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Input-Output Modeling: Proceedings of the Third IIASA Task Force Meeting

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INPUT – OUTPUT MODELING

PROCEEDINGS OF THE THIRD IIASA TASK FORCE MEETING
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Maurizio Grassini and Anatoli Smyshlyaev, Editors

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PREFACE

Input-output modeling at IIASA has a relatively short but interesting history. It started in 1979 with the pioneering efforts of Clopper Almon, who visited IIASA from the University of Maryland to lay the foundations of cooperation with the INFORUM project. The primary goals of the research at IIASA were to further develop econometric input-output models, to link individual national models on the basis of the software developed by Almon's team, and to build a collaborative network of scientists in different National Member Organization (NMO) countries who would contribute these models to IIASA. These aims were successfully realized within a few years by the installation at IIASA of 18 national models (varying in size and complexity), the dissemination of the necessary software to numerous institutions, and the linkage of three national models (of France, Belgium, and the Federal Republic of Germany). The software package, called SLIMFORP, has been transferred to practically all NMOs and implemented on a variety of computers to help scientists in their economic research. All these developments have demonstrated the usefulness of the initiative and a large network of scientists from NMO countries and elsewhere has been established.

An important role in this input-output work has been played by the annual conferences of the IIASA-INFORUM "family", which have been held at IIASA since 1980. During these three-day meetings, scholars from different countries have presented their experience on input-output analysis and forecasting, mostly within or based on the INFORUM framework. Potential users and prospective collaborators with the INFORUM project have normally also been invited. Therefore the papers in this volume represent the latest state in input-output research at IIASA and elsewhere in the INFORUM network. Additional information can be found in other IIASA publications, for example the working and collaborative papers by Clopper Almon, Douglas Nyhus, Ulrike Sichra, and Maurizio Ciaschini, which are available on request.

Now IIASA is embarking on a new project entitled "Patterns of Economic Structural Change and Industrial Adjustment", in which the input-output modeling work (previously under the umbrella of the System and Decision Sciences research area) will play an important part. It is therefore important to select approaches from previous IIASA work and the ongoing research of our collaborators on the INFORUM project that will contribute significantly to a better understanding of the deep structural changes currently affecting national economies. We sincerely hope that many of the scientists whose papers are included in this volume will collaborate fruitfully with the new IIASA project.

Anatoli Smyshlyaev
Patterns of Economic Structural
Change and Industrial Adjustment

CONTENTS

INTRODUCTION

Maurizio Grassini and Anatoli Smyshlyaev
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The papers presented in this volume differ somewhat in their styles of presentation; many have been significantly revised since they were delivered at the Task Force meeting in September 1982 and we hope that the way in which they are now organized will help readers to discover common problems and explore the possibilities of joint research in the future. The papers have been divided into two groups. The first, on linkage of input-output models, also includes a few papers on models ready for linkage but not yet linked; the second group deals with the analysis and investigation of structural change, but also contains a description of recent developments in the INFORUM family of models.

The first part of the volume opens with a contribution from Nyhus (USA) dealing with the linkage of seven national models (of the USA, Canada, Japan, the FRG, France, Italy, and Belgium) by means of the INFORUM international trade model. The aim of this exercise is twofold; firstly, it proves that the linkage of input-output models of different scales is feasible using this trade model, and secondly, the assumption that the evolution of foreign demand is confined to a "smaller" rest of the world - based on an estimate of the multiplier effect of the intra-country trade - can easily be tested.

The international part of the Belgian INFORUM model presented by Tahon and Vanwynsberghe (Belgium) is an attempt to define a suitable international scenario for a single national model. Because no comprehensive world model is available, it is usual to rely upon standard assumptions on the growth rate of international demand as a driving hypothesis for simulations of national economies. However, when an economy is relatively open, that is to say where the international trade portion of total GDP is high, this kind of assumption tends to be too rough and a more detailed description of the inflows and outflows for services and goods is called for. This is because in such cases the performances of international trading partners strongly influence the trade pattern of the national economy studied.

The four Nordic countries - Denmark, Finland, Norway, and Sweden - are easily classified as small, open economies; moreover, they are geographically close to each other and there is a considerable amount of intercountry trade between them. In order to study the structural interdependence of the Nordic economies, input-output models of the four countries are going to be linked by means of a complete international model of the IIASA/INFORUM type. The work completed, and that remaining to be done, is described in the three papers by Bjerkholt and Sand (Norway), by Olsson and Sundberg (Sweden), and by Thage (Denmark). Bjerkholt and Sand outline the model system in use in Norwegian economic planning and the proposed framework of the Nordic INFORUM system of models. Particular attention is devoted to the role of the Norwegian model as a supporting tool within the Advisory Board of the Ministry of Finance. The determination of total import demand for each country and each commodity group will form the core of the trade model, and Olsson and Sundberg present the analytical structure of the import equations. In the third paper of this group, Thage reports the first contributions to the project from Denmark. He describes the construction of a statistical data base providing the necessary code for understanding the connection between international trade data classifications and branches of the Danish input-output table.

The paper by Martellato (Italy) deals with the problem of disaggregating macroeconomic models, in terms of the trade-off between detailed descriptions of either the process of production or the spatial dimension of an economy. If the first step in disaggregation is a sectoral representation of the process of production - which leads essentially to modern input-output modeling - sooner or later spatial or regional disparities will represent the major constraints on reaching a better understanding of economic phenomena. Ways of integrating a spatial dimension into a national input-output model are analyzed, together with the relationships between capacity constraints and trading behavior. The construction of a regional input-output model within the framework of a national model is discussed, using as an example IIASA's case study of Tuscany. In further work related to the Tuscany study, Laura Grassini (Italy) reports on the estimation of the demand equations system in the biregional model of Tuscany and the rest of Italy. She describes the data sources used and explains how data from different sources have been reconciled; after a brief outline of the model's structure she discusses the treatment of income elasticity and presents some preliminary results.

Continuing with regional input-output models, Münzenmaier (FRG) suggests a method for evaluating the division of labor in a national economy, with reference to a case study of the Federal Republic of Germany and the federal state of Baden-Württemberg. Input-output tables provided by the Federal Statistical Office as an economic statistical basis refer to different levels of jurisdiction. Due to lack of information concerning intraregional trade, the integration of a region into the national economy cannot be analyzed in the framework of a standard multiregional model. A parallel analysis of the impact of export demand over regional and national economies is shown to be useful for measuring the direct and indirect consequences for a single region of integration within a national economy.

National econometric models based upon input-output tables are now growing in complexity and A. and L. Tomaszewicz and Welfe (Poland) present here an input-output model for the Polish economy.

It is an INFORUM-type model whose construction is underway. Among the preliminary results presented, it is interesting to note the attention given to the impact of technical coefficient changes on production forecasts. When defining scenarios, the empirical results emphasize the importance of structural changes in the production processes. Without knowledge of these changes, activity levels would otherwise have to be explained by somewhat questionable procedures, such as "residual methods".

The Finnish Long Range Model System presented in two papers, the first by Forssell, Mäenpää, and Svento (Finland) and the second by Svento alone, is an example of modeling price, output, and income within an input-output framework. This method permits a richer set of control rules to be used when simulating economic growth on the basis of different economic theories.

The paper by Ciaschini (Italy) discusses the use of modern input-output models as simulation tools for policy making. He describes the construction of part of the Interindustrial Italian Model (INTIMO), a modern model of the INFORUM family, and presents the results of various simulations. In particular, he shows how a simple investment theory was used to estimate sectoral investment functions, and under which input-output technical coefficients were made to change according to forecast patterns.

The section closes with a paper from Almon (USA) presenting the most advanced model of the INFORUM group. Here attention is focussed on the price-income block recently developed; knowledge of the real side of the model, which is the fundamental requirement for membership of the INFORUM group, is largely taken for granted. The paper is divided into two parts. First, a number of observations on income-side modeling are addressed to anyone contemplating the implementation of such a model; they comprise an interesting set of comments on the accounting framework and the structural equations. Second, a few simulations with the most recent version of the model are presented; these concern the evaluation of the effects of tax cuts, increased defense spending, and increased transfer payments, all of which are currently possible US economic policy measures.

The second part of the volume contains papers dealing mainly with structural changes in economics: the authors discuss investment behavior, labor productivity, patterns of consumer behavior, and changes in technical coefficients. The first three papers, by Barbera (USA), Schmoranz (Austria), and Bell (UK) present results of factor input investigations within the input-output framework. Both Barbera and Schmoranz use a "revised perpetual inventory model" to estimate capital stock data necessary for modeling investments and labor inputs. Schmoranz gives only preliminary results for capital stock, whereas Barbera has estimated fairly developed models for the behavior of both investment and labor productivity. Gross investment is divided into two compound groups, net and replacement investments, that improve our understanding of various important issues. In his paper, Bell deals only with labor inputs for the UK economy. He concludes that labor productivity on the level of industries cannot be uniformly represented by the simplified models usually applied at the macroeconomic level. All three papers seem to be well balanced in terms of methodology, econometrics, and empirical results. Though developed in different countries, the three papers are complementary and clearly pinpoint the most crucial issues in factor demand modeling.

The next three papers differ considerably in their approaches despite the fact that they all study energy-economy interactions. The paper by Beutel (FRG) shows how to transform a standard input-output model to a linear optimization formulation, with a substantial gain in the information supplied to users. The rectangular presentation of intermediate flows makes it possible to disaggregate energy inputs to whatever degree required and then to apply the model for intercountry comparisons. Alessandroni (Italy) and Koprinkov (Bulgaria) show the impact of different energy-input coefficients on overall economic growth. Their preliminary results illustrate the need for energy submodels to be incorporated properly into an input-output scheme of forecasting because rough estimates of energy-economy interactions should be supported by engineering data.

The papers by Dimitrov (Bulgaria) and Csepinszky (Hungary) consider possible approaches to developing a consistent dynamic input-output model using a poor data base. It is shown that at the most aggregated level various methodological problems must be solved to arrive at a dynamic or semidynamic model. Both authors present results of their econometric analysis at a fairly aggregated level. However, it should be pointed out that Csepinszky's division model is a simplified version of the very detailed description of the Hungarian economy developed in the Hungarian Statistical Office.

The use of an input-output model for deriving aggregate characteristics of future overall structural changes is given in a paper by Ludwig, Biebler, and Kraft (GDR), which studies how long-term structural changes are reflected in terms of either full labor, energy, or capital requirements.

Three papers are devoted to changes in technical coefficients. Skolka (Austria) considers problems of the comparability of two input-output tables for the Austrian economy (1964, 1976). He uses these data to distinguish significant interindustry interactions that will later be studied further to analyze the role of prices in the changes. This topic is also discussed in the paper by Erber and Stäglin (FRG). They use different techniques to simulate the behavior of the technical coefficients, and also give a short survey of the related problems under investigation using the Disaggregated Bonn Forecasting Model II. These two papers contribute significantly to our understanding of the limits on using outdated technical coefficients in forecasting. Kárász (Czechoslovakia) describes the system of Czechoslovakian input-output data that permits him to apply the Markov-chain technique to examine the stability of technical coefficients and to use these results in forecasting.

Two papers discuss the problems of national model development using a limited data base. Rainer (Austria) emphasizes the important role of trade and transport margins in empirically oriented input-output research. These margins are especially important for a small, open economy and also play a significant role in the linking of the real and price sides of an input-output model. The author gives details of the estimation process being used for Austrian economic data. Smyshlyaev (USSR) shows that it is both possible and useful to combine input-output data with current statistics. One of the main reasons is that econometric techniques circumvent some difficult methodological problems in model reconstruction and make it possible to apply estimates from historical data in forecasting.

The next two papers present much more formal treatments of the same subject area: estimation techniques and the problems of limited or inconsistent data bases. Weale (UK) discusses the validity of the estimation techniques used on data containing errors. He has applied a technique used in balancing national accounts to the derivation of price indices. The procedure is also illustrated with reference to a small input-output system for the UK. Harrigan (UK) also considers the reconciliation of related but inconsistent data sets. His paper gives a short overview of the problems and techniques used previously and a new approach developed by the author is applied to consumption data for Scotland.

When dealing with a comprehensive input-output model, it is necessary to ensure the efficient representation of all possible relations between economic indicators. In the final paper of the volume, Sekerka (Czechoslovakia) describes the possibilities of the so-called "system for systems design" in input-output related studies.

PART ONE

LINKING SEVEN INPUT–OUTPUT MODELS OF THE INFORUM SYSTEM

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This paper reports on an exercise in which seven of the input-output models of the INFORUM system are linked together, industry by industry, year by year, through bilateral export demand indexes. The seven are: USA, Canada, Japan, West Germany, France, Italy and Belgium. The explicit linking equation used has been described previously in a more limited exercise (Nyhus, Almon, 1981).

For clarity, the linking mechanism is briefly described. We relate the exports of one country directly to the domestic demands in customer countries for the product. More precisely, we estimate the equation

$$(1)\ X(t) = (a + b \sum_k w_k\ D_k(t)/D_k(0))\ (d/f)^n$$

where

X is one country's exports of a particular commodity

w_k is the fraction ot these exports which went to country k in the base year ot the country

D_k is the index of domestic demand (output + imports - exports) in country k

f/d is a moving average of foreign and domestic prices

n is the price elasticity of demand.

In the foreign price index, f, foreign domestic prices of the customer countries are combined with weights proportional to the share of global exports of that customer country -- i.e. f is an index based on competitor's prices. By taking moving averages of prices, we allow for delay in the response of exports to changes in relative prices. The estimation of the weights in these moving averages is taken from [Nyhus, 1975]. Of course, products seldom have the same definition in the input-output tables of different countries, so the D_k do not, in fact, match X perfectly in definition. In some cases, several sectors in a customer country will be combined to give a single D_k; in other cases, a single input-output sector domestic demand in the buying country has to serve as the D_k for several sectors in the exporting country's table.

Summary descriptions of the models are given in Table 1 below.

Table 1

Model Description

Country	Number of Sectors	Base Year
USA	78	1977
Canada	94	1971
Japan	67	1975
West Germany	53	1975
France	88	1976
Italy	45	1975
Belgium	51	1970

To see the linking at work, a simple assumption is common to all of the models was changed. In the alternative solution, the assumed rate of growth of domestic demand for all countries <u>not</u> in the group of seven linked countries is two percent per year faster than in the base solution in the period 1985-1990, specifically, they are summarized in Table 2.

Table 2

Alternative Industrial Growth Rate Assumptions

Country	Base	Alternative
Netherlands		
Through 1981	Actual	Actual
82-85	2.2	2.2
85-88	2.2	4.2
88-90	1.9	3.9
United Kingdom		
Through 1981	Actual	Actual
82-83	1.0	1.0
83-85	2.0	2.0
85-88	2.0	4.0
88-90	2.1	4.1
Rest of the World		
Through 1981	Actual	Actual
82-83	1.0	1.0
83-85	2.8	2.8
85-90	3.0	5.0

The "base" solution was derived by iterating the models in the following manner; The foreign demands for the American model were derived using whatever previous forecasts were available for the other six plus the "base" assumptions, for the non-seven. Next, the Canadian model was solved using the American model's results, the "base" assumptions and the other five results. This process continued, following the order of the countries in Table 1, until the last model, Belgium, was solved. The whole process was repeated for each of the seven, each time utilizing the most recent results. Four such overall iterations were done. The results of the last iteration became the "base" solution. The "alternative" solution was arrived at in an almost identical manner. The slight difference appeared during the first iteration. In this first iteration of the seven models, only the alternative non-seven assumptions were different. Thus, the solution of the Belgium model, the last of the seven, on the first iteration utilized the "base" solution for the other six and the alternative for the non-seven. In this manner we can distinguish the "direct" (i.e. non-seven) effects from the indirect (i.e. interactive) effects. Table 3 shows some of the principal aggregate results for each of the seven countries.

The first column, labeled "Base 85-90", shows the continuously compounded annual growth rate for 1985-1990 of the row items. The second column, labeled "Direct 85-90", shows the results at the end of the first iteration. The third column, labeled "Alternative 85-90", shows the results of the fourth iteration. The fourth column shows the proportion of the total change of the row items, alternative-base, which can be attributed to the direct result. The last column, labeled "Change in Exports", shows the change in the export growth rate under the alternative.

A principal result is that the system converged quickly. More will be said on this is point later. Another result is that the effects were quite varied, ranging from only a .36% per year increase in export growth in Canada to 1.83% per year growth in Japan. A principle factor contributing to this difference is that the non-seven factor in foreign demand is relatively high for Japan (on average 60%) and low for Canada (25%). While demands in the seven increased, they were in every case less than the two percent increase assumed by the non-seven. That however cannot explain all of the differences. The non-seven effect is

Table 3
Effects of Alternative Assumptions Concerning
Non-seven Growth

	Base 85-90	Direct 85-90	Alternative 85-90	% Direct	Change Exports
USA					
GNP	2.55	2.63	2.67	91	
Exports	2.73	3.77	3.88	90	1.15
Canada					
GNP	3.38	3.43	3.45	68	
Exports	2.98	3.21	3.34	63	.36
Japan					
GNP	3.95	4.18	4.20	92	
Exports	3.99	5.68	5.82	92	1.83
West Germany					
GNP	1.85	2.26	2.32	87	
Exports	3.05	4.34	4.53	87	1.48
France					
GNP	2.49	2.66	2.70	81	
Exports	2.65	3.26	3.42	78	1.77
Italy					
GNP	3.12	3.68	2.75	88	
Exports	4.02	5.59	5.81	87	1.79
Belgium					
GNP	1.88	2.18	2.26	78	
Exports	3.09	4.00	4.30	75	1.21

shown in the direct column. The American and Japanese models have approximately the same proportion of direct effects. The differences in export growth rate changes occur for two reasons. The export demand elasticities estimated for Japan are higher than those estimated for comparable American products. American exports are primarily in two areas -- food and machinery. Food, in particular, is unlikely to be affected much by the direct effect since incomes in the other six models were left unchanged. Hence, the direct effect would show demands rising for American food in the U.K., but the indirect effects on French de ands for American food would be small. Therefore, we have reason to feel that the direct effects are correct for the U.S.model but underestimate the indirect income in the other models.

Table 4

Running Statistics of the Models

Iteration / Country	Clock (Min.) 1	2	3	4	CPU (Sec.) 1	2	3	4
USA	63	50	160	46	1674	1750	1865	1865
Canada	9	9	32	9	354	408	426	409
Japan	31	25	23	23	990	1028	1019	1019
Germany	6	5	4	5	143	176	173	176
France	12	9	8	9	344	395	388	391
Italy	3	5	4	4	134	172	166	167
Belgium	4	8	6	4	207	265	259	267

	Minimum Clock (Iteration)	Average CPU (2-4)
USA	46 (4)	1820
Canada	9 (4)	424
Japan	23 (4)	1022
Germany	5 (4)	174
France	8 (3)	391
Italy	4 (3)	168
Belgium	4 (4)	264
SUM	99	SUM 4263 (71 min.3 sec.)

The iterative procedure for linking the models had several practical aspects. The models, for most efficient linkage, should be on the disk simultanteously. How much CPU is needed per iteration? How much clock time? For a small machine such as the Prime 550, how is other work affected? How much disk space is necessary? In what order should the models be solved? Table 4 shows some of the major statistics on the computations. The clock time portion of Table 4 shows the wide differences in the running time of the models. The U.S. model is a fully integrated, closed model generating outputs, prices, and incomes. Therefore each side, output, price, and income, is computed several times for each year of the forecast. In contrast, the Belgium model contains only a real side and has fewer sectors. The Japanese model also iterates between its real and price sides until a solution is found. The large variation in clock times for different iterations of the same model is explained by the system load at the time. Iteration 3 began at 1:30 p.m. and ended at about 5:30 p.m. The afternoon was busy and system response was felt to be "slow". Iteration 4 was done when almost no other work was being performed. The CPU times varied little between iterations. Iteration 1 is less because there was no need to extract the information about results from the other models. Utilizing the data for iteration 2-4, we have a minimum clock time of 99 minutes and an average CPU time of 71 minutes. In short, the system solves relatively quickly. Finally, 44 megabytes of our 300 megabyte disk were necessary for the storage of all the programs and data for the seven models.

Table 5
GNP in Successive Iterations

Country \ Iteration	1	2	3	4
USA	2652.01	2653.47	2653.69	2653.70
Canada	1806.24	1807.93	1808.06	1808.06
Japan	2979.71	2982.37	2982.63	2982.66
W.Germany	16850.35	16891.85	16898.90	16898.90
France	18453.12	18387.80	18489.66	18489.54
Italy	2085.79	2093.22	2093.42	2093.38
Belgium	20530.67	20616.25	20616.81	20616.73

Table 5, showing the evolution of the 1990 GNP by country by iteration, shows that only three iterations would be sufficient. The difference between iterations 3 and 4 is minimal.

The country order of solution for quickest convergence can be determined by utilizing a total bilateral trade matrix for the seven together with their trade with the rest of the world. Treating domestic use as known we find that we should solve the U.S. model first since 98.2% of output is known, a higher figure than for any other country. Next, we treat exports to the U.S. as known for each of the remaining countries and find the one with the highest known rate. In this case it is Japan. We add exports to Japan to the known amount of those remaining. The next highest known rate now is for Canada. We continue on, finding in order: Italy, West Germany, France and finally Belgium. The order is, to be sure, very dependent on which country models are included and which are left out.

The result of the exercise is that this particular form of linkage can be done and produces meaningful, sensible results. There is good reason to believe that linkage of the models can continue and that the costs entailed are not unduly large.

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THE INTERNATIONAL PART OF THE BELGIAN INFORUM MODEL

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1. INTRODUCTION

Exports of goods and services constitute almost 63% of Belgian GDP and are, together with imports, among the major determinants of national economic development. All forecasting for the Belgian economy is therefore to a great extent influenced by the type and the nature of the export and import equations, the way they depend on demand and competitive factors, and the evolution of the trade shares of the major import and export partners.

About 70% of Belgium's exports goes to the EEC (the FRG and France each account for about 20% and the Netherlands for 15%). Another 12% has its destination elsewhere in Europe so that only about 17% moves outside Europe. Due to the relatively high proportion of raw materials and sources of energy in total imports, the European share in Belgian imports is somewhat lower (63% from the EEC and 73% from Europe as a whole).

Forecasting future international trade flows for a small and open economy such as that of Belgium is therefore a complex problem. As the INFORUM world trade model is not yet fully operational, the following shortcut method was adopted as an interim solution.

2. THE INFORUM EXPORT-IMPORT EQUATIONS

We take as our starting point the normal INFORUM equations

$$M_{.b,i} = (a_i+b_iD_i)(P_{M_{bi}}/P_{D_{bi}})^{-\lambda_i} \tag{1}$$

$$E_{b.,i} = (c_i+d_iF_i)(P_{E_{bi}}/P_{W_i})^{-\varepsilon_i} \tag{2}$$

where

$i = 1,2,\ldots,n$	=	sector or industry in the I/O model producing goods or services;
b	=	subscript denoting Belgian;
$M_{.b,i}$	=	total Belgian imports of goods or services of type i;
$E_{b.,i}$	=	total Belgian exports of goods or services of type i;
D_i, F_i	=	domestic and foreign demand for goods or services of type i;
λ_i,ε_i	=	import and export price elasticities for goods or services of type i;
$P_{M_{bi}}$	=	import price for goods or services of type i expressed in Belgian francs (import price of similar products of type i (competitive imports));
$P_{D_{bi}}$	=	domestic producer price for goods or services of type i;
$P_{E_{bi}}$	=	export price for goods or services of type i for Belgium;
P_{W_i}	=	price on the world market (excluding Belgium) for goods or services of type i (competitive market price for Belgian exporters expressed in Belgian francs).

The classical INFORUM approach (Vanwynsberghe et al. 1977) uses time trend extrapolations for the relative prices $P_{M_{bi}}/P_{D_{bi}}$ and $P_{E_{bi}}/P_{W_i}$ based on historical patterns.

The demand terms D and F are defined as follows:

$$D_i = Q_i + M_i - E_i \tag{3}$$

where Q_i = output or production of sector i and F_i depends on:

$I_{j,k}$ = industrial production in country j for sector k (index) based on OECD statistics;

$I_{j,.}$ = total industrial production of country j;

$E^0_{bj,i}$ = export flow of product i to country j(j≠b) in the base year;

such that

$$F_i = \sum_{\substack{j=1 \\ j \neq b}}^{r} (E^0_{bj,i}/E_{b.,i}) \cdot I_{j,k} \tag{4}$$

where k is the OECD sector corresponding to the Belgian sector i, and j(j=1,2,...,r) denotes the regions, in this case the FRG, France, Holland, Italy, the UK, the US, Canada, Japan, and the rest of the world.

The terms $I_{j,k}$ or $I_{j,.}$ are either obtained from other models or from a regression of $I_{j,k}$ on $I_{j,.}$ and an exogenous estimate for $I_{j,.}$

$$I_{j,k} = a + b\, I_{j,.} \tag{5}$$

3. THE DEMAND FACTOR EXTENSION (Nyhus, 1975)

The extension used in the Belgian model for the foreign demand factor F_i uses information on OECD trade flows for the I/O sectors for all regions j(j=1,2,...,r) and on price deflators by country from local sources. The domestic demand factor D_i is basically treated as described in Vanwynsberghe et al.

(1977, pp. 21-23) and in Appendix II of this paper; for certain particularly important sectors D_i is defined slightly differently to take into account the phenomena of straightforward reexport and import-processing-export.

Using this information, historical market shares for the Belgian exports by sector are calculated as follows:

$$e^t_{bj,i} = E^t_{bj,i}/E^t_{b.,i} \quad \text{(for } j=1,2,\ldots,r \text{ and } j \neq b) \tag{6}$$

and then extrapolated:

$$e^t_{bj,i} = a_i + b_i\ f(t) + c_i\ .\ e^{t-1}_{bj,i} \tag{7}$$

where f(t) is a decreasing function of time t to slow down the time component.

Each year, market shares are adjusted so that

$$\sum_{\substack{j=1 \\ j \neq b}}^{r} e^t_{bj,i} = 1 \tag{8}$$

Note that the future export market share does not depend on relative prices. This approach, although permissible for already existing systems, was abandoned because of its interdependence with forecasts of future relative prices (as discussed further below).

4. THE RELATIVE PRICE EXTENSIONS

Four prices have to be forecast for each trading sector i:

1. The competitive market price P_{W_i};
2. The Belgian import price $P_{M_{bi}}$;
3. The Belgian domestic producer price $P_{D_{bi}}$;
4. The Belgian export price $P_{E_{bi}}$.

4.1 The competitive market price P_{W_i}

This price will depend on the price of product i in each of the regions j(j=1,2,...,r) weighted by the regional shares of the world market (excluding Belgium) for product i:

$$e^t_{\overline{W}_{j,i}} = E^t_{j.,i} / \sum_j E^t_{j.,i} \qquad (j=1,2,...,r \text{ and } j \neq b) \qquad (9)$$

and

$$\sum_{j=1}^{r} e^t_{\overline{W}_{j,i}} = 1 \qquad (10)$$

The competitive market price for past periods is defined as:

$$P^t_{W_i} = \sum_j P_{E_{ji}} \cdot e^t_{\overline{W}_{j,i}} \cdot EXR(B/j)^t \qquad (11)$$

with EXR(B/j) representing the exchange rate of country j in Belgian francs per local currency unit.

The corresponding future $P^t_{W_i}$ price depends on two sets of factors. First, the labor cost per unit of output in each region j in local currency and the oil price in local currency per barrel (as a substitute or proxy for imports of raw materials) together define the local price $P^t_{E_{ji}}$ in *local currency*. Second, the exchange rate assumptions and the trade patterns $e^t_{\overline{W}_{j,i}}$ translate these prices into one effective price in *Belgian francs*.

However, in order to reduce the number of forecast variables, the following shortcut procedure is followed:

a. the labor cost per unit of output in each region is weighted by the trade patterns $e^t_{\overline{W}_{j,.}}$ (local currency);

b. the oil price in local currency is also weighted as in point a;

c. the $P^t_{W_i}$ values expressed in Belgian francs are converted into local currencies by the weighted currency effect using the $e^t_{\overline{W}_{j,i}}$ values, thus giving new values $P'^t_{W_i}$;

d. the $P'^t_{W_i}$ values are then regressed on (a) and (b) using percentage changes of the variables in current and past years;

e. the $P^t_{W_i}$ values are calculated using the trade-weighted exchange rate effect on the forecast $P'^t_{W_i}$ values.

The labor cost per unit of output in each region and the oil price (in $/barrel) are exogenous variables. Future $e^t_{\overline{W}_{j,i}}$ patterns are extrapolated using a similar procedure to eqn.(7).

Appendix I gives examples of $e^t_{\overline{W}_j}$ patterns for chemicals, wood and furniture.

4.2 The Belgian import price $P_{M_{bi}}$

This price will depend on the price of product i in each of the regions j(j=1,2,...,r) weighted by the market shares of the Belgian imports of product i:

$$e^t_{jb} = E^t_{jb,i}/M_{.b,i} \tag{12}$$

The import price is defined as:

$$P^t_{M_{bi}} = \sum_j P^t_{ji} \cdot e^t_{jb} \, EXR(B/j)^t \tag{13}$$

with EXR(B/j) representing the exchange rate of country j in Belgian francs per local currency unit.

The forecasting of future $P^t_{M_{bi}}$ prices is analogous to that of the competitive market price (described above), except that the trade patterns are different. See Appendix I for examples.

4.3 The Belgian domestic producer price $P_{D_{bi}}$

The domestic producer price will depend first on the unit labor cost in Belgium and second on the cost of imported (raw) materials. As we have abundant and very detailed information on the latter, we prefer to make use of it instead of the rough oil-proxy method used elsewhere.

In fact, we know the imported shares of the intermediate inputs of sector i (from the base I/O year) and we have already forecast the import prices $P_{M_{bi}}$ for all types of imported goods. Therefore, we calculate the costs $C^t_{M_i}$ of imported materials for sector i as follows:

$$C^t_{M_i} = \sum_{\ell=1}^{n} x_{M_{\ell i}} \cdot P^t_{M_\ell} \tag{14}$$

with

$$x_{M_{\ell i}} = X_{M_{\ell i}} / Q_i \tag{15}$$

where $X_{M_{\ell i}}$ represents the imported intermediate flow of goods of type ℓ by sector i.

Then we regress the prices $P_{D_{bi}}$ on the Belgian unit labor cost and on the costs $C^t_{M_i}$ using percentage changes for current and past years. See Appendix I for examples.

4.4 The Belgian export price $P_{E_{bi}}$

When explaining and forecasting Belgian export prices we distinguish two different situations:

1. cases where the Belgian exporting industry is a price *maker;*

2. cases where the Belgian exporting industry is a price *taker.*

In the first situation, export prices are explained and forecast in the same way as the producer prices, using the same cost price elements (unit labor cost and costs of imported materials).

In the second situation, where the Belgian exporting industry is a price taker, the competitive market price P_{W_i} will enter into the equation and will be adjusted to some extent by the producer price $P_{D_{bi}}$.

Once again, the regression equations used are based on percentage changes of the variables in current and past years.

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APPENDIX I: MARKET SHARES

Market shares for Belgian exports (e^t_{bj}): Sector 29: Wood/Furniture

JAAR	NR	CANADA	USA	JAPAN	FRANCE	BRD	ITALIA	NDL	UK	OVERIGE
1963	29	0.04	0.36	0.02	51.41	14.02	3.92	23.64	0.88	5.72
1964	29	0.02	0.40	0.05	44.47	15.28	3.05	32.61	0.41	3.72
1965	29	0.02	0.33	0.04	41.95	17.65	1.95	33.55	0.30	4.21
1966	29	0.06	0.57	0.05	41.06	19.84	1.40	33.21	0.15	3.65
1967	29	0.04	1.58	0.17	42.93	15.48	1.97	33.03	0.28	4.51
1968	29	0.14	0.41	0.06	50.21	14.52	1.14	29.86	0.26	3.39
1969	29	0.09	0.50	0.05	57.00	13.65	0.94	23.32	0.13	4.32
1970	29	0.08	0.45	0.05	39.73	23.63	1.09	29.38	0.26	5.34
1971	29	0.05	1.02	0.02	40.69	23.73	0.85	29.73	0.75	3.17
1972	29	0.11	1.30	0.02	36.27	27.05	0.76	29.43	1.76	3.29
1973	29	0.13	1.16	0.10	36.51	27.81	0.66	27.33	2.69	3.61
1974	29	0.09	0.57	0.22	39.55	25.03	0.64	27.60	2.36	3.95
1975	29	0.19	0.56	0.10	35.98	27.59	0.42	26.97	3.62	4.57
1976	29	0.02	0.66	0.03	40.53	22.54	0.83	27.96	3.45	3.99
1977	29	0.10	0.73	0.07	40.90	21.70	0.97	28.14	3.40	3.98
1978	29	0.09	0.75	0.08	40.64	21.98	0.96	28.12	3.42	3.96
1979	29	0.10	0.76	0.08	40.19	22.53	0.89	28.01	3.50	3.94
1980	29	0.10	0.78	0.08	39.70	23.14	0.79	27.86	3.62	3.91
1981	29	0.11	0.80	0.09	39.20	23.76	0.69	27.69	3.78	3.89
1982	29	0.11	0.81	0.09	38.68	24.38	0.58	27.51	3.98	3.86
1983	29	0.12	0.83	0.09	38.15	24.99	0.47	27.32	4.20	3.83
1984	29	0.12	0.84	0.09	37.61	25.59	0.36	27.13	4.44	3.80
1985	29	0.13	0.86	0.10	37.07	26.19	0.25	26.93	4.71	3.77
1986	29	0.13	0.87	0.10	36.52	26.77	0.15	26.73	4.99	3.74
1987	29	0.13	0.89	0.10	35.97	27.36	0.04	26.52	5.27	3.71
1988	29	0.14	0.90	0.10	35.40	27.91	0.00	26.29	5.57	3.67
1989	29	0.14	0.92	0.11	34.82	28.46	0.00	26.06	5.86	3.64
1990	29	0.15	0.93	0.11	34.24	28.99	0.00	25.82	6.16	3.60

Market shares for Belgian imports (e^t_{jb}): Sector 29: Wood/Furniture

JAAR	NR	CANADA	USA	JAPAN	FRANCE	BRD	ITALIA	NDL	UK	OVERIGE
1963	29	0.02	1.29	0.47	12.38	31.65	2.92	37.71	3.31	10.26
1964	29	0.03	0.84	0.29	11.82	36.34	3.03	33.23	3.12	11.31
1965	29	0.03	0.76	0.26	11.53	40.56	4.04	30.16	3.45	9.22
1966	29	0.02	0.41	0.25	13.30	41.52	5.42	28.47	2.57	8.02
1967	29	0.01	0.45	0.37	15.95	42.31	5.93	25.66	2.67	6.73
1968	29	0.01	0.45	0.21	14.49	46.12	6.81	23.26	2.05	6.58
1969	29	0.03	0.29	0.23	12.94	55.63	6.87	16.33	1.60	6.07
1970	29	0.02	0.26	0.19	16.30	47.96	8.41	18.15	2.41	6.28
1971	29	0.16	0.37	0.11	18.46	45.56	8.29	18.07	2.45	6.52
1972	29	0.01	0.48	0.14	18.68	44.61	8.42	18.40	2.64	6.62
1973	29	0.00	0.51	0.08	19.18	44.77	7.67	18.34	2.11	7.33
1974	29	0.00	0.39	0.06	17.18	43.88	9.89	17.68	2.71	8.20
1975	29	0.02	0.33	0.06	17.48	44.67	9.99	17.89	2.60	6.95
1976	29	0.00	0.27	0.05	16.82	47.13	7.90	18.23	2.46	7.13
1977	29	0.00	0.38	0.18	16.53	47.55	7.63	18.10	2.46	7.16
1978	29	0.00	0.42	0.15	16.60	47.50	7.80	17.90	2.48	7.15
1979	29	0.00	0.42	0.16	16.79	47.30	8.06	17.68	2.48	7.11
1980	29	0.00	0.40	0.14	17.03	47.09	8.34	17.49	2.47	7.05
1981	29	0.00	0.37	0.13	17.29	46.87	8.61	17.30	2.45	6.96
1982	29	0.00	0.35	0.12	17.55	46.66	8.89	17.14	2.42	6.87
1983	29	0.00	0.32	0.10	17.81	46.45	9.17	16.98	2.40	6.77
1984	29	0.00	0.30	0.09	18.07	46.24	9.45	16.83	2.37	6.66
1985	29	0.00	0.27	0.08	18.33	46.03	9.72	16.68	2.34	6.55
1986	29	0.00	0.25	0.06	18.58	45.82	10.00	16.54	2.31	6.43
1987	29	0.00	0.22	0.05	18.84	45.61	10.27	16.41	2.29	6.31
1988	29	0.00	0.19	0.04	19.09	45.41	10.54	16.27	2.26	6.19
1989	29	0.00	0.17	0.02	19.34	45.20	10.81	16.15	2.23	6.07
1990	29	0.00	0.14	0.01	19.59	44.99	11.08	16.02	2.20	5.95

Market shares on the world market ($e^t_{W_j}$): Sector 29: Wood/Furniture

JAAR	NR	CANADA	USA	JAPAN	FRANCE	BRD	ITALIA	NDL	UK	OVERIGE
1963	29	1.23	11.93	2.87	10.39	21.23	7.20	6.74	10.31	28.11
1964	29	1.58	10.37	2.75	8.59	22.38	7.18	6.99	9.48	30.68
1965	29	1.73	9.49	3.19	7.52	22.40	8.50	6.96	9.28	30.93
1966	29	1.41	9.43	3.25	7.38	23.74	9.80	7.11	8.34	29.53
1967	29	1.33	9.48	3.39	6.97	24.43	10.38	6.65	7.39	29.97
1968	29	1.70	7.98	3.22	6.39	28.11	11.85	5.66	6.32	28.76
1969	29	2.65	6.33	3.21	6.03	28.95	12.48	5.50	6.45	28.40
1970	29	3.26	5.44	3.05	6.46	27.65	12.22	5.68	6.20	30.03
1971	29	2.82	4.61	3.16	7.02	28.35	11.78	5.94	6.20	30.11
1972	29	2.48	4.46	3.10	6.81	26.81	12.50	5.94	5.61	32.29
1973	29	2.38	4.40	1.96	6.94	26.90	12.05	6.17	4.73	34.46
1974	29	2.15	5.29	1.42	6.63	26.87	12.91	5.70	5.65	33.37
1975	29	1.70	5.37	1.26	7.35	26.05	14.71	5.52	6.66	31.38
1976	29	1.45	4.80	0.92	7.14	27.34	14.02	5.66	6.29	32.38
1977	29	1.50	4.48	0.86	7.11	27.97	13.66	5.85	6.14	32.44
1978	29	1.60	4.25	0.85	7.13	28.28	13.52	5.95	6.08	32.34
1979	29	1.69	4.07	0.83	7.16	28.42	13.52	5.98	6.07	32.26
1980	29	1.75	3.93	0.79	7.20	28.43	13.61	5.95	6.10	32.23
1981	29	1.80	3.83	0.73	7.24	28.36	13.75	5.90	6.17	32.22
1982	29	1.82	3.76	0.64	7.28	28.22	13.93	5.84	6.26	32.23
1983	29	1.82	3.73	0.54	7.33	28.03	14.13	5.78	6.39	32.25
1984	29	1.81	3.73	0.42	7.38	27.80	14.35	5.71	6.53	32.27
1985	29	1.79	3.75	0.28	7.42	27.54	14.57	5.64	6.70	32.30
1986	29	1.76	3.81	0.13	7.47	27.26	14.79	5.57	6.89	32.32
1987	29	1.72	3.88	0.00	7.51	26.94	15.01	5.50	7.09	32.33
1988	29	1.68	3.98	0.00	7.55	26.58	15.20	5.42	7.30	32.30
1989	29	1.62	4.09	0.00	7.57	26.19	15.39	5.34	7.51	32.27
1990	29	1.56	4.22	0.00	7.60	25.79	15.58	5.27	7.73	32.25

Market shares for Belgian exports (e^t_{bj}): Sector 14: Chemical Prod.

JAAR	NR	CANADA	USA	JAPAN	FRANCE	BRD	ITALIA	NDL	UK	OVERIGE
1963	14	0.40	6.24	3.32	17.07	11.50	2.32	15.77	5.38	38.01
1964	14	0.41	5.77	2.24	20.04	11.91	2.58	16.71	5.04	35.31
1965	14	0.36	3.98	1.75	18.83	13.37	3.23	15.84	4.65	37.99
1966	14	0.45	4.03	1.68	21.51	15.33	3.31	16.90	3.63	33.15
1967	14	0.39	4.04	1.61	22.84	15.54	3.30	17.37	3.92	31.00
1968	14	0.59	2.62	1.72	25.28	17.87	3.39	15.37	3.59	29.57
1969	14	0.33	1.51	1.45	22.89	20.80	4.13	17.27	3.06	28.56
1970	14	0.41	2.87	1.78	20.83	21.73	4.29	17.03	3.36	27.70
1971	14	0.43	2.00	1.30	21.35	25.53	3.79	15.91	3.06	26.63
1972	14	0.38	2.51	1.04	20.62	24.28	4.46	14.97	3.19	28.54
1973	14	0.42	2.48	1.30	20.76	23.48	4.54	15.67	3.64	27.71
1974	14	0.58	3.22	0.81	19.95	23.36	4.42	13.54	4.40	29.72
1975	14	0.65	3.63	0.62	19.59	24.67	4.41	13.74	3.92	28.76
1976	14	0.46	3.33	1.13	20.16	23.59	3.93	14.99	3.76	28.65
1977	14	0.46	3.16	1.26	20.49	23.12	3.90	15.32	3.68	28.60
1978	14	0.46	3.06	1.27	20.65	23.02	3.97	15.36	3.65	28.56
1979	14	0.47	3.01	1.22	20.68	23.15	4.05	15.29	3.64	28.48
1980	14	0.48	3.00	1.16	20.62	23.40	4.14	15.20	3.65	28.36
1981	14	0.48	3.01	1.09	20.50	23.73	4.22	15.08	3.66	28.22
1982	14	0.49	3.03	1.02	20.34	24.12	4.31	14.96	3.69	28.05
1983	14	0.50	3.06	0.95	20.16	24.53	4.39	14.84	3.71	27.86
1984	14	0.50	3.10	0.87	19.96	24.97	4.47	14.72	3.75	27.66
1985	14	0.51	3.15	0.80	19.75	25.41	4.55	14.59	3.78	27.46
1986	14	0.52	3.19	0.73	19.53	25.86	4.64	14.47	3.81	27.24
1987	14	0.52	3.25	0.66	19.32	26.32	4.72	14.34	3.84	27.03
1988	14	0.53	3.30	0.59	19.10	26.78	4.80	14.22	3.88	26.81
1989	14	0.54	3.35	0.52	18.88	27.24	4.88	14.09	3.91	26.60
1990	14	0.54	3.40	0.44	18.65	27.69	4.96	13.97	3.95	26.38

Market shares on the world market ($e^{t}_{w_j}$): Sector 14: Chemical Prod.

JAAR	NR	CANADA	USA	JAPAN	FRANCE	BRD	ITALIA	NDL	UK	OVERIGE
1963	14	3.40	22.88	3.23	8.59	16.74	4.12	5.21	10.89	24.95
1964	14	3.29	24.00	3.37	8.61	16.63	4.23	5.20	10.44	24.24
1965	14	3.38	21.86	4.37	9.14	16.86	4.69	5.64	10.16	23.91
1966	14	3.44	21.91	4.71	9.22	17.58	4.44	5.70	9.69	23.30
1967	14	3.41	21.75	4.44	9.51	18.35	4.26	6.21	9.42	22.64
1968	14	3.08	21.01	4.36	8.69	17.61	4.10	6.20	8.48	26.46
1969	14	3.22	20.11	4.86	9.04	18.74	4.15	6.77	9.16	23.95
1970	14	3.33	19.38	4.91	8.00	20.70	3.79	8.24	8.97	22.69
1971	14	3.30	18.11	5.78	8.11	21.01	3.98	8.10	9.55	22.06
1972	14	2.98	16.77	6.03	8.63	21.48	3.94	9.61	9.17	21.39
1973	14	2.55	16.54	5.25	9.01	22.71	3.72	9.63	8.72	21.86
1974	14	2.36	16.47	6.56	8.75	19.44	4.55	10.54	9.10	22.23
1975	14	2.40	18.06	6.68	9.50	17.84	4.33	10.06	9.01	22.10
1976	14	2.58	19.41	5.01	9.21	18.38	4.27	8.69	9.20	23.24
1977	14	2.67	19.36	5.26	9.06	18.70	4.22	8.38	9.24	23.11
1978	14	2.71	19.11	5.38	9.01	18.93	4.20	8.45	9.23	22.98
1979	14	2.71	18.81	5.51	8.99	19.09	4.19	8.64	9.21	22.85
1980	14	2.70	18.51	5.64	8.98	19.20	4.18	8.88	9.18	22.73
1981	14	2.67	18.22	5.77	8.99	19.28	4.17	9.13	9.15	22.62
1982	14	2.64	17.94	5.91	9.00	19.33	4.17	9.39	9.12	22.51
1983	14	2.60	17.65	6.05	9.01	19.38	4.16	9.65	9.09	22.41
1984	14	2.55	17.37	6.18	9.02	19.42	4.16	9.92	9.07	22.31
1985	14	2.50	17.09	6.32	9.04	19.45	4.16	10.19	9.04	22.21
1986	14	2.45	16.81	6.46	9.05	19.47	4.15	10.46	9.02	22.11
1987	14	2.40	16.53	6.60	9.07	19.50	4.15	10.73	9.00	22.02
1988	14	2.35	16.25	6.74	9.09	19.52	4.15	11.01	8.97	21.92
1989	14	2.30	15.97	6.88	9.11	19.55	4.15	11.28	8.95	21.82
1990	14	2.25	15.69	7.02	9.12	19.57	4.14	11.55	8.93	21.72

Market shares for Belgian imports (e^{t}_{jb}): Sector 14: Chemical Prod.

JAAR	NR	CANADA	USA	JAPAN	FRANCE	BRD	ITALIA	NDL	UK	OVERIGE
1963	14	0.71	22.78	0.60	15.72	20.85	2.37	12.57	7.82	16.57
1964	14	0.76	25.75	0.57	15.05	21.00	2.84	11.97	7.19	14.85
1965	14	0.64	23.45	1.13	15.82	22.28	2.93	11.95	7.03	14.75
1966	14	0.78	20.60	1.09	17.49	24.92	2.43	13.22	6.07	13.41
1967	14	0.68	19.04	0.90	19.35	25.56	2.33	13.81	6.02	12.31
1968	14	0.69	20.38	0.80	18.26	25.48	2.56	14.58	5.33	11.92
1969	14	0.70	20.50	0.84	16.87	24.02	3.20	15.68	6.20	11.98
1970	14	0.79	23.94	0.85	14.09	23.77	3.11	16.34	6.30	10.82
1971	14	0.74	17.80	0.87	15.57	25.47	2.94	17.14	7.41	12.05
1972	14	1.19	15.47	1.27	15.28	25.17	2.84	19.62	7.46	11.70
1973	14	1.21	14.54	0.91	15.43	27.25	2.94	20.34	7.41	9.96
1974	14	0.46	14.15	1.31	14.53	23.23	3.77	19.93	10.56	12.07
1975	14	0.74	15.62	1.09	14.13	21.98	3.63	19.32	10.48	13.00
1976	14	0.62	18.04	0.77	14.78	22.54	3.00	17.85	9.91	12.48
1977	14	0.81	18.22	1.02	15.05	22.78	3.03	17.31	9.60	12.18
1978	14	0.79	18.05	0.98	15.19	23.07	3.05	17.35	9.48	12.04
1979	14	0.80	17.69	1.01	15.18	23.23	3.09	17.58	9.46	11.97
1980	14	0.81	17.28	1.02	15.09	23.32	3.13	17.89	9.52	11.93
1981	14	0.81	16.86	1.04	14.96	23.36	3.17	18.24	9.64	11.92
1982	14	0.82	16.44	1.05	14.79	23.35	3.22	18.60	9.80	11.92
1983	14	0.83	16.01	1.07	14.61	23.32	3.26	18.97	10.00	11.94
1984	14	0.83	15.58	1.09	14.42	23.27	3.29	19.33	10.23	11.95
1985	14	0.84	15.14	1.10	14.23	23.20	3.33	19.70	10.47	11.97
1986	14	0.85	14.71	1.12	14.03	23.13	3.37	20.06	10.73	12.00
1987	14	0.85	14.28	1.14	13.83	23.04	3.41	20.42	11.00	12.02
1988	14	0.86	13.85	1.15	13.63	22.96	3.45	20.78	11.28	12.04
1989	14	0.86	13.42	1.17	13.43	22.87	3.49	21.14	11.56	12.06
1990	14	0.87	12.99	1.18	13.23	22.78	3.53	21.49	11.84	12.08

APPENDIX II: CLASSICAL IMPORT EQUATION FOR BELGIUM

The import equations are of the form:

$$M_i = (a_i + b_i D_i)(P_{fi}/P_{di})^{n_i}$$

where

M_i = imports of commodity i in year t;

D_i = demand for commodity i (discussed further below);

P_{di} = domestic (Belgian) price index for commodity i;

P_{fi} = an index of foreign prices for commodity i.

More precisely, P_{fi} is a weighted average of prices in the other eight countries for commodities as similar as available statistics allow to commodity i in Belgium. The prices include the effects of exchange rate changes but not the effect of tariff changes. The weight on the price of a given country is equal to that country's share in Belgian imports of commodity i in 1970. The shares, however, are taken from the trade model and refer to the shares of the other countries in that model in Belgium's imports of the combination of commodities in that model which most nearly match Belgian product i.

In most equations, D_i is domestic demand, defined as output less exports plus imports. This is the definition used in all the other models, but for Belgium it soon became apparent that, for some important sectors such as Chemicals and Machinery, the imports depended on the exports of the same product group. Chemical inputs, for example, into chemical exports might well be imported. This sequence of import-processing-export, all within the same commodity classification, is so common in Belgium that we had to change the definition of D to include exports for a number of products. Indeed, to go further, in a few cases we found that the sequence was merely one of import and reexport with no processing at all. Agriculture (1), Diamonds (34), and Chemicals (14) displayed this last property to some degree. Data from studies done at the University of Louvain indicated just where the reexport and import-processing-export phenomena were crucial. Thus, in our final equation D_i and M_i were redefined as follows:

$$D_i = (1-a_{ii})\ Q_i + M_i^* - dX$$

and

$$M_i^* = M_i - a_{ii}^{imp}\ Q_i - e_i X$$

where

a_{ii}^{imp} = that proportion of the input-output diagonal coefficient (the sales of an industry to itself) which were imports for processing for the export market;

d = a variable, either zero or one, which tells us whether export demand enters into the demand for imports in a way other than through the diagonal term;

e = the proportion of exports which were actually reexports.

The import equation was then estimated using M^* as the dependent variable.

THE USE OF A NORDIC SYSTEM OF INPUT–OUTPUT MODELS IN NORWEGIAN ECONOMIC PLANNING

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1. Introduction

There is a long tradition in use of input-output models for economic planning in Norway. Input-output models play a central role in short-to medium term economic planning and policy-making as well as in long-run projections.

The Norwegian tradition in economic planning going back to the immediate postwar period have distinctive features. The philosophy and practice of economic planning in Norway has developed in response to the national political and economic background but to a considerable extent it is also due to the pervasive influence of the late professor Ragnar Frisch over several vintages of Norwegian economists. The model tools for short-term analysis, in particular the MODIS model to be described below, are thus rather different from short-term models developed and used in other countries.

Since the 1960s the exports and imports figures of the Norwegian economy have been steadily above forty per cent of GDP. The foreign trade thus is of central importance for the national economic development. The economic models in use have until recently dealt with foreign trade in a detailed but simple way relying more on expert assessment than econometric estimates. In the 1960s and the early 1970s the development of Norway's foreign trade caused little ground for concern. Although rapid changes in industrial structure took place as a consequence of the lowering of trade barriers in a period of high and sustained economic growth the Norwegian economy seemed to adapt very well to changing trade patterns. International booms and recessions in this period caused only small cyclical effects in the Norwegian economy. The expert assessments used in the economic planning were quite reliable with a marked tendency to underestimate the fast growth in exports. The urgency of improved model tools for foreign trade was thus not very strongly felt.

In the 1970s, however, the expert assessments turned out to be gravely overoptimistic, even in the short run. In retrospect the explanation is partly a neglect of domestic production costs as a factor in export performance and partly incorrect assumptions about the development in major trading countries.

The four Nordic countries Denmark, Finland, Norway and Sweden constitute a group of small open economies with a considerable amount of intragroup trade. (The smallest Nordic country, Iceland, with about one per cent of the total Nordic population is for practical reasons left completely out of consideration in the following). In spite of the fact that one country (Denmark) is a member of the EEC, two countries (Denmark and Norway) belong to NATO, there are strong political and popular affiliations between the countries. The establishment of a Nordic INFORUM system of input-output models is very much in line with ideas expressed in many quarters of the need and desirability of increased Nordic economic co-operation. There exists no major study of structural dependencies of the Nordic economies. The idea of an international INFORUM system of input-output models seems to fit very well into the Nordic context.

2. The model system in use in Norwegian economic planning

The MODIS model is the main tool for macroeconomic planning in Norway. The model is used in short and medium term analysis and forecasting. The model originated around 1960 but has undergone successive reconstructions and extensions. The present version, MODIS IV, has been in operation since 1973.

The reliance on the MODIS model marks Norway out has having a different approach to the use of models in economic planning than many other countries. The MODIS model can be characterized, in general terms, as a very disaggregated and detailed macroeconomic model with an elaborated input-output structure, but with less economic behaviour embodied in the formal relations of the model than is common in macroeconomic short-term models.

The core of the model is an input-output framework comprising about 200 commodities and 150 industries. Final demand is specified as several hundred separate items including about 50 items of private consumption demand. The specifications of the model are closely related to the Norwegian national accounts. The results of the model include complete accounts consistent with the definitions of the national account. The model has 2 000 - 2 500 exogenous variables and about 5 000 endogenous variables. Because of the accounting aspect of the model the stated number of endogenous variables may give an exaggerated indication of the size of the model.

The model is subdivided in two main parts, the quantity part and the price part. A somewhat simplified description of the working of the model runs as follows. In the quantity part all final demand except private consumption is exogenous. Exports by commodity (about 100) are thus wholly exogenous. Imports are determined endogenously by a matrix of import shares, by imported commodity and by industry or final demand category. The use of a matrix of import shares thus gives a differential treatment not only of each imported commodity (more than 100) but also between the import requirements of different receivers within the economy. It has been found empirically that the import shares for a great number of commodities vary considerably between receivers. The model is thus well equipped to take care of import changes due to changes in the composition of production and final demand. The import shares are constant coefficients which can be adjusted exogenously by across-the-board changes for each commodity. The input of labour in industries is determined by labour requirement functions basically consisting of an estimated labour coefficient and exogenously given productivity growth.

The price part incorporates the hypothesis of a dichotomy between exposed and sheltered industries. Prices of exposed commodities as well as export and import prices are exogenous. The prices of sheltered commodities are determined by adding up intermediate and primary costs in a simultaneous equation system. Wage rates and indirect tax and subsidy rates are exogenous. Gross profits (operating surplus plus depreciation) in sheltered industries vary proportionally with wage costs with proportionality coefficients subject to exogenous adjustment. The price part includes the same detailed representation of the market shares of imported commodities as described above.

The model has a very detailed representation of fiscal items, i.e. direct and indirect tax rates, government expenditures etc., of less interest in the present context.

The model may be assessed on the basis of this somewhat superficial description. For a proper assessment of the benefits of its use it is necessary to take into account how the model is used within its administrative environment. The openness of the model and the requirements for its use in terms of exogenous data, are in the fact effective barriers which prevent widespread use of the model for full-scale forecasting. The model has been designed and constructed by the Central Bureau of Statistics to be used by the Ministry of Finance for planning and policy-making and only someone with the staff resources and expert knowledge similar to those of the Minstry will be able to use the model to full advantage. The model is generally available for any interested user but is seldom used for forecasting without explicit or tacit support by the Ministry.

The philosophy underlying the use of the model by the Ministry of Finance is that the disadvantages of working with an incomplete, open-ended model are outweighted by the advantages of being able to draw upon expert knowledge and only partly formalized models from various sources within the government administration. The model is used as an integrating tool which provides overall consistency in definitions and balance equations as well as taking well care of some central behavioural relationships of the economy. To serve in this role the iterative use of the model is crucial. The model is used in sequential runs where each computes a main alternative as well as side alternatives expressing partial deviations from the main alternative. The side alternatives may express alternative uses of policy instruments or alternative assumptions of exogenous variables, for instance for exports. Thus, through sequential runs one aims at recovering the loss in simultaneity that follows from using an incomplete model.

For the foreign trade sector, in particular, the model forecasts gain from being based on detailed assessments of import prices, export prices and export volumes. On the other hand there are no built-in mechanisms which lead from changes in import prices to changes in import shares of intermediate inputs in production. Likewise, the export volumes are not derived within the context of a complete behavioural description of the firms constituting an industrial sector. The exogenous foreign trade variables are, of course, neither deduced from an international model giving a consistent picture of the development of Norway as well as her main trading partners.

The MODIS model has a number of support models connected with it. Some support models are used to prepare exogenous input, others to derive results in more detail or to check and corroborate the overall macroeconomic consistency of the results. The support models cover i.a. financial flows, tax incidence and tax revenues, social security system, energy flows, external competitiveness and export shares. The support model developed to check the external competitiveness, called KONK, has an input-output structure consisting of only four aggregate industries, three exposed industry groups and one industry aggregate for all sheltered industries. In the model the changes in the price indices of exposed industries, i.e. the export prices as well as the domestic import-competing industries, are determined as weighted averages of a unit costs and a representative world market price index. The cost structure of the industries are connected through the input-output structure of intermediate goods. Changes in wage levels and productivities are exogenous as in MODIS. On the basis of the price forecasts the changes in market shares both for exports and imports are derived straightforwardly as the product of estimated or assumed elasticities and time-weighted differences between Norwegian and world market prices.

The development of this support model of MODIS should perhaps not be considered as more than a way of systematizing the preparation of exogenous estimates for MODIS and also as a means of exploring consequences of changes in competitiveness from a MODIS reference path. No real effort has been put into the estimation of the coefficients of the model and preliminary tests of some crucial parts of the model have not been too promising. The model has been found useful within the Ministry of Finance, however, as a way of structuring the problem of forecasting the foreign trade development. The model has an extension (GLOBKONK) that forecasts the world market prices by means of a model simulating the cost structure of the same industries in Norway's trading partners. In the international extension of KONK unit labour costs are transformed to commodity prices and the results from the combined models can be considered as a transformation into commodity prices of a relative unit labour cost comparison. This extension is logically interesting but has so far neither been corroborated empirically nor tested in practice.

While the KONK model originated in the user environment of MODIS and MSG another model project of more econometric content, called MODEX, has been developed by the Central Bureau of Statistics. The MODEX model aims at explaining Norwegian exports. In the model the volume and price of Norwegian exports are determined from variables representing costs (unit labour costs) and production level (GNP) of 14 other OECD countries. The model has been estimated only for one commodity aggregate, namely manufacuring goods (SITC 5-9, excl. 68 and 735).

In the MODEX model there is a simultaneous system of price equations in which each country's export price is determined as a function of an index of production costs and a competitive price which is a doubly weighted sum of all export prices. In the reduced form of this system each of the export prices is a function of all cost indices. In another set of equations of the model the volume of imports of each country is determined as a function of the GNP and the ratio of the domestic price level to an import price index defined as a weighted sum of export prices (adjusted for the difference between fob and cif prices and customs duties). The import volumes are again weighted and summed to the Norwegian export market which together with the ratio of the Norwegian export price index and the competitive price for Norwegian exports determine the volume of Norwegian exports.

The MODIS model is too large to be included in an international system of models even if full simultaneity is not attempted. The model is too cumbersome and costly to solve to be part of an iterative solution of a system of models. There exists, however, a recently developed aggregate version of the MODIS model called MODAG.

The MODAG model has about 30 industries. The current version is quite similar to MODIS. Further development is going on to make the model less open and with more behavioural relations and short-run dynamics than in the MODIS model. The development work comprises factor demand, credit flows, foreign trade, and price and income determination.

For long-run projections the model in use is the MSG model which in its current version (MSG-4) uses the same input-output table as the MODAG model. The MSG model originated in a study by professor Leif Johansen, published in 1959. Since 1973 the model has been developed further in successive versions by the Central Bureau of Statistics. The MSG model is a general equilibrium model built around an input-output framework. Each industry has a neoclassical production function in labour, capital, energy, and materials. The demand for labour and capital services by the industries is derived assuming full mobility of productive resources. The available resources are thus always fully utilized in the model. The total productive capacity is exogenously given as growth in total labour force, growth in total capital stock and coefficients of neutral technical change. The nominal price level is determined by exogenous wage rates. The relative prices of commodities and the returns on capital are determined within the model as equilibrium prices of the respective markets. The volume of imports is determined endogenously via a matrix of import shares as described above for MODIS IV. The volume of exports is exogenously determined. The import prices are exogenously given and have to be independently forecasted while the implicit assumption about export prices is that they are the same as the domestic prices. The MSG model is used within a similar administrative framework as the MODIS model.

3. The need for improved forecasts of foreign trade

The Norwegian economy with export and import figures steadily above forty per cent of GDP, obviously qualify as small, open economy. The total value of exports in 1981 was 157 billion kroner corresponding to 48 percent of GDP. Of the total value of exports 68 percent consisted of goods, of which 45 percent came from crude oil and natural gas. The export basket of goods has traditionally been based on certain industries comprising fish prosessing, pulp and paper, basic chemicals, and primary metals with increased diversification over time. From a modest beginning in 1971, the crude oil and natural gas export have grown rapidly and are today almost as important as the export of all other goods. Shipping is still an important part of Norwegian exports and counted for 63 percent of the export of services in 1981.

Norwegian imports amounted to 131 billion kroner in 1981 i.e. 40 percent of GDP. Both traditional imports and import intensive investments in the oil sector increased rapidly in the middle 1970s, and total imports peaked in 1976 with 51 percent of GDP. Import of goods amounted to 69 percent of the total value of imports.

Due to the rapid growth in the Norwegian production of crude oil and natural gas and the rising oil prices, the balance of payments has had a positive surplus on current account since 1980 after years with

Table 1. Norwegian commodity imports in 1980 by MODAG-classification and exporting country (group of countries). Mill.kr.

		Den-mark	Fin-land	Sweden	United King-dom	West Germany	Other OECD	Develop. count-ries	Total[1)]
11	Agricultural products	114	11	120	28	40	931	145	1 396
12	Forestry products	10	5	208	2	5	12	25	287
13	Fishery products	6	1	4	4	1	31	9	56
32	Coal	0	0	0	39	19	188	3	321
33	Other ores and minerals	4	1	144	80	35	141	36	452
16	Food products	424	31	335	174	190	963	234	2 397
17	Beverages and tobacco	22	58	9	72	17	229	10	425
18	Textiles and wearing apparel	939	796	799	1 016	663	2 144	742	7 267
26	Wood and wood products	322	198	1 291	30	88	170	149	2 310
34	Paper and paper products	73	454	943	57	119	219	179	2 046
37	Industrial chemicals	67	49	386	410	715	1 518	1 087	4 337
41	Petrol	0	0	282	131	220	648	76	1 392
42	Fuel oils etc.	167	0	566	266	215	1 196	343	2 960
27	Non industrial chemicals etc.	960	244	1 808	1 968	1 961	4 128	334	11 568
43	Metals	251	305	1 451	867	1 391	2 992	92	7 517
45	Metal products, machinery and equipment	1 234	736	4 540	2 022	4 723	7 000	185	20 569
50	Ships and oil platforms	117	77	423	189	298	938	275	2 691
28	Printing and publishing	96	86	231	76	87	130	4	715
71	Electric power	82	0	150	0	0	0	0	241
66	Crude oil and natural gas	0	0	0	4 501	0	0	3 101	7 725
	Non-Competitive imports								
00	Food and agricultural products	220	59	16	103	36	800	1 295	2 543
01	Raw materials	30	0	25	6	8	152	319	576
02	Industrial products	14	13	261	214	763	2 435	4	3 772
	T o t a l	5 152	3 124	13 992	12 255	11 594	26 965	8 657	83 563

1) COMECON is included in the total.

heavy deficits. The surplus amounted to almost 14 billion kroner in 1981. High expected export of oil and gas in the future will allow a considerable import surplus in other commodities in years to come.

The foreign trade pattern between Norway and other countries have been relatively stable in spite of instability of world commodity markets and increased uncertainty in the international economic development in general during the last decade. The foreign trade statistics of Norway specify trade by exporting or importing country in the SITC commodity classification. Tables 1 and 2 show the exports and imports in 1980 by country (group of countries) in the commodity classification of the MODAG model. About 90 percent of the trade took place with OECD-countries. The main trade partners beside the Nordic countries were the United Kingdom and West Germany.

Although the foreign trade sector plays an important role in the Norwegian economy, little emphasis has been placed - as explained above - on export and import relations in the macroeconomic models in current use for economic planning and policy-making in Norway. The main models in use are the MODIS model for short and medium term planning and the MSG model for long term projections. Especially in the MODIS model the treatment of the foreign trade sector is very detailed and explicitly made so in an attempt to benefit from expert assesment of world market development. Imports are determined edogenously by a matrix of import shares, while exports are wholly exogenous.

In an attempt to evaluate the model in use, the forecasts from the "national budgets" , the annual economic plans, were combined with the observed values for the foreign trade in the 1970s. Table 3 shows the forecasts and observed percentage changes in columes of exports and imports by type of goods and services. The same figures are shown also for another important demand component, namely private consumption and for gross domestic product. By comparing forecasts and observed values one may see to what extent the economic developments occuring in the period were expected by policy makers or forseen by experts. The forecasts shown in table 3 have been made midyear in preparing the national budget for the coming year, while the observed values are taken from the national accounts.

The main impression from the table is that there have been very great discrepancies between forecasts and observed values both for imports and exports. The average absolute deviation between forecasts and observed

Table 2. Norwegian commodity exports in 1980 by MODAG-classification and importing country (group of countries). Mill.kr.

		Den-mark	Fin-land	Sweden	United King-dom	West Germany	Other OECD	Develop. count-ries	Total[1)]
11	Agricultural products	13	13	13	41	96	180	10	368
12	Forestry products	6	0	155	2	6	1	1	171
13	Fishery products	71	3	44	37	72	119	4	351
32	Coal	1	0	1	0	19	0	1	22
33	Other ores and minerals	9	37	55	64	508	260	6	1 035
16	Food products	220	312	943	781	261	1 932	1 207	5 793
17	Beverages and tobacco	3	12	32	1	7	19	2	76
18	Textiles and wearing apparel	168	101	414	104	119	231	25	1 204
26	Wood and wood products	121	6	212	142	164	293	42	983
34	Paper and paper products	245	12	161	949	742	1 216	696	4 389
37	Industrial chemicals	751	145	873	526	312	685	778	4 127
41	Petrol	138	0	129	270	35	256	1	829
42	Fuel oils etc.	411	37	430	214	154	504	40	1 790
27	Non industrial chemicals etc.	298	133	962	349	347	1 150	387	3 732
43	Metals	587	234	1 276	2 003	3 065	4 195	944	12 620
45	Metal products, machinery and equipment	577	392	1 979	527	616	2 084	944	7 285
50	Ships and oil platforms	66	21	277	249	67	309	1 404	2 405
28	Printing and publishing	16	7	38	2	4	9	2	78
71	Electric power	87	0	175	0	0	0	0	262
66	Crude oil and natural gas	0	0	297	31 556	8 870	676	0	41 399
	Non-Competitive imports								
00	Food and agricultural products	2	0	10	0	1	0	0	13
01	Raw materials	1	0	1	1	1	0	1	5
02	Industrial products	6	13	10	15	4	212	3	268
	Total	3 797	1 478	8 487	37 833	15 470	14 331	6 498	89 205

1) COMECON is included in the total.

annual percentage changes in the period 1972-81 is calculated to 6.6 percent for commodity exports and 4.4 for commodity imports. Even in these figures there is a certain amount of uncertainty left out because oil and gas, ships and equipment for oil production are excluded. The forecasts for services are even more imprecise. A closer examination of the table reveals an extremely bad forecast for the imports of commodities in 1978. Imports went down with almost 10 percent this year while the predicted value was a growth of nearly 4 percent. A great deal of the discrepancies between observed values and forecasts are due to contractive policy measures introduced during the year in 1978. This change in policy led to a drop in private consumption and also to a lower growth rate in GDP than predicted. If this year is left out, the average absolute error for imports of commodities is reduced to 3.3 percent. This is yet more reliable than the export forecasts, but more than twice as high as the error for private consumption and GDP.

The export forecasts were overoptimistic in the 1970s until 1978 when the predictions turned out to be more reliable. In retrospect one may say that the expert assessments failed partly in the assumption about the development abroad and partly in misjudging the domestic production costs as a factor behind export performance. The optimistic export forecasts also led to overestimation of the growth rate of GDP.

From examination of table 3 it is rather obvious that the imprecise forecasts of foreign trade have caused considerable uncertainty in the short term outlook of the economy. Efforts in improving the analytical tools are therefore highly welcomed by the planning authorities.

4. Outline of a Nordic INFORUM system of models

The basic ideas behind the INFORUM-IIASA international system of input-output models were stated by its founder professor Clopper Almon in a paper for the Seventh International Conference on Input-Output Techniques in 1979 in three main points:

- connection through international trade
- similarity in input and output conventions
- freedom for diversity in internal structure.

As a result of this initiative today about 20 institutions from different countries participate in the model system. The final goal of linking models contributed by national partners still lies ahead, but will hopefully be achived in the near future.

Table 3. Export and import of commodities and services, private consumption, and gross domestic product. Annual percentage changes in volume (forecasts and observed values), and current values for 1981

	Bill.kr 1981	1972/73	1973/74	1974/75	1975/76	1976/77	1977/78	1978/79	1979/80	1980/81	e
Export of commodities[1) ...	53.9										
Forecasts		11.8	6.1	5.2	16.6	12.0	3.0	5.0	4.0	0.1	6.6
Observed		10.3	-1.0	-12.5	11.0	-2.7	4.3	9.2	-0.5	3.0	
Export of services[2)	9.3										
Forecasts		10.1	28.1	23.9	9.9	10.3	6.4	4.7	4.4	7.9	7.9
Observed		13.0	20.9	14.3	10.7	-5.4	-6.9	17.7	0.3	3.3	
Import of commodities[3) ...	84.9										
Forecasts		10.0	9.5	7.5	7.9	10.2	3.9	6.0	3.7	0.4	4.4
Observed		14.5	10.5	1.1	10.0	7.5	-9.8	4.7	9.0	-2.5	
Import of services[4)	9.3										
Forecasts		11.6	5.1	5.9	5.8	7.5	6.1	5.0	2.5	1.8	6.3
Observed		9.1	10.0	17.7	10.3	6.5	14.3	-10.4	-0.8	-3.5	
Private Consumption	155.5										
Forecasts		4.3	4.0	5.7	4.3	4.4	3.2	0.0	2.1	2.0	1.7
Observed		2.9	3.9	5.1	6.1	6.9	-1.6	3.2	2.2	1.5	
Gross Domestic Product	328.0										
Forecasts		4.6	5.4	6.2	7.0	8.0	6.8	1.8	4.2	1.0	1.5
Observed		4.1	5.2	4.2	6.8	3.6	4.5	5.1	3.9	0.8	

1) Excl. oil and gas, ships, and equipment for oil production.
2) Excl. tourism, gross receipts from shipping, and oil drilling and pipeline services.
3) Excl. ships and equipment for oil production.
4) Excl. tourism and gross expenditure for shipping and oil drilling.

e = Average absolute error between forecasted and observed annual percentage changes in the period 1972-81.

The international INFORUM system of models fits very well into the Norwegian framework of planning models, and there is a growing interest and concern for modeling the foreign trade sector of the models in use. Such a system can improve assessments of foreign trade if the forecasts of the national models in the systen are reliable and the linking mechanism represents foreign trade relations in a satisfactory way.

As part of this more comprehensive international project, government and research institutions in the Nordic countries (Denmark, Finland, Norway and Sweden) decided at a meeting in February 1982 to try to establish a Nordic INFORUM system of models or a submodel to the INFORUM system, called NORDHAND. The four Nordic countries constitute a group of small open economies with a considerable amount of mutual trade. The model system in preparation is also planned to include foreign trade with other countries. The data base will comprise 12 countries or groups of countries. In addition to the four Nordic countries, these are the United Kingdom, West Germany, the United States, Canada, Japan, other OECD-members, Comecon and developing countries.

The Nordic system of models will be based on a grouping of commodities that comprise 34 groups. These groups are defined so as to be aggregates of the 119 SITC-commodities in the INFORUM-system. (See the Danish paper for details.) National input-output models for the respective countries will be joined in the model system. The use and further development of the national models will be left to the participating institutions. The model system will include a trade model, and the first approach to this will be quite simple.

With differing commodity specifications in the national models, transformation matrices for converting exports and imports to the common grouping of commodities must be established. The transformation matrices must also take into account the necessity of transforming the national trade figures to a common currency. The data work in the first phase if this project will be to establish time series for market share matrices for exports and imports by commodity and countries. The model system will also include export relations for exports to some countries outside the Nordic area.

The national model for Norway will be the model MODAG in current use in the Central Bureau of Statistics. The model is described earlier in this paper, and the current version is quite similar to MODIS. The Swedish model, named ISMOD, comprises 28 industries with a detailed representation of different technologies in the manufacturing sector. The national model for Finland, named FMS, also comprises about 30 industries. The model system consists of various submodels with the input-output production-consumption model and the price model as the main parts. The Danish model is the most disaggregated one with 117 industries. The model is at present simple with regard to economic content, but development is going on to make the model less open and with more behavioural relations.

The outline of a trade model presented below is meant as a first step in creating a more comprehensive trade model. The model presented is a simple market share model. The national models are represented here only through a vector of activity levels, comprising production as well as final demand, and a vector of commodity exports.

Imports of each commodity represented in the national model is estimated from an import share matrix.

(1) $B^k = T_B^k \cdot m_k \cdot A^k$ Vector of imports by commodity in country k

where A^k = activity levels of country k,

m_k = import share matrix for country k, and

T_B^k = transformation matrix for imports in country number k, i.e. for transforming national commodity classification to the common Nordic list of commodities.

The inter-Nordic trade structure is described by a matrix M, each element of which is a vector:

M_{kl} = market shares by commodity of country k in the imports of country l.

The exports of country k can now be determined from other countries' imports as

(2) $$X^k = \sum_{l \neq k} M_{kl} \circ B_l$$

By means of a transformation matrix T_x^k, X^k can be transformed to the commodity classification of the national model.

The trade matrix M, which really is threedimensional, can be constructed from foreign trade statistics, such as given for Norway in tables 1 and 2 above. Implicit in the above reasoning is that for a given year import of a given commodity by country i from country j is equal to the export of the same commodity from country j to country i. This is not necessarily the case in the foreign trade statistics. There are several reasons why deviations from this may occur. The statistical data will have to be reconciled to fulfill this condition. There is now work going on to compile time series for the elements in M.

The model (2) will be applied to inter-Nordic trade. Exports to countries outside the Nordic countries will be estimated from export relations very much like the short-cut link in the INFORUM system of models described in the Status Report, December 1980.

(3) $$X_L^k = (\alpha + \beta \sum_j M_{kj} \cdot D_j) \left(\frac{P_k}{P_L}\right)^{\gamma}$$

where X_L^k = exports from country k to the rest of the world,

D_j = domestic demand in country j,

p_k = export price for country k,

p_L = weighted average of exportprices for export-competing countries, and

γ = price elasticity

In the ongoing work with the Nordic INFORUM project the export relations (3) will be estimated for four aggregate commodity groups, food, raw materials, energy and manufactures. This strong aggregation is mainly due to data problems, but also because of the difficulty of achiving reliable estimates to be used in the model forecasts.

The Nordic INFORUM system of models as outlined above, will be operating in the following way:

Each Nordic country carries out model calculations of the national I-O model based on estimation of exports to the world outside the Nordic countries and preliminary guesses of the Nordic trade. From the preliminary estimation of imports in the national models, the inter Nordic exports can be determined. Differences between the "guesstimates" and the model calculated exports, requires another round of national I-O model calculations - and so on until the discrepancies are acceptable. The evaluation of the overall model results may of course lead to further model calculations based on revised market shares, changes in other exogeneous variables or policy instruments. The iteration procedure presupposes that each participant in the project easily can run the national model, and that the model results quickly can be distributed to the others. No steps have been taken so far to make the national models run on the same computer or be programmed in a common language. In connection with further development of the model system, and specially in the case of linking the Nordic system to the international INFORUM system of models, these problems will have to be resolved and SLIMFOR might then be the nearest alternative.[1)]

The use of the national models in the model system are an important feature of the project. And the success of the project is highly dependent on them. Specially in the use and analysis of the model results it is necessary that the national models are built upon a common framework and that the economic behaviour in the formal relations are of approximately the same type covering equal parts of the economy.

The model system will benefit from operational I-O models in use in the Nordic countries. The system will secure consistent trade between them, and the feed-back effects will be fully taken care of. The detailed specification of exports and imports and the integration into the input-output structure of production and demand, make the model system very well suited in analyzing structural changes and the development of trade.

The model system are, of course, very simple in economic content, and should be further developed, first of all by introducing prices or cost indices to endogenize the inter-Nordic market shares. At present the shares are treated exogenously given, and changes have to come from trends in time series, or intuitive assessments of the impact of the use of policy instruments or other factors.

1) The SLIMFOR program is converted to a NORD 10 computer in the Central Bureau of Statistics, and a slim version of MODAG is implemented.

A TRADE MODEL FOR THE NORDIC COUNTRIES

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1. General background

At a meeting in Oslo, February 1982, representatives of the largest Nordic countries (Denmark, Finland, Norway and Sweden) decided to build a sub-model for the trade between their countries, aiming at later possibly be connected to a complete international model of the IIASA/INFORUM type. The nordic model, however, should also be possible to use seperately.

It was decided to disaggregate the model into 34 commodity sectors (see list in Appendix 2). With the help of transformation matrices these sectors will be linked to the individual countries' own national input-output models. The design and aggregation levels of the latter is not intended to be considered in this model project.

Determination of total import demand for each country and each commodity group will be the core of the trade model. This import demand will then be distributed among the other Nordic producer countries and countries outside, by a type of market share functions. Having determined all these imports, the different Nordic countries exports to each other will also be determined, in principle. In order to arrive at total exports, the exports to the non-Nordic countries also have to be determined.

2. Outline of the equation system

Each Nordic country's total import volume of each commodity group is a function of the total demand for the commodity group within the country and the relation between the average import price and the price of home-produced goods:[1]

(1) $$M_{ij} = m_{ij}\ (C_{ij},\ PM_{ij}/PH_{ij})$$

It is possible to include in the import function also such factors as capacity utilisation, trends and so on.

This total import demand is distributed among different producer countries: each of the other Nordic countries and the rest of the world. Imports from a certain Nordic country is a function of the import country's total import demand and the price of the imports from the producer country in question, in relation to the average import price:

(2) $$M_{ijk} = m_{ijk}\ (M_{ij}, PM_{ijk}/PM_{ij})$$

Where of course $M_{ijk} = 0$ for $j = k$.

Imports from rest of the world is determined as a residual:

(3) $$M_{ij5} = M_{ij} - \sum_{k=1}^{4} M_{ijk}$$

Turning now to export volumes, one country's imports from another is, theoretically, equal to the exports from the latter to the former:

(4) $$X_{ikj} = M_{ijk}$$

In practice, however, differences exist, owing to different principles of registration, periodising and so on. Therefore a more general function is employed:

(5) $$X_{ikj} = x_{ijk}\ (M_{ijk})$$

1) The symbols are fully explained in Appendix 1.

Exports to the rest of the world could be taken as a function of world trade and the relation between the seller country's export price and the world market price:

$$(6) \qquad X_{ik5} = x_{ik5}\,(W,\ PX_{ik5}/PX_{i5})$$

This equation could be made more sophisticated, especially if world trade can be divided among commodity groups and among countries (regions).

Total volume of exports will now be

$$(7) \qquad X_{ik} = \sum_{j=1}^{5} X_{ikj}$$

A country's average import price could be viewed as a weighted average of the prices of imports from different countries:

$$(8) \qquad PM_{ij} = \sum_{k=1}^{5} v_{ijk}\ PM_{ijk}$$

where the weights reflect the seller countries shares in the buyer country's imports.

Theoretically, the price of the imports to one country from another is equal to the price of the exports from the latter to the former. Substituting PX_{ikj} for PM_{ijk} in (9) would yield

$$(9) \qquad PM_{ij} = \sum_{k=1}^{5} v_{ijk}\ PX_{ikj}$$

In practice this would not hold, depending on valuation (cif-fob), different index formulas, periodising, etc. This would motivate a more general function

$$(10) \qquad PM_{ij} = p_{ij}\left[\sum_{k=1}^{5} v_{ijk}, PX_{ikj}\right]$$

In the import function (2) above, the variable PM_{ijk} is also used. In practice PM_{ijk} is often not observable. Therefore, it is assumed that the "translation function" p_{ij} also holds for each seller country, that is that $p_{ijk} = p_{ij}$ for all k:

(11) $$PM_{ijk} = p_{ij} (PX_{ikj})$$

The question of measuring PX_{ikj} is considered below, relation (13).

As for export prices, it could be assumed that each Nordic country's average export price for a commodity is a function of its domestic producer price:

(12) $$PX_{ik} = r_{ik} (PH_{ik})$$

A more complicated model might be desired, where PX_{ik} and perhaps also PH_{ik} were dependent on prices in other countries, linked with the help of import prices.

In the import price equations (10) above the variable PX_{ikj} is used. This one is not so easy to measure, however. It is generally assumed that a seller country's export price index (if not the absolute price levels) is the same for exports to all countries:

(13) $$PX_{ikj} = PX_{ik}$$

Export prices for the world outside the Nordic countries might prove to be difficult to observe. As an approximation the Nordic countries' average import price could be used for each commodity group:

(14) $$PX_{i5} = \sum_{j=1}^{4} u_j \, PM_{ij}$$

where the weights u_j reflect the size of the Nordic countries imports.

Appendix 1: Symbols and definitions

The model contain 5 countries, which as importers are designed by j and as exporters by k:

$j = 1, 2, 3, 4, 5$
$k = 1, 2, 3, 4, 5$

where it could be taken that

1 = Denmark
2 = Finland
3 = Norway
4 = Sweden
5 = rest of the world

The rest of the world is planned to be divided further into regions but this has no important consequences for the model structure.

The model contain 34 commodity groups, designed by i:

$i = 1, 2, \ldots\ldots, 34$

The following variables are employed:

M^{*}_{ijk} = imports in current prices, US \$, commodity group i, to country j, from country k.

M^{*}_{ij} = imports in current prices, US \$, commodity group i, to country j, total.

X^{*}_{ikj} = exports in current prices, US \$, commodity group i, from country k, to country j

X^{*}_{ik} = exports in current prices, US \$, commodity group i, from country k, total.

PM_{ijk} = price index, US \$, 1975 = 1,00 for country j:s imports from country k of commodity group i

PM_{ij} = price index, US \$, 1975 = 1,00 for country j:s total imports of commodity group i

PX_{ijk} = price index, US $, 1975 = 1,00 for country k:s exports to country j of commodity group i

PX_{ik} = price index, US $, 1975 = 1,00 for country k:s total exports of commodity group i

$M_{ijk} = M_{ijk}/PM_{ijk}$

$M_{ij} = M_{ij}/PM_{ij}$

$X_{ijk} = X_{ijk}/PX_{ijk}$

$X_{ij} = X_{ij}/PX_{ij}$

} corresponding trade flows in constant prices

C_{ij} = total consumption in constant prices of commodity i in country j.

W = world trade in constant prices

m_{ij}, m_{ijk}, x_{ijk}, p_{ij}, r_{ik} means functions, as understood by the text

v_{ijk}, u_j means weights, as understood by the text.

Appendix 2: Grouping of foreign trade

Sector	Group
1	Agricultural products
2	Fishery products
3	Forestry products
4	Iron ore
5	Crude oil
6	Other ores and minerals
7	Food products
8	Beverages and tobacco
9	Textiles
10	Clothing, leather and skin products and footwear
11	Sawn and planed wood
12	Furniture, also of metals
13	Other wood products

(cont.)

Sector	Group
14	Wood pulp
15	Paper and paper products
16	Printing and publishing
17	Petroleum products
18	Rubber products
19	Primary chemicals and plastics
20	Other chemicals and plastic products
21	Non-metallic mineral building materials
22	Glass and ceramic products
23	Iron and steel
24	Non-ferrous metals
25	Metal products
26	Non-electric machinery
27	Electrical machinery
28	Motor vehicles
29	Ships, oil rigs etc.
30	Other transport equipment
31	Precision instruments, watches
32	Other manufacturing products
33	Electric power
34	Gas

DIRECT AND INDIRECT IMPORT CONTENTS BY COMMODITY GROUPS AND COUNTRIES: DENMARK 1978

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Introduction.

As part of a more comprehensive international project initiated by the International Institute for Applied Systems Analysis (IIASA) and the Inforum group at University of Maryland, USA[1], the Scandinavian countries (Norway, Sweden, Finland and Denmark) decided in the beginning of 1982 to start work on a submodel for these countries, the aim of which is to describe and explain foreing trade between them[2]. To take into account the further international plans for the project, foreign trade with other countries has also been subdivided by country or country group. Annex 1 sets out the 12 countries or regions considered.

Commodities are subdivided into 34 groups, cf. annex 2. These groups are defined so as to be aggregates of the more detailed commodity classification (comprising 119 groups) aimed at in the international project. The 34-group classification has been worked out by the Swedish participants.

1. Cf. "An international system of input-output models:Status reports". IIASA December 1980, and "An international system of national input-outputmodels" by Clopper Almon , University of Maryland. 1981.

2. Cf. "Report from the Oslo-seminar on a Nordic trade model, February 1-3, 1982. (In Norwegian only).

This paper contains calculations for just one year, 1978, and only data from Danish national statistics have been used. The calculations must be considered as the first Danish contribution to the project .
Apart from the results which are in no way without interest, it illustrates our facilities for handling data and calculations of this kind.

Data.

The most detailed input-output table for Denmark for the year 1978 has been used in all calculations. It contains 117 branches, cf. annex 4. In the SNA terminology it is an industry x industry table constructed on the assumption of an industry technology. The price concept applied is basic values, and imports have been separated from domestic production and are shown in matrices of the same dimensions (and using the same classefications) as domestic production[3].

By means of the tapes with detailed foreign trade statistics and a reemployment of the procedures used for first constructing the input-output table it is now possible to subdivide the export column into columns for each of the 12 country groups. For practical reasons it is assumed that reexports from each branch has the same distribution on countries as exports of commodities produced in Denmark.

The foreign trade statistics show figures at f.o.b. prices for exports. In the input-output table the price concept is basic values so that trade margins and indirect taxes net are shown as exports of services and primary inputs respectively. The same construction is made at the 12 country level by assuming that trade margins and indirect taxes, net, are - for each branch - identical for all countries.

Total imports (classified at the 117 branch level) are subdivided on countries by an identical procedure, but in this case there is no problem with the price level, as c.i.f. values are also basic values. Also there is no problem about reexports in this connection.

After this extensions the input-output table now contains 12 export columns instead of one.

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3. Cf. Bent Thage: "Techniques in the compilation of Danish input-output tables: A new approach to the treatment of imports" Paper prepared for the IARIW conference 1981 and shortly to be published in a collection of papers from Springer Verlag, edited by J. Skolka.

The subdivided imports are not incorporated in the input-output table, but is used at a later stage of the calculations.

The calculations.

For each category of final demand (private consumption, collective consumption, fixed capital formation in machinery and equipment, transport equipment and construction respectively, and the 12 categories of exports) it is now possible to calculate direct and indirect contents of imports.

In the first round this results in an import vector of the dimension 117 x 1. In the second round this is subdivided on delivering countries and in the third round aggregated to the 34 commodity groups agreed upon between the Nordic countries, i.e. a matrix of the dimension 34 x 12 for each of the 17 categories of final demand.

In the following the techniques in the calculation is briefly set out. Basicly it is a very simple use of a quantity input-output model.

The symbols used are:

A (dim. 117 x 117). The coefficient matrix for input of domestic output.

M (dim. 117 x 117). The coefficient matrix for input of imports.

F (dim. 117 x 17). The coefficient matrix for final demand of domestic output. f_j is a column in F.

E (dim. 117 x 17). The coefficient matrix for direct imports to final demand. e_j is a column in E.

B (dim. 117 x 12). Matrix of absolute figures showing imports by branch (commodity group) and country.

G (dim. 34 x 117). Aggregation matrix which aggregates the 117 branch-defined commodity groups approximately into the 34 commodity groups used in the project, cf annex 3.

I and i are unit matrix and vector of appropriate dimensions.

All other symbols are defined by the formulaes where they first appear.

After each formulae in the calculations the dimension of the result is shown.

(1)	$m_j = M(I-A)^{-1}f_j + e_j$	$j=1,2,..,17.$	(117 x 1)
(2)	$C = \widehat{(Bi)}^{-1}B$		(117 x 12)
(3)	$M_j = \widehat{m}_j C$		(117 x 12)
(4)	$T_j = GM_j$		(34 x 12)

In (1) the direct and indirect import contents is calculated per kr. final demand in purchasers values. In (2) matrix B is transformed into a coefficient matrix by being scaled with the horizontal sums. So C shows the distribution on countries for each of the 117 branch-defined commodity groups.

The calculation in (3) is based on the assumption that imports of a branch-defined commodity group has the same distribution on delivering countries no matter what category of final demand has caused it. There is no immediate way of checking this assumption.

In (4) the matrix M_j is aggregated into the 34 commodity grouping, giving the resulting matrix T_j. In this connection it must be recalled, that the 117 groups are branch-defined, and we want to aggregate them to commodity-defined groups. In principle this is not possible without setting up a complete transformation matrix. To avoid this difficult and time consuming task an approximate aggregation is carried out. The aggregation key is shown in the first column of annex 3. This means that matrix G only contains ones and zeroes.

To test the classification errors caused by applying this approximate key the absolute figures obtained from the aggregation from branches

are for both imports and exports compared with the ideal aggregation using the key in annex 2 directly on the figures in the foreign trade statistics. The outcome shown i annex 3 is not as bad as might have been feared which is probably due to the relatively high level of aggregation (compared to the 117 branches). In the cases showing the biggest differences the principles used in defining the ideal commodity classification might be called in question, cf. the footnotes in annex 3.

The results

The results are given in 17 tables of the type defined by matrix T_j. In order not to overload this paper with figures only four of these tables are reproduced. That is the tables for the effects of 1 mill. kr. of private consumption (table A1) and the tables for the effects of exports of 1 mill. kr. to Norway, Sweden and Finland (table B1-B3). These as well as the other 12 tables have been summarized in table 1 and table 2. These tables show the distribution of direct and indirect imports according to delivering country, whereas the commodity details have been left out. The first column in table 1 is made up of the column sums of table A1 etc.

The sum row in table 1 shows that total import requirements are heavely dependent upon the category of final demand, varying from 6,6 per cent for collective consumption to 63,4 per cent for transport equipment. The distribution on countries does show considerably less variation. The most outspoken variation is found between the groups for fixed capital formation. The general picture is that about 20 per cent is supplied from the Nordic countries and about 50 per cent from the EEC-countries.

Table 2 is of most immediate interest with respect to international repercussions of an increase of Danish exports to specific countries or groups of countries. In interpreting the figures it must be recalled that the 1 mill. kr. exports in each case has the same commodity composition as total exports to the country refered to in 1978. If we assumed an export increase from an individual branch the picture might be quite different. Such calculations - as well as many others - might easily be carried out using the available data.

The diagonal elements in table 2 show the first round repercussion on the importing country. For instance an increase of Danish exports to Norway of 1 mill. kr. would cause an increase of Norwegian exports to Denmark of 15.206 kr., i.e. 1,5 per cent of the initial increase. The highest first round effect is found for West Germany with 6,4 per cent.

Total import contents in exports do not vary as much between different country categories as between categories of domestic demand (cf. table 1). The highest total is for Sweden with 41,6 per cent, and the lowest for UK with 31,2 per cent. The distribution of imports on delivering countries show even less variation than in table 1, and the general picture from that table is refound in table 2.

Table 1 Direct and indirect import requirements by country caused by domestic final demand. 1978. (kr. per mill. kr. final demand)

Countries or country groups, cf. annex 1	A1 Private consumption	pct.	A2 Collective consumption	pct.	A3 Gross fixed capital formation in machinery a. equipment	pct.	A4 Gross fixed capital formation in transp. equipment	pct.	A5 Gross fixed capital formation in construction	pct.
1 Norway	7055	4	2672	4	18264	3	32631	5	7521	4
2 Sweden	22053	12	9295	14	72231	14	79938	13	34502	20
3 Finland	4950	3	2112	3	10846	2	7056	1	8640	5
4 Fareo Islands, Greenland	1023	1	191	0	196	0	4595	1	189	0
1-4 Nordic countries	35081	19	14270	22	101537	19	124220	20	50852	30
5 UK	22241	12	8683	13	61385	12	55365	9	17042	10
6 West Germany ...	32057	17	12439	19	161676	31	169643	27	39411	23
7 Other EEC countries	35464	19	12106	18	81835	16	115994	18	26944	16
5-7 EEC-countries ..	89762	49	33228	50	304896	58	341002	54	83397	49
8 USA, Canada, Japan	14831	8	5336	8	58202	11	130484	21	13363	8
9 Other OECD countries	10362	6	3731	6	40217	8	24278	4	8514	5
10 Centrally planned economies in Europe	10107	5	3676	6	9763	2	10475	2	6870	4
11 OPEC countries .	7266	4	2459	4	1729	0	997	0	3101	2
12 Other countries	17143	9	3281	5	7332	1	2635	0	5359	3
1-12 Total	184551	100	65982	100	523677	100	634092	100	171455	100

Table 2 Direct and indirect import requirements by country or country group caused by exports grouped in the same way. 1978.
(kr. per mill. kr. exports)

Country see table 1	B1 Norway	pct.	B2 Sweden	pct.	B3 Finland	pct.	B4 Faroe Islands and Greenland	pct.	B5 United Kingdom	pct.	B6 West Germany	pct.
1	15206	4	16302	4	15102	4	15037	4	17694	6	15973	5
2	46486	12	43165	10	43784	12	45435	12	32959	11	40000	13
3	13844	4	11415	3	16169	5	11801	3	9020	3	21983	7
4	1407	0	3856	1	2362	1	969	0	4424	1	5847	2
1-4	76943	20	74738	18	77417	22	73242	19	64097	21	83803	26
5	44546	12	59872	14	39894	11	52289	13	30465	10	29038	9
6	79831	21	73092	18	78757	22	74715	19	64593	21	63634	20
7	70797	19	61837	15	65059	18	61216	16	46891	15	49290	16
5-7	195174	52	194801	47	183710	52	188220	48	141949	45	141962	45
8	26525	7	28003	7	28982	8	27668	7	28422	9	29049	9
9	24537	7	19111	5	21351	6	18917	5	13776	4	14808	5
10	16705	4	36563	9	12929	4	29527	8	13758	4	11214	4
11	11258	3	37736	9	8600	3	29330	8	12780	4	7941	2
12	25742	7	25170	6	21823	6	23611	6	37600	12	28660	9
1-12	376884	100	416123	100	354812	100	390516	100	312381	100	317436	100

Table 2 (continued)

Country see table 1	B7 Other EEC	pct.	B8 United States, Canada and Japan	pct.	B9 Other OECD countries	pct.	B10 Centrally planned economies in Europe	pct.	B11 OPEC countries	pct.	B12 Other countries	pct.
1	16346	5	15948	5	15859	5	16417	5	17351	5	17946	6
2	38899	12	38558	12	46694	14	37791	11	42147	13	39588	12
3	18484	6	28396	9	21455	6	8777	3	9274	3	7711	2
4	5536	2	5758	2	8276	2	4457	1	1879	1	1987	1
1-4	79265	25	88660	28	92284	27	67442	20	70651	22	67232	20
5	29258	9	27538	9	33944	10	32484	10	32249	10	33781	10
6	65240	21	64790	20	73341	21	74972	23	76512	24	80546	24
7	48561	16	47967	15	57786	17	53868	16	56470	17	59095	18
5-7	143059	46	140295	44	165071	48	161324	49	165231	51	173422	52
8	29454	9	29278	9	31609	9	36098	11	29367	9	34262	10
9	14891	5	15505	5	19170	6	16168	5	16592	5	17488	5
10	10905	3	9475	3	10957	3	11272	3	10900	3	10235	3
11	8003	3	6021	2	5620	2	8953	3	7091	2	6254	2
12	27213	9	28503	9	19933	6	30133	9	27036	8	23337	7
1-12	312789	100	317737	100	344641	100	331392	100	326868	100	332231	100

Concluding remarks

Even through direct and indirect imports in average make up 30-35 per cent of categories of final demand, including categories of export demand, it is found that first round repercussion on exports from any individual foreign country to Denmark is rather limited.

As Danish shares in total imports of other countries are in most cases considerable smaller than these countries share in total Danish imports, it is obvious that the second round effect, understood as the further increase in Danish exports to a particular country caused by the first round increase of Danish exports to that country, is quite negligible. For instance the first round effect of an increase in Danish exports of 1 mill. kr. to Sweden is an increase in Swedish exports to Denmark of 43.000 kr. If we assume, rather optimistically, that 4 per cent of this amount directly and indirectly will be imported from Denmark, the second round effect will be less than 2.000 kr.

Disregarding the effect of the model assumptions, and especially the use of average coefficients all over, there are two reasons why the calculated effects must be considered to be minimum effects.

Firstly the model is partial in the respect that it does not take into account the further effects on domestic demand of an increase in exports. When exports increase, domestic private consumption and capital formation must be assumed to increase as well, and this will give rise to further increases in imports. Let us again exemplify this with the Swedish case. If we assume that an export increase of 1 mill. kr. will cause an increase in private consumption of 200.000 kr. and in capital formation of 100.000 kr., it can be found from table 1, that this will give rise to a further increase in imports from Sweden of about 12.000 kr. (calculated as 2,2 per cent of 200.000 kr. + 7,2 per cent of 100.000 kr.). So taking into account this effect adds about 30 per cent to the first round effect of 43.000 kr.

Secondly the model is, as already illustrated above, not able to catch the second round effect from one single foreing country and even less total effects from the world economy as a whole. No doubt these effects are quite small when caused by an isolated Danish expansion of domestic demand or exports, but the main interest from a Danish viewpoint in establishing a world wide model would not be attached to this question but rather to the effects on the Danish economy of what is happening in the rest of the world or regions of the world.

The above observations lead us to the following conclusions. Even through it is quite interesting to study the results of simple combinations of input-output calculations and foreign trade statistics for an individual country, it is obvious that the full advantage of such detailed studies can only be obtained, when they are linked internationally for the world as a whole or regions of the world, and when they are combined with behavioural relationships both for the individual ecomomy and for trade between countries.

ANNEX 1. Countries and groups of countries in NORDHAND.

The three-digit numbers given below are the country codes used in the Danish foreign trade statistics (1978).

Group	Code and countries
1.	028: Norway
2.	030: Sweden
3.	032: Finland
4.	025, 406: Faeroe Islands, Greenland
5.	006 : United Kingdom
6.	004 : FRG
7.	001, 002, 003, 005, 007, 008, 050: Other EEC countries, including Greece.
8.	401, 404, 732: USA, Canada, Japan
9.	800, 804, 038, 024, 040, 042, 036, 052, 048: Other OECD countries
10.	056, 058, 060, 062, 064, 066, 068, 070: Centrally-planned economies in Europe
11.	700, 288, 208, 500, 616, 612, 314, 636, 216, 644, 632, 647, 484: OPEC countries
12.	Other countries

Note: Compared to the grouping of countries agreed upon with Norway, Sweden and Finland, three groups have been subdivided here. Group 7 has been separated out from "other OECD countries", to make calculations of effects for all EEC countries possible. Group 11 (OPEC countries) has been separated out because of the special interest in this group. Group 4 (Faeroe Islands and Greenland) has been separated out from "other countries", as some countries might consider foreign trade with these areas as trade with Denmark.

ANNEX 2. Grouping of foreign trade based on the SITC.[a]

NORDHAND sector	Name	SITC rev.1	SITC rev.2	ISIC approx.
1	Agricultural	00	00	11
	products	025	025	
		041	041	
		042	042	
		043	043	
		044	044	
		045	045	
		051	054	
		052	057	
		054	061	
		061	071	
		071	121	
		121	212	
		212	222	
		22	223	
		29	29	
		94	94	
2	Fish	031	03 (excl.037)	13
	products			
3	Forestry	241	245	12
	products	242	246	
		631.83	247	
4	Iron ore	281	281	2301
5	Crude oil	331.01	333	22
6	Other ores and	27	27	2 (excl.22,230)
	minerals	283	286	
		285	287	
		286	289	
		32	32	
7	Food	01	01	311
	products	02 (excl.025)	02 (excl.025)	312
		032	037	
		046	046	
		047	047	
		048	048	
		053	056	
		055	058	
		062	062	
		07 (excl.071)	07 (excl.071)	
		08	08	
		09	09	
		211	211	
		4	4	

a Worked out by Hans Olsson and Lennart Sundberg, Statens Industriverk, Sweden.

ANNEX 2 (continued)

NORDHAND sector	Name	SITC rev.1	SITC rev.2	ISIC approx.
8	Beverages and tobacco	11 122	11 122	313 314
9	Textiles	26 (excl.266) 65 841.25 841.43 841.44	26 (excl.266,267) 65 845 846	321
10	Clothing, leather and skin products, and footwear	61 83 84 (excl.841.25, 841.43, 841.44) 85	61 83 84 (excl.845, 846) 85	322 323 324
11	Sawn and planed wood	243	248	3311(part of [b])
12	Furniture, incl. metal furniture	82	82	332 3812
13	Other wood products	244 63 (excl.631.83)	244 63	33 (excl.3311 (part of),332)
14	Wood pulp	25	25	341 (part of [c])
15	Paper and paper products	64	64	341 (remainder)
16	Printing and publishing	892	892	342
17	Petroleum products	331.02 332 52	334 335	353 354
18	Rubber products	62	62	355
19	Primary chemicals and plastics	23 266 51 58	23 266 267 51 52 58	351

[b] 33111 according to the Swedish classification of economic sectors (SNI).

[c] 34111 according to the Swedish classification of economic sectors (SNI).

ANNEX 2 (continued)

NORDHAND sector	Name	SITC rev.1	SITC rev.2	ISIC approx.
20	Other chemicals and plastic products	53 54 55 56 57 59 862 893	53 54 55 56 57 59 882 893	352 356
21	Nonmetallic mineral building materials	661 662 663 664	661 662 663 664	36 (part of)
22	Glass and ceramic products	665 666	665 666	36 (part of)
23	Iron and steel	282 67	282 67	371
24	Nonferrous metals	284 68	288 68	372
25	Metal products	69 81	69 81	381
26	Nonelectrical machinery	71 95	71 (excl.716) 72 73 74 75 95	382
27	Electrical machinery	72 (excl.729.5) 891.1	716 76 77	383
28	Motor vehicles	732 (excl.732.9)	781 782 783 784	3843
29	Ships, oil rigs, etc.	735	793	3841

ANNEX 2 (continued)

NORDHAND sector	Name	SITC rev.1	SITC rev.2	ISIC approx.
30	Other transport equipment	731 732.9 733 734	785 786 791 792	384 (excl.3841, 3843)
31	Precision instruments, watches, etc.	729.5 861 864 891 (excl.891.1)	87 88 (excl.882,883) 898	385
32	Other manufactured products	667 894 895 897 899	667 894 895 897 899	39
33	Electric power	35	35	4101
34	Gas	34	34	4102

ANNEX 3. I/O Aggregation key and comparisons of branch and commodity aggregation.

		Aggregation of branches cf. Annex 4	IMPORTS 1978		EXPORTS[4] 1978	
			In aggregated IO branches Mill. kr.	According to key in Annex 2 Mill. kr.	In aggregated IO branches Mill. kr. (basic)	According to key in Annex 2 Mill. kr. (f.o.b.)
1	Agricultural products	1-4	5212[1]	5774	4068[1]	5306
2	Fishery products	6	811	900	1166	2604
3	Forestry products	5	45	46	122	85
4	Iron ore	.	.	5	0	1
5	Crude oil	7	5394[2]	4299	1	0
6	Other ores and minerals	8	363[2]	1623	103	291
7	Food products	9-26	5378[1]	4365	22420[1] [5]	17494
8	Beverages and tobacco	27-29	798	688	880	818
9	Textiles	30-33	3658	3895	2036	2322
10	Clothing, leather and skin prod. and footwear	34-36	2514	2552	1230	1182
11	Wood products excl. furniture (11+13)	37	2440	2276	943	1023
12	Furniture (also of metals)	38,72	867	729	1470	1602
13		.	.			
14	Pulp, paper and paper products (14+15)	39,40	2519	2836	552	662
15		.	.			
16	Printing and publish.	41-49	752	478	434	368
17	Petroleum products	57-58	6991	6782	1556	1518
18	Rubber products	59-60	763	800	318	265
19	Primary chemicals and plastics	50-52	5342	4304	2495	1864
20	Other chemicals and plastic products	53-56,61	2879	4128	3103	4077
21	Non-metallic mineral building materials	64-67	738[3]	849	741[3]	823
22	Glass and ceramic prod.	62-63	782[3]	347	443[3]	280
23	Iron and steel	68-69	2144	3863	872	1315
24	Non-ferrous metals	70-71	901	1483	327	618
25	Metal products	73-75	4922	2421	2376	1832
26	Non-electric machinery	76-80	8749	9668	7388	9672
27	Electrical machinery	81-84	5426	5437	3092	3687
28	Other transport equipm. than 29 (28+30)	86,87	5849	6024	730	1029
29	Ships, oil rigs etc.	85	1846	1330	2062	1576
30		.				
31	Precision instruments, watches	88	1460	1697	1405	1497

ANNEX 3 (continued)

	Aggregation of branches cf. Annex 4	IMPORTS 1978		EXPORTS[4] 1978	
		In aggregated IO branches Mill. kr.	According to key in Annex 2 Mill. kr.	In aggregated IO branches Mill. kr. (basic)	According to key in Annex 2 Mill. kr. (f.o.b.)
32 Other manufacturing products	89-90	1119	1114	1015	1175
33 Electric power	81	330	330	73	73
34 Gas	92	81[2]	76	40[2]	17
Total		81073	81119	63461	65076

Notes:

1. In branch aggregation is sugar (SITC 061) in group 7, and in annex 2 aggregation in group 1. (Imports 258 mill. kr. and exports 474 mill. kr.)
2. In branch aggregation is coal in group 5 and coke in group 34, and in annex 2 aggregation are both in group 6. (Imports 1187 mill. kr.)
3. In branch aggregation is all glass in group 22, whereas glass for building purposes (imports 296 mill. kr.) in annex 2 aggregation is placed in 21.
4. The comparison of exports according to the two aggregations gives only a rough picture, as the branch aggregated figures are basic values and the annex 2 aggregation is based on figures at f.o.b. values. Generally figures in the last column are expected to be higher, as they do include trade margins. Total trade margins are 5897 mill. kr.
5. The big difference in exports for food products (group 7) is caused by the EEC subsidy system. Total subsidies, net on exports are 4245 mill. kr.

ANNEX 4. Branches in the detailed Danish input-output table.

1	Agriculture	32	Knitting mills
2	Horiculture	33	Cordage, rope and twine industries
3	Fur farming, etc.	34	Manufacture of wearing apparel
4	Agricultural services	35	Manufacture of leather products
5	Forestry and logging	36	Manufacture of footwear
6	Fishing	37	Manuf.of wood products,excl.furnit.
7	Extraction of coal, oil and gas	38	Manuf. of wooden furniture, etc.
8	Other mining	39	Manuf. of pulp, paper, paperboard
9	Slaughtering etc.of pigs and cattle	40	Manuf.of paper containers,wallpaper
10	Poultry killing, dressing, packing	41	Reproducing and composing services
11	Dairies	42	Book printing
12	Processed cheese, condensed milk	43	Offset printing
13	Ice cream manufacturing	44	Other printing
14	Processing of fruits and vegetables	45	Bookbinding
15	Processing of fish	46	Newspaper printing and publishing
16	Oil mills	47	Book and art publishing
17	Margarine manufacturing	48	Magazine publishing
18	Fish meal manufacturing	49	Other publishing
19	Grain mill products	50	Manuf.of basic industrial chemicals
20	Bread factories	51	Manuf.of fertilizers and pesticides
21	Cake factories	52	Manuf.of basic plastic materials
22	Bakeries	53	Manuf.of paints and varnishes
23	Sugar factories and refineries	54	Manufacture of drugs and medicines
24	Chocolate and sugar confectionery	55	Manufacture of soap and cosmetics
25	Manufacture of food products n.e.c.	56	Manuf. of chemical products n.e.c.
26	Manuf. of prepared animal feeds	57	Petroleum refineries
27	Distilling and blending spirits	58	Manuf.of asphalt and roofing mater.
28	Breweries	59	Tyre and tube industries
29	Tobacco manufactures	60	Manuf. of rubber products n.e.c.
30	Spinning, weaving etc. textiles	61	Manuf. of plastic products n.e.c.
31	Manuf. of made-up textile goods	62	Manuf. of earthenware and pottery

ANNEX 4 (continued)

63	Manuf. of glass and glass products	91	Electric light and power
64	Manuf. of structural clay products	92	Gas manufacture and distribution
65	Manuf. of cement, lime and plaster	93	Steam and hot water supply
66	Concrete products and stone cutting	94	Water works and supply
67	Non-metallic mineral products n.e.c.	95	Construction
68	Iron and steel works	96	Wholesale trade
69	Iron and steel casting	97	Retail trade
70	Non-ferrous metal works	98	Restaurants and hotels
71	Non-ferrous metal casting	99	Railway and bus transport, etc.
72	Manufacture of metal furniture	100	Other land transport
73	Manuf. of structural metal products	101	Ocean and coastal water transport
74	Manuf. of metal cans and containers	102	Supporting services to water trsp.
75	Manuf.of other fabr. metal products	103	Air transport
76	Manuf. of agricultural machinery	104	Services allied to transport, etc.
77	Manufacture of industrial machinery	105	Communication
78	Repair of machinery	106	Financial institutions
79	Manufacture of household machinery	107	Insurance
80	Manuf.of refrigerators, accessories	108	Dwellings
81	Manuf.of telecommunication equipm.	109	Business services
82	Manuf.of electrical home appliances	110	Education, market services
83	Manuf.of accumulators and batteries	111	Health, market services
84	Manuf. of other electrical supplies	112	Recreational and cultural services
85	Ship building and repairing	113	Repair of motor vehicles
86	Railroad and automobile equipment	114	Household services
87	Manufacture of cycles, mopeds, etc.	115	Domestic services
88	Professional and measuring equipm.	116	Private non-profit institutions
89	Manufacture of jewellery, etc.	117	Producers of government services
90	Manuf.of toys, sporting goods, etc.		

