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INTRODUCTION

The reference level approach [1] has been shown to be an appropriate tool for studying conflicting objectives in practical decision situations [2]. A software package (Dynamic Interactive Decision Analysis and Support System or DIDASS) based on this approach has been developed at IIASA to analyze linear and non-linear multiple-criteria optimization problems .

This paper describes another experiment with the reference level approach, this time with the energy supply model MESSAGE [3]. In its original form , MESSAGE is a dynamic linear programming model with the (single) objective of minimizing the total discounted costs of meeting a set of energy demands over a given time horizon. The experiment described here shows that it is possible to take into account more than one objective and thus to study the interplay between costs and other factors such as import dependence,the need to develop infrastructure, and so on.

The main purpose of this paper is to describe the use of a new methodology; the data defining the MESSAGE run serve only to illustrate the method and their policy implications are therefore not discussed here.

PROBLEM FORMULATION

To test whether the reference level approach could be used to generate efficient energy policies, we used the energy supply model MESSAGE to study energy supply policies for the countries of the European Economic Community (EEC) [4] over the period 1980-2030. The main aim of the model is to meet the predicted demand for secondary energy by manipulating the vector of annual consumption of resources, the vector of energy production, and the annual increase in energy-producing capacity. The feasible set is determined mainly by strategies for the supply of primary energy resources via a menu of possible technologies (see Figure 1).

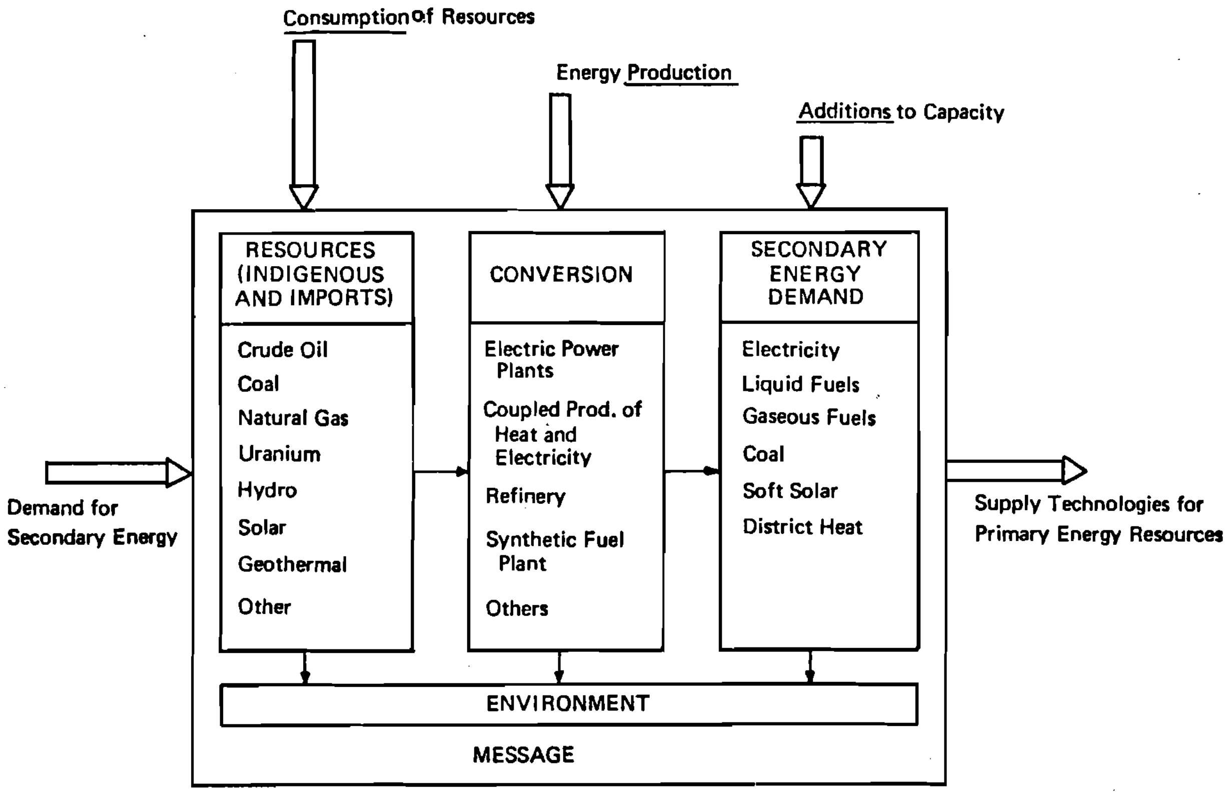

Figure 1. Schema of the energy supply model MESSAGE.

The resulting optimization problem can be formulated as a standard dynamic linear program. A detailed description of the formulation is given in the Appendix.

1. State Equations:

$$y(t+1) = \sum_{i=1}^{\nu} \bar{A}(t-n_i)\, y(t-n_i) + \sum_{j=1}^{\mu} \bar{B}(t-m_j) u(t-m_j) \qquad (1)$$

where:

$t=0,1,\ldots,T-1$

y is a vector of state variables

u is a vector of control variables

$\bar{A},\bar{B}$ are matrices of input data,

$(n_1, \ldots, n_\nu),(m_1, \ldots, m_\mu)$ are sets of integers which characterize time lags in state and/or control variables

T is the length of the planning period

2. Constraints:

$$\bar{G}(t)y(t) + \bar{D}(t)u(t) \le f(t) \qquad (2)$$

where:

$t=0,1,\ldots,T$

$\bar{G}$, $\bar{D}$ are matrices of input data,

f is a vector of input data

3. Bounds:

Upper and lower bounds on the control variables $u(t)$ and on the state variables $y(t)$ are also specified:

$$L(t) \le \begin{bmatrix} u(t) \\ y(t) \end{bmatrix} \le U(t) \qquad (3)$$

where $t = 0,1,\ldots,T$.

