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Adaptability of Nonlinear Equilibrium Models to Central Planning

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ABSTRACT

Linear multisectoral models have for long been applied in the Hungarian national economic planning. Price-quantity correspondences and interaction, however, cannot easily be taken into account in the traditional linear framework. Computable general equilibrium modelers in the West have developed techniques which use extensively price-quantity interdependences. However, since they are usually presented with the controversial strict neoclassical interpretation, the possibility of their adaptation to socialist planning models has been concealed. This paper reflects on some results of a research investigating the possible adaptation of equilibrium modeling techniques to central planning models.

ADAPTABILITY OF NONLINEAR EQUILIBRIUM MODELS TO CENTRAL PLANNING *

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1. INTRODUCTION

Linear multisectoral input-output and programming models have become more or less integrated into the complex process of planning in many socialist (centrally planned) economies. These models concentrate on the production and use of economic resources and commodities at some level of aggregation. Similar models are also used in both western and developing countries, the differences in the economic environment and data sources being reflected in the specification and purpose of the models. The use of linear models has been paralleled by the development of more complex, nonlinear models, most of which come under the general heading of computable or applied general equilibrium models.

The basic ideas of a multisectoral general equilibrium growth model were first suggested by Johansen [10] in 1959, although full-scale implementation of large, nonlinear models has become computationally feasible only lately.

* This paper is based on a research initiated by the author. This research is a combined effort of experts in the Hungarian Planning Office, Karl Marx University of Economics (Budapest) and the International Institute for Applied Systems Analysis (Laxenburg, Austria). References 5, 13 and 16 contain a more detailed discussion of most of the issues only touched upon here.

Recent applications are described in references 1, 6, 7 and 8; models of this type developed at the International Institute for Applied Systems Analysis (IIASA) are discussed in references 3, 4, 11 and 16. Some of these models have been designed to capture the interrelationships between economic, spatial, and demographic processes.

The structure of general equilibrium models, the estimation procedures applied, and the theoretical explanations associated with them generally follow the neoclassical tradition quite closely. The neoclassical approach has often been criticized and even rejected in both the East and West, and this partly explains the apparent lack of interest of central planning modelers in these models. It is not at all obvious whether the models, or some of the techniques of applied general equilibrium modeling, could be adapted for central planning processes.

The main purpose of this paper therefore is to highlight the possibility, and expected benefits, of incorporating nonlinear multisectoral models of the general equilibrium type into the planning methodology of socialist (centrally planned) economies.

First we will briefly review the major characteristics of applied general equilibrium analysis and compare them with those of a typical planning model. Next, with the help of a simple illustrative model focusing on foreign trade relationships, we will provide some examples of why and how techniques of general equilibrium models can be adopted for central planning. Finally, the possible advantages of such adaption are considered in a somewhat wider context.

2. GENERAL EQUILIBRIUM VERSUS OPTIMAL PLANNING MODELS

General competitive equilibrium *theory** provides an abstract partial model of the economic systems centered around the law of supply and demand, and rational economic behavior. The

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* We will confine our attention to competitive or Walrasian general equilibrium models, which provide a basis for less classical equilibrium models and various disequilibrium models.

abstract economic theory of general equilibrium takes many important elements of the economy as *data* and sets out to define and determine the equilibrium within this postulated environment in which only prices control economic decisions.

Applied general equilibrium models adopt a *relative* point of view and try to estimate the likely consequences of various changes in the economic environment by comparing the "base equilibrium solution" with the solutions computed on the basis of these changes. A typical approach may be summarized as follows. A formal model of the necessary and sufficient conditions for general equilibrium is developed. The observed state of the economy is considered to be in equilibrium (base solution), and many of the parameters of the model are statistically estimated on the basis of this assumption. Next, by classifying the economic variables as endogenous or exogenous, the impact of assumed changes in the exogenous variables is analyzed in terms of the models solution. Thus, the equilibrium framework is used to evaluate, consistently and in quatitative terms, the direction of change of certain crucial interdependent economic variables. A distinctive feature of general equilibrium analysis is that it takes into account simultaneously real, price (cost) and financial variables and their interaction.

