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ABSTRACT

The treatment of foreign trade has a great influence on the results obtained from multisectoral macroeconomic models. This manifests itself clearly in the problem of overspecialized solutions, which arises in most of the models currently in use. This unwanted phenomenon is treated differently in the two main classes of models: programming models and general equilibrium models.

The paper discusses the theoretical and methodological problems related to this issue using a special comparative framework, in terms both of the above two classes of applied models and in terms of *laissez-faire* equilibrium and planner's optimum. Attention is focussed on alternative export specifications and optimum tariff problems. The optimum tariff problem is discussed from the point of view of both large (the usual case) and small open economies. The argument is illustrated by numerical results based on two models of the Hungarian economy.

MULTISECTORAL MACROECONOMIC MODELS AND OPTIMUM TARIFFS

1. INTRODUCTION

Multisectoral planning or economic policy analysis models can be roughly classified into three main categories: statistical (econometric) input-output models, mathematical programming models, and applied (computable) general equilibrium models. This classification is, however, loosing more and more of its relevance, because there is a strong tendency toward integrating and combining the various approaches in complex models. Nevertheless, in this paper we adopt the above classification and focus our attention on the second and third classes of model in a special comparative framework.

Planning models of the mathematical (linear) programming type are well known and have been extensively applied to development planning problems throughout the world. No detailed references are needed, or will be given here, thus avoiding the dangers of overselectivity or bias. Computable general equilibrium (CGE) modeling is a relatively recent development, although its roots go back at least twenty-five years). Despite the early pioneering work of Johansen (1959), who constructed a linearized multisectoral general equilibrium growth model, there was no real breakthrough in the field until the second half of the 1970s.* To date, numerous papers, journal articles, and books have reported on such modeling efforts in various countries and have described applications to a wide range of economic development issues. Without being in any way exhaustive, we refer here to some of the more concentrated or sustained efforts. Thus, for example, we have the work associated with the World Bank as represented by Adelman and Robinson (1978) who introduced the term "CGE", Taylor *et al.* (1980), Ginsburgh and Waelbroeck (1981), and Dervis, de Melo, and Robinson (1983); then there is the work of the IMPACT project in Australia, as outlined, for example, by Dixon *et al.* (1977); and finally the research done at IIASA, as reported by Bergman and Pór (1980, 1983), Kelley and Williamson (1980), Karlström (1980), Shishido (1981), and Zalai (1982a, 1982b), among others.

CGE models (CGEMs) closely follow the neoclassical general equilibrium tradition and are usually interpreted as models that imitate market behavior. The estimation of many parameters of these models is also based on indirect methods derived from neoclassical economic theory. It is interesting to note in this context that the appearance of CGEMs seems to have undermined the '*detente*' between macroeconomic modelers in East and West, which was a marked characteristic of the era when linear input-output and programming models were almost exclusively used. In my view, the CGEMs have just made more clearly visible some profound conceptual and methodological differences between modeling in East and West; these differences were there all along, but were hidden by their common mathematical structure (see Table 1 for a condensed summary of some major differences).

Modelers from centrally planned countries have usually viewed as harmless intellectual games the efforts of their western colleagues to give sophisticated theoretical respectability or

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*One of the problems that made full-scale application of nonlinear general equilibrium models infeasible for so long was the lack of efficient solution algorithms. Now, however, there are several solution algorithms available for general equilibrium models, some of them tailored to specific models. See, for example, Scarf and Hansen (1973), Manne, Chao and Wilson (1980), Keyzer (1982), and Bergman and Pór (1983).

Table 1. Major features of computable general equilibrium (CGEM) and optimal planning (OPM) models.

Aspect	CGEM	OPM
Base of comparison	Observed state (counter-factual simulation)	Provisional plan (counter-plan simulation)
Characteristic types of variables	Real, price, cost, financial	Mainly real, some financial assets
Functional relation-ships based on	Neoclassical economics (e.g., production functions, demand functions)	Pragmatic considerations (e.g., fixed norms, struc-tures)
Data bases	Statistics (*ex post*)	Plan information (*ex ante*)
Parameter estimation techniques	Direct and indirect econometric estimation (mostly single-point data estimates)	Mixed methods, heavy reliance on experts from various fields
Decision criteria	Individual profit and utility maximization	Overall consistency and efficiency
Special allocational limits reflected by	Varying rates-of-return require-ments, taxes (indirect)	Special bounds on variables (direct)
Mathematical form	Nonlinear equation system, locally unique solutions (assumed)	Linear inequalities with alternative overall objec-tive functions

interpretation to what the socialist planners considered equations or inequalities dictated by pragmatic commonsense considerations. Indeed, this explains partly why modelers in the East have completely ignored the CGEMs. They were seen as the result of taking these "games" to extremes. Moreover, the models are squarely based in mainstream Western economics, which has not only come in for criticism in both East and West as largely irrelevant theory but is also considered as completely alien and ideologically adversary to socialist (Marxist) economic theory.

I have tried to show in some related papers* that the conceptual gaps are not as wide as they may appear. Much of the neoclassical "mist" surrounding CGEMs can be dispelled, and most of the models can be discussed in purely pragmatic terms as natural extensions of structurally similar programming models. Indeed, since all of these macroeconomic models deal with "economic agents" (sectors, large consumer groups, etc.), which are collections of agents of individual decision-making authority, one may even question the theoretical validity of interpreting CGE models in terms of the adopted (neoclassical) *microeconomic* theory. Thus, the purely pragmatic reinterpretation is not only feasible but might even be viewed as desirable. In this respect, my attitude towards the CGEMs is markedly different from that of my colleagues in the West. They seem to follow just the opposite line of reasoning and try to see linear programming models as primitive, early examples of Walrasian general equilibrium models.

This paper is basically concerned with these and related issues in the specific context of foreign trade as it is typically treated in the multisectoral macroeconomic models discussed above. We start (in Section 2) by discussing the problem of overspecialization and how it is dealt with in different types of models. Our main aim is to show examples (rigid versus flexible bounds) of what we mean by the pragmatic reinterpretation of some elements of the neoclassical-based CGEMs. Section 3 will be devoted to the problem of optimum tariff. This well-known

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*See the list of references. This paper draws heavily on Zalai (1982b).

theoretical problem seems to have been completely overlooked in applied general equilibrium models. In our special comparative framework (planner's optimum versus *laissez-faire* equilibrium) it clearly shows up as a possible qualitative difference between optimal programming models and CGEMs. We will argue that the usual adoption of Armington's (1969) assumption in CGE models turns otherwise "small" (in the usual sense) economies into "large" ones (in the sense of international trade theory). This not only gives rise naturally to the problem of optimum tariffs, but it also brings in terms-of-trade effects that can hardly be justified on empirical grounds. It will be shown that slight modification of the neoclassical model can lead to an optimum-tariff kind of phenomenon even in theoretically "small" economies, which does not seem to have been discussed in the literature to date. Finally, Section 4 provides some numerical illustrations of the theoretical arguments. These are based on two models of the Hungarian economy*. The simulation results are clear examples of the order of magnitude of effects introduced into the macroeconomic models by assuming less than perfectly elastic export demand.

2. FOREIGN TRADE IN MULTISECTORAL MODELS: RIGID VERSUS FLEXIBLE BOUNDS

The pure, 'theoretical' resource allocation constraints of most multisectoral macroeconomic models currently in use tend to produce highly overspecialized solutions. Overspecialization manifests itself in the existence of only a small number of producing and/or exporting sectors and little or no intrasectoral trade. In view of the fact that even in the most detailed models, the sectors represent rather aggregated product groups, such overspecialized solutions cannot be defended on practical grounds. Thus, model builders tried to find ways of avoiding unrealistic solutions.

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*The models have been developed under the joint auspicies of IIASA and the Hungarian Planning Office. The author gratefully acknowledges the valuable assistance of his colleagues and, in particular, that of Gy. Boda, I. Csekö, A. Pór, J. Sivák, and A. Tihanyi.

In planning models of the linear programming type the main means of preventing overspecialization is the extensive use of special constraints on individual variables or groups of variables. The use of such bounds in planning models in not universally approved.* One of the main criticisms is that they are *ad hoc*, arbitrary restrictions, and moreover they can also distort the shadow prices. An alternative approach favored by some model builders involves the introduction of more complicated nonlinear relationships into the model, perhaps in a piecewise linear fashion.

