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A DYNAMIC MULTISECTOR MODEL WITH ENDOGENOUS FORMATION OF CAPACITIES AND EQUILIBRIUM PRICES: An Application to the Swedish Economy

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PREFACE

In a recent project organized by the Regional Development Group, a comprehensive assessment was made of multisectoral models with a multiregional specification [see Multiregional Economic Modeling: Practice and Prospect, Issaev, Nijkamp, Rietveld and Snickars (eds), North-Holland 1982]. The current paper presents a multisectoral model recently developed as a tool for studying the structural change of the Swedish economy. The model is formulated for the national level and contains some new contributions by integrating capacity formation, investment behavior and price formation in an equilibrium framework.

In its present version, the model may be of interest for other IIASA projects. Moreover, its structure is such that it should be possible to develop a multiregional extension of the model. From this point of view, the paper provides a basis for further research within the Regional Development Group.

Börje Johansson
Acting Leader
Regional Development Group
IIASA

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CONTENTS

1. INTRODUCTION: Medium-term Equilibria in a Multisectoral Model[1]

1.1 Equilibrium Solutions of Multisectoral Models

The most coherent way of describing equilibrium solutions in a multisectoral setting can be found in models adhering to the von Neumann type of closed systems [see for example von Neumann (1945), Morishima (1970), Brody (1970)]. Among non-closed model versions, the MSG type of framework has been widely used for long-term applied analysis [see Johansen (1974)]. The model described in this paper combines elements from these two traditions in a computable, equilibrium type medium-term model.

In the model presented, capital formation is determined endogenously as an integral part of a dynamic capacity change process. Consumption and international trade is modeled in a way similar to that which can be found in MSG models.

To illuminate the difference between the present model and an MSG model, let us just point at the following properties which distinguish our approach from that usually followed in

[1]The model presented has been called MACROINVEST in certain applications.

MSG models. In the latter one may note that (i) the input of labor and capital is determined by a production function, (ii) production is limited by the available amount of labor and capital which makes investments exogenously given, (iii) capital is completely malleable. In addition, the composition of the capital goods is usually identical between sectors.

The differences indicated above may be related to the production and investment theory on which our model is founded. One of its fundamental characteristics is a consistency criterion which ensures that in every solution production capacities are never below production levels. Simultaneously production capacities are strictly related to investment processes in the model.

1.2 Assumptions about Production and Investment

One important background to the model is a vintage type of production theory. A formal presentation of such a theory was given early by Johansen (1959). In our setting this approach means that each sector is composed of different techniques, each with fixed input coefficients, and each with an associated capacity limit. Capacities and techniques change as a result of capital formation and capacity removal, and these processes are influenced by changes in price and wage patterns.

In the model investments are simultaneously determined by a profit condition and a capacity requirement. The latter means that investments have to fill the gap between the demand for output and existing capacities. At the same time, investments in a sector are carried through only if the ratio between the profits and the investment costs in the sector exceed or equal a sector-specific "rate of return". Profits and investment costs are calculated in terms of prices and wages generated by the model.

In summary, the model presented has (i) a sector-specific non-homogenous "capital" concept, (ii) an endogenous process of capital formation, and (iii) non-malleable capital. Moreover, each individual technique displays constant returns to scale within its capacity limit. The aggregate production function of each sector is characterized by variable returns to scale, both in the short and medium term. On the aggregate level each sector will also have a variable input structure.

The paper is structured in the following way. Section 2 introduces the assumptions about production techniques, technical change, and investments. Different ways of interlinking medium-term periods are also discussed.

In section 3 the capacity change process is characterized. The concepts of capacity demand and propensity to invest are introduced and explained. The notion of capacity change equilibrium is defined.

Section 4 analyzes the general equilibrium properties of a model solution. The algorithm utilized to solve the model is described.

In two appendices the technical details of the model are presented and the features of model solutions are illustrated.

2. TECHNOLOGY ASSUMPTIONS AND TIME PERSPECTIVES

2.1 Commodities, Sectors and Prices

The core of the model structure we shall present consists of an input-output description of production activities. For this core two types of classifications are utilized: one in terms of commodities and one in terms of production sectors. To make this distinction clear we denote

$$x_i^v = \text{amount of commodity type } i = 1,\dots,n$$
$$x_i = \text{activity level in sector } i = 1,\dots,n \qquad (2.1)$$

Commodities are related to sectors by means of the matrix $U = \{u_{ij}\}$

$$x_i = \sum_j u_{ij} x_j^v \ ; \ \sum_i u_{ij} = 1 \qquad (2.2)$$

Then if p_i denotes the price level in sector i, the price of commodity j, must satisfy the following condition

$$p_j = \sum_i u_{ij} p_i^a \tag{2.3}$$

We shall call p_i commodity price and p_j^a producers' sector price.

The average technologies in the economy may be described by the following coefficients

$$\begin{aligned} a_{ij} &= \text{input of commodity i per unit output in an average process in sector j} \\ l_j &= \text{input of labor per unit output in an average process in sector j} \\ k_{ij} &= \text{input of commodity i per creation of one average unit of capacity in sector j} \end{aligned} \tag{2.4}$$

Let $\Delta x_j/T$ denote the annual capacity increase in sector j. Then we may describe the quantity balance of the economy as:[1)]

$$x_i^v = \sum_j a_{ij} x_j + \sum_j k_{ij} \Delta x_j/T + D_i \tag{2.5}$$

where D_i represents the remaining part of final demand for commodity i.

