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PREFACE

In November 1982, the International Institute for Applied Systems Analysis (IIASA) and the Committee for Systems Analysis of the USSR Academy of Sciences cosponsored a Workshop on Adaptation and Optimization in Moscow.

The purpose of the Workshop was to discuss the aspects of optimization and adaptive control concerned with systems that operate under conditions of uncertainty. There is great interest in this area in the countries represented at IIASA because most of the systems studied at IIASA (e.g., economic, technolcgical and environmental systems) are of this type. Control in such systems often has a dual purpose: (i) to identify and refine the system model; (ii) to achieve the ultimate aim of the control process. The implementation of these control procedures is closely connected with the related optimization processes. Problems of optimization under incomplete information may also be approached from the areas of multicriteria optimization, game theory and interactive decision analysis.

This volume contains abstracts of the thirty papers presented at the Workshop. They may be roughly classified by subject into the following groups:

- optimization and estimation of dynamical systems under uncertainty (papers by J.-P. Aubin; B.N. Pshenichnyi; M. Pavon; A.B. Kuržanskii; M.I. Gusev; F.L. Chernousko and A.A. Lyubushin; A.A. Melikjan and A.I. Ovseevitsh).

- stochastic optimization, estimation and identification of stochastic processes (papers by R.J.-B. Wets; Y. Ermoliev; Ya.Z. Tsypkin; L. Nazareth; R.S. Liptzer; A.A. Gaivoronski; N.A. Kuznetsov, P.I. Kitsul and A.I. Yashin; V.I. Arkin, E.L. Presman and I.M. Sonin).

- applications of adaptive control in ecology and engineering (papers by V.V. Ivanischev et al.; V. Sragovich; C. Walters; J.F. Koonce; E. Stolyarova; L.A. Kairukstis; D.O. Logofet and Yu.M. Svirezhev; V.K. Bogotov et al.; V.V. Solodovnikov, V.P. Kolesnik and O.N. Zhdanov).

- decision making and applications in economics and the social sciences (papers by V.L. Makarov and V.A. Vasiljev; V.P. Busygin; V.G. Sokolov; V.L. Volkovich; E. Markarian; I.V. Evstigneev).

Further information on the results reported in these abstracts may be obtained from the authors.

ANDRZEJ WIERZBICKI
Chairman
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CONTENTS

SEQUENTIAL CONTROL WITH INCOMPLETE DATA AS A BRANCH OF GENERAL ADAPTIVE SYSTEMS THEORY

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Control problems such as resource allocation in economics, control in adaptive systems and clinical trials in medicine have two main features: (i) decisions are made sequentially; (ii) the dynamics of the system are not completely known, depending on the (unknown) values of certain parameters.

We model such systems using a dynamic statistical decision model with elements $\{\Theta, X, A, p, r\}$, where Θ is the parameter set, X represents the state space (x_n is "the state" of the system at time n), A is a set of actions (controls), $p_\theta(y|xa)$ is the conditional distribution of the system at the next time point given the current state x and action a, and r(xa) represents a reward.

A strategy π is defined in the usual way as a sequence $\{\pi_n\}$ of transition probabilities $\pi_n(\cdot|x_1, a_1, \ldots, x_n)$ on A. Let E_θ^π be the mathematical expectation corresponding to strategy π and parameter θ. We consider the expected total return $V_{\theta T}^\pi = E_\theta^\pi \sum_{i=1}^{T} r(x_i a_i)$. If Θ contains more than one element, we consider the *a priori* distribution $\xi(d\theta)$ on Θ and define $V_T^\pi(\xi) = \int_\Theta V_{\theta T}^\pi \xi(d\theta)$. The main questions faced in these models are whether Bayes and minimax strategies exist, whether a least favorable distribution exists, and whether the maximum total return converges as $T \to \infty$.

Another interesting question more typical of the adaptive approach is whether the *a posteriori* distribution $\xi(n, d\theta)$ converges to the distribution concentrated on the "true value" of the parameter (the problem of identification).

We will now derive some results for the so-called "multi-armed bandit" problem. This is a particular case of the general model in which $X = \{0,1\}$, $A = \{d^1, \ldots, d^m\}$, $p_\theta(1|xd^j) = 1 - p_\theta(0|xd^j) = \lambda_\theta^j$, and $r(x, d^j) = x$, $x = 0,1$. We shall take $\Theta = \{\theta_1, \ldots, \theta_N\}$.

The matrix $\{\lambda^j_{\theta_i}\} = \{\lambda^j_i\} = \Lambda$ is called the matrix of hypotheses and the *a priori* distribution is $\xi = (\xi_1, \ldots, \xi_N)$.

If $m = N = 2$ and $\lambda^1_1 = \lambda^2_2 = a$, $\lambda^2_1 = \lambda^1_2 = b$ we have a classical "two-armed bandit" problem [1]; if $m = N = 2$ and $\lambda^2_1 = \lambda^2_2$ we have a "one-armed bandit" problem [2]. We are interested in the asymptotic behavior of $V_T(\xi) = \sup V^\pi_T(\xi)$ as $T \to \infty$. This is equivalent to studying the asymptotic behavior of $W_T(\xi) = T \sum_{i=1}^N \xi_i (\max_j \lambda^j_i) - V_T(\xi)$.

THEOREM 1. The matrices of hypotheses Λ *can be divided into two classes,* I *and* II. *If* $\Lambda \in \mathrm{I}$, *then* $W_T(\xi) \to \infty$; if $\Lambda \in \mathrm{II}$, *then* $W_\infty(\xi) < \infty$ *and at infinite time there exists an optimal stationary strategy* π^* *(i.e., there exists a function* $r(\xi)$ $(r(\xi) = d^1, \ldots, d^m))$ *such that* $\pi^*(k) = r(\xi(k))$ *and* $W^{\pi^*}_\infty(\xi) = \inf_\pi W^\pi_\infty(\xi) = \lim_{T\to\infty} W_T(\xi)$. *In both cases* $W_T(\xi)/T \to 0$.

It is relatively simple to discover the class to which a given matrix belongs. For instance, if we have $\lambda^j_i \neq \lambda^k_i \; \forall i, j \neq k$, then $\Lambda \in \mathrm{II}$ and $P^\pi_{\theta_i}[\xi_i(n) \to 1] = 1$. If $m = N = 2$ and iff we have the "one-armed bandit" situation, then $\Lambda \in \mathrm{I}$.

We now consider the case of continuous time. In discrete time we observe a sequence of Bernouilli trials with parameters λ^j_i if $\theta = \theta_i$ and we adopt control d^j. In continuous time, however, we consider a Poisson process with intensity λ^j_i. We have found the explicit form of $W_\infty(\xi)$ for the case $m = N = 2$ and the associated optimal strategy for $T = \infty$.

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DIFFERENTIAL INCLUSIONS AND VIABILITY THEORY

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The need for greater understanding of dynamical processes in a number of fields has motivated mathematicians to study dynamical systems with velocities which are not uniquely determined by the state of the system. This requires the replacement of differential equations

$$\dot{x} = f(x)$$

by differential inclusions

$$\dot{x} \in F(x) \quad ,$$

where F is a set-valued map associating a set of feasible velocities with each state of the system.

Having done this, it becomes important to devise mechanisms by which certain special trajectories can be selected from the set of all possible trajectories.

One class of such mechanisms is provided by optimal control theory: in this case only trajectories that optimize a functional on the space of all such trajectories can be selected.

These mechanisms do not seem to provide a suitable description of the evolution of what we shall call the "macrosystems" arising in economics and the social sciences. Such macrosystems do not appear to have any goals, nor the desire to optimize any particular criterion. However, they do try to remain "alive" or "viable" in the sense of satisfying certain binding constraints. To discover the class of mechanisms by which this may be achieved is the object of Viability Theory.

Let X be the (finite-dimensional) state space and $K \subset X$ be the set of feasible states. It is assumed that K is locally compact, which allows K to be either open or closed. Let F be a set-valued map from K to X.

A solution x(t) of the initial value problem for the differential inclusion

$$\dot{x}(t) \in F(x(t)) \quad , \quad x(0) = x_0 \quad , \quad x_0 \in K \tag{1}$$

is said to be viable if

$$\forall t \geq 0 \quad , \quad x(t) \in K \tag{2}$$

and locally viable if

$$\exists\, t_x 0 \text{ such that } \forall t \in [0, t_x 0] \quad , \quad x(t) \in K \quad .$$

For $x \in K$ let

$$T_K(x) = \bigcap_{\varepsilon>0} \; \bigcap_{\alpha>0} \; \bigcup_{0<h<\alpha} \left(\frac{1}{h}(K-x)+\varepsilon B\right) \quad , \tag{3}$$

where $T_K(x)$ is called the contingent cone to K at x.

Necessary and sufficient conditions for the existence of a viable solution are given in the following theorem.

<u>THEOREM</u> *(Viability Theorem). Let* $K \subset X$ *be locally compact and* F *be upper semicontinuous with nonempty compact convex values. The necessary and sufficient condition for the existence of a local viable solution to the differential inclusion (1) for every initial state in* K *is then*

$$\forall x \in K \qquad F(x) \cap T_K(x) \neq \emptyset \quad . \tag{4}$$

When K is closed and F is bounded, (4) implies the existence of a viable solution for every initial state.

When K is convex and compact, the assumptions of the Viability Theorem imply that the dynamical system has an equilibrium point or stationary solution, i.e., a solution $\bar{x} \in K$ to the inclusion

$$0 \in F(\bar{x}) \quad . \tag{5}$$

Viability Theory also provides the feedbacks (concealed in both the dynamics and the viability constraints) which relate the state of the system to its control. As long as the state of the system lies within the viability domain any regulatory control will work. If, at the boundary of the viability domain, the chosen velocity pushes the trajectory back into the interior of the domain then we keep the same regulatory control. If the chosen velocity is "outward" at the boundary, however, we must either: (i) find another regulatory control such that the new associated velocity is "inward", or (ii) operate on the viability domain, enlarging it in such a way that the state of the system lies in the interior of the new viability domain.

When these two strategies for "structural change" fail, the trajectory "dies", i.e., it is no longer viable.

Among other applications, viability theory can explain the evolution of prices as a mechanism for decentralization in a simple economic system.

METHODS AND ALGORITHMS FOR THE STATISTICAL TREATMENT AND REDUCTION OF REDUNDANT INFORMATION FLOWS IN ADAPTIVE SYSTEMS

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Holling has suggested various principles which can be used to define the resilience of ecological systems. Until quite recently, attention has been focussed on deterministic methods for constructing mathematical models of resilient systems.

The paper provides an outline of methods and algorithms for the statistical treatment and reduction of redundant information flows. It is hoped that these techniques may provide the basis for constructing models of economic, ecological and other complex systems.