It is undoubtedly true that the individual constraints account for the inadequacy of the chosen model, reflecting our lack of knowledge and modeling ability. On the other hand, however, this problem, i.e., the arbitrariness of certain elements, is common to all present economic models. In some models this is quite apparent, while in others it is partially hidden behind an elegant mathematical facade. Thus, for example, the use of nonlinear relationships (rather than individual bounds) to deal with overspecialization can just be seen as introducing another type of arbitrariness into the model. Moreover, for plan coordination models at least, most of the individual bounds are based on partial, presumably rather careful analysis of the underlying phenomena in the traditional planning process; it is doubtful that this expertise could be replaced by some simple modeling device.

To avoid this argument's becoming one-sided, we must make a brief mention of some points which will be discussed in more detail only later. It will be argued that the real choice is not between expert judgment and individual bounds, on the one hand, and nonlinear, econometrically estimated relationships, on the other. If reliable econometric estimates cannot be hoped for, the parameters of the nonlinear forms in question might just as well be based on expert judgment as are the individual bounds in the other solution. What is more important, in our view, is the fact that the use of nonlinear relationships may result in

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*See Taylor (1975) for a more complete treatment of auxiliary constraints and their criticism. Also see Ginsburgh and Waelbroeck (1981).

macroeconomic models that are able to produce less distorted accounting (shadow) prices, which, in turn, may be a useful source of information for price and cost planning, or project evaluation. We will try to show that these nonlinear functions can, in most cases, be viewed as *flexible bounds* on certain variables.

For the sake of simplicity we will use an extremely stylized, textbook type of model. We will assume that there is only one sector whose net output ($\overline{Y}$) is given (determined by available resources). The only allocation problem is to divide $\overline{Y}$ into domestic use (C_d) and exports (Z). Exported goods will be exchanged for an imported commodity which is assumed to be a perfect substitute for the home commodity. Intermediate use will be neglected.

Following a simple linear programming approach, export ($\overline{P}_E$) and import ($\overline{P}_M$) prices will be treated as (exogenously given) parameters of the model. Introducing M for the amount of imports purchased and C_m for the amount of imports used, our optimal resource allocation problem can be formulated in the following way

$$C = C_d + C_m \rightarrow \max$$

$$C_d + Z \leqq \overline{Y} \qquad (P_d)$$

$$C_m \leqq M \qquad (P_m)$$

$$\overline{P}_M M - \overline{P}_E Z \leqq 0 \qquad (V)$$

$$C_d, C_m, Z, M \geqq 0$$

where P_d, P_m, and V are the dual variables associated with the constraints, i.e., the shadow prices of domestic output, imports, and foreign currency, respectively.

The solution of the above problem obviously depends only on the relation of $\overline{P}_E$ and $\overline{P}_M$, i.e., on the terms of trade. The problem of overspecialization is illustrated here very clearly. If the terms of trade are favorable ($\overline{P}_E > \overline{P}_M$), then everything will be exported ($Z = \overline{Y}$) and only imported goods consumed ($C_d = 0$, $C_m = M = \overline{P}_E\ Z/\overline{P}_M$). However, if the terms of trade are unfavorable the optimal policy will be autarky.

Let us assume for a moment that the terms of trade are favorable at prices $\overline{P}_E$ and $\overline{P}_M$. The model builders will be aware of the fact that $\overline{P}_E$ is only an approximate value of the unit export price, and that at such a price the export markets could not absorb more than, say, an amount $\overline{Z}$ of exports. Introducing $\overline{Z}$ as an individual upper bound on Z would prevent the model producing a completely overspecialized solution. $\overline{Z}$ would clearly be binding* and the solution would be

$$Z = \overline{Z} \qquad C_d = \overline{Y} - \overline{Z} \qquad C_m = M = \overline{P}_E\ \overline{Z}/\overline{P}_M$$

It is also easy to see that the optimal values of the dual variables will be

$$P_d = P_m = V\ \overline{P}_M = 1 \quad , \quad t = V\ \overline{P}_E - P_d$$

where t is the shadow price of the individual bound, $\overline{Z}$.

We could therefore say that, in this simple situation, commodity prices are determined by the world market price of the substitute commodity; the higher export price is neutralized by an appropriate tax (t) on exports, which is determined as the shadow price of the individual export constraint.

The analysis of this hypothetical planning model should not stop here, however, for we know that $\overline{Z}$ is a constraint on export at given export prices $\overline{P}_E$. If we changed $\overline{P}_E$, would $\overline{Z}$ change too? Suppose that, at least within certain limits, the answer is yes, i.e., a decrease in the export price ($\overline{P}_E$) would increase the capacity for absorption of exports ($\overline{Z}$). In other

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*This is why we use the word "completely" in the preceding sentence. Instead of $\overline{Y}$, $\overline{Z}$ will now be the upper limit. This strong bound on Z will not qualitatively change the solution.

words, the economy faces decreasing marginal export revenue or, what amounts to the same thing, less than perfectly elastic export demand. Let $D(P_E)$ be the export demand function. Instead of the rigid, fixed export bound ($\bar{Z}$) we could therefore use the following *flexible constraint:*

$$Z \leqq D(P_E)$$

simultaneously treating P_E as a variable in the balance of payments constraint. This would, however, turn our linear programming problem into a nonlinear one, which is generally more difficult to solve. To keep the linear programming framework intact we could adopt a piecewise linearization technique, as suggested, for example, by Srinivasan (1975).

Most linear programming models used for national resource allocation will contain individual bounds on imports as well as on exports. Typically, the ratio of imported goods used to domestic products used (m) will be forced to obey some constraints. In our original model the ratio $m = C_m/C_d$ is not constrained, and so we shall introduce m^+ and m^- as upper and lower bounds (respectively) on m. Our previous programming model will now have to be augmented by two additional constraints, which can be written jointly as

$$m^- C_d \leqq C_m \leqq m^+ C_d$$

Let t_m^- and t_m^+ denote the corresponding shadow prices. As a result of the modifications in the primal problem the dual constraints corresponding to C_d and C_m also have to be modified, as follows:

$$P_d = 1 - t_m^- m^- + t_m^+ m^+$$

$$P_m = 1 + t_m^- - t_m^+$$

Computable general equilibrium models usually adopt a different approach. There the dependence of the import share (m) is usually an explicit, continuous, smooth function of the ratio of the prices of domestic and imported commodities. In most cases, constant elasticity functions are used, such as the following:

$$m = m_o \left(\frac{P_d}{P_m} \right)^{\mu}$$

The difference in treatment is not as crucial as it may seem at first glance. In the linear programming case, observe that if the lower limit on imports is binding (neglecting degenerate solutions), then we will have $t_m^- > 0$ and $P_d < 1$, $P_m > 1$. If the upper limit is binding then $t_m^+ > 0$ and $P_d > 1$, $P_m < 1$. Otherwise $P_m = P_d$. Reversing the argument leads to the following conclusion. If the shadow price of the domestic commodity is less than that of the imported commodity, then we will not import more than the minimum required. If the shadow price of the domestic commodity is more than that of the imported commodity, we will import as much as possible. Otherwise the import volume will be determined by other considerations. We can write this formally as

$$m = m(P_d, P_m) = \begin{cases} m^- & \text{if } P_d/P_m < 1 \\ (m^-, m^+) & \text{if } P_d/P_m = 1 \\ m^+ & \text{if } P_d/P_m > 1 \end{cases}$$

Thus, the import share can formally be treated as a function of relative prices as in a computable general equilibrium model, although in this case the function is not smooth (see Figure 1).

We want to emphasize that the difference in the treatment of import restrictions between linear programming models and computable equilibrium models can once again be seen as the difference between *fixed* (rigid) and *flexible* individual bounds. The relative-(shadow or equilibrium) price-dependent import share implies a variable (flexible) individual bound on imports. The larger the gap between the shadow prices of the domestic and imported commodities the larger the deviation from the observed (or planned) import ratio (m_o).