2.2 Production Techniques and Capacities

Applied input-output analysis usually describes the production techniques of a sector with one average and constant input vector. For each sector, we shall make use of a specification of different techniques k = 1,2,... At a given point in time each technique is characterized by its own input coefficients l_j^k, $\{a_{ij}^k\}$ and a production capacity $\bar{x}_j^k$. Hence, we may describe the sector aggregate as

$$\begin{aligned} \bar{x}_j &= \Sigma \bar{x}_j^k \\ a_{ij} &= \sum_k a_{ij}^k \bar{x}_j^k / \bar{x}_j \\ l_j &= \sum_k l_j^k \bar{x}_j^k / \bar{x}_j \end{aligned} \tag{2.6}$$

[1)]In the sequel we shall denote the capacity increase in a T-year period by Δx. Hence, $\Delta x/T$ describes the annual increase.

We make the vintage type assumption that the input coefficients of each technique are fixed. The implications of 2.6 are obvious:

> *Remark 1:* For each individual production technique the scale elasticity is constant within the capacity limit of the technique and no substitution of input factors is possible. At the same time the aggregate technique of each sector has a variable scale elasticity, and factor substitution can occur. These are short run properties of the aggregate production function.

Changes in returns to scale and input compositions emerge as different combinations $x_j^1, \ldots x_j^k, \ldots$ are chosen within the constraints $x_j^k \leq \bar{x}_j^k$ for all k.[1]

In order to illuminate substitution possibilities and the associated choice of techniques we need technique-specified expressions for value added and gross profit per unit output, F_j^k and π_j^k, respectively. As can be seen these are defined for each given composition of prices and wages

$$\begin{aligned} F_j^k &= p_j^a - \sum_i p_i a_{ij}^k \\ \pi_j^k &= F_j^k - w_j l_j^k \end{aligned} \tag{2.7}$$

where w_j denotes the wage level in sector j. For each given set of commodity prices one may order the techniques according to falling profits as shown in Figure 1. As prices change the order will change and this gives rise to altered incentives for selecting techniques also in the short run.

2.3 New and Old Production Techniques

Our model is designed to capture the decision problem of investors who at each time t=0 make capacity decisions in a medium-term perspective. That is, they decide about capacity change between time t=0 and t=T. For each such opportunity to

[1] Such variations are described in detail by Johansen (1972) and Hildenbrand (1981). See also Johansson and Holmberg (1982).

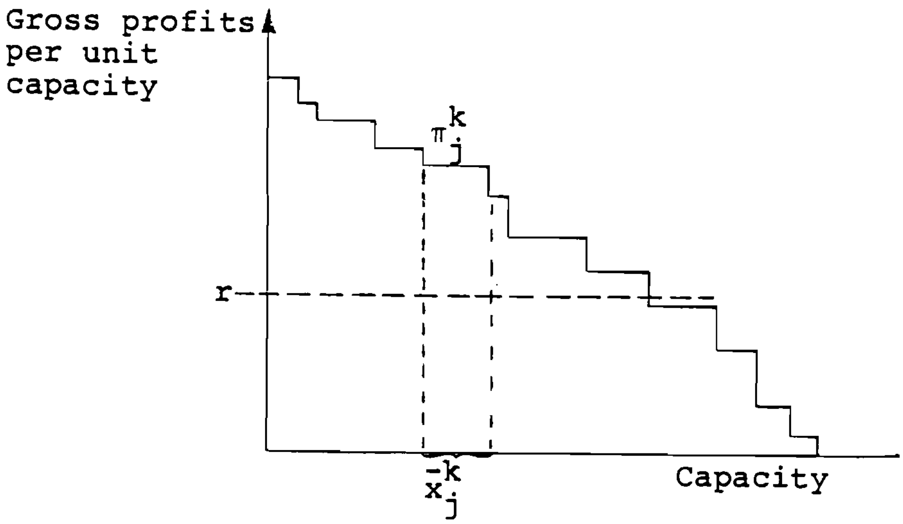


Figure 1. Distribution of gross profits per unit capacity.

decide a new production technique is available for each sector. This technique is signified by k=*, and for sector j the following input coefficients are associated with such a technique

l_j^* = labor input coefficient of the new technique

a_{ij}^* = input-output coefficient associated with the new technique (2.8)

For each price and wage structure a given profit π_j^* per unit capacity is associated with the technique introduced in (2.8).

Consider now a production technique k. By $\bar{x}_j^k(0)$ we denote the capacity corresponding to this technique at time t=0. Let d_j^k be the capacity removed from $\bar{x}_j^k(0)$ during the medium-term period [0,T]. Then the remaining capacity at time t=T becomes $\bar{x}_j^k(0)-d_j^k$. Let Δx_j denote the capacity created during the period. Then the capacity level at time T becomes

$$x_j = \Delta x_j + \sum_k (\bar{x}_j^k(0) - d_j^k)$$
$$\Delta x_j \geq 0 \tag{2.9}$$

Let us assume that the capacity Δx_j is distributed over the old techniques so that Δx_j^k is the new capacity in production units belonging to technique class k. Let l_j^{k*} and $\{a_{ij}^{k*}\}$ be the input coefficients attached to the capacity Δx_j^k. The input coefficients at time T are then obtained as

$$\begin{aligned} l_j^k(T) &= (l_j^{k*}\Delta x_j^k + l_j^k(0)[\bar{x}_j^k(0) - d_j^k])/x_j^k \\ a_{ij}^k(T) &= (a_{ij}^{k*}\Delta x_j^k + a_{ij}^k(0)[\bar{x}_j^k(0) - d_j^k])/x_j^k \end{aligned} \tag{2.10}$$

where $x_j^k = \Delta x_j^k + \bar{x}_j^k(0) - d_j^k, \Delta x_j^k \geq 0$. We shall assume that the new technique associated with Δx_j is additive over techniques so that

$$\left.\begin{aligned} &\sum_k l_j^{k*}\Delta x_j^k = l_j^*\Delta x_j \\ &\sum_k\sum_i a_{ij}^{k*}\Delta x_j^k = \sum_i a_{ij}^*\Delta x_j \\ &\sum_k \Delta x_j^k = \Delta x_j \end{aligned}\right\} \text{ for each } j \tag{2.11}$$

The assumption in (2.11) means that when one single investment period [0,T] is studied, then the analysis may focus entirely on the average (aggregate) new technique $l_j^*,\{a_{ij}^*\}$ and the aggregate capacity Δx_j irrespective of how this average technique and the corresponding capacity is distributed over technique classes.