Because such a vast amount of information is obtained in the investigation of complex systems, some means of reversible or irreversible information compression is required. In addition, the reliability of the decisions and conclusions resulting from these analyses depends directly on the accuracy with which the incoming information is treated.

A new adaptive algorithm may be used to increase the reliability of the incoming information in an irreversibly compressed subsystem. This is done by removing single or grouped pieces of information from the aggregate information flow, and comparing the differences in the flow with a threshold value which is either fixed or determined while treating the information.

The efficiency of this algorithm is demonstrated by a practical example.

A detailed description of a new adaptive method for making the initial information stationary is given. This method makes it possible to identify non-stationary components of the information flow with minimum error, and is especially useful in studies of economic systems, which often involve variable initial data.

Much attention is paid to new methods of defining probabilistic distribution laws for information flows; these are essential, for instance, in sociological investigations into the qualitative structure of a national population.

The report describes various methods of determining the errors in estimated distribution functions and probability densities for information flows with different statistical properties. An algorithm has been developed which assesses the probabilistic characteristics of information flows during the course of the analysis, reducing the errors to minimal values.

In a quasireversible compression subsystem, which reduces the amount of information by a factor of between twenty and one hundred, attention is focussed on controlling the feedback with regard to the inflow and outflow of information. A versatile method of choosing the parameters for such subsystems has been developed, which is based on the principles of minimum information loss and maximum utilization of the capacity of the adaptive system.

Optimal values of parameters have been calculated for specific systems. A new method of controlling feedback is presented which reduces information treatment errors by a factor of three or more.

To sum up, the methods and algorithms discussed in the paper decrease the volume of information analyzed in adaptive systems while increasing the reliability and accuracy of the final results.

INTERACTION MODELS: RESEARCH DIRECTIONS AND METHODS OF SOLUTION

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The various different types of interaction model can be associated, by a standard procedure, with game formulations such that the models have the same solutions (cooperative and noncooperative) as the corresponding games. The specificity of the game-theoretic interpretations, and the identification of situations of practical importance in which these games have a structure sufficiently simple for analysis, represents one of the most important areas of research in this field.

One example of such a situation is provided by a model of a two-level industrial planning system proposed by A.G. Aganbegyan and K.A. Bagrinovsky. The coordinated solutions of such a model can be found, for example, by a simple iterative procedure. Another example is given by exchange models which have surplus demand functions with the property of gross replaceability.

The results of our analysis of interaction models are discussed in the paper. Such models are used to coordinate solutions in long-term planning problems and in situations similar to those mentioned above.

However, the conditions which produce games with a simple structure are sometimes found to be too restrictive. This fact forces us to consider methods for finding solutions to wider classes of games. This can be very complicated. As a result, the so-called combinatorial methods, which are based on the ideas of combinatorial topology, were introduced and are still being developed. These methods have been used with success to solve practical problems of small dimension. Their main virtue is their universal character, which allows them to be applied to a wide variety of interaction problems. The cost of this "universality" is their great unwieldiness of solution, even when the problem is of comparatively small dimension. As a result, various heuristic methods for solving these models are being designed.

The second part of the paper deals with the design of such algorithms and their behavior in numerical experiments.

SOME METHODS OF ESTIMATION AND OPTIMIZATION IN DYNAMICAL SYSTEMS

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We consider general dynamical systems of the form

$$\dot{x} = f(x,u,t) \quad , \quad u(t) \in U \quad , \quad x(t_0) = x^0 \quad , \quad t_0 \leq t \leq T \ , \qquad (1)$$

where x is a state vector, u is a vector of controls or disturbances, U is a set of constraints imposed on u, and x^0 is a vector of initial values. We consider two problems in connection with this system:

(a) estimation of the attainable set D(t) (D(t) is the set of all possible states which can be attained for arbitrary feasible functions u(t));

(b) optimization of the terminal state of system (1), i.e., determination of the u(t) which minimizes (or maximizes) the functional

$$J = F(x(T)) \quad . \qquad (2)$$

If u(t) in (1) is a control function then internal bounds for the attainable set D(t) in problem (a) are essential. In this case problem (b) is a normal optimal control problem with terminal cost functional (2).

If, on the other hand, u(t) in (1) is a vector of disturbances, then external bounds for D(t) in problem (a) are important. Problem (b) can now be considered as determination of the "worst" possible disturbance (in the sense of functional (2)).

The two-sided ellipsoid estimate method [1] is developed for problem (a). This method consists in constructing external and internal bounds for D(t):

$$E^-(t) \subset D(t) \subset E^+(t) \quad . \qquad (3)$$

Explicit nonlinear systems of ordinary differential equations are obtained which describe the evolution of the centers and matrices of ellipsoids $E^{-}(t)$, $E^{+}(t)$. These ellipsoids are optimal in the sense of their volumes. To obtain estimates (3) it is only necessary to integrate the systems mentioned above using the given initial data. This method can be applied to discrete-time problems, optimization and multicriteria problems, etc.

There are many numerical methods that could be used to solve problem (b), in particular gradient methods. We consider the method of successive approximations [2], which is based on Pontryagin's maximum principle and has some advantages compared with the gradient method. Some new efficient modifications of this method are described, and their applications demonstrated. For example, this method was used to compare and evaluate different strategies in adaptive systems in the presence of disturbances.

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ASPECTS OF OPTIMIZATION AND ADAPTATION

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The rapid changes taking place in the world today emphasize the need for methods capable of dealing with the uncertainties inherent in virtually all modern systems. We cannot know, or measure, everything. In addition, systems are now undergoing disturbances which may be unlike anything they have experienced before. To cope with such uncertainty we must develop new adaptive mechanisms.

According to classical theory, a system should be thoroughly investigated before it can be optimized. However, such an investigation is generally ruled out under conditions of uncertainty due to the possibility of unexpected changes in both system and environment. Adaptation must therefore be a continuous process, a series of steps with an additional piece of information becoming available at each step. During this process, various estimation and prediction procedures are used to update the system parameters, and an optimization technique is employed to update the control law.

The process of adaptation is often associated only with the short-term (control) actions taken after an observation of the current state of the system has become available. However, in practice it is possible to identify problems which require adaptability over the long term (engineering design, allocation of resources, investment strategies, etc.) as well as those requiring only short-term adjustment (flying an aeroplane, marketing, inventory policy, etc.). Thus, to deal with uncertainty successfully we need to develop approaches (models, computational methods) which integrate a long-term strategy with short-term adaptive actions. In other words, we need approaches which combine the idea of preparing for several possible futures (anticipatory optimization) with that of learning from experience (adaptation).

As uncertainty is such a broad concept, it is possible (and indeed useful) to approach it in a number of different ways. One rather general approach is to assign a measure of confidence (which can be interpreted as a probabilistic measure) to various unknown parameters. This technique can be applied to stochastic optimization problems with partially known distribution functions and incomplete observations of unknown parameters; in this case evaluation of control policy and information collection must take place repeatedly and with systematic adjustments.

The problems outlined above can very rarely be solved using traditional optimization techniques. Most of these techniques require the evaluation of multiple integrals which characterize the random properties of the system as functions of the control variables: attempts to evaluate these directly fail for systems of dimensionality greater than three. In addition, most of the existing methods have been developed for off-line control and adaptation is essentially an on-line process. Thus, new methods are needed to deal with adaptive processes. One approach could be to reduce stochastic problems to deterministic problems through approximation schemes. There are also promising ideas based on the use of direct stochastic procedures employing available random observations. Methods of this type can be regarded as a sort of formalized process of trial and error, and in fact this cannot be avoided when dealing with real uncertainties.

We shall now consider some typical problems and describe possible solution techniques currently being developed at IIASA.

Short-term actions. Nonmonotonic techniques. Consider the simplest case, in which we have to minimize the loss function

$$F(x) = f(x,\alpha) \quad ,$$

where $\alpha \in R^k$ is a vector of unknown parameters and $x \in R^n$ is a vector of control variables. Function $F(x)$ cannot be optimized directly because of the unknown parameters α. Suppose that at each iteration $s = 0,1,\ldots$ an observation h^s is available which has the form of a direct observation of the parameter vector, i.e.,

$$Eh^s = \alpha \quad .$$

By using h^s at iteration s we can obtain a statistical estimate α^s of α such that $\alpha^s \to \alpha$ with probability 1. In this case F(x) must be replaced at iteration s by

$$F^s(x) = f(x,\alpha^s) \quad ,$$

where $F^s(x) \to F(x)$ with probability 1 for $s \to \infty$. The question is: can we use the sequence of functions $F^s(x)$ to find the minimum of F(x)? One possibility is to use the procedure

$$x^{s+1} = x^s - \rho_s F_x^s(x^s) \quad , \qquad s = 0,1,\ldots, \tag{1}$$

where $F_x^s(x)$ is the gradient of $F^s(x)$ or its analogue (for non-differentiable functions), and ρ_s is a step-size multiplier. This procedure, together with a procedure for calculating α^s, allows us to carry out the optimization while simultaneously estimating α. The properties of such procedures are described in [1]. It should be emphasized, however, that the behavior of approximations $F^s(x^s)$ is not necessarily monotonic, that is, $F^s(x^s)$ might be greater than $F^{s-1}(x^{s-1})$, independent of the choice of ρ_s. Therefore, even the simplest case of on-line optimization with unknown parameters requires the development of nonmonotonic optimization techniques.

Long-term actions. Suppose that we have to choose x before observing α and that the probabilistic measure $dH(\alpha)$ can be assigned to α. The expected loss function is

$$F(x) = E_\alpha f(x,\alpha) = \int f(x,\alpha)\,dH(\alpha) \quad .$$

The problem now is to minimize F(x) with respect to the feasible decision variables x. The main difficulty here is concerned with the evaluation of F(x) and its derivatives. The stochastic approximation method and its generalizations (see, for instance, [2]) avoids these difficulties, since it provides a means of minimizing F(x) using information on the random functions $f(x,\alpha)$

only. The approximation schemes for such problems are discussed in [3,4].

Partially known distribution functions. If $H(\alpha)$ is only partially known, i.e., $H \in W$, where W is the class of feasible distributions, then the following minimax problem is of interest:

$$\min_{x} \max_{H \in W} \int f(x,\alpha)\, dH(\alpha) \quad .$$

Computational methods for such problems are described in [5].

In general, most problems will involve all of the above-mentioned difficulties simultaneously.

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ADAPTIVE CONTROL PROBLEMS IN PROBABILISTIC MODELS OF ECONOMIC SYSTEMS

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We consider probabilistic economic models which reflect the ability of the economic system to adapt to randomly varying conditions. (The random character arises, for example, from changes in the environment or in the economic situation). Mathematical techniques based on the theory of stochastic optimization are used to construct these models.

The paper consists of two parts. In the first section, simple control models are examined. These models provide a natural (for economic problems) formalization of the most important concepts in the theory of adaptive systems, which include the following:

- the degree of adaptivity of an economic system
- the degree to which information is used in the control process
- the degree of flexibility of the control strategy
- the cost of information in economic problems.