The use and philosophy of macroeconomic planning models could be summarized in the following way. Suppose that at some stage in the planning process the coordinating unit decides to summarize the calculations made so far, and as a result some provisional values of the sectoral outputs, inputs, consumption, etc., are therefore made available. The coordinating unit wishes to know whether these more or less seperately planned figures represent a consistent and balanced picture, and if not, how this could be rectified. The unit also wishes to check how certain changes in one part of the plan would affect other parts of the provisional plan and its efficiency. In Hungary, formal models support the process of checking the consistency, sensibility and efficiency of a draft plan (coordination process). (See for example, reference 2 regarding current planning modeling practice in Hungary.)

An economy-wide planning model, built into and upon the traditional planning methodology of a socialist country, would differ from the outlined general equilibrium models in several respects. First, it would almost exclusively contain "real" variables and relations reflecting physical constraints on allocation. Second, because the prices used in a planning model are either constant or planned, being predicted more or less independently from "real" processes, the interdependence of the real and value (prices, taxes, rate of return requirements, etc.) variables would not be considered explicitly in the model. Third, most mathematical planning models are closely related to and rely upon traditional or nonmathematical planning. This means, among other things, that the values of the exogenous variables and parameters and also certain upper and/or lower target values for some of the endogenous variables would not be derived directly from statistical observations, but would be based on figures given by traditional planners. (This is not to say, however, that more or less sophisticated statistical estimation techniques would not be combined with experts' "guesstimates" in traditional planning.) Finally, planning modelers in socialist countries tend to concentrate more on the problems of how to fit their models into the actual process of planning and make them practically applicable and useful. Therefore, applied planning models tend to be both theoretically and methodologically simpler than those in the development planning literature.

Table 1 summarizes the major features of the two modeling approaches. The list is far from complete and also it includes a few conflicting or alien features. Thus, the question as to whether models of the computable general equilibrium type could be used in more or less the same function in planning as the linear programming ones, is not trivial.

We do not have enough time or place here to go into details (for such, see references 16 and 17) but the answer is affirmative. Certain techniques and certain types of models can be viewed as natural extentions of the linear planning techniques developed to date. Their study and adaption appear to open new paths for central planning modeling practice. Before highlighting

Table 1. MAJOR FEATURES OF APPLIED GENERAL EQUILIBRIUM (AGEM) AND OPTIMAL PLANNING (OPM) MODELS

Aspect	AGEM	OPM
Base of comparison	observed state (counter-factual simulation)	provisional plan (counter-plan simulation)
Characteristic types of variables	real, price, cost, financial	mainly real, some financial assets
Functional relationships based on	neoclassical theory (e.g. production functions, demand functions)	pragmatic considerations (e.g. fixed norms, structures)
Data basis	statistics (ex post)	plan information (ex ante)
Parameter estimation techniques	direct and indirect econometric estimation	mixed methods, heavy reliance on experts of various fields
Decision criteria	individual profit and utility maximization	overall consistency and efficiency
Special allocational limits reflected by	varying rates of return requirements (indirect)	individual bounds on variables (direct)
Mathematical form	nonlinear equation system, locally unique solutions (assumed)	linear inequalities with alternative overall objective functions.

the possible advantages of such adaption, we would like to illustrate how one can reinterpret the neoclassical forms and adapt them to planning models.

3. ILLUSTRATION: FOREIGN TRADE IN THE TWO VERSIONS OF MACROECONOMIC MODELS

With the following simple example we try to facilitate our discussion and the comparison of programming and general equilibrium approach. We will concentrate our attention on the treatment of export and import in different multisectoral models. For the sake of simplicity we will use an extremely stylized, textbook type of a model. We will assume that there is only one sector whose net output ($\bar{Y}$) is given (determined by available resources). The only allocation problem is to divide $\bar{Y}$ into domestic use (C_d) and export (Z). Export will be exchanged for an imported commodity which is assumed to be a perfect substitute for home commodity. Intermediate use will be neglected.