REGIONAL INPUT–OUTPUT MODELS AND INTERREGIONAL TRADE IN THE FRAMEWORK OF A NATIONAL MODEL

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1. PROBLEM STATEMENT

It is often argued that input-output models are much better than conventional macromodels because they present a more detailed view of the process of production. 'Disaggregated is beautiful' has accordingly become a type of faith for many model builders. But what and how disaggregate is--with few exceptions--simply the production account of the economy. The income account and the corresponding income distribution process is, for instance, a much less favorite object of the disaggregation effort.

Another often forgotten, but interesting, perspective for disaggregation is the spatial one. Considering that, at least two important arguments can be found in favor of a spatial or regional disaggregation of an economic model, one may wonder if a point could be found where the incremental benefit of a more deeper sectoral disaggregation is smaller than the incremental gain of a regional breakdown of the national model.

If A and B (see Figure 1) are assumed to imply the same budget (A' and B' do the same at a lower level) the maximum of insight is obtained giving a premium to the regional breakdown (other wise for M'). The two arguments are the following.

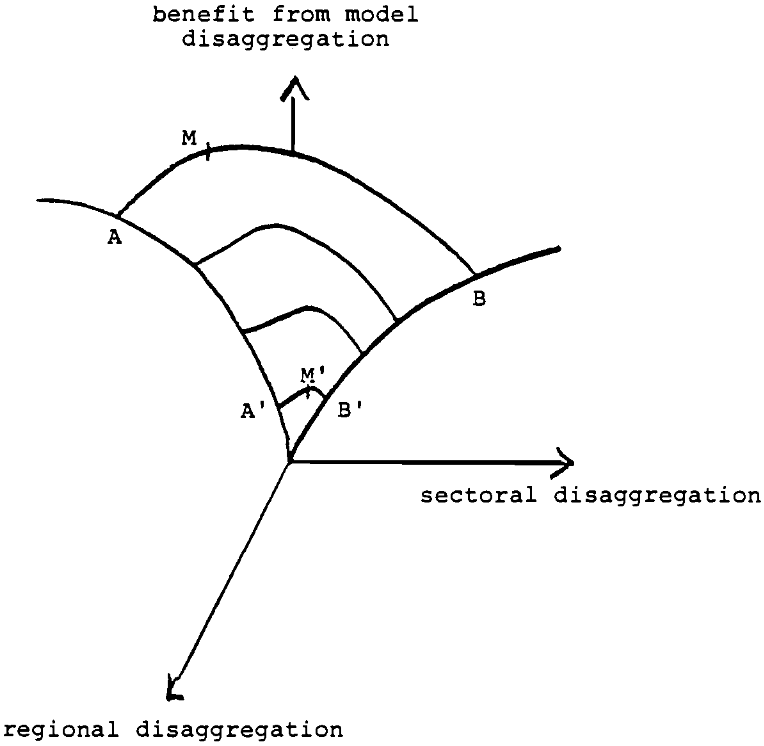


Figure 1. The efficiency frontier for disaggregated models.

If one can think of an economic system with strong spatial disparities, one can easily find arguments which seem to support the hypothesis that the regional allocation of economic activity is not neutral. If the disparities have a negative feedback on the national performance (an example will be given in the next section) not only regional policies are in order, but a model is needed for the assessment of disparities and the conduct of policies.

But also if these feedbacks are weak or absent the spatial allocation of economic activities is not irrelevant. As soon as a local level of government is present (as Italy) there is an obvious interest in tracing back the regional impacts of national and regional policies.

2. HOW TO CONFER SPATIAL DIMENSION TO A NATIONAL INPUT-OUTPUT MODEL

In this section we will dwell on the problems connected with the introduction of the regional dimension in a national input-output model. To make a more fruitful analysis we will discuss mainly with reference to the data constraints given to the system of models used for the Tuscany case study by IIASA (Cavalieri, et al. 1982b). The system of models assumes the availability of an input-output econometric model for Italy. This national model--Interindustrial Italian Model (INTIMO)--is well documented (see M. Grassini 1982b; M. Grassini 1982a; L. Grassini 1982; Ciaschini 1982), so the basic features of this model are assumed to be sufficiently known.

When regional data is not available, regional parameters cannot be estimated; it means that the national model will supply the regional model system with proxies. But the national model has other uses: it provides spatially invariant variables and gives a consistency constraint to the regional variables.

The constraint given by the national module is not absolute because some of the national variables cannot be assumed to invariant with respect to their spatial setup. This problem leads us to favor a two-level (national-regional) system, not strictly hierarchical. To make this point clear, it is better to start from a short description of the whole set of possible solutions.

If we distinguish the multiregional from the interregional models and the integrated from the nonintegrated model (Courbis 1982b) we can define four categories of two-level systems:

	INTEGRATED	NONINTEGRATED
INTERREGIONAL	1	2
MULTIREGIONAL	3	4

The first category (interregional and integrated national-regional models) is built up with model in which single sectors and/or variables of single regions directly interact. Furthermore, the national variables obtained by summation of regionally defined variables have an impact on the national variables upon which regional variables are defined. These types of models are 'closed' at the national level (Courbis 1982b) with a feedback. The parameters of this circular relation (regions—nations-regions) have to be estimated. A classical example is the positive correlation between regional dispersion of the unemployment rate and the national unemployment rate coupled with the positive effect of the latter on the rate of national wage increase. Given the structural disparities of regional productivity growth, the resulting spread of unit labor costs gives an explanation of the different regional unemployment rates. These types of models are quite difficult to implement, but as the well known Regional National Model--REGINA--demonstrates (Courbis 1982a), they are very useful for an effective analysis of the dynamics of the economic system.

If a neutrality assumption for the spatial factors is made (one may ignore the feedbacks of regional or national variables in this case), the choice is between categories two and four. In a multisectoral context the construction of a system of the fourth category implies the availability of regional tables which will be linked to the national table, but not each other.

If the ambitions of the analysis are higher one can turn to an interregional input-output model. This kind of model will be a bottom-up system if the solutions of the regional models are not constrained by the solution of the national model. More often, the interregional model is built up as a top-down system. In this case the regional outcomes are consistent with the national aggregates. A change in one region affects the other regions but not the aggregate because the national model works as a consistency frame for the regional accounts.

This is the approach used in the Tuscany case study where the intraregional input-output table for that region has been embedded in the national table. The aim of the resulting biregional model (where the table for the second region is obtained by difference) is the analysis of the local impacts of the trade linkages of Tuscany with the rest of Italy and the rest of the world. With the indirect technique described in the block diagram (see Figure 2), we have guessed the interregional export flows. From this vector the trade coefficients relating to interregional exports to total regional demand in each region (there is no distinction between the import content of intermediate consumption, final consumption, fixed investment, etc.) are immediately computed. With these coefficients, the full interregional input-output table is produced.

At this point it is possible to perform a comparison between the solutions obtained under the assumptions (A) and (B) (see Figure 2). Assumption (A) implies minimal regional interaction and no cross haulings, while (B) implies flows on both directions. We have to remark that under (B), not the trade balance nor the level of production changes in the reference year. Things could change, on the contrary, for the forecast years if the maintained hypothesis is (B). The trade balance (and with them, the public deficit on the net transfers) as well as the levels of production and employment will change in an unpredictable way (see Appendix) (Martellato 1982b). It has to be noted that the gain of information obtained in solution (B) is due to a direct survey in Tuscany. This is not the unique method of construction of an interregional input-output model. As it is well known, often maximization models are used to fill the statistical gap of solution (A) with respect to (B) (Batten 1982).

3. CAPACITY CONSTRAINTS AND THE STABILITY OF THE TRADING BEHAVIOR

It has been often asserted that trade coefficients of input-output models are not very stable, but change, presumably in a nonrandom way over time (Moses 1955). The problems of making endogenous such parameters in an input-output model has consequently been tackled as a problem to find a general equilibrium between production, interregional trade, and location (Moses 1960; Andersson 1980).

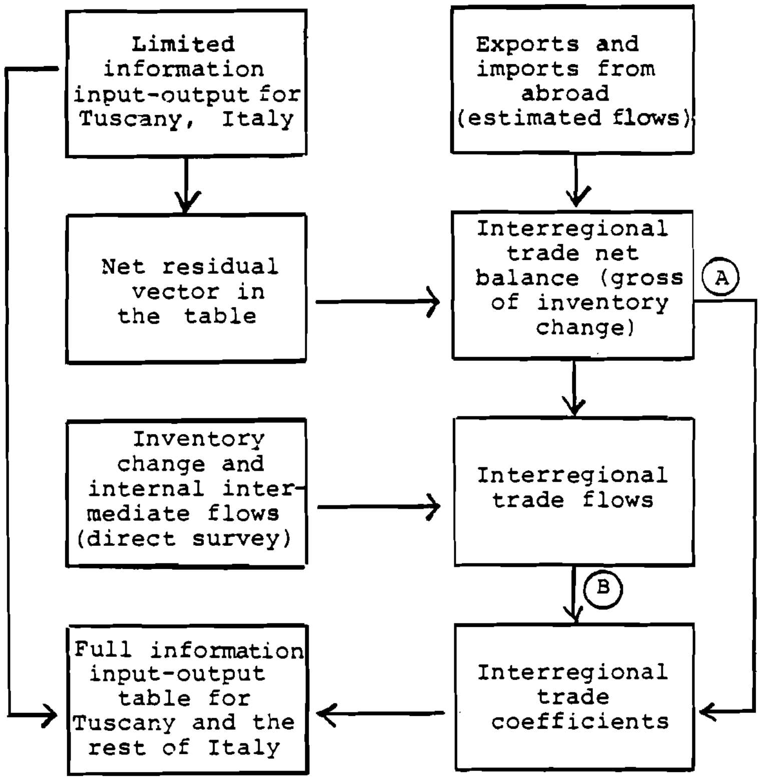


Figure 2. Information flows for the top-down biregional input-output model of Tuscany.

As far as the production level, however, is purely demand determined, we cannot be sure that the reached equilibrium solution is also feasible as regards existing production capacity. We have accordingly decided to take into consideration the existing capacity in the determination both of fixed investment and production. This implies that for a given location of capacity and final demand the import behavior of the system of regions has to change with a system of prices acting (implicitly) as an accommodating variable. In the short period we then assume that firms have to face the given current level of demand with fixed levels of installed capacity and employment. In the short period firms have to manage, accordingly, their trade of quantities of goods and their prices in order to keep the level of actual production inside a type of snake where the ceiling is the current capacity level and the floor is the minimum profitable level of production for the current hired workers. In the medium term they can modify the shape of the snake making appropriate new fixed instruments combining--again consistently--the desired and the existing capacity levels (you cannot, after all, increase your future capacity without importing more if there is not room for more fixed investment, which is a probable event when capacity is already saturated).

The outcome of this idea of a consistent model for production and trade is to make the regional and foreign trade parameters a function of the location of total demand capacity rather than of prices.

For a given level of capacity any increase of total demand in a region can both (i) increase or decrease its import coefficient from the rest of the country (accordingly, its relative degree of unused capacity), and (ii) increase its import coefficient from abroad (if idle capacity is zero within the whole national system). The import content (from the rest of the country) of a given amount of demand increases if the degree of unused capacity is relatively lower in the region. Otherwise, the import coefficient will decrease. This does not occur if the degree of unused capacity is equalized over the regions, in this case the regional trade coefficients will be stable. Only if capacity is fully utilized there will be an increase in the import coefficient from abroad.

The assumption being that, foreign markets are taken into consideration by firms only after the national markets because of higher costs. Another assumption is on the flexibility of the transport network. An increase of total demand reduces overcapacity and, by assumption, it strengthens regional interaction. This event is possible, however, only if the transport network is perfectly 'flexible'. A third implication of our way of modeling regional interaction is the tendency of equalization of the relative regional overcapacity. Assume an increase of demand in all regions: the export coefficients of shipping regions with relative higher overcapacity increase and vice versa.

When imports substitute for unfeasible local production they are considered as competitive. We have considered, accordingly, competitive all the interregional imports and part of the foreign imports. Foreign competitive imports are essentially incremental competitive imports.

In the reference year, all foreign imports (mw) are taken as complementary and, consequently, related to total demand. This is simply an assumption because in those import flows one can find also competive imports. In the following, new competitive imports are added if there is excess demand:

$$mw = MB[Ax + q] + \max[(x - \bar{x}), 0]; \tag{1}$$

A = Leontief matrix (see the Appendix);

B = Chenery-Moses matrix (tridiagonal matrix) for regional trade;

M = diagonal matrix for foreign imports;

x = production vector by sector and region;

q = local final demand by sector and region;

$\bar{x}$ = capacity by sector and region.

The last term is equal to zero when capacity is not fully utilized (or if the model is solved for the reference year), otherwise it is positive.

The equations for regional exports (er) are defined in compact form using a $\bar{B}$ matrix where the principal diagonal of B has been set to zero and defining e as the foreign export vector:

$$er = \bar{B}[Ax + q + e] \quad . \tag{2}$$

The parameters of the matrix B (and also of $\bar{B}$) change as they have to record the shifting import content of total demand. A function which seems to show the desired properties is the following (Martellato 1982a):

$$\bar{B} = \{_{tr}b_i\} = \frac{1}{1+(\bar{x}_i/x_i)\,\rho\,\exp\{|_t z_i - {}_t\bar{z}_i| \,/\, |z' - z^0|\}} \quad , \tag{3}$$

where

$${}_t z_i - {}_t \bar{z}_i = \frac{{}_t x_i}{{}_r x_i} - \frac{{}_t \bar{x}_i}{{}_r \bar{x}_i} \quad .$$

The parameters are ρ and z', z^0. The cusp profile of this equation for the export coefficient of region t to region r (sector i) reaches its maximum-maximorum, $[1/(1+\rho)]$, when $|{}_t z_i - {}_t \bar{z}_i| = 0$ and $\bar{x}_i = x_i$. When there is overcapacity somewhere with the regional system $(\bar{x}_i > \bar{x}_i)$, the cusp is equal to $[q/(1 + \bar{x}_i \rho / x_i)]$.

Assume now an increase in total demand of region t, ${}_t B[Ax + q + e]$, when there is overcapacity in the same region. This event should increase its production. If we accept the idea that also its share in the total national production increases, we must expect an increase of its export coefficient and/or a decrease of its import coefficient.

Our function will allow this result only if ${}_t z_i < {}_t \bar{z}_i$, that is, only if its demand ${}_t x_i$ is below its normalized capacity. From the second row of (3) we have indeed ${}_t x_i < {}_t \bar{x}_i \; {}_r x_i / {}_r \bar{x}_i$.

The nonlinear interregional input-output model is then formed by the equation (3), the definition (4) for each sector i, and the equations (5):

$$B(x,\bar{x})_i = \begin{bmatrix} 1 & \\ & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} {}_{rt}b_i & \\ & {}_{tr}b_i \end{bmatrix} \quad ,$$

$$\bar{B}(x,\bar{x})_i = \begin{bmatrix} & 1 \\ 1 & \end{bmatrix} \cdot \begin{bmatrix} {}_{rt}b_i & \\ & {}_{tr}b_i \end{bmatrix} \quad . \tag{4}$$

$$x = B(x,\bar{x})[Ax+q+e] - mw \quad ,$$

$$mw = MB(x,\bar{x})[Ax+q] + \max[(x-\bar{x}),0] \quad , \tag{5}$$

$$er = \bar{B}(x,\bar{x})[Ax+q+e] \quad .$$

The model, solvable by iteration, implies that $x \le \bar{x}$ in each sector and region. The level of effective demand can, of course, exceed the capacity available. We note, in this case, that assuming one period of gestation lag of the new investment, the resulting capacity gap has two distinct effects. The first effect is an increase in the desired level of capacity which induces new investment. The second is an adjustment, in the same year, of the import coefficients from the rest of the country and, eventually, of the world.

4. THE MODEL SYSTEM FOR THE TUSCANY CASE STUDY

The interregional input-output model (5) outlined in the preceding section is actually only the core of the more detailed system of models described in Cavalieri et al. (1982b). The system has several modules. Besides the interregional one just discussed, these are distinct submodels for private investment, foreign exports, private consumption, and public expenditure. These three activities are all considered, almost in part, endogenous in the system.

Fixed investment is certainly the more difficult to deal with. The submodel is, at the same time, the most ambitious of all three because investment can be made endogenous in an input-output context only in the medium-term (Johansson et al. 1982). We are accordingly obliged to build a special medium-term module for the computation of investment and capacity and to link it with the basic short-term system in a rather elaborate way (Cavalieri et al. 1982b).

In Figure 3 the feedbacks between the two modules are sketched. The medium-term feeds back to the other with the current private fixed investment ($i(t)$) as a function of current capacity) and with the level of capacity of the next year ($\bar{x}(t+1)$) as a lagged function of current investment). The short-term module gives to the medium-term module the flow of final demand $f(t)$ net of fixed investment for five years. The actual path of capacity, which the current level is a function of its lagged level as $\bar{x}(t+1) = \phi[\psi(\bar{x}(t), f(t))]$ is characterized by a changing year growth rate even if it is modeled in the medium-term. We have to say that this part of the project is still under way which means that all the problems have not yet been solved.

Foreign exports have to get special attention (Cavalieri et al. 1982a) because their relevance for Tuscany. The four leading exporting sectors for Tuscany have nonlinear functions where the level of export is a function of relative prices and relative demand pressure. The remaining exporting sectors have,

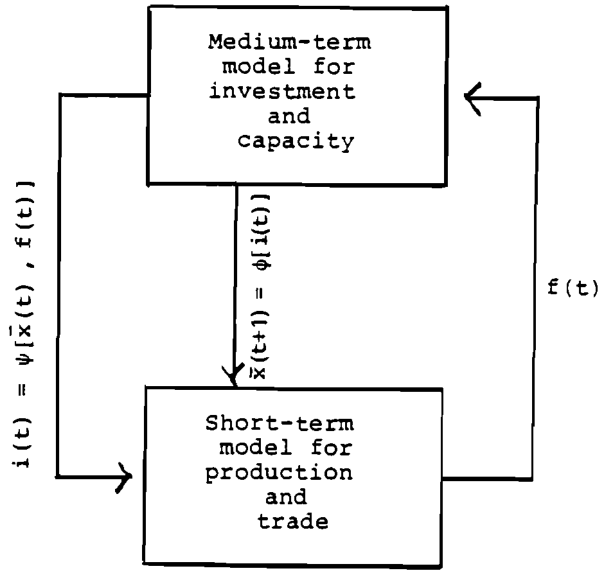


Figure 3. Linkages between the medium- and the short-term interregional input-output models.

in the two regions, a fixed share of the amount obtained by INTIMO, the national model.

As regards final private consumption and public expenditure, we have to take into account that they are interlinked because the redistributive policy of the public sector (Maltinti and Petretto 1982). This consideration has induced us to take into account not only the distribution of value added, but also of the fiscal policy. In its reduced form, the consumption equations relate consumption by sector and region (c_i^r) to production per sector and region (x_j^r and Δx_j^r) via two disposable income matrices (h_{ij}^{1r}, h_{ij}^{2r}) plus an additive term (k_i^r). The equation is

$$c_i^r = k_i^r + \sum_j (h_{ij}^{1r} x_j^r + h_{ij}^{2r} \Delta x_j^r) \quad .$$

It has to be underlined that all these parameters k and h^1, h^2 do change according to the fiscal policy.

Public expenditure has also received special attention. Because the different sectoral-spatial impacts, the total amount of public expenditure g will be splitted according to a matrix of weights a_{ij} (one for consumption and one for investment), $g_{ij} = a_{ij}g$, in order to trace back the effects of different patterns of public expenditure. The weights a_{ij} have been estimated with a direct survey of the public sector in Tuscany.

The last equations of the model system have a monitoring purpose. The unemployment rate in the two regions is first considered. The level of employment is computed over the level of production for a given trend of sectoral labor productivity. Labor supply is obtained from a demographic-migration model and a function defining the participation rates.

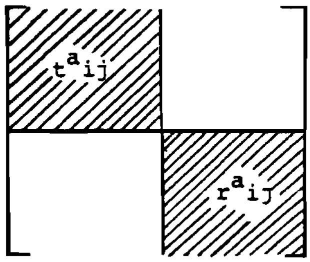
The second monitoring variable is the foreign trade balance. The regional trade balance has its own interest, of course, but it is the resulting national balance which should be consistent with official known figures or with the results of the national model. If the average unemployment rate and/or the foreign trade balance do not match with those benchmarks we have to decide if a revision of our senarios for the exogenous variables used in the forecast is necessary.

When the resulting foreign trade balance does not match that obtained with the national model a consistency problem arises. If the latter is taken as a constraint a revision of the exogenous demand is in order as usually happens in the two-gap models when the foreign pay restrictions limits growth. In this case the growth rate is essentially endogenous. This is exactly the opposite of our intention. Our model is expected to give the local impacts of the growth rate implicit in the path of exogenous demand and the economic policies followed by local and central authorities so no endogenous revision of parameters and/or exogenous vectors is provided.

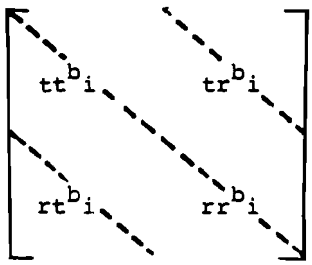
APPENDIX

The structure of the matrices of coefficients, used in section 3, for the system with n=31 sectors and k=2 regions, is as follows:

The Leontief matrix for our biregional system is a is a block diagonal matrix of total input coefficients.

A = 

The Chenery-Moses matrix contains the regional trade coefficients. Its column sums are equal to one, and it has three non-null diagonals.

B = 

The coefficient $_{it}b_i$ gives the percentage of total demand in region t satisfied with local production. The coefficient $_{rt}b_i$ gives the percentage of the same demand satisfied with imports from other regions (r). The matrix $\bar{B}$ used in the expression (2) is equal to B with $_{tt}b_i$ and $_{rr}b_i$ set equal to zero everywhere. The matrix M is a diagonal matrix giving the import (from abroad) content of total demand in each sector and region (complementary assumption).

Let us now consider the effects on the forecasted level of production of a change in the regional trade parameters of matrix B. Because of the nature of the matrix B we have (assuming $M_{ii}=0$ for simplicity): (1) new elementary multipliers $(I-BA)^{-1}$, (2) constant total multiplier $\sum_i \sum_j (I-BA)^{-1}$ = constant, and consequently (3) new rows totals $\sum_j (I-BA)^{-1}$.

The general conclusion is that the impact on the level of production (and employment) depend on the structure of final demand which is a type of weighting vector for the multipliers $(I-BA)^{-1}$. The impact on the trade balance depends on the resulting level of total demand in the different regions and sectors.

More formally: let the old Leontief matrix be $(I-BA) = Z$ and the new one $(I-B^*A) = Z^*$, where B^* is the new trade pattern. The result $Z^{*-1} \geq Z^{-1}$ can be proved only if $Z^* \leq Z$. This is not our case because $B^*_{ii} \leq B_{ii}$ implies that $\sum_i B^*_{ij} \geq \sum_i B_{ij}$, for $i \neq j$, $\forall_j$.

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THE ESTIMATION OF A DEMAND EQUATIONS SYSTEM IN A REGIONAL INPUT–OUTPUT MODEL: THE TUSCANY CASE STUDY. PRELIMINARY RESULTS

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The Tuscany Case Study (TCS), a research project carried on at the International Institute for Applied System Analysis (IIASA), is a biregional model based upon an input-output framework. The biregional structure is determined by Tuscany and the rest of the country: Italy. The global structure of the TCS can be found in Cavalieri,Martellato, Snickars (1982); the core of the model, the biregional input-output scheme, is described in Martellato (1982).

A multiregional model covering a national economy must find a coherence check and even a source of information in a theoretically compatible national model. This need is coupled with INTIMO (Interindustry Italian Model)*, a modern input-output model of Italy partially developed at IIASA within the INFORUM family models**.

The TCS is performed for impact analyses and medium-term forecasts and a specific attention has been devoted to the final demand components. As far as the private consumption expenditure is concerned, a regional estimate of a demand equations system

* The INTIMO project is supported by IIASA, IRPET (Istituto Regionale per la Programmazione Economica della Toscana), ENI (Ente Nazionale Idrocarburi) and is directed by Prof. M. Grassini (University of Siena).

** The INFORUM (Interindustry Forecasting Project University of Maryland) project is founded and directed by Prof. C. Almon (University of Maryland).

has been considered. Since the macroeconomic aggregates for Tuscany represent about 10 percent of the national aggregates, a 'regional specific structure' of private consumption expenditure is expected, while for the rest of Italy the parametric structure of the national demand equations system is expected to be a plausible approximation.

The model proposed by Almon (1979), already estimated for the INTIMO model (Grassini 1981; Grassini and Ciaschini 1981) as for many other models of the INFORUM family, has been adopted.

1. THE DATA

The data are described from different sources: (a) from family budget data related to central Italy, (b) from family budget data specific for Tuscany, (c) regional account (RA) data, and (d) national account (NA) data.

Due to the sample size, family budget data related to Central Italy, surveyed in 1978, has been used for estimating income elasticities. These data have been considered the statistical information closer to Tuscany to detect peculiarities about such behavioral parameters.

A time series of family budget data for Tuscany (1973-1980) has been used to construct a time series of per capita expenditures on 40 items corresponding to the items listed in the private consumption expenditures of the NA scheme.

RA data have provided time series (1973-1980) on total private consumption for Tuscany*. Matching this time series with the previous one, a time series for 40 items of private consumption expenditures at the current price has been obtained. Unfortunately, regional deflators for such detailed information are not available. Assuming that the price dynamics is homogenous all over the country, the national deflators on private consumption expenditures have been used to turn the previous time series into constant price values.

* For 1980 we have used estimated values by applying the national rate of change to the values of 1979.

Finally, RA data have been used for the construction of the time series (1972-1980) on regional disposable income* as described in Maltinti and Petretto (1982).

2. THE MODEL

This applied demand system is composed by equations of the following form

$$q_i = f_i(.)\ g_i(.) \quad , \tag{1}$$

where

$i = 1,2,\ldots,n$

n is the number of commodities,

q_i is consumption per capita in constant price of good i,

$f_i(.)$ is a function of consumption determinants of good i out of prices,

$g_i(.)$ is a function on (relative) prices

The function $f_i(.)$ has the following form

$$f_i(.) = b_{0i} + b_{1i}\ y/\bar{p} + b_{2i}\ \Delta(y/\bar{p}) + b_{3i}\ t \tag{2}$$

where

y is income,

t is trend,

$\bar{p}$ is the price index computed as follows:

$\bar{p} = \Pi_j p_j^{s_j}$ with s_j as the budget share of expenditure for good i at the base year,

$\Delta(y/p)$ is the first difference of real income.

The function $g_i(.)$ for the i.th item is of the form

* For 1980 we have used estimated values by applying the national rate of change to the values of 1979.

$$g_i(.) = \Pi_j p_j^{c_{ij}} \quad . \tag{3}$$

The restrictions on parameters of expression (1) are:

(a) adding up in the base year

$\Sigma_i b_{2i} = \Sigma_i b_{3i} = 0$ and $\Sigma_i b_{1i} = 1$,

(b) homogeneity of degree zero in income and prices in the base year

$\Sigma_j c_{ij} = 0$,

(c) the Slutsky condition imposed in the base year implies

$$c_{ij} \frac{q_i}{p_j} = c_{ji} \frac{q_j}{p_i} \ , \tag{4}$$

so

$$\lambda_{ij} = \frac{c_{ij}}{p_j q_j} = \frac{c_{ji}}{p_i q_i} = \lambda_{ji} \ , \tag{5}$$

and if we consider that $p_i q_i = s_i$, then

$$c_{ij} = \lambda_{ij} s_j \quad . \tag{6}$$

A further reduction on parameters in estimating the model can be obtained by introducing the concept of groups and subgroups of items. The grouping criteria is based upon complementarity and substitutability between goods. The hypotheses are that the same λ prevails within the same group (or subgroup) and the same λ prevails among the groups which are closely related by definition. A detailed description of the model can be found in Almon (1979); the estimation for Italy is presented in Grassini (1981).

3. INCOME ELASTICITY

The income elasticity is an 'exogenous' information for estimating the model which allows the computation of parameter b_{1i} (Almon 1966). The observations used, derived from the ISTAT (1979) family budget data, are grouped into 40 items and 19 income classes according to increasing values of monthly family income (total expenditure). For each class t (t=1,2,...,T) of extremes (R'_{t-1}, R'_t) the arithmetic means of family income, R_t, expenditure on each item i, C_{it}, (i=1,2,...,n) and family size, d_t, are available. The information about d_t allows the computation of per capita income and expenditure as follows

$$R^*_t = R_t / d_t \quad , \quad C^*_{it} = C_{it} / d_t \quad . \tag{7}$$

Two methods have been considered: the first has been proposed by Almon (1966), the second derives income elasticities from concentration curves (Kakwani 1980).

The expression for calculating Engel elasticity η_i for an item i according to the method proposed by Almon is of the following form:

$$\eta_i = \frac{\Sigma_t k_{it} R^*_t N_t}{\Sigma_t C^*_{it} N_t} \quad , \tag{8}$$

where k_{it} is defined as

$$k_{it} = \frac{C^*_{it+1} - C^*_t}{R^*_{t+1} - R^*_t} \quad . \tag{9}$$

This method assumes that if aggregate income increases by α% per capita income increases by the same amount and the individuals recorded in a given expenditure class t, with extremes $(R^{*\prime}_{t-1}; R^{*\prime}_t)$, with income greater than R^*_t will move their own consumption pattern toward the one of the individuals belonging to the next higher class t+1 with income less than R^*_{t+1}; the Engel curve underlying η_i is of the form described in Figure 1, where $R^{*\prime}_{t-1}$ is the per capita value which, in our case, is unknown.

Some considerations must be made about this assumption. The first is that the method may give unsatisfactory results if consumption patterns are very different from class to class. This can be a specific problem of sampling data structure. The treatment of grouping data requires the assumption that individuals belonging to the same class have the same consumption pattern. In fact the ISTAT classification records, into each class, those families that are 'more homogeneous' with respect to the number

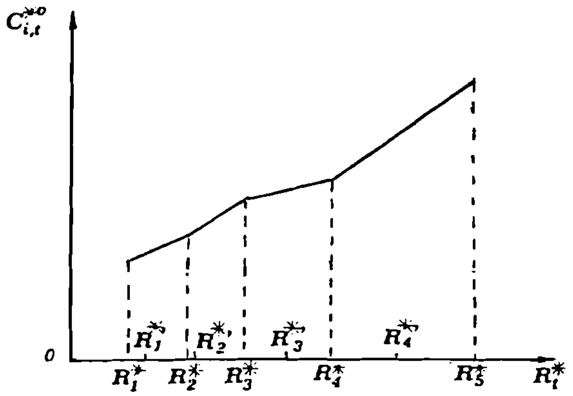


Figure 1. Scatter of per capita expenditure $C^{*}_{i,t}$ versus per capita income R^{*}_{t}.

of components owing to the effect of the high correlation between total expenditure and family size. Furthermore, it has been shown that this method, that was applied to the ISTAT family budget data, does not take into account the concentration of expenditures between income classes, so that it may give elasticity estimates too far from any theoretical expected value (L. Grassini 1982).

The second method assumes that income and expenditure are not equally distributed within every class t and the elasticity for a given value of income is calculated in terms of the concentration indexes of income and expenditure (Kakwani 1980).

If we express function (1) as

$$q_i = [a_i(t) + b_{1i}(y/\bar{p})]\, g_i\,(p_1,p_2,\ldots,p_n) \quad , \qquad (10)$$

where $a_i(t)$ contains the constant term, other non-income, non-price factors and the first difference of income as a term indicating incomplete adjustment to income levels, the equation is reduced to a linear form at the base point where all the prices are set to unity. According to Kakwani (1980), we can compute elasticity μ_i of the linear function $f_i(y)$ at the mean value of per capita income by the following expression

$$\mu_i = \frac{I_i}{I} \quad , \tag{11}$$

where I_i and I are respectively the concentration indexes for expenditure on good i and for income. The concentration index can be calculated without specifying the form of the concentration curve, by an approximate method, as follows:

$$\hat{I}_i = \sum_{t=1}^{T-1} h_t q_{i,t+1} - \sum_{t=1}^{T-1} h_{t+1} q_{i,t} \quad , \tag{12}$$

and

$$\hat{I} = \sum_{1}^{T-1} h_t q_{t+1} - \sum_{1}^{T-1} h_{t+1} q_t \quad , \tag{13}$$

where

$$\begin{aligned} h_t &= \sum_1^t M_z \quad , \quad q_{i,t} = \sum_1^t C^*_{i,z} M_z \Big/ \sum_1^T C^*_{i,t} M_t \quad , \\ M_t &= N_t \Big/ \sum_1^T N_t \quad , \quad q_t = \sum_1^t R^*_z M_z \Big/ \sum_1^T R^*_t M_t \quad . \end{aligned} \tag{14}$$

It can be shown that

$$\sum_i^n \hat{I}_i C^*_i / R^* = \hat{I} \quad , \tag{15}$$

where C^*_i and R^* are the arithmetic means of per capita expenditure and income, so μ_i estimated by (12) and (13) verifies the adding-up criterion at the mean value of income.*

Owing to the specific structure of data, we have adopted this more sophisticated method for estimating elasticities.

*This method generally underestimates the value of the index because it refers to within-class equidistribution.

4. ESTIMATION AND RESULTS

According to Almon (1979) and Grassini (1981), the equation for an item i belonging to the group G can be written in the following form:

$$q_i = f_i(y,t)\,(p_i/\bar{p}_G)^{-\lambda'_G}\,(p_i/\bar{p})^{-\lambda_0} \quad , \qquad (15)$$

where $\bar{p}_G$ is the price index within the group G, λ_0 is the λ_{ij} for those items that do not belong to the group G and

$$\lambda'_G = (\lambda_G - \lambda_0)\,s_G \quad ,$$

with λ_G and s_G as λ_{ij} and sum of expenditure shares for those items belonging to the group G.

The algorithm of calculus for the estimation assumes b_{i1}, from the estimation of income elasticity, and λ_0 as given and proceeds with a method of non-linear estimation.

If we carry out a Taylor series expansion of q_i about the point $b_{oi} = b_{2i} = b_{3i} = b_{4i} = \lambda'_G = 0$ and curtail the expansion at the first derivatives we obtain

$$\hat{q}_i = \sum_{r=0}^{4} \frac{\partial \hat{q}_i}{\partial b_r}\, b_{ri} + \frac{\partial \hat{q}_i}{\partial \lambda_G}\,\lambda_G + \frac{\partial \hat{q}_i}{\partial \lambda_o}\,\lambda_o \quad , \qquad (17)$$

where the unknown parameters, not b_{1i} and λ_o, are estimated by means of the least squares method. The estimated values represent the new point about it the Taylor series is calculated. This procedure goes on until the sum of squared residuals is irrelevant with respect to a given value.

Table 1 shows the results of the estimation for 40 items which have been grouped according to the scheme actually used in the INTIMO model.

1. Food

Subgroup 1: Bread and cereals, fruits and vegetables, potatoes, soft drinks, sugar;

Table 1. Preliminary results of the estimation of demand functions for Tuscany.

foods

sector	subgroup	commodity	income elasticity	time in % of last yr.	own	price elasticities group	subgroup	general	mape	rho
1	2	bread and cereals	0.166	0.2	-0.327	0.007	0.160	0.004	4.6	0.15
2	3	meat	0.414	3.1	-0.168	0.022	0.207	0.013	3.8	0.25
3	3	fish	0.264	2.7	-0.358	0.002	0.017	0.001	7.3	-0.44
4	3	dairy	0.271	0.1	-0.317	0.006	0.058	0.004	7.7	0.11
5	4	butter, margarine, oil	0.467	-6.7	-0.225	0.005	0.281	0.003	6.2	-0.29
6	2	fruits and vegetable	0.361	5.5	-0.325	0.007	0.162	0.004	3.8	-0.12
7	2	potatoes	0.188	5.3	-0.469	0.001	0.018	0.000	12.1	0.41
8	2	sugar	0.173	3.1	-0.460	0.001	0.027	0.001	7.4	0.05
9	4	coffe, tea, cocoa	0.309	13.8	-0.448	0.001	0.058	0.001	10.2	-0.01
10	4	other foods	0.723	2.7	-0.455	0.001	0.051	0.001	5.2	-0.17
11	2	soft drinks	0.926	-3.1	-0.475	0.001	0.012	0.000	13.7	0.06

alcohol and tobacco

sector	subgroup	commodity	income elasticity	time in % of last yr.	own	price elasticities group	subgroup	general	mape	rho
12		alcoholic drinks	0.535	-2.2	-0.056	-0.055		0.003	8.9	0.31
13		tobacco	0.504	1.7	-0.040	-0.039		0.002	3.9	0.16

clothing

sector	subgroup	commodity	income elasticity	time in % of last yr.	own	price elasticities group	subgroup	general	mape	rho
14		clothing and repairs	1.348	2.9	-0.071	-0.071		0.008	5.5	0.46
15		shoes and repairs	1.039	-9.1	-0.019	-0.019		0.002	8.5	0.26

housing

sector	subgroup	commodity	income elasticity	time in % of last yr.	own	price elasticities group	subgroup	general	mape	rho
16		housing rent	0.467	3.0	-0.100	0.101		0.014	2.3	-0.43
17		fuel and electricity	0.956	4.9	-0.185	0.016		0.002	5.1	0.31

continued

durables

sector	subgroup	commodity	income elasticity	time in % of last yr.	price elasticities own	group	subgroup	general	aape	rho
18		forniture	2.967	3.0	-0.014	-0.013		0.002	27.8	0.16
19		textiles	1.852	-18.4	-0.008	-0.007		0.001	8.5	-0.43
20		household appliances	1.882	-10.0	-0.010	-0.009		0.001	6.5	-0.21
21		glasswork and pottery	1.611	-6.8	-0.006	-0.005		0.001	7.1	0.22
22		non durables and services	0.640	7.9	-0.032	-0.031		0.004	14.3	0.32
32		radio and tv sets	1.431	6.1	-0.024	-0.023		0.003	19.0	0.46

health

sector	subgroup	commodity	income elasticity	time in % of last yr.	price elasticities own	group	subgroup	general	aape	rho
24		medicine	0.565	5.1	-0.092	-0.002		0.000	16.7	0.52
25		medical appliances	1.807	-13.2	-0.092	-0.002		0.000	13.2	-0.43
26		medical care	1.634	9.4	-0.094	-0.004		0.000	8.7	0.17
27		hospital care	2.250	16.1	-0.091	-0.001		0.000	****	0.01

transportation

sector	subgroup	commodity	income elasticity	time in % of last yr.	price elasticities own	group	subgroup	general	aape	rho
28		means of transortation	3.306	3.3	-0.026	-0.006		0.001	38.4	-0.34
29		user cost of transportation	1.280	0.9	-0.080	-0.060		0.009	5.5	0.05
30		transport services	0.979	3.8	-0.023	-0.003		0.001	7.8	-0.23

culture and recreation

sector	subgroup	commodity	income elasticity	time in % of last yr.	price elasticities own	group	subgroup	general	aape	rho
31		comunication	1.280	4.5	-0.875	0.117		0.001	7.7	0.48
33		newspapers and books	1.205	5.2	-0.650	0.342		0.002	14.8	0.08
34		education	1.205	5.2	-0.765	0.227		0.002	4.2	0.13
35		entarteinment and recreation	2.108	-8.2	-0.967	0.025		0.000	9.2	0.07
37		hotels, cafe, restaurants	1.506	4.1	-0.805	0.187		0.001	11.8	0.21

other goods

sector	subgroup	commodity	income elasticity	time in % of last yr.	price elasticities own	group	subgroup	general	aape	rho
23		domestic servant	2.379	-27.2	-0.198	0.004		0.000	20.5	-0.44
36		personel toilet articles	0.904	2.8	-0.169	0.033		0.003	8.6	0.14
38		other goods	1.355	-36.7	-0.131	0.071		0.006	8.1	0.41
39		financial services and insur	1.581	3.0	-0.200	0.002		0.000	9.0	0.14
40		other services	2.033	11.6	-0.201	0.001		0.000	8.9	-0.56

Subgroup 2: Meat, fish, dairy;

Subgroup 3: Butter, margarine and oil, coffee, tea, cocoa, other food;

2. Alcohol and Tobacco:

   Alcoholic drinks, tobacco;

3. Clothing:

   Clothing and repairs, shoes and repairs;

4. Housing:

   Housing rent, fuel and electricity;

5. Durables:

   Furniture, textiles, household appliances, glasswork, and pottery, nondurables and services, radio and television sets;

6. Health:

   Medicine, medical appliances, medical care, hospital care;

7. Transportation:

   Means of transportation, user cost of transportation, transportation services;

8. Culture and Recreation:

   Communication, newspaper and books, education, entertainment and recreation, hotels, cafes, restaurants;

9. Other Goods:

   Domestic servants, personal toilet articles, other goods, financial services and insurance, other services.

The income elasticities computed with respect to total expenditure are scaled assuming a propensity to consumption about 0.76; they are converted into linear income coefficients at the mean income and expenditure of the year 1978. For the estimation of the model the value of disposable income estimated for Tuscany was used and λ_0 has been put equal to 0.1.

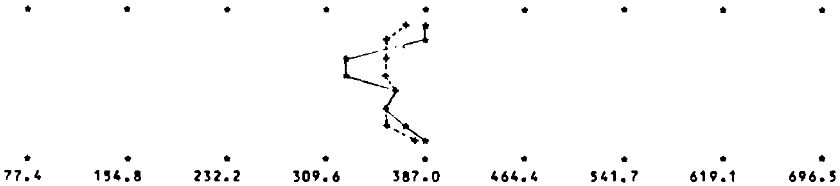
These preliminary results (see Table 1 and Table 2) must be very carefully analyzed. The own price elasticities are all negative with a range of variation between -0.006 and -0.97. Complementarity is present within the groups Clothing, Transportation, Alcohol and Tobacco, Health and Durables. It is not present in Food where all the goods appear within the subgroups independent from one to the other. An unexpected degree of substitutability is even present in the group Housing. The interdependence among the items in Other Goods is very small.

The value of the average absolute percentage error (AAPE) is somewhere a little disappointing and often the plots are not reassuring about the fitting obtained. Some words must be said about the group Health that has a 'starred' AAPE. The matching of the family budget and regional account (RA) data shows some problems about the interpretation of the results. On one side, data on monthly family expenditure for 40 items gives necessary information about the composition of consumption expenditure and they have been used to split RA total family consumption into 40 items since such data is not available from the RA system. A bridge matrix derived directly from the national one used for the INTIMO model will create, within the model, the required coherence between 40 consumption items and 31 producer sectors of the Tuscany economy. On the other hand, those data sources are based on different schemes and assumptions. In the RA system the total expenditure contains public administration health expenditure as if it was made by families. So, if we split RA total family expenditure applying the shares derived from family budget data, we distribute part of public expenditure on health on the other items which are not contained in the item Health. Expenditures which are not determined by consumer units are included in their choice process. The small incidence of expenditure for Health within the family budget, about 0.93%, explains the values of the estimated elasticities and the AAPE index. In fact the AAPE index is sensitive to observed values of expenditure which are about zero and the item Hospital care expenditure presents very small values for the years 1976 and 1977.

Table 2. Preliminary results for Tuscany.

bread and cereals

date	actl=*	predic=+	miss=p-a*
73	389.19	374.21	-14.98
74	387.31	358.30	-29.01
75	326.37	363.49	37.12
76	332.09	358.86	26.77
77	365.20	364.79	-0.42
78	357.21	359.28	2.07
79	374.15	359.03	-15.12
80	386.96	380.99	-5.97
date	actl=*	predic=+	miss=p-a*

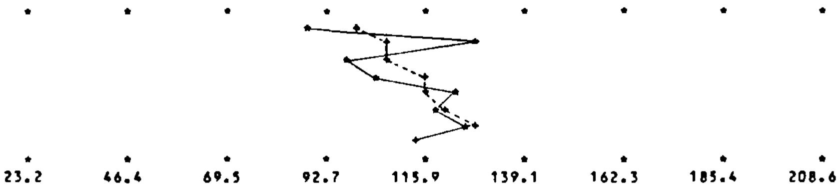


meat

date	actl=*	predic=+	miss=p-a*
73	1146.48	1098.18	-48.31
74	1157.00	1109.93	-47.06
75	1074.60	1135.58	60.98
76	1070.96	1140.96	70.00
77	1189.30	1201.26	11.95
78	1189.25	1224.84	35.59
79	1333.35	1265.97	-67.38
80	1395.68	1381.29	-14.39
date	actl=*	predic=+	miss=p-a*

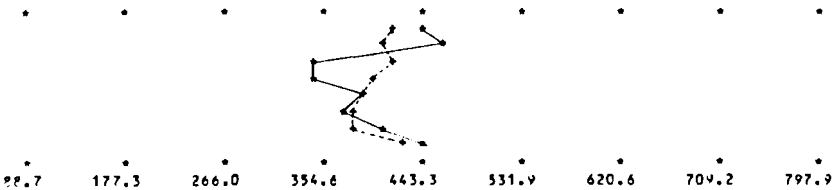
fish

date	actl=*	predic=+	miss=p-a*
73	90.10	100.39	10.29
74	129.41	106.75	-22.65
75	98.67	107.10	8.43
76	106.07	116.29	10.22
77	124.71	116.07	-8.64
78	119.79	121.84	2.05
79	127.03	128.86	1.83
80	115.90	114.23	-1.67
date	actl=*	predic=+	miss=p-a*



dairy

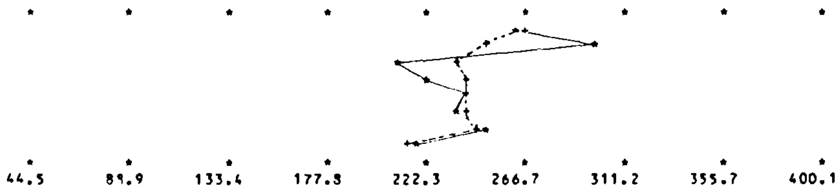
date	actl=*	predic=+	miss=p-a*
73	445.43	421.99	-23.44
74	466.09	410.73	-55.35
75	353.33	418.01	64.69
76	354.58	399.35	44.77
77	391.73	394.12	2.39
78	379.29	389.89	9.60
79	415.52	384.73	-30.79
80	443.26	430.72	-12.54
date	actl=*	predic=+	miss=p-a*



continued..

butter, margarine, oil

date	actl=*	predic=+	miss=p-a*
73	262.34	270.57	8.22
74	298.57	250.79	-47.78
75	209.85	235.76	25.91
76	222.97	244.08	21.11
77	244.08	243.86	-0.22
78	236.51	242.90	6.39
79	252.15	244.76	-7.39
80	222.29	215.40	-6.89
date	actl=*	predic=+	miss=p-a*
			0.



fruits and vegetable

date	actl=*	predic=+	miss=p-a*
73	364.35	359.74	-4.62
74	388.94	371.72	-17.22
75	374.18	389.96	15.79
76	368.50	408.66	40.17
77	452.12	423.81	-28.31
78	435.04	424.38	-10.66
79	436.82	435.83	-1.00
80	474.51	479.42	4.91
date	actl=*	predic=+	miss=p-a*
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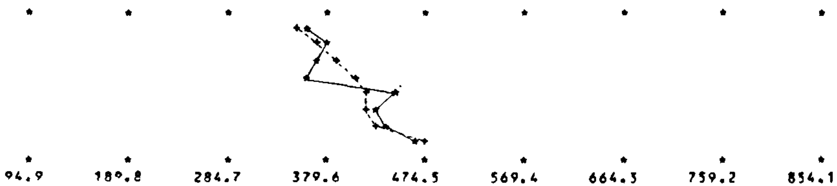
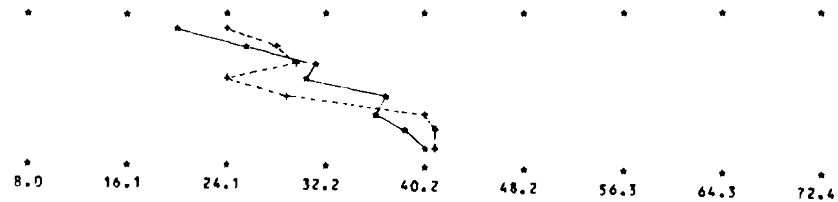


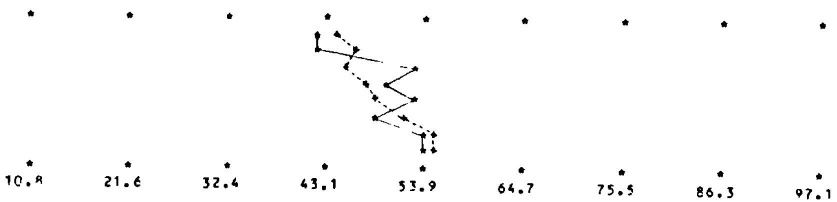
potatoes

date	actl=*	predic=+	miss=p-a*
73	20.22	24.19	3.97
74	26.05	28.48	2.43
75	32.15	29.96	-2.19
76	30.79	24.33	-6.46
77	37.59	29.51	-8.08
78	36.74	40.28	3.54
79	39.16	41.62	2.46
80	40.21	41.42	1.22
date	actl=*	predic=+	miss=p-a*
			0.



sugar

date	actl=*	predic=+	miss=p-a*
73	42.22	45.22	3.00
74	42.25	47.30	5.05
75	53.00	45.41	-7.59
76	50.19	47.54	-2.65
77	53.51	48.82	-4.69
78	48.71	51.89	3.18
79	54.30	55.44	1.14
80	53.93	55.68	1.75
date	actl=*	predic=+	miss=p-a*
			0.



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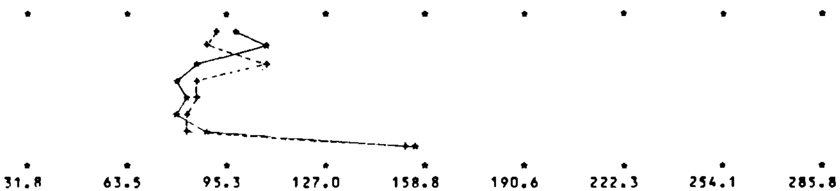
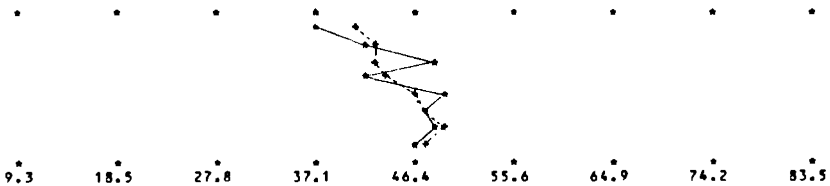
caffe, tea, cocoa

date	actl=*	predic=+	miss=p-a*
73	100.13	93.52	-6.61
74	110.27	90.43	-19.84
75	86.40	108.43	22.04
76	79.58	88.43	8.85
77	83.43	86.70	3.27
78	81.04	84.04	3.00
79	90.81	82.60	-8.21
80	158.80	152.68	-6.12
date	actl=*	predic=+	miss=p-a*



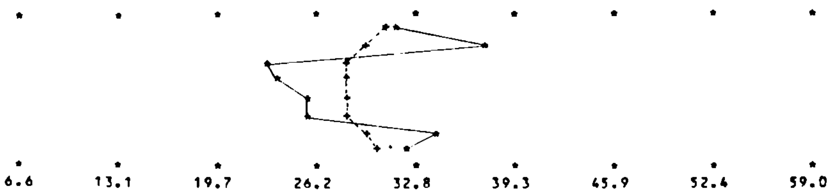
other foods

date	actl=*	predic=+	miss=p-a*
73	37.25	40.82	3.57
74	42.60	43.48	0.88
75	49.02	43.19	-5.83
76	42.29	44.43	2.14
77	49.73	46.38	-3.36
78	47.51	47.60	0.09
79	48.40	49.87	1.47
80	46.36	47.93	1.57
date	actl=*	predic=+	miss=p-a*



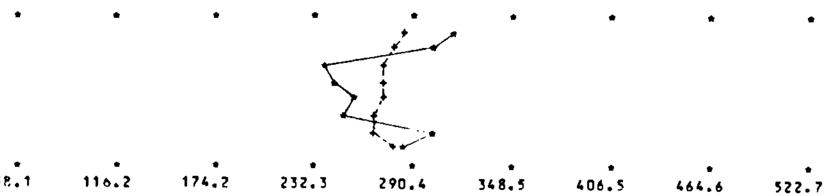
soft drinks

date	actl=*	predic=+	miss=p-a*
73	31.90	30.81	-1.09
74	37.90	30.03	-7.87
75	23.08	28.52	5.44
76	23.65	28.32	4.67
77	25.90	28.56	2.65
78	25.67	28.81	3.14
79	34.28	29.74	-4.54
80	32.75	30.77	-1.99
date	actl=*	predic=+	miss=p-a*



alcoholic drinks

date	actl=*	predic=+	miss=p-a*
73	313.80	287.49	-26.31
74	307.13	281.42	-25.70
75	243.77	278.62	34.85
76	245.62	273.45	27.84
77	257.28	273.63	16.35
78	250.93	269.27	18.34
79	304.93	269.51	-35.42
80	290.39	280.16	-10.23
date	actl=*	predic=+	miss=p-a*



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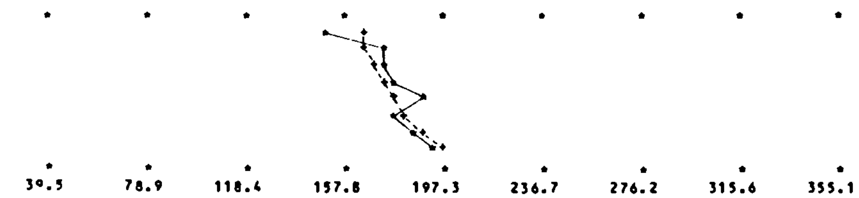
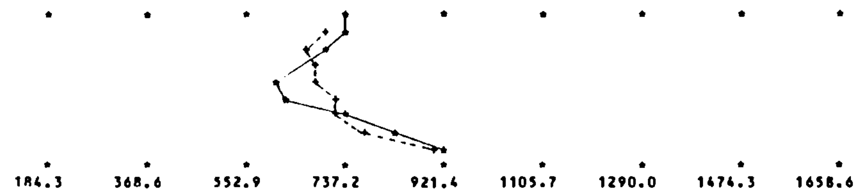
tobacco

date	actl=*	predic=+	miss=p-a*
73	152.38	166.24	13.86
74	175.11	168.90	-6.21
75	176.80	171.73	-5.07
76	179.48	175.55	-3.93
77	192.58	180.40	-12.19
78	178.36	184.80	6.44
79	186.49	192.32	5.82
80	197.27	198.65	1.38
date	actl=*	predic=+	miss=p-a*



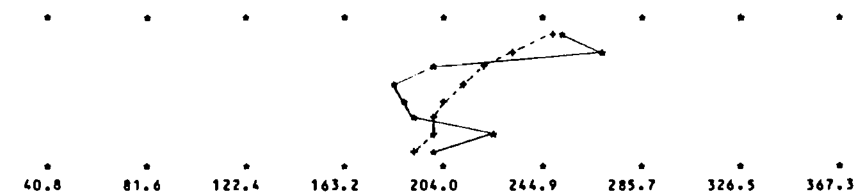
clothing and repairs

date	actl=*	predic=+	miss=p-a*
73	744.21	712.21	-32.01
74	713.37	680.35	-33.02
75	695.85	690.17	-5.68
76	616.96	682.94	65.99
77	631.37	719.48	88.10
78	742.77	735.25	-7.52
79	841.90	784.44	-57.46
80	921.44	903.46	-17.98
date	actl=*	predic=+	miss=p-a*



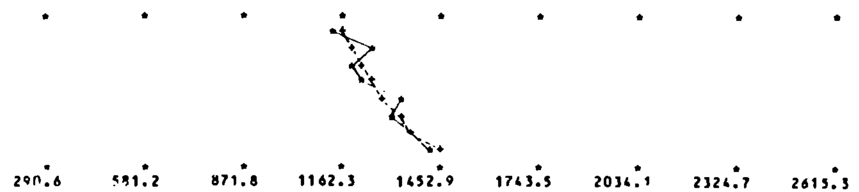
shoes and repairs

date	actl=*	predic=+	miss=p-a*
73	256.89	249.99	-7.90
74	269.82	235.80	-34.02
75	202.62	223.18	20.56
76	184.89	214.76	29.88
77	190.16	206.50	16.34
78	195.70	201.78	6.08
79	225.37	203.88	-21.99
80	204.04	195.26	-8.78
date	actl=*	predic=+	miss=p-a*



housing rent

date	actl=*	predic=+	miss=p-a*
73	1134.74	1172.13	37.39
74	1260.56	1195.27	-65.29
75	1197.28	1223.78	26.49
76	1222.28	1254.55	32.27
77	1349.49	1297.75	-51.74
78	1333.37	1337.28	3.91
79	1376.92	1393.06	16.15
80	1452.92	1453.92	1.00
date	actl=*	predic=+	miss=p-a*



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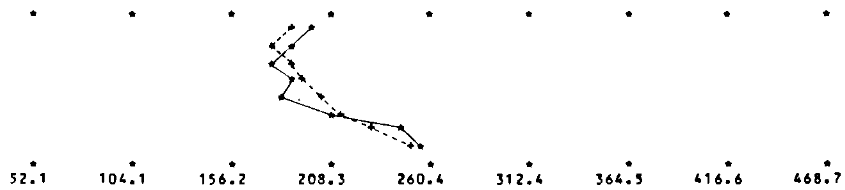
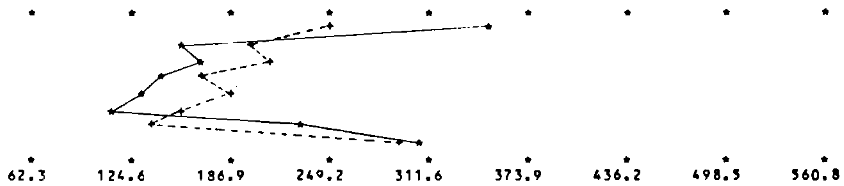
fuel and electricity

date	actl=*	predic=+	miss=p-a*
73	198.46	187.75	-10.71
74	189.53	181.25	-8.28
75	180.98	190.42	9.44
76	190.97	194.32	3.35
77	183.04	206.98	23.93
78	211.12	215.74	4.62
79	245.20	230.10	-15.10
80	260.37	253.17	-7.21
date	actl=*	predic=+	miss=p-a*
			0.



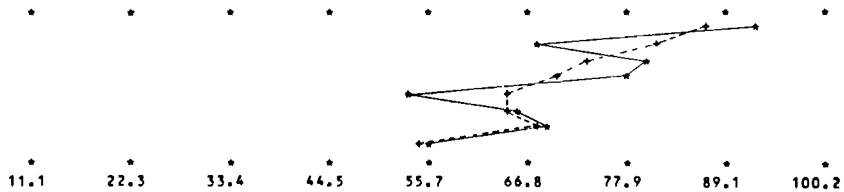
forniture

date	actl=*	predic=+	miss=p-a*
73	353.82	251.30	-102.52
74	158.93	202.32	43.39
75	171.09	211.89	40.81
76	148.00	168.98	20.98
77	132.57	191.96	59.39
78	115.86	161.02	45.16
79	233.73	141.32	-92.41
80	311.56	297.50	-14.06
date	actl=*	predic=+	miss=p-a*
			0.



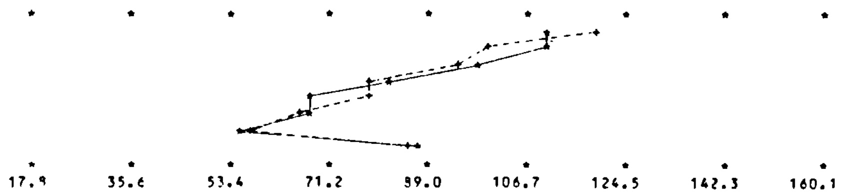
textiles

date	actl=*	predic=+	miss=p-a*
73	92.59	87.42	-5.17
74	68.94	81.71	12.77
75	80.69	73.53	-7.16
76	79.00	70.73	-8.27
77	54.40	65.22	10.83
78	65.90	64.78	-1.12
79	69.37	68.19	-1.68
80	55.67	55.49	-0.18
date	actl=*	predic=+	miss=p-a*
			0.



household appliances

date	actl=*	predic=+	miss=p-a*
73	110.51	119.30	8.79
74	111.10	101.01	-10.10
75	99.24	95.61	-3.63
76	83.02	79.69	-3.33
77	67.71	78.45	10.74
78	68.54	66.21	-2.33
79	55.31	58.12	2.81
80	88.95	86.00	-2.95
date	actl=*	predic=+	miss=p-a*
			0.



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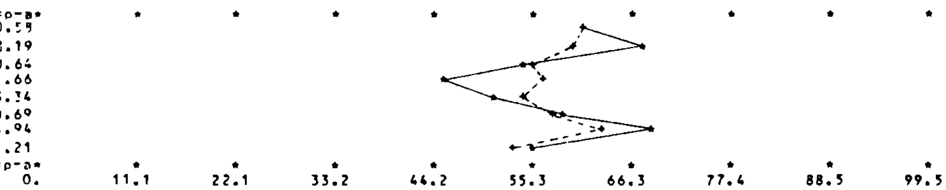
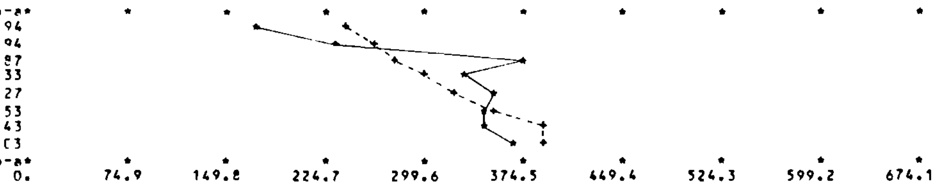
glasswork and pottery

date	actl=*	predic=+	miss=p-a*
73	61.51	60.92	-0.59
74	68.01	59.83	-8.19
75	55.15	55.30	0.64
76	45.41	57.07	11.66
77	51.70	55.05	3.34
78	58.77	58.08	-0.69
79	69.06	64.12	-4.94
80	55.28	54.08	-1.21
date	actl=*	predic=+	miss=p-a*



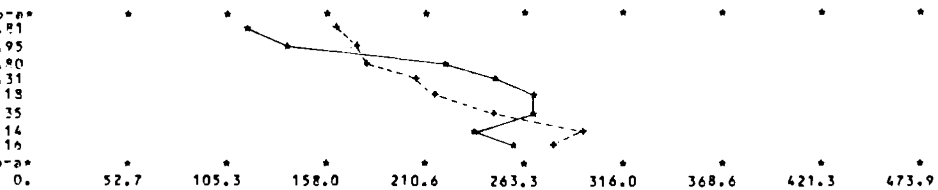
non durables and services

date	actl=*	predic=+	miss=p-a*
73	175.01	242.95	67.94
74	234.07	264.01	29.94
75	376.86	277.98	-98.87
76	332.80	304.47	-28.33
77	358.23	322.96	-35.27
78	346.86	353.39	6.53
79	351.90	391.32	39.43
80	374.50	392.54	18.03
date	actl=*	predic=+	miss=p-a*



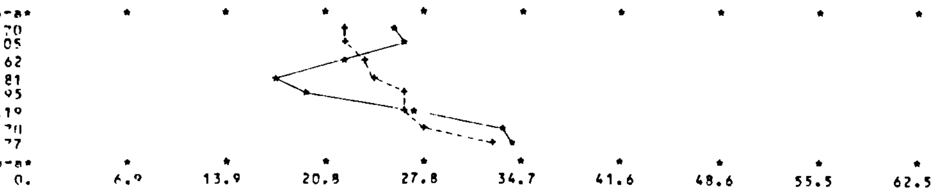
radio and tv sets

date	actl=*	predic=+	miss=p-a*
73	120.85	163.67	42.81
74	141.96	177.91	35.95
75	226.28	182.48	-43.80
76	251.41	207.10	-44.31
77	269.44	220.26	-49.18
78	268.74	252.39	-16.35
79	240.74	296.88	56.14
80	267.30	281.46	13.16
date	actl=*	predic=+	miss=p-a*



medicine

date	actl=*	predic=+	miss=p-a*
73	26.32	22.62	-3.70
74	26.78	22.73	-4.05
75	22.43	24.06	1.62
76	17.51	24.32	6.81
77	19.43	26.38	6.95
78	27.11	26.92	-0.19
79	33.73	28.03	-5.70
80	34.70	32.97	-1.77
date	actl=*	predic=+	miss=p-a*



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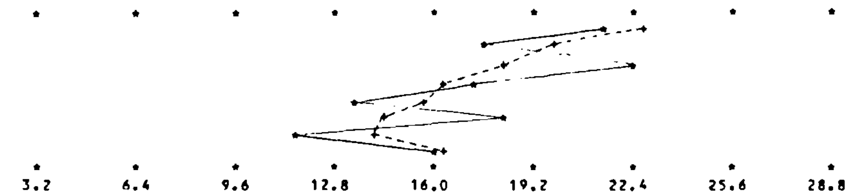
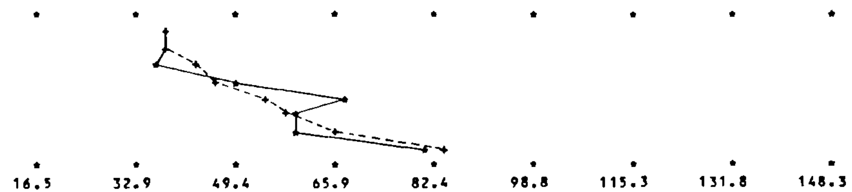
medical appliances

date	actl=*	predic=+	miss=p-a*
73	21.74	22.87	1.13
74	17.76	20.03	2.29
75	22.42	18.51	-3.91
76	17.52	16.43	-1.09
77	13.46	15.91	2.45
78	18.30	14.44	-3.86
79	11.66	14.15	2.49
80	16.00	16.48	0.48
date	actl=*	predic=+	miss=p-a*



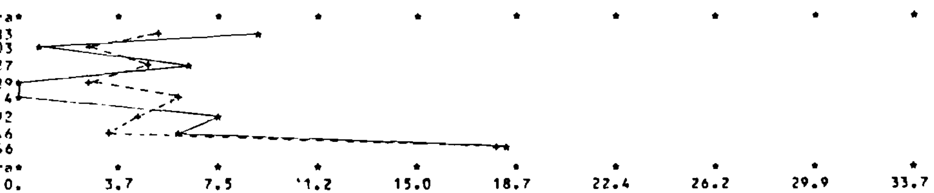
medical care

date	actl=*	predic=+	miss=p-a*
73	38.00	39.44	1.43
74	38.64	39.53	0.89
75	36.68	44.22	7.54
76	50.96	46.48	-4.43
77	68.09	54.47	-13.62
78	59.53	59.09	-0.44
79	60.46	67.11	6.65
80	82.37	84.41	2.04
date	actl=*	predic=+	miss=p-a*



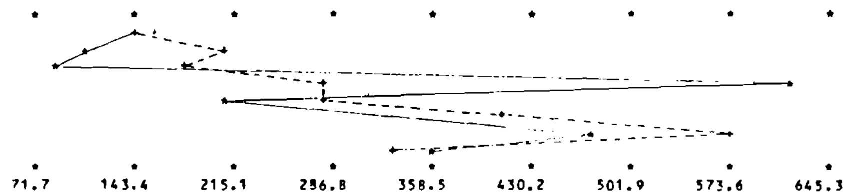
hospital care

date	actl=*	predic=+	miss=p-a*
73	9.21	5.38	-3.83
74	0.78	2.81	2.03
75	6.56	5.19	-1.37
76	0.01	2.91	2.89
77	0.01	6.15	6.14
78	7.57	4.65	-2.92
79	6.17	3.71	-2.46
80	18.70	18.24	-0.46
date	actl=*	predic=+	miss=p-a*



means of transportation

date	actl=*	predic=+	miss=p-a*
73	150.39	146.73	-3.66
74	114.10	209.87	95.77
75	89.69	181.88	92.19
76	620.19	284.83	-335.36
77	209.47	283.50	74.02
78	413.82	412.07	-1.75
79	477.01	579.76	101.74
80	352.49	333.92	-24.57
date	actl=*	predic=+	miss=p-a*



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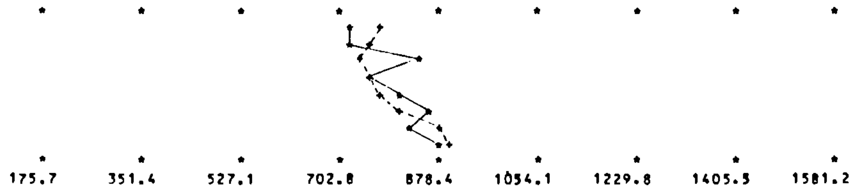
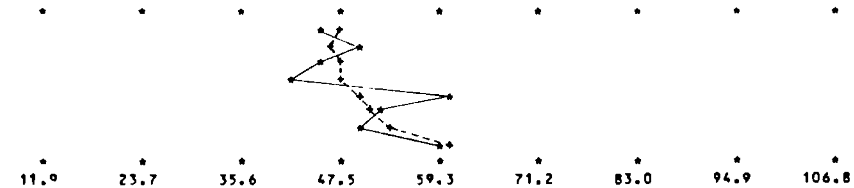
user cost of transportation

date	actl=*	predic=+	miss=p-a*
73	720.79	782.18	61.39
74	734.56	761.51	26.95
75	846.86	753.69	-93.16
76	764.57	762.16	-2.41
77	808.46	780.89	-27.57
78	877.74	823.29	-54.45
79	828.17	893.72	65.55
80	878.44	901.37	22.92
date	actl=*	predic=+	miss=p-a*



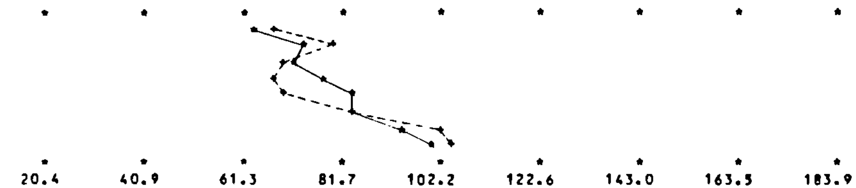
transport services

date	actl=*	predic=+	miss=p-a*
73	45.36	48.15	2.79
74	50.17	46.69	-3.48
75	45.97	47.84	1.87
76	41.60	47.55	5.95
77	60.75	50.12	-10.62
78	53.31	51.51	-1.80
79	50.96	54.25	3.29
80	59.32	61.28	1.96
date	actl=*	predic=+	miss=p-a*



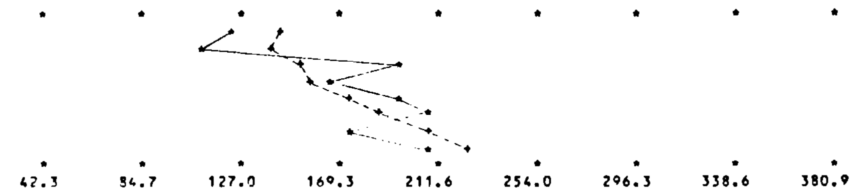
comunication

date	actl=*	predic=+	miss=p-a*
73	63.93	68.36	4.44
74	74.83	81.55	6.73
75	72.41	69.81	-2.61
76	77.90	68.05	-9.84
77	85.69	70.52	-15.17
78	84.63	84.65	0.02
79	95.65	103.98	8.33
80	102.18	105.50	3.33
date	actl=*	predic=+	miss=p-a*



newspapers and books

date	actl=*	predic=+	miss=p-a*
73	123.04	147.25	24.21
74	110.81	140.27	29.47
75	198.84	152.90	-46.04
76	165.48	157.96	-7.52
77	194.84	174.03	-20.80
78	209.83	189.25	-21.58
79	177.53	207.56	30.03
80	211.63	225.12	13.49
date	actl=*	predic=+	miss=p-a*



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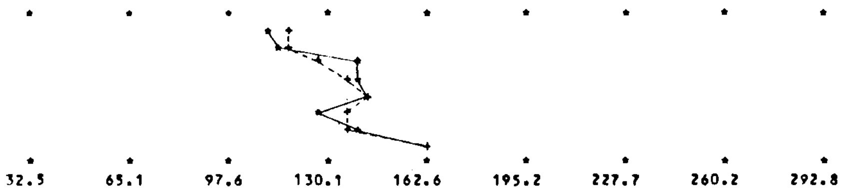
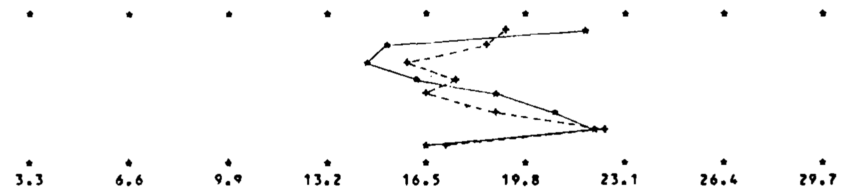
education

date	actl=*	predic=+	miss=p-a*
73	112.84	120.04	7.20
74	114.69	119.85	5.17
75	142.52	128.32	-14.20
76	143.00	137.39	-5.61
77	145.84	145.82	-0.02
78	127.59	137.10	9.51
79	141.54	139.79	-1.76
80	162.64	163.48	0.83
date	actl=*	predic=+	miss=p-a*



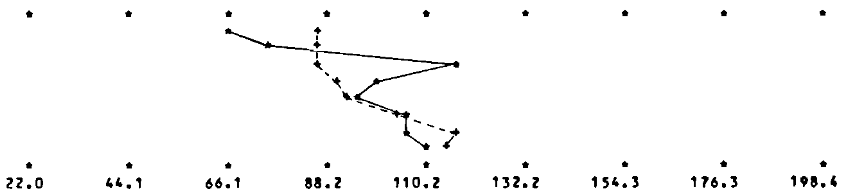
entertainment and recreation

date	actl=*	predic=+	miss=p-a*
73	21.98	19.30	-2.68
74	15.52	18.54	3.02
75	14.76	16.00	1.24
76	16.46	17.73	1.27
77	18.88	16.82	-2.07
78	20.83	19.02	-1.81
79	22.46	22.73	0.27
80	16.53	17.38	0.85
date	actl=*	predic=+	miss=p-a*



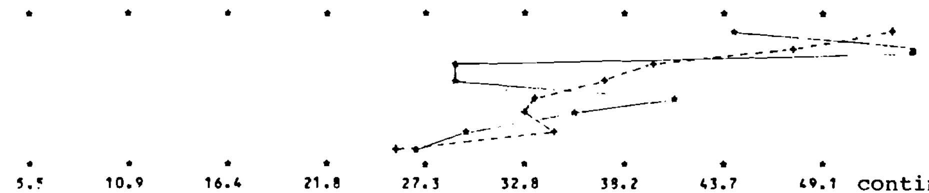
hotels, cafe, restaurants

date	actl=*	predic=+	miss=p-a*
73	67.94	86.46	18.51
74	76.61	85.96	9.35
75	118.72	86.58	-32.14
76	99.40	91.85	-7.55
77	96.95	94.09	-2.85
78	106.76	104.51	-2.25
79	105.89	117.78	11.89
80	110.20	114.94	4.74
date	actl=*	predic=+	miss=p-a*



domastic servant

date	actl=*	predic=+	miss=p-a*
73	44.45	53.50	9.04
74	68.93	47.75	-21.18
75	29.40	40.39	10.99
76	29.30	37.64	8.34
77	41.29	33.56	-7.73
78	35.54	32.91	-2.63
79	29.83	34.57	4.74
80	27.30	25.92	-1.39
date	actl=*	predic=+	miss=p-a*



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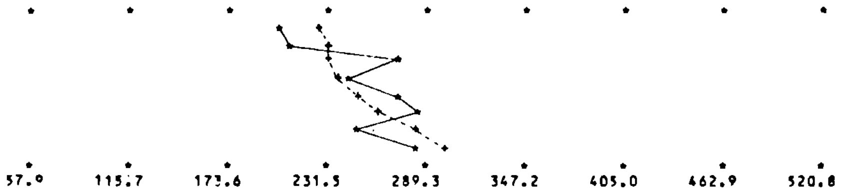
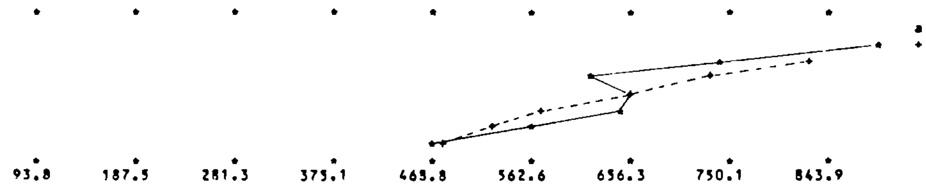
personal toilet articles

date	actl=*	predic=+	miss=p-a*
73	205.63	229.16	23.53
74	211.23	232.54	21.31
75	273.57	236.24	-37.34
76	247.38	241.69	-5.69
77	275.36	252.23	-23.13
78	283.93	263.49	-20.44
79	254.52	284.03	29.51
80	289.32	301.51	12.19
date	actl=*	predic=+	miss=p-a*



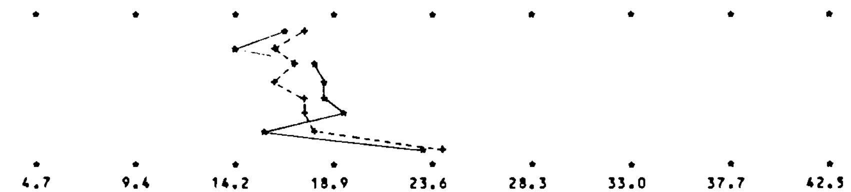
other goods

date	actl=*	predic=+	miss=p-a*
73	1203.64	1081.88	-121.76
74	893.13	934.72	41.59
75	744.34	829.43	85.09
76	624.97	731.64	106.67
77	664.90	657.09	-7.81
78	648.41	575.38	-73.03
79	563.84	525.36	-38.49
80	468.82	479.45	10.64
date	actl=*	predic=+	miss=p-a*



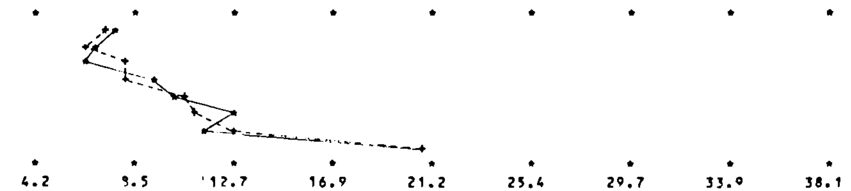
financial services and insur

date	actl=*	predic=+	miss=p-a*
73	16.78	17.65	0.88
74	14.39	16.49	2.10
75	18.11	17.21	-0.91
76	18.70	16.20	-2.50
77	18.43	17.56	-0.88
78	19.40	17.52	-1.88
79	15.74	18.38	2.64
80	23.59	24.18	0.59
date	actl=*	predic=+	miss=p-a*



other services

date	actl=*	predic=+	miss=p-a*
73	7.88	7.59	-0.29
74	6.80	6.70	-0.10
75	6.52	8.36	1.83
76	9.57	8.23	-1.34
77	10.51	10.67	0.16
78	12.74	11.34	-1.40
79	11.52	12.80	1.27
80	21.15	21.07	-0.12
date	actl=*	predic=+	miss=p-a*



Apart from some unexpected value of AAPE's, it is interesting to look at the estimated parameter b_{3i}, the time coefficient, which is reported in percentage with respect to the value of consumption of the last year. It is very important for forecasting aims to obtain small values of that parameter; otherwise, it can indicate a removal of the estimated curve from the true trend. In fact, estimated high values of b_{3i} mean that income and prices are not able to describe adequately consumption expenditure and so even the estimated income elasticities appear unappropriate. The worse results, in terms of AAPE and b_{3i}, have been obtained for expenditures on durables, for which the sample referring to a part of Italy, is suspected to be underdimensioned.

These results, obtained from a preliminary application, show that it is necessary to search for more refined data since the expenditure functions seem to work reasonably well. Furthermore, other estimation experiments might be performed, with different assumptions about starting λ_0 and the grouping criteria.

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SOME PROBLEMS OF REGIONAL INPUT–OUTPUT ANALYSES DEMONSTRATED WITH EXPORT DEPENDENCE IN BADEN–WÜRTTEMBERG

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1. Input-Output Techniques in the State of Baden-Württemberg

In accordance with the political organization of the Federal Republic of Germany (FRG) certain sectors of economic politics fall under the jurisdiction of the individual states. A consequence of this is that input-output tables have been compiled not only for the FRG as a whole, but also for some of the individual federal states as well. For the state of Baden-Württemberg input-output tables exist in the versions A and B for the year 1972, with 41 functionally defined production sectors (commodity by commodity tables); see /2/.

The most important data for the processing and the construction industries were gathered through a special survey /1/. In addition more aggregated input-output tables at 1970 prices with 14 production sectors were compiled for the years 1972, 1974 and 1976 in the version B. The regional matrix of Baden-Württemberg for the year 1974 is coordinated with a corresponding national matrix, compiled at current prices for 60 production sectors by the Federal Statistical Office /5/, and corrected to 1970 prices for 14 production sectors by the author. The method

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of compiling and price adjustment is described in /4/. Because of the enormous problems not only of estimating input-output tables for successive years on the basis of only partly complete information, but also of price adjustment, it is obvious that the data quality of the regional and national matrices compiled in this way, is not as accurate as that of the mentioned source matrices. Nevertheless for the successive empirical analyses we will refer to these input-output tables of 1974 because of the importance of having coordinated regional and national matrices.

2. Some Economic Data of Baden-Württemberg

The data of the mentioned input-output tables point to some characteristics of the economic structures of Baden-Württemberg. Table 1 shows that the economy of Baden-Württemberg is more concentrated on the production of investment goods - i.e. especially machinery, vehicles, electrical engineering, precision engineering and optics, hardware and metal goods -, and on timber, paper, leather and textiles and on the building and construction industries than that of the FRG as a whole. Thus the economies of the other federal states on an average are more concentrated on the service economy (including trade, transportation and the public sector) on the one hand, and on the winning of raw materials (i.e. agriculture, forestry and mining) as well as the basic industries (chemicals, building materials, mineral oil refining, iron, steel and metals) on the other. These differences in the economic structures are also reflected in the export structures. Whereas for example 67 per cent of the exports from Baden-Württemberg consist of machinery, vehicles, products of electrical or precision engineering, optics, hardware, metal goods and so on, the exports of these goods amount to only 45 per cent for the FRG as a whole. These differences in export structures explain largely why the export quota of Baden-Württemberg (12.9 per cent) exceeds that of the FRG as a whole (11.5 per cent).

Table 1: Some data of the economic structure of Baden-Württemberg (BW) and the Federal Republic of Germany (FRG) 1974

Production sector [1]	Gross value added				Exports				Export quota	
	BW		FRG		BW		FRG		BW	FRG
	mill. DM	p.c.	mill. DM	p.c.	mill. DM	p.c.	mill. DM	p.c.	p.c.	
1 Agriculture	3 560	3,0	24 912	3,3	62	0,2	1 089	0,6	0,7	1,9
2 Energy, mining	3 245	2,7	29 257	3,9	182	0,6	3 248	1,7	3,9	5,5
3 Chemicals, building mat.	7 956	6,7	71 574	9,5	2 676	8,8	30 918	16,6	14,9	19,1
4 Iron, steel, metals	3 397	2,9	23 805	3,2	1 445	4,7	20 177	10,9	18,4	15,7
5 Machinery, vehicles	14 747	12,4	62 244	8,3	13 117	43,0	53 537	28,8	38,1	34,9
6 Electrical engineering	13 149	11,1	58 237	7,8	7 313	24,0	29 292	15,8	27,1	25,2
7 Timber, paper, textiles	9 610	8,1	46 754	6,2	3 282	10,8	13 761	7,4	14,9	12,0
8 Food, beverages	5 306	4,5	39 568	5,3	524	1,7	6 285	3,4	3,4	5,3
9 Construction	10 088	8,5	51 153	6,8	222	0,7	1 179	0,6	1,1	1,0
10 Trade and commerce	12 735	10,7	80 635	10,7	670	2,2	11 588	6,2	3,7	9,5
11 Transportation	5 482	4,6	44 192	5,9	745	2,4	9 887	5,3	8,8	14,6
12 Other services	17 150	14,5	136 978	18,2	284	0,9	4 656	2,5	0,9	2,0
13 Private organizations	1 183	1,0	10 674	1,4	-	-	-	-	-	-
14 Public sector	11 013	9,3	70 837	9,4	-	-	205	0,1	-	0,1
All sectors	118 621	100	750 820	100	30 522	100	185 822	100	12,9	11,5

1) See glossary in the appendix

3. Export Dependence, Direct and Indirect

3.1 Special Regional Aspects

Questions of differences in export dependence between Baden-Württemberg and the FRG as a whole (or the other federal states respectively) will be treated comprehensively in the following. For this purpose we will calculate the indirect export dependence of the production sectors of Baden-Württemberg and of the FRG. For this we use the open static Leontief model (quantity version)

$$x_e = (I - A)^{-1} y_e \tag{1}$$

where y_e = vector of the (direct) exports
I = unit matrix
A = matrix of the input coefficients
x_e = vector of the direct and indirect exports.

It is useful to remember that the iterative solution procedure for this model is as follows:

$$x_e = y_e + Ay_e + A^2y_e + \dots + A^ny_e \tag{2}$$

in the iteration steps 1 2 .... n.

For $n \to \infty$ we arrive at the general solution shown in (1).

Applying this model to calculate the export dependence in Baden-Württemberg and the FRG, and using the export vectors and the input coefficient matrices of Baden-Württemberg on the one hand and of the FRG on the other, we get the results shown in table 2. We see that in 1974 the direct and indirect export quota of Baden-Württemberg amounts to 17.7 per cent, and that this is less than that of the FRG as a whole (22.7 per cent). This may seem surprising because as mentioned the direct export quota of Baden-Württemberg is 1.4 percentage points higher than that of the FRG as a whole. Indeed we see a considerable difference between the indirect export quotas of Baden-Württemberg (4.8 per

Table 2: Direct and indirect export dependence in Baden-Württemberg and the Federal Republic of Germany 1974

Production sector[1]	Export dependence of Baden-Württemberg total	Baden-Württemberg direct	Baden-Württemberg indirect	Federal Republic of Germany total	Federal Republic of Germany direct	Federal Republic of Germany indirect
	Exports in mill. DM					
1 Agriculture	468	62	406	6 303	1 089	5 214
2 Energy, mining	612	182	430	17 041	3 248	13 793
3 Chemicals, building mat.	3 674	2 676	998	56 803	30 918	25 885
4 Iron, steel, metals	3 107	1 445	1 662	69 049	20 177	48 872
5 Machinery, vehicles	15 042	13 117	1 925	69 118	53 537	15 581
6 Electrical engineering	8 552	7 313	1 239	42 885	29 292	13 593
7 Timber, paper, textiles	4 304	3 282	1 022	25 517	13 761	11 756
8 Food, beverages	723	524	199	10 980	6 285	4 695
9 Construction	320	222	98	2 115	1 179	936
10 Trade and commerce	1 474	670	804	24 635	11 588	13 047
11 Transportations	1 608	745	863	18 742	9 887	8 855
12 Other services	1 857	284	1 573	23 276	4 656	18 620
13 Private organizations	6	0	6	99	0	99
14 Public sector	124	0	124	1 159	205	954
All sectors	41 871	30 522	11 349	367 722	185 822	181 900
	Export quota in per cent					
1 Agriculture	5,4	0,7	4,7	10,8	1,9	8,9
2 Energy, mining	13,2	3,9	9,3	28,9	5,5	23,4
3 Chemicals, building mat.	20,4	14,9	5,5	35,1	19,1	16,0
4 Iron, steel, metals	39,5	18,4	21,2	53,8	15,7	38,1
5 Machinery, vehicles	43,7	38,1	5,6	45,0	34,9	10,1
6 Electrical engineering	31,7	27,1	4,6	36,9	25,2	11,7
7 Timber, paper, textiles	19,6	14,9	4,7	22,3	12,0	10,3
8 Food, beverages	4,7	3,4	1,3	9,2	5,3	3,9
9 Construction	1,6	1,1	0,5	1,8	1,0	0,8
10 Trade and commerce	8,2	3,7	4,5	20,1	9,5	10,6
11 Transportations	18,9	8,8	10,2	27,6	14,6	13,0
12 Other services	6,1	0,9	5,2	9,8	2,0	7,9
13 Private organizations	0,4	0	0,4	0,7	0	0,7
14 Public sector	0,6	0	0,6	0,8	0,1	0,6
All sectors	17,7	12,9	4,8	22,7	11,5	11,2

1) See glossary in the appendix

cent) and the FRG (11.2 per cent). Nevertheless we cannot conclude from these results a much reduced demand for intermediate inputs for Baden-Württemberg in relation to the other states of the FRG. This would be most implausible as the exports from Baden-Württemberg as mentioned are more concentrated on finished products (especially investment goods), that usually require intermediate goods in greater quantities than the production of raw materials or the products of basic industries. The figures reflect rather the use of the model, more accurately the regional distinction in the inclusion of indirect effects.

For demonstration purposes the national economy of a state 'n' can be subdivided into two regions 'a' and 'b' to analyse the effects of indirect export dependence in these regions. In figure 1 we see the (direct) exports and the two initial steps of the indirect effects of export dependence - according to equation (2) - in a state 'n', now subdivided into all possible effects for the two regions 'a' and 'b'. The lines 1 to 4 show the effects of the exports from region 'a'. For the first two steps these exports can induce supplying effects in region 'a' only (line 1), in region 'b' only (line 4) or both in region 'a' and 'b' (lines 2 and 3). In the same manner indirect effects in regional distinction exist for the exports from region 'b' (lines 5 to 8). Of course all these direct and indirect export effects are part of the economy of the state 'n' i.e. all intermediate inputs imported from the economies of other states are left out of consideration.

The open static Leontief model for the state 'n'

(3) $$x_e^{nn} = (I - A^n)^{-1} y_e^n \qquad \text{(model I)}$$

where y_e^n = vector of the direct exports from the state 'n'

A^n = matrix of the input coefficients of the state 'n'

x_e^{nn} = vector of direct and indirect exports induced by exports from the state 'n' and effective in the state 'n'

Figure 1
Export induced supplying effects for a national economy with 2 regions 'a' and 'b'

Line	Direct exports from region	Export induced supplying effects in step 1 for region	Export induced supplying effects in step 2 for region
1	a	a	a
2		a	b
3		b	b
4		b	a
5	b	b	b
6		b	a
7		a	a
8		a	b

gives the results of all indirect effects that are presented in figure 1 for the first two steps according to equation (2) and also the effects of the following steps. The appropriate indirect effects are given exactly in

(4) $$z_e^{nn} = x_e^{nn} - y_e^n$$

where z_e^{nn} = vector of the indirect exports induced by the export from state 'n' and active in state 'n'

x_e^{nn} and y_e^{n} as are defined in equation (3).

An open static Leontief model for the region 'a'

(5) $$x_e^{aa} = (I - A^a)^{-1} y_e^a \qquad \text{(model II)}$$

where y_e^a = vector of the direct exports from the region 'a'

A^a = matrix of the input coefficients of the region 'a'

x_e^{aa} = vector of direct and indirect exports induced by exports from the region 'a' and effective in the region 'a'

gives the vector of the indirect effects

(6) $$z_e^{aa} = x_e^{aa} - y_e^a.$$

Looking at the first two supplying steps shown in figure 1, this vector includes both steps in line 1 and the first step in line 2. The second step of line 4 is not included, although it is induced by exports from region 'a' and concerns the supplying industries in this region too. Likewise in model II the effects are not included that are induced by exports from region 'a' and are effective for the supplying industries of region 'b', or that are induced by exports from region 'b' and are effective in the supplying industries of region 'a' respectively. In the same manner there are problems for the successive steps 3,4, etc.

Thus model II does not include three kinds of indirect effects, induced by exports from region 'a' or 'b', that should be considered when discussing indirect export effects in region 'a' or 'b', and that are in some sense important for comparison with results of corresponding national analyses. In particular these are the following effects:

1) Certain indirect effects that are induced by the exports from region 'a' and that are effective for the supplying industries of this region. Parts of these effects are not included in the results of model II, namely when before an appropriate supplying effect step in the series of direct and indirect effects at least one supplying effect is included which is active for industries of region 'b'. Figure 1 gives an example of this in line 4, effect of second step.

2) All indirect effects that are induced by the exports from region 'b' and that are effective for the supplying industries of region 'a'. From the point of view of the whole national economy these effects are clearly export induced, and they are effective in region 'a'. Nevertheless they are neither included in the results of model II nor in a corresponding model based on the export vector and the input coeffient matrix for the exporting region 'b' (y_e^b, A^b). In figure 1 these effects are demonstrated in line 7 (two effects) and in the lines 6 and 8 (one effect each).

3) All indirect effects that are induced by the exports from region 'a' and that are effective for the supplying industries of region 'b'. From the point of view of the national economy these effects are export induced, but they are neither included in model II for region 'a' nor in the corresponding model for region 'b'. Figure 1 shows these effects for the first two steps in line 3 (two effects) and in the lines 2 and 4 (one effect each).

3.2 Possibilities for Solving the Regional Problems

The following passages show some possibilities of estimating these regional effects in their order of magnitude.
It is comparatively simple and comprehensible to estimate the effects mentioned in 3). The open static Leontief model

(7) $$x_e^{na} = (I - A^n)^{-1} y_e^a \qquad \text{(model III)}$$

where y_e^a = vector of the direct exports from the region 'a'

A^n = matrix of the input coefficients of the state 'n'

x_e^{na} = vector of the direct and indirect exports induced by exports from region 'a' and effective in the state 'n'

gives information of the indirect exports in the national economy of the state 'n' induced by the exports of region 'a'. Precisely the appropriate vector of the indirect effects is defined by

(8) $$z_e^{na} = x_e^{na} - y_e^a.$$

Subtracting the results of model II (equation (6)) from these results we get an order of magnitude of the effects induced by the exports from region 'a' and effective in the other region of the state 'n', i.e. region 'b'

(9) $$z_e^{ba} = z_e^{na} - z_e^{aa}$$

where z_e^{ba} = vector of the indirect exports induced by exports from region 'a' and effective in region 'b'

z_e^{na}, z_e^{aa} as are defined in the equations (8) and (6).

As mentioned in 1) of section 3.1 it should be taken into consideration however that only some parts of indirect export effects that are both induced and effective in the region 'a' can be estimated with model II. We must remember this later on.

The effects described in 2) can be computed indirectly with the help of model I. The indirect effects induced by the exports from the state 'n' and effective in the state 'n' thus gained - see the equations (3) and (4) - can be divided into those that are effective in region 'a' and those in region 'b'. This can be done in an auxiliary fashion by taking the regional quotas of sectoral gross output

(10 a) $$q_i^a = x_i^a / x_i^n$$

(10 b) $$q_i^b = x_i^b / x_i^n$$

with $i = 1, 2, \ldots, m$

where x_i^n = gross output of production sector 'i' in the state 'n'

x_i^a = gross output of production sector 'i' in the region 'a'

x_i^b = gross output of production sector 'i' in the region 'b'.

We know that using regional quotas of sectoral gross output is not as appropriate as using regional quotas of sectoral intermediate outputs. These quotas can only be obtained when comparable input-output tables are available for both regions. Unfortunately this is not the case in the FRG.

Using the quotas shown in equation (10 a) and (10 b) the vector z_e^{nn} giving the results of indirect export induced effects in the state 'n' - see equation (4) - can be divided into the vectors z_e^{an} and z_e^{bn}. These vectors express respectively the indirect effects that are induced by the exports of the state 'n' and are effective for the supplying industries of both regions 'a' or 'b'. The elements of these vectors are defined

(11 a) $$z_{ei}^{an} = z_{ei}^{nn} \, q_i^a$$

(11 b) $$z_{ei}^{bn} = z_{ei}^{nn} \, q_i^b$$

with $i = 1, 2, \ldots, m$

where z_{ei}^{nn}, z_{ei}^{an}, z_{ei}^{bn} are the elements of the appropriate vectors which express the indirect effects of the production sector 'i'

q_i^a, q_i^b as are defined in the equations (10 a) and (10 b).

To estimate the effects described in 2) of section 3.1 the indirect export induced effects active in region 'a' and induced by the exports from region 'a' too - i.e. z_e^{aa}, see equation (6) - are subtracted from the effects given in the vector z_e^{an} (equation (11 a))

(12) $$z_e^{ab} = z_e^{an} - z_e^{aa}$$

where z_e^{ab} = vector of the indirect exports induced by exports from region 'b' and effective in region 'a'.

In addition, with the help of the results of the previous calculations we are now able to estimate the indirect effects that are both operational in, and induced by the exports from region 'b'. For this purpose the following operation is done

(13) $$z_e^{bb} = z_e^{bn} - z_e^{ba}$$

where z_e^{bb} = vector of the indirect exports induced by exports from region 'b' and effective in region 'b'

z_e^{bn}, z_e^{ba} as are defined in the equations (11 b) and (9).

As a provisional result we have four vectors of indirect effects induced by the exports from the regions 'a' and 'b' of a state 'n', and effective for the supplying industries of these two regions. These vectors were computed by the procedures shown in the following equations

(6) $\longrightarrow z_e^{aa}$

(9) $\longrightarrow z_e^{ba}$

(12) $\longrightarrow z_e^{ab}$

(13) $\longrightarrow z_e^{bb}$

in which generally

(14) z_e^{kl} = vector of the indirect exports induced by exports from region 'l' and effective in the supplying industries of region 'k'.

We must remember however, that as mentioned in 1) of section 3.1 the vector z_e^{aa} computed with model II does not include some streams of the indirect effects active in region 'a' and induced by exports from region 'a'. The most important of these effects are included in the results of the vector

(15) z_e^{aba} = vector of the indirect effects active in region 'a' and induced by the first supplying effect of region 'b', which has been induced by the exports from region 'a' in its turn.

As can be seen this vector only expresses the effects induced by the first supplying step of region 'b'. Therefore its results do not include all relevant effects, but apparently the most important ones.

This vector z_e^{aba} can be used not only to correct the results of the vector z_e^{aa}, but also the results of the other vectors, because these vectors are computed with the help of the 'regional' vector z_e^{aa} on the basis of relevant 'national' vectors. This is shown in the equations (9), (12) and (13).
By calling

(16) z_e^{kl*} = 'corrected' vector of indirect exports induced by exports from region 'l' and effective in the supplying industries of region 'k'

we get the equations

(17) $z_e^{aa*} = z_e^{aa} + z_e^{aba}$

(18) $z_e^{ba*} = z_e^{ba} - z_e^{aba}$

(19) $z_e^{ab*} = z_e^{ab} - z_e^{aba}$

(20) $z_e^{bb*} = z_e^{bb} + z_e^{aba}$.

We will try now to give some points of the order of magnitude of these effects. The first supplying step of the indirect effects in the vector z_e^{aba} is induced by exports from region 'a' and is effective for the industries in region 'b'. Its values can be computed with the model

(21) $z_{e1}^{ba} = A^{ab}\ y_e^{a}$

where y_e^a = vector of the exports from region 'a'

A^{ab} = matrix of the 'supplying coefficients' in reference to supplies made by industries in region 'b', and delivered to industries in region 'a'. The matrix is computed by dividing the elements containing the supplies from region 'b' into region 'a', by the gross output of the appropriate receiving production sector of region 'a'

z_{e1}^{ba} = vector of the first supplying effect induced by exports from region 'a' and effective in region 'b'.

This first indirect effect active in region 'b' induces in its turn further indirect effects; according to equation (2) the second and following supplying steps. These can be computed to ascertain their effectiveness for state 'n' as a whole with the model

$$(22) \qquad x_e^{nba} = (I - A^n)^{-1} z_{e1}^{ba}$$

where x_e^{nba} = vector of the direct and indirect effects in state 'n' induced by the first indirect effect in region 'b', which has itself been induced by exports from region 'a'

z_{e1}^{ba} and A^n as are defined in the equations (3) and (21).

In further steps we can compute the vector

$$(23) \qquad z_e^{nba} = x_e^{nba} - z_{e1}^{ba}$$

where z_e^{nba} = vector of the indirect effects in state 'n' induced by the first indirect effect in region 'b', which has itself been induced by exports from region 'a'

and distribute its results between the regions 'a' and 'b' by using regional quotas of the sectoral gross output again.
In particular we get

$$(24) \qquad z_{ei}^{aba} = z_{ei}^{nba} q_i^a$$

where z_{ei}^{aba} are the elements of the vector z_e^{aba} defined in (15)

z_{ei}^{nba} are the elements of the vector z_e^{nba} defined in (23)

q_i^a as is defined in equation (10 a).

We must admit that the results of these operations can only give an order of the magnitude of the effects which are not included in model II. The particular effects are not computed which are induced by the second and the following steps - i.e. z_{e2}^{ba}, z_{e3}^{ba}, ..., -, and there are also problems in distributing the various effects between the two regions.

4. Some Results

4.2 Preliminary remarks

The following examples show some results concerning the export dependence of Baden-Württemberg more thoroughly. Calling the FRG as a whole the state 'n', Baden-Württemberg the region 'a', and the other federal states as a whole the region 'b', we can estimate the various results of regional export dependence expressed by the vectors that were defined and explained in chapter 3. For calculation purposes we use the input-output tables of the FRG and Baden-Württemberg for 1974 which were mentioned in chapter 1. It should be noted that it is intentional that only the export dependence is discussed here, because the export vector is the only vector of an input-output table that does not include imports or supplies from other economies. Consequently it is possible to calculate this vector for 'region b' (or the other federal states respectively) although no input-output table for this region is available, but only for 'state n' (FRG) and 'region a' (Baden-Württemberg). This aspect has special importance for calculating the results of equation (9) with the use of the equations (6) and (8) or the results of equation (12) with the use of the equation (4), (6) and (11) respectively. Finally it has been possible to estimate in equation (13) the indirect effects induced by the exports from region 'b' and active for supplying industries in region 'b', without having an input-output table for this region.

In presenting the results we begin with estimating the order of magnitude of vector z_{e}^{aba} - see (15) and (24). For Baden-Württemberg the sum of the effects represented by this vector amount to about 0,75 billion DM. That is about 6.6 per cent of the indirect effects shown in table 2, column 1 i.e. the indirect effects induced by the exports from Baden-Württemberg and effective for the supplying industries of this state are underestimated using model II by at least 6.6 per cent. For this reason only the corrected results will be refered to in the following, i.e. the results of the equations (17) to (20). In this respect the results of this paper differ from those of the earlier analysis /3/, where these problems of certain missing effects were no more than mentioned. Finally the analysis of the present paper is based on coordinated national and regional input-output tables and of a more recent date.

4.2 The Export Dependence in Baden-Württemberg and the Other States of the FRG

Table 3 shows the values of the export induced supplying effects for Baden-Württemberg and the other federal states. We see that the exports from Baden-Württemberg induce supplying effects that ar more effective in the other states of the FRG (17.9 billion DM) than in Baden-Württemberg itself (12.1 billion DM). Moreover these supplying effects for the other states (17.9 billion DM) are more extensive than those that are induced by the exports from the other states and active in Baden-Württemberg (10.9 billion DM).

To come back to the statements given in chapter 2 and in the beginning of section 3.1, a comparison between the indirect export dependence of a national state and that of one of its region must consider the indirect export dependence of the other regions too, whether for their exporting or their supplying industries.

Otherwise the figures in table 3 give an impression of the characteristics of the division of labour between Baden-Württemberg and the other federal states. As mentioned the industries of Baden-Württemberg induce more supplying effects in other states (17.9 billion DM) than in their own state (12.1 billion DM), or than do the other states in Baden-Württemberg (10.9 billion DM). These differences and especially the enormous magnitude of the supplying effects for industries in the remaining federal states account for the economic structure of Baden-Württemberg that has been already mentioned in chapter

Table 3: Direct and indirect export dependence of Baden-Württemberg (BW) and the other states of the Federal Republik of Germany 1974

Production sector[1]	Supplying effects induced by the exports from Baden-Württemberg and effective in BW	Baden-Württemberg: other states	other states: BW	other states: other states
	million DM			
1 Agriculture	466	147	299	4 302
2 Energy, mining	472	1 448	614	11 259
3 Chemicals, building mat.	1 088	2 812	1 785	20 200
4 Iron, steel, metals	1 740	6 525	1 251	39 356
5 Machinery, vehicles	1 987	1 321	1 508	10 765
6 Electrical engineering	1 307	1 534	1 856	8 896
7 Timber, paper, textiles	1 099	1 029	1 153	8 475
8 Food, beverages	242	297	363	3 793
9 Construction	106	19	49	762
10 Trade and commerce	868	1 179	1 042	9 958
11 Transportations	900	498	208	7 249
12 Other services	1 682	1 091	720	15 127
13 Private organizations	7	6	4	82
14 Public sector	130	22	2	800
All sectors	12 094	17 928	10 854	141 024
	Regional distribution in per cent			
1 Agriculture	76,0	24,0	6,5	93,5
2 Energy, mining	24,6	75,4	5,2	94,8
3 Chemicals, building mat.	27,9	72,1	8,1	91,9
4 Iron, steel, metals	21,1	78,9	3,1	96,9
5 Machinery, vehicles	60,1	39,9	12,3	87,7
6 Electrical engineering	46,0	54,0	17,3	82,7
7 Timber, paper, textiles	51,6	48,4	12,0	88,0
8 Food, beverages	44,9	55,1	8,7	91,3
9 Construction	84,8	15,2	6,0	94,0
10 Trade and commerce	42,4	57,6	9,5	90,5
11 Transportations	64,4	35,6	2,8	97,2
12 Other services	60,7	39,3	4,5	95,5
13 Private organizations	53,8	46,2	4,7	95,3
14 Public sector	85,5	14,5	0,2	99,8
All sectors	40,3	59,7	7,1	92,9

1) See glossary in the appendix

2. The concentration on the production of export intensive investment goods involves an enormous demand for basic goods - especially iron, steel and other metals and also chemical goods -, as well as for raw materials too. In accordance with the national division of labour in the FRG, the basic goods are produced mainly in the other federal states. The figures in table 1 give an impression of these economic structures. In consequence the demand in Baden-Württemberg for intermediate products produced in other federal states is above average.

A look at the individual production sectors in table 3 confirms this. Of the typical supplying sectors energy and mining or the production of basic goods (i.e. chemicals etc., iron, steel and metals) the supplying effects induced by the exports from Baden-Württemberg are very much lower for its own industries than for those of the other states. The corresponding industries of Baden-Württemberg profit from these supplies for only 21 to 28 per cent (see table 3, column 1). Only for certain investment goods within the processing industries are these supplying effects for Baden-Württemberg considerably greater than for the other states: of the intermediate machinery and vehicle products 60 per cent are delivered from industries in Baden-Württemberg, 40 per cent from the other states. With timber, paper, leather and textiles the relation is 52 to 48 per cent. The even greater quotas in favour of its own state existing for agriculture and forestry (76 per cent), building and construction (85 per cent), transportation (64 per cent) and services as a whole (62 per cent) are the consequence of their regionally more limited radius of action.

Altogether the high supplying effects in favour of energy, mining and the production of basic goods in the other states dominate the total regional distribution (60 per cent in favour of the other states - see table 3, column 2).

In the same way we see a relative concentration in the production of intermediate investment and consumption goods in the deliveries from Baden-Württemberg to the other states (table 3, column 3). The supplying industries of Baden-Württemberg are very important in this sense for electrical goods (17 per cent of all supplying effects induced by exports of the other states concern this state - see table 3, column 3), for machinery and vehicles, (12 per cent) and for timber, paper and textiles (12 per cent). But as the export induced demand of intermediate basic goods and raw materials is generally greater the result is that Baden-Württemberg has only 7 per cent of all German supplying effects induced by exports from the other federal states (see table 3, column 3). Consequently Baden-Württemberg is as a supplier to other federal states not so important as conversely.

5. Summary

This paper points to some problems of regional input-output analyses that are important for comparison with corresponding national analyses. As shown in the example of indirect export dependence, the open static Leontief model gives results of supplying effects for a region that do not include all relevant effects for this region. Especially in comparing the corresponding results of the nation as a whole, it is necessary to take into consideration the economies of other regions of this nation too. Thus the following two effects should be computed for comparison purposes: Firstly, the supplying effects induced by exports from other regions and effective in the relevant region, and secondly, the supplying effects induced by the exports from this region and effective in the other regions of the nation. The computation of these effects is possible with the use of the open static Leontief model on the basis of coordinated input-output tables for the nation and at least one of its regions. In the case of Baden-Württemberg and the other states of the FRG the values of these supplying effects are enormous and cannot be neglected. Moreover the results of the analyses give an interesting impression of the interregional streams in the FRG and indicate the characteristics of the division of labour in its national economy.

Appendix

Glossary of terms

No	Production sector	
1	Agriculture	Agriculture, forestry, fishing and gardening
2	Energy, mining	Electricity, gas and water, coal mining, iron ore mining, potash and rock salt mining, mineral oil extraction, mining n.e.s.
3	Chemicals, building mat.	Building materials, chemicals, mineral oil refining, rubber and asbestos manufactures, fine ceramics, glass, plastics manufactures
4	Iron, steel, metals	Iron and steel, iron and steel foundries, steel drawing and cold rolling mills, nonferrous metals, steel forging
5	Machinery, vehicles	Constructional steel, machinery, vehicles, aerospace, shipbuilding
6	Electrical engineering	Electrical engineering, precision engineering and optics, hardware and metal goods, musical instruments, toys, jewelry und sport articles
7	Timber, paper, textiles	Saw mills and wood processing, cellulose and paper, timber manufactures, paper and board manufactures, printing and duplicating, leather, textiles, clothing
8	Food, beverages	Grain milling, edible oils and margarine, sugar, brewing and malting, tobacco manufactures, other food and beverages
9	Construction	Construction
10	Trade and commerce	Wholesaling, retailing
11	Transportation	Railway, shipping, waterways and harbours, other transport, communications (Bundespost)
12	Other services	Banks and insurance, rented dwellings, services n.e.s.
13	Private organizations	Private households, private non-profit organizations
14	Public sector	Public sector (incl. social insurance)

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AN INFORUM-TYPE INPUT–OUTPUT MODEL FOR THE POLISH ECONOMY: PRELIMINARY EMPIRICAL RESULTS

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1. Disaggregation level of the model, statistical data

The input-output model in its present version comprises 31 branches of material production sector[1/]. In final demand - apart from branch division - the following categories were distinguished: personal consumption expenditures are examined in 20 groupes, investments are divided into three kinds - machinery, buildings and constructions, and others. The acceptance of such disaggregation level of the model was conditioned by many factors. However, it is not the final version. The possibilities of further disaggregation of the model are discussed below.

Input-output matrix coefficients /A matrix/ and also bridge /conversion/ matrices referring to the branch structure of consumption and investments were taken in testing

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1/ Material services are also included in industry input. The sector of non-material services was not considered. The appearance of the 1977 balance in SNA version this year will enable us to include non-material services in the model.

calculations from the input-output table for 1977. Two factors determined the choice of this table: 1/ it was the latest of the constructed tables, and 2/ the elements of final demand and other values to be generated in the model are so far available in statistical publications in 1977 prices.