It is worth saying a few words about the last of these notions. Both restrictions on information and constraints on resources are real and important factors in economics. However, usually only the constraints on resources are considered in economic analysis. It is known that the Lagrange multipliers which correspond to constraints on resources in economic optimization problems can be interpreted as the costs of these resources. It turns out that restrictions on information can be connected with estimates of the effectiveness of information in a similar way, using the theory of stochastic extremal problems. Restrictions on information are formulated as linear constraints in the space of admissible stochastic plans. A simplified version of the Rockafellar-Wets method is used to construct the corresponding Lagrange multipliers.

The second part of the report deals with a class of probabilistic models with incomplete information concerning the time horizon and the ultimate aims of control.

STOCHASTIC PROGRAMMING METHODS FOR ADAPTATION PROBLEMS

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We are concerned with adaptive control problems of the following type. Consider a controlled system operating in discrete time $s = 0,1,\dots$. The performance index of the system during time period s is described by the function

$$F^s(u,a) = Ef(u,a,w) \quad ,$$

where $u \in R^n$ is a vector of controls which belong to set $U \subseteq R^n$, $a \in R^m$ is a vector of uncontrolled variables with unknown values, and w represents random variables. Parameters a change over time, taking values a^s during time period s. In addition, we make observations of the random vector h^s at times s so that we can estimate a^s:

$$h^s = \psi^s(u^s,a^s) + \eta^s \quad , \qquad E\eta^s = 0 \quad , \tag{1}$$

where η^s is noise. We have to choose a sequence of controls u^s, $u^s \in U$, which minimizes the objective function G, where G is derived from the performance indexes f^s.

For example, if we consider adaptation in a stochastic dynamical system over a finite time period, then G may be of the form

$$G = \sum_{s=0}^{N} Ef^s(u^s,a^s,w) \quad , \tag{2}$$

where $f^s(u^s,a^s,w) = W^s(x^s,u^s)$. In this case the state of the system is denoted by x^s, the dynamics are described by

$$x^{s+1} = \phi^s(x^s,u^s,a^s) + v^s \quad , \qquad Ev^s = 0 \quad ,$$

and we observe the following function of state and unknown parameters:

$$h^s = \tau^s(x^s, a^s) + \eta^s \quad , \qquad E\eta^s = 0 \quad ,$$

where V and η represent random noises.

In this case it is impossible to choose the sequence of controls *a priori* because of the uncontrollable changes in parameters a^s. It is therefore necessary to alter the future sequence of controls as observations come in and to estimate current controls on line.

The above formal description embraces many different adaptive control problems, and can be used in the control of various economic and technological systems.

Until quite recently methods for the optimal control of such systems had been developed only for the special case in which both the (stochastic or deterministic) relations between a^s and a^{s+1} and the distribution functions of the random noises are known. Dynamic programming techniques were then applied, resulting in a laborious numerical method which was successful only in relatively simple cases. For this reason suboptimal approaches are of great importance when dealing with these systems [1].

One such approach is presented in this note. It is based on stochastic programming methods and works successfully even when the distributions of random variables and the laws governing the changes in parameters are unknown.

We wish to choose sequence u^s on line in such a way that the following equation is satisfied (in some probabilistic sense):

$$\overline{\lim_{s\to\infty}} \left(F(u^s, a^s) - \min_{u\in U} F^s(u, a^s)\right) = 0 \quad . \tag{3}$$

<u>Passive adaptive algorithm</u>. Using observations h^s we construct an identification functional $\Phi^s(u^s, a^s, z)$ which has a minimum a^s with respect to z when u^s and a^s are fixed. For example, under certain conditions we could use

$$\Phi^s(u^s, a^s, z) = E\| h^s - \phi^s(u^s, z)\|^2 \quad . \tag{4}$$

The estimates z^s of the unknown varying parameters a^s are calculated using the equations

$$z^{s+1} = z^s - \rho_s \xi^s$$

$$E(\xi^s | u^s, a^s, z^s) = \Phi_z^s(u^s, a^s, z^s) \quad , \qquad (5)$$

where ρ_s is the step size.

These estimates are used in the following optimization algorithm:

$$u^{s+1} = \pi_U(u^s - \delta_s f_u^s(u^s, z^s, w^s)) \quad , \qquad (6)$$

where π_U is a projection operator on set U.

When $\|a^{s+1}-a^s\| \to 0$ step sizes ρ_s and δ_s can be chosen such that (3) is satisfied. When $\overline{\lim} \|a^{s+1}-a^s\| = \bar{a}$ then (3) is satisfied only approximately; the smaller the value of $\bar{a}$ the more precisely is (3) fulfilled. Estimates of

$$\overline{\lim}\, E(F(u^s, a^s) - \min_{u \in U} F(u, a^s))$$

are given for this case together with the preferred values of the step sizes ρ_s and δ_s.

We call this a passive adaptive algorithm because the controls are used only to minimize the objective function and not to improve the identification process.

<u>Active adaptive algorithm</u>. Feldbaum [2] was the first to recognize the dual nature of control in adaptive systems, i.e., that control should be used to obtain extra information about the system as well as for optimization purposes. But his optimal dual control algorithms are too complex to calculate controls in most cases and require knowledge of the noise distributions. To overcome these problems we have developed suboptimal methods based on the stochastic programming approach [3].

We take the optimal control u^s to be the sum of two variables:

$$u^s = \tilde{u}^s + y^s \quad ,$$

where $\tilde{u}^s$ minimizes the performance index f^s and y^s is especially designed for identification purposes. This y^s is a random variable with distribution function defined by optimal experiment design methods.

We use the following algorithm:

$$z^{s+1} = z^s - \rho_s \xi^s$$

$$\tilde{u}^{s+1} = \pi_U(\tilde{u}^s - \delta_s f_u^s(\tilde{u}^s + y^s, z^s, w^s)) \qquad (7)$$

$$E(\xi^s | u^s, a^s, z^s, y^s) = \Phi_z^s(u^s + y^s, a^s, z^s) \quad .$$

This active adaptive algorithm can be applied to many more cases than its passive counterpart.

Convergence conditions are derived and the mean distance to solution if these conditions are violated is estimated.

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DYNAMIC PROCEDURES FOR SYNTHESIZING CONTROL AND ESTIMATION SYSTEMS UNDER UNCERTAINTY

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We consider the linear control system $\dot{x} = Ax + Bu + Cv$, $x(t_0) = x^0$, where u is a known input and v an unknown disturbance. The state of the system $x(t_1)$ is estimated from information obtained by observing the vector $y(t) = G(t)x(t) + F(t)\xi(t)$, $t_0 \leq t \leq t_1$, where $\xi(t)$ is the observation error. It is assumed that *a priori* information on the initial state x^0 and the disturbances $v(\cdot)$, $\xi(\cdot)$ is restricted to the inclusion $\{x^0, v(\cdot), \xi(\cdot)\} \in W$ [1,2]. The problem of selecting the series of coordinate measurements (matrix G(t) in the measurement equation) that provides the best vector estimate of the set of feasible states of the system under the restrictions imposed by the available information is examined [2,3]. This set of feasible states is called the information domain. The following two cases are considered: (i) all directions in the state space are assumed to be equivalent (ii) the estimation is performed in given directions, the values of the projections of the information domains along these directions being ordered lexicographically. Under quadratic constraints on disturbances, the problems may be reduced to optimal control problems involving Riccati equations with vector-valued criteria. Necessary and sufficient conditions for optimality are given. The structure of the optimal solutions and the dependence of the centers of the information domains on the observations are examined. The problem of adaptation in the observation process is discussed.

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A SYSTEM FOR THE DESIGN OF ALTERNATIVE MODELS BASED ON IDEOGRAPHIC REPRESENTATION LANGUAGES

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The concept of "adaptive control" was first introduced in connection with the control problems arising in industrial systems. Over the last few years, however, this concept has spread to fields such as economics, sociology and ecology. These systems all share the capacity to respond in a direction other than that intended by the control [1]. The use of an adaptive approach is one way of effectively controlling such systems [2]. The adaptive approach, as described in [2], assumes the use of some scientific and organizational measures. A set of alternative models is then proposed, based on these measures. The efficiency of the adaptive approach is assessed on the basis of the time taken to verify each new model. We propose a system (SYPSAM) which supports the creation of alternative models [3]. This system consists of three subsystems. These are:

- a system for representing information about the problem area, based on a special ideographic language
- a system for building the model
- a system for making decisions on the basic ideographic models.

The structure of the system SYPSAM and various results in the field of model design are discussed in the paper.

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DEVELOPMENT OF FOREST SECTOR MODELS FOR THE LITHUANIAN SSR

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Natural resource use and environmental management in the Lithuanian SSR is being examined as part of an international program called "MAB". The fundamental structure of a general model system for optimizing regional development has been determined. This was done on the basis of investigations carried out by the Lithuanian Academy of Sciences using experience gained at IIASA. The Lithuanian Research Institute of Forestry is working on forest sector modeling, with the aim of establishing forest sector development indices for the whole system of natural resource utilization and environmental management. This study could be a component of the forest sector studies currently underway at IIASA.

The main purpose of forest sector modeling in the Lithuanian SSR is to build a system of models for analysis, prediction and projection of forest sector development. The models are being constructed at three levels: intersectoral, sectoral and intra-sectoral. The following problems are addressed:

1. Prediction and optimization of forest sector production in a model of total production. The basis for this is a regional model of the total production of the Lithuanian SSR which was constructed at the Institute of Economics of the Lithuanian Academy of Sciences. It consists of:

- an objective function which maximizes the annual growth of net output
- a sectoral balance equation
- an industrial production function
- a capital investment function
- a standard limit on production accumulation
- formulae for the determination of the annual growth of net output.

The model gives economic indices for 32 sectors of the economy of the republic. Wood growing and wood processing constitute the forest sector. Calculations up to the year 2005 have been performed.

2. Determination of the optimum forest area using an intersectoral land use optimization model. Such a model is presently under development. It consists of:

- an objective function which maximizes the estimated income from production
- limits on the availability of land
- restrictions on the demands of individual sectors.

Data for two sectors (forest management and agriculture) have been computerized in a form suitable for linear programming. The model will be completed by including more of the factors that determine land use.

3. Optimization of forest growing in a particular area by identifying different types of forest:

- commercial forests
- forests for recreational use
- forested nature reserves
- game forests (for hunting)

and so on.

4. The development of optimization models for these special types of forests. For example, models which calculate the most productive stands of trees have been derived for commercial forests. Computer programs based on these models are used to plan the volume of intermediate cutting and the necessary thinning.