4. *Planning Period:*

The planning period is fixed (T) and the initial state of the energy system is also given:

$$y(0) = y^0 \tag{4}$$

5. *Criteria Functions:*

The performance function for the scalar case has the general form:

$$J(u) = (a(T),y(T)) + \sum_{t=0}^{T-1} \left[(a(t),y(t)) + (b(t),u(t)) \right] \tag{5}$$

where a and b are input vectors.

The following scalar objective function, which reflects the total discounted costs of energy supply, was originally used in MESSAGE [5]:

$$J(u(t)) = \sum_{t=1}^{T} \left\{ \beta_1(t)(\alpha_1(t),x(t)) + \beta_2(t)(\alpha_2(t),z(t)) + \beta_3(t)(\alpha_3(t),r(t)) \right\} \rightarrow \min \tag{6}$$

where:

$T = 11$

$J(u(t)) = J(x(t),z(t),r(t))$

$x(t)$ is the vector of energy production

$z(t)$ is the vector of annual increase in energy-producing capacity

$r(t)$ is the vector of annual consumption of resources

β_i are discount factors

α_i are vectors of annual cost coefficients

The solution of eqn. (6) under conditions (1) - (4) will be described as problem S.

To improve our analysis of the decision situation we decided not simply to minimize a single aggregated function at the end of the planning period but to minimize the trajectory of certain interesting criteria. As a test we considered the problem of simultaneous minimization of the undiscounted costs $J_{cost}(t)$, the amount of coal extracted $r_{coal}(t)$, and the volume of oil imported $r_{oil}(t)$ for each time period. This leads to the following vector of 33 criteria:

$$\begin{bmatrix} J_{cost}(t) \; ; \; t=1,2,\ldots,11 \\ r_{coal}(t) \; ; \; t=1,2,\ldots,11 \\ r_{oil}(t) \; ; \; t=1,2,\ldots,11 \end{bmatrix} \tag{7}$$

where:

$$J_{cost}(t) = \left\{ (\alpha_1(t),x(t)) + (\alpha_2(t),z(t)) + (\alpha_3(t),r(t)) \right\}$$

$r_{coal}(t)$ and $r_{oil}(t)$ are subvectors of the vector $r(t)$.

The minimization of vector (7) under constraints (1)-(4) will be described as problem M1. This represents a situation in which we wish to minimize both current costs and the use of fossil fuels in the production of energy. We also analyzed a slightly different problem in which both the overall costs (6) and the amount of coal extracted and oil imported are minimized. This gives an objective vector with 23 components:

$$\begin{bmatrix} J(u) \\ r_{coal}(t) \; ; \; t=1,2,\ldots,11 \\ r_{oil}(t) \; ; \; t=1,2,\ldots,11 \end{bmatrix} \tag{8}$$

The minimization of vector (8) under constraints (1)-(4) will be denoted as problem M2.

The general mathematical formulation of the linear multiple-criteria problems M1 and M2 discussed above is as follows:

Let A be in $R^{m \times n}$, C in $R^{p \times n}$, and b in R^m. If q is the vector of criteria (such as (7) or (8)) and x the joint vector of states y and controls u:

$$\begin{aligned} Cx &= q \rightarrow \min \\ Ax &= b \\ x &\geq 0 \end{aligned} \tag{9}$$

The reference or aspiration level approach is then used to analyze problem (9).

REFERENCE LEVEL APPROACH

The reference (or aspiration) level or trajectory is a suggestion $\bar{q}$ made by the decision maker reflecting in some sense the outcomes desired by him; in this case the trajectory of oil imported, coal extracted, and costs over the planning period 1980-2030. According to Wierzbicki [1], we must first define a partial ordering in the objective space that corresponds to the nature of the problem. This means that for two trajectories q_A and q_B we may say for example that trajectory q_A is not worse than q_B if, $q_A(t) \leq q_B(t)$ for all $t \in [0,T]$. When specifying the reference trajectory $\bar{q}$ we introduce a relative ordering in the objective space - we can determine which trajectories are better or worse as than given reference trajectory $\bar{q}$ (see Figure 2). There are, of course, trajectories that are neither better nor worse.

The reference trajectory optimization problem can then be formulated as follows:

Given the reference trajectory $\bar{q}$, find a Pareto-optimal trajectory $\hat{q}$ which is attainable and in some sense related to the reference trajectory $\bar{q}$.

In principle , two situations can arise :

(a) *Reference trajectory $\bar{q}$ is attainable , i.e., there is an admissible decision q for which $q = \bar{q}$ (i.e., there is a feasible x for which $Cx = q$).*

(b) *Reference trajectory $\bar{q}$ is not attainable, i.e., for every admissible decision q is unequal to $\bar{q}$.*

Figure 3 illustrates the two situations (a) and (b) for the static two-dimensional case. In problems (7) and (8), the dimensionality of the problem is increased according to the number of time steps.

