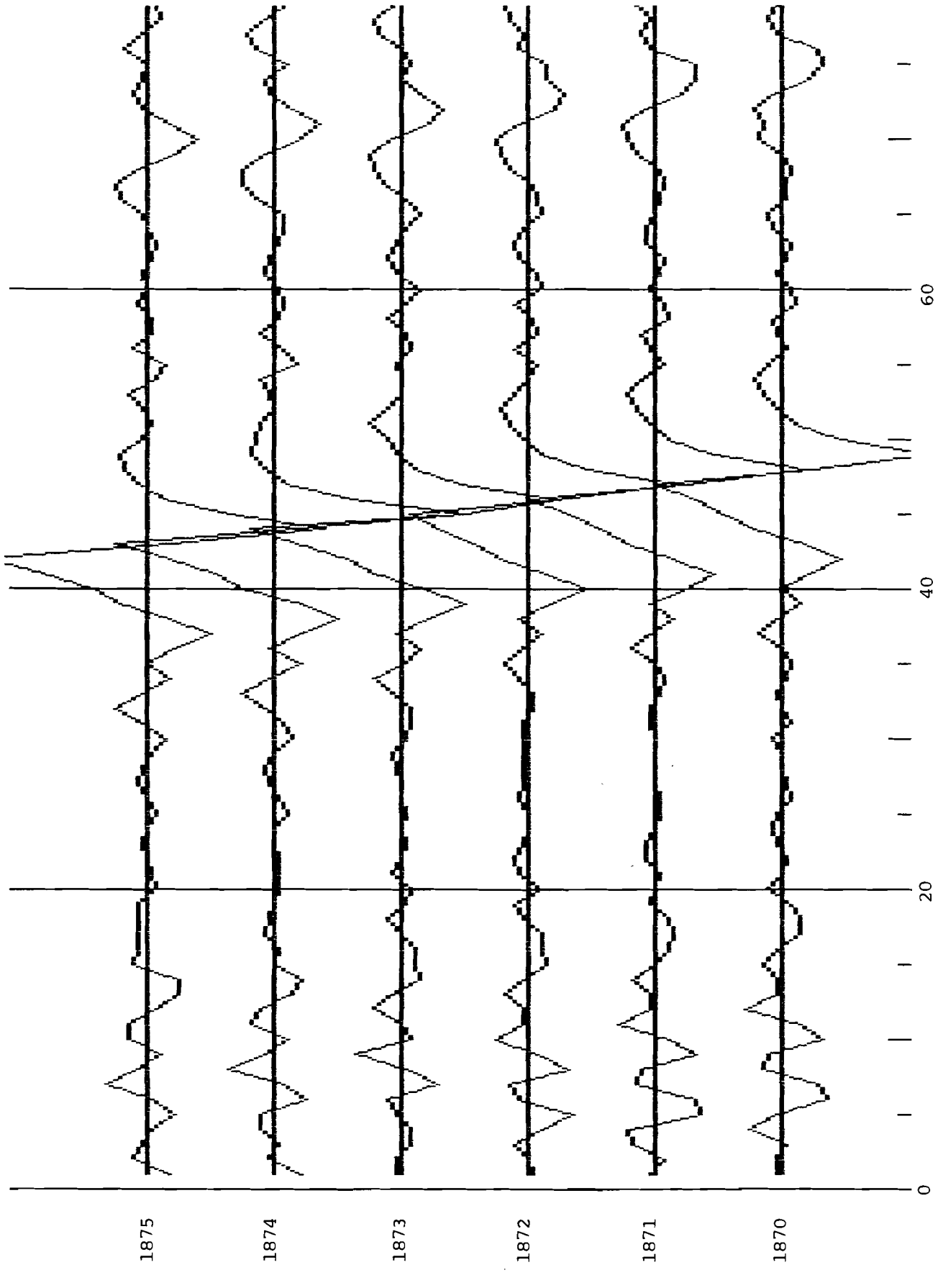

Figure 9: k_c-residuals, $k_c^o - \hat{k}_c$, by cohorts: 1870-75.

