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CONTENTS

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Howard Kunreuther and Mark Pauly

I. INTRODUCTION

The effect of asymmetric information between buyers and sellers on product quality was first explored by Akerlof (1970) in his pathbreaking paper on the market for "lemons." He showed that if all purchasers have imperfect information on quality, then a market for the product may not exist, or if it does function it may not be efficient. These results have led to a number of papers concerning insurance and labor markets under different assumptions regarding how agents discriminate between "pro-

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ducts" of varying quality. (See Pauly 1974, Rothschild and Stiglitz 1976, Wilson 1977, Miyasaki 1977, and Spence 1978).

These treatments assume that all of the imperfectly informed agents have identical levels of knowledge of product quality. In contrast, this paper will consider situations where some agents learn over time about the quality of a particular good. However, this knowledge is private or agent-specific and may be costly for others to obtain. For example, firms may learn about the differential skills of their labor force by observing their productivity; other firms do not have easy access to this information. Insurance firms learn about the risk characteristics of their customers by observing claims records; they will not share these data costlessly with their competition. We are interested in characterizing the nature of the market equilibrium when agents have such private knowledge on the endowed qualities of a good. Our analysis is undertaken in the context of insurance markets, although it is applicable to those situations where sellers cannot easily communicate their product's special qualities to prospective buyers even though the current purchasers have at least partially observed these features.

The following problem is first analyzed in detail. Suppose that a set of customers has been with a specific insurance firm for t years, during which time the firm has collected information on their claims experience. The insurer naturally will not make these data available to other firms, and consumers are unable to furnish verified histories. Not having direct knowledge of each customer's risk class, the insurance firm utilizes claims data to set premiums. What is the schedule of profit-maximizing rates at which no customer will have an incentive to purchase insurance elsewhere in