Information contained in the remaining tables does not seem to be of any considerable use in constructing input-output and conversion coefficients. These tables are set yearly from 1966 to 1975. The 1977 balance mentioned above is the succeeding one. They are all presented according to the prices used and the treatment of imports in three variants: in final recipient prices, in producer prices with imported goods distinguished or not distinguished. The main disadvantage is that balances are constructed in current prices of the given year only; besides, different level of branch classification causes that branch disaggregation level - common for all years - includes only 15 branches of material production sector. Moreover, comparability is made difficult by a considerable change of branch classification on the turn of the sixties. The publishing of the 1980 table /the 1982 Year-Book/ does not help much. Though it is the most recent table the structural relationships of the national economy are rather typical for the seventies. Economic crisis resulting in forced structural changes /and technological ones to a certain degree/ occurred later. General change of price levels should also be taken into account - it was introduced this year. While pointing at all these facts we want to emphasize that forecasting input-output coefficients change is extremely difficult in this situation. All mechanical methods /forecasting based on coefficient changes functions and other adjustment methods/ may appear deceptive in relation to big qualitative changes in structural relationships between branches.

Elements of final demand together with other endogenous variables entering the econometric submodel and also exogenous variables included in it are shown in the 1977 prices which ensures their conformability with the data included in the 1977 balance. At present it is impossible to take into account the general change of price levels introduced this year and that causes certain conventionality of debates regarding final recipients' demand and also brings the necessity of reestimation of respective equations. However, general trends of final recipients' demand and its limitations will be shown. The huge total increase of domestic prices did not change deeply the relative prices of both foodstaffs and manufactured goods. Time series for parameters estimation in the econometric part of the model cover the period from 1960 to 1980.

2. Testing computations based on the input-output model

The input-output model with 31 branches and 24 final demand categories and respective matrices fixed for 1977 was used to test the accuracy of computations of total output/and material costs /for the years 1978, 1979 and 1980 that were performed using the SLIMFORP program.

The aim of these calculations, apart from using the SLIMFORP program in the Polish model was to define the degree of influence of accepting the assumption of coefficients stability on the obtained results regarding production forecasts by comparing them to the respective actual values.

Such comparison was possible first of all for the years

1978 and 1979 for which the actual values of variables referring to final demand were taken . The data referring to exports and imports in fixed domestic prices were take exceptionally from the data bank compiled in the Institute of Econometrics and Statistics of Łódź University especially for econometric models of Polish economy constructed there 3/.

While performing computations we did not yet have official statistical publications for 1980 therefore the values referring to final demand for this year are preliminary 4/.

The results relating to net material product appeared to be overestimated as compared to the observed values. Net material product, for example, increased by 3% in 1978 while the forecasts based on the model gave a result of 6.6%. Similarly in 1979 - 3% in relation to 0.5% based on the model.

For total output the following relations were observed: actual rates of growth in 1978 and 1979 were respectively 4.7% and 2.7% while those based on the model - 7.8% and 0%.

The comparison of growth rate of total output for some branches and groupes of branches in 1979 is shown below:

rate of growth in % for:	Forecast	Actual value
fuel and power engineering complex	1.8	-3.7
electric and machinery industry	+2.4	-4

3/ exports and imports in constant domestic prices were computed according to realised exchange rate for 1977. Branch structure of aggregates was based on domestic prices, it was obtained by referring the structure in domestic prices of 1977 to the structure in exchange zloty in 1978 and 1979

4/ apart from the 1981 Year-Book that includes data for 1980 /published in April this year/ the 1982 Year-Book has been published now and it includes data for 1981

food industry	2	0
building industry	-2.9	-1
agriculture	0	2.8
transport and communication	7.7	10.7

Taking into consideration those forecasting errors that are due to inaacurate estimates of exports and imports it should be noted that the divergence between forecasts and observed values results from accepting constant A matrix and conversion coefficients. In 1979 the rate of growth of total output determined on the basis of the model is lower for the whole economy rather than observed. As regards net material product the opposite occurs. Considering this result it may be claimed that the use of materials per unit of output increased which might be due to the forced substitution of imported and domestic raw-materials and semifinished products as well as spare parts being in short supply. Particularly big divergencies between forecasts and reality refer to total output in industries. They confirm the assumption of great increase of the usage of materials in industries.

The conclusion obtained from the conducted experiment is explicit. We face the problem of introducing necessary changes of coefficients in the input-output model.

3. Some remarks on input-output coefficients forecasting

In our studies on including changes of input-output coefficients that were conducted within the W-2 model of the Polish economy[1] various methods were used. The range of studies in relation to some methods was described in[2].

Let us quote the methods that have been used so far:

1. Indirect method based on correcting forecasts of economic values obtained with the use of the input-output model with constant coefficients. This correction consisted in adding certain residuals to the equation which generated this value. This residual shows the difference between the observed value and the forecast of this value /so called residual method/.

2. Biproportional methods of RAS type and its modifications consist mainly in introducing trends of some coefficients in fixed elements in base matrix.

3. Methods of regression analysis consisting in estimation of parameter vectors βi and γi of balance equations

$$E_{it} = \underline{Q}_t'(\underline{\beta}_i + \underline{\gamma}_i t) + \eta_{it},$$

/E_{it} - the difference between total input and final output,
Q_t - total output,
η_{it} - random term supposed as a white noise for the given i,
i - industry index,
t - year index;
the vector $\underline{\alpha}_{it} = \underline{\beta}_i + t\,\underline{\gamma}_i$ is the i-th row of A matrix in the year t /.

As can be seen, while using this method one need not know any matrix of input-output flows, the knowledge of total and final output vectors is sufficient. Formally the problem of defining forecasts was to determine the minimum quadratic form under conditions of linear inequalities in order to guarantee non-negativity of a_{ij} and productivity of A matrix. In order to estimate mean square errors of estimates the Monte Carlo experiment was used. It takes into account the fact that the set of constrains which appeared to be "active" in our sample/they were satisfied as equalities/is random.

In our experiments of comparing forecasts - mainly of net output - obtained with the use of various methods of adjustments procedures introducing changes of coefficients, the input-output tables for 1971- 75 were used. The third method has not been tested empirically yet. The most general conclusions that can be drawn from the conducted experiments are as follows: more satisfying net output forecasts were obtained by correcting rather immediately the coefficients[5/] than by correcting the forecasts obtained with constant coefficients with the use of the residual method.

Forecasting coefficients with the use of biproportional methods or other balanced techniques using the matrix of the given year as a base matrix means correcting base coefficients by minimizing the difference between the elements of base matrix and the forecast matrix. In this situation big changes of coefficients in the forecast period may not be shown correctly. Therefore, various modifications of correcting techniques are used; they consist in fixing values of some coefficients before the correcting procedure is applied. Most frequently, they are the so called important /sensitive/ coefficients. There are many methods of defining important coefficients. In our studies these coefficients are distinguished first of all because of their influence on the value of net output or total output /for example how many percent an input coefficient may change such that the output of any sector does not change more than one percent/ we also try to range the importance according to various criteria.

5/ We have in mind the coefficients of conversion type linking net output with the categories of final demand. This conversion matrix is obtained by multiplying several matrices, among others $/I-A/^{-1}$

Thus the main problem is in fact the accurate forecasting of some coefficients only. Our experiments with trend functions made for the seventies, when there were not big changes in structural relationships, appeared to be unsatisfactory, chiefly because of too short time series of data concerning coefficients. For this reason we did not try to construct the equations explaining the coefficients' changes where suitable explanatory variables are introduced. In fact, the insufficiency of statistical base was the stimulus to develop the method presented above as the third method. This method seems to be of much use when the coefficients are expected to be subject to regular changes in time.

In our present situation, considering important coefficients only, may solve the problem. Limiting our studies to some coefficients only, will enable us to make deep analysis of economic and technological causes of their changes.

4. *Final demand categories equations*

The INFORUM-type model is a typical example of the demand determined model. In these models the system of input-output equations that generates production requirements is most important as production capacities are assumed to be unlimited and thus the realization of final and intermediate demand secured. In supply determined models production functions are of main importance /they generate the ceilings of production levels that can be obtained given input-output constrains/, then allocation equations of this production among production recipients and final recipients. These are obviously extreme approaches which show extreme economic situations. Production

Production capacities of some branches and sectors can be practically accepted as unlimited while those of others can not, generating bottle-necks often causing further under-utilization of capacities of the first ones /provided they are complementary input/.

Such situation is typical for the Polish crisis. Thus from this point of view it seems that the formal description of functioning of the Polish economy does not have to be based on typical supply determined models, not mentioning cognitive and informational advantages of the model that would generate production requirements.

Considering the above remarks the INFORUM-type model for the Polish economy may be elaborated in at least three variants.

Variant 1 - of typical type, i.e. generating requirements for production by industries only. Accurate forecasts based on this model should refer to the past being historical, counterfactual ones.

Variant 2 - keeping of the demand determined structure of the input-output model by generating final recipients demand by means of such equations that include explicitely capacity utilization constrains. In fact this model would generate the feasible levels of production.

Variant 3 - basically corresponding to Variant 1 but creating possibilities of passing from requirements to factual realization on the basis of analysis and modeling relations between requirements and factual realization /e.g. analogous to residual models/.

Our studies are concentrated rather on Variant 2. Variant 3, however, seems to be interesting, too, because it gives some information about the requirements for production by industries. Studies of final demand and other economic categories submodels used in Variant 2 are based on those to be found in the macro--econometric model of Polish economy, the so called W-5 Model.[x/] The equations of this model are extended and partly adjusted in order to include the block concerning final demand and some of its determinants into the INFORUM-type Model for Poland using more detailed structure both by branches and types of final demand categories.

The equations explaining consumers' expenditures C are linear approximations of consumers' demand functions. The typical specification includes real personal income, relative prices and lagged expenditures /to observe inertia/ as explanatory variables. The equation explaining demand for durables takes into account also savings /their increase being competitive/. To allow for excess demand either dummies or special disequilibria indicators were used. We thus have

$$CB = f/Y,\ PCB/PC,\ CB_{-1},\ UCB\ /,$$

where C - consumers'real expenditures, Y - real personal income PC - deflator of consumers' expenditures, UC - dummy variable, the symbol B - stands for B-group of expenditures.

The equations explaining investment outlays are composed of a system which starts with initials estimates of requirements generated by different industries JD. They are then adjusted at the macro-level according to the policy assumptions on the distribution of national income into consumption and accumulation /and net investment/. At the end they are allocated among

x/ W. Welfe is the author of this model

industries and commodity groups 6/. The typical equation explaining the requirements for investment outlays reflects the assumption that the new investment projects depend on the expected increase of production capacities /this being generated by an autoreggressive process/ and the modernization process replacements etc. depend on the desired level of fixed assets being the function of planned output. No users' capital cost has been included so far because of the neutral role of prices. Thus we have

$$JKD = h/\ XK,\ KK_{-1},\ JKD_{-1},\ \ UJKD/,$$

where JD - investment outlays, X - net output, K - fixed assets, UJ - dummy, and K stands for branch of industry; the variables are in constant prices

Hence $JD = \sum JKD$.

Having globally adjusted total investment outlays J the allocation among branches follows according to a simple scheme:

$$JK = a_0 + a_1 J + a_2\ UJK + u$$

where u - disturbance term. The allocation parameters may be modified too, observing some consistency rules.

The changes of inventories are explained taking into account their both functions - they are proportional to the change in the activity levels and take into account the disequilibria in the markets for consumers' and producers' goods.

A system of price equations was developed . The producers' prices are main category in this system. They depend on the

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6/ Model W-5 generates also the supply of investment goods both domestic and imported. It assumes that the restrictions on the supply side determine the actual investment with regard to buildings and constructions whereas the constrains on the demand side-the actual level of investment with regard to machinery equipment.

fabrication costs, including prices of domestic and imported raw-materials and fuel /technical coefficients being kept constant/ and labour costs. They also reflect the financial as well as market pressures /the centrally administered price changes are introduced using dummies/.

The wages are partly explained by productivity of labour, partly by inflationary pressures. The administered wage increases /reforms/ are also taken into account - using dummies. Given exogenously employment we arrive at wage bill and nominal personal income. This last category together with retail prices generated from the price system are used to determine the consumers' demand.

5. Trends of further development of the model

The development of the model will aim at disaggregation of final demand elements and improving of modeling them. Further branch disaggregation, though possible, does not seem to be of much use. The accepted disaggregation by branches corresponds to the division used in statistical surveys and it enables us both immediate use of statistical data and making possible comparisons of forecasts obtained from the model with actual values and also with the results of computations based on other models.

As far as final demand is concerned the number of categories of personal consumption expenditures will be kept at the same level. However, more detailed presentation of government expenditures/ in 7 groups/ is possible, government expenditures being the exogenous variable in the model yet. There is also a possibility of presenting investments requirements not only by types and branches separately but also in a type-branch classification. Stocks are also foreseen to be divided into 5 groups by types.

As regards exports and imports, 6 groups of goods are distinguished according to SITC classification. The CMEA classification system uses different classification /also 6 groups/. As the rules of the foreign trade within and outside the CMEA countries are quite special, functions of exports and imports for this group of countries are specified separately. Thus in order to ensure the comparability of values referring to foreign trade occurring in the Polish Model with other models of INFORUM -type, at least two bridge matrices will have to be determined. The first will transform the CMEA classifications into the SITC classification according to the Polish surveys, and the other from branch division into the SITC classification but for 119 goods using the aggregated information divided into 6 groups first.

Further integration with the W-5 econometric model is foreseen. It will consist, among others, in including blocks concerning financial processes from the model, and, on the other hand, in including the input-output block in the W-5 model.

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THE FINNISH LONG-RANGE MODEL SYSTEM

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1. General characteristics

The Finnish long-range model system (FMS) is developed for studing growth possibilities of the Finnish economy. The model system is composed of three submodels: the price model, the output model and the income model. The system is solved iteratively for the terminal year of the growth period examined. The development between the initial and terminal years follows geometrical growth. The system has a steady state equilibrium solution. Special emphasis is given to income distribution and technological change. The system is more suitable for simulation than forecasting purposes.

The price model is an extended input-output price model. The production functions are clay-clay vintage type with embodied and disembodied technical change for labour input coefficients. Input coefficients of intermediate inputs are constant. Final profit rates of industries are solved in the model system, but wage rates are given. The ruling price of an industry is assumed to be determined according to the technology of the newest vintage.

The output model is an open dynamic input-output model. Outputs of 30 industries are solved by equations for intermediate and final demand. Input-output coefficients are constant. Final demand deliveries from industries are solved by demand equations for fixed capital formation, private and government consumption expenditures and imports. Exports are exogeneously determined.

The income model has two parts: the functional income distribution and the institutional income distribution. The functional income distribution consists of the items of value-added of industries -

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wages and salaries and operating surplus. In the institutional part of the income model the redistribution is solved using the income-outlay tables of SNA. The disposable incomes of various institutional sectors are derived using these tables. The sectors are: corporate enterprises, households, general government and non-profit institutions. The use of disposable incomes between saving and consumption is also solved here.

Available statistical data affects the structure of the model. The starting point for industrial classification in the model was the latest published input-output table of the Finnish economy. It was then 1970 and had 65 industries. Since it is easier to aggregate than to disaggregate a detailed industrial classification was preferred in the beginning. However, problems in collecting timeseries data and the revision of SNA forced us finally to use only 30 industries. The newest input-output table available is now for 1978, which is the initial year of calculations.

The basic logic of FMS is described in diagram 1. The equilibrium solution is found iteratively. The demand of exports (E) is first evaluated outside the model system. The iteration starts by making an initial guess about the final values of the main endogeneous variables: relative prices, disposable incomes of consuming sectors, investments (I) and import (M) shares. Using these guesses the first estimates for different demand categories are calculated. Gross outputs are then solved using an open input-output model. Since investments depend on growth rates of outputs there must be a feedback loop between demand, output and investments. Four rounds of this feedback loop has proven to be enough in evaluating investment demand. Using gross output and price estimates income items and savings (S) can be solved. The guesses of incomes can now be replaced with these new ones. New import shares are also derived.

The equilibrium condition of the model is $S \equiv I$ or $M \equiv E$. Since the various parts of the equilibrium condition are determined through different mechanisms, there is not any a priori reason for them to be equal. Investments are determined by the growth rates of gross outputs and saving

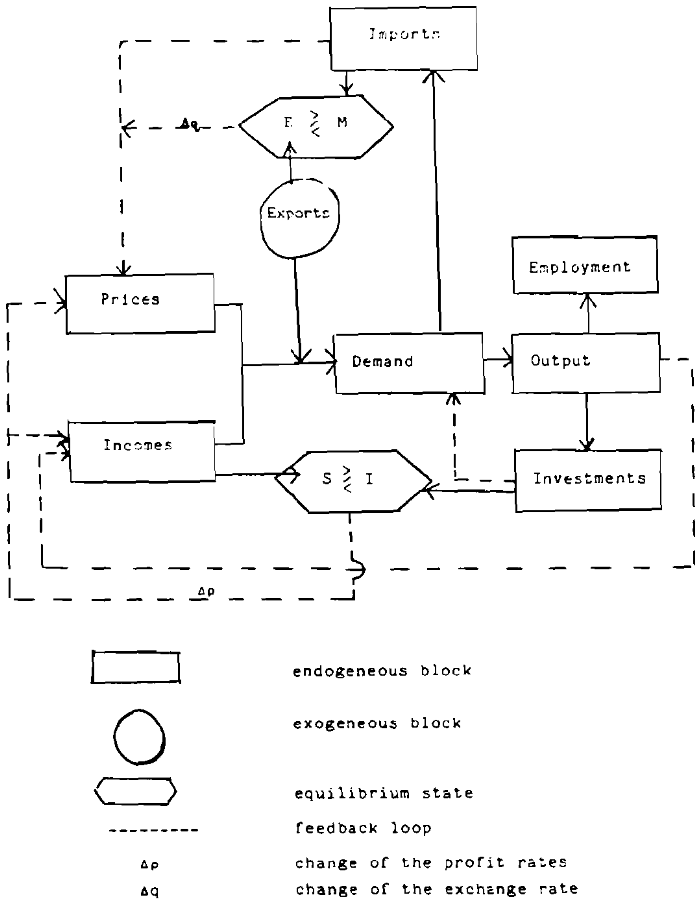


Diagram 1. Basic logic of FMS.

is a function of disposable incomes of institutional sectors. Export are axogeneous and imports are determined by domestic demand. The equilibrating variable is first chosen. Its value is changed according to the disequilibrium and new prices are calculated for the next iteration. Iteration is continued as long as the equilibrium condititions are reached.

The process of finding the equilibrium solution can be viewed from three different angles: that of domestic product market, that of foreign balance and that of income policy. In the first case the equilibrating variable is the general level of profit rates. If investments are bigger than savings due high growth rate of exports the financial possibilities for investments become weaker. Consequently, the required profitability of investments must increase: $\Delta\rho > 0$ and investments declain. When the profit rates increase, the share of profits in functional incomes also increases - since a greater part of profits is saved, saving increases in the economy. The gap between I and S is thus reduced. With a fixed exchange rate and fixed policy the solution implies $M \equiv E$ in the current account of the rest of the world.

When the exchange rate is chosen as an equilibrium variable profit rates and policy parameters are kept constant. Equality between exports and imports is then sought for. If there is a deficit in the current account the Finnish currency must be devalueted. In a surplus situation revaluation is the proper policy. When the equality between M and E is reached this way the implication is now $S \equiv I$.

When the equilibrium process is looked from the income policy point of view the reasoning is as follows. If investments are bigger than savings domestic demand has to be reduced. This can be achieved by changing the distribution of disposable incomes in favour of those sectors whose savings rates are greater. If, on the other hand, there seems to be too much saving in the economy the income distribution has to be changed in favour of demand.[1)]

2. The price model[(2]

Price equations are constructed on the basis of column identities of the input-output table. They describe formulation of prices from a point of view of costs. These equations are as follows:

$$(1) \quad p = A'(I-\hat{m})p + A'\hat{m}\,\hat{q}^{M}p + A^{NM'}q^{NM}(\imath' x_0)^{-1}x_0' p + \hat{\rho}K(I + \hat{t}^{I})^{-1}A^{I'}[I - (I - \hat{q}^{M})\hat{m}]\,p + \hat{t}\,p + \hat{w}h$$

1) The basic logic has been developed in Mäenpää (1979) and (1982)

2) Mäenpää (1982) p. 5.13-5.18.

where p = a column vector of price levels of output in year t,

A = a matrix of technical coefficients

$\hat{m}$ = a diagonal matrix of competitive import shares in industries in year t,

q^M = a diagonal matrix of rate of competitive import price to price level of output of corresponding domestic industry.

A^{NM} = a matrix of input coefficients of non-competitive imports (raw oil, natural gas, coal)

q^{NM} = a vector of rates of non-competitive import prices to average price level of production prices (at factor costs)

$\hat{p}$ = a diagonal matrix of profit rates of industries

K = a matrix of capital-output ratios of industries,

$\hat{t}^I$ = a diagonal matrix of indirect taxrates on investment-goods.

$\hat{t}$ = a diagonal matrix of indirect tax rates paid by industries per unit of output,

$\hat{w}$ = a diagonal matrix of wage rates of industries in 1978, wage per hour is constant in the model,

h = a vector of labour input coefficients of industries in year t: $h_{jt} = (1+a_j)^t(1+b_j)^t h_{j0}$, where a_j is the the rate of investment embodied technical progress in the same industry. These parameters were estimated with the parameters c_j and k_j for vintages of equipments in year 1959-1977.

A^I = an industry x type of capital good coefficient matrix

x_0 = a vector of gross outputs in base year.

Prices are assumed to be determined by the newest vintage. Labour-coefficients are then those of the year t in the model. Price levels may be solved when profit rates, wage rates and production technology (A,h) is known. At first profit rates are assumed to be the same in the industries as they were 1971-1977. Their general level is, however, changed in order to find the equilibrium solution ($I \equiv S$) in the model system.

The economic life of equipment is endogenous in the model system. It changes between 14 and 34 years among industries. The economic life of equipment is determined by the equality of money value of value added and wage costs. Equipment will remain in use until it has a positive yield. When the yield of equipment of a certain vintage become negative this eguipment will be scrapped. Its capacity output is depreciated from total capacity output available in industry. This will then determine the amount of reinvestment demand of the industry (Diagram 2). There is a direct relationship between profit rates and the economic life span. Increasing of profit rates makes the life longer since prices increase as well. The main variables of the price model are determined by vintage type production functions.

The production functions used are clay-clay vintage production functions with embodied and disembodied technical change.[1] The production function has three different tasks in the model system. Labour-input demand, investment demand and the components of value added are solved using the parameters of the vintage function. The basic assumptions are: labour-input coefficients diminish by -100a percentage yearly, the capital coefficients change 100c percentage yearly, and disembodied technical change is a trend parameter $(1 + b)^t$. The wage rates are assumed to be equal in all vintages in use within an industry.

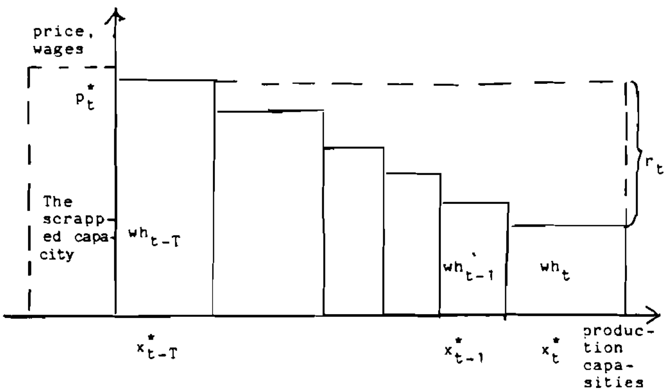


Diagram 2. Profits, prices and economic life of capacities

1) The description is based on Mäenpää (1982), p. 4.7-4.16.

x_t^* = capasity installed in year t
w = wage rate
h_{t-T} = labour-input coefficient in year t-T
T = economic life of capital
r_t = profit rate at year t
p_t^* = value-added price.

Combining the assumptions it is possible to formulate the following non-linear system of equations for each industry for estimation of the parameters.

$$\begin{cases} x_t = \sum_{v=0}^{T(t)} x_{t-v}^* \quad \frac{1}{k_0} \sum_{v=0}^{T(t)} (1+c)^{t-v} i_{t-v} \\ l_t = (1+b)^t \sum_{v=0}^{T} (1+a)^{t-v} h_0 x_{t-v}^* \\ x_{t-v}^* = (1+c)^{t-v} (1/k_0)\ i_{t-v} \\ T = \frac{\log(w_t/p_t^*) + \log(h_0)}{\log(1+a)} + [1 + \frac{\log(1+b)}{\log(1+a)}]\ t \end{cases}$$

where T = economic life of capital
k_0 = capital-output ratio in the base year
h_0 = labour-input coefficient in the base year
l_t = total labour demand of the industry in year t
i_{t-v} = investment in year t-v
a, b, c, h_0, k_0 = parameters to be estimated
x_t = capacity output

The final results of the estimates are presented in Appendix I. These estimates are used when gross fixed capital formation and labour demand are evaluated.

The price model is described in diagram three.

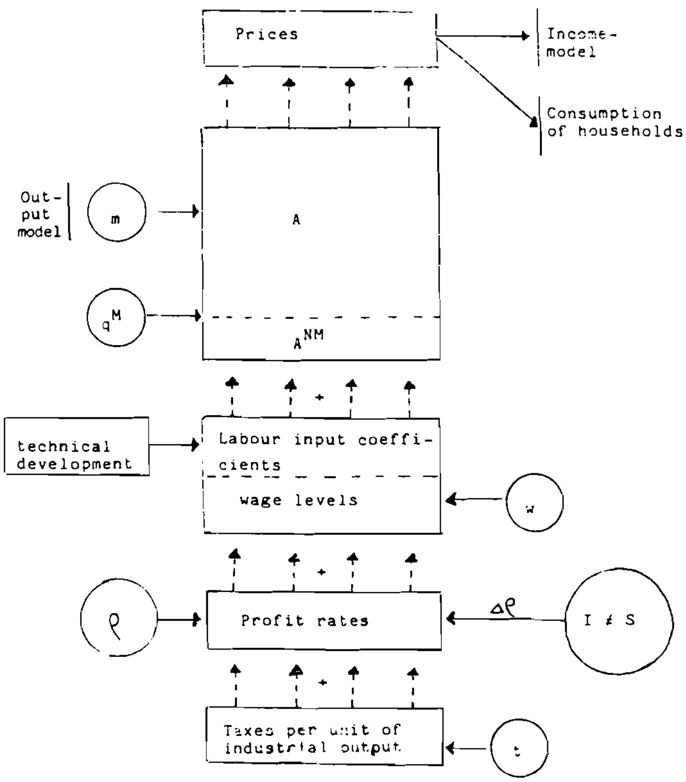


Diagram 3. The price model

3. The output model

The output equations are constructed on the basis of row identities of the input-output table. They are as follows

$$(2) \qquad x = Ax + x^{I} + x^{C} + x^{G} + x^{N} + x^{E} - x^{M}$$

where x is a column vector of domestic output of industries,

A is a matrix of input-output coefficients; inputs include both domestic and imported competitive products,

x^I, x^C, x^G, x^N, x^E — are column vectors of products delivered by industries for gross fixed capital formation (x^I), consumption expenditures of households (x^C), government expenditures (x^G), non-profit institutions (x^N) and exports (x^E),

x^M — is a column vector of competitive imports classified according to corresponding industry.

Gross fixed capital formation of industries is decomposed into four product groups:

1. Residential buildings
2. Non-residential buildings
3. Other constructions
4. Machinery, transport and other equipment

Investments on machinery, transport and other equipment of industry j are solved as follows:

$$(3) \qquad i^E_{jt} = (1+c_j)^t \, k^E_{j0} \, (\Delta x_j + x_{j,\, t-T_j})$$

where i^E_{jt} = investments on machinery, transport and other equipment

Δx_j = change of output in year t+1 along the average growth path

$x_{j,t-T_j}$ = the oldest vintage in operation in year t and reinvested that year to appear as production capacity next year.

k^E_{j0} = capital-output ratio in base year

Investments on non-residential buildings (i^c) and other construction (i^{oc}) are explained as follows

$$(4) \qquad i^c_j = k^c_j \, (\Delta x_j + x_{j,t-40})$$

$$i^{oc}_j = k^{oc}_j (\Delta x_j + x_{j,t-40})$$

where k_j^c = capital-output ratio on non-residential buildings

k_j^{oc} = capital-output ratio on other construction.

The economic life on non-residential buildings and other construction is exogenously set to 40 years. Capital-output ratios are assumed constant.

Investments on residential buildings (i^R) are derived analogously:

(5) $$i^R = k^R (\Delta x_R + x_{R,\ t-50})$$

where k^R = capital-output ratio on residential buildings.

x_R = gross output of letting and oper. of dwellings.

Gross output of letting and operating of dwellings and use of owner-occupied dwellings (x_R) is equal to demand of gross rent fuel and power by households.

Each product group is summed up by industries and investments made in other activities (solved analogously) are added. Investments in product groups are finally decomposed into delivery demands of industries by a bridge matrix A^I: $x^I = A^I i$, where i = total investments by type of capital goods.

Consumption expenditures of households[1]

The total consumption expenditures of households are decomposed into demands of purpose categories with parameters of a Linear Expenditure System type demand system. The parameters were estimated for the system:

(6) $$\hat{p}_t q_t = \hat{p}_t c + (b^o + b^1\lambda)(C_t - p_t c) + e_t,$$

where $\hat{p}_t$ = diagonal matrix of consumer prices in year t (15 x 15)

q_t = vector of consumed quantities in year t (15 x 1)

c, b^o, b^1 = vectors of parameters to be estimated (15 x 1)

1) Svento (1979) and (1982b)

λ = trend factor

e_t = vector of error terms in year t.

C_t = total consumption expenditures in current prices

The estimation period used was 1960-78 and the estimation method was OLS. The trend parameter of clothings has been revised upwards to prevent too rapid declain. The transformation from purpose categories into final demand deliveries from industries is done using a fixed coefficient transformation matrix: $x^C = A^C q$. Demand for gross rent, fuel and power is exogenous.

Government consumption expenditures

The demand deliveries of industries to the general government are solved using fixed input coefficients. Gross output of general government is derived by adding the purchases of households and enterprises from the general government to consumption expenditures of the general government (see chapter four).

Competitive imports[1]

Competitive imports are solved using import shares of industries. The following equation has been estimated:

$$m_{it} = b_{0i} d_{it}^{b_{1i}} (q^M)^{b_{2i}} , \tag{7}$$

where m_{it} = $x_{it}^M / (x_{it} + x_{it}^M - x_{it}^E)$ is the import share of industry i in year t

x_{it}^M = import of commodities by industry i in year t

x_{it}^E = export of commodities from industry i in year t

d_{it} = $x_{it} + x_{it}^M - x_{it}^E$ = domestic demand of commodity i in year t

1) Kärrymäki (1979)

$q^M \quad = p^M_{it}/p_{it}$

p^M_{it} = price index of imports of commodity i in year t

p_{it} = price index of domestic production of industry i in year t

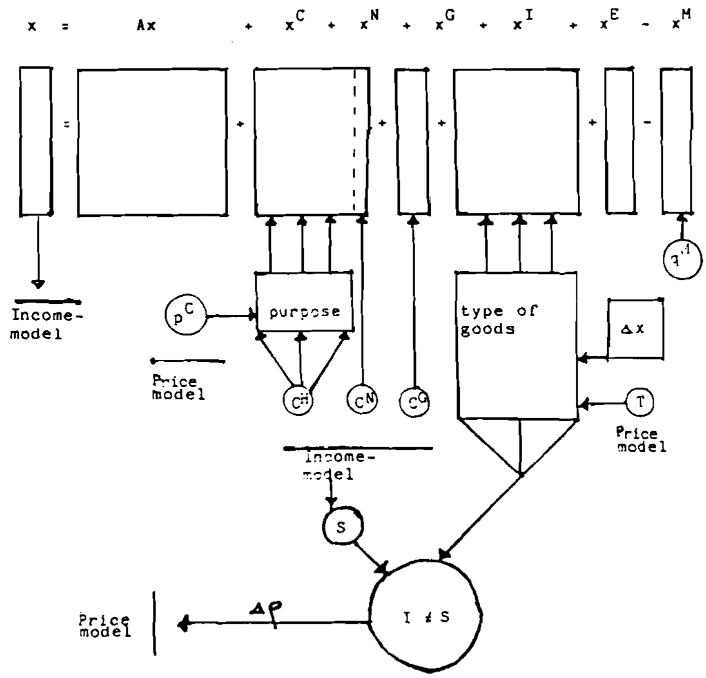
b_{0i}, b_{1i}, b_{2i} = parameters to be estimated. Expected signs being $b_{1i} > 0$, $b_{2i} < 0$.

Relative prices between imports and domestic output are exogenously given. Parameter estimates were calculated for 1959-1975.

The output model can be described by the aid of diagram 1.

Diagram 4

The output model



4. The income model[1]

In the income model the following items are solved.

1. Domestic factor incomes from production activities
2. Redistribution of incomes through transfers among institutional sectors to get disposable incomes in each sector and
3. Distribution of disposable incomes between consumption and saving in institutional sectors.

Domestic factor incomes are calculated using production and price information from output and price models.

Wages and salaries are calculated as follows:

(8) $W = w'l$

where W = sum of wages and salaries

l = a vector of labour input of industries in year t in industry j it is

$$l_{jt} = (1+b)^{t} \sum_{v=0}^{Tj} (1 + a_{j})^{t-v} h_{j0}\, x^{*}_{t-v}$$

where h_{j0} is the labour-input coefficient in the base year and x^{*}_{t-v} capasity installed in year t-v.

Employers' contributions to social security schemes are included in wage levels.

Gross operating surplus is equal to $(p^{*}x - W)$, where p^{*} = value added price. Wages and salaries are subtraced from the sum of value added. Depreciation is assumed to be a constant ratio of the gross operating surplus.

Factor incomes abroad (net) are added to domestic items using a given rate on the sum of domestic factor incomes. In order to find out disposable incomes of different institutional sectors two accounting frameworks are used. They are for outlays and incomes.[2]

1) Mäenpää (1982), Svento (1982a) and (1982b)

2) For a more detailed description of the redistribution see Svento (1982b).

Total outlays in each sector (corporate enterprises, households, general government, non-profit institutions) are equal to the sum of different transfers paid: requited current transfers + indirect taxes + direct taxes + social security contributions + social security benefits + other transfers and disposable incomes. On the other hand total incomes are equal to the sum of recieved transfers (classification is the same as in outlays) and factor incomes. Since outlays equal incomes in each sector disposable income = (transfers recieved - transfers paid) + factor incomes.

Distribution coefficients are constructed for outlays and incomes by sectors as well as by recieved and paid transfers. Coefficients may be assumed constant or used as policy parameters. Disposable incomes in different sectors are solved by means of these coefficients and factor incomes.

The saving rate of households is exogeneous and the consumption function used is a proportional function, where average and marginal propensities to consume are equal:

(9) $$C_t^H = (1-s_w)Y_t^{dH},$$

where C_t^H = total consumption of households in year t

Y_t^{dH} = disposable incomes of households in year t

$(1-s_w)$ = average = marginal propensity to consume in households.

The function is very simple. However, the empirical evidence on the long run development of household's savings doesn't justify strong conclusions.

In Finland the general government has been a net lender. Accordingly, the net saving rate is set as a policy variable. The net saving rate of the general government is defined followingly:

$$s^{nG} = \frac{N^G}{Y^{dG}+D^G-I^G}$$

where s^{nG} = net saving rate of the general government

N^G = net lending of the general government

Y^{dG} = disposable income of the general government

D^G = consumption of fixed capital in the public sector

I^G = gross fixed capital formation of the public sector

The consumption expenditures of the general government can now be solved followingly:

$$(10) \quad C_t^G = (1 - s_t^{nG})(Y_t^{dG} + D_t^G - I_t^G).$$

The solution uses the iterative nature of the model system. Investments and consumption of fixed capital depend on gross output of producers of government services in the preceding iteration. Gross output for the iteration in question is then calculated by adding the purchases of households and enterprises from general government to consumption expenditures. Labour input demand, gross fixed investments, consumption of fixed capital and delivery demands from industries are then derived from gross output with fixed labour-input, capital-output and input coefficients.

5. A basic scenario for the development of the Finnish economy

The model system was applied for calculating a basic development scenario of the Finnish economy from 1978 to 1990. The following assumptions were then made:

Price model

- observed changes of energy input-coefficients between 1978 and 1980 were made to 1978 coefficients
- other input coefficients are 1978 coefficients

- observed changes of relative prices of raw oil imports between 1978 and 1980 were made. There after relative price is assumed to increase 3 % annually
- wage level of industries is at 1978 level, profit rates between industries are those prevailed 1971-77, tax rates are those in 1978

Output model

- exports, which is an exogenously determined variable in the model, is assumed to increase 5.2 % annually. Industrial differences around this average growth rate were evaluated according to observed growth rates in the 70's and forecasts made elsewhere
- relative prices between imported competitive products and corresponding domestic industry are assumed to be constant.

Income model

- saving ratios of institutional sectors are the same as in 1970-78
- shares of transfers are the same as 1978

It must be stressed that the results are tentative and should not in any case be taken as a 'project's view' of the probable development. All details have not been checked to the final degree and changes are still quite probable.

Table 1. Expenditure on the gross domestic product in purchasers' values (th. mln. Fmk., 1978 prices).

Item	1978 value	1978 structure	1990 value	1990 structure	annual rate of growth(%)
Private consumption	78.7	56.3	108.5	54.5	2.7
Government consumption	26.3	18.8	28.9	14.5	0.8
Private capital formation	28.2	20.1	36.6	18.4	2.2
Gov.ment capital formation	4.6	3.3	2.4	1.2	-5.3
Exports	43.1	30.8	79.8	40.1	5.3
Imports	-37.7	-26.9	-58.3	-29.3	3.7
Discrepancy	-3.3	-2.3	1.2	0.6	0.0
Gross domestic product	139.9	100.0	199.0	100.0	3.0

Diagram 5

The income model

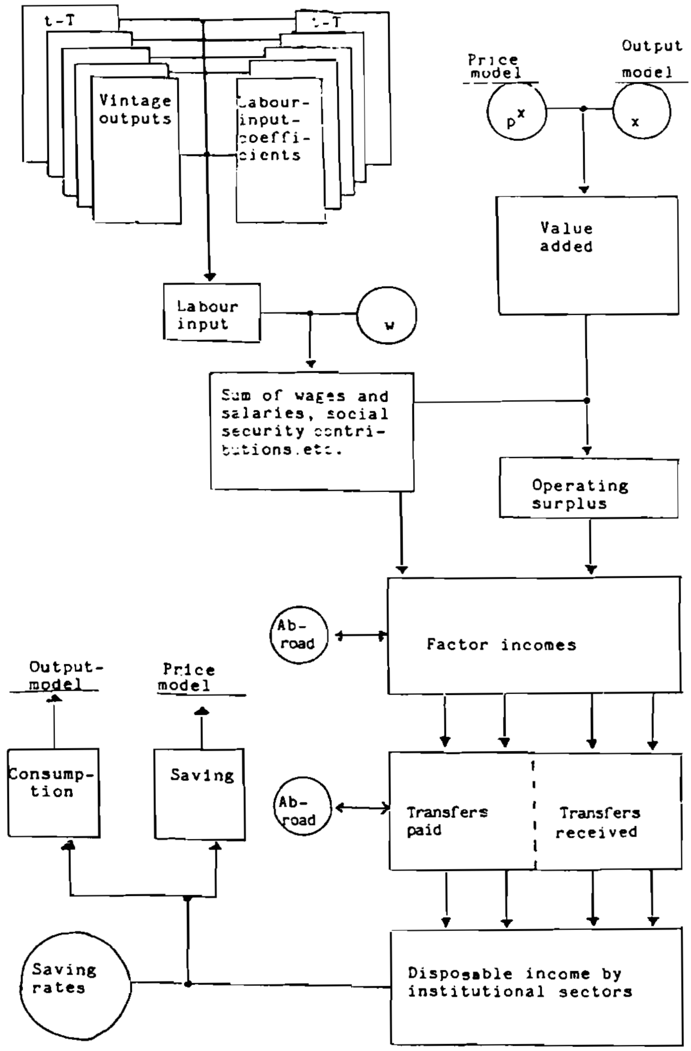


Table 2. Gross output (mln. Fmk. 1978), labour input (mln. h.), and economic life-years, by industry.

Activities	Gross 1978	Output 1990	Annual rate of growth (%)	Labour 1978	Input 1990	Annual rate of growth(%)	Producer Prices	Economic Life
01. Agriculture, hunting, fishing	11898	15282	2.1	644	446	-3.0	.73	20.
02. Forestry and logging	5726	8051	2.9	86	40	-6.2	.72	16.
03. Mining and quarrying	957	1527	3.4	13	11	-1.2	.96	16.
04. Food manufacturing	19441	24205	1.8	101	85	-1.5	.74	33.
05. Beverage and tobacco industries	1741	2958	4.5	10	14	2.8	.72	30.
06. Textile, wearing apparels and leather industries	6514	9339	3.0	116	103	-.9	.71	23.
07. Sawing, planing and preserving	4380	5951	2.6	40	38	-.5	.75	26.
08. Other manufacture of wood	3969	5214	2.3	64	40	-3.8	.68	23.
09. Pulp mills	5649	8171	3.1	25	25	.0	.85	33.
10. Manufacture of paper and paperboard and of pulp, paper and paperboard articles	11298	18427	4.2	64	57	-.9	.74	27.
11. Printing and publishing	5191	7712	3.4	58	60	.2	.70	17.
12. Manufacture of chemicals	4278	8841	6.2	23	32	2.7	.82	30.
13. Manufacture of chemical, rubber and plastic products	3316	4535	2.6	40	36	-1.0	.77	23.
14. Petroleum refineries and miscellaneous products of petroleum and coal	6822	7734	1.1	6	6	.3	1.56	34.
15. Pottery, glass and earthenware prod.	3090	4299	2.8	26	28	-2.2	.76	25.
16. Basic metal industries	7218	11433	3.9	34	51	3.5	.97	34.
17. Manufacture of metalproducts and machinery	11596	22977	5.9	151	202	2.4	.79	19.
18. Manufacture of electrical machinery related products	4304	8351	5.7	61	74	1.7	.81	19.
19. Manufacture of transport equipment	5155	12028	7.3	62	73	1.4	.68	26.
20. Other manufacturing industries	616	1127	5.2	10	9	-.6	.64	25.
21. Electricity, gas and water	12505	17161	2.7	45	41	-.8	.86	34.
22. Building	18641	19176	.2	229	136	-4.2	.74	25.
23. Other constructions	6485	6441	-.1	99	62	-3.8	.75	21.
24. Trade	19526	22794	1.3	463	287	-3.9	.64	22.
25. Restaurants and hotels	5483	9106	4.3	117	119	.1	.74	21.
26. Transport	14727	19672	2.4	213	194	.8	.79	20.
27. Communications	3606	4635	2.1	67	51	-2.2	.55	25.
28. Letting and operating of dwellings and use of owner-occupied dwellings	14696	20906	3.0	32	46	3.0	.80	0.
29. Other real estate, financing, insurance and business services	12673	16364	2.2	155	172	.9	.77	18.
30. Private social and personal services	6354	9725	3.6	150	148	-.1	.70	21.
31. Producers of non-profit services	3112	3462	.9	162	169	.4	.84	0.
32. Producers of government services	29317	32324	.8	582	642	.8	.87	0.
Gross output	270283	369827	2.6	3961	3497	-1.0	.79	24.

Table 3. Consumption expenditures by households (mln. Fmk., 1978 prices).

Item	1978 value	1978 structure	1990 value	1990 structure	annual rate of growth (%)
01. Food	17332	22.7	19939	19.3	1.2
02. Beverages and tobacco	5446	7.1	7237	7.0	2.4
03. Clothing and footwear	3802	5.0	4125	4.0	.7
04. Gross rent, fuel and power	14696	19.2	20906	20.2	3.0
05. Furniture, furnishing and household equipment and operation	5051	6.6	6506	6.3	2.1
06. Medical care and health expenses (incl. personal cleanness)	3173	4.2	4296	4.2	2.6
07. Personal transport equipment	8865	11.6	10592	10.2	1.5
08. Other transport	2633	3.4	2956	2.9	1.0
09. Communication	913	1.2	1961	1.9	6.6
10. Recreation, culture and education (incl. goods n.e.c. and packaged tours)	5590	7.3	11208	10.8	6.0
11. Books, papers and magazines	1656	2.2	2473	2.4	3.4
12. Expenditure in restaurants and hotels	4782	6.3	7893	7.6	4.3
13. Services n.e.c.	535	.7	590	.6	.8
14. Purchases from producers of government services	1294	1.7	1710	1.7	2.3
15. Purchases from producers of non-profit services	799	1.0	970	.9	1.6
16. Purchases abroad	1720	2.3	2656	2.6	3.7
17. Purchases by non-resident households	-1861	-2.4	-2515	-2.4	2.5
Consumption expenditure of household	76420	100.0	103505	100.0	2.6

Table 4. Exports of goods and services (mln. Fmk. 1978), by industry.

Industry	1978 value	1978 structure	1990 value	1990 structure	annual rate of growth (%)
01. Agriculture, hunting, fishing	834	2.0	796	1.1	-.4
02. Forestry and logging	53	.1	50	.1	-.6
03. Mining and quarrying	128	.3	127	.2	-.1
04. Food manufacturing	1345	3.3	2004	2.7	3.4
05. Beverage and tobacco industries	127	.3	281	.4	6.8
06. Textile, wearing apparels and leather industries	2489	6.1	5388	7.2	6.6
07. Sawing, planing and preserving	2811	6.9	3197	4.3	1.1
08. Other manufacture of wood	1661	3.9	1807	2.4	1.0
09. Pulp mills	1838	4.5	1823	2.4	-.1
10. Manufacture of paper and paperboard and of pulp. paper and paperboard articles	8041	19.7	13683	18.2	4.5
11. Printing and publishing	232	.6	584	.8	8.0
12. Manufacture of chemicals	1125	2.8	2827	3.8	8.0
13. Manufacture of chemical, rubber and plastic products	742	1.8	1864	2.5	8.0
14. Petroleum refineries and miscellaneous products of petroleum and coal	885	2.2	878	1.2	-.1
15. Pottery, glass and earthenware products	366	.9	751	1.0	6.2
16. Basic metal industries	1879	4.6	2123	2.8	1.1
17. Manufacture of metalproducts and machinery	3999	9.8	11408	15.2	9.1
18. Manufacture of electrical machinery and related products	1296	3.2	3696	4.9	9.1
19. Manufacture of transport equipment	3764	9.2	8315	11.1	6.8
20. Other manufacturing industries	271	.7	575	.8	6.5
21. Electricity, gas and water	28	.1	28	.0	-.1
22. Building	11	.0	20	.0	5.2
23. Other constructions	0	.0	0	.0	.0
24. Trade	1514	3.7	2782	3.7	5.2
25. Restaurants and hotels	107	.3	196	.3	5.2
26. Transport	4252	10.4	7812	10.4	5.2
27. Communications	23	.1	42	.1	5.2
29. Other real estate, financing, insurance and business services	1067	2.6	1961	2.6	5.2
30. Private social and personal services	51	.1	93	.1	5.2
Exports by industries	40879	100.0	75110	100.0	5.2

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Appendix I. The parameters of the vintage production function used in FMS.

Industry	$(1+a)$	$(1+b)$	h_0	$(1+c)$	k_0^E	k^C	k^{0C}	w	ρ_0
1. Agric. + fisheries	0.9652	0.9873	0.03686	1.01	2.25906	1.29585	1.02887	5.92	0.0680
2. Forestry	0.9824	0.9400	0.01168	1.02	0.98081	0.02927	1.83456	24.65	0.0322
3. Mining	0.9354	1.0334	0.00780	1.03	2.84310	0.79116	0.43689	28.23	0.0911
4. Food manufact.	0.9638	1.0000	0.00293	1.00	0.52509	0.23946	0.02774	21.41	0.1624
5. Bevs. + tobacco	0.9440	1.0000	0.00385	1.00	1.49583	0.79472	0.04019	25.65	0.1272
6. Textiles	0.9727	0.9825	0.01417	1.00	0.49569	0.32753	0.00556	18.75	0.2154
7. Sawing	0.9736	1.0000	0.00592	1.00	0.56344	0.17138	0.05053	25.18	0.1767
8. Other wood manufact.	0.9757	0.9688	0.01220	1.00	1.03061	0.32358	0.02039	23.03	0.1031
9. Pulp mills	0.9594	1.0000	0.00242	1.00	2.12363	0.45331	0.21002	33.04	0.0653
10. Paper manufact.	0.9600	1.0000	0.00315	1.00	1.20226	0.35187	0.04837	31.49	0.0981
11. Printing + publish.	0.9707	1.0000	0.00886	1.00	0.39251	0.14798	0.00369	28.80	0.2896
12. Man. of chemicals	0.9455	1.0000	0.00331	1.00	1.25197	0.40947	0.03867	27.09	0.1523
13. Chemical products	0.9534	1.0000	0.00779	1.00	0.73031	0.46449	0.00107	27.33	0.2682
14. Petroleum refin.	0.9443	1.0000	0.00054	1.00	0.56342	0.05916	0.40128	31.49	0.1272
15. Pottery + glass	0.9577	0.9811	0.00773	1.00	0.80992	0.41890	0.03021	25.50	0.1888
16. Basic metals	0.9748	1.0000	0.00434	1.00	1.30097	0.37916	0.09480	28.68	0.1133
17. Metal products	0.9638	1.0000	0.01066	1.00	0.51032	0.40680	0.00363	26.64	0.2622
18. Elec. machinery	0.9698	1.0000	0.01026	1.00	0.39221	0.43040	0.00424	27.42	0.2417
19. Transport equip.	0.9809	0.9742	0.00822	0.99	0.29685	0.25872	0.13195	28.51	0.1800
20. Other manufact.	0.9807	0.9386	0.01206	1.00	0.31154	0.61937	0.03345	24.31	0.1381
21. Elec., gas, water	0.9435	1.0000	0.00180	1.00	1.74648	0.60832	1.75459	29.51	0.0674
22. Building	0.9875	0.9654	0.01067	0.99	0.17987	0.04485	0.00000	28.26	0.3970
23. Other construct.	0.9825	0.9818	0.01181	1.00	0.85471	0.15904	0.00000	24.05	0.1080
24. Trade	0.9763	0.9721	0.01753	1.00	0.75450	0.78012	0.01738	21.57	0.1197
25. Restaur. + hotels	0.9783	0.9832	0.01714	1.02	0.44998	0.37433	0.00000	17.58	0.1517
26. Transport	0.9581	0.9961	0.01124	0.98	2.05130	0.20171	3.18592	24.10	0.0561
27. Communications	0.9757	0.9749	0.01449	1.00	1.06252	0.41245	6.19578	26.85	0.0317
29. Financing	0.9525	1.0349	0.00811	1.02	0.21020	0.93440	0.00000	30.39	0.3255
30. Soc. + pers. serv.	0.9722	1.0000	0.01683	1.00	1.74515	1.69231	0.26501	21.09	0.0746

THE INCOME BLOCK OF THE FINNISH LONG-RANGE MODEL SYSTEM

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1. The position of the income block in FMS[1]

The income block has a very special role in FMS. FMS has a firm theoretical background. The model is based on the post-Keynesian theory of growth and income distribution. The core of this theory conserns the relationships among the rate of profit, income distribution and economic growth. Assuming different savings rates from different sources of income or for different receivers of income, it is possible to find out an income distribution and a rate of profit that produce the savings needed for steady state equilibrium investments.[2] The theory has, however, been developed assuming no government intervention. In the paper we shall model the institutional income distribution in such a way that the features of the original theory can be maintained while government has been included. We shall not, however, go into all the theoretically interesting questions of such an extension in this connection.

A complete income block should have three main parts. The first part should give domestic factor incomes from production activities. The second part should redistribute these and give disposable incomes. The third should then divide disposable incomes into various uses.

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1) For a more detailed description of FMS see Forssell O. - Mäenpää I. - Svento R. (1983). FMS = Finnish Long-Range Model System.

2) See f.e. Sen, A. (1970).

The basic logic of FMS and the position of the income block in FMS can be illustrated by the aid of the following diagram.

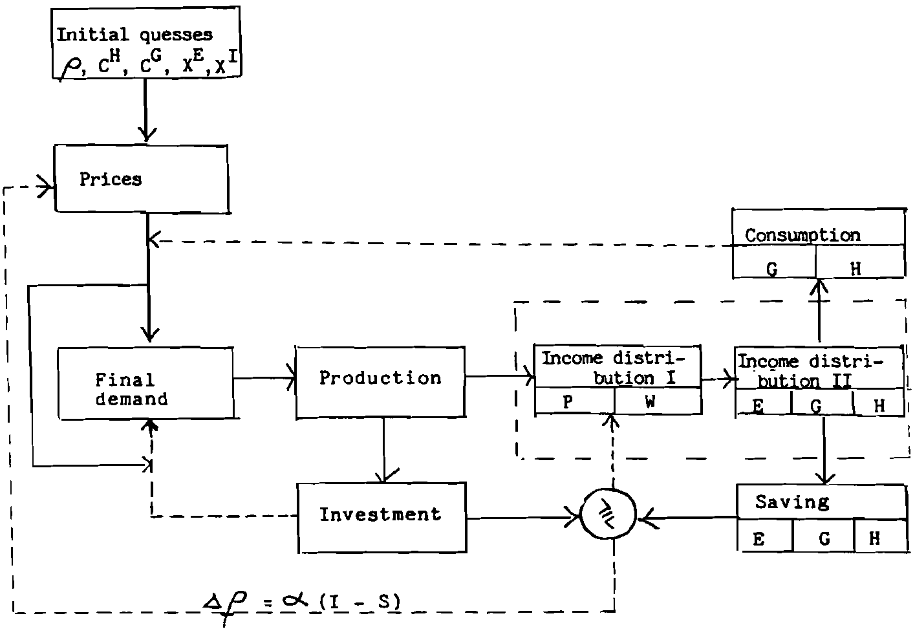

Diagram 1. The income block in FMS. The symbols used are

ρ = general level of profit rates
C^H = consumption expenditures of households
C^G = public consumption expenditures
X^E = vector of exports by industry
X^I = vector of investments by industry
P = operating surplus
W = wages, salaries and employers' contributions to social security schemes
E = corporate enterprises
G = general government
H = households.

FMS is a steady state equilibrium model. The equilibrium solution for the end year is found out iteratively. The development between base and end years follows geometrical growth. We start with an initial guess for the general level of the profit rates, personal and public consumptions expenditures, investments and exports. Combining corresponding profit rates with time dependent labour-input coefficients we can solve the price model. Prices combined to the consumption guesses give the final demand items. The input-output model is then solved to give gross outputs. Gross outputs are then used to solve the investments and the functional income distribution. From the functional income items disposable incomes of various institutional sectors[1] are received. From different savings rates of the institutional sectors we get sectoral consumptions and savings. Since investments depend on the growth rates of gross outputs and savings on the income distribution there is no a priori reason for them to be equal. The difference between investments and savings determine the change in the general level of profit rates. We can proceed to the next iteration. Iteration is continued as long as $S = I$.[2]

If the export growth is such that investments tend to become bigger than savings we raise the profit rates - only the most profitable investments can be financed. As a consequence, the prices increase and the volume of demand reduces so that growth and investments diminish. On the other hand, since profit rates have increased the income distribution will change im favour of profits. Thus savings are increased.

Economic policy has a twofold job in the model. Since the solution can very well be an unemployment equilibrium, investments need not be at the fullemployment level. The problem is to find this level and the income distribution that produces it.

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1) Non-profit institutions has here been omitted for reasons of simplicity.

2) The solution could, of course, as well be viewed from the foreign balance side with the exchange rate as a variable.
The description of the basic logic is based on Mäenpää (1982).

2. The structure of the income block in FMS

The structure of the income block has further been described in diagram two. The various parts of the income block have been presented on the right hand side of the diagram. The connections to the rest of the model from the income block have been depicted on the left hand side.

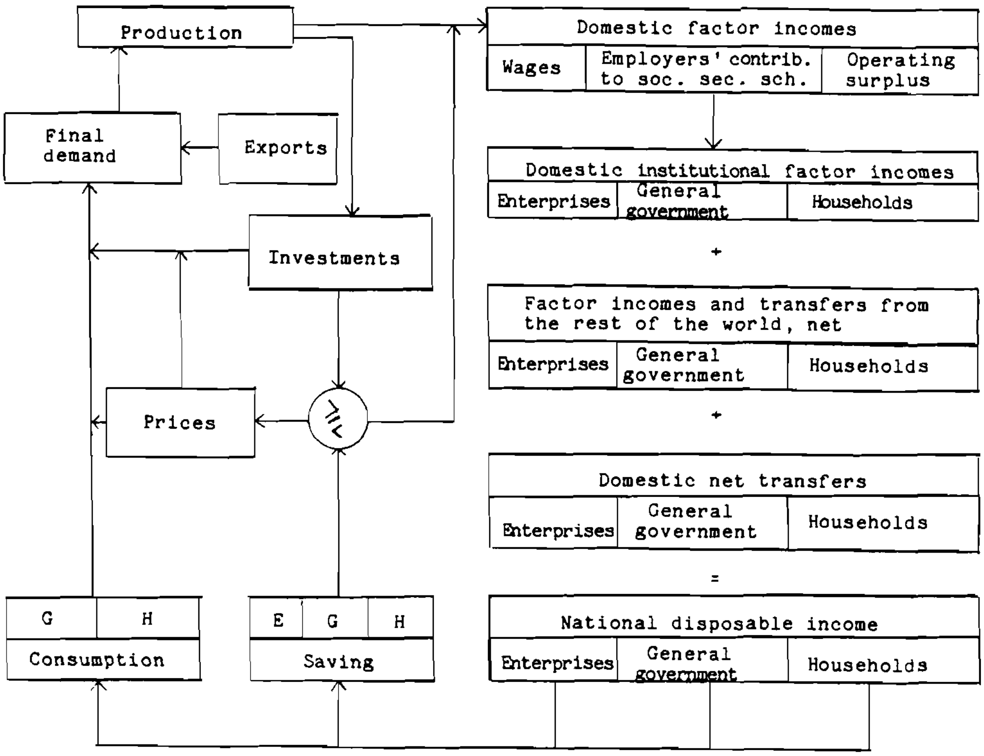

Diagram 2. The structure of the income block in FMS.

Nominal wage rates are fixed in the model system. Wages, salaries and employers' contributions to sss. are obtained by multiplying the industrial labour demands by the corresponding wage rate and summing over industries. Gross profits are then obtained when wages are subtracted from value-added, where the value-added prices depend on the profit rates. Operating surplus is then obtained by subtracting a fixed share as depreciation.(1

Domestic factor incomes are then divided into domestic institutional factor incomes using fixed base year shares. Foreign factor incomes and transfers are also fixed on the base year shares.

In what follows we shall concentrate on the final part of the chain - the problem of the change from domestic institutional factor incomes to national disposable income. This part of the process is usually not solved in an analytical or comprehensive way. In the following we shall critically examine one possible analytical solution of this problem. This solution is based on the income-outlay tables of the SNA(2. This is why we shall first have a look at the nature of these tables. Then we shall derive the model and analyse its characteristics.

2. The income-outlay tables in SNA

The institutional accounting has greatly improved in RSNA(3. There are two main sides in the institutional accounting framework: the income-outlay accounts and the capital finance accounts. The structure of an income-outlay account for the whole economy for any particular year can be described as in the following scheme. The account has two sides: current disbursements and current receipts. On the rows of the account we have different transfers and on the columns the different institutional sectors of the economy.

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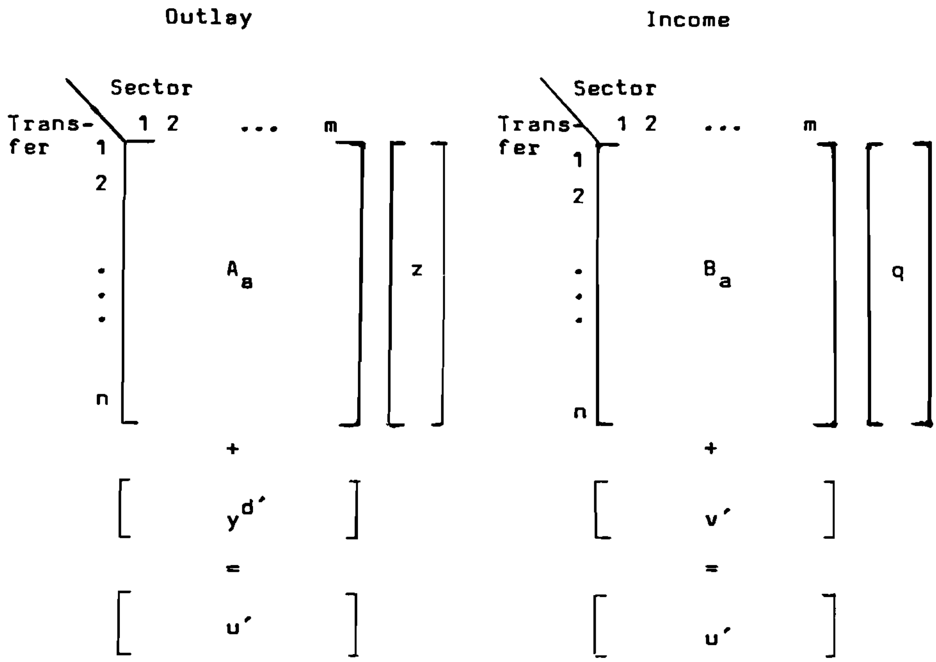
1) For a more detailed description of the functional income distribution see Mäenpää (1982).

2) A System of National Accounts, Studies in Methods (1968)

3) This description is based on Finnish accounts, see Revised National Accounts for 1960-1978 (1981) and Sourama H. - Saariaho O. (1980).



where A_a = nxm table of current disbursements

B_a = nxm table of current receipts

y^d = mx1 vector of sectoral disposable incomes

v = mx1 vector of institutional factor incomes by sector

z = nx1 vector of current disbursements total by transfer

q = nx1 current receipts total by transfer

u = mx1 current disbursements total by sector

= current receipts total by sector

Diagram 3. The structure of the income-outlay accounts.

The principle of double accounting is being fulfilled in the accounts. Any transfer paid by any sector has a receiver on the income side, unless, of course, if the transfer is paid to (or received from) the rest of the world. The classification of the transfers has been presented in Appendix I.

The classification is the same for both sides except for indirect taxes and subsidies. In the outlayside general government pays subsidies and in the income side receives indirect taxes. Subsidies can, however, be interpreted as negative indirect taxes. The level of aggregation can be freely chosen, as long as it is the same for both sides.

Seven sectors have been distinguished: enterprises, financial institutions, central government, local government, social security funds, non-profit institutions and households.

3. Analytical solutions of the institutional redistribution

Let us also have i_1 = mx1 and i_2 = nx1 unit vectors. The following identities hold in the income-outlay accounts:

$$A_a' i_2 + y^d = u$$

$$B_a' i_2 + v = u$$

or

$$y^d = (B_a' - A_a') i_2 + v.$$

The last equation simply states that net transfers plus factor incomes equal disposable incomes of each sector. We need several equations if we want to have all the levels of various transfers. Instead we can ask, would it be possible and sensible to use the information of the income-outlay structure in relative shares. We are then looking for a way of expressing the tables in a coefficient form. Let us analyse the possibility first.

Let there be four matrices of coefficients as follows:

$$A = A_a \hat{u}^{-1}$$
$$B = B_a \hat{u}^{-1}$$
$$C = A_a' \hat{z}^{-1}$$
$$D = B_a' \hat{q}^{-1}$$

where ´ denotes transpose and ^ diagonal matrix. Using these matrices and the basic identities, it is possible to derive the following specification:[1]

(1) $$y^d = (DB - CA)u + v.$$

This specification is still not very nice since we need the u vector. We can, of course, relate u to v in the same way they are related in the year from wich A,B,C and D come from. This, however, is not very desirable from the policy point of view. Economic policy operates through the coefficients in A,B,C and D thus changing the relations between u and v or u and y^d.

The column sum of A and B give, respectively, the shares of all transfers paid or received from total income. Using this information it is possible to solve for u using A or B in the following way:

$$u = (\widehat{i_1 - A'i_2})^{-1}y^d$$

$$u = (\widehat{i_1 - B'i_2})^{-1}v.$$

Inserting into (1) and solving for y^d we have two possible specifications for an income-outlay model:

(2) $$y^d = [I - (DB - CA)(\widehat{i_1 - A'i_2})^{-1}]^{-1}v$$

(3) $$y^d = [I + (DB - CA)(\widehat{i_1 - B'i_2})^{-1}]\ v.$$

Using (2) or (3) it is possible to solve for the distribution of disposable incomes over various sectors of the economy. Knowing the sensitivity of inverse matrices to changes in initial elements one would be tempted to be more doubtfull with respect to specification (2). However, it is difficult to draw conclusions of the models analytically. We have to look for the dependencies at the empirical level. This way we can also look for an answer to the

1) For full derivation see Svento, R.(1982).

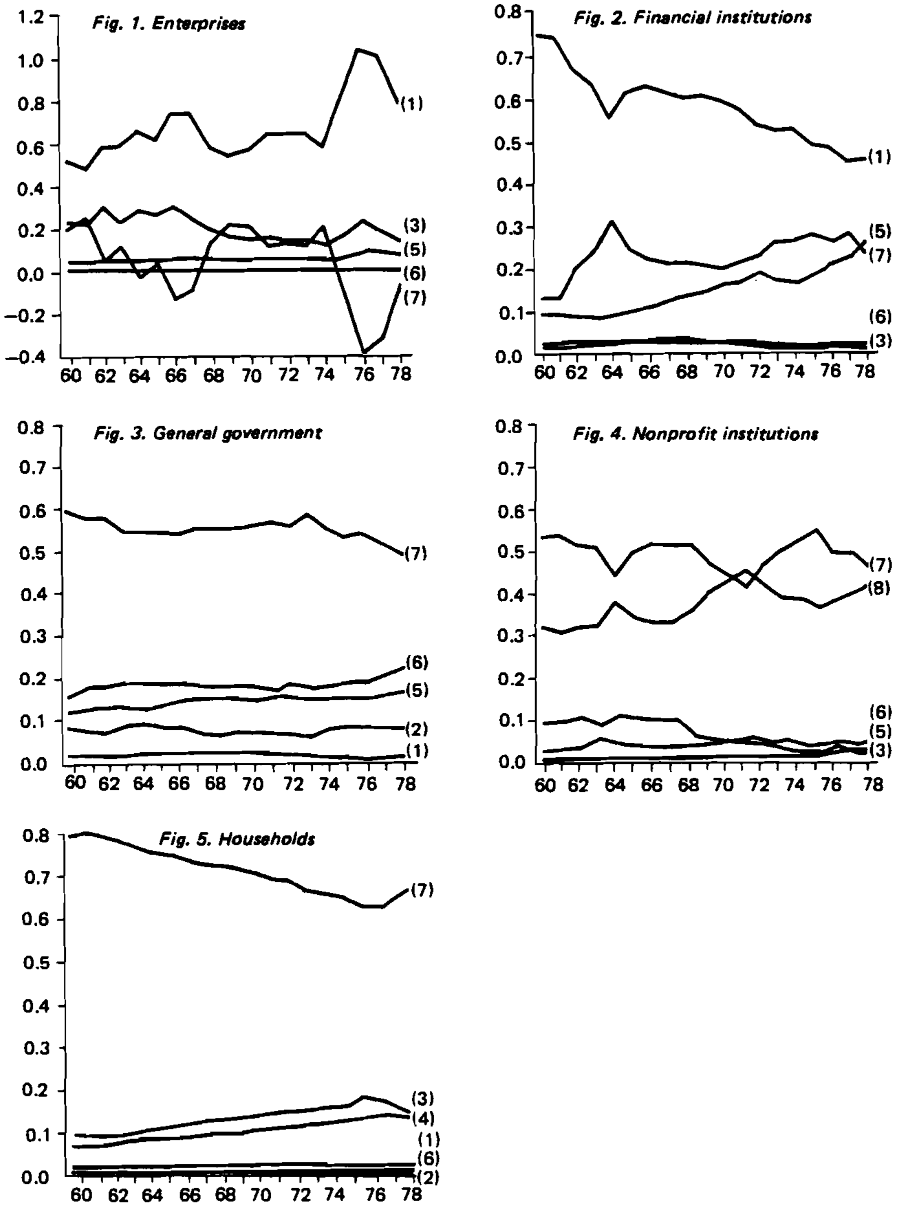
sensibility side of the question we posed above. As we shall see, there are severe difficulties related to both specifications.

4. Development of the income-outlay coefficients

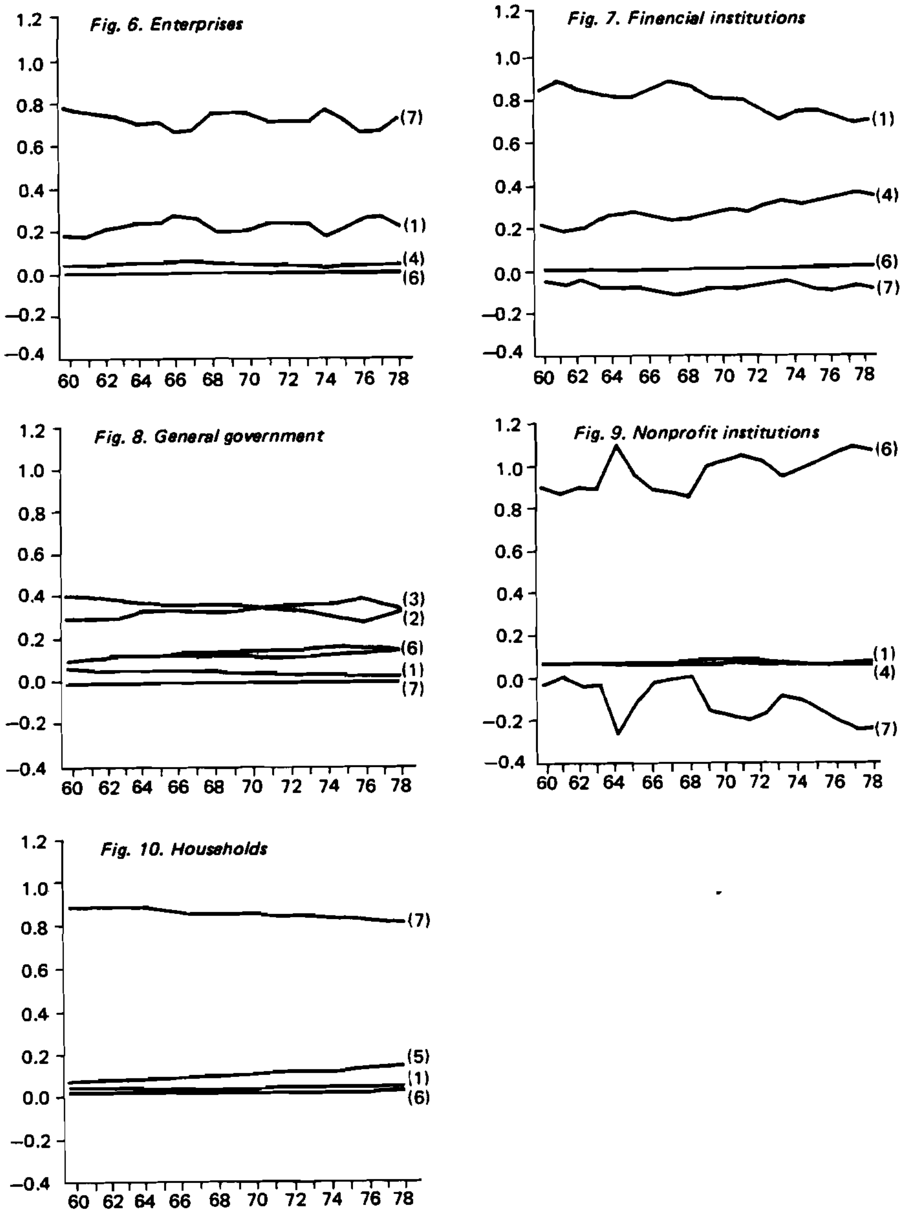
The basic question we have to ask, when thinking about the validity of (2) or (3), is whether the coefficients in A, B, C and D are dependent on the levels of factor incomes and if such dependencies exist, can we deduce any causal relations behind these. If we find any systematic relationships between the coefficients and the activity of the economy we would have to work these dependencies out before using(2) or (3) in forecasting purposes. We shall approach this question by having a look at the time-series of the coefficients in the Finnish data. We have used the following sectoral classification: enterprises , financial institutions, general government, non-profit institutions and households. The transfers are classified followingly:

1. Requited current transfers
2. Indirect taxes
3. Direct taxes
4. Contributions to social security schemes
5. Social security benefits
6. Other transfers.

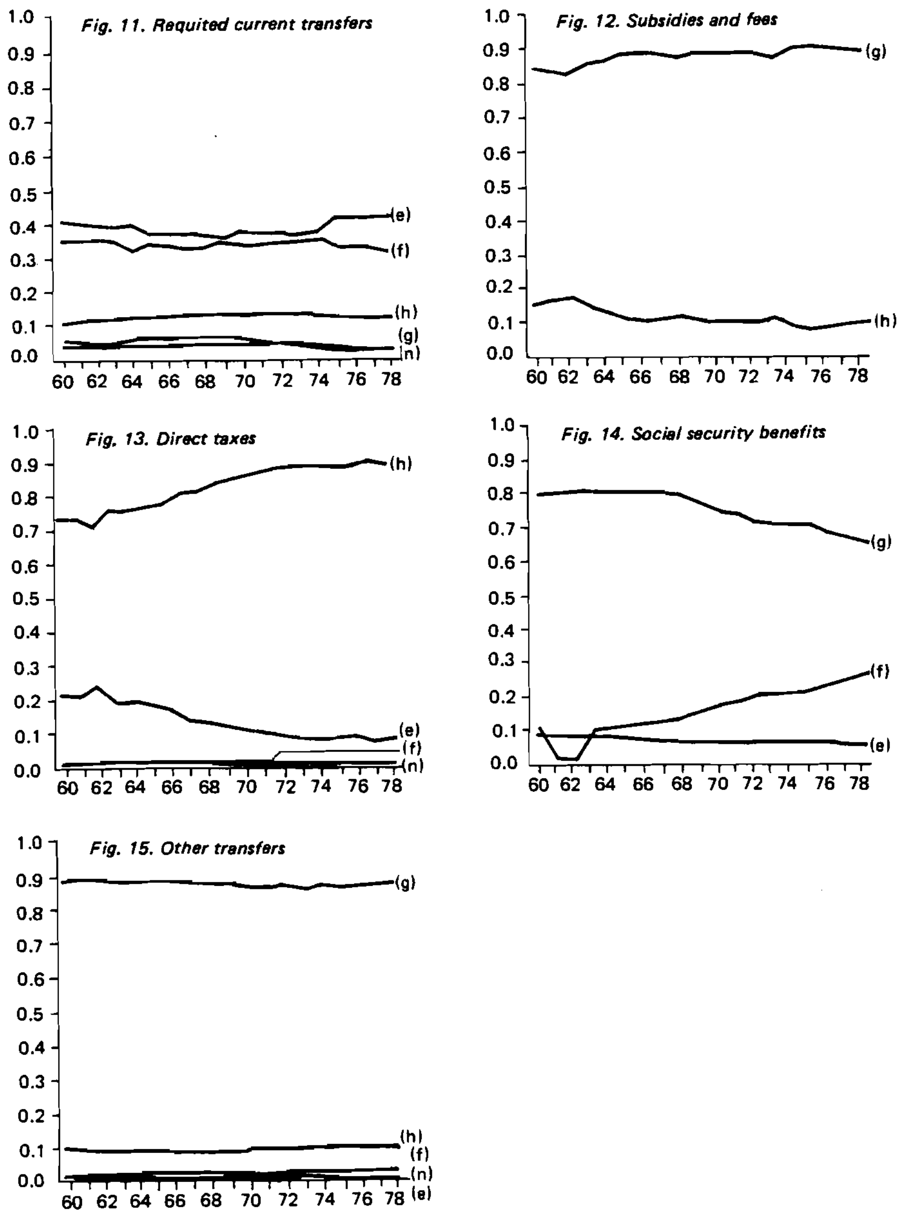
In figures 1 - 18 we have the time-series of the coefficients in matrices A, B, C and D. The share of disposable income and factor income have been added to A and B figures respectively.



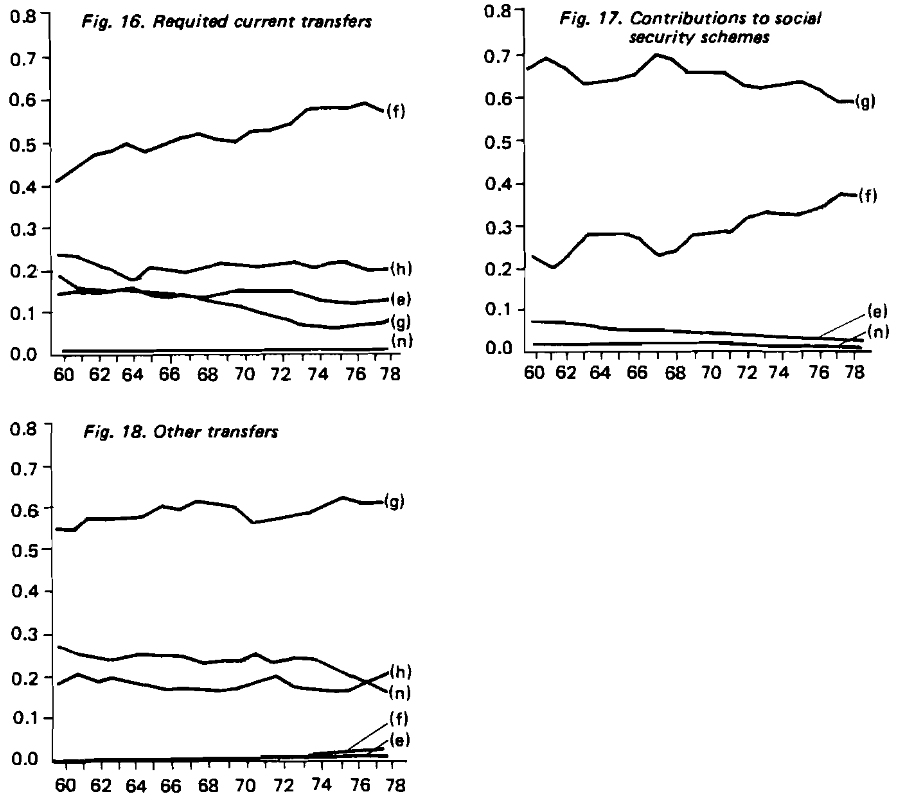
FIGURES 1-5. Outlay coefficients for the years 1960-1978: (1) requited current transfers, (2) indirect taxes, (3) direct taxes, (4) contributions to social security schemes, (5) social security benefits, (6) other transfers, (7) disposable income.



FIGURES 6-10. Income coefficients for the years 1960-1978: (1) requited current transfers, (2) indirect taxes, (3) direct taxes, (4) contributions to social security schemes, (5) social security benefits, (6) other transfers, (7) factor income.



FIGURES 11-15. Values of the outlay coefficient C for the years 1960-1978: (e) enterprises, (f) financial institutions, (g) general government, (n) nonprofit institutions, (h) households.



FIGURES 16-18. Values of the income coefficient D for the years 1960-1978: (e) enterprises, (f) financial institutions, (g) general government, (n) nonprofit institutions, (h) households.

On the basis of figures 1 - 18 we can draw following conclusions.

1^o There exists a strong inverse dependency between the shares of requited current transfers and disposable incomes in matrix A (figures 1-5). This is obvious with respect to enterprises, financial institutions and non-profit institutions.

2^o The development of the A-coefficients of general government has been stable. What needs to be noticed is the fact that after 1973 other transfers have increased their share while the share of disposable income has diminished.

3^o With respect to households there has been a clear tendency until 1976. The tax burden has constantly increased at disposable income's expence. After 1976 the tax rates have been checked and the development has reversed.

4^o The development of the coefficients in the income side of the account has been much more stable than that in the outlay side. The exception is non-profit institutions (figures 6-10).

5^o The elements of matrices C and D (figures 11-15 and 16-18, only non-zero or non-unity elements have been presented) have been rather stable.

Conclusion one above imlies that the inverse matrix in model (2) becomes very dependent of the variation of elements in A. It is possible to use this specification only if the exact relationship between the level of factor incomes and the elements in the first row of A are worked out. This problem is not so obvious if we aggregate enterprises and financial institutions.

Conclusion two poses the following question: how can we be sure about the consistency of the system? One would expect that a rising share of other transfers in the outlay structure of general government would have some implications on the income side. However, this does not seem to be the case. There exists a one more basic identily in the system. Namely, that factor incomes + net indirect taxes = disposable incomes + net property income and transfers to the rest of the world. This implies that a consistency error affects both the sectoral distribution and the level of total national income.

5. An iterative solution of the institutional redistribution

The analytical solutions (2) and (3) are valuable in revealing the essential features of the problem. Their practical fruitfulness is, however, questionable. From the practical point of view the following iterative algorithm can be better. Let there be still another matrix of coefficients B^* defined followingly: $B^* = \hat{z}^{-1}B_a$. In other words B^* is a matrix the elements of which show the shares of each receiving sector from total paid transfers. Let also r = vector of total received transfers by sector.

The basic idea behind the iterative solution is simple. We make an initial quess of the shares of received transfers and receive total incomes using these guesses. Since total incomes = total use of incomes we can calculate various transfers paid (A_a) using the share matrix A. The next step is to calculate row sums of paid transfers (z). Using B^* we have a new value for the received transfers. Indirect taxes must be added and new total incomes received. The process has converged when total incomes of various sectors do not change from previous round.

Formally the procedure can be expressed in the following way. The addition of indirect taxes has been excluded in favour of the more essential features. It is also better to relate the initial guess of the total income of the general government to tax revenue than to factor income.

0. $u_1 = (i_1 - \hat{B}' i_2)^{-1} v$

1. $A_a = A\hat{u}_t$ (t is the number of iteration, 1 in the first round, of course)

3. $z = A\hat{u}_t i_1 = A_a i_1 \leftarrow A_a = Au_{t+1}$

4. $B_a = B^* \hat{z}$

5. $r = i_2' B_a$

6. $u_{t+1} = v + r$

7. $u_{t+1} = u_t$? — No: $A_a = Au_{t+1}$, return to step 3; Yes: step 8

8. $y^d = (B_a - A_a)' i_2 + v.$

The process has some clear advantages over the analytical solutions. Exact consistency can be achieved. Information from outside the system can be incorporated. For instance the effects of the change of the system of social security benefits has been taken into account. It is also easy to include economic policy measures. The direct taxation system is proportional with respect to growth. Five rounds of the iteration have been shown to be enough.

In the table one we have the income and outlay accounts in year 1990 according to the basic scenario of FMS.[1] The solution method was the iterative method. A and B^* matrices are those of year 1978. It must be stressed that the results are tentative.

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1) For a description of the assumptions behind the basic scenario see Forssell - Mäenpää - Svento (1982). Some minor differences in the parameters exist between the above mentioned and the basic scenario adopted here.

Table 1: Income and Outlay Accounts in year 1990 in the basic scenario of FMS - th.mln.Fmk. Current prices.[1)]

Current Disbursements

Transfer	Sector: Corporate Enterprises	General Government	Non-profit Institutions	House-holds	Total
Reg. curr. transf.	23.9	1.4	1.3	3.8	30.3
Compulsory fees	.0	.0	.0	.7	.7
Direct taxes	2.7	.0	.1	21.8	24.7
Contrib. to sos.s.s.	.0	.0	.0	19.0	19.0
Soc.sec.benefits	8.4	14.9	.1	.0	23.4
Other transfers	.6	16.7	.2	1.7	19.2
Disposable income	2.8	34.6	1.4	91.6	130.3
Consumption	.0	29.7	2.6	85.9	118.2
Saving	2.8	4.9	-1.3	5.7	12.1
Total	38.5	67.5	3.0	138.6	247.7

Current Receipts

	Sector: Corporate Enterprises	General Government	Non-profit Institutions	House holds	Total
Req. curr. transf.	18.9	2.2	.2	5.4	26.7
Indirect taxes, net	.0	17.9	.0	.0	17.9
Direct taxes	.0	24.7	.0	.0	24.7
Contrib.to sos.s.s.	7.5	11.3	.2	.0	19.0
Soc.sec.benefits	.0	.0	.0	23.4	23.4
Other transfers	.6	11.5	3.1	3.8	19.0
Factor incomes	11.5	.0	-.5	106.0	117.0
Total	38.5	67.5	3.0	138.6	247.7

1) The sums may look small for being current price based. However, the FMS price model gives prices in relation to fixed wage levels. Thus prices are below one.

In table two we have the effects on the disposable income distribution of an ± 5%-unit increase in the direct tax rate of enterprises and households.

Table 2. The effects of an ± 5%-unit increase of the tax rates of enterprises and households

Sector	t_o^e -0.05	t_o^e +0.05	t_o^h -0.05	t_o^h +0.05
Enterprises	3.7	3.0	3.4	3.3
Gen.gov.	26.6	27.3	24.0	30.0
Non-profit inst.	.9	.9	.7	1.1
Households	68.8	68.9	71.9	65.6
	100.0	100.0	100.0	100.0

The yearly growth rates of the volume of public consumption in the above runs are 0.65, 0.94, 0.09 and 1.42. In the basic scenario it is 0.80.

The low growth of the public sector is due to assumed high level of net lending plus higher than average price increase in producing government services.

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Appendix I. The classification of the transfers

Requited current transfers
Withdrawals from entrepreneurial income
Property income
Interest
Dividends
Other
Net casualty insurance preminiums
Casualty insurance claims
Unrequited current Transfers
Indirect taxes or subsidies
Direct taxes
Compulsary fees and fines and penalties
Contributions to social security schemes
The emplyers´
The insured persons´
Social security benefits
Unfunded employee welfare contributions, imputed
Unfunded employee welfare benefits
Social assistance grants
Other current transfers
From/to general government
From/to other domestic sectors
From/to the rest of the world.

MODERN INPUT–OUTPUT MODELS AS SIMULATION TOOLS FOR POLICY MAKING

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1. INTRODUCTION

Current policy issues require economic models to play the role of national decision schemes (Caffè 1977, Rey 1965). Since the problems we face today are more complex and the policymaker's role more fragmented than formerly, it has become increasingly necessary to have a coherent scheme for forecasting and simulating alternative types of economic behavior. This naturally implies that the methodological principles underlying economic model building should be carefully examined. Many of the fundamental dichotomies assumed in the past for the sake of simplification appear to be inappropriate for present-day policy problems.

The main distinction between stabilization and growth models is in their statistical and mathematical basis, from which it is easy to find a unique mathematical generating trends and a unique statistical cause generating fluctuations. However, when consiering these models from an economic viewpoint, it is more difficult to find a unique cause generating trends and fluctuations (Hicks 1965). Such a distinction, be it explicit or implicit, is based on the idea that stabilization problems should be dealt with by short run demand-oriented models and growth problems by medium run supply-oriented models (Fox et. al 1973). The ultimate implication of such a methodological approach is to neglect the interaction between stabilization and growth aspects, omitting a consistency criterion coordinating short- and medium-term policies.

There has been a tradition of 'macro' model building in which the demand side is privileged. However, recent events have focused interest on economic variables defined in more detail and have emphasized the need for policies to be specified at a greater level of disaggregation but consistent with the macro level. Macro models provide information on each final demand component, such as imports, exports, and domestic consumption but do not describe the structure of each variable. Yet the sectoral composition of these components is often crucial in indicating the pattern of either technological or behavioral change in the economy.

This issue seems to reveal the indadequacy of the concept of the macro-variable (Pasinetti 1975). The internal dynamics of such variables seem to compromise not only the very concept of macro-variables but also their macro inter-relations (Spaventa and Pasinetti 1970). Nor is the solution to be found in disaggregating macro models in a nonsystematic way, such as by introducing additional sectoral equations or splitting the macro results by means of a given set of weights.

To deal with these and other issues Almon (1982) proposes that modern input-output models be used as rational decision schemes for economic policy making. This implies changing the way of looking at the economic process. Although it does not mean that macro aspects of the economy should be ignored, they are no longer considered central to the explanation of the individual's economic behavior. Rather they are the result of an aggregation of the behavior that has been defined and simulated at a more detailed level, for example the level of the input-output sector for total output and intermediate demand, the items of expenditure from household budgets for final consumption, and the appropriate disaggregation for each particular item for the remaining items of final demand.

Such a framework can be used to address a set of issues that are currently relevant to policymaking. In the past these issues were not tackled satisfactorily for a number of reasons.

First, a great part of interest was devoted to the aggregate control of expenditure and taxation. Second, there was a lack of flexible computing programs for estimating sectoral behavioral equations and for operating multisectoral simulation models. Finally, theoretical advantages were not so developed to tackle conveniently the integration of the input-output side with the demand side.

2. MACROECONOMIC and INPUT-OUTPUT MODELS

Steady progress in economic modeling has been stimulated by the increasing complexity of economic problems. For some time analytical tools have been developed independently in two methodological frameworks: input-output models and macroeconomic models.

Traditional input-output models have influenced the field of applied modeling in two ways. First, they have stressed the need to refer to the economic system by means of detailed categories. For such a purpose the producing sector is defined as a component of the system having a homogeneous output for a given technology. Second, they made it clear that production must satisfy not only final demand but also intermediate demand, which can be identified when the technical coefficients (such as those indicating the intermediate demand for the output of a certain sector) have been defined. The main contribution of traditional input-output models is that they allow the list of final demands to be transformed into a vector of sectoral outputs.

Given a vector x representing n outputs, a vector f representing the list m of final demands and a (n x n) matrix A of technical coefficients, the problem of the supply/demand equilibrium is solved by finding a value of vector x such that the following relation is fulfilled:

$$x = Ax + f \quad , \qquad (1)$$

or

$$x_i = \sum_{j=1}^{n} a_{ij}x_j + f_i \quad , \qquad i = 1,\ldots,n \quad , \tag{2}$$

the coefficients a_{ij} were traditionally considered as constants.

Less importance has been devoted to the vector f of final demand. It represents the total final demand for the specific good produced by each sector. Thus, the disaggregation of the final demand components, in general, does not allow their behavioral functions to be adequately specified.

Conversely, macro models have completely ignored the interindustrial aspects since they emphasize Gross Domestic Product only. Nevertheless, they were able to specify the behavirol functions for each demand component with great accuracy.

The supply-demand equilibrium macro-relation is represented by:

$$Y = C(\bullet) + I(\bullet) + G(\bullet) + X(\bullet) - M(\bullet) \quad , \tag{3}$$

where Y represents GDP (Siesto 1977), $C(\bullet)$ is the consumption function, $I(\bullet)$ is the investment function, $G(\bullet)$ is public expenditure, and $X(\bullet)$ and $M(\bullet)$ are exports and imports, respectively. Each final demand component is explained by a set of variables, denoted by $(\bullet)$, among which Y may also appear. The only point of intersection between the two schemes is:

$$\begin{aligned} Y &= \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}x_j \quad , \\ C + I + G + X - M &= \sum_{i=1}^{n} f_i \quad . \end{aligned} \tag{4}$$

The points of contact between the two approaches have steadily increased and in particular input-output models have begun to explain the final demand formation process without compromising on the multisectoral approach.

The Interindustrial Italian Model--INTIMO (Ciaschini and Grassini 1981)--is a modern input-output model of the INFORUM family

(Almon 1974 and 1981, Young and Almon 1978, Nyhus 1981). The final demand components are explained by behavioral equations econometrically estimated. Each final demand component is explained at a level of disaggregation which allows for a correct specification of the sectoral demand functions. The disaggregated consumption vector is composed of n_1 expenditure items according to the items appearing in the household budget accounts. In fact, the effects of the consumer's behavior through those items can be correctly observed. Investments (F_2) are explained in terms of the n_2 investing industries, and so on for the remaining components of final demand. In this way we obtain:

$$\begin{aligned} F_1 &= F_1(\ldots) \ , \\ F_2 &= F_2(\ldots) \ , \\ &\vdots \\ F_k &= F_k(\ldots) \ , \end{aligned} \tag{5}$$

where F_1 is the vector of disaggregated consumption functions, F_2 is the vector of disaggregated investments, and so on up to the k^{th} component of final demand.

The multisectoral supply-demand relation is to be fulfilled at the input-output level. We therefore need to transform consistently the $F_1,\ldots,F_k$ demand vector and to do so we make use of bridge matrices $B_1(t)$ - $B_k(t)$ such that

$$\begin{aligned} f_1 &= B_1(t)F_1 \ , \\ f_2 &= B_2(t)F_2 \ , \\ f_k &= B_k(t)F_k \ , \end{aligned} \tag{6}$$

The B matrices express the consistency between the input-output accounts and the final demand accounts. In this model the equilibrium relation analogous to (1) and (2) is given by:

$$\begin{bmatrix} F_1 \\ \cdot \\ \cdot \\ \cdot \\ F_k \\ X \end{bmatrix} = \begin{bmatrix} & & C_1 \\ & & \cdot \\ & 0 & \cdot \\ & & \cdot \\ & & C_k \\ B_1(t), & \ldots, B_k(t) & A(t) \end{bmatrix} \begin{bmatrix} F_1 \\ \cdot \\ \cdot \\ \cdot \\ F_k \\ X \end{bmatrix} + D_1 + D_2 \ , \qquad (7)$$

where

$$D_1 = \begin{bmatrix} H_{11} & H_{12} \\ & \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} F_1 \\ \vdots \\ F_k \\ X \end{bmatrix} \ , \qquad (8)$$

$$D_2 = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} Z \ . \qquad (9)$$

Equation (7) shows the simultaneity in the simulation of the model. The $B_1, \ldots, B_k$ bridge matrices allow the purchasing sectors to be connected to the producing sectors. The supply demand equation is solved at the input-output level. This means that we can obtain the solution for final demand according to the purchasing sectors and to the input-output sectors. While the first result allows a change in the demand structure to be analyzed effectively, the other provides information on the destination of output at the input-output level.

$C_1, \ldots, C_k$ matrices represent the parametric structure, econometrically estimated, of the simultaneous relationship between the final demand vectors and sectoral output. Equation (8) shows the lagged effect and equation (9) the exogenous variable effect.

Such is the logical scheme that connects matrices and variables within the model. We now give a detailed example of how demand functions are introduced in the input-output structure, of the type of *a priori* information that can be provided for the model, and of the type of result that can be expected.

3. THE INTEGRATION OF DEMAND: THE ROLE OF INVESTMENT FUNCTIONS

The integration of interindustrial and demand aspects, achieved by means of equation (7), enables us to construct a flow table between the intermediate and final sectors that is much richer in information than traditional flow tables (Ciaschini 1982; M. Grassini 1932; and L. Grassini 1981). Table 1 presents the flow table for the INTIMO model.

Table 1. The flow table for the INTIMO model.

A	B	C	D	E	F	G
INTERME-DIATE FLOWS	CONSUMP-TION	INVEST-MENT	INVENTORIES	PRIVATE AND PUBLIC EXPEN-DITURES	IMPORT	EXPORT

Each row of the table referes to a product of the input-output list and each column refers to a purchasing sector. Such sectors, summing to 114, are specified as in Table 2.

Table 2. The flow table for the INTIMO model: purchasing sectors.

MATRIX	PURCHASING SECTOR	CONTENT
A	44	Intermediate demands
B	40	Expenditure items in household budgets
C	23	Investment by investing sector
D	1	Inventory change
E	4	Public administration and private social institution expenditures: 1. Health 2. Education 3. Other public expenditures 4. Private institutions
F	1	Imports
G	1	Exports

Table 2 shows the type of item for which the INTIMO model produces information for each year along the time horizon. The computational algorithm constructs such tables by solving equation (7) iteratively. A given output vector for the input-output sectors is transformed into a vector of total output consistent with final demand equations x. With such a vector and with a vector of exogenous variables y, the set of final demand vectors F_i, i = 1,...,k is determined. These demands are transformed into the input-output demand vectors $f_1,\ldots,f_k$. Then, using the technical coefficients, we can determine the new vector of total output $\hat{x}_{IO}$. If significant differences are found between the vectors x_{IO} and $\hat{x}_{IO}$, the procdure is repeated. Within such a loop there exists a further loop that determines the total output vector given the final demand vector.

Intermediate and final demand can be determined simultaneously on the basis of total output because some final demand equations, such as the investment equations, show total output among their arguments. The logical scheme of such a process is shown in Figure 1. The sectoral investment function used is the following. Total investment I is given by expansion investment V and substitution investment S, so that

$$I = V + S \quad . \tag{10}$$

Substitution investment is given by a replacement rate that is r times the capital stock K.

$$S = rK \quad , \tag{11}$$

where the capital stock K is determined as the capital-output ratio k times the smooth output $\overline{Q}$:

$$K = k\overline{Q} \quad . \tag{12}$$

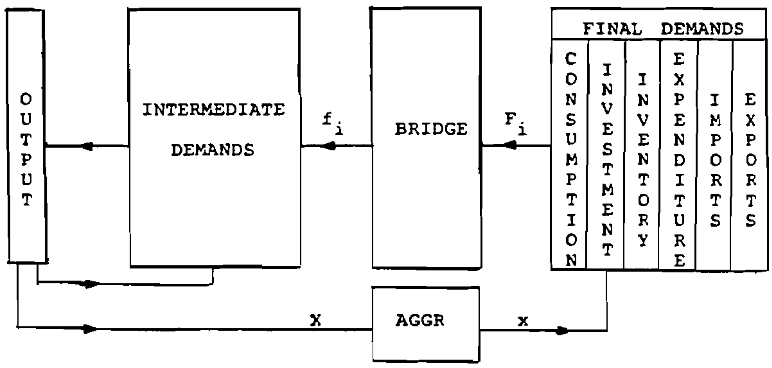

Figure 1. Scheme of the simulation procedure.

Expansion investment is equal to the capital-output ratio k times a distributed lag on changes in output:

$$E = k \sum_{i=0}^{n} w_i \; \Delta Q_{t-i} \quad , \tag{13}$$

where

$$\sum w_i = 1 \quad .$$

The sectoral investment function is then given by:

$$I_t = rk\overline{Q} + k \sum_{i=1}^{n} w_i Q_{t-1} \quad . \tag{14}$$

At this stage the capital cost is not considered within the arguments of the sectoral investment functions. Even if such an element were to be taken into consideration, we do not have available reliable sectoral data on such a variable. Given the limited length of the variable series, the hypothesis of equality between the marginal and the average capital-output ratio was preferred to a more elaborate one.

In the estimation

$$\overline{Q} = 0.5Q_t + 0.3Q_{t-1} + 0.2Q_{t-2} \quad , \tag{15}$$

and

$$I_t = rk\overline{Q} + k(w_0 \Delta Q_t + w_1 \Delta Q_{t-1} + w_2 \Delta Q_{t-2}) \quad , \tag{16}$$

where w_2 is not estimated but calculated according to:

$$w_2 = 1 - w_0 - w_1 \quad . \tag{17}$$

The sectoral investment function estimated for 23 investing industries is then given by:

$$I_t = k(r\overline{Q} + \Delta Q_{t-2}) + kw_0(\Delta Q_t - \Delta Q_{t-2}) + kw_1(\Delta Q_{t-1} - \Delta Q_{t-2}) \quad . \tag{18}$$

The statistical data base for the regression is given by:

1. Investment by producing and investing sectors for 23 investing sectors from 1970 to 1979 in constant and current prices (ISTAT 1970 - 1980).
2. Total output for 44 input-output sectors from 1966 to 1979 determined on the basis of the industrial production index and services' value only.

The relation (18) was imposed on available data, assuming a replacement rate of 10 percent and a distributed lag of the third and fourth order. The results obtained are summarrized in Tables 3 and 4. The estimation was performed earlier (Ciaschini 1981), but has been repeated since better information on total output prior to 1970 for the industrial sectors is now available. The goodness-of-fit, in terms of the average absolute percentage error (AAPE), is slightly better in the 4 period lag estimation. However, in such a case the percentage of negative w_i is higher. Some sectoral functions show a reasonable fit. For one third of the sector the AAPE is less than 10 percent, in the second third it is between 10 and 20 percent and in the final third it is greater than 20 percent. All the capital-output ratios show a standard error that makes the estimation look reasonable on statistical grounds, but in at least one third of the w_i estimations the capital-output ratio seems to be too low.

Additional estimations were performed allowing the value of r to vary parametrically. The results relating to the goodness-of-fit in terms of the AAPE are shown in Table 5.

The 23 sectoral investment equations are an example of how a final demand component was introduced consistently in an input-output scheme. For the remaining items of final demand see Ciaschini and Grassini (1982) and Alessandroni (1981).

4. EXOGENOUS INFORMATION

Having introduced the final demand components into the input-output structure (Almon 1979, Nyhus and Almon 1977), we need to define how the model deals with external information.

From a system's viewpoint external inputs may affect the

-- exogenous variables,
-- endogenous variables,
-- parametric structure of the model.

Table 3. Results of estimating sectoral investment functions given a replacement rate of 10 percent and a third order distributed lag.

SECTORS	see	rho	aape	cap/out	t-test	dq(t)	t-test	dq(t-2)	t-test
1 agriculture,for,fishery	220.47	0.499	9.69	1.149	28.21	0.355	1.83	0.549	3.31
2 energy	150.04	0.274	5.86	0.978	46.04	0.354	5.10	0.235	3.41
3 ferrous&non ferrous ores	416.70	0.653	32.44	0.743	7.34	0.568	1.83	0.090	0.27
4 non metal min.&prod.	78.36	0.064	13.61	0.654	19.60	0.540	4.42	-0.135	-0.75
5 chemical products	349.64	0.802	30.24	0.690	10.08	0.191	1.15	0.310	2.21
6 nonmachinery metal products	69.38	0.039	16.28	0.376	14.83	0.269	3.06	0.017	0.14
7 agricultural machinery	73.72	0.144	16.59	0.336	15.91	0.111	2.77	0.056	1.27
8 office, precis,opt instr.	24.61	0.425	25.57	0.331	9.86	0.147	4.08	0.034	0.66
9 electrical goods	61.33	0.330	16.05	0.378	17.16	0.103	1.70	0.154	2.85
10 transport equipment	84.25	0.346	10.38	0.689	27.28	0.437	6.33	0.128	1.92
11 food,beverages&tobacco	54.70	0.373	9.21	0.166	27.59	0.085	4.50	0.048	2.70
12 textiles,clothing,shoes	107.02	0.445	17.40	0.221	13.88	0.089	2.56	0.051	1.48
13 paper&printing	21.77	0.072	7.39	0.285	35.41	0.150	8.44	0.065	3.30
14 rubber & plastics	70.86	0.770	24.26	0.434	9.81	0.272	2.65	0.023	0.21
15 wood & furniture	41.34	0.447	22.17	0.107	13.85	0.089	4.06	-0.034	-1.06
16 construction	41.52	0.505	10.39	0.144	23.67	0.197	5.82	-0.129	-2.88
17 trade	239.99	0.703	12.36	0.575	20.49	0.518	3.44	0.000	0.00
18 hotels & restaurants	42.84	0.468	9.14	0.418	29.68	0.193	2.84	-0.078	-0.81
19 transport	115.73	0.081	5.08	1.448	54.02	0.667	7.21	0.120	1.05
20 communications	256.87	0.693	24.39	3.134	13.03	0.218	0.15	1.339	1.52
21 banking & insurance	88.61	0.772	18.09	0.389	16.98	0.051	0.35	0.185	1.42
22 other services	955.44	0.392	6.99	3.165	31.79	1.684	1.75	1.529	2.01
23 other non dest. serv.	360.74	0.710	10.24	1.389	22.44	1.382	1.67	-0.365	-0.27

Table 4. Results of estimating sectoral investment functions given a replacement role of 10 percent and a forth order distributed lag.

INVESTING SECTORS	SEE	RHO	AAPE	CAP/OUTP	t-test	DQ(t)	t-test	DQ(t-1)	t-test	DQ(t-3)	t-test
1 agriculture,for,fishery	171.17	0.313	7.25	1.139	35.73	0.356	2.37	0.213	1.16	0.363	2.56
2 energy	145.89	0.216	5.74	0.877	47.13	0.386	4.82	0.241	3.57	-0.060	-0.75
3 ferrous&non ferrous ores	389.43	0.760	35.19	0.762	7.94	0.574	1.98	-0.001	-0.00	0.321	1.20
4 non metal min.&prod.	74.63	0.191	12.46	0.655	20.61	0.649	4.10	-0.285	-1.26	-0.119	-1.01
5 chemical products	319.22	0.788	26.36	0.720	10.91	0.022	0.11	0.226	1.59	0.311	1.41
6 nonmachinery metal products	69.12	0.080	16.24	0.376	14.88	0.286	2.64	-0.000	-0.00	-0.022	-0.27
7 agricultural machinery	61.02	0.291	13.25	0.335	19.15	0.032	0.64	0.065	1.77	0.096	2.14
8 office, precis,opt instr.	22.85	0.682	21.99	0.342	10.53	0.118	2.91	0.042	0.86	0.064	1.26
9 electrical goods	48.38	0.586	10.27	0.380	21.86	0.019	0.32	0.101	2.12	0.139	2.46
10 transport equipment	79.38	0.467	9.97	0.689.	28.97	0.347	3.37	0.106	1.61	0.106	1.12
11 food,beverages&tobacco	52.44	0.279	9.13	0.165	27.02	0.103	3.93	0.053	2.97	-0.029	-0.93
12 textiles,clothing,shoes	104.78	0.475	17.24	0.228	12.01	0.058	0.98	0.039	0.99	0.065	0.65
13 paper&printing	13.96	-0.098	4.84	0.288	55.18	0.133	10.7	0.055	4.20	0.048	3.78
14 rubber & plastics	59.16	0.642	15.84	0.418	11.10	0.448	3.72	0.052	0.57	-0.296	-2.08
15 wood & furniture	33.61	0.622	17.41	0.105	16.35	0.141	4.87	-0.081	2.44	-0.077	-2.26
16 construction	39.87	0.639	10.80	0.143	24.32	0.170	3.87	-0.087	-1.39	0.030	0.91
17 trade	239.33	0.706	12.30	0.577	19.51	0.510	3.29	0.003	0.01	0.038	0.23
18 hotels & restaurants	22.99	0.528	3.78	0.420	55.52	0.143	3.79	-0.023	-0.44	0.167	4.97
19 transport	93.25	-0.157	4.07	1.460	65.78	0.504	4.92	0.180	1.89	0.232	2.32
20 communications	168.42	0.379	15.73	3.198	20.16	0.729	0.77	-0.193	-0.27	2.268	3.64
21 banking & insurance	88.44	0.767	18.01	0.388	16.51	0.069	0.40	0.189	1.44	-0.026	-0.19
22 other services	883.32	0.138	7.16	3.149	33.94	1.745	1.96	2.040	2.53	-0.926	-1.30
23 other non dest. serv.	280.40	0.255	8.27	1.345	26.33	1.897	2.82	-0.492	-0.47	-1.862	-2.55

Table 5. The Average Absolute Percentage Error for sectorial investment functions with various depreciation rates.

INVESTING SECTORS	DEPRECIATION RATES 0.025	0.050	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.500
1 agriculture,for,fishery	21.530	14.812	9.685	8.535	7.876	7.386	7.042	6.858	6.709	6.393
2 energy	17.427	10.667	5.863	5.342	4.938	4.616	4.357	4.539	4.690	6.434
3 ferrous&non ferrous ores	34.342	31.819	32.442	32.896	33.201	33.416	33.574	33.693	33.787	34.165
4 non metal min.&prod.	16.674	13.317	13.698	14.103	14.459	14.727	14.935	15.101	15.237	15.879
5 chemical products	32.809	31.590	30.243	29.847	29.547	29.318	29.239	29.249	29.258	29.307
6 nonmachinery metal products	29.516	21.158	16.278	16.103	16.142	16.171	16.193	16.210	16.224	16.291
7 agricultural machinery	26.272	21.466	16.587	15.199	14.163	13.361	12.723	12.202	11.770	9.655
8 office, precis,opt instr.	30.745	28.587	25.573	24.502	23.650	23.055	22.558	22.136	21.774	19.815
9 electrical goods	24.838	20.690	16.054	14.653	13.583	12.859	12.298	11.837	11.452	9.511
10 transport equipment	13.484	11.331	10.384	10.871	11.292	11.777	12.186	12.517	12.789	14.105
11 food,beverages&tobacco	10.054	9.045	9.208	9.525	9.757	9.935	10.076	10.190	10.284	10.744
12 textiles,clothing,shoes	27.241	21.374	17.397	16.543	15.915	15.435	15.055	14.748	14.494	13.616
13 paper&printing	17.391	12.294	7.386	6.109	5.232	4.978	4.982	4.985	5.167	6.310
14 rubber & plastics	27.514	26.003	24.257	23.714	23.294	23.023	22.848	22.848	22.888	24.056
15 wood & furniture	30.147	26.201	22.165	21.010	20.145	19.474	18.938	18.501	18.137	16.407
16 construction	26.057	16.325	10.392	9.306	8.530	7.947	7.491	7.126	6.828	6.332
17 trade	24.140	17.606	12.359	11.121	10.375	9.815	9.379	9.030	8.745	7.390
18 hotels & restaurants	26.296	17.702	9.137	7.362	6.758	6.640	6.696	6.969	7.262	8.912
19 transport	13.157	8.901	5.080	4.127	3.424	3.280	3.337	3.489	3.710	4.836
20 communications	35.502	30.556	24.385	22.397	20.843	19.806	18.976	18.287	17.707	15.155
21 banking & insurance	19.007	18.506	18.094	17.974	17.884	17.813	17.757	17.710	17.672	17.477
22 other services	10.114	7.388	6.991	7.263	7.788	8.218	8.556	8.829	9.054	10.143
23 other non dest. serv.	12.825	11.275	10.239	9.969	9.774	9.648	9.604	9.569	9.541	9.510

With respect to the exogenous variables, this consists mainly in defining the trajectories of a set of exogenously determined variables either as being under the control of a decision maker or as being outside the set of variables the model can influence. In this sense they constitute the traditional exogenous variables, i.e. instruments and data, of the policy problem (Tinbergen 1952).

The effects on the endogenous variables consist in the possibility of substituting the simulated values with observations. This turns out to be particularly useful for the forecasting period that starts with the base year, i.e., the year in which the forecast begins. As we approach the current year from the base year, the available statistical data become gradually less numerous. Thus, the statistical data covering the current year is incomplete.

For all such periods of the forecasting horizon, the model takes the observed values and simulates those values for which there are no statistical data. Only the total output vector cannot be imposed on the observed values but should be simulated. Thus, the initial values for the endogenous variables are always the most recent ones. If there are no data available for a particular variable, it is simulated according to the most recent observations on the other variables.