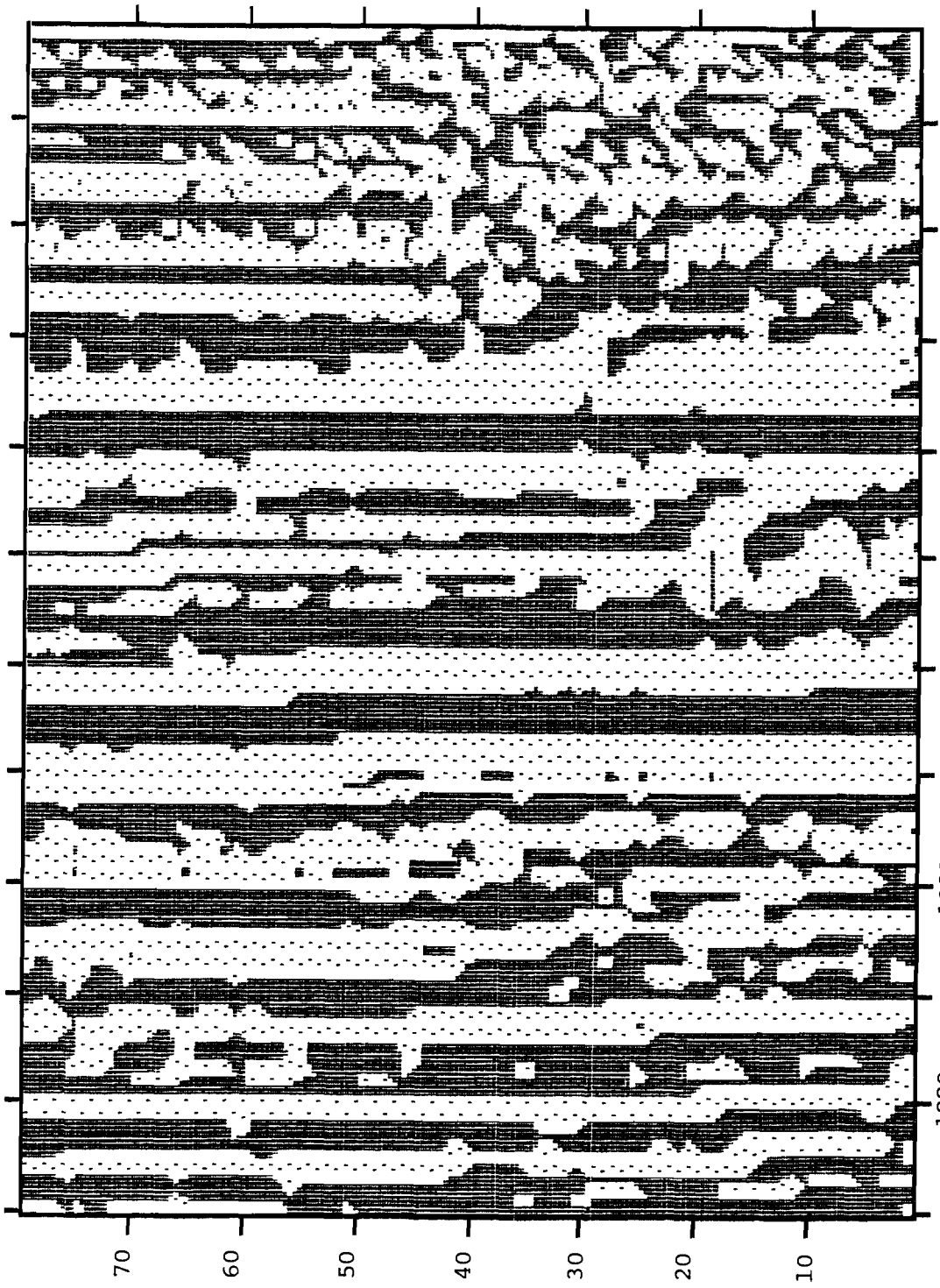

Figure 10: "Lexis" map of k_c-residuals (unfavorable periods are dark, favorable ones are light).