Following the traditional linear programming approach, export ($\bar{P}_E$) and import ($\bar{P}_M$) prices will be treated as (exogenously given) parameters of the model. Introducing M for import purchased and C_m for import used, our optional resource allocation problem can be formulated in the following simple way.

$$C = C_d + C_m \rightarrow \max$$

$$C_d + Z \leqq \bar{Y} \qquad (P_d)$$

$$C_m \leqq M \qquad (P_m)$$

$$\bar{P}_M M - \bar{P}_E Z \leqq 0 \qquad (V)$$

$$C_d, C_m, Z, M \geqq 0$$

The solution of the above problem depends clearly on the relation of $\bar{P}_E$ and $\bar{P}_M$, i.e., on the terms of trade. The problem of overspecialization* appears here in a very clear way. If the terms of trade are favorable ($\bar{P}_E > \bar{P}_M$) then everything will be exported ($Z = \bar{Y}$) and only imported goods consumed ($C_d = 0$, $C_m = \dot{M} = \bar{P}_E\ Z/\bar{P}_M$). In case of unfavourable terms of trade, the optimal policy will be that of autarchy.

Let us assume for a moment that the terms of trade are favorable at prices $\bar{P}_E$ and $\bar{P}_M$. The model builders will be aware of the fact that $\bar{P}_E$ is only an approximate value of the unit export price, and that at such a price the export markets could not absorb more than, say, $\bar{Z}$ amount of export. Adding $\bar{Z}$ to the model as an individual upper bound on Z would prevent it producing a completely overspecialized solution. $\bar{Z}$ would be clearly binding and the solution would be

$$Z = \bar{Z} \qquad C_d = \bar{Y} - \bar{Z} \qquad C_m = M = \bar{P}_E\ \bar{Z}/\bar{P}_M$$

It is also easy to see that the optimal values of the dual variables will be

$$P_d = P_m = V\ \bar{P}_M = 1 \quad , \quad t = V\ \bar{P}_E - P_d$$

where t is the shadow price of the individual bound, $\bar{Z}$.

The analysis of this hypothetical planning model would not stop here, for we know that $\bar{Z}$ is a constraint on export at given $\bar{P}_E$ export prices. What if we changed $\bar{P}_E$, would $\bar{Z}$ also change? Suppose that, at least within certain limits, the answer is yes, i.e., a decrease in the export price ($\bar{P}_E$) would increase the export absorbtive capacity ($\bar{Z}$). In other words, the modeled economy faces less than perfectly elastic export demand. Let $D(P_E)$ be the export demand function. Instead of the rigid, fixed export bound ($\bar{Z}$) we could thus use the following *flexible constraint:*

$$Z \leqq D(P_E)$$

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* See, for example, reference 14 on the problem of overspecialization and on the use of individual bounds in macroeconomic models.

treating at the same time P_E as a variable in the balance of payment constraint.

As is known, one will usually find in linear programming models of nationwide resource allocation individual bounds in import as well. Typically, it is the ratio of import to the domestic source (m) which is forced into some bounds. In our case

$$m = \frac{C_m}{C_d}$$

was not constrained. Let us introduce m^+ and m^- as upper and lower bounds on m. In such a case our previous programming model will have to be augmented by two additional constraints. These might be written together as

$$m^- C_d \leqq C_m \leqq m^+ C_d$$

Let t_m^- and t_m^+ denote the corresponding new shadow prices. As a result of the above modifications in the primal problem the dual constraints corresponding to C_d and C_m will have to be modified in the following way

$$P_d = 1 - t_m^- m^- + t_m^+ m^+$$

$$P_m = 1 + t_m^- - t_m^+$$

The computable models of general equilibrium usually follow a different approach. There, the dependence of the import share (m) is usually an explicit and continuous, smooth function of the relative prices of the domestic and imported commodities. In most case, constant elasticity functions are used, such as

$$m = m_o \left(\frac{P_d}{P_m}\right)^{\mu}$$

In the linear programming case observe that if the lower limit on import is binding (neglecting degenerated solutions), then we will have $t_m^- > 0$ and $P_d < 1$, $P_m > 1$. If the upper limit is binding then $t_m^+ > 0$ and $P_d < 1$, $P_m \quad 1$. Otherwise $P_m = P_d$. Reversing the argument we could say the following. If the shadow price of the domestic commodity is less than that of the imported commodity, then we will not import more than the minimum required. In the opposite case, we will import as much as possible. Otherwise the import volume will be determined by other considerations. Formally

$$m = m(P_d, P_m) = \left\{ \begin{array}{ll} m^- & \text{if } P_d/P_m \quad 1 \\ (m^-, m^+) & \text{if } P_d/P_m = 1 \\ m^+ & \text{if } P_d/P_m \quad 1 \end{array} \right\}$$

Thus, the import share can be treated formally as a function of relative prices in this case too. The function in this case is, however, not a smooth one. (See Figure 1.)

