International Institute for
Applied Systems Analysis
IIASA www.iiasa.ac.at

Visual Q-Analysis: A Case Study of Future Computer Systems Developments

Sugiyama, K. and Toda, M.

IIASA Collaborative Paper
August 1982

VISUAL Q-ANALYSIS: A CASE STUDY OF FUTURE COMPUTER SYSTEMS DEVELOPMENTS

Kozo Sugiyama*
Mitsuhiko Toda**

August 1982
CP-82-51

* International Institute for Applied Systems Analysis, Laxenburg, Austria
** International Institute for Advanced Study of Social Information Science, Fujitsu Limited, Numazu, Shizuoka, Japan

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
2361 Laxenburg, Austria

ABSTRACT

A novel method called Visual Q-Analysis (VQA) is proposed to analyze structures of complex systems. This method is based upon Atkin's Q-analysis where the structure of a system is represented by simplicial complex in topology and analyzed in terms of q-connectivity.

Two different types of hierarchies, Q-hierarchy and F-hierarchy, are introduced and algorithms to obtain these are given. In order to draw these hierarchies in a visually understandable form SKETCH system developed by the authors are used. The Q-hierarchy visualizes a hierarchical q-connectivity structure among all the simplices and the F-hierarchy expresses a structure of face-sharing among the simplices in the complex. By inspecting their drawings we can grasp the structural information embedded in the complex.

This method is applied to a structural study of technological development of future computers of Japan in terms of relationships between social needs and technological requirements (seeds). Results of the application not only show the effectiveness of VQA to support in planning technological developments but also suggest wide applicabilities of VQA to various other fields.

CONTENTS

VISUAL Q-ANALYSIS: A CASE STUDY OF FUTURE COMPUTER SYSTEMS DEVELOPMENTS

Kozo Sugiyama and Mitsuhiko Toda

1. INTRODUCTION

Efficient methodologies for analyzing structures of complex systems such as social systems have been developed independently by Atkin (1972a, 1972b, 1977) and Warfield (1973, 1974, 1976). The former is called Q-analysis (QA)*, whereas the latter is known as interpretive structural modeling (ISM). Both methodologies are based upon a notion of mathematical relations. However, they differ not only in algorithms but also in other fundamentals, e.g., viewpoints of models and data, purpose, the mathematical theories upon which they are based.

In QA the structure of a system is represented by binary relations between two finite (well-defined) sets arranged on a hierarchy of data sets such as the following:

Level	*Sets*	*Types*	
$N+2$	---- L ----	$P^3(S)$	(power set of $P^2(S)$)
$N+1$	A ----- B -----	$P^2(S)$	(power set of $P(S)$)
	μ | \ Θ		
N	X --λ-- Y ----- Z	$P(S)$	(power set of S)
	|		
$N-1$	P -----	S	(a set of elements)

*Note that in this paper Atkin's work excluding his ***dynamics*** is called QA and an analysis obtaining Q-vector is referred as Q-vector analysis, although Atkin himself calls Q-vector analysis QA.

This hierarchy (H) of data sets A, B, L, ... is defined by the mathematical relations λ, μ, ... in a basis of set covering; i.e., if X is N-level set and A is a corresponding $(N + 1)$-level set, then A must be a cover set for X. (It should be noted here that these relations are considered "hard" data, which have become *known* through intuitive experiences.) Then each relation is converted into a geometrical structure in a multi-dimensional space which is called a simplicial complex in topology, and the structure of the simplicial complex is analyzed in terms of q-connectivity among simplices included in the complex with the aid of computers. This complex is regarded as the backcloth against which any activity in the system is carried on. As a major result of QA a vector called Q-vector is obtained. This vector can be considered as a useful measure of the global structure of the complex. In a local view special structures, e.g., holes are found which can express important structural features. Moreover, the eccentricity of simplex is introduced as a measure for integrating the 'q-connected' relation of the individual simplex to the rest of the complex.

In ISM, "beginning with a mental model (individually or collectively held), a particular transitive contextual relation among a set of system elements is embedded in a binary matrix model with computer assistance. This matrix model can be partitioned in various ways and one or more digraphs can be extracted from the matrix model. The digraphs can then be replaced by substituting verbal statements of elements for their numerical representations. The structures then are tested against the prevailing mental model, whereupon the latter may be improved. In the process it is usually necessary to make corrections in the structures, which can also be done with computer assistance. This iterative learning process terminates when a suitable *interpretive structural model* is achieved, whereupon it serves as a basis for documentation and communication of the substantive ideas represented by the structures (Warfield 1974)."

Diagrams of QA and ISM processes are shown in Figure 1(a) and 1(b), respectively. Conceptual differences between these two methodologies are clearly seen in the figure:

(i) A hierarchical structure of the data sets is presumed in the former, while a hierarchical structure of system elements is extracted from a matrix model in the latter.

(ii) In the former, relations (or incidence matrices) embedded by examining intuitive experiences are regarded as "hard" data, which are based on well-defined sets, while in the latter the matrix model is corrected iteratively by comparing a tentative interpretive structural model with a mental model and therefore the matrix model is considered intrinsically "soft" data. These differences are summarized in Table 1.

It is interesting to make the conceptual process of QA more flexible from a pragmatic viewpoint, referring to the ISM process, i.e., the process of QA is altered into an interactive one as represented in Figure 1(c). This is because we want to apply QA to problems where soft data must be treated, such as planning, assessment, etc. In such an interactive

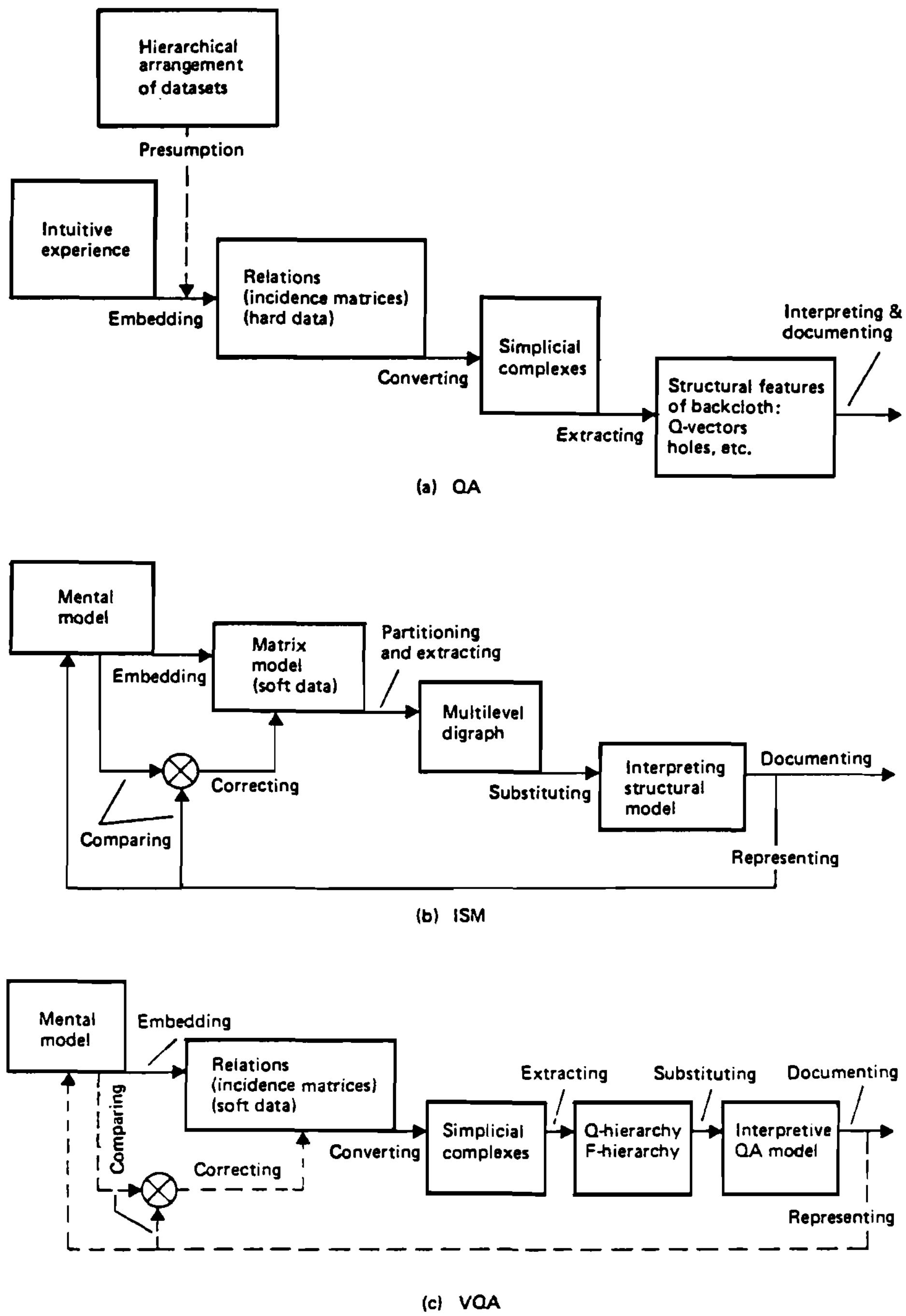


Figure 1. Conceptual diagrams of the processes of (a) QA, (b) ISM, and (c) VQA.

Table 1. Summary of differences between QA and ISM.

Item	QA	ISM
Fundamental model structure	hierarchical arrangement of data sets relations between two finite sets (converted into simplicial complexes)	a transitive relation among a set of system elements (converted into a multilevel digraph)
Data	hard data	soft data
Process of analysis (or purpose)	precise interpretation of structural features of backcloth	hierarchical modeling through the interactive learning process
Major results	Q-vectors (global view) holes (local view) eccentricity etc.	interpretive structural model (hierarchical model)
Mathematical theory based on	topology	graph theory

process a tentative structural model should be tested against a mental model and therefore a method for grasping a structure of the tentative model immediately is needed in both global and local senses. For this purpose, a visual representation of the structure is very convenient, if, of course, it is done in a visually understandable way.

In this paper we present a method called visual Q-analysis (VQA) where two different types of multilevel digraphs (or hierarchies), Q-hierarchy and F-hierarchy, are extracted from a simplicial complex and then these digraphs are drawn in a visually understandable form. The drawing of Q-hierarchy can visualize not only the results of Q-vector analysis but also a hierarchical q-connectivity structure among all simplices. The drawing of F-hierarchy can express a more detailed structure or a structure of face-sharing among simplices such as nearness, chain and loop in the complex. As a measure of integration we also define the *concentricity* other than the *eccentricity* defined by Atkin. By inspecting the combination of these drawings as well as measures, Q-vector, eccentricity and concentricity, we can know well structural features of the complex. In developing the algorithms for VQA we are indebted to Atkin (1977) and Warfield (1974), and in drawing the hierarchies we use the SKETCH system, which has been developed by Sugiyama, Tagawa, and Toda (1981a, 1981b). A flow diagram of procedures from an incidence matrix to a visually understandable map of hierarchy is shown in Figure 2.

Notations and algorithms to obtain Q-hierarchy and F-hierarchy are described in Section 2 and a simple example is presented in Section 3 to illustrate the algorithms and to explain how to interpret the results.

