



Resources, Trade, and Debt

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FOREWORD

This Collaborative Paper is one of a series embodying the outcome of a workshop and conference on *Economic Structural Change: Analytical Issues*, held at IIASA in July and August of 1983. The conference and workshop formed part of the continuing IIASA program on Patterns of Economic Structural Change and Industrial Adjustment.

Structural change was interpreted very broadly: the topics covered included the nature and causes of changes in different sectors of the world economy, the relationship between international markets and national economies, and issues of organization and incentives in large economic systems.

There is a general consensus that important economic structural changes are occurring in the world economy. There are, however, several alternative approaches to measuring these changes, to modeling the process, and to devising appropriate responses in terms of policy measures and institutional redesign. Other interesting questions concern the role of the international economic system in transmitting such changes, and the merits of alternative modes of economic organization in responding to structural change. All of these issues were addressed by participants in the workshop and conference, and will be the focus of the continuation of the research program's work.

Geoffrey Heal
Anatoli Smyshlyaev
Ernö Zalai



ABSTRACT

The paper studies a two-region economy that has two sectors and three factors of production: oil, capital, and labor. The South exports oil in exchange for industrial goods from the North. There is a net capital inflow to the South. This equals the difference between its export revenues and import costs, and represents the South's indebtedness. This overseas borrowing finances the development of the oil sector: increased borrowing leads to higher oil supplies, to new levels of consumption and a new distribution of income in the South, and to new levels of industrial exports from the North. The paper studies the macro impacts of changes in the values of the debt on both the borrowing and the lending regions.



RESOURCES, TRADE, AND DEBT*

Graciela Chichilnisky**, Geoffrey Heal**, and D. McLeod***

1. INTRODUCTION

A great deal of attention has been given recently to the debt problems of developing countries, most notably Argentina, Brazil, Ecuador, and Mexico. Their debts currently total about 300 billion US dollars, of which Mexico's share is about one third. Ecuador and Mexico are particularly interesting cases because their current difficulties follow a period of concentration on oil exports, an activity which was widely recommended, and which it was generally thought would improve rather than worsen their balance-of-payments conditions.

Experience has not fulfilled these expectations. It is now clear that the relationship between resource export policies and debt is rather complex, and poses a challenge to the economist. In the case of Mexico, it is generally accepted that much of the borrowing was used to finance the development of its oil export sector. Sterner (1982) shows that about 30% of Mexico's outstanding debt was used to finance investment in PEMEX, the national oil company. It appears therefore that there exists a link between borrowing and oil exports, and the macroeconomic impacts of borrowing and of resource exports must be jointly analyzed and balanced against each other. It is the purpose of this paper to explore these links.

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A first concern is the impact of increasing oil exports on the exporting country. This was studied in Chichilnisky (1981a) within a two-region, two-good, and three-factor general equilibrium model. It was shown there that increasing oil exports may have either a positive or a negative impact on the terms of trade, on domestic consumption, and on the distribution of income of the oil exporter, depending on the structure of the economy. The degree of dualism in production and the initial levels of wages and profits played a crucial role in determining the outcome.

In this paper we extend Chichilnisky's model to study the impact of debt on the resource-exporting economy. The model is extended to allow for an imbalance in the trade account, which is matched by an inflow of overseas investment or a financial transfer. This imbalance represents the debt owed to foreigners, and is directed towards the expansion of oil supplies. Except for the wedge between export revenues and import costs, which represents the debt, the model is consistent with a standard competitive general equilibrium specification. Prices of all goods and factors in the two regions are free to adjust to market conditions.

The introduction of the debt wedge changes the main relations in the model: the operation of Walras' Law or the national income identity in both countries is altered. Overseas investments lead to changes in oil supplies and consequently most variables adjust. As the debt increases, a new equilibrium emerges with different prices and levels of imports and exports. There are also changes in all domestic variables in both South and North: real wages, profits, domestic use of industrial and consumption goods, and employment of the factors labor, capital, and oil. This allows us to trace the impact of the debt on the major macro variables of the two countries. The model could also be used to examine the impact of rescheduling, i.e. repaying the debt over a different time period, or of repaying it at a different rate of interest.

Following the macroeconomic impact analysis, two main questions emerge: the first is, who benefits and who loses from the accumulation of debt; and the second is, whether there exist debt-management policies that could make both countries better off, after taking fully into account the recycling effect of borrowing funds on imports from the lender. For example, at present 25-30% of all machine-tool exports of the US are purchased by Mexico, and a similar proportion of all US exports are purchased by Latin America.

The interest of the results lies in part in their simplicity and in part in the fact that they account fully for the impact of the debt on all markets simultaneously. Fairly simple analytical solutions are obtained to the rather complex questions posed. This is of course at the cost of somewhat stylized assumptions.

We describe conditions under which increasing the debt leads the country to export more oil. In certain cases, this leads to lower prices of oil, lower volumes of industrial imports, lower real wages, and higher profits in the oil-exporting country. In other cases, the results are reversed, and real wages, consumption, and terms of trade all improve in the exporting country. The outcome depends on the technologies of the South and on the initial prices.

We also examine conditions under which the economy of the North actually benefits in macroeconomic terms from its loan to the South: because of lower oil prices, the consumption of both goods increases in the North when the transfer or loan increases. This occurs mainly because the transfer leads to an improvement in the terms of trade in the North, and because its production system is integrated and efficient. This result is reminiscent of the argument that British investment overseas in the nineteenth century benefited the

country by developing overseas supplies of food and raw material, thus making these supplies more elastic, keeping down prices, and improving the UK's terms of trade. Essentially we are specifying here conditions for overseas investment in material supplies to benefit the investing country even before any financial returns are paid, or in the case of a loan, before the loan is repaid.

The rest of the paper is organized as follows. To provide some empirical background, we begin by reviewing the case of Mexico. We then present the North-South model with debt, after which we prove the main theorems. The conclusions summarize the results, and an appendix shows that, although the model contains 33 independent equations, its comparative static properties can be understood by studying a single implicit functional relationship between one endogenous variable (the terms of trade of oil for industrial goods) and one exogenous parameter (the value of the debt).

2. EMPIRICAL BACKGROUND: THE CASE OF MEXICO

In this section we review briefly the empirical material relating to a number of the issues to be discussed below. The focus is on the case of Mexico, which is an important exemplar of the phenomena under examination.

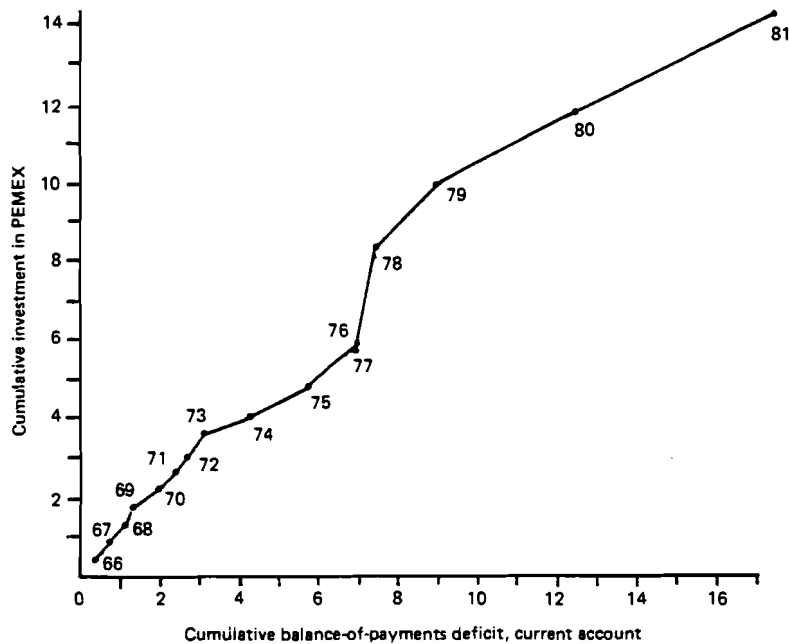


FIGURE 1 The relationship between Mexico's cumulative balance-of-payments deficit and investment in PEMEX, 1966-1981. (All figures are in billions of 1970 US dollars.)