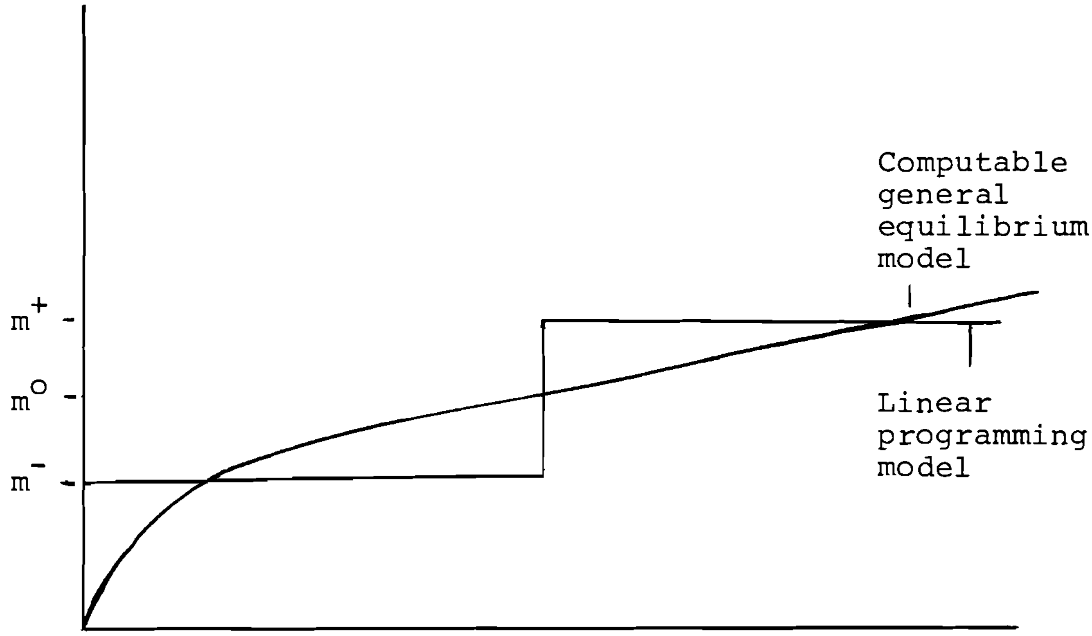

Figure 1. Import share functions.

In fact, allowing for a smooth variation of the import share around its proposed level in a planning model makes at least as much sense as the usual import restrictions. Smooth import share functions could again be incorporated into an otherwise linear model without destroying its linear character, through the use of piecewise linearization*. In many cases, however, it might turn out to be more advantageous to transform the model into either nonlinear programming form or computable general equilibrium form.

To close this section, we shall examine the effect of replacing the fixed bounds in our example with flexible ones. Suppose we have a linear programming model with fixed individual bounds on both exports and import shares:

$$C = C_m + C_d \to \max!$$

$$C_d + Z \leqq \overline{Y} \qquad (P_d)$$

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*Ginsburgh and Waelbroeck (1981) give examples showing how piecewise linear (nonlinear) relationships can be introduced into linear programming models and outline some applications.

$$C_m \leqq M \qquad (P_m)$$

$$\bar{P}_M\ M - \bar{P}_E\ Z \leqq 0 \qquad (V)$$

$$m^-\ C_d \leqq C_m \leqq m^+\ C_d \qquad (t_m^-,\ t_m^+)$$

$$Z \leqq \bar{Z} \qquad (t)$$

If we want to replace the fixed individual bounds by flexible ones, as described earlier, we can rewrite the above linear model in nonlinear form by replacing the objective function with one reflecting import limitations* and introducing an export demand function as before. These changes yield the following model (using constant elasticity forms):

$$C = (h_m\ C_m^{-\eta} + h_d\ C_d^{-\eta})^{-1/\eta}$$

$$C_d + Z \leqq \bar{Y} \qquad (P_d)$$

$$C_m \leqq M \qquad (P_m)$$

$$\bar{P}_M M - DZ^{(1+\varepsilon)/\varepsilon} \leqq 0 \qquad (V)$$

Parameter D in the foreign trade balance is a constant term obtained by solving the export demand function for P_E:

$$Z = e_o \left(\frac{P_E}{\bar{P}_{WE}} \right)^{\varepsilon}$$

where $\bar{P}_{WE}$ is the export price charged by competitors (exogenous variable) and e_o is a scaling parameter.

With reasonable values for the parameters, we can expect to obtain an interior solution. By interpreting P_d, P_m, and V

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*This objective function should be viewed as the planners' preference function with respect to the composition of total source (domestic versus imported). Parameter η could be based on expert judgment concerning the ease of substitutability (technological and institutional), whereas h_m and h_d can be estimated from knowing the planned (target) import share.

as Lagrangian multipliers for the corresponding constraints, the first-order necessary (Kuhn-Tucker) conditions for a maximum can be stated as follows:

$$P_d = \frac{\partial C}{\partial C_d} \tag{1.1}$$

$$P_m = \frac{\partial C}{\partial C_m} \tag{1.2}$$

$$P_m = V\,\bar{P}_m \tag{1.3}$$

$$P_d = \left(\frac{1+\varepsilon}{\varepsilon}\right) DVZ^{1/\varepsilon} = \left(\frac{1+\varepsilon}{\varepsilon}\right) V\,P_E \tag{1.4}$$

One can show that conditions (1.1) and (1.2) actually yield the import share function

$$m = m_o \left(\frac{P_d}{P_m}\right)^{\mu}$$

It is also fairly easy to see that we can replace the above programming model by the following system of simultaneous equations:

$$(2.1)\quad P_m = V\,\bar{P}_M \qquad\qquad Z = a\left(\frac{P_E}{\bar{P}_{WE}}\right)^{\varepsilon}\quad (2.5)$$

$$(2.2)\quad P_E = \left(\frac{\varepsilon}{1+\varepsilon}\right)\frac{P_d}{V} \qquad\qquad C_d + Z = \bar{Y}\quad (2.6)$$

$$(2.3)\quad m = m_o\left(\frac{P_d}{P_m}\right)^{\mu} \qquad\qquad C_m = M\quad (2.7)$$

$$(2.4)\quad C_m = m\,C_d \qquad\qquad \bar{P}_M\,M - P_E\,Z = 0\quad (2.8)$$

This is already very close to a typical specification of a computable general equilibrium model. The argument underlying a CGE formulation of the same resource allocation problem can be summarized as follows. Suppose that there are four collections of economic agents: suppliers and buyers in the home country and those in the rest of the world. Each set contains enough individual agents to ensure that none of them can have a significant influence on prices (they are all price takers). Suppliers of the domestically produced commodity (total available amount $\bar{Y}$) can choose whether to sell at home or abroad. They are assumed to be perfectly elastic, and thus, if at equilibrium they sell on both home and foreign markets, the prices on the two markets must be equal:

$$P_d = V \, P_E$$

Supplies from the rest of the world are also assumed to be perfectly elastic with no supply constraint (i.e., the home country is small). The price of the imported commodity is set exogenously at level $\bar{P}_M$. Following Armington's (1969) assumption of regionally differentiated commodities, demand in both the home country and the rest of the world is assumed to be less than perfectly elastic. Similarly, the demand of the rest of the world for the commodity exported by the home country is assumed to be less than perfectly elastic.

We can represent the conditions for a competitive equilibrium with the following system of equations, in which the endogenous variables are m, C_d, C_m, M, Z, P_d, P_m, P_E, and V.

Price Identities

$$P_m = V \, \bar{P}_M \qquad (3.1)$$

$$P_E = \frac{P_d}{V} \qquad (3.2)$$

Demand Functions

$$m = m_o \left(\frac{P_d}{P_m} \right)^{\mu} \qquad (3.3)$$

$$C_m = m_o C_d \qquad (3.4)$$

$$Z = e_o \left(\frac{P_E}{\overline{P}_{WE}} \right)^{\varepsilon} \tag{3.5}$$

Market Clearing Conditions

$$C_d + Z = \overline{Y} \tag{3.6}$$

$$C_m = M \tag{3.7}$$

Current Account Balance

$$\overline{P}_M M - P_E Z = 0 \tag{3.8}$$

It is also easy to see that all equations are homogeneous and of degree zero in P_d, P_m, and V, so that one of these variables can be chosen freely. We therefore have eight equations in eight variables, which, under the usual assumptions on the parameters, will have a unique solution. As can be readily seen, the two sets of equations, i.e., those characterizing the planners' optimum and the *laissez-faire* equilibrium differ only in one pair of equations.

3. OPTIMUM TARIFF IN APPLIED MODELS

In the previous section we have discussed some foreign trade issues as they appear in multisectoral macroeconomic models designed for numerical simulation. We have basically developed two simple theoretical models for comparison. One is a nonlinear programming model, obtained from its more traditional linear counterpart by introducing flexible rather than rigid individual bounds on export and import activities. The other model is an equation system representing the necessary conditions for a purely competitive (*laissez-faire*) equilibrium. We have also seen that this equation system and the first-order necessary (Kuhn-Tucker) conditions for the optimum in the programming model are almost, but not completely, identical.

The difference between the two sets of conditions is not a surprising one, in the light of the theoretical literature on international trade. This phenomenon has long been recognized as the "optimum tariff" problem (see, for example, Dixit and Norman 1981) or as the difference between the planner's optimum (welfare optimum) and the pure competitive (*laissez-faire*) equilibrium (see, for example, Srinivasan 1982). It is also well known that in many situations a welfare optimum solution can be sustained as a competitive equilibrium regulated by appropriate "optimum" taxes or subsidies, or through direct government intervention.