2.4 Time Perspectives and Development Paths

For each medium-term period [0,T] the value of an arbitrary variable at time t=0 is denoted by $\xi(0)$ and at time t=T by $\xi(=\xi(T))$. In the applications described in the following sections the model simulates economic change in terms of interlinked sequences of T-year periods [0,T],[T,2T],..., as described in Figure 2a. However, a more elaborate form of dynamics as suggested in Johansson and Persson (1983) may also be considered. In that case the sequences are [0,T],[1,T+1],..., as illustrated in Figure 2b. This implies that the investors have a moving T-year time horizon, and medium and short-term economic adjustments may be interlinked.

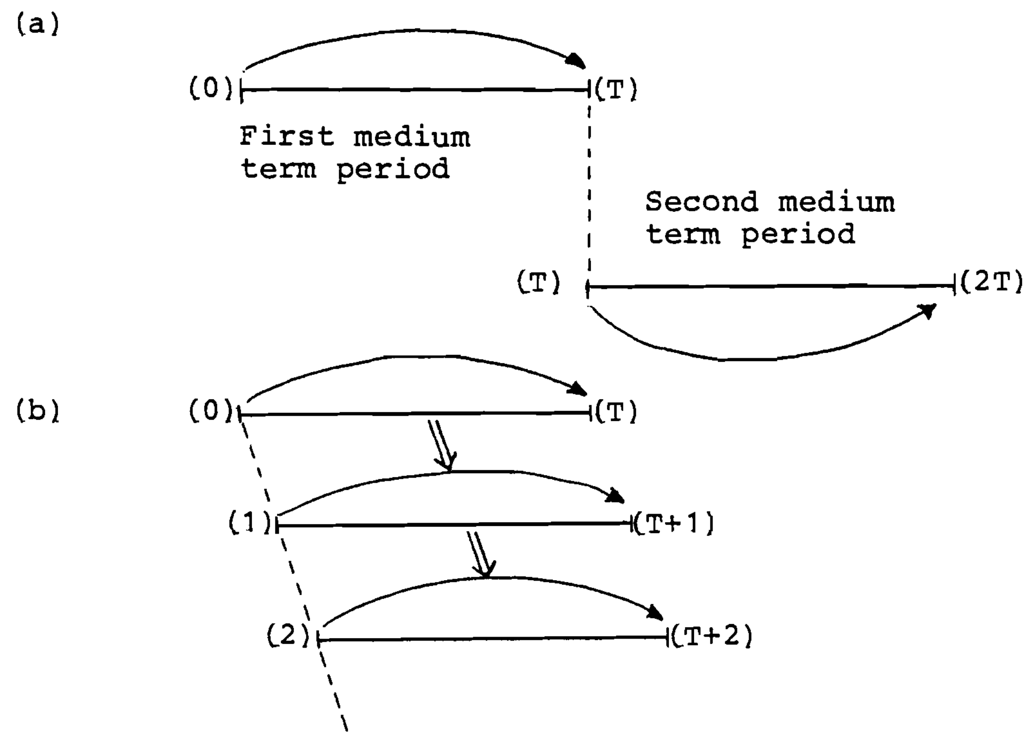


Figure 2. Two alternative ways of interlinking medium-term period.

The model notations only distinguish between the initial time, at which $\xi(0)$ is specified, and the terminal time, at which ξ is specified. This is sufficient, since the analysis of each medium-term period is carried through on the basis of an assumption about the form of the path between $\xi(0)$ and ξ for each relevant variable. Of course, this presupposes a consistency between the different paths. This approach means, for example, that if ξ represents the level of profits in year T, there is a unique path $\xi(1)\ldots,\xi(T)$ of annual profits corresponding to ξ. In this way an equilibrium solution for the terminal year also implies a given path towards the solution. However, the present version of the model is not designed for analyzing the economic outcome in each intermediate year $1,\ldots,T-1$. Also, note that the solution for year T may be interpreted in two different ways. One corresponds to case (a) in Figure 2. In this case the solution implicitly constitutes a development path for the medium-term period, generated by the equilibrium in year T. The alternative interpretation relates to case (b) in Figure 2, and means that an expected economic state in year T generates investment decisions in the beginning of the period and also determines the capacity levels in the first year. Simultaneously, in year 1 new expectations have to be found with regard to time T+1 so that the process can be repeated recursively. Only case (a) is elaborated in the paper.

Let us finally state that the fundamental feature of the model is a consistency property. At each time the economic system cannot produce more than is feasible with regard to given capacity limits. And capacities are created by the economic system itself.