In the Introduction we mentioned that the accumulation of Mexican debt is generally believed to have been associated with investment in PEMEX. Figure 1 presents data on this association. Mexico's cumulative balance-of-payments deficit on current account is measured horizontally. The vertical axis represents cumulative investment in PEMEX. All figures are in billions of 1970 US dollars, and data sources are given after the tables below. It is clear from Figure 1 that there is an almost one-to-one association between the cumulative payments deficit and investments in PEMEX: on average, the cumulative deficit slightly exceeds investment in PEMEX, but the two move very closely indeed. This is confirmed by the regression in Table 1. It therefore seems justifiable to claim that investment in PEMEX was financed by the payments deficit, and indeed this provides the empirical justification for an important assumption in the model that follows.

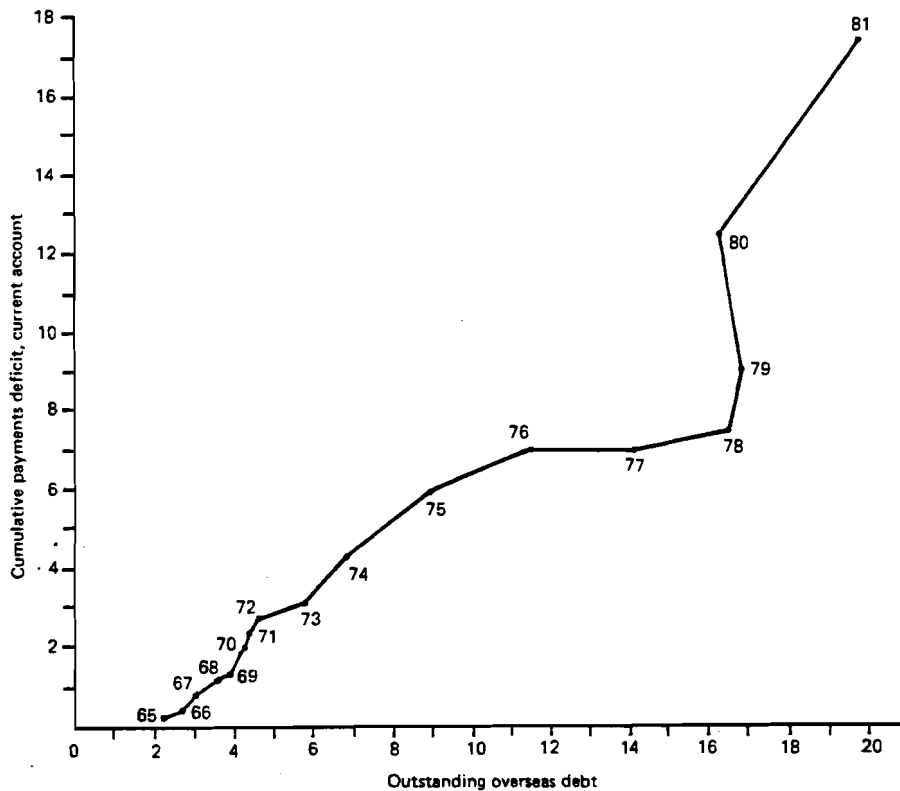


FIGURE 2 The relationship between Mexico's outstanding overseas debt and her cumulative payments deficit, 1965-1981. (All figures are in billions of 1970 US dollars.)

What is the relationship between Mexico's cumulative current account payments deficit, and her outstanding foreign debt? Figure 2 addresses this issue. Except for the period 1976-1979, these variables moved together, with the debt consistently some US\$2-3 billion in excess of the cumulative deficit. (Figures are again in billions of 1970 US dollars.) This interpretation of the graph is supported by the regression in Table 2, and is consistent with the fact that there was substantial private overseas borrowing by Mexican citizens which was then used for the acquisition of overseas assets and which added to the accumulation of overseas debt. In the model which follows, this borrowing to

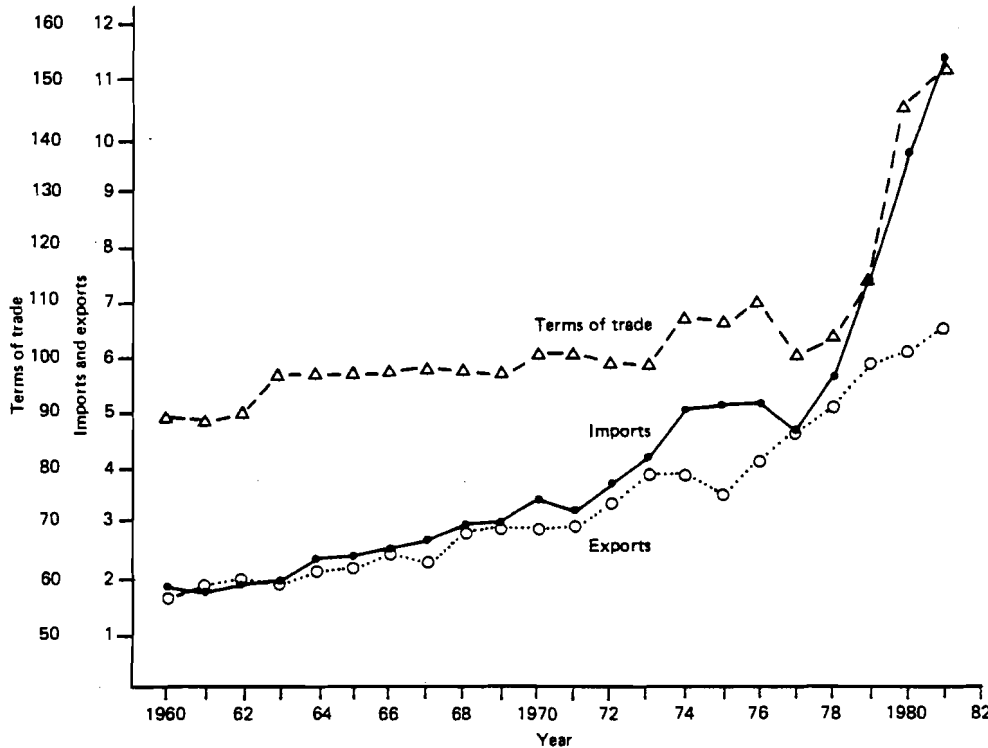


FIGURE 3 Mexican imports, exports, and terms of trade, 1960-1982. (Exports and imports are in billions of 1970 US dollars.)

acquire overseas assets is neglected: it is assumed that indebtedness is equal to the cumulative balance-of-payments deficit, and is used entirely to finance investment in the oil sector. Obviously, this is a good approximation to the data for Mexico; furthermore, it seems likely that borrowing to finance the private acquisition of overseas assets had little macroeconomic impact within Mexico. The important macroeconomic changes were driven by investment in the oil sector, and by the consequent changes in oil output and oil exports. In any case, we shall argue below that when the overseas investment by Mexicans is taken into account, the results are likely to be reinforced.