It is worth noting here that essentially the same restrictions on import could not have been implicitly achieved by introducing a piecewise linear objective function. Such an objective function in a planning model could be viewed as the planners preference (utility) function with respect to the domestic-import composition of available sources. (See Figure 2.)

We would like to emphasize that the difference between the import restriction modes in the case of linear programming and computable equilibrium models can again be seen as the one between *fixed* (rigid) and *flexible* individual bounds. The relative (shadow or equilibrium) price dependent import share implies a variable (flexible) individual bound on import: the larger the gap between the domestic and import commodity (shadow) prices, the large the deviation from the observed (or planned) import ratio (m_o). In fact, allowing for a smooth variation of the import share around its planned level in a plan coordination model makes at least as much sense, if not more, that the usual rigid restrictions.

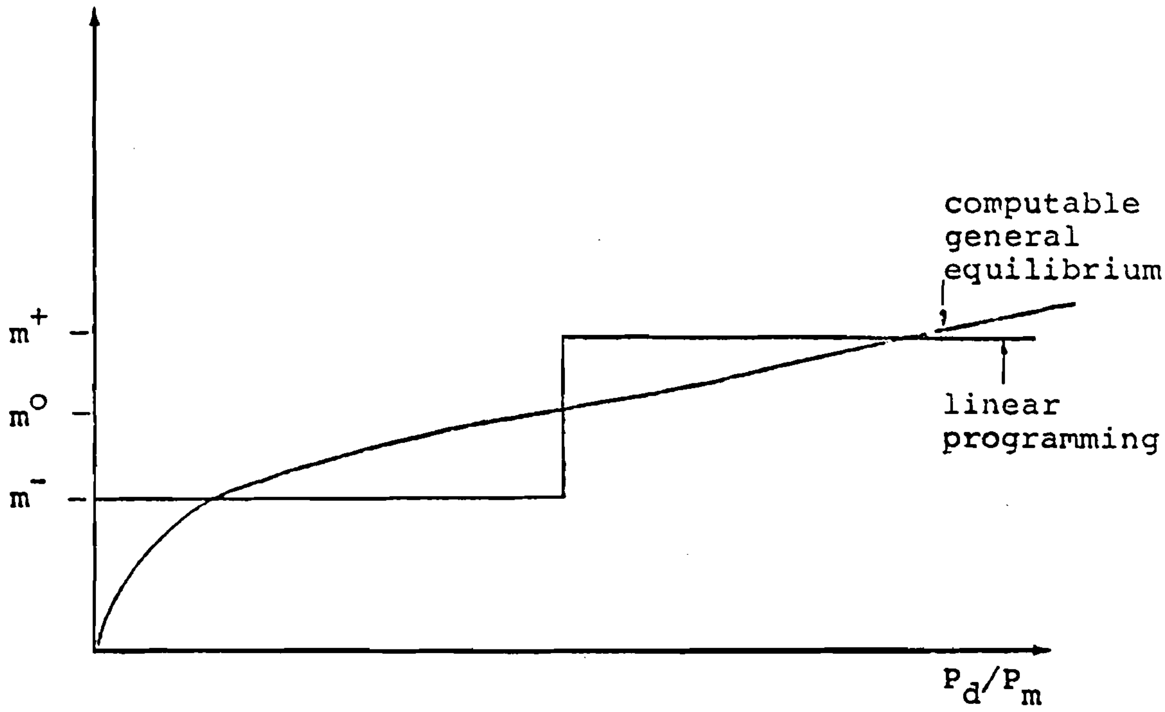

Figure 1. Import Share Functions

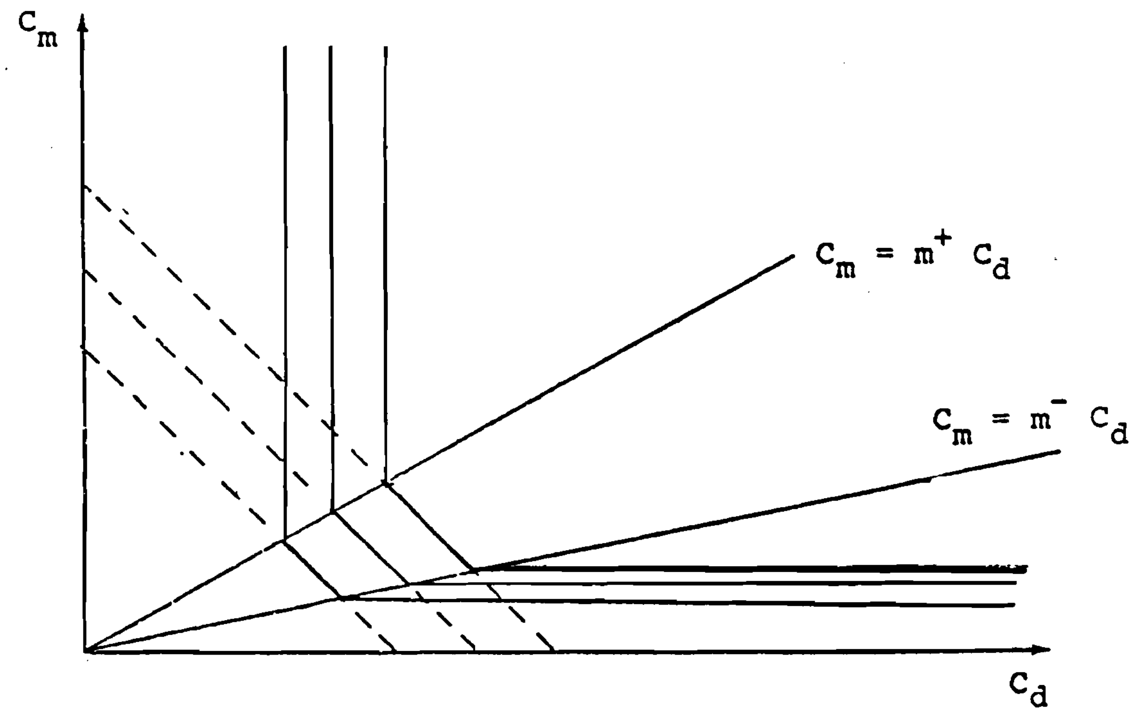

Figure 2. Import restriction built into the objective function

We complete our example by replacing fixed bounds with flexible ones. Suppose we have a linear programming model with fixed individual bounds, both on export and on import shares. Let us repeat the model in full here

$$C = C_m + C_d \rightarrow \max$$