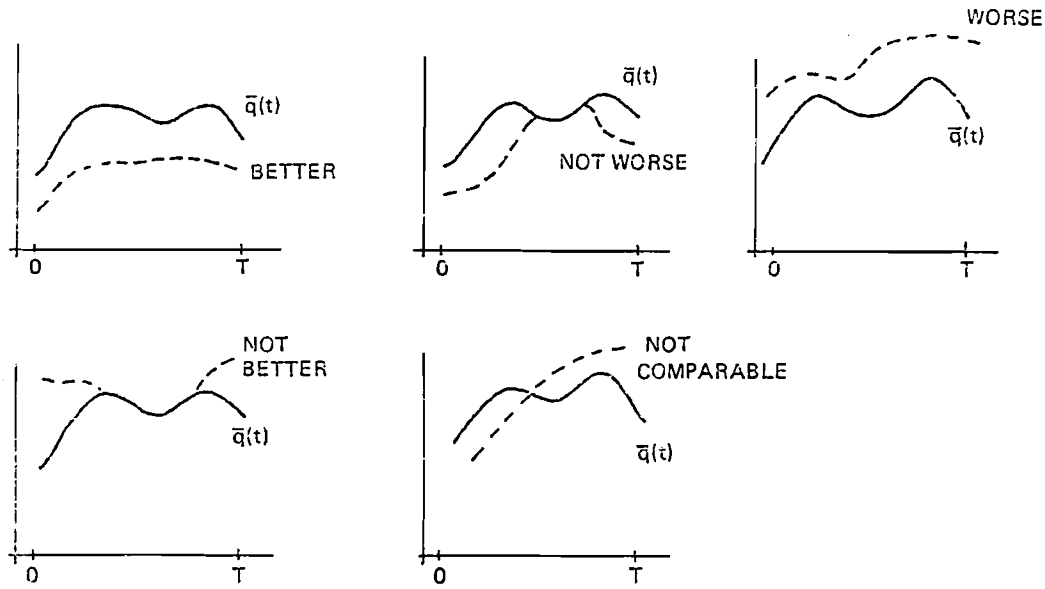

Figure 2. Ordering in the trajectory space.

Since the reference trajectory expresses the outcomes that are desirable for the decision maker, it is reasonable to propose the following solution:

(I) *It is reasonable to require that the method proposes only non-improvable decisions , i.e. only such objectives* $\hat{q}$ *that the set of attainable objectives better than* $\hat{q}$ *is empty (objectives in the Pareto-set, dashed line in Figure 3).*

(II) *In the case (a) it is reasonable to improve all components of the perfomance vector as much as possible but in a sense equitably , that is to maximize a "utility"* $-s(q-\bar{q})$ *of improving* q *over* $\bar{q}$.

(III) *In the case (b) it is reasonable to find the attainable objective in the Pareto-set that is in a sense "nearest" to* $\bar{q}$, *that is to minimize a "distance"* $s(q-\bar{q})$ *for all* $q \in \Omega_p$.

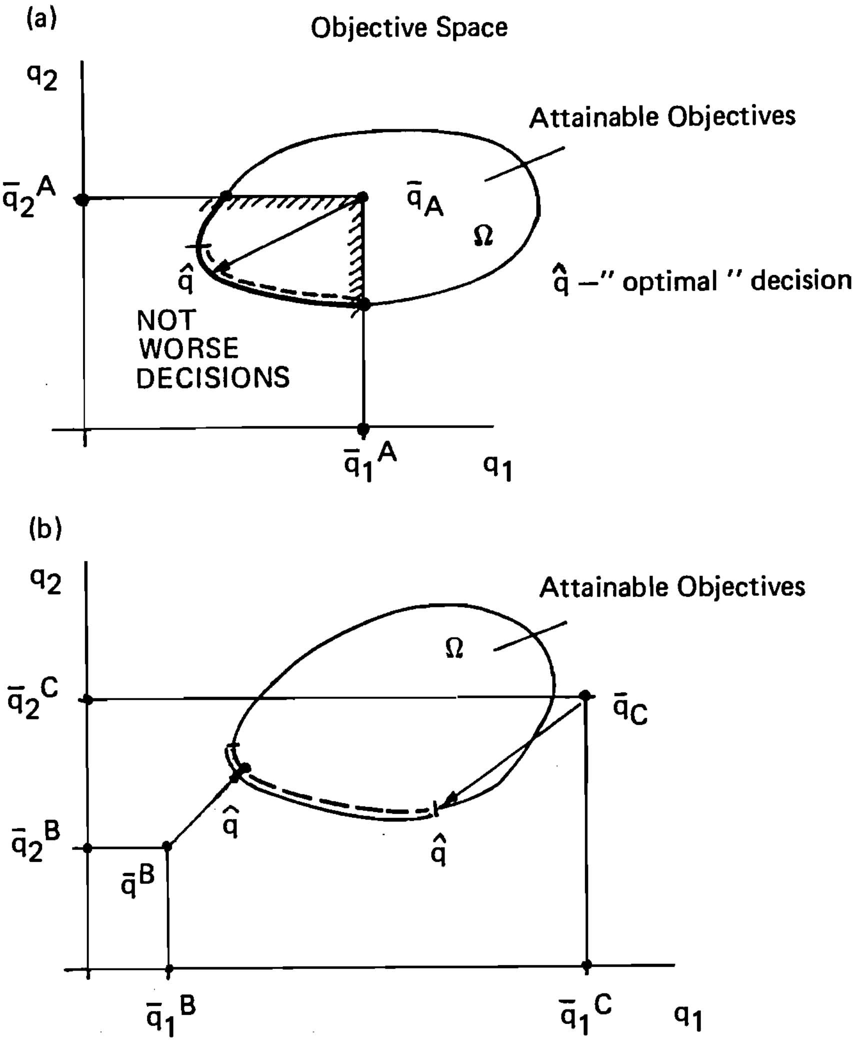

Figure 3. Interpretation of the reference level approach in the objective space ($\bar{q}_A$ is an attainable reference point, $\bar{q}_B$ and $\bar{q}_C$ are unattainable reference points).