In Section 4 a case study is shown to demonstrate the effectiveness of VQA. A national project to develop the fifth generation computer systems has been initiated this year (1982) in Japan. In order to plan the project, extensive surveys and investigations were carried out by a

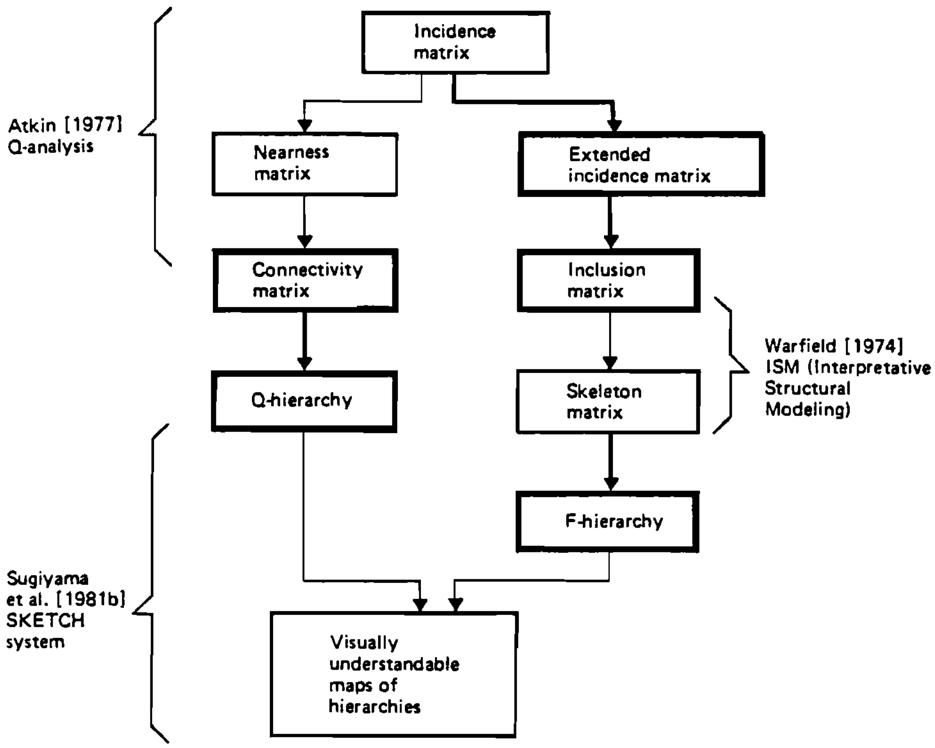


Figure 2. Flow diagram of procedures used in VQA. Arrows show flows of procedures, and boxes show matrices, hierarchies, or maps. The bold-faced arrows and boxes indicate new concepts and algorithms proposed in the paper.

special committee during two fiscal years (1979-1980); as a first approach by the committee social needs (bottlenecks) and technological seeds expected in 1990s were identified and connected into a relation in a systematic way. We analyzed the structure of developments of the future computer systems in terms of q-connectivity in the relation. Results not only show the effectiveness of VQA for the planning of technological developments, but also suggest wide applicability of VQA to various other fields.

The work presented in this paper is based upon the previous work by the present authors; Toda, Sugiyama, and Tagawa (1981), Sugiyama, Toda and Tagawa (1981) and Toda and Sugiyama (1982).

2. NOTATIONS AND ALGORITHMS

2.1. Binary Relation and Incidence Matrix

Let λ be a binary relation between two finite sets $x = \{x_1, x_2, \cdots, x_m\}$ and $Y = \{y_1, y_2, \cdots, y_n\}$, i.e., $\lambda \subset X \times Y$. We write $x_i \lambda y_j$ if x_i is λ-related to y_j, i.e., $(x_i, y_j) \in \lambda$. For a given relation $\lambda \subset X \times Y$ an inverse relation $\lambda^{-1} \subset Y \times X$ is defined by

$$(y_j, x_i) \in \lambda^{-1} \Longleftrightarrow (x_i, y_j) \in \lambda.$$

Such a λ-relation between X and Y can be represented by an incidence matrix A defined by

$$A = (a_{ij})$$

$$a_{ij} = \begin{cases} 1 & \text{if } x_i \lambda y_j \\ 0 & \text{otherwise} \end{cases} \qquad (2.1)$$

$$i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n.$$

This is summarized in the notation

$$\begin{array}{c|c} \lambda & Y \\ \hline X & A \end{array} \qquad (2.2)$$

2.2. Simplex, Face and Simplicial Complex

When a $(p+1)$-subset of Y is λ-related to $x_i \in X$, we call the $(p+1)$-subset of Y a p-simplex of x_i. If a $(p+1)$-subset is $\{y_1^{(i)}, y_2^{(i)}, \ldots, y_{p+1}^{(i)}\}$, the p-simplex is denoted by

$$\sigma_p(x_i) = \langle y_1^{(i)}, y_2^{(i)}, \ldots, y_{p+1}^{(i)} \rangle. \qquad (2.3)$$

We sometimes write $\sigma(x_i)$ or σ_p instead of $\sigma_p(x_i)$ for simplicity.

Every $(q+1)$-subset of the $(p+1)$-subset of Y $(q \le p)$ is also λ-related to x_i and is therefore another simplex, i.e., a q-simplex. This q-simplex, σ_q, is said to be a face of $\sigma_p(x_i)$, and this is written as

$$\sigma_q < \sigma_p . \tag{2.4}$$

A collection of simplices $\{\sigma(x_i); = 1,2,\ldots,m\}$ is called a simplicial complex $K_X(Y;\lambda)$ (or $K_X(Y)$). Similarly λ^{-1} gives us the conjugate complex $K_Y(X;\lambda^{-1})$ (or $K_Y(X)$). In $K_X(Y)$, X and Y are called sets of name elements and vertex elements respectively.

We say that p is the dimension of σ_p. The dimension of simplicial complex K is the largest value of dimensions of simplices, i.e., $\max\{p \mid \sigma_p \in K\}$. The dimension of K is written by dim K. We denote the empty set by the (-1)-simplex, σ_{-1}.

2.3. Nearness Matrix and Connectivity Matrix

When an r-face is shared by $\sigma(x_i)$ and $\sigma(x_j)$, we say that they are r-near and write this as

$$\sigma(x_i) \cap \sigma(x_j) = \sigma_r \tag{2.5}$$

where r is called nearness between $\sigma(x_i)$ and $\sigma(x_j)$. If $\sigma(x_i)$ and $\sigma(x_j)$ are r-near, they are also t-near for $t=0,1,\ldots,r-1$. A nearness matrix B is introduced by

$$\begin{aligned} B &= (b_{ij}) \\ b_{ij} &= \max\{r \mid \sigma(x_i)\cap\sigma(x_j)=\sigma_r, \sigma(x_i), \sigma(x_j)\in K_X(Y)\} \\ &\quad i,j=1,2,\ldots,m. \end{aligned} \tag{2.6}$$

Given two simplices σ_p and σ_r in K, we shall say they are joined by a chain of connection if there exists a finite sequence of simplices

$$\sigma_{\alpha_1}, \sigma_{\alpha_2}, \ldots, \sigma_{\alpha_h}$$

such that (i) $\sigma_{\alpha_1} < \sigma_p$, (ii) $\sigma_{\alpha_h} < \sigma_r$, and (iii) $\sigma_{\alpha_i} \cap \sigma_{\alpha_{i+1}} = \sigma_{\beta_1}$, $i=1,2,\ldots,h-1$. We call this sequence a chain of q-connection (or a q-connectivity) if $q = \min\{\alpha_1, \beta_1, \ldots, \beta_{h-1}, \alpha_h\}$. The chain is denoted by $[\sigma_p, \sigma_r]_q$ and we say that σ_p and σ_r are q-connected. If σ_p and σ_r are q-connected, they are also t-connected for $t=0,1,,,,q-1$. When there exist M different chains between σ_p and σ_r in K, i.e., $[\sigma_p, \sigma_r]_{q_k}$, $k=1,\ldots,M$, we call the value given by $\max_k \cdot \max\{q_k \mid [\sigma_p, \sigma_r]_{q_k}\}$ the degree of connectivity between σ_p and σ_r.

Here we define a connectivity matrix C by

$$\begin{aligned} C &= (c_{ij}) \\ c_{ij} &= \text{the degree of connectivity between } \sigma(x_i) \\ &\quad \text{and } \sigma(x_j) \text{ in } K_X(Y), \quad i,j=1,2,\ldots,m. \end{aligned} \tag{2.7}$$

This connectivity matrix C is calculated by the following:

(i) Put $c_{ij}^{(1)} = b_{ij}$ for $i,j=1,2,\ldots,m$.

(ii) Calculate $c_{ij}^{(m)}$ according to

$$c_{ij}^{(l+1)} = \max_k \{\min(c_{ik}^{(l)}, c_{kj}^{(1)})\}. \quad (2.8)$$

(iii) Put $c_{ij} = c_{ij}^{(m)}$ for $i,j=1,2,\ldots,m$.

2.4. Q-hierarchy

Consider a relation γ_q ("is q-connected with") defined on simplices $\{\sigma(x_i); i=1,2,\ldots,m\}$ of $K_X(Y)$. This relation is an equivalence relation (Atkin 1977:156). Sets of the equivalence classes of this relation are denoted by

$$U^{(q)} = \{U_\alpha^{(q)}, \alpha=1,2,\ldots,Q_q\}, \quad q=0,1,\ldots,n-1, \quad (2.9)$$

where Q_q is the number of the classes for a fixed q and corresponds to a component of Atkin's Q-vector $Q=(Q_d, Q_{d-1},\ldots,Q_q,\ldots,Q_0)$ where $d=\dim K_X(Y)$. When the power set of $\{\sigma(x_i); i=1,2,\ldots,m\}$ is written by P, $U_\alpha^{(q)} \in P$ or $U^{(q)} \subset P$.

A Q-hierarchy H_Q is a multilevel digraph defined by

$$H_Q = (V, T, \psi) \quad (2.10)$$

where V is a set of nodes, T is a set of edges and ψ is a level assignment (Harary et al. 1965) defined by

$$V = U^{(n-1)} \cup U^{(n-2)} \cup \cdots \cup U^{(1)} \cup U^{(0)},$$

$$T = \{(v_i, v_j) \mid v_i \subseteq v_j, v_i \in U^{(q)}, v_j \in U^{(q-1)}, q=1,\ldots,n-1\},$$

$$\psi : V \rightarrow J = \{0,1,\ldots,n-1\} \text{ such that } \psi(U_\alpha^{(q)})=q \text{ for } U_\alpha^{(q)} \in V.$$

Theorem: If all the smplices of $K_X(Y)$ are 0-connected, H_Q is a tree.

Proof: Since all the smplices of $K_X(Y)$ are 0-connected, there is one equivalence class when $q=0$, i.e., $U^{(0)}=\{U_1^{(0)}\}$ which is the root of a tree. From the definition any node can be incident only to nodes of adjacent levels. Any node, $U_\alpha^{(q)}$, cannot be incident to more than one node of the lower adjacent level, since if it is incident to $U_\mu^{(q-1)}$ and $U_\eta^{(q-1)}$, then

$$U_\alpha^{(q)} \subseteq U_\mu^{(q-1)} \quad \text{and} \quad U_\alpha^{(q)} \subseteq U_\eta^{(q-1)}$$

which contradict the fact that $U_\mu^{(q-1)}$ and $U_\eta^{(q-1)}$ are disjoint which comes from a property of equivalence classes.