3. CAPACITY CHANGE EQUILIBRIUM

3.1 Profits and Capacity Removal

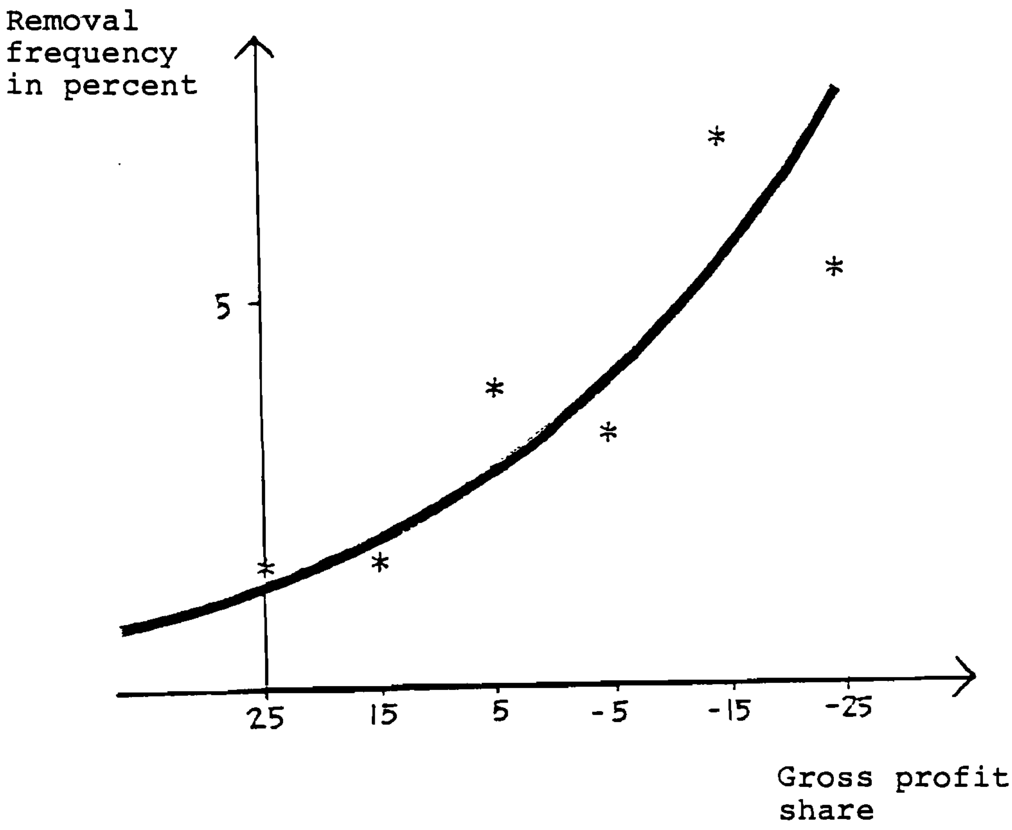
A standard assumption about firms and production units in a market economy is that shut down occurs when the gross profits or quasi-rents cease to be positive. This assumption has been especially stressed in vintage type production theory. Accepting this assumption as an approximation, the following expression was estimated for each sector:

$$\lambda_j^k = \delta_j^o \exp\{\delta_j^1 \sigma_j^k\} \tag{3.1}$$

where λ_j^k denotes the average annual removal rate given the wage share $\sigma_j^k = w_j l_j^k / F_j^k$, calculated from (2.7). Estimated positive parameters are δ_j^o and δ_j^1. From (3.1) one may calculate the total capacity removal during the period which yields

$$d_j^k = (\lambda_j + T\lambda_j^k)\bar{x}_j^k(0) \tag{3.2}$$

where $\lambda_j \geq 0$ signifies such reduction of capacities which is invariant with respect to the wage share σ_j.



Remark: Observe that the gross profit share $\pi_j^k/F_j^k = 1 - w_j l_j^k/F_j^k$.

Parameter values: $\delta_j^o = 0{,}11; \delta_j^1 = 3.4$

Figure 3. Illustration of a removal function. Annual removal frequency in percent (manufacturing of wood products, Sweden 1969-77).

Remark 2: The assumption expressed by (2.11) implies that one does not have to consider the distribution of new capacities (embodying new technologies) over existing production units (technique classes) when solving the model for a given medium-term period. However, when transforming such a solution to a starting-point for the subsequent period the calculations in (2.10) become essential. The reason for this is that the removal functions must be applied to the profit shares which obtain when the input coefficients are changed according to (2.10).

3.2 Investment Demand and Propensity to Invest

Consider the equation system (2.5), in which $x_i^v = \Sigma a_{ij}x_j + \Sigma k_{ij}\Delta x_j/T + D_i$. For given world market prices and a given initial structure of established production techniques, the system in (2.5) may be solved in terms of production capacities with the help of the matrix U. The solution is obtained contingent on a price vector $p = (p_1,...,p_n)$, a vector of wage levels $w = (w_1,...,w_n)$, and the aggregate disposable income y. In this way, every vector (p,w,y) induces (i) an aggregate capacity removal $d_j = \Sigma d_j^k$ in each sector, and (ii) a minimum level of capacities $\bar{x} = (\bar{x}_1,...,\bar{x}_n)$. From this we can derive a corresponding vector $\Delta x = (\Delta x_1,...,\Delta x_n)$ of capacity increments such that[1]

$$\Delta x_j = \max\{0, \bar{x}_j + d_j - \bar{x}_j(0)\}, \text{ all } j \qquad (3.3)$$

Formula (3.3) describes the demand for new capacity. Making use of the investment coefficients, k_{ij} in (2.4), one may determine the associated demand for investment deliveries $k_{ij}\Delta x_j$.

Let us now assume that the capacity increase during the period is linear. Then sector j's demand for investment deliveries, I_j, has the following value:

$$I_j = \sum_i p_i k_{ij}\Delta x_j/T \qquad (3.4)$$

The annual costs of investment, I_j, must be compared with the associated profits. At the same time we shall, for each sector, introduce a medium-term propensity to invest. Such an estimated "propensity" is denoted by a parameter α_j which shows how much of sector j's profits investors associated with sector j are prepared to use for capital formation.

---

[1] Compare formula (2.9).

According to (2.7) the profits in sector j, π_j, may be specified as follows:

$$\pi_j = \sum_k \pi_j^k (\bar{x}_j^k(0) - d_j^k) + \pi_j^* \Delta x_j \qquad (3.5)$$

At equilibrium the propensity to invest, profits, and costs of investment must be in balance. Therefore, we can define a capacity change equilibrium as

$$I_j = \alpha_j \pi_j \qquad (3.6)$$

Combining (3.4) and (3.5) one can see that $\pi_j^* \Delta x_j / I_j$ refers to a standard notion of the rate of return on investment in sector j, while π_j / I_j reflects a more intricate form of "rate of return".[1]

$$\pi_j / I_j = 1/\alpha_j$$

Observe that α_j has been estimated during a specific period during which each sector was facing a given cost for its capital funds. Let r_j denote the corresponding rate of interest with regard to sector j. We should assume that the estimated parameter α_j reflects the interest rate, r_j, which existed during the estimation period. In this sense we could write $\alpha_j = \alpha_j(r_j)$ with $\partial\alpha_j/\partial r_j < 0$.