The basic technical problem is to determine whether the situation is represented by case (a) or case (b) for a specified $\bar{q}$. To avoid this difficulty, the concept of an achievement scalarizing function has been introduced by Wierzbicki. The properties of the achievement scalarizing function are such that the result of the minimization:

$$\min_{q \in \Omega} \; s(q - \bar{q})$$

satisfies all the requirements (I)-(III) specified above. The general properties of such functions are discussed by Wierzbicki elsewhere [1], [6] and[7].

The following form of the achievement scalarizing function $s(q-\bar{q})$ has the advantage that minimization results in a linear programming formulation [8]:

$$s(q-\bar{q}) = -\min_{q\in\Omega}\left\{\rho\min_i(q_i-\bar{q}_i)\ ;\ \sum_{i=1}^{p}(q_i-\bar{q}_i)\right\}-\sum_{i=1}^{p}\varepsilon_i(q_i-\bar{q}_i) \tag{10}$$

Here ρ is an arbitrary penalty coefficient which is greater than or equal to p and $\varepsilon=(\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_p)$ is a nonnegative vector of parameters (this guarantees strict Pareto-optimality if $\varepsilon>0$).

We also define $w_i=(q_i-\bar{q}_i)/\gamma_i$ for $i=1,2,\ldots,p$ where $\gamma_i\approx\bar{q}_i$ and γ_i (different from 0)is a scaling factor, chosen by the user. This scaling factor is introduced not in order to weight different objectives, but to make their influence independent of their physical units and their scale.

The set $S_{\hat{s}}(\bar{q})\equiv\left\{q\ ;\ s(w)\geq\hat{s}\ ,\ w=(q-\bar{q})\gamma^{-1}\right\}$, for a given scalar $\hat{s}$, is called the level set of the scalarizing function; here γ is a diagonal matrix of scaling factors γ_i. The influence of scaling factors is illustrated in Figure 4 for function (10) and the case $\rho=p$, $\varepsilon=0$.

Using these definitions, the problem of minimizing of (10) over the attainable points q can be formulated as a linear programming problem. For this we denote $w=(q-\bar{q})\gamma^{-1}=(Cx-\bar{q})\gamma^{-1}$ and introduce an auxiliary decision variable $y=z+\varepsilon w$. The resulting LP is :

$$\min s(w) = \min_{\substack{w\in W\\ y\in R}}\left\{y-\varepsilon w \mid -y-pw_i\leq 0\ ,\ \text{for } all\ i,\ -y-\sum_i w_i\leq 0\right\} \tag{11}$$

where $W\equiv\left\{w \mid -\gamma w+Cx=\bar{q}\ ,\ Ax=b\ ,\ x\geq 0\right\}$ is the feasible set for w. This problem can be solved using any commercial LP system.

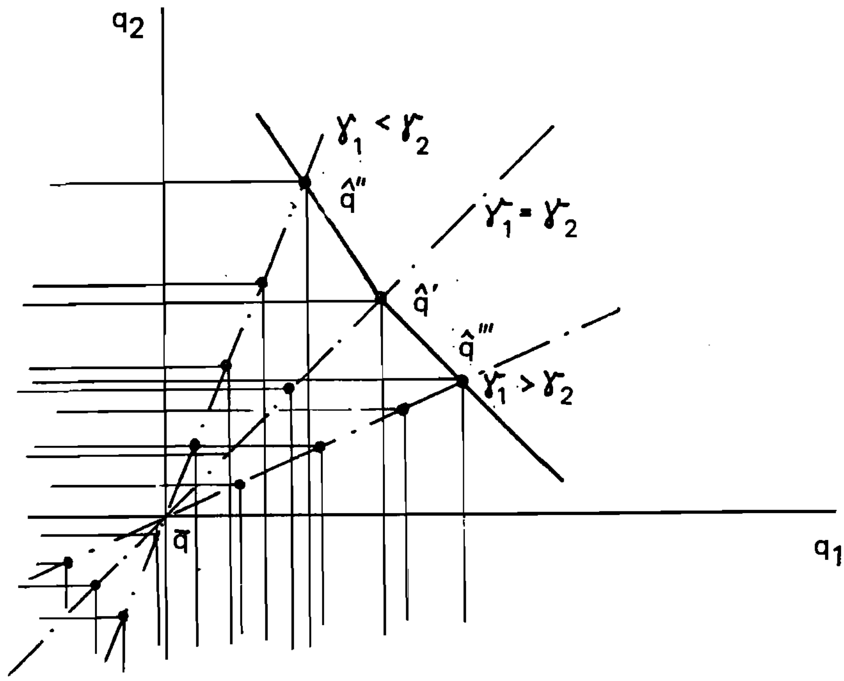

Figure 4. Level sets for achievement scalarizing function (10) for ε=0, ρ=p, and various scaling factors.

COMPUTER IMPLEMENTATION

The software for the energy supply model MESSAGE [3] has been combined with the DIDASS package for linear multiple-criteria reference point optimization to produce a system capable of solving the problems outlined above. The combined structure of the energy model and the multiple-criteria software is given in Figure 5.