Although the problem has been discussed at length in the theoretical literature, it has not been recognized as a possible source of concern in computable general equilibrium models. It is not clear why this is so. Perhaps the unfortunate notion of a "small open economy" is partly responsible. Many of the computable models were designed for small economies and the adoption of Armington's assumption was dictated only by a pragmatic concern with overspecialization. Perhaps it was not apparent that the adoption of such an innocent assumption would change the otherwise small economy into a "large" one. Another partial explanation may lie in the ideological values associated with the concepts of pure competition and monopoly power ("it would be unfair if a country made use of its monopoly power in international trade").

Whatever the case, it remains a fact that in this respect the multisectoral planning models of the programming type differ from those of general equilibrium type. The former seek optimum, whereas the latter seek equilibrium solution. In most cases it is easy to alter the general equilibrium model and its solution algorithm so as to derive the planner's optimum instead of the *laissez-faire* equilibrium. Thus a choice must be made. This choice is usually quite important because, as will be seen in the next section, it can qualitatively affect the solution.

We will also show that the optimum may be different from the *laissez-faire* equilibrium, even if the economy is "small and open", in the sense of facing exogenously given terms of trade.

This side of the optimum tariff problem does not seem to have been discussed in the literature but nevertheless appears to be of interest. It can be associated with short-run inflexibility in export supply, and may give rise to both taxes and subsidies (not only to taxes as in the classical optimum tariff problem).

3.1 Optimum and Equilibrium: Perfectly Elastic Supply

Let us examine the equation systems characterizing the optimal solution (equations 2.1 to 2.8), and the competitive equilibrium (equations 3.1 to 3.8). We see that they differ in only one pair of equations, namely, equations (2.2) and (3.2):

$$P_d = V\ P_E$$

$$P_d = (1 + t_e)\ V\ P_E = \left(\frac{1 + \varepsilon}{\varepsilon}\right) V\ P_E$$

The difference can be explained by the following familiar argument. The optimum can be achieved in an otherwise fully competitive system by introducing an *ad valorem* tax, t_e, on exports. Since supply is assumed to be perfectly elastic, domestic suppliers will offer their products abroad at a price rate $[\varepsilon/(1 + \varepsilon)\ P_d/V]$ (expressed in foreign currency), generating an equilibrium export demand equal to its optimal volume*.

It is also useful to look at the difference between the two solutions from a different angle. Recall that the planner's optimum can be determined by solving the following programming problem**:

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*It is interesting to note that most econometric estimates of export elasticities lie between the values -1 and -3 (see, for example, Houthakker and Magee (1969), Hickman and Lau (1973), Sato (1977), Goldstein and Khan (1978), Stone (1979), Browne (1982)). Such values are usually adopted in numerical general equilibrium models too. Observe that $\varepsilon = -1.5$ implies a tax rate of 200 percent (i.e., two-thirds of the revenue is taxed away!); $\varepsilon = -2$ corresponds to 100 percent; $\varepsilon = -3$ to 50 percent, and so on.

**We know that $C_m = M$ in the optimal solution and therefore our programming problem has been reduced to only three variables and two constraints. The other variables and equations can, of course, also be derived from this model.

$$C = (h_d\, C_d^{-\eta} + h_m\, C_m^{-\eta})^{-1/\eta} \rightarrow \max$$

$$C_d + Z \leqq \overline{Y} \qquad (P_d)$$

$$\overline{P}_M M - e_o^{-1/\varepsilon}\, \overline{P}_{WE} Z^{(1+\varepsilon)/\varepsilon} \leqq 0 \qquad (V)$$

$$C_d,\ C_m,\ Z \geqq 0$$

It is fairly easy to see that the pure competitive solution can be found by means of a parametric programming problem of the following form:

$$C - (h_d\, C_d^{-\eta} + h_m\, C_m^{-\eta})^{-1/\eta} \rightarrow \max$$

$$C_d + Z \leqq \overline{Y} \qquad (P_d)$$

$$\overline{P}_M\, C_m - \left(\frac{\varepsilon}{1+\varepsilon}\right) e_o^{-1/\varepsilon}\, \overline{P}_{WE} Z^{(1+\varepsilon)/\varepsilon} \leqq k \qquad (V)$$

$$C_d,\ C_m,\ Z \leqq 0$$

The underlying idea is very simple. The planner's optimum model has been modified in such a way that its dual satisfies the equilibrium pricing requirements. This has been achieved simply by multiplying the export term in the foreign currency constraint by $\varepsilon/(1 + \varepsilon)$ in order to offset the "monopoly distortion" effect. This change, however, alters the meaning of the given constraint, which was the current account balance. One should, therefore, vary the left-hand side (k) parametrically until the solution (C_m and Z, in particular) also satisfies the original current account condition*.

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*Lundgren (1982) proposed an algorithm of this type for solving a special type of multisectoral equilibrium model which could incorporate nonsmooth relationships.

Figure 2 throws more light on the nature of the competitive equilibrium solution. The horizontal axis is primarily a measure of Z, but the difference between $\overline{Y}$ and Z also yields C_d. The vertical axis measures C_m. Thus, we can represent the indifference curves (involving C_m and C_d), the balance of payment condition, and the second constraint of the programming problem all on the same figure.

The curve from 0 to d = 0 represents the export-import combinations fulfilling the current account requirement. Notice that the only difference between the latter and the second constraint in the programming model at k = 0 is that the export term is multiplied by the constant $\varepsilon/(1 + \varepsilon)$, which is assumed to be greater than 1. Hence, the points satisfying this latter constraint are found on the curve from 0 to k = 0, which lies above and is steeper than the current account curve. Thus the optimal solution of the programming problem at k = 0 clearly cannot meet the current account requirement. If we change k parametrically then the optimal solutions will lie on the curve $S\overline{Y}$. The competitive equilibrium solution is found where this latter curve intersects the current account curve. For an optimal solution the indifference curve and the current account constraint must be tangential to each other (see Figure 3). In the case of competitive equilibrium the two curves intersect and a small movement along the current account curve toward the origin would increase the value of the objective (utility) function. Hence the pure competitive equilibrium cannot be optimal.

The above argument has also demonstrated how nonlinear programming methods can be used to compute equilibrium solutions for certain types of models. In the case of most general equilibrium models, however, the solution algorithm is tailored to the specific model and therefore will probably be more efficient than some general-purpose algorithm. Thus, it may be better to keep the equilibrium-searching algorithm. As we have shown, it is usually quite easy to alter the specification and solution algorithm of the equilibrium model (by introducing a tax on exports, for example) to obtain an optimal solution.