$$C_d + Z \leqq \bar{Y} \qquad (P_d)$$

$$C_m \leqq M \qquad (P_m)$$

$$\bar{P}_M M - \bar{P}_E Z \leqq 0 \qquad (V)$$

$$m^- C_d \leqq C_m \leqq m^+ C_d \qquad (t_m^-, t_m^+)$$

$$Z \leqq \bar{Z} \qquad (T_c)$$

If we want to replace the fixed individual bounds by flexible ones, in the manner described and discussed earlier, we can proceed in the following way. We can rewrite the above linear model into a nonlinear one by replacing the objective function with one reflecting import limitations and introducing an export demand function as before. These replacements will yield the following model (using constant elasticity forms):

$$C = (h_m C_m^{-\eta} + h_d C_d^{-\eta})^{-1/\eta} \rightarrow \max$$

$$C_d + Z \leqq \bar{Y} \qquad (P_d)$$

$$C_m \leqq M \qquad (P_m)$$

$$\bar{P}_M \cdot M - D \cdot Z^{1+\varepsilon/\varepsilon} \leqq 0 \qquad (V)$$

For lack of place we cannot show here how the parameters h_m, h_d and η can be determined from m_o and μ (the parameters of the import share function) and *vice versa*. Parameter D in the foreign trade balance is a constant term derived by solving the following export demand function for P_E

$$Z = e_o (P_E)^{\varepsilon}$$

In case of a "real" model it might be difficult to handle nonlinearities. In such cases, piecewise linear approximations could save the linear character of the model. (See reference 9 for more details of such a approach). We want to turn the reader's attention to an alternative approach.