The aim of Figure 5 is to explain how a model (e.g., the energy supply model) may be used in conjunction with an interactive multiple-criteria analysis procedure. The left-hand side of Figure 5 gives the usual stages in a computer

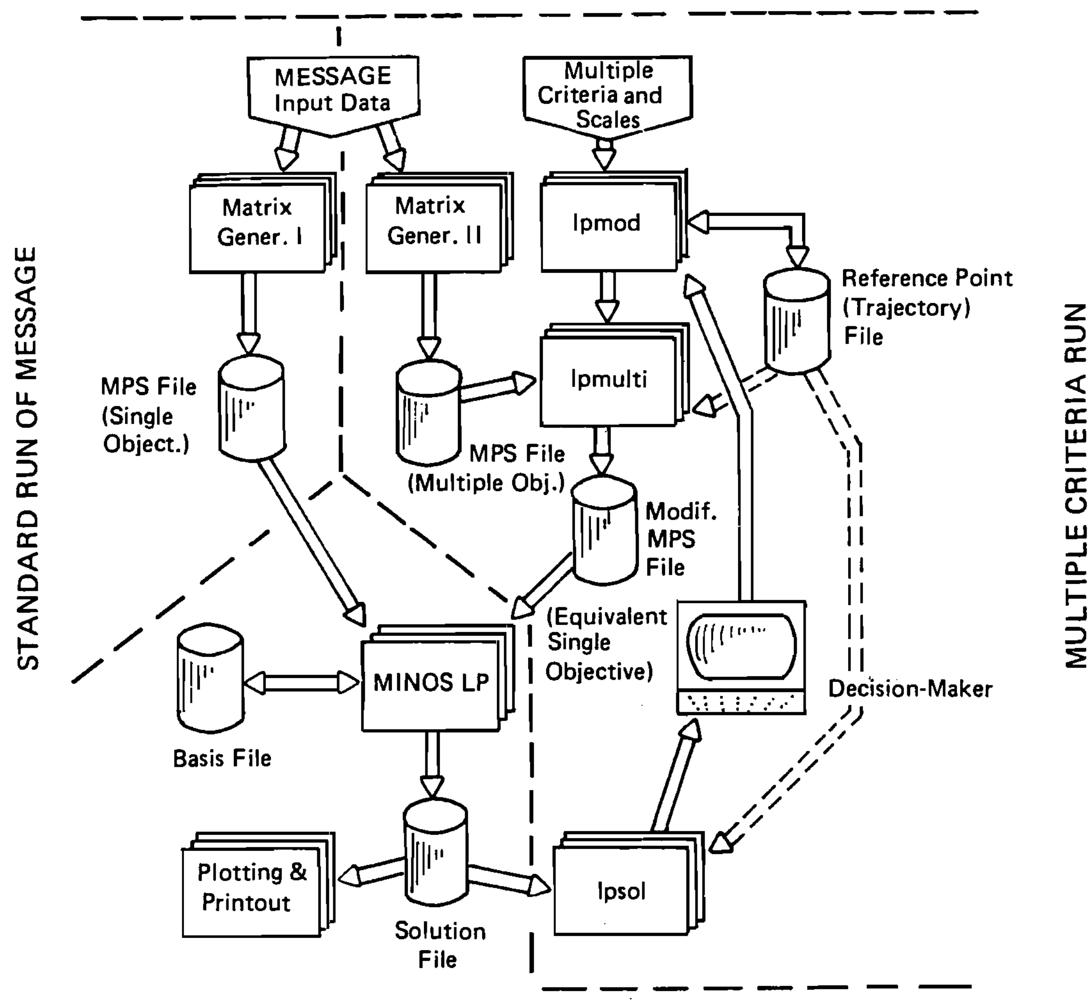

Figure 5. The combined structure of the energy model and the DIDASS package for the interactive generation of efficient energy supply strategies.

run of MESSAGE. In the combined case, however, the MPS input format file must be prepared according to the formulation of the multiple-criteria problem: for large models such as MESSAGE, the original matrix generator must be altered (Matrix Gener. II) to modify the MPS input format file in this way.

The right-hand side of Figure 5 illustrates the multiple-criteria optimization procedure. This begins with an interactive "editor" which is used to define the trajectories of various criteria and to manipulate the reference trajectories and the scaling factors (lpmod).

In the next step, the preprocessor (lpmulti in Figure 5) converts the prepared MPS input format file into a single-criterion equivalent (11). This single-criterion problem is solved with the MINOS system [9]. A postprocessor (lpsol in Figure 5) extracts selected information from the LP system output file, computes the values of the objectives and displays the information to the decision maker. The decision maker can then change the reference trajectories on the basis of this information, and possibly on the basis of experience gained in previous sessions, to generate new efficient energy supply strategies which he can analyze in the next iteration .

COMPUTATIONAL EXPERIENCE

We tested the combined software by applying it to one of several scenario runs for the EEC-countries [4] for the planning period 1980-2030 under the conditions of problems M1 and M2.

The first main result was that it was necessary to scale the components of the objective vector so that the numerical values of the components are of the same order of magnitude (independent of their physical unit). If this is not done the solution of (11) is dominated by the trajectory whose components have the largest numerical values and the other trajectories are virtually insensitive to changes in their reference trajectories.