The effect of exogenous information on the parameter structure allows for a time-change in the technological coefficients and bridge-matrices. This is possible because of the flexibility of the computing routines, which enables us to include time-varying technological coefficients. The trajectories of changes in the exogenous coefficients can be forecasted and imposed on the model. For this purpose we can assume that technological coefficient C varies over time so that the present change is proportional, together with constant b, to the distance between the actual value of C and a given constant value a. In algebraic terms:

$$\frac{1}{c}\frac{dc}{dt} = b\ (a-C) \quad , \qquad (19)$$

which admits as a solution the logistic curve

$$c_t = a \;/\; (1+Ae^{-bat}) \quad , \tag{20}$$

where A is an integration constant.

For estimation purposes, equation (20) can be written as

$$\log\left(\frac{a}{C_t}-1\right) = \log A - bat, \text{ if } \frac{a}{C_t} \geq 1 \tag{21}$$

or

$$\log\left(1-\frac{a}{c_t}\right) = \log(-A) - bat, \text{ if } \frac{a}{c_t} \leq 1 \quad .$$

Equation (20) is used for coefficients with increasing values, whereas equation (21) is used for those with declining values.

Unfortunately, we have only one flow matrix for intermediate goods. We therefore apply (20) and (21) to a complete row of the matrix rather than to each coefficient. In this way we are able to identify the dependepent variable C_{it} as an index that shows the volume of intermediate goods provided by a sector for the whole economy as a percentage of the total volume of intermediate goods produced by that sector.

$$C_{it} = U_{it} / V_{it} \quad , \tag{22}$$

where

$$U_{it} = (1-a_{ij})\; x_{it} - F_i \quad , \tag{23}$$

$$V_{it} = \sum_{j=1}^{n} a_{ii} x_{it} \quad . \tag{24}$$

Such a method of introducing changes in the coefficients is not exhaustive because the price substutution effect on intermediate goods is neglected. This effect can be dealt with by means of Leontief generalized production function (Dewiert 1971), once the price formation process has been modeled. Work on this aspect of the model, which is underway (Ciaschini 1982), is based on Belzer (1978).

The effects of the exogenous information considered above can be classified (Figure 2) in relation to economic policy according to:

(1) assumptions,
(2) demand controls,
(3) structural hypothesis,
(4) forecasting hypothesis.

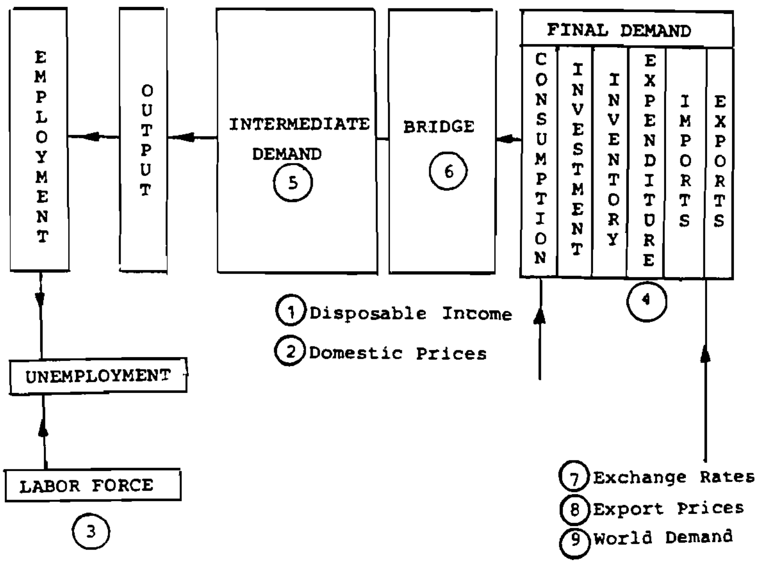

Figure 2. The impact of exogenous information.

The assumptions are represented by the set of variables that makes the output section of the model independent of the price and income side. If the former operates autonomously, we have to specify the trends in domestic prices (2), and in disposable income (1). We have also to forecast the labor force (3).

The demand controls mainly relate to simulation of the effects of different public expenditure paths (5). The disposable income trajectory can also be used in simulating different trends in taxation.

The structural hypothesis allows exogenous changes in the elements of the intermediate coefficients (5) and in the bridge matrices (6) to be taken into account in the model.

The forecasting hypothesis allows us to include in the model information on the exchange rate (7), the vector of international prices for competing exports (8), and world demand (9).

All this exogenous information enables us to formulate a detailed scenario which forms the basis of the forecast. The results obtained are thus a function of the scenario that has been chosen.

5. INFORMATION PRODUCED BY THE MODEL

The exogenous inputs affect the macro and sectoral variables in the model. Having defined a base scenario that takes into account the hypothesis of change in the technological structure of the economic system by means of (20) and (21) and a trajectory of energy demands consistent with the national energy plan, we obtained the macro results shown in Table 6. This table presents the forecasts of the macro variables in the supply-demand equation for a 10-year period together with the associated macro assumptions.

We should stress that these macro results have been obtained using a procedure that aggregates the sectoral results. First, the sectoral forecasts are obtained; they are then aggregated into the macro variables. This process is dependent on the model

Table 6. Aggregated results for a simulation of the base scenario.

AGGREGATED RESULTS (prices 1975) growth rates

	75-79	79-81	81-82	82-83	82-85	82-88	82-90	83-84	85-90	75-90
GROSS DOMESTIC PRODUCT	4.11	1.90	-2.32	3.18	4.56	3.38	3.29	5.52	2.53	2.96
Private Consumption	7.44	2.20	0.12	3.41	3.30	2.94	2.81	3.30	2.52	2.72
Foodstuffs	1.98	1.41	0.39	1.75	1.73	1.60	1.56	1.73	1.45	1.57
Goods	4.10	2.49	-0.11	3.74	3.52	3.03	2.86	3.48	2.47	2.94
Services	4.26	2.69	0.09	4.63	4.52	4.02	3.84	4.54	3.44	3.55
Fixed Investment	1.76	4.64	-12.18	-0.40	7.39	3.80	3.71	12.99	1.20	2.25
Inventory Change										
Exports	9.29	0.71	-1.51	3.34	5.10	4.53	4.49	5.82	4.12	4.86
Goods	9.93	0.70	-0.86	3.58	5.45	4.78	4.74	6.21	4.31	5.21
Services	5.59	0.78	-5.69	1.74	2.75	2.78	2.71	3.18	2.69	2.66
Imports	6.13	0.93	-3.56	3.57	5.21	3.53	3.47	6.52	2.43	3.37
Goods	6.20	0.81	-3.78	3.59	5.31	3.56	3.50	6.69	2.42	3.38
Services	5.41	2.04	-1.56	3.31	4.28	3.27	3.18	4.95	2.53	3.31
Public and Social Private Expenditure	2.29	1.93	2.07	2.03	1.99	2.01	2.00	1.99	2.00	2.07
Government	2.30	1.93	2.08	2.04	2.00	2.02	2.00	2.00	2.00	2.08
Education	2.23	1.93	2.09	2.04	2.00	2.02	2.00	2.00	2.00	2.06
Health	2.34	1.92	2.05	2.01	1.97	2.00	1.98	1.97	1.98	2.07
Social Private Expenditure	2.18	2.07	2.02	1.98	1.94	2.00	1.98	1.94	2.01	2.05
Employment	0.67	0.64	-1.38	-1.13	1.57	1.16	1.08	2.50	0.79	0.75
ASSUMPTIONS										
Disposable Income(per cap)	3.46	0.00	2.65	3.34	3.24	2.87	2.74	3.23	2.45	2.56
Unemployment	5.01	21.40	40.33	19.93	-15.91	-9.78	-9.33	-21.87	-5.38	1.90
Exchange Rate	2.94	1.96	1.69	1.67	1.64	1.45	1.39	1.64	1.24	1.90
Foreing Demand	5.26	-1.90	0.96	1.74	2.75	2.78	2.71	3.18	2.69	2.66

operating at the sectoral level without the assistance of the macro part that 'drives' it.

The unemployment variable is the result of the difference between the labor force forecast, which is exogenously given, and total employed. Such a variable provides a first check at the macro level of the consistency of the model inputs.

A detailed analysis of the results produced in sumulations (see Tables 6 and 7), thus showing the results on three different levels: the macro level, the purchasing sector, and the selling sector.

The forecasting method using the scenario approach not only enables us to evaluate the effects of exogenous inputs on the set of specific trajectories, it also allows us to take full advantage of the information generated by comparing the results of various scenario hypotheses. Table 7 presents the macro results of the 'base' scenario plus scenarios ALT 1 and ALT 2.

In ALT 1 the energy hypothesis was maintained while assuming a constant technological structure. In ALT 2 the technical coefficient change was maintained while dropping the energy hypothesis.

Table 7 summarizes the results obtained from the three scenarios and compares the average growth rates for the aggregated results over the periods 1985 - 1990 and 1975 - 1990.

Note that the average growth rate in consumption and total exports is the same in all three simulations. This is because in the model consumption depends on disposable income and relative domestic price trend assumptions, which were kept the same for all the simulations. The effects of prices on the consumption structure can be simulated when the price side of the model is complete. The interaction between output and prices can thus be adequately taken into account. Exports depend on the forecasting hypothesis related to the exchange rate, world demand, and the vector of international prices for competing exports, which were also kept the same for all three simulations. The average growth rate of Gross Domestic Product throughout the forecasted period is almost

Table 7. The aggregated results obtained from the base scenario and ALTs 1 and 2.

	(base) 75-81	(base) 81-82	(base) 82-85	(base) 85-90	(alt1) 85-90	(alt2) 85-90	(base) 75-90	(alt1) 75-90	(alt2) 75-90
GROSS DOMESTIC PRODUCT	3.41	-2.32	4.56	2.53	2.50	2.49	2.96	2.96	2.95
Private Consumption	3.03	0.12	3.30	2.52	2.52	2.52	2.72	2.72	2.72
Foodstuffs	1.79	0.39	1.73	1.45	1.45	1.45	1.57	1.57	1.57
Goods	3.56	-0.11	3.52	2.47	2.47	2.47	2.94	2.94	2.94
Services	3.74	0.09	4.52	3.44	3.44	3.44	3.55	3.55	3.55
Fixed Investment	2.72	-12.18	7.89	1.20	1.49	1.49	2.25	2.59	2.61
Inventory Change									
Exports	6.43	-1.51	5.10	4.12	4.12	4.12	4.86	4.86	4.86
Goods	6.85	-0.86	5.45	4.31	4.31	4.31	5.21	5.21	5.21
Services	3.98	-5.69	2.75	2.69	2.69	2.69	2.66	2.66	2.66
Imports	4.39	-3.56	5.21	2.43	2.72	2.76	3.37	3.70	3.76
Goods	4.40	-3.78	5.31	2.42	2.72	2.77	3.38	3.73	3.79
Services	4.29	-1.56	4.28	2.53	2.71	2.74	3.31	3.42	3.46
Public and Private Social Expenditure	2.17	2.07	1.99	2.00	2.00	2.01	2.07	2.07	2.05
Administration	2.18	2.09	2.00	2.00	2.00	2.01	2.08	2.08	2.05
Education	2.13	2.09	2.00	2.00	2.00	2.01	2.06	2.06	2.05
Health Care	2.20	2.05	1.97	1.98	1.98	2.01	2.07	2.07	2.05
Social Private Expenditures	2.14	2.02	1.94	2.01	2.01	2.01	2.05	2.05	2.05
EMPLOYMENT	0.66	-1.38	1.57	0.79	0.95	0.88	0.75	0.91	0.88
ASSUMPTIONS									
Disposable income (per cap)	2.31	2.65	3.24	2.45	2.45	2.45	2.56	2.56	2.56
Unemployment	10.47	40.33	-15.91	-5.38	-21.86	-14.51	1.90	-6.43	-3.78
Exchange Rate	2.62	1.69	1.64	1.24	1.24	1.24	1.90	1.90	1.90
Foreign Demand	2.87	0.96	2.75	2.69	2.69	2.69	2.66	2.66	2.66

identical in all three simulations. Such a growth rate is compatible with the different unemployment growth rates of the three scenarios and is explained by the fact that the sectoral structure of total output appears to be significantly more important than GDP in determining employment through productivity equations.

The sectoral results show the growth rates for sector output, employment, consumption and investment. The complete set of such tables is of particular interest for policy forecasting since it describes the growth in the sectoral structure of the most relevant economic variables. These results can also be used for defining new scenarios and for verifying the consistency of those already defined. The tables indicate how the dynamics of the macro variables sectoral composition affects both their structure and level. By measuring the time change in the sectoral composition of the relevant economic variables, we are able to evaluate the simultaneous effect of changes in the technological and behavioral structure of the given economic system. This is one of the main issues of present-day policy making.

6. CONCLUSIONS

In recent years economic policy problems have outgrown the instruments designed to support the policy makers's activity. Modern input-output models constitute an attempt to provide schemes for dealing with such problems.

The applications that can be made of the theoretical results obtained go through two main stages: (i) the integration of the input-output side of the model with the demand side so that sectoral demand equations can be consistently specified in the real part of the model, and (ii) the formulation of the price side of the model so that all information on sectoral prices and value added components can be conveniently exploited.

In this paper some characteristics of a real part of a modern input-output model for Italy have been described and some results of the simulations presented. In particular it has been shown how a simple investment theory was used for estimating sectoral investment functions and under which assumptions the input-output technical coefficients were made to change according to forecasted patterns.

A price side is also being developed for the Italian economy so that the relative price vectors shall be simulated simultaneously with the remaining endogeneous variables. This shall improve the effectiveness of the whole scheme; for example, in the case of the price substitution effect on technical coefficients, the availability of a price side is essential for the endogenous determination of the coefficient change.

The theoretical and applicative improvements that can be attained are heavily influenced by the quality and coherence of the statistical data available. An increasing effort is required to the data sources in order that a greater quantity of information on input-output data, as well as sectoral demands be provided in a greater detail and with an higher degree of coherence.

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THE PRICE–INCOME BLOCK OF THE US INFORUM MODEL

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The income side of the US INFORUM model has progressed substantially in the last year. The model now includes as standard output:

- Value-added at the level of 42 industries.
  This value added further subdivided among:
    - labor income
    - capital income
    - indirect business taxes
  the capital income is further divided among
    - corporate profits
    - net interest
    - depreciation
    - proprietor income
    - inventory valuation adjustment
- Prices consistent with this value added
- National income and product accounts which summarize the value added data and, with a few adjustments computed at the macro level, show the connection between GNP, net national product, national income, personal income, disposable income, savings and consumption.
- Government receipts and expenditures, in the same detail as shown in the national accounts. The Federal and the State and Local levels of government are shown separately.
- Interest rates.

Samples of some of the tables now produced are shown in the appendix of this paper.

In this paper, I want, in the first place, to make a few observations on income--side modeling which may prove helpful to others undertaking such work. These remarks will concern both the accounting framework and the structual equations. Secondly, I want to show a few simulations with the present model.

The Accounting Framework

In the USA Inforum model, as in its cousins elsewhere, both the rows and columns of the I-O matrix are defined on the basis of products. A single number for value added is available for each product for the year of the table. There is, however, no time series for this value-added-by-product data, nor is there any subdivision of this value added among types of income such as wages, profits, interest, depreciation, indirect taxes, and so on. There is, however, information, known in the USA as the "GPO data," that gives annual time series for value added in 62 industries, each divided among some 13 types of income. These series are produced by the same office that produces the National Income and Product Accounts (NIPA), and they are consistent with the income side of the NIPA. The 62 industries are defined on the basis of establishments -- not companies, fortunately -- but also not products, alas. The sum of the GPO is not precisely GNP but differs from it by the statistical discrepancy, DN, reported in the NIPA thus:

$$GNP = GPO + DN.$$

These GPO data naturally form the basis of our income modelling. The builders of the I-O matrix, however, did not provide a bridge to the GPO. We have had to make that bridge ourselves. First, we made use of the "mix" or "secondary" matrix included as part of the I-O matrix to allocate products to industries. Then we undid a number of "redefinitions" made by the makers of the I-O table. (Construction activities of industrial establishments, for example, had been "redefined" into the construction industry. We put them back with their establishments.) When all the allocations were done, some GPO industries had received too much, some too little. Suppose, as has recently been the case, that the statistical discrepancy, DN, is negative so that GNP < GPO. Then the total allocations of products will be less than total GPO. We then look at every GPO industry which has had too much allocated to it and move the excess allocations to some similar, plausible industry which is short on allocations. This reallocation is done by judgment, not algorithm. After it is completed, no GPO industry has too much allocated to it, though a number have too little, and the sum of these shortfalls is exactly DN. In this way, in the course of making the product-industry bridge, we find out which industries "make" statistical discrepancy.

In fact, our task was slightly more complicated. In the course of making our 1977 update of the 1972 table, it became apparent that the detailed components of Personal Consumption Expenditure in the NIPA were inconsistent with production, export, and import data on numerous products. (The NIPA statisticians refuse to use these data.) The same was true of Producer durable equipment. Some of the differences are positive, some negative, but they do not cancel out exactly. In fact, they made the statistical discrepancy somewhat worse. We balanced the 1977 table to our final demands, so the sum of the value added by product summed to our GNP, not the official one. The statistical discrepancy allocated as described above was therefore the difference between GPO and our GNP.

Figure 1 shows schematically the accounting framework. All 0's in the figure are 0 by definition. Note the two columns of statistical discrepancy, the one on the right containing the negative of NIPA discrepancy in the discrepancy row. The one beside it, DI, shows the INFORUM discrepancies; the entry in its discrepancy row balances the others to give the entire column a zero total.

In forecasting, the GNP column is forecast in constant prices; the DI column, except for the last element, is derived from the consumption and investment portions of the GNP column, also in constant prices. The last element is calculated to make the columns sum to zero in constant prices. The NIPA discrepancy is specified exogenously in constant prices. Then the outputs are calculated in constant prices. For each GPO industry, these outputs are then weighted together by the fraction of each of them allocated to that industry in the base year. This weighted sum we call real value added weighted output (REVAWO) of the industry. The REVAWO for a GPO industry then becomes one of the major determinants of income in the industry. This total income is then allocated back to products in proportion to each product's contribution to REVAWO of the industry in the year being forecast. From these values added by product, V, current prices, p, are computed by the equation

$$p = pA + v$$

where A is the usual I-O coefficient matrix, and

$$v_i = V_i/q_i$$

the q_i being the previously calculated product outputs in base-year prices. (See below for the complications introduced by imports.) In particular, a current price is derived for statistical discrepancy. The value of the entire DI column at these prices -- remember it sums to to zero in base year prices -- when added to the value of the GNP and -DN columns in current prices gives GPO in current prices. The "statistical discrepancy" in the forecast is therefore DN - DI, both in current prices.

This last assertion makes uses of the cornerstone theorem of input-output modelling with value added and prices. It is simple and obvious, but perhaps needs to be stated. Suppose that we know the vector of final demands, say y, in some jumping-off year, presently 1981. This y is expressed, however, in prices of 1977, the base year of the table. Suppose further that we know the total value-added in each sector, Vi, in 1981 in current prices. Finally suppose that $\sum_i$ Vi = GNP in 1981 prices. If, with some old A matrix, no matter how wrong or out-of-date, we compute outputs, q, by solving $q = Aq + y$ and with these q's compute unit value-added in each industry, $v_i = V_i/q_i$, and

Figure 1

		Total Sales				SD	GNP	DI	-DN
Total Cost			140	100	80	12	167	0	5
	1	140	10	20	10	0	110	-10	0
Product	2	100	20	10	20	0	45	5	0
	3	80	30	30	10	0	12	-2	0
Stat.Dis.		12	0	0	0	0	0	+7	5 = -DN
Value Added			80	40	40	12			

⇓

						REVAWO	Labor	Capital	Tax
GPO industry:	1	70	20	5	7	102	65	45	7
	2	10	20	35	5	70	60	40	5

GNP - 167 GPO = 172

then compute prices by solving

$$p = pA + v$$

(where p and v are row vectors) then when these prices are used to evaluate y, the final demand vector, the total value, py, will be exactly GNP. The proof is very simple:

$$q = Aq + q \Longrightarrow pq = pAq + py$$
$$p = pA + v \Longrightarrow pq = pAq + vq$$
$$\text{so } py = vq = V_. = GNP.$$

It must be emphasized that the proposition in no way depends upon having the correct A matrix, y vector, or v vector. Any disparity found between GNP calculated from the income side and GNP calculated from the product side cannot be attributed to bad data, but only to faulty computation. This fact led to the uncovering of several errors in our programming.

On the other hand, it is important to realize what this proposition does not say. Suppose the y vector is subdivided into a consumption vector y_C, an investment vector y_I, and a government vector y_G. Suppose we know these vectors perfectly and also the current price amounts for these components of GNP, C, I, and G. We are not then guaranteed that

$$py_C = C \text{ or } py_I = I \text{ or } py_G = G,$$

only that $py_C + py_I + py_G = C + I + G$. That is, the current price values calculated by the model for the components of GNP will not, in general, be correct.

Our treatment of this problem is cosmetic: calculate $C - py_C$ for 1981, add this amount to py_C to get C in 1981 and subsequent years. Thus, we take the constant price accounts as fundamental and simply "touch-up" the current accounts to have them fit smoothly with the published accounts of the jumping-off year.

There are, in practice, two complications to this cornerstone proposition. One we have already discussed, the statistical discrepancy. The second is imports. INFORUM-type models usually have a negative vector, -m, of imports for all uses. Conceptually, at least, we can divide the A matrix into a domestic part, D, and an imported part, M. Let us continue to use y to represent final demands for both imported and domestic products in price of the base year of the table; q to represent domestic output in base year prices; p, the index of domestic prices; and f, the index of foreign prices. The p and f indexes are both equal to 1.0 in the base year of the table. Then q satisfies

$$q = Dq + Mq + y - m$$

and p satisfies

$$p = pD + fM + v$$

Pre-multiplying the first of these equations by p and post-multiplying the second by q gives

$$pq = pDq + pMq + py - pm$$
$$pq = pDq + fMq + vq$$

or

$$vq = py + pMq - fMq - pm$$
$$= p(y - (m - Mq)) - fMq$$

In words, this equation reads

GNP on income side = value at domestic prices of final demand for domestic goods less value at foreign prices of imported goods.

For the USA, no M matrix exists; we assume that, across a particular row, imports are the same share of all flows. If we arrange these shares in a diagonal matrix S, we can write

imports in final demand = $(m - Mq) = Sy$

$$vq = p(y - Sy) - f(m - Sy)$$
$$= (p\ (I - S) + fS)y - fm$$

In words, this equation says

GNP from income side = total final demands evaluated at the average of foreign and domestic prices _less_ the value of all imports in foreign prices.

That all seems obvious enough in retrospect, but I must confess that we initially evaluated y in domestic prices and subtracted m evaluated in foreign prices. In fact, this whole discussion seems simple in retrospect, but it cost us much labor to work out the scheme and get it programmed correctly. Truth to tell, I'm not positive it is correct yet.

The Principal Characteristics of the Income Side

Before setting out to estimate a complex economic model, it is important to have fairly clearly in mind what broad relations need to be preserved in the forecasts. Otherwise, it is easy to get lost in the search for individually close fitting equations and come up with a _pot pouri_ of equations which fail to work well together. Here is my personal list of the relations which need to be preservered and some account of how we have gone about assuring that they are.

1. The model must be able to track growth of the labor force. Over long periods, most market economies do a remarkably good job of providing work for those who are qualified for and seeking employment. Yet our model, like most others, determines employment from demand. If the model is to follow the growth of the labor force, any unemployment which develops must somehow stimulate aggregate demand. Now in the first place,unemployment holds down the growth in wage rates, and that is an influence in the wrong direction. What counters the perverse influence? In the first place, personal savings are sensitive to unemployment. Increases in unemployment reduce savings, presumably because the unemployed are drawing down their savings. The reciprocal of the unemployment rate appears in the savings equation, so there is little chance that the model will "overheat" and drive unemployment negative, a frequent occurance before the reciprocal was used in the equation. Secondly, this same reciprocal of unemployment appears in the return to capital equations. Here, an increase in unemployment reduces return to capital and, in particular, profits. This reduction holds down prices and increases purchasing power of wages and other income. Only a small part of profits finds its way into disposable income, so the reduction in prices stimulates demand more than the reduction in dividends after tax curtails it. Finally, increases in unemployment increase unemployment insurance payments. This effect is useful mainly on the down

side; it will not prevent the economy from going into negative unemployment.

I show here only the savings equation, which appears as Figure 2. It includes one- and two- year lags on the reciprocal of the unemployment rate. In addition, the cube of the rate of chance of disposable income per capita has a marked stabilizing effect in the current year. The ratio of transfers to disposable income shows that less savings are done from this source of income. Finally, the ratio of automobile purchases to disposable income is a very strong variable; it shows that consumers do not consider automobile purchases to be primarily consumption, but rather a switch from one kind of asset to another.

2. Money supply, measured by M2, must somehow feed into prices in such a way that the ratio of M2/GNP remains about constant. (This ratio is now about what it has been for the last twenty-five years.) Moreover, relative wages should not be affected in the long-run by inflation. In order to achieve the second characteristic, a single basic equation is estimated for manufacturing, then a second one, for non-manufacturing, is estimated under the constraint that its index can deviate from that of manufacturing only by transient factors. Similarly, wage rates in each industry are then estimated relative to one of these two. And again, only transient factors, such as the rate of change of output, are allowed to effect these ratios.

The basic equation for manufacturing wages has a dependent variable, AW, (for Adjusted Wages) composed as follows:

+ The rate of change of compensation per man hour
- The rate of change of M2 to GNP in constant prices (GNP$)
- The rate of change of aggregate productivity (GNP$/EMPLOYMENT)

averaged over the three preceding years.

Basically AW is the hourly compensation -- including all fringe benefits -- less the excess of money growth over real growth, less productivity growth. AW should differ from zero only by transient factors. The ones we have included are the lagged values of the first difference of the rate of change of M2/GNP$ ($ = in constant prices), the excess of the rate of change of import prices over the rate of change of M2/GNP$, the rate of change of the social security tax rate, and the rate of change of the minimum wage. If no constraints are put on the coefficients of the last two variables, they may well not be transients. Also, if an intercept is allowed in the equation, it is not transient. Consequently, in the equation used, we have imposed the constraints of no intercept and zero sum for the coefficients of changes in the social security tax and minimum wages. Figure 3 shows the equation estimated with these constraints; Figure 4 shows it estimated without the constraints. Of course, the fit without constraints is the closer. In this case, it is considerably better. But if we use it, we will find that a change in the minimum wage causes a change in the M2 velocity of money, contrary to all past experience. Here we have a good example of a case in which we have to give up closeness of fit in the historical period in order to get a forecasting model with plausible macroeconomic properties.

There is not space here to consider in detail the other components of income. Suffice it to say that the manufacturing wage is the main determinant of the absolute level of wages. Return to capital depends on the price index for capital goods, unemployment rates, capacity utilization, imports relative to domestic demand, interest rates, and a few factors peculiar to particular industries.

3. Long-term inflation rates should be passed on to long-term interest rates at least point-for-point, perhaps more, because of taxes. Because of this concern, long-term interest rates are primary in this model. They are somewhat influenced also by monetary conditions. Short-term rates are determined relative to long-term rates, with monetary conditions playing a major role.

* THE SAVINGS RATE

NV = 6 SEE = 0.4650 RSQR = 0.8439 RBARSQR = 0.7815
RHO =-0.056 DW = 2.111 AAPE = 5.18

VARIABLE	REGRES-COEF	T-VALUE	ELASTICITY	MEXPLAVAL	MEAN
INTERCEPT	4.828875	4.63	0.718	53.05	1.0000
1/UNEMP(T-1)	1.943586	1.15	0.092	4.03	0.3198
1/UNEMP(T-2)	1.921692	1.07	0.091	3.53	0.3183
(PC INCOME)**3	0.011733	5.19	0.060	63.72	34.6883
TRANSFERS/DY	-11.592780	-2.61	-0.209	19.47	0.1212
AUTOS/DY	-81.174058	-8.02	-0.426	123.96	0.0353
SAVRAT(T-1)	0.669327	5.50	0.674	69.98	6.7715
SAVRAT(T-1)	0.669327	5.50	0.674	69.98	6.7715
SAVRAT	DEPENDENT VARIABLE - - - - - -				6.72921

DATE	ACTUAL	PREDIC	MISS
	IS *	IS +	IS A-P
60	5.60	5.62	-0.03
61	6.30	5.77	0.53
62	6.03	5.90	0.12
63	5.41	5.30	0.11
64	6.72	7.12	-0.39
65	7.09	7.11	-0.02
66	7.01	7.12	-0.10
67	8.10	7.31	0.79
68	7.08	7.87	-0.79
69	6.36	7.30	-0.94
70	8.04	7.66	0.38
71	8.08	7.69	0.39
72	6.50	7.08	-0.57
73	8.64	8.12	0.53
74	8.54	8.11	0.43
75	8.61	7.91	0.70
76	6.91	7.32	-0.40
77	5.65	5.82	-0.17
78	5.22	5.46	-0.24
79	5.25	5.24	0.01
80	5.56	5.58	-0.01
81	5.33	5.65	-0.32
	IS *	IS +	IS A-P

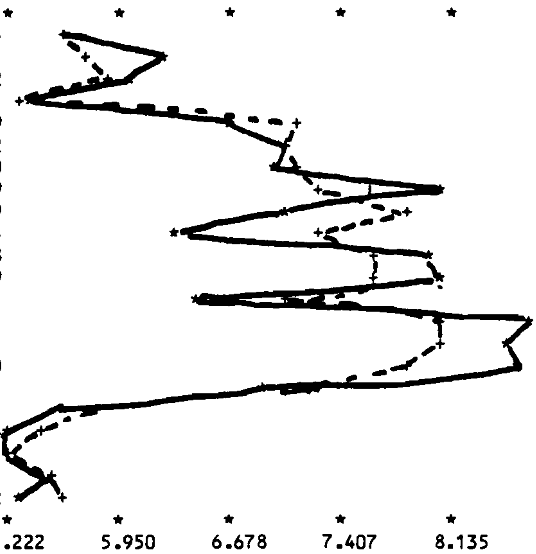


FIGURE 2

* ANNUAL MONEY WAGE EQUATION BASED ON THE RATIO OF M2 TO REAL GNP

** CONSTRAINED TO HAVE ONLY TRANSIENT EFFECTS **

NV = 10 SEE = 1.6090 RSQR = 0.6758 RBARSQR = 0.3515

RHO = 0.536 DW = 0.928 AAPE = 83.76

VARIABLE	REGRES-COEF	T-VALUE	ELASTICITY	MEXPLAVAL	MEAN
FDPCMG(T-1)	-0.531292	-1.81	-0.168	12.89	0.1796
FDPCMG(T-2)	-0.293239	-1.12	-0.130	5.09	0.2524
FDPCMG(T-3)	-0.187828	-0.73	-0.127	2.23	0.3827
PCSOCR	0.056801	0.76	0.454	2.40	4.5455
PCSOCR(T-1)	0.028646	0.41	0.256	0.69	5.0746
PCSOCR(T-2)	-0.083733	-1.18	-0.739	5.67	5.0117
PCPIM	0.093172	1.33	0.445	7.14	2.7137
PCPIM(T-1)	0.113217	1.54	0.391	9.45	1.9606
PCMIN(T-1)	0.099997	1.44	1.280	8.34	7.2737
PCMIN(T-2)	-0.021352	-0.35	-0.256	0.50	6.8001
PCMIN(T-3)	-0.065716	-0.99	-0.698	4.02	6.0377
ADJ WAGES MFG	DEPENDENT VARIABLE - - - - - -				0.56820

DATE	ACTUAL IS *	PREDIC IS +	MISS IS A-P *
60	4.00	2.70	1.30
61	-0.60	-1.58	0.98
62	-2.37	-2.51	0.14
63	-1.34	-0.28	-1.06
64	-2.94	-1.24	-1.71
65	-3.55	-1.17	-2.37
66	-0.33	0.59	-0.92
67	1.72	1.85	-0.13
68	1.19	0.04	1.15
69	2.10	0.12	1.98
70	2.60	-1.36	3.96
71	1.59	-0.47	2.06
72	-3.89	-2.26	-1.63
73	-1.07	-0.15	-0.92
74	4.29	5.67	-1.38
75	4.44	4.45	-0.00
76	-2.76	-2.29	-0.47
77	0.67	-0.77	1.44
78	-0.77	0.41	-1.18
79	4.34	4.15	0.19
80	4.62	2.56	2.06
	IS *	IS +	IS A-P *

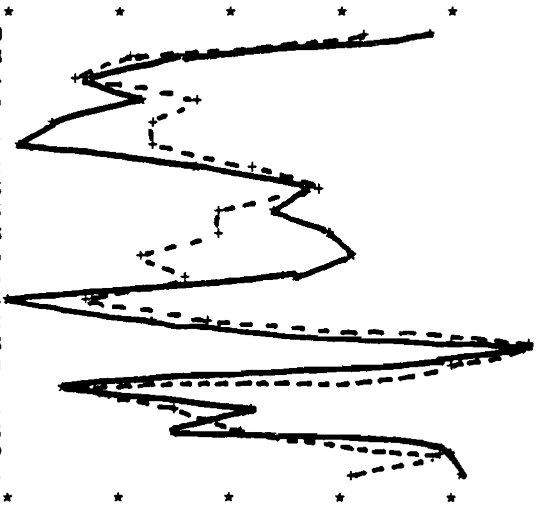


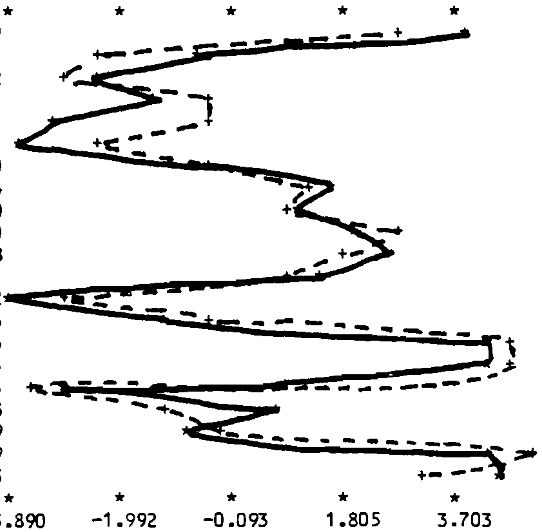
Figure 3

* ANNUAL MONEY WAGE EQUATION BASED ON THE RATIO OF M2 TO REAL GNP

NV = 11 SEE = 1.0573 RSQR = 0.8600 RBARSQR = 0.6888
RHO = 0.394 DW = 1.212 AAPE = 54.81

VARIABLE	REGRES-COEF	T-VALUE	ELASTICITY	MEXPLAVAL	MEAN
INTERCEPT	-3.741214	-2.67	-6.584	33.86	1.0000
FDPCMG(T-1)	-0.654381	-2.92	-0.207	39.52	0.1796
FDPCMG(T-2)	-0.344038	-1.65	-0.153	14.09	0.2524
FDPCMG(T-3)	-0.300434	-1.49	-0.202	11.70	0.3827
PCSOCR	0.167495	2.19	1.340	23.85	4.5455
PCSOCR(T-1)	0.164729	2.14	1.471	22.79	5.0746
PCSOCR(T-2)	-0.011015	-0.15	-0.097	0.13	5.0117
PCPIM	0.090370	1.67	0.432	14.44	2.7137
PCPIM(T-1)	0.155562	2.69	0.537	34.21	1.9606
PCMIN(T-1)	0.206602	3.27	2.645	48.01	7.2737
PCMIN(T-2)	0.103605	1.74	1.240	15.67	6.8001
PCMIN(T-3)	0.054513	0.90	0.579	4.42	6.0377
ADJ WAGES MFG	DEPENDENT VARIABLE ------				0.56820

DATE	ACTUAL IS *	PREDIC IS +	MISS IS A-P *
60	4.00	2.91	1.09
61	-0.60	-2.21	1.62
62	-2.37	-2.89	0.52
63	-1.34	-0.47	-0.87
64	-2.94	-0.33	-2.61
65	-3.55	-2.33	-1.22
66	-0.33	-0.33	-0.00
67	1.72	1.28	0.44
68	1.19	0.90	0.30
69	2.10	2.80	-0.70
70	2.60	1.92	0.68
71	1.59	0.98	0.61
72	-3.89	-2.87	-1.02
73	-1.07	-0.41	-0.66
74	4.29	4.65	-0.36
75	4.44	4.77	-0.33
76	-2.76	-3.39	0.63
77	0.67	-1.06	1.73
78	-0.77	-0.28	-0.49
79	4.34	5.03	-0.69
80	4.62	3.27	1.35
	IS *	IS +	IS A-P *



Simulations of Tax Cuts, Increased Defense, and Increased Transfer Payments

The tables at the end of this paper show 8 of the several hundred pages of printing produced by the model. These 8 have been selected to emphasize the macro aspects of the model, especially those concerning income, for they are the recent additions. Page S-3 shows the GNP accounts in constant 1972 dollars. The addenda shows a number of other macro variables, such as the unemployment rate, the index of hourly labor compensation, M2, the AAA corporate bond rate. Page S-7 shows a number deflators, indexes of hourly compensation, energy prices, and financial variables. Page S-9 gives the relation between GNP and personal income; Page S-11 shows the composition of personal income and its relation to disposable income. Page S-13 shows the receipts and expenditures of the federal government; Page S-15 shows the same for state and local governments.

Up to this point all of the tables could have come from any well-developed macro model. The point here, however, is that they are, in fact, an integral part of a comprehensive input-output model. Wage and salary disbursements on page S-11, for example, do not come from a single macro equation, but from compensation of employees in 46 different industries. These tables are summaries of the model, not controls on it. The fact that we are dealing with an input-output model appears plainly, on pages S-31 and S-32 where we see the industry outputs.

The numbers in these tables show the economy in 1982 and then 1985 and 1990 under four different assumptions. The "Base" case was our best judgment in June of 1982. At that time we expected that the 1983 income tax cut, though already enacted, would not go into effect. It now appears that it will, but that there will be offsetting increases in other taxes. We have not yet put the new bill into the model.

Against this base forecast, we ran three alternative assumptions.
REBATE -- a five percent reduction in the federal income tax rate
INCDEF -- an increase in defense spending of $20 - $25 billion per year (current prices) beginning in 1984.
INCTRP -- an increase in transfer payments by $20 billion per year per year after 1984.

All three alternatives have approximately the same initial cost to the government. And all three have about the same impact on unemployment in 1985 and 1990; in the latter year the alternatives lower unemployment from about 4.1 percent in the base to 3.8 in the alternatives. Likewise, the real GNP is very similar in 1972 $. The multiplier is clearest for the increased defense. In 1990, the increase in defense spending between BASE and INCDEF is 8.3 billion 1972 dollars. The GNP$ increased by only 7.0 billion, so the multiplier is less than 1, though still considerably above 0, which must certainly be its ultimate, long-run value. The tax cut and the increase in transfers have almost exactly the same effect on real GNP and employment, but not on the budget deficit. The tax cut is costing $7 billion more in 1990 than is the increase in transfers. The increase in defense is between these two in its impact on the deficit. Labor compensation is increased $ 38 billion, or 1 percent, more by increasing defense than by the other two alternatives. Consumer prices, however, were also 1 percent higher in this alternative, so there was no increase in purchasing power. Personal consumption expenditures are essentially the same for INCDEF as for the base case. Output of durable goods were one percent higher under the increased defense assumption than under the other two; in other areas, outputs are virtually identical.

These results are frankly somewhat puzzling to me. They seem to say that over a period as long as eight years and in an economy as tight as four percent unemployment, we can still have more of both guns and butter, or by cutting taxes and running deficits we can still increase real output. I am aware that many do not find this result surprising; it is not unusual for the macro models to show long-run multipliers as high as two or three. Our results are more sensible than that, but I still think we need a careful review of the elements in the model that put on the brakes as it nears full employment.

Directions of Work on the US Model

We are working on the US model in several directions. First, from the programming side, we are redesigning the way it takes its starting information so that it can start in any year for which we have either historical data or a previous forecast. This capability would be useful both for historical simulations and tests and also for running alternatives that do not differ from one another until some future year. At present, the model always begins in 1977, the base year of the table, although much actual data is read in up through 1981. This ability is common in macro model simulation programs, such as our LS package. The core of the Inforum model's logic dates back prior to disk hardware, and this shortcoming is a remnant of those days. It is high time we got rid of it.

Secondly, we are converting the 200-sector model to have all the same income side as does the 78-sector model discussed here. We are also working on the calculation of outputs for 400 products via a "skirt" on the 200 sector model. Thirdly, a project on the role of monetary and financial variables in the model is underway. And finally, we are improving the federal government sector to make it easier to put in changes in tax policies that change tax rates differently at different levels of income. Needless to say, we are also very much concerned with developing new specific applications for individual clients.

Page S-3. Gross national product (1972$).

	(BASE) 1982	(BASE) 1985	(REBATE) 1985	(INCDEF) 1985	(INCTRP) 1985	(BASE) 1990	(REBATE) 1990	(INCDEF) 1990	(INCTRP) 1990
Gross National Product	1558.25	1685.56	1696.32	1695.42	1696.74	1909.25	1914.67	1916.27	1915.44
Personal Consumption Expenditures	991.95	1065.90	1074.48	1068.26	1074.87	1207.13	1213.40	1208.98	1213.87
Durables	148.15	154.76	157.35	155.23	157.36	178.30	180.02	178.83	180.16
Non-durables	372.13	391.55	393.99	392.21	394.18	426.67	428.54	427.31	428.78
Services	471.67	519.59	523.14	520.81	523.33	602.15	604.83	602.84	604.92
Gross Private Domestic Investment	216.27	237.63	240.74	238.60	240.75	265.86	264.66	265.84	264.98
Structures	95.57	101.44	102.07	99.68	101.91	108.43	107.50	107.37	107.55
Residential	47.93	51.84	52.10	50.28	51.93	54.14	53.73	53.55	53.73
Non-residential	47.64	49.61	49.97	49.40	49.98	54.28	53.77	53.82	53.82
Producers' durable equipment	113.18	128.47	130.58	131.21	130.74	147.16	147.20	148.06	147.43
Inventory change	7.52	7.71	8.09	7.72	8.10	10.27	9.96	10.41	10.00
Exports of goods & services	167.84	186.30	186.86	187.06	186.88	227.21	227.90	226.21	227.96
Merchandise (producers' prices)	80.56	89.01	89.06	88.87	89.08	109.21	109.49	108.45	109.51
Transportation, trade, services	35.13	38.85	38.87	38.81	38.87	46.24	46.32	46.03	46.33
Rest of world	52.15	58.44	58.92	59.38	58.94	71.77	72.10	71.73	72.13
Imports of goods & services	108.03	115.71	117.19	118.24	117.20	133.77	134.11	135.87	134.20
Merchandise(domestic port price)	71.46	76.78	77.68	77.80	77.70	89.61	89.81	90.59	89.88
Petroleum & natural gas	7.73	8.11	8.14	8.19	8.14	8.66	8.68	8.74	8.68
Transportation, trade, services	19.44	20.68	20.84	21.35	20.81	23.63	23.66	24.27	23.66
Rest of world	17.13	18.25	18.68	19.09	18.68	20.53	20.64	21.01	20.66
Government Purchases	290.22	311.44	311.44	319.74	311.44	342.82	342.83	351.12	342.83
Federal	108.77	120.55	120.55	128.85	120.55	138.33	138.33	146.62	138.33
Defense	77.32	89.94	89.94	98.23	89.94	101.35	101.35	109.64	101.35
Compensation of employees	32.54	34.06	34.06	34.06	34.06	34.26	34.26	34.26	34.26
Structures	1.76	2.10	2.09	2.10	2.09	2.42	2.42	2.42	2.42
Other	43.03	53.79	53.79	62.08	53.79	64.66	64.66	72.96	64.66
Non-defense	31.45	30.61	30.61	30.61	30.61	36.98	36.98	36.98	36.98
Compensation of employees	14.37	14.44	14.44	14.44	14.44	16.30	16.30	16.30	16.30
Structures	3.94	4.05	4.05	4.05	4.05	4.55	4.54	4.54	4.54
Other	13.14	12.11	12.12	12.12	12.12	16.13	16.14	16.14	16.14
State and local	181.45	190.89	190.89	190.89	190.89	204.49	204.50	204.50	204.50
Education	73.59	74.83	74.83	74.83	74.83	79.48	79.49	79.49	79.49
Compensation of employees	56.35	57.48	57.48	57.48	57.48	59.36	59.36	59.36	59.36
Structures	6.89	7.30	7.29	7.29	7.29	8.99	8.98	8.98	8.98
Other	10.34	10.05	10.05	10.06	10.05	11.13	11.14	11.15	11.14
Other	107.86	116.06	116.06	116.06	116.06	125.01	125.01	125.02	125.01
Compensation of employees	47.78	48.91	48.91	48.91	48.91	50.79	50.79	50.79	50.79
Structures	19.48	20.35	20.33	20.22	20.34	21.26	21.11	20.99	21.12
Other	40.59	46.79	46.82	46.93	46.81	52.96	53.11	53.23	53.10
Addenda:									
Unemployment rate	7.36	6.32	5.76	5.82	5.74	4.11	3.81	3.74	3.79
GNP / Civilian jobs	20.60	21.15	21.16	21.16	21.16	21.96	21.96	21.96	21.96
(GNP-Govt) / Private jobs	18.87	19.36	19.38	19.26	19.39	20.14	20.14	20.03	20.15
PCE deflator	2.03	2.48	2.47	2.49	2.47	3.37	3.36	3.39	3.36
Index, unit compensation, mfg	153.07	192.28	191.99	194.10	191.83	263.97	262.80	265.58	262.74
Index, unit compensation, oth	154.94	195.17	195.10	196.57	194.98	272.58	271.68	274.44	271.58
M2 (billions of CUR$)	1905.61	2422.50	2422.50	2422.50	2422.50	3613.95	3613.95	3613.95	3613.95
Disp. income per capita (1972$)	4692.82	4989.58	5053.67	5025.10	5047.45	5413.82	5475.35	5446.60	5468.19
Savings rate	6.63	8.39	8.84	8.83	8.70	8.66	9.24	9.08	9.08
AAA Corporate bond rate	12.47	11.91	12.13	12.44	12.17	12.37	12.53	12.68	12.53

Page S-7. Price indexes and financial variables.

	(BASE) 1982	(BASE) 1985	(REBATE) 1985	(INCDEF) 1985	(INCTRP) 1985	(BASE) 1990	(REBATE) 1990	(INCDEF) 1990	(INCTRP) 1990
IMPLICIT DEFLATORS									
Gross National Product	2.05	2.51	2.51	2.52	2.51	3.41	3.41	3.43	3.41
Personal consumption expenditures	2.03	2.48	2.47	2.49	2.47	3.37	3.36	3.39	3.36
Residential structures	2.36	2.94	2.94	2.96	2.94	4.13	4.12	4.15	4.12
Non-residential structures	2.49	3.15	3.15	3.17	3.15	4.51	4.50	4.53	4.50
Producers' durable equipment	2.05	2.48	2.48	2.50	2.48	3.30	3.29	3.32	3.29
Exports, merchandise	2.55	3.07	3.07	3.09	3.07	4.07	4.07	4.10	4.06
Imports, merchandise	3.41	4.15	4.15	4.15	4.15	5.50	5.50	5.50	5.50
Federal defense	2.14	2.58	2.58	2.59	2.58	3.43	3.43	3.45	3.42
Federal non-defense	2.12	2.64	2.63	2.65	2.64	3.65	3.64	3.67	3.64
State & local education	2.25	2.81	2.81	2.83	2.82	4.04	4.02	4.06	4.02
State & local other govt	2.24	2.81	2.81	2.83	2.81	3.99	3.97	4.00	3.97
COMPENSATION PER MAN-HOUR INDEXES									
Manufacturing	153.07	192.28	191.99	194.10	191.83	263.97	262.80	265.58	262.74
Non-manufacturing	154.94	195.17	195.10	196.57	194.98	272.58	271.68	274.44	271.58
LABOR PRODUCTIVITY (GNP/JOBS)	20.60	21.15	21.16	21.16	21.16	21.96	21.96	21.96	21.96
ENERGY PRICE INDEXES									
Domestic crude oil ($/bbl)	25.00	29.50	29.50	29.50	29.50	40.00	40.00	40.00	40.00
Foreign crude oil ($/bbl)	25.01	29.51	29.51	29.51	29.51	40.01	40.01	40.01	40.01
FINANCIAL VARIABLES									
AAA Corporate bond rate	12.47	11.91	12.13	12.44	12.17	12.37	12.53	12.68	12.53
Commercial paper rate	11.87	12.29	12.66	13.15	12.67	12.78	12.88	13.20	12.89
Mortgage rate	12.84	12.31	12.52	12.80	12.55	12.74	12.89	13.03	12.89
Interest rate on Federal debt	14.43	13.59	13.84	14.21	13.89	14.14	14.35	14.51	14.35
Average rate paid by S&L govt	8.50	8.12	8.24	8.39	8.24	8.40	8.56	8.66	8.55
Average rate received by S&L govt	10.33	9.95	10.08	10.28	10.11	10.23	10.33	10.42	10.33
Real rate of interest (ex ante)	2.57	2.57	2.57	2.57	2.57	2.57	2.57	2.57	2.57
M2 (billions of current$)	1905.61	2422.50	2422.50	2422.50	2422.50	3613.95	3613.95	3613.95	3613.95
Ratio of M2 to real GNP	1.23	1.44	1.43	1.43	1.43	1.89	1.89	1.89	1.89
Ratio of M2 to nominal GNP	0.60	0.57	0.57	0.57	0.57	0.55	0.55	0.55	0.55
Savings rate	6.63	8.39	8.84	8.83	8.70	8.66	9.24	9.08	9.08

Page S-9. GNP, NNP, national income, personal income.

	(BASE) 1982	(BASE) 1985	(REBATE) 1985	(INCDEF) 1985	(INCTRP) 1985	(BASE) 1990	(REBATE) 1990	(INCDEF) 1990	(INCTRP) 1990
Gross National Product	3193.85	4236.27	4258.01	4283.26	4257.49	6526.03	6530.76	6587.78	6531.06
-: Capital consumption allowances with capital consumption adj.	323.02	424.41	426.07	427.86	426.36	658.19	660.18	662.03	660.31
=: Net National Product	2888.83	3835.29	3853.91	3878.20	3853.63	5893.13	5897.91	5951.45	5898.10
-: Indirect business tax and nontax liability	241.91	314.77	316.42	317.63	316.50	477.26	478.84	481.42	478.92
Business transfer payments	12.23	14.96	14.71	14.79	14.70	19.24	19.35	19.43	19.36
Statistical discrepancy									
+: Subsidies less current surplus of govt enterprises	5.33	6.78	6.81	6.83	6.82	10.03	10.05	10.07	10.06
=: National Income	2640.28	3512.65	3529.89	3552.91	3529.55	5407.10	5410.22	5461.12	5410.32
-: Corporate profits with IVA and capital consumption adj.	281.57	359.76	364.50	360.60	364.88	573.90	576.15	579.36	576.48
Net interest	219.50	318.06	317.80	322.88	317.54	503.18	503.64	508.12	503.77
Contributions for social insur.	263.62	357.02	358.51	361.34	358.46	571.73	571.49	577.29	571.46
Wage accruals less disbursements	-0.18	-0.18	-0.18	-0.18	-0.18	-0.18	-0.18	-0.18	-0.18
+: Govt transfer payments to person	351.84	471.84	469.90	472.90	492.74	730.02	727.62	732.89	758.80
Personal interest income	352.58	486.40	490.77	498.53	490.22	756.33	772.51	780.40	770.23
Personal dividend income	71.68	104.14	104.85	104.67	104.88	178.97	180.41	180.00	180.46
Business transfer payments	12.23	14.96	14.71	14.79	14.70	19.24	19.35	19.43	19.36
Error	-42.00	-30.60	-30.60	-30.60	-30.60	0.00	0.00	0.00	0.00
=: Personal Income	2624.07	3527.42	3541.62	3571.28	3563.51	5447.41	5463.39	5513.63	5492.03

Page S-11. Personal income - sources and disposition.

	(BASE) 1982	(BASE) 1985	(REBATE) 1985	(INCDEF) 1985	(INCTRP) 1985	(BASE) 1990	(REBATE) 1990	(INCDEF) 1990	(INCTRP) 1990
Personal Income	2624.07	3527.42	3541.62	3571.28	3563.51	5447.41	5463.39	5513.63	5492.03
Wage and salary disbursements	1629.92	2158.48	2167.22	2183.84	2166.91	3269.14	3267.86	3299.81	3267.68
Other labor income	159.45	212.11	213.09	214.97	213.06	323.75	323.61	327.24	323.59
Proprietors' income w. IVA&CCADJ	155.66	214.15	215.61	217.31	215.51	350.96	352.00	355.61	351.81
Farm	24.17	25.91	26.32	26.31	26.31	39.95	40.29	40.92	40.22
Nonfarm	131.48	188.25	189.29	191.00	189.20	311.01	311.71	314.69	311.59
Rental income of persons w. CCADJ	31.74	31.03	31.83	31.93	31.81	36.00	36.92	37.78	36.96
Dividends	71.68	104.14	104.85	104.67	104.88	178.97	180.41	180.00	180.46
Personal interest income	352.58	486.40	490.77	498.53	490.22	756.33	772.51	780.40	770.23
Transfer payments	364.07	486.80	484.61	487.69	507.44	749.27	746.97	752.32	778.16
Federal	299.65	400.03	398.17	400.72	421.04	616.44	614.21	618.62	645.42
State and local	52.19	71.81	71.73	72.18	71.70	113.59	113.41	114.27	113.37
Business transfer payments	12.23	14.96	14.71	14.79	14.70	19.24	19.35	19.43	19.36
:-Pers contrib to social insurance	101.12	137.92	138.60	139.90	138.58	221.49	221.38	224.02	221.36
Error	-42.00	-30.60	-30.60	-30.60	-30.60	0.00	0.00	0.00	0.00
-:Personal tax and nontax payments	406.06	562.11	541.38	569.38	568.24	879.72	850.87	890.62	887.19
Federal income taxes	295.99	408.29	386.85	413.45	412.58	632.83	603.19	640.58	638.09
=: Disposable Income	2218.01	2965.31	3000.24	3001.90	2995.27	4567.69	4612.52	4623.00	4604.83
-: Personal Outlays									
Consumption expenditures	2014.37	2648.64	2665.32	2667.49	2665.66	4070.69	4087.09	4101.76	4087.24
Interest paid by consumers to businesses	54.56	68.79	69.07	69.63	68.99	102.45	102.10	103.00	102.12
Personal transfer payments to foreigners (net)	1.69	2.42	2.42	2.42	2.42	3.65	3.65	3.65	3.65
=: Personal Savings	147.03	248.68	265.12	264.86	260.37	395.25	425.88	419.32	418.00
ADDENDA:									
Disposable Income (1972$), Total	1088.03	1189.51	1204.79	1197.98	1203.31	1348.58	1363.91	1356.75	1362.13
Per capita	4692.82	4989.58	5053.67	5025.10	5047.45	5413.82	5475.35	5446.60	5468.19
Population (mid-period, millions)	231.85	238.40	238.40	238.40	238.40	249.10	249.10	249.10	249.10
Personal savings as % of disposable personal income (less interest paid to business and transfer payments to foreigners)	6.63	8.39	8.84	8.83	8.70	8.66	9.24	9.08	9.08
Total taxes / Personal income	15.47	15.94	15.29	15.94	15.95	16.15	15.57	16.15	16.15
Federal Deficit, NIPA	-83.33	-74.57	-96.15	-96.08	-93.26	-52.23	-97.23	-92.01	-90.72

Page S-13. Federal government receipts and expenditures.

	(BASE) 1982	(BASE) 1985	(REBATE) 1985	(INCDEF) 1985	(INCTRP) 1985	(BASE) 1990	(REBATE) 1990	(INCDEF) 1990	(INCTRP) 1990
RECEIPTS	641.91	875.60	857.48	886.86	883.12	1377.34	1349.21	1393.53	1384.23
Personal tax and non-tax receipts	303.79	418.53	397.12	423.80	422.90	648.32	618.72	656.25	653.69
Corporate profits tax	85.45	114.96	116.69	116.68	116.58	179.52	180.95	182.15	181.02
Indirect business tax and nontax accruals	36.65	47.69	47.94	48.12	47.95	72.30	72.54	72.94	72.56
Contributions for social insurance	216.02	294.42	295.74	298.26	295.70	477.20	476.99	482.20	476.96
EXPENDITURES	725.24	950.17	953.63	982.95	976.38	1429.57	1446.44	1485.54	1474.96
Purchases of Goods and Services	232.94	319.16	319.06	343.08	319.08	504.58	503.77	536.11	503.67
National defense	165.34	236.44	236.34	260.03	236.34	363.80	363.21	394.93	363.14
Compensation of employees	63.98	83.95	83.95	83.95	83.95	120.04	120.04	120.04	120.04
Other	101.35	152.48	152.38	176.08	152.39	243.76	243.17	274.89	243.11
Nondefense	67.60	82.73	82.72	83.05	82.74	140.78	140.56	141.18	140.53
Compensation of employees	30.93	39.50	39.50	39.50	39.50	64.32	64.32	64.32	64.32
Other	36.67	43.23	43.22	43.55	43.24	76.46	76.24	76.86	76.21
Transfer Payments	306.73	408.69	406.81	409.42	429.68	628.20	625.95	630.45	657.16
To persons	299.65	400.03	398.17	400.72	421.04	616.44	614.21	618.62	645.42
Old age benefits	142.02	182.78	182.60	183.71	194.78	269.35	268.94	270.95	285.55
Hospital & medical	45.56	68.30	68.23	68.66	72.13	124.11	123.92	124.86	129.23
Unemployment	18.29	21.17	19.69	19.91	19.61	20.02	18.70	18.63	18.61
Retirement: Fed civ & RR	24.99	33.07	33.04	33.24	35.18	51.46	51.38	51.76	54.29
Vet life insur, workmen comp.	2.11	2.58	2.58	2.60	2.58	3.51	3.50	3.53	3.50
Military retirement	15.46	21.02	21.00	21.13	22.31	33.59	33.54	33.79	35.32
Veterans benefits	21.24	29.70	29.67	29.86	31.39	46.24	46.16	46.51	48.51
Food stamps	9.14	12.60	12.59	12.67	12.58	20.81	20.78	20.94	20.77
Other	20.84	28.80	28.77	28.95	30.49	47.36	47.28	47.64	49.63
To foreigners	7.08	8.65	8.65	8.70	8.64	11.76	11.74	11.83	11.74
Grants-in-Aid to S&L Govt	95.11	119.44	119.44	119.44	119.44	160.00	160.00	160.00	160.00
Net Interest Paid	80.42	96.60	102.13	104.86	102.00	138.95	158.94	161.26	156.36
Interest paid	96.84	113.01	118.54	121.27	118.41	155.96	175.95	178.27	173.37
Interest received	16.41	16.41	16.41	16.41	16.41	17.01	17.01	17.01	17.01
Subsidies less Current Surplus of Govt Enterprises	10.08	6.32	6.24	6.19	6.22	-2.11	-2.17	-2.23	-2.19
Surplus or Deficit (-), NIPA	-83.33	-74.57	-96.15	-96.08	-93.26	-52.23	-97.23	-92.01	-90.72
Social insurance funds	-13.70	-16.17	-12.58	-11.77	-34.38	-6.62	-2.45	-0.75	-44.07
Other funds	-69.63	-58.41	-83.57	-84.32	-58.88	-45.61	-94.78	-91.26	-46.66
Debt of Federal Government	1099.10	1321.64	1379.83	1375.28	1371.31	1708.62	1938.05	1936.33	1904.78
Debt from Federal loans	310.00	310.00	310.00	310.00	310.00	320.00	320.00	320.00	320.00

Page S-15. State and local government receipts and expenditures.

	(BASE) 1982	(BASE) 1985	(REBATE) 1985	(INCDEF) 1985	(INCTRP) 1985	(BASE) 1990	(REBATE) 1990	(INCDEF) 1990	(INCTRP) 1990
RECEIPTS	466.31	614.46	617.00	619.70	618.13	924.46	926.73	931.97	928.17
Personal tax and nontax receipts	102.27	143.58	144.27	145.58	145.34	231.40	232.15	234.37	233.50
Corporate profits tax	16.08	21.75	22.05	22.09	22.04	33.58	33.79	34.02	33.81
Indirect business tax and nontax accruals	205.26	267.08	268.48	269.51	268.55	404.95	406.29	408.49	406.37
Contributions for social insurance	47.60	62.61	62.77	63.07	62.76	94.53	94.50	95.09	94.50
Federal grants-in-aid	95.11	119.44	119.44	119.44	119.44	160.00	160.00	160.00	160.00
EXPENDITURES	441.39	603.75	603.64	605.76	603.67	953.02	951.42	955.93	950.91
Purchases of goods and services	394.31	524.00	523.98	525.69	524.13	803.63	802.30	805.25	802.14
Compensation of employees	217.93	282.38	282.38	282.38	282.38	420.31	420.31	420.31	420.31
Other	176.38	241.61	241.59	243.31	241.75	383.32	381.98	384.93	381.82
Transfer payments to persons	52.19	71.81	71.73	72.18	71.70	113.59	113.41	114.27	113.37
From social insurance funds	21.80	29.32	29.29	29.47	29.27	46.13	46.06	46.41	46.05
Direct relief	23.29	32.74	32.71	32.92	32.69	51.69	51.61	52.00	51.59
Other	7.10	9.75	9.74	9.80	9.73	15.77	15.74	15.86	15.74
Net interest paid	-0.16	7.72	7.60	7.49	7.48	23.99	23.81	24.43	23.48
Interest paid	26.49	31.08	31.58	32.20	31.58	41.02	41.84	42.41	41.82
Interest received	26.66	23.36	23.98	24.71	24.10	17.03	18.03	17.97	18.34
Subsidies less current surplus of govt enterprises	-4.75	0.47	0.57	0.64	0.60	12.14	12.22	12.30	12.25
Surplus or deficit (-)	24.93	10.71	13.37	13.95	14.46	-28.56	-24.69	-23.96	-22.74
Social insurance funds	51.20	77.93	78.57	79.23	78.53	150.53	153.11	155.44	153.06
Other funds	-26.27	-67.22	-65.20	-65.29	-64.06	-179.09	-177.80	-179.40	-175.80
Debt of S&L government	377.14	450.00	450.00	450.00	450.00	550.00	550.00	550.00	550.00

Page S-31. Output by producing sector (1977$).

		(BASE) 1982	(BASE) 1985	(REBATE) 1985	(INCDEF) 1985	(INCTRP) 1985	(BASE) 1990	(REBATE) 1990	(INCDEF) 1990	(INCTRP) 1990
1	AGRICULTURE, FORESTRY, FISHERY	144.03	146.86	147.38	146.91	147.42	157.01	157.36	156.98	157.44
	MINING	89.92	96.15	96.70	97.07	96.75	105.51	105.75	106.25	105.79
2	IRON ORE MINING	3.21	3.29	3.31	3.31	3.31	3.56	3.57	3.57	3.57
3	NONFERROUS METALS MINING	4.43	4.89	4.94	4.96	4.94	5.42	5.42	5.44	5.42
4	COAL MINING	20.53	22.70	22.82	22.85	22.84	26.67	26.74	26.79	26.75
5	NATURAL GAS EXTRACTION	19.54	20.36	20.50	20.56	20.51	21.25	21.30	21.39	21.31
6	CRUDE PETROLEUM	34.59	36.62	36.80	37.10	36.82	39.27	39.36	39.71	39.38
7	NON-METALLIC MINING	7.63	8.30	8.33	8.30	8.33	9.35	9.36	9.36	9.36
8	CONSTRUCTION	102.63	107.99	108.50	107.09	108.41	116.87	116.32	116.18	116.37
	NON-DURABLES	730.93	776.84	781.54	780.91	781.83	862.30	865.18	865.48	865.62
9	FOOD & TOBACCO	223.69	231.52	232.46	231.81	232.57	246.92	247.65	247.03	247.83
10	TEXTILES, EXC. KNITS	38.87	41.48	41.95	41.64	41.96	46.80	47.05	46.88	47.07
11	KNITTING	11.87	12.88	13.02	12.94	13.03	15.05	15.12	15.08	15.13
12	APPAREL, HOUSEHOLD TEXTILES	49.19	53.18	53.75	53.43	53.77	61.39	61.70	61.59	61.72
13	PAPER	56.38	60.27	60.69	60.52	60.70	68.28	68.55	68.42	68.59
14	PRINTING & PUBLISHING	53.75	57.34	57.73	57.78	57.76	63.51	63.78	63.91	63.80
15	AGRICULTURAL FERTILIZERS	14.47	15.21	15.27	15.22	15.28	17.13	17.19	17.11	17.20
16	OTHER CHEMICALS	125.48	137.05	137.84	137.91	137.87	159.19	159.64	159.86	159.71
17	PETROLEUM REFINING	103.25	109.43	109.97	110.88	110.02	117.27	117.55	118.60	117.60
	18 FUEL OIL	23.88	24.75	24.89	25.13	24.90	25.14	25.18	25.46	25.19
19	RUBBER PRODUCTS	17.86	19.23	19.36	19.32	19.38	22.06	22.13	22.15	22.14
20	PLASTIC PRODUCTS	28.99	31.92	32.15	32.12	32.16	37.10	37.19	37.24	37.21
21	SHOES AND LEATHER	7.13	7.34	7.34	7.34	7.34	7.62	7.62	7.62	7.62
	DURABLES	723.09	798.45	805.67	812.11	805.82	928.23	929.59	938.46	930.28
22	LUMBER	41.25	43.80	44.04	43.32	43.99	48.11	48.06	47.88	48.07
23	FURNITURE	17.43	18.37	18.64	18.48	18.63	20.28	20.40	20.35	20.40
24	STONE, CLAY, GLASS	36.56	39.60	39.86	39.50	39.83	44.39	44.33	44.36	44.35
25	FERROUS METALS	77.29	81.38	81.99	82.03	82.02	87.48	87.58	87.85	87.66
26	COPPER	9.74	10.05	10.14	10.18	10.15	10.67	10.69	10.73	10.70
27	OTHER NONFERROUS METALS	36.96	40.99	41.32	41.56	41.33	48.88	48.96	49.20	48.99
28	METAL PRODUCTS	94.74	102.42	103.23	103.30	103.24	117.44	117.39	118.00	117.48
	NON-ELEC MACHINERY	141.96	156.43	157.76	158.66	157.88	181.82	181.97	182.77	182.13
29	ENGINES AND TURBINES	12.26	13.71	13.81	13.92	13.82	16.18	16.21	16.27	16.23
30	AGRICULTURAL MACHINERY	12.18	12.62	12.74	12.68	12.76	13.61	13.68	13.63	13.70
31	CONSTR, MINING, OILFIELD EQ	21.28	22.36	22.43	22.37	22.46	25.07	25.01	25.04	25.07
32	METALWORKING MACHINERY	14.18	15.28	15.46	15.73	15.47	16.68	16.66	16.87	16.67
33	SPECIAL INDUSTRY MACHINERY	8.38	9.46	9.56	9.58	9.57	10.76	10.76	10.78	10.77
34	MISC NON-ELECTRICAL MACH.	35.84	39.67	40.03	40.37	40.05	45.23	45.23	45.61	45.27
35	COMPUTERS	22.01	25.75	25.99	26.31	26.00	34.01	34.10	34.22	34.11
36	OTHER OFFICE EQUIPMENT	2.79	3.10	3.14	3.15	3.14	3.66	3.67	3.68	3.67
37	SERVICE INDUSTRY MACHINERY	13.04	14.46	14.60	14.54	14.60	16.62	16.64	16.66	16.65
	ELECTRICAL MACHINERY	108.39	125.45	126.49	129.26	126.53	149.07	149.31	151.73	149.39
38	COMMUNIC EQ, ELECTRONIC COMP	47.57	57.75	58.13	60.81	58.15	72.56	72.60	74.97	72.63
39	ELEC INDL APP & DISTRIB EQ	21.64	24.71	24.97	25.27	24.98	27.64	27.64	27.83	27.65
40	HOUSEHOLD APPLIANCES	12.27	13.16	13.28	13.18	13.30	14.50	14.58	14.52	14.59
41	ELEC LIGHTING & WIRING EQ	18.32	20.55	20.75	20.71	20.75	23.18	23.24	23.22	23.26
42	TV SETS, RADIOS, PHONOGRAPHS	8.60	9.28	9.36	9.29	9.36	11.19	11.25	11.19	11.26
	TRANSPORTATION EQ	167.15	188.44	190.85	194.48	190.88	226.43	227.14	231.85	227.41
43	MOTOR VEHICLES	110.73	122.26	124.40	123.25	124.45	148.09	148.68	148.77	148.93
44	AEROSPACE	39.80	46.56	46.68	50.51	46.66	55.06	55.13	58.79	55.14
45	SHIPS, BOATS	8.87	10.72	10.77	11.70	10.76	13.21	13.24	14.15	13.23
46	OTHER TRANSP. EQUIP	7.75	8.90	9.00	9.02	9.01	10.07	10.10	10.13	10.10

Page S-32. Output by producing sector (1977$) (continued).

		(BASE) 1982	(BASE) 1985	(REBATE) 1985	(INCDEF) 1985	(INCTRP) 1985	(BASE) 1990	(REBATE) 1990	(INCDEF) 1990	(INCTRP) 1990
47	INSTRUMENTS	30.60	35.44	35.73	35.92	35.75	42.71	42.86	43.01	42.86
48	MISC. MANUFACTURING	21.83	23.78	23.97	23.86	23.97	27.46	27.61	27.50	27.60
	TRANSPORTATION	146.33	160.56	161.53	161.97	161.52	186.45	186.98	187.59	187.03
49	RAILROADS	24.75	25.77	25.93	25.91	25.94	27.71	27.78	27.80	27.80
50	TRUCKING, HWY PASS TRANSIT	69.34	76.87	77.33	77.48	77.34	90.59	90.82	91.12	90.86
51	WATER TRANSPORT	17.33	18.98	19.04	19.24	19.04	22.10	22.15	22.29	22.16
52	AIR TRANSPORT	28.40	31.81	32.05	32.16	32.03	37.87	38.01	38.16	38.01
53	PIPELINE	3.44	3.73	3.74	3.77	3.74	4.12	4.13	4.15	4.13
54	TRANSPORTAION SERVICES	3.06	3.41	3.43	3.43	3.43	4.06	4.08	4.06	4.08
	UTILITIES	226.64	249.26	250.88	251.11	250.93	290.10	290.73	291.37	290.87
55	COMMUNICATIONS SERVICES	81.56	92.92	93.55	93.74	93.54	114.05	114.27	114.56	114.32
56	ELECTRIC UTILILITIES	82.15	89.91	90.50	90.53	90.55	104.11	104.40	104.59	104.45
57	GAS UTILITY	51.71	54.73	55.07	55.06	55.08	59.69	59.77	59.90	59.80
58	WATER AND SANITATION	11.23	11.70	11.77	11.78	11.77	12.25	12.29	12.33	12.30
59	WHOLESALE TRADE	197.13	215.17	216.74	216.45	216.83	246.52	247.25	247.41	247.41
60	RETAIL TRADE	206.11	220.13	222.20	220.57	222.28	247.42	248.77	247.87	248.91
61	EATING & DRINKING PLACES	91.99	96.77	97.69	97.27	97.66	105.64	106.25	106.09	106.28
62	FINANCE & INSURANCE	140.38	151.39	152.43	151.95	152.47	168.42	169.13	168.77	169.11
63	REAL ESTATE	153.70	168.98	170.04	169.47	169.99	188.53	189.03	189.05	189.03
64	OWNER-OCCUPIED HOUSING	171.62	196.97	197.91	197.39	198.32	244.49	245.24	244.38	245.47
	SERVICES	496.38	546.44	550.29	549.69	550.18	636.14	638.99	639.48	638.99
65	HOTELS, REPAIRS EXC AUTO	48.28	50.68	51.09	51.09	51.09	54.55	54.79	54.86	54.80
66	BUSINESS SERVICES	195.06	222.28	223.75	224.46	223.74	271.61	272.37	273.59	272.42
67	AUTOMOBILE REPAIRS	52.23	56.35	56.79	56.57	56.82	64.54	64.76	64.74	64.81
68	MOVIES AND AMUSEMENTS	22.65	24.12	24.40	24.20	24.31	26.54	26.81	26.62	26.72
69	MEDICINE, EDUCATION, NPO	178.16	193.01	194.27	193.38	194.23	218.89	220.26	219.66	220.24
70	FED & S&L GOVT ENTERPRISES	30.45	32.65	32.87	32.91	32.87	36.46	36.59	36.70	36.60
71	NON COMPETITIVE IMPORTS									
72	DOMESTIC SERVANTS	5.10	4.83	4.90	4.83	4.87	5.03	5.04	5.02	5.04
73	UNIMPORTANT INDUSTRY	10.22	10.92	11.01	11.00	11.02	12.18	12.22	12.24	12.23
74	SCRAP AND USED	5.35	4.91	5.03	4.98	5.03	4.53	4.58	4.57	4.60
75	REST OF THE WORLD INDUSTRY	42.71	50.32	50.24	50.40	50.29	64.86	65.18	64.12	65.20
76	GOVERNMENT INDUSTRY	214.43	219.83	219.83	219.83	219.83	228.16	228.16	228.16	228.16
77	INFORUM STAT. DISCREPANCY	-14.65	-15.78	-15.97	-15.83	-15.98	-18.63	-18.73	-18.65	-18.75
78	NIPA STAT. DISCREPANCY									

PART TWO

A MODEL OF NET INVESTMENT, REPLACEMENT INVESTMENT, AND LABOR REQUIREMENTS

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1. Introduction

This paper presents some empirical results of estimating a factor demand model for each of 53 industries which make up the U.S. Economy. The model consists of a gross investment equation and a labor requirements per unit of output equation. The gross investment equation consists of two components: one which is designed to explain net investment, and one intended to explain replacement investment. The parameters which compose the replacement part of gross investment simultaneously imply equipment depreciation patterns by industry. The estimation of the model also generates own and cross price elasticities for capital and labor. The prices which appear in the model are capital costs, labor costs, and energy prices. The purpose of the model is to generate long run forecasts of investment and average labor productivity within the context of a large Input-Output model of the U.S. economy.

The next section contains a derivation of the model while section three contains a brief discussion of the method of estimation. In section four, patterns of depreciation by industry and elasticity estimates are discussed. The final section contains a summary, and discusses future extentions and modifications of the model.

2. Theory

We assume for each industry a twice differentiable production function relating the flow of output, Q, to a vector of three inputs:

capital (K), labor (L), and energy (E). Further, we assume the production function is characterized by constant returns to scale, with factor augumenting technical change growing at exponential rates, a_1, a_2 and a_3 for capital, labor and energy, respectively; and disembodied technical change growing at an exponential rate given by a_D. The production function may be written as

$$Q = Q(Ke^{a_1 t}, L\ e^{a_2 t}, Ee^{a_3 t})\ e^{a_D t} \tag{1}$$

Each industry is viewed as seeking to minimize aggregate cost subject to the aggregate production function in (1). Then, according to Duality theory, there exists a dual cost function to the production function (1) which gives the minimum total cost of producing a given output, Q, subject to an exogenous vector of input prices. The cost function we have chosen for this study is the Generalized Leontief Cost Function (GLC) suggested by Diewert:[1]

$$C(P_t, Q_t, Z_t) = Q_t * (P_t^{.5}\ BP_t^{.5} + P_t'A\ Z_t)\ e^{-a_D t} \tag{2}$$

where

B = 3 x 3 symmetric matrix of constants;

Q = output;

P = (P_K, P_2, P_E);

Z = vector of nonprice determinants of cost; and

A = 4 x n matrix of constants with any possible sign pattern short of generating negative costs.

Shepard's Lemma may be applied to derive static, cost minimizing factor demand equations which relate the inputs which are the arguments of (1) to relative prices and output. Taking the derivative of (2), then, gives the following factor demand equations:

$$X_{it} = e^{-a_D t} * Q * \{\sum_j b_{ij} (P_j/P_i)^{.5} + \sum_j a_{ij} Z_j\} \qquad i=1,3 \qquad (3)$$

where

$b_{ij} = b_{ji}$;

$X_i = \{Ke^{a_1 t}, Le^{a_2 t}, Ee^{a_3 t}\}$; and

P_i and Z_j are previously defined.

The long run own and cross price elasticites are then given by the formula

$$E_{ij}(t) = \begin{cases} e^{a_i^* t} (-.5/X_i)\, Q \sum_{i \neq j} b_{ij} (P_j/P_i)^{.5} & i = j \\ e^{-a_i^* t} (.5/X_i)\, Q\, b_{ij} (P_j/P_i)^{.5} & i \neq j \end{cases}$$

where $a_i^* = a_i + a_D$, and all the right hand side variables are time dimensioned. The Allen elasticity of substitution is given by $\sigma_{ij} = E_{ij}/S_j$ where S_j is the cost share of the j^{th} input. Estimates of E_{ij} evaluated at 1977 prices are presented below as well as σ_{KL}.[2]

The equations actually estimated are modifications of (3). First, the desired capital stock equation must be converted into a desired net investment equation. By taking time derivatives and rearranging terms, we have

$$N_t^* = \Delta Q_t * (K/Q)_t + e^{-a_1^* t} Q \sum_j b_{Kj} \Delta (P_j/P_K)^{.5} - a_1^* K_t, \qquad (4)$$

where N_t^* is desired net investment and all the values on the right hand side are "expected" values. Actual net investment responds to observed changes in prices and output with a lag. Therefore, we modify (4) as follows:

$$N_t = (K/Q)_{t-1}^* \sum_\ell W_\ell \, \Delta \, Q_{t-\ell} + e^{-a_1^* t} Q_t \sum_j \sum_{\ell=0}^{m_{Kj}} \beta_\ell^{Kj} \, \Delta \, (P_j/P_K)_{t-\ell}^{.5} \qquad (5)$$

where

$$\sum_\ell W_\ell = 1 ;$$

$$\sum_\ell \beta_\ell^{Kj} = b_{Kj}$$

n = Length of lag on changes in output

m_{Kj} = Length of lag on changes in the k^{th} relative price, and

N_t = actual net investment.

To derive the labor requirements equation, we divide (2) by Q and introduce lags:

$$(L/Q)_t = e^{-a_2^* t} \sum_j \sum_{\ell=0}^{m_{Lj}} \beta_\ell^{Lj} (P_j/P_L)_{t-\ell}^{.5} + (L/Q)_{t-1}^* \sum_{i=0}^{3} V_i \, \Delta \, Q_{t-i} \qquad (6)$$

where

$$\sum_\ell \beta_\ell^{Lj} = b_{Lj}$$

and

$$\sum_i V_i = 0$$

$$\beta_\ell^{LK} = \beta_\ell^{KL} \qquad \ell = 1, \ldots M_{LK} \; .$$

The last equality is not required by the cost function but was imposed, nevertheless, to insure what we believed was reasonable dynamic behavior. In addition, the right hand side of (6) consists of variables introduced to capture the cyclical behavior of (L/Q) which we did not expect to be captured by relative price movements. See (2) for further discussion of both of these topics.

To round out the model, we need to derive an expression for replacement investment. Replacement investment is defined as that investment designed to maintain the existing capital stock. It results from the fact that, over time, equipment is either being discarded are is simply losing productive capacity, and so, some positive equipment purchases are necessary merely to keep the size of the capital stock from declining. The rate of depreciation is defined as the sum of the rate of discards and the rate at which existing equipment loses productive capacity. Therefore, like with other work on investment behavior, this study posits that replacement investment is determined by speeds and patterns of physical depreciation of the capital stock. However, our model differs from others by *estimating* rather than assuming a pattern of depreciation.

The most widely used pattern of depreciation is that of geometric decay, where the rate of decay is a function of the inverse of the average service life of the capital equipment. If we let be the rate of retention,[3] then $(1-\lambda)$ is the rate of decay, and we may readily construct a capital stock series implied by the geometric decay pattern as follows:

$$K(t) = I(t) + \lambda * K(t-1) .$$ [4]

Depreciation and, hence, replacment investment is then given by

$$D(t) = (1-\lambda)*K(t-1).$$

An alternative approach is to create a second, fictitious, class of capital (or "bucket"), into which the depreciation out of the first class of capital falls. Thus, if $K_1(t)$ is the first class of capital at time t and $K_2(t)$ is the second class capital at time t, then we have

$$K_1(t) = I(t) + \lambda * K_1(t-1)$$

$$K_2(t) = (1-\lambda)*K_1(t-1) + \lambda * K_2(t-1)$$

The total capital stock at time t is then defined as

$$K(t) = K_1(t) + K_2(t).$$

With this scheme, depreciation is given by

$$D(t) = (1-\lambda)*K_2(t-1).$$

This pattern suggests that, at first, depreciation of equipment is very slow, then increases to a maximum, then recedes. See Figure 1 for a comparison of depreciation under the two schemes.

The approach in this study is a generalization of the "two bucket" approach just outlined, which allows the geometric pattern as a special case. We define three buckets as follows:

$$B_1(t) = I(t) + \lambda * B_1(t-1)$$

$$B_2(t) = (1-\lambda)*B_1(t-1) + \lambda*B_2(t-1)$$

$$B_3(t) = (1-\lambda)*B_2(t-1) + \lambda * B_3(t-1)$$

where I(t) is gross investment and λ is an appropriately selected "spill rate".[5] Now define three "spills" from the three buckets as follow:

$$D_1(t) = (1-\lambda) * B_1(t-1)$$

$$D_2(t) = (1-\lambda) * B_2(t-1)$$

$$D_3(t) = (1-\lambda) * B_3(t-1)$$

Then depreciation is defined as

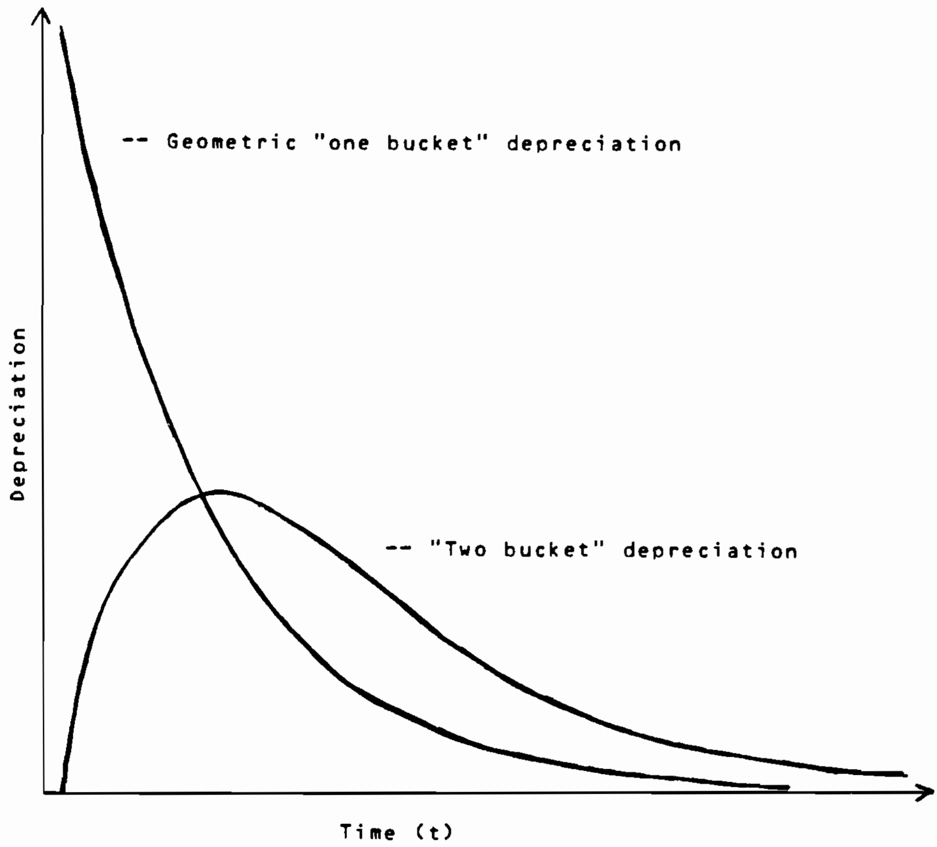

Figure 1. Depreciation pattern for one dollar's worth of equipment.

$$R_t = \sum_i d_i D_i(t) \qquad (7)$$

where

$$\sum_i d_i = 1.$$

The d_i's enter linearly into the regression as part of (5). Consequently, the d_i's may be easily estimated and may vary by industry. Should $d_1 = 1$ and $d_2 = d_3 = 0$, then the implied depreciation is geometric. If $d_2 = 1$, and $d_1 = d_3 = 0$, then the pattern is that which results from the two bucket scheme described previously. If all three d's are nonzero, the implied depreciation pattern would be some intermediate case.[6]

It is clear why the sum of the d's must be unity, for only in that case will each dollar of capital investment be depreciated once and only once. As one dollar in investment passes through the buckets, d_1 percent is written off as it leaves B_1, d_2 percent is written off as it leaves B_2 and the remaining d_3 percent is written off as it passes out of B_3. If the sum of the d's were less than unity, not all of the dollar's worth of capital would depreciate; while, should the sum of the d's be greater than one, the total depreciation would be greater than the original investment.

With this method of determining depreciation, a straight-forward expression for the capital stock results. Since all investment goes into B_1, all of B_1 must be part of the capital stock. Now, recall that the fraction d_1 of the spill from B_1 counts as depreciation, so that $1 - d$, is the fraction of the spill which represents capital held for a while in B_2. Hence, $(1-d_1) * B_2$ represents capital stock held in B_2.

Similarly, d_2 represents that portion of the spill from B_2 which counts as depreciation. Consequently, $(1 - d_1 - d_2) * B_3$ is that portion of B_3 which represents capital stock. The total capital stock expression, then, is given by the following sum:

$$K(t) = B_1 + (1-d_1) B_2 + (1-d_1-d_2) B_3.$$

The model which is finally estimated combines (5) and (7) into a gross investment equation, and (6) as the labor requirements equation.

3. Estimation Considerations

Since the model described in the previous section is designed primarily as a forecasting tool, careful attention was given to the long run properties of the equations. Consequently, we found the need to impose a number of theoretical priors upon the estimation.

First, we required that both capital and labor respond inversely to their own prices, so that E_{KK} and E_{LL} were not permited to be positive. If left unconstrained, many industries showed evidence that capital formation, for example, increased as the cost of capital increased.

Second, we required that capital and labor not be complements, so that the cross price elasticities between capital and labor, E_{KL} and E_{LK}, were contrained to be non-negative. There was no constraints placed upon the sign of the capital-energy and labor-energy elasticity.

Third, we did not permit the estimated weights used to generate replacement investment to be negative. Otherwise, it would have been possible for the estimated weights to imply that equipment depreciaton at some point during the service life of the equipment could actually be negative, which would make little sense.

An additional group of modificatons involved specifying the distributed lags on changes in output and the various relative price variables. A search for the best lag structure for the independent variables began with an estimation of all lags without restrictions except those implied by the cost function. It was clear from the beginning that unconstrained lags would not give reasonable results. We therefore found it necessary to impose some structure on the distributed lags.

The distributed lag on the capital-labor relative price was allowed a length of up to four years without further constraints, except that each coefficient be positive, consistent with our elasticity restriction. This decision was based upon early experimentation in which OLS estimates clearly showed that the lag structures differed significantly from industry to industry. As noted in the previous section, the distributed lag on the capital-labor relative price in the investment equation is required to be identical to that in the employment equation.

Instantaneous adjustment of labor requirements to the energy-labor relative price was required. The contemporaneous price proved superior to a broad range of lag structures tried. The lag structure on the energy-capial relative price was required to be a five-year moving average. Unconstrained OLS estimates tended to be U shaped and frequently changed signs. The chosen lag structure appeared to work best for the industry data, and conforms closely to our prior view that should the size of the optimal capital stock change in response to energy price changes, it would require an extended period of time for the complete adjustment to take place.

The lag on change in output in the investment equation was allowed a maximum length of five years; lag weights were required to lie on a second degree polynomial, and to be declining in the fifth year. The pattern worked well in a previous study of investment behavior at the industry level using the CES production function.[7]

We chose, finally, to introduce an additional trend variable in the employment equation which begins in 1970 with the value of one. There were two reasons for allowing the trend growth in employment per unit of output to change in the estimation. First, a review of the data suggested that there was, indeed, a distinctive shift in the trend growth in productivity starting around 1970. Second, an earlier version of a model which allowed for a constant trend coefficient throughout the history generated what appeared to be unreasonably high labor-energy price elasticities. In effect, practically all of the slowdown in productivity growth which occurred since 1970 was attributed to higher energy prices. To be more confident with this result, we allowed for a modified trend in the 1970's to account for other influences on productivity which might have been improperly captured in the energy price variable. The energy price elasticities presented in the next section are significantly lower in absolute value than the same elasticities estimated with just one trend coefficient.

The presence of inequality constraints required the use of quadratic programming techniques to arrive at the parameter estimates. The program was supplied by the INFORUM project and adapted by this author for the present study.

4. Results

The model derived in section 2 combined with the constraints outlined in section 3 was estimated for 53 industries which make up the U.S. economy. The entire system is displayed in (8). The titles of the 53 industries are presented in Appendix A. A detailed discussion of the derivation of the data may be found in (2).

Table 1 presents evidence of the substitution possibilities among the inputs as well as measures of fit and serial correlation.[8] Looking at the first industry, AGRICULTURE,FORESTRY,FISHERY, and reading across the CAPITAL row, we observe, first, the elasticity of capital with respect to its own price, P_K. Then we see the elasticity of capital with respect to the price of labor, P_L, and the price of energy, P_E. The SIGMA column gives the Allen elasticity of substitution, the only elasticity which we were able to compute with the data in hand. This elasticity measures the percentage change in the capital labor ratio in response to a percentage change in the ratio of the price of labor to the price of capital, holding all other input prices an output constant. Next, we see the cost share column ,CSTSHR, which together with the capital-labor cross price elasticity gives the AES for capital and labor. FIT is the root mean squared error expressed as a percentage of the mean of the dependent variable. The smaller this number, the better the fit. The final column, RHO, is the coefficient of serial correlation and is used in the forecast.

Table 1 sheds light on the substitution possibilities between capital and energy. There are 16 industries in which capital and energy are substitutes. Among the industries most sensitive to energy prices in this way are AGRICULTURE (1), CONSTRUCTION (4), AGRICULTURE

$$I_t = \left(\frac{K}{Q^*}\right)_{t-1} \sum_{i=0}^{4} W_i \, \Delta \, Q_{t-i} + e^{-a_K t} \, Q_t \left\{ \sum_{i=0}^{3} \beta_i^{KL} \, \Delta\left(\frac{P_L}{P_K}\right)_{t-i} \quad b_{KE} \, \Delta\left(\frac{P_E}{P_K}\right)^{*.5}_{t} \right\}$$

$$- \, a_K \, K_{t-1} + \sum_{i=1}^{3} d_i \, D_i(t)$$

$$\left(\frac{L}{Q}\right)_t = e^{-a_1 t_1} \, e^{a_2 t_2} \left\{ \sum_{i=0}^{3} \beta_i^{LK} \left(\frac{P_K}{P_L}\right)_{t-i} + b_{LE} \left(\frac{P_E}{P_L}\right)^{.5} \right\} + \left(\frac{L}{Q}\right)^{*}_{t-1} * \sum_{i=0}^{3} V_i \, \underline{\Delta} \, Q_{t-i}$$

where

$$\sum_{i=0}^{4} W_i = 1 \quad ;$$

$$W_i \geq 0 \qquad i=0, \ldots 4 \quad ;$$

$$\beta_i^{KL} = \beta_i^{LK} \geq 0 \tag{8}$$

$$\sum_{i=0}^{3} V_i = 0$$

$$\sum_i d_i = 1$$

$$d_i \geq 0 \qquad i=1, \ldots 3$$

and

$$Q^* = \sum_{i=0}^{3} Q_{t-i}/4$$

$$\Delta\left(\frac{P_E}{P_K}\right)^{*.5}_{t} = \sum_{i=0}^{4} \Delta\left(\frac{P_E}{P_K}\right)^{.5}_{t-i} /5$$

$$\left(\frac{L}{Q}\right)^{*}_{t} = \sum_{i=0}^{3} \left(\frac{L}{Q}\right)_{t-i} /4$$

$$K_t = B_1(t) + B_2(t) + B_3(t)$$

$$t_1 = t - 1946 \qquad t = 1947, \ldots 77$$

$$t_2 = \begin{cases} 0 & t < 1970 \\ t-1969 & t = 1970, \ldots 77 \end{cases}$$

Table 1. Elasticities.

	PK	PL	PE	SIGMA	CSTSHR	FIT	RHO
1 FARMS AGR. SERVICES,FORESTRY,FISHERY							
CAPITAL	-0.515	0.199	0.316	*	*	11.6	0.504
LABOR	0.825	-0.385	-0.440	0.995	0.200	6.9	0.755
2 CRUDE PETROLEUM AND NATURAL GAS (4)							
CAPITAL	-0.161	0.000	0.161	*	*	21.8	0.604
LABOR	0.000	-0.902	0.902	0.000	0.161	4.2	0.608
3 MINING (2,3,5)							
CAPITAL	-0.248	0.018	0.230	*	*	32.0	0.802
LABOR	0.267	-0.557	0.290	0.054	0.337	3.0	0.439
4 CONSTRUCTION (6)							
CAPITAL	-0.260	0.045	0.216	*	*	14.0	0.523
LABOR	0.225	-0.000	-0.224	0.068	0.657	4.6	0.645
5 FOOD, TOBACCO (7)							
CAPITAL	-0.042	0.015	0.028	*	*	9.8	0.500
LABOR	0.082	-0.000	-0.082	0.110	0.133	3.2	0.788
6 TEXTILES (8)							
CAPITAL	-0.000	0.222	-0.222	*	*	13.8	0.570
LABOR	0.879	-0.142	-0.737	0.836	0.265	8.4	0.741
7 KNITTING, HOSIERY (9)							
CAPITAL	0.000	0.432	-0.432	*	*	21.5	0.500
LABOR	0.319	-0.353	0.034	2.124	0.203	3.9	0.426
8 APPAREL AND HOUSEHOLD TEXTILES (10)							
CAPITAL	-0.000	0.265	-0.265	*	*	12.4	0.500
LABOR	0.400	-0.000	-0.400	1.118	0.237	5.0	0.594

Table 1. Elasticities (continued).

	PK	PL	PE	SIGMA	CSTSHR	FIT	RHO
9 PAPER (11)							
CAPITAL	-0.003	0.005	-0.001	*	*	13.5	0.502
LABOR	0.182	-0.161	-0.021	0.020	0.242	1.7	0.389
10 PRINTING (12)							
CAPITAL	-0.000	0.130	-0.130	*	*	7.3	0.547
LABOR	0.477	-0.000	-0.476	0.362	0.359	3.8	0.777
11 AGRICULTURE FERTILIZERS (13)							
CAPITAL	-0.718	0.035	0.683	*	*	23.8	0.518
LABOR	0.969	-0.994	0.024	0.325	0.108	5.6	0.129
12 OTHER CHEMICALS (14)							
CAPITAL	-0.233	0.000	0.233	*	*	12.2	0.581
LABOR	0.000	-0.115	0.115	0.000	0.190	2.8	0.482
13 PETROLEUM REFINING & FUEL OIL (15,16							
CAPITAL	-0.251	0.006	0.245	*	*	30.8	0.515
LABOR	0.132	-0.251	0.119	0.140	0.042	3.0	0.559
14 RUBBER AND PLASTIC PRODUCTS (17,18)							
CAPITAL	0.000	0.011	-0.011	*	*	9.8	0.577
LABOR	0.248	-0.531	0.283	0.037	0.289	3.7	0.572
15 FOOTWEAR AND LEATHER (19)							
CAPITAL	-0.045	0.062	-0.016	*	*	8.0	0.505
LABOR	0.143	-0.003	-0.139	0.186	0.331	2.5	0.190
16 LUMBER (20)							
CAPITAL	-0.000	0.054	-0.054	*	*	10.8	0.501
LABOR	0.257	-0.000	-0.257	0.247	0.219	4.2	-0.067

Table 1. Elasticities (continued).

	PK	PL	PE	SIGMA	CSTSHR	FIT	RHO
17 FUNITURE (21)							
CAPITAL	0.000	0.241	-0.241	*	*	12.9	0.500
LABOR	0.867	-0.699	-0.168	0.730	0.331	3.3	0.661
18 STONE,CLAY & GLASS (22)							
CAPITAL	0.000	0.015	-0.015	*	*	14.7	0.506
LABOR	0.453	-0.436	-0.017	0.048	0.319	1.4	-0.324
19 IRON AND STEEL (23)							
CAPITAL	-0.000	0.015	-0.015	*	*	38.5	0.519
LABOR	0.531	-0.000	-0.531	0.058	0.257	4.9	0.636
20 NON-FERROUS METALS (24,25)							
CAPITAL	0.000	0.014	-0.014	*	*	25.4	0.572
LABOR	0.814	-0.919	0.105	0.072	0.191	2.7	0.380
21 METAL PRODUCTS (26)							
CAPITAL	0.000	0.067	-0.067	*	*	10.4	0.517
LABOR	0.532	0.000	-0.532	0.215	0.309	3.5	0.599
22 ENGINES & TURBINS (27)							
CAPITAL	0.000	0.040	-0.040	*	*	16.1	0.518
LABOR	0.682	-1.272	0.590	0.173	0.228	8.5	0.380
23 AGRICULTURE MACHINERY (28)							
CAPITAL	0.000	0.276	-0.276	*	*	28.2	0.566
LABOR	1.030	0.000	-1.031	1.146	0.241	9.4	0.500

Table 1. Elasticities (continued).

	PK	PL	PE	SIGMA	CSTSHR	FIT	RHO
25 METALWORKING MACHINERY (30)							
CAPITAL	-0.000	0.049	-0.049	*	*	18.8	0.513
LABOR	0.588	-0.654	0.066	0.113	0.436	3.3	0.285
27 SPECIAL INDUCTRY MACHINERY (31)							
CAPITAL	-0.000	0.143	-0.143	*	*	21.3	0.522
LABOR	0.637	-0.080	-0.557	0.390	0.367	8.9	0.710
28 MISC.NONELEC. MACHINERY (29,32)							
CAPITAL	-0.000	0.091	-0.091	*	*	11.2	0.500
LABOR	0.558	-0.762	0.204	0.264	0.346	2.6	0.247
29 COMPUTERS & OTHER OFFICE MACHINERY (							
CAPITAL	-0.000	0.275	-0.275	*	*	28.3	0.500
LABOR	2.114	-1.802	-0.312	0.890	0.309	9.7	0.635
30 SERVICE INDUCTRY MACHINERY (35)							
CAPITAL	-0.000	0.415	-0.415	*	*	19.2	0.504
LABOR	1.129	-0.200	-0.929	1.734	0.239	5.9	0.605
31 COMMUNICATIONS MACHINERY (36)							
CAPITAL	-0.000	0.796	-0.796	*	*	18.5	0.520
LABOR	1.655	-0.738	-0.917	2.131	0.374	7.7	0.421
32 HEAVY ELECTRICAL MACHINERY (37)							
CAPITAL	0.000	0.248	-0.248	*	*	12.2	0.530
LABOR	1.102	0.000	-1.102	0.814	0.305	7.3	0.677

Table 1. Elasticities (continued).

	PK	PL	PE	SIGMA	CSTSHR	FIT	RHO
33 HOUSEHOLD APPLIANCES (38)							
CAPITAL	-0.000	0.613	-0.612	*	*	22.3	0.500
LABOR	2.195	-0.500	-1.695	2.320	0.264	10.8	0.801
34 ELECTRICAL LIGHTING & WIRING EQUIP (							
CAPITAL	-0.000	0.046	-0.046	*	*	14.8	0.525
LABOR	0.941	-1.361	0.420	0.126	0.366	2.6	0.306
35 RADIO,T.V.RECEIVING,PHONOGRAPH (40)							
CAPITAL	-0.147	0.885	-0.738	*	*	21.7	0.554
LABOR	1.781	-1.445	-0.336	3.183	0.278	12.4	0.469
36 MOTOR VEHICLES (41)							
CAPITAL	-0.000	0.043	-0.043	*	*	34.8	0.510
LABOR	0.411	-0.212	-0.199	0.213	0.202	14.8	0.962
37 AEROSPACE (42)							
CAPITAL	-0.628	0.005	0.623	*	*	42.4	0.506
LABOR	0.018	-0.018	-0.001	0.015	0.334	8.4	0.562
38 SHIPS & BOATS (43)							
CAPITAL	-0.243	0.193	0.050	*	*	33.2	0.610
LABOR	0.632	-0.613	-0.019	0.464	0.415	4.4	0.225
39 OTHER TRANSPORTATION EQUIP. (44)							
CAPITAL	-0.000	0.609	-0.609	*	*	31.3	0.547
LABOR	0.718	-0.000	-0.718	1.185	0.514	22.6	0.752
40 INSTRUMENTS (45)							
CAPITAL	-0.000	0.187	-0.187	*	*	9.1	0.503
LABOR	0.746	-0.358	-0.388	0.476	0.394	4.7	0.662

Table 1. Elasticities (continued).

		PK	PL	PE	SIGMA	CSTSHR	FIT	RHO
41 MISC. MFG. (46)								
	CAPITAL	0.000	0.107	-0.107	*	*	12.4	0.501
	LABOR	0.403	-0.253	-0.149	0.386	0.277	2.8	0.211
42 RAILROADS (47)								
	CAPITAL	-0.011	0.016	-0.005	*	*	13.5	0.523
	LABOR	0.630	-0.975	0.345	0.030	0.528	1.6	0.118
43 AIR TRANSPORT (50)								
	CAPITAL	0.000	0.008	-0.008	*	*	23.9	0.522
	LABOR	0.845	-1.064	0.219	0.022	0.380	3.5	0.487
44 TRUCKING AND OTHER TRANSPORT (48,49,								
	CAPITAL	0.000	0.015	-0.015	*	*	18.7	0.633
	LABOR	0.142	0.000	-0.142	0.034	0.431	2.5	0.580
45 COMMUNICATIONS SERVICES (53)								
	CAPITAL	0.000	0.003	-0.003	*	*	16.2	0.524
	LABOR	0.123	-0.000	-0.123	0.008	0.414	3.7	0.775
46 ELECTRIC UTILITIES (54)								
	CAPITAL	-0.000	0.002	-0.002	*	*	12.1	0.518
	LABOR	0.124	-0.196	0.072	0.007	0.209	1.4	0.509
47 GAS,WATER & SANITATION (55,56)								
	CAPITAL	-0.734	0.000	0.734	*	*	24.8	0.555
	LABOR	0.000	-0.261	0.261	0.000	0.067	1.6	0.116
48 WHOLESALE & RETAIL TRADE (57,58)								
	CAPITAL	-0.138	0.089	0.049	*	*	7.1	0.510
	LABOR	0.152	-0.297	0.145	0.148	0.604	1.1	0.206

Table 1. Elasticities (continued).

	PK	PL	PE	SIGMA	CSTSHR	FIT	RHO
49 FINANCE & INSURANCE (60)							
CAPITAL	0.000	0.015	-0.015	*	*	10.5	0.500
LABOR	0.052	-0.002	-0.050	0.031	0.471	1.4	0.235
50 REAL ESTATE (61)							
CAPITAL	-0.157	0.059	0.098	*	*	20.8	0.505
LABOR	0.930	-0.545	-0.385	0.729	0.080	6.4	0.595
51 HOTELS & REPAIRS MINUS AUTO (63)							
CAPITAL	-0.115	0.017	0.098	*	*	16.2	0.501
LABOR	0.049	0.000	-0.049	0.029	0.595	1.9	0.366
52 BUSINESS SERVICES (64)							
CAPITAL	-0.000	0.013	-0.013	*	*	8.9	0.506
LABOR	0.056	-0.000	-0.056	0.030	0.451	2.5	0.552
53 AUTO REPAIR (65)							
CAPITAL	0.000	0.006	-0.006	*	*	15.5	0.501
LABOR	0.111	-0.286	0.175	0.026	0.247	4.2	0.614
54 MOVIES & AMUSEMENTS (66)							
CAPITAL	-0.175	0.003	0.172	*	*	17.7	0.507
LABOR	0.157	0.000	-0.157	0.008	0.415	3.5	0.703
55 MEDICAL & ED. SERVICES (67)							
CAPITAL	-0.147	0.020	0.128	*	*	7.1	0.509
LABOR	0.236	-0.359	0.123	0.034	0.581	1.2	0.224

FERTILIZERS (11), OTHER CHEMICALS (12), PETROLEUM REFINING (13), AEROSPACE (37), and GAS,WATER AND SANITATION (47), all with cross price elasticites greater than 0.2. Of the remaining 9 industries in which capital and energy are substitutes, three have elasticities between 0.1 and 0.2, and the remainder have elasticities less than 0.1.

There are 37 industries which show evidence of varying degrees of capital-energy complementarity. The most sensitive among these industries are OTHER TRANSPORTATION (39) and RADIO, T.V. (35) with elasticities greater than 0.5 in absolute value. Of the remaining industries nine have elasticities greater than 0.2 while 26 have elasticities less than 0.1.

Table 1 also provides information about the relationship between labor and energy by industry. There are 19 industries in which labor and energy are substitues; that is, higher energy prices lead to more labor employment, and lower average labor productivity. Consequently, higher energy prices relative to the wage rate played a role in the productivity slowdown in the 1973-77 period in a minority of the industries. For those industries where labor and energy are substitutes, two have elasticities greater than 0.5, while twelve have elasticities between 0.1 and 0.5. The most sensitive of the 19 industries are CRUDE PETROLEUM (2), ENGINES & TURBINES (22), and RAILROADS (42).

There are 33 industries in which labor and energy are complements. Therefore, in these industries higher energy prices lead to reduced employment for a given level of output. Some of the industries which are most sensitive in this way are TESTILES (6), IRON & STEEL (19), AGRICULTURE MACHINERY (23), and SERVICE INDUSTRY MACHINERY (30). Of the remaining 29 industries, 17 have elasticities greater than 0.1 in absolute value.

As mentioned earlier, own price elasticities of capital and labor are required to be non-positive, while the cross price elasticities between capital and labor are required to be non-negative. Nevertheless, the magnitudes of these elasticities supply some useful information. There are four industries in which capital's own price elasticity is greater than 0.5 in absolute value: AGRICULTURE (1), AGRICULTURE FERTILIZERS (11), AEROSPACE (37), and GAS,WATER AND SANITATION (47), while there are 18 industries where the same is true of labor's own price elasticity. In addition, of the 42 industries in which at least one of the own price elasticities is non-zero, 32 show labor's own price elasticity is larger in absolute value than capital's. There is, consequently, evidence that labor is more adjustable in production than is capital.

Finally, there are three industries in which the elasticity of substitution is zero, 28 in which it is between zero and .25, seven in which it is between .25 and 0.5; five in which it is between 0.5 and 1.0, and 10 in which it is greater than unity. The industries with the highest elasticities of substitution between capital and labor are KNITTING, HOSIERY (7), COMMUNICATIONS MACHINERY (31), HOUSEHOLD APPLIANCES (33), and RADIO,T.V. (35). There seems to be little evidence to support the Cobb-Douglas function at the industry level.

5. Summary

This report contains the results of estimating factor demand equations derived from the Generalized Leontief Cost Function. A flexible method of estimating replacement investment was included which implied various patterns of equipment depreciation by industry.

We see from these results that both labor employment and equipment investment are sensitive to relative price movements. Capital appears as a complement with energy in most industries as does labor. The results show, further, that capital and labor are substitutable in production in most industries. Finally, the results strongly suggest that the assumption of geometrically declining depreciation is inappropriate for most industries.

Future extentions of the model will involve introducing an energy demand equation into the system once energy consumption data by industry is in hand. Further work on the replacement part of gross investment is called for to correct the deficiency in the method mentioned in footnote three. Finally, the method for estimating replacement investment remains insensitive the market conditions, while there may very well be conditions under which the firm may choose, for a time, not to replace some depreciated capital.

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NOTES

1. See (3) for a discussion of the cost function. The parameter estimates which make up the matrix B may not take on an unrestricted set of values in order that is remain a "well behaved" cost function. For an introduction to the theoretical aspects of the cost function, see (2).
2. At the time when this research was completed, we did not have in hand data on energy shares in total cost by industry. Consequently, we could not compute $_{KE}$ and $_{LE}$, although we were able to compute E_{KE} and E_{LE}.
3. The rate of retention is defined as the fraction of equipment purchased in the present period which remains productive in the next period.
4. This equation states, simply, that this period's capital is the sum of this year's investment and the fraction of the previous years capital which has been retained.
5. The "spill rates" are related to the assumed average service life of equipment. Specifically, we want capital to "spill" from the bucket system at a rate which implies the initial guess at the average service life. See (2) for a derivation of the spill rate. One shortcoming of the depreciation approach outlined in this report is that the estimated average service life will be different from the initial guess, and so, the estimated pattern of depreciation may be improperly dependent upon the initial guess of the average service life. Research is being undertaken at this time to correct this deficiency.

6. It can be easily shown that if any of the d_i's equals one, the pattern of depreciation follows a Pascal distribution of degree i. Note that the geometric distribution is Pascal of degree one.

7. See (1) for earlier results.

8. Only one aspect of the empirical results are discussed in report: the elasticity estimates. We would like to note, however, that there was substantial evidence that the geometric depreciation pattern would have been an inappropriate assumption for most industries. For a complete discussion, see (2).

SECTORAL CAPITAL STOCK ESTIMATES FOR AUSTRIA

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1. Introduction

Capital theory in general and problems of measurement of capital in particular comprise many complex and controversial issues which have been the subject of interesting dicussions by the proponents of the different approaches. The following paper concentrates on major issues in capital theory to which - to a greater or lesser extent - empirical relevance must be attributed.

The main principles guiding the computations presented below depend largely on the purpose of these computations. Capital stock data play a key role in growth accounting, in production and investment functions, in analyses of distribution of factor income, and the distribution of wealth between individual institutional sectors. The aim for the computations presented below is the establishment of reliable disaggregated capital stock data that can be incorporated into a medium-term multisectoral model of the Austrian economy.

The second chapter deals with theoretical questions. What do we mean by "capital" and how can we measure it? Though an unequivocal answer cannot be given, it is possible to derive some interesting results from this theoretical but by no means esoteric issue (ROBINSON). Empirically oriented scientists should consider the problems raised in this discussion, even though their practical relevance may appear to be hidden behind a theoretical smoke-screen.

In the second chapter the problems related to the relationsship between quality changes, price indexation problems and capital stock figures, which effect the results of our computation, are discussed. Hedonic price indices do not fully reflect the heterogeneity of the problems involved. The chapter finishes with a short description of the role of capital in different multisectoral models.

The third chapter is devoted to computational aspects. We start with the basic model (perpetual inventory model) developed by GOLDSMITH (1951) and proceed to a revised perpetual inventory model developed by a group of scientists at the

University of Maryland (ALMON et al. 1974), which we shall call the ALMON-model. We then consider one of the most crucial variables in capital stock estimates, namely the lifetime variable.
The fourth chapter presents capital stock estimates together with some derived main economic indicators.