5. The creation of a data bank for the Lithuanian forest sector.

6. Evaluation of the impact of forest management systems on the environment (soil erosion, etc.).

7. Investigation of the feedback relations linking economic and other indices to the general system of models.

8. Optimization of the balance of demands and possible supply of forest products.

To achieve all of the above goals will require further collaboration between the Lithuanian Academy of Sciences and IIASA.

COMMENTS ON THE CONSTRUCTION OF INTERACTIVE, DECISION-AIDING MODELS

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The problem of using models of renewable resource systems in the formulation of policy is one of the main areas of interest in the Adaptive Resource Policy (ARP) project at IIASA. This problem arises, in part, from a fundamental communication gap between traditional analysts and decision makers. Often a decision maker relies on intuitive notions and verbal arguments that are not simply related to the complex models that analysts frequently generate. Discussions of optimal policies are also constrained by the basic incompatibility of the optimal solutions derived by the analyst and the kinds of trade-off, often external to the problem under consideration, that most decision makers face. These difficulties arise to a large degree from certain fundamental characteristics of renewable resource problems.

Two features separate renewable resource problems from others in which optimal control theory has enjoyed greater success. Firstly, the system in which these problems arise are large and complex, and the control options are therefore far more complicated. Secondly, uncertainty is a dominant component of such problems, and itself has both theoretical and identification/estimation elements. These processes have large stochastic variations, some measurement error, and limited (less than 30 to 40) observations. The theory of renewable resources has no way of dealing with these difficulties, and often specialists may offer several competing hypotheses about population regulation.

For these kinds of decision our group continues to use the principles of adaptive environmental assessment and management introduced by Holling [1] as the basis of interactive model development. This method requires the organization of a workshop involving policy makers, managers, and technical experts. A small modeling team works with this group to produce a dynamic model of the problem under study. In this way, the goals of the policy makers help guide the level of model resolution, and the trial-and-error exploration of the model brings the problem into sharper focus. Priorities for additional information and experimentation are by-products of this process.

Experience with this procedure has raised two key issues. Firstly, technical information that is not readily reduced to the verbal and qualitative arguments generally used by decision makers cannot be explicitly used in the decision-making process. If these summaries are explicit products of the model, control of the resource can be viewed in the context of the actual trade-off decisions in policy formulation. Secondly, model compression and simplification are critical in producing a model meeting this requirement. Simple model structures, such as those based on linear differential equations, may not be appropriate and should not be forced onto the problem. Rather, model simplification should be pursued in individual cases. Our experience suggests that most decision-making models can be reduced in complexity and dimensionality, at least retrospectively. Techniques for model simplification would thus seem to be important areas of future research.

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ADAPTIVITY AND GAME-THEORETIC CONTROL

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One of the basic problems of adaptive systems theory is to design a dual-purpose control for a system with uncertain parameters. The first purpose is to identify or refine the model - if this is not done the control process may not be able to achieve its second goal: to direct the system along a chosen course. Such a "dual" control process may be constructed through a feedback procedure based on available information on the system as the latter evolves over time.

On the other hand, the main object of control in uncertain systems is to construct closed-loop dynamic control strategies that ensure a specific guaranteed result over all possible combinations of uncertainties. These strategies are constructed using available parameter measurements. In the absence of statistical information on the uncertain parameters, solutions can be found through game-theoretic dynamic procedures.

The fact that a significant number of adaptive control problems may also be treated using the theory of guaranteed control shows that there is a close connection between these branches of control theory. One such problem is outlined below.

Assume that the system dynamics may be described by a differential inclusion

$$\dot{x} \in F(t,x,u) \quad , \quad t \geq t_0 \quad , \quad x(t_0) \in X^0 \subseteq R^n \quad , \qquad (1)$$

where $F(t,x,u)$ is a given multivalued mapping that characterizes the presence of uncertainty in the system, X^0 is a given starting set, and u is the control action. In particular we may have either

$$F(t,x,u) = A(t)x + Q(t) \quad ,$$

or

$$F(t,x,u) = A(t,Q(t))x + u \quad ,$$

where the set $Q(t)$ is a given range of uncertainties. The current information is described by an equation of observations

$$y(t) \in G(t,x,u) \quad , \tag{2}$$

where $G(t,x,u)$ is a multivalued function and in particular $G(t,x,u) = G(t,u)x + R(t)$, set $R(t)$ being given. Knowledge of $y_t(\cdot) = y(\tau)$, $t_0 \le \tau \le t$, allows us to define a set $X(t,y_t(\cdot),u_t(\cdot),X^0)$ of information domains in the phase space formed by the ends of all trajectories $x(\tau)$, $t_0 \le \tau \le t$, that start at X^0 and are consistent with expressions (1) and (2).

The problem is to construct, from a given class U, a control strategy $u(t,y_t(\cdot)) \in U$ such that

$$\inf_{\{u(t,y_t(\cdot))\}} \sup_{\{y_t(\cdot)\} \in Y_u} \sup_{x \in X(\theta,y_t(\cdot),u(t,y_t(\cdot)),X^0)} \Phi(x) \le \nu \tag{3}$$

subject to

$$X(t,y_t(\cdot),u(t,y_t(\cdot)),X^0) \subseteq M \quad \text{for } t \ge \tau^* \ge t_0 \quad .$$

Here Y_u is the set of measurements that yield nonempty sets $X(t,y_t(\cdot),u_t(\cdot),X^0)$ and $\Phi(x) \ge 0$ is a given function.

With $\Phi(x) = \|x-c\|^2$, where terminal point c is given, it is possible to determine a value ν^0 and an adaptive control strategy $u^0(t,y_t(\cdot))$ that satisfies the inclusions

$$X(\theta,y_t(\cdot),u^0(t,y_t(\cdot)),X^0) \subseteq S_{\nu^0}(c) \quad , \tag{4}$$

$$X(t,y_t(\cdot),u^0(t,y_t(\cdot)),X^0) \subseteq M \quad , \qquad t \ge \tau^* > t_0 \quad ,$$

where $S_r(c)$ is a ball of radius r with center c in Euclidian space R^n.

To solve this problem we have to combine an optimal feedback estimation procedure (including evolution equations for the domains $X(t,y_t(\cdot),u_t(\cdot),X^0)$) with a strategy $u^0(t,y_t(\cdot))$ that satisfies inclusions (4). The latter is constructed via solution of problem (3) using methods of game-theoretic guaranteed control.

This approach has applications in technical, biological and economic systems.

The given class of problems is closely connected with optimization under nonscalar criteria with partial ordering introduced on the sets $X(t,y_t(\cdot),u_t(\cdot),X^0)$. A typical problem of this kind (an adaptive observation problem for linear systems with quadratic constraints) is described by M. Gusev (see p. 22 of this volume).

IDENTIFIABILITY IN BAYESIAN ADAPTIVE SYSTEMS

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The design of models of complex social, economic, biological and technical systems with complete or incomplete data requires reliable procedures for identifying parameters. Specialists in these fields often use the Bayesian approach to construct effective estimation procedures of this type. However, it appears that in many practical cases even Bayesian procedures do not lead to the correct solution of the identification problem, due to the structure of the system and the peculiarities of the measurement scheme. A dynamic system with unknown parameters may be described as identifiable if the corresponding Bayesian estimation algorithms converge (in some sense) to the real parameter values. It turns out that it is possible to find necessary and sufficient conditions for the convergence of Bayesian estimation algorithms for a wide class of observation processes. The conditions for convergence are equivalent to the singularity conditions for a particular family of probabilistic measures. These singularity conditions, in their turn, may be specified for any particular type of observation process. The main theorems dealing with the relations between the identifiability properties of estimation algorithms and singularity properties are given. Examples in which the identifiability conditions may be checked before starting the observation or measurement process are described. These include observation processes governed by stochastic differential equations, by difference stochastic equations or by stochastic integro-differential equations. The latter correspond to random processes with piecewise continuous sampling paths. In particular, it is easy to check necessary and sufficient conditions for the convergence of Bayesian estimation algorithms in an adaptive Kalman filter scheme.

ON APPROXIMATION OF STOCHASTIC PROCESSES

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A number of stochastic optimization problems can be solved relatively easily if the stochastic system may be described by Ito differential equations. Thus, the approximation of stochastic processes by Markov differential equations is of considerable importance.

Conditions for weak convergence of measures corresponding to the sequence of semimartingales X^n, $n \geq 1$, for the diffusion process $X = (X_t)_{t \geq 0}$ with Ito differential

$$dX_t = a(t,X_t)dt + b(t,X_t)dw_t$$

are derived.

Necessary and sufficient conditions for weak convergence are stated, based on triplets of predictable characteristics of semimartingales X^n, $n \geq 1$.

These results can be used in filtering problems, in stochastic descriptions of complex systems and in identification.

MATHEMATICAL MODELS OF ECOLOGICAL SYSTEMS AND THEIR USE IN MANAGEMENT AND DECISION MAKING

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The mathematical theory of optimal control is almost a classical branch of modern mathematics, with its powerful methods and success in solving problems in many different fields. In ecology and ecological management, however, we encounter problems with specific features which invalidate the assumptions of classical optimal control theory. In addition, many traditional criteria fail when applied to ecological problems. There is thus a need to develop new approaches and methods capable of solving optimization problems in ecology.

Mathematical models of populations and ecosystems serve as a basis for these optimization problems, the problem formulation inevitably inheriting features of the mathematical apparatus used to construct the model.

While an analytical model allows traditional optimality formulations, a simulation model, as a rule, does not, due to its complexity and the many variables and factors involved. The search for optimal controls in this case is reduced to experimenting with the model, usually in an interactive regime.

A number of examples from the Laboratory of Mathematical Ecology (Computer Center, USSR Academy of Sciences) illustrate the above situation, and support the idea that progress in the theory of population and community dynamics can be of considerable use in constructing optimization models in ecology.

These examples include:

- A theory of qualitative stability of model ecosystems. This involves the notion of sign-stability of a community matrix and is based on crude preliminary investigations of the patterns of species interaction within a cotton agrobiocenosis. Some general recommendations for biological pest control problems were obtained.

- A mathematical theory of food chains which provides new insights into the optimal nutritional structure of agrobiocenosis and into fertilizer application and rational harvesting policies (antarctic krill, for example).

- A simulation model of the dynamics of the ecosystem in a freshwater fishpond. This can be used to formulate an adaptive management and control policy (via nutrition, aeration and mineral fertilization) capable of dealing with changing environmental conditions.

- A model of the global carbon cycle that has been constructed within the framework of a more general model of dynamical processes in the global biosphere. It considers an "atmosphere-plants-soil" system in each cell of a geographical grid ($4^{\circ} \times 5^{\circ}$), and could be used to evaluate the course of biospheric evolution under various scenarios (e.g., different levels of CO_2 release).

While analytical models provide a general approach to management and control problems, more detailed simulation models for particular ecosystems can be used to formulate adaptive policies for management and control under changing conditions.