In problem M1 we experimented with different scaling factors for the cost terms because the numerical values for coal extraction and oil imports are of the same order of magnitude, while the figures for costs are greater by a factor of 10^4 . We therefore used three different sets of scaling factors for the first eleven components of vector (7):

(I) $\gamma_1 = \gamma_2 = \cdots = \gamma_{11} = 10^2$; $\gamma_{12} = \gamma_{13} = \cdots = \gamma_{33} = 1$

(II) $\gamma_1 = \gamma_2 = \cdots = \gamma_{11} = 10^3$; $\gamma_{12} = \gamma_{13} = \cdots = \gamma_{33} = 1$

(III) $\gamma_1 = \gamma_2 = \cdots = \gamma_{11} = 10^4$; $\gamma_{12} = \gamma_{13} = \cdots = \gamma_{33} = 1$

The problem M1 is solved for the three sets of scaling factors (I-III) and for given reference trajectories for costs, coal extraction, and oil imports. Figure 6 illustrates the reference trajectories and the corresponding efficient trajectories (Response) obtained in each of the three cases.

For case (I) the coal and oil trajectories (Figure 6b, 6c) are affected only very slightly by the corresponding reference trajectories, the coal response even reaching the upper bound (Figure 6b). The solution is fully dominated by the cost response and follows the cost reference trajectory. Increasing the values $\gamma_1 = \gamma_2 = \cdots = \gamma_{11}$ reduces the influence of the cost terms, and for case (III) the coal and oil responses follow the corresponding reference trajectories exactly , with a slight vertical displacement (see Figure 6b, 6c).

The trajectories s in Figure 6 indicate the solution of problem S with the scalar objective function (6) - it is interesting to compare this with the multicriteria case.

The problem described above consists of 711 rows and 761 columns. One run of the equivalent single LP problem on a VAX without an old basis from a previous session takes about 90 min CPU time; if an old basis is available the LP solution takes between 25 sec and 12 min CPU time. Using the current version of the preprocessor (lpmulti), the modification of the MPS input format file takes from 47 sec to 51 sec CPU time.

We also analyzed problem M2 using the new software. Figure 7 presents the

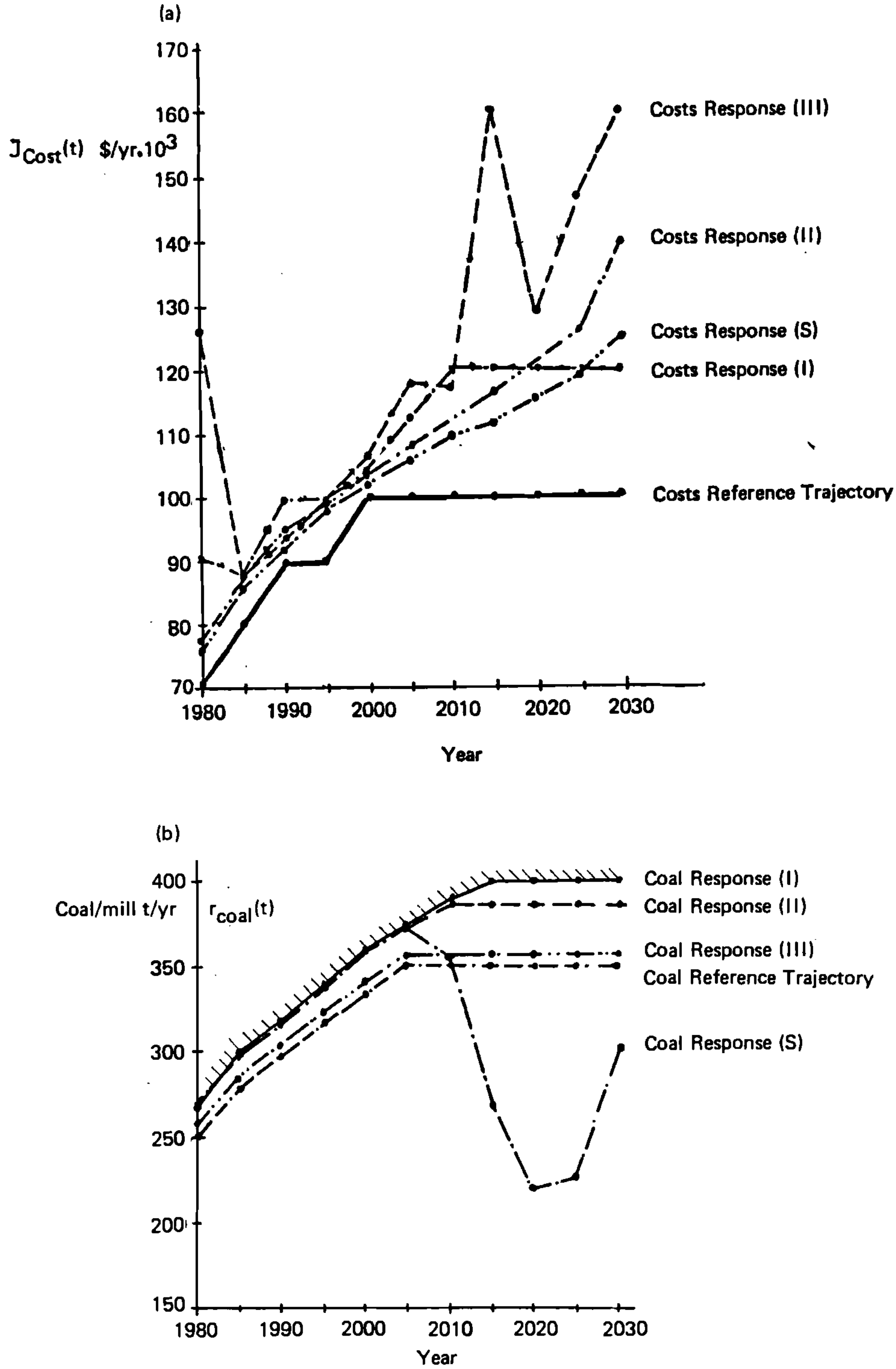

Figure 6. Experiments with different scaling factors γ_i for the cost terms (see set (I)-(III)) in problem M1 with $p = \rho = 33$, $\varepsilon = 10^{-4}$. Here s is the solution of problem S with the scalar objective function (6), given for comparison.