2. Theoretical and Empirical Concepts of Capital

2.1 The Definition of Capital

The question what is meant by "capital", especially in the context of an aggregate production function, is one of the fundamental controversies in economic theory. However, it seems that this dispute has been quieting down somewhat, although no agreement has yet been reached and probably never will be reached. The roots of the controversy are complex. It started with ROBINSON'S (1953) complaints that traditional (orthodox) economic theory does not fully reflect the ambiguity in the concept of capital. In typically British understatement she qualified this issue: "The question (i.e. what is meant by capital, note by the author) is certainly not an easy one to answer", and in an article written as late as 1977 she wrote retrospectively: "Capital theory was regarded as an esoteric doctrine which had no application to any question of general interest" (reprinted in ROBINSON 1978, p. 119).

SOLOW, to whom she was addressing her remarks qualified "capital" as a proxy for something, which had empirical significance only and no place in rigorous theory (SOLOW 1956). Capital has thus only operative meaning, for example when explaining marginal productivity or the development of factor shares.

One of the reasons of the intensity of this controversy was the fact that it is not limited to economic theory itself. It is certainly a question of psychology, tradition, or one's general view of social processes or ideology[1). From that it follows that this dispute will never be solved but nevertheless there is a growing need for capital stock data. Therefore, USHER's qualification

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1) See e.g. HARCOURT: "It is my strong impression that if one were to be told whether an economist was fundamentally sympathetic or hostile to basic capitalist institutions, especially private property and the related rights to income streams, or whether he were a hawk or a dove in his views on the Vietnam War, one could predict with a considerable degree of accuracy both, his general approach in economic theory and which side he would be on in the present controversies".(HARCOURT 1972, p. 13.)

that "... there is widespread agreement that the working definition of real capital in equation (1)[1] is only an approximation, but there is less than full agreement on what the definition is supposed to approximate "(USHER 1976, p. 13) may turn out to remain valid forever.

In conceptual terms, three distinctive interpretations of "capital"can be distinguished: (i) capital as a fund of money, (ii) capital as a proxy for a collection of human or non-human, tangible or non tangible assets, and finally (iii) capital as a value concept, including available funds, assets, discounted future income streams, etc. The decision which interpretation is most appropriate largely depends on the problem raised and any recommendation in this respect - if at all possible - will have to be related to the specific topic of investigation.

Capital as a "fund of money" emphasizes its disposability. It enables the entrepreneur to initiate a process of production. Disposable money capital is needed because production takes time and the stream of expenditures and returns is not balanced at any moment of time. This interpretation goes back to the classical economists (SMITH, MARX, JEVONS) and later BÖHM-BAWERK (1884), WICKSELL (1934), von HAYEK (1941) and HICKS (1939, 1965, 1973). The "Neo-Austrian" approach of HICKS (see BURMEISTER 1974) rests on an arbitrary definition of the production period (in HICKS terminology "construction period", 1973, p. 15) which similiar to a "black box" reduces the production process to observable units of labour-inputs and product-outputs. This led BURMEISTER to conclude: "Clearly such a discription is incomplete and fatally so - if one of our primary concerns is capital theory and we cannot observe any capital goods in the economy". (BURMEISTER 1974, p. 416).

The second concept emphasizes the periodized production property of all sorts of assets. It encompasses items like machinery, equipment, cars, buildings, roads etc. Some authors regard non-physical assets, especially human capital, as an integral part of productive capacity. This broader concept includes

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1) This equation is $$K^t = \sum_i P_i^o \quad K_i^t$$

where K^t is total capital stock, P_i^o are blocks of capital stocks and K_i^t are units of i type of capital goods in period t.

items in "material existence" (in CLARK's sense), and this is the variable commonly used in most empirical analyses. The aggregate capital variable, however, is a composition of different capital goods and there is nearly complete agreement among theorists that aggregation is possible only under very restrictive assumptions (SOLOW 1956). In addition, the conceptial framework for capital measurement used in production functions suggests the following conditions; (i) capital should be defined with reference to the use of commodities and not to their inherent characteristics, (ii) it should include all assets used in the production process and finally (iii) it should be defined for periods of production and not accounting periods. It is this last condition which turns out to be impractical in empirical work. In the System of National Accounts (UN 1968) a time period (accounting period) of one year is proposed, which however is an arbitrary procedure from the point of view of "production period".

Finally, the third concept of capital rests on the wealth aspect of an economy. It includes all available funds, assets and discounted future income streams. Contrary to the foregoing concept, the wealth or value concept allows aggregation over different items.

For our computations the production capacity concept is used. Capital in this context is defined as the sum total of homogenous production capacities, and thus is a derived production factor. By this definition the heterogeneity of different capital goods and different vintages is not neglected but can be considered as a plausible approximation of reality. Furthermore, it can be translated into quantitative terms and allows aggregation over different items.

2.2 The Measurement of Capital

The controversy about the concept of capital arose out of the discrepancies between theory and applied measurement procedures. "In each seperate context, an appropriate measurement should be applied, not least because some of the recent controversy concerning capital appears to have arisen as much from a confusion of the issues of statistical measurement as from problems of economic interpretation" (WARD 1976, p. 15). The following table 1, presents relevant capital stock or flow measures to be used for various problems of economic analysis.

Table 1: Objectives of capital measurement and the capital concept required. Source: (WARD 1976, p. 16).

OBJECTIVES AND PROBLEMS	CAPITAL MEASURE REQUIRED
1. Growth accounting and explanations of economic development in terms of the different contributions of the various factors of production.	Equivalent capital services; production factor values at constant base period prices.
2. The determination of a potential output trend in an attempt to measure cyclical fluctuations and quantify demand pressures.	Gross capital stock available valued at a given reference year's average prices.
3. The determination of factor income shares in output.	Capital services valued in base period prices.
4. Factor inputs and long-term projections.	Gross capital stocks and projected capital services.
5. The role of technical progress.	Capital services valued in base period prices.
6. Choice of techniques.	Gross capital stock: capital services and the value of the discounted future income flow of current capital stock at present year prices.
7. Forecasting the future demand for capital goods.	Gross capital stock and potential capital services at the same base year prices.
8. Sector and national balance sheets integrated in a system of national accounts, i.e. the relationship between stocks at the beginning and end of a period and the flows occuring within that period.	Net capital stock at current replacement cost.
9. Alternative cost evaluations; marginal rates of substitution.	Addition to gross capital stock.
10. Replacement of capital.	Retirements valued in current prices.
11. Manpower utilization and labour productivity; the relationship of capital to labour.	Gross capital stock and capital services.

The underlying intention of our computations was to incorporate stock data into a medium-term multisectoral model and from that followed that it real gross capital stocks were to be estimated.

2.3 <u>Quality Changes and Price Index Problems</u>

Since investment data are Only availabe in nominal (value) terms, an appropriate deflator is needed to derive real investment and consequently capital stock series. The pricing of capital goods, however, may prove to be the Achilles' heel in the measurement of real capital (USHER 1980, p. 10), or in the words of KENNEDY and THIRLWALL (1972, p. 29),there is no obvious "price" of capital goods that can be used for deriving a measure of the volume of capital from value figures. The conclusion from this qualification is to find a proxy variable for the value, and here the basic question is whether capital should be valued in terms of its costs (backward looking concept) or in terms of its contribution to present or future production (forward looking concept). If one adopts the latter approach then the question arises, how changes in the efficiency of capital as well as in utilisation have to be considered. An overestimation of price changes, which maybe interpreted as neglect of technical progress, leads to an underestimation of the latest vintages of capital and vice versa. This,in turn, overestimates capital productivity and underestimates capital intensity.

On a multisectoral basis this problem reaches a new dimension because it poses the question of the origin of productivity increases, i.e. whether improvements in technology should be attributed to the capital goods sectors or to the sectors using them. Costless quality improvements in capital goods lead to productivity increases in the sectors using these goods and are reflected thus either in price or profit movements.

Another relevant issue is the time dimension of productivity changes. "By measuring real capital according to its capacity to contribute to output one would be including in measured output all quality changes in capital goods at the time those goods were produced instead of including the effects of costless quality changes at the time the improved capital goods were used" (HIBBERT et al. 1977, p. 129). Consequently, the effect of knowledge production, the ultimate source of technology improvement, and its "economic" rate of return, cannot be measured on a disaggregated level.

An interesting proposal was put forward by HICKS (1973). Since capital can be interpreted as foregone consumption, the consumer price deflator would be the most appropriate deflator for capital. This would mean that technical progress is again included in the deflator and that it could not be measured in real terms.

If we have decided which concept we consider appropriate, then the question remains how to evaluate the changes in quality reflected in longer durability, higher precision, lower operating costs etc. The following alternatives are open for use:

(i) The market prices of capital goods are related to their underlying attributes, whose quantitative weight could be estimated by regression analysis or similiar methods (see e.g. GRILICHES 1971 b).

(ii) The producer of the capital good is requested to decide how much of the observed price change refers to quality change, and

(iii) adjustment is made by experts.

For our computation the price problem was no issue since the deflators are published by the Austrian Statistical Office. With their help we constructed capital stock at constant prices, necessary for fitting production functions, determining capital coefficients and capital intensities, both at current prices, replacement costs or historical costs. The latter two concepts are necessary for the establishment of sectoral balance sheets and the analysis of stock flow relationships and portfolio selection analysis.

2.4 Capital Stocks in Multisectoral Models

Though the capital controversy arose from the use of this concept in aggregated production functions, its relevance in the context of multisectoral models is increasing, too. Therefore, a short digression to HICKS' alternative capital models seems appropriate. HICKS (1973, p. 2-12) distinguishes between three models according to their degree of disintegration; (i) the method of sectoral[1)]

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1) In this context "sectoral" does not refer to the institutional context.

disintegration, (ii) the capital model of von NEUMANN (1945-46), and (iii) the "neo-Austrian" model of BÖHM-BAWERK (1984) and HICKS (1939, 1965, 1973) himself.

The first model distinguishes two categories of firms; those which produce capital goods e.g. in the form of machines, and those which use these capital goods in the process of producing consumer goods, thus "allowing the accounting distinction between consumption and investment" being converted into an institutional (industrial, sectoral) division. One conclusion from this classification could be to consider the interest rate as the determining factor of the economic structure.

The second model is based on von NEUMANNS' pioneering work on a model of general economic equilibrium (v. NEUMANN 1945-46). The main point of v. NEUMANN'S model is the determination of prices in a"quasi-stationary state" (CHAMPERNOWNE 1945-46) and though capital goods are explicitly mentioned and introduced in equation (1) of v. NEUMANNS paper, their main feature, namely durability, is simply defined away by describing capital goods at different times as different goods. Thus a clear specification of what is meant by capital goods is avoided.

The third model, the so called "neo-Austrian model of capital" translates the production process into a series of dated inputs and, at the end of the process, units of output. Since capital can be regarded as "frozen labour" the inputs are reduced to the primary factors labour and time, the latter variable indicating the agree of "roundaboutness", anouther measure of capital intensity. The main result is the specification of the relationship between the latter variable and the rate of interest.

For the empirical economist working with multisector models the question remains how this approach can be related to the dynamic LEONTIEF model (LEONTIEF 1953, 1966). From the point of view of statistical analysis, LEONTIEF'S model is the most appropriate one. Its main features are distinct from those of models type (i) and (ii) described above, although some similarities cannot be overlooked. It therefore seems appropriate to consider it as another type of model.

But what is the relevance of these models to empirical stock estimates or what can we learn from them? Model (i) is certainly most inappropriate for two reasons: First, once the sectoral categorisation in consumption and capital goods industries is established, the definition and quantification of capital is simply a problem of measuring the output of the industries in question. On the other hand, this completely ignores the possible dual character of goods - i.e. a commodity viewed as an investment or as consumption good. Second, the model excludes the possibility of joint production and must, on these grounds, be considered too restrictive.

Von NEUMANN'S model though it explicitly introduces capital goods, treats them like any other input. The emphasis lies on processes, or more accurately, on the selection procedure of alternative processes. Capital goods at different points of time are different goods, though they are physically the same. They are different, because they may have different prices. In the absence of markets for these goods, which we can realistically assume to exist for the majorty of them, von NEUMANN'S model establishes the rules which determine their equilibrium prices.

Another controversial issue in empirical capital theory is the interpretation of capital as a stock or flow variable respectively. In von NEUMANN's model capital is a stock variable which, however, changes its identity, in succeeding production periods. This leads us to the dynamic LEONTIEF-model and its treatment of capital. LEONTIEF (1966) starts with the balance equation

$$x_t - A_t x_t - B_{t+1} \ (x_{t+1} - x_t) = C_t \qquad (1)$$

where x_t and x_{t+1}presents output at year t and t+1; c_t are deliveries to final demand, A_t are technical flow coefficients representing direct current input requirements and B_t are capital coefficients. Substituting G_t for $(I-A_t + B_{t+1})$, one gets

$$G_t x_t - B_{t+1} \ x_{t+1} = C_t \qquad (2)$$

System (2) can be extended to (m + 1) periods, which gives

$$\begin{bmatrix} G_{-m} - B_{-m+1} & & & & \\ & G_{-m-1} - B_{-m+2} & & & \\ & \ddots & \ddots & & \\ & & G_{-2} - B_{-1} & & \\ & & & G_{-1} - B_{0} & \\ & & & & G_{0} \end{bmatrix} \begin{bmatrix} x_{-m} \\ x_{-m+1} \\ \vdots \\ x_{-2} \\ x_{-1} \\ x_{0} \end{bmatrix} = \begin{bmatrix} c_{-m} \\ c_{-m+1} \\ \vdots \\ c_{-2} \\ c_{-1} \\ c_{0} \end{bmatrix} \tag{3}$$

and solving for c_t, one gets

$$\begin{bmatrix} x_{-m} \\ \vdots \\ x_{-2} \\ x_{-1} \\ x_{0} \end{bmatrix} = \begin{bmatrix} G_{-m}^{-1} \ldots R_{-m} \ldots R_{-3}R_{-2}G_{-1}^{-1} & R_{-m} \ldots R_{-3}R_{-2}R_{-1}G_{0}^{-1} \\ \vdots & \vdots \\ R_{-2}G_{-1}^{-1} & R_{-2}R_{-1}G_{0}^{-1} \\ G_{-1}^{-1} & R_{-1}G_{0}^{-1} \\ & G_{0}^{-1} \end{bmatrix} \begin{bmatrix} c_{-m} \\ \vdots \\ c_{-2} \\ c_{-1} \\ c_{0}' \end{bmatrix} \tag{4}$$

The first term of RHS of (4) may be interpreted as a time related structure to produce a given vector c_t. The intertemporal links are defined through the B_t's. The time subscript indicates the possibility of capital-embodied technological change. The capital concept, one can see at once, is a technological one, because the idea of productive capacity is introduced. On this basis the valuation or pricing of capital goods constitutes no problem.

3. The Construction of Sectoral Capital Stock Data

3.1 The Basic Model

The basic model defines gross capital stock by

$$K_t^g = K_{t-1}^g + I_t - R_t \tag{5}$$

where K_t is gross capital stock at the end of period t, I_t is gross investment during period t and R_t is retirement in the same period. Assuming a retirement rate ρ , we get

$$R_t = \rho K_{t-1}^g \tag{6}$$

successively lagging (6) and substituting for (5), we have

$$K_t^g = \sum_{i=o}^{m} (1-\rho)^i I_{t-1} + (1-\rho)^{m-1} K_{m-1}^g \tag{7}$$

Similiarly we define net capital stock

$$K_t^n = K_{t-1}^n + I_t - D_t \tag{8}$$

where D_t stands for depreciation. Again assuming a linear relationship between D_t and net capital stock,

$$D_t = \delta K_{t-1}^n \tag{9}$$

and further

$$K_t^n = I_t + (1-\delta) K_{t-1}^n \tag{10}$$

substituting for (8)

$$K_t^n = \sum_{i=o}^{k} (1-\delta)^i I_{t-i} + (1-\delta)^{m-1} K_{t-k-1}^n \tag{11}$$

The two models (7) and (11) are identical except for the parameters ρ and δ. They either represent retirement or depreciation. Let us look at ρ more closely, which we can reformulate as a weight (survival) function, in which the parameter depends on the vintage of the capital goods. In other words, it indicates the share of gross capital stock of a vintage still in use at a specific time.

Let us define g(i) as the survival function, then

$$o \leq g(i) \leq 1 \quad \text{holds for} \quad 1 \leq i \leq m+1 \tag{12}$$

with m the maximal survival period of an asset. In addition, the following conditions hold:

$$\begin{aligned} g(1) &= 1 \\ g(i) &\geq g(i+1) \\ g(m) &\geq 0 \\ g(m+1) &= 0 \end{aligned} \tag{13}$$

The mortality function h(i) is defined by

$$h(i) = g(i) - g(i+1) \tag{14}$$

Model (7) in combination with (13) has been extensively applied in Germany (KIRNER 1968, BAUMGART, KRENGEL 1970) and Austria (SCHENK, FINK 1976, PRUCHA 1976). In both countries, a logistic survival function of the form

$$g(i) = (1 + e^{s(i)})^{-1} \tag{15}$$

with

$$s(i) = \frac{a}{(m-i+1)} + \frac{b}{1-i} \tag{16}$$

has been used. Replacing a by c . p and b by c . q, we get

$$g(i) = (1+e^{\frac{c}{m}\left(\frac{p.m}{m-i+1} + \frac{q.m}{1-i}\right)})^{-1} \tag{17}$$

Function (17) has been tested empirically for locomotives and cars of different production times with the result that p and q are fairly close to o . 5 and c/m to 2.2, (KIRNER to 1968), which gives

$$g(i) = (1 + e^{1.1\left(\frac{m}{m-i+1} + \frac{m}{1-i}\right)})^{-1} \tag{18}$$

The following figures 1 and 2 show this survival function for different assumptions for m and the parameters p and q.

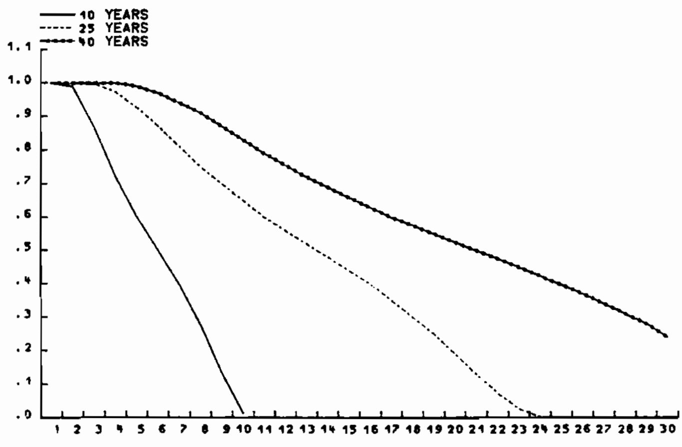

Fig. 1: Survival functions with different lifetime assumptions.

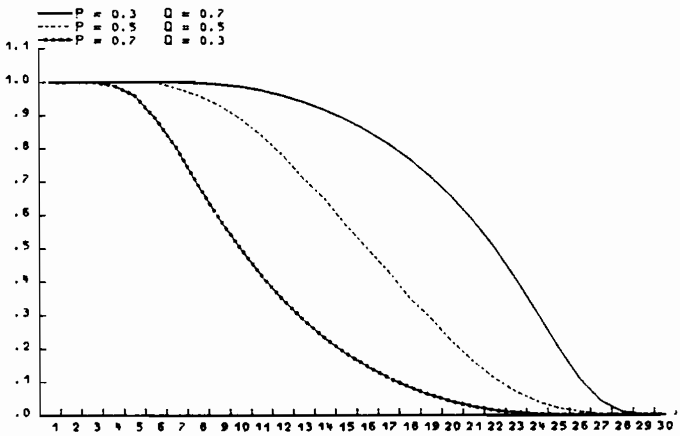

Fig. 2: Survival functions with different assumptions for p and q.

Quantitative estimates of g(i) are certainly rough, but they can nevertheless be considererd more appropriate than linear or exponential functions. This method, however demands long-term series of investment data, which usually cannot be supplied. This, in turn, constitutes a severe constraint on empirical estimates.

3.2 A Revised Perpetual Inventory Model

The following model is an approximation of the basic perpetual inventory model with an exponential mortality function. It was constructed by ALMON et al. (1974), and makes only modest demands on the length of time series. Total capital stock is composed of two "classes" of stock:

$$K_t = K_t^1 + K_t^2 \qquad (19)$$

The first class K_t^1, or the book value of capital, increases with investment and decreases with depriciation.

$$\begin{aligned} K_t^1 &= K_{t-1}^1 + I_t - s_t K_{t-1}^1 \\ &= (1-s_t) K_{t-1}^1 + I_t \end{aligned} \qquad (20)$$

The second class, the still reserves, is filled up by depreciation of the first category and reduces with retirements.

$$
\begin{aligned}
K_t^2 &= K_{t-1}^2 + s_t\, K_{t-1}^1 - s_t\, K_{t-1}^2 \\
&= (1-s_t)\, K_{t-1}^2 + s_t\, K_{t-1}^1
\end{aligned}
\tag{21}
$$

s_t is the "double declining balance rate" of depreciation and is determined by the physical life of capital.

$$
s_t = \frac{2}{m_t} \tag{22}
$$

The following figure 3 shows the two classes and their total.

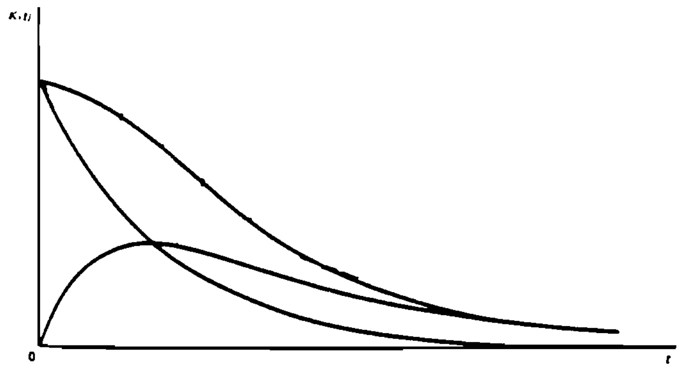

Fig. 3: Retention curve of capital.

3.3 The Lifetimes of Capital Assets

The assumption of lifetimes of capital assets is a crucial variable in capital stock computations, as can be seen from fig. 1. However, the definition of an asset's life, either in economic or purely technical terms, is not unambigwous and leaves wide scope to individual quantification. Some authors conclude that "... there is virtually no hard information either about the mean length of life of a particular type of asset or about its stability over time." (HIBBERT et al. 1977, p. 123).

Basically two concepts can be distinguished; a technically defined service life and an economically defined life. The first one aims at the technically feasible production period of a specific capital good, irrespective of economic factors. In their most simple form, all capital goods of one period continously enter the production process until they abruptly break down (one-hoss-shay assumption). The economic service life is determined by the time span, in which the capital good in use is most economic among alternative ones. Changes in relative prices, in the tax structure, or other economic factors may exert an influence on the actual length of time.

The OECD (WARD 1976, p. 36) proposes three different methods for estimation procedures:

(i) Actual observation of the interval between the date of installation and the date of final retirement of specific assets;

(ii) Use of enterprise balance sheet data at different periods and known intervening investment outlays and depreciation rates applied to particular groups of assets; and the

(iii) Use of the standard income tax or corporate depreciation rates to obtain the implied lifetimes.

Each of these methods has its own drawbacks. BARNA e.g. concludes that "neither the date of birth nor the date of death are always uniquely definiable" (1963, p.86). FELDSTEIN and FOOT (1971), as well as GRIFFIN (1979) suggest that assumptions on lifetime cannot be made independently of price effects thus closing the circle of economically based service lives.

Most important is the fact that knowledge of lifetime actually turns out to be correct or incorrect when it is not needed any more, because the particular capital good has disappeared from the production process. In addition, technical progress not only changes the character of the goods, making compiled information on service lives obsolete, but it also increases the potential substitutable new capital goods, before the estimated lifetime has been reached. Therefore estimates of economic lives are regarded as the weakest aspect of the simulation exercise" (WARD 1976, p. 36). A rough picture of the consequences of alternative service life assumptions is given in the following table 2 which shows capital stock in relation to this assumptions.

Analyses of scrappings and retirement series showed that the relation between the average length of life and the standard deviation was relatively constant, in other words, the form of the mortality function did not change substantially in the past. Information on new technology, however, led the Federal Statistical Office in Germany to assume that the service life is gradually growing shorter. "The results of the 1969 enquiry conducted by the Ifo-Institute of Economic Research in Munich among close to 3.000 manufacturing enterprises also confirm the trend towards a shorter lifetime" (LÜTZEL 1977, p. 69). According to this enquiry, 60 percent of the respondents stated that the service life of their machines decreased over the past 10 to 15 years, and approximately 80 percent of the enterprises expected a further reduction over the next 10 to 15 years. Accelerating technical progress was usually given as the main reason". (MÜLLER 1973, p. 36). TENGBLAD and WESTERLUND (1976) found some evidence in Sweden that in periods of rapid technological and economic development the durability of capital assets decreases, similar to findings of the Bureau of Economic Analysis (1976)[1], which expects the utilisation rate to be the prime factor influencing lifetime. GROES (1976), on the other hand, suggests a lengthenig of lifetime when liquidity or proftis are low.

For the computations presented below, constant service lifetimes were assumed for two reasons; firstly, hard economic data or facts for a decrasing lifetime were not available. In additon, concerning the factors mentioned above, some sectors would experience variable service lives, while others would have constant ones, depending on their dynamic development. Reliable data on this sectoral level, however, are not available at all. Secondly, if technical progress is the factor influencing the lifetime variable then the economic and not the technical

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1) Quoted in OECD (1982).

Tab. 2: Consequence of different assumptions of lifetime on capital stock figures for 1980.

Kapitalstock Anfangsbestand \ Lebensdauer in Jahren	10	15	20	25	30	35	40	45
40	88.28 (0.65)	114.29 (0.84)	131.48 (0.97)	142.43 (1.05)	149.81 (1.10)	154.98 (1.14)	158.74 (1.17)	161.54 (1.19
45	88.84 (0.65)	115.80 (0.85)	133.79 (0.98)	145.32 (1.07)	153.13 (1.13)	158.61 (1.17)	162.59 (1.19)	165.57 (1.22)
50	89.41 (0.66)	117.31 (0.86)	136.11 (1.0)	148.22 (1.09)	156.44 (1.15)	162.23 (1.19)	166.44 (1.22)	169.60 (1.25)
55	89.97 (0.66)	118.81 (0.87)	138.43 (1.02)	151.12 (1.11)	159.76 (1.17)	165.95 (1.22)	170.29 (1.25)	173.62 (1.28)
60	90.53 (0.67)	120.32 (0.88)	140.74 (1.03)	154.02 (1.13)	163.07 (1.20)	169.47 (1.25)	174.14 (1.28)	177.65 (1.31)

Tab. 3: Lifetimes of sectoral capital stocks for equipment and construction.

Nr.	Sektoren	Ausrüstung		Bau	
		Lebens-dauer	Kapitalst. 64 nom.	Lebens-dauer	Kapitalst. 64 nom.
01	Land-u. Forstwirtschaft	15	39.820	67	87.080
02	Bergbau	20	5.050	40	1.794
03	Nahrungs-u.Genußmittel	22	8.940	40	4.360
04	Textil-u. Bekleidung	17	11.070	40	5.409
05	Holz	15	1.300	40	0.870
06	Papier	18	9.040	40	3.524
07 08	Chemie (ohne Erdöl) Erdöl	18	14.380	40	4.510
09	Nicht-Metallmineralien	17	5.760	40	2.740
10	Grundmetalle	20	14.055	40	5.170
11	Metallverarbeitung	20	18.680	40	7.760
12	Energie	18	41.500	40	34.760
13	Bauwesen	8	9.060	40	5.760
14	Handel	20	33.680	40	46.260
15	Gastgewerbe	20	1.700	40	23.000
16	Verkehr (exkl. Straßen)	25	40.550	40	54.735
16a	Straßen			∞	50.650
17	Vermögensverwaltung	20	1.700	40	347.850
18	Sonstige Dienste	20	3.940	40	5.880
19	Öffentliche Dienste	15	19.530	40	63.380

lifetime is affected. Since the capital concept used for our computations is a technical concept, economic factors could not be considered in the capital variable.

The following table 3 shows lifetime assumptions on a disaggregated level for two categories of capital assets, namely machinery and equipment, as well as buildings and other construction works. The main source has been expert advice though lives used for tax purposes have also been considered.

3.4 Sectoral Stock Data

Model (19) - (21) has been applied for our sectoral capital stock estimates. Three problems have to be solved before starting computational work. These are (i) the selection of (sectoral) price indices for capital stock series, (ii) the adjoinment of leased capital goods and finally (iii) the relation of K^1 to K^2 in the revised perpetual inventory method (ALMON-Model).

The problems involved in the construction of price indices have been dealt with extensively in the works of GRILICHES (1971b). The "hedonic price indices" are based on a bundle of characteristics, attributed to each individual capital asset. Thus, this method accounts for a changing and technically more advanced bundle of characteristics which should be reflected in the price index of this asset. If one accepts this approach as theoretically correct and statistically possible, the results are statistically different price indices for almost every category of assets (i.e. sectors) and this in turn does not only unnecessarily extend the data requirements but may also affect the aggregate investment deflator in a curious way. To avoide this difficulty, a single investment good deflator is used for the two investment categories, namely equipment and construction. Possible sectoral differences were neglected and this may exert an influence on real capital stock, capital intensity and -productivity.

In addition, capital assets can be valued according to the replacement cost or historical cost concept (WARD 1976, COEN 1980). According to the replacement cost concept, nominal capital stock at time t, K_t^n is defined by

$$K_t^n = K_t^r \cdot PI_t \tag{23}$$

with K_t^r real capital stock and

PI_t the Deflator of current investment goods.

Capital stock at historical costs K_t^m is given by

$$K_t^m = \sum_{i=0}^{t} (1-\rho)^i . PI_i . K_i^r \tag{24}$$

If we assume $PI_t \leq PI_{t+1}$, from

$$\sum_{i=0}^{m} (1-\rho)^i \, P_{t-i} \leq P_t \sum_{i=0}^{m} (1-\rho)^i \tag{25}$$

it follows, that $K_t^n \leq K_t^m$.

Point (ii) stresses the problem of physical capital goods. This question may become more important in the case of a considerable share of leased capital goods. According to the underlying production -based concept, leased fixed assets ought to be allocated to those sectors that use them, because our main interest is the relationship between the inputs, i.e. capital input, and the output in each individual production sector. In wealth distribution analysis, however, the ownership of the assets is important.

The revised perpetual inventory method requires information on the relationship of K^1 and K^2 for the basis year. Assuming a constant retirement rate and a constant growth rate r, this relationship is given by

$$\lim_{t \to \infty} \frac{K_t^1}{K_t^2} = \frac{r+\rho}{\rho} \tag{26}$$

4. The Results

The following tables show the results of the computations. Tables 3 - 4, contain the sectoral stock data for equipment, construction, and sum total. Table 5 shows the stock data broad economic sectors, namely the primary, secondary and tertiary sectors. The figures indicate an average growth of 4.2 percent for the period 1965/1980 for total capital stock, the figure for the period 1965/1973 is 4.3 percent and for 1973/1980 it is 4.0 percent. Table 6, finally shows the real capital/output rates for the broad economic sectors.

Tabelle 4: Real Capital stocks for sectors 01 - 09, 1965/1980.

Year \ Sectors	01	02	03	04	05	06	07	08	09
65	224.416	11.288	27.900	29.404	6.324	21.710	35.025	16.963	32.085
66	229.133	11.368	30.018	30.108	8.249	22.219	36.965	19.464	33.224
67	234.032	11.231	32.093	30.580	9.460	23.262	38.695	21.817	34.196
68	238.208	11.236	35.257	31.463	10.921	23.875	40.888	23.706	34.814
69	243.077	11.292	37.919	32.734	12.466	24.635	44.189	25.029	35.412
70	247.883	11.404	40.432	34.009	14.706	26.031	47.248	26.517	36.502
71	252.771	11.473	43.635	35.374	17.434	28.028	51.279	28.636	37.810
72	258.052	11.559	47.806	36.720	20.614	29.493	55.570	31.763	40.219
73	259.979	11.480	50.686	38.010	23.619	30.693	58.908	33.882	44.820
74	262.572	11.511	53.629	38.656	26.061	32.022	61.924	35.729	48.609
75	264.855	11.730	56.250	38.877	27.310	33.142	64.698	36.633	50.939
76	267.321	11.968	59.175	39.330	29.181	34.313	67.960	37.220	51.703
77	270.240	11.971	62.364	39.647	30.963	35.944	71.786	38.293	53.730
78	273.139	12.040	65.314	39.703	32.145	38.174	75.554	39.259	54.239
79	275.483	12.589	67.907	39.867	33.440	38.956	78.337	40.278	55.681
80	278.452	12.736	70.593	40.245	35.015	39.911	81.661	41.686	58.172

Tab. 5: Real Capital stocks for sectors 11 - 19, 1965/1980

Years \ Sectors	11	12	13	14	15	16	17	18	19
65	49.208	133.817	27.756	140.284	46.938	169.559	633.520	20.047	151.750
66	53.192	138.754	29.325	144.127	49.826	176.687	652.259	22.404	158.943
67	56.564	144.323	30.083	148.083	52.920	182.356	671.326	25.117	167.710
68	59.296	149.294	30.550	152.364	56.314	187.207	690.953	28.273	176.756
69	62.499	152.261	31.667	156.928	60.049	193.267	710.117	31.819	185.576
70	67.140	155.669	33.437	161.981	63.988	200.165	730.165	35.626	195.146
71	73.240	159.505	36.112	167.429	68.499	208.947	754.186	39.883	206.965
72	79.975	165.773	40.274	172.821	73.754	219.024	782.111	43.812	221.039
73	86.184	172.060	43.471	178.668	79.556	230.595	814.400	47.510	235.386
74	92.592	181.291	45.740	184.917	85.107	241.803	847.602	51.345	250.278
75	96.519	189.678	46.786	190.505	90.618	253.605	879.868	55.495	266.743
76	101.303	197.871	48.047	196.317	96.552	266.344	914.138	59.458	281.680
77	106.843	205.484	49.550	203.151	103.163	279.337	948.081	63.613	296.739
78	111.556	213.187	49.939	209.827	110.798	292.750	980.312	67.860	311.822
79	116.753	218.168	50.316	217.479	119.422	306.677	1011.121	72.218	325.970
80	122.918	222.560	50.957	226.154	128.515	320.010	1039.140	76.411	338.039

Table 6: Real capital/Output ratio, 1965/1980

years \ Sectors	Primary	Secondary	Tertiary	Total
65	8.374	2.226	5.739	4.165
66	8.341	2.202	5.705	4.121
67	7.478	2.263	5.764	4.166
68	7.652	2.239	5.773	4.153
69	7.736	2.149	5.767	4.088
70	7.599	2.101	5.646	3.994
71	8.386	2.092	5.620	3.976
72	8.500	2.089	5.599	3.935
73	8.051	2.100	5.623	3.933
74	7.814	2.141	5.630	3.965
75	7.523	2.328	5.747	4.148
76	7.335	2.307	5.726	4.128
77	7.700	2.290	5.697	4.118
78	7.304	2.359	5.875	4.255
79	7.226	2.282	5.833	4.207
80	6.996	2.286	5.847	4.215

Tab. 7 : Real capital stocks for broad economic sectors, 1965/1980

1) excluding roads 2) including roads

Prim. Primary Sector
Sec. Secundary Sector
Tert. Tertiary Sector

years	Equipment				Construction						Equip. + Constr.	
	Prim.	Sec.	Tert.	Total	Prim.	Sec.	Tert.[1]	Tert.[2]	Total[1]	Total[2]	Total[1]	Total[2]
65	69 344	243 298	177 973	490.615	155.072	148.102	984.124	1088.333	1287.379	1391 588	1777.994	1882 283
66	72 212	254.652	189 934	516.798	156.920	158.234	1014.312	1127.005	1329.466	1442 160	1846.264	1950 950
67	74 897	265.207	200 934	541.038	159 135	167 698	1046.579	1169.069	1373 411	1495 902	1914 449	2036 940
68	76 656	275 741	211 728	564.126	161 551	175.559	1080 139	1212.424	1417 250	1549.534	1981 376	2113 660
69	78 858	286 686	223.482	589 025	164.219	183 416	1114 274	1257.138	1461 910	1604 774	2050 935	2193 799
70	81 564	299 323	235 994	616 880	166 319	193.771	1151.079	1305.480	1511 168	1665 570	2128 048	2282 450
71	83 535	315 576	250 861	649 972	169 236	206 950	1195.049	1362 313	1571.235	1738 500	2221 207	2388 472
72	85 722	336 910	268 199	690 830	172.330	222 855	1244 361	1424 319	1639.546	1819.504	2330 376	2518 334
73	86 042	357 342	286.920	730 305	173.936	236 478	1299.195	1491.373	1709 609	1901.788	2439 914	2632 093
74	86 964	377 290	305 676	769.929	175.608	250.472	1355.376	1561.764	1781.456	1987.845	2551.385	2757 774
75	87 642	390 938	325.756	804.337	177.213	261.624	1411.077	1631.198	1849 913	2070 034	2654 250	2874 378
76	88 704	404.525	345 750	838 979	178.617	273 548	1468.738	1700.000	1920 903	2152.965	2759 881	2991 944
77	89 979	420 633	367 725	878.338	180.260	285.943	1526 358	1769 978	1992 560	2236.181	2870.898	3114 519
78	91 165	435 484	392 520	919.169	181.974	295 626	1580 049	1837 598	2058 449	2315.198	2977 618	3234 367
79	92 251	447 418	417 771	957 440	183 232	304 872	1635.117	1904 840	2123 221	2392 945	3080 661	3350 385
80	93 993	463 054	443 752	1000 799	184.460	313.408	1684.516	1966 570	2182 384	2464 438	3183 103	3465 237

INV.QUOTE INGES REAL

IQR (A) INV.QUOTE INGES REAL
(P) PERCENTUAL CHANGES

Inv./QNP	
abs.	perc. change
.247	2.017
.255	3.530
.249	-2.577
.245	-1.293
.244	-.512
.251	2.960
.272	8.255
.287	5.577
.275	-4.325
.275	.023
.262	-4.604
.260	-.728
.262	.785
.251	-4.204
.249	-.771
.247	-.935

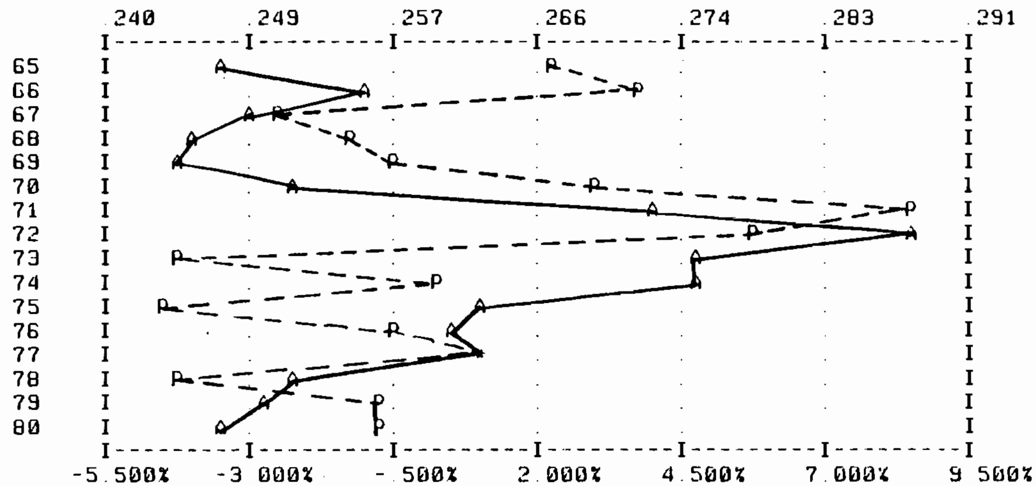

Figure 4 : Investment/QNP ratio for 1965/1980, absolute and inpercentage change.

KKR (A) KAP.KOEFF.INSGES REAL
(P) PERCENTUAL CHANGES

Year	abs.	in p.c.
65	4.165	.000
66	4.121	-1.074
67	4.166	1.093
68	4.153	-.299
69	4.088	-1.581
70	3.994	-2.292
71	3.976	-.444
72	3.935	-1.043
73	3.933	-.035
74	3.965	.800
75	4.148	4.607
76	4.128	-.465
77	4.118	-.252
78	4.255	3.328
79	4.202	-1.241
80	4.215	.295

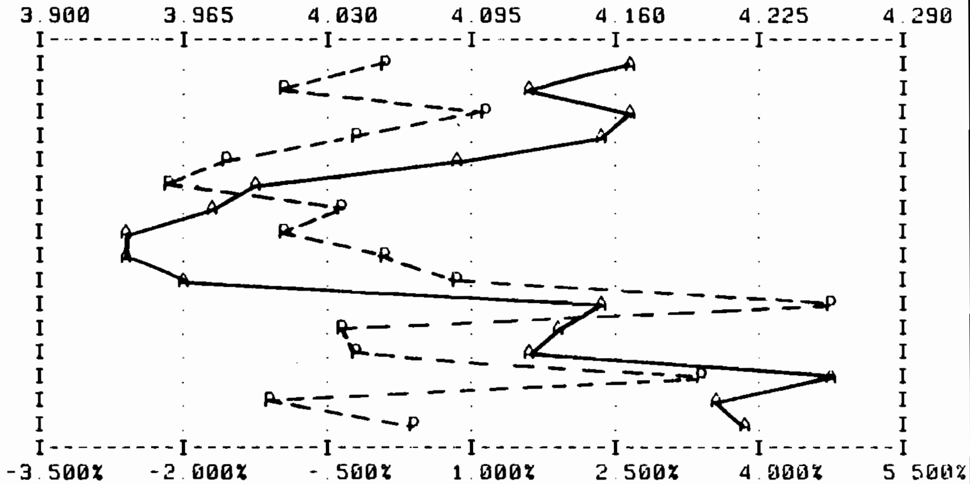

Figure 5 : Capital/Output ratio 1965/1980, absolute and in percentage change

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EMPLOYMENT EQUATIONS IN THE UK MODEL

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INTRODUCTION

Employment or productivity equations are not normally included in the first version of an INFORUM model. Yet the addition of these equations can have a profound effect on the properties of such a model. Their effects can feed back through:

1. Unit costs - levels of productivity in industry are a major determinant of unit costs. These, in turn, affect levels of relative prices which will determine the development of demand in both the domestic and foreign markets. To bring all these influences into a model would normally require the development of a full price side. At a more rudimentary stage, a partial account of developments in relative prices can be taken through the import and export equations.

2. Labour market balance - the balance of supply and demand in the labour market will influence the price of labour which again will feed through to unit costs.

3. Policy - many instruments of government policy are designed to operate through the labour market. Direct taxes, payroll taxes, etc., vary the price of labour as perceived by buyer and seller and therefore exert an influence on demand and supply.

The condition of the labour market itself is also of direct interest to policy makers, particularly at the present time when most industrialised countries face severe problems of unemployment. For example, in the UK, recent gains in productivity have exacerbated the immediate problem of unemployment. Whether the increased competitiveness which has resulted from this improved productivity will result in an increase in output which will more than compensate for the present level of layoffs remains in some doubt.

There have been dramatic movements in output, employment and productivity in the UK in the last three years. Manufacturing employment has fallen even more sharply than output and consequently productivity has risen. This contradicts the conventional view of the labour market which tends to assume that there will be a degree of labour hoarding during a recession.

APPROACHES TO MODELLING EMPLOYMENT AND PRODUCTIVITY

There are two seemingly distinct approaches to the modelling of employment and productivity relationships.

The *first approach* stresses the need to model employment and productivity from a secure base in the microeconomic theory of production. The seminal work in this area is that of Brechling (1965) who first estimated a factor demand equation consistent with the Cobb-Douglas production function.

Since then, a great deal of refinement has been added to this work. Composite sets of factor demand equations have been estimated simultaneously - see for example Nadiri and Rosen (1969). More flexible functional forms have been used for the underlying production relationships. The duality between production and cost relationships has been exploited to yield factor demand equations specified in terms of relative input prices.

This literature suggests that one should carefully consider the implications of the choice of any particular method of estimation for an INFORUM system. One major question which needs to be resolved is whether all factor demand equations should be estimated simultaneously.

Such an approach can usefully exploit cross-equation information to improve the efficiency of the estimation procedure. Of course efficiency gains can easily be offset by the costs of misspecification - this is the essence of Peterson's (1979) argument against the systems approach. He argues:

(a) that cross-equation constraints on the cross and price and substitution effects, namely that the substitution matrix should be symmetric and negative semi-definite, are only valid in the case of the static factor demand system. This has been demonstrated by Treadway (1971).

(b) that there is a considerable difference in the gestation lag between employment and capital decisions. Employment decisions can be and are revised in a shorter period than decisions to replace the capital stock. Thus the proper specification should be recursive with the direction of causation running from capital to labour. This argument is similar to that put forward by Hart and Sharot (1978) regarding the demand for employment and that for hours.

There are some considerations which perhaps might modify this argument, namely:

(i) capital comes in a whole variety of forms: buildings are certainly subject to long gestation lags - yet the same is not true for many forms of plant and equipment - it is not entirely clear that the whole spectrum of gestation lags are subject to longer decision lags than employment.

(ii) the utilization of capital can be varied at relatively short notice - so can the utilisation of labour through overtime and short time working - this is perhaps the decision which, by implication, should come last in any system of recursive factor demand equations. It is also perhaps worth noting that since capital and labour utilisation are complementary it is worth considering the use of an hours variable as an indicator of short term capacity utilisation.

Yet all such systems essentially rely on an initial static formulation of producer behaviour. Adjustment costs (normally in a quadratic form) are tagged on to the static optimising equations in order to provide some dynamic terms in each equation and give a more satisfactory representation of the stochastic properties of the dependent variable.

The *second approach* contrasts sharply with the first in that it was not primarily developed from a microeconomic basis, but rather tends to rely on some observed regularities at the macroeconomic level. One of the most common of these is the so-called "Verdoorn's Law" following Verdoorn (1949) which states that there exists:

> "a fairly constant relation over a long period between the growth of labour productivity and the volume of industrial production" (Verdoorn 1949)

Sectors which are growing rapidly will tend to have a younger and more technologically advanced capital stock: they will be able to realise greater economies of scale. This is the basis of the argument put forward by Verdoorn to support his empirical findings which appeared to support the thesis stated above.

In the UK this argument was taken up by Lord Kaldor who claimed it as the second of his three laws concerning the growth of sectoral output and productivity in advanced countries. Kaldor believed that there were substantial economies of scale in manufacturing which could be demonstrated by a regression of the form:

$$p = a + bq \tag{1}$$

where p and q are the growth rates of productivity and output respectively. Kaldor found that his results generally showed that the parameter b took a value close to 0.5 indicating significant returns to scale. Further the direction of causation implies that what is necessary to stimulate productivity is a significant boost to demand and thence to output. In effect this argument provided a rationale for the advocacy of strongly stimulatory fiscal policies as a basis of economic management.

Yet it is not entirely clear that this is the appropriate direction of causation. It could be the case that exogenous productivity growth leads to gains in output. Under this scenario presumably output prices should remain fairly constant or even fall since improved productivity should lead to a downward shift of the supply curve. Presumably the consumer electronics market is a prime example of just such an effect. Caves' (1968) study of the UK economy argued, from information on price behavior, that the direction of causation tended to run from productivity to output.

The simple INFORUM productivity relationships are an expression of the Verdoorn relationship with the direction of causation running from output to productivity. Let us summarise the implications of building such a series of relationships into a model.

First, the simple productivity relationships cannot take account of variations in input prices as an influence on demand for the factors of production. Thus an increase in the price of capital vis-à-vis that of labour will not solve the problem of unemployment (assuming labour and capital are substitutes - this may be questionable for certain categories of labour).

Second, the relationships take no explicit account of the role of capital in determining productivity. It is presumably implicit in the relationships that the capital stock grows at a sufficient rate to accommodate the output growth. Or perhaps there is a constant capital/output ratio such that the relationship runs from the capital stock to productivity rather than from output to productivity. Any attempt to distinguish between these hypotheses is usually hampered by the lack of sectoral capital stock data.

Third, the short term dynamics of the relationships are unclear. Without any microeconomic framework, there is little on which to base any dynamic specification. Thus it is difficult to assess whether the recent rapid rise in productivity in the UK at a time of rapidly declining manufacturing output is merely a short term fluctuation or a more serious challenge to the

Verdoorn relationship. Without some adequate dynamic specification, it is clear that the productivity/output relationships will not generate successful short-term predictions.

EMPIRICAL CONVERGENCE OF THE TWO APPROACHES

So far we have considered the two approaches to employment functions as being distinct. Yet the empirical specifications which emerge from them are not entirely distinctive. Consider the following argument. Equation (1) can be re-expressed as:

$$e = f + gq \qquad (2)$$

where e, the rate of change of employment, is equal to p - q. Now, approximately e by log E - log E (1) and q by log Q - log Q (1), where E and Q are, respectively, levels of employment and output, we have

$$\log E = f + g \log Q - g \log Q(1) + \log E(-1) \qquad (3)$$

which can be contrasted with the type of function normally used in short term employment equations:

$$\log E = a + b \log Q + c \log E(-1) \qquad (4)$$

The major differences between (3) and (4) are:

(1) The assumption that the coefficient on log E (-1) is unity in the Verdoorn relationship whereas it is c in the employment function. Normally we would expect $0 \leq c \leq 1$ to maintain the stability of the equation.

(2) The inclusion of a lagged output term in the Verdoorn relationship. This is a trivial difference since the use of lagged output terms is fairly common in the employment equation literature. It is normally rationalised on the grounds that what governs entrepreneurial decisions is not actual output but *expected* output. It is often argued that expectations are simply extrapolations of past experience, thus justifying the use of lagged

output terms in the specification. It should be noted, however, that such an approach is anathema to the rational expectations school who would argue that such expectations schemes can lead to systematic (and therefore suboptimal) errors in the forecasts of future output.

It is apparent that one can easily devise a sufficiently general specification such that the Verdoorn and the employment function specifications can be nested within it. Further, since neither of these relationships, for a variety of reasons includes a factor price term, one can extend the specification further to potentially encompass long-run real wage effects. This is the approach which is currently being followed at the Fraser Institute. Results for a fairly general autoregressive process for manufacturing industry as a whole are shown below. The limitation on log length was solely determined by degrees of freedom considerations.

$$E = \underset{(-1.99)}{-4.44} + \underset{(2.84)}{1.18}\,E(-1) \underset{(-0.01)}{-0.07}\,E(-2) \underset{(-3.42)}{-0.30}\,RW \underset{(-0.23)}{-0.04}\,RW(-1)$$

$$+ \underset{(7.30)}{0.72}\,Q \underset{(-0.06)}{-0.02}\,Q(-1)$$

where all variables are in logarithms. Figures in brackets are t statistics. This more general form of specification follows observed employment patterns more closely. This is not surprising given that it is essentially a reduced form equation which is compatible with a wide number of behavioural hypotheses. Whether it is important to discriminate between these for forecasting purposes is a moot point. At present a number of restrictions are being tested although the signs and sizes of this unrestricted version seem fairly satisfactory.

While this is the direction in which work is presently going, some further results are reported in this paper which describe attempts to see whether there exist simpler modifications to the Verdoorn relationship which describe the recent employment experience of the UK adequately.

The underlying argument is that UK industry has, for a considerable period of time, operated below the frontier of its production possibility curve. Productivity did grow gradually during the sixties but showed no discernible trend during the seventies. However, although output fell dramatically in the period 1979-1982, productivity rose substantially. This behaviour might lead one to suspect that what determines productivity growth is the change in output, *whatever* the direction of that change. As output falls, firms are forced to draw on previously unrealised reservoirs of productivity in order to survive. In contrast, when output rises the usual processes of economies of scale serve to improve productivity levels. Only when output is stagnant will there be no change in productivity levels.

This suggests a specification of the form

$$p = a + b\,|q| \tag{5}$$

where $|q|$ is the absolute value of the change in output. Results of this equation for various sectors of UK manufacturing industry are shown alongside those for equation (1), the standard INFORUM productivity equation, in the Appendix.

Now the specification (5), though heuristically appealing, imposes one major restriction on the data. The response of productivity to a given percentage fall in output is constrained to be the same as that for an *increase* of the same magnitude. There is no a priori reason to believe that these reactions should be of identical size. To allow slightly more flexibility, a third specification was tried which made the growth of productivity a quadratic function of the growth of output as shown in (6)

$$p = a + bq + cq^2 \tag{6}$$

In the regression the growth in labour input per unit of output has been used as the dependent variable, following INFORUM precedents. Thus the various possibilities for the relationships are shown in Figure 1.

The results are mixed. No specification is uniformly superior across industries, suggesting as one might expect that returns to scale and potential gains from previously unrealised efficiency improvements are spread fairly diversely throughout industry. One finding is almost completely uniform however: nearly all the equations underpredict the rise in productivity in 1981. The degree of unanimity is disturbing, suggesting that perhaps there is an omitted variable or variables in all the equations which would give a more random pattern of residuals in 1981. Hence our current experiments with real wage variables.

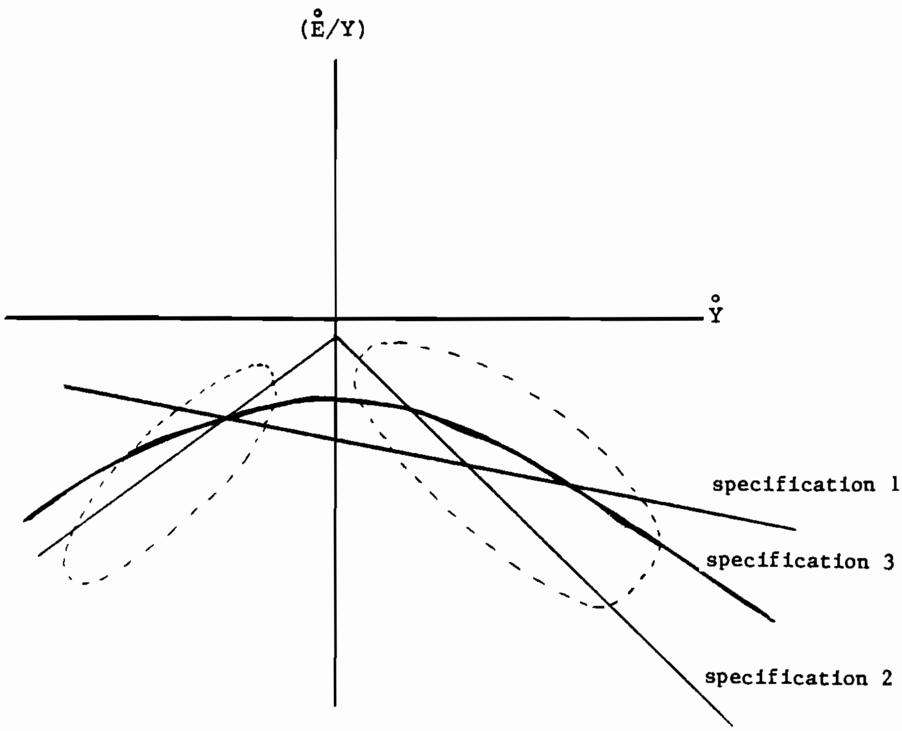

FIGURE 1.

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APPENDIX

Industry	Spec	a	b	c	Rsq.	DW	1981
Manufacturing	1	-0.02	-0.47		0.45	1.55	-0.666
		(-3.7)	(-4.03)				
	2	-0.02	-0.31		0.09	1.94	-0.021
		(-1.52)	(-1.42)				
	3	-0.02	-0.46	-2.97	0.48	1.66	-0.062
		(-2.41)	(-3.39)	(-1.03)			
Mining & Quar	1	-0.05	-0.86		0.95	1.48	0.002
		(-7.51)	(-19.38)				
	2	-0.02	-0.63		0.31	2.19	-0.040
		(-0.59)	(-2.96)				
	3	-0.05	-0.89	0.15	0.95	1.52	0.005
		(-6.95)	(-14.28)	(0.65)			
Food,Drink & T	1	-0.02	-0.30		0.09	1.89	-0.035
		(-3.87)	(-1.40)				
	2	0.01	-1.18		0.42	1.41	-0.022
		(0.13)	(-3.83)				
	3	-0.01	0.23	-30.23	0.47	1.49	-0.016
		(-2.17)	(1.05)	(-3.68)			
Coal & Petrol	1	-0.02	-0.84		0.63	1.85	-0.018
		(-2.08)	(-5.82)				
	2	-0.07	0.63		0.09	1.65	-0.053
		(-2.33)	(1.42)				
	3	-0.04	-0.76	3.25	0.68	1.81	-0.017
		(-2.70)	(-5.20)	(1.67)			
Chemicals	1	-0.01	-0.89		0.75	1.55	-0.078
		(-1.04)	(-7.65)				
	2	-0.07	-0.65		0.15	2.31	-0.060
		(0.32)	(-1.85)				
	3	-0.00	-0.86	-1.39	0.75	1.52	-0.082
		(0.38)	(-6.76)	(-0.82)			
Metal Manuf.	1	-0.02	-0.89		0.69	1.29	-0.193
		(-1.97)	(-6.66)				
	2	-0.06	0.69		0.26	2.02	-0.229
		(-2.44)	(2.66)				
	3	-0.03	-0.76	0.24	0.69	1.23	-0.194
		(-1.89)	(-4.71)	(0.34)			
Mech.Eng.	1	-0.02	-0.18		0.07	1.26	-0.004
		(-2.00)	(1.21)				
	2	-0.00	-0.31		0.07	1.26	-0.004
		(-0.32)	(-1.24)				
	3	-0.01	-0.15	-2.34	0.11	1.15	-0.014
		(-0.89)	(-1.04)	(-0.98)			

Industry	Spec	a	b	c	Rsq.	DW	1981
Inst Eng	1	-0.035	-0.36		0.22	1.63	-0.056
		(-3.52)	(-2.42)				
	2	-0.01	-0.72		0.45	1.67	-0.005
		(-0.81)	(-4.06)				
	3	-0.01	0.08	-6.20	0.48	1.59	-0.002
		(-3.15)	(0.41)	(-2.99)			
Elec Eng	1	-0.03	-0.34		0.19	1.34	-0.046
		(-2.96)	(-2.0)				
	2	-0.00	-0.73		0.39	0.94	-0.016
		(-0.19)	(-3.55)				
	3	-0.02	-0.02	-5.93	0.40	1.08	-0.019
		(-2.62)	(0.12)	(-2.53)			
Shipbuilding	1	-0.02	-0.67		0.59	0.92	-0.053
		(-3.34)	(-5.42)				
	2	-0.04	0.78		0.43	1.60	-0.035
		(3.65)	(3.88)				
	3	-0.02	-0.95	-3.70	0.62	0.90	-0.062
		(-2.06)	(-3.60)	(-1.22)			
Vehicles	1	-0.01	-0.50		0.56	1.30	-0.074
		(-2.03	(-5.09)				
	2	-0.01	-0.05		0.01	1.80	-0.016
		(-0.51)	(-0.14)				
	3	-0.01	-0.62	-2.78	0.60	1.27	-0.058
		(-0.62)	(-5.33)	(-1.29)			
Metal Gds nes	1	-0.01	-0.55		0.58	1.61	-0.107
		(-0.66)	(-5.23)				
	2	-0.01	-0.14		0.01	1.87	-0.028
		(-0.30)	(-0.49)				
	3	-0.00	-0.56	-0.93	0.59	1.75	-0.097
		(0.002)	(-5.25)	(-0.85)			
Textiles	1	-0.04	-0.59		0.59	1.87	-0.077
		(-4.75)	(-5.35)				
	2	-0.04	0.16		0.02	2.06	-0.027
		(-2.37)	(0.64)				
	3	-0.03	-0.65	-0.98	0.61	1.97	-0.075
		(-3.69)	(5.28)	(-1.01)			
Leather Goods	1	-0.021	-0.73		0.74	0.29	-0.073
		(-3.16)	(-7.46)				
	2	-0.04	0.75		0.47	2.41	-0.056
		(-3.18)	(4.22)				
	3	-0.02	-0.76	-0.18	0.74	2.28	-0.074
		(-3.02)	(-4.39)	(-0.19)			

Industry	Spec	a	b	c	Rsq.	DW	1981
Clothing & Ft	1	-0.02	-0.47		0.49	1.86	-0.045
		(-4.66)	(-4.42)				
	2	-0.04	0.25		0.61	2.00	-0.012
		(-3.20)	(1.14)				
	3	-0.02	-0.49	-0.02	0.49	1.86	-0.045
		(-3.52)	(-3.49)	(-0.14)			
Brcks,Po,Gl&C	1	-0.02	-0.55		0.59	1.69	-0.075
		(-3.60)	(-5.38)				
	2	-0.01	-0.45		0.16	1.94	0.001
		(-0.58)	(-1.94)				
	3	-0.02	-0.51	-1.52	0.62	1.90	-0.064
		(-2.22)	(-4.79)	(-1.14)			
Timber & Furn	1	-0.01	-0.66		0.82	2.55	-0.032
		(-2.15)	(-9.58)				
	2	-0.03	0.17		0.02	2.28	0.030
		(-1.33)	(0.66)				
	3	-0.02	-0.66	0.72	0.83	2.56	-0.034
		(-2.43)	(-9.59)	(1.15)			
Paper & Print	1	-0.01	-0.73		0.71	1.24	-0.049
		(-1.57)	(-7.00)				
	2	-0.02	0.02		0.00	1.88	-0.001
		(-1.16)	(0.01)				
	3	-0.01	-0.74	-0.29	0.71	1.21	-0.050
		(-1.11)	(-6.57)	(-0.23)			
Other Manuf	1	-0.02	-0.42		0.58	1.78	-0.038
		(-3.18)	(-5.26)				
	2	-0.00	-0.48		0.30	1.44	0.047
		(-0.28)	(-2.95)				
	3	-0.01	-0.36	-1.50	0.63	1.71	-0.019
		(-1.81)	(-4.27)	(-1.61)			
Construction	1	-0.01	-0.56		0.42	1.20	-0.047
		(-1.32)	(-3.81)				
	2	-0.02	0.14		0.01	1.39	0.011
		(-1.10)	(0.47)				
	3	-0.01	-0.59	-1.20	0.43	1.04	-0.037
		(-0.67)	(-3.68)	(-0.57)			
Utilities	1	-0.01	-1.11		0.56	0.58	-0.019
		(-0.10)	(-5.06)				
	2	0.01	-1.21		0.44	0.87	-0.025
		(0.42)	(-3.95)				
	3	-0.00	-1.19	1.42	0.56	0.61	-0.020
		(-0.88)	(-2.62)	(0.21)			

INPUT–OUTPUT ANALYSIS AND LINEAR PROGRAMMING: THE GENERAL INPUT–OUTPUT MODEL

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A. Introduction

With his first publications LEONTIEF (1936, 1937, 1941) founded input-output analysis. DANTZIG (1951) developed the simplex method to solve linear optimization problems. DORFMAN, SAMUELSON and SOLOW (1958) proved that there is a strong relationship between input-output analysis and linear programming. Although further books and articles on this subject have been published [1)], it is obvious that in empirical input-output analysis the instruments of operations research are widely neglected.

Mathematically, input-output analysis can be regarded as a special statement of linear programming models. Both approaches are methodologies to solve linear equations systems. The subject of this paper will be to demonstrate that the common price and quantity models of input-output analysis can be transferred into linear programming models with a substantial gain of information for the users. An essential advantage of a linear programming input-output model is the fact that this approach includes all information of the four quadrants of an input-output table. Therefore, it comprises the entire production system simultaneously. In contrast, the traditional quantity and price models encompass only two quadrants of an input-output table at a time.

1) See for example KOOPMANS (1951), SCHUMANN (1968), BOMSDORF (1977), PASINETTI (1977), and BEUTEL/MORDTER (1982).

One of the most important facts about linear programming is that to every programming problem there corresponds a dual problem. If the original problem, called the primal problem, is a maximum problem then the dual problem is a minimum problem. The general input-output model in the linear programming version therefore comprises simultaneously the primal quantity model and the dual price model of input-output analysis. A second major advantage of this methodology is the fact that linear programming is not restricted to square matrices [2]. In other words, in an input-output model the number of commodities does not have to equal the number of activities. In the empirical application of this approach in the second part of this study we will benefit from this distinctive feature. The subject will be to analyse the substitution of production activities within the European Communities.

B. Input-Output Analysis and Linear Programming

For linear models, the traditional approach of input-output analysis is best expressed by an application of linear programming. We begin with a simple numerical example. For a closed economy the following input-output table will be given in value terms:

Table 1: Input-Output Table

Millions of DM	Agri culture 1	Coal 2	Elec-tricity 3	In-dustry 4	Servi-ces 5	Final Demand 6	Output 7
1 Agriculture	10	60	5	9	12	4	100
2 Coal	20	30	40	30	30	50	200
3 Electricity	10	20	20	90	60	200	400
4 Industry	30	12	24	120	90	324	600
5 Services	6	24	12	21	15	222	300
6 Labour	24	54	299	330	93	-	800
7 Input	100	200	400	600	300	800	1600

2) See MATUSZEWSKI (1972) on conventional rectangular input-output models.

The primal version of the customary input-output models (quantity model) is used to project commodity outputs for an exogeneously given final demand. The dual version of the national input-output models (price model) is often implemented to analyze changes in commodity prices which are caused by increases or decreases in wages or other prices for primary inputs. Both models can be formulated in physical units and value units[3]:

<u>Primal Model in physical units:</u>	<u>Primal Model in value units:</u>	
$Q = FQ + Z$	$X = AX + Y$	(1)
$Q - FQ = Z$	$X - AX = Y$	(2)
$(I - F)Q = Z$	$(I - A)X = Y$	(3)
$Q = (I-F)^{-1} \cdot Z$	$X = (I - A)^{-1} \cdot Y$	(4)

<u>Dual model in value units:</u>	<u>Dual model in value units:</u>	
$P = F'P + W$	$V = A'V + U$	(5)
$P - F'P = W$	$V - A'V = U$	(6)
$(I - F')P = W$	$(I - A')V = U$	(7)
$P = (I - F')^{-1} \cdot W$	$V = (I - A')^{-1} \cdot U$	(8)

The notation is defined as follows:

F = matrix of technical input coefficients for commodities in physical units
Q = vector of commodity outputs in physical units
Z = vector of final demands in physical units
A = matrix of technical input coefficients for commodities in value units
X = vector of commodity outputs in value units
Y = vector of final demands in value units
P = vector of actual prices
V = vector of relative prices
W = vector of value added per unit of output in physical units
U = vector of value added per unit of output in value units

The basic quantity model of input-output analysis only comprises two quadrants of an input-output table, namely the first quadrant for intermediate production and the second quadrant for final demand. With the next conventional step we will determine for our example in table 1 the activity levels of production for an exogeneously given final demand. The primal model in value units AX + Y = X will be:

3) The following notation is based on POLENSKE (1980), p. 123.

$$\begin{bmatrix} 0.1000 & 0.3000 & 0.0125 & 0.0150 & 0.0400 \\ 0.2000 & 0.1500 & 0.1000 & 0.0500 & 0.1000 \\ 0.1000 & 0.1000 & 0.0500 & 0.1500 & 0.2000 \\ 0.3000 & 0.0600 & 0.0600 & 0.2000 & 0.3000 \\ 0.0600 & 0.1200 & 0.0300 & 0.0350 & 0.0500 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 4.00 \\ 50.00 \\ 200.00 \\ 324.00 \\ 222.00 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

This approach leads to the following solution $X = (I-A)^{-1} Y$:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1.2603 & 0.4792 & 0.0762 & 0.0741 & 0.1430 \\ 0.3847 & 1,3749 & 0.1658 & 0,1347 & 0.2384 \\ 0,2988 & 0,3049 & 1,1157 & 0.2495 & 0.3584 \\ 0.5836 & 0.3912 & 0.1496 & 1.3362 & 0.5192 \\ 0.1591 & 0.2280 & 0.o665 & 0.0788 & 1.1222 \end{bmatrix} \cdot \begin{bmatrix} 4.00 \\ 50.00 \\ 200.00 \\ 324.00 \\ 222.00 \end{bmatrix} = \begin{bmatrix} 100.00 \\ 200.00 \\ 400.00 \\ 600.00 \\ 300.00 \end{bmatrix}$$

In contrast, the general input-output model of activity analysis includes all four quadrants of an input-output table, namely quadrant 1 for the intermediate production, quadrant 2 for the final demands, quadrant 3 for value added in production activities, and quadrant 4 for value added in final demand activities.

At the beginning the challenge is to formulate a linear program which leads to the same solution as the preceding conventional primal model of input-output analysis. The Leontief System I will be:

Maximize

$Z = P \cdot X$ Objective Function (9)

subject to

$X - AX \leqq Y$ Constraints (10)

and

$X \geqq 0$ Nonnegativity (11)

The notation is defined as follows:

A = matrix of technical input coefficients for commodities and primary resources (capital, labour)
X = vector of commodity outputs and employment of primary resources
Y = vector of final demand including leisure or unused capacities of capital
P = vector of prices

For this system the objective is to maximize revenue subject to the constraints of an exogeneously given final demand and a given technology of intermediate production. Under these conditions the objective to maximize revenue implies the objective to maximize profits. Due to the price model of input-output analysis the ficticious prices of all commodities will be $p_j=1.0$ if all input coefficients (commodities and primary ressources) sum up to $a_{.j}=1.0$ for each production activity. Therefore the vector P in the objective function (9) is a vector of unit prices for the different commodities. Furthermore, the traditional quantity model of input-output analysis with exogeneous final demand implies the hidden assumption that the economy does not have to face restricted capacities of primary inputs. Therefore, the amount of labour is not restricted in our next example.

Table 2: Leontief System I

	Activities					Slack Variables (Final Demand)						Re-striction
	Coal	Elec-tricity	Agri-culture	In-dustry	Ser-vices	Commo-dity 1	Commo-dity 2	Commo-dity 3	Commo-dity 4	Commo-dity 5	La-bour	
	1	2	3	4	5	6	7	8	9	10	11	12
Statement:												
1 Coal	0.9000	-0.3000	-0.0125	-0.0150	-0.0400	1.0000						4.0
2 Electricity	-0.2000	0.8500	-0.1000	-0.0500	-0.1000		1.0000					50.0
3 Agriculture	-0.1000	-0.1000	0.9500	-0.1500	-0.2000			1.0000				200.0
4 Industry	-0.3000	-0.0600	-0.0600	0.8000	-0.3000				1.0000			324.0
5 Services	-0.0600	-0.1200	-0.0300	-0.0350	0.9500					1.0000		222.0
6 Labour	-0.2400	-0.2700	-0.7475	-0.5500	-0.3100						1.0000	0.0
7 Objective	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000							0.0
Solution:												
1 Coal	1.0000					1.2603	0.4792	0.0762	0.0741	0.1430		100.00
2 Electricity		1.0000				0.3847	1.3749	0.1658	0.1347	0.2384		200.00
3 Agriculture			1.0000			0.2988	0.3049	1.1157	0.2495	0.3584		400.00
4 Industry				1.0000		0.5836	0.3912	0.1496	1.3362	0.5192		600.00
5 Services					1.0000	0.1591	0.2280	0.0665	0.0788	1.1222		300.00
6 Labour						1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	800.00
7 Objective						2.6865	2.7782	1.5739	1.8733	2.3811		1600.00

At first sight it is obvious that the competing data of the linear programming model in columns 6-10 correspond to the inverse matrix in equation (4). In addition we can find some more interesting information in row 6 and row 7 of the same columns. The results in row 6 can be interpreted as the shadow prices of the different commodities. They are identical with the cost prices in competitive markets or the labour content of goods and services which result from an application of the price model. Finally, in row 7 the column sum of the inverse matrix is shown. These figures represent important economic multipliers for the activity levels of production.

With the next step we would like to introduce an important result of modern consumer theory[4]. Due to this theory consumers are far more interested in carrying out certain consumption activities than in just consuming certain quantities of goods and services. In order to carry out these consumption activities the consumers require - similarly to the production activities - certain inputs in constant proportions. For our simple example we purposely consider only one vector of final demand. The general assumption is now that all economic activities (production and consumption) are facing constant input coefficients in the short run.

In the following Leontief System II the capacity of labour is restricted to the given level of table 1 (L^o=800). The objective is to maximize final demand in the given structure. The corresponding value in the objective function (P_6=1.0) can be regarded as the "numéraire" of the economic system.

The Leontief System II has the following structure:

Maximize

$$Z = P X \qquad (12)$$

subject to

$$AX - X \leq 0 \qquad (13)$$

$$BX \leq L^o \qquad (14)$$

and

$$X \geq 0 \qquad (15)$$

The notation is defined as follows:

A = matrix of technical input coefficients for commodities
B = vector of technical input coefficients for labour
X_o= vector of commodity outputs
L^o= capacity of labour
P = vector of prices

4) See LANCASTER (1971).

Table 3: Leontief System II

	Activities						Slack Variables (Change of Inventories)						Re-striction
	Coal	Elec-tricity	Agri-culture	In-dustry	Ser-vices	Final Demand	Commo-dity 1	Commo-dity 2	Commo-dity 3	Commo-dity 4	Commo-dity 5	La-bour	
	1	2	3	4	5	6	7	8	9	10	11	12	13
Statement:													
1 Coal	-0.9000	0.3000	0.0125	0.0150	0.0400	0.0050	1.0000						0.00
2 Electricity	0.2000	-0.8500	0.1000	0.0500	0.1000	0.0625		1.0000					0.00
3 Agriculture	0.1000	0.1000	-0.9500	0.1500	0.2000	0.2500			1.0000				0.00
4 Industry	0.3000	0.0600	0.0600	-0.8000	0.3000	0.4050				1.0000			0.00
5 Services	0.0600	0.1200	0.0300	0.0350	-0.9500	0.2775					1.0000		0.00
6 Labour	0.2400	0.2700	0.7475	0.5500	0.3100							1.0000	800.00
7 Objective						-1.0000							0.00
Solution:													
1 Final Demand						1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	800.00
2 Coal	1.0000						-1.1353	-0.3542	0.0488	0.0509	-0.0180	0.1250	100.00
3 Electricity		1.0000					-0.1347	-1.1249	0.0842	0.1153	0.0116	0.2500	200.00
4 Agriculture			1.0000				0.2012	0.1951	-0.6157	0.2505	0.1416	0.5000	400.00
5 Industry				1.0000			0.1664	0.3588	0.6004	-0.5862	0.2308	0.7500	600.00
6 Services					1.0000		0.2159	0.1470	0.3085	0.2962	-0.7472	0.3750	300.00
7 Objective							1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	800.00

Again the inverse production coefficients of input-output analysis can be derived from the competing linear programming data[5]. Now each cumulative (inverse) production coefficient can be split into two components which represent the capacity effect of primary resources and the structural effect of final demand. The first set of competing data in columns 7-11 of table 3 include the results of a marginal deviation of the given structure of final demand under the constraint that the capacity of primary resources remains unchanged. It is obvious that under these conditions certain production activities can expect gains, others must expect losses in production. In contrast to these results in column 12 the effect is shown which results from a marginal increase of the capacity of primary resources under the constraint that the structure of final demand remain unchanged. Under these conditions all production activities can expect gains in production. The difference of both structural components will lead us again to the well-known inverse of equation (4)[6].

5) See BEUTEL/MORDTER (1982), pp. 233.

6) For example, the elements of the inverse can be computed in the following way: competing data for labour minus competing data for commodities equals element of the inverse matrix, for example 0.1250 - (-1.1353) = 1.2603 or 0.1250 - (-0.3542) = 0.4792 for the first two elements.

The main characteristics of the general input-output model can be summarized in the following way:

o Primal and dual model

An essential advantage of the linear programming input-output model is the fact that it comprises the entire production system. It evaluates all information of an input-output table simultaneously. The primal quantity model and the dual price model are included at the same time.

o Shadow prices and cumulative production coefficients

The cumulative (inverse) production coefficients can be derived from the competing programming data. In addition the solution includes shadow prices for the different commodities.

o Rectangular input-output models

Rectangular input-output systems can be analyzed with linear programming input-output models. In this way no valuable information is lost by aggregation to square matrices.

o Substitution

According to the Leontief production functions no substitution among the inputs will be allowed. However, the substitution of techniques can be analyzed with the general input-output model.

In this context we would like to refer to the theorem of nonsubstitution[7]. According to this theorem only one production activity out of many techniques is efficient to produce a certain commodity independent of the structural components of final demand if only one primary input and no joint production exist. In a situation with more than one primary resource the structure of final demand has a definite influcence on the choice of techniques. In our example and in the following empirical application of the general input-output model we therefore aggregated the different components of value added into one primary input.

7) See SAMUELSON (1951) and CHIPMAN (1953).

C. Empirical Application

The discussion of energy problems is currently encountering limits. An essential reason for this is the lack of adequate statistics and instruments. The Commission of the European Communities therefore asked the Federal Statistical Office of the Federal Republic of Germany and the Ifo Institute for Economic Research in Munich to establish, in close cooperation, input-output tables of energy flows. Relevant studies were simultaneously conducted in six other EEC countries (France, Italy, Belgium, Netherlands, United Kingdom, and Denmark)[8].

The input-output tables of energy flows do not show energy flows only. In value units they are complete input-output tables comprising all production activities with an appropriate aggregation of 45 sectors for analyzing energy problems. A special feature of the input-output tables of energy flows consists in presenting all flows of energy sources in physical units (terajoule) and value units (mill. DM). The physical unit "joule" is a unit which measures the actual energy content of each energy flow. At the same time it ensures that the quantities of different energy sources can be aggregated by column and row (see fig. 1).

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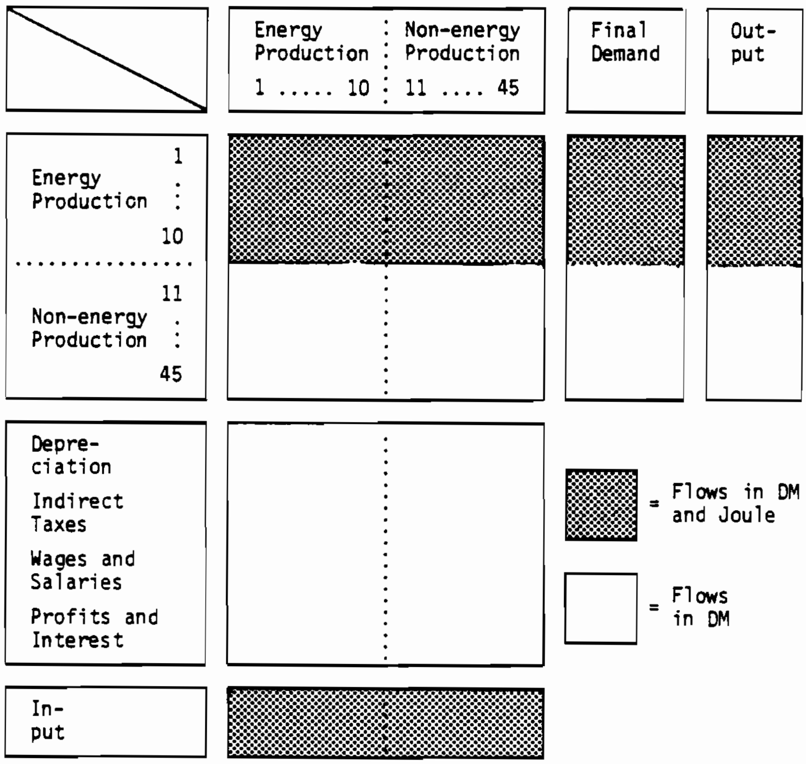
8) The input-output tabels of energy flows will be published by the national statistical offices. See for instance STATISTISCHES BUNDESAMT (1982) or STATISTICAL OFFICE OF THE EUROPEAN COMMUNITIES (1982).

Figure 1: Input-Output Table of Energy Flows



With the input-output tables of energy flows a statistical data base is made available which allows to determine the total energy content of goods and services in physical units and value units[9]. These estimates are very important for energy forecasting.

In our empirical application it will be shown how mixed input-output systems in terms of physical and value units can be analysed with the general input-output model. The objective will be to investigate structural differences in the energy production and energy use of France, Italy and the Federal Republic of Germany[10].

9) See BEUTEL/STAHMER (1982).

10) At the moment input-output tables of 1975 energy flows are only available for the following countries: Italy, France, Denmark, and the Federal Republic of Germany

With the help of the following programming model we want to examine if there are significant differences in the efficiency of the production functions within the European Communities. This can be done in two ways. One approach would be to examine all inputs in value units which have a quantity character (intermediate inputs, depreciation, wages and salaries). This procedure would be an analysis of cost functions which represent the underlying production functions[11]. A second and very promising approach is to use a data base which is as close as it can be to the concept of production functions.

With the following linear program we will analyze input-output tables with mixed data in terms of quantities and values. All flows of energy sources will be given in quantity units (terajoule), those of all other inputs in value units (mill. DM).

The structure of the optimization model for deriving efficient production activities corresponds to the Leontief system II. The statement will be presented in table 4. It is important to keep in mind that the input coefficients of the model are defined in different dimensions (joule/joule, DM/joule, joule/DM, DM/DM).

According to the theorem of nonsubstitution the sample of efficient production activities is independent of the structure of final demand if there is just one constraint of primary inputs. In our comparative analysis we aggregated the different components of value added (depreciation, indirect taxes, wages and salaries, profits and interest) to one vector of primary inputs. As constraint of the system we chose the amount of gross value added for the Federal Republic of Germany in the year 1975 (967.831 bill. DM). The objective is to maximize final demand in the given structure for the Federal Republic of Germany. The vector in column 136 of the linear program is therefore determined by input coefficients for final demand. There is no valuation problem in the objective function. The reason is that we just need a "numeraire" (p=1.0) for the vector of input coefficients for final demand. We would get the same results for efficiency if we entered the corresponding data for France, Italy or a fantasy country. The matrices for intermediate inputs include domestic and foreign inputs.

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11) We cannot follow this approach in this study because France didn't publish the different components of value added. The results for Germany were published in BEUTEL/MORDTER (1982).

Table 4: Evaluation of Production Activities for France, Italy, and the Federal Republic of Germany in the Year 1975

	West-Germany Energy Production 1 10	West-Germany Non-energy Production 11 45	France Energy Production 46 55	France Non-energy Production 56 90	Italy Energy Production 91 ... 100	Italy Non-energy Production 101 .. 135	Final Demand FRG 136	Dummy Variables Energy Production 137 .. 146	Dummy Variables Non-Energy Production 147 .. 181	Dummy Variables Primary Inputs 182	Re-striction 183
Energy Inputs 1 : 10	A-I	C	A-I	C	A-I	C	G	1 ⋱ 1			
Non-energy Inputs 11 : 45	B	D-I	B	D-I	B	D-1	H		1 ⋱ 1		
Primary Inputs 46	E	F	E	F	E	F				1	L^0
Objective Function 47							P				Max

Legend:

A = matrix of technical input coefficients for the requirements of energy in energy production activities (joule/joule)
B = matrix of input coefficients for the requirements of non-energy inputs in the energy production activities (DM/joule)
C = matrix of input coefficients for the requirements of energy in non-energy production activities (joule/DM)
D = matrix of input coefficients for the requirements of non-energy inputs in non-energy production activities (DM/DM)
E = vector of input coefficients for the requirements of primary inputs in energy production activities (DM/joule)
F = vector of input coefficients for the requirements of primary inputs in non-energy production activities (DM/DM)
G = vector of input coefficients for the requirements of energy in final demand activities (joule/DM)
H = vector of input coefficients for the requirements of non-energy inputs in final demand activities (DM/DM)
L^0= capacity constraint of primary inputs (DM)
P = valuation of final demand activity (numéraire)
I = unit matrix

Table 5: Efficient Production Activities for France, Italy and the Federal Republic of Germany

Production Activities	Shadow Prices in mill. DM		
	West-Germany	France	Italy
1 Coal	0.6	-	x
2 Lignite	-	1.6	1.4
3 Coke	1.3	0.1	-
4 Crude oil	0.3	-	0.7
5 Petroleum products .	0.8	0.5	-
6 Natural gas	2.1	14.0	-
7 Electricity	3.4	-	1.5
8 Produced gas	-	4.5	1.3
9 Steam, hot water ...	1.3	-	x
10 Nuclear fuels	x	-	x
11 Water	-	9.7	29.3
12 Agriculture	-	16.2	21.1
13 Iron and steel	-	72.7	27.6
14 Non-EGKS products ..	-	1.6	4.7
15 Non-ferrous metals .	-	26.9	12.3
16 Aluminium	28.9	-	5.1
17 Cement	-	3.9	93.6
18 Glass	-	7.4	16.9
19 Ceramics	12.3	-	40.8
20 Other minerals	-	9.9	14.1
21 Chemical products ..	11.8	4.0	-
22 Metal products	4.6	7.7	-
23 Machinery	-	4.0	2.5
24 Electrical products	-	2.0	2.1
25 Motor vehicles	-	2.7	2.7
26 Other vehicles	0.5	2.7	-
27 Food	0.6	4.4	-
28 Textiles	-	7.5	5.4
29 Leather	-	5.0	0.8
30 Wood	-	11.7	8.0
31 Paper	-	20.7	52.2
32 Printing	-	4.6	1.4
33 Synthetics	-	7.3	4.6
34 Other products	-	7.8	0.6
35 Buildings	-	8.7	5.5
36 Repairs, recovery ..	-	5.6	1.9
37 Trade. restaurant ..	9.6	-	1.8
38 Railroad	-	25.5	48.1
39 Road transportation	152.6	-	120.3
40 Pipelines	-	52.5	120.2
41 Inland navigation ..	285.8	-	69.3
42 Maritime transport..	-	5.0	41.5
43 Aviation	48.8	-	10.1
44 Private services ...	7.3	8.8	-
45 Public services	-	0.1	2.6

Legend: - = efficient production activity
x = production activity not existent

Source: Input-Output Tables of Energy Flows for France, Italy and the Federal Republic of Germany in the Year 1975.
Calculations by the Ifo Institute for Economic Research.

Before discussing the solution of the optimizing model we can summarize the most important details about the input statement as follows:

- o Energy flows are given in physical units, all other flows in value units.
- o The production functions of production activities and final demand activities are represented by input coefficients. They have different dimensions in terms of quantity and value.
- o In a production system with more than one primary input a structural change of final demand can result in a reswitching of techniques. To avoid this phenomenon the different components of value added are aggregated to one vector of primary inputs.
- o The levels of production and demand activities are determined endogeneously. With the given level of gross value added the system has only one exogeneous constraint.
- o The objective is to maximize final demand in a given structure. Because of this given structure there is no valuation problem in the objective function.

If we start with a model of input coefficients which are only defined in value units the optimal solution is trivial. All production activities are efficient and their shadow prices will be zero. The reason for this result is the fact that in this case all monetary input coefficients of an activity sum up to unity.

For a mixed system of input coefficients in value units and quantity units we get a different result. The optimal solution in table 5 shows significant differences in the efficiency of production activity within the European Communities. Due to the statement of the model this concept of efficiency is restricted to the efficient use of energy.

The shadow prices in table 5 have the following interpretation. Only those production activities which are not efficient have shadow prices. The French production function for coal (sektor 1) is the most efficient one. If the vector of final demand were produced with the West-German production function the value of final demand would be reduced by 0.6 million DM. In Italy there is no production activity for coal. On the other hand the West-German production function for cement (sektor 17) is more efficient than those of France (3.9 mill. DM) and Italy (9.36 mill. DM). Out of a total of 45 production activities 26 efficient production activities are located in West-Germany, 11 in France and 8 in Italy. The welfare gain of an efficient use of energy sources in these three countries amounts to 28.837 billion DM [12].

With these results we don't want to recommend changing the structure of production within the European Communities. It is obvious that in many cases this is impossible because of the regional character of production. The main objective of our approach is to derive quantitative measures for the structural differences of energy use in the EEC.

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12) In the statement of the model gross value added has a value of 967.831 billion DM. In the optimal solution final demand has a value of 996.668 billion DM. Using just monetary input coefficients the value of final demand would remain at the value of the constraint.

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THE EFFECTS OF THE ITALIAN ENERGY PLAN ON SECTORAL OUTPUTS AND INVESTMENT AND ON PRIVATE CONSUMPTION

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This paper examines the interaction between the energy system and the economy. It is part of a research project of the Italian Hydrocarbons Agency (ENI) which investigates the application of INTIMO to national energy planning. The Italian group of the INFORUM project collaborates with ENI on the research.

This preliminary report sets out the potentialities of a multisectorial model such as INTIMO when applied to energy analysis. A few integrations and modifications to the model are then suggested to improve its performance in interpreting and forecasting energy phenomena. The results of two simulations are presented. The scenarios are those implied by the Italian energy plan or have been inferred from forecasting models and studies of ENI.

I wish to thank Clopper Almon for his invaluable suggestions and comments. I am also grateful to Douglas Nyhus, Maurizio Grassini and Maurizio Ciaschini for their assistance in running the simulations. But I am of course entirely responsible for any remaining error or shortcoming.

ENI does not necessarily agree with the views expressed in this paper.

1. INTRODUCTION

The events of the last few years have emphasized the importance of energy for the economic growth of industrialized countries. Italy, in particular, which depends for 67 percent of its energy needs on imported crude oil, has seen its potentialities for growth severely curtailed by the problem of energy (◊). Actually the increase in the dollar price of crude oil and the coincidental appreciation of the US dollar affected adversely the balance of trade and the rate of inflation. It was therefore necessary to resort to restrictive policies in order to contain domestic demand. Moreover the global recession further penalized the growth of the Italian economy in which exports play a fundamental role. Unemployment also increased as a consequence of the business slowdown.

The problems mentioned above call for the definition of medium-to-long term policies, acting on both the supply of and the demand for energy, to support the optimal reallocation of the factors of production that follows a change in relative prices.

During the last few years a large number of models have been built, mainly in the United States, to forecast the demand for energy in each segment and to assess the reaction of consumers to changes in the price of energy products and to other instruments of economic and energy policies /1/. Supply models giving indications on the mix of primary energy sources and on the best technology to meet a given demand have also been developed /2/ /3/ /4/ and so have detailed partial-equilibrium models of specific energy industries (gas, coal, electric power, etc.) /5/ /6/ /7/.

(◊) The figure is for 1980.

Nonetheless energy policies at both firm and government levels must be elaborated using detailed scenarios of economic growth and must then be evaluated in a general-equilibrium context on the grounds of their impact on the whole economy. It is desirable to ascertain how leading economic aggregates such as GNP, output and employment in each industry, the balance of payments and the rate of inflation, are affected by changes in the price of energy products and by the investments required to realize the energy plans. It is thus necessary to develop instruments to describe the energy sector in full details and to give a complete representation of its interactions with the economy.

Aggregate models based on the Keynesian theory of effective demand are unfit for this purpose and it is therefore necessary to resort to multisectorial models combining interindustry flows with the generation of income and final demand. Models of this kind (e.g. the Wharton Annual Energy Model /8/, the Hudson-Jorgenson model /9/, the Norwegian MSG-E model /10/) have been used by government agencies, often with the integration of detailed energy models, to evaluate energy policies.

The development of INT.I.MO. (INTerindustry Italian MOdel) by a research staff directed by Professor Grassini on the facilities of SOGESTA and IRPET has supplied Italy with a powerful instrument to make medium-to-long term intersectorial forecasts, to define the framework of national energy planning and to analyze the impact of energy policies on the economy.

2. ENERGY IN INTIMO (◊)

2.1 Level of Aggregation

Fig. 1 shows the level of disaggregation used by INTIMO to describe the energy system. It has been adopted because use has been made of the 44-sector input-output table of the Italian Institute of Statistics (ISTAT) for 1975 including five energy sectors and of national accounts data on household expenditures and business investments.

The level of disaggregation of the energy sectors (which is similar to the subdivision of energy products) is not adequate for energy analysis. It is in fact difficult to assess the impact of specific energy policies on the price of and the demand for energy products. It is similarly difficult to integrate INTIMO with energy models because the latter are characterized by a more detailed description of energy sectors and energy products.

Moreover, due to the coexistence of different production functions in the same energy sector, a variation in the mix of products within the sector will alter the cost structure. Similarly it will modify, sometimes significantly, the technical coefficients of energy products because the "energy-intensive" sectors are in fact engaged in different productions each of which requires different amounts of energy.

Besides, the induction of imports by, say, an expansion of the economy would be different for products such as gas and electricity which nonetheless are aggregated in the same sector.

(◊) This paper analyzes the energy aspects only. Interested readers should refer to /11/ and /12/ for a general description of INTIMO and for the preliminary results for the estimated equations. Fig. 2 shows the general structure of the model (a special emphasis is given to energy aspects).

INDUSTRIES

(OUTPUT, EMPLOYMENT, WAGES & PRICES)

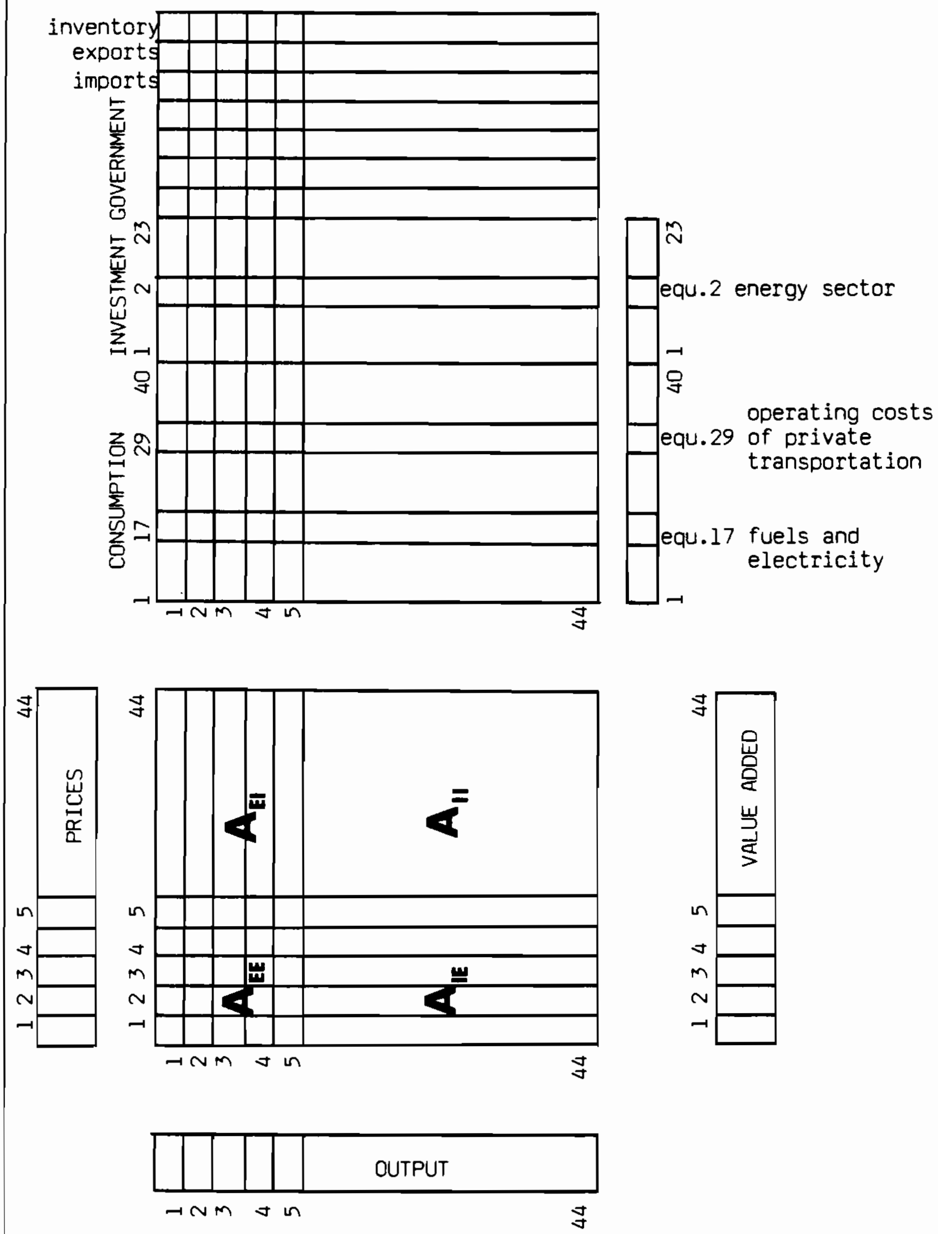
1. coal and lignite
2. products of coking
3. crude oil, natural gas & refined petroleum products
4. electricity, manufactured gas & water
5. nuclear fuels

FINAL DEMAND

household expenditures	1. fuels and electricity 2. fuels and other operating costs of private transportation
investments	1. energy products
foreign trade	1. 2. 3. 4. 5. (see industries)
government education & research health collective consumption	1. 2. 3. 4. 5. (see industries)

Fig. 1 - The energy sectors in INTIMO

Fig. 2 - INTIMO overview



Thus it seems necessary to adopt a higher level of disaggregation of the energy sectors. Different products (gas-electricity; petroleum production-petroleum refining) should be distinguished and so should different technologies (nuclear power, hydroelectricity, etc.) Similarly it seems desirable to distinguish joints products -- at least into subgroupings -- by introducing the corresponding rows and same number of dummy columns into the input-output matrix.

As for the disaggregation of "energy-intensive" sectors into more or less homogeneous subsectors (like the ISIS table for 1975 /13/) it is necessary to check the availability of diasggregated data and to appraise the option of increasing the model in size.

2.2 Cost Structure of the Energy Sectors

The A matrix can be partioned into the following four matrices (see Fig. 2):

- A_{EE} energy inputs in energy sectors
- A_{IE} intermediate nonenergy inputs in energy sectors
- A_{EI} energy inputs in nonenergy sectors
- A_{II} intermediate nonenergy inputs in nonenergy sectors.

The matrices A_{EE} and A_{IE} give the cost of intermediate energy and nonenergy inputs required to support one unit of output of the energy industries. The cost of primary-input requirements is given in the value-added matrix.

The transformation of energy products and the losses occuring during transformation, transport and distribution account for 80 percent of the input requirements given by the elements of A_{EE} (◊). Only the remaining 20 percent is used up by applications common to other industries (space heating, lighting, electromotive force, process heat, etc.) It is therefore preferable to consider the elements of A_{EE} as belonging to the columns rather than the rows.

(◊) the main transformations are: coal (sector 2) into coke (sector 3), crude oil (sector 4) into refined petroleum products (sector 4) and coal, fuel oil (sector 4), gas (sector 5) and nuclear fuels (sector 6) into electricity (sector 5).

Important variations in the matrices will come about approximately during the next ten years. First, the mix between the methods for generating electricity will be modified because coal and nuclear plants provide electricity at a lower cost and because there is a need to diversify the energy sources and to reduce the dependence on crude oil. Second, new technologies such as coal gasification and coal liquefaction will be adopted and the adoption of cogeneration of heat and electricity and district heating will be extended.

It is possible to single out different technologies in INTIMO by using statistical data for the existing technologies and engineering data for the new technologies. Thus the shares of different technologies can be defined exogenously using the indications of the national energy plan or optimization models of energy supply (the MARKAL model is available at ENI).

The reconstruction of the elements of the energy columns can be accomplished outside the model or within the model by bringing in the new technologies and then reconstructing the cost structure in each period on the basis of the shares which are exogenous to the model.

2.3 Consumption of Energy by Nonenergy Sectors

The matrix A_{EI} gives the cost of the energy inputs required to support one unit of output of the nonenergy industries. It is made up by the intersections of rows 2, 3, 4, 5 and 6 and columns 1 (agriculture), 7 to 28 (nonenergy industries), 31 to 33 (transport), 29 to 30 and 34 to 44 (services).

The energy balance imputes the consumption of motor fuels by firms to the transport sector. On the contrary, INTIMO rightfully imputes the consumption to the sector to which the firm belongs.

Solid, liquid and gaseous fuels used by a firm to generate electricity are subtracted from the consumption of energy of tne sector to which it belongs and are imputed to the only sector generating and distributing electricity (sector 5). This procedure is likely to cause the following problems:

1. The cogeneration of electricity and heat is not taken into account. Given the amounts of heat and electricity, cogeneration reduces the need for

primary energy sources. During 1978, 89 percent of the electricity generated outside the electric sector, that is about 20 percent of all thermoelectricity, was generated by heat-recovering plants saving about 500-billion-lire worth of energy products.

2. If investment is appraised on the basis of output, the investments sustained by firms not belonging to the electric sector -- but nonetheless engaged in the production of electricity -- would be imputed to the electric sector.

3. The electricity generated outside the electric sector, but conventionally imputed to it, must then be "sold" to the sector that produced it. The value of this inexistent transaction is probably made equal to the value of the fuels employed to generate the electricity. The cost of intermediate inputs required by each sector is therefore left unchanged. However the rate conventionally assumed for the electricity produced by nonelectric sectors affects the rate applied to their global consumption.

   When electricity and hot water are jointly produced it is necessary to take account of the sale of the hot water. The problem of determining the price of the joint products is considered in /14/.

However, the main problem is the variation in the coefficients of the A_{EI} matrix.

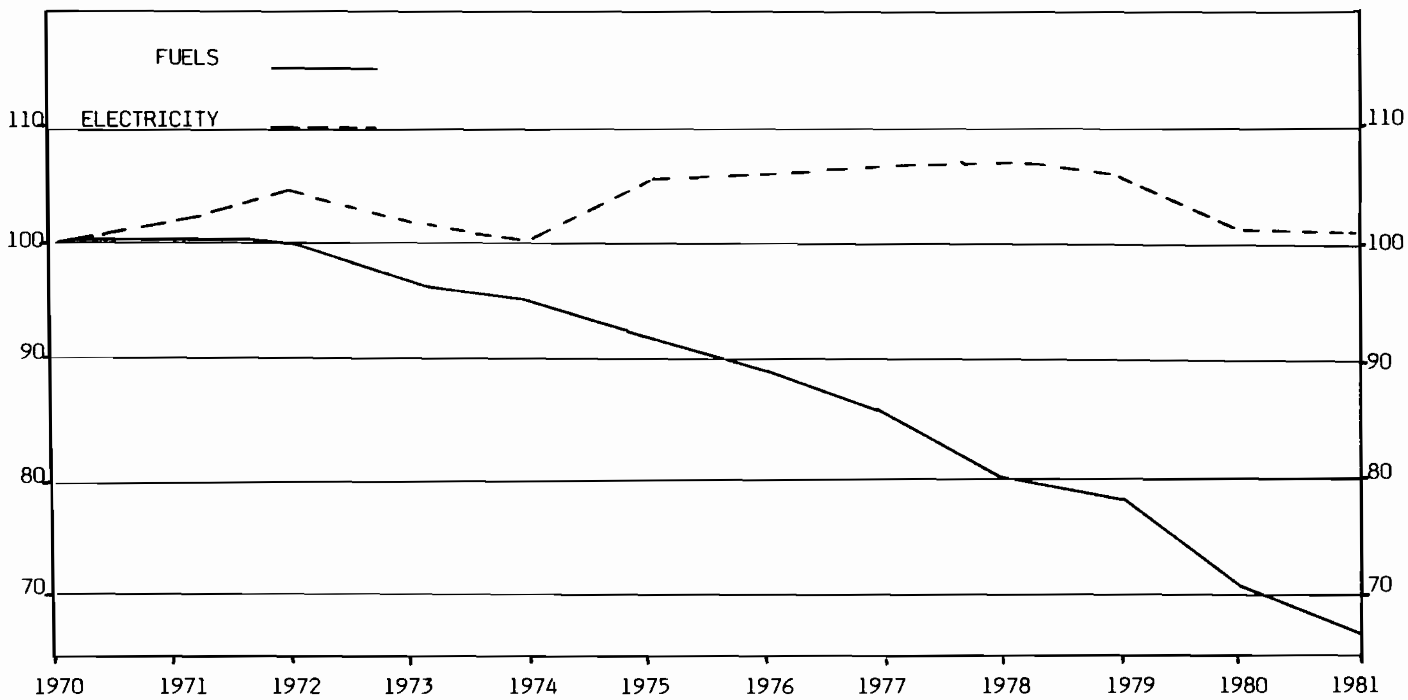
Energy inputs, like other intermediate and primary inputs, undergo variations as production technologies and relative prices change.

Fig. 3 shows the behavior of the specific consumption of fuels and electricity by the Italian industry from 1970 to 1981. Fig. 4 is limited to seven sectors from 1970 to 1979.

The following remarks can be made:

1. From 1973 there has been a sensible reduction in the specific consumption of fuels by the industry as a whole, while the specific consumption of electricity has remained stationary during the years taken into consideration.

Fig. 3 - specific consumption of fuels and electricity by the Italian industry from 1970 to 1981 (1970 = 100)



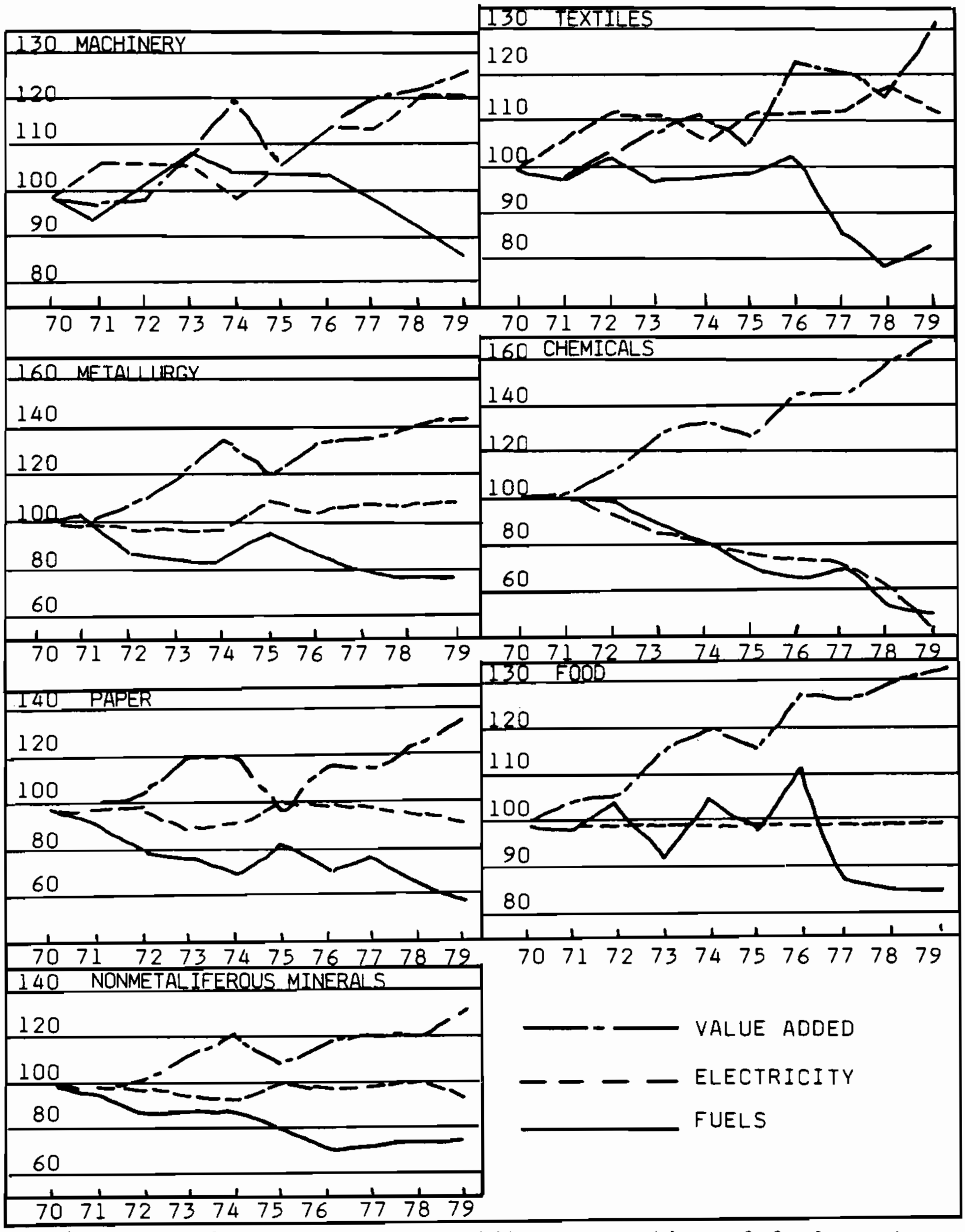


Fig. 4 - value added and specific consumption of fuels and electricity in seven sectors of the Italian industry from 1970 to 1979 (1970 = 100)

2. The trend is common to all sectors except the chemical industry which has reduced its specific consumption of both fuels and electricity. But more than 80 percent of this reduction can be explained by a variation in the product mix within the sector. The output of the primary chemical industry, which has a specific consumption of energy ten times larger than the secondary chemical industry, has been more or less stagnant since 1974 while the output of the latter has increased during the whole period. The importance of disaggregation for this analysis comes out in what has been said.

3. For a number of sectors the reduction begun before 1973 chiefly due to modifications in production techniques (◊). For these sectors the reduction is linked to the innovation in the capital stock brought about by an increase in demand.

4. For some other sectors (e.g. metallurgy and paper industry) the specific consumption of energy increased when ouput decreased (e.g. in 1975). This could well mean that some of the industrial uses of energy are fixed and thus unrelated to the level of output.

We have two approaches to deal with variations in coefficients. The first is to update and project the coefficients through logistic time trends or expert information. With this technique the variations in coefficients are still exogenous to the model. The second approach is to use proper firm-behavior models so that variations in coefficients become endogenous.

In the current release of INTIMO, the technical coefficients of the AM matrix are projected using logistic curves estimated for each row rather than for each element because data on the latter are lacking.

In the simulations presented in this paper the technical coefficients of energy products are modified according to interviews with experts of the different sectors.

(◊) In iron metallurgy adoption of continuous casting; in nonmetelliferous-mineral industry adoption of dry processing.

These techniques are aimed at the improvement of the forecasting performance. For evaluating alternative energy policies it would be desirable that the energy- and nonenergy-input coefficients be endogenous to the model. It would equally be desirable that the structure of production be consistent with the structure of relative prices which is determined endogenously.

2.4 Consumption of Energy by Households

In INTIMO two of the forty categories of household expenditures are relative to energy products:

17 FUELS AND ELECTRICITY

expenditures for
- space heating
- water heating
- lighting
- utilization of electric appliances
- cooking

29 OPERATING COSTS OF PRIVATE TRANSPORTATION

expenditures for
- fuels
- servicing, insurance, etc.

The consumption equations estimated for Italy (1970 to 1980) /15/ using the specification suggested by Almon /16/ give the following elasticities and trend:

Category	Income Elasticity	Price Elasticity	Trend % per year
17	.943	-.014	+2.0
29	1.346	-.288	+1.4

Columns 17 and 29 of the bridge matrix (44x40) split the expenditures included in categories 17 and 29 into four energy sectors and agriculture (firewood). Expenditures included in category 29 are also splitted into other sectors related to the operation of private transportation.