COORDINATION OF INTERESTS IN ECONOMIC SYSTEMS WITH EXTERNALITIES

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External effects caused by the dependence of various actors' preferences on the state of the system as a whole are responsible for the essential "complexity" of modern mathematical economic models. The traditional approach to such models leads to solutions which are not satisfactory from the point of view of collective rationality. The little-studied problem of the way in which actors adapt in such complicated economic systems requires special attention.

As a basic model we take the exchange model $\varepsilon = \langle N, \{X_i, w^{(i)}, u_i\}_{i\in N}\rangle$, where N is a set of actors, $X_i \subseteq R^{\ell}, i \in N$ are their consumption sets, $w^{(i)} \in X_i$, $i \in N$ are the initial states, and $u_i: \prod_{i\in N} X_i \to R$, $i \in N$ are utility functions.

Our analogue to classical equilibrium is based on the idea of the information market [1]: the state $\bar{z} = (\bar{x}^{(i)}) \in \Pi X_i$ is said to be an I-equilibrium state if there exist $\bar{p} \in R^{\ell}$, $\bar{p}^{(i)} = (\bar{p}_k^{(i)})_{k\in N} \in (R^{\ell})^N$ such that (a) $u_i(\bar{z}) = \max\{u_i(z) \mid \bar{p}^{(i)} z \leq \bar{p}w^{(i)}, z \in \Pi X_i\}$, $i \in N$; (b) $\sum_{i\in N} \bar{p}_k^{(i)} = \bar{p}$, $k \in N$; (c) $\sum_{i\in N} \bar{x}^{(i)} = \sum_{i\in N} w^{(i)}$. The components $p_k^{(i)}$ of the vector $p^{(i)}$ are interpreted as prices of goods, and represent the information available to actor i about the consumption of actor k. The set $IW(\varepsilon)$ of I-equilibria is non-empty for a wide class of models ε, and $IW(\varepsilon)$ belongs to the Pareto boundary $\theta(\varepsilon)$ of the economy ε.

We also consider principles of collective rationality based on the notion of a system of contracts

$$V = \{x_r(S)\} = \{(x_r^{(i)})_{i\in S} \mid x_r^{(i)} \in R^{\ell}, \sum_{i\in S} x_r^{(i)} = 0, r \in R_{S,V} = \{1,\dots,k_{S,V}\}, S \subset N\}$$

and on the following dominance relation $<_\varepsilon$ between them: $V' = \{x'_r(S)\}$ dominates $V = \{x_r(S)\}$ $(V <_\varepsilon V')$ if there exist $S_0 \subseteq N$, $\lambda_S^r \in [0,1]$ and $x'_{n_{S_0,V}+1}(S_0)$ such that $R_{S_0,V'} = R_{S_0,V} \cup \{n_{S_0,V}+1\}$, $R_{S,V'} = R_{S,V}(S+S_0)$, $\lambda_S^r = 1$ $(S \cap S_0 = \emptyset)$, $x'_r(S) = \lambda_r^S x_r(S)$, and

$$u_i\left(w^{(i)} + \sum_{S,r \in R_{S,V'} | i \in S} x_r'^{(i)}\right) > u_i\left(w^{(i)} + \sum_{S,r \in R_{S,V} | i \in S} x_r^{(i)}\right), \quad i \in S_0 .$$

We state the relationships between $IW(\varepsilon)$, the set $D_0(\varepsilon)$ of contractual states of ε (corresponding to non-dominated systems of contracts) and several variants of the core $C(\varepsilon)$ of economy ε. We also prove the existence of a generalized NM-solution [2], which is understood as a set of states of ε such that the overall system of contracts corresponding to it is internally stable with respect to $<_\varepsilon$, and for any system $V \notin U$ there exists a monotonic sequence $V = V_1 <_\varepsilon V_2 \cdots <_\varepsilon V_m$ such that $V_m \in U$. It is shown that the intersection of all the generalized NM-solutions of ε coincides with $D_0(\varepsilon)$.

Two different ways of defining replicas of ε are proposed, and some conditions for the contractibility of the cores of these replicas to the set of ε-equilibria are stated. We formulate problems concerning the influence of current states of ε upon the actors' adaptively changing preferences. These formulations are closely connected with the asymptotes of the replicas of ε.

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ADAPTATION, OPTIMIZATION AND CULTUROLOGY

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Culturology is a newly developing discipline which proposes the study of culture as a total system. The idea of culturology was first suggested by the American anthropologist L.A. White, although his variant of this discipline has no management potential because of his fatalistic interpretation of culture. In actual fact culturology has great management potential because culture may be seen as a specific suprabiological mode of human activity and thus as a mechanism for adaptation and optimization within society.

Adaptation and optimization are manifestations of the class of phenomena connected with the capacity of self-organizing systems to adjust themselves to changes in their environment. But adaptation is also a wider phenomenon, one aspect of which is optimization. We would argue that not all adaptive processes are optimal because the survival of a system (which is the purpose of adaptation) can be secured by meeting only the minimum requirements for survival; in contrast, optimization processes must by their very nature be adaptive.

We interpret the optimization of particular human activity as optimization of the culture supporting this activity.

Until recently, these systemic qualities of culture received little research attention. This is largely because in the past these optimization processes have taken place virtually autonomously in various spheres of human activity, directed only by the criteria specific to these spheres. Using the example of L. White, it can be said that the aircraft and automobile industries developed faster aeroplanes and cars only because their advancing technology permitted them to do so. These optimization processes were not carried out under very broad criteria. Ecological criteria should play a special role in such processes because of their immediate connection with the survival of human society. Considered from this point of view, ecological criteria have an integrative role because they provide some definite orientation for optimization processes in different spheres of human activity and thus connect them to the overall adaptive goals of the system.

This leads to the need to understand the systemic qualities of culture as part of a universal mechanism for adaptation and optimization. And this, in its turn, explains the main management stimuli in the development of culturology and its importance in the simulation of social processes.

THE INVARIANT MANIFOLDS OF HAMILTONIAN SYSTEMS IN THE SINGULARITY THEORY OF THE BELLMAN-ISAACS (HAMILTON-JACOBY) EQUATION

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We are concerned with the theory of optimal control and differential games. The purpose of this contribution is to find a scalar function V(x) of phase vector $x \in R^n$ (the Bellman function or value of the game) which satisfies the Bellman-Isaacs (Hamilton-Jacoby) equation $F(x,p) = 0$, $p = p(x) = \partial V/\partial x$ at smooth points. The main difficulty in finding this function lies in the determination of singular discontinuity manifolds for V(x) or its gradient p; the latter case is known as weak discontinuity.

The optimality condition for the surface of a weak discontinuity of specific type is obtained. The problem of constructing the surface of a discontinuity is thus reduced to the Cauchy problem with a nonfixed boundary hypersurface. We have a fixed manifold of smaller dimension on the unknown boundary. Sufficient conditions are obtained for the existence and uniqueness of the solution of the problem described above. The discontinuity hypersurface in (x,V,p)-space defines an invariant manifold of an appropriate Hamiltonian system. A number of game-theoretical problems are solved using this technique.

SOME ALGORITHMIC APPROACHES FOR SOLVING TWO-STAGE STOCHASTIC PROGRAMS

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Here we describe research in progress (carried out jointly with Roger J.-B. Wets) on the development and implementation of algorithms for solving two-stage stochastic programs. The form of the problem we are concerned with is as follows:

Find x such that

$$\left.\begin{array}{l} Ax = b \quad , \quad Tx = \chi \quad , \quad x \geq 0 \\ \text{and} \\ z = cx + E_{\omega}\{\Psi(\chi,\omega)\} \text{ is minimized} \end{array}\right\} , \qquad (1)$$

where

$$\Psi(\chi,\omega) = \min_{y \in G} \{q(y) \,|\, Wy = p(\omega) - \chi\} \quad . \qquad (2)$$

Here A, T, W are fixed matrices and c is a fixed vector, all of appropriate dimensions, $q(y)$ is a convex function, G is a convex set usually taken to be the non-negative orthant, and $p(\omega)$ is a stochastic right-hand side with known distribution function. (There are of course more general forms of (1) in which other quantities are stochastic. The characteristics of this problem are fully discussed by Wets [1].) The fact that T is fixed permits us to introduce the variable χ, which we think of as a tender. The extent to which this tender χ matches a particular realization of the vector y in part determines the cost, through the recourse problem (2).

The equivalent deterministic form of (1) is

$$\text{minimize } z = cx + \overline{\Psi}(\chi)$$

subject to

$$Ax = b \quad , \quad Tx - \chi = 0 \quad , \quad x \geq 0 \quad , \qquad (3)$$

where $\overline{\Psi}(\chi)$ is a convex, but in general non-smooth, function, which is finite on its feasible domain if the induced constraints are included in Ax = b. The precision to which $\overline{\Psi}(\chi)$ can be economically computed depends upon the particular form of the recourse problem (2).

Two main approaches to the solution of (1), when expressed in its equivalent deterministic form (3), are outlined. These are based upon:

(a) Inner linearization of the objective function using the generalized programming approach of Wolfe. Each iteration of the algorithm optimizes a linear program defined over the convex hull of a set of tenders $\chi^1,\ldots,\chi^\nu$. One or more tenders are then added to the set by solving an unconstrained minimization problem formed from $\overline{\Psi}(\chi)$ and the dual variables of the linear program, and the process repeated.

(b) An extension of the reduced gradient method to problems with a non-smooth objective function (the reduced subgradient method). Some antecedents of this approach (method of bounded variables, convex-simplex method for non-smooth optimization) are mentioned.

Details of the above can be found in [2]. For a more general discussion of stochastic programs see also [3].

Our progress in implementing some of these ideas is outlined within the context of our overall view of implementation. This is described more fully in [4].

The development of effective routines for solving two-stage, or more generally multi-stage, programs with recourse (surely a central problem of stochastic programming and the most natural extension of linear programming) requires the bringing together of a variety of techniques. These include techniques that stem from an analysis of mathematical stochastic programs, techniques from non-smooth optimization [5], implementation techniques from large-scale mathematical programming that are efficient and numerically sound (as exemplified, for instance, by the MINOS code of Murtagh and Saunders [6]).

We are currently developing implementations of the methods outlined above which are primarily geared toward algorithmic experimentation but in a realistic problem setting. It is hoped that these implementations will eventually evolve into user-oriented software; but much more research into algorithms for the two-stage problem is needed before the field settles down sufficiently to make it worth undertaking the development of user-oriented software in any systematic way.

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MARKOVIAN REPRESENTATION PROBLEMS AND THEIR APPLICATION

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In many problems of estimation and optimal control it is necessary to represent a given non-Markov stochastic process $\{y(t), t\in T\}$ by means of the following relation

$$y(t) = h(x(t)) \quad , \tag{1}$$

where $x(t)$ is a Markov process.

The problem of determining an $x(t)$ of minimum dimension and a function h such that (1) holds has been the subject of recent developments in stochastic optimization theory.