(a) Trajectories for the undiscounted costs $J_{cost}(t)$;

(b) Trajectories for the use of coal $r_{coal}(t)$;

(c) Trajectories of oil import policies $r_{oil}(t)$.

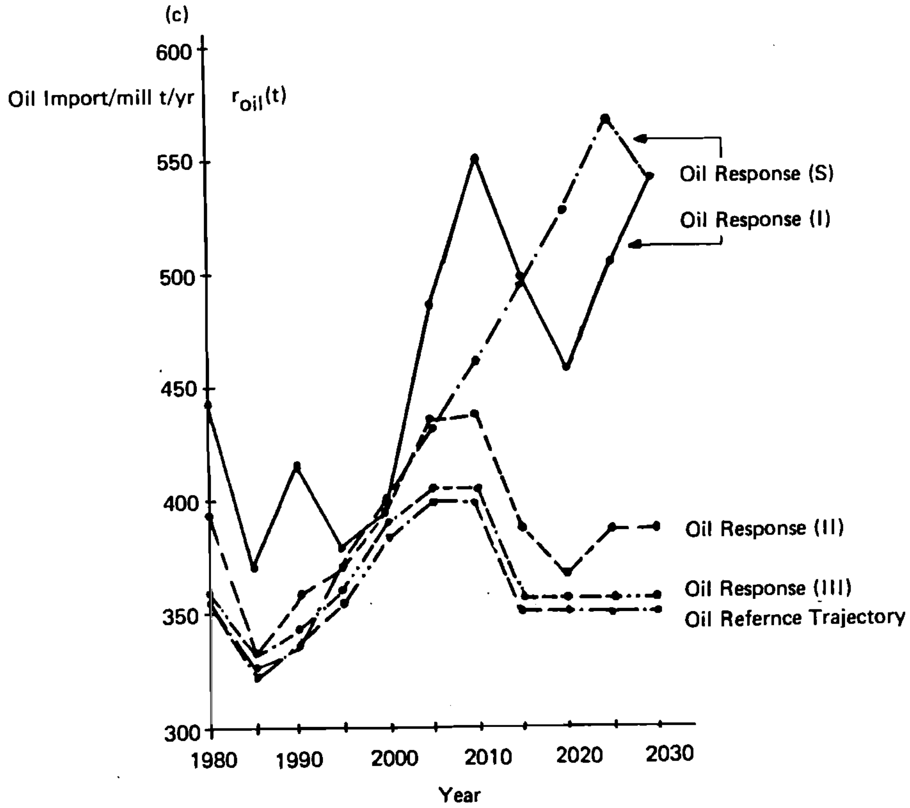

Figure 6. (continued)

results obtained for M2 assuming the same reference trajectories for coal extraction and oil imports as in problem M1 (see Figure 6b, 6c). The scaling factors corresponding to vector (8) are as follows: $\gamma_1 = 10^4$, $\gamma_2 = \gamma_3 = \cdots = \gamma_{23} = 1$. The reference point for the cost function is the scalar solution (s), which is also illustrated for the other objectives . The reference trajectories can be interpreted as follows. After a transition period ending in 2015, the decision maker wishes oil imports to a level off at 350 mill. t/year and coal extraction to remain approximately constant just below the upper bound. The reference point for the overall cost of supplying energy is assumed to be given by the scalar solution. At the scale used in Figure 7,the responses of the efficient trajectories for coal and oil appear to be identical with the reference trajectories; they actually