The bridge matrix of ISTAT is built on a market-price basis and market prices must thus be converted into producer prices. This is done by subtracting the margins of trade and transport. The global markup of the trade and transport sectors on total household expenditures is used to calculate the margins (this is evident in the input-output table for 1975). For energy products the energy tables also give the values at producer prices. These can be used for a direct conversion of the bridge matrix thus avoiding errors.

The following points can be made about the way the current release of INTIMO treats the household consumption of energy:

1. Separability is assumed between the global demand for energy for household uses (explained by a behavorial equation) and the choice of the mix of energy products (assumed constant). The same assumption is made for the operation of private transportation.

   This assumption is made by a number of authors (including Pindyck, Boughman and Jorgenson) /17/ /5/ /9/. Because there are no substitutes for a number of household uses of energy (electricity is required for some of them) it is preferable not to make this assumption for the global consumption of energy. Some authors suggest to separate the consumption of electricity from the consumption of fuels.

2. The consumption functions have a static specification and are in reduced form.

   Models based on consumer theory must possess a number of formal attributes /18/. Because of these formal restrictions it is sometimes impossible to treat dynamic aspects properly. Dynamic analysis is essential to the study of energy consumption because the adjustment in the stock of energy-using durables to changes in energy supply conditions is not istantaneous.

The explanatory variables are price and income but variations in the stock of energy-using durables are not taken into account. Problems of identification arise and it is generally impossible to get the structural coefficients. Furthermore it is not possible to assess how much of the change in demand, due to variations in prices and income, passes through the efficiency and the utilization level of the stock of energy-using durables. This aspect is particularly important if the application of the model is not restricted to forecasting but encompasses the evaluation of alternative energy policies.

Moreover a static specification implies that price effects exhaust in one period. This assumption would be plausible only if the equation includes the stock of energy-using durables as an explanatory variable. A static specification can also lead to distorted estimation of the price elasticities.

3. Even if the separability hypothesis is assumed we cannot rightfully treat the columns 17 and 29 of the bridge matrix as a set of constants. In fact the coefficients of column 17 refer to products for which substitution is possible (in many final uses substitution depends upon the relative prices and the supply conditions). Fig. 5 shows the shares of solid, liquid and gaseous fuels and of electricity (1975 prices) in the consumption of energy and the relative prices of energy products plotted against time. It is easy to see that variations in shares are significantly correlated to variations in prices.

   The components of column 29 are complements but nonetheless we can still have relative variations. This is because some of the expenditures included in category 29 depend exclusively upon the stock of automobiles (e.g. insurance), others also depend on the level of utilization and still others also on efficiency (e.g. fuels). Therefore if the utilization level of automobiles changes or if mileage is increased the relationship between the components of column 29 is likely to change.

The following are partial solutions requiring only limited modifications to the model.

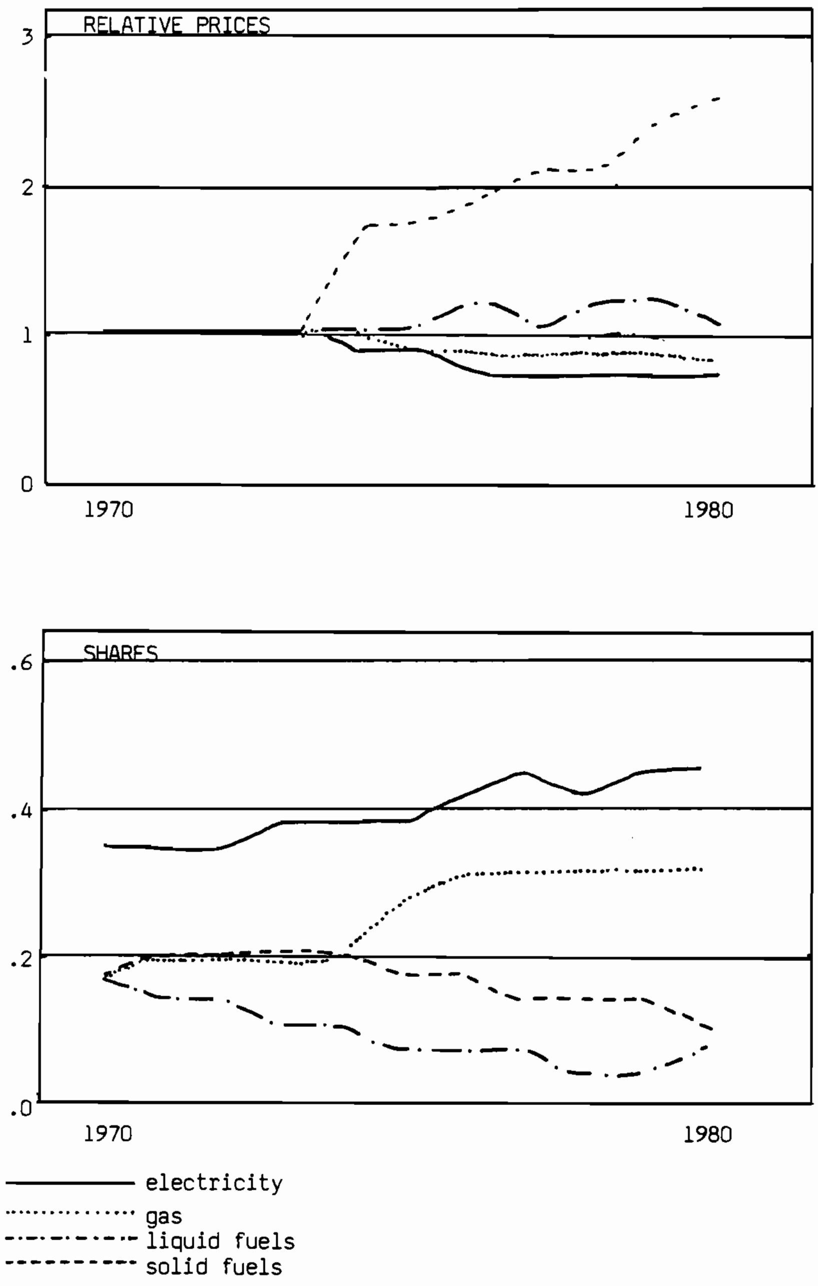


Fig. 5 - Household energy consumption: shares and relative prices of solid, liquid and gaseous fuels and electricity

1. The current number of equations is retained but variations in columns 17 and 29 of the bridge matrix and thus variations in the energy-product mix are allowed for. Variations can be determined either exogenously (e.g. from the national energy plan) or endogenously as a function of relative prices.

2. The number of categories into which household expenditures are subdivided is increased to 43 by substituting the consumption of each of four energy products (solid, liquid and gaseous fuels and electricity) to the global consumption of energy. Substitution between energy products would then be explained by the household-expenditure equations (this already occurs in the INFORUM models of France, Germany, Belgium, etc.) The number of columns in the bridge matrix would have to be increased to 43 and the problems arising from variations in the mix would be restricted to solid fuels.

A solution intermediate to 1. and 2. is to separate fuels from electricity in the consumption equations and to explain the variation in the mix of fuels in the bridge matrix.

Alternatively energy submodels can be built and linked to INTIMO. The submodel should explain the consumption of energy for household uses and the consumption of fuels for automobiles. In this case two problems would arise: (1) finding the data and (2) linking the energy submodel to the system of functions. At any rate the dynamic aspects should be given adequate consideration so that constraints imposed by the stock of energy-using durables can be taken into account.

ENI uses a simulation model following the MEDEE approach to forecast the demand for energy in each segment and for each final use /19/. The consumption of energy is reconstructed on the basis of the efficiency and the utilization level of the stock of energy-using durables, determined by using time trends and expert information /20/.

2.5 Investment in the Energy Sectors

INTIMO distinguishes 23 buying sectors. For each of these an econometric equation is estimated /21/. Investments -- included in each buying sector -- are broken down into 44 selling sectors by means of a bridge matrix (44x23) built for 1975.

Investments of the energy sectors are aggregated into a single buying sector and subdivided by selling sector by the means of column 2 of the bridge matrix.

The quantity and the mix of capital goods are explained by the global output of the energy sectors. They do not depend on the relative incidence of the different sources, characterized by different capital output ratios and by different subdivisions by selling sector.

A solution suggested in /22/ and /23/ is to use the B matrix of the Leontief dynamic model for reconstructing the investment of the energy sectors.

Net investment can be reconstructed using the following equation

$$I_e(t) = \sum_{\tau=t}^{t+\hat{\tau}} F(\tau - t)\, \Delta \mathrm{OUTPUT}(\tau)$$

where $I_e(t)$ is the vector of direct investment of the energy sectors subdivided by selling sector

Δ OUTPUT(τ) is the vector of variations in the output of the energy industries

$F(\tau - t)$ is the matrix of the coefficients giving the requirement of capital goods to put into function the additional capacity of the energy industries during the year $\tau (t \leq \tau \leq t + \tau)$

$\hat{\tau}$ is the lead time

F can be reconstructed as a function of production capacity on the basis of (1) capital output ratios of each industry and technology of the energy sector, (2) the subdivision by selling sector of each kind of investment and (3) the distribution of investment costs over the years of construction.

For the time being the capital output ratios and the subdivision by selling sector have been estimated only for the following industries and technologies:

1. Coal mining
2. Production of crude oil and natural gas
3. Petroleum refining
4. Pipelines
5. Thermoelectric plants
6. Nuclear plants
7. Hydroelectric plants
8. Electric power distribution
9. Coal gasification
10. Coal liquefaction

The results have been obtained, as first approximations, by means of the capital coefficients computed by the Brookhaven National Laboratory /22/ in 1967 dollars (per 10^6 BTUs) and classified into 110 sectors. Priorly these coefficients have been aggregated into the 44 sectors of INTIMO and converted into 1975 lire.

By the means of the composition of the capital stock, the investment provided for by the national energy plan has been subdivided by selling sector for 1981 to 1983 and for 1984 to 1990. The results given in Table 1 show that the subdivision does not differ significantly from the mix given by the 1975 bridge matrix.

Once the F matrix is built, it will be possible to assess the impact of energy policies on the demand for capital goods.

For the time being, expenditures for plants and equipment directed to energy conservation and sustained by households and business are not taken into account.

Sector	National energy plan 1981-83	National energy plan 1984-90	1975 matrix
PROD AGRICOLTURA SILVICOLTURA PESCA	0.00000	0.00000	0.038
CARBONE LIGNITE AGGLOMERATI	0.00000	0.00000	0.000
PROD DELLA COKEFAZIONE	0.00000	0.00000	0.000
PETROLIO GAS NAT PROD PETROLIO	0.00000	0.00000	0.000
ELETTRICITA GAS ACQUA	0.00000	0.00000	0.000
COMBUSTIBILI NUCLEARI	0.00000	0.00000	0.000
MINERALI METAL FERROSI E NON	1.51914	1.13920	0.000
MINERALI E PROD NON METALLIFERI	0.13746	0.11831	0.294
PROD CHIMICI E FARMACEUTICI	0.45823	1.12157	0.000
PROD IN METALLO	3.47117	4.80882	2.195
MACCHINE AGRICOLE E INDUSTRIALI	10.21200	12.32393	11.837
MACCHINE UFFICIO OTTICA E SIMILI	0.81740	0.73059	1.722
MATERIALI E FORNITURE ELETTRICHE	5.43508	4.95900	4.818
AUTOVEICOLI E MOTORI	0.65228	0.53735	1.621
ALTRI MEZZI DI TRASPORTO	0.90638	1.40159	1.447
CARNI FRESCHE E CONSERVATE	0.00000	0.00000	0.000
LATTE E PROD DERIVATI	0.00000	0.00000	0.000
ALTRI PROD ALIMENTARI	0.00000	0.00000	0.000
BEVANDE ALCOLICHE E NON	0.00000	0.00000	0.000
TABACCHI LAVORATI	0.00000	0.00000	0.000
PRODOTTI TESSILI E ABBIGLIAMENTO	0.00841	0.00632	0.000
CUOIO E CALZATURE	0.00066	0.00057	0.070
LEGNO E MOBILI IN LEGNO	0.99550	0.77648	0.697
CARTA CARTOTECNICA EDITORIA	0.03863	0.02765	0.000
GOMMA E MATERIE PLASTICHE	0.05491	0.04502	0.242
ALTRE INDUST MANIFATTURIERE	0.24039	0.19536	0.015
COSTRUZIONE E OPERE PUBBLICHE	72.33201	68.72110	70.794
BENI RECUPERO E RIPARAZIONI	0.00013	0.00032	0.000
COMMERCIO	1.11792	1.11023	3.505
ALBERGHI E PUBBLICI ESERCIZI	0.00000	0.00000	0.000
TRASPORTI INTERNI	0.85097	0.84613	0.741
TRASPORTI MARITTIMI ED AEREI	0.03188	0.05050	0.003
ATTIVITA CONNESSE TRASPORTI	0.00436	0.00306	0.000
COMUNICAZIONI	0.00013	0.00032	0.000
CREDITO E ASSICURAZIONI	0.00033	0.00081	0.000
SERVIZI ALLE IMPRESE	0.35246	0.28179	0.000
LOCAZIONE DI FABBRICATI	0.36206	0.79385	0.000
SERVIZI DI INSEGNAMENTO	0.00000	0.00000	0.000
SERVIZI SANITARI	0.00000	0.00000	0.000
SERVIZI RICREAT E CULTUR	0.00000	0.00000	0.000
SERVIZI GENERALI P A	0.00000	0.00000	0.000
SERVIZI INSEGNAMENTO P A	0.00000	0.00000	0.000
SERVIZI SANITARI P A	0.00000	0.00000	0.000
SERVIZI DOMESTICI E ALTRI	0.00000	0.00000	0.000

Table 1 - composition of the investments provided for by the Italian energy plan compared to the coefficients of the bridge matrix for 1975

2.5 Foreign Trade and Domestic Production of Energy

Sector	Product	1975 Imports (billions of 1975 lire)	1975 Exports
2	COAL & LIGNITE	523.0	.8
3	COKE	12.8	54.7
4	CRUDE OIL	5354.6	0.0
	DERIVATES	810.7	1235.8
5	NATURAL GAS	162.4	0.0
	ELECTRICITY	60.4	20.2
	MANUFACTURED GAS	17.0	23.9
6	NUCLEAR FUELS	15.3	5.3

Source: ISTAT

The table shows that foreign trade in energy products is primarily made up by:

- IMPORTS OF PRIMARY ENERGY SOURCES
  - crude oil
  - coal
  - natural gas
- EXCHANGE OF PETROLEUM DERIVATES necessary to balance domestic demand and domestic supply of refined petroleum products.

The import and export equations for the five energy sectors of INTIMO have been estimated /24//25/. The results of the import equations follow

Sectors	price elasticity	demand elasticity
2 COAL	constant share of domestic demand	
3 COKE	constant share of domestic demand	
4 PETROLEUM & DERIVATES	0	≃ 1
5 ELECTRICITY & GAS DISTRIBUTION	0	≃ 3
6 NUCLEAR FUELS	constant share of domestic demand	

It should be no surprise that the price elasticity in sector 4 and sector 5 is zero and that the elasticity to demand in sector 4 is almost one. This is because with the current level of disaggregation, the imports (M) of sector 4 are almost entirely made up by crude oil and the output (P) exclusively by refined petroleum derivates. Therefore if domestic demand is $D = P + M$ then we can write $P \simeq kM$ from which $D = kM + M$ and $M/D \simeq M/(kM+M) = 1/(k+1) = \text{constant}$ because of the unit elasticity to demand.

The high elasticity to demand in sector 5 can be explained by the sensible increase in natural gas imports (from almost zero in 1970 to 162 billion lire in 1975) while domestic output has remained practically constant.

Because primary energy sources are noncompetitive imports it would be correct, once domestic output is given, to calculate them as residuals. This is especially important if the disaggregation of the energy sector is carried out.

3. SIMULATIONS

The purpose of the simulation experiments is to check the possibility of converting the energy scenario for 1981 to 1990 -- given by the national energy plan and ENI models /22/ -- into inputs for INTIMO so that its impact on the whole economy can be assessed.

No modification has been made to INTIMO before running the simulations. However, the inputs to the model have been processed externally thus taking advantage of the software option for accessing each component of final demand and the structural matrices.

Note that the energy scenario has not been modified according to the results of INTIMO because the link between INTIMO and energy models has not yet been implemented.

The results of the following two simulations are presented in Table 5.

1. BASIC SIMULATION

The main assumptions are reported in Table 2. The coefficients are modified by using logistic curves estimated for each row.

2. SIMULATION WITH ENERGY SCENARIO

In this simulation the projections up to 1990 of the national energy plan are brought in after having been integrated with the forecasts of ENI models.

In particular:

- matrix A_{EE} The coefficients representing the transformation of coal into coke and crude oil into refined derivates and the losses occuring during the transformation and distribution of electricity are left unchanged.

		1975	1980	1985	1990
1	POPULATION (millions)	56.01	57.30	57.40	57.50
2	PERCAPITA DISPOSABLE INCOME (thousands of 1975 lire)	1913.00	2272.00	2486.00	2810.00
3	LABOR FORCE (thousands)	20450.00	21500.00	22375.00	23000.00
4	UNEMPLOYMENT (thousands)	573.70	922.00	1421.14	2008.46
9	EXCHANGE RATE (1975=100)	100.00	132.00	152.50	150.00
14	AVERAGE PROPENSITY TO CONSUME (percent)	77.49	76.28	77.13	77.25
20	FOREIGN DEMAND (quantity index 1975=100)	100.00	124.51	141.98	165.72

Table 2 - Main assumptions of the basic simulation

The coefficients of the inputs to the electric sector are modified to take into account the variations in the mix of fuels employed to generate electricity given by the national energy plan.

- matrix A_{IE} The matrix is left unchanged
- matrix A_{EI} The matrix is varied on the basis of information obtained by ENI by interviewing experts of the eight sectors of manufacturing industry, of the transport sector and the services sector (see Table 3).
- matrix BR The column 17 is allowed to vary to take into account the expected variation in the mix (increase in the shares of gas, electricity and coal and decrease in the share of liquid fuels). The column 29 is allowed to vary because we expect a 8 percent increase in mileage in the period 1981 to 1990. Therefore the share of fuels in the global expenditure for the operation of private transportation will fall (see Table 4).
- matrix BM The column 2 is varied on the basis of the composition of investment provided for by the national energy plan (see Table 1).
- investment The investments indicated in the national energy plan are 2,500 billions of 1975 lire per year from 1981 to 1983 and 3,100 billions per year from 1984 to 1990.
- household expenditures The expenditures for fuels and electricity (cat. 17) are estimated at 4,650 billions of 1975 lire by ENI models (1990). The forecasts are based on the expected efficiency, utilization level and stock of energy-using durables. The expenditures for the operation of private transportation are calculated on the basis of projections of mobility and of the stock of automobiles. The expected reduction in the consumption of motor fuels is taken into account.
- public sector ENI forecasts are used.

INDUSTRY (◊)	FUELS		ELECTRICITY	
	1980	1990	1980	1990
(7) METALLURGY	100	89	100	108
(8) NONMETALLIFEROUS MINERALS	100	82	100	100
(10-11-12-13-14-15) MACHINERY	100	82	100	105
(16-17-18-19-20) FOOD	100	82	100	104
(21) TEXTILES & CLOTHING	100	80	100	107
(9) CHEMICALS & PETROCHEMICALS	100	90	100	100
(24) PAPER & PRINTING	100	82	100	100
(22-23-25-26) OTHER MANUFACTURING	100	93	100	90

Table 3 - Energy intensity in the main industrial branches

(◊) corresponding INTIMO sectors are shown in parenthesis.

Sector		Bridge matrix 1975	1980	Projections (◊) 1985	1990
Column 17					
1	firewood	0.009396	0.006371	0.005471	0.004802
2	coal	0.003141	0.002730	0.002671	0.002344
3	coke	0.003989	0.002874	0.002800	0.002457
4	petroleum products	0.373528	0.246716	0.203956	0.165915
5	gas & electricity	0.496591	0.656023	0.693553	0.751127
29	trade	0.097958	0.073831	0.072181	0.063345
31	transport	0.013550	0.010201	0.010018	0.008792
32	transport	0.001848	0.001391	0.001284	0.001127
Column 29					
4	petroleum products	0.5178	0.4678		0.4586

Table 4 - Coefficients of the BR matrix and projections for 1985 and 1990

(◊) using the Italian national energy plan.

RESULTS OF BASIC SIMULATION	81-83	83-85	85-90	75-90
GROSS DOMESTIC PRODUCT	1.75	3.03	2.47	3.16
TOTAL OUTPUT	1.92	2.97	2.39	3.35
ADMINISTRATION	2.07	1.99	2.01	2.05
EDUCATION	2.07	1.99	2.01	2.05
HEALTH CARE	2.07	1.99	2.01	2.05
COLLECTIVE CONSUMPTION	2.00	1.92	2.01	2.05
PRIVATE CONSUMPTION	1.76	3.25	2.52	2.72
EXPORTS	5.15	2.24	2.98	4.74
IMPORTS	-1.82	3.87	2.74	2.73
INVENTORY CHANGE	-40.64	30.36	2.27	0.00
FIXED INVESTMENT	-4.92	4.24	2.26	2.77
EMPLOYMENT	0.40	0.55	0.07	0.41
RESULTS OF SIMULATION WITH THE ENERGY SCENARIO				
GROSS DOMESTIC PRODUCT	1.96	2.89	2.47	3.14
TOTAL OUTPUT	2.07	2.70	2.31	3.22
ADMINISTRATION	2.06	1.98	2.00	2.08
EDUCATION	2.06	1.98	2.00	2.06
HEALTH CARE	2.03	1.95	1.98	2.07
COLLECTIVE CONSUMPTION	2.00	1.92	2.01	2.05
PRIVATE CONSUMPTION	1.76	3.25	2.52	2.72
EXPORTS	5.15	2.24	2.98	4.74
IMPORTS	-1.15	3.49	2.80	2.71
INVENTORY CHANGE	-29.49	18.67	3.53	0.00
FIXED INVESTMENT	-3.69	3.50	2.30	2.62
EMPLOYMENT	0.30	0.37	0.04	0.36

Table 5 - Results of simulations

The results of the two simulations are very much alike at least as far as aggregated values are concerned. The only appreciable differences arising from the introduction of the energy scenario are the following.

1. Employment increases at a slower rate during the simulation period because of a decrease in the outputs and the investments of the energy sectors.

2. The increase in imports is smaller.

It should be emphasized that the results have been obtained by using only the real side of the model. Thus only changes in quantities are taken into account. Variations in costs and prices arising from energy policies have no influence on the results of the simulations.

4. CONCLUSIONS

The potency of multisectorial models in the evaluation of energy policies -- with special emphasis given to sectorial effects -- has emerged during the last few years.

The INTIMO model of the Italian economy -- developed within the framework of the INFORUM project -- can be applied to the analysis of the national energy plan and of the scenarios produced by energy models available at ENI.

An application of INTIMO to energy analysis has been suggested in this paper with the chief purpose of checking the potentialities of the model in this field.

Once the price side of the model, which is currently being implemented, is made available it will be possible to make relative prices endogenous to the model. Therefore the performance of the model will be improved because it will then be possible to evaluate the effects of changes in relative prices arising from energy policies. For example it will be possible to assess the effect of a reduction in energy costs sustained by firms on the competitiveness of Italian exports.

The link between INTIMO and energy models is also being implemented. The link will assure consistency between economic scenarios and energy scenarios. The disaggregation of energy sectors, which is presently being considered, will facilitate the integration.

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INPUT–OUTPUT ANALYSIS OF ENERGY–MACROECONOMY INTERACTIONS

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1. INTRODUCTION

The I/O basic model is described by the equation

$$AX + Y = X,$$

where

A - square matrix of the I/O coefficients $a_{i,j}$;

X - vector of outputs, where $X = (x_1,..,x_j,..,x_n)$;

Y - vector of final demands, where $Y = (y_1.., y_i.., y_n)$;

$i, j = 1, 2,...,n$.(indexes of sectors).

Thus $X = (I - A)^{-1}. Y,$

$$X = f\ (A,Y). \qquad (1)$$

When Y is fixed and A vary

$$X = f\ (A) \qquad (2)$$

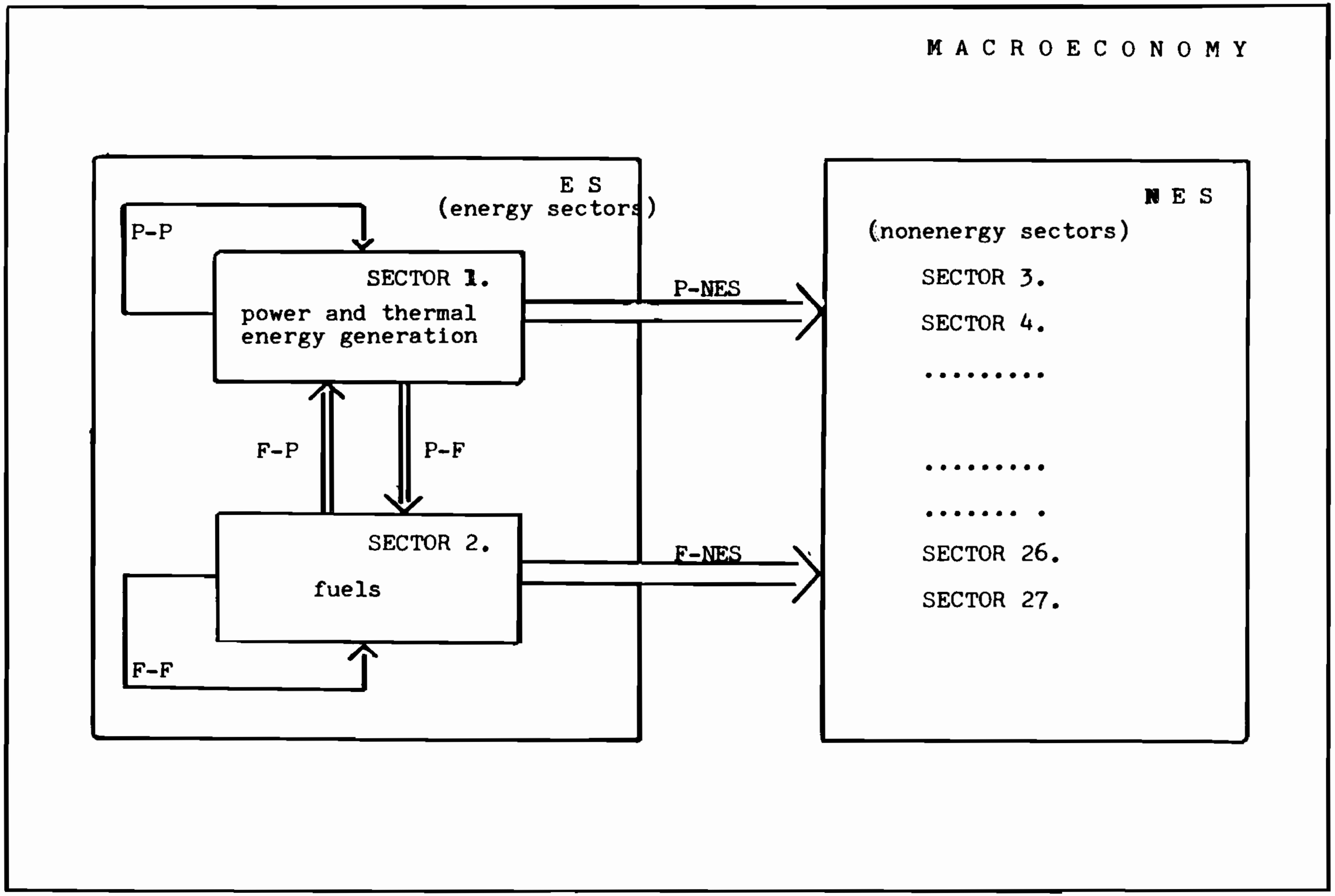
By using matrix A as an agrument it must be defined which one of the coefficients $a_{i,j}$ should vary and how. Then the variation of X should be related to the variation of the argument A, thus getting some quantificated indicator of certain interactions, which could be named "input-output elasticity":

$$\text{I/O elasticity} = \frac{\Delta x_k\ (\%)}{\Delta a_{k,1}\ (\%)} . \qquad (3)$$

There are no mathematical problems when all coefficients $a_{i,j}$ are fixed and only $a_{k,1}$ vary. No mathematical problems occur even if more than one of the coefficients $a_{i,j}$ vary, too, but methodological problems do. The present paper is an attempt to solve some of them.

2. METHODOLOGY of the I/O ANALYSIS of the ENERGY-MACRO-ECONOMY INTERACTIONS

The Bulgarian I/O table prepared for the INFORUM-project collaboration with IIASA includes 27 sectors. Two of them are energy sectors: power and thermal energy generation (sector 1) and fuels (sector 2). If interpreting these sectors only as producers all nonzero coefficients $a_{1,j}$ and $a_{2,j}$ could be a subject of variety according to a certain methodological assumptions. The backfeeds described by the coefficients $a_{i,1}$ and $a_{i,2}$, except for $a_{2,1}$ and $a_{1,2}$, are not treated here due to their slight significance in the case of the Bulgarian I/O table. In order to clarify the methodological assumptions of the experiment the system of the energy-macroeconomy interactions must be described here in terms of flows and intensities (figure 1). It is a static interpretation of energy-macroeconomy interactions because investment flows are not treated. Thus the next six flows compile the system of interactions:


M A C R O E C O N O M Y
E S
(energy sectors)
P-P
SECTOR 1.
power and thermal
energy generation
F-P
P-F
SECTOR 2.
fuels
F-F
P-NES
F-NES
N E S
(nonenergy sectors)
SECTOR 3.
SECTOR 4.
SECTOR 26.
SECTOR 27.

F I G U R E 1.

FLOW	COMMENTARY	FLOW INTENSITY
1. P-P	power to power	$a_{1,1}$
2. F-P	fuels to power	$a_{2,1}$
3. F-F	fuels to fuels	$a_{2,2}$
4. P-F	power to fuels	$a_{1,2}$
5. P-NES	power to non-energy sectors	$(o , o , a_{1,3}, \dots \quad \dots, a_{1,27})$
6. F-NES	fuels to non-energy sectors	$(o , o , a_{2,3}, \dots \quad \dots, a_{2,27})$

There are too many factors (economical, technical, etc.) creating flow intensity variations, which could be a subject of special investigation, so they are not treated in the present paper. We consider only the methodological problem of measuring the impact of flow intensity variations onto the outputs of the sectors and the whole economy. For the present purpose equation (3) should be interpreted in a wider aspect than the elasticity of outputs based on a single coefficient's variation but on a flow intensity variation. Furthermore, the flows must be combined in a certain way in order to simulate the flow intensities of the production and distribution processes. Thus the above mentioned six flows could be grouped into the following five scenarios:

DISTRIBUTION OF POWER AND FUELS

scenario	flows included
A.	P-NES
B.	F-NES
C.	P-NES, F-NES

PRODUCTION OF POWER AND FUELS

scenario	flows included
D.	P-P, P-F, F-P, F-F

PRODUCTION AND DISTRIBUTION OF POWER AND FUELS

E.	P-P, P-F, F-P, F-F, P-NES, F-NES

The next step is to generate the very flow intensity variations by scenarios. Two types of variations are possible:

First: an universal ratio $\Delta a_{i,j}(\%) = \text{const}$ for all coefficients within one of the scenarios.

Second: an individual ratio $\Delta a_{i,j}(\%) \neq \text{const}$ for the coefficients within one of the scenarios.

The first type of variations equal to 25% were implemented in the experiment. Neither the sign (+ or -), nor the value of the variation have any influence onto the value of I/O elasticity related to a certain scenario. In Table I. are given the energy and total outputs' elasticity values by scenarios. When analizing these elastisity values one decision maker proceeds from the criteria which values differ from the indifferent elasticity value of 1.

TABLE 1.

I/O ELASTICITIES TO ENERGY FLOW INTENSITIES

OUTPUTS	SCENARIOS				
	A	B	C	D	E
SECTOR 1. power and thermal energy generation	0,60	0,12	0,64	0,04	0,80
SECTOR 2. fuels	0,08	0,04	0,76	0,68	1,20
TOTAL OUTPUTS	0,04	0,04	0,12	0,08	0,16

3. CONCLUSION.

When analizing the results of the experiment given in Table 1. one can easily mark the only intensifying contour given in scenario E. The final point of this contour is the fuels' sector, which I/O elasticity is 1,2. If the total energy consumption of the sectors does slow down by 1%, the output of the fuels' sector does by 1,2%.

The present methodology could be implemented on the base of I/O table series, thus obtaining elasticity series. They could be very usefull data for the forecasting activity, particularly when planning a pricing policy of the energy production.

The present experiment was run by using the SLIMFORP at PDP 11/7o here in IIASA on the base of the available I/O data for the years 1963 and 1978 prepared for the INFORUM model of Bulgaria. The SLIMFORP is a rather handy instrument for such computations. By means of the input file entitled "Matfixes" the five scenarios were run without any complications. This experiment is planned to be repeated at IBM/37o on the base of the up-to-date available matrix A-series for the years 198o, 1985 and 199o.

INVESTMENT IN AN INPUT–OUTPUT MODEL (IOM)

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The development and improvement of IOM is connected mainly with sector nomenclature and with methodology on receipt of necessary data in practice now. There is some success of using classical sheme of IOM in the field of economic analysis and forecasting. This is related to first quadrant of the model, while the otheres and particularly the fourth do not obtain suitable development. It does not mean that is not necessary to use these quadrants. In the opposite, many economic problems require a more detailed information in complex, about value-material composition of the product and his distribution, repartition and final utilization on a large nomenclature. In principle this is possible to give us the sheme of IOM. The necessity of a dynamic aspect of economic problems requires a dynamic variant of IOM in addition.

An earnest disadvantage and obstacle to the development and improvement of IOM is absent of interrelation between the model and the national accounts system. This fact is a result from distinction in the statistical methodology, incl. scope and structure, on sectoral and national level. In our country, the data needed about IOM is based on relatively independent isolated statistical forms. The result of that is impossibility to get a correspondence of data on sectoral and national level. Sometimes there is an inconsistency between the same information on sectoral and national level about all parts of the model, not only for bad-developed parts.

They made some experiments to build an economic information system on the basis of IOM in different levels, specialy about planning, in our country and in some other socialistic countries. These efforts do not realise yet up to now in our country. The IOM frames are narrow about all the necessary and collectable economic information.

The IOM dynamic theory is connected with the capital coefficient b_{ij} above all, which is the stock coefficient for capital goods produced by sector i and used in sector j. The meaning of capital coefficient is a new produced capital goods of definite kind (producing sector), which is needed to increase the volume of production of definite kind (consuming sector) on the next unit of time (e.g. the next year). "Leontief formulated a dynamic version of the model by adding a term in the change in the output vector premultiplied by a matrix of capital coefficients".*)

The quasidynamisation of the model we can get to assume some exogenous value of the variables, which do not result from the model directly. For instance the change of the flow coefficient a_{ij} for intermediate goods, produced by sector i and used in sector j, does not follow from the IOM. It must be done externally.

The future of the labore cost, the labore intensification, the consumption level and etc., does not follow from the consumption now and they do not define the dynamics of the IOM. The nonproductive sphere and the services are not present in the model. The import of capital goods does not reflect in the IOM. About the small countries with open economy, foreign trade factor has a special meaning to define the means of production. Their volumes and structures limit the production

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*) Richard Stone, Where are we now ? A short account of the development of input-output studies and their present trends, Seventh International Conference on Input-Output Techniques; Innsbruck, Austria, April 1979, p. 5 .

on the next units of time.The IOM does not give us answers many of the questions in these cases.

If the investment vector of the second quadrant has a matrix form and contents,which shows a flow of capital goods between the sectors,it was not enough to limit the production changes on the next units of time,under a condition that is a single factor of growth only.This is a result from the gestation lag,which is impossibly to include in the capital matrix directly.At the same time it is important to know what kind is the investment and it relation with the lag: new capital stocks,a reconstruction and a modernization (the disposition of capital investment) or a building,machines and equipments (the technological structure of capital investment).Then is possibly to receive more realistic capital matrix in a period of time t .

In our country the capital matrix is not make by the Committee of Social Information System (Statistical office) or the planning organs at same time.That is why this matrix must be to build now from an irrelevent economic data and to use analytical methods to restore losing information on the stages of statistical observation and grouping of data.There are many sources of mistakes in this case.What imperfections has the information,which is used about?

In the first place the statistics "tears" data about the investment process from the production to the consumption of capital goods.More concrete: when machines and equipments are produced or imported and building is made at same time, i.e. the source of capital goods is explicitly done,there is not information string with the place of capital goods consumption in a macro level and the opposite.The capital investment vector of IOM (the final demand quadrant) includs capital goods for material and nomaterial production spheres,i.e. they are not divide.In the vector of capital investment consumption they are separate.There is an inconsistency between these two vectors.

On capital goods production they include a capitalized repair and an improvement and on the capital goods consumption - not.In the second case it is done separatly.That is mean that this activity (the volume of this kind of production) is difficulted to separate in the main capital goods production sector s - construction and machinary building industry,in a macro level.

There are problems to find sector's origin of the capital goods import.At the same time it is not known what kind of the antiimport production is needed to import machines and equipments.

The investment process is examined mainly from it clear financial aspect,with a view to resourcing and from capital construction activity,but not from value-material composition of the investment process as a whole.

In the second place there is a sector di(s)parity in capital goods production and capital goods consumption sectors.The lose of capital investment is about 3-4 per cent annually.This is not enough to make sense of a difference between the annuall volume of capital goods production and consumption.It is possibly to explain that with a gestation lag and with a lose of capital investment.But this insists: producing to be more than consuming.In our country for the most part of years in the period 1970-1980 the situation was opposed.Probably the explanation of that is foreign trade inderect late effect, which does not only compensate the smalest volume of consumption,but ensuers the most volume of capital goods consumption over the volume of capital goods production average 4-5 per cent annually.

Consequently,all the more that before overcoming the difficulties of capital investment distribution and to get a capital matrix,there are many problems to find values of marginal vectors in this matrix and to balance the sum of its elements.One of the basic elementary condition of the analytical methodes for determination elements of the capital matrix is the annual volume of the capital investment production will be equal to the capital investment consumption.As previously mentioned,this is not realised in our practice.We knew the others such cases in the statistics,we can to assume that the annual volume of the capital investment consumption is "more exact" quantity and on the basis of that to get the same volume of the capital investment production.This is possible to realise about a big period of time (e.g. about five,ten or more years) or about every year separatly (the volumes of a capital investment production and a capital investment consumption by sectors are different and this is naturally,when we bear in mind a specificity of capital goods production and consumption by sectors and the flows between its).

We show for example in Table 1 for Bulgaria in 1979 the marginal vector structures of capital matrix on three sectors nomenclature (industry - 1,construction - 2 and other sectors of material production - 3),the sum of which is equaled on the basis of a capital investment consumption:

Table 1

The Marginal Vector Structures of
Capital Investment Matrix for Bulgaria in 1979

Sectors Producing	Consuming 1	2	3	Total sum
1				0,4494
2		k		0,5124
3				0,0382
Total sum	0,5596	0,0409	0,3995	1,0000

Note: The capitalized repair and improvement is included in the both vectors.Capital goods are in the material production sphere only.The consumption is from home made production only,without imported capital goods.The total sums are equaled on the basis of the capital investment consumption total sum.

The task of estimation the capital coefficients in a dynamic IOM reduces to search a distribution of capital investments, e.g. the matrix **k** on Table 1 in our case. One of the first approximation to this matrix is so called the matrix of normal relative share, which is founded on multiplication the corresponding values of marginal vectors:

$$k_{ij} = k_{i.} \times k_{.j} \quad .$$

In our case this matrix is:

$$\mathbf{k} = \begin{bmatrix} 0,2515 & 0,0184 & 0,1795 \\ 0,2867 & 0,0210 & 0,2047 \\ 0,0214 & 0,0015 & 0,0153 \end{bmatrix} \quad .$$

The matrix **k** is used to find one variant of the capital coefficients matrix B_t^n (See Table 2).

One of the other formal procedure about finding the k_{ij} is to use the North-wester angle method. We can do no small at n^2 distributions (the every cage of matrix takes place of the cage in matrix north-wester angle and it row and column accordingly. The others do not change its places, in a simple variant of distribution. The full formal variant demands too much distributions – $/n(n-1)!/^2$). On the basis of the distributed relative shares from North-wester angle method we calculate average relative share $\bar{k}_{ij}$, which is used to get the other variant of a capital coefficients matrix B_t^{nw}, see Table 2 .

We had examined the other formal procedure, but we did not got a satisfactory results. This was the method of the least-squares*). In our case the values k_{ij} was received from the known quantities k_{il} and k_{lj}, where $l \geq \max(i,j)$. About index l we had choosed the time. This factor had a

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*) This method was presented about other purposes from french scientist Academician M. Freshét (Tableax de correlation et distance de deux lois, Ljubljana, Yugoslavia, 1956) and from yugoslav scientist Prof. Br. Ivanović (Discriminaciona analiza sa primenom u ekonomskim istraživanjima, Beograd, 1963, pp 22-31) .

a determinant meaning about these satisfactory results probably.

At the same time about the capital coefficients matrix we used so called Column proportionality method (Method I and II), which had offered from swdish scientist Dr David F.Batten*).

First of all we got three matrices **B** about 1979 in three sectors nomenclature (B_t^{BI} , B_t^{BII1} and B_t^{BII2}), which did not differ essentially from B_t^{n} and B_t^{nw} (See Table 2) . But this method (I and II) keeps the all actual values of the marginal vector $k_{i.}$, but does not of the marginal vector $k_{.j}$ (the sums are the same). With the purpose to equalize the elements of $k_{.j}$ with its actual values, we use the results and calculate the matrix of so called delta-coefficients, where

$$\delta_{ij} = \frac{k_{ij}}{k_{i.} \times k_{.j}}$$

(every δ_{ij} has a value one in the matrix of normal relative share). The delta-coefficients are measure to degree of coherence between separate sectors in a line to capital investment production, distribution and consumption. The calculated δ_{ij} do not essentially differ from one. In practice they are one for Method II and for Method I are:

$$\delta_{ij} = \begin{bmatrix} 1,1679 & 0,8288 & 0,5912 \\ 0,8505 & 1,1901 & 1,3558 \\ 1,0307 & 0,4622 & 1,0492 \end{bmatrix} .$$

The delta-coefficients we use to correct the matrix **k** of normal relative share. On the basis of this matrix we get a new matrix B_t^{nBI} (See Table 2) . The matrix B_t^{nBI} ensures an equalization of the elements of vector $k_{.j}$ to its actual values. In this case, the values of vector $k_{i.}$ come near to its actual values, but the differences are not so big than in case of matrix B_t^{BI}, about the vector $k_{.j}$.

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*) David F.Batten, The Estimation of Capital Coefficients in Dynamic Input-Output Models, Department of Economics, University of Gothenburg, pp 23 .

In addition to that, the equalization of the vector $k_{\cdot j}$ elements to its actual values is on principle much better, than in case of the vector $k_{i\cdot}$.

For illustration in the next Table 2 are given the values of capital coefficients matrix B , getting from different methods, for Bulgaria in 1979 on three sectors nomenclature:

Table 2

Computed Capital Coefficients for Bulgaria, 1979

Elements of matrix B	B^{n}	B^{nw}	B^{BI}	B^{nBI}	B^{BII1}	B^{BII2}
b_{11}	0,7867	0,8599	1,1214	0,9187	0,7127	0,8633
b_{12}	0,9270	0,4884	1,1719	0,7660	0,4205	1,3203
b_{13}	1,2619	1,1586	0,4753	0,7458	1,4987	1,0347
b_{21}	0,8968	0,8744	0,9315	0,7626	0,8128	0,9846
b_{22}	1,0587	0,9324	1,9148	1,2598	0,4795	1,5057
b_{23}	1,4391	1,5010	1,2420	1,9509	1,7091	1,1799
b_{31}	0,0669	0,0131	0,0841	0,0691	0,0605	0,0733
b_{32}	0,0756	0,6397	0,0533	0,0356	0,0357	0,1122
b_{33}	0,1076	0,1490	0,0720	0,1132	0,1273	0,0879

These results have an illustrative disposition only. The more definitive conclusions we will can do in the more extended variants, about which we prepare the necessary economic information now.

On this paper we discussed some problems of investment in an IOM. There are many difficulties, which must overcome step by step. The next purpose is to research the possibilities to use these results about economic analysis and forecasting. Our task here is more limited.

CONSTRUCTING A COMPREHENSIVE INPUT–OUTPUT SYSTEM FOR THE HUNGARIAN ECONOMY

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Input-output tables have been constructed and analyses based on them performed in many countries. The use of these tables has developed across a rather wide empirical base due to the relative simplicity of input-output theory as compared to other methods utilized in economic practice. In many ways the tables have proved to be very useful tools in the analysis of essential economic phenomena from new viewpoints. However, we have now reached a point where both a widening of the empirical data base and utilization of more theoretical results are desirable and therefore some further methodological development is needed. This paper gives a brief account of the first steps in realizing these aspirations for input-output work in Hungary. To illustrate possible ways of improving an oversimplified, open, static, input-output system, a five-sector model will be presented. This model can be considered as a highly aggregated version of an earlier model containing 15-20 sectors.

Before continuing it seems appropriate to briefly look back on the history of input-output table construction in Hungary and to describe the empirical data base presently available. In Hungary input-output tables have been constructed systematically since the early sixties. Interindustry relations have been determined in every fourth or fifth year within the framework of a large input-output table (with about 100 sectors) and within these periods in a smaller input-output table (15-20 sectors) for each year. Input-output table construction procedures were integrated into the general national accounting system in 1968. Since that time column and row sums in input-output tables as well as the values of the corresponding main economic categories appearing in other balances of the national accounting system have been identical. Thus, throughout the seventies, series of 26-sector input-output tables were available at current and at comparable producer's prices. This provided a very good opportunity to widen the empirical base by utilizing the newly available data series and to experiment with the resulting system;

Our work to date has concentrated on improving a number of aspects of the treatment of an open, static, input-output system:

- Variables of final demand components (consumption, investment, and exports) are not treated as exogenous to the system; rather, they are estimated by econometric functions, and their parameters are intrinsic parts of the system;

- Relations between investments and sectoral production are determined either analogously to input-output coefficients or by econometric functions;

- Value added formed in the economy and the net product of each industry are connected to each other by a constant ratio throughout the whole period of observation;

- Together with these constant ratios, value added is the explanatory variable in each function describing the final demand components;

- To avoid an over-complex treatment of imported goods in the final demand functions, the investigations are based on so-called "A" version input-output tables. In these tables the gross output of an industry contains not only the value of its own production but also the value of complementary imports.

Two major problems arise immediately in such an investigation:

The first problem concerns the question of whether the investment functions should be constructed from the supply-side or demand-side viewpoint. To define the investment functions from both the supply and the demand side seems to be the most satisfactory solution, so that results obtained from each type of function can be compared and thoroughly examined before any decisions are taken. This was the approach followed here. Results indicated that in the seventies sectoral output determined the extent of investment activity. Since supply was clearly more important than demand over the period studied, the investment functions were eventually drawn up from the point of view of the supply side only.

The second problem arises from the fact that the input-output table series on which the research is based can be compiled in either current or comparative prices. Price movements can be taken into account either explicitly or implicitly. Of course, in the first case, price functions must be defined in the model in addition to quantity-type functions. There is also the possibility of including both quantity and price variables in the functions simultaneously. When input-output table series at comparative prices serve as the data base there is no need to take price relations into account explicitly. First of all, however, a question must be answered: do price movements in the real economy play a role or not? The choice between the two solutions depends on the answer to this question. As a matter of fact, price movements did not play an essential role during the seventies in the Hungarian economy. The behavior of industries was not significantly influenced by price movements. Having examined price relations thoroughly, it was decided that it was unnecessary to take them into account explicitly.

As mentioned above, the empirical data base used consisted of input-output tables constructed in the Central Statistical Office of Hungary for the years 1970-1979. In order to carry out Durbin-Watson tests for consumption, investment, and export functions, time series of the appropriate variables were extended back to 1965. Data necessary for this purpose were obtained from national accounting and investment statistics.

An input-output table for Hungary during the period 1970-1979 is given in Table 1. Gross investment by sector during the same period is shown in Table 2, while Table 3 explains how the functions for each component of final demand are defined.

Using this empirical data base, two types of input-output system have been defined: one static and one dynamic. Both have opened up new ways of utilizing input-output techniques in modern economic analysis.

The *dynamic system* can be mathematically formulated as follows:

$$Bx(t+1) = (I-A+B-F)x(t) - f(t)$$

or

$$x(t+1) = [I + B^{-1}(I-A-F)]x(t) - B^{-1}f(t)$$

$$(t = 1970, 1971, \ldots, 1979)$$

where

$x(t+1)$ = a vector of gross industrial output $X_i(t+1)$ for period t+1;

$x(t)$ = the same vector, but for period t;

B = a matrix of gross investment implemented, with the characteristic element:

$b_{ij} = I_{ij}(t)/\Delta X_j(t)$;

TABLE 1. Input-output table for Hungary, 1970-1979 (ten-year averages). All values are in billions (10^9) of forints.

	Manufacturing	Construction	Agriculture, forestry, water	Transportation, trade	Other	Together	Consumption	Gross investment	Increase in stocks	Exports	Final demand	Resources
Manufacturing	312.71	46.87	51.84	23.32	33.27	468.01	150.54	61.19	8.73	151.97	372.43	840.44
Construction	9.89	3.97	0.92	2.92	9.00	26.70	5.78	86.49		1.16	93.43	120.13
Agriculture, forestry, water	76.47	2.86	45.47	1.98	0.97	127.75	40.06	4.05	1.25	23.90	69.26	197.01
Transportation, trade	38.73	14.24	5.47	13.81	9.16	81.41	51.29	2.93	0.60	10.22	65.04	146.45
Other	6.76	1.57	1.54	3.59	4.45	17.91	98.22		0.03		98.25	116.16
Together	444.56	69.51	105.24	45.62	56.85	721.78	345.89	154.66	10.61	187.25	698.41	1420.19
Depreciation	24.29	2.53	13.13	11.57	11.34	62.86						
Wages	66.50	25.97	56.47	30.41	41.75	221.10						
Net income	109.98	21.71	10.77	53.32	6.22	202.00						
Imports	195.11	4.1	11.40	5.53		212.45						
Resources	840.44	120.13	197.01	146.45	116.16	1420.19						

TABLE 2. Gross investment implemented in Hungary, 1970-1979 (ten-year averages). All values are in billions (10^9) of forints.

	Manufacturing	Construction	Agriculture, forestry, water	Transportation, trade	Other	Together
Manufacturing	25.76	2.43	10.70	9.82	3.51	52.22
Construction	19.42	3.32	11.50	11.59	35.64	81.47
Agriculture, forestry, water			4.05			4.05
Transportation, trade	1.45	0.11	0.60	0.56	0.21	2.93
Other	0.50	0.06	0.28	0.23	0.39	1.46
Together	47.13	5.92	27.13	22.20	39.75	142.13

TABLE 3. Characteristics of final demand functions.

	a	b	s^2	s_b	s_a	R	DW
1. Consumption functions:							
Manufacturing	34.2296	0.1665	5.8566	0.0066	4.6575	0.9938	1.38
Construction	0.3323	0.0078	0.0577	0.0007	0.4622	0.9731	1.09
Agriculture, forestry, water	26.0534	0.0201	0.3660	0.0016	0.7045	0.9741	1.42
Transportation, trade	23,2378	0.0402	2.4674	0.0043	3.0231	0.9576	1.13
Other	18.1843	0.1146	4.9463	0.0060	4.2803	0.98905	1.46
2. Investment functions:							
Manufacturing	-20.5159	0.1170	7.8035	0.0076	5.3762	0.9836	1.02
Construction	- 3.1274	0.1283	4.1614	0.0055	3.9260	0.9926	1.38
3. Export functions							
Manufacturing	-58.5069	0.3014	97.0025	0.0268	18.9550	0.9699	1.04
Agriculture, forestry, water	- 5.6868	0.0424	7.0509	0.0072	5.1104	0.9009	1.50
Transportation, trade	- 5.1798	0.0220	0.5716	0.0021	1.4551	0.96696	1.21

Abbreviations:

a and b - estimates of constants in linear relationships;
s^2 - estimate of the theoretical error variance;
s_b - estimate of standard error of b;
s_a - estimate of standard error of a;
R - correlation coefficient;
DW - Durbin-Watson statistic.

A = the input matrix, with the characteristic element: $a_{ij} = X_{ij}(t)/X_j(t)$;

F = a matrix of linear combinations of parameters in the final demand functions;

f(t) = a vector of constants in the final demand functions.

Parameters have been estimated for consumption functions:

$$C_i(t) = c_i V(t) + d_i$$

and for export functions:

$$E_i(t) = g_i V(t) + h_i$$

The weighting for linear combinations is given by:

$$v_j = V_j/X_j$$

and

$$V_j = X_j - \sum_{i=1}^{5} X_{ij}$$

Thus the form of the matrix F is:

$$\begin{bmatrix} (c_1+g_1)v_1 & . & . & . & (c_1+g_1)v_5 \\ . & & & & . \\ . & & & & . \\ . & & & & . \\ (c_5+g_5)v_1 & . & . & . & (c_5+g_5)v_5 \end{bmatrix}$$

The form of the vector f(t) is:

$$\begin{bmatrix} d_1 + h_1 \\ \cdot \\ \cdot \\ \cdot \\ d_5 + h_5 \end{bmatrix}$$

Results computed from the Hungarian input-output tables based on this dynamic system are shown in Tables 4 and 5.

Turning now to the improved *static system*, this can be represented as follows:

$$X = (I-T)^{-1}t$$

where T is the matrix of linear combinations of parameters in the final demand functions. Parameters have been estimated in this case not only for consumption and export functions (in a similar way to the dynamic system) but for the investment functions too:

$$B_i(t) = b_i V(t) + \ell_i$$

Matrix T will be, mutatis mutandis, constructed in the same way as $[I+B^{-1}(I-A-F)]$ above. Thus, its form will be as follows:

$$\begin{bmatrix} a_{11} + (c_1+g_1+b_1)\ v_1 & \cdots & a_{15} + (c_1+g_1+b_1)\ v_5 \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{51} + (c_5+g_5+b_5)\ v_1 & \cdots & a_{55} + (c_5+g_5+b_5)\ v_5 \end{bmatrix}$$

while the vector of constant ℓ is:

TABLE 4. Results computed for the matrix $[I + B^{-1}(I-A-F)]$ for Hungary, 1970-1979.

	Manufacturing	Construction	Agriculture, forestry, water	Transportation, trade	Other
Manufacturing	-540.3285055	-515.2193215	-545.3281876	-1225.0301430	8109.0650540
Construction	52.0850834	52.7385898	40.0819893	-75.0988422	-510.5740595
Agriculture, forestry, water	0.2137214	-0.0889529	1.3139559	-0.1002852	-0.0713967
Transportation, trade	240.7585528	227.4567992	244.9497189	612.5807792	-3700.7613482
Other	-28.8341124	-27.1286178	-28.6249294	-61.6052515	426.4678342

TABLE 5. Results computed for the vector $[B^{-1} f(t)]$ for Hungary, 1970-1979.

Manufacturing	139059.5
Construction	-12305.1
Agriculture, forestry, water	38.4
Transportation, trade	62250.2
Other	7378.6

TABLE 6. Results computed for the matrix $[(I-T)^{-1}]$ for Hungary, 1970-1979.

	Manufacturing	Construction	Agriculture forestry,water	Transportation, trade	Other
Manufacturing	33.045221850	32.079604690	31.930895910	31.662533370	31.926282900
Construction	4.344687511	5.369020534	4.336269929	4.349511536	4.413934489
Agriculture, forestry, water	6.200698808	6.119720417	7.368285179	6.055562393	6.082354274
Transportation, trade	4.472105596	4.555192288	4.450055247	5.50323605	4.510411642
Other	3.496635814	3.503939736	3.496129825	3.510240633	4.528068014

TABLE 7. Results computed for the vector ℓ for Hungary, 1970-1979.

Manufacturing	-36.06312975
Construction	-1.63511689
Agriculture, forestry, water	25.66665531
Transportation, trade	21.58807179
Other	18.21432443

$$\begin{bmatrix} d_1 + h_1 + \ell_1 \\ \cdot \\ \cdot \\ \cdot \\ d_5 + h_5 + \ell_5 \end{bmatrix}$$

Returning to the input-output table for Hungary and using the improved static system we obtain the computed results shown in Tables 6 and 7.

MODELING THE STRUCTURAL CHANGE OF CONSUMPTION AND THE INPUT OF PRIMARY RESOURCES IN THE GDR ECONOMY BY A TIME-SERIES AND INPUT–OUTPUT APPROACH

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1. Introduction

The material basis for the existence and development of society is reproduction. Its first and last phases are production and consumption. The material goods and services for the satisfaction of needs, which are produced in the phase of production, are used in the phase of consumption. The final purpose of production - the steadely improving satisfaction of material and cultural needs of the people - results from the main economic principle of socialism.

Changes in the structure of needs result in structural changes of production, in changes within the needed labour force and fixed capital as well as in the provision of further economic primary resources. This regards to the production sectors, which are producing directly goods and services for the satisfaction of needs as well as the preliminary production stages.

Planning this process on an macro-economic level, the effects of the change in consumption on the structures of production and primary resources must be known. Thus one has to link the vector of consumption with the individual production sectors and the primary resources. Input-output tables are the analytical technique for such investigations. By their help one can trace back the flow of production from sectors near to consumption to those far from it.

Subject of the present paper is the analysis of the relationship between consumption, production, input of labour force, fixed capital and primary energy carriers according to actual input-output tables and a first approach to foracast the development of the structure of consumption and its effects on the structures of production and resources for future periods. The estimation of future consumption structures was made by the help of econometric functions. Input-output tables for the years 1968, 1972 and 1977, the retail turnover according to main groups of goods, the private and the social consumption in time-series according to the national account were available. The test calculations relate to the years 1985, 1990, 1995 and 2000.

2. Development of consumption according to complexes of needs

2.1. Characterization of the consumption structure and its hitherto existing development

Thousands of different goods and services are consumed. Therefore investigations for the short-term planning of economy must be rather concrete. Whereas, for the long-term planning one needs investigations with less but more aggregated items. As there are certain dependences between the satisfaction of different needs (e. g. accomodation-furniture, motor car-fuel ...), it is possible to unite different goods and services to homogeneous consumption entities, called complexes of needs. Moreover, consumption theories have proved that bigger aggregates of needs correlate with such economic variables as the national income.[1] The formation of complexes of needs being relatively independent of each other provides the possibility to define additive utility functions which correspond well with the linear equation approach of static input-output models.[2]

When forming such complexes of needs one has to take into account other facts too. Thus, the horizon in time of the investigation, the degree of homogeneity of the aggregated goods and services, the unambiguousness necessary when attributing the goods and services to the needs, the organizational structure of the economy, the kind of registering and interpreting statistical data as well as other factors play a role. Each classification of goods and services according to the complexes of needs is up to a certain degree limited and a compromise solution.

There have been already several approaches for forming complexes of needs.[3] We proceed from the following ones:

1 - nutrition
2 - clothing
3 - accommodation
4 - health care
5 - education/culture/recreation/communication
6 - transportation
7 - general public needs

These complexes of needs are easily to survey and develop relatively independently. Thus continuous changes of the structures of needs and of the consumption habits can better be investigated and guided by measures than in case of isolated measurement of single products. In this way, the definition of concepts for the development and the determination of priorities within the satisfaction of needs are facilitated for a longer period. Proceeding from the basic equation of the standard static input-output model

$$y = (I - A)x \qquad (1)$$

at first, the components of the consumption vector are attributed to the complexes of needs. On the basis of different statistical documents it is estimated which goods and services have to be attributed to the individual complexes of needs.[4] We use the following designations:

$A = (a_{ij})_{nxn}$ - matrix of input-output coefficients;

$x = (x_i)_{nx1}$ - vector of gross domestic output;

$y = (y_i)_{nx1}$ - vector of final demand;

$c = (c_i)_{nx1}$ - vector of consumption;

$c_k = (c_{ik})_{nx1}$ - vector of 'supplies' of the sectors i to the complex of needs k;

$z_k = (z_{ik})_{nx1}$ - vector of shares of c_{ik} in the production of consumer goods of the sectors i.

To calculate the complexes of needs we proceed from the relation:

$$c_k = \mathrm{diag}(c)z_k \tag{2}$$

The investigation is based on the time independent constant share coefficients z_{ik}. The components of the partial vector of consumption c_k are furthermore summed up.

Table 1 presents a survey on the effects of the different growth rates of the production of consumer goods and material services on the structure of consumption of the GDR in the 70s. These changes agree with the regularity in the upward development of needs. Thue one can state an above average rise in the provision of such consumer goods and services which serve to satisfy those complexes of needs as health care, transportation, general public needs, education/culture/recreation/communication, and accommodation. The growth rates do not differ substantially. The complex of needs 'health care' has the highest growth rate of more than 9 percent annually. Regarding the complex of needs 'accommodation' one has to consider that the data do not include the final product for residential construction and modernization of housing as it belongs to the investment. There has been a below than average rise in the provision of consumer goods and services for the complexes of needs 'nutrition' and 'clothing'.

In the GDR, the relative share of those needs which - in a certain sense - can be characterized as primary or basic needs decreased within the entire structure of consumption in the 70s. Such regularity was also observed in earlier periods. However, there were tendencies in some subgroups which counteracted a stronger decline. The contrary tendency took place in the case for those complexes of needs which can be characterized - in the same limited sense - as secondary or higher needs; they arise from the social development of the people and from the development of social institutions.

Table 1

Structural change of consumption according to complexes of needs in the GDR in the 70s.

Complex of needs	Structure of consumption in percent	
	1972	1977
Nutrition	45,4	41,5
Clothing	11,7	10,9
Accommodation[1]	18,4	19,8
Primary needs	75,5	72,2
Health care[1]	4,7	5,7
Education[1]/culture/recreation/communication	7,2	7,8
Transportation	8,1	9,3
General public needs	4,5	5,0
Secondary needs	24,5	27,8
Total consumption	100,0	100,0

1 without non-productive investments in residential construction, health service and education system.

The structural shift between the primary and the secondary needs was caused by the relative drop of growth in those goods and services belonging to the complex of needs 'nutrition'.

2.2. Results of the econometric forecastings for the structure of consumption

Two different methods were used for these forecastings:

1. The forecasting for the complexes of needs by the help of trend functions which were calculated on the basis of the shares of the complexes of needs in the consumption of 1968, 1972 and 1977.

2. The calculation of the complexes of needs on the basis of an estimation of the consumption vector by the help of econometric functions.

As in the GDR there are available input-output tables for only 3 years of statistical investigations with different nomenclature, we had only a rather short time-series at hand for calculating the trend functions. Even if a statement on the basis of only 3 values cannot provide a confidence coefficient, clues to an estimation of the quality of forecasts with other methods are given. The following trend functions were calculated:

$$v_{kt} = f(t), \text{ where} \tag{3}$$

v_{kt} - is the share of the complex of needs k in the consumption in percent in the period t.

The forecasts on the basis of trend functions provided results which indicate relatively strong structural changes of consumption according to the complexes of needs. Because of the low confidence coefficient and of the necessary knowledge of the entire vector of consumption for the linkage with input-output tables, forecasts were also made with econometric functions.

The linkage of the consumption functions with the input-output model takes place with this approach through the retail turnover. This approach is justified as the retail turnover includes 80 percent of the private consumption. However, this approach has to overcome some difficulties which are caused by the different registration of statistical data through the national accounts and the commercial statistics. The vector of final demand c for the entire consumption was calculated as follows:

$$c_t = c_t^p + c_t^s \, , \tag{4}$$

$$c_t^p = (\alpha)^{-1} c_t^s + c_t^r \, . \tag{5}$$

The following designations mean:

$c_t^q = (c_{it}^q)_{nx1}$; $q \in \{p, s, e, r\}$.

p - private consumption;
s - social consumption;
e - retail turnover;
r - remaining private consumption;
i - main groups of products;
t - time index, t_o = 1977.

$$\alpha = (\alpha_i)_{nx1}, \text{ where } \quad \alpha_i = \frac{c_{it_o}^e}{c_{it_o}^p} . \tag{6}$$

$$c_t^r = (c_{it}^r)_{nx1}, \text{ where } \quad c_{it}^r = \begin{cases} 0, \text{ if } \alpha_i \neq 0, \\ \dfrac{c_{it_o}^r}{\sum_i c_{it_o}^p} C_t^p, \text{ if } \alpha_i = 0 . \end{cases} \tag{7}$$

$$C_t^p = f(t), \text{ where} \tag{8}$$

C_t^p - private consumption according to national accounts.

The elements c_{it}^e of the vector c_t^e were calculated by the help of consumption functions based on main groups of goods:

$$c_t^e = \beta C_t , \tag{9}$$

$$C_{jt} = f(Y_t) , \tag{10}$$

$$Y_t = f(t). \tag{11}$$

Here applies:

$C_t = (C_{jt})_{10x1}$,

C_{jt} - consumption in the main group of goods j (j = 1,...10) according to the commercial statistics in period t, e.g. food and beverages, clothing and the like;

$$\beta = (\beta_{ij})_{n\times 10}\ , \text{ where } \beta_{ij} = \frac{c^{e}_{it_o}}{C_{jt_o}}\ ; \qquad (12)$$

Y - net income of population.

The amount of social consumption up to the year 2000 was estimated by the help of a trend function. For determining its internal structure according to main groups of products we took the share coefficients from 1977.

To forecast the main groups of goods, the net income, the private and the social consumption we had to choose a type of function suitable for both the values of statistical tests (coefficient of determination and value of F-test) and the forecast. Beside the linear type of function, we estimated the potency function, the exponential function, the semi-logarithmic function, the hyperbolic function and the Törnquist function.

Proceeding from the values of statistical tests and from the forecasts we chose the following types of functions for the main groups of goods:

a) the Törnquist function in the form of:

$$\frac{1}{C_j} = \frac{1}{a_j} + \frac{b_j}{a_j}\frac{1}{Y} + \mu_j \qquad \text{for} \qquad (13)$$

- food;
- footwear, fancy goods and saddlery;
- textiles for clothing and linen, ready-made upper apparel;
- knitted goods, underwear, household linen, haberdashery;
- furniture, musical instruments, toys, sports goods;
- products of hygiene and health, chemicals.

b) the potency function in its linearized form:

$$\ln C_j = \ln a_j + b_j \ln Y + \mu_j \qquad \text{for} \qquad (14)$$

- alcoholic drinks, tabacco, coffee, tea, cocoa products;
- electroacoustics, photographic and optical instruments, jewelry, road vehicles.

c) the exponential function in its linearized form:

$$\ln C_j = \ln a_j + b_j Y + \mu_j \qquad \text{for} \qquad (15)$$

- household and technical consumer goods;
- building materials, timber, fuels.

As there mean:

C_j - consumption in main groups of goods j for j = 1,...,10;
a_j, b_j - parameters of the regression functions;
μ_j - coefficients of disturbancy.

For forecasting the net income and also the private and social consumption we used the linear approach of the trend function, thus assuming a constant volume of growth.

The values of statistical tests of the differently calculated types of functions for the single main groups of goods do not differ from each other essentially, therefore it is difficult to make an unambiguous analysis of the content on the basis of the chosen types of functions. The Törnquist function was chosen for a number of main groups of goods not only because of the better values of statistical tests but also because of their - as we think - good values of forecast. Moreover, this type of function involves limits of saturation which seemed to be justified for a number of main groups of goods. The exponential function for the item 'household and technical consumer goods' can be explained by the growing mechanization of households.

The calculated coefficients of elasticity ϱ showed the value $\varrho < 1$ in the main group 'food' as known for the Engel curves. The same resulted from the analysis of the main group 'knitted goods', whereas $\varrho > 1$ was found for the remaining main groups of goods.[5] The high coefficient of elasticity for the main groups of goods 'building materials, timber, fuels' and 'electroacoustics, photographic and optical instruments, jewelry, road vehicles' is probably due to the high growth rate of weekend houses (building materials, timber), the growing number of private cars and electroacoustic equipment. The number of private cars per 100 housholds was 8.2 in 1965 but more than 37 in 1980.[6]

The forecast of consumption involved a growing volume of goods and services with all subgroups. Only the growth rates differed. On this basis the consumption structure according to complexes of needs was calculated for the years 1985, 1990, 1995 and 2000 (see Tables 2 and 3). By analysing the results it turns out that those changes within the above mentioned structure of consumption will continue that have already been observed for the period from 1968 until 1977:

- The shares of the primary needs within the entire consumption is dropping. But within the needs the share of the complex of needs 'accommodation' is growing due to the high mechanization of the households.

- The share of the secondary needs within the entire consumption is increasing.

Table 2

Development of consumption structure estimated on the basis of econometric functions

Complex of needs	Structure of consumption in percent		
	1985	1990	1995
Nutrition	39.1	36.8	34.3
Clothing	10.9	10.6	10.3
Accommodation	21.1	22.5	24.0
Primary needs	71.1	69.9	68.6
Health care	6.2	6.7	7.3
Education/culture/recreation/ communication	8.1	8.3	8.5
Transportation	9.3	9.5	9.6
General public needs	5.3	5.6	6.0
Secondary needs	28.9	30.1	31.4
Total consumption	100.0	100.0	100.0

Regarding tha development of the calculated consumption structure for the years 1968-2000, that is a period of about 30 years, one can say:

- The ranking of the shares of the complexes of needs within the consumption does nearly not change. The basic needs 'nutrition', 'accommodation', 'clothing' cover the highest ranks. Only then the complex of needs 'transportation' as a secondary need follows. Only for the year 2000 the estimatione show an exchange of the ranks between the complexes of needs 'clothing' and 'transportation'.

- There is an obvious tendency of an equalization of the shares of the complexes of needs except the complexes 'nutrition' and 'accommodation'.

In principle, the forecast of the shares of the complexes of needs within the entire consumption made by the help of simple trend functions resulted in the same tendencies: a drop of the share of primary needs (except the complex of needs 'accommodation') and a rise of the share of secondary needs. The differences between the both estimations were of gradual nature.

Table 3

Results of the forecast of the consumption structure for the year 2000 according to different methods

Complex of needs	Structure of consumption in percent	
	Trend estimation	Estimation according to consumtion functions
Nutrition	25.3	31.1
Clothing	7.2	9.8
Accommodation	27.0	25.8
Primary needs	59.5	66.7
Health care	8.7	7.8
Education/culture/recreation/communication	12.6	9.1
Transportation	11.5	10.0
General public needs	7.7	6.4
Secondary needs	40.5	33.3
Total consumption	100.0	100.0

Table 3 shows that there are major differences between the estimations of the complexes of needs 'nutrition', 'clothing', 'education/culture/recreation/communication'. The comparision makes clear that the linkage of econometric functions with the consumption vector of the input-output table is a practicable way - although it has some imperfections - for the forecast of the complexes of needs and it seems to provide realistic estmmates compared with the simple trend calculations.

3. Effects of the Consumption change on the structure of production and primary resources

3.1. Development of the input structure of gross product, labour force and fixed capital

The classification of consumption according to complexes of needs and their forecast up to the year 2000 allows to draw conclusione from the input of gross product, labour force, fixed capital and primary energy carriers to satisfy the complexes of needs. The present investigation tries to analyse the re-allocation of the social total labour to the different complexes of needs only in dependence on the forecast development of consumption and its parts. Contrary to the estimation of the complexes of needs, no forecasts can be made from the calculations of the adequate total input of labour and fixed capital. It only serves to found the necessity of changes in production, especially changes of technology on the basis of the scientific-technological progress. Thus, it will reveal that measures in production itself have a stronger influence on the employment of primary resources for satisfying the consumptive needs than changes in the consumption structure do.

For all complexes of needs holds:

$$\hat{x}_k = (I - A)^{-1} c_k \ , \qquad (16)$$

$$\hat{l}_k = \text{diag}(l)\hat{x}_k \ , \qquad (17)$$

$$\hat{g}_k = \mathrm{diag}(g)\hat{x}_k \; . \tag{18}$$

The designations mean:

$\hat{x}_k = (\hat{x}_{ik})_{nx1}$ - vector of the total gross input for the complex of needs k;

$l = (l_i)_{1xn}$ - vector of the direct sectoral labour input per output unit;

$\hat{l}_k = (\hat{l}_{ik})_{nx1}$ - vector of the total labour input for the complex of needs k;

$g = (g_i)_{1xn}$ - vector of the fixed capital input per output unit;

$\hat{g}_k = (\hat{g}_{ik})_{nx1}$ - vector of the total fixed capital input for the complex of needs k.

The vectors $\hat{x}_k$, $\hat{l}_k$ and $\hat{g}_k$ are summarized by addition of their components. From the formulas (17) to (18) derives that the input of primary resources for the satisfaction of the complexes of needs is mediated through the calculation of the total gross output for the consumption. As not only the direct labour input and the direct fixed capital input per output unit (l_i and g_i) of the basic year 1977 but also the coefficients of the direct material input (matrix A) of that year were the basis for calculating the total input of primary resources in the years from 1980 to 2000, these essential tendencies become obvious, which are common with the mentioned input due to the growth of the gross output.

Table 4 shows that the development of the gross product after 1977 would agree essentially with the development of the complexes of needs. Before 1977 this nearly parallel development was hidden by changes in the coefficients of the direct material input. Calculations have shown that the gross product input per consumption unit referring to the complexes of needs and their aggregates is developing with an essentially less dynamics after 1977 than before, although the change of the internal structure of the different partial vectors of consumption is going on with the same speed. According to the model equations, structural changes of consumption due to complexes of needs and complexes of needs themselves are causing after

Table 4

Development of the structure of total gross product input as a whole and in relation to the consumption units according to complexes of needs - an estimation

Complex of needs	Gross input in percent			Gross input per consumption unit		
	1968	1977	2000	1968	1977	2000
Nutrition	52.2	47.2	35.1	2.46	2.99	2.90
Clothing	10.4	8.9	8.2	2.08	2.15	2.13
Accommodation	16.2	18.0	24.5	2.09	2.40	2.43
Primary needs	78.8	74.1	67.8	2.32	2.70	2.61
Health care	4.0	5.2	7.6	2.02	2.43	2.49
Education/recreation/ culture/communication	5.7	6.8	8.3	2.02	2.28	2.34
Transportation	7.6	9.1	10.1	2.07	2.59	2.60
General public needs	3.9	4.8	6.2	2.17	2.51	2.49
Secondary needs	21.2	25.9	32.2	2.07	2.46	2.48
Total consumption	100.0	100.0	100.0	2.26	2.64	2.56

1977 far less changes of the gross product input per consumption unit than before, where both the structure of consumption and the direct input coefficients were subjected to changes.

The increase of efficiency of the applied primary resources, which is necessary to secure the forecast growth of consumption materially, has to be estimated. Therefore one must pass from the item gross product to the items total labour input and total fixed capital. Their volume was calculated by the formulas (17) and (18) from the development of the sectoral gross outputs.

Table 5 reflects the development of the structure of necessary labour force for satisfying the growing consumption in the years after 1977 with unchanged technology: While in 1977 about 5 million labour force were still sufficient to secure the material consumption, in the year 2000-according to the above mentioned unchanged technology-there should be more than double the labour force. However, within the next 20 years, an essential growth of labour force potential will not be available

Table 5

Estimated development of the structure of the total input of labour force for consumption

Complex of needs	Labour input in percent		
	1968	1977	2000
Nutrition	50.2	46.1	35.9
Clothing	11.9	9.8	9.4
Accommodation	16.4	17.5	23.0
Primary needs	78.5	73.4	68.3
Health care	3.7	4.4	6.1
Education/culture/recreation/ communikation	6.2	8.3	9.8
Transportation	7.8	9.1	9.8
General public needs	3.8	4.8	6.0
Secondary needs	21.5	26.6	31.7
Total consumption	100.0	100.0	100.0

in the GDR. Therefore we are faced with the task to compensate the demand of additional labour force by an increase of labour productivity. This means to increase labour productivity within 23 years by about 132 percent. A projection of the rise of labour productivity within the last years revealed an annual growth of about 5 percent which corresponds to an annual rise of 3.7 persent and fits into the frame of the hitherto achieved results.

The average rise of labour productivity will be implemented differently in the framework of the various complexes of needs. Before 1977, in the production of goods and services for the single complexes of needs, the annual growth rates of labour productivity differed between 0.2 and 9 percent.

Regarding the single complexes of needs, the unequal development will lead to the fact that the structure of labour input in the production of goods and services will differ from the data in Table 5 due to the effects of technological changes; whereas the data in Table 5 were calculated by only regarding the effects of structural changes of consumption.

Also the development of the total fixed capital input for the satisfaction of the complexes of needs turns out parallely to their growth according to the assumptions made (see Table 6).

Table 6

Estimated development of the structure of the total fixed capital input for consumption

Complex of needs	Fixed capital input in percent		
	1968	1977	2000
Nutrition	45.9	42.8	30.2
Clothing	9.4	7.6	6.6
Accommodation	17.3	19.8	28.2
Primary needs	72.6	70.2	65.0
Health care	4.8	5.7	8.2
Education/culture/recreation/ communication	7.4	7.7	8.6
Transportation	10.7	10.9	10.7
General public needs	4.5	5.5	7.5
Secondary needs	27.4	29.8	35.0
Total consumption	100.0	100.0	100.0

The total fixed capital input per consumption unit drops because of internal structure shifts in the complexes of needs 'food', 'clothing', 'transportation' and it increases slightly in the other complexes of needs. The tendencies of a decline of the fixed capital input per consumtion unit are, however, not strong enough to prevent a slight increase of this item in the aggregate 'primary needs' and in consumption as a whole. From that fact and from the development of the total amount of consumption results more than a doubling of the fixed capital stock for the year 2000. Regarding the average investment volume for the material realization of consumtion of the last years, only 1.75 percent annually could be depreciated from the present stock to accumulate the estimated value in the year 2000. This depreciation rate seems to be too small to make fully efficient the scientific-technological progress, which is embodied in the means of production. This means that the capital-output ratio has to be decreased considerably on the macroeconomic level because of the only slightly changing investment volume. To comply with this requirement also depends more on the technological and economic changes than on structural changes in consumption.

3.2. Approach to the analysis of the input of primary energy carriers

The allocation of the entire social labour depends not only on the proportions of the needs to be satisfied and on the technological and organizational stage of production but also on the kind and amount of available raw material and energy carriers. The shortage of some fossil energy carriers caused also in the GDR's economy structural changes in the second half of the 70s. To analyse this development the relationship between consumption according to complexes of needs and primary energy consumption is investigated more in detail.

An analysis of the primary energy consumption by the help of input-output tables of a medium aggregation level becomes more complicate by the fact that the consumption of specific energy carriers is not expressed as utilization of a primary resource (in the 3rd quadrant) but as material consumption (in the 1st quadrant). Hence according to the basic equations of the input-output table the supply of primary energy carriers as products of single branches is limited only by the availability of listed primary resources (labour force and fixed capital). Because of this different registration, the total input of primary energy carriers for consumption cannot be calculated analogously to the total input of labour and fixed capital. However, by disaggregation of those branches, in which primary energy carriers are extracted, it is possible to fix the total input of primary energy carriers to satisfy the different complexes of needs in terms of gross product. This approach includes double countings, but nevertheless the special structure of the input coefficients of the branches supplying primary energy allows to use these total inputs as a suitable approximation for the total input of the primary energy in value units.

The following designations mean:

$\tilde{A} = (\tilde{a}_{ij})_{(n+z)x(n+z)}$ - augmented matrix of input-output coefficients formed by separating the branches of primary energy from matrix A;

$W = (w_{ij})_{(n+z)xn}$ - matrix of coefficients aggregating the columne of $(I-\tilde{A})^{-1}$ to nomenclature of A;

$H = (h_{ij})_{zx(n+z)}$ - matrix by which all coefficients not related to primary energy are eliminated from an initial matrix;

$E = (e_{ij})_{zxn}$ - matrix of the total input coefficients of all those sectors supplying primary energy;

$e_k = (e_{ik})_{zx1}$ - vector of the total input of primary energy carriers for the complex of needs k in gross product units.

The following equations apply:

$$E = H(I - \tilde{A})^{-1}W \;, \qquad (19)$$

$$e_k = E\, c_k \;. \qquad (20)$$

The investigation revealed that in 1977 the input of primary energy carriers varied, as expected, according to complexes of needs. In general, it was found that the satisfaction of the secondary needs compared with those of the primary neede required the 1.7fold amount of primary energy carriers per value unit of consumer goods and servicee.

The forecasts reveal that the demand of primary energy carriers, at constant coefficients of direct input, especially of the energy input per output unit, would rise parallely to the single complexes of needs. Within 20 years, the primary energy input per consumption unit would rise altogether by 14 percent, whereas there exist no great differences between the complexes of needs. The absolute difference between the primary energy inputs per consumption unit, however, are so evident that references to energy conserving potentials can be derived. Within the total consumption, the complexes of needs 'health care', 'transportation', general public needs' and 'accommodation' reveal over-average energy inputs per consumption unit.

4. Outlook

The presented results of structures of consumption, production and primary resources are a first attempt to link input-output tables with econometric functions. On this basis, proceeding from estimated developments in the consumption structure according to complexes of needs, necessary changes in production could be founded quantitatively. The preconditions include strong simplifications of reality. This regards especially to the assumed constancy of the input-output matrix A and of the level of labour productivity and fixed capital efficiency, the neglection of investment flows, and also the economic and methodical assumptions made when estimating econometric functions. These assumptions must be reduced step by step. Moreover, it will be necessary to develop approaches for evaluating the quality of the results. Steps in that direction are international comparisons and comparable calculations through other model constructions.

References

1 See H. Wold/L. Jureen, Demand Analysis - A Study in Econometrics, New York 1953, p. 20, 226.

2 See H.S. Houthakker, Additive Preferences, in: Econometrica, 1960, p. 244.

3 Differences in comparison with former publications of the authors result from the realignment of the complexes of needs. See U. Ludwig/M. Kraft/J. Behr, On the Structural Change of the Relationship Between Consumption and Primary Resources in the GDR Economy, in: A. Smyshlyaev (Ed.), Prodeedings of Task Force Meeting on Input-Output Modeling (1981), IIASA Collaborative Papers, June 1982, CP-82-32, p. 239-253. J. Behr/M. Kraft/U. Ludwig, The Utilization of Input-Output Analysis for Planning Basic Structures Affecting the Satisfaction of Needs in the German Democratic Republic, Seventh International Conference on Input-Output Techniques, Innsbruck 1979. Gerd Knobloch, Bedürfnisbefriedigung - Produktionsstruktur - Grundproportionen, Berlin 1974, S. 22-38 (Forschungsberichte des Zentralinstituts für Wirtschaftswissenschaften der Akademie der Wissenschaften der DDR, Nr. 10).

4 In contrast with the system of national accounts (SNA) in this paper the attribution of goods and services to complexes of needs is built upon the flow of the so-called material products and services (MPS) going to the housholds and to the government. Hardly there arises any difficulty of understanding, in principle, if the satisfaction of private needs is considered such as nutrition, clothing, accommodation, transportation and communication. But this may happen, if such needs as health care, education, culture, recreation and general public needs are considered which are satisfied to a high degree collectively. In these cases the income of the employees (doctors and medical staff, teachers etc.) is not taken into account. Thus the complex of general public needs mustn't mixed up with the traditionally used term of government expenditure.

5 See also Manfred Wölfling, Ein ökonometrisches Modell der Volkswirtschaft der DDR, Berlin 1977, S. 75 (Forschungsberichte des Zentralinstituts für Wirtschaftswissenschaften der Akademie der Wissenschaften der DDR, Nr. 21).

6 See Statistisches Jahrbuch der Deutschen Demokratischen Republik 1981 , Berlin 1981 , S. 276.

IMPORTANT INPUT COEFFICIENTS IN AUSTRIAN INPUT–OUTPUT TABLES FOR 1964 AND 1976

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The Austrian Institute for Economic Research1) and the Institute for Advanced Studies ("Austrian Institutes") are preparing two versions of a dynamic "INFORUM" input-output model for Austria. One version is based on the 1964 input-output table for Austria, and the other one the 1976 table. Both tables have an identical breakdown into 19 industries (see table 1). They are valued at current, "producers prices net of value added tax". This pricing system was directly used in the statistical compilation of the 1976 input-output table. The 1964 table was repriced, i.e. the 1964 turnover tax was excluded from all flows with the help of an input-output price model2). (A project to construct a 1964 input-output table valued at 1976 constant prices, funded by the National Bank of Austria, started only recently. Its results are expected for late 1983.)

Prediction of changes in input coefficients is one of the difficult problems in constructing input-output models. One way to solve it is to interview scientists and technologists about future technology and to translate their assessment of technical progress into changes in input-output coefficients. For this the input-output table has to be dissaggregated into great detail. The other approach is to analyse past changes in input coefficients, and to extrapolate them into the future. Because the "Austrian Institutes" do not have the capacity to use the former time consuming and costly approach, and because the input-output tables used in their models are highly aggregated, they must follow the

other path. However, the conditions for this are not very good either. The two available input-output tables are valued at current prices. Between 1964 and 1976 there was fast inflation combined with pronounced shifts in relative prices of energy and services. The purpose of the investigation is to gain experience which could be helpful in projecting coefficient changes in the two Austrian versions of the INFORUM model.

Table 1

Classification of industries in the dynamic input-output model of "Austrian institutes"

Industry	ISIC groups[1)]
1. Agriculture and forestry	1
2. Mining and quarrying	2 minus 22, 2901
3. Manufacture of food, beverages, and tobacco	31
4. Textile, wearing apparel, and leather industries	32
5. Manufacture of wood and wood products	33, 3902, 3903
6. Manufacture of paper and paper products, printing, and publishing	34, 9592
7. Manufacture of chemicals	35 minus 353
8. Crude petroleum, natural gas, and petroleum refineries	22, 353
9. Manufacture of nonmetallic mineral products	36, 2901
10. Basic metal industries	37
11. Manufacture of fabricated metal products, machinery, and equipment	38
12. Electricity, gas, and water	4
13. Construction	5
14. Trade	61, 62
15. Restaurants and hotels	63
16. Transport, storage, and communications	7
17. Financing, insurance, real estate, and business services	8 minus 833
18. Social and personal services	9 minus 91, 9592, 96
19. Public administration and defence	91

1/ International Standard Industrial Classification of All Economic Activities, Rev. 2, United Nations, New York, 1968.

1. Input Coefficients and Measures of their Change

The well known definition of the input coefficient is as follows:

(1) $$a_{ij} = \frac{X_{ij}}{X_j}$$

where:

X_{ij} = intermediate inputs from industry i into industry j

X_j = gross output value of industry j

The origin of the inputs is not taken into account. If, however, domestically produced and imported flows are accounted for separately in the input-output table, the following breakdown of input coefficients can be used3):

a) For domestically produced inputs:

(2) $$a^D_{ij} = \frac{X^D_{ij}}{X_j}$$

b) For the imported inputs4):

(3) $$a^M_{ij} = \frac{X^M_{ij}}{X_j}$$

(D stands for domestically produced and M for imported inputs.)

The input coefficients of industry j can be written as elements of a column vector a_j . If the valued added coefficient is also included in the vector, the sum of the elements is unity. The value added coefficient is defined as follows:

(4) $$a_{vj} = \frac{V_j}{X_j}$$

where:

V_j = value added in industry j

Any change in a_{vj} thus influences all a_{ij} . The input coefficients alone cannot show the stability of relations between intermediate inputs themselves. This can be seen only with the "technological coefficients" which otherwise are used seldom in input-output analysis. They are defined as follows:

(5) $$t_{ij} = \frac{X_{ij}}{X_{.j}}.$$

where:

$X_{.j}$ = total intermediate inputs in industry j

Two tables for one country with an identical industry breakdown, but refering to two different years, can differ in the values of their input coefficients for the following five reasons:

a) Different statistical methodology of data compilation of the input-output tables (e.g. minor differences in industry definitions, or in treatment of secondary output etc.). A very important difference can be caused by the transition from the 1958 to the 1968 national accounting systems5), which differ, inter alia, in valuation of the input-output transactions (producers′ prices and approximate basic values respectively):

b) Differences in price levels, if tables are valued at current prices.

c) Shares of value added in the gross output values (i.e. the value added coefficients) can change due to shifts in inputs of primary production factors, (such as labour and capital) per unit of output. The value added share could also be influenced by changes in the remuneration of primary factors, i.e. in wage and profit rates. Such changes fall, however, under point b above). A change of the value added coefficient influences the values of the coefficients for intermediate inputs.

d) The composition of intermediate inputs changes if raw materials, energy, half-finished products or services are needed in smaller or in larger quantities per unit of output.

e) Domestically produced and imported goods are not always substituable and also not valued at the same price. (This is true in particular for small input-output tables such as those used by the "Austrian Institutes".) Changes in relations between domestically produced and imported inputs may also influence input coefficients as defined by (1).

The last three causes of changes in input coefficients, as listed above, can have their roots either in technology change or in a change of the composition of the output of the industry (output mix).

A change in technology, i.e. in the way goods and services are produced, can be a consequence of technical innovations (either in production methods or in the nature of the final product), or of a decrease or increase in the volume of output (economies or diseconomies of scale), or of shifts in relative prices of inputs.

The output of most industries is composed of a large number of similar goods. These are fabricated in slightly different ways and do not serve exactly the same purpose. The assumption of input-output theory, that industry output is homogenous, is never fulfilled, not even in most detailed tables. A shift in the (intermediate or final) demand for individual products changes the output mix and affects both the input and technological coefficients.

If there are two comparable input-output tables for years 0 and 1, the change in an input-output coefficient can be measured by its absolute or relative difference i.e. either as:

(6) $$a^{0,1}(a_{ij}) = a_{ij}^{1} - a_{ij}^{0}$$

or as:

(7a) $$r^{0,1}(a_{ij}) = \frac{a_{ij}^{1} - a_{ij}^{0}}{a_{ij}^{0}}$$

In input-output calculations, however, the most common problem is that the accuracy of forecasting suffers because instead of the true value of the input coefficient in the year 1 - which is not known - its value in the year 0 is used. The relative difference is then measured as:

(7b) $$r^{1,0}(a_{ij}) = \frac{a_{ij}^{0} - a_{ij}^{1}}{a_{ij}^{1}}$$

Absolute differences are positively correlated with the absolute size of the coefficients, and their total in the column vector is zero. Relative differences are inversely correlated, i.e. they are often small for the large and large for the small coefficients.

A sophisticated measure of the importance of a change in the input coefficients is the indicator of "tolerable limits". It defines the maximum rate of change in a_{ij} for which the maximal resulting change in gross output volume in no sector exceeds one per cent. The indicator takes into account the whole network of interrelations in which the coefficient a_{ij} takes part through the matrix inverse. Its definition is as follows6):

(8) $$d_{ij} = \frac{1}{a_{ij}\,[b_{ji} + 100 \max (b_{pi}/X_p) X_j]}$$

where:

a_{ij} = input coefficient for which the tolerable limits are calculated

b_{ij}, b_{pi} = elements of the inverse matrix

X_j, X_p = gross output values of industries j and p respectively

The tolerable limits (8) and the relative differences of input coefficients are expressed in the same units, i.e. as percentages of the value of the input coefficient. They can be therefore compared. Their ratio will be called "maximum relative error" (i.e. for the calculation of gross output values):

(9a) $$e_{ij}^{0,1} = \frac{r^{0,1}(a_{ij})}{d_{ij}^{0}}$$

or

(9b) $$e_{ij}^{1,0} = \frac{r^{1,0}(a_{ij})}{d_{ij}^{1}}$$

If the coefficients (as defined above by the formuale 1-9) are calculated for a pair of comparable input-output tables, their "important" values have to be identified. In the past "importance" has depended on the personal judgement of individual authors7). In this paper, an attempt will be made to use the parameters of the frequency distributions to identify important values of coefficients.

2. Important Indicators in the Austrian Input-Output Tables for 1964 and 1976

Coefficients and measures of change, defined in Chapter 1., were calculated for the 1964 and 1976 input-output tables for Austria. For all indicators, such characteristics of their frequency distribution (the mean, standard deviation, skewness and kurtosis), were calculated. Frequency distributions were constructed in a unified framework (the interval's length is 0.2 of the standard deviation). For each indicator three sets were analysed: all values (including zero and negative values), positive values, and logarithms of positive values.

2.1 Maximum relative errors

Indicators of maximum relative errors were calculated both for the 1964 and 1976 input-output tables. Only the 1976 values are reproduced (Table 3), with their frequency distribution (App. 1) and the characteristics of the frequency distributions of both the 1964 and 1976 sets (table 2). The meaning of the final product of this procedure is as follows: Gross output values are assumed to be calculated with the 1976 matrix of input coefficients, in which one and only one coefficient has its 1964 value. The indicator gives the maximum percentage error in the gross output value of any of the industries caused by the substitution of the 1976 by the 1964 input coefficient value. The maximum relative error is large either (i) if the relative difference between the 1964 and 1976 input coefficient's value is large or (ii) if the 1976 value of the tolerable limit is small. In most (but not necessarilly all) cases the maximum error arises in the calculation of the gross output value of the delivering sector (i.e. in the row i of the indicator ε_{ij}).

One can see in Table 2 that the frequency distributions of the 1964 and 1976 maximum errors are symmetric. The average maximum error is close to zero (0.090 per cent for the 1964 and 0.050 per cent for the 1976 table), the standard deviation is over 3 per cent (somehow higher in the 1976 case), the frequency distribution of the 1964 values is skewed to the left and that of the 1976 values to the right, and both distributions are very sharp.

Table 2

Statistical Characteristics of the Frequency Distributions of the "Maximum Relative Errors"

	Input-output tables	
	1964	1976
Mean	0.090	0.050
Standard Deviation	3.068	3.509
Skewness	3.681	- 3.912
Kurtosis	37.621	54.592

The frequency distribution in App. 1 can help to identify the "important" maximum errors. It will be ex ante assumed that errors of about ten per cent cells of the table should be considered as important. In App. 1 one can find that values outside the interval given by the mean plus or minus 0.8 standard deviations meet this requirement. Sixteen negative errors are below the lower boundary (i.e. below -2.7572), 22 positive errors are above the upper boundary (i.e. above +2.8572). These 38 important values are boxed in Table 3.

2.2 Tolerable limits

The treatment of the tolerable limits is similar to that of the maximum relative errors, but results for both 1964 and 1976 are reproduced. The frequency distributions were constructed for the logarithms of the tolerable limits (i.e. of the non-zero values), because small absolute values are too frequent.

The characteristics of the distributions of the logarithms of the tolerable limits are given in Table 4. The average tolerable limit (recalculated from the logarithms) is equal to 0.9550 in 1964 and 1.1320 in 1976; both are close to unity. The standard deviations are similar for both 1964 and 1976. The logarithmic distributions are almost symmetric (see the skewness) and flat (kurtosis).

Small values for tolerable limits are important; the smaller the indicator, the greater the error in the calculation of gross output values caused by a change in the input coefficient. The frontier of importance was deliberately fixed at the mean minus 1.2 times the standard deviation. One then gets 30 important limits both for the 1964 and 1976 input-output tables. (A shift of the boundary closer to the mean, i.e. to the mean minus one standard deviation, would increase the number of important values to 48 in 1964 and 42 in 1976). If the boundaries are recalculated to the arithmetic scale, they are equal to

Table 3

Maximum relative errors calculated from the 1976 input-output table for Austria (important values are boxed).

Sector[a]	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
(1)	2.90	0.14	18.38	2.26	3.71	−0.25	0.84		0.20	0.02	0.04	−0.00	−0.11	−0.20	4.53	0.01		0.15
(2)	−0.69	−6.24	1.19	0.46	0.03	3.19	3.48		8.72	−4.34	2.76	19.25	1.86	−0.51	1.99	9.17		1.32
(3)	−3.42		−5.76	0.37	0.04	0.07	0.40	−0.08	0.02	0.14	0.08	0.02		−0.05	7.82	0.06		0.30
(4)	−0.14	0.03	0.21	4.64	0.14	0.59	2.40	−0.00	0.09	0.22	2.51	0.01	−0.07	2.24	−0.08	0.42	0.00	0.17
(5)	−0.06	0.03	0.30	−0.44	1.94	2.80	−0.10	0.05	−0.30	−0.17	2.54	−0.37	1.61	−0.65	−0.41	−0.18	−0.38	−0.68
(6)	0.31	0.05	−0.19	0.55	0.50	−5.03	4.77	0.05	1.56	0.44	2.92	−0.10	1.62	2.40	−0.18	0.83	−0.84	1.09
(7)	−2.63	−0.12	−3.59	−1.89	−1.09	−0.77	−8.36	−0.99	−1.69	−0.92	4.46	−0.22	−4.06	0.39	−0.84	1.97	−0.06	−1.06
(8)	−3.88	−0.50	−1.52	−0.68	−0.78	−0.33	4.24	−39.82	0.10	1.58	1.49	−22.64	−0.80	−1.76	−1.98	1.11	0.04	2.20
(9)	−0.00	0.18	−1.02	−0.33	−0.16	0.71	0.19	0.08	−9.07	1.54	3.03	0.16	8.82	−1.50	−1.58	0.13	−0.10	0.01
(10)	0.01	0.21	0.56	0.17	−0.02	0.33	0.29	0.28	0.28	11.62	9.58	0.06	−5.24	−1.92	0.48	0.31	−0.01	1.16
(11)	−0.21	−0.21	−0.73	−0.31	−0.78	−0.12	−0.26	−0.62	−0.10	−2.31	−5.56	−0.75	−1.51	−0.99	−0.18	0.42	−0.16	−0.27
(12)	−0.94	−0.31	−2.23	−0.55	−0.51	−0.41	−0.63	−0.64	−1.02	−1.03	0.84	1.50	0.26	0.04	−1.68	−0.44	−0.01	0.41
(13)	0.11	0.02	0.21	0.20	0.04	0.07	0.18	−0.13	0.18	−0.00	0.65	−0.16	1.34	1.39	0.11	0.51	1.72	−0.05
(14)	−1.14	−0.01	−1.40	−0.89	−0.70	−0.11	−2.30	−0.41	−0.37	0.25	0.81	−0.04	−2.33	−0.62	−0.43	−0.48	−1.47	−0.44
(15)	−0.02	0.00	0.29	0.01	0.09	0.09	0.28	−0.00	0.09	0.17	1.74	0.01	0.54	1.25	0.05	−0.06	0.19	0.09
(16)	−0.00	−0.32	2.53	−0.31	0.10	−0.47	0.17	−0.75	0.52	1.04	2.69	0.50	3.39	6.93	−0.04	−8.85	−0.30	−0.86
(17)	0.25	−0.05	0.92	0.27	0.04	0.48	0.53	0.45	0.08	1.40	1.77	0.77	0.02	−12.25	−2.24	−2.19	−0.82	−0.48
(18)	0.37	−0.15	3.05	0.34	0.00	0.38	0.58	−0.21	0.05	0.52	2.56	0.17	0.29	0.66	0.93	−0.15	2.11	0.31
(19)	−0.08	−0.04	−0.38	−0.23	−0.15	−0.00	−0.16	−0.03	−0.05	0.10	−0.22	0.01	−0.15	−0.15	−0.02	−0.22	−1.96	−0.15

[a] Sector numbers and names are as follows: (1) agriculture, (2) mining, (3) food, (4) textiles and apparel, (5) wood and wood products, (6) paper and publishing, (7) chemicals, (8) petroleum, (9) nonmetallic minerals, (10) basic metals, (11) metal products, (12) electricity, gas, and water, (13) construction, (14) trade, (15) restaurants and hotels, (16) transport and communications, (17) financing and business services, (18) personal services, (19) public administration.

Table 4

Statistical Characteristics of Frequency Distributions of Logarithms of the Tolerable Limits

	Input-output tables	
	1964	1976
Mean	-0.046	0.124
Standard Deviation	1.793	1.889
Skewness	0.329	0.300
Kurtosis	-0.065	0.086

0.1111 for the 1964 and to 0.1173 for the 1976 sets. This means that those coefficients for which a deviation of about 11 per cent from their correct value causes a one per cent error in the input-output calculation of the gross output value are important (i.e. very sensitive). Important tolerable limits smaller than the two almost identical critical values are boxed in Tables 5 and 6 respectively.

2.3 Results for other indicators and their interrelation

Similar analysis was carried out for most of the indicators defined above in Chapter 1, i.e. for the relative differences of the 1964 and 1976 input coefficients, for the absolute values of the input coefficients, for the price changes between 1964 and 1976 and for the import shares of the 1964 and 1976 coefficients. (Detailed results were given in an earlier version of this paper presented at the Task Force Meeting in September 1982).

Table 5

Tolerable limits of the input coefficients calculated from the 1964 input-output table for Austria (important values are boxed).

Sector[a]	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
(1)	0.26	6.01	*0.01*	0.26	*0.09*	0.35	1.39		7.04	25.66	17.46		5.22	94.08	0.17	94.08		2.93
(2)	54.24	*0.09*	0.46	1.71	11.50	0.19	*0.10*	16.53	0.12	*0.01*	0.55	*0.05*	0.88	11.97	0.49	0.15	117.35	1.07
(3)	0.26		*0.05*	0.71	13.10	9.20	1.90		24.33	6.81	16.22	48.66		25.94	*0.06*	17.02		1.62
(4)	30.81	20.80	2.66	*0.01*	1.47	1.25	0.54	30.80	14.10	4.02	0.60	26.41	13.21	0.53	2.60	2.98	61.61	1.79
(5)	2.78	3.68	1.48	1.81	*0.04*	0.26	3.56	19.63	4.52	1.84	0.26	58.90	0.15	0.75	1.62	1.68	0.81	0.89
(6)	0.90	5.10	0.13	0.22	0.66	*0.03*	0.11	8.98	0.41	0.93	0.19	16.46	0.41	*0.08*	0.78	0.47	0.32	0.13
(7)	*0.08*	1.61	0.27	*0.06*	0.21	0.21	*0.02*	2.28	0.45	0.17	*0.08*	5.08	0.14	0.33	0.83	0.20	2.74	0.19
(8)	*0.06*	0.66	0.15	0.32	0.79	0.40	*0.09*	*0.01*	0.12	*0.09*	0.13	*0.06*	0.17	0.12	0.27	*0.05*	1.90	0.16
(9)	0.63	2.43	0.45	7.94	1.39	1.09	0.75	14.57	*0.07*	0.33	0.20	2.39	*0.02*	1.92	0.71	2.06	1.61	3.41
(10)	22.09	2.03	1.31	3.57	3.78	2.61	1.61	5.75	0.79	*0.01*	*0.02*	13.24	0.12	44.19	1.76	1.33		0.98
(11)	0.28	3.55	0.62	0.75	0.68	1.29	1.04	4.71	0.81	0.49	*0.03*	1.42	0.16	0.54	2.33	0.28	4.90	0.78
(12)	0.52	0.76	0.41	0.48	0.73	0.29	0.17	4.20	0.32	0.15	0.19	0.26	0.56	0.22	0.32	0.25	0.18	0.43
(13)	2.22	4.92	0.72	1.32	5.49	2.63	2.14	9.81	2.68	5.14	0.66	1.93	0.38	0.57	0.75	1.03	0.16	1.74
(14)	0.56	3.18	0.50	0.63	1.08	1.79	1.09	12.81	1.80	0.62	0.18	2.38	0.26	0.54	0.40	1.16	1.20	0.79
(15)		20.69	1.65	2.98	3.15	5.86	1.72	82.00	4.42	4.55	0.48	54.69	1.16	0.55	4.39	0.67	1.02	1.35
(16)	1.21	2.06	0.13	0.30	0.40	0.58	0.31	3.69	0.38	0.32	0.17	1.20	0.26	*0.07*	0.79	*0.07*	0.46	0.54
(17)	0.34	1.54	0.15	0.17	0.47	0.45	0.40	1.27	0.65	0.32	0.12	0.80	0.18	*0.09*	0.33	0.22	0.22	0.30
(18)	0.39	1.30	0.13	0.31	1.47	1.19	0.44	6.34	1.94	1.25	0.18	4.51	0.73	0.26	0.32	1.65	0.13	0.16
(19)	1.17	14.51	1.12	1.87	5.31	9.46	2.98	108.87	3.69	4.53	1.43	18.14	2.90	3.56	31.10	24.19	0.30	5.58

[a]See footnote to Table 3 for sector names.

Table 6

Tolerable limits of the input coefficients calculated from the 1976 input-output table for Austria (important values are boxed).

Sector[a]	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
(1)	1.25	22.75	*0.02*	0.76	*0.07*	0.24	2.04		87.26	40.27	15.86	130.87	2.19	4.51	0.23	261.79		2.24
(2)	1.41	*0.05*	0.77	16.97	11.31	0.28	*0.07*		0.34	*0.01*	2.18	*0.06*	22.62	1.50	1.09	1.11		4.84
(3)	0.15		*0.03*	1.31	19.17	16.87	2.30	11.80	41.71	39.40	54.55	70.89		8.86	*0.09*	64.45		1.62
(4)	5.76	64.55	4.03	*0.02*	1.11	1.94	0.96	8.72	64.56	8.96	1.48	16.98	4.74	0.84	1.48	8.72	32.27	1.25
(5)	3.45	6.30	4.40	1.28	*0.05*	1.70	2.29	20.07	1.92	1.56	0.50	2.53	0.16	0.45	0.99	1.26	0.49	0.52
(6)	2.03	8.90	0.14	0.37	0.80	*0.02*	0.13	4.54	0.83	1.56	0.23	5.14	0.60	*0.07*	0.61	0.58	0.17	0.12
(7)	*0.10*	1.87	0.16	*0.09*	0.19	0.22	*0.02*	0.54	0.27	0.18	*0.10*	2.22	*0.08*	0.36	0.52	0.36	1.95	0.15
(8)	*0.09*	0.81	0.20	0.51	0.61	0.55	0.26	*0.00*	0.19	0.21	0.21	*0.02*	0.19	0.12	0.23	*0.08*	2.36	0.42
(9)	0.89	6.81	0.33	2.37	1.04	4.95	0.63	12.11	*0.04*	0.76	0.28	2.29	*0.01*	0.45	0.32	2.34	0.96	2.83
(10)	58.21	6.29	14.55	349.30	3.32	43.65	2.06	9.97	1.00	*0.02*	*0.01*	24.93	*0.06*	0.51	11.26	2.01	58.20	31.75
(11)	0.46	2.56	0.54	0.98	0.49	1.38	0.79	0.98	0.88	0.26	*0.02*	0.64	0.13	0.35	1.82	0.36	2.51	0.66
(12)	0.56	0.96	0.27	0.66	0.65	0.36	0.17	0.96	0.29	0.18	0.25	0.45	0.76	0.25	0.25	0.28	0.17	0.64
(13)	7.80	9.84	1.38	5.50	9.50	4.62	3.51	3.10	8.00	6.77	1.07	1.35	0.82	2.92	1.02	2.96	0.19	1.58
(14)	0.47	4.84	0.36	0.58	0.66	1.81	0.30	1.78	1.19	1.07	0.20	1.94	0.16	0.40	0.39	0.79	0.40	0.60
(15)	36.37	36.39	5.07	5.20	4.50	17.46	2.34	29.09	7.66	54.60	1.13	87.32	2.10	1.09	5.92	0.60	0.86	1.32
(16)	1.82	1.44	0.26	0.41	0.40	0.48	0.26	0.73	0.47	0.58	0.21	1.63	0.88	*0.09*	0.75	*0.04*	0.30	0.33
(17)	0.89	2.44	0.34	0.42	0.65	1.02	0.59	1.25	0.96	1.77	0.18	2.31	0.21	*0.04*	0.22	0.17	0.18	0.30
(18)	0.97	1.55	0.48	0.71	1.71	3.53	0.55	1.95	2.58	13.51	0.29	11.94	0.91	0.31	0.58	1.39	0.15	0.17
(19)	2.19	13.00	1.13	2.18	3.51	14.42	2.22	20.98	4.21	31.83	1.21	27.15	2.31	2.60	23.67	3.92	0.19	3.40

[a] See footnote to Table 3 for sector names.

Interrelations between important values of all these indicators are shown in Table 7. The first two columns contain the numbers of rows and columns (which refer to the industry classification in Table 1) of the 38 important "relative maximum errors" (see Table 3). The following columns indicate which other indicators corresponding to these cells were classified as important.

One can see a strong link between important values of the relative maximum errors and important values of the tolerable limits, and also of important absolute values of the input coefficients, but weak links between important values of the maximum relative errors and important relative differences of the coefficients, important price changes and important import shares.

2.4 Frequency of Important Indicators by Industries

Important changes in input coefficients in the Austrian input-output tables between 1964 and 1976 do not affect the 19 industries equally. If one looks at the distribution of the important values of indicators the following picture emerges.

In Table 3, 38 maximum relative errors were identified. If their distribution by industries were equal, each column and each row of the table should contain two important entries. Two important entries, however, can be found in seven columns row only. One column (metal products) contains five important values, four important values can be found in three columns (food, chemicals and construction). At the other hand two columns (financing, business and personal services) contain no important maximum relative errors. The number of important maximum errors in columns reflects the intensity of technological change. Its low intensity in services and high intensity in the metalworking and chemical industries are not surprising, the intensity of technical change in food manufacturing and construction is, however, unexpected.

Table 7

Interrelations between the "maximum relative errors" for the 1976 input-output table and other important indicators of the 1964 and 1976 input-output tables.

Location in the input-output table: row	column	1976 maximum relative errors	1964 tolerable limits: 1964	1976	1964/1976 relative differences	Input coefficients: 1964	1976	Products of the 1964/1976 deflators	Import shares of intermediate inputs Input-Output Table: 1964	1976	Ratios 1976/1964
1	1	x			x						x
3	1	x					x				
8	1	x	x	x					x		
2	2	x	x	x		x	x				
1	3	x	x	x		x	x				
3	3	x	x	x		x	x				
7	3	x								x	
18	3	x									
4	4	x	x	x		x	x				
1	5	x	x	x		x	x				
2	6	x									x
6	6	x	x	x		x	x				
2	7	x	x	x					x	x	x
6	7	x				x					
7	7	x	x	x		x	x				
8	7	x	x					x	x	x	
8	8	x	x	x		x	x				
2	9	x							x	x	
9	9	x	x	x		x	x				
2	10	x	x	x		x	x		x	x	
10	10	x	x	x		x	x				
6	11	x									
7	11	x	x	x						x	
9	11	x									
10	11	x	x	x		x	x				
11	11	x	x	x		x	x				
2	12	x	x	x							x
8	12	x	x	x		x	x			x	
7	13	x		x				x			
9	13	x	x	x		x	x				
10	13	x		x							
16	13	x			x						
16	14	x	x	x		x					
17	14	x	x	x		x	x				x
1	15	x				x					x
3	15	x	x	x		x	x				
2	16	x			x				x	x	
16	16	x	x	x		x	x				
Total items		38	24	25	3	21	19	2	6	8	6

The distribution of important maximum relative errors by rows is more uneven. Seven important values are in the row of mining, four in agriculture, chemicals and petroleum. On the other hand the following six rows contain no important maximum relative error: wood and wood products, electricity, gas and water, construction, trade, restaurants and hotels, and public administration. Among the four rows with many important values three industries are sources of raw materials, the fourth one- petroleum products- was affected by the development of the oil prices after 1973.