If $\{y(t), t\in Z\}$ (where Z denotes the integers) is a Gaussian stationary process with zero mean, $y(t)$ may be represented in the following way

$$\left.\begin{aligned} y(t) &= Hx(t) \\ x(t+1) &= Fx(t) + w(t) \end{aligned}\right\} \tag{2}$$

where $w(t)$ is white noise.

The problem of determining the matrices H, F may also be interpreted as a system identification problem.

The paper presents new methods of interpolation and extrapolation for multivariable stationary discrete-time Gaussian processes. These methods are derived using concepts and techniques from stochastic optimization theory. New representations of the optimal interpolator and the interpolation error variance are given. In particular it is shown that the interpolation estimate is characterized by two steady-stage Kalman filters, one evolving forward and the other backward in time.

ADAPTATION, DIFFERENTIAL GAMES AND SEMI-GROUPS OF MULTIVALUED MAPPINGS

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Adaptation is a term which covers a large number of notions and processes, as can be seen by looking through the numerous papers and monographs on the subject. It is used for identification of controls, extraction of signals from background noise, pattern recognition and many other purposes. Even the construction of a minimization algorithm may be considered as an adaptive process. Probably the most that can be said about the abstract term "adaptation" is that it implies adjustment to varying conditions in pursuit of some definite goal.

In view of the above, there seems little hope of building a reasonable theory capable of covering all aspects of adaptation. To develop a theory of any practical use it seems necessary to restrict the area of investigation. One such theory is that of differential games, which displays many characteristics of the adaptation process:

1. The definition of differential games includes dynamical systems which, on the one hand, can be influenced by the environment, and, on the other, can be controlled by a player.

2. The principal problem in differential games is that of available information and its processing.

3. The behavior of the players is guided by the desire to attain a certain objective.

The theory of differential games is now quite well-developed. Important results have been obtained, illustrating the fact that reasonable restriction of the area of investigation enables us to make significant advances in that area.

We consider an approach to the solution of differential games which is based on the construction of a semi-group of multivalued mappings characterizing the game. This approach allows us to combine classical dynamic systems theory with the theory of multivalued mappings.

We then define game strategies, propose a method of constructing the semi-group of multivalued mappings, and investigate some individual cases.

Finally, we consider games in which the goal can only be achieved by the combined efforts of all players, not by any one player individually.

ADAPTATION AND RELIABILITY OF DEVELOPMENT PLANS AND ECONOMIC SYSTEM MODELS

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The increasing complexity and dynamical properties of economic systems requires that we pay greater attention to their adaptive properties. Most economico-mathematical planning models assume deterministic initial information, although this assumption is not usually justified in practice. This noncoincidence can be considered to be an effect of disturbances. The introduction of an adaptive decision-making process makes it possible to link the following stages: prognosis, selection of a course of action, observation during implementation, control following implementation, and optimal correction of the real trajectory of the system.

To optimize an economic plan under conditions of uncertainty and with probabilistic initial information requires:

- the selection of a plan with the greatest possible reliability
- the existence of quantitative estimates for this reliability and for the degree of deviation of the real values of output parameters
- some means of guaranteeing the conditions necessary for reliability and adaptivity.

A NEW PROCEDURE FOR COMPUTER-AIDED DESIGN OF NONLINEAR HIGH-ORDER CONTROL SYSTEMS UNDER CONDITIONS OF UNCERTAINTY

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We describe a new procedure and program package for the analysis and development of nonlinear, time-optimal, high-order, hierarchical control systems with inequality constraints under conditions of uncertainty. This problem is of great theoretical and practical importance in research on complex technical and economic systems.

All of the approaches known to us for the development of such systems are characterized by a rapid increase in computation time and complexity as the order of the controlled system increases. Thus, it is practically impossible to obtain any solution for a third-order nonlinear system, even with the aid of large computers.

We propose a new approach for the solution of such problems. This approach is based on the substitution of a finite high-order state space by a combination of two spaces - a functional infinite state space and a finite state space of lower order than the original space.

The most important feature of this substitution method is the quasiproportional dependence of the amount of computation required on the order of the substitute system.

This is ensured by the transformation of the minimal time control problem involving high-order system G

$$G: X \times V \times U \to X \quad , \quad X \subset R^n \quad , \quad V \subset R^r \quad , \quad U \subset R^1$$

by a recursive sequence of problems involving subsystems G_i of lower order n_i:

$$G_i : X_i \times V_i \times (Y_i \times T) \to X_i , \; X_i \subset R^{n_i} , \; n_i < n , \; Y_i , T \subset R^1 \quad ,$$

where $X_i, V_i, Y_i \times T$, $i = 1, \ldots, k$, $n_1 + \ldots + n_k = n$, are compact sets.

The program package for the proposed procedure is written in FORTRAN and runs on an ES-1033 (a computer of the same type as an IBM 360) with operating system OS ES.

The package consists of program modules for basic functions (there are now 15 basic modules in the form of text files), a monitor (organizing program) and accompanying documentation.

The program is compiled by the user. It addresses the monitor, which calls and connects the modules required without any further instructions from the user.

Required input: state equations and associated integration steps, constraints on state variables, external disturbances and control inputs, termination parameters, approximation accuracy of switching curves, and auxiliary variables which make it possible to change the parameters of the functions.

Package output: parameters of the optimal control law, solution of minimum time control problem and auxiliary text information. The package has a storage capacity of 60 KB.

A computer performing 10^5 operations per second would require 2-5 minutes to deal with a typical sixth-order nonlinear system consisting of three levels with constrained input resources.

STATE OF THE ART OF ADAPTIVE CONTROL THEORY

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Classical control theory is concerned with the following problem: given an object of control O and an aim of control Z, to develop a control U which leads object O to aim Z. If there are errors in the definition of O the control U may fail.

Adaptive control is concerned with classes of objects K = {O}. In this case the problem is to design a control U_{ad} which will lead all the objects from K to their goals, assuming that there is no information on the objects during the control process.

The classical branches of adaptive control theory are:

1. Finite automata theory (notably the work of Zetlin)
2. Stochastic approximation theory
3. Theory of optimal regulators
4. Methods of adaptive stabilization for systems of differential equations

The following new branches have recently been investigated:

1. Stability problems in systems of difference equations
2. Optimization problems involving finite homogeneous controlled chains with rewards
3. Optimization problems involving certain classes of discrete-time Markov processes
4. Optimization of certain non-Markov processes, in particular optimization of functionals defined on nonobservable finite Markov chains
5. Stability problems in systems of differential equations.

Research in the last of these areas seems to be well underway.

Of the many adaptive control problems remaining to be solved, that of optimization involving the main classes of Markov processes seems to be the most important.

STUDIES IN APPLIED ECOLOGY

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Several ecological projects dealing with, for example, forest management, the fisheries industry, and models of plant growth connected with crop production are currently in progress at the Computer Center of the Academy of Sciences in Moscow.

Our forestry project is being carried out in collaboration with a group in Finland, and involves the development of a system of models and mathematical methods for analysis and control of forest exploitation. These include a forest industry model and a multiobjective forest management model in which the volume of trees felled is treated as a control variable.

We are also looking at the fisheries industry, using a complex model of the Volga-Caspian region of the USSR. The water use situation in this region is very complicated due to the limited local water resources and relatively high level of economic development. Industry and agriculture have to compete for water, since their water uses often conflict. For example, hydroelectric power stations normally consume a great deal of water in the winter, and as a result the fisheries suffer from lack of water in the spring and summer. The hydroelectric schemes severely restrict the spawning grounds and cause a marked change in the hydrological regime of the river-sea system.

The regional model mentioned above takes into account the interactions between the various sectors of the national economy in the region, and is designed to find an optimal distribution of water among the main users, according to some criterion.

The author is working on the fish module of a larger model connected with the rest of the system through the hydrological regimes of the river. This fish module is basically a model for optimizing the fish population in a regulated river.

The simplified version of the model can be summarized by the following equations:

$$\frac{\partial x}{\partial z} + \frac{1}{v}\frac{\partial x}{\partial t} = - W(x,t,u(t),z) \quad ,$$

$$y(t,0,z) = \int_{\tau\ min}^{\tau\ max} \beta(t,\tau)W(x,t,u(t),z)d\tau \quad ,$$

$$\frac{\partial y}{\partial t} + \frac{\partial y}{\partial z} = - \phi\ (y,t,z,\tau,u(t)), \ J = \int_{t_0}^{T}\int_{0}^{L}\int_{\tau_1}^{\tau_2} y(t,\tau,z)dtd\tau dz \rightarrow max \ .$$

Here x represents the number of adult fish swimming upstream and finding their spawning grounds appropriated; v is the velocity of the fish relative to the banks of the river; z is a distance measured along the river-bed, where z = 0 corresponds to the mouth of the river and z = L to the hydroelectric dam at Volgograd. The function W is the spawning strategy, which depends on several factors, the most important of which is the hydrological regime u(t) of the river. This is the control function of the model. Spawning efficiency is closely connected with the hydrological regime through the flooding of spawning grounds by water from the hydroelectric dam. The so-called birth-rate equation is defined by the fertility coefficient $\beta(t,\tau)$ (the quantity of spawn produced by one fish). The young fish develop and grow in accordance with the survival equation, which reflects their chances of surviving long enough to make the journey downstream to the sea. Hazards that they face include lack of water, floods, and predators.

The parameters of the model were identified by simulation. The optimization model is now being studied in two different ways: by straightforward numerical methods and analytically through Miliutin-Dubovitsky theory.

OPTIMIZATION UNDER UNCERTAINTY

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Many problems in recognition, estimation, identification, control and design reduce to minimization of the mean loss function

$$J(\theta) = E\{Q(\xi,\theta)\} \to \min_{\theta\in R^N} \quad , \tag{1}$$

where E is an expectation operator, $Q(\xi,\theta)$ is a loss function, ξ is random noise, and θ is a vector of parameters [1]. The objective function J is not generally known exactly; only noise-corrupted measurements of this function or its gradient are available. In this case we observe the vectors

$$y_n(\theta) = \nabla_\theta Q(\xi_n,\theta) = \nabla J(\theta) + \xi_n \quad , \quad n = 1,2,\ldots,\xi_n \in R^N \ , \tag{2}$$

where n represents time. The noises ξ_n are, as a rule, assumed to be independent and distributed identically with density $p_0(\xi)$ such that $p_0(\xi) = p_0(-\xi)$. To solve the optimization problem we have to find an optimal solution $\theta^* = \arg\min_{\theta\in R^N} J(\theta)$ and/or minimal mean losses $J^* = \min_{\theta\in R^N} J(\theta)$ by means of an optimization algorithm.