Absorbed Q- hierarchy: We introduce an absorbed Q-hierarchy H'_Q, since a drawing of H'_Q is expected to be more effective to represent the structure of q-connectivity among smplices than H_Q.

Let $H_Q(V')$ be a subgraph of H_Q defined by $H_Q(V')=(V' < T(V'))$ where $V' \subset V$ and $T(V')=\{(v_i,v_j)\in T \mid v_i,v_j \in V'\}$. The removal of a node v_i induces the hierarchy $H_Q(V-\{v_i\})$ by deleting node v_i and its incident edges from H_Q. To absorb a node v_i, remove v_i and add (v_j,v_k) to $H_q(V-\{v_i\})$ iff $(v_j,v_i)\in T$ and $(v_i,v_k)\in T$.

We absorb v_j in H_Q to obtain $H_Q^{(1)}=(V^{(1)},T^{(1)},\psi)$ if the conditions

(i) $(v_i,v_j)\in T$ and $(v_j,v_k)\in T$

(ii) $v_j = v_i$ (or two equivalence classes corresponding to v_i and v_j consist of the same subsets of simplices)

are satisfied in H_Q. Same procedures are carried out in $H_Q^{(1)},H_Q^{(2)},\ldots$ iteratively until there are no nodes which satisfy the conditions. Finally we obtain a hierarchy which is called an absorbed Q-hierarchy and written by $H'_Q=(V',T',\psi)$.

It is clear from the above procedure to generate H'_Q that the sum of nodes at level q and the number of edges traversing level q is equal to Q_q of the Atkin's Q-vector.

Algorithm to obtain an absorbed Q- hierarchy: An absorbed Q-hierarchy is calculated from a connectivity matrix C. As the index sets of the rows and columns of C we use $\{v_1,v_2,\ldots,v_m\}$ instead of $\{\sigma(x_1),\ldots,\sigma(x_m)\}$ for simplicity. As a work matrix we introduce a square matrix C^* where index sets of rows and columns are $\{v_1,\ldots,v_m\}$.

Step 1: $V\leftarrow\{v_1,\ldots,v_m\}$; ($v_i$ corresponds to $\sigma(x_i)$); $T\leftarrow$ empty set; $\psi(v_i)\leftarrow\dim[\sigma(x_i)]$ for $i=1,\ldots,m$; $q\leftarrow\max\{c_{ij} \mid i\neq j; i,j=1,\ldots,m\}$; $C^*\leftarrow C$; $t\leftarrow m$.

Step 2: Obtain the equivalence classes $\{U_\alpha;\alpha=1,\ldots,r\}$ of the relation "is q-connected with" for the current q by inspecting C^* (The cardinality of U_α must be greater than 1).

Step 3: $\alpha\leftarrow 1$.

Step 4: Let $U_\alpha=\{u_1,\ldots,u_\rho\}$ and obtain $U'_\alpha=\{u_k \mid \dim[\sigma(u_k)]=q, u_k\in U_\alpha\}$.

Step 5: $t\leftarrow t+1$; rename u_1 to v_t and delete rows and columns corresponding to u_k for $k=2,\ldots,\rho$ in C^*; $V\leftarrow V\cup v_t - U'_\alpha$; $T\leftarrow T\cup(u_k,v_t)$ for $u_k\in U_\alpha - U'_\alpha$; $\psi(v_t)\leftarrow q$.

Step 6: If $\alpha<r$, then go to Step 4 with $\alpha\leftarrow\alpha+1$.

Step 7: If $q>0$, then go to Step 2 with $q\leftarrow q-1$; otherwise calculations are terminated.

2.5. Extended Incidence Matrix

Each entry of nearness matrix expresses the dimensions of the face shared by two simplices in K. Each row of an extended incidence matrix, which we introduce here, expresses not only the face shared by two simplices in K, but also sequences of faces shared by higher dimensional faces.

To define an l-th order extended incidence matrix $E^{(l)}$, we introduce the following notations;

(i) $X^{(l)} = \{x_1, \ldots, x_{m_l}\}$

(ii) $E^{(l)} = (e_{ij}^{(l)})$

$$e_{ij}^{(l)} = \begin{cases} 1 & \text{if } y_j \in \sigma(x_i) \\ 0 & \text{otherwise} \end{cases} \qquad (2.11)$$

$$i=1,\ldots,m_l; j=1,\ldots,n; l=1,\ldots,L$$

where the sets of row and column indices of $E^{(l)}$ are $X^{(l)}$ and Y respectively.

Algorithm to obtain $E^{(l)}$ from a given incidence matrix A

Step 1: $E^{(0)} \leftarrow A$; $X^{(0)} \leftarrow X$; $l \leftarrow 0$; $m_{-1} \leftarrow 0$; $m_0 \leftarrow m$.

Step 2: $X^{(l+1)} \leftarrow X^{(l)}$; $E^{(l+1)} \leftarrow E^{(l)}$; $m_{l+1} \leftarrow m_l$; $i \leftarrow m_{l-1}+1$.

Step 3: $j \leftarrow i+1$

Step 4: $\sigma \leftarrow$ the maximum-dimensional face shared by $\sigma(x_i)$ and $\sigma(x_j)$.

Step 5: If σ is empty or σ is identical to some one of $\{\sigma(x_\tau) \mid \tau=1,\ldots,m_{l+1}\}$, then go to Step 7.

Step 6: $m_{l+1} \leftarrow m_{l+1}+1$; $X^{(l+1)} \leftarrow X^{(l+1)} \cup X_{m_{l+1}}$;

$$e_{m_{l+1}\nu}^{(l+1)} = \begin{cases} 1 & \text{if } y_\nu \in \sigma \\ 0 & \text{otherwise} \end{cases}$$

for $\nu=1,\ldots,n$.

Step 7: If $j<m_l$, then go to Step 3 with $j \leftarrow j+1$.

Step 8: If $j<m_l-1$, then go to Step 2 with $i \leftarrow i+1$.

Step 9: If $m_{l+1}>m_l+1$, then go to Step 2 with $l \leftarrow l+1$.

Step 10: $L \leftarrow l$; $N \leftarrow m_l$; and calculations are terminated.

2.6. Inclusion Matrix and its Skeleton Matrix

We define an inclusion matrix D by

$$D = (d_{ij})$$

$$d_{ij} = \begin{cases} 1 & \text{if } \sigma(x_i) > \sigma(x_j) \text{ in } E^{(L)} \\ 0 & \text{otherwise} \end{cases} \qquad (2.12)$$

$$i,j=1,2,\ldots,N.$$

It should be noed that a relation "is included by" is transitive.

Warfield (1974) has presented a method to obtain a skeleton matrix from a binary square matrix (called an adjacency matrix) through a reacheability matrix and condensation matrix. The process to obtain the skeleton matrix from the inclusion matrix is easier than Warfield's process since the inclusion matrix is a reachability matrix and therefore has no cycles because of its transitive nature. Here the skeleton matrix obtained from D is written by $S=(s_{ij}),\ i,j=1,2,..,N$.

2.7. F-hierarchy

An F-hierarchy H_F is a multilevel digraph defined by

$$H_F = (X^{(L)},\ F,\ \varphi) \tag{2.13}$$

where

$X^{(L)} = \{x_1, \ldots, x_N\}$

$F = \{(x_i, x_j) \mid s_{ij}=1,\ i \neq j, i,j=1,\ldots,N\}$

φ : $X^{(L)} \rightarrow J=\{0,1,\ldots,n-1\}$ such that $\varphi(x_i)=\dim[\sigma(x_i)]$ for $i=1,..,N$; φ is called a level assignment (Haray et al 1965).

2.8. Visual Representations of Hierarchies

Sugiyama, Tagawa & Toda (1981a, b) have developed methods to draw visually understandable (or readable) maps of multilevel digraphs automatically with the use of computers, where common aspects of the readability are considered.

Generally speaking, it is difficlut to grasp the structure of digraph readily unless vertices are laid out in some regular form (e.g., culstered layout) and/or unless edges are drawn in such a form that paths can be readily traced by human eyes. In the case of hierarchies, the former, regular layout of vertices, is identified as the following readability element:

Element A: "Hierarchical" layout of vertices

The latter, traceability of pathes, is broken down into the following four readability elements:

Element B: "Less-crossings" of lines (The greatest difficlty in tracing paths is line crossings.)

Element C: "Straightness" of lines (It is easy to trace straight lines.)

Element D: "Close" layout of vertices connected to each other (It is desirable that paths are short.)

Element E: "Balanced" layout of lines coming into or going from a vertex (This means that structural information on branching and joining of paths is drawn clearly.)

A program for drawings coded by FORTRAN is called "SKETCH system" which has also the function to draw characters labelling nodes. Q-hierarchy and F-hierarchy are represented in a visually understandable form by using SKETCH system.

2.9. Eccentricity and Concentricity

Each simplex in a complex corresponds to some substantive event in the problem context. Therefore it is usefl to introdce measures expressing the situation on which each smplex is put within the whole structure. In other words it is desirable to know an integrated relationship between each smplex and the rest simplices of the complex. The meaning of the situation is different according to the problem context, e.g., "independent," "special," "influential," etc.

As one of measures Atkin (1972b) has defined *eccentricity* $Ecc(\sigma)$ of a smplex σ:

$$Ecc(\sigma) = \frac{\hat{q} - \check{q}}{\check{q} + 1} \tag{2.14}$$

where $\hat{q}$=dimσ and $\check{q}$ is the largest value of q for which σ is q-connected to some other simplex in a complex. We introduce a new measure called *concentricity* $Con(\sigma)$ of a simplex (or face) σ in $K_X(Y)$ for the convenience to the application described in Section 4:

$$Con(\sigma) = \sum_{y_j \in \sigma} \dim \sigma(y_j) / (\dim\sigma + 1) \tag{2.15}$$

Here it should be noted that

(i) both measures, eccentricity and concentricity, are relative values,

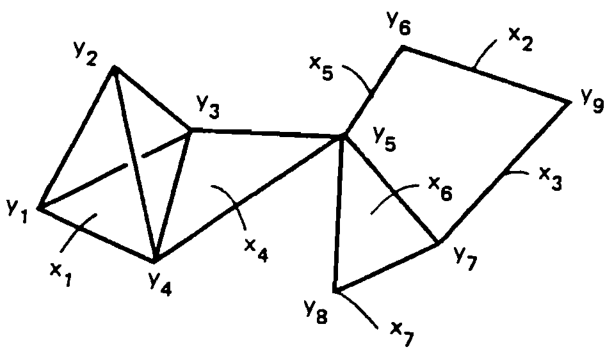
(ii) the eccentricity depends upon another single simplex whereas the concentricity depends upon the whole structure.