From Figure 1 it is clear that Mexico's cumulative deficit has risen over time. Figure 3 gives more insight into this: it shows the movements of imports, exports and the terms of trade (import prices relative to export prices) over the period 1960-1982. Exports rose steadily over the period. Imports moved roughly in parallel until 1973 when, with the exception of 1977, they moved ahead dramatically. As the terms of trade moved very similarly to imports in the last ten years, it seems that the rise in imports reflected an increase in prices rather than in volume of imports. The regression in Table 3 documents this point. So as Mexico borrowed and entered the world oil market as a seller, the terms of trade moved sharply against it, leading to a rapid deterioration in the balance of payments. The theory of the next section suggests that this was certainly not a coincidence, but was indeed to be expected.

TABLE 1 OLS regression of investment in PEMEX, *PMXI*, on the cumulative current-account deficit, *CCA*.

$$PMXI = 0.598 + 0.844CCA$$

(1.96) (19.36)

t-statistics in parentheses. $R^2 = 0.964$.

TABLE 2 OLS regression of cumulative current account deficit, *CCA*, on outstanding overseas debt, *D*.

$$CCA = -1.46 + 0.753D$$

(-2.03) (10.76)

t-statistics in parentheses. $R^2 = 0.885$.

TABLE 3 OLS regression of total imports, *IMP*, against the terms of trade, *TT*.

$$IMP = -11.65 + 0.153TT$$

(-10.78) (14.85)

t-statistics in parentheses. $R^2 = 0.916$.

Data Sources:

All regressions cover the period 1965-1981.

PMXI Statistics on the Mexican Economy, NAFINSA, 1981.

CCA World Tables, 1981, World Bank.

D 1965-1972 International Financial Statistics Yearbook 1982.
1972-1981 Francisco Carrada-Bravo, "The Dynamics of Foreign Debt and Energy Policy: The Case of Mexico." Mimeo, Department of Economics, University of California at Los Angeles.

IMP World Tables, 1981, World Bank.

TT World Tables, 1981, World Bank.

3. THE NORTH-SOUTH MODEL WITH DEBT

In this section we present the model, which is an extension of that of Chichilnisky (1981a). There are two regions, the North and the South. Each produces two goods, denoted B and I , with three factors of production, capital K , labor L , and oil ϑ . The South exports an input, oil, in exchange for a good, the "industrial" good I . The "basic" good B is not traded internationally.

We first specify the model for one region, namely the South. In what follows, the subscripts S and D will be used to denote supply and demand, and the superscripts N and S to denote variables or parameters referring to the North and South, respectively. All variables or parameters without a superscript refer to the South. The superscripts B and I after a factor (e.g. L^B , K^I) denote the amount of that factor used in sector B or I , respectively.

The basic good is produced according to the relation

$$B_S = \min \left[L^B / a_1, \vartheta^B / b_1, K^B / c_1 \right] \quad (1)$$

and the industrial good according to

$$I_S = \min \left[L^I / a_2, \vartheta^I / b_2, K^I / c_2 \right] \quad (2)$$

Labor and capital supplies are responsive to their rewards:

$$L_S = \alpha w / p_B, \quad \alpha > 0 \quad (3)$$

where w is the wage and p_B the price of B , and

$$K_S = \beta \tau, \quad \beta > 0 \quad (4)$$

where τ is the rate of profit. p_I and p_ϑ will stand for the prices of industrial goods and of oil, respectively. The demand for B derives from wage income

$$p_B B_D = wL \quad (5)$$

The South produces oil (within given bounds), without using either domestic capital or labor. We shall assume that it uses the overseas borrowing or financial transfer FT to increase its oil supplies

$$\vartheta_S = \vartheta_S(FT), \quad \partial \vartheta_S / \partial FT > 0 \quad (6)$$

This completes the behavioral specification for the South.

The equilibrium conditions for the South are:

$$B_S = B_D \quad (7)$$

where B is not traded internationally,

$$I_D = I_S + M_I^S \quad (8)$$

where M_I^S denotes the South's imports of I ,

$$\vartheta_S^S = \vartheta_D + X_\vartheta^S \quad (9)$$

where X_{ϕ}^S denotes oil exports by the South,

$$K_S = K_D \quad (10)$$

$$L_S = L_D \quad (11)$$

$$L_D = B_S a_1 + I_S a_2 \quad (12)$$

$$K_D = B_S c_1 + I_S c_2 \quad (13)$$

$$\vartheta_D = B_S b_1 + I_S b_2 \quad (14)$$

and the payments condition

$$p_{\phi} X_{\phi}^S = p_I M_I^S - FT \quad (15)$$

Note that FT could be either positive or negative, depending on the relative magnitudes of the debt service and the financial credit. However, as will be seen below, the effect of a transfer (FT positive) is not symmetric with that of a repayment (FT negative), because of the irreversibility of the investment in the oil sector. We assume that the entire financial transfer FT is used to purchase industrial goods to augment the supply of oil. This means that the new industrial investment in the oil sector is paid for by foreign loans. Hence, oil supplies ϑ_S change as the debt level changes; the debt is assumed to increase with increases in the level of the transfer (FT positive), but obviously, it does not decrease when FT is negative, since the debt is not paid by selling the oil production equipment. The balance-of-payments condition (15) is that imports of industrial goods exceed export revenues by FT . As the demand for the basic good B comes entirely from wage income (eqn. 5), the national income identity ((16) below) implies that the demand for industrial goods comes from the profit income τK , oil revenues $p_{\phi} X_{\phi}^S$, and the borrowing FT , with the last of these going to the oil sector. In the North we make a corresponding assumption, namely that the financial transfer to the South is taken from income that would otherwise have purchased industrial goods, so that the North's demand for industrial goods is $\tau K - FT$.