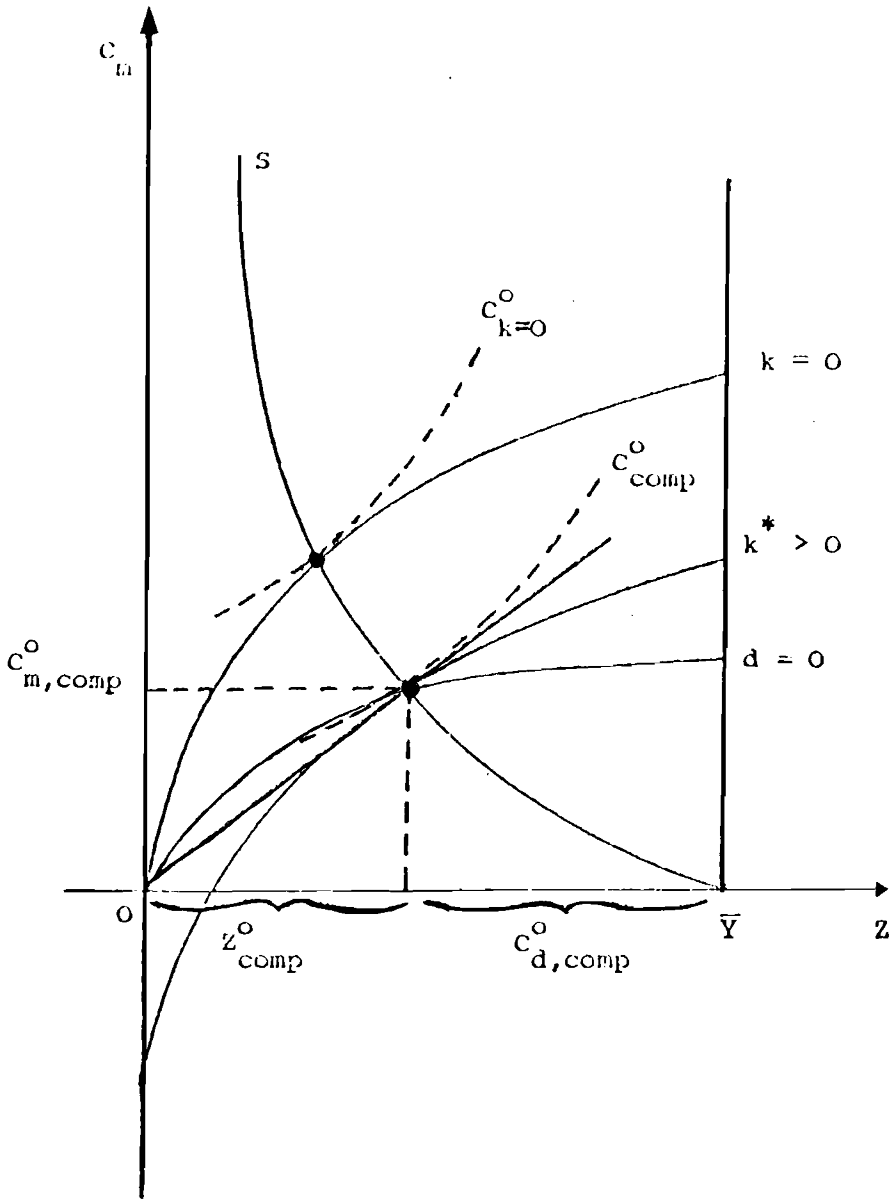

Figure 2. Finding the equilibrium solution by parametric programming.

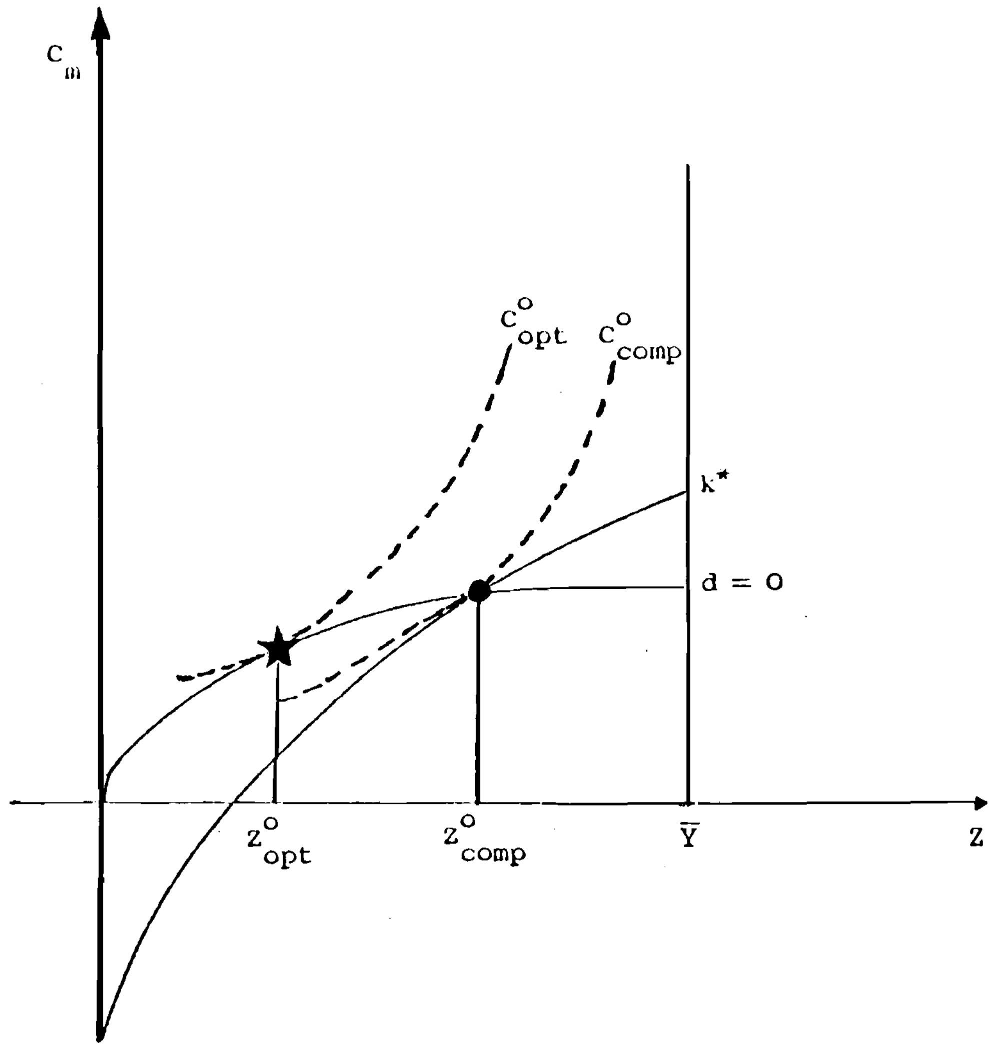

Figure 3. Planners optimum (★) and pure competitive equilibrium (●).

It is sometimes difficult to tell whether the more complicated empirical models are perfectly consistent with neoclassical competitive equilibrium theory, and thus it may happen that the introduction of tariffs will not produce the "best" solution. It may also be difficult to define a welfare function which could be used to check whether there was any improvement on introducing tariffs (when, for example, there is more than one consumer). In such cases special optimization techniques might be used to determine the "second best" solution.

3.2 Optimum Tariffs in a Small Economy: Imperfectly Elastic Export Supply

So far we have examined the usual optimum tariff argument within a special framework. The optimum tariff situation is generally associated with large economies (which have a kind of monopoly power over their export prices and potential buyers), but we have seen that it is not necessarily limited to such "large" economies, at least not in the usual sense. This claim may, however, be rejected on the grounds that it is simply a question of definition (that a small economy is defined as a price-taker on the world market!). Some readers, on the other hand, may wonder why the optimum tariff argument always leads only to taxes on exports and never to subsidies. Indeed, in practice we generally find a complicated system involving both taxes and subsidies regulating foreign trade.

For both of the above reasons it is interesting to see that optimum tariff situations *do* arise in small open economies, too. We will also show that this is a case in which not only taxes but also subsidies may emerge as a means of optimal regulation.

Let us now consider a small open economy as defined in conventional international trade theory, once again using an abstract theoretical model to highlight the problem. We assume that there is only one commodity involved in a pure exchange situation, that world market prices ($\overline{P}_E$ and $\overline{P}_M$) are given exogenously, and we make use of Armington's assumption only in describing demand in the home country. Figure 4 illustrates the problem to be investigated.