3. ILLUSTRATIONS OF ALGORITHMS

We illustrate the algorithms described in Section 2 by using an incidence matrix A

$$A = \begin{array}{c|ccccccccc} \lambda & y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 & y_8 & y_9 \\ \hline x_1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ x_2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ x_3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ x_4 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ x_5 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ x_6 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ x_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array} \tag{3.1}$$

for example. A geometrical representation of a smplicial complex $K_K(Y;\lambda)$ is shown as



where x_i denotes $\sigma(x_i)$. The simplices constituting the complex are

$$\begin{aligned} \sigma(x_1) &= < y_1\, y_2\, y_3\, y_4 > \\ \sigma(x_2) &= < y_6\, y_9 > \\ \sigma(x_3) &= < y_7\, y_9 > \\ \sigma(x_4) &= < y_3\, y_4\, y_5 > \\ \sigma(x_5) &= < y_5\, y_6 > \\ \sigma(x_6) &= < y_5\, y_7\, y_8 > \\ \sigma(x_7) &= < y_8 >. \end{aligned} \tag{3.2}$$

3.1. Q-hierarchy

The nearness matrix B can be obtained readily from A or $K_Y(X)$ according to (2.6)

$$B = \begin{array}{c|ccccccc|} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline x_1 & 3 & -1 & -1 & 1 & -1 & -1 & -1 \\ x_2 & -1 & 1 & 0 & -1 & 0 & -1 & -1 \\ x_3 & -1 & 0 & 1 & -1 & -1 & 0 & -1 \\ x_4 & 1 & -1 & -1 & 2 & 0 & 0 & -1 \\ x_5 & -1 & 0 & -1 & 0 & 1 & 0 & -1 \\ x_6 & -1 & -1 & 0 & 0 & 0 & 2 & 0 \\ x_7 & -1 & -1 & -1 & -1 & -1 & 0 & 0 \\ \hline \end{array} \tag{3.3}$$

from which we obtain the connectivity matrix C after some calculation using (2.8) as

$$C = \begin{array}{c|ccccccc|} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline x_1 & 3 & 0 & 0 & 1 & 0 & 0 & 0 \\ x_2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ x_3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ x_4 & 1 & 0 & 0 & 2 & 0 & 0 & 0 \\ x_5 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ x_6 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ x_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array} \tag{3.4}$$

Since the algorithm to obtain an absorbed Q-hierarchy from C is complicated, we shall explain it step by step by presenting the results of each step:

Step 1: $V=\{v_1,\ldots,v_7\}$; $T=\{\ \}$; $\psi(v_1)=3$, $\psi(v_2)=1$, $\psi(v_3)=1$, $\psi(v_4)=2$, $\psi(v_5)=1$, $\psi(v_6)=2$, $\psi(v_7)=0$; $q=1$; $C^*=C$; $t=7$.

Step 2: $U_1=\{v_1,v_4\}$; $r=1$.

Step 3: $\alpha=1$.

Step 4: As dimensions of $\sigma(x_1)$ and $\sigma(x_4)$ are not equal to $q(=1)$, $U'_1=\{\ \}$.

Step 5: $t=8$; $V=\{v_1,\ldots,v_7,v_8\}$; $T=\{(v_1,v_8),(v_4,v_8)\}$; $\psi(v_8)=1$;

$$C^* = \begin{array}{c|cccccc|} & v_8 & v_2 & v_3 & v_5 & v_6 & v_7 \\ \hline v_8 & 1 & 0 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 1 & 0 & 0 & 0 & 0 \\ v_3 & 0 & 0 & 1 & 0 & 0 & 0 \\ v_5 & 0 & 0 & 0 & 1 & 0 & 0 \\ v_6 & 0 & 0 & 0 & 0 & 2 & 0 \\ v_7 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array}$$

Step 6: As $\alpha \not< r$, go to Step 7.

Step 7: As $q=1>0$, go to Step 2 with $q=0$.

Step 2: $U_1=\{v_8,v_2,v_3,v_5,v_6,v_7\}$; $r=1$.

Step 3: $\alpha=1$.

Step 4: $U'_1=\{v_7\}$.

Step 5: $t=9$; $V=\{v_1,\ldots,v_6,v_8,v_9\}$;
$T=\{(v_1,v_8),(v_4,v_8),(v_2,v_9),(v_3,v_1),(v_5,v_9),(v_6,v_9),(v_8,v_9)\}$;
$\psi(v_9)=0$;

$$C^* = \begin{array}{c|c|} & v_9 \\ \hline v_9 & 0 \\ \hline \end{array}$$

Step 6: As $\alpha \not< r$, go to Step 7.

Step 7: As $q=0$, calculations are terminated.

As the results we obtain

$$H'_Q = (V, T, \psi) \tag{3.5}$$

where $V=\{v_1,v_2,v_3,v_4,v_5,v_6,v_8,v_9\}$; $T=\{(v_1,v_8),(v_2,v_9),(v_3,v_9),(v_4,v_8),(v_5,v_9),(v_6,v_9),(v_8,v_9)\}$; $\psi(v_1)=3, \psi(v_2)=1, \psi(v_3)=1, \psi(v_4)=2, \psi(v_5)=1, \psi(v_6)=2, \psi(v_8)=1, \psi(v_9)=0$.

3.2. F-hierarchy

Calculations to get an extended incidence matrix $E^{(l)}$ are terminated when $l=1(=L)$. The result of the calculations is

$$E^{(1)} = \begin{array}{r|ccccccccc|} & y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 & y_8 & y_9 \\ \hline x_1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ x_2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ x_3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ x_4 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ x_5 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ x_6 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ x_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \hline x_8(x_1,x_4) & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ x_9(x_2,x_3) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ x_{10}(x_2,x_5) & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ x_{11}(x_3,x_6) & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ x_{12}(x_4,x_5) & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline \end{array} \quad \left.\begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}\right\} E^{(0)} \qquad (3.6)$$

where $m_0=7, m_1=12(=N)$ and a row index $x_i(x_j,x_k)$ means that x_i is the name of a simplex corresponding to the row newly generated by an AND operation, $\sigma(x_j) \cap \sigma(x_k)$. From $E^{(1)}$ we can obtain inclusion matrix D easily as

$$D = \begin{array}{r|cccccccccccc|} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} & x_{12} \\ \hline x_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ x_2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ x_3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ x_4 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ x_5 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ x_6 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ x_7 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ x_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ x_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ x_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ x_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ x_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hline \end{array} \qquad (3.7)$$

According to the algorithm by Warfield (1974), we eliminate 1-entries in D and get a skeleton matrix S. In this case there are no 1-entries which should be eliminated, i.e., $S = D$.

From S we have readily H_F,

$$H_F = (X^{(L)}, F, \varphi)$$

where

$$X^{(L)} = \{x_1, x_2, \ldots, x_{12}\};$$

$$F = \{(x_1,x_8),(x_2,x_9),(x_2,x_{10}),(x_3,x_9),(x_3,x_{11}),(x_4,x_8),(x_4,x_{12}),(x_5,x_{10}), (x_5,x_{12}),(x_6,x_7),(x_6,x_{11}),(x_6,x_{12})\}$$

$\varphi(x_1) = 3$, $\varphi(x_2) = 1$, $\varphi(x_3) = 1$, $\varphi(x_4) = 2$, $\varphi(x_5) = 1$, $\varphi(x_6) = 2$,

$\varphi(x_7) = 0$, $\varphi(x_8) = 1$, $\varphi(x_9) = 0$, $\varphi(x_{10}) = 0$, $\varphi(x_{11}) = 0$, $\varphi(x_{12}) = 0$.

3.3. Eccentricity and Concentricity

According to (2.14) and (2.15) eccentricity and concentricity for each simplex in $K_X(Y)$ are calculated:

	Ecc	*Con*		*Ecc*	*Con*
x_1	1	0.5	x_5	1	1.5
x_2	1	1	x_6	2	1.33
x_3	1	1	x_7	0	1
x_4	0.5	1.33			

As known from definitions the eccentricity means "independent" while the concentricity means "influential." Therefore, in general the values of these measures change in an opposite sense. Nevertheless, both measures of x_6 show high values. This is because the eccentricity depends upon another single simplex whereas the concentricity the whole structure.

3.4. Interpretation of Maps

Obtained Q-hierarchy and F-hierarchy are drawn by the SKETCH system, and their maps are shown in Figure 3(a) and 3(b), respectively. Form these maps and calculated measures we can easily recognize structural features of complices as follows:

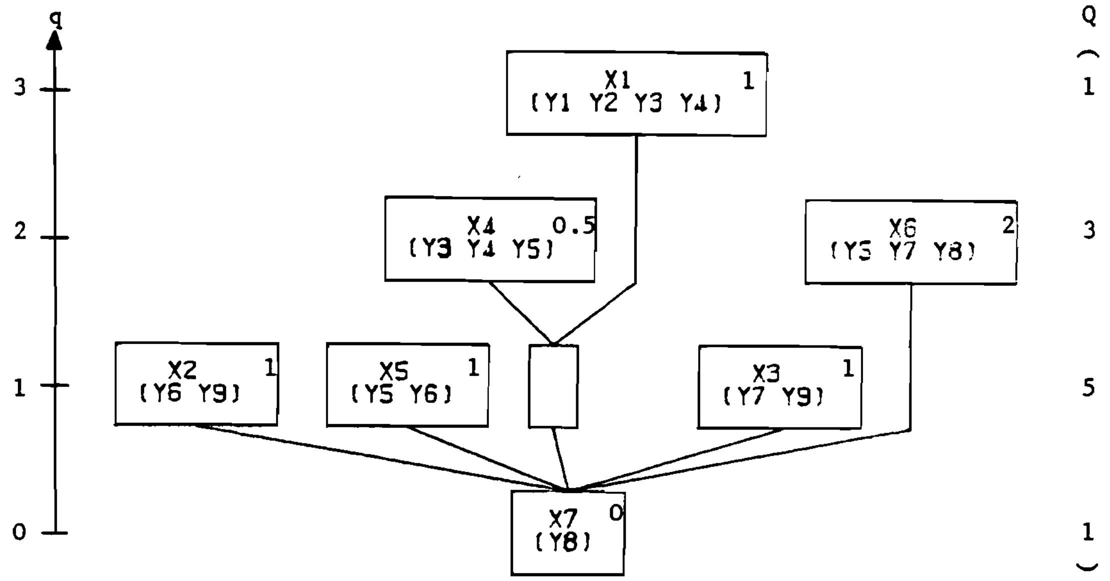
(1) In both figures simplices are drawn as boxes labeled with names of simplices and names of nodes constituting the simplices, whereas the dummy boxes in Figure 3(a) are non-labeled and the boxes of faces in Figure 3(b) are labeled only with the names of the nodes constituting the faces. Each box except the dummy is configurated in a level corresponding to the dimension of the simplex or the face.