In an equilibrium situation, Walras' Law or the national income identity of the South is always satisfied (see e.g. Chichilnisky 1981a), i.e.

$$p_B B_D + p_I I_D = \omega L + \tau K + p_{\phi} \vartheta + FT \quad (16)$$

where $\vartheta = \vartheta_S$ is, as in (6), a function of FT . Equation (16) can also be rewritten as

$$p_B B_S + p_I (I_S + M_I^S) = \omega L + \tau K + p_{\phi} (\vartheta_D + X_{\phi}^S) + NF \quad (16)'$$

The model of the North consists of the same 15 equations, but with possibly different parameters α , β , a_1 , a_2 , b_1 , b_2 , c_1 , c_2 . The following equation now substitutes for the original eqn. (6):

$$\vartheta_S = 0 \quad (6)'$$

and, of course, the equations corresponding to (8) and (9) reflect the fact that the North imports oil and exports industrial goods. In a world trade equilibrium the prices of the traded goods must be equal:

$$p_{\phi}^S = p_{\phi}^N \quad (17)$$

$$p_I^S = p_I^N \quad (18)$$

and traded quantities must also match:

$$X_{\mathfrak{G}}^S = M_{\mathfrak{G}}^N \quad (19)$$

$$X_I^N = M_I^S \quad (20)$$

where X_I^N and $M_{\mathfrak{G}}^N$ represent, respectively, the North's exports of I and imports of oil. There are therefore two sets of eight exogenous parameters each, one set for the North and the other for the South. Each set contains $\alpha, \beta, a_1, a_2, b_1, b_2, c_1,$ and c_2 . These parameters are generally different in the two regions. We shall make certain stylized assumptions to simplify computations: α is large in the South and relatively smaller in the North, indicating that labor is more "abundant" in the South. The corresponding parameter for capital exhibits the opposite behavior: β is larger in the North than in the South. We shall also assume that c_1 is small in the South, i.e. the production of basic goods uses little capital, and a_2 is small in the North, i.e. Northern industry uses little labor. There are a total of 33 independent equations for the complete North-South system: thirty correspond to two sets of (1) through (15), one set for each region, and three equations arise from the international trade conditions (17) through (20), since of these four, as usual, only three are linearly independent. There are 17 endogenously determined variables each in the North and in the South: $p_I, p_{\mathfrak{G}}, p_B, w, \tau, L_S, L_D, K_S, K_D, B_S, B_D, I_S, I_D, M_I^S, \vartheta_S, \vartheta_D,$ and $X_{\mathfrak{G}}^S$. Finally, we have the transfer FT , making a total of 35 endogenous variables for the complete North-South system. We therefore have 33 equations in 35 unknowns. When we choose the numeraire ($p_{\mathfrak{G}} = 1$) an equilibrium is determined up to one variable. If we fix exogenously one variable, the equilibrium is (locally) unique. We choose this variable to be the value of the transfer FT . The transfer or loan thus becomes a policy variable. In the Appendix we show how to compute explicitly a solution to the model, i.e. a value for each of the endogenous variables, for each policy sector FT . In particular, we show that by successive substitutions the more important properties of the model can be obtained from the study of a single equation, giving an implicit relationship between the financial transfer FT and the price of industrial goods relative to oil.

There are a number of determinants whose signs are important in the following sections, which determine factor intensities in the different sectors. In total we have the following technical input-output coefficients:

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix}$$

in each region. The determinants to be used are:

$$D = a_1c_2 - a_2c_1 \quad M = c_1b_2 - b_1c_2 \quad Q = a_2b_1 - a_1b_2$$

The assumptions are:

$$D^N > 0, \quad D^S > 0, \quad M^S < 0, \quad Q^N < 0$$

The positivity of the determinant D implies that the basic goods sector is relatively more labor intensive and the industrial goods sector relatively more capital intensive. The assumption (made above) that the basic goods sector uses very little capital in the South implies that c_1^S is small and therefore that $M^S < 0$. The industrial goods sector in the North was assumed to use little labor: hence a_2^N is small and $Q^N < 0$. The above assumptions on the signs of

the various determinants are maintained at all points below unless there is an explicit statement to the contrary.

4. MAIN RESULTS: TRADE AND DEBT

This section studies the impact of a change in the net transfer FT on the economies of the North and the South. Before going on to the algebra, it seems useful to explain the economics of this impact.

An increase in the transfer FT increases oil supplies ϑ_S , since the South invests borrowed funds in expanding the oil sector. At the new equilibrium, corresponding to higher FT , the total amount of oil utilized in the North and in the South therefore increases. This in turn alters the supplies of both goods in each region, possibly in different proportions. The composition of the product changes in both regions.

The changes in supplies lead to new equilibrium prices for the two goods. The prices of the factors labor and capital also change as relatively more or less labor and capital are employed. This implies that total income in the North and in the South are different at the new equilibrium. The results in this section give simple sufficient conditions for determining the signs of each of these effects.

The first theorem gives conditions under which an increase in oil supplies decreases the price of oil with respect to that of the industrial good. While it is intuitively plausible that the price of oil should drop as supplies increase, this is not always true. The second theorem gives conditions under which the relative price of oil *increases* as the transfer increases oil supplies. Whether one or the other result obtains, depends on the relative strength of supply and demand effects, and the general equilibrium solutions trace this in detail. The results are obtained from various assumptions on technologies and initial prices.

The next step is to explore the general equilibrium impacts of an increase in the relative price of industrial goods. The rate of profit rises both in the North and in the South. In the North, the rate of profit and the real wage move together, because the North's economy is rather homogeneous. Therefore, both wage and profit income increase in the North, and we show that there is also an increase in the consumption of both goods, even allowing for the loss of national income due to the transfer. All this occurs because the transfer has improved significantly the North's terms of trade.

In the South, because of the rather different technologies in the two sectors, the real wage moves in the opposite direction to the rate of profit. The transfer increases oil supplies and oil exports, but oil revenues in terms of industrial goods imported are reduced. Wage income and domestic consumption of basics decrease as well. If one sought to improve wage income without negatively affecting industrial consumption in the South, the economy of the South would have to be made more homogeneous.

The second theorem explores a different set of assumptions, and arrives at rather different conclusions. Now the transfer increases oil supplies, but it also increases the relative price of oil with respect to industrial goods. As the terms of trade of the South improve, its macro variables react differently, and so do the variables in the North. The conditions under which one or the other result obtains are therefore quite relevant for policy, and should be determined empirically. The simulations in the next section are a first move in this direction.

A factor that plays an important role in determining the results of an increase in the transfer FT is the sign of the expression

$$\Delta = \left[c_2 / D - 2w / p_B \right] \alpha$$

where D is the determinant of the matrix

$$\begin{pmatrix} a_1 & c_1 \\ a_2 & c_2 \end{pmatrix}$$

The role and interpretation of this term have been discussed elsewhere (Chichilnisky 1981a,b). Basically, the sign of this expression determines whether income effects will dominate price effects, so that increases in supplies will be proportionately larger or smaller than increases in demand as prices change. We refer to an economy as *dual* if $c_2 / D < 2w / p_B$, since a large D would have this interpretation. Conversely, the economy is *homogeneous* if $c_2 / D > 2w / p_B$. It should be noted that this condition can be written so as to be independent of the particular units of measurement used.