Observe finally that the equilibrium condition in (3.6) consists of n equations with 2n unknown price and wage variables. The condition gives simultaneous requirements for prices, wages and capacities.

4. GENERAL EQUILIBRIUM PROPERTIES

4.1 Demand Components in the Model

The demand components of the model are summarized by the right hand side of the equation $x_i^v = \Sigma a_{ij} x_j + \Sigma k_{ij} \Delta x_j / T + D_i$ in formula (2.5). The demand variable D_i may be divided into the following separate components with regard to commodity i:

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[1] In equilibrium $1/\alpha_j$ is the sum of the two ratios $\Sigma \pi_j^k x_j^k / I_j$ and $\pi_j^* \Delta x_j / I_j$.

$$\begin{aligned} D_i &= c_i + g_i + h_{ig} + e_i - m_i \\ c_i &= \text{private consumption} \\ g_i &= \text{deliveries to public consumption} \\ h_{ig} &= \text{deliveries to public investments} \\ e_i &= \text{exports} \\ m_i &= \text{imports} \end{aligned} \tag{4.1}$$

Private consumption, exports and imports are determined endogenously while deliveries to the public sector are exogenously given. A detailed specification of the variables in (4.1) is given in Appendix 1. For given world market prices D_i may be expressed as $D_i = D_i(p,x_i,y)$, since then we have that

$$\begin{aligned} e_i &= E_i(p_i) \quad ; \ \partial E_i/\partial p_i \leq 0 \\ m_i &= M_i(p_i,x_i) \ ; \ \partial M_i/\partial p_i \geq 0 \\ & \qquad\qquad\quad \partial M_i/\partial x_i > 0 \\ c_i &= C_i(p,y) \quad ; \ \partial C_i/\partial y > 0 \\ & \partial C_i/\partial p_i < \partial C_i/\partial p_j < 0 \end{aligned} \tag{4.2}$$

where y represents disposable income, and where the signs refer to estimated functions as presented in the appendix.

4.2 Characterization of Equilibrium Solutions

Consider the following function, $G(x)$, which summarizes the demand in the economy in terms of sector production, $x = (x_i,\ldots,x_n)$:

$$G(x) = U[Ax + K(x) + D], \tag{4.3}$$

where $A = \{a_{ij}\}$, $U = \{u_{ij}\}$, $D = \{D_i\}$, $K(x) = \{k_{ij}(x)\}$, and according to (2.9) and (3.3)-(3.4) $k_{ij}(x) = k_{ij}[x_j + d_j - \bar{x}_j(0)]/T$

Balance between supply and demand is obtained when $x = G(x)$. Two additional constraints are attached to this balance. The first concerns the balance of trade;

$$\bar{\beta} = \beta$$
$$\beta = \sum_i p_i^w (e_i - m_i) \tag{4.4}$$

where p_i^w denotes the exogenous world market price with regard to commodity i. The second constraint requires that total employment equals an exogenously given (full employment) level $\bar{L}$. This yields

$$\bar{L} = \sum L_j$$
$$L_j = \sum_k l_j^k x_j^k + l_j^* \Delta x_j \tag{4.5}$$

Suppose now that labor supply functions have been estimated so that $L_j = L_j(w_j)$, then the second part of (4.5) gives a determination of wage levels in a way which corresponds to the way in which investments and prices interact in (3.6). Then it only remains to determine the general wage level such that $\bar{L} = \sum L_j$.

Currently the model is applied without labor supply functions. Therefore, a fixed wage structure $\bar{w} = (\bar{w}_1,\ldots,\bar{w}_n)$ is determined exogenously as a part of the model calibration. The actual wage levels are then obtained through a multiplication with the general wage level ω (scalar) so that $w = \omega\bar{w}$.

An equilibrium can now be defined as a price vector $\tilde{p} = (\tilde{p}_1,\ldots,\tilde{p}_n)$, a wage level $\tilde{\omega}$ and an aggregate disposable income $\tilde{y}$ such that the following balances are fulfilled

$$
\begin{aligned}
&(i)\begin{cases} x = G(x) \\ x = \Delta x + \bar{x}(0) - d \end{cases}\\
&(ii)\begin{cases} \tilde{p}^a = \tilde{p}^a UA + \tilde{\pi} + \tilde{w}l \\ \tilde{\pi}_j = (1/\alpha_j)\sum \tilde{p}_i k_{ij}\Delta x_j/T, \text{ all } j \end{cases}\\
&(iii)\ \bar{\beta} = \sum_i p_i^w (e_i - m_i)\\
&(iv)\ \bar{L} = \sum_j L_j
\end{aligned}
\tag{4.6}
$$

where $x,\Delta x,\bar{x}(0),d,\pi,w,l$ denote the vectors of the corresponding variables and where $\tilde{p}^a$ denotes the vector of sector prices which from (2.3) satisfy the equation:

$$\tilde{p}_j = \sum_i u_{ij} \tilde{p}_i^a \text{ , all } j$$

Remark 3: The equilibrium condition in (4.6) is described as a system, (i) - (iv), of 2n + 2 equations in 2n + 2 dependent variables. These are $x_1, \ldots, x_n$; $p_1, \ldots, p_n, \omega, y$. The equations in (i) are defined for every structure of (p, ω, y) while (ii) is defined for given values of the quantity variables and the associated techniques.

4.3 Solving the Model

The iterative algorithm utilized to solve the model is depicted in Figure 4. By describing the different steps of the iterative scheme, we can also illustrate the operation of the overall market mechanism.