(2) From the map of Q-hierarchy we can readily observe:

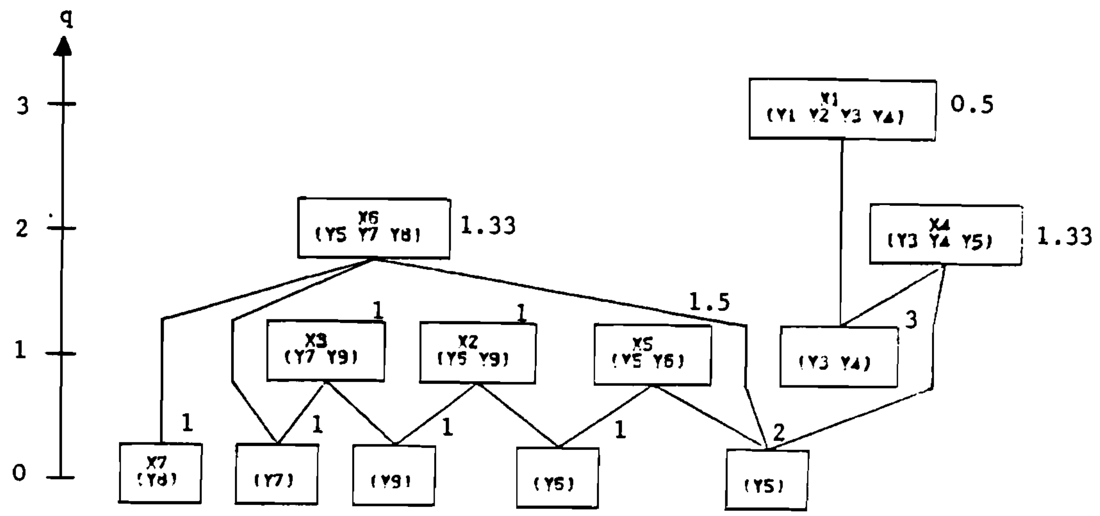
(i) ***connectivity:*** Edges going out from boxes x_1 and x_4 join at a dummy box at the level $q=1$; therefore simplices x_1 and x_4 are 1-connected. Similarly all simplices are 0-connected.

(ii) ***the result of Q- vector analysis:*** By inspecting simplices and paths traversing at each level (these correspond to equivalence classes), we know that the map of the Q-hierarchy is a direct expression of the result of Q-vector analysis, that is

$$
\begin{array}{lll}
q = 3 : & \sigma(x_1) & Q_3 = 1 \\
q = 2 : & \sigma(x_4), \sigma(x_1), \sigma(x_6) & Q_2 = 3 \\
q = 1 : & \sigma(x_2), \sigma(x_5), \{\sigma(x_4), \sigma(x_1)\} & \\
 & \sigma(x_3), \sigma(x_6) & Q_1 = 5 \\
q = 0 : & \{\sigma(x_2), \sigma(x_5), \sigma(x_4), \sigma(x_1), \sigma(x_3), & \\
 & \sigma(x_6), \sigma(x_7)\} & Q_4 = 1
\end{array}
$$



(a)



(b)

Figure 3. Visual representations of hierarchies which are drawn by SKETCH system. (a) Q-hierarchy H_Q and eccentricity (b) F-hierarchy H_F and concentricity

Q-vector : $Q = \{1, 3, 5, 1\}$.
Obstruction vector : $\hat{Q} = \{1, 2, 4, 0\}$.
Q-vector is shown along the ordinate of Figure 3(a).

(iii) ***eccentricity:*** For example, we can easily see in Figure 3(a) that bottom-q ($\check{q}$) and top-q ($\hat{q}$) of the simplex x_4 are vely, therefore the eccentricity ($=(\hat{q}-\check{q})/(\check{q}+1)$) of x_4 is 0.5. The eccentricities of simplices also are shown in Figure 3(a), where the eccentricities of simplices x_6 and x_7 are largest and least respectively; this means that $\sigma(x_6)$ is most independent upon the remainder and $\sigma(x_7)$ most dependent (in this case $\sigma(x_7)$ is a face of $\sigma(x_6)$).

(3) From the map of the F-hierarchy the following can be observed:

(i) ***face and nearness:*** For example, a face and a nearness between simplices x_1 and x_4 are $\langle y_3 y_4 \rangle$ and 1 respectively.

(ii) ***chain, loop and hole:*** All paths with two simplices at both ends express chains. Special chains which have the same simplex at both ends are called loops; for example, we can readily observe a loop $\overline{x_6-x_5-x_2-x_3-x_6-x_5}$. This loop is at the same time a hole. Thus Figure 3(b) helps us to find holes, which are pointed out to be very important structural features by Atkin.

(iii) ***concentricity:*** Concentricities of simplices and faces also are shown in Figure 3(b), where we can see x_4 is most influential among simplices.

4. APPLICATION TO THE TECHNOLOGICAL DEVELOPMENT OF FUTURE COMPUTERS IN THE 1990S

The committee for investigations on the fifth generation computers of the Japan Information Processing Development Center (JIPDEC) investigated during two fiscal years (1979-1980):

(1) What type of computer systems should be developed for Japan as so-called fifth generation computer systems (FGCS) in 1990s.

(2) How development projects should be promoted as a national effort with international cooperation.

This committee consisted of three subcommittees, (i) social environment (first year) and system-integration (second year) subcommittee, (ii) basic theory subcommittee, and (iii) architecture subcommittee. Through the wide-scope investigations the committee clarified requirements for FGCS and precise subjects of R & D, and proposed a plan for the development of FGCS including organization and schedule, where the image (or target) of FGCS was settled as *knowledge information processing systems* which can play a role of the key tool of information technology in almost all social activities such as economies, industries, academies, administration, defense, international relation, education, culture, human life of the people, etc. The procedure of the investigations and the requirements for FGCS are shown in Figure 4 and Table 2 respectively. Results of these investigations were reported in the form of big volumes of documents (JIPDEC 1980, 1981a) and were presented at the International Conference on Fifth Generation Computer Systems held on October 19-22, 1981 in

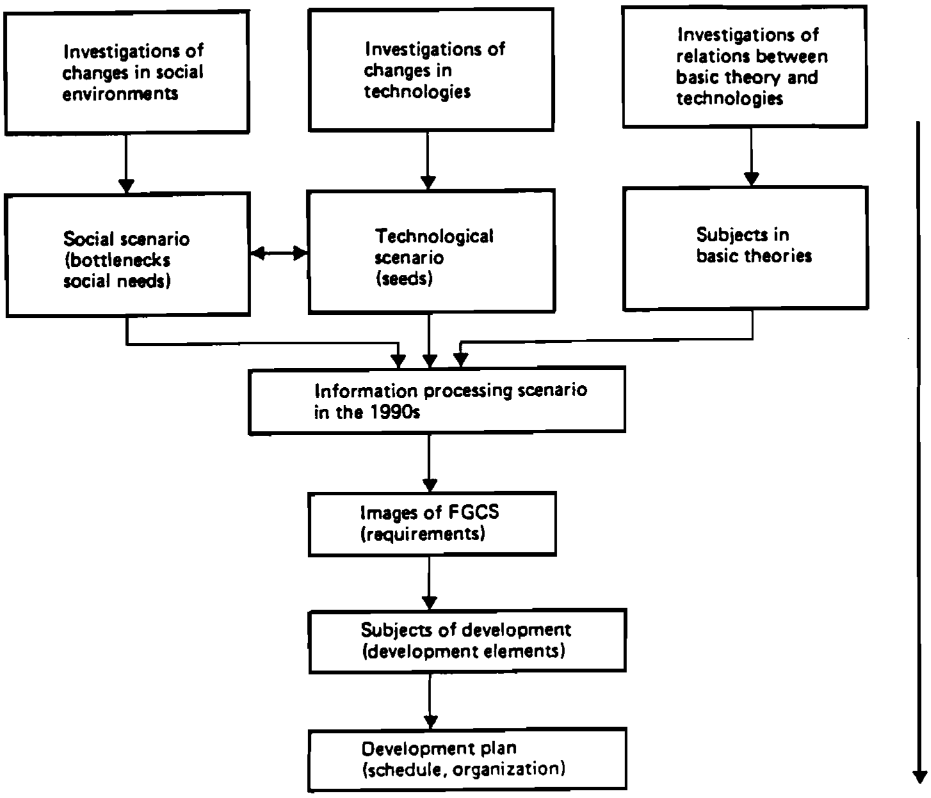


Figure 4. Procedure in the investigations of FGCS.

Tsukuba (JIPDEC 1981b). Under a basis of these results the ten-year national project has been initiated in 1982.

The first subcommittee reported on a relationship between social needs which Japan will face to attain a desirable society and computer technologies (seeds) which should be developed to satisfy the needs. In this section we carry out to

(i) describe the work done by the subcommittee in terms of VQA,

(ii) extract Q- and F-hierarchies, and calculate eccentricity and concentricity from the needs/seeds relation,

(iii) interpret structures of the hierarchies in a viewpoint of technological development.

It seems that the usefulness of VQA exists in the following:

(1) An integration-type technology is defined as a technology which can meet various needs by combining and integrating elementary technologies. The development of such a technology is considered to play an important role in developments of advanced

Table 2. Requirements for FGCS (Source: JIPDEC 1981b).

Item	Theme	Requirement
Conversational function	Natural media for I/O	- Symbol, Table, Picture - Natural language - Pattern to handle
	Conversational performance	- Ambiguous question - Guided function to confirm question - Error correcting - Learning for specified people
Graphics	Graphic processing	- Drafting supported by machine - Graphic reading - Document including graphics and photos
	3 dimension image processing	- 3 dimension graphics
	2 dimension pattern recognition	- Signature - Finger print
Language and voice	Translation by machine	- Multi-language documentation - Simultaneous interpretation
	Dialogue with computer	- Voice and/or natural language Q & A function - Programming by natural language
	Sentence processing	- Input by voice and/or natural language
Knowledge base	Simulation of modeling	- Optimization of simulation model - Decision making - Layout of the cell
	Specific system	- Forecasting - Management strategy - Consensus support
	Basic function	- Rule finding from raw materials - Summary function for getting used data - Discovery, storage and reconstruction of knowledge

Item	Theme	Requirement
Software	Programming	- Easy programming - Methodology for large scale software (Module structure) - Auto-programming by program base
	Software system	- Integration of functions (Coordinate CAD and simulation) - Flexible software system
Data base	Data base system	- Expandability, Flexibility - Auto designing and correcting of data module
	User interface	- Usable by non-professional
Communication network	High performance network	- Intelligent electronic mail - Remote conference
Distributed database	Q & A function	- Efficiency - Virtuality
I/O device	Graphic terminal	- Electronic drafter - 3D I/O - Portable I/O
	Voice operated I/O	- Operatable by non-professional
Processor/ Main memory	Super high speed processor	- $10^3 \sim 10^5$ MFLOPS, 1M ~ 20MW
	3D model processor	- 100M ~ 1GB on-line database machine
	Small size array processor	- 7 freedom manipulator
	Personal work station	- 2MIPS 0.5 ~ 5MB with 100MB/1msec disk
2ry memory	Large disc	- Several 10GB/10msec
	Small disc	- 100MB/1msec
	DASD	- 100TeraB (character file) 50 Million frame light disk

Item	Theme	Requirement
Computer system	Modular computer	- Assembled by user
	Reliability, Availability	- Self-recover and maintenance
	Distributed function system	- General purpose CPU & simulator jointed through 2ry file
Database	Performance	- Capacity $10^2 \sim 10^4$/today's - Special $10 \sim 10^5$/today's
	Specified database	- For design, figure, pattern distributed system
Communication network	Multi function database	- 200,000 picture/day
	Quality	- Optical fiber, digital network
	Distributed database	- Broadcast network with high performance - High precision clock - Protection for dead lock

technologies, e.g., information, electronics, etc., as well as developments of elementary technologies themselves. VQA is an adequate tool to express an structure of their integration.