Theorem 1. *Consider a North-South economy as defined above. Assume the economy of the North to be homogeneous ($c_2 / D > 2w / p_B$) and that of the South to be dual ($c_2 / D < 2w / p_B$). Suppose that at the initial equilibrium the price of industrial goods and the rate of profit are relatively high in the North ($p_I > b_2$ and $2r > a_1 / D$). Labor is relatively abundant in the South (α large) and capital relatively abundant in the North (β large). In this case an increase in the transfer FT to the South has the following consequences:*

- (i) *Oil supplies and oil exports increase in the South.*
- (ii) *The North exports, and the South imports, fewer industrial goods. However, the terms of trade move in favor of the North (p_I increases) so much that its export revenues rise. There is a corresponding fall in oil export revenues of the South denominated in terms of its import I .*
- (iii) *Profits and real wages rise in the North, so much that its consumption of both goods increases.*
- (iv) *In the South, profits rise, but employment, real wages, and consumption of basics all fall.*

Proof. We consider first the market-clearing condition in the oil market:

$$X_D^S = M_D^N \quad (21)$$

From (6), (9), and (6)', this equals:

$$\vartheta_D^S(FT) - \vartheta_D^S = \vartheta_D^N \quad (22)$$

From (14),

$$\vartheta_D = b_1 B_S + b_2 I_S \quad (23)$$

and from inverting (12) and (13) we obtain:

$$\vartheta_D = \frac{b_1}{D}(c_2 L - a_2 K) + \frac{b_2}{D}(a_1 K - c_1 L) \quad (24)$$

In view of (3) and (4), we may rewrite (22):

$$\vartheta_S(FT) + \frac{\alpha}{D} \frac{w}{p_B} M + \frac{\beta\tau}{D} Q = - \frac{\alpha^N}{D^N} \left(\frac{w}{p_B} \right) M^N - \frac{\beta^N \tau^N}{D^N} Q \quad (25)$$

where M and Q are the determinants defined above. Equation (25) gives an implicit relation between real wages and the rates of profits in both regions, and the transfer FT , which we denote as

$$\varphi \left[\tau^N, \tau^S, (w/p_B)^N, (w/p_B)^S, FT \right] = 0 \quad (26)$$

Since factor prices are functions of commodity prices (see Appendix eqn. A.7), we obtain from substitution of (A.7) into (25) a function linking the transfer FT to the prices of B and I :

$$\begin{aligned} \vartheta_S(FT) + \frac{\alpha^M}{D^2 p_B} (c_2 p_B^S - c_1 p_I + M) + \frac{\beta Q}{D^2} (p_I a_1 - p_B^S a_2 + Q) \\ + \frac{\alpha^N M^N}{(D^N)^2 p_B^N} (c_2^N p_B^N - c_1^N p_I + M^N) + \frac{\beta^N Q}{(D^N)^2} (p_I a_1^N - p_B a_2 + Q) = 0 \end{aligned} \quad (27)$$

Equation (27) is an implicit function of the form

$$\Gamma(FT, p_I, p_B^N, p_B^S) = 0$$

However, the prices of basics p_B^S and p_B^N (which may be different since basics are not traded) are themselves functions of the price of industrial goods p_I in equilibrium.

From the Appendix eqn (A.13) we obtain:

$$p_B^N = p_B^N(p_I) \quad \text{and} \quad p_B^S = p_B^S(p_I)$$

Therefore, eqn. (27) is actually an implicit function of p_I and FT *only*

$$\Gamma(FT, p_I) = \Gamma(FT, p_I, p_B^S(p_I), p_B^N(p_I)) = 0 \quad (28)$$

It is then possible to differentiate implicitly across equilibria and obtain $\partial p_I / \partial FT$, or equivalently its reciprocal

$$\frac{\partial FT}{\partial p_I} = - \left[\frac{\partial \Gamma}{\partial p_I} \right] / \left[\frac{\partial \Gamma}{\partial FT} \right] \quad (29)$$

This equation represents the change in the price of industrial goods that follows an increase in the transfer FT . By (27) and (6),

$$\frac{\partial \Gamma}{\partial FT} = \frac{\partial \vartheta_S}{\partial FT} > 0$$

Therefore the sign of (29) is always that of $-\partial \Gamma / \partial p_I$.

We may now compute the derivative $-\partial\Gamma/\partial p_I$. From (27) and (28) we obtain:

$$\begin{aligned}
 -\frac{\partial\Gamma}{\partial p_I} = & -\frac{\partial p_B^S}{\partial p_I} \left(-\frac{\alpha M}{D^2 p_B^2} (M - c_1 p_I) - \frac{a_2 \beta Q}{D^2} \right) \\
 & + \frac{\alpha c_1 M}{D^2 p_B} - \frac{\beta Q a_1}{D^2} \\
 & -\frac{\partial p_B^N}{\partial p_I} \left(-\frac{\alpha^N M^N}{(D^N)^2 (p_B^N)^2} (M^N - c_1^N p_I) - \frac{a_2^N \beta^N Q^N}{(D^N)^2} \right) \\
 & + \frac{\alpha^N c_1^N M^N}{(D^N)^2 p_B^N} - \frac{\beta^N Q^N a_1^N}{(D^N)^2}
 \end{aligned} \tag{30}$$

From expression (30) we may compute the changes in p_I as FT changes, provided we know the signs of the derivatives $\partial p_B^S/\partial p_I$ and $\partial p_B^N/\partial p_I$ across equilibria.

The next step is therefore to compute the signs of the derivatives of the price of basic goods with respect to the price of industrial goods across equilibria in each region. For this we utilize the expression relating the real wage and the rate of profit in each region, derived from the market clearing condition $B_S - B_D = 0$:

$$\frac{\alpha c_2}{D} \frac{w}{p_B} - \frac{\beta a_2 \tau}{D} - \alpha \left(\frac{w}{p_B} \right)^2 = 0 \tag{31}$$

(see Appendix eqn. (A.11)), and also the equations relating factor prices to commodity prices:

$$\tau = \frac{p_I a_1 - p_B a_2 + Q}{D} \tag{32}$$

$$\frac{w}{p_B} = \frac{p_B c_2 - p_I c_1 + M}{D p_B} \tag{33}$$

(see Appendix eqn. (A.7)). Equation (31) is an implicit expression between real wages and profits in each region, denoted $\Lambda(w/p_B, \tau) = 0$. Since eqns. (31) and (32) give real wages and profits as functions of commodity prices, (31) actually gives an implicit relation between commodity prices in each region, denoted

$$\psi(p_B, p_I) = \Lambda \left[\frac{w}{p_B}(p_B, p_I), \tau(p_B, p_I) \right] = 0 \tag{34}$$

From (34), by the implicit function theorem, in each region:

$$\begin{aligned}
 \frac{\partial p_B}{\partial p_I} &= - \left[\frac{\partial \psi}{\partial p_I} \right] / \left[\frac{\partial \psi}{\partial p_B} \right] \\
 &= - \left[\frac{\partial \psi}{\partial (w/p_B)} \frac{\partial (w/p_B)}{\partial p_I} + \frac{\partial \psi}{\partial \tau} \frac{\partial \tau}{\partial p_I} \right] / \left[\frac{\partial \psi}{\partial (w/p_B)} \frac{\partial (w/p_B)}{\partial p_B} + \frac{\partial \psi}{\partial \tau} \frac{\partial \tau}{\partial p_B} \right]
 \end{aligned} \tag{35}$$