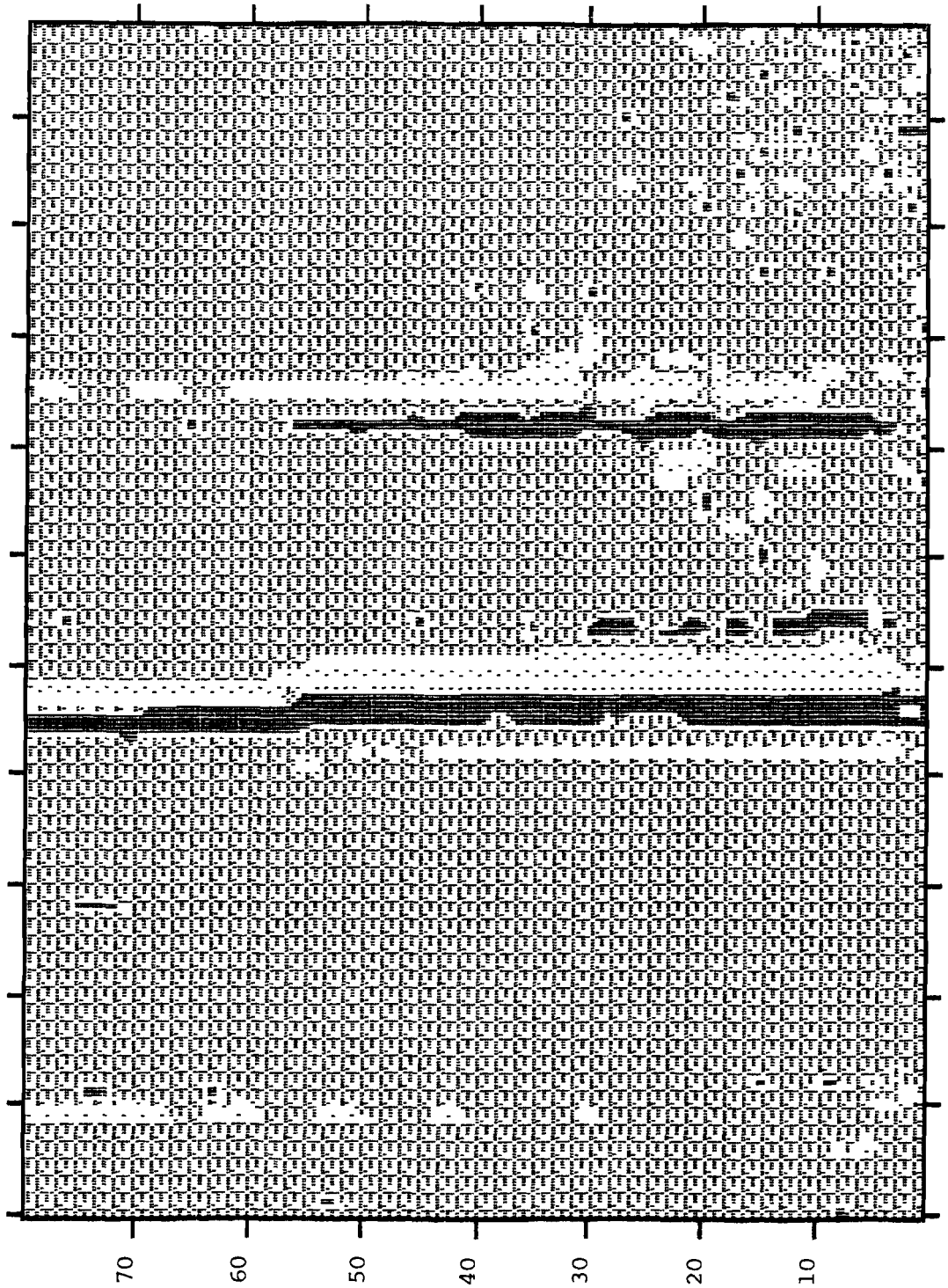

Figure 11: "Lexis" map of large k_c-residuals (> 0.1 or < -0.1)

Figure 12: $\hat{p}$ estimate: shows inadequacies of current averaging techniques.