Let the exogenous parts of the demand components be given. Then for given values of (p, ω, y) all capacity removals and all D_i - components are determined. This means that the intermediary and investment deliveries can be obtained through the iterative procedure

$$x^{(n+1)} = G(x^{(n)}) \tag{4.7}$$

where G is defined in (4.3), and n denotes the n'th iteration step. The sequence $\{x^{(n)}\}$ converges for the given demand structure. Retaining the initial prices, the disposable income y is changed so that the employment condition is satisfied. By changing y the private consumption is altered, since $\partial C_i / \partial y > 0$. The variations in consumption generates variations in total demand. In this way the demand for labor is controlled.

The change in demand due to the employment target generates a change in the demand for capacity and investments. In order to realize these investments, the gross profits of each sector, π_j, must reach a level such that the capacity change condition in (3.6) is satisfied. Let $\{\tilde{\pi}_j\}$ denote these gross profits. With a given wage level ω, the prices are then obtained as

$$p^a = p^a UA + \tilde{\pi} + wl \tag{4.8}$$

where $p^a = \{p_i^a\}$ denotes sector prices, $w = (\omega\bar{w}_1,\dots,\omega\bar{w}_n)$, $l = (l_1,\dots,l_m)$. A solution to (4.8) is obtained by means of the same type of iteration scheme as that described in (4.7).

The new prices obtained in (4.8) are now inserted into export and import functions, and the trade balance condition is checked. The domestic price level is changed by keeping $\{\tilde{\pi}_j\}$ constant and at the same time varying the wage level ω. These variations are continued until the desired trade balance is obtained. At this step the most recent price structure is compared with the initial price pattern. If the two price vectors are not the same the process is repeated, now with the most recent values of (p,ω,y) as the starting point. The first basic step is then once again (4.7).

One should observe that the input-output matrix A and the labor input vector l are changed when old capacities are removed and new are introduced. In this way also the technology structure is endogenously determined in the medium-term perspective.

One may also remark that the algorithm is converging fast at each partial step. On the average 5 overall iterations are necessary to obtain the overall equilibrium solution[1]. No formal characterization has yet been established with regard to which necessary and sufficient conditions the model structure must satisfy in order to guarantee the convergence of the algorithm.

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[1] Including compilation, a solution is obtained in about 5 seconds CPU-time on an IBM 3033.

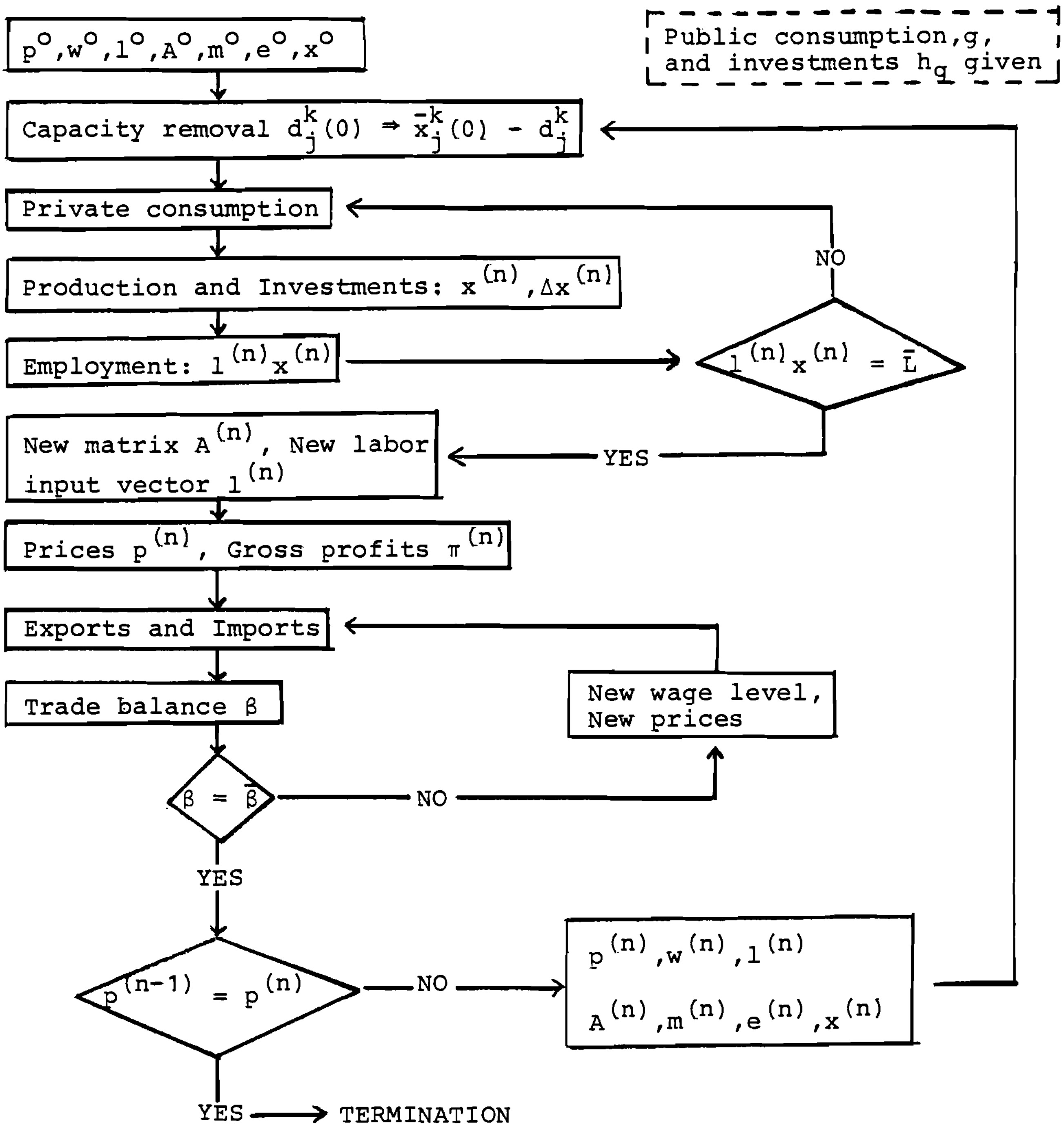

Figure 4. Solution algorithm.