Furthermore, from (32) and (33) we find that the partial derivatives

$$\frac{\partial(w/p_B)}{\partial p_I} = -\frac{c_1}{Dp_B} < 0 \quad (36)$$

$$\frac{\partial r}{\partial p_I} = \frac{a_1}{D} > 0 \quad (37)$$

$$\frac{\partial(w/p_B)}{\partial p_B} = \frac{p_I c_1 - M}{Dp_B^2} > 0 \quad \text{when } p_I > b_2 \quad (38)$$

and

$$\frac{\partial r}{\partial p_B} = -\frac{a_2}{D} < 0 \quad (39)$$

Therefore we obtain from (35) and (39)

$$\frac{\partial p_B}{\partial p_I} = \left[\frac{c_1}{Dp_B} \Delta + \frac{a_1 \beta a_2}{D^2} \right] / \left[\Delta \frac{(p_I c_1 - M)}{Dp_B^2} + \frac{a_2^2 \beta}{D^2} \right] \quad (40)$$

where

$$\Delta = \alpha(c_2/D - 2w/p_B)$$

From relation (40) we may now determine the sign of $\partial p_B / \partial p_I$ in both the North and the South. First note that $\partial p_B / \partial p_I$ is always positive in the North since $p_I > b_2$, so that $p_I c_1 - M > 0$, and $\Delta > 0$ by assumption. In the South $\Delta < 0$, but β is rather small. Therefore, (40) is also positive in the South. With this information we may now return to eqn. (27) and compute $pg / \partial p_I$. As α is large in the South and β is large in the North, we have from eqn. (30) that the expression for $-pg / \partial p_I$ is dominated by the following terms:

$$\frac{\alpha M}{D^2 p_B^2} (M - c_1 p_I) \frac{\partial p_B^S}{\partial p_I} + \frac{\alpha c_1 M}{D^2 p_B} + \frac{\partial p_B^N}{\partial p_I} \frac{a_2^N \beta^N Q^N}{(D^N)^2} - \frac{\beta^N Q^N a_1^N}{(D^N)^2} \quad (41)$$

Here $M - c_1 p_I = c_1 b_2 - b_1 c_2 - c_1 p_I$ is negative as c_1 is small in the South. Hence the first term is positive (because $M^S < 0$) and dominates the second, which is multiplied by c_1 . As $Q^N < 0$, the third term is negative and the fourth positive. But a_2 is small in the North, so that the fourth term dominates. Hence we have that

$$-\frac{pg}{\partial p_I} > 0$$

This implies that the price of industrial goods p_I rises as the transfer to the South increases, i.e.

$$\frac{\partial p_I}{\partial FT} > 0 \quad (42)$$

We next study the movements of the rate of return in the North r^N as p_I changes. From the national income identity

$$p_I I_D^N = rK - FT$$

As $I_D^N = I_S^N - X_I^N$ and $p_I X_I^N = X_D^S = \vartheta_D^N$.

$$p_I I_S^N = \tau K + \vartheta_D^N - FT$$

In the North, β is large. We can therefore neglect terms other than those in β , giving

$$p_I = \left[(-Q/D) + \tau \right] / (a_1/D)$$

with

$$\frac{\partial p_I}{\partial \tau} = D/a_1 > 0 \quad (43)$$

Hence as FT rises, p_I rises and the profit rate in the North τ^N rises. Knowing how τ^N moves enables us to find the sign of the change in the real wage in the North. We can rewrite the market-clearing condition for the B market, eqn. (7), as

$$\frac{\alpha c_2 w}{D p_B} - \frac{\beta a_2 r}{D} - \alpha \left[\frac{w}{p_B} \right]^2 = 0$$

(see Appendix eqn. (A.11)). Implicit differentiation gives:

$$\frac{\partial(w/p_B)}{\partial \tau} = \frac{-a_2 \beta}{D \Delta} \quad (44)$$

where $\Delta = \alpha(c_2/D - 2w/p_B)$. As $\Delta < 0$ in the North by assumption, we have that

$$\frac{\partial(w/p_B)}{\partial \tau} > 0 \quad (45)$$

in the North. Hence an increase in FT raises the real wage in the North, as well as the profit rate. The next step is to show that the consumption levels of B and I rise in the North.

$$I_D^N = \tau K - FT = \beta \tau^2 - FT$$

$$\frac{\partial I_D^N}{\partial FT} = 2\beta \tau \frac{\partial \tau^N}{\partial FT} - 1 \quad (46)$$

which is positive for large β . Also,

$$B_D^N = wL/p_B = \alpha(w/p_B)^2 \quad (47)$$

so that B_D^N also rises with FT by (45), (42), and (43). We have now proven point (iii) of Theorem 1.

Next we study the response of trade patterns to FT . We have, by inverting (12) and (13),

$$X_I^N = I_S^N - I_D^N = \frac{a_1 K}{D} - \frac{c_1 L}{D} - \tau K + FT$$

From (3) and (4)

$$X_I^N = \frac{a_1}{D} \beta \tau - \frac{c_1 w}{D p_B} - \beta \tau^2 + FT$$

Hence

$$\frac{\partial X_I^N}{\partial \tau} = \beta \left(\frac{a_1}{D} - 2r \right) - \frac{c_1}{D} \frac{\partial(w/p_B)}{\partial \tau} + \frac{\partial FT}{\partial p_I} \frac{\partial p_I}{\partial \tau}$$

By the conditions of the theorem, the first term is negative. By (45) the second term is negative, and by (44) it contains β . As β is large, these terms dominate, and

$$\frac{\partial X_I^N}{\partial \tau} < 0 \quad (48)$$

i.e. the North's exports of the industrial good fall as FT and hence τ^N rise. This implies, of course, that the South's imports of industrial goods fall,

$$\frac{\partial M_I^S}{\partial \tau^N} < 0 \quad (49)$$

We next check what happens to the volume of oil traded. This equals oil demanded in the North, ϑ_D^N , which from Appendix eqn. (A.3) is

$$- \frac{\alpha \omega M}{D p_B} - \frac{\beta \tau Q}{D}$$

Here β is large and Q is negative, by assumption. τ rises, by (43). Hence

$$\frac{\partial \vartheta_D^N}{\partial FT} = \frac{\partial X_D^S}{\partial FT} > 0 \quad (50)$$

This proves points (i) and (ii) of Theorem 1.

What remains is to study the behavior of the Southern economy. We first show that τ^S rises with FT . This is done by showing that $\partial M_I^S / \partial \tau^S < 0$. As

$$\frac{\partial M_I^S}{\partial \tau^S} = \frac{\partial M_I^S}{\partial \tau^N} \frac{\partial \tau^N}{\partial \tau^S}$$

this will imply from (49) that $\partial \tau^N / \partial \tau^S > 0$, which in conjunction with (42) and (43) gives the desired result.

$$\begin{aligned} M_I^S &= I_D^S - I_S^S \\ &= \tau K + \vartheta_S + FT - I_S^S \\ &= \beta \tau^2 - \frac{\beta \tau a_1}{D} + \frac{c_1 \alpha \omega}{D p_B} + \vartheta_S + FT \end{aligned}$$

$$\begin{aligned} \frac{\partial M_I^S}{\partial \tau^S} &= \beta(2\tau - a_1/D) + (c_1 \alpha / D) \left(\frac{\partial(w/p_B)}{\partial \tau} \right) + \left(\frac{\partial \vartheta_S}{\partial M_I^S} + \frac{\partial FT}{\partial M_I^S} \right) \frac{\partial M_I^S}{\partial \tau^S} \\ \frac{\partial M_I^S}{\partial \tau^S} \left(1 - \frac{\partial \vartheta_S}{\partial M_I^S} - \frac{\partial FT}{\partial M_I^S} \right) &= \beta(2\tau - a_1/D) + c_1 \alpha / D \left(\frac{\partial(w/p_B)}{\partial \tau} \right) \end{aligned}$$

Now

$$\frac{\partial \vartheta_S}{\partial M_I^S} = \frac{\partial \vartheta_S}{\partial \tau^N} \frac{\partial \tau^N}{\partial M_I^S} < 0$$

by (49), (42), (43), and (6). Similarly, $\partial FT / \partial M_I^S < 0$. By (44), $\partial(w / p_B) / \partial r < 0$ in the South. As by assumption α^S is large, this establishes that

$$\frac{\partial M_I^S}{\partial r^S} < 0 \quad \text{so that} \quad \frac{\partial r^S}{\partial FT} > 0 \quad (51)$$

It now follows from (44) and the fact that $\Delta^S < 0$ by assumption, that real wages in the South fall with FT . It follows immediately from (3) and (5) that employment and the consumption of basics also fall.