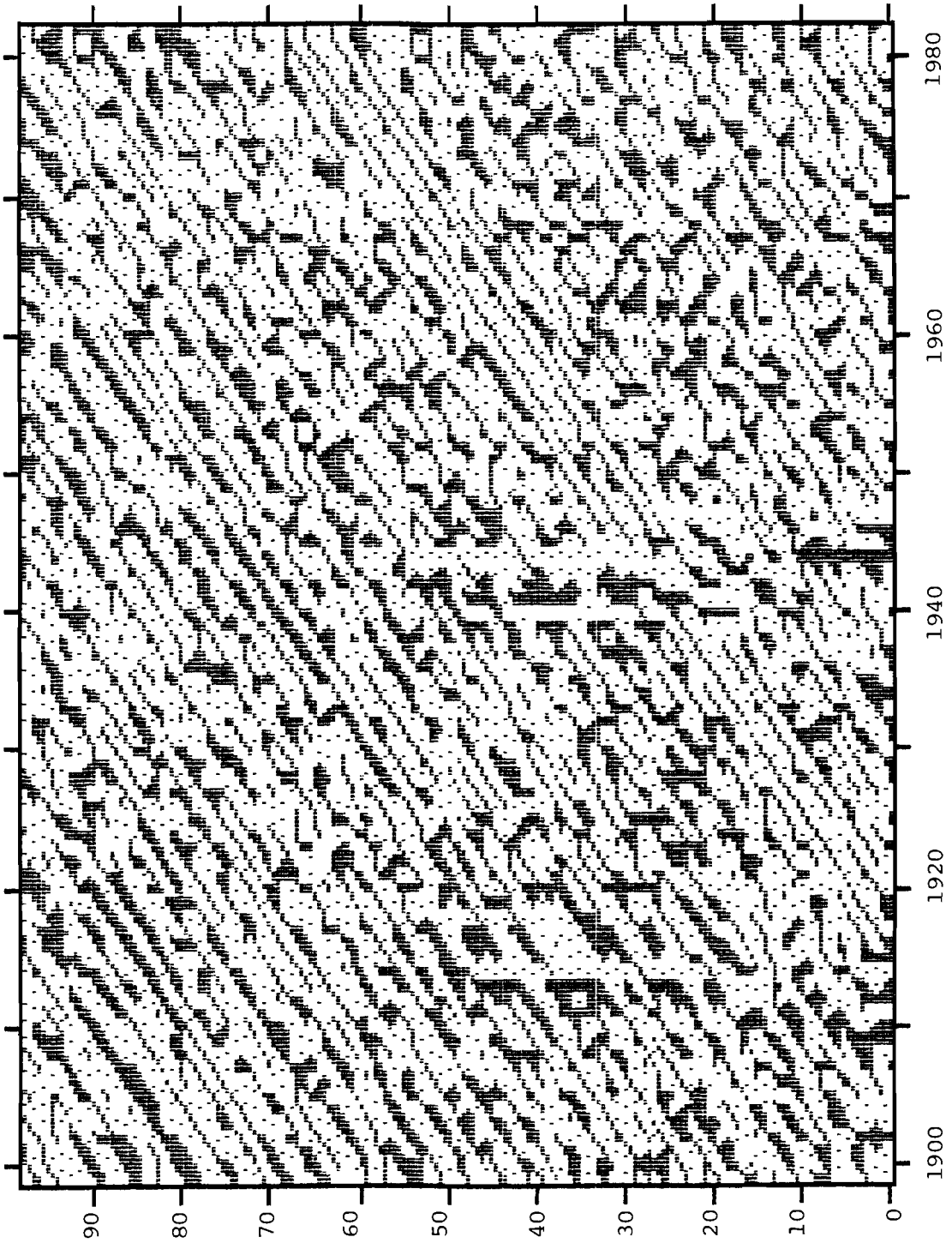

Figure 13: "Lexis" map of k_p-residuals for French males.

$$\rho^0 = k_p^o - k_c^o$$

$$= \hat{k}_p - \hat{k}_c + (k_p^o - \hat{k}_p) - (k_c^o - \hat{k}_c)$$

$$= \hat{\rho} + (k_p^o - \hat{k}_p) - (k_c^o - \hat{k}_c)$$

$$= \text{health progress} + \text{cohort effects} + \text{period effects} \; . \qquad (10)$$

6. DISCUSSION

The technique, as it now stands, appears to be very useful in discerning consistent mortality differences between adjacent birth cohorts and periods of time. Several modifications suggest themselves which might extend the usefulness of the technique.

By considering single-year cohorts, we are limiting ourselves to cohort differences which must come from birth. For instance, it may be reasonable to believe that children who are born in a particularly unfavorable year could be so heavily affected by the lack of proper nutrition, poor living conditions, etc., while still in the womb or during the first months of life, that they could be disadvantaged through a long period compared to the cohort which was already a year old at the time of the unfavorable conditions. That is to say, the difference in susceptibility between age 0 and age 1 might truly be great enough that cohort differences could be observed.

On the other hand, we might be interested in whether cohort differentiation can occur later in life as well. It seems unreasonable, however, to believe that someone age 19 could somehow be more susceptible to adverse conditions than someone age 20 (an exception might be the case of a one-year war where only those 20 and over were involved in combat, but the distinction would surely never be so clear). It does seem possible, though, that one five- or ten-year age-group might be more susceptible. Especially in the case of war, the age-groups who are most heavily involved in fighting may subsequently be disadvantaged compared with adjacent age-groups, who may have avoided active combat. Horiuchi (1983), in a similar analysis, finds that it is the 5-year cohort of men who were around age 15 at the end of the two World Wars (in the affected countries) who are the most disadvantaged later in life, although these men were too young to have actually fought in the wars.