<u>Optimization algorithms</u>. If an optimal solution θ^* exists and is unique, then it is usually found using stochastic approximation algorithms [2,3] of the form

$$\theta_n = \theta_{n-1} - \gamma_n y_n(\theta_{n-1}) \quad , \quad n = 1,2,\ldots,|\theta_0| < \infty \quad . \tag{3}$$

More general algorithms are obtained if the scalar γ_n in (3) is replaced by a matrix Γ_n:

$$\theta_n = \theta_{n-1} - \Gamma_n y_n(\theta_{n-1}) \quad , \quad n = 1,2,\ldots,|\theta_0| < \infty \quad .$$

In particular it can be assumed that $\Gamma_n = \Gamma n^{-1}$, where Γ is a symmetrical positive definite matrix.

Optimal algorithms. The rate of convergence of an algorithm is estimated using an asymptotic matrix of error covariances (AMEC)

$$V = \lim_{n\to\infty} nE\{(\theta_n-\theta^*)(\theta_n-\theta^*)^T\} \quad ,$$

where the smaller the matrix $V(V_1 > V_2$ if $V_1 - V_2 > 0$ is a positive definite matrix), the higher the rate of convergence. If $\Gamma_n = \Gamma n^{-1}$ the AMEC satisfies the matrix equation

$$[(I/2)-\Gamma\nabla^2 J(\theta^*)]V + V[(I/2)-\Gamma\nabla^2 J(\theta^*)] = \Gamma E\{\xi\xi^T\}\Gamma \quad ,$$

where I is the identity matrix and $\nabla^2 J(\theta^*)$ is a Hessian matrix. The AMEC $V = V(\Gamma)$ attains a minimum for $\Gamma = \Gamma^0 = [\nabla^2 J(\theta^*)]^{-1}$ which means that

$$V_{min} = V(\Gamma^0) = [\nabla^2 J(\theta^*)]^{-1} E\{\xi\xi^T\}[\nabla^2 J(\theta^*)]^{-1} \quad .$$

One algorithm, optimal in terms of Γ, which displays the maximum rate of convergence is the Newton algorithm [3,4]:

$$\theta_n = \theta_{n-1} - n^{-1}[\nabla^2 J(\theta^*)]^{-1} y_n(\theta_{n-1}) \quad , \quad n = 1,2,\ldots,|\theta_0| < \infty . \quad (4)$$

A more general form can be obtained through nonlinear transformation of observations:

$$\theta_n = \theta_{n-1} - \Gamma_n \phi(y_n(\theta_{n-1})) \; , \; \Gamma_n = \Gamma n^{-1}, \; n = 1,2,\ldots,|\theta_0| < \infty , \quad (5)$$

where $\phi: R^N \to R^N$ is an odd function [5]. The AMEC now reaches a minimum when $\Gamma^0 = E\{\nabla\phi(\xi)\}^{-1}[\nabla^2 J(\theta^*)]^{-1}$ and the optimal algorithm takes the form

$$\theta_n = \theta_{n-1} - n^{-1} E\{\nabla\phi(\xi)\}^{-1}[\nabla^2 J(\theta^*)]^{-1}\phi(y_n(\theta_{n-1})) \; ,$$

$$n = 1,2,\ldots,|\theta_0| < \infty \quad . \quad (6)$$

Setting $\phi(\xi) = \xi$ we have $E\{\nabla\phi(\xi)\} = I$ and algorithm (6) coincides with (4).

Absolutely optimal algorithms. Each transformation of observations ϕ is associated with its own optimal algorithm (6), and $V_{min} = V(\phi, p_0)$, where $p_0(\xi)$ is the error distribution density.

Setting

$$\phi_0(y) = -\ln p_0(\xi)\Big|_{\xi=y} = -\ln p_0(y) \tag{7}$$

we have

$$V(\phi_0, p_0) = [\nabla^2 J(\theta^*)]^{-1} \Pi_F^{-1}(p_0)\, [\nabla^2 J(\theta^*)]^{-1} \leq V(\phi, p_0) \quad \forall\phi , \tag{8}$$

where

$$\Pi_F(p_0) = E\{\nabla\phi_0(\xi)\nabla^T\phi_0(\xi)\} = -E\{\nabla\ln p_0(\xi)\nabla^T \ln p_0(\xi)\} \tag{9}$$

is a Fisher information matrix.

From (8) it follows that the AMEC $V(\phi_0, p_0)$ of the algorithm

$$\theta_n = \theta_{n-1} + n^{-1}[\Pi_F(p_0)\, \nabla^2 J(\theta^*)]^{-1} \nabla\ln p_0(y_n(\theta_{n-1})) \quad ,$$

$$n = 1,2,\ldots, |\theta_0| < \infty \tag{10}$$

is the smallest possible [5]. Such algorithms are referred to as absolutely optimal. This algorithm converges faster than (6) and (4) because it takes into account the *a priori* information on the noise through an optimal nonlinear transformation $\phi_0(y)$ determined in equation (7).

Algorithms which are absolutely optimal on a class. If the noise distribution density $p_0(\xi)$ is known only to belong to a certain class $p_0(\xi) \in \mathcal{P}$ then, determining the least favorable distribution density $p_*(\xi)$ on this class using

$$\Pi_F(p_*) \leq \Pi_F(p_0) \qquad \forall p_0(\xi) \in \mathcal{P} \quad , \tag{11}$$

we have

$$\phi_*(y) = -\nabla \ell n\ p_*(\xi)\Big|_{\xi=y} = -\nabla \ell n\ p_*(y) \quad . \tag{12}$$

Algorithms which make use of this nonlinear transformation of observations, such as

$$\theta_n = \theta_{n-1} + n^{-1}[I_F(p_*)\nabla^2 J(\theta^*)]^{-1}\nabla \ell n\ p_*(y_n(\theta_{n-1})) \quad ,$$

$$n = 1,2,\ldots,|\theta_0| < \infty \quad , \tag{13}$$

are absolutely optimal on class P, and for these the AMEC $V(\phi,p)$ satisfies the conditions

$$V(\phi,p) \le V(\phi_*,p_*) \le V(\phi,p_*) \qquad \forall p_0(\xi) \in P \ , \quad \forall\phi \quad . \tag{14}$$

Algorithms which are absolutely optimal on a class are stable and not very sensitive to relaxation of the assumption on the noise distribution density which defines the optimal nonlinear transformation from the true distribution density [6]. Such algorithms are referred to as robust [7,8].

Algorithms such as (13) which are absolutely optimal on a class are robust because, whatever the actual noise distribution density $p_0(\xi) \in P$, the estimates they produce are guaranteed to be of a certain accuracy.

If the distribution density class reduces to a single distribution density, then algorithm (13) reduces to the absolutely optimal algorithm (10).

Criterial algorithms. The basic goal of the optimization algorithms described above is to determine the optimal solution θ^*. Very often, however, it is useful to determine the minimum value of mean losses $J = \min_{\theta\in R^N} J(\theta)$ rather than the optimal solution θ^*, which is not necessarily unique and may not even exist. We shall refer to algorithms which solve this problem as criterial. In what follows we will be concerned with mean losses which meet the condition

$$\nabla^T J(\theta) H \nabla J(\theta) \geq 2(J(\theta) - J^*) \quad . \qquad (15)$$

The performance of criterial algorithms, i.e., the rate of convergence of $E\{J(\theta_{n-1})\}$ to J^* is estimated in terms of asymptotic deviation (AD)

$$v(\phi,p) = \lim_{n\to\infty} n(E\{J(\theta_n)\} - J^*) \leq v_*(\phi,p) \quad ,$$

where $v_*(\phi,p)$ is some accessible AD upper bound $v(\phi,p)$.

Absolutely optimal criterial algorithms are represented in the form

$$\theta_n = \theta_{n-1} + n^{-1} H I_F^{-1}(p_0) \nabla \ln p_0(y_n(\theta_{n-1})) \ , \quad n = 1,2,\ldots, |\theta_0| < \infty.$$

The optimal transformation of observations $\phi_0(y)$ is determined from (7).

For these algorithms we have

$$v(\phi_0,p_0) \leq v_*(\phi,p_0) \quad .$$

If it is only known that $p_0(\xi) \in P$ and there exists a $p_*(\xi)$ which satisfies condition (10), then

$$v_*(\phi_*,p_0) \leq v_*(\phi_*,p_*) \leq v_*(\phi,p_*) \qquad \forall p \in P \quad , \quad \forall \phi \quad ,$$

where the optimal transformation of observations $\phi_*(y)$ is determined from (11), and the algorithm

$$\theta_n = \theta_{n-1} + n^{-1} H I_F^{-1}(p_*) \nabla \ln p_*(y_n(\theta_{n-1})) \ , \quad n = 1,2,\ldots, |\theta_0| < \infty$$

is a criterial algorithm which is absolutely optimal on P [8]. It differs from algorithm (13) only in that the inverse Hessian $[\nabla^2 J(\theta^*)]^{-1}$ is replaced by the matrix H which characterizes the *a priori* information on the mean loss function (1).

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DESIGN OF INTERACTIVE PROCEDURES FOR SOLUTION OF MULTICRITERIA OPTIMIZATION PROBLEMS

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Interactive procedures have recently been used quite widely in the solution of multicriteria optimization problems. Many of these procedures require the decision maker to indicate his preferences on a set of criteria, allowing him to modify these preferences through the interactive dialogue. In virtually all cases the set of admissible solutions of the multicriteria optimization problem is assumed to remain unchanged throughout the interactive procedure. However, experience gained at the Institute of Cybernetics of the Ukrainian Academy of Sciences has shown that the domain of admissible solutions can vary if informal methods are used to try to improve the model parameters. This is sometimes done because the initial formulation of the multicriteria optimization problem cannot take into account all possible variants of model parameters and modifications are occasionally necessary. It is not usually desirable to introduce variable model parameters into the initial formulation of the multicriteria optimization problem because it would then be necessary to class them with the variables determining the solution of the problem. This would mean that optimal values for each criterion would be obtained for different parameter values, i.e., for different sets of admissible solutions. Thus, the set of admissible solutions is varied only in the search for compromise solutions with the criteria values chosen by the decision maker. This can be taken into account in the design of interactive procedures in a number of different ways:

- The decision maker may modify his preferences by proposing a new "ideal" solution at each step of the interactive dialogue.

- Formation of a new domain of admissible solutions with the possibility of varying the parameters of the models which define this domain (system optimization in multicriteria problems).

- Changes in the decision maker's preferences result in the formation of a new domain of admissible solutions at each step of the interactive dialogue.

We have investigated the sequence of problems which must be solved to construct such interactive procedures and looked at the conditions under which they converge to efficient solutions with criteria values acceptable to the decision maker. Since the algorithms used to implement these procedures depend considerably on the nature of the criterion functions and constraints describing the domain of admissible solutions, we have concentrated on algorithms for solution of multicriteria linear programming problems.