Two other aspects in Table 3 are also of interest. One is the ranking of the maximum relative errors. The highest value is that for intraindustry deliveries in the petroleum industry. The second ranking value is that for deliveries of petroleum to electricity, the third one for deliveries of mining (i.e. of coal) to electricity, the fourth one for agricultural inputs to food manufacturing and the fifth (and also the last value over 10 per cent), the inputs of financing and business services into trade. Negative maximum relative errors mean that the use of the 1964 coefficients in input-output calculations for 1976 would cause an underestimation of the gross output values (in most cases of the gross output value of the delivering industry). For positive errors, the opposite is true. Another interesting aspect is the high number of important errors on the main diagonal: 11 out of 19 main diagonal cells contain important maximum relative errors.

Table 7 has shown a strong link between the allocations of important maximum relative errors and of important tolerable limits. Thirty important values of tolerable limits were selected for both the 1964 and 1976 input-output tables. One can see in Tables 5 and 6 respectively that they are allocated evenly by columns. In both tables financial, business and personal services have no important tolerable limits. Three important limits can be found in both years in metal products and trade, in 1964 also in chemical and petroleum industries and in 1976 in construction. The distribution by rows is uneven. Input coefficients in three industries delivering raw materials and energy are very sensitive: mining (four important limits) chemicals (four important limits in 1964 and five in 1976) and petroleum (six important limits in 1964 and four in 1976). On the other hand the rows of electricity, gas and water, construction and trade have no important item.

As for maximum relative errors, 11 important tolerable values (out of 19) are located on the main diagonal in both years. The lowest value of tolerable limit is that for the interindustry flows in the petroleum industry. Other very sensitive input coefficients are for inputs of agriculture into food manufacturing and of basic metals into metal products and intraindustry inputs in textiles, chemicals and basic metals.

The following few paragraphs refer to indicators for which detailed results are not given in the paper. Input coefficients were such a set of indicators. The distribution of their important values by industries is even both by columns and rows. Twelve main diagonal coefficients are large, and there were also 19 large value added coefficients.

Relative differences between 1976 and 1964 input coefficients are large mainly for small and not sensititive items. They are more frequent in the columns of agriculture, basic metals, trade and transport, and row-wise are they concentrated in mining, basic metals, agriculture and food (which have ten, seven, six and five important entries respectively). Seven important changes are due to a missing coefficient in one of the two input-output tables and are caused by methodological differences in the compilation of the 1964 and 1976 input-output tables for Austria.

Changes in relative prices were measured by products of value added deflators by industries. The important values are concentrated in the columns and rows of the chemical industry (six items in both), and non-metallic mineral industry (four cases in both) and in the rows of personal services and of public administration (there are five important values in each of these rows). They reflect the most pronounced divergencies in price development between 1964 and 1976. On the one hand there are the low 1976/1964 price indices in the chemical industry (0.97) and non-metallic minerals (1.29), and on the other hand the high indices in private services (2.63) and public administration (2.77). The divergencies among price increases by industries are related only in two cases to an important maximum relative error (see Table 7): for inputs of petroleum products into chemicals and for inputs of chemicals into construction. In both cases, however, the values of tolerable limits show also a high sensitivity of the coefficient.

The last sets of indicators calculated from the Austrian input-output tablee are the 1964 and 1976 import shares in intermediate deliveries and their ratios. Important import shares are distributed evenly by columns (they are more frequent only in the textile and chemical industries), but are concentrated in the rows of mining (in both years), of petroleum products (1964) and of chemicals and textiles (1976).

The allocation of important values of various indicators in the input-output tables for Austria provides some help in further evaluation of changes in input coefficients between 1964 and 1976. Deliveries by certain industries are more sensitive to changes in input coefficients than deliveries by other industries, and some industries have undergone profound changes between 1964 and 1976 in the composition of intermediate inputs while other industries have a rather stable input structure.

2.5 Industries with unstable input structures

Four industries had an unstable input structure between 1964 and 1976: food manufacturing, chemicals, metal products and construction.

The maximum relative errors (see Table 3) in the food industry show strong decreases in inputs of agriculture and of personal services on the one hand and increases in intraindustry flows and in inputs of chemical products. The important maximum relative errors (Table 3) for the chemical industry show an increase in intraindustry flows and a decline in inputs of mining, paper and petroleum.

The pattern of change of input coefficients in the metalworking industry is one sided: there was an increase in the intraindustry deliveries due to higher specialisation both of domestic enterprises and also between the domestic and foreign firms: the import share increased by a factor of 1.28. There was a decline of inputs of basic metals, chemicals, non-metallic minerals and paper. These shifts could be partly caused by an increase in the value added coefficient from 0.41 in 1964 to 0.46 in 1976. Another plausible explanation, which however cannot be proved here, could be a change in the output mix of the Austrian metalworking industry.

In construction, the maximum relative errors (Table 3) show a decline in inputs of non-metallic minerals and of transportation services and an increase in inputs of chemicals and of basic metals. Traditional construction materials (which have to be transported in huge volumes) were substituted by metals and chemical products.

2.6 Industries Sensitive to Shifts in Intermediate Demand on their Production

Gross output estimates for four industries are sensitive to changes in the input coefficients between 1964 and 1976. These are agriculture, mining, chemicals and petroleum. All are important suppliers of raw materials, and two of them, mining and petroleum, also have high import shares.

Mining is the most sensitive sector. In Table 3 one finds in the row of mining seven important maximum relative errors. The intraindustry deliveries and the deliveries to basic metals increased, and deliveries to paper and chemical industries, non-metallic minerals, electricity, gas and water and transport and communications declined (in the last two cases due to lower demand for coal.) Four cases could be ex ante identified by important tolerable limits (from both the 1964 and 1976 tables): the intraindustry flows and the inputs in chemicals, basic metals and electricity, gas and water. The import shares of mining are very high and almost constant.

The import shares in the petroleum industry are also high, but they are not comparable due to accounting differences between the 1964 and 1976 input-output tables. Important maximum relative errors were found in four cells of the row of petroleum: an increase of inputs in agriculture, in electricity, gas and water, and increase of the intraindustry flows, and a decline in chemicals. The increases in agriculture and electricity are quite understandable: in the latter case a substitution for products of mining (i.e. for coal) took place. The change in the chemical industry is less clear, and it could be caused by shifts in output mix. The intraindustry coefficient on the main diagonal of the petroleum industry is the most sensitive input coefficient in both 1964 and 1976. Its correct estimation decides the correct prediction of the gross output value of the petroleum industry, and is related to the prediction of the value added. The main diagonal coefficient is obviously complementary to the value added coefficient. If it is too high and the gross output value is overestimated, the value added coefficient must be lower and the value added estimate closer to its correct level. (The complementarity of the main diagonal and value added coefficients in the petroleum industry can be seen easily proved. Their values were in 1964 0.3953 and 0.5383 and in 1976 0.5650 and 0.3083. The sum of both coefficients was 0.9232 in 1964 and 0.8733 in 1976.)

The row of the use of agricultural products contains (in Table 3) four important maximum relative errors. They indicate a decline in the demand for agricultural output in agriculture, food industry, wood and wood products and restaurants and hotels. In agriculture they decline seems to be outweighed by an increase of inputs from the food industry (i.e. animal feed), and that in the food industry by higher intraindustry flows and higher use of chemical products. In restaurants and hotels they are accompanied by a parallel decline in intermediate demand for food.

The row of chemical industry in Table 3 contains four important maximum relative errors. They indicate increased demand in food processing and in construction, increased intermediate flows and a decline in the use of chemicals in metal products. The increases are not surprising, and the decline in the metal working industry could be perhaps explained by changes in its output mix.

2.7 Stable industries

Contrary to several industries in which the patterns of inputs or of output changed between 1964 and 1976, the following three industries remained stable: public administration, financing business and personal services. All three industries have, however, undergone very rapid increases in output prices. The 1964 table at constant 1976 prices, which is under preparation, will show if the pattern of their inputs and outputs in real terms was so stable as in nominal terms.

2.8 Other industries

Other industries will be briefly mentioned in the same order in which they are ranked in the input-output table. In the textiles and apparel industry the only important maximum relative error is caused by a decline in the intraindustry flows. Wood and wood products industry had a stable demand for its output. Technology was also stable, the only important change being a decline of inputs of timber. The paper and publishing industry had also a relatively stable technology, the only important changes being an increase in intraindustry deliveries and a decline in inputs from mining. An important decline in the demand for output occured in the chemical and metalworking industries. Two important changes in the input structure of non-metallic minerals were complementary: a decline in inputs of mining and an increase in intraindustry inputs. Significant declines in the demand for non-metallic minerals were recorded in metalworking industry and in construction. The reason in the former case was probably a change in the output mix, in the latter case a substitution of traditional construction materials by metals. The input structure of basic metals has undergone two important changes: an increase of inputs from mining and a decline of intraindustry flows. Demand for basic metals declined in the metal-working industry (perhaps due to shifts in the output mix) and increased in construction (substitution of traditional construction materials).

The following three industries had a rather stable pattern of demand for their products, but have undergone some changes in their input structures. These are electricity, gas and water, trade, restaurants and hotels. Most interesting is the decline in the share of agricultural and food processing products in restaurants and hotels, which was balanced by an increase in the value added coefficient from 0.34 in 1964 to 0.44 in 1976. These shifts were at least partly caused by differences in price developments. The prices of inputs from agriculture and food processing increased much less than prices of the output restaurants and hotels: the 1976/1964 price indices were 1.41 and 1.44 for these two inputs and 2.56 for the output. In transport and communications the inputs from mining (i.e. of coal) declined and the intraindustry flows increased. The demand on transportation services declined in construction (probably due to lower use of traditional construction materials) and in trade. The input pattern of trade shows a complementary shift: a decline in the inputs of transportation services and an increase of demand for financial services. The former change could be, inter alia, caused by the increasing weight of large business enterprises. The latter change reflects probably an increase in the demand for banking services.

3. Tentative Conclusions

Matrices of input coefficients from the 1964 and 1976 for Austria input-output tables were analyzed, the purpose being to obtain findings which could be helpful in predicting future changes in input coefficients in the "Austrian Institutes" versions of the INFORUM model.

Since a 1964 input-output table at 1976 constant prices is not yet available, only a few tentative methodological and substantial conclusions can be made.

The methodological conclusions can be summarized as follows:

1. The indicator of tolerable limits is a good tool for identification of input coefficients the changes of which can significantly influence the outcome of the input-output calculations. Other indicators (such as the absolute size of the input coefficients, differences in the increases in relative prices, or in import shares) are much less useful.

2. Methodological differences in and quality of statistical compilation of input-output tables can cause significant changes in input coefficients. The Austrian 1976 input-output table is much more reliable than the 1964 table. Hopefully the 1964 table at 1976 prices will be of the same quality.

3. Special attention should be paid to main diagonal and to value added coefficients. Coefficients on the main diagonal influence strongly the outcome of input-output calculations and may be also complementary to the value added coefficients.

4. The investigation could not show the impact of changes in relative prices on the variability of input coefficients. The strongest impact should be due to the rapid growth of relative price levels in the service industries. The 1964 input-output table at 1976 prices, which is under construction, will allow assessment of the influence of price developments on the stability of input coefficients in real terms, the substitution effects of price increases and the transmission of higher input prices into higher output prices.

The following tentative substantive conclusion can be made:

1. The chemical and metalworking industries were the most affected by changes in input coefficients between 1964 and 1976. The shifts in their input structures cannot be easily explained, and may originate in changes in the output mix.

2. Four industries i.e. mining, agriculture, chemicals and petroleum, were influenced by changes in demand for their output. Two of them, mining and petroleum have also a high import share. The estimates of domestic gross output of all these four industries, but in particular of the latter two, can be subject to relative by large errors.

3. The services industries show a relatively stable pattern both of inputs and of the demand for their output in nominal terms. This, however, will probably not be true for coefficients in real terms.

4. For most other industries it was possible to identify a small number of significant potential errors in input-output calculations. Most them could be detected ex ante by the indicator of tolerable limits.

Footnotes:

1) The author would like to thank Mr.J.Richter for helpful comments.

2) See also: J.Skolka: "Außenhandelsverflechtung der österreichischen Wirtschaft: Ein Input-Output Vergleich zwischen 1964 und 1976", WIFO Monatsberichte Nr.10, 1981, pp.594-604.

3) Both are used in the "Austrian Institutes" version of the INFORUM Model.

4) In input-output tables imports are valued at customs clearance prices which include custom duties. These are, however, a part of domestic value added.

5) United Nations: "A System of National Accounts and Supporting Tables", New York, 1958; United Nations: "A System of National Accounts" New York, 1968.

6) J.Sherman, W.J.Morrison: Adjustment of an Inverse Matrix Corresponding to a change in One Element of a Given Matrix, Annals of Mathematical Statistics.

7) For example the following criteria were used for the selection of important input coefficients from the 1959 set of the standardized input-output tables for the ECE countries:

"A. The so-called tolerable limits for each cell of the first quadrant of the input-output tables of individual countries and the average tolerable limits have been calculated;

B. In the individual columns of individual tables items have been picked out in descending order of size until their total exceeded 90 per cent of the respective column total, i.e. the total of the first quadrant column. The selected items are considered as important;

C. In the individual rows of individual tables items have been picked out in descending order of size until their total exceeded 90 per cent of the respective row total, i.e. the total of the first quadrant row. The selected items are considered as important.

The critical value of the tolerable limit in the A- case was set at 1.0. (J.Jilek: "The Selection of Most Important Input Coefficients", Economic Bulletin for Europe, Vol.23, No.1, 1971.)

Appendix 1.

Frequency Distribution of the Errors and Tolerable Limits

Intervals: (Deviations from the Mean in Terms of the Standard Deviation)	Maximum Relative Errors for 1976	Logarithms of the Tolerable Limits	
		1964	1976
below -3.0	3	-	-
-3.0 to -2.8	-	-	-
-2.8 to -2.6	-	-	1
-2.6 to -2.4	2	-	1
-2.4 to -2.2	1	3	-
-2.2 to -2.0	-	4	5
-2.0 to -1.8	-	3	4
-1.8 to -1.6	2	3	4
-1.6 to -1.4	3	8	8
-1.4 to -1.2	1	9	7
-1.2 to -1.0	3	18	12
-1.0 to -0.8	1	26	25
-0.8 to -0.6	7	24	26
-0.6 to -0.4	13	23	26
-0.4 to -0.2	27	29	30
-0.2 to 0.0	144	27	32
0.0 to 0.2	88	25	26
0.2 to 0.4	17	31	34
0.4 to 0.6	15	17	15
0.6 to 0.8	12	16	14
0.8 to 1.0	7	18	9
1.0 to 1.2	2	7	11
1.2 to 1.4	4	5	10
1.4 to 1.6	-	11	9
1.6 to 1.8	-	7	7
1.8 to 2.0	1	8	6
2.0 to 2.2	-	2	8
2.2 to 2.4	1	4	2
2.4 to 2.6	2	3	1
2.6 to 2.8	2	2	0
2.8 to 3.0	-	0	1
over to 3.0	3	0	1
total	361	333	335

IMPROVEMENT OF FORECASTS BY USING VARIABLE INPUT COEFFICIENTS

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Introduction.

Since the development of input-output models it has been a common observation that the assumed linear limitational production theory underlying the model is not as robust for comparisons of the coefficient structure for different time periods as should be the case to neglect the variablity, because of minor importance. More and more it became obvious that the whole structure of the matrix changed from period to period because of technological progress in the sectoral production techniques, varying factor prices, changing institutional patterns of e.g. government subsidies or taxes, aggregation problems of changing product mixes, different degrees of capacity utilization during growth and business cycles or economies of scale (cf.e.g. Timmermann, Focke, Ruiz (1976)). One important target of input-output modelling has always been the ability to forecast a whole input-output structure or to use this structure for policy simulation. Therefore the question how to model properly this changing structure has been at the center of many conferences from the beginning.

The ability to build applicable models had always rested heavily on the available data base which took a long time to set up. Up to now it is nearly an impossible undertaking to set up something like a time series of input-output tables for a time period of about ten to twenty years, which rests on original data alone. This situation is traditionally overcome by using a mathematical model for updating an input-output table with partial information, at least the column and row sums for different years should be available. The common methods to get with this information a formal consistent updated matrix are the RAS method (Leontief (1941), Stone(1962)) or the MODOP method (Stäglin (1972); Schintke (1973)) used at the German Institute of Economic Research (DIW) and other institutions (further methods cf.e.g. HÜBLER(1974)).

The problem to find the adequate methods to model the variable input-output structure were tackled by different approaches.

Firstly, there are a number of models which used a simple time trend updating technique. To guarantee positive values for the intermediate input coefficients, which are computed from an estimated time trend relationship between every coefficient and a time variable, mostly logistic trend functions are used (cf.e.g.Almon, Buckler, Horwitz, Reimbold (1974); Rettig (1980)). This kind of input coefficient modelling still rests on rather mechanistic considerations of the changing technological structure. Hardly this kind of models are able to track cyclical patterns or turning points of input coefficient variability, and the economic interpretation of the estimated trends is rather arbitrary.

Secondly, another approach tried to overcome these weaknesses. The models of Hudson-Jorgenson (1974) and Krelle (cf. Krelle (1964); Krelle, Kübler (1976); Kübler (1977); Frerichs (1975); Kiy (1981)) belong to this class. They try to give an explicit economic explanation for the changing input-output structure. Changing price ratios, capacity utilization, changing capital-labour ratios are variables which are used to explain the variation of input coefficients.

The present paper gives some results on the investigation of some central questions obtained from experiments with the Disaggregated Bonn Forecasting Model II (Kiy (1981)):

How much could be gained by using variable input coefficients, which are explained by economic variables, in comparison with constant input coefficients ?

Is it possible to predict a whole coefficient matrix with this kind of models in a reliable way so that large errors (e.g. more then 10%) in the matrix structure can be avoided ?

How large is the influence of the prediction errors of the input coefficients on the values of sectoral gross production ?

Since the Disaggregated Bonn Forecasting Model contains four matrices of intermediate sectoral flows as there are the domestic and the import input coefficients and the domestic and the import investment coefficients, it is important to know, how are the relative efficiency of the explained structures compared with each other ?

Before presenting the results, we give a short survey of the model and its statistical data base.

<u>The Disaggregated Bonn Forecasting Model.</u>

The Disaggregated Bonn Forecasting Model is a fully integrated econometric model, it explains the interindustrial structure for domestic and import input and investment coefficients as well as all components of the final demand, it contains a labour market submodel, a transfer table of the transfer system and endogenous price equations, too. The model consists of about 1856 equations with 992 definitional and 864 behavioural equations. The following table gives a short overwiew of the different sections and the number of equations used for explanation in it:

sections	definitional equations	structural equations	sum
definitional framework: national product, sectoral production, imports and profits	255	0	255
consumption, investment, exports	106	85	191
primary inputs (without wages)	42	44	86
labour market	29	52	81
domestic input coefficients	78	246	324
import input coefficients	103	149	252
domestic investment coefficients	63	171	234
import investment coefficients	29	61	90
prices, price relations	155	22	177
transfer tabel and investment financing	22	21	43
other variables: depreciation, capital stocks, capacity utilization, production capacity, disposable income	110	13	123
sum	992	864	1856

The largest part of the model consists of the four coefficient matrices for input and investment. All these coefficients were estimated on the basis of one standard form of a behavioural equation, which was modified by changing the lag structure of the explanatory variables, by omission of not significant variables, or in some few cases with supplementary dummy variables. The standard form is:

$$a_{i,j,t} = \beta_1 \, (K_j/A_j)_{t-k}^{\beta_2} \, (p_i^M/p_i)_{t-q}^{\beta_3} \, C_{t-r}^{\beta_4} \, u_t$$

$a_{i,j,t}$ = input or investment coefficient in period t

$(K_j/A_j)_{t-k}$ = capital-labour ratio in period t-k

$(p_i^M/p_i)_{t-q}$ = import price to domestic price ratio in period t-q

C_{t-r} = capacity utilization at period t-r

u_t = error term

β_l = model parameter l = 1,2,3,4

It is used for 18 production sectors as being described on page 8 of this paper.

In the following tables we give a survey of the specification for all domestic and import input coefficients. We use the following symbols for the explaining variables:

- Q = capital-labour ratio
- P = import price to domestic price ratio
- D = capacity utilization
- * = dummy variable
- ////// = the coefficient is definitional zero
- CONST = the coefficient ist constant

sectors	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	Q	Q	P Q	Q	Q	Q	P	Q	Q	Q	Q	P Q	Q	Q	P Q	Q	Q
	D	D	D	D	D	D	D	D	D	D	D	D	D			D	D
2	Q	Q	Q	P Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q
	D			*	D		D	D *	D	D	D	D	D		D	D	D
3	P Q		P Q	P Q	P Q	P Q	P Q	P Q	P Q	P Q	Q	Q	Q	Q	Q	Q	Q
		D	D	D		D		D	D	D		D	*	*		D	D
4	Q	Q	Q	Q	P Q	P Q	Q	Q	P	Q	Q	P	Q	Q	Q	//////	Q
	D	D	D	D	D	D	D	D	D	D	D	D	D	*	D	//////	D
5	Q	P Q	Q	Q	P Q	Q	Q	Q	Q	Q	P Q	Q	P Q		Q	Q	Q
		D	D	D	D	D	D		D	D	D	D	D	*	D *		D
6	Q	P Q	Q	P Q	P Q	Q	P Q	Q	Q	P Q	Q	Q	Q	Q	Q		Q
	D	D	D	D	D	D	D	D	D	D	D *	D				*	D
7	P	Q	P	Q	Q	P Q	P	Q	Q	Q	Q	Q	Q	Q	P Q	Q	Q
	D	D	D	D	D		D		D	D	D	D	D	D *	D	D	D
8	Q	Q	Q	Q	Q	Q	Q	P Q	Q	Q	Q	P		Q	P Q	Q	Q
	D			D			D	D		D		D	D		D		D
9	Q	Q	Q		Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	
	D	D	D	D	D	D	D			D	D	D	*	D		D	D
10	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q		Q	Q	Q	Q	Q	Q
	D	D		D	D	D	D	D	D		D	D	D	D	D	D	D
11	Q	Q	P Q	Q	Q	Q	Q	Q	P	Q	Q	Q	Q	Q	Q	Q	P Q
	D			D	D	D	D	D	D	D	D		D	D	D	D	
12	P Q		Q	Q	P Q	Q	P Q	Q	Q		Q	Q	Q	Q	Q	Q	
	D	D	D	D		D		D	D	D	D	D	*	D *	D	D	D
13	Q	Q	Q	Q	CONST	Q	Q	Q	Q	Q	Q	Q	//////	//////	//////	//////	//////
	D	D	D			D	D		D		D	*	//////	//////	//////	//////	//////
14	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q		//////	//////	//////	Q	Q
				*	D	D	D	D	D	D		*	//////	//////	//////		*
15	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	Q	//////	//////	//////	Q	Q
	D	D	D	D	D	D	D	D	D	D	D	D	//////	//////	//////		*
16	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	Q
	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	*
17	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	Q	Q	Q	//////
	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////		D		//////

tabel of the specifications for the domestic input coefficients

sectors	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	Q	//////	P Q	//////	//////	CONST	P Q	Q	Q	//////	Q	P		Q	P Q	Q	Q
	D	//////	D	//////	//////		D			//////	D	D	D +	D	D	D	
2	Q	Q	Q	P Q	Q	Q	Q	Q	//////	Q	Q	Q	//////	//////	//////	//////	//////
			D	D	D	D		D	//////	D	D	+	//////	//////	//////	//////	//////
3	P Q	Q	P Q	P Q	P Q	P Q	P Q	P Q	P Q	P Q	P Q	Q			Q	P Q	Q
	D	D	D	D	D	D	D	D	D			D	D +	D +	+	D	D
4	Q	//////	Q	Q	P Q	P Q	Q	//////	P Q	//////	Q	P Q	//////	//////	//////	//////	//////
		//////	D	D	D	D	D	//////	D	//////		D	//////	//////	//////	//////	//////
5	Q	P Q	Q	Q	P Q	Q	Q	Q	Q	Q	P	Q	P Q	//////	//////	//////	Q
			D		D	D	D	D	D	D	D	D	D	//////	//////	//////	
6	Q	P Q	Q	P Q	P Q	Q	P	Q	Q	P Q	Q	Q	Q	//////	Q		Q
	D						D		D	D	D	D	D	//////	D	D	D
7	P Q	Q	P Q	CONST	Q	P	P	Q	Q	Q	Q	Q		Q	P	Q	Q
	D	D	D		D	D	D	D	D	D	D	D	D +		D	D	D
8	Q	//////	Q	//////	//////	CONST	CONST	P Q	//////	//////	Q	P Q	Q	Q	P Q	Q	Q
	D	//////	D	//////	//////				//////	//////		D	+			D	
9		//////	//////	//////	Q	Q	//////	//////	CONST	//////	Q	Q	Q	Q	Q	//////	
	D	//////	//////	//////		D	//////	//////		//////		D	D +	+	D	//////	D +
10	//////	//////	//////	//////	//////	//////	//////	//////	//////	Q	//////	//////	//////	//////	//////	//////	//////
	//////	//////	//////	//////	//////	//////	//////	//////	//////	+	//////	//////	//////	//////	//////	//////	//////
11	Q	Q	P	Q	Q	Q	Q	Q	P Q	Q	Q	Q		Q	Q	//////	P Q
		D	D	D	D	D	D	+			D	D	D +	D	D	//////	
12	P Q	Q	Q	Q	P	Q	P Q	Q	Q	Q	Q	CONST	Q			Q	Q
	D	D	D	D	D	D	D	D	D		+	+	D +	D +	D +	+	+
13 +14 +	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	Q	//////	//////	//////	//////
15 + 16	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	D	//////	//////	//////	//////
17	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////
	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////	//////

tabel of the specifications for the import input coeffcients

Statistical basis.

The main statistical basis of the Disaggregated Bonn Forecasting Model II for the FRG consists of annual input-output tables with 14 production sectors, nine final demand vectors and seven primary inputs, corresponding imports matrices and investment tables. These tables were compiled by the DIW for the years 1960 through 1974 (cf.Baumgart, Stäglin, Weiß, Wessels (1979); Stäglin, Weiß, Wessels (1979)).

In the context of this paper it might be useful to underline that all possible data for final demand and primary inputs, listed in the second and third quadrants in the tables, were derived from the official National Accounts of the Federal Statistical Office. Where possible, data for the first quadrant were determined from original statistics. Of its 196 (14 x 14) intermediate input-output elements, 104 were completely based on statistical obeservations and a further 29 partially so (cf.table of page 7). These statistically based elements account for about one third of the intermediate transactions within the first quadrant. The missing intermediate inputs were obtained by means of MODOP programmed at the DIW ,using the already existing input-output tables with 56 production sectors for the years 1962, 1967, and 1972 as bench marks. This procedure was also used in constructing the annual matrices of gross fixed capital formation while the yearly import matrices were obtained by aggregation of the DIW's annual 56 sector import matrices. The import tables themselves were compiled by use of a computer program developed at the DIW which sorts the some 8000 items of the Classification of Commodities in Foreign Trade into the 56 sector classification with a key, reflecting the percentage distribution among production and final demand sectors for each commodity class.

All these tables were available in current and in constant prices. To get the input-output tables in 1970 prices, price adjustments were carried out row by row. After a separate row deflation of the 56 sector import matrices, price indices were first calculated for 56 production sectors differentiating for domestic deliveries and exports before being weighted for the 14 sectors.

In addition to this input-output set of the DIW, the following information basis necessary to implement the Disaggregated Forecasting Model II was supplemented by the research group in Bonn (Frerichs, Kübler, Jäger, Hellmuth (1979); Kiy (1980)): A disaggregation of the public sector into federal government (Bund), state government (Länder), local government (Gemeinden) and social insurance that results in the enlargement of the 14 production resctors to 17 branches); the separation of private households and private non-profit organizations leading to the maximum of 18 production sectors, a compilation of consolidated transfer tables for five big sectors; an estimation of sectoral gross and net fixed capital accounts; a collection of detailed data for the labour market and additional macroeconomic data, i.e. interest rates, population figures, exchange rates, and volumes of world trade.

I/////I = estimated with MODOP
I   I = based on statistical observations
I^^^^^I = partially estimated with MODOP

sectors	1	2	3	4	5	6	7	8	9	10	11	12	13-16	17-18
1														
2														
3		^^^^^	^^^^^	^^^^^	^^^^^	^^^^^	^^^^^	^^^^^	^^^^^	^^^^^	^^^^^	^^^^^		
4		//////	//////	//////	^^^^^	//////	//////	//////	//////	//////	//////	//////		
5		^^^^^	^^^^^	^^^^^	^^^^^	^^^^^	^^^^^	^^^^^	^^^^^	^^^^^	^^^^^	^^^^^		
6		//////	//////	//////	^^^^^	//////	//////	//////	//////	//////	//////	//////		
7		//////	//////	//////	^^^^^	//////	//////	//////	//////	//////	//////	//////		
8			^^^^^					//////		//////		//////		
9														
10		//////	//////	//////	^^^^^	//////	//////	//////	//////	//////	//////	//////		
11		//////	//////	//////	^^^^^	//////	//////	//////	//////	//////	//////	//////		
12		//////	//////	//////	^^^^^	//////	//////	//////	//////	//////	//////	//////		
13 - 16														
17 - 18														

years 1963-1966,1968-1971,1973,1974

tabel of the compilation of intermediate transactions in the input-output tables

The following table reflects the final disaggregation of the production part of the model:

		Classification of the production sectors
1	agriculture	agriculture, forestry, fishing, gardening
2	energy, mining	electricity, gas and water, coal mining, iron ore mining, potash and rock salt mining, mineral oil extraction, mining n.e.s.
3	chemicals	chemicals, building materials, mineral oil refining, rubber and asbestos manufactures, fine ceramics, glass, plastics manufactures
4	iron, steel, NFM	iron and steel, iron and steel foundries, steel drawing and cold rolling mills, non-ferrous metals, steel forging
5	constr.steel, vehicles	constructional steel, machinery, vehicles, aerospace, shipbuilding
6	electrical goods	electrical engineering, precision engineering and optics, hardware and metal goods, musical instruments, toys, jewelry and sport articles
7	paper, textiles	saw mills and wood processing, cellulose and paper, timber manufactures, paper and board manufactures, printing and duplicating, leather, textiles, clothing
8	food	grain milling, edible oils and margarine, sugar, brewing and malting, tobacco manufactures, other food and beverages
9	construction	construction
10	trade, commerce	wholesaling, retailing
11	transport, communications	railway, shipping, waterways and harbours, other transport, communication (Bundespost)
12	other services	banks and insurances, rented dwellings, services n.e.s.
13	federal gov.	federal government
14	state gov.	state government
15	local gov.	local government
16	social insurance	social insurance
17	n.p.organizations	non-profit organizations
18	private HH	private households

Simulation results of the Disaggregated Bonn Forecasting Model II

After having given a short survey of the Disaggregated Bonn Forecasting Model II in its present state and the available data base, we return to the first question of the possible advantage of variable input coefficients compared with constant coefficients. Since the Disaggregated Bonn Forecasting Model II is based on variable input coefficients it is easy to reduce the model to the simpler approach of constant coefficients. Before going into further details it should be pointed out, that the following results are based on computations, which were done at the Bonn University and the German Institute of Economic Research and will be presented in greater detail in a forthcoming article (Erber, Kiy, Pischner (1983)).

For the comparison two different approaches were used:

1. The input coefficients were fixed at the 1970 values.
2. The input coefficients were set equal to its average value computed from the time series data for the corresponding simulation period (1960-1974).

Both approaches seem to be favourable assumptions for the hypothesis that constant input coefficients are equal efficient as variable ones (Hansen (1981)).

To isolate the effect of the predictive performance of the input coefficients in the model, the final demand variables were not explained by the endogenous behavioural equations but were kept exogenous. In a second step the combination of the different hypothesis for input (investment) coefficients and endogenous final demand were investigated. The following tables give a survey of the distribution of simulation errors for the four coefficient matrices.

set of variables	hypothesis	percentage errors			
		0 % - 5 %	5 % - 10 %	10% - 20 %	> 20 %
domestic input coefficients	variable	2021	727	368	95
		62.9 %	22.6 %	11.5 %	3.0 %
	const.70	1015	622	717	857
		31.6 %	19.4 %	22.3 %	26.7 %
	const.AV	966	708	833	704
	(60-74)	30.1 %	22.0 %	26.0 %	21.9 %
import input coefficients	variable	729	537	435	301
		36.5 %	26.8 %	21.7 %	15.0 %
	const.70	477	236	348	941
		23.8 %	11.8 %	17.4 %	47.0 %
	const.AV	328	302	479	893
	(60-74)	16.4 %	15.1 %	23.9 %	44.5 %

tabel of the distribution of the simulation errors for the input coefficients with exogenous final demand

set of variables	hypothesis	percentage errors			
		0 % - 5 %	5 % - 10 %	10% - 20 %	> 20 %
domestic investment coefficients	variable	1362 53.5 %	626 24.6 %	381 14.9 %	179 7.0 %
	const.70	839 32.9 %	526 20.6 %	649 25.5 %	534 21.0 %
	const.AV (60-74)	740 29.0 %	552 21.7 %	679 26.7 %	577 22.6 %
import investment coefficients	variable	191 23.3 %	178 21.8 %	232 28.3 %	218 26.6 %
	const.70	106 12.9 %	54 6.6 %	100 12.2 %	559 68.3 %
	const.AV (60-74)	69 8.4 %	82 10.0 %	120 14.7 %	548 66.9 %

tabel of the distribution of the simulation errors for the investment coefficients with exogenous final demand

From this two tables we can see that the variable input and investment coefficients are uniformly superior to the constant coefficients hypothesis. Surprisingly the average values of the input and investment coefficients give no better performance than the 1970 fixed values. The domestic input and investment coefficients give about 30 % better results than the corresponding import coefficients, if we take the error bound of 5 % deviation as a criterium. This could be explained by the different quality of the data base and the imperfections of the model to catch up the important characteristics of foreign trade properly.

Since the relative comparison of variable with constant input coefficients gives us a positive impression of the econometric approach to input and investment coefficient modelling, there is an obvious lack of abolute efficiency in simulation accuracy. If it is our target to give reliable forecasts of input and investment coefficient matrices, it seems to be not very satisfactory to have 37.1 % simulation errors with more than 5 % deviation from the original data for the domestic input coefficients to 76.7 % simulation errors for the import investment coefficients of the same category. The estimated model still rests on a time period where a smooth growth process without the disruptions of the two oil crisis was going on, so that it would be not surprising to get worse results, if we change the time horizon to the years up to 1980 or 1981. From this experiences it seems that it is not very probable to expect relatively accurate forecasts of whole input-output structures in the near future.

After evaluating the coefficient structure let us now look at the resulting simulation errors for the nominal and real gross production for the different sectors. The following tables summarizes the results of an ex post simulation of the whole model for the period 1963-1974.

	production sectors	percentage errors		
		0 % - 5 %	5 % - 10 %	> 10 %
1	agriculture	11	-	-
2	energy, mining	10	1	-
3	chemicals	9	2	-
4	iron, steel NFM	6	5	-
5	constr.steel,vehicles	6	5	-
6	electrical goods	7	3	-
7	paper, textiles	9	2	-
8	food	11	-	-
9	construction	9	2	-
10	trade, commerce	11	-	-
11	transport, communicatn.	10	1	-
12	other services	11	-	-
13	federal gov.	10	1	-
14	state gov.	7	4	-
15	local gov.	7	4	-
16	social insurance	7	1	3
17	non profit org.	4	6	1
18	private households	7	3	1
	total	163	41	5
	in %	78.0 %	19.6 %	2.4 %

tabel of simulation errors for the real gross production

	production sectors	percentage errors		
		0 % - 5 %	5 % - 10 %	> 10 %
1	agriculture	11	-	-
2	energy, mining	7	4	-
3	chemicals	9	2	-
4	iron, steel NFM	7	2	2
5	constr.steel,vehicles	7	2	2
6	electrical goods	8	2	1
7	paper, textiles	10	1	-
8	food	11	-	-
9	construction	9	2	-
10	trade, commerce	10	1	-
11	transport, communicatn.	11	0	-
12	other services	11	-	-
13	federal gov.	9	2	-
14	state gov.	11	-	-
15	local gov.	7	4	-
16	social insurance	5	4	2
17	non profit org.	10	1	-
18	private households	11	-	-
	total	175	27	7
	in %	83.7 %	12.9 %	3.3 %

tabel of simulation errors for the nominal gross production

From the two tabels we see that even if the input coefficient structure is not well predicted the corresponding sectoral gross production values are quite reliable. The aggregate of all sectoral nominal as well as real gross production values have all simulation errors less than 5 %. So an error compensating process occurs if we start from the most detailed structure of the input and investment coefficients and compute from them the simulated final and intermediary demand values, the sectoral gross production and finally the aggregate gross production. The question comes up, whether this can be explained on an underlying theoretical basis, which explaines why and when this error compensation happens. Before we try to give an answer to this question we want to present the resulting simulation errors of the variable input and investment coefficients for the Disaggregated Bonn Forecasting Model II.

	production sectors	percentage errors		
		0 % - 5 %	5 % - 10 %	> 10 %
1	agriculture	68	52	67
		36.4 %	27.8 %	35.8 %
2	energy, mining	117	45	25
		62.6 %	24.0 %	13.4 %
3	chemicals	110	49	28
		58.8 %	26.2 %	15.0 %
4	iron, steel NFM	106	41	29
		60.2 %	23.3 %	16.5 %
5	constr.steel,	115	50	22
	vehicles	61.5 %	26.7 %	11.8 %
6	electrical goods	107	44	36
		57.2 %	23.5 %	19.3 %
7	paper, textiles	111	50	26
		59.4 %	26.7 %	13.9 %
8	food	115	41	31
		61.5 %	21.9 %	16.6 %
9	construction	82	50	55
		43.9 %	26.7 %	29.4 %
10	trade, commerce	115	51	21
		61.5 %	27.3 %	11.2 %
11	transport, communications	129	44	14
		69.0 %	23.5 %	7.5 %
12	other services	116	40	31
		62.0 %	21.4 %	16.6 %
13	federal gov.	65	39	28
		49.2 %	29.6 %	21.2 %
14	state gov.	73	44	37
		47.4 %	28.6 %	24.0 %
15	local gov.	87	49	18
		56.5 %	31.8 %	11.7 %
16	social insurance	3	4	4
		27.2 %	36.4 %	36.4 %
17	non profit org.	16	11	6
		48.5 %	33.3 %	18.2 %
	total	1 535	704	478
	in %	56.5 %	25.9 %	17.6 %

tabel of simulation errors for the domestic input coefficients

Firstly, the tables show that the dynamic simulation of the whole model leads to increasing simulation errors because the feedback of the endogenous final and intermediate demand causes additional errors. The results are therefore more realistic for the evaluation of the forecasting target, since during ex ante forecasts the whole model rests only on the exogenous variables.

Secondly, we see that for the domestic input coefficients the agriculture, construction, all government, social security, non profit organization and private households sectors perform rather unsatisfactory. One reason for this could be that these sectors are not adequately explained by using a neoclassical theory of the firm as the theoretical fundament for explanantion. More sectoral specifics have to be regarded to model these sectors properly.

production sectors		percentage errors		
		0 % - 5 %	5 % - 10 %	) 10 %
1	agriculture	60	40	43
		41.9 %	28.0 %	30.1 %
2	energy, mining	32	21	68
		26.4 %	17.4 %	56.2 %
3	chemicals	65	49	73
		34.8 %	26.2 %	39.0 %
4	iron, steel NFM	32	30	37
		32.3 %	30.3 %	37.4 %
5	constr.steel,	44	24	86
	vehicles	28.6 %	15.6 %	55.8 %
6	electrical goods	57	40	79
		32.4 %	22.7 %	44.9 %
7	paper, textiles	73	42	72
		39.0 %	22.5 %	38.5 %
8	food	55	34	43
		41.7 %	25.7 %	32.6 %
9	construction	19	18	51
		21.6 %	20.5 %	57.9 %
10	trade, commerce	2	3	6
		18.2 %	27.3 %	54.5 %
11	transport, commu-	77	61	38
	nications	43.7 %	37.7 %	21.6 %
12	other services	65	40	82
		34.8 %	21.4 %	43.8 %
13+14	government	4	3	4
15+16		36.4 %	27.2 %	36.4 %
	total	585	405	682
	in %	35.0 %	24.2 %	40.8 %

tabel of simulation errors for the import input coefficients

	production sectors	percentage errors		
		0 % - 5 %	5 % - 10 %	> 10 %
1	agriculture	57	40	43
		30.5 %	23.0 %	46.5 %
2	energy, mining	78	50	70
		39.4 %	25.2 %	35.4 %
3	chemicals	78	63	57
		39.4 %	31.8 %	28.8 %
4	iron, steel NFM	32	30	37
		40.1 %	37.4 %	22.5 %
5	constr.steel, vehicles	50	15	12
		64.9 %	13.5 %	15.6 %
6	electrical goods	96	63	39
		48.5 %	31.8 %	19.7 %
7	paper, textiles	113	54	31
		57.0 %	27.3 %	15.7 %
8	food	3	2	6
		27.3 %	18.2 %	54.5 %
9	construction	72	31	18
		59.5 %	25.6 %	14.3 %
10	trade, commerce	111	56	20
		59.4 %	29.9 %	10.7 %
11	transport, commu-nications	56	44	65
		33.9 %	26.7 %	39.4 %
12	other services	65	40	82
		47.5 %	24.2 %	28.3 %
13	federal gov.	2	1	8
		18.2 %	9.1 %	72.7 %
	total	885	540	511
	in %	45.7 %	27.9 %	26.4 %

tabel of simulation errors for the domestic investment coefficients

	production sectors	percentage errors		
		0 % - 5 %	5 % - 10 %	> 10 %
4	iron, steel NFM	16	23	115
		10.4 %	14.9 %	74.7 %
5	constr.steel, vehicles	38	60	89
		20.3 %	32.1 %	47.6 %
6	electrical goods	59	31	97
		31.5 %	16.6 %	51.9 %
7	paper, textiles	36	41	88
		21.8 %	24.9 %	53.3 %
	total	149	155	389
	in %	21.5 %	22.4 %	56.1 %

tabel of simulation errors of the import investment coefficients

After having demonstratedt he relationship between coefficient structure and gross production values, we try to give a theoretical foundation for the observed error compensation or the minor sensitivity of gross production on errors in a large number of input coefficients. The following rests on the work of Schintke (1976), who wrote his thesis on sensitivity analysis of input-output matrices. He developed a method how to get a first impression of error propagation in an input-output table.you can compute error bounds for input coefficients, if you fix a maximum percentage error for all gross production values. These error bounds still rest on the assumption that the final demand does not contain errors and there are no simultaneous measurement errors in the input coefficients. From an econometric point of view these assumptions do not hold, but using this simple method gives a first impression to what degree the single input coefficients influence the predictive performance of the sectoral gross production variables. The effect of simultaneous coefficient changes on the gross production variables seems to be after all approximately additive (cf.Schintke (1976)), so that it still makes sense to use this error bounds for a first investigation. The following table gives the maximum percentage error, which could occur to cause an 1% error in at least one gross production variable. The range of maximum percentage errors is from 0 to 100 % and is divided into 10 subclasses of 10 % range each. Coefficients which might change more than 100 % without causing an error of 1 % in gross production are marked by a '>' sign. They are called irrelevant coefficients. So every value in the table should be interpreted in the following way: If we take for example the coefficient $a_{7,7}$ its error bound in the table is 1. This value times 10 gives the maximum percentage error which could occur for this coefficient without causing an error of more than 1 % in any gross production variable. The lower the maximum error bound, the more important is the coefficient for the computation of the gross production values.

///// = coefficient is zero and maximum error bound is undefined
> = maximum error bound is larger than 100 %
i * 10 = maximum percentage error bound

sectors	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	9	>	10	>	>	>	2	1	>	>	>	2	>	>	8	>	>
2	3	1	1	1	2	4	3	4	8	5	3	2	>	9	3	>	>
3	3	3	1	4	2	2	1	4	1	6	3	4	8	>	8	4	>
4	8	4	4	1	1	1	9	>	2	>	8	9	9	>	>	/////	>
5	5	7	>	6	1	9	>	>	4	7	5	6	3	>	>	>	>
6	>	6	3	5	1	1	5	7	3	9	6	4	3	>	>	>	>
7	>	>	2	>	3	3	1	4	3	6	7	2	5	8	7	>	>
8	2	>	5	>	>	>	>	1	>	>	>	1	>	>	>	>	>
9	>	>	>	>	>	>	>	>	3	>	>	2	3	>	6	>	>
10	8	>	3	5	3	4	3	4	4	1	10	5	4	>	>	5	>
11	8	4	1	3	3	3	2	3	3	1	1	2	5	7	6	7	>
12	6	9	2	7	3	4	4	5	>	3	6	1	>	5	8	2	10
13	>	>	>	>	>	>	>	>	>	>	>	>	/////	/////	/////	/////	/////
14	>	>	>	>	>	>	>	>	>	>	>	5	/////	/////	/////	4	>
15	>	>	>	>	>	>	>	6	>	7	>	2	/////	/////	/////	1	>
16	/////	/////	/////	/////	/////	/////	/////	/////	/////	/////	/////	/////	/////	/////	/////	/////	>
17	/////	/////	/////	/////	/////	/////	/////	/////	/////	/////	/////	/////	3	1	1	1	/////

tabel of the error bounds for the domestic input coefficients

Evaluating the simulation errors of the coefficients matrices with respect to their maximum error bounds shows that the robustness of the Disaggregated Bonn Forecasting Model II rests on the good performance of the important input coefficients in the dynamic model simulation of this matrix. Only 3.9 % of all simulation errors in a dynamic ex post simulation with the variable domestic input coefficients exceeded their limits, compared to 6.2 % for a dynamic simulation with constant coefficients (AV) or 12.3 % with constant coefficients (1970).

Summary.

The paper investigates two major problems concerning the efficiency of forecasts by using variable instead of constant input coefficients for a sectoral disaggregated econometric model.

The authors conclude from experiences with the Disaggregated Bonn Forecasting Model that on the one hand there are significant improvements of the average predictive performance using variable input coefficients, but on the other the increased efficiency is not as large as to guarantee a reliable forecast of the whole input-output structure. This weak accuracy cast some light on the common usage of constant input coefficients for input-output structures, which is even worse when used for prediction of a whole structure.

This result might be satisfactory, if we are not interested in the prediction of the coefficient matrix but on the prediction of the values of gross production (in constant prices), which we can calculate from it. These forecasts showed to be much more reliable then the forecasted coefficient matrix itself. One reason could be a general error compensating process, but the more important reason seems to be that only few of the prediction errors of the input coefficient values could influence the prediction error of the gross production values considerably. This can be shown by sensitivity analysis of the transformation of errors of single coefficients to the gross production values. Therefore it would be sufficient to get reliable predictions of the important input coefficients to guarantee a good predictive performance for the gross production values. The investigation of the Disaggregated Bonn Forecasting Model has shown that the robustness of its predictive performance for the gross production values is based on this mechanism.

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ON THE APPLICATION OF MARKOV CHAINS TO INPUT–OUTPUT DYNAMICS

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1. Introduction

Markov chains are an effective and generally used tool for the solution of many kinds of tasks in the sphere of practical life, exact and social sciences. The similarity between the properties of Markov chains and some characteristics of the input-output model has also made possible their application to the solution of some tasks of input-output analysis e.g. [4], [5].

The system of Czechoslovak input-output tables provides a suitable data base for different investigations into the input-output relations in national economy. For the needs of these investigations a lot of adequate tools has been created, of which the Markov chains may represent one of the possible applicable approaches. Our contribution is an attempt to investigate some questions within this problem area. Its aim is to present some possibilities of applying Markov chains in the sphere of examining the stability and forecasting input-output relations, with concrete verification under the conditions given by the data base of input-output tables of the Czechoslovak economics.

2. Basic Characteristics of the Input-Output Tables of Czechoslovakia

The input-output tables of Czechoslovakia with more than 400 branches of production are one of the largest in the world. In our tables the coverage of material and non-material consumption by domestic production and imported production is consistently differentiated, and at the same time the principles of the supplier and customer branches of production, or spheres of final use are observed. Such a representation quantifies the complicated relations among domestic and foreign markets and creates suitable preconditions for the analysis of specific questions concerning external relations of the economy.

The input-output tables of Czechoslovakia are constructed on two basic price levels: in producers´ prices and in users´ prices.

A specific feature of the input-output tables of Czechoslovakia is the construction of these tables by the method of gross turnover, that means including the part of intradepartmental turnover. A table of this type enables the representation and analysis of the inner economic relations of economic units, as well as the evaluation of their efficiency. It makes further possible an analysis of the exploitation and efficiency of the production factors, first of all the capital fund and manpower. At present it is relatively easy to transform this table by means of recalculation algorithms and a computer to a table constructed by the method of gross output and to facilitate thus, in general features, the connection between this table and the macroeconomic indicators of the national economy balance sheets.

The additional quadrants of input-output tables - capital funds and manpower - provide data on these production factors divided according to the individual branches of production. They are a starting-point for the analysis of efficiency of these factors within the framework of their inclusion into the production process.

The first works connected with the construction of input-output tables in Czechoslovakia were earied out at the beginning of the sixties and they had an experimental character. They concerned the construction of a medium-size /96 branches/ input-output table for the year 1962. Further input-output tables were constructed on the basis of regular statistical surveys for the years 1967, 1973 and 1977, and their size is more than 400 branches of production. Recalculation input-output tables for the years 1969 and 1970 were constructed for bridging the time gaps between the periods 1967 and 1973, as well as small-size input-output tables for the period 1970--1980. At present another big-size input-output table for the year 1982 is being worked out.

The input-output tables of Czechoslovakia are supplemented by basic parameters/technical coefficients, complex coefficients, distribution coefficients, coefficients of capital requirement, labour requirement and some derived indicators/ which, on the one hand, represent certain basic analytical knowledge and, on the other hand, they may be directly used for analytical, forecasting and planning purposes.

The above-mentioned characteristics of the Czechoslovak input-output tables give us a certain image of the possibilities of their practical applications. Recently there has been a more and more intensive and complex utilization of input-output tables for different analytical and forecasting purposes, which means that the examination of the dynamics of development of input-output relations is of great importance for the Czechoslovak input-output tables as well.

3. Examination of the Stability of Input-Output Relations

We proceed from the assumption that the examined input-output relations of the given economic system are represented by corresponding characteristics which will be denoted by us as a non-negative matrix $S = \|s_{ij}\|$, for i, j = 1, 2, ..., n. The most typical example of such a matrix is the first quadrant of the input-output table, the matrix or submatrix of the coefficients which have been derived from it, etc.

Let us suppose that input-output relations changes in time. Let us denote the matrix representing the input-output relations in the time t as $S(t) = \|s_{ij}(t)\|$ for i, j = 1, 2, ..., n. In that case the development of input-output relations in the time-set T may be described by matrixes of S (t) for all $t \in T$. Let us denote $D(t_1, t_2) = S(t_1) - S(t_2)$ as the difference matrix for all $t_1, t_2 \in T$. In practice this difference matrix in most cases happens to be a non-zero matrix.

In general it is quite natural that the greater the difference between the periods t_1, t_2, the more the elements of this difference matrix are different from zero. Such a development may be influenced by many factors, e. g. the technical development of products, a change in the production technology, the development of the inner structure of the branch, price changes, adjustment changes of a methodological character, etc. It is quite evident that the zero difference matrix corresponds again to a constant development, i. e. the input-output relations undergo no changes.

We shall say that the input-output relations in the time-set T will be the more stable, the nearer the elements of the matrixes $D(t_1, t_2)$ are to zero for all $t_1, t_2 \in T$. That means that the elements of the matrixes $D(t_1, t_2)$ completely characterize the stability of input-output relations, while the deviation value of these elements from zero, within the framework of which the input-output relations are still considered to be stable, depends on the given economic system and the given economic system and the concrete aims of the investigation.

For the measurement of the stability of input-output relations different synthetical characteristics may be used. For our

purpose we shall use a coefficient which we shall denote as β. Its construction is as follows: Let $T = \{ t_1, \dots, t_k \}$, where k is a natural number larger than 1, and
$S(t_1) = \| s_{ij}(t_1) \|$, ..., $S(t_k) = \| s_{ij}(t_k) \|$ for
i, j = 1, 2, ..., n are matrixes representing input-output relations, then

$$= \frac{\sum_i \sum_j \left(|s_{ij}(t_1) - s_{ij}(t_2)| + \dots + |s_{ij}(t_{k-1}) - s_{ij}(t_k)| \right)}{\sum_i \sum_j s_{ij}(t_k)} \qquad /1/$$

It is evident that $\beta \geq 0$. The growth of the value β signals a growth of the unstability of input-output relations and, vice versa, to a smaller value of this coefficient it is necessary to ascribe a larger stability of input-output relations, where in the case of $\beta = 0$ the input-output relations are of a constant character.

Our aim in this part of the paper will be to examine the stability of input-output relations expressed by:
- the coefficient β, and
- a homogeneity used within the framework of the apparatus of Markov chains.

A Markov chain is completely determined by the probability distribution of the initial state /vector p (0)/, and by the probabilities of transition /the matrix of transition probabilities P/. In homogeneous Markov chains /that means that the matrix P does not change in time/, the matrix of transition probabilities in the period t /i. e. P (t) / is equal to the t-th power of the matrix of transition probabilities; and the probability distribution of the state of the system which is being modelled, in the time t /i. e. p (t)/, is determined by the product of the initial state and the t-th power of the matrix of transition probabilities. This means that it holds:

$$P(t) = P^t \quad \text{and} \qquad /2/$$

$$p(t) = p(0)\, P^t \qquad /3/$$

where t is a natural number.

Let us assume that the development of input-output relations can be described by a Markov chain. Then the hypothesis may be stated that there exists a connection between the stability in the development of input-output relations and the homogeneity of the corresponding Markov chain. Let us try to verify this hypothesis empirically.

First of all it is necessary to assign to the matrixes $S(t) = \|s_{ij}(t)\|$ the matrixes of transition probabilities $P(t) = \|p_{ij}(t)\|$ for i, j 1, 2, ..., n and all $t \in T$, where $T = \{1, 2, \ldots, r\}$. It means that

$$p_{ij}(t) = \frac{s_{ij}(t)}{\sum_j s_{ij}(t)} \qquad \text{for } t = 1, 2, \ldots, r \qquad /4/$$

We want to find out whether these matrixes P(t) are homogeneous, i. e. whether they are independent of t. Therefore we assume the H_o hypothesis

$$H_o : P(t) = P \qquad \text{for } t = 1, 2, \ldots, r$$

The hypothesis may be verified in accordance with the procedure mentioned in [1], where at first the matrix $P = \|p_{ij}\|$ is constructed, where

$$p_{ij} = \frac{\sum_t s_{ij}(t)}{\sum_t \sum_j s_{ij}(t)} \qquad \text{a} \qquad /5/$$

then the statistics is computed:

$$\gamma = \sum_{t=2}^{r} \sum_{i=1}^{n} \sum_{j=1}^{n} s_{ij}(t) \ln \frac{s_{ij}(t)}{s_{ij}^{(t-1)} p_{ij}} \qquad /6/$$

which has a χ^2 distribution with $(r-1)[n(n-1)]$ degrees of freedom. We accept the H_o hypothesis at the given significance level α, if it does not fall into the critical region of the χ^2 distribution.

For the purpose of carrying out the above-mentioned procedure serving the examination of the stability of input-output relations we choose the first quadrant of the input-output table as a representative of the input-output relations, and we denote the indicators of this quadrant by the matrix $X(t) = \| x_{ij}\ (t) \|$ for $i, j, = 1,2\ \ldots, n$ and $t \in T$.

For the derivation of the characteristics describing these input-output relations /i. e. the matrix $S(t)= \| s_{ij}(t) \|$ / we shall use six different procedures $W_1, \ldots, W_6$ which are defined in the following way:

$$W_1 : s_{ij}(t)= x_{ij}(t)$$

$$W_2 : s_{ij}(t)= \frac{x_{ij}(t)}{\sum\limits_i x_{ij}(t)}$$

$$W_3 : s_{ij}(t)= \frac{x_{ij}(t)}{\sum\limits_j x_{ij}(t)} \qquad /7/$$

$$W_4 : s_{ij}(t)= a_{ij}(t)$$

$$W_5 : s_{ij}(t)= d_{ij}(t)$$

$$W_6 : s_{ij}(t)= b_{ij}(t)$$

for $t \in T$ and $i, j = 1, 2, \ldots, n$ where $a_{ij}(t), d_{ij}(t)$ and $b_{ij}(t)$ are technical, distribution and complex coefficients.

As a data base for the concrete computation the time series of indicators of input-output tables of the Czechoslovak economy has been used, in producers´ prices, with a variant with imports included, with the production indicator gross output in constant prices of the year 1967 for the period $T = 1962, 1967, 1969, 1970, 1973$ in an aggregation into 23 branches, i. e. $n = 23$.

We have tested the time homogeneity of the matrixes of transition probabilities $P(t)$ on the basis of a procedure characterized by the relations (4-6). It follows from the χ^2

distribution that the value of χ^2_α in the case of 2024 degrees of freedom and at the significance level α = 0,001 is equal to 1833,044. The computed values of statistics γ and the coefficient β determined by the relation /1/ for the individual types of procedures for the construction of the matrixes P/T/ are shown in Table 1.

Table 1

Procedure	Statistics γ	Coefficient β
W_1	5921701,210	3,432
W_2	1344,631	2,596
W_3	1105,685	3,128
W_4	430,831	2,936
W_5	351,434	2,847
W_6	848,935	2,626

If follows from the above table that in the sense of the H_o hypothesis test it can be stated about the time homogeneity of the matrixes of transition probabilities that the procedures W_2 - W_6 mention matrixes of transition probabilities homogeneous as to time, that means that the matrixes S/t/ acquired by these procedures are independent of time /stable/ in the sense of a homogeneous Markov chain. The matrix which has been received by the procedure W_1 does not satisfy the H_o hypothesis, i. e. it is not homogeneous as to time and it is also least stable in the sense of the coefficient β . This means that the input-output relations whose development is not homogeneous as to time in the sense of the Markov chains are, at the same time, the least stable also in the sense of the coefficient β .

The above-mentioned results show us that, apart from the absolute indicators of the first quadrant of the input-output table, the development of all coefficients under review, first of all the technical, distribution and complex ones, is stable in the Markov sense, and that this stability is the greatest in the area of the distribution structure of the production. The values of the coefficient β which are a certain kind of expression of the dispersion of the examined characteristics in time, indicate, on the other hand, the greatest stability in the area of material costs /the W_2 procedure/. These facts may be also considered to be results which, in a certain way, are of a complementary character. That means that the stability of development in the area of the distribution structure of the production in the Markov sense calls forth a certain stability in the area of material costs, and vice versa. This fact may also represent a certain impulse for a deeper research not only of the development of technological structures, but also of the dynamics of distribution relations.

4. Prediction of Input-Output Relations

As soon as the assumption of the homogeneity of development of input-output relations in the sense of the Markov chains, as characterized in the preceding part of our paper, is ful - filled, real prerequisites for the application of the relations /2/, /3/ are created concerning the prediction of certain types of input-output relations in the future. Choosing these principles as a starting-point, two basic aims may be set:

a/ the prediction of the matrix S/t/, and
b/ the prediction of characteristics of input-output relations described by the vector p/t/.

For fulfilling both purposes the construction of a suitable matrix of transition probabilities P is necessary which is homogeneous.

Let us denote the procedure on the basis of which it is possible to assign the corresponding matrix of transition probabilities P to the given structure S, by the symbol W. Formally we shall denote this assignment in the following way:

$W : S \longrightarrow P$

The prediction of the structure matrix for the period t, i. e. for S(t) may be acquired by means of the t-th power of the corresponding matrix of transition probabilities P^t and a inverse procedure W^{-1} by the application of which we get from the matrix P^t the corresponding matrix of the structure S(t). The whole procedure of predicting the structure matrix may be graphically expressed by the following flow chart:

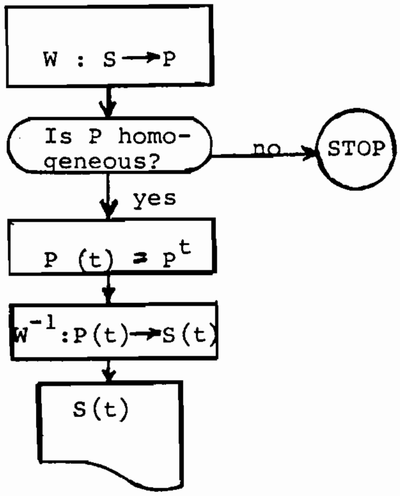

Fig. 1

It follows from the above diagram that the only problem in carrying out the prediction is the determination of the suitable procedures W and W^{-1}, while the test of homogeneity is done by the methodology characterized in the preceding part.

For the prediction of the characteristics of structure described by the vector p(0), apart from the matrix P, also the construction of this vector is necessary. Let us denote the procedure on the basis of which the corresponding vector p(0) with elements $p_i(0)$ may be assigned, in the period chosen as a starting-point, to the given structure S, by the symbol V. The vector p(0)has the following properties:

$p_i(0) \in \langle 0,1\rangle$, for all i, and

$\sum_i p_i(0) = 1$

Formally we shall denote the assignment V as follows:

$V : S \rightarrow p(0)$

The prediction of the vector p for the period t may be thus achieved on the basis of the recurrent relation $p(t) = p(0)\,P^t$ for $t \in T$. The whole procedure of this prediction may be graphically expressed by the flow chart in Fig. 2.

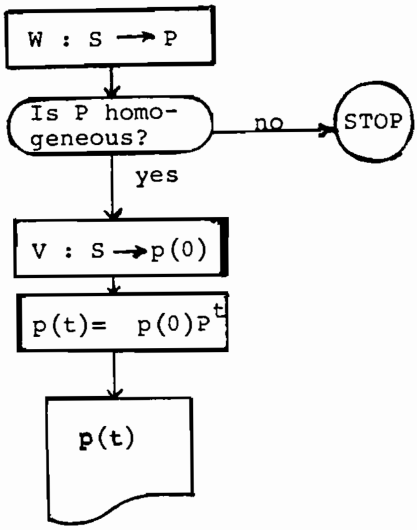

Fig. 2

It is evident that the vector p(0) may be a representative of the structure of a certain indicator in the period 0. It may represent, for instance, the structure of the vector of production, material consumption, investments, etc.That means that in the case of the existence of a suitable prediction of the corresponding aggregate indicators this vector may be used for the disagregation of this aggregate indicator into a structure, and thus, as a matter of fact, a certain prediction of the corresponding indicators may be acquired in the given detailed breakdown.

Let us denote the vector of indicators in the period t, which we want to predict in this period, by the symbol $u(t)$ with elements $u_i(t)$, where the vector $p(t)$ represents the corresponding prediction of its structure and $x(t)$ is the prediction of the corresponding aggregate indicator, i. e. $x(t) = \sum_i u_i(t)$. Then the prediction $u(t)$ is given:

$$u_i(t) = p_i(t) \cdot x(t) \qquad \text{for all } i \text{ and } t \in T \qquad /8/$$

For the prediction of the indicators of the matrix of the first quadrant of the input-output table $X(t) = \|x_{ij}(t)\|$ and the suitable vectors of indicators we have used the above-mentioned procedures, using the data base described in the preceding part.

As the procedure W in the prediction of the indicators of the matrix $X(t)$ the procedures $W_2, \ldots, W_6$ have been used according to the relation /7/, whereby we have used in every procedure the same inverse procedure defined in the following way:

$$W^{-1} : X(t) = p^t \cdot v(t) \qquad \text{for } t \in T \qquad /9/$$

where $v(t)$ denotes the vector of the row sums of the first quadrant i. e. the volumes of supply for material consumption from supplier branches in the time t. The computation was carried out for t = 1,..., 5 which represent the individual years of the set T, i. e. 1962, 1967, 1969, 1970 and 1973, where the year 1962 was taken as the zero period. As a measure of precision for the above-mentioned ex-post application in comparison with the initial data we used the average values of the determination indexes computed for the above-mentioned time horizon. Their values are given in Table 2.

Table 2

Procedures	W_2	W_3	W_4	W_5	W_6
Average values of determination indexes	0,332	0,227	0,330	0,230	0,368

It is evident from the above table that the precision of the estimation of the matrix of **first** quadrant carried out as has been described above is insufficient in the case of every procedure used. That means that although the development of input-output relations in the area of the first quadrant of the input-output table given by the coefficients used is stable in the Markov sense, but it is not possible to describe with sufficiently precision the development of the structure of the indicators of the first quadrant computed by using these coefficients, by a Markov chain. Within the framework of these results it has been shown, however, that the Markov character of development of these input-output relations is the largest in that case if we take the complex coefficients as our starting-point.

On the vasis of an analysis of the predicability of the vectors of the each directions of final use /i. e. the second quadrant of the input-output table/ and of the elements of the value added /i. e. the third quadrant of the input-output table/ including the vector of production, while using the procedure characterized in this part of our paper /Fig. 2 and relation 8//, we have come to the conclusion that only the vectors of production and material consumption are predictable with sufficient precision. During the application of the above-mentioned methodology the procedure W_5 for acquiring the necessary matrix of transition probabilities has proved to be the most precise one, holding that $p_i(0) = u_i(0) \cdot (\sum_i u_i(0))^{-1}$, where $u_i(0)$ denotes the individual component parts of the corresponding predicted vectors for the zero period /the year 1962/. For characterizing the precision of the above computation we used, similarly as in the preceding case, as a measure of precision the average values of the indexes of determination computed for the time horizon which has been used. Their values are given in Table 3.

Table 3

Vector	Procedure	Average values of determination indexes
Production	W_5	0,828
Material consumption	W_5	0,901

5. Conclusion

Our experience has shown that it is possible to apply the apparatus of homogeneous Markov chains in certain spheres of examining the stability of development and the predictions of input-output relations of a concrete economic system.

Empirical experience at the level of the Czechoslovak economy has shown that the stability of development of the input--output relations is in certain cases closely connected with the homogeneity of the transition probabilities of the corresponding Markov chain, where the measures of stability which have been used provide us a picture, completing itself in a certain way, about the character of development of the input-output relations under review. Further it has been shown that by the application of the apparatus of Markov chains the vectors of production and material consumption are predictable with sufficient precision, and that the time horizon of prediction to be used is about 4-5 years. An eventual reaching of a higher precision of computations and the possibility of predicting other vectors, or matrixes would require a further development of the method which has been used, first of all in the sphere of competent procedures and matrixes of transition probabilities.

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ESTIMATING TRADE MARGIN MATRICES IN A MAKE-USE FRAMEWORK

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Trade and transport margins play an important role in the process of compiling IO-tables. In order to reconcile total supply and total demand, trade and transport margins have to be disaggregated by commodities. A margin matrix has to be estimated showing the margins by commodities and using industries and final demand. Furthermore, margin matrices do not only play an ancillary role in the compilation of IO-tables but are quite often of specific relevance for some aspects of IO-analysis and IO-modeling.

The task of compiling such margin matrices is connected with a variety of data problems. Considering the quantitative magnitude involved, any estimation process should be based on economically plausible assumptions. In its first part the paper tries to outline the alternative approaches. The second part describes the estimation process applied in Austria for the compilation of the trade margin matrices of the make-side of the make-use data system in some detail.

INTRODUCTION

1. Trade and transport margins play an important role in the process of compiling IO-tables. Basic data on intermediate demand as well as on final demand are usually available in purchasers' prices (incl.margins)

whereas data on total output by commodities are surveyed at producers' prices. (Basic data on international trade has its own specific valuation.) In order to reconcile total supply and total demand two data sets have to be estimated:

- trade and transport margins have to be disaggregated by commodities, thus showing the total "burden" of margins by commodities; these data show the margins produced by commodities and by industries (this matrix of margins produced is part of the make-matrix of a make-use system);

- starting from these data of total trade and transport margins by commodities a full margin matrix has to be estimated, which shows the margins by commodities and by using industries and final demand (input margin matrix); this matrix is part of the use-matrix.

  The deduction of this matrix from the use-matrix at purchasers' prices leads directly to the matrix at producers' prices.

The procedure of estimating two kinds of margin matrices is conceptually the same when compiling a square IO-table or when compiling a make-use data system. [1)] In the latter system the matrix by commodities and industries of the produced margins, which has to be estimated first, is shown as a part of the make-matrix, whereas in a square table this information is not shown separately (see annex I).

2. The key problem in the estimation of these data sets lies in the fact that in most countries neither the full matrix of distributive margins nor even the control data (margins by commodities) can be observed

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1) In a make-use system there are no problems in showing margins produced by non-trade industries and non-transport industries (non-characteristic output); there the connection between buying industries and related margin industries is not necessary, but only between buying industries and commodity margins, irrespective of the producers of the margins.

directly. What is usually available is some information on gross margins produced by industries (primarily in trade and transport) and on "channels" of distributive services.

Unfortunately - with a view to the poor data situation - the orders of magnitude involved are by no means negligible. For example, in the Canadian Table for 1977 /5/ the wholesale and retail sale margins amount to 7 % of total supply, in Austria 1976 this ratio is nearly 10 %. If the supply of those commodities that can be traded (agricultural products and manufactured products) is considered alone, the importance of the margins is much higher. For Canada this ratio is close to 12 %, for Austria about 16 %.

Due to the lack of data, the involved magnitude of the margins and the estimation problems it becomes clear that the margin data sets in IO-tables are often referred to as beeing of comparably low quality, and are sometimes used as a buffer to allocate some of the differences that inevitably occur when compiling IO-tables.

3. However, an underestimation of the margin "burden" for one commodity (given the totals) leads to an overestimation for some other commodities (and vice versa). Consequently this results in an overestimation (underestimation) of the use of this commodity either by industries or by final demand. Any inadequate attribution of the margins to commodities and using industries will lead to an inhomogeneous valuation of the input flows and to a biased depiction of the technologies of the various industries.

4. For some applications of IO-techniques, the details as such of the input margin matrix are relevant. In evaluating changes in technology the differences in "margin burden" should be taken into account explicitly. For example, any substitution of coal for electricity will lower the total trade input of the receiving sector. For the integration of behavioral equations of final demand components (which are usually valued at purchasers' prices), a bridge matrix between producers' and purchasers' prices is necessary.

5. In the following it is attempted to describe the procedure of estimating trade margins by commodities for the 1976 Austrian table. Although the data situation for this specific year was quite favorable (compared with the situation in other countries), there remained a considerable degree of uncertainty in this step, which is so relevant for the overall reliability of the IO-table.

The estimation of trade margin matrices of the make-side is part of the IO-statistical project of the Austrian Central Statistical Office (reference year 1976). In this project a make-use system fully in line with the rev. SNA 1968 /1/ is being implemented.

THE DATA BASIS

6. From the 1976 industrial census, which covered manufacturing, trade, transport and other services, gross margins by industries were available, for the trade branches as well as for the non-trading industries. A sample survey in the trade branches and in some parts of the non-trading industries provided some information on the wholesale and retail sale turnover by commodities. For the rest of the non-trade industries the commodity structure of the trade turnover was estimated on the basis of expert information. These data made possible the set-up of a turnover-matrix, "industries by turnover-type and commodities", the turnover type being wholesale or retail sales: 1)

$$T = \left(\begin{array}{c|c} T_{WW} & T_{WR} \\ \hline T_{RW} & T_{RR} \\ \hline T_{NW} & T_{NR} \end{array}\right)$$

1) It should be noted that the commodity classification corresponds with the industry-classification.

The first suscript stands for the type of industry:

W ..... wholesale trade branches
R ..... retail trade branches
N ..... other industries

The second suscript for the type of turnover:

W ..... wholesale turnover
R ..... retail sale turnover

7. In the turnover matrix about 160 industries are distinguished, 21 of which are wholesale trade branches and 24 are retail trade branches. About 120 commodities (incl. dummy-commodities) built the commodity dimension of the turnover-matrix T. 1) According to the SNA trade activities are treated as non-characteristic outputs for all industries other than trade.

To give some impression of the magnitude of the trade turnovers and gross margins, the following table summarizes the Austrian data, 1976 in mio AS (for some details see annex II):

	wholesale turnover	retail sale turnover	$\sum$	gross margins
wholesale trade branches	321.514	12.964	334.478	58.182
retail trade branches	3.842	189.146	192.988	51.312
other industries	45.074	18.040	63.114	13.884
$\sum$	370.430	220.150	590.580	123.378

1) Part of the turnover matrix is published in N. Rainer /4/.

Illustration of the involved magnitude can best be done by comparing the order of GDP for 1976 which amounted to 724.750 mio AS.

The above table shows that the bulk of the trade turnover is concentrated in the trade branches:

- 88 % of the wholesale and
- 92 % of the retail sales turnover.

It can also be seen that 4 % of the total trade turnovers of the wholesale trade branches have been retail sales, and 2 % of the total trade turnovers of the retail trade branches have been wholesale turnovers. [1] It should be noted that these data are the results of surveys based on establishments. If the basic statistical material would have been based on enterprises, the share of non-characteristic output would have been higher.

8. The turnover matrix shows a high number of off-diagonal elements that have not been expected beforehand. If one looks at the trade branches there is no single trade branche which is only trading in one commodity. [2] There are only a few trade branches where the main commodity accounts for more than 90 % of the total wholesale or retail sale turnover. In many cases there are 3 to 5 commodities which together represent a share of 60 - 90 % of the turnover.

   This does not apply to the non-trading industries to the same extent. In the manufacturing sector these industries can often be observed to be trading in the same product mix which they are producing.

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1) Seen on the level of individual branches this percentage is often much higher, for example: wholesale with clothing 12 %, wholesale with metalproducts, kitchen utensils, glass and china products 8 %, wholesale with vehicles 6 %. The same applies to the retail trade branches, even if the overall percentage is half of the one of the wholesale trade branches.

2) Of course, these findings depend on the industry and commodity classification used.

9. Therefore, as a consequence of this relatively high degree of dispersion we can identify many commodities which are traded by 10 or more industries; more than three forths of the commodities are traded by 5 industries at least.

It should be noted that the concept of characteristic and non-characteristic output according to SNA can be applied to the trade turnover-type as a whole as well as to the commodity structrue of the trade turnover. So, for example one would say that for a wholesale trade branche retail trade activities are non-characteristic. But one would also say that wholesale trading with sewing-machines by the wholesale branch "wholesale with vehicles" is a non-characteristic trade activity too. So, the high degree of dispersion of the turnover-matrix is to some extent due to both kinds of "non-characteristic" trade activities.

10. The gross margins by industries were given by the census in manufacturing, trade, transport and other services. These data can be written in the form of a vector g:

$$g = \begin{pmatrix} g_W \\ \hline g_R \\ \hline g_N \end{pmatrix}$$

The relation of these data to the corresponding trade turnovers results in the markup rates by industries. [1)] The average markup rate over all industries amounted to 21 %, over wholesale trade branches to 17 % and over retail branches to 27 %. Within the trade branches a wide range of markup rates can be seen:

9 - 38 % in the wholesale and
11 - 47 % in the retail trade branches

1) In the following markup rates are defined as margins in absolute values divided by the corresponding turnovers.

METHODS OF COMPILATION MARGIN MATRICES

11. Starting from this data background the aim of our investigation is a matrix M showing the margins produced by industry, commodity and turnover-type:

$$M = \begin{pmatrix} M_{WW} & M_{WR} \\ M_{RW} & M_{RR} \\ M_{NW} & M_{NR} \end{pmatrix}$$

The matrix M has the same dimension as matrix T. (The inclusion of matrix M in the make-use system as a whole can be seen in annex I .)

For this purpose two different assumptions of how margins are to be explained could be imagined:

- assumption of industry-specific margins
- assumption of commodity-specific margins.

The first assumption implies that each industry has a specific markup rate, irrespective of the commodities it trades. The second approach assumes that each commodity has a specific markup rate, irrespective of the industries which are trading with this commodity, but though depending on the turnover type. The two assumptions of how trade margins could be explained stand for two methods of estimating trade margin matrices of the make-side, respectively.

Industry-specific margins

12. This approach has the advantage that no computational problems are to be expected.

If p denotes a vector of the markup rates by industries:

$$p = \begin{pmatrix} p_W \\ \hline p_R \\ \hline p_N \end{pmatrix}$$

equation (1) leads directly to M

(1) $$M = \hat{p} \,.\, T, \text{ where } p = \widehat{(T \,.\, i)}^{-1} \,.\, g$$

According to the underlying assumption the overall markup rate for a certain commodity is the weigthed average of markup rates of those industries that are trading with this commodity.

13. This procedure might lead to the result that the markup rates for one commodity differ widely over industries. Such procedure means, for instance, that sugar confectionary, which one can buy at petrol stations has the same markup than that of the average markup of the petrol stations. A result which will often not seem to be very plausible.

A second disadvantage lies in the fact that for data limitation within a certain industry normally no distinction between wholesale and retail sales can be made. (This applies only to those industries which engage in both wholesale and retail sales.) If as usual we assume that the wholesale markup rates are smaller than that of the retail sales this assumption of industry-specific margins necessarily leads to biased estimates. Of course such undesired results are the bigger the higher the ratio of non-characteristic trade turnover is. As mentioned above such ratios amount often from 5 to 12 % in the trade branches.

Commodity-specific margins

14. The alternative hypothesis is based on commodity-specific markup rates. If we define a vector c of these commodity-specific markup rates

$$c = \begin{pmatrix} c_W \\ \hline c_R \end{pmatrix}$$

where c_W stands for the commodity markup rates when the sales are of the wholesale type, and c_R when it is a retail sale; multiplying the turnover matrix with vector c leads to the vector of gross margins by industries

(2) $$T \cdot c = g$$

Solving equation (2) by c

(3) $$c = T^{-1} \cdot g$$

one achieves the vector of commodity-specific markup rates.

Matrix M could then be obtained by

(4) $$\begin{aligned} M &= T \cdot \hat{c} \\ &= T \cdot \widehat{T^{-1} \cdot g} \end{aligned}$$

Gross margins by industries are the result of the weigthed margins of the commodities that are traded.

15. In order to estimate M by means of equation (4), T needs to be a sqare matrix. Since in most cases the commodity dimension will be more detailed than the breakdown by industries a certain aggregation over commodities is necessary. If some industries in matrix T have a similar structure of commodities, difficulties in inverting T may arise. If equation (4) does not lead to "plausible" results this of course is an indication that the underlying assumption is unrealistic in these respects.

THE AUSTRIAN APPROACH OF ESTIMATION TRADE MARGIN MATRICES [1)]

16. The present version of the trade margin matrix for the Austrian 1976

1) The basic idea of the Austrian approach can be found in J.Richter /2/. For description of the margin matrix system see N.Rainer /3/.

IO-table was estimated in a stepwise procedure. It was assumed that both methodological assumptions are of importance, but more emphasis was laid on the hypothesis of commodity specific markup rates, although the attempt of solving the problem by equation (4) did not yield satisfactory results. Industry specific margins seemed to be more realistic in the non-trade industries, especially in the case of wholesale turnovers. Their trade turnovers are usually not very important relative to the production of goods and services. Non-trade industries are often trading because of broadening their commodity supply or because they don't produce some of the accessoires for their products.

For the bulk of trade turnovers the commodity-specific assumption seemed to be more appropriate. One can argue that the functioning of the market tends to support this hypothesis. Regulated prices of all types (which still are of certain importance in Austria) also fit well into the framework of identical markups, irrespective of the trading industries.

This commodity "philosophy" and the principle of using all available information built the background of the Austrian approach for the transformation of matrix T into matrix M.

17. The following iterative procedure was applied:

$$(5) \qquad T \cdot \hat{c}_k = M_k \qquad\qquad k = 1, \ldots\ldots, n$$

Vector c is again the vector of commodity-specific markup rates.

Whenever possible the elements of c were estimated on the basis of separate information outside the already mentioned data sets. In particular the following sources were used:

- regulated prices: when prices are fixed on each - or at least some - levels of the delivery chain, one can derive the corresponding markups very easy;

- individual items of price indices: comparing the prices of corresponding items of different prices indices (for example wholesale price - consumer price index) allows to compile markups, too;

- expert information (ministries, Chamber of Commerce, etc.).

For the remaining elements of vector c specific industry markup rates can be used as a proxy for the unknown commodity markup rate.

It is clear that these sources of information are normally of different quality. Therefore, the approach started from the best information available, i.e.. the margins of those commodities with by government regulated prices. In Austria (1976) this applies for some food products (like milk, sugar, bread, flour), the bulk of pharmaceutics, oil refinery products and some others.

All the "best quality" information given exogenously was used to build vector c_1 :

(6) $$T \cdot \hat{c}_1 = M_1$$

Matrix M_1 of course would still be very incomplete, because exogenous information of "best quality" will normally be available only for a fewer number of the commodities, but the following test had to be done:

(7) $$M_1 \cdot i \leq g$$

The remaining gross margins (g_1) by industries (in absolute terms and in percentages) were of high interest for checking the consistency of the basic data sets T and g with the exogenous given information:

(8) $$g_1 = g - M_1 \cdot i$$

(9) $$p_1 = \widehat{(T_1 \cdot i)}^{-1} \cdot g_1$$

T_1 is the remaining turnover matrix of the same dimension as T, but with zeros in the columns of these commodities, for which in c_1 the elements are $\neq 0$.

18. The next step, c_2, was set up using external information of second best quality. After testing the consistency of c_2 in analogy to equation (8) and (9) information of third class quality was used for vector c_3 and so on. Although there remained some commodities in those industries, for which the corresponding elements in vector c are still zero, negative values in p_k are again indications for an inadequacy of the commodity-specific assumption, the external information or even the turnover structure. In all these cases the underlying data have to be examined and - if necessary - adjustments be done.

19. In the Austrian case for one third of the commodities external information as mentioned above were found. (These commodities accounted for 40 % of the total trade turnovers.) For the rest of the commodities the industry markups had to serve as indicators for the magnitude of the commodity markup. For these steps the following principles were applied:

- only the - after the respective last iterative step - remaining industry markups were used
- the turnover ratio of the commodity in question related to the total remaining turnover of the industry had to be relatively high (at best 90 % or more), and
- the turnover ratio of the commodity in question in the industry related to the total turnover of that commodity also had to be very high.

In order to achieve the goal that both the market share of a commodity is very high in the industry under consideration and the product-mix concentrates in this specific commodity, an attempt was made to rearrange the trade branches in a kind of triangular order. So, proceeding along the triangular order meant that a proxy markup rate was first taken for those commodities, for which the abovementioned conditions were fullfilled best.
These proxy commodity markup rates were applied to all turn-

overs of the respective commodity, irrespective of the industries that were trading in this commodity. So, generally speaking the assumption of commodity specific markups was applied in these cases, too.

20. In a last step, after markup rates have been deducted for all commodities, no zero elements remain in vector c_n

(10) $$M_n = T . \hat{c}_n ,$$

but M_n still might not be consistent with the gross margins by industries:

(11) $$M_n . i \lesseqgtr g$$

Confrontation of $M_n . i$ and g provides valuable insight into the reliability of the underlying data and adequacy of background philosophy of commodity specific markups.

These differences were balanced on a prorata basis that left those margin values unchanged,for which information on markup rates of high quality could be assumed. Although this transformation procedure seems to be highly subject to the quality of external information and use of industry margins,in the Austrian case the remaining differences - especially in the trade branches - were moderate (in most cases about 2 to 8 %).

21. The balancing on a prorata basis can be interpreted as taking into consideration industry-specific margins. An illustrative example might be the general stores (768.2 of the Austrian industry classification) which contain supermarkets too. In this industry the calculated gross margin ($M_n . i$) was much higher than the survey margin. That implies that if the general stores had the some markups for the different commodities as the other industries a much greater gross margin would result. So the balancing of the difference on a prorata basis reduced all commodity margins proportionallv. The influence of the

industry margin is, therefore, to be seen in a downward grading of the commodity margins, leaving the "hierarchy" of the commodity margins unchanged. So,for example the relation of clothing margins to food margins is the same in the case of general stores as in all other branches.

Annex III shows an illustrative example of the resulting dispersion of commodity markup rates within one trade branch . Such different markups are the result of the application of the assumption of commodity specific markup rates.

CONCLUSIONS

22. The description of the Austrian method should not lead to the opinion that there did not arise some other specific problems. The degree of uncertainty remained quite remarkable. Sometimes it seemed that the external information were not enough representative and quite often commodities were not enough homogeneous in respect of markups. Besides, although based on a rather broad sample survey, the turnover matrix proved to be biased in some cases, which might be due to misunderstanding of the used commodity classification by the respondents. (Such a survey took place for the first time.) Finally, the problem of distinction between wholesale and retail sale turnover turned out to be of some importance in certain branches. There are branches of trade industries where this is no problem, but in others for which one can observe new channels of deliveries, this distinction becomes more and more obsolete.

23. Summing up, the method applied in Austria for the transformation of trade turnovers to margins by commodities can be characterized as follows:

- all industries are allowed to have wholesale and retail trade activities

- besides the gross margins by industries the data basis included a matrix of trade turnovers by industries, commodities and type of turnover

- the transformation to the margin matrix was primarily based on the assumption of commodity-specific markup rates.

It need not be mentioned that the compilation of margin outputs by commodities is only the first step of the trade margin data set for IO-tables and that good information and plausible methods for estimating the input trade margins are needed to make sure that the envisaged aims of IO-tables could be achieved.

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Annex I: System Table *)

		Goods & Services: Commodities	Goods & Services: Margins: wholesale	Goods & Services: Margins: retail sale	Goods & Services: Margins: transport	Goods & Services: Other: non-commodity	Goods & Services: Other: services produced for own use	Activities: wholesale trade branches	Activities: retail trade branches	Activities: other industries	Activities: producers of government services	Activities: producers of private non-profit services	Final demand: Private consumption	Final demand: Fixed capital formation	Final demand: Inventories	Final demand: Exports	Final demand: Government and private non-profit final consumpt.exp.	Total demand	Total output
Goods & Services	Commodities							U_{WC}	U_{RC}	U_{NC}	U_{GC}	U_{PC}	C_C	F_C	V_C	E_C		D_C	
Goods & Services: Margins	wholesale							U_{WW}	U_{RW}	U_{NW}	U_{GW}	U_{PW}	C_W	F_W	V_W	E_W		D_W	
Goods & Services: Margins	retail sale							U_{WR}	U_{RR}	U_{NR}	U_{GR}	U_{PR}	C_R	F_R	V_R	E_R		D_R	
Goods & Services: Margins	transport							U_{WT}	U_{RT}	U_{NT}	U_{GT}	U_{PT}	C_T	F_T	V_T	E_T		D_T	
Goods & Services: Other	non-commodity							U_{WN}	U_{RN}	U_{NN}	U_{GN}	U_{PN}	C_N					D_N	
Goods & Services: Other	services produced for own use																G_U	D_O	
Activities	wholesale trade branches	M_{WC}	M_{WW}	M_{WR}	M_{WT}														O_W
Activities	retail trade branches	M_{RC}	M_{RW}	M_{RR}	M_{RT}														O_R
Activities	other industries	M_{NC}	M_{NW}	M_{NR}	M_{NT}														O_N
Activities	producers of government services					M_{GN}	M_{GO}												O_G
Activities	producers of privat non-profit services					M_{PN}	M_{PO}												O_P
Primary input	Imports, incl. import duties	K_C			K_T														
Primary input	Value added							V_W	V_R	V_N	V_G	V_P							
Total supply		S_C	S_W	S_R	S_T	S_N	S_U												
Total input								I_W	I_R	I_N	I_G	I_P							

*) Note that this system table is simplified in some ways, thus concentrating on the margin matrices. So, for example, tax matrices are not distinguished as well as import matrices on the use side.

Annex II: Austrian classification of the wholesale and retail trade branches
(The data on turnover and gross margin refer to 1976, mio AS)

		Wholesale turnover	Retail sale turnover	Gross margin
711	Wholesale with agricultural products and cattle	55.438	1.183	5.167
712	Wholesale with textiles	3.558	174	896
713	Wholesale with skins, hides and leather	1.317	53	308
714	Wholesale with wood, construction material and glass	19.221	1.038	4.180
715	Wholesale with iron and non-ferrous metals	12.722	136	1.787
716	Wholesale with fuel and oil refinery products	52.415	3.349	5.924
719	Wholesale with other raw material	8.679	32	1.294
721	Wholesale with food	43.741	1.796	5.705
722	Wholesale with wine and spirituous liquors	3.064	223	1.259
723	Wholesale with tobacco	983	384	75
724	Wholesale with clothing, embroideries and bed-linen	2.896	383	906
725	Wholesale with shoes and leather goods	1.520	29	288
726	Wholesale with medicaments, cosmetics, lotions and detergents	8.740	151	2.245
731	Wholesale with farming machines	4.344	22	1.064
732	Wholesale with electrical appliances	16.322	347	4.505
733	Wholesale with vehicles	24.303	1.406	5.134
734	Wholesale with other machines	24.056	372	7.553
735	Wholesale with furniture and home textiles	4.351	131	1.402
736	Wholesale with metal products, kitchen utensils, glass and china products	16.127	1.327	4.003
737	Wholesale with paper, paper products and office materials	6.180	120	1.345
739	Wholesale with other goods and products	11.537	308	3.142
$\sum$	Wholesale trade branches	321.514	12.964	58.182

741	Retail sale with food	865	51.291	11.446
742	Retail sale with tobacco	6	10.383	1.670
743	Retail sale with textiles and clothing	352	20.044	7.231
744	Retail sale with shoes	98	5.345	1.943
745	Retail sale with leather and leather products	6	784	284
746	Retail sale with medicaments	51	5.490	2.089
747	Retail sale with cosmetics, lotions and other chemicals	116	3.955	1.381
751	Retail sale with furniture and home textiles	109	10.198	3.674
752	Retail sale with metal products, kitchen utensils, glass and china products	269	3.984	1.349
753	Retail sale with rubber products and plastic products	4	478	207
754	Retail sale with vehicles	795	18.981	3.890
755	Retail sale with sewing-machines and office machines	75	1.376	588
756	Retail sale with optical goods and precision tools	36	2.404	960
757	Retail sale with electrical appliances	75	6.605	1.918
761	Retail sale with paper products and office materials	81	1.706	563
762	Retail sale with books and journals	152	3.615	1.552
763	Retail sale with watches and jewelleries	13	2.612	1.082
764	Retail sale with toys, sporting goods and musical instruments	28	3.084	1.062
765	Retail sale with fuels	411	3.821	709
766	Retail sale with oil refinery products	70	10.258	1.096
767	Retail sale with flowers	3	687	324
768.1	Department stores and mail-order houses	2	9.252	3.197
768.2	General stores	91	9.617	1.917
769	Retail sale with other goods and products	134	3.176	1.180
$\sum$	Retail sale branches	3.842	189.146	51.312
$\sum\sum$	Trade branches	325.356	202.110	109.494

Annex III: An illustrative example of the dispersion of the resulting commodity mark-ups whithin one trade branch (mio AS): 721: Wholesale with food.

	Commodity	Wholesale turnover	Wholesale margin	Mark-up in %
010	Agricultural products	4.411	438	9.9
072	Office materials	41	11	26.8
240	Salt	231	30	13.0
311	Meat	2.912	492	16.9
312	Fruit and vegetables products	1.875	408	21.8
313	Flour	746	34	4.6
314	Bakeries and confectioneries	1.137	240	21.1
315	Milk and milk products	12.069	784	6.5
316	Sugar	803	36	4.5
317	Chocolate and chocalate products	1.926	271	14.1
319	Other food [1)]	15.521	2.089	13.5
321	Beer	107	28	26.0
322	Wine and spirituous liquors	624	160	25.6
323	Non-alkoholic drinks	417	115	27.6
372	Wood products	26	6	23.1
412	Paper and paper products	189	54	28.6
448	Plastic products	4	1	25.0
454	Cosmetics, lotions and detergents	489	98	20.0
459	Other chemicals	14	2	14.3
476	China products	9	2	22.2
480	Glass products	3	1	33.0
531	Tools, cutleries and other cutting tools	18	4	22.2
533	Sheet metal products	32	7	21.9
541	Farming machines	137	33	24.1
$\sum$	Wholesale activities of 721	43.741	5.344	12.2

Commodity		Retail sale turnover	Retail margin	Mark-up in %
010	Agricultural products	160	47	29.4
240	Salt	42	8	19.0
311	Meat	256	62	24.2
312	Fruit and vegetables products	60	14	23.3
313	Flour	101	9	8.9
314	Bakeries and confectioneries	114	18	15.8
315	Milk and milk products	118	13	11.0
316	Sugar	84	4	4.8
317	Chocolate and chocolate products	85	22	25.9
319	Other food 1)	511	99	19.4
321	Beer	60	14	23.3
322	Wine and spirituous liquors	84	20	23.8
323	Non-alcoholic drinks	105	25	23.8
454	Cosmetics, lotions and detergents	16	5	31.2
$\sum$	Retail sale activities of 721	1.796	360	20.0
$\sum\sum$	Trade activities of 721	45.537	5.705	12.5

1) Like farinaceous products, coffee, tea, margarine, spice, deep-freeze food.

AN INPUT–OUTPUT MODEL FOR THE USSR ECONOMY BASED ON THE 1972 SURVEY

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1. INTRODUCTION

The goal of this study, which is presently being carried out at IIASA, is to provide data for the USSR input-output model. The main sources are the annual statistical yearbooks of the USSR, Narodnoe Khozyaistvo (Narkhoz), which contain data compiled by the Central Statistical Board of the USSR Council of Ministries (CSB). Data for the basic year and time series for outputs, primary inputs, and final demand components are needed in order to develop the econometric model for long-range forecasting.

We have chosen 1972 as the base year and estimated, as far as possible, the values and parameters of a disaggregated 85-industry model.

The outline of the paper is as follows. Section 2 describes the available data with some references to the methodology adopted in the USSR statistics. Section 3 contains an explanation of the approach used to estimate industry gross output values. Section 4 deals with the estimation of final demands. Section 5 briefly describes the primary inputs estimated on the basis of time-series and input-output data and reports preliminary results obtained using these data in a model of interindustry interactions. Section 6 discusses the level of aggregation and a preliminary analysis of the reconstructed table, while Section 7 draws some general conclusions and points the way for future work.

2. DATA AND METHODOLOGY

The reported input-output tables of the USSR economy are used by the statistical agencies and planning offices to study interindustry relationships as part of the process of socialist production. The first input-output table was compiled and published for 83 branches of the production sphere for 1959. During the last 20 years, similar surveys have been carried out for 1966, 1972, and 1977. Together with the distribution of production, these tables cover primary inputs (labor force and fixed capital assets). The number of sectors has been changed slightly and the methodology has been improved. There have been many papers discussing the first and second USSR input-output tables but, due to rapid structural changes within the USSR economy, they are already somewhat out-of-date.

A few words should be said about the main features of the statistical methodology. Economic activities are divided into two clusters - "productive" and "nonproductive" spheres. The first kind of economic activity creates value (as proved by Marx) while the second only uses this value for redistribution between economic agents. Therefore, such branches as public (passenger) transportation, health care, education, the sciences, and management are in the nonproductive sphere. These activities are not therefore shown on the supply side of the input-output table.

The results of the input-output survey are deliberately consistent with the system of national accounting adopted in the USSR. Each survey is the result of comprehensive investigations carried out by the CSB:

> "In order to obtain data necessary for reporting input-output tables the Central Statistical Board of the USSR conducts periodically (once in 5 years) special surveys of industrial, construction and agricultural enterprises

and organizations on production inputs (with detailed nomenclature), capital funds and other indexes. In so doing the survey covers industrial and constructional enterprises and organizations which produce more than 90 per cent of all production. For other industries (some branches of food and light industries, agriculture, transportation, trade, some nonmanufacturing industries) the survey is sampling. Survey questionnaires for industrial enterprises contain production input data both for the whole output of the enterprise and for the principal product of the industry ("net" industry). The latter data are the basis for the tabulation of reporting input-output tables by "net" industries. In the process of reporting input-output tables compilation there are widely used, along with single-time survey data, other statistical materials existing in the statistical bodies as a result of current and annual reporting tabulations, specifically the materials of annual statistical input-output tables compiled by Central Statistical Board of the USSR." (Eidelman, 1982).

The 1972 input-output table was prepared for 112 sectors ("net" industries), of which 97 represent industrial activities (mining and manufacturing industries), five construction sectors, two agricultural sectors, forestry, railway transportation, other kinds of transportation, communications, three trade service sectors, and one sector for other productive activities.

As is well known, input-output tables in value terms can be constructed in terms of two sets of prices, either "consumers'" or "producers'" prices, and there are arguments in favor of each. The majority of input-output tables in western countries are in "consumers'" prices. There are some doubts about the reliability of the data quoted in "producers'" prices when they have actually been compiled from primary inputs in consumers' prices.

According to Eidelman (1964):

"In the current conditions of forming economic relationships, input-output tables should be constructed in the first place in purchasers' prices. This is necessary, not only as a result of practical considerations (largely, conformity with the existing system of planning production expenses and turnover) but also because it is essential to have a detailed picture of the relationships and proportions of the national economy in the prices that are really operative, that is in those prices at which material goods are realised and consumed."

The definition of output used in USSR statistical methodology is that of "gross turnover", i.e. intraindustry flows are included in gross output value. Therefore, an intraindustry (diagonal) element is higher when the ith industry is more aggregated, i.e., there is a correlation within a set of homogenous industries between the values of these coefficients and aggregation levels. For example, we find that for the aggregated industry "Ferrous and Nonferrous Metals" this technical coefficient is about 0.40.

The results of surveys for the 1972 input-output table were published in three consecutive yearbooks (Narkhoz, 1973-1975). The first contains intermediate flows for 85 industries and a few technical coefficients corresponding to the distribution of ferrous metals, coal, electricity, and basic chemicals. The values of selected inverse matrix coefficients for the same industries are also given, plus data on the relationship between aggregated industries such as fuels, machinery, metals, electricity, and agriculture, and highly aggregated measures of the structure of the gross national product.

The second yearbook (Narkhoz, 1974) covers data on 26 types of fixed capital assets, which generally correspond to the classifications "machine-building industry" and "construction" in the input-output tables. However, these values are shown for 84 industries. In contrast to the first yearbook, agriculture is divided into two branches and forestry is shown separately from other industries; but the item "other manufacturing industries" does appear in the second yearbook, although it is not shown in the intermediate flows matrix.

The third yearbook (Narkhoz, 1975) gives the labor force inputs implied in intermediate flows for 85 industries, i.e., it corresponds to the data published in the first yearbook.

However, these data are still insufficient to enable us to construct a satisfactory input-output model. First of all the values of gross industry outputs are not published, so that the technical coefficients a_{ij} [1/] cannot be found. Almost no data exist for the composition of final demand, with the exception of the distribution of aggregated industries' outputs, and labor inputs by sector are also not shown.

Nevertheless, these sources provide some background for the development of an input-output model when the information they contain is supplemented by statistical data currently available. Time-series on most aspects of personal consumption expenditures exist on an annual basis, while trade turnover data in current prices and the corresponding price indices are also published. Gross fixed capital formation values are available, at least for the aggregated industries. The values of exports and imports in the commodities classification are also available.

These data sources will be referred to as "survey" and "current statistics" respectively. Titles of industries will only be used when clarifying a procedure; otherwise, the numbers corresponding to the order in which the industries appear in published surveys are used. (The industries and the corresponding numbers are listed in Appendix Table A1.)

The first problem is to estimate the values of all industries' gross outputs. The original calculations made by CSB covered 112 industries. Therefore, it is obvious that at least some of the industries listed in the Appendix have been aggregated and that intermediate flows corresponding to others, for example, "other manufacturing industries", have possibly been omitted. The explanation is that the "construction" industry was surveyed for five distinct types of construction and then transportation and communications were separated, so that there are slight discrepancies between the original and the published survey data. We know the titles of only 79 manufacturing and mining industries, but we do not know how many industries from the original 97 have been aggregated into "other manufacturing industries". However,

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1/ $a_{ij} = x_{ij}/x_j$, where x_{ij} is the intermediate delivery of the ith industry to the jth industry and x_j is the gross output of the jth industry.

there are a few indications as to which "industries" presented in the yearbook are in fact the result of aggregation. Some titles show us the level of aggregation and frequently technical coefficients are given for identifiable subindustries (like electricity requirements for production of ferrous ore, synthetic fiber, etc.). It is also easy to distinguish aggregate industries by dividing the corresponding rows of intermediate flows in value terms (x_{ij}) by the same flows expressed in labor units (L_{ij}). If an "industry" is an aggregate of smaller sub-industries, then this ratio ($t_{ij} = x_{ij}/L^{\ell}_{ij}$) will change across columns. This is the case for industries 1, 20, 29, 34, 46, 68, and 78. The titles of some of the industries permit us to distinguish the corresponding original industries noted in the survey, for example: 1 - ferrous metals, nonferrous metals, ferrous ores, nonferrous ores, etc.; 20 - boring equipment, metallurgical equipment, oil and gas equipment, mining equipment, etc. This means that 15 industries (mining and manufacturing) have been combined into seven.

Assuming that industry 1 corresponds to five originally surveyed industries and that industry 20 corresponds to a further seven such industries, then the total number of defined industries is 79 + 11 = 90. In chemicals there should be at least one in "other chemicals", namely synthetic fiber, and there might be more (tires and chemical/pharmaceutical) that are shown in current data as separate industries. In transportation equipment (29) there should be at least two to three, because current data are divided into two groups -- railways and others -- and the same is probably true for other metal goods (34). In other words, it is clear that all industries do appear somewhere in the published data.

It is important to distinguish aggregate industries from the rest because the main sources of gross output values in our study are inverse Leontief coefficient values. These have been found for the disaggregated versions and therefore will not correspond exactly to the values that are obtained for the developed aggregated versions.

3. ESTIMATION OF INDUSTRIES' GROSS OUTPUT VALUES

As mentioned above, data on values of industries' gross outputs in an input-output framework (in consumers' prices) are also not published in current statistics. Only very aggregated indicators of the structure of the gross social product corresponding to the branches of the USSR economy are given. From this can be derived the figures for construction, transportation and communication, and agriculture (e.g. for 1972, 77.9, 29.5, and 114.4 x 10^9 roubles respectively).

Since the 1972 input-output table is given in purchasers' prices, the values of the transportation, communications, and trade sectors can also be calculated as sums of the corresponding intermediate flows or from the value of coal deliveries by using the technical coefficient given in Narkhoz (1973).

Note that interindustry flows related to "other manufacturing industries" are not shown in the first yearbook. We assume therefore that there may be a gap in the statistics (see Table 1). Thus, the value of transportation and communications services delivered to "other manufacturing industries" is rather large - 0.8 x 10^9 roubles - and its output will be correspondingly large. It seems reasonable to assume that the output for the

Table 1. Alternative estimates of the value of the gross output of the transportation and communications sector.

Source	Value (10^9 roubles)
Gross social product	29.50
Sum of intermediate flows	28.70
Technical coefficient*	29.04

* This value is biased because the coefficient is given for transportation only, whereas the flow (coal to transportation and communication) is for both branches. Assuming that the major part of coal is consumed by transportation and that the technical coefficient for "coal to communications" is rather small, one would find that the estimates were more consistent.

other manufacturing industries, which accounts for part of the fixed productive assets, is quite large (36.7×10^9 roubles out of the manufacturing and mining industry total of 65.7×10^9 roubles, or 14%) and can consume 0.8×10^9 roubles of transportation and communications services. Sverdlick (1980) has assumed that these other manufacturing industries should be associated with heavy (basic) industries (see Table 2). This also supports our conclusion that all industries are represented in the published data.

The gross output values for 42 industries can be, and in fact have been, calculated very simply by dividing intermediate flows by the corresponding technical coefficients. As mentioned above, a few input-output coefficients have been published for four products - ferrous metals, coal, electricity, and basic chemicals.

The gross output figures of various other industries have been calculated as sums of intermediate flows because these products are only (or mainly) used as inputs to other productive sectors of the USSR economy. For a few of these industries, we increased the values of x_{ij} by between 1% and 5% according to the available data on personal consumption and production in physical terms[1/]. The gross output values for 15 other industries have also been calculated first as sums of intermediate flows then increased in proportion to the percentage difference between exports and imports in physical terms to correspond to the observed physical output.

After following these various steps we found that the outputs of only 26 industries were still unknown. Therefore, in the absence of direct information, we decided to apply regressions to estimate these missing values. The procedure consists of two stages: first we found the values of the inverse matrix coefficients corresponding to electricity flows; and second we found the gross outputs for each remaining industry.

1/ This means, however, that the consumer's prices are equal along the row.

Table 2. Transportation and communications flows associated with aggregated industries (10^9 roubles).

Source	Heavy	Light	Food	Agric.	Constr.	Trans.	Trade	Others	Total
Sverdlick	24.30	0.90	2.30	1.40	0.10	-	0.2	0.3	29.5
Input-output table	23.55	0.87	2.33	1.36	0.09	0.03	0.2	0.3	28.7

Let us consider the basic input-output relationship:

$$\sum_j a_{ij}x_j + y_i = x_i \qquad i = 1,2,\ldots,85$$

where x_j = gross output value for jth industry,

y_i = final demand value for ith industry,

a_{ij} = technical coefficients ($a_{ij} = x_{ij}/x_j$),

x_{ij} = intermediate flow from ith to jth industry.

This can also be written as:

$$\sum_j b_{ij}y_j = x_i$$

where b_{ij} = element of the inverse matrix $(E - A)^{-1}$ corresponding to the ith supplying industry and the jth consumer.

Let us divide sector b_{11j} (index 11 corresponds to electricity on the supply side) into two parts $(\hat{b}, \beta)$, where $\hat{b}_{11k}$ are the published values and $\beta_{11\ell}$ are unknown values. Multiplying this vector by the columns a_{ij}, where j corresponds to known industry gross output values or estimates we have made ourselves, we obtain a set of 59 equations:

$$\sum_k \hat{b}_{11k} * a_{kj} + \sum_\ell \beta_{11\ell} * a_{\ell j} = \beta_{11j}$$

and if j = 11 then

$$\sum_k b_{11k} * a_{k11} + \sum_\ell \beta_{11\ell} * a_{\ell 11} = \beta_{11.11} - 1$$

This can be rewritten as:

$$(1-a_{ji})\ \beta_{11} = (\sum_k \hat{b}_{11k} a_{kj} + \sum_{\ell \neq j} \beta_{11\ell} a_{\ell j}).$$

The set of k indices contains 40 members. This means that we need to find 45 unknowns $\beta_{11\ell}$ from 59 equations. This is quite possible as many lower and upper boundaries can be imposed on these values, i.e.

$$\underline{\beta}_{11\ell} \leq \beta_{11\ell} \leq \overline{\beta}_{11\ell}$$

These boundaries can be calculated in the following way. As the lower boundary one can take $\Sigma_k \hat{b}_{11k} * a_{k\ell}$; and the upper limits on $\beta_{11\ell}$ do not exceed the previous sum plus the term $\max_{k,k\neq 11}\{\hat{b}_{11k}\} \cdot \Sigma_\ell a_{\ell j}$, where specific values of max {} can be chosen for particular industries[1/].

1/ Some numerical examples show that such boundaries are rather close. The sum of small technical coefficients (less than 1% each) corresponding to the unknown $\beta_{11\ell}$ is denoted $\Sigma a_{\ell j}$.

<u>Example: industry 43</u>

$$.115 = \Sigma \hat{b}_{11.k} * a_{k43} + \sum_{\ell \neq 43} \beta_{11\ell} * a_{\ell 43} \leq 0.115 + 0.055 * \sum_\ell a_{\ell 43}$$

where 0.055 is $\max_k\{\hat{b}_{11,k}\}$ chosen for the 43rd industry

$$.123 \leq b_{11.43} \leq 0.134$$

<u>Example: industry 38</u>

$$0.073 = \Sigma \hat{b}_{11k} * a_{k38} + \sum_{\ell \neq 38} \beta_{11\ell} * a_{\ell 38} \leq 0.073 + 0.07 * \Sigma a_{\ell 38}$$

$$.0738 \leq b_{11.38} \leq 0.080.$$

This looks very simple, but some problems need first to be solved. The first relates to the value of $b_{11.1}$ for the aggregated industry "ores, ferrous and nonferrous metals". It is obvious that the values of x_{1j} corresponding to the aggregated industry "ferrous and nonferrous metal products and ores" are larger than the calculated values $a_{1j} * x_j$ if the a_{1j} correspond only to the ferrous metals industry. As far as we know, the intermediate flows x(1,j) and the major part of the industry itself correspond to ferrous metals through given technical coefficients; therefore two input coefficients related to this industry are given ($a_{1.1,j}$ and $a_{1.2,j}$). In order to find out the corresponding inverse coefficients $\beta_{11,1.1}$ and $\beta_{11,1.2}$ we use the regression for 17 machine-building industries (indices 12-23, 25-28, 30-35). For many of them, only a few significant coefficients fall in the column, e.g., transportation, trade, and electricity. Upper and lower boundaries for the corresponding coefficients are 0.03-0.04, 0.01-0.02, 1.02-1.04.

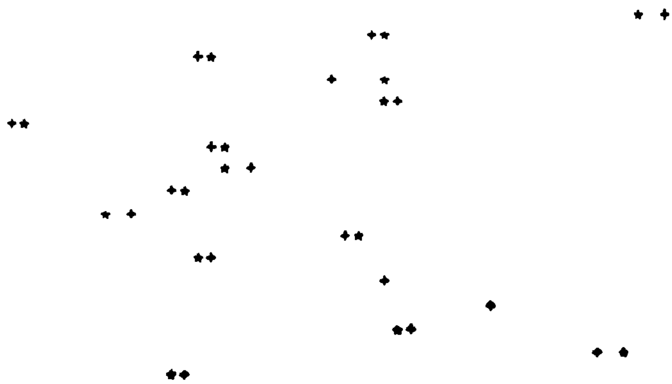
Table 3. Alternative estimates of inverse electricity coefficients for the ferrous and nonferrous metals industry.

Variables expressed in a_{ij} terms	Direction estimation		Estimation under constraints on $b_{11.11}$, $b_{11.81}$, $b_{11.82}$	
	Estimate	t-statistic	Estimate	t-statistic
Intercept	0.0058	2.5		
Residual	0.0287	1.6	0.0770	5.3
Ferrous metals	0.0646	17.4	0.0660	17.5
Nonferrous metals	0.1147	3.3	0.1770	4.5
Electricity	1.0414	10.7	1.03 (assumed)	
Transportation and communications	0.0048	0.2	0.0375 (assumed)	
Trade	0.0303	0.8	0.0160 (assumed)	
Sum of squared residuals	0.000021		0.000051	

Comparison of the regression estimates for $\hat{\beta}_{11,1.1}$ and $\hat{\beta}_{11,1.2}$ show that the values 0.066 and 0.177 are acceptable. All small inputs, corresponding to the unknown $\beta_{11\ell}$ values, have been summed so that an average parameter for other insignificant inputs has been estimated.