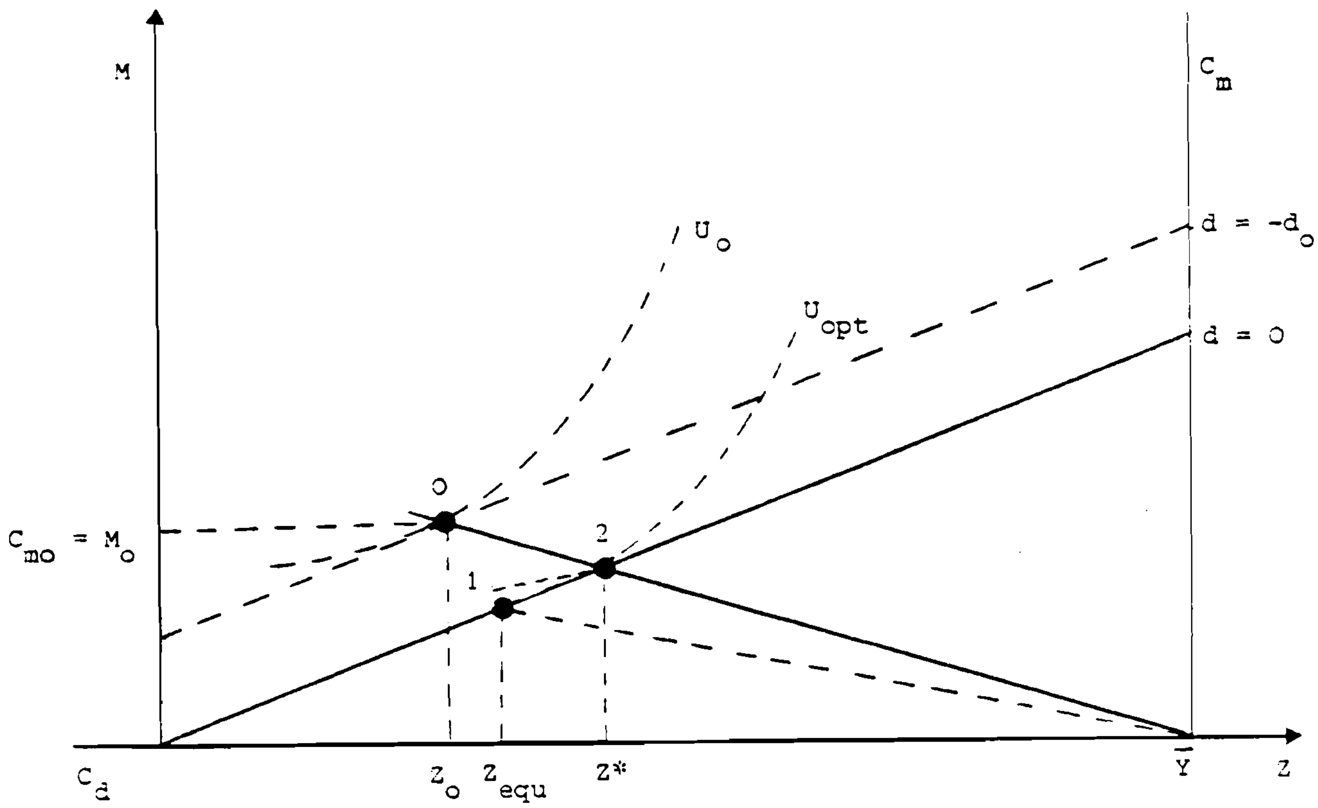

Figure 4. Base (0), *laissez-faire* equilibrium (1) and planners' optimum (2) in a small open economy.

To add a realistic flavor to our abstract problem, let us assume the following familiar situation. After some major deterioration in her terms of trade, the home country adopts a policy of borrowing instead of curtailing domestic consumption. This leads to a (base) situation in which the current account shows a deficit (d_o), but otherwise the economy is (internally) in a state of *laissez-faire* equilibrium (parts and curves labeled with o subscripts in Figure 4). For the sake of simplicity, we also assume that this situation has already existed for sufficiently long to allow the country in question to accommodate herself fully to the new set of world market prices. Thus, the domestic price ratios are exactly the same as the world market price ratios (see equations 4.1 and 4.2).

The above assumptions imply that the following conditions are fulfilled in the base case:

$$(4.1)\quad P_{do} = V_o \, \overline{P}_E \qquad\qquad C_{mo} = m_o \, C_{do} \quad (4.4)$$

$$(4.2)\quad P_{mo} = V_o \, \overline{P}_M \qquad\qquad C_{do} + Z_o = \overline{Y} \quad (4.5)$$

$$(4.3)\quad m_o = \overline{m} \left(\frac{P_{do}}{P_{mo}}\right)^{\mu} \qquad\qquad \overline{P}_M \, C_{mo} - \overline{P}_E \, Z_o = d_o \quad (4.6)$$

Here we have used the subscript o to refer to the base case; all other notation is the same as before. We thus have seven endogenous variables (C_d, C_m, Z, m, P_d, P_m, V) and six equations characterizing the base competitive equilibrium (as usual, relative prices are indeterminate).

One of our assumptions needs special consideration. We have assumed that long-run adjustment has brought about "equalization" of international and domestic prices, i.e., export supply is perfectly elastic in the long run. However, this does not necessarily mean that export supply is perfectly elastic in the shorter run, too. These two assumptions are not contradictory. Let us assume that the short-run export supply function is given by the following constant elasticity form*

$$Z = Z_o \left(\frac{P_d}{V \, \overline{P}_E}\right)^{\alpha} \qquad (4.7)$$

Assume now that we want to assess what would happen in the short run if the government wanted to restore external equilibrium. Suppose that, to achieve this, the government stops borrowing, thus cutting down on the supply of foreign currency (d = 0), but otherwise follows a *laissez-faire* strategy. The resulting short-run equilibrium can be calculated by solving equations (4.2) - (4.7) with a new target of zero for the current account balance.

The only structural difference between the two sets of equilibrium conditions if the replacement of equation (4.1) by (4.7). This difference is due to the assumed divergence of

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*Since $P_d = V\overline{P}_E$ in the base case, the scaling constant must be equal to Z_o.

short- and long-run export supply adjustment: export supply is assumed to be perfectly elastic in the long run, and imperfectly elastic in the short run. (Observe that the two equations are in effect equivalent when α approaches minus infinity.)

It is easily seen that the long-run equilibrium, i.e., the solution of equations (4.1) - (4.6) for $d_o = 0$, is Pareto superior to the short-run equilibrium; it is in fact the optimal solution in the absence of friction in export supply adjustment. Under normal assumptions on the values of the parameters, the different solutions will be as shown in Figure 4. What happens is the following. Foreign currency becomes scarcer, resulting in a higher exchange rate and, as a consequence, higher domestic prices for both domestically produced and imported commodities. However, since export supply is less than perfectly elastic, the domestic price of the home produced commodity will not, in the short run, increase at the same rate as the exchange rate and the price of imports. Thus, in the short-run *laissez-faire* equilibrium the consumption of imported commodities will be reduced more than that of domestic commodities (m decreases). In the optimal case, on the other hand, because of the (assumed) linear homogeneity of the utility function, consumption of both commodities will decrease by the same proportion (as would happen in the long-run *laissez-faire* equilibrium). Of course, prices in the optimal case will also increase proportionally.

Thus, the optimal state of the economy (which is the same here as the long-run equilibrium) is different from the short-run equilibrium*. The *laissez-faire* equilibrium is less efficient than the optimum solution due to the imperfect adjustment of the export supply. This friction could, however, be overcome by appropriate export subsidies, which must be sufficient to increase the amount of goods exported to the optimal level (Z^*). Given the short-run supply function and the optimal solution, the optimal rate of subsidy (ψ^*) can be easily determined. To

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*Observe that the distinction between long- and short-run equilibrium is not essential to our discussion. All we really need to show is that the economy would be better off if supply were perfectly elastic, and that such a state is attainable under suitable regulation (in the realm of the model).

this end observe that $P_d^* = V^* \bar{P}_E$, if prices are set according to the optimality conditions. Thus, introducing the subsidy (ψ) into the determination of supply will result in the following relationship

$$Z = Z_o \psi^{-\alpha}$$

From this we can determine the optimum rate of subsidy as

$$\psi^* = \left(\frac{Z^*}{Z_o}\right)^{-1/\alpha}$$

which is indeed greater than 1 since according to our assumptions $Z^* > Z_o$ and $\alpha < 0$.

We should perhaps make a few comments concerning the above analysis. First of all, the above arrangement could only work if the government collected the money needed for the subsidy through some form of taxation. Thus, in general, this solution implies a redistribution of income which may have unwanted effects. However, this cannot be taken into account in our simplified model.

A second remark concerns the possibility of generalizing our analysis. It is fairly easy to show that the above result can be extended to the case of the large open economy, i.e., an economy facing a downward-sloping demand curve. In this case, the usual optimum tariff argument and the above argument can simply be combined: this means that the optimum tariff derived from the demand relationship must be multiplied by the tariff implied by the supply function

$$t^* = \left(\frac{1 + \varepsilon}{\varepsilon}\right)\left(\frac{Z^*}{a}\right)^{-1/\alpha}$$

where ε and α are the demand and supply elasticities as before, and a is the scale factor in the supply function (Z_o before). Thus, in this case, the tax implied by pure demand (frictionless supply) considerations might be reduced or even offset by the subsidy dictated by supply constraints.

Thirdly, we would like to call attention to one of our specific assumptions and point out the possibility of a supply-implied tax instead of a subsidy. This would arise if our comparative static example resulted in a decrease rather than an increase in exports (as could happen if, for example, the given country borrowed more from abroad). This is especially important in the more complex analyses involving many sectors and different types of assumed exogenous changes, where the different sectors would probably produce a variety of different combinations of taxes and/or subsidies based on export demand and supply considerations.

Finally, we have to do justice to *neoclassical* optimum tariff theory. It is clear that our introduction of the export supply function is not strictly consistent with the usual neoclassical way of thinking and reasoning. The basis of neoclassical theory is that every action of economic agents can be explained by assuming optimizing behavior. Thus, for example, the export supply function is usually derived by assuming joint production of domestic and export commodities, and profit-maximizing producers. In such a case a supply-related optimum tariff would probably not emerge and so it is not surprising that this case is not discussed in the strictly neoclassical literature.

On the other hand, however, we do not think that general equilibrium models can or should be based strictly on neoclassical theory. It is a question of personal taste whether one prefers an equilibrium model which is strictly consistent with neoclassical theory or one which is not. The export supply function, for example, can be introduced into a model in a *non*-neoclassical way, simply to reflect *non*-instantaneous adjustment to changing situations (frictions other than those implied by technological restrictions).