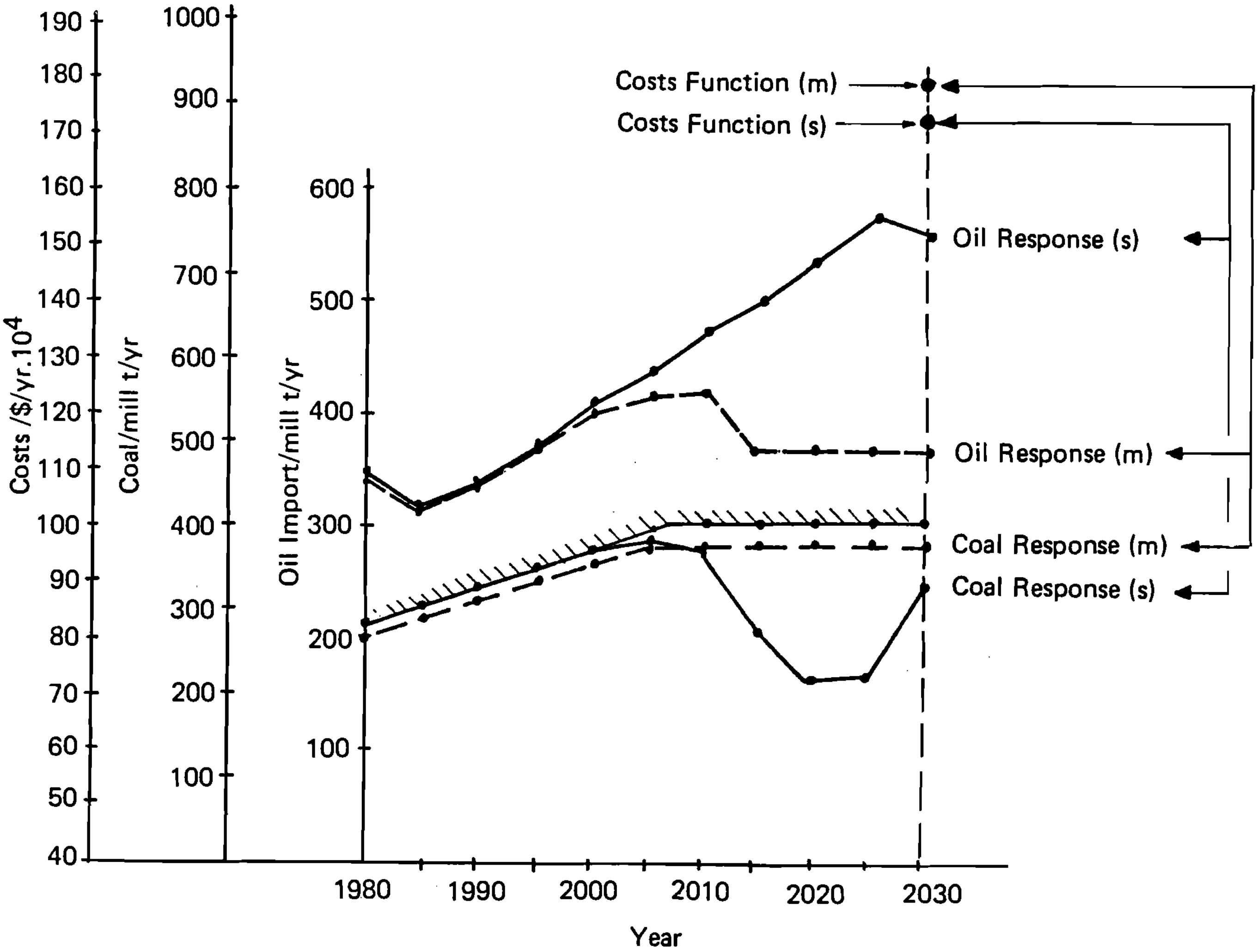

Figure 7. Efficient trajectories for the problem M2.

differ by a constant value of approximately 1%. The resulting overall costs are of course higher than in the scalar case. After studying the solution on the basis of plots and printouts the decision maker will either be satisfied with this strategy or he will not; if he is not satisfied he should change the reference trajectories and/or the scaling factors before starting the next session.

SUMMARY AND CONCLUSIONS

This application has once again shown the reference level approach to be a useful tool for analyzing situations with conflicting objectives. In addition the program package DIDASS seems to be flexible enough to allow good control of the behavior of the attainable trajectories.

Further work should be done to improve the "user-friendliness" of the software. There are three ways of achieving this:

- speeding up the modification of the MPS input format file by improving the preprocessor (lpmulti)
- speeding up the solution of the equivalent LP problem
- including the history of the interactive decision-making process by displaying the sequences of references and obtained objectives visually.

ACKNOWLEDGMENTS

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APPENDIX

This appendix gives examples of equations of the type (1) and (2) taken from the energy supply model.

I. State Equation:

Capacities of Technologies:

$$c(t)=c(t-1)+5z(t)-5z(t-6) \quad , \quad t=1,2,\ldots,11$$

where:

z is the vector of annual additions to capacity

$t-6$ reflects a 30-year service life

Resource Balances:

$$s(t)=s(t-1)-5r(t) \quad , \quad t=1,2,\ldots,11$$

where:

s is a vector of reserves (stocks) of primary energy carriers or man-made fuels

r is a vector of annual consumptions of primary energy

II. Constraints:

Demand/Supply Balance:

$$Dx(t) \geq d(t) + H(t) \quad , \quad t=1,2,\ldots,11$$

where:

D is a matrix describing supply/demand paths

x is a vector of annual supply activities

d is a vector of annual secondary energy demand (exogenous inputs)

H is a matrix of coefficients for secondary energy inputs to technologies

Capacity Utilization

$$B_i x(t) \leq c(t) \quad , \quad i=1,2,\ldots,n \quad t=1,2,\ldots,11$$

where:

B_i are matrices defining load regions and the availability of technologies in the load regions, ?i = 1,2,...,n? (input data)

Build-Up Constraint:

$$z(t) \le \delta z(t-1) + g \quad , \quad t = 1,2,\ldots,11$$

where:

δ is a diagonal matrix of growth parameters (input)

g is a vector of startup values allowing z to reach positive values from zero

Build-Up Constraint:

$$\sum_{i \in I_1} z_i(t) \le GUB(t)$$

where:

$GUB(t)$ is a vector of absolute upper limits (input data)

I_1 is a subset of the set of technologies

Resource Consumption:

$$G\,r(t) \ge Q_1 x(t) + Q_2 z(t) - Q_3 z(t-6)$$

where:

G is a binary matrix which aggregates resource categories

Q_1, Q_2, Q_3 are matrices of parameters describing the specific consumption of resources by conversion technologies (input)

Resource Extraction

$$G_1 r(t) \le p(t) \quad , \quad t = 1,2,\ldots,11$$

where:

G_1 is a matrix for aggregating indigenous resource categories (input data)

p is a vector of annual production limits for each type of resource (exogenous inputs)