A PROTOTYPE PROBLEM IN DUAL CONTROL OF NATURAL RESOURCES

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We regularly encounter cases in renewable resource management where (1) there is large uncertainty about system response, but this may be represented in terms of a few major hypotheses or models; (2) the uncertainties are connected with responses at larger spatial/temporal scales than are practical to study experimentally; and (3) at least some control actions are differentially informative in that the expected response to them is different under different hypotheses, i.e., the controls potentially constitute a sequential experiment.

A simple way to formulate these problems is as Markov decision processes. This leads to the following problem statement.

State dynamics and response to control. Describe the system at time t as being in one of a set of possible discrete states, $S_t = 1,...,n_s$. Let the control possibilities also be contained within a discrete set $U_t = 1,...,n_u$. The alternative models $m = 1,...,n_m$ must then specify the probabilities $p(S_{t+1}|S_t,U_t,m)$ of reaching each state S_{t+1} if control U_t is applied to state S_t during the interval t, t+1, assuming that hypothesis m holds.

An "informative action for S_t" is then defined as any situation where the action U_t is such that $p(S_{t+1}|S_t,U_t,m) \neq p(S_{t+1}|S_t,U_t,m')$ for at least one S_{t+1} and pair m,m'.

We can also represent the above model in terms of a set of matrices $P_m(U)$, $m = 1,...,n_m$, $U = 1,...,n_u$, where $P_{ij} = p(i_{t+1}|j_t,U,m)$. We assume in the following that these matrices are specified at the outset, usually by discretization of dynamic models representing the alternative hypotheses.

Information state and learning. Suppose the models are assigned degrees of credibility or probabilities of being correct, so that at time t the decision maker places probability $B_t(m)$ ($m = 1,\ldots,n_m-1$, $\sum_{m=1}^{n_m} B_t(m) = 1$) on model m being correct. Then he should update this information state after the interval t,t+1 by using Bayes theorem with his observation of S_{t+1} (and U_t, S_t):

$$B_{t+1}(m) = \frac{p(S_{t+1}|S_t,U_t,m)B_t(m)}{\sum_{m'=1}^{n_m} p(S_{t+1}|S_t,U_t,m')B_t(m')} \qquad (m = 1,\ldots,n_m) . \qquad (1)$$

Here we will sidestep the issue of pure prior probabilities $B_0(m)$ by considering the policy that is optimum at $t >> 0$ without ever specifying the origin of B_t (i.e., use backward recursion from some distant endpoint $T >> t$).

In what follows it is not essential that the information state be updated using (1). We will seek the policy that is optimum for whatever rule is used to update the $B_t(m)$, assuming only that the same rule will be applied consistently in the future. Note that under the assumption that the "true" model is actually in the set $m = 1,\ldots,n_m$, equation (1) provides sufficient statistics for all information contained in the historical sequence $S_0,U_0,S_1,U_1,\ldots,S_t,U_t$ (there is no reason to retain these individual observations as explicit dimensions of the information state description).

Optimization objective. We suppose a short-term payoff measure (yield or net revenue or whatever) that depends on the state transition, the control, and the correct model: $v(S_{t+1},S_t,U_t,m)$. The global objective from time t forward would then be to maximize the expectation:

$$V_t = E(v_t + \lambda v_{t+1} + \lambda^2 v_{t+2} + \ldots \lambda^T v_{t+T}) \quad , \qquad (2)$$

where $0 \leq \lambda \leq 1$ is a discount measure. T may be considered infinite in practice, so we seek a stationary feedback policy

$u(S_t, B_t(1), \ldots, B_t(n_m-1))$ that gives the optimum action as a function of the current system and information state.

Assuming that a feedback policy $U(S,\underline{B})$ does exist, the optimization problem can be formulated more understandably in terms of the basic equation of dynamic programming, as follows.

At every stage t and for every state $S_t, \underline{B}_t$, choose U_t to maximize the expected value $\hat{\hat{V}}$ of total returns (stage t and forward) across all stochastic possibilities:

$$\hat{\hat{V}}(S_t, \underline{B}_t) = \sum_m B_t(m) \hat{V}(S_t, U_t, m) \quad , \tag{3}$$

where

$$\hat{V}(S_t, U_t, m) = \sum_{S_{t+1}} p(S_{t+1} | S_t, U_t, m) [v(S_{t+1}, S_t, U_t, m) + V^*(S_{t+1}, \underline{B}_{t+1}, m)] \; . \tag{4}$$

Here the value function $V^*(S_t, \underline{B}_t, m)$ is the maximum expected total value from time t forward if the state is S_t, the information state is $\underline{B}_t$, and the true model is m. Note that the decision maker at time t will not know m, and so should choose that U_t^* which maximizes $\hat{\hat{V}}$. Then given $V^*(S_{t+1}, \cdot)$, it is easy to compute $V^*(S_t, \cdot)$ from

$$V^*(S_t, \underline{B}_t, m) = \hat{V}(S_t, U_t^*, m) \quad . \tag{5}$$

The maximization of $\hat{\hat{V}}$ can be thought of as a local objective for the decision maker at time t (if he is an expected-value maximizer). Note that he will implicitly take the effects of his decision on the information state available to the decision maker at t+1 into account if he uses equation (1) or any other "filter" to compute $\underline{B}_{t+1}$ when evaluating the $V^*(S_{t+1}, \underline{B}_{t+1}, m)$ component of $\hat{V}$.

It is not clear whether the maximization of $\hat{\hat{V}}$ sequentially (by backward recursion dynamic programming) is equivalent to the global maximization of V_t in equation (2). Equation (3) explicitly recognizes that each decision maker in the sequence t+1,...,T will reevaluate the odds $B_t(m)$ to place on the alternative models, and modify his behavior accordingly. It is conceivable that a global decision maker (with the power to commit all future decision makers) could do better by requiring his successors to follow an open-loop experimental regime for some time without becoming "confused" along the way by random changes in B(m). We would appreciate methodological comments on this matter.

Solution methods and early experience. The only option we have seriously explored is value iteration dynamic programming. This can be done easily, even using micro-computers, for small problems ($n_s = 5$, $n_c = 5$, $n_m = 2$). The only serious computational burden is the interpolation of $V^*(S_{t+1}, \underline{B}_{t+1}, m)$ in the $\underline{B}$ dimensions (which are continuous on {0,1}).

It appears that the B dimensions of V^* can be discretized quite coarsely (5-6 levels between 0 and 1). V_t^* seems to vary smoothly with respect to each B_m, provided t << T; for any m, V_t^* is maximum when $B_t(m) = 1$, and minimum when $B_t(m) = 0$.

Most problems appear to converge (the values of $U(S,\underline{B})$ stop changing) after about ten backward steps, even if $\lambda = 1$. In cases for which there is a "super-informative" decision choice U^{**} that instantly sends all the B's to 0 or 1, but is not optimum for either model ($n_m = 2$), this choice is taken as optimum unless the B's are all near 0 or 1 in the first place. In other words, it is quite possible for the optimum decision to be one that none of the competing hypotheses would prescribe.

Thoughts on the form of optimal dual control policies. By solving some simple examples, we seek to gain some qualitative understanding of $U(S,\underline{B})$ that can be used to guide policy design in more complex cases. Analysis of the simple model outlined above has already led to some potentially useful observations.

In particular, we can specify more clearly where it is not worth implementing an actively adaptive, probing policy. Experimentation is not worthwhile when any of the following conditions is met:

1. All decisions are equally informative (no dual effects, as in linear-quadratic control problems, or when all models imply very different responses).

<u>or</u> 2. All models imply the same optimal policy (alternative ecological hypotheses often specify differences in behavior at extreme states, but not under "normal" operating conditions; it is not worth probing to see if a disaster <u>could</u>, but should not, be induced).

<u>or</u> 3. Predicted performance is the same under all hypotheses at the feasible decision choices (all models are effectively the same with respect to control).

These conditions imply that experimentation may be needed for only a few special types of problems, namely those where:

1. At least one alternative model m^* implies a distinctive optimal decision.

<u>and</u> 2. Following the optimal decisions implied by the other models will not provide any information as to whether m^* is correct (m^* must predict a performance as high as that predicted by other models if "their" optimal decisions were chosen; m^*'s optimum payoff must therefore be higher than theirs (otherwise it would not imply a different optimum decision)).

More simply, it would appear that dual control is worth considering only when there is a credible hypothesis m^* that predicts some opportunity to improve payoffs by moving outside the (uninformative) set of recent decision choices. Many natural resource problems fall into this category, especially when historical policy choices have been deliberately conservative.

MODELING AND SOLUTION STRATEGIES FOR STOCHASTIC OPTIMIZATION PROBLEMS WITH INCOMPLETE INFORMATION

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The problem of finding the solution x^* of a minimization (optimization) problem when the objective function f (cost function, performance criterion,...) is explicitly available, reduces to solving the inclusion

$$x^* \in \operatorname{argmin} f = \{x | f(x) \leq \inf f\} \quad . \tag{1}$$

There are no real conceptual difficulties here: it is only necessary to find an efficient procedure that yields x^*. Depending on the properties, in particular on the complexity, of the function f, there is a rich collection of methods for doing exactly this. Most of these methods depend on the possibility of obtaining (at small computational cost) the values and/or gradient of the function, or, better still, some second-order information.

However, when optimal decisions must be reached in an environment bedevilled with uncertainties, not only does the formulation of the model demand a deeper probing of the aspiration criteria in order to establish the appropriate form of the model, but also conceptual questions must be raised in connection with the significance and reliability of the data available about the random phenomena. In addition, major obstacles must be overcome in the calculation of "optimal" decisions, if this is viewed as <u>the</u> solution to the problem in question. The model and the type of solution sought predetermine the appropriate solution strategies, and thus there is a strong interaction between these various aspects of the problem.

Suppose we have a cost function given by

$$(x,\xi) \longrightarrow f(x,\xi) : R^n \times \Xi \longrightarrow R \quad ,$$

where ξ are random parameters whose real values will only be revealed after a decision x has been selected. By Ξ we denote the sample space of ξ, i.e., the set of possible values of the random variables. If the decision maker is indifferent to risk and confident that he has sufficient knowledge about the distribution P of the random events, the optimal decision can be reached by finding an x^* satisfying

$$x^* \in \operatorname{argmin} F \quad , \tag{2}$$

where

$$F(x) = E\{f(x,\xi)\} = \int f(x,\xi)P(d\xi) \quad . \tag{3}$$

In theory every procedure developed to solve (1) could be used to solve (2). However, the implementation of these methods demands easy access to function values, gradients, and even Hessians. Given the limitations of multivariate calculus, these quantities can often only be obtained with reasonable accuracy at very high cost, which may far exceed any gain derived from knowledge of the optimal solution.

However, ignoring these numerical difficulties for the time being, this type of approach may only be used when the problem at hand satisfies all the conditions outlined above.

We consider alternative formulations, their relations to (2), the problem of insufficient data to determine P, and approximation schemes for solving (2).