The following graph shows the goodness of fit for these 17 engineering industries.

actual	fitted
0.6340e-01	0.6542e-01
0.4670e-01	0.4584e-01
0.3490e-01	0.3349e-01
0.4590e-01	0.4285e-01
0.4620e-01	0.4733e-01
0.2190e-01	0.2039e-01
0.3510e-01	0.3494e-01
0.3510e-01	0.3770e-01
0.3260e-01	0.3142e-01
0.2720e-01	0.2951e-01
0.4480e-01	0.4370e-01
0.3330e-01	0.3477e-01
0.4610e-01	0.4593e-01
0.5340e-01	0.5321e-01
0.4690e-01	0.4856e-01
0.6310e-01	0.6068e-01
0.3140e-01	0.3227e-01



The next step is to divide the set of $\beta_{11\ell}$ into two subsets, the first consisting of those ℓ values that correspond to known industry gross values and the second to the unknown values. This is very important to the calculations because in the equation

$$\sum_k \hat{b}_{11k} a_{kj} + \sum_\ell \beta_{11\ell} a_{\ell j} = \beta_{11j}$$

the unknown parameter is on both sides, i.e., it is multiplied by $(1-a_{jj})$ - the largest absolute value in the vector of explanatory variables.

The second subset - the unknown coefficients - has been divided into a number of clusters by the origin of each entry:

Group 1: 20, 29, 34: unknown aggregated machine-building industries;
Group 2: 45, 46: chemical products, also aggregated;
Group 3: 47-50: wood products;
Group 4: 61-68: clothing, footwear, textiles, etc.;
Group 5: 69-73, 76, 78: food processing.

The value of $b_{11,8}$ (8 = gas industry) was found, by calculation of $\Sigma b_{11j} * a_{11j}$, to be equal to 0.035 (this value was also obtained by a regression procedure).

We then separated out industry 47 because its electricity requirements are rather different from those of other industries included in this cluster.

However, it is impossible to distinguish the values $\beta_{11,20}$, $\beta_{11,29}$, and $\beta_{11,34}$ from the aggregated estimate, which is larger than any of the corresponding values for the machine-building industries. Therefore, we calculated gross outputs for these three industries by a regression procedure in which all significant aggregated inputs were explanatory variables: fixed capital assets, intermediate flows of ferrous and nonferrous metals (x_{ij}), electricity, chemical products, transportation and communications cost, trade, and internal consumption x_{ii} (the sample covers all machine-building industries). In other words:

$$x(j) = \gamma_\ell + \gamma_1 * x_{jj} + \gamma_2 * K_j + \gamma_3 * x_{1,j}$$

$$+ \gamma_4 * x_{11,j} + \gamma_5 * x_{84,j} + \varepsilon_j$$

where K_j = fixed capital assets.

Then the number of regressions was increased to 62 industries and we estimated some other $\beta_{11,j}$ values at a high level of significance.

Applying these significant estimates and the known $\hat{b}_{11,j}$ values, we can find the output values for 65 industries. Unfortunately, it was impossible to estimate by regression the values of $\beta_{11\ell}$ for some light and food-processing industries (61-78). Only the lower boundaries on the values of their gross outputs can be calculated:

$$\hat{b}_{11k}x_{kj} + \Sigma\hat{\beta}_{11\ell}x_{\ell j} = \underline{x}_j$$

where $\hat{\beta}_{11\ell}$ are estimates of the $\beta_{11\ell}$ values.

However, these boundaries are of very little value because intraindustry flows are very high. But there are also upper boundaries calculated as sums of intermediate flows and consumer expenditures, represented by the retail trade statistics. There are few figures available to estimate exports and imports and these calculations are discussed further below. Here we can only say that all X(i) have been found so that the inverse matrix coefficients $\hat{b}_{11j}$, $\hat{b}_{5j}$, and $\hat{b}_{39j}$ given in the yearbook are very much closer to the estimated ones, i.e., to *ex post* results, when we invert the 85 x 85 matrix a_{ij}.

4. FINAL DEMAND

There are three sources of data for consumption, namely investments, exports, and imports. Other elements of final demand can be estimated as the residuals. These data are: time-series of trade turnover by commodity classification (see Appendix Tables A1 and A2), investments in aggregated industries, and exports and imports by commodity classification. The figures on the structure of national income (savings and consumption) are also helpful.

4.1. Personal and Social Consumption

When dealing with such disaggregated input-output models it is possible to distribute commodities by industries, i.e., to find a so-called "bridge" matrix, but sometimes we simply assume that the corresponding industries have equal shares in this commodity group. This procedure is helpful, and seems to introduce no serious bias, in obtaining the most important values for industries 61-78.

Obviously, there cannot be exact comparability with the aggregated figures on national income, because such elements as electrical consumption, turnover of second-hand shops, amortization of houses, etc., are not available. Nevertheless, the calculated sum is reasonably close to the data presented in the balance of national income.

The figures published by Sverdlick (1980) (also corresponding to the yearbook) throw some light on the structure of consumption for manufactured goods and unprocessed agricultural products. Sverdlick's results, together with the actual structure used here, are shown in Table 4.

Table 4. The structure of consumption in the USSR. All values are in billions (10^9) of roubles.

Category	After Sverdlick (1980)			This study
	Personal	Social	Total	Total
Heavy industry	26.2	13.8	40.0	33.6
Light industry	44.5	1.8	46.3	43.1
Food	88.3	3.2	91.5	90.2
Others	2.9	0.2	3.1	3.1
(Total manufactured)	(161.9)	(19.0)	(180.9)	(170.0)
Agricultural products (unprocessed)	28.4	0.7	29.1	29.1
Amortization	7.8	7.6	15.4	-
Total	198.1	27.3	225.4	-

If we exclude the figure of 15.4×10^9 roubles for amortization, then Sverdlick's results lead to an aggregated consumption value of 210×10^9 roubles. Our calculations, however, give a somewhat different value. The total turnover of manufactured goods is calculated as 170.0×10^9 roubles; adding to this 29.1×10^9 roubles for unprocessed agricultural products we reach a new total of only 200×10^9 roubles, rather than Sverdlick's 210×10^9. The contributions for electricity, water, and gas that are not included in our turnover figures, together with the slightly different structure adopted here, may well contribute to the discrepancy between the two figures.

4.2 Investment

Data on gross fixed capital formation are given in the Yearbooks; for 1972 there was a total of 94.3 x 10^9 roubles broken down as follows:

Mining and manufacturing industries	33.09
Agriculture	18.10
Transportation and communications	9.62
Construction	3.60
Housing	14.63
Trade, nonmaterial sphere	15.22

There are also data available on the distribution of investment between manufacturing industries; one minor but significant point is that investment in collective farms is excluded from these latter data so that the two sets are unfortunately incompatible. However, the total for mining and manufacturing industries for 1972 was 32.4 x 10^9 roubles, disaggregated as follows:

Electricity	3.42
Coal	1.71
Oil, mining and refinery	3.00
Gas	1.23
Ferrous metals	2.37
Chemicals	2.76
Machine building	7.00
Wood products	1.20
Cellulose and paper	0.44
Basic construction materials	1.98
Textiles, footwear, leather	1.48
Food processing	2.50
Other manufacturing industries	3.30

Wherever data on the structure of capital stock are available, one can distribute the aggregated values between the corresponding industries in proportion to the capital stock within each aggregated cluster. This is also the simplest way of finding the share of equipment, assuming that it is close to this share in the capital stock. There are also helpful data available on the share of equipment for the mining and manufacturing sector in total and for ten aggregate industries, as well as for agriculture and for transportation. The differences in the shares of equipment in investment across industries are very large, so that the average (and very stable) overall ratio is not a good indicator of structural changes in the USSR economy.

4.3 Exports and Imports

The foreign trade turnover of the USSR totalled 26.044×10^9 roubles in 1972; exports accounted for 12.735 and imports for 13.309×10^9 roubles. Some published data cover the composition of both items. The structure of USSR foreign trade over the period 1965-1973 is shown in Table 5. The information in Table 5 is based on so-called foreign trade prices rather than domestic prices. However, by assuming exchange rates between domestic and foreign trade prices of 1.4:1 for exported goods and 2.2:1 for imported goods, Sverdlick estimated the structure in domestic prices as 17.5×10^9 roubles for exports and 29.6×10^9 roubles for imports. He also estimated the sectoral breakdown of these values between branches as follows:

	Exports	Imports
Heavy manufacturing	15.0	15.2
Textiles, footwear	1.0	7.5
Food processing	0.9	3.9
Agriculture	0.7	3.0

These calculations have helped us to approximately reestimate the current statistics on foreign trade in domestic prices and to use them in the input-output framework.

Table 5. Structure of USSR foreign trade, 1965-1973 (percentages of total).

Commodity groups	Exports			Imports		
	1965	1970	1973	1965	1970	1973
Machines, equipment, transportation tools	20.0	21.5	21.8	34.0	35.5	34.3
Fuels and electricity	17.2	15.6	19.2	2.5	2.0	3.4
Ores, metals	21.6	19.6	17.1	8.8	9.6	9.6
Chemicals, fertilizers	2.9	3.5	3.0	6.2	5.7	4.3
Wood, pulp and paper	7.3	6.5	6.4	1.9	2.1	1.6
Textiles, raw materials, and semimanufactured goods	5.1	3.4	3.3	4.4	4.8	3.7
Furs	0.7	0.4	0.3			
Food products and raw materials for food products	8.4	8.4	5.6	21.3	15.8	20.2
Manufactured goods for personal consumption	2.4	2.7	3.0	14.1	18.3	15.9
Others	14.4	18.4	20.3	6.8	6.2	7.0
Total	100.0	100.0	100.0	100.0	100.0	100.0

5. PRIMARY INPUTS

To forecast investment demand by sector we need time-series for fixed productive capital assets and labor force consistent with the input-output methodology for the base year.

The CSB has published data on capital inputs for all 85 industries, distinguishing all types of equipment (types of equipment correspond to the classification of machine-building industries in the input-output table) and also building stocks. These data are also broadly consistent with the current statistical data, at least for the aggregated industries. Thus, the time series for capital inputs can be used for estimation.

Intermediate flows expressed in labor units (L_{ij}) have also been published for 84 industries. In cases where we have estimated the value of gross industry outputs, we can also find the labor input by applying the ratio x_{ij}/L_{ij} for homogenous industries. For heterogenous (aggregated) sectors, we have used weighted average ratios. For the mining and manufacturing industries at least, the estimates for 1972 are rather close to the current statistical data. For example, the real industry "electricity" employed 655,000 compared to the input-output estimate of 600,000, and in the case of the "basic building materials" industry the figures are 2,070,000 and 2,105,000, respectively.

However, this approach cannot be applied to "construction" because both x_{79j} and L_{79j} are equal to zero. The only source of data here is the current statistics. The "transportation and communications" sector raised the problem of comparability because in practise the activities of this sector are only partly associated with the material productive sphere, and passenger transportation is not represented in the input-output framework. Therefore, we have used data from the current statistics.

The estimation of the labor force in "agriculture" is also a complicated issue. First of all there are no direct data from which to develop time-series. The demand side of the USSR labor force balance (CMEA Statistics, 1980) gives only percentage figures for 1972, which are as follows:

I.	Material productive sphere	79.1
	I.1 Mining, manufacturing, and construction	38.0
	I.2 Agriculture and forestry	24.3
	I.3 Transportation and communications	8.4
	I.4 Trade and others	8.4
II.	Nonproductive spheres	20.9

Approximate measurement of the labor force employed in "mining, manufacturing and construction"[1/] enables us to estimate the agricultural labor force at a minimum of 27.1 million. This is consistent with data published on the estimation of the temporary labor force used in agriculture.

These calculations also provide a basis for testing the accuracy of estimation of industry output values. The data on fixed productive capital assets were also very helpful in distributing investment properly between aggregated industries and allocating it between equipment and other uses.

6. AGGREGATION AND PRELIMINARY ANALYSIS OF THE RECONSTRUCTED TABLE

The model developed for 84 industries (where agriculture is treated as one sector) can already be used in simulations and forecasting, but a complete analysis of its properties has not yet been performed. We have quite small coefficients in each column so that any predictions should be supported by engineering studies. For example, for such disaggregated industries as 2, 3, and 4, only five to seven coefficients exceed 0.01 and approximately 90% of the others are each less than 0.003. The machine-building industries should certainly be aggregated because it is not possible to predict their share in the investment related to aggregated industries; once again, the columns contain only six to ten coefficients larger than 0.01. However, the interactions within chemicals, basic building materials, and wood and paper products appear to be reasonably disaggregated. It is important that we can now directly connect the structure of personal consumption expenditure with that of the food processing, textiles, and footwear industries.

The aggregated version can be expressed in the same format as the model of interindustry interactions, for which we already have time-series of the most significant coefficients. The

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1/ This total does not include members of collective farms working in these nonagricultural sectors.

level of aggregation on the demand side can be related to the available data on rates of growth of industries (the CSB publishes time-series for 24 mining and manufacturing industries). The a_{ij} matrix for 19 industries is given in Table 7. The aggregation scheme used was as follows:

Primary industries	Aggregated industries
1-4	1
5	2
6,7	3
8	4
9,10	5
11	6
12-35,37	7
36	8
38-46	9
47-52	10
53-59	11
60	12
61-68	13
69-78	14
79	15
80,84,85	16
81	17
82	18
83	19

These results are consistent with Sverdlick's very aggregated version shown in Table 8.

In the aggregation of annual data we can rely on the available time-series data on gross industry outputs, fixed productive capital assets, and manpower. For example, annual rates of growth are published in the CMEA Statistics (1980) for 20 industries in machine-building, 11 in chemistry, 10 in light industries, and 12 in food processing. For many industries there is a strict relation between rates of output growth in physical units and in

Table 7. Technical coefficients for the aggregated 19-sector model.

Sector	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	.452	.008	.002	.001	.012	.001	.135	.071	.043	.011	.059	.022	.001	.002	.054	.001	.005	.001	.001
2	.084	.273	.000	.000	.000	.170	.001	.006	.002	.006	.017	.002	.001	.002	.002	.002	.005	.004	.001
3	.012	.003	.237	.034	.034	.115	.005	.015	.018	.019	.028	.027	.001	.005	.020	.019	.093	.005	.003
4	.015	.000	.005	.029	.000	.077	.003	.002	.015	.001	.015	.022	.000	.001	.001	.000	.002	.001	.000
5	.000	.000	.	.002	.152	.018	.000	.000	.000	.001	.001	.000	.000	.000	.000	.001	.000	.000	.000
6	.042	.029	.027	.013	.029	.012	.014	.019	.058	.018	.029	.021	.004	.004	.007	.003	.028	.011	.004
7	.019	.021	.003	.002	.039	.008	.154	.221	.017	.024	.018	.014	.003	.003	.083	.018	.026	.010	.003
8	.012	.004	.003	.001	.002	.011	.002	.004	.006	.004	.007	.012	.000	.001	.012	.016	.007	.005	.
9	.015	.011	.010	.005	.032	.005	.035	.027	.277	.029	.017	.069	.027	.003	.018	.021	.023	.004	.011
10	.006	.030	.001	.001	.011	.002	.013	.009	.035	.312	.015	.037	.004	.009	.061	.005	.007	.012	.062
11	.002	.004	.000	.000	.002	.001	.003	.004	.005	.003	.160	.011	.000	.001	.222	.002	.001	.004	.000
12	.000	.000	.000	.000	.000	.000	.002	.001	.005	.006	.001	.031	.000	.001	.006	.000	.000	.003	.000
13	.005	.004	.001	.000	.007	.002	.008	.010	.041	.026	.006	.008	.395	.006	.008	.004	.007	.011	.012
14	.001	.001	.001	.002	.001	.001	.002	.001	.042	.002	.002	.002	.005	.238	.002	.036	.000	.026	.000
15	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
16	.000	.000	.000	.000	.000	.000	.000	.000	.001	.019	.000	.000	.087	.400	.001	.202	.000	.010	.009
17	.078	.147	.164	.415	.084	.002	.034	.002	.060	.131	.258	.071	.009	.019	.001	.012	.001	.008	.056
18	.019	.027	.039	.022	.	.	.019	.	.044	.035	.009	.067	.028	.086	.	.038	.	.	.095
19	.021	.000	.000	.000	.000	.001	.001	.004	.003	.003	.002	.002	.001	.001	.002	.001	.	.005	.001

Table 8. Aggregated input-output table for the USSR economy, 1972 (10^9 roubles) (after Sverdlick, 1980)

Sector	Sector 1	2	3	4	5	6	7	8	Total	Con-sumption	Gross invst./ Extens. repair	Ex-ports	Others	Total output
1. Heavy Industry	140.2	4.2	4.1	38.1	13.3	5.4	1.5	0.6	207.4	40.0	37.2	15.0	22.3	307.7
2. Textiles	2.8	41.4	0.8	0.6	0.4	0.2	0.3	0.1	46.6	46.3	-	1.0	1.1	87.5
3. Food	1.3	0.5	30.0	0.1	4.3	-	0.7	-	36.9	91.5	-	0.9	-0.5	124.9
4. Construction	-	-	-	-	-	-	-	-	-	-	77.4	-	-	77.4
5. Agriculture	2.4	8.9	50.4	-	23.9	-	0.2	-	85.8	29.1	1.1	0.7	1.0	114.7
6. Transport and Communication	24.3	0.9	2.3	0.1	1.4	-	0.2	0.3	29.5	-	-	-	-	29.5
7. Trade and distribution etc.	6.3	2.9	0.9	-	4.6	-	-	0.5	15.2	-	-	-	-	25.2
8. Others	1.6	0.1	0.1	0.2	0.1	-	0.1	0.1	2.3	3.1	-	-	-	5.4
9. Total material production	178.9	58.9	88.6	39.1	48.0	5.6	3.0	1.6	423.7	210.0	115.7	17.6	23.9	772.3
10. Depreciation	17.7	0.8	1.6	3.6	7.1	4.4	1.9	-	37.1	15.4				
11. Output	307.7	87.5	124.9	77.4	114.7	29.5	25.2	5.4	772.3	70.6				
12. Imports	15.2	7.5	3.9	-	3.0	-	-	-	29.6					
Total consumption	322.9	95.0	128.8	77.4	117.7	29.5	25.2	5.4	801.9	70.6				

value terms, so that various material balances, for example, an energy balance, can be developed and used within the input-output framework.

7. CONCLUSIONS AND FUTURE ACTIVITIES

This study has demonstrated the value of simple econometric techniques in the reconstruction of input-output data, and the use of regression estimates provides a basis for quantitative conclusions.

The disaggregated input-output model of the USSR economy enables us to distinguish the importance of changes in final demand on industrial structure. Some proportional shares in gross domestic products that remain steady over time are the result of interrelated fundamental shifts in investment policy, consumption patterns, and foreign trade policy. Such changes are easy to analyze within the straightforward framework of the input-output model. This means that we should distinguish, in as much detail as possible, entries on the demand side. The ground rules are rather simple: all industries that are covered by current statistics on the production (in physical units) of major commodities should be presented separately, and aggregated industries should be associated with real economic activities managed by different ministries. For example, basic building materials are consumed mostly by construction, but the substitution between these products is clear from the analysis of their production (and distribution) in terms of physical units. The same is true for textiles of different kinds (cotton, silk, and wool). The import of agricultural products, such as grain, is reported in physical units for all products; to gain useful insights into the process of interaction between industries it is necessary to estimate such imports on a more disaggregated level. This approach has been successfully used in the study of such relationships during the development of a model of interindustry interactions (Yaramenko et al, 1981, 1983).

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APPENDIX

Table A1. Sectoral classification and sources of the gross output value estimates for each industry.

Sector no.	Sector	Source of estimate
1	Ferrous and non-ferrous metals	Σx_{ij} + Exports - Imports
2	Coking coal	$a_{5.2}$, $a_{11.2}$ and $a_{39.2}$
3	Refractory materials	$a_{11.4}$ and $a_{5.4}$
4	Metal goods	$a_{11.4}$ and $a_{5.4}$
5	Coal	$a_{11.5}$
6	Oil extraction	$a_{11.6}$
7	Oil refining	$a_{11.7}$
8	Gas industry	Regression/lower and upper limits
9	Peat	$a_{11.9}$
10	Shale oil	$a_{11.10}$
11	Electrical and heat energy	$a_{5.11}$

Sector no.	Sector	Source of estimate
12	Electrical machinery	$a_{11.12}$ and $a_{39.12}$
13	Electrotechnical industry equipment	$a_{11.13}$ and $a_{39.13}$
14	Cables	$a_{11.14}$
15	Metalwork, lathes, etc.	$a_{11.15}$
16	Forging and pressing equip.	$a_{11.16}$
17	Foundry equipment	$a_{11.17}$ and $a_{5.17}$
18	Tools	$a_{11.18}$
19	Instruments, etc.	$a_{11.19}$
20	Boring equipment, etc.	Regression for machinery
21	Pumps and refrigeration equipment	$a_{11.21}$
22	Lumber, paper equipment	$a_{11.22}$
23	Light industry equipment	$a_{11.23}$
24	Food industry equipment	Regression for machinery
25	Printing industry equipment	$a_{11.25}$
26	Lifting and transportation equipment	$a_{11.26}$
27	Equipment for construction and road-building	$a_{11.27}$
28	Equipment for construction material	$a_{11.28}$
29	Transportation machinery	Regression for machinery
30	Vehicles	$a_{11.30}$

Sector no.	Sector	Source of estimate
31	Tractors and agricultural machinery	$a_{11.31}$
32	Bearings	$a_{11.32}$
33	Sanitary technology	$a_{11.33}$ and $a_{5.33}$
34	Other metal goods	Regression for machinery
35	Metal constructions	$a_{11.35}$
36	Machinery repairs	$a_{11.36}$
37	Abrasives	$a_{11.37}$ and $a_{5.37}$
38	Mineral chemicals	$a_{5.38}$
39	Basic chemicals	$a_{11.39}$ and $a_{5.39}$
40	Aniline dyestuffs	$a_{11.40}$ and $a_{39.40}$
41	Synthetic resins and plastics	$a_{39.41}$
42	Artificial fibers	$a_{11.42}$ and $a_{39.42}$
43	Synthetic organic materials, etc.	$a_{39.43}$
44	Paints	$a_{39.44}$
45	Rubber, asbestos	Regression
46	Other chemicals	Regression
47	Woodworking	Regression
48	Saw mills, etc.	Regression
49	Furniture	Regression
50	Other wood	Regression
51	Paper, cellulose	$a_{11.51}$, $a_{5.51}$, and $a_{39.51}$
52	Wood, chemicals	$a_{5.52}$ and $a_{39.52}$

Sector no.	Sector	Source of estimate
53	Cement	$a_{5.53}$ and $a_{11.53}$
54	Prefabricated concrete	Regression and Σx_{ij}
55	Wall materials, tiles	Regression and Σx_{ij}
56	Asbestos and slate	Regression and Σx_{ij}
57	Roofing felt	$\sum_j x_{ij}$
58	Construction ceramics	$\sum_j x_{ij}$
59	Other building materials	$\sum_j x_{ij}$
60	Glass, porcelain	$a_{11.60}$ and $a_{5.60}$
61	Cotton, textiles	Regression
62	Silk piece goods	Regression
63	Woollens	Regression and Σx_{ij} + final demand
64	Linen textiles	Regression and Σx_{ij} + final demand
65	Knitted goods	Regression
66	Other textiles	Regression and Σx_{ij} + final demand
67	Clothing	Regression and Σx_{ij} + final demand
68	Leather, fur, footwear, etc.	Regression
69	Fish and fish products	Regression and Σx_{ij} + final demand
70	Meat and meat products	Regression and Σx_{ij} + final demand
71	Milk and milk products	Regression and Σx_{ij} + final demand
72	Sugar	Regression and Σx_{ij} + final demand
73	Flour	Regression

Sector no.	Sector	Source of estimate
74	Bread and bread products	Σx_{ij} + final demand
75	Confectionery	Σx_{ij} + final demand
76	Butter, fats	Σx_{ij} + final demand
77	Fruit and vegetables	Σx_{ij} + final demand
78	Other food products	Σx_{ij} + final demand
79	Construction	Gross output values from balance of social product
80	Agriculture	Gross output values from balance of social product
81	Crops	Gross output values from balance of social product
82	Livestock	Gross output values from balance of social product
83	Forestry	Σx_{ij}
84	Transport and communications	Σx_{ij} and $a_{11,84}$
85	Trade and distribution	Σx_{ij}
86	Other sectors of material production	Regression

TABLE A2. Sectoral classification of retail turnover.

Sector no.	Sector	Commodity	Value (10^9 roubles)
49	Furniture	Furniture	3.380
51	Paper, cellulose	Paper, notebooks	1.125
60	Glass, porcelain	Wall glass, glass and porcelain	0.934
61	Cotton, textiles	Cotton fabrics	1.679
62	Silk piece goods	Silk fabrics	1.752
63	Woollens	Wool fabrics	1.565
64	Linen textiles	Linen fabrics	0.365
65	Knitted goods	Knitted goods / socks, pants	9.639
66	Other textiles	Carpets	0.814
67	Clothing	Clothing	15.915
68	Leather, fur, footwear, etc.	Fur	0.673
		Footwear	8.290
		Hats	0.342
69	Fish and fish products	Fish	2.228
		Fish products	0.842
70	Meat and meat products	Meat	14.043
		Meat products	0.648
71	Milk and milk products, excl. butter	Milk	5.370
		Milk products	0.908
72	Sugar	Sugar	5.703
73	Flour	Flour	1.367
		Cereals	2.075
74	Bread and bread products	Bread	7.287
		Macaroni	0.644
75	Confectionery	Confectionery	6.588
76	Butter, fats	Butter, fats	5.273
77	Fruit and vegetables	Canned fruit and vegetables	1.789

Sector no.	Sector	Commodity	Value (10^9 roubles)
78	Other food products	Other goods	30.922
		Tea	0.685
		Salt	0.135
		Tobacco	3.122
19		Watches	0.653
		Radios	3.265
13		Electrical goods	2.263
30		Automobiles, bicycles	1.153
48		Matches	0.143
34		Metal dishes	1.153
		Beds	0.141
7		Kerosine	0.108
46		Other chemicals	1.250
		Soaps, perfume	1.161
		Synthetic soap	0.579

DEFLATORS IN INPUT–OUTPUT TABLES

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1. Introduction

Inconsistencies in data sources have been widely recognised as a source of difficulty in economic modelling. Errors in data cast doubt on the validity of some of the estimation methods used and may lead to serious distortion in some estimated parameters. On the pricing side it has long been known that published price indices are inconsistent with profit data, although the discrepancy is open to a variety of interpretations. This paper applies a technique used in balancing national accounts to the derivation of price indices based on published data and consistent with current and constant price national accounts. Price differentials for goods sold in different markets can be derived and restrictions on these differentials imposed. The procedure is illustrated with reference to a small input-output system for the UK, although it will eventually be applied to the disaggregate data base used by the Cambridge Growth Project.

2. The Balancing Technique

Techniques for balancing input-output tables and other systems of accounts have developed considerably. They can be dated back to Stone, Champernowne and Meade (1942), whose approach is followed in Stone (1982), Byron (1978) and van der Ploeg (1982). This method, differing from the better known RAS (Bacharach, 1971), has also found application as a means of updating input-output coefficients (Lawson (1981)) and can from a Bayesian point of view, be seen as a means of updating data on the basis of the additional information that the data must satisfy accounting constraints.

Assume that we have a vector of observations x_0 which should satisfy the accounting constraint $Ax_0 = 0$ (any set of linear constraints can be defined in this way by suitable choice of the constraint matrix A). In practice because of measurement errors etc. this is unlikely to be the case and we should try to find a vector x as close to x_0 as possible such that the constraint $Ax = 0$ is satisfied. More specifically a suitable definition of closeness should reflect the precision with which the estimates are known. One would tolerate a large variation in a variable which was estimated with a large degree of imprecision but only a small change in a variable thought to be known with considerable accuracy. If the variance matrix V, of the observations is known we may seek to minimise $(x - x_0)'V^{-1}(x - x_0)$ subject to the constraint $Ax = 0$. In the simple case, where the standard deviation of each estimate is assumed proportional to its magnitude, percentage variations in the observations are equally penalised.

The problem can be expressed as the Lagrangian

$$\text{Min } L \equiv (x - x_0)'V^{-1}(x - x_0) - \lambda Ax$$

$$(x - x_0)'V^{-1} = \lambda A$$

$$x' - x_0' = \lambda AV$$

$$x'A' - x_0'A' = \lambda AVA'$$

$$\lambda = - x_0'A'(AVA')^{-1}$$

$$x = x_0 - VA'(AVA')^{-1}Ax_0$$

and it will be noted the solution is independent of any scaling factor in V - only relative variances and covariances matter.

3. Constant and Current Price Accounts

The above procedure can be used to balance any set of national accounts in volume or value terms. The accounts may include a detailed institutional breakdown or, as in the volume case, only a summary GDP account. The rest of this paper looks at applications of the balancing technique to the production accounts. A means of deriving volume flows for one year based on the input-output table for another year is first considered. These flows are then revalued to current prices and balanced again yielding new deflators.

Thus, it is assumed, in illustrating the technique, that value accounts are derived from volume accounts and price estimates, although in developing this approach it would be desirable to use specific information about the derivation of each volume and value entry, provided by a statistical office. This is because in some cases value estimates are derived from volume estimates and deflators while in others the volume estimates may be derived.

Deflators in national accounts link current and constant price goods flows and are therefore Paasche indicies. However published price indices such as the UK Wholesale Price Index (CSO, 1980) are Laspeyres indices and may therefore tend to be upwardly biassed estimates of the deflators. In this example the initial aggregate deflators for commodity flows are calculated on a Paasche basis from disaggregate wholesale price indices. If the balancing technique is used at a disaggregate level the bias in the price indices is likely to be much less important. Thus both the initial and final deflators can be regarded as Paasche indices.

Since there is no restriction on these final deflators, starting out from a position in which all home sales of a particular commodity have the same price will probably lead to sales from any row to each column being at different relative prices. These different implicit prices are derived from "best-estimate" volume and value cells and can therefore be regarded as indicating that, for reasons of commodity composition or commercial practice prices have moved differentially from the base year.

In the example considered in this paper the accounts are relatively consolidated and therefore it might be possible to analyse each individual price relative to the row total. However in the production accounts used by the Cambridge Growth Project there would be too many prices to do this.

4. Variances and Covariances

In both the price and the volume data there are likely to be two distinct sources of error. Full input-output volume tables are not likely to be available for every year. Instead initial estimates have to be constructed from estimates of column totals such as industry or commodity output or estimates of flows to categories of final demand. These column totals are used with estimates of the converter, absorbtion and make matrices expressed in coefficient form in order to produce intitial estimates of volume flows.

Lawson (1981) discusses various techniques for forecasting these coefficients and estimating their variance. In the simplest case, used subsequently, they are derived from published input-output tables relating to a different year and are assumed independent with standard deviations proportional to their size. Errors in any individual estimate may arise from uncertainty about both the coefficients and the column totals. The initial estimate of the variance matrix of the voluмne flows, V_{x_o}, is derived as the sum of these components. Algebraically

x_{o_i} = element i of the vector of volume flows

c_k = estimate of kth column total

$V_{kk'}$ = $cov(c_k, c_{k'})$

a_{ik} = ith coefficient relating volume flow i to column total k.

$$x_{o_i} = a_{ik}c_k$$

$$V_{a_{ij}} = cov(a_{ik}, a_{jk'})$$ (Kendall and Stuart, 1976)

$$V_{x_{o_{ij}}} = a_{ik}a_{jk'}V_{kk'} + c_k c_{k'} V_{a_{ij}}$$

Thus covariances can arise either because two elements are derived from interdependent column totals, or because the estimates of two coefficients are not independent.

The same point arises with prices. Estimates of absorbtion and output prices are typically built up (in UK at least) from estimates of prices of imports, prices of exports and prices of home sales of domestic output. Thus, for example, the price of home absorbtion must be expected to show some covariance with the price of imports, and the price of commodity output will show some covariance with the price of exports and with the price of home absorbtion. The fact that these prices are derived in this way as linear combinations of published prices may be regarded as directly analogous to the fact that elements of the volume table are derived from column totals. However one must also give some weight to the possibility that all prices of home absorbtion of any single commodity are not the same. It may be the case that goods are marked up in some markets where demand is buoyant. Further there is the obvious point that, however fine the classification, the price indices relate to bundles of goods rather than individual commodities. Different categories of demand will buy different goods from the same basket, and their relative prices may have diverged since the base period. This uncertainty about individual prices can be compared with the doubt about individual coefficients in the volume case.

Where π_k are observed prices relating to specific flows (such as exports, imports or home sales to the home market) the price of each volume flow will be derived from these as

$$p_i = \Sigma W_{ik}\pi_k$$

with W_{ik} the elements of a weighting matrix W derived from the volume estimates. The variance V_p of p, the price vector is derived as

$$V_p = WV_\pi W' + V^*_p$$

where V_π is the variance of the observed price vector π and errors arising from imprecision in measuring the weights are neglected. In practice price indices are insensitive to small changes in weights whereas measurement problems may be important. V^*_p represents the component of variance arising from uncertainty about the suitability of p_i as an appropriate price index for flow i.

If we initially assume that the variance of x_o the initial volume estimates is V_{x_o} then the updated variance of x the updated volume estimates is

$$V_x = M_x V_{x_0} M'_{x_0} = M_x V_{x_0}$$

where $M_x = I - V_{x_0} A'(AV_{x_0} A')^{-1} A$ with constraint matrix A.

Theil (1958) points out that the imposition of the accounting restriction must reduce the variance of the estimates.

Since we also assume that prices are estimated independently of volumes (one could assume prices and values independently measured but it would be surprising if volume and value estimates were independent), the price variance matrix is V_p described above. Under these circumstances the initial estimate of the value cells, y_0, derived as the price estimate multiplied by the balanced volume estimate has variance

$$V_{y_{0\,ij}} = p_i p_j V_{x_{ij}} + x_i x_j V_{p_{ij}}$$

This can be regarded as derived from two components, uncertainty about volumes and uncertainty about prices. There are now two possibilities. The first is to derive an updated variance matrix by adjusting taking into account uncertainty about volume and price. This yields a new variance matrix V_y as

$$V_y = (I - V_{y_o} A'(AV_{y_o} A')^{-1}A)V_{y_o}$$

and the new variance matrix of the implicit deflators may be estimated approximately (Kendall and Stuart, 1976) as

$$\text{cov}(p_i,p_j) = \frac{\text{cov}(y_i,y_j)}{x_i x_j} - \frac{y_j \text{cov}(y_i,x_j)}{x_i x_j^2} - \frac{y_i \text{cov}(y_j,x_i)}{x_i^2 x_j} + \frac{y_i y_j \text{cov}(x_i,x_j)}{x_i^2 x_j^2}$$

where $p_i = \dfrac{y_i}{x_i}$

and the covariances of x and y are derived by expressing V_y in terms of V_x and V_p.

However following this approach one can no longer regard the volume and value estimates as linked by uncertainly determined implicit deflators. Part of the work of balancing has been done by taking account of the quantity as well as the price uncertainty in the value estimates although the value estimates were in fact derived from "best estimate" volume accounts. To restore balance in this way would generate unreasonable movement in the implicit deflators and exaggerate their posterior unreliability. It would be more satisfactory to merely adjust the value accounts with reference to uncertainty in the implicit deflators. Thus we denote

$$V^P_{y_0\,ij} = x_i x_j V_{p_{ij}}$$

as representing the uncertainty attributable to imprecision in measuring prices and derive

$$y = (I - V^P_{y_0} A'(AV^P_{y_0} A')^{-1}A)y_0 = M^P_y \cdot y_0$$

and

$$V_y = M^P_y \cdot V_{y_0}$$

The adjusted variance only takes into account the enhanced precision with which prices are determined as a result of balancing.

Finally it is noted that **constraints** on the final data which are already present in the prior estimates can be maintained by the specification of the variance matrix as well as by suitable specification of the restriction matrix. As an example suppose the final estimate y, should satisfy the restriction

$$a'y = 0$$

and that the data are constructed so that the initial estimates y_0 also satisfy the restriction

$$a'y_0 = 0 .$$

If in addition the variance matrix V satisfies the restriction

$$a'V = 0$$

then it is seen that, since

$$y = (I - VA'(AVA')^{-1}A)y_0$$

where A represents the accounting constraints,

$$a'y = 0$$

and the updated variance matrix

$$V_0 = (I - VA'(AVA')^{-1}A)$$

also satisfies the restriction $a'V_o = 0$.

This approach may provide a more compact way of imposing restrictions. If it is desired to restrict one variable to be a multiple of another and the only information on which prior estimates can be obtained is on the basis of this restriction, then specifying the variance matrix so that unit correlation is maintained, may be a more convenient way of doing this than by adding a row to the restriction matrix. It is clear that this second procedure also leads to the updated variance matrix satisfying the restriction.

5. An Example- The UK Production Account for 1978

This section demonstrates the application of the techniques to the construction of volume and value production accounts for the UK in 1978. The accounts are prepared on a consolidated basis with just four commodities/ industries and only one type of factor income identified. They are derived from an input-output table for the UK for 1975 together with data in the National Accounts (National Income and Expenditure, 1980) and the data base of the Cambridge Growth Project.

Although a series of input-output tables has been produced for the UK (CSO, 1954, 1963, 1968, 1974) one suspects that some of the major variations in coefficients may have occurred as much because different people compiled them, using different underlying principles, as because technical change has taken place. Thus the prior estimates of the 1978 volume table (Appendix, Table 1) are derived on the assumption that the make matrix coefficients, the input coefficients and the coefficients of the final demand converters are constant. The residuals are shown in this table.

The volume accounts are balanced on the basis of the assumption that standard errors are constant. However the assumed standard error of stock additions is 30 times the base level, while those of indirect taxes and trade flows are $\frac{1}{4}$ and $\frac{1}{2}$ of this base. While the data are estimated from column totals, some allowance is given to the possibility that the coefficients have a margin of error attached to them. It is arbitrarily assumed that the independent error in each coefficient is $\sqrt{3}$ times that derived from uncertainty about the column totals. If no independent variance were allowed, the whole burden of the adjustment would, as noted above, fall on the column totals. These assumptions about errors may be considerably improved on if national accountants provide advice on the reliability of their data of the type shown in Maurice (ed.) (1967).

These balanced volume accounts are shown in Table 2. As would be expected, given the assumptions made, the estimate of stock change has been substantially modified. An increase of the order of ½% also takes place in the volume estimate of total value added. Different assumptions of reliability would obviously change these adjustments. However the main aim of this illustration is to look at the modifications, which take place in the deflators. These are likely to be little affected by the precise pattern of the volume adjustments and the issue is therefore not pursued further here.

The balanced volume accounts are reflated to current prices using the deflators shown in Table 3. The initial estimates of the current price accounts are presented with the associated residual errors in Table 4. The standard errors of the observed price indices for goods flows are assumed to be 3% and those of the other prices 1%. This is because these goods price indices relate partly to list prices and are not calculated specifically for use as national accounts deflators. It is again assumed that the independent error in each price is $\sqrt{3}$ times the error arising from the source price indices. Substantial scope is therefore given for relative prices to vary even though they start out being equal. The absolute magnitude of the standard errors only becomes important in the subsequent section.

Table 5 presents the balanced value accounts. It is seen that the final estimates of the deflators (Table 6) are substantially different from their starting values (Table 3). The home absorbtion prices now cover fairly wide ranges. That of construction has been less affected than the others because construction is a more homogeneous good. The price of manufacturing output sold to investment in vehicles seems to have risen rather more than that sold as an input to construction, since 1975. Fairly large movements in the services deflators are also observed, reflecting severe data problems.

It must be noted that these deflators are derived from reflated balanced volume accounts. The residuals shown in Table 3 are therefore a consequence of pricing problems and the adjustments to the value figures and associated movements in deflators show the changes needed to fit value accounts to a given set of volume accounts. This is consistent with the assumption that value accounts are derived from volume accounts and prices.

6. Testing Hypotheses on Relative Prices

The above sections have focussed on a statistical problem - a technique for balancing accounts is applied to volume and value production accounts and in the light of this initial estimates of deflators can also be updated. But the results so derived can be used in investigating questions of an economic as well as a statistical nature. In this example deflators such as the price of home absorption, which started off being the same in all markets, have now diverged. Initially the price of manufactures was 150.5 in all home markets, but after the balancing exercise it ranges between 139.4 and 163.0. An obvious question one should investigate is whether the prices are significantly different, or can one choose deflators without breaking the accounting contstraints so that the price indices of manufactured goods sold to two different categories of demand are in fact the same. One might wish to examine the proposition that all types of consumption buy at one price while all types of investment buy at a different uniform price. This could be compared with the hypothesis that the accounts could be balanced with all the price differentials occurring in sales of commodity 4. In a more disaggregated model this could be interpreted as firms facing different distribution margins on goods they buy in, and distribution margins differing on different categories of final demand. A further possibility would be that all prices of home absorption and commodity output are the same. In this case the variance would arise solely from uncertainty about the published price indices, and the restriction could be imposed through specification of the covariance matrix. Value accounts satisfying this restriction on absorbtion and commodity output prices are shown in Table 7 with associated deflators in Table 8.

These approaches contrast with that of Coutts, Godley and Nordhaus (1978) who comment on the discrepancy between published price indices and those implicit price indices derived by adding the cost of unit inputs based on fixed in put-output coefficients to unit input taces and factor incomes. They attribute this discrepancy to the fact that the wholesale price indices measure list prices while the implicit deflators are based on costs (including actual profit) and therefore reflect prices actually paid. They further attempt to see whether the difference between the two can be explained on the assumption that actual prices are marked above list prices during a boom and depressed below them in a slump. However their approach assumes that the discrepancy arises purely from differences in the observed price of output and does not reflect the possibility that profits may be incorrectly measured, or that there may be other accounting errors.

The imposition of the restrictions of course substantially changes the estimated value accounts and the question of determining whether the change is statistically significant or not must arise. This of course depends on the magnitude of the variance matrix and not just on the relative magnitudes of the elements (which is the case in the construction of the adjusted estimates themselves), and also depends on whether an accounting matrix for just one year is being restricted or whether the restriction is imposed simultaneously on a time series of accounting matrices.

If the revised estimates are y with variance matrix V then the distribution of variable values y_1 about y must be considered. V is of course a singular matrix since, in a situation where each element of y is allowed some independent variation its rank will be the number of variables less the number of independent restrictions. Consider a transformation P such that rank (PV) = rank (V) but PVP' is non-singular. Then

$$(y_1 - y)' P'(PVP')^{-1} P(y_1 - y) \sim \chi^2$$

with rank (V) degrees of freedom, on the assumption that the individual errors are normally distributed.

Applying this test to the case where all the absorption prices are restricted to be equal it is found in 1978 that $\chi^2 = 276$ which is significant at a 5% level. It would remain so even if the standard errors were of almost twice the assumed magnitude.

More generally one must consider the effect of imposing restrictions on the adjusted estimates. In the case where a restriction is "true" it has the effect of reducing the variance of the estimate. The explanation is exactly the same as that showing that balancing reduced the variance. For the accounting restrictions, while they must always be correct are not the only ones which need be correct.

However the mere fact that a restriction is not rejected does not automatically imply that it is correct. The results present by Theil (1958) in the context of regression carry over to this situation, for both regression and this adjustment technique are the minimisation of sums of squares.

Suppose the restriction $Ay = 0$ is imposed when in fact the true restriction is $Ay = \Delta$.

Under these circumstances the estimates are biased by

$$VA'(AVA')^{-1}A\Delta$$

and the new variance matrix is

$$V - VA'(AVA')^{-1}\left[I - \Delta\Delta'(AVA')^{-1}\right]AV$$

As in the case of regression the imposition of an incorrect restriction may nevertheless lead to a reduction in the variance of the updated and biased estimators compared with the unrestricted case. For the change in the variance is substantially influenced by the matrix in square brackets. If the original estimates are considered reasonably precise then the imposition of an incorrect restriction is likely to lead to this being negative and thus to the overall variance being increased. In practice one should be reluctant to impose restrictions which are rejected by the χ^2 test.

7. Future Work

The previous sections have demonstrated an application of an old technique which has been relatively little used until recently. Work on the Cambridge Growth Project (Barker, van der Ploeg and Weale, (1982)) is now being directed towards constructing a time-series of accounting matrices for the UK expressed in current and constant prices at the level of disaggregation used in the Growth Project Model by means of the techniques described here. In this context it will be possible to examine price differentials, although in order to make the problem practical it will be necessary to impose restrictions on relative prices so as to limit the number of observed differentials. In this paper it has been assumed that value accounts are derived from volume accounts and price indices, which implies one particular approach to the adjustment problem. However this approach, while it illustrates the techniques and allows the derivation of price differentials, is not completely satisfactory. Further work should also investigate the simultaneous adjustment of the volume and value accounts with suitable estimates of the covariance between the two. While such an approach would change the level of estimates in volume and value terms it is unlikely to affect the conclusions reached about the importance of price differentials.

8. Conclusions

A technique used previously to balance accounts has been applied to the problem of determining consistent volume and value accounts, and illustrated using a set of 4-sector production accounts for the UK. The technique makes use of all the information available so as to minimise the final variance of the estimates. Using extensions of the technique it becomes possible to test hypotheses about prices of goods sold to different categories of demand in a consistent accounting framework. The limited evidence of the UK suggests that prices of home absorption (relative to 1975) in 1978 differed in different sectors of demand and prices of commodity output depended on the industries producing the goods concerned; economists should attempt to investigate this further and construct time series of differentials which may be examined econometrically.

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APPENDIX

Table 1. Unbalanced volume accounts.

1978 PRODUCTION ACCOUNTS 1975 PRICES

	1	2	3	4	5	6	7	8	9	10	11	12	13
1Commodities Primary	0	0	0	0	1926	9166	315	193	0	2	107	2346	221
2 Manuf & Util	0	0	0	0	2753	39114	4853	8443	0	1877	2415	9118	3594
3 Construction	0	0	0	0	307	438	2989	566	0	178	816	0	1643
4 Services	0	0	0	0	1368	12687	1184	13931	0	5	2072	5746	777
5Industries Primary	10703	50	54	89	0	0	0	0	0	0	0	0	0
6 Manuf & Util	37	88759	693	4217	0	0	0	0	0	0	0	0	0
7 Construction	0	0	16518	23	0	0	0	0	0	0	0	0	0
8 Services	6	1253	48	58920	0	0	0	0	0	0	0	0	0
9Ownership of Dwellings	0	0	0	0	0	0	0	0	0	0	0	0	6471
10Defence	0	0	0	0	0	0	0	0	0	0	0	0	0
11Other Government	0	0	0	0	0	0	0	0	0	0	0	0	0
12Cons. Food, Drink & Tab.	0	0	0	0	0	0	0	0	0	0	0	0	0
13 Fuel and Housing	0	0	0	0	0	0	0	0	0	0	0	0	0
14 Durables	0	0	0	0	0	0	0	0	0	0	0	0	0
15 Services	0	0	0	0	0	0	0	0	0	0	0	0	0
16Inv Stocks	0	0	0	0	0	0	0	0	0	0	0	0	0
17 Buildings	0	0	0	0	0	0	0	0	0	0	0	0	0
18 Plant & Machinery	0	0	0	0	0	0	0	0	0	0	0	0	0
19 Vehicles	0	0	0	0	0	0	0	0	0	0	0	0	0
20Exports	0	0	0	0	0	0	0	0	0	0	0	0	0
21Imports	4817	20749	269	5226	0	0	0	0	0	468	0	0	0
22Final Demand	0	0	0	0	0	0	0	0	0	0	0	0	0
23Indirect Taxes	69	521	0	0	-675	737	119	1476	552	158	604	3142	81
24Value Added	0	0	0	0	5219	31564	7080	35618	5919	2325	12838	0	0
25Total	15632	111332	17582	68475	10898	93706	16540	60227	6471	5013	18852	20352	12787
26Residual	373	2909	-203	-623	-2	0	1	0	0	1	-1	0	-64

	14	15	16	17	18	19	20	21	22	23	24	25
1Commodities Primary	0	130	55	49	0	0	1495	0	0	0	0	16005
2 Manuf & Util	7336	3798	597	257	6605	2489	20992	0	0	0	0	114241
3 Construction	0	40	191	9506	405	0	300	0	0	0	0	17379
4 Services	5172	14809	0	1136	479	-568[1]	9054	0	0	0	0	67852
5Industries Primary	0	0	0	0	0	0	0	0	0	0	0	10896
6 Manuf & Util	0	0	0	0	0	0	0	0	0	0	0	93706
7 Construction	0	0	0	0	0	0	0	0	0	0	0	16541
8 Services	0	0	0	0	0	0	0	0	0	0	0	60227
9Ownership of Dwellings	0	0	0	0	0	0	0	0	0	0	0	6471
10Defence	0	0	0	0	0	0	0	0	5014	0	0	5014
11Other Government	0	0	0	0	0	0	0	0	18851	0	0	18851
12Cons. Food, Drink & Tab.	0	0	0	0	0	0	0	0	20352	0	0	20352
13 Fuel and Housing	0	0	0	0	0	0	0	0	12723	0	0	12723
14 Durables	0	0	0	0	0	0	0	0	13521	0	0	13521
15 Services	0	0	0	0	0	0	0	0	21478	0	0	21478
16Inv Stocks	0	0	0	0	0	0	0	0	843	0	0	843
17 Buildings	0	0	0	0	0	0	0	0	11054	0	0	11054
18 Plant & Machinery	0	0	0	0	0	0	0	0	7579	0	0	7579
19 Vehicles	0	0	0	0	0	0	0	0	2169	0	0	2169
20Exports	0	0	0	0	0	0	0	0	32034	0	0	32034
21Imports	0	241	0	0	0	0	0	0	0	0	0	31770
22Final Demand	0	0	0	0	0	0	0	31770	0	11919	101929	145618
23Indirect Taxes	1013	2459	0	106	90	248	193	0	0	0	0	10893
24Value Added	0	0	0	0	0	0	0	0	0	0	0	100563
25Total	13521	21477	843	11054	7579	2169	32034	31770	145618	11919	101929	0
26Residual	0	1	0	0	0	0	0	0	0	-1026	-1366	-0

1. Sales by final buyers are included with services throughout

Table 2. Balanced volume accounts.

1978 PRODUCTION ACCOUNTS 1975 PRICES

	1	2	3	4	5	6	7	8	9	10	11	12	13
1Commodities Primary	0	0	0	0	1931	9058	313	194	0	2	107	2371	220
2 Manuf & Util	0	0	0	0	2779	39518	4787	8423	0	1873	2417	9181	3572
3 Construction	0	0	0	0	308	444	2976	571	0	178	819	0	1644
4 Services	0	0	0	0	1380	13071	1177	14056	0	5	2081	5859	776
5Industries Primary	10868	50	54	89	0	0	0	0	0	0	0	0	0
6 Manuf & Util	37	91008	702	4258	0	0	0	0	0	0	0	0	0
7 Construction	0	0	16369	22	0	0	0	0	0	0	0	0	0
8 Services	6	1265	48	59318	0	0	0	0	0	0	0	0	0
9Ownership of Dwellings	0	0	0	0	0	0	0	0	0	0	0	0	6492
10Defence	0	0	0	0	0	0	0	0	0	0	0	0	0
11Other Government	0	0	0	0	0	0	0	0	0	0	0	0	0
12Cons. Food, Drink & Tab.	0	0	0	0	0	0	0	0	0	0	0	0	0
13 Fuel and Housing	0	0	0	0	0	0	0	0	0	0	0	0	0
14 Durables	0	0	0	0	0	0	0	0	0	0	0	0	0
15 Services	0	0	0	0	0	0	0	0	0	0	0	0	0
16Inv Stocks	0	0	0	0	0	0	0	0	0	0	0	0	0
17 Buildings	0	0	0	0	0	0	0	0	0	0	0	0	0
18 Plant & Machinery	0	0	0	0	0	0	0	0	0	0	0	0	0
19 Vehicles	0	0	0	0	0	0	0	0	0	0	0	0	0
20Exports	0	0	0	0	0	0	0	0	0	0	0	0	0
21Imports	4915	21015	268	5188	0	0	0	0	0	468	0	0	0
22Final Demand	0	0	0	0	0	0	0	0	0	0	0	0	0
23Indirect Taxes	69	532	0	0	-664	763	118	1546	562	158	616	3467	81
24Value Added	0	0	0	0	5326	33150	7018	35846	5930	2326	12886	0	0
25Total	15896	113871	17443	68877	11062	96007	16392	60639	6492	5012	18929	20878	12788

	14	15	16	17	18	19	20	21	22	23	24	25
1Commodities Primary	0	132	30	49	0	0	1485	0	0	0	0	15896
2 Manuf & Util	7318	3853	164	257	6560	2487	20676	0	0	0	0	113872
3 Construction	0	40	180	9574	404	0	300	0	0	0	0	17443
4 Services	5208	15071	0	1139	478	-568	9140	0	0	0	0	68877
5Industries Primary	0	0	0	0	0	0	0	0	0	0	0	11061
6 Manuf & Util	0	0	0	0	0	0	0	0	0	0	0	96006
7 Construction	0	0	0	0	0	0	0	0	0	0	0	16392
8 Services	0	0	0	0	0	0	0	0	0	0	0	60639
9Ownership of Dwellings	0	0	0	0	0	0	0	0	0	0	0	6492
10Defence	0	0	0	0	0	0	0	0	5012	0	0	5012
11Other Government	0	0	0	0	0	0	0	0	18929	0	0	18929
12Cons. Food, Drink & Tab.	0	0	0	0	0	0	0	0	20878	0	0	20878
13 Fuel and Housing	0	0	0	0	0	0	0	0	12788	0	0	12788
14 Durables	0	0	0	0	0	0	0	0	13573	0	0	13573
15 Services	0	0	0	0	0	0	0	0	22011	0	0	22011
16Inv Stocks	0	0	0	0	0	0	0	0	375	0	0	375
17 Buildings	0	0	0	0	0	0	0	0	11127	0	0	11127
18 Plant & Machinery	0	0	0	0	0	0	0	0	7533	0	0	7533
19 Vehicles	0	0	0	0	0	0	0	0	2168	0	0	2168
20Exports	0	0	0	0	0	0	0	0	31795	0	0	31795
21Imports	0	245	0	0	0	0	0	0	0	0	0	32101
22Final Demand	0	0	0	0	0	0	0	32101	0	11604	102486	146192
23Indirect Taxes	1046	2667	0	106	90	249	193	0	0	0	0	11604
24Value Added	0	0	0	0	0	0	0	0	0	0	0	102486
25Total	13573	22011	375	11127	7533	2168	31795	32101	146192	11604	102486	0

Table 3. Initial deflators.

1978 PRODUCTION ACCOUNTS IMPLICIT DEFLATORS

	1	2	3	4	5	6	7	8	9	10	11	12	13
1Commodities Primary	0.0	0.0	0.0	0.0	140.1	140.1	140.1	140.1	0.0	140.1	140.1	140.1	140.1
2 Manuf & Util	0.0	0.0	0.0	0.0	150.5	150.5	150.5	150.5	0.0	150.5	150.5	150.5	150.5
3 Construction	0.0	0.0	0.0	0.0	131.1	131.1	131.1	131.1	0.0	131.1	131.1	0.0	131.1
4 Services	0.0	0.0	0.0	0.0	150.8	150.8	150.8	150.8	0.0	150.8	150.8	150.8	150.8
5Industries Primary	142.3	151.5	131.3	150.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6 Manuf & Util	142.3	151.5	131.3	150.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7 Construction	0.0	0.0	131.3	150.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8 Services	142.3	151.5	131.3	150.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9Ownership of Dwellings	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	141.7
10Defence	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11Other Government	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12Cons. Food, Drink & Tab.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13 Fuel and Housing	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14 Durables	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15 Services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16Inv Stocks	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17 Buildings	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18 Plant & Machinery	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19 Vehicles	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20Exports	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21Imports	136.1	144.4	134.6	144.2	0.0	0.0	0.0	0.0	0.0	143.0	0.0	0.0	0.0
22Final Demand	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23Indirect Taxes	164.4	164.4	0.0	0.0	164.4	164.4	164.4	164.4	164.4	164.4	164.4	164.4	164.4
24Value Added	0.0	0.0	0.0	0.0	129.3	148.0	128.5	141.7	146.1	141.7	141.7	0.0	0.0

	14	15	16	17	18	19	20	21	22	23	24
1Commodities Primary	0.0	140.1	140.1	140.1	0.0	0.0	143.0	0.0	0.0	0.0	0.0
2 Manuf & Util	150.5	150.5	150.5	150.5	150.5	150.5	148.4	0.0	0.0	0.0	0.0
3 Construction	0.0	131.1	131.1	131.1	131.1	0.0	147.0	0.0	0.0	0.0	0.0
4 Services	150.8	150.8	0.0	150.8	150.8	150.8	146.1	0.0	0.0	0.0	0.0
5Industries Primary	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6 Manuf & Util	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7 Construction	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8 Services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9Ownership of Dwellings	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10Defence	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	148.8	0.0	0.0
11Other Government	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	135.1	0.0	0.0
12Cons. Food, Drink & Tab.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	144.1	0.0	0.0
13 Fuel and Housing	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	148.6	0.0	0.0
14 Durables	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	142.1	0.0	0.0
15 Services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	144.1	0.0	0.0
16Inv Stocks	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	127.0	0.0	0.0
17 Buildings	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	131.6	0.0	0.0
18 Plant & Machinery	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	150.3	0.0	0.0
19 Vehicles	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	173.4	0.0	0.0
20Exports	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	148.0	0.0	0.0
21Imports	0.0	143.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
22Final Demand	0.0	0.0	0.0	0.0	0.0	0.0	0.0	143.0	0.0	164.4	141.7
23Indirect Taxes	164.4	164.4	0.0	164.4	164.4	164.4	164.4	0.0	0.0	0.0	0.0
24Value Added	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 4. Initial value estimates.

1978 PRODUCTION ACCOUNTS CURRENT PRICES

		1	2	3	4	5	6	7	8	9	10	11	12	13
1Commodities	Primary	0	0	0	0	2705	12687	438	272	0	2	150	3321	309
2	Manuf & Util	0	0	0	0	4184	59488	7206	12679	0	2820	3639	13820	5378
3	Construction	0	0	0	0	404	583	3901	748	0	233	1074	0	2154
4	Services	0	0	0	0	2081	19711	1775	21197	0	7	3138	8835	1171
5Industries	Primary	15460	76	71	134	0	0	0	0	0	0	0	0	0
6	Manuf & Util	53	137861	922	6416	0	0	0	0	0	0	0	0	0
7	Construction	0	0	21489	34	0	0	0	0	0	0	0	0	0
8	Services	8	1917	63	89377	0	0	0	0	0	0	0	0	0
9Ownership of Dwellings		0	0	0	0	0	0	0	0	0	0	0	0	9200
10Defence		0	0	0	0	0	0	0	0	0	0	0	0	0
11Other Government		0	0	0	0	0	0	0	0	0	0	0	0	0
12Cons. Food, Drink & Tab.		0	0	0	0	0	0	0	0	0	0	0	0	0
13	Fuel and Housing	0	0	0	0	0	0	0	0	0	0	0	0	0
14	Durables	0	0	0	0	0	0	0	0	0	0	0	0	0
15	Services	0	0	0	0	0	0	0	0	0	0	0	0	0
16Inv	Stocks	0	0	0	0	0	0	0	0	0	0	0	0	0
17	Buildings	0	0	0	0	0	0	0	0	0	0	0	0	0
18	Plant & Machinery	0	0	0	0	0	0	0	0	0	0	0	0	0
19	Vehicles	0	0	0	0	0	0	0	0	0	0	0	0	0
20Exports		0	0	0	0	0	0	0	0	0	0	0	0	0
21Imports		6690	30346	361	7481	0	0	0	0	0	669	0	0	0
22Final Demand		0	0	0	0	0	0	0	0	0	0	0	0	0
23Indirect Taxes		114	874	0	0	-1092	1254	195	2542	924	261	1013	5700	133
24Value Added		0	0	0	0	6887	49062	9018	50795	8403	3296	18260	0	0
25Total		22327	171076	22908	103444	15170	142787	22536	88235	9587	7291	27276	31677	18347
26Residual		-18	-103	0	-6	572	2466	-1012	3131	-387	166	-1703	-1591	656

		14	15	16	17	18	19	20	21	22	23	24	25
1Commodities	Primary	0	185	43	68	0	0	2123	0	0	0	0	22308
2	Manuf & Util	11017	5800	247	387	9875	3744	30683	0	0	0	0	170973
3	Construction	0	53	235	12547	530	0	441	0	0	0	0	22908
4	Services	7853	22727	0	1717	721	-857	13354	0	0	0	0	103437
5Industries	Primary	0	0	0	0	0	0	0	0	0	0	0	15742
6	Manuf & Util	0	0	0	0	0	0	0	0	0	0	0	145253
7	Construction	0	0	0	0	0	0	0	0	0	0	0	21524
8	Services	0	0	0	0	0	0	0	0	0	0	0	91367
9Ownership of Dwellings		0	0	0	0	0	0	0	0	0	0	0	9200
10Defence		0	0	0	0	0	0	0	0	7458	0	0	7458
11Other Government		0	0	0	0	0	0	0	0	25573	0	0	25573
12Cons. Food, Drink & Tab.		0	0	0	0	0	0	0	0	30086	0	0	30086
13	Fuel and Housing	0	0	0	0	0	0	0	0	19003	0	0	19003
14	Durables	0	0	0	0	0	0	0	0	19287	0	0	19287
15	Services	0	0	0	0	0	0	0	0	31718	0	0	31718
16Inv	Stocks	0	0	0	0	0	0	0	0	476	0	0	476
17	Buildings	0	0	0	0	0	0	0	0	14643	0	0	14643
18	Plant & Machinery	0	0	0	0	0	0	0	0	11322	0	0	11322
19	Vehicles	0	0	0	0	0	0	0	0	3760	0	0	3760
20Exports		0	0	0	0	0	0	0	0	47056	0	0	47056
21Imports		0	351	0	0	0	0	0	0	0	0	0	45900
22Final Demand		0	0	0	0	0	0	0	45905	0	19078	145222	210206
23Indirect Taxes		1720	4386	0	175	148	410	317	0	0	0	0	19078
24Value Added		0	0	0	0	0	0	0	0	0	0	0	145981
25Total		20591	33504	526	14897	11274	3297	46920	45905	210386	19078	145222	0
26Residual		-1303	-1785	-50	-254	47	462	136	-5	-180	0	759	-0

Table 5. Balanced value accounts.

1978 PRODUCTION ACCOUNTS CURRENT PRICES

		1	2	3	4	5	6	7	8	9	10	11	12	13
1Commodities	Primary	0	0	0	0	2670	12309	430	268	0	2	147	3235	304
2	Manuf & Util	0	0	0	0	4161	58710	6680	12391	0	2843	3489	13173	5396
3	Construction	0	0	0	0	407	586	3893	754	0	234	1075	0	2232
4	Services	0	0	0	0	2008	19148	1668	20298	0	7	2921	8281	1127
5Industries	Primary	14804	74	71	129	0	0	0	0	0	0	0	0	0
6	Manuf & Util	52	134460	912	6126	0	0	0	0	0	0	0	0	0
7	Construction	0	0	21719	33	0	0	0	0	0	0	0	0	0
8	Services	8	1887	63	85280	0	0	0	0	0	0	0	0	0
9Ownership of Dwellings		0	0	0	0	0	0	0	0	0	0	0	0	9445
10Defence		0	0	0	0	0	0	0	0	0	0	0	0	0
11Other Government		0	0	0	0	0	0	0	0	0	0	0	0	0
12Cons. Food, Drink & Tob.		0	0	0	0	0	0	0	0	0	0	0	0	0
13	Fuel and Housing	0	0	0	0	0	0	0	0	0	0	0	0	0
14	Durables	0	0	0	0	0	0	0	0	0	0	0	0	0
15	Services	0	0	0	0	0	0	0	0	0	0	0	0	0
16Inv	Stocks	0	0	0	0	0	0	0	0	0	0	0	0	0
17	Buildings	0	0	0	0	0	0	0	0	0	0	0	0	0
18	Plant & Machinery	0	0	0	0	0	0	0	0	0	0	0	0	0
19	Vehicles	0	0	0	0	0	0	0	0	0	0	0	0	0
20Exports		0	0	0	0	0	0	0	0	0	0	0	0	0
21Imports		6788	30592	359	7415	0	0	0	0	0	671	0	0	0
22Final Demand		0	0	0	0	0	0	0	0	0	0	0	0	0
23Indirect Taxes		114	874	0	0	-1091	1254	195	2543	922	261	1011	5690	133
24Value Added		0	0	0	0	6924	49541	8885	50984	8522	3320	17809	0	0
25Total		21768	167889	23126	98984	15080	141551	21752	87240	9445	7340	26455	30382	18639

		14	15	16	17	18	19	20	21	22	23	24	25
1Commodities	Primary	0	182	42	67	0	0	2106	0	0	0	0	21768
2	Manuf & Util	10458	5659	229	380	9876	4054	30382	0	0	0	0	167888
3	Construction	0	53	217	12693	533	0	444	0	0	0	0	23126
4	Services	7305	21212	0	1618	691	-793	13490	0	0	0	0	98984
5Industries	Primary	0	0	0	0	0	0	0	0	0	0	0	15080
6	Manuf & Util	0	0	0	0	0	0	0	0	0	0	0	141551
7	Construction	0	0	0	0	0	0	0	0	0	0	0	21752
8	Services	0	0	0	0	0	0	0	0	0	0	0	87240
9Ownership of Dwellings		0	0	0	0	0	0	0	0	0	0	0	9445
10Defence		0	0	0	0	0	0	0	0	7340	0	0	7340
11Other Government		0	0	0	0	0	0	0	0	26455	0	0	26455
12Cons. Food, Drink & Tob.		0	0	0	0	0	0	0	0	30382	0	0	30382
13	Fuel and Housing	0	0	0	0	0	0	0	0	18639	0	0	18639
14	Durables	0	0	0	0	0	0	0	0	19482	0	0	19482
15	Services	0	0	0	0	0	0	0	0	31844	0	0	31844
16Inv	Stocks	0	0	0	0	0	0	0	0	488	0	0	488
17	Buildings	0	0	0	0	0	0	0	0	14935	0	0	14935
18	Plant & Machinery	0	0	0	0	0	0	0	0	11249	0	0	11249
19	Vehicles	0	0	0	0	0	0	0	0	3673	0	0	3673
20Exports		0	0	0	0	0	0	0	0	46740	0	0	46740
21Imports		0	352	0	0	0	0	0	0	0	0	0	46179
22Final Demand		0	0	0	0	0	0	0	46179	0	19065	145987	211231
23Indirect Taxes		1718	4384	0	175	148	412	317	0	0	0	0	19065
24Value Added		0	0	0	0	0	0	0	0	0	0	0	145987
25Total		19482	31844	488	14935	11249	3673	46740	46179	211231	19065	145987	0

Table 6. Deflators.

1978 PRODUCTION ACCOUNTS IMPLICIT DEFLATORS

	1	2	3	4	5	6	7	8	9	10	11	12	13
1Commodities Primary	0.0	0.0	0.0	0.0	138.3	135.9	137.4	137.8	0.0	137.9	137.7	136.5	138.0
2 Manuf & Util	0.0	0.0	0.0	0.0	149.7	148.6	139.5	147.1	0.0	151.7	144.3	143.5	151.0
3 Construction	0.0	0.0	0.0	0.0	131.8	131.9	130.8	132.1	0.0	131.8	131.2	0.0	135.8
4 Services	0.0	0.0	0.0	0.0	145.5	146.5	141.7	144.4	0.0	144.2	140.4	141.3	145.2
5Industries Primary	136.2	148.9	131.1	144.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6 Manuf & Util	139.4	147.7	129.9	143.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7 Construction	0.0	0.0	132.7	144.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8 Services	139.4	149.1	131.2	143.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9Ownership of Dwellings	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	145.5
10Defence	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11Other Government	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12Cons. Food, Drink & Tob.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13 Fuel and Housing	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14 Durables	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15 Services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16Inv Stocks	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17 Buildings	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18 Plant & Machinery	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19 Vehicles	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20Exports	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21Imports	138.1	145.6	133.9	142.9	0.0	0.0	0.0	0.0	0.0	143.4	0.0	0.0	0.0
22Final Demand	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23Indirect Taxes	164.4	164.4	0.0	0.0	164.3	164.4	164.3	164.4	164.1	164.5	164.2	164.1	164.4
24Value Added	0.0	0.0	0.0	0.0	130.0	149.4	126.6	142.2	143.7	142.7	138.2	0.0	0.0
25Total	136.9	147.4	132.6	143.7	136.3	147.4	132.7	143.9	145.5	146.4	139.8	145.5	145.7

	14	15	16	17	18	19	20	21.	22	23	24	25
1Commodities Primary	0.0	137.8	136.7	137.8	0.0	0.0	141.9	0.0	0.0	0.0	0.0	136.9
2 Manuf & Util	142.9	146.9	139.4	147.6	150.5	163.0	146.9	0.0	0.0	0.0	0.0	147.4
3 Construction	0.0	131.1	120.7	132.6	132.0	0.0	148.0	0.0	0.0	0.0	0.0	132.6
4 Services	140.3	140.7	0.0	142.1	144.5	139.7	147.6	0.0	0.0	0.0	0.0	143.7
5Industries Primary	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	136.3
6 Manuf & Util	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	147.4
7 Construction	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	132.7
8 Services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	143.9
9Ownership of Dwellings	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	145.5
10Defence	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	146.4	0.0	0.0	146.4
11Other Government	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	139.8	0.0	0.0	139.8
12Cons. Food, Drink & Tob.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	145.5	0.0	0.0	145.5
13 Fuel and Housing	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	145.8	0.0	0.0	145.8
14 Durables	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	143.5	0.0	0.0	143.5
15 Services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	144.7	0.0	0.0	144.7
16Inv Stocks	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	130.2	0.0	0.0	130.2
17 Buildings	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	134.2	0.0	0.0	134.2
18 Plant & Machinery	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	149.3	0.0	0.0	149.3
19 Vehicles	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	169.4	0.0	0.0	169.4
20Exports	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	147.0	0.0	0.0	147.0
21Imports	0.0	143.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	143.9
22Final Demand	0.0	0.0	0.0	0.0	0.0	0.0	0.0	143.9	0.0	164.3	142.4	144.5
23Indirect Taxes	164.3	164.3	0.0	164.4	164.4	164.8	164.4	0.0	0.0	0.0	0.0	164.3
24Value Added	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	142.4
25Total	143.5	144.7	130.2	134.2	149.3	169.4	147.0	143.9	144.5	164.3	142.4	0.0

Table 7. Balanced value accounts with restricted deflators.

1978 PRODUCTION ACCOUNTS CURRENT PRICES

	1	2	3	4	5	6	7	8	9	10	11	12	13
1Commodities Primary	0	0	0	0	2550	11963	413	256	0	2	141	3131	291
2 Manuf & Util	0	0	0	0	4156	59090	7158	12594	0	2801	3615	13728	5342
3 Construction	0	0	0	0	414	597	3998	767	0	239	1101	0	2208
4 Services	0	0	0	0	1909	18081	1629	19445	0	6	2878	8105	1074
5Industries Primary	14572	74	73	126	0	0	0	0	0	0	0	0	0
6 Manuf & Util	50	134537	945	6021	0	0	0	0	0	0	0	0	0
7 Construction	0	0	22038	32	0	0	0	0	0	0	0	0	0
8 Services	8	1871	65	83884	0	0	0	0	0	0	0	0	0
9Ownership of Dwellings	0	0	0	0	0	0	0	0	0	0	0	0	9242
10Defence	0	0	0	0	0	0	0	0	0	0	0	0	0
11Other Government	0	0	0	0	0	0	0	0	0	0	0	0	0
12Cons. Food, Drink & Tob.	0	0	0	0	0	0	0	0	0	0	0	0	0
13 Fuel and Housing	0	0	0	0	0	0	0	0	0	0	0	0	0
14 Durables	0	0	0	0	0	0	0	0	0	0	0	0	0
15 Services	0	0	0	0	0	0	0	0	0	0	0	0	0
16Inv Stocks	0	0	0	0	0	0	0	0	0	0	0	0	0
17 Buildings	0	0	0	0	0	0	0	0	0	0	0	0	0
18 Plant & Machinery	0	0	0	0	0	0	0	0	0	0	0	0	0
19 Vehicles	0	0	0	0	0	0	0	0	0	0	0	0	0
20Exports	0	0	0	0	0	0	0	0	0	0	0	0	0
21Imports	6416	31368	356	7067	0	0	0	0	0	674	0	0	0
22Final Demand	0	0	0	0	0	0	0	0	0	0	0	0	0
23Indirect Taxes	114	873	0	0	-1092	1254	195	2538	920	261	1011	5682	133
24Value Added	0	0	0	0	6907	50567	8676	50226	8322	3329	17862	0	0
25Total	21161	168724	23479	97133	14846	141556	22070	85829	9242	7315	26611	30647	18293

	14	15	16	17	18	19	20	21	22	23	24	25
1Commodities Primary	0	175	40	64	0	0	2128	0	0	0	0	21161
2 Manuf A Util	10943	5761	245	385	9810	3719	29376	0	0	0	0	168729
3 Construction	0	54	241	12862	543	0	448	0	0	0	0	23479
4 Services	7204	20848	0	1575	661	-786	14497	0	0	0	0	97132
5Industries Primary	0	0	0	0	0	0	0	0	0	0	0	14846
6 Manuf & Util	0	0	0	0	0	0	0	0	0	0	0	141554
7 Construction	0	0	0	0	0	0	0	0	0	0	0	22071
8 Servicas	0	0	0	0	0	0	0	0	0	0	0	85829
9Ownership of Dwellings	0	0	0	0	0	0	0	0	0	0	0	9242
10Defence	0	0	0	0	0	0	0	0	7315	0	0	7315
11Other Government	0	0	0	0	0	0	0	0	26611	0	0	26611
12Cons. Food, Drink & Tob.	0	0	0	0	0	0	0	0	30647	0	0	30647
13 Fuel and Housing	0	0	0	0	0	0	0	0	18293	0	0	18293
14 Durables	0	0	0	0	0	0	0	0	19863	0	0	19863
15 Services	0	0	0	0	0	0	0	0	31583	0	0	31583
16Inv Stocks	0	0	0	0	0	0	0	0	528	0	0	528
17 Buildings	0	0	0	0	0	0	0	0	15063	0	0	15063
18 Plant & Machinery	0	0	0	0	0	0	0	0	11163	0	0	11163
19 Vehicles	0	0	0	0	0	0	0	0	3348	0	0	3348
20Exports	0	0	0	0	0	0	0	0	46767	0	0	46767
21Imports	0	354	0	0	0	0	0	0	0	0	0	46237
22Final Demand	0	0	0	0	0	0	0	46240	0	19054	145891	211185
23Indirect Taxes	1715	4389	0	175	148	415	317	0	0	0	0	19053
24Value Added	0	0	0	0	0	0	0	0	0	0	0	145891
25Total	19864	31583	528	15063	11163	3348	46768	46240	211186	19054	145891	0

Table 8. Restricted deflators.

1978 PRODUCTION ACCOUNTS IMPLICIT DEFLATORS

	1	2	3	4	5	6	7	8	9	10	11	12	13
1Commodities Primary	0.0	0.0	0.0	0.0	132.1	132.1	132.1	132.1	0.0	132.1	132.1	132.1	132.1
2 Manuf & Util	0.0	0.0	0.0	0.0	149.5	149.5	149.5	149.5	0.0	149.5	149.5	149.5	149.5
3 Construction	0.0	0.0	0.0	0.0	134.3	134.3	134.3	134.3	0.0	134.3	134.3	0.0	134.3
4 Services	0.0	0.0	0.0	0.0	138.3	138.3	138.3	138.3	0.0	138.3	138.3	138.3	138.3
5Industries Primary	134.1	147.8	134.6	141.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6 Manuf & Util	134.1	147.8	134.6	141.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7 Construction	0.0	0.0	134.6	141.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8 Services	134.1	147.8	134.6	141.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9Ownership of Dwellings	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	142.4
10Defence	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11Other Government	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12Cons. Food, Drink & Tob.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13 Fuel and Housing	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14 Durables	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15 Services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16Inv Stocks	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17 Buildings	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18 Plant & Machinery	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19 Vehicles	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20Exports	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21Imports	130.5	149.3	132.7	136.2	0.0	0.0	0.0	0.0	0.0	144.0	0.0	0.0	0.0
22Final Demand	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23Indirect Taxes	164.3	164.1	0.0	0.0	164.4	164.5	164.3	164.2	163.6	164.5	164.1	163.9	164.4
24Value Added	0.0	0.0	0.0	0.0	129.7	152.5	123.6	140.1	140.3	143.1	138.6	0.0	0.0
25Total	133.1	148.2	134.6	141.0	134.2	147.4	134.6	141.5	142.4	145.9	140.6	146.8	143.0

	14	15	16	17	18	19	20	21	22	23	24	25
1Commodities Primary	0.0	132.1	132.1	132.1	0.0	0.0	143.3	0.0	0.0	0.0	0.0	133.1
2 Manuf & Util	149.5	149.5	149.5	149.5	149.5	149.5	142.1	0.0	0.0	0.0	0.0	148.2
3 Construction	0.0	134.3	134.3	134.3	134.3	0.0	149.6	0.0	0.0	0.0	0.0	134.6
4 Services	138.3	138.3	0.0	138.3	138.3	138.3	158.6	0.0	0.0	0.0	0.0	141.0
5Industries Primary	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	134.2
6 Manuf & Util	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	147.4
7 Construction	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	134.6
8 Services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	141.5
9Ownership of Dwellings	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	142.4
10Defence	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	145.9	0.0	0.0	145.9
11Other Government	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	140.6	0.0	0.0	140.6
12Cons. Food, Drink & Tob.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	146.8	0.0	0.0	146.8
13 Fuel and Housing	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	143.0	0.0	0.0	143.0
14 Durables	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	146.3	0.0	0.0	146.3
15 Services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	143.5	0.0	0.0	143.5
16Inv Stocks	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	140.8	0.0	0.0	140.8
17 Buildings	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	135.4	0.0	0.0	135.4
18 Plant & Machinery	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	148.2	0.0	0.0	148.2
19 Vehicles	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	154.4	0.0	0.0	154.4
20Exports	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	147.1	0.0	0.0	147.1
21Imports	0.0	144.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	144.0
22Final Demand	0.0	0.0	0.0	0.0	0.0	0.0	0.0	144.0	0.0	164.2	142.4	144.5
23Indirect Taxes	164.0	164.5	0.0	164.3	164.4	166.4	164.4	0.0	0.0	0.0	0.0	164.2
24Value Added	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	142.4
25Total	146.3	143.5	140.8	135.4	148.2	154.4	147.1	144.0	144.5	164.2	142.4	0.0

A QUADRATIC PROGRAMMING APPROACH TO DATA RECONCILIATION: CONTRASTS WITH RAS

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The topic of this paper is the reconciliation of related but inconsistent data sets. Perhaps, the best known context where the explicit revision of model structure takes place with a view to the accomodation of external data is in the updating of input-output tables (Allen and Gossling (1975)). For this, and other kindred problems, the solution technique which has widest currency is the RAS method of Stone (1961). The popularity of the RAS method in this respect is due largely to its properties of both preserving signs between prior and posterior estimates and of preserving the zero/non-zero pattern of flows. While for some applications these attributes of the method are desirable, they are, at the same time, unnecessarily restrictive and limited in scope. An altogether more comprehensive approach to the problem would appear to be provided by mathematical programming techniques. To date, however, adoption of mathematical programming methods has been handicapped by the unrealistic nature of the solutions produced by linear programming algorithms (see Minassian (1979)) and by the computational problems posed by non-linear (quadratic) specifications. Whilst there is no plausible way of circumventing the problem of 'corner solutions' in linear programming formulations, the computational problems posed by quadratic programming specifications are more apparent than real. In Harrigan and Buchanan (1982) an algorithm of Hildreth (1957) is used to solve a quadratic programming problem which has 1,290 non-zero variables, 2,580 bounds on variables and 104 constraints. Here, we summarise the problem statement and solution technique described in Harrigan and Buchanan and illustrate the concepts with an application to Scottish consumption data.

DATA RECONCILIATION: A QUADRATIC PROGRAMMING FORMULATION

The problem which is under consideration may be expressed as:

$$(1) \quad z = \min \tfrac{1}{2} \sum_{ij} (b_{ij}-\tilde{b}_{ij})^2 . w_{ij} + \sum_{j} (u_j-\tilde{u}_j)^2 . w_j + \sum_{i} (v_i-\tilde{v}_i)^2 . w_i + \sum_{T} (c_T-\tilde{c}_T)^2 . w_T$$

st

$$\underline{u}_j \leq \sum_i b_{ij} \leq \overline{u}_j \qquad \forall_j$$

$$\underline{v}_i \leq \sum_j b_{ij} \leq \overline{v}_i \qquad \forall_i$$

$$\underline{c}_T \leq \sum_{ij\epsilon T} b_{ij} \leq \overline{c}_T \qquad \forall_T$$

$$\underline{b}_{ij} \leq b_{ij} \leq \overline{b}_{ij} \qquad \forall_{ij}$$

where the $b_{ij's} \epsilon \{b_{ij}\}$ are the elements of the matrix to be estimated, the u_j's are the outcome column sums of $\{b_{ij}\}$ and the v_j's are the outcome row sums of $\{b_{ij}\}$. A tilde ($\sim$) in (1) represents an initial best estimate of a variable; an upper bar represents its upper bound value and a lower bar, its lower bound value. The value, w, represents objective function weights and the indices, T, refer to the linear aggregates of the elements of $\{b_{ij}\}$, other than row and column sums, for which exogeneous data are available.

The following characteristics differentiate (1) from the RAS formulation of Stone (op cit):

(a) The use of a quadratic loss minimand as opposed to a minimum information gain entropy function (Theil (1967))

(b) The (optional) inclusion of constraints in addition to row and column sum constraints.

(c) The (optional) use of inequality rather than equality operators.

(d) The (optional) imposition of lower and upper bounds on all variables (other than ϕ and min (u_j, v_i)).

(e) The (optional) treatment of linear aggregates as variables rather than as constants.

(f) An additive rather than multiplicative structural form solution.

While the generalised RAS method of Lecomber (1964) and Allen and Lecomber (1975) can accomodate (e) and permits variables to be 'more or less' subject to biproportional balancing, it allows neither specific lower or upper bounds to be imposed on variables nor the ranging of constraints. Additionally, the generalised RAS method can only encompass row and column sum constraints and is not suited for use where information on aggregates of elements in the interior of the matrix is available.

To the extent that (b) - (e) extend and subsume the capabilities of RAS, (1) is a more powerful statement of a data reconciliation problem. It remains therefore to analyse the significance of properties (a) and (f). The use of a quadratic rather than entropy minimand in (1) is dictated solely by computational considerations. The extremal conditions on a quadratic programme yields a linear system of equations whereas the extremal conditions on a programme with an entropy minimand yields a non-linear system of equations. To date, the solution of the linear system has proved more tractable than solution of its non-linear entropy counterpart, though progress is being made with the latter (Ericksson (1981)). Additionally, since the leading quadratic term of the usual Taylor series expansion to the information gain function is in fact the well known χ^2, it would appear that in practical application quadratic and entropy solutions are in fact likely to closely resemble one another. The use of an explicit quadratic loss function in (1) is not, therefore, as restrictive as at first it may seem.

It is the use of a quadratic rather than entropy minimand which yields the differences in structural form solutions between (1) and the entropy based RAS. The well known RAS relationship connecting base and updated values has been interpreted in terms of the operation of row substitution and column fabrication effects and has been used extensively for forecasting (Lecomber(1975)). If one wishes to make parallel assumptions, the quadratic programming solution can also be used in this way: the lagrangeans on the row constraints acting as substitution multipliers, the lagrangeans on the columns as fabrication multipliers, with the additional lagrangeans capturing element specific aspects of coefficient change (see Harrigan and Buchanan (op cit)). Since the economic rationale underpinning such a procedure is just as weak as that underlying RAS projections we will pursue this line of inquiry no further for the present.

The formulation (1) is superior to RAS and generalised RAS in circumstances both where information is scarce and of poor quality and in circumstances where extensive and good quality information is available. In the former case, the ability to range constraints in (1) means that the variable reliability of data may be explicitly catered for. Initial best estimates of relevant aggregates may be imposed and the posterior distribution of values around this estimate controlled through the use of weights and lower and upper bounds. Ranges on variables should vary inversely with the 'certainty' attached to an estimate, while the weights which penalise solution deviations from best estimates should be directly related to the same. While the notion of 'certainty' in this context is somewhat nebulous it is better that analysts be forced to make explicit their data judgements rather than veiling them in a series of ad hoc and ex-post manipulations such as those which normally characterise the derivation of balanced controls.

Where data are in abundance, (1) imparts to the analyst much greater freedom in determining the outcome solution structure of $\{b_{ij}\}$ than does RAS or generalised RAS. For example, with (1), there exists scope to limit precisely the values taken both by individual cells and aggregates of cells in the interior of the matrix. This presents the possibility for use of information from a wide variety of official and unofficial sources. As Lamel et al (1974) and Morrison and Thuman (1980) point out, data on aggregates of elements may be gleaned from production census, industry studies and accounts, industry research councils, chambers of commerce, and from 'ex-ante' surveys in the mould of Fisher (1975). The formulation (1), however, generalises the methods of Lamel et al and Morrison and Thuman to the extent that it uses inequality rather than equality constraints.

The motive force underpinning (1) is more than adequately summarised by the sentiments of Lecomber (1975):

> "Central to all these suggestions is the idea that as far as possible all available information should be used, even that which is not fully reliable or appropriate and that, where possible, a subjective assessment of the reliability of the information should be taken into account in a more or less formal way in the estimation procedure. <u>If all information has been taken into account, then there will be no occasion for rejecting the results as implausible, for such a judgement must depend on further information so far ignored.</u>" (op cit p.22).

The formulation (1) more nearly attains this desideratum than the existent alternatives.

In the next section some brief words on the solution of (1) will be made.

LARGE SCALE QUADRATIC PROGRAMMING SOLUTION STRATEGIES

It is somewhat ironical that while economists and other social scientists have been bemoaning the difficulties involved in solving large scale quadratic programmes, an efficient algorithm for this purpose has existed for the last twenty-five years. One reason for the relative obscurity of the altorithm of Hildreth (op.cit.) would seem to be that the problems which interested mathematicians were better solved by pivoting methods, which lead to a direct solution, than by asympototic interative methods such as Hildreth's. Recently, Bachem and Korte (1978) and Cottle (1981) have rediscovered Hildreth's algorithm and Cottle and Duvall (1982) have elaborated a solution method which appears to subsume iterative matrix balancing techniques as a special case - though they don't claim this property of their method. Another potentially interesting method of solution lies in the use of linear complementary problem algorithms. Research on this latter approach is now underway at the University of Strathclyde. Here we will confine attention to Hildreth's method.

The problem (1) can be rewritten as

$$(2)\quad z = \min \tfrac{1}{2} b' Wb + b'q$$
$$st \quad Ab = a$$
$$\ell \leq b \leq u$$

where W is a positive definite weighting matrix, q is a vector of constants, a is a vector of right hand side values, ℓ and u are vectors of lower and upper bounds, respectively, and b is a vector of solution values. The details of the transition from (1) to (2) are given in Harrigan and Buchanan (op.cit.). To solve (2) the strategy is to derive its conjugate dual and solve for the lagrangean multipliers in the dual. Substitution of these multipliers in

(3) $\quad b = Pa + PAW^{-1} (q - \mu + \eta) - W^{-1} (q - \mu + \eta) \mid \mu, \eta \geq 0$

yields the solution for b, where

(4) $\quad P = W^{-1} A' D^{-1}$

(5) $\quad D = A W^{-1} A'$

and μ and η are the Lagrangean multipliers on the lower and upper bounds of (2). The interested reader is again referred to Harrigan and Buchanan (op.cit.) for a derivation of (3).

To obtain μ and η involves the solution of a system of linear inequalities. Hildreth's method proceeds to solve this sytem by a modified Gauss Seidel technique where each Gauss Seidel iterate is projected back onto the boundary of the feasible region if it violates the non-negativity constraints on μ and η. Covergence of the algorithm to the correct solution is assured (see Hildreth (op.cit.)). Bachem and Korte's and Cottle's algorithms utilise an identical rationale but solve the system using block Gauss-Seidel and block over-relaxation techniques, respectively. These latter techniques are likely to lead to accelerated covergence. It should be noted, however, that Bachem and Korte, Cottle, Cottle and Duvall (op.cits.) discuss their algorithms only in relation to capacitated transportation problems. The introduction of ranged inqualities in (1) requires modification of their problem statement and corresponding algebra.

With regard to software, FORTRAN 77 code is available both for the generation of matrices and data required for the problem and for the solution of the problem itself. The software will be published shortly in Harrigan (1982). Early experiences have shown it to be robust though it is perhaps not yet as efficient as it might be. The code makes full use of sparse matrix techniques and for a problem of the size quoted in the introduction can be expected to use less than 50K of store and take an average about 35 CPU seconds to solve on an ICL 2988; this time should be divided by a factor of between two to three for comparison with machines such as an ICL 2980 or IBM 370. Other, 'pivoting' codes are presently being tested by a colleague of the author and initial results indicate that these are also efficient.

In the next section below a short illustration of a quadratic programming data reconciliation problem will be given.

THE ESTIMATION OF A SCOTTISH CONSUMPTION MATRIX

There exists an extensive literature on the estimation of consumption functions; no attempt will be made here either to review this literature or to add to it. The intention is simply to illustrate one practical application in which the problem (1) has been defined and solved.

In the most recent version of the Scottish medium term model there are forty-three intermediate sectors and twelve consumption categories. Strictly speaking there are eleven bona fide consumption categories plus a taxes less subsidies column. This last column can, of course, contain negative elements.

The basic data problem faced in the estimation of a Scottish consumption matrix was that no disaggregation of flows by commodity existed for the different consumption categories. Controls were available, however, for consumption by commodity and consumption by category for 1973. While these controls yielded total regional consumption estimates that were close to each other, a small discrepancy remained. Other information which was available included a United Kingdom consumption matrix for 1974, and knowledge of one or two elements of the Scottish matrix.

As an initial estimate of the unknown Scottish matrix, a column proportioned UK matrix was derived and applied to the consumption by category controls for Scotland. This yielded a matrix which resulted in a serious mismatching of consumption flows with the known commodity controls. Revisions to the estimates then took place to try and reconcile these differences. The estimates which emerged from this process, however, were still inconsistent with the known controls. At this stage, the programming algorithm was employed.

The information used to constrain the estimation procedure were the commodity and consumption category controls. Additionally, sign constraints were imposed on individual elements so that the prior estimated negative values could not become non-negative and the prior estimated positive values could not be revised to values less than zero in the solution. Ranges of $\pm$ 2.5% were applied to all controls to ensure feasibility. Without these ranges, row and column sum controls would have been inconsistent.

In the sense that the results generated by the programming algorithm were 'plausible', it was judged to be a success in the estimation of the Scottish consumption matrix. No actual benchmark existed against which accuracy could be compared. It converged in under fifty interations and took less than 5 CPU seconds on an ICL 2988. It is worthwhile recalling that the RAS method could not have handled the problem: in the first instance, RAS will not coverge where there are negative elements, and secondly, some ad hoc adjustment to the controls would have been necessary to make the problem consistent.

CONCLUSIONS

This short paper has advocated the use of quadratic programming methods of the solution of data matrix problems. The emphasis has been very much on a comparison of the capabilities of the method vis à vis RAS. A more technical discussion of the issues involved is given in Harrigan and Buchanan (op.cit.) and in Harrigan (op.cit.) relevant FORTRAN 77 codes are detailed. Preliminary numerical experience with solution algorithms have been satisfactory and suggest problems with up to 5,000 variables and 1,000 constraints may be solved in most time sharing computing environments. Future work will take place with a view to ascertaining the sensitivity of algorithm results to different 'baskets' of information and it is also hoped to develop an entropy analogue of (1).

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USE OF THE INFORMATION SYSTEM FOR SYSTEMS DESIGN AND ANALYSIS IN INPUT–OUTPUT MODELLING

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1. INTRODUCTION

Let us consider a model of an economic system in which we have controlling levels consisting of various subjects that organize the production and consumption processes. These subjects use different classifications of products. The classifications of products are in mutual relations.

Let us consider the subject at the given controlling level. For this subject we can formulate a relation for the distribution of its production and relations for consumption of the products that are produced in the system and products that are imported into the system and are not produced in the system. We can divide the consumption into consumption depending on production and consumption that does not depend on production. It is possible to determine the part of the consumption that depends on production by using norms of the consumption per unit of production. While seeking equilibrium in this model the use of procedures of agregation and disaggregation is suitable. These procedures are described e.g. in the article [3]. I have dealt with special computation procedure within a simplified system in article [5].

In the process of preparing and then utilining the system of models in management it is always necessary to have information concerning the items of individual classifications of products, the relations of items in different classifications, lists of subjects according to management levels, information about relations between subjects on different management levels and further data corresponding with items of classification of products, subjects and other elements of the system. This information makes aggregation and disaggregation procedures and other procedures concerning the utilization of a given system of models easy.

While creating and filling the given system of models we can manage without computer as long as the classifications of products contain only some tens of items and the number of considered subjects is small. If, however, some classifications of products contain hundreds of items and the number of subjects on the lowest managing level has the order of hundreds, then it is suitable to make the model more concrete and maintain it by using computer.

Designing the concrete system of models however makes heavy demands on

- management and organisation work
- design procedure documentation
- documentation on system structure and on changes during designing
- following the consistency of links etc.

Therefore I have concentrated on the utilization of computer in this area. I have designed the information system that enables making analyses and gaining necessary information for any system during its designing. This system is tested on the EC 1040 computer. The programs are written in PL/I language and so the system can be implemented on any computer which has the PL/I compiler. It is possible to code the programs of the system in another programming language. This system being open enables further generalisation and the introduction of further functions without outstanding changes.

I believe it is possible to use this system in the project INFORUM, too.

The idea of my approach is based on the fact that the designer first defines a model of a system and then defines and maintaines the successivelly in steps concrete model in the framework of the model of the system.

2. MODEL OF A SYSTEM

2.1 INTRODUCTION

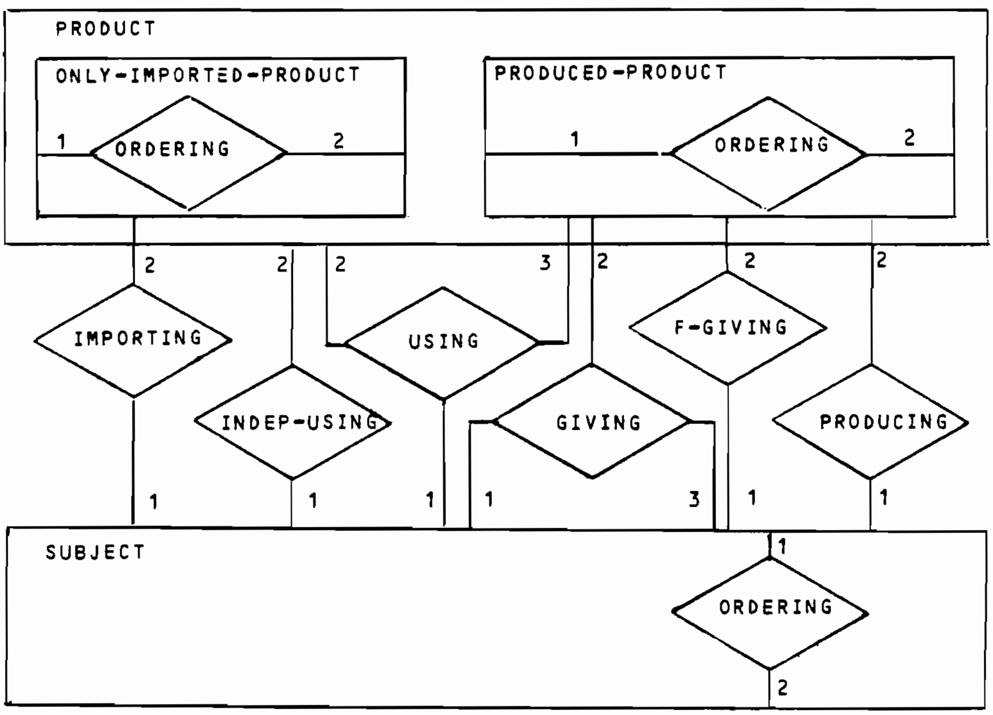
When defining a system, model types of system elements are introduced which enable accumulating of elements will selected identical properties into groups. Sometimes it is suitable to introduce sets of element types. Types of descriptions by which the meaning and function of system elements can be described are also introduced. Different relations can be between elements in the concrete system. The types of these relations are introduced when the model of the system is defined. Each relation type may have several variants which differ in the order of operands. E.g. the type or relation CONSISTING has the variants 1 CONSISTS OF 2 and 2 IS CONTAINED IN 1. If the number of operands of a relation is larger than two, other types can be derived from a given type of relation by leaving out some of the operands.

2.2 STATEMENTS FOR DETERMINING A SYSTEM MODEL

The types of elements, types of descriptions, sets of types of elements and types of relations are defined by names in statements DEFF, DEFT, DEFS and DEFR. While defining variants of relations, words are determined, which are used for the creation, maintenance and utilization of information about models /reserved words/. More names and reserved words /synonyms/ can be used with one meaning and some reserved words can be omitted /optional reserved words/. Names and words /further only words/ are introduced in the information about a model for the first time when they are used in the text which contains statements. During the introduction of words, synonyms can be stated directly after the word in parantheses. Individual synonyms are separated by a comma. The use of synonyma in statements is equivalent to the use of words. In the output texts the words before the parantheses are used. The introduced word can be used in the further text.

2.2 EXAMPLES OF A MODEL

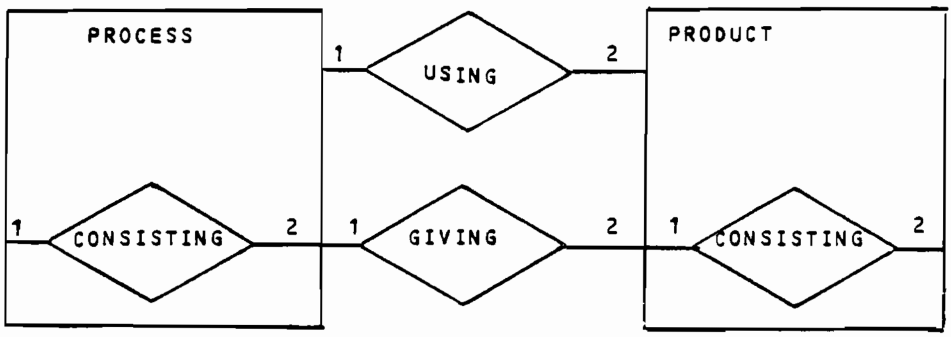
Let us illustrate a model of a system on examples from the region of input-output analysis. We shall begin with a diagram. An oblong will mean a type of object or a set of types of objects. Relations with their variants will be represented by symbols ◇ and lines from one type of object /or set of types/ to another. Numbers denote the order of operands in the relations.
In the following diagram the model verbally described in part 1 is represented.



The statements for determining this model are

```
*MODEL;
DEFE: ONLY_IMPORTED_PRODUCT (IPROD);
      PRODUCED_PRODUCT  (PR, GOODS);
      SUBJECT (ENTERPRICE, SUBJ);
DEFS: PRODUCT (PROD)= PR, IPROD;
DEFR:
      PRODUCING <SUBJ, PR> =
         1 PRODUCES  2 ;
      F_GIVING <SUBJ,PR> =
         1 GIVES 2 <FOR> FINAL (F);
      GIVING <SUBJ, PR, SUBJ> =
         1 GIVES 2 TO 3 ,
         3 OBTAINS 2 FROM 1 ,
      USING <SUBJ, PROD, PR> =
         1 USES 2 <FOR>  PRODUCTION (PRION) 3;
      INDEP_USING <SUBJ, PROD> =
         1 USES 2  INDEPEDENTLY_AT_PRODUCTION  ;
      IMPORTING <SUBJ, IPROD> =
         1 IMPORTS (IMP)2 ,
         2 <IS>  IMPORTED (IMPD) 1 ;
      ORDERING  <IPROD, IPROD> <PR, PR> <SUBJ, SUBJ> =
         1 CONSISTS ( CSTS ) <OF> 2 ,
         2 <IS> CONTAINED (CNTD)<IN> 1 ;
*END;
```

The further description concerns the simpler example of the input output model. We shall consider the diagram.



The statements for determining the model are

```
*MODEL,
DEFE:  PROCESS (PROC);
       PRODUCT (GOODS,PR);
DEFR:
       USING <PROC,PR> =
         1   USES   2 ,
         2  <IS> USED  <IN>  1 ;
       GIVING <PROC,PR> =
         1    GIVES   2 ,
         2    <IS> GIVEN  <BY>  1 ;
       CONSISTING <PR,PR> <PROC,PROC> =
         1    CONSISTS   (CSTS) <OF>  2 ,
         2     <IS>  CONTAINED  (CNTD) <IN>  1 ;
*END;
```

In the framework of this model we shall make a further explanation.

3. A SYSTEM

3.1 INTRODUCTION

A concrete system is determined by its elements and the relations between elements. Elements are determined by a name or names which can be substituted /synonyms/. Elements are associated with a type which was defined when the model was being defined. Properties of function and states of elements can be stored as descriptive texts.

Elements can be in mutual relationships the types of which were defined during defining the model. Each relation can be supplemented by some text that is reprezented by a set of symbols.

A system is usually created in steps. First the basic elements and relations among them are defined. If they are contained in the model, states of elements are determined. Then the defined states, elements and relations are determined in more detail either by refining or complementing further facts. This refining and complementing triggers creating new relations and elements. So that new elements and relations can then be complemented or perhaps some elements or relations deleted or modified according to new knowledge. At the end of this process the elements of the system should be divided into elementary further indivisible /on the given level of detail/ elements and all relations between elements should be registered.

The procedure by which introduction and maintenance of information on the system which is being designed is ensured consists of statements the form of which is determined standardly.

Statements are formed with the following components
- words determining statements
- compulsory and arbitrary
  words which were determined when the model was being defined
- names of elements
- descriptions
- texts asigned to relations
- separators
- notes

Besides defining objects, relations and descriptive texts the introduction of further operations appears suitable:
- deleting an element of a given name
- joining elements into one element
- deleting the synonym of an element
- deleting a certain type of descriptive texts

- deleting relations of a certain type
- deleting a relation
- asigning a text to a relation
- deleting the text of a relation

Deleting an object, a relation or a descriptive text is performed by marking the appropriate record of the appropriate data file. The data, however, stay recorded in the computer. When a large number of data are deleted it is suitable to reorganize them so that deleted data are not recorded any more.

3.2 EXAMPLE

At the end of part 2 we defined the model for the system we are considering. Now let us start to define a concrete system by the statements

```
*DEFINE;
PRODUCTION: PROC;
   CSTS INDUSTRIAL-PRODUCTION    (IP),
        NONINDUSTRIAL-PRODUCTION(NP) ;
CONSUMPTION: PROC;
   CSTS CONSUMPTION-FOR-PRODUCTION (CP),
        OTHER-CONSUMPTION  (OC);
CP: CSTS  IP,NP;
IMPORT: PROC;
IP:     GIVES INDUSTRIAL-PRODUCTS     IPROD  <'1000'>  ;
NP:     GIVES NONINDUSTRIAL-PRODUCTS  NPROD  <'500'>   ;
IMPORT: GIVES IMPORTED-PRODUCTS       IMP    <'300'>   ;
IMP:    CSTS  IPROD, NPROD    <'100', '200'>;
IP:     USES IPROD, NPROD     <'500', '300'>;
NP:     USES IPROD, NPROD     <'200', '100'>;
OC:     USES IPROD, NPROD     <'400', '300'>;
*END,
```

It is possible to demonstrate the concrete economics model mentioned above by the following table:

	PRODUCTION		IMPORT	CONSUMPTION		
				CP		OC
	IP	NP		IP	NP	
IPROD	1000		100	500	200	400
NPROD		500	200	300	100	300

In the next step of modelling we are able to define the distribution of the OTHER-CONSUMPTION by the statements:

```
*DEFINE;
OC: CSTS  INDIVIDUAL-CONSUMPTION  (IC),
          PUBLIC-CONSUMPTION  (PC),
          INVESTMENTS  (INV),
          EXPORTS  (EXP),
          OTHER-CONSUMPT  (OTC);
*END;
```

In this way we can continue in the process of designing the concrete model in the framework of the model from the end of part 2.

4. OUTPUT REPORTS

4.1 SELECTING

The introduced information system can be printed in different reports. Reports usually use selected date. Elements are chosen in two ways. In the first further elements are added to those which have been already chosen, in the second the relevant elements are selected from those ones chosen before. There are three types of retrieval: direct retrieval, retrieval on type basis, retrieval on relation basis.

The choice of types of descriptive texts is realized by the statement CHOOSE-T and the choice of types of relations by the statement CHOOSE-R. Application of these statements cancels the selection performed before.

4.2 REPORTS

From information about the system well-arranged reports may be obtained, which make analysis easy end give documentation on the progress of designing. Next we present some reports to give clear image of which reports it is possible to obtain.

An example of the ELEMENTS report is shown in Table 1 and an example of the RELATIONS report /after the first step/ is given in Table 2.

Table 1. An example of the ELEMENTS report.

NAME	TYPE	SYNONYMS
1 CONSUMPTION	PROCESS	
2 CONSUMPTION-FOR-PRODUCTION	PROCESS	CP
3 EXPORTS	PROCESS	EXP
4 IMPORT	PROCESS	
5 IMPORTED-PRODUCTS	PRODUCT	IMP
6 INDIVIDUAL-CONSUMPTION	PROCESS	IC
7 INDUSTRIAL-PRODUCTION	PROCESS	IP
8 INDUSTRIAL-PRODUCTS	PRODUCT	IPROD
9 INVESTMENTS	PROCESS	INV
10 OTHER-CONSUMPT	PROCESS	OTC
11 OTHER-CONSUMPTION	PROCESS	OC
12 NONINDUSTRIAL-PRODUCTION	PROCESS	NP
13 NONINDUSTRIAL-PRODUCTS	PRODUCT	NPROD
14 PRODUCTION	PROCESS	
15 PUBLIC-CONSUMPTION	PROCESS	PC

Table 2. An example of the RELATIONS report /after the first step/.

RELATION: CONSISTING

VARIANTS:

1 CONSISTS OF 2

2 IS CONTAINED IN 1

LIST

1	1	PRODUCTION	2	INDUSTRIAL-PRODUCTION
2	1	PRODUCTION	2	NONINDUSTRIAL-PRODUCTION
3	1	CONSUMPTION	2	CONSUMPTION-FOR-PRODUCTION
4	1	CONSUMPTION	2	OTHER-CONSUMPTION
5	1	CONSUMPTION-FOR-PRODUCTION	2	INDUSTRIAL-PRODUCTION
6	1	CONSUMPTION-FOR-PRODUCTION	2	NONINDUSTRIAL-PRODUCTION
7	1	IMPORT	2	INDUSTRIAL-PRODUCTS
	TEXT: 400			
8	1	IMPORT	2	NONINDUSTRIAL-PRODUCTS
	TEXT: 200			

STRUCTURE REPORT

Let us consider the set R of ordered pairs of elements (x_i, x_j). To given element x_1, for which there does not exist a pair $(x_o, x_1) \in R$, let us construct a sequence of elements $x_1, x_2, \ldots, x_n, \ldots$, for which $(x_j, x_{j+1}) \in R$. To element x_i of this sequence we shall asign a number of level i.

If more than one sequence begins with the element x_1, which has a number of level 1, then this data item is stated in the report once only. If more than one sequence begin with the data x_1 and x_2 with numbers of levels 1 and 2 then the data item x_2 with a number of level 2 is stated once only etc. The order of elements of the same level within one set of sequences beginning with the element x_1 and the order of the elements x_1 is determined by the alphabetical order of the names of elements.

The number of considered levels can be limited.

The set of pairs of elements R is determined by the parameter VR. We present the use of this parameter in the example. Let us consider the first variant of the relation which has been defined by the direction

1 CONSISTS OF 2

The first operand of this relations is the first operand of the relation being written and the second operand is the second operand of the relation being written.

The parameter

VR = CONSISTING (1) <1,2>

gives the result that the first operand of the relation being written is taken as the first member of the pair and the second member of the pair is taken as the second operand of the relation being written. The example of the STRUCTURE report if we choose elements that have the type PROCESS is given in Table 3.

Table 3. An example of the STRUCTURE report.

```
RELATIONS
1 CONSISTING   (1)  <1,2>
  1 CONSISTS 2
STRUCTURES

NAMES                                            TYPE

   1 CONSUMPTION                                 PROCESS
     2 CONSUMPTION-FOR-PRODUCTION                PROCESS
        3 INDUSTRIAL-PRODUCTION                  PROCESS
        3 NONINDUSTRIAL-PRODUCTION               PROCESS
     2  OTHER-CONSUMPTION                        PROCESS
        3 EXPORTS                                PROCESS
        3 INDIVIDUAL-CONSUMPTION                 PROCESS
        3 INVESTMENTS                            PROCESS
        3 OTHER-CONSUMPT                         PROCESS
        3 PUBLIC-CONSUMPTION                     PROCESS

   1 PRODUCTION                                  PROCESS
     2 INDUSTRIAL-PRODUCTION                     PROCESS
     2 NONINDUSTRIAL-PRODUCTION                  PROCESS
```

Table 4. An example of the CONCATENATION ("CHAINING") report.

```
RELATIONS 1
1 USING(2)<1,2>
  2 IS USED IN 1

RELATIONS 2
1 GIVING(1)<1,2>
  1 GIVES 2

CHAINING
*****                        IMPORT                      *****
                                                         1 IMPORTED-PRODUCTS
*****                        INDUSTRIAL-PRODUCTION       *****
1 INDUSTRIAL-PRODUCTS                                    1 INDUSTRIAL-PRODUCTS
2 NONINDUSTRIAL-PRODUCTS

*****                        NONINDUSTRIAL-PRODUCTION    *****
1 INDUSTRIAL-PRODUCTS                                    1 NONINDUSTRIAL-PRODUCTS
2 NONINDUSTRIAL-PRODUCTS

*****                        OTHER-CONSUMPTION           *****
1 INDUSTRIAL-PRODUCTS
2 NONINDUSTRIAL-PRODUCTS
```

CONCATENATION REPORT

Let us consider the set R_1 of ordered pairs (x,y) and the set R_2 of ordered pairs (y,z). To given element y we shall determine such elements $x_1^y,\ldots,x_s^y$ that $(x_i^y, y)\in R_1$ and such elements $z_1^y,\ldots,z_t^y$ that $(y,z_i^y)\in R_2$. Let us assume that the identifiers of the elements x_i^y $i=1,\ldots,s$ are in the alphabetical order and that the identifiers of the elements z_i^y $i=1,\ldots,t$ are also in the alphabetical order, too. We shall order the elements y alphabetically.

The specification of the sets R1 and R2 is the same as the specification of the set R in STRUCTURES report.

If we choose the relation USING and GIVING and the parameter VR1 = USING(2)<1,2> and VR2 = GIVING (1)<1,2> in our example we obtain the report shown in Table 4.

REMAINING REPORTS

From the database information it is possible to obtain other reports.

5. CONCLUSION

From knowledge experienced during the use of the system given above it is possible to propose other suggestions for extending of its functions. It is possible to consider about an interface to other systems that deal with the computation aspects of economic models, e.g. the SLIMFORP system.

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