(2) R & D of advanced technologies are usually accompanied by high risks or high uncertainties, where we can hardly obtain reliable quantitative data to evaluate plans for the R & D, especially in the early stage. VQA can be utilized in such cases that we can expect only qualitative relations.

(3) The FGCS project is characterized as a kind of social development project because the impact is expected to be extensive and enormous when the new machine is installed in the future society. Consequently it is necessary to grasp the interrelationship between technology and society in the wide view and to present them to the people in an easily understandable way. VQA might offer an effective means for this purpose.

(4) Although in the computer field the seeds have been and are developing the needs, we should not lack the conjugate views of needs-pull and seeds-push, especially in the national project. The conjugate complices can represent them.

4.1. Data Structure —Needs/Seeds Relation

The subcommittee (JIPDEC 1980, 1981b) found out many kinds of bottlenecks to realize the society desired in the next decade, standing on the situation of Japan in 1980 as follows:

GNP: 10% of the world
area: 0.3% of the world
population: 2.7% of the world
little natural resources and energy
homogeneous in race and culture
low birth rate and high education

The bottlenecks were organized as a three-level hierarchy shown in Figure 5. (These bottlenecks were replaced with the words "social needs.") On the top of the hierarchy the subcommittee specified the following five items ($B = B_1, \ldots, B_5$), which are mainly based upon a paper by H. Karatsu in JIPDEC (1981b).

B_1: improvement in the fields of low productivity such as office, engineering design, agriculture and fishery, medicine, education, public service, government

In the field of manufacturing, of course, we have much more expectation to realize high level quality and productivity by introducing the intelligent robot and no man factory.

B_2: internationalization of Japan through transferring our experience to the world and maintaining competitive power in international trades by overcoming shortage of energy and natural resources

In order to live more prosperously and peacefully, we must endeavor to transfer the result and know-how of our experience for the past several decades all over the world and to hold the priority in such industries as electronics and precision machinery which fit to Japan. The newly developed computer should become the effective tool to optimize operation of all kinds of energy consuming systems, to process the data to search the natural resources, to do R & D of the new energy, etc.

B_3: coping with problems generated by a structural change of Japanese society such as increase in old generations, change in attitudes of young generation toward labor, change in industrial structure, etc.

The intelligent robot and no-man factory can be useful to cope with the decrease in labor population and relieve for labor from unwilling and dirty work, and life-time education system can help people learn and adapt to new jobs. Medical care information system also can work well for medicine and welfare of aged people.

B_4: advancing an information society

Our society is expected to depend on the computer more and more aiming at increases in efficiencies of social services such as medicine, education, etc., enrichments of services which can be helpful for diverse needs of individuals, and improvement of various information gaps such as regional, generation, international.

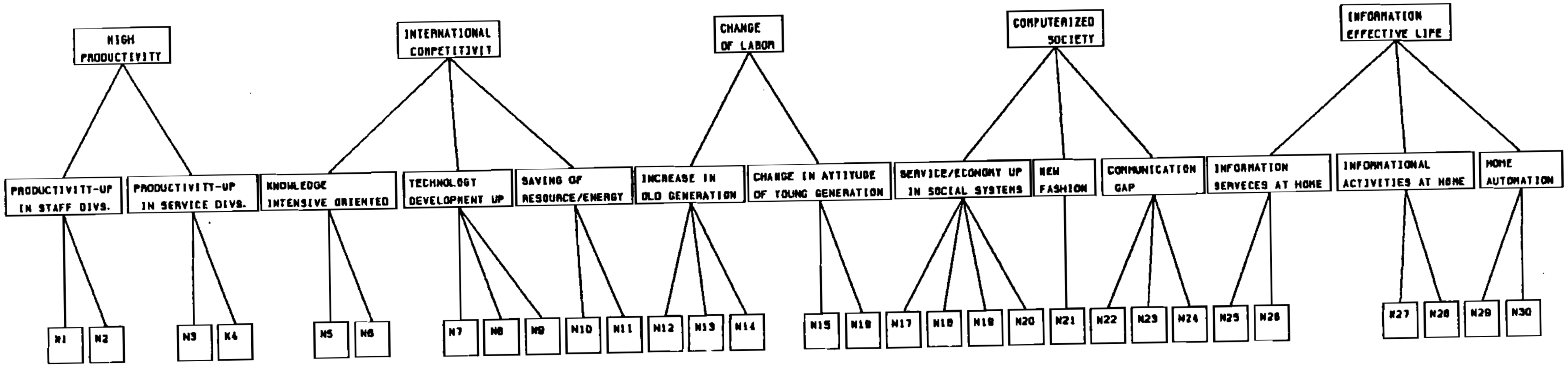


Figure 5. Hierarchy of social needs (drawn by SKETCH system).

B_5: advancing an information effective life in the future

Japan is characterized as the highly structured society which is operated efficiently and organized with high educated people in a limited and crowded area. This means that a bit of mismatching between the people and society might bring hazardous effect upon each other. Interactive video home terminals might be one solution to soften the stress. FGCS must be the machine that should work to fit the human beings intimately.

These five items (B) were broken down into 13 items ($C=\{C_1, \ldots, C_{13}\}$) and then C was broken down into 30 items ($N=\{N_1, \ldots, N_{30}\}$). N was further broken down into elementary methods ($M=\{M_1, \ldots, M_{55}\}$) which realize needs ($N$). On the other hand, from a viewpoint of computer technology, computer applications ($A=\{A_1, \ldots, A_{16}\}$) such as decision support systems, office automation, very large database, industrial robot, computer-aided design, etc. which will become more important in the future society were selected and then A was broken down into elementary technologies (seeds: $S=\{S_1, \ldots S_{37}\}$). Thirty subitems (needs: N) were related to thirty-seven seeds (S) through referring a relation between M and A. The data structure described above is shown in Figure 6, where it should be noed that all data sets and relations are "soft" data.

In Table 3 the identified needs (N) and seeds (S) are represented. Table 4 shows the simplices of the simplicial complex $K_N(S)$ obtained from the relation between N and S.

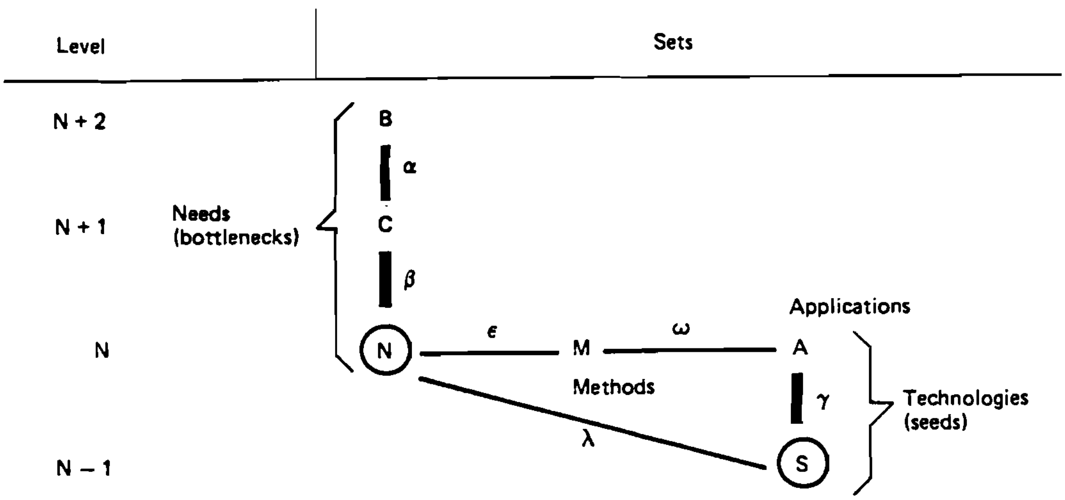


Figure 6. Hierarchy of data sets in investigations of FGCS.

Table 3. Needs and seeds identified.

N_1	decision support systems for business	S_1	super computers
N_2	office automation for business	S_2	parallel processing architecture
N_3	office automation for services sectors	S_3	highly-integrated microcomputer
N_4	VLDB for service sectors	S_4	very large storage technology
N_5	intelligence machines; robots and CAM	S_5	packaged software
N_6	diversity in custom made products	S_6	computer games
N_7	IR for technology information	S_7	computer arts
N_8	design automation	S_8	simulation technology
N_9	laboratory automation	S_9	information processing services
N_{10}	control for saving energy	S_{10}	software development services
N_{11}	conversion of industrial structure; new information industries	S_{11}	TSS services
N_{12}	medical electronics for medical offices	S_{12}	facility management systems
N_{13}	education for senior citizens	S_{13}	computer network services
N_{14}	health care industries	S_{14}	computer lease
N_{15}	personal computers for young people	S_{15}	digital network
N_{16}	robots for dirty work	S_{16}	picture and image transmission
N_{17}	medical service systems	S_{17}	VIDEO technology
N_{18}	CAI for personal education	S_{18}	distributed processing
N_{19}	disaster prevention IR system	S_{19}	distributed database
N_{20}	crime prevention IR system	S_{20}	documents database
N_{21}	personal computers for customized applications	S_{21}	engineering database
N_{22}	computer network for regional information services	S_{22}	protection of security
N_{23}	CAI for communication among generations	S_{23}	associative retrieval
N_{24}	language translation	S_{24}	data services
N_{25}	reservation at home	S_{25}	picture processing
N_{26}	telemail and teletext	S_{26}	picture and image processing
N_{27}	CAI for learning at home	S_{27}	character and picture recognition
N_{28}	hobby computers	S_{28}	optical processing
N_{29}	program-controlled electric devices	S_{29}	Japanese language processing
N_{30}	home automation	S_{30}	natural language processing
		S_{31}	intelligent terminals
		S_{32}	terminal for counter services
		S_{33}	health care equipments
		S_{34}	senser base
		S_{35}	built-in microcomputer
		S_{36}	database
		S_{37}	man-machine interface