With reasonable values for the parameters, an interior solution to the programming problem can be expected. By interpreting P_d, P_m and V as Lagrangian multipliers for the corresponding constraints, the dual part of the first order neccessary (Kuhn-Tucker) conditions for a maximum can be stated as follows:

$$P_d = \frac{\partial C}{\partial C_d} \tag{1}$$

$$P_m = \frac{\partial C}{\partial C_m} \tag{2}$$

$$P_m = V\ \bar{P}_M \tag{3}$$

$$P_d = \frac{1+\varepsilon}{\varepsilon}\, D \cdot V \cdot Z^{1/\varepsilon} = \frac{1+\varepsilon}{\varepsilon}\, V\ P_E \tag{4}$$

We can show that conditions (1) and (2) will, in fact, yield the import share function

$$m = m_o \left(\frac{P_d}{P_m}\right)^{\mu}$$

It is also fairly easy to see that we can replace the above programming model by the following simultaneous equations system

$$m = m_o \left(\frac{P_d}{P_m}\right)^{\mu} \qquad (11) \qquad\qquad Z = a \left(\frac{P_E}{\bar{P}_{WE}}\right)^{\mu} \qquad (15)$$

$$P_m = V\ \bar{P}_M \qquad (12) \qquad\qquad C_d + Z = \bar{Y} \qquad (16)$$

$$C_m = m\ C_d \qquad (13) \qquad\qquad C_m = M \qquad (17)$$

$$P_d = \frac{1+\varepsilon}{\varepsilon}\, V\, P_E \qquad (14) \qquad\qquad \bar{P}_M M - P_E Z = 0 \qquad (18)$$

This latter form is almost identical with a typical computable general equilibrium model specification. The only difference from a competitive equilibrium model is in equation (14). In the latter, we would only have $P_d = V P_E$. The difference can be viewed as that between a *planners optimum* and a *laissez-faire* equilibrium. (For details see reference 17).

We close this subsection with a brief discussion on the derived equation system. Counting the variables (m, C_d, C_m, M, Z, P_m, P_d, P_E, V) we find that there is one more variable than equations. This might lead to overdetermination problems. However, observe that all the equations are homogenous of degree zero in variables P_m, P_d and V. Thus the level of all of them can be set freely.

The solution of a general equilibrium equation system needs special algorithms. These will be discussed in a separate paper by A. Por (see reference 12). We want to mention here only our experiences with a model containing 19 sectors and close to 500 variables. To get a solution needs on average less than a minute on an ICL System 4/70 computer.*

4. CONCLUSIONS

Both here and in the earlier papers we have shown that a certain class of multisectoral general equilibrium models, by proper reinterpretation of their elements, can be adopted to support planning in socialist countries. We have also demonstrated how certain nonlinear formulations of substitution possibilities could be utilized in macroprogramming models in order to keep the model relatively small and generate meaningful dual solutions.

One major advantage of the equilibrium framework is that it makes the dual side of the model less distorted while explicitly taking into account the interaction of real and value

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* For a more detailed description of the model and its solution algorithm, consult reference 13.

variables. Thus, it may help planning modelers to achieve a better linkage between plans for real and value processes. These two main planning functions are usually quite separate from each other in both traditional planning and modeling. Changes in relative prices, costs, tariffs, etc., are not properly reflected in physical allocation models, while the effects of production, import/export, and consumption decisions are not always taken into consideration in price planning models.

The mixed, primal-dual formulation of the resource allocation problem requires and also make it possible to reinterpret the notion of efficiency (shadow) prices. On the one hand, the mixed form allows the model builder to explicitly introduce shadow-price dependent resource allocation decisions into his model. In our simple model, it was quite easy to see how the efficiency-price-dependent foreign trade decisions related to the programming problem formulation. In more complex models, such price-dependent (mixed primal-dual) decision rules can be used in describing consumption and resource use alternatives, etc. (see reference 16). The general equilibrium (mixed primal-dual) formulation allows also for combining econometrically estimated, price dependent macrofunctions with the optimal resource allocation approach.