This completes the proof of Theorem 1.

Theorem 2. *Suppose $M^S > 0$, i.e. $c_1 b_2 - b_1 c_2 > 0$ in the South. Let p_B be small and $p_I > b_2^S$ at the initial equilibrium, with all other conditions as in Theorem 1. Then an increase in the financial transfer to the South has the opposite effects to those established in Theorem 1: it leads to a fall in p_I , the price of the industrial good, and a relative increase in the price of oil, even though oil supplies have increased. The oil exporter's terms of trade therefore improve. In addition, oil exports and the rate of profit in the South decrease. The North exports more industrial goods. Real wages, employment, and consumption of basics increase in the South. In the North, the rate of profit and the real wage decrease.*

Proof. As in the proof of Theorem 1, the sign of $\partial FT / \partial p_I$ equals that of $\partial r / \partial p_I$. This is given in eqn. (30), or approximately in (41). The latter may also be written as

$$\frac{\alpha M}{D^2 p_B} \left[\frac{\partial p_B}{\partial p_I} (M - c_1 p_I) \frac{1}{p_B} + c_1 \right] + \frac{\beta^N Q^N}{(D^N)^2} \left[\frac{\partial p_B^N}{\partial p_I} a_2^N - a_1^N \right]$$

Now note from (40) that for large β^N ,

$$\frac{\partial p_B}{\partial p_I} \sim \frac{a_1}{a_2}$$

Hence the second term above is zero and (41) can be expressed as

$$\left[c_1 \left\{ (b_2 - p_I) \frac{\partial p_B}{\partial p_I} \frac{1}{p_B} + 1 \right\} - \frac{\partial p_B}{\partial p_I} \frac{b_1}{p_B} c_2 \right] \frac{\alpha M}{D^2 p_B} \quad (41)$$

Under the conditions of Theorem 2, this is negative, proving that the oil exporter's terms of trade improve, i.e. p_I falls with FT .

The rest of the theorem follows immediately. Inequality (43) implies that the profit rate in the North falls, and (44) implies that real wages in the North fall. Inequality (48) tells us that the North's exports (and the South's imports) of industrial goods will increase, and from (50) we then know that oil exports of the South fall. (52) establishes that the rate of profit in the South falls, and using (44) again proves that real wages, employment, and consumption of basic goods rise in the South. This completes the proof.

The main difference in the conditions of Theorems 1 and 2, which reverse the results, are first, the sign of M^S and second, the impact that the transfer has on the relative price of industrial goods. The sign of M^S is positive in Theorem 2, and negative in Theorem 1. It seems more plausible that M^S should be negative, since this happens when the basic goods sector in the

South uses few capital inputs. Theorem 2 assumes, instead, that the basic goods sector is more capital intensive. The impact of the transfer on prices seems also more plausible in Theorem 1. There, the transfer increases oil supplies, and this leads to lower oil prices. In Theorem 2, the transfer also increases oil supplies, but this leads to higher oil prices. Clearly, an empirical analysis of the actual conditions is needed to evaluate the results, but, *a priori*, the conditions in Theorem 1 appear more intuitively natural than those in Theorem 2.

A final point is the stability of the equilibria under the standard Walrasian adjustment process in which prices increase with excess demand, and decrease with excess supply. This is a rather specialized issue since the model has constant returns to scale. The Walrasian stability of a closely related model (Chichilnisky 1981b) has been studied in Heal and McLeod (1983) and the interested reader is referred to that paper for a detailed analysis.

5. CONCLUSIONS

We have considered a situation where an inflow of capital investment into a country's oil sector has allowed that country to run a deficit on its balance of trade. The capital inflow is, of course, matched by an accumulation of indebtedness to foreigners. An inflow of foreign capital, whether used for consumption or for investment, inevitably affects the internal equilibrium of the receiving country. Consumption patterns, production patterns, and prices all change. The same is true of the lending country: it changes its consumption pattern by making a loan, and for this reason, and because the equilibrium of its trading partner changes, its own domestic equilibrium alters. A crucial factor in determining these macro effects of a loan is the change in relative prices (oil prices, industrial prices, and prices of basic goods that are not traded). A loan must be of a significant size before having a measurable impact on prices, and the cases we discussed here, where the loan is of the order of 100 billion US dollars, certainly fit this description.

It is clear, then, that it is a complex matter to trace the full impacts of a loan from one trading country to another. Our model has enabled us to identify these impacts in a rather simple fashion, because of our somewhat stylized assumptions, and to assess the gains and the losses arising from such a loan for different groups within the lending and borrowing countries. One important feature to emerge is that the loan may have a beneficial effect on the equilibrium of the lending country. This happens when the borrowed funds are used to increase oil supplies, leading to more abundant oil, increased oil exports, and lower oil prices. The terms of trade of the lending country improve, and this leads to higher levels of consumption of both goods in the lending country. Theorem 1 establishes the conditions under which the welfare level in the lending country will rise as a result. In making a social cost-benefit analysis of such a loan, this is a point that should clearly be considered; there is a social return to the loan over and above the rate of interest paid on it. It is possible that even if a major rescheduling that delayed repayment were to happen, the lending country as a whole could nevertheless benefit. Private financial institutions making the loan might of course be strained in such a situation. There could then be an argument in favor of the government compensating banks in the case of temporary losses, in view of the positive externalities that their actions have generated for the rest of the economy. Obviously, such a policy would require very careful analysis of the macro effects and of the international markets concerned.

Similar issues apply to the receiving country. The borrowing sector may benefit in commercial terms from the loan, but a social cost-benefit analysis of the loan should also take into account its effects on the overall economic equilibrium. As Theorem 1 shows, these could be substantially negative, if there has been overspecialization in one sector thus leading to lower terms of trade for the country, with correspondingly negative welfare effects. In summary, the fact that a loan, if large, may affect the equilibrium pattern of prices and quantities in both countries means that it will have macroeconomic consequences going far beyond its impacts on the profits of the borrowing and lending institutions.

Theorems 1 and 2 have indicated two very different possible outcomes. In one case, the effects are beneficial to the lending and harmful to the borrowing country, while in the other case the opposite is true. The distinguishing feature is the effect of the loan on the oil exporter's terms of trade. In the first case, they worsen, and in the second, they improve. Which of these two outcomes occurs depends on the patterns of factor intensities in the receiving country and the initial price levels. Once these are known, thus establishing whether the loan improves or worsens the receiver's terms of trade, everything else can be traced. Experience indicates that over the last three years, the terms of trade of oil exporters have worsened. While many factors have contributed to this price movement, this suggests that a policy of borrowing to invest in the oil sector might not have been the most favorable to the oil exporter. However, such a policy could be favorable to the lender; it yields more oil at lower prices. Such macro outcomes should be computed when discussing the present situation. The calculus of the debt must go beyond the financial aspects, and must include the macroeconomic effects on prices, imports, and exports of both countries.