APPENDIX 1: TECHNICAL PROPERTIES OF THE MODEL

In its current version the model has 28 sectors of which 20 consist of industrial subsectors comprising the mining and manufacturing industries. With regard to these 20 sectors, the estimation of production techniques, investment behavior, capacity removal etc., has been based on individual data for 10,000-11,000 production units (establishments). The current data base covers the period 1968-1980. In the operative data base these units are grouped into technique classes. The data base allows for a disaggregation of the 20 sectors into about 80-100 sectors. The available input-output table for the whole economy can be expanded from 28 to 88 sectors.

A1.1 Domestic Demand Components

Public Consumption and Employment

The consumption of commodities (goods and services) in the public sector is determined by the following types of equations

$$g_i = \bar{g}_i(1 - f)G + h_{ig} \text{ , } i=1,\ldots,n$$

where $\bar{g}_i$ is a fixed coefficient and where f denotes the ratio between value added in the public sector and total public con-

consumption. The latter is denoted by G. The delivery of commodity i to investment projects in the public sector is denoted by h_{ig}. The demand for labor in the public sector is obtained by dividing value added in the sector by an estimated labor productivity coefficient.

Private Consumption

The private consumption is decomposed into 10 different aggregate commodity groups, $V_1,\ldots,V_{10}$. For a given disposable income y and given prices $p_1^v,\ldots,p_{10}^v$ corresponding to the commodity groups, the consumption of commodity group $j = 1,\ldots,10$ is obtained from a linear expenditure system such that

$$V_j = \gamma_j + [\beta_j/p_j^v][y - \sum_{k=1}^{10} \gamma_k p_k^v], \tag{A.1}$$

where γ_j and β_j are estimated parameters. The consumption given by the system in (A.1) is distributed over the 28 commodities of the model by means of coefficients c_{ij} such that $\sum_i c_{ij} = 1$.

Private consumption of commodity i is therefore obtained as

$$c_i = \sum_{j=1}^{10} c_{ij}V_j \quad , i = 1,\ldots,28 \tag{A.2}$$

and the commodity prices, $(p_1,\ldots,p_{28})$, are transformed in an analogous way to commodity group prices so that

$$p_j^v = \sum_{i=1}^{28} p_i c_{ij} \tag{A.3}$$

The model is calibrated for the base year so that all prices are given unit value.

One should observe that the system in (A.1)-(A.3) has a form which is appropriate for a multisector model with endogenous price formation. A basic consistency requirement in such a model is that the disposable income, y, equals the consumption expenditure. If $\Sigma\beta_i = 1$, it follows directly from (A.1)-(A.3) that

$$y = \sum_{j=1}^{10} p_j^v V_j = \sum_{i=1}^{28} p_i c_i \tag{A.4}$$

Alternative consumption functions can often be expressed in the following form

$$c_i = \bar{c}_i \prod_j p_j^{e_{ij}} y^{\eta_i} \quad , \; e_{ii} < 0 \tag{A.5}$$

Such functions will generally not satisfy the consistency criterion in (A.4).

Suppose that the function in (A.5) is initially calibrated so that $\Sigma p_j c_j = y$. Consider then an alteration of the income level, y, and the price structure such that the consistency requirement in (A.4) is violated. A standard procedure of remedying such an inconsistency is to recalibrate each function so that c_i is replaced by $\hat{c}_i$, where the latter is assumed to be valid after the alteration:

$$\begin{aligned} \hat{c}_i &= k c_i \\ k &= y/\Sigma p_i c_i \end{aligned} \tag{A.6}$$

One should now note that also if k in (A.6) is close to unity, the accuracy of the procedure may be questioned. To see this, let us first simplify by setting $e_{ij} = 0$ for $j \neq i$ and then differentiate the recalibrated consumption function, $\hat{c}_i$ in (A.6), with respect to p_i which yields

$$\partial \hat{c}_i / \partial p_i = c_i k [e_{ii}/p_i - c_i(e_{ii}+1)/\Sigma p_j c_j]$$

The own price elasticity, $\hat{e}_{ii}$, of the recalibrated function can therefore be expressed as

$$\hat{e}_{ii} = e_{ii} - [e_{ii}+1] p_i c_i / \Sigma p_j c_j$$

Obviously, the elasticity remains unchanged after the recalibration only if $e_{ii} = -1$.

Hence, if the procedure in (A.6) is utilized for a case in which $e_{ii} \neq -1$, then every recalibration implies a shift for another consumption function than the one initially estimated and introduced in (A.5).

Cost of Living Index

Since the model determines both consumption and price levels one may deliberate the possibility of calculating the effects a solution has on the real income and the cost of living. The linear expenditure system is based on assumptions which simplify such calculations. Referring to (A.1) we may define

$$\rho = [y - \sum_k \gamma_k p_k^v]/y$$

$$s_k = \gamma_k p_k^v / \sum_k \gamma_k p_k^v$$

Let $\{\tilde{p}_k^v\}$ be the base year prices. Then the index of cost of living, $I(p^v)$, becomes [see Theil (1980) p23]:

$$I(p^v) = \rho \prod_{k=1}^{10} [p_k^v/\tilde{p}_k^v]^{\beta k} + (1-\rho)\Sigma s_k [p_k^v/\tilde{p}_k^v]$$

where a solution $\{p_k^v\}$ is compared with base year prices $\{\tilde{p}_k^v\}$.