4. NUMERICAL ILLUSTRATIONS

We will now present the results of some numerical simulations. Two models have been used for the purposes of illustration.

One of the models* is rather detailed; it distinguishes 19 sectors. Commodities are distinguished according to their sectoral origin and each sectoral commodity is further classified into three categories: domestically produced, competitive and noncompetitive import. In import and export activities, dollar and rouble trade relations are treated separately. The share of domestic source and competitive (dollar and rouble) import changes as a function of their selective prices. Export is specified in alternative ways (pure supply, pure demand, equilibrium of supply and demand and planner's optimum) as indicated below in Table 2.

Production technology is described by a Johansen-type specification, i.e., the use of sectoral commodities is proportional to the output (Leontief technology), whereas labor and capital usage are specified by linear, homogeneous (Cobb-Douglas), smooth production relationships (production capacity functions).

Gross investment is treated as a special sectoral activity. Demand for investment is the sum of replacement and net investment (replacement rate is different from the rate of amortization!). Production (supply) of new capital goods is represented by fixed-coefficient technology.

The remainder of the final use (termed simply as consumption) is divided into a fixed and a variable part. In the runs presented here, the fixed (minimum) part is the observed 1976 (base) consumption. In order to be able to measure and compare efficiency (optimality) of various solutions easily and unambiguously the sectoral composition of the variable (excess) part of consumption is fixed, thus leaving only the level of excess consumption as variable. This treatment leads to a special demand system, formally very close to the more usual LES systems.

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*The model is a version of the computable general equilibrium model developed for experimental purposes by the author in collaboration with experts from the Hungarian Planning Office. A more detailed description of the model can be found in Zalai (1980). The author wishes to acknowledge the valuable assistance in preparing the numerical model and its solution algorithm given by Gy. Boda, I. Csekő, F-né Hennel, L. László, A. Pór, S. Poviliaitis, J. Sivák, A. Tihanyi and L. Zeöld.

Price formation rules closely follow the input-output tradition, except that the cost of labor and capital is derived on the basis of the needed cost minimizing assumption. Prices are formed on a cost-plus-profit mark-up basis, where the exogenous profit rates are the observed ones (one of the non-neoclassical features of the model).

The parameters and exogenous variables of the model are mostly evaluated on the basis of the 1976 Hungarian statistical input-output tables (using single data point estimates), and partly guesstimated (especially the foreign trade elasticities).

The other model is in many respects a simplified and aggregated version of the first. Only 3 sectors are distinguished. Foreign trade is represented only by one export and one import variable in each sector. In the various runs the volume and price of export in the service sectors (third sector) are kept constant at the base level.

The second model is made more neoclassical by treating import and domestic commodities as less than perfect substitutes, according to Armington's proposition. (In the previous model the assumptions of perfect substitutability but less than perfect adjustment mechanism gave rise to basically the same import functions.) This and some other features make the smaller model similar to the ones used for simulations in Western or developing economies. Consumption of the composite (domestic and imported) commodity is, for example, determined by an LES demand structure. In fact, the only deviation from the standard neoclassical general equilibrium specification is that the export supply functions (if used) are supposed to reflect institutional rather than technological adjustment frictions. Therefore, exported and domestically sold commodities are considered perfect substitutes.

First we will present the *results of the more aggregated model*. In this case we have adopted a rather simple simulation framework which can be summarized as follows. The observed (1976) state of the economy was considered the *base* solution. It was assumed, as usual, that these data reflect certain partial equilibria (e.g., rational decisions under the given price regime), but that they describe, in general, a distorted general equilibrium.

For the sake of simplicity we assumed that the major distortions manifested themselves in the prices, namely in the sectorally different rates of return on the primary resources. Thus, we set out to analyze the effect of introducing an economically more sound (competitive) price system, in which the amount of profit (net income) is determined according to uniform (normative) net rate of return requirement on both labor and capital.

We have generated 8 solutions. They differ from each other only in the export treatment. First we calculated the results with *four* alternative export specifications: a neoclassical, i.e., pure export demand case (Dem), pure export supply case (Sup), export supply and demand equilibrium case (Equ), and optimum tariff case (Opt). (Table 2 shows the export specification in the three versions of equilibrium.) In order to illustrate the effect of the size of export elasticities we have repeated each run at larger absolute values of the elasticities, as shown below:

	Small Elasticities (1)		Large Elasticities (2)	
Sector	Supply	Demand	Supply	Demand
1	- 0.5	- 1.5	- 5.0	- 6.0
2	- 2.5	- 3.0	- 4.0	- 8.0

The set of smaller elasticities is representative for the numerical models used in practice. Tables 3 and 4 summarize the alternative solutions in terms of some characteristic variables. Most of the analysis can be left to the reader, since the figures speak for themselves. To amplify some conclusions we prepared Table 5 which contains only the most relevant information.

Table 5 gives some insights into the working of the general equilibrium models typically used. First of all, due to the input-output structure producers' prices are rather stable (see Table 4). Therefore the relative price-dependent variables (like export, import share) will generally follow the same pattern of change in the various solutions. Only the optimal solution is an exception to this general observation, where we can see qualitatively different solutions.

Table 2. The effect of the elasticity of export demand and supply on model specification.

Demand	Supply: Perfectly elastic[a] ($\alpha = -\infty$)	Supply: Imperfectly elastic ($-\infty > \alpha > 0$)	Supply: Perfectly inelastic ($\alpha = 0$)
Perfectly elastic[b] ($\varepsilon = -\infty$)	*Standard Model* $P_E = \frac{P_d}{V} = \bar{P}_{WE}$ Z determined by other factors (overspecialization may occur)	*Pure Supply* $P_E = \bar{P}_{WE}$ $Z = a\left(\frac{P_d}{V\,\bar{P}_{WE}}\right)^{\alpha}$	*Rigid Supply Constraint* (a) $P_E \leqq \bar{P}_{WE}$ $Z \leqq a$
Imperfectly elastic ($-\infty > \varepsilon > 0$)	*Pure Demand* $P_E = \frac{P_d}{V}$ $Z = e_o\left(\frac{P_d}{V\,\bar{P}_{WE}}\right)^{\varepsilon}$	*Supply-Demand Equilibrium* $P_E = \left[\left(\frac{a}{e_o}\right)\left(\frac{P_d}{V}\right)^{\alpha}\bar{P}_{WE}^{\varepsilon}\right]^{1/(\alpha+\varepsilon)}$ $Z = \left(a^{\varepsilon}\;e_o^{\alpha}\right)^{1/(\alpha+\varepsilon)}\left(\frac{P_d}{V\,\bar{P}_{WE}}\right)^{\alpha\varepsilon/(\alpha+\varepsilon)}$	*Fixed Export Supply* $P_E = \left(\frac{a}{e_o}\right)^{1/\varepsilon}\bar{P}_{WE}^{\varepsilon}$ $Z = a$
Perfectly inelastic ($\varepsilon = 0$)	*Rigid Demand Constraint* (e_o) $P_E \geqq \frac{P_d}{V}$ $Z \geqq e_o$	*Fixed Export Demand* $P_E = \left(\frac{a}{e_o}\right)^{1/\alpha}\frac{P_d}{V}$ $Z = e_o$	*Both Fixed* No adjustment possible

[a] Export price follows domestic price.
[b] Export price follows world market price.

Table 3. Major real variable in various runs (small model).

Sector	Dem1	Sup1	Equ1	Tar1	Dem2	Sup2	Equ2	Tar2
RELATIVE CHANGES IN EXPORT (base = 1.)								
1	0.1733	0.900	0.926	0.222	0.286	0.375	0.588	0.221
2	1.068	1.046	1.035	0.720	1.181	1.137	1.093	1.039
3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Total	1.016	1.023	1.017	0.662	1.041	1.019	1.015	0.917
RELATIVE CHANGES IN IMPORT (base = 1.)								
1	1.103	1.104	1.100	0.961	1.108	1.100	1.098	1.043
2	0.991	0.991	0.984	0.767	1.000	0.988	0.982	0.897
3	1.334	1.334	1.332	0.978	1.343	1.325	1.317	1.195
Total	1.020	1.020	1.013	0.807	1.028	1.017	1.012	0.931
EXCESS CONSUMPTION (base = 0.)								
1	566.500	272.600	183.800	2832.600	579.600	736.200	496.800	898.800
2	1753.000	844.400	568.400	8271.000	1794.400	2273.000	1533.500	2689.900
3	1549.600	745.200	503.300	8255.800	1584.800	2017.700	1362.400	2449.500
Total	3869.100	1862.200	1255.500	19359.400	3958.500	5026.800	3392.800	6038.100

Table 4. Major price variables in various runs (small model).