Table 4. Simplices of the simplicial complex $K_N(S)$

$\sigma(N_1) = \langle S_{23}, S_{25}, S_{29}, S_{36} \rangle$	$\sigma(N_2) = \langle S_{20}, S_{23}, S_{25}, S_{28}, S_{29} \rangle$
$\sigma(N_3) = \langle S_{18}, S_{25}, S_{27}, S_{29}, S_{31}, S_{32} \rangle$	$\sigma(N_4) = \langle S_4, S_{19}, S_{22}, S_{27}, S_{29} \rangle$
$\sigma(N_5) = \langle S_5, S_{35} \rangle$	$\sigma(N_6) = \langle S_2, S_{21}, S_{26} \rangle$
$\sigma(N_7) = \langle S_{23}, S_{30}, S_{36} \rangle$	$\sigma(N_8) = \langle S_2, S_{21}, S_{26} \rangle$
$\sigma(N_9) = \langle S_1, S_{34} \rangle$	$\sigma(N_{10}) = \langle S_{34}, S_{35} \rangle$
$\sigma(N_{11}) = \langle S_9, S_{10}, S_{11}, S_{12}, S_{13}, S_{14}, S_{24} \rangle$	$\sigma(N_{12}) = \langle S_{22}, S_{23}, S_{26}, S_{31}, S_{34}, S_{36} \rangle$
$\sigma(N_{13}) = \langle S_{25}, S_{29}, S_{35}, S_{36}, S_{37} \rangle$	$\sigma(N_{14}) = \langle S_{33} \rangle$
$\sigma(N_{15}) = \langle S_3, S_5, S_{35} \rangle$	$\sigma(N_{16}) = \langle S_{34}, S_{35}, S_{36} \rangle$
$\sigma(N_{17}) = \langle S_{22}, S_{23}, S_{26}, S_{34}, S_{35}, S_{36} \rangle$	$\sigma(N_{18}) = \langle S_{25}, S_{29}, S_{35}, S_{37} \rangle$
$\sigma(N_{19}) = \langle S_8, S_{34} \rangle$	$\sigma(N_{20}) = \langle S_{22}, S_{27}, S_{36} \rangle$
$\sigma(N_{21}) = \langle S_3, S_5, S_{35} \rangle$	$\sigma(N_{22}) = \langle S_{15}, S_{16} \rangle$
$\sigma(N_{23}) = \langle S_6, S_7, S_{35} \rangle$	$\sigma(N_{24}) = \langle S_1, S_4 \rangle$
$\sigma(N_{25}) = \langle S_{15}, S_{19} \rangle$	$\sigma(N_{26}) = \langle S_{15}, S_{31} \rangle$
$\sigma(N_{27}) = \langle S_{17}, S_{31} \rangle$	$\sigma(N_{28}) = \langle S_5, S_{26}, S_{35} \rangle$
$\sigma(N_{29}) = \langle S_{35} \rangle$	$\sigma(N_{30}) = \langle S_{34}, S_{35} \rangle$.

4.2. Interpretations of the Structures

From the simplicial complex $K_N(S)$ we calculate the absorbed Q-hierarchy H'_Q and F-hierarchy H_F and they are drawn by using the SKETCH system. Their drawings are shown in Figures 7 and 8, respectively. In Figure 7 we can see that the complex $K_N(S)$ is separated into three parts, N_{11}, N_{14} and the other simplices which are connected. When we denote Atkin's Q-vector corresponding to these parts by $Q^{(1)}$, $Q^{(2)}$, $Q^{(3)}$ respectively, they are obtained from Figure 7 easily;

$$Q^{(1)} = (\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ )$$

$$Q^{(2)} = (\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ )$$

$$Q^{(3)} = (\ 0\ 3\ 5\ 6\ 10\ 10\ 1\ )$$

The Q-vector for the whole $K_N(S)$ is obtained by summing up $Q^{(i)}$'s element-wise;

$$Q = (\ \overset{6}{1}\ 4\ 6\ 7\ 11\ 11\ \overset{0}{3}\ )$$

From Figure 7 we can understand visually connectedness among needs by observing the joining of paths from top to bottom. For example, N_{13} and N_{18} are 3-connected and N_1, N_2, N_{13}, and N_{18} are 2-connected to each other. These needs constitute a closely related group in the sense that for each of these we can find another need that shares at least three seeds (2-connected in $K_N(S)$) with the former need. Of these N_{13} and N_{18} are more closely related than the others, since they share four seeds (3-connected). Indeed, N_{18} requires a subset of the seeds required by N_{13} (N_{18} is a face of N_{13} in $K_N(S)$). Another closely related group is N_{12} and N_{17}, which require the same five seeds pertaining to medical electronics. Similarly, we can trace paths of the tree in Figure 7 to visually grasp

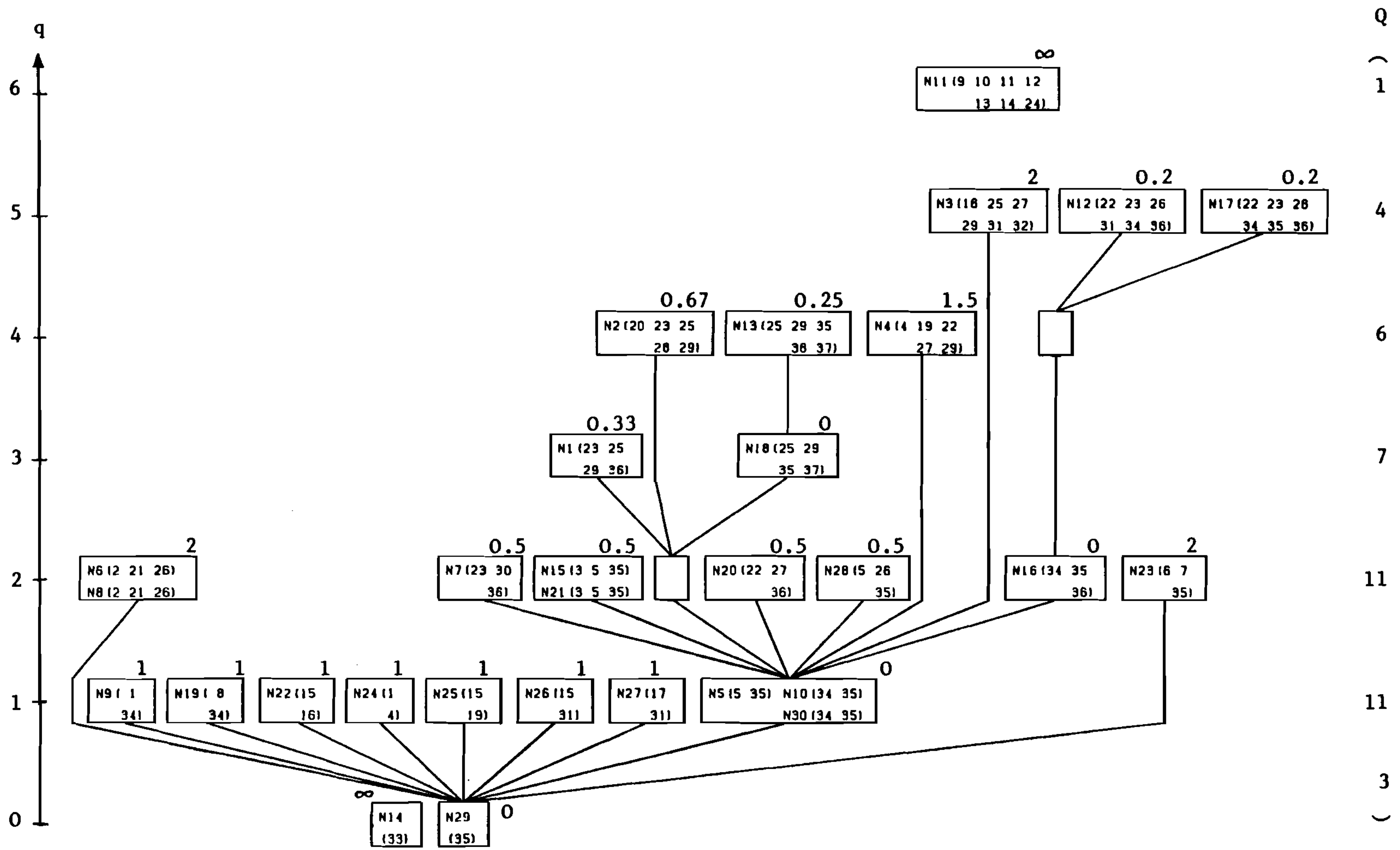


Figure 7. Q-hierarchy H'_Q and eccentricity for $K_N(S)$.

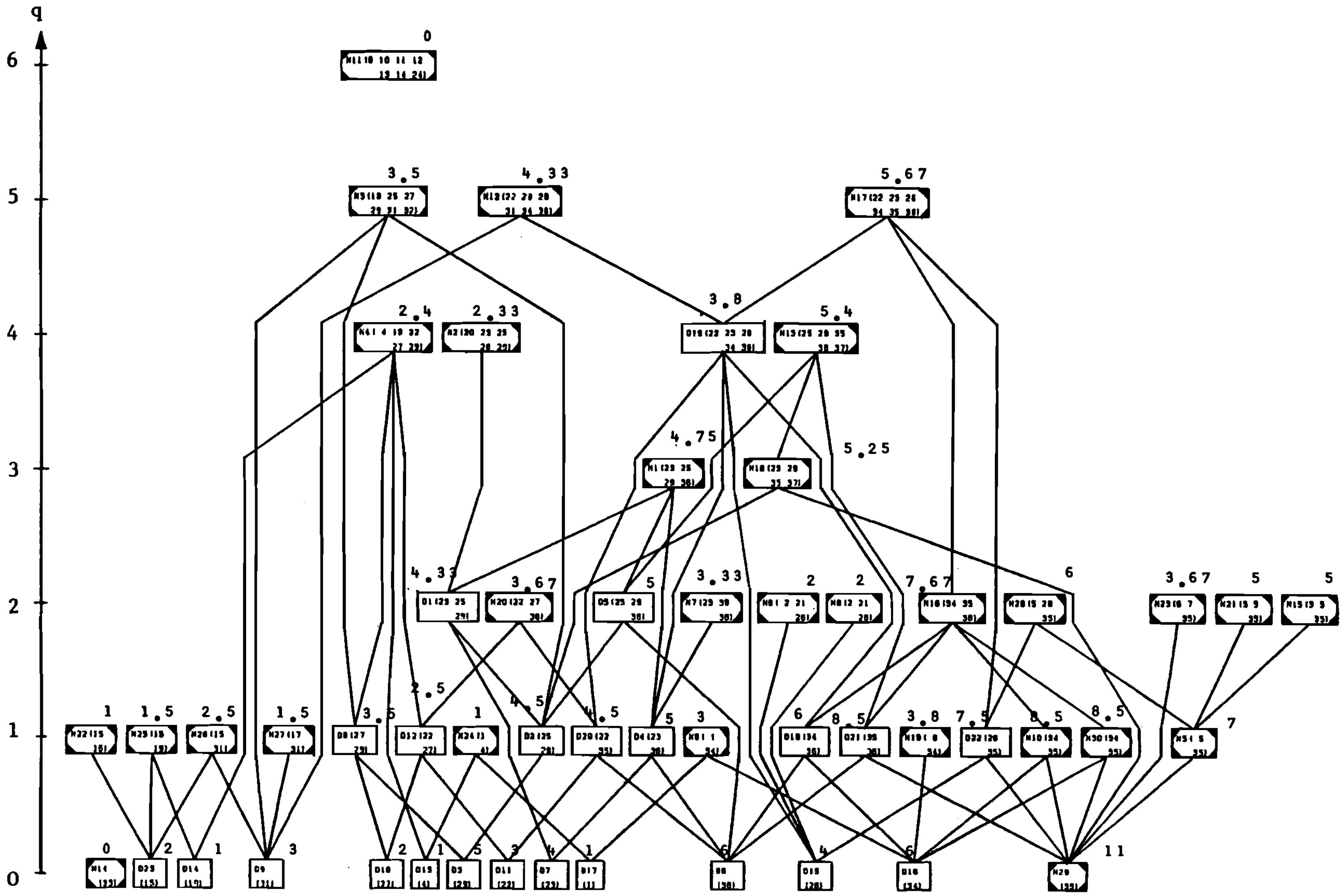


Figure 8. F-hierarchy H_F and concentricity for $F_N(S)$.

groups of needs which are related with various degree of proximity. These facts mean that this figure shows a hierarchical structure of clusters among needs. It is remarkable that these clusters among needs do not conform at all to the needs-oriented clusters which are shown in Figure 5. This fact implies that social needs cannot be separated but are closely related to each other in terms of technological seeds which realize the needs. We can also observe that N_{11} and N_{14} are singular needs in terms of technological development, since they share no seeds with the other needs.