In a similar way we may also calculate the index of real income [see Theil (1980) p24]. The income index, $y(p^v)$, becomes

$$y(p^v) = (\rho - \tilde{\rho}) + \rho\left[\frac{y/\tilde{y}}{\Pi [p_k^v/\tilde{p}_k^v]^{\beta_k}} - 1\right] + 1$$

where $\tilde{p}_k^v$, $\tilde{\rho}$, and $\tilde{y}$ refer to the base year.

A1.2 International Trade

The export of commodity j, e_j, is determined through an estimated function

$$e_j = e_j^o [p_j/p_j^w]^{e_j^1} \exp\{e_j^2 t\} \qquad (A.7)$$

where t denotes time, p_j^w the world market price and p_j the domestic price of commodity j. The parameters e_j^1 and e_j^2 are econometrically estimated while e_j^o is determined by the base year calibration procedure. Through the calibration the base year prices satisfy $p_i = p_i^w = 1$.

The import of commodity j, m_j, is determined through the following relationship

$$m_j = m_j^o [p_j/p_j^w]^{m_j^1} \exp\{m_j^2 t\}(1+s_j)x_j^v \qquad (A.8)$$

where m_j^1 and m_j^2 are estimated parameters, s_j a coefficient expressing tax and subsidy rates with regard to commodity j. The parameter m_j^o is calibrated in such a way that consistency is obtained in the base year for normalized base year prices.

The scenarios/projections of the model are obtained contingent on prespecified requirements on the balance of trade. The constraint which has been utilized is

$$\bar{\beta} = \sum_i p_i^w (e_i - m_i)$$

APPENDIX 2: ILLUSTRATION OF SCENARIOS OBTAINED WITH THE MODEL

The rationale for this appendix is merely to illustrate the nature of the model by presenting a selected sample of various outputs which the model generates. One important feature of the model results is the possibility to distinguish between several types of prices and thereby also different kinds of fixed price evaluations. The presentation describes how three different scenarios were generated for the Swedish economy with regard to the period 1980-1990. Then we illustrate some effects of the equilibrium projections on trade and capital formation. All scenarios described have the character equilibrium impacts of different balance of trade targets.

A2.1 Basic Assumptions for Three Development Scenarios 1980-1990

The background for all three scenarios presented here is a comparatively slow change of public consumption and public investments. The development of the world market is represented by a price scenario describing the prices which Swedish exporters and importers are expected to meet on the world market during the eighties. In summary these price projections are more favorable for industries producing machinery and other forms of equipment than for industry sectors like mining, steel production, and segments of the forest industry.

Three different scenarios have been obtained by specifying three alternative requirements for the balance of trade. The labor market condition is the same in all cases. The ratio between export incomes and import expenditures are for each of the alternatives (I,II,III), related to the export/import ratio 1980 in the following way:

Alternative I: + 6 percent
Alternative II: + 10 percent
Alternative III: + 20 percent

The third alternative turns out to be extreme in the sense that both investments and consumption have to be pressed down so as to satisfy the foreign trade condition. As seen in Table A2:1 the second alternative is more balanced, while alternative I has the character of "laissez-faire".

Table A2:1. The Swedish economy 1980-1990
Three development alternatives

	Alternative		
	I	II	III
Annual change in percent:			
- total production	+1.3	+1.8	+1.5
- private consumption	+2.5	+1.6	+0.3
- volume of export	+2.4	+3.8	+4.4
Ratio between the average for the period and the level 1980 (percent):			
- total investments	103	110	104
- building investments	100	102	97
- housing consumption	106	105	103

A2.2 Capital Formation and Capacity Change in Three Scenarios

The distinction between production, consumption and capacity is obvious if the housing sector capacity in Table A2:1 is compared with housing consumption in Table A2:2

Table A2:2 Capacity change and investment share

	Alternative I	Alternative II	Alternative III
Ratio between capacity 1990 and 1980 in percent			
- the whole economy	109	114	111
- the building industry	97	98	94
- the housing sector	102	101	99
Investments/value added in percent (whole economy)	23.8	24.6	23.8
Building investments divided by total investments (percent)	56.8	54.6	55.0

Table A2:3 Distribution of building investments 1980-1990.

	Distribution in percent 1980	1990 according to alternative I	II	III
Manufacture of machinery & equipm.	21	33	38	39
Forest industries	13	23	22	23
Food industries	15	10	9	7
Chemical production	6	10	10	11
Agriculture, forestry and other manufacturing industries	45	24	21	20
	100	100	100	100
Ratio between the average for the period and the level 1980		126	147	144

A2.3 Illustration of Trade Scenarios

In the following three tables two industries have been selected to illustrate how the foreign trade is changing in the three scenarios. The degree of specialization, which is calculated in Tables A2:5 and A2:6 is defined as

$$\text{Degree of specialization} = \frac{\text{Export volume} - \text{Import volume}}{\text{Export volume} + \text{Import volume}}$$

Table A2:4 Manufacturing of Wood products and Mineral products.

	Alternative I	II	III
Ratio between the capacity 1990 and 1980 in percent			
- Wood products	111.0	115.5	111.0
- Mineral products	86.7	94.5	100.0
Level of exports compared with altern. I in percent			
- Wood products	100	113	117
- Mineral products	100	124	132
Ratio between domestic and world market price			
- Wood products	1.05	0.93	0.90
- Mineral products	1.26	1.12	1.08

Table A2:5 Trade scenarios for Wood products. Fixed prices (1975)

	Level 1980	Scenario level 1990 according to altern. II	III
Volume of exports	5001	7480	7753
Volume of imports	1730	2203	2033
Export surplus	3271	5277	5720
Degree of specialization	49%	54%	58.5%

Table A2:6 Trade scenarios for Mineral products. Fixed prices (1975).

	Level 1980	Scenario level 1990 according to altern. II	III
Volume of exports	1022	1226	1302
Volume of imports	1422	1962	1842
Export surplus	-400	-736	-540
Degree of specialization	-16%	-23%	-17%

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