On the other hand, the equilibrium formulation makes it possible to incorporate price-formation rules that reflect the actual process more accurately than the shadow prices of (linear) programming models. For example, even with constant returns to scale, it is possible to define prices that do contain profits (mark-up). One can also take into account changes in taxes and tariffs and see how these would affect the allocation decisions.

These comments suggest that the possible use of general equilibrium models is manifold and not limited to coordinating a plan. In fact, we believe that these models could also be used for either *ex post* or *ex ante* simulation of various issues of concern to planners. Using statistical estimates of the model parameters, structurally similar models (especially their multiperiod extensions) could be tested in the forecasting phase of planning. In the central planning context, it seems promising to combine such models with reference path optimization techniques (see reference 15). Research in this direction is currently underway both at the International Institute for Applied Systems Analysis and in the Hungarian Planning Office.

REFERENCES

/1/ Adelman, I. and S. Robinson. 1978. Income Distribution Policy in Developing Countries: A Case Study of Korea. Stanford University Press. Stanford, California.

/2/ Augusztinovics, Maria, et al. 1981. Macroeconomic Models in the Preparation of the 6. Five Year Plan. (Mimeo, in Hungarian) OT-2224/XIV.

/3/ Bergman, L. 1978. Energy Policy in a Small Open Economy: The Case of Sweden. RR-78-16. International Institute for Applied Systems Analysis, Laxenburg, Austria.

/4/ Bergman, L. and A. Por. 1980. A Quantitative General Equilibrium Model of the Swedish Economy. WP-80-04. International Institute for Applied Systems Analysis, Laxenburg, Austria.

/5/ Boda, Gy., I. Csekő, F-ne Hannel, L. Laszlo, S. Poviliaitis. 1982. Input and Output System of a General Equilibrium Model (mimeo, in Hungarian), Budapest, Hungary, OT-XVIII/120.

/6/ De Melo, J. 1978. A Simulation of Development. Strategies in an Economy-Wide Policy Model. World Bank/IBRD/, Washington, D.C.

/7/ Dervis, K. and S. Robinson. 1978. The Foreign Exchange Gap, Growth and Industrial Strategy in Turkey: 1973-1983. WP.306, World Bank, IBRD, Washington, D.C.

/8/ Dixon, P.B., B.R. Parmenter, G.T. Ryland and J. Sutton. 1977. ORANI, A General Equilibrium Model of the Australian Economy: Current Specification and Illustrations for Use for Policy Analysis. First Progress Report of the IMPACT Project, Vol.2, Australian Government Publishing Service, Canberra.

/9/ Ginsburgh, V. and J. Waelbroek. 1981. Activity Analysis and General Equilibrium Modelling, North-Holland Publishing Company, Amsterdam.

/10/ Johansen, L. 1959. A Multisectoral Study of Economic Growth. North-Holland Publishing Company, Amsterdam.

/11/ Kelley, A.C. and J.G. Williamson. 1980. Modeling Urbanization and Economic Growth. RR-80-22. International Institute for Applied Systems Analysis, Laxenburg, Austria.

/12/ Por, A. 1982. Some Recent Mathematical Techniques Available for for Economic Modeling (in Hungarian). Paper to be presented at the Session.

/13/ Sivak, F. and A. Tihanyi. 1982. On the Solution of Nonlinear Macroeconomic Models. (Mimeo, in Hungarian). Budapest. OT-Ig/520

/14/ Taylor, L. 1975. The Theoretical Foundations and Technical Implications. Edited by C.R. Blitzer, P.C. Clark and L. Taylor. Economy-Wide Models and Development. Oxford University Press, Oxford.

/15/ Wierzbicki A. 1979. The Use of Reference Objectives in Multiobjective Optimization: Theoretical Implications and Practical Experience. WP-79-65 . International Institute for Applied Systems Analysis, Laxenburg, Austria.

/16/ Zalai, E. 1980. A nonlinear multisectoral model for Hungary: General equilibrium versus optimal planning approach. WP-80-148. International Institute for Applied Systems Analysis, Laxenburg, Austria.

/17/ Zalai, E. 1982. Nation-wide Economic Models:Foreign Trade, Optimum and Equilibrium.(Forthcoming WP). International Institute for Applied Systems Analysis, Laxenburg, Austria.