Some work should also be done to facilitate comparison of birth cohorts born more than one year apart. We might want to know, for instance, which was the more disadvantaged cohort, 1917 or 1919? We need a means of measuring cumulative effects for cohorts. A simple average of the k_p-residuals for cohorts could be made, and then a running total could be kept so that all cohorts could be compared on the same level. Even more instructive would be averages over different age ranges; for example, before age 45 and after, in order to observe whether cohort variations change with age.

Other possible extensions of the model would include application to other kinds of demographic data. The prime candidate would seem to be marriage rates; or, if the proper data were available, perhaps parity-specific fertility rates. The essential requirement is that the age-structure of the event change only slowly over time. We recall that this is what insures the stability of the k_p and k_c curves.

One necessary improvement which has already been noted is the question of averages. There may exist more robust methods (running medians, for example) of estimating $\hat{k}_p$ and $\hat{k}_c$. The essential requirement is that $\hat{k}_p$ and $\hat{k}_c$ should show absolutely no signs of cohort or period variation. As noted, improving these two estimates will automatically improve the estimate, $\hat{\rho}$.

If we could then properly estimate $\hat{\rho}$, this would open up a whole new field of research. As already observed, the crucial question in the traditional age-period-cohort identification problem is that of how much of the long-term trend in mortality should be attributed to periods and how much to cohorts. An accurate estimate of this long-term trend would be a useful starting point in any analysis.

Another extension of the technique which is called for is a means of estimating the statistical significance of the cohort and period variations found. Very little work has been done in this direction, but even a simple approach can be quite instructive. For example, is it significant that one cohort is advantaged compared with another over a 10-year interval? Of course, if two cohorts are in an exactly equal relationship, the probability in any one period that one particular cohort would appear advantaged over the other one, due to mere stochastic variation, is one-half. The probability that the same cohort would appear advantaged over 10 successive periods, under the hypothesis of equality, is

$$\frac{1}{2^{10}} = \frac{1}{1024} < 0.001.$$

If we observe such a string of positive residuals, then, we may be fairly certain

that they are not due to random fluctuations in mortality, but rather that they represent real differences (at least within the data!) between cohorts.

This last parenthetical comment reflects the continuing skepticism of the author concerning the possibility of systematic data errors which may have, in some unknown way, created the observed results. In the Italian data, especially for the cohorts from 1862-1925 and for the periods before 1957, the results seem too regular and show little evidence of stochastic variation. The plots of the k_p-residuals for the cohorts, 1870-75, were more regular than could easily be believed. Some of this may be due to the slight amount of smoothing that was applied to the cohort data, but this could not explain the high degree of regularity observed. It also seems rather mysterious that 1957 should mark both the end of the 5-year cyclical pattern and the beginning of a more random scheme of mortality (but one which shows, nevertheless, a tendency toward cohort- and period-specific mortality variations). As noted, 1957 corresponds, neither to a change in demographic technique, nor to a national census.

In support of the method is the fact that it has also been successfully applied to French mortality data for the periods, 1899-1982, and none of the same inconsistencies have been noted. The results appear more random (in many ways similar to the later Italian data), but they still show the usefulness of the technique for observing cohort and period variations. The graphs for French males which are comparable to Figures 6 and 10 are shown in Figures 13 and 14. When we examine the k_p-residuals in the cohort direction, the curves are not smooth, as in Figure 8, but more jagged, as we might expect. Nevertheless, many cohorts show a clear tendency to be either advantaged or disadvantaged, as in the Italian data.

Furthermore, in support of the Italian data itself, it seems impossible that the mortality for a cohort could be systematically over- or underestimated for the life of the cohort. First, it seems unreasonable that, in any one census, every other age group would be overcounted while the others would be undercounted. Secondly, censuses were conducted approximately every ten years (in 1881, 1891, 1901, 1911, 1921, 1931, 1951, 1961, 1971, and 1981), so any systematic under- or overcount could not affect the denominator in the q_x^0 rates for more than about 10 years. Lastly, beginning in 1928, all deaths were reported by single-year-of-age, so there is little possibility that it could be a problem with the numerators.