It is important to emphasize that we have studied the consequences of granting a loan before this was repaid. The repayments will not have effects that are simply equal and opposite to those of the granting of the loan. The asymmetry arises because, when the loan is made, it is invested or consumed in sectors different than those that will pay the debt. For instance, in this paper the debt was used to build up the production capacity of the oil sector. However, when the loan is repaid, this will not of course coincide with running down this capacity. Investment is irreversible, and capital stock and machines depreciate. The loan will be repaid by running a balance-of-trade surplus. The effects of running a trade surplus at a constant capacity level in the oil sector are not the opposite of those running a trade deficit and using the capital inflow to expand oil-producing capacity. As a matter of fact, both could affect the major macro variables in the same direction. This distinction between receiving and repaying a loan will be developed further in a subsequent paper.

Finally, we point out a connection between the problem that we have studied here and the extensive literature on the transfer problem in international economics. This literature is concerned with the possibility that a transfer of resources from one agent or country to another may benefit the donor and harm the recipient. This issue has so far been studied only in the context of a barter economy without production in the case of perfectly competitive general equilibrium models. For surveys of these results, see Chichilnisky (1980), Jones (1983), and Geanakoplos and Heal (1983). Our present Theorem 1 provides an example of the transfer paradox in a production economy: resources are transferred from lender to borrower, and the lender gains as a result (Theorem 1), even though the receiver expands its production capacity.

APPENDIX

This appendix gives an explicit analytic solution to the model, and presents the results of numerical simulations on the effects of rescheduling the debt reported in the paper.

In order to solve the model we consider first the equation equating oil exported with oil imported:

$$X_{\vartheta}^S = M_{\vartheta}^N \quad (\text{A.1})$$

In view of (6), (9), and (6)', this equals

$$\vartheta_S(FT) - \vartheta_D = \vartheta_D^N \quad (\text{A.2})$$

where the left-hand-side variables are from the South. From (14), (12), and (13)

$$\begin{aligned} \vartheta_D &= \frac{b_1}{D}(c_2L - a_2K) + \frac{b_2}{D}(a_1K - c_1L) \\ &= -\frac{\alpha}{D} \frac{w}{p_B} M - \frac{\beta\tau}{D} Q \end{aligned} \quad (\text{A.3})$$

where

$$M = c_1b_2 - b_1c_2 \quad Q = a_2b_1 - a_1b_2$$

Therefore, we may rewrite (A.2) as

$$\vartheta_S(FT) - \frac{\alpha}{D} \frac{w}{p_B} M - \frac{\beta\tau}{D} Q = \frac{\alpha^N}{D^N} \left(\frac{w}{p_B} \right)^N M^N - \frac{\beta^N \tau^N Q^N}{D^N} \quad (\text{A.4})$$

(A.4) is therefore an implicit equation in five variables, which we denote

$$\varphi(FT, \tau, \frac{w}{p_B}, \tau^N, (\frac{w}{p_B})^N) = 0 \quad (\text{A.5})$$

Our next step is to write the rate of profit τ and the wage w/p_B in the two regions as functions of the prices of basic and industrial goods, p_B and p_I . Recall that oil is the numeraire ($p_{\vartheta} = 1$). From the production functions (1) and (2) we obtain the associated competitive price equations

$$p_B = a_1w + b_1p_{\vartheta} + c_1\tau \quad (\text{A.6})$$

$$p_I = a_2w + b_2p_{\vartheta} + c_2\tau$$

i.e.

$$\begin{bmatrix} p_B - b_1 \\ p_I - b_2 \end{bmatrix} = \begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix} \begin{bmatrix} w \\ \tau \end{bmatrix}$$

since $p_{\vartheta} = 1$. We therefore obtain the factor-commodity price relations:

$$\begin{aligned}
 w &= \frac{c_2 p_B - c_1 p_I + M}{D} \\
 \frac{w}{p_B} &= \frac{(p_B - b_1)c_2 + (b_2 - p_I)c_1}{D p_B} \\
 r &= \frac{(b_1 - p_B)a_2 + a_1(p_I - b_2)}{D} = \frac{a_1 p_I - a_2 p_B + Q}{D}
 \end{aligned} \tag{A.7}$$

Substituting w/p_B and r from (A.7) into (A.5), we obtain a new implicit function, in four rather than five variables:

$$\psi(FT, p_I, p_B^N, p_B^S) = 0 \tag{A.8}$$

Recall that p_B^N may be different from p_B^S because B is not traded internationally. The last step is to substitute p_B^N and p_B^S as functions of p_I into (A.8). This will lead to an implicit function in two variables

$$\chi(FT, p_I) = 0 \tag{A.9}$$

Since FT is an exogenously given parameter, (A.9) is an analytic solution to the model: from (A.9) we may compute the equilibrium level of industrial prices $p_I^*(FT)$. It is easy to check that once p_I^* is known, we may solve for the equilibrium values of all other endogenous variables. This will be explained below.

Now, in order to obtain $p_B = p_B(p_I)$, we use another market-clearing condition, this time in the B -market:

$$B_S = B_D \tag{A.10}$$

From (12) and (13) this can be written as

$$\frac{c_2 L - a_2 K}{D} = \frac{w}{p_B} L \tag{A.11}$$

or

$$\alpha \frac{c_2}{D} \frac{w}{p_B} - \frac{\beta a_2 r}{D} - \alpha \left(\frac{w}{p_B} \right)^2 = 0$$

from which we obtain

$$\frac{w}{p_B} = \frac{c_2}{D} \pm \left(\frac{c_2^2}{4D^2} - \frac{\beta a_2 r}{D\alpha} \right)^{\frac{1}{2}} \tag{A.12}$$

a two-branched function relating w/p_B and r . The different parameter values will determine which is the appropriate branch in (A.12).

Using again the factor-commodity price relations, (A.12) yields an implicit relation between p_B and p_I , as desired:

$$\frac{c_2}{\alpha D p_B} \pm \frac{1}{p_B} \left[\frac{c_2^2}{4D^2} - \frac{\beta a_2}{D^2 \alpha} (Q - a_2 p_B + a_1 p_I) \right]^{\frac{1}{2}} - \frac{M}{D p_B} - \frac{c_2}{D} + \frac{c_1 p_I}{D p_B} = 0 \tag{A.13}$$

Substituting (A.13) into (A.8), we obtain the desired relation (A.9) between FT and p_I

$$\chi(FT, p_I) = 0$$

From (A.9) we may then compute $p_I^* = p_I^*(FT)$. From (A.13) we obtain $p_B^*(N)$ and $p_B^*(S)$, and from these three equilibrium prices we obtain the equilibrium rates of profit $\tau^*(N)$ and $\tau^*(S)$, and of real wages, $(w/p_B)^*(N)$ and $(w/p_B)^*(S)$. From these we obtain supply of labor and capital in the North and the South, and using the inversion of (12) and (13) we obtain the output of B and I in both regions. From the national income identity we may compute demand for I in the South, which determines imports from the North, and from (40), exports of oil from the South. From (14) we obtain oil demanded in the South, thus completing the computation of the equilibrium.

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