Sector	Dem1	Sup1	Equ1	Tar1	Dem2	Sup2	Equ2	Tar2
DOMESTIC PRICE INDICES								
1	1.053	1.053	1.053	1.029	1.053	1.052	1.052	1.022
2	0.837	0.837	0.837	0.843	0.837	0.837	0.837	0.831
3	1.134	1.134	1.133	1.073	1.134	1.132	1.131	1.121
EXPORT PRICE INDICES								
1	1.230	1.000	1.052	2.724	1.232	1.000	1.092	1.286
2	0.978	1.000	0.989	1.116	0.979	1.000	0.989	0.995
3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
DOMESTIC PRICE PER EXCHANGE RATE								
1	1.230	1.236	1.226	0.908	1.232	1.217	1.215	1.072
2	0.978	0.982	0.975	0.744	0.979	0.969	0.967	0.871
3	1.325	1.331	1.319	0.947	1.327	1.309	1.306	1.175

Table 5. Summary of simulation results with alternative export specifications (small model) (percentage changes).

	Dem	Sup	Equ	Opt
SMALL ELASTICITIES				
Total export	+ 1.6	+ 2.3	+ 1.7	- 33.8
in sec. 1	- 27	- 10	- 8	- 78
in sec. 2	+ 7	+ 5	+ 4	- 28
Total import	+ 2	+ 2	+ 1.3	- 19.3
Total excess consumption	+ 1.1	+ 0.5	+ 0.3	+ 5.3
Term of trade	+ 0.6	0	+ 0.3	+ 18.5
Exchange rate	- 14	- 15	- 14	+ 13
LARGE ELASTICITIES				
Total export	+ 4.1	+ 1.9	+ 1.5	- 8.3
in sec. 1	- 71	- 62	- 41	- 78
in sec. 2	+ 18	+ 14	+ 9	+ 4
Total import	+ 2.8	+ 1.7	+ 1.2	- 6.9
Total excess consumption	+ 1.1	+ 1.4	+ 0.9	+ 1.6
Term of trade	- 1.0	0	+ 0.2	+ 0.5
Exchange rate	- 15	- 14	- 13	- 5

It is also evident that the size of the elasticities has a real influence on the size *order* of changes. If they are relatively small the changes are larger and *vice versa*. This effect is visible even if we compare only the demand, supply and equilibrium solutions in one (small or large) class of elasticities. As shown in Table 1, equilibrium elasticities are the smallest of all, and in this particular example we have chosen the supply elasticities smaller than the demand ones. These show up in the respective orders of change in the exports. Thus, *the larger the elasticities the larger the room for the forces of comparative advantage in structural adjustment* (allocative efficiency).

However, the above positive effects of larger elasticities are *counterbalanced by the terms-of-trade effects* brought in by the same demand elasticities. Thus, for example, in the pure export demand case these two effects offset each other. The increased allocative efficiency is offset by a 1.6% simultaneous deterioration of the terms of trade (from +0.6 to -1.0), and the increase of consumption remains the same (1.1).

The *terms-of-trade effects* brought in by the demand elasticities can best be seen in the case of *optimal tariff solutions* which takes them to an extreme. When the elasticities are small the optimizing logic of the solutions generates an 18.5% (!) gain in the terms of trade, and this is the real source of the outstanding welfare improvement (+5.3% increase in consumption). With large elasticities this effect is only marginal as compared to the allocative efficiency. This explains why the various solutions are so close to each other in the case of larger elasticities.

It is also worth noting that the *laissez-faire* solutions and *optimal* solutions *qualitatively* differ in their economic policy implications. The former ones suggest a more open foreign trade policy: both total exports and total imports increase in all six solutions. The optimal solutions, on the other hand, suggest rather severe import-export restrictions.

Finally, as a matter of special interest, we would like to report on some specifics of the *optimal tariff* solution. As we have discussed in the theoretical part of this paper, the optimal tariff solution works in the following way. The exchange rate will be corrected by taxes or subsidies in regulating exports. All the *laissez-faire* solutions suggested a 13%-15% *revaluation* of the exchange rate. (This can be explained by the ca 16% decrease in the price of the major exporting sector, number 2.) As opposed to this, the optimal tariff solution implied a 13% *devaluation* in the case of small elasticities and only 5% revaluation in the other case. This immediately explains why imports are reduced in both cases. To discourage exports, on the other hand, *export taxes* have to be introduced. Their order of *magnitude* in the first two sectors are 98% (!) and 42% when elasticities are small and 40% and 11.7% when they are high. (If supply were perfectly elastic the corresponding figures would be

67% and 33% in the first case, and 17% and 12.5% in the second. Thus, except for the last figure, the supply effect adds to that of demand.) These results clearly call into question the relevance of optimal tariff argument in the case of small (constant) demand elasticities.

We think this example convincingly illustrates the point that the question of export demand specification and especially the size of demand elasticities commonly used in computable general equilibrium models must be critically re-examined. The results obtained from the more complex and disaggregated model show that our findings are not overexaggerated by the small model. The simulation framework in this case was somewhat different. The question we asked from this model was the following. Suppose Hungary wanted to achieve a zero balance in her dollar trade in 1976, what structural changes would be needed? Again, we calculated four solutions differing only with respect to the export specification. Some additional specifics of the calculations should be mentioned before presenting the main results. First, the balance of trade was assumed to be restored at the cost of a more or less uniform decrease of consumption. Second, rouble trade and terms of trade were kept constant. Third, profit rates were assumed to remain the same.

The details of these model solutions are not too interesting and might be somewhat misleading. Therefore we decided to show here only some of the main indicators (Table 6). However, the results are perfectly adequate for illustrating the results of the alternative export specifications discussed and the differences between the *laissez-faire* and planners' optimum solutions.

Table 6. Main indicators (large model) (base = 100)

	Dem	Sup	Equ	Opt
Total dollar export	128.18	116.51	123.90	108.74
Total dollar import	97.35	98.44	95.55	89.05
Total trade/GDP ratio*	84.81	82.90	83.57	79.45
Final consumption	92.04	95.52	92.75	94.63
Dollar terms of trade	89.89	100.00	91.27	96.92
Dollar exchange rate	111.21	108.87	125.39	188.31

*base = 80.42

4. CONCLUDING REMARKS

In the first part of this paper we argued that the rigid individual bounds on export and import activities typical of programming-type macroeconomic models can be usefully replaced by flexible bounds. This replacement was, in fact, carried out using some tools borrowed from similar models of the computable general equilibrium type. The choice of parameters in the neoclassical export and import functions is at least as crucial as the choice of the size of individual bounds, and this is clearly demonstrated in the numerical simulations. Thus, since these parameters cannot be estimated any more reliably than the individual bounds can be determined, there is some degree of arbitrariness in both cases.

Our numerical examples also illustrate the terms-of-trade effects introduced by export demand functions. It is important to emphasize that in many cases these effects are unrealistic and unwanted. The smaller the elasticities, the larger the terms-of-trade effects. Small elasticities, however, usually arise only because the observed changes in exports are small, especially when compared to changes in relative prices.

It is, therefore, crucial to distinguish between and possibly separate the changes in the terms-of-trade and the changes in the speed of export adjustment. The special advantage of introducing both demand and supply functions lies, in part, in this area. Small supply elasticities imply small shifts in exports (if needed), while the size of the demand elasticity can more accurately reflect the assumed changes in the terms-of-trade.

A major problem with the most commonly used export and import functions is their constant-elasticity form. Even if one could rely on the econometric estimates of these elasticities, they would give an accurate representation of supply and demand behavior only in a relatively small area of the observed pattern. Another problem with constant elasticities is that the effects of increases and decreases in relative prices are treated symmetrically. It is rather unrealistic to assume that, say, a 10% increased in exports will produce a change in relative prices of the same size as a 10% decrease in exports.

One would intuitively think that the export demand would be much more elastic with respect to an increase in prices than to a decrease in prices. It would therefore seem reasonable to replace the constant elasticity forms by nonsymmetric forms with variable elasticities. Since observations usually lie within a narrow range, it is extremely difficult to make econometric estimates of such functions. The only possibility seems to be the combination of econometric estimates with qualitative export judgments.

On the whole, our numerical simulations demonstrate that the treatment of foreign trade in a multisectoral macromodel has a very great influence on the final results of the model. This is not very surprising since these models operate on the basis of resource reallocation. The freedom in reallocating resources in an open economy depends greatly on the potential for foreign trade. Thus, it is very important to devise an accurate representation of this potential: it seems that the currently available techniques are not sufficiently sophisticated to handle these problems adequately.

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