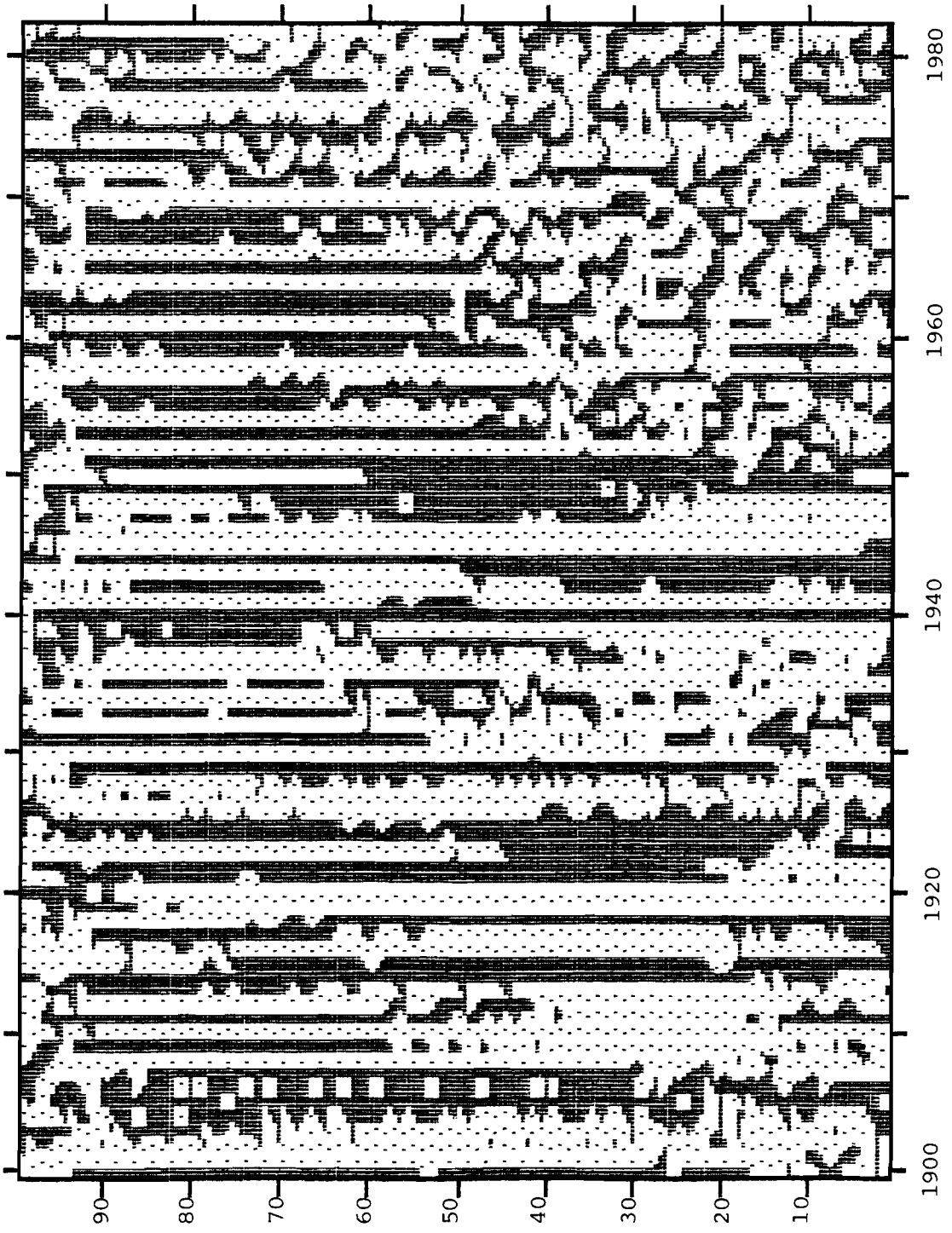

Figure 14: "Lexis" map of k_c-residuals for French males.

7. CONCLUSIONS

It seems, then, that analyzing historical mortality data by single-year-of-age and single-year-of-time may prove to be a useful technique. There appear to be cohort and period variations in mortality which often operate on a time-scale of one or two years, so this finer analysis seems to be required.

The technique of examining the derivatives on the mortality surface, $\log\mu$, proves to be a very powerful one, both for discovering data inconsistencies and for analyzing the data itself. It is an excellent means of discerning cohort and period variations in mortality and may eventually be a successful means of decomposing observed mortality trends into three components: the long-term, underlying trend, and the cohort and period deviations from that trend.

More study is needed on the finer points of applying the technique, on extension of the technique to allow greater cohort and period comparison, on decomposition of the long-term trend of mortality at various ages, on application of the technique to other types of demographic data, and, finally, on the nature of the data collection itself to insure against data-related errors of analysis.

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