On the other hand, Figure 8 tells us hierarchical structures of faces each pair of needs share. Therefore it might be said that Figure 8 shows what strategies for developments should be taken in the following way:

(i) Needs in higher levels are composite needs which require many seeds. Consequently they are considered as targets.

(ii) On the other hand, nodes in the bottom level* are considered to be starting points, and the larger is the number of needs which share a seed, the more significantly the seed is desired to be developed in terms of the propagation of technologies. The same thing also holds in the upper levels where combinations of seeds are compared.

(iii) Therefore if we inspect Figure 8 carefully, we can find an efficient strategy to attain a target need.

For example, let us suppose that we intend to satisfy N_{17} (medical service systems) which consists of six seeds, S_{22}, S_{23}, S_{26}, S_{34}, S_{35}, and S_{36}. Moreover we suppose that these seeds are independent of each other. Then our problem is that in what order these seeds should be developed. First, since S_{35} (built-in microcomputer) is a need (N_{29}) itself and it has seven upward edges which is the largest number of edges of nodes in the bottom level, S_{35} might be developed. More upward edges from a seed generally mean that the seed is more likely to contribute to the fulfillment of other needs when additional seeds are developed to combine with the seed. We might choose S_{34} (sensor base) secondly because it has five upward edges and the pair of S_{34} and S_{35} is effective to two needs N_{10} and N_{30}. Next we might develop S_{36} because it has five upward edges, and the seeds subset $\{S_{34}, S_{25}, S_{36}\}$ are effective to a need N_{18}. Then, according to similar considerations the other seeds might be developed in the order of S_{26}, S_{23}, and S_{22}.

Following the order of development of these seeds requested for the target N_{17}, one will thus have more opportunities to meet other needs in the course of the development, since Figure 8 illustrates cross support relations of the needs through their requiring seeds.

* Note that Figure 8 does not illustrate a seed that is required for only one need which requires more than one seed.

We can obtain more observations from Figure 8 useful for planning technological developments for the fifth generation computers. One of the observations that can be elicited by observing local structures of relations in Figure 8 is the importance of combinations of seeds. For example, the dummy node D_2 representing combinations of S_{25} and S_{29} has four upward edges. This means that picture processing (S_{25}) and Japanese language processing (S_{29}) technologies will be jointly utilized by various needs. Similar interpretations may be obtained for the combination of S_{23} and S_{36}, and that of S_5 and S_{35} which are represented by nodes D_4 and N_5 respectively having three upward edges.

The above-described observations become clearer when we consider two measures for a complex, i.e., eccentricity and concentricity, together with the drawings of both hierarchies. We calculate the eccentricity and the concentricity for $K_N(S)$:

	Ecc	*Con*		*Ecc*	*Con*		*Ecc*	*Con*
N_1	0.33	4.75	N_{11}	∞	0	N_{21}	0.5	5
N_2	0.67	2.6	N_{12}	0.2	4.33	N_{22}	1	1
N_3	2	2.33	N_{13}	0.25	5.4	N_{23}	2	3.67
N_4	1.5	2.4	N_{14}	∞	0	N_{24}	1	1
N_5	0	7	N_{15}	0.5	5	N_{25}	1	1.5
N_6	2	2	N_{16}	0	7.67	N_{26}	1	2.5
N_7	0.5	3.33	N_{17}	0.2	5.67	N_{27}	1	1.5
N_8	2	2	N_{18}	0	5.25	N_{28}	0.5	6
N_9	1	3.5	N_{19}	1	3	N_{29}	0	11
N_{10}	0	8.5	N_{20}	0.5	3.67	N_{30}	0	8.5
D_1	-	4.33	D_9	-	3	D_{17}	-	1
D_2	-	4.5	D_{10}	-	2	D_{18}	-	6
D_3	-	5	D_{11}	-	3	D_{19}	-	3.8
D_4	-	5	D_{12}	-	2.5	D_{20}	-	4.5
D_5	-	5	D_{13}	-	1	D_{21}	-	8.5
D_6	-	6	D_{14}	-	1	D_{22}	-	7.5
D_7	-	4	D_{15}	-	4	D_{23}	-	2
D_8	-	3.5	D_{16}	-	6			

These values are shown in Figure 7 (eccentricity) and Figure 8 (concentricity).

The eccentricity of simplices N_{11} and N_{14} is ∞, which means that the simplices are separated or completely "independent" upon the remainder of $K_N(S)$. On the other hand the eccentricity of simplices N_5, N_{10}, N_{16}, N_{18}, N_{29}, and N_{30} is 0, which means that the simplices at the sametime are faces of other simplices or completely "dependent." In Figure 8 we can find that the above intuitive considerations on the strategy for developments are wholly supported by values of the concentricity; i.e., the order of development $S_{35} \rightarrow S_{34} \rightarrow S_{36} \rightarrow S_{26} \rightarrow S_{23} \rightarrow S_{22}$ to attain the need N_{17} is quite reasonable in a viewpoint of the concentricity or

Order	Concentricity	Simplex (or face) attained
S_{35}	11	$N_{29} = <S_{35}>$
$\downarrow S_{34}$	8.5	$N_{10} = <S_{34}, S_{35}>$
$\downarrow S_{36}$	7.67	$N_{18} = <S_{34}, S_{35}, S_{36}>$
$\downarrow S_{26}$	(4	$D_{15} = <S_{26}>$)
$\downarrow S_{23}$	4	$D_{7} = <S_{23}>$
$\downarrow S_{22}$	5.67	$N_{17} = <S_{22}, S_{23}, S_{26}, S_{34}, S_{35}, S_{36}>$

where all values of the concentricity are high and according to a decreasing order. Therefore we can conclude that a path $N_{29} \rightarrow N_{10}$ (or N_{30}) $\rightarrow N_{18} \rightarrow N_{17}$ is the most important ***strategic core*** for the FGCS project in terms of the concentricity.

It is, of course, true that strategies should be determined by considering various other conditions such as costs, manpower, time, etc.* Nevertheless, Figure 8 is very useful as an overview of a structure of the relations between needs and seeds.

We have also drawn the Q-hierarchy and the F-hierarchy for the simplicial complex $K_S(N)$ by the SKETCH system. These hierarchies are conjugates of those for $K_N(S)$ and, as such, they are useful in understanding the global and local structures of the relations of needs and seeds by complementing Figures 7 and 8 or in a viewpoint of *needs- pull*.

5. CONCLUDING REMARKS

We have proposed a novel method called visual Q-analysis (VQA) to analyze structures of complex systems, beginning with the discussion on relationships among Atkin's Q-analysis, Warfield's interpretive structural modeling and our method. We have given definitions of Q-hierarchy and F-hierarchy, and algorithms to obtain them from a given relation. We have applied VQA to a structural study of technological developments of future computers of Japan in 1990s. A relation between social needs toward a desirable society and technologies (seeds) to support in realizing the society are analyzed and represented by using Q- and F-hierarchies and we have discussed on how to utilize drawings of their hierarchies. In this work following conclusions are obtained:

(1) The drawings of Q- and F-hierarchies visualize a hierarchical structure among simplices of a simplicial complex, and help us effectively grasp the structural information embedded in the complex.

(2) Results of the application tell us the effectiveness of VQA in
   (i) clustering the needs (seeds) in terms of the seeds (needs)
   (ii) supporting for planning technological developments.

In a national (neutral) project, comprehensive view of the project and consensus among people are especially required. For these purposes VQA might be utilized. In other words, the drawings of the hierarchies might be useful as a kind of map which help us recognize the state of

* For this purpose ***dynamics of patterns*** in Atkin's Q-analysis is expected to be useful.

progress of the project, where iterative modifications will be indispensable through the feedback loop represented in Figure 1(c).

Finally, the following studies are envisaged for future research:

(1) Extension of VQA toward a quantitative method where costs, manpower, time, resources, information, knowledge, etc. are considered as the factors for a decision making in technology developments.

(2) Investigating further applications of VQA to various other fields such as holistic recognition of IIASA new research plan by considering relations between projects and methodologies (or scientists), which suggests the connectedness and the flexibility of the research organization.

REFERENCES

Atkin, R.H. 1972a. From cohomology in physics to q-connectivity in social science. *International Journal of Man- Machine Studies*, 4(2):139-167.

Atkin, R.H. 1972b. *Mathematical studies in human affairs*. London: Allen & Unwin.

Atkin, R.H. 1977. *Combinatorial connectivities in social systems*. Basel: Birkhauser Verlag.

Harary, F., R.Z. Norman, and D. Cartwright. 1965. *An introduction to the theory of directed graphs*. New York: Wiley.

Japan Information Processing Development Center (JIPDEC). 1980. First report on investigations of fifth generation computer systems. (in Japanese) Tokyo: Japan Information Processing Development Center.

Japan Information Processing Development Center (JIPDEC). 1981a. Second report on investigations of fifth generation computer systems. (in Japanese) Tokyo: Japan Information Processing Development Center.

Japan Information Processing Development Center (JIPDEC). 1981b. Proceedings of the International Conference on Fifth Generation Computer Systems held October 19-22, 1981 in Tsukuba. Tokyo:

Japan Information Processing Development Center.

Sugiyama, K., M, Toda and S. Tagawa. 1981. Visual Q-analysis and its application to a structural study of technology innovations. Research Report No. 19. p.28 Numazu, Japan: International Institute for Advanced Study of Social Information Science (IIAS-SIS), Fujitsu, Ltd.

Sugiyama, K., S Tagawa, and M. Toda. 1981a. Visual representations of structural information, Algorithms for automatic drawing of hierarchical structures and its applications. Japanese Report No. 2, p.41. Numazu, Japan: IIAS-SIS, Fujitsu Ltd. (in Japanese)

Sugiyama, K., S. Tagawa, and M. Toda. 1981b. Methods for visual understanding of hierarchical systems structures. *IEEE Transactions Syst. Man, Cybern.*, SMC-11(2):109-125.

Toda, M., and K. Sugiyama. 1982. A structural study of the planning of integration-typed technological developments — a case study on the fifth generation computer systems development. Proceedings of the 11th Symposium on Technology Assessment. (in Japanese)

Toda, M., K. Sugiyama, and S. Togawa. 1981. An application of Visual Q-analysis to the integration-typed technological developments and methods for representing the results. Proceedings of the 21st conference of the Society of Instrument and Control Engineers. pp. 73-74. (in Japanese)

Warfield, J.N. 1973. On arranging elements of a hierarchy in graphic form. *IEEE Trans. Syst., Man, Cybern.*, SMC-3:121-132.

Warfield, J.N. 1974. Toward interpretation of complex structural models. *IEEE Trans. Syst. Man, Cybern.*, SMC-4(5):405-417.

Warfield, J.N. 1976. *Societal Systems: Planning Policy, and Complexity*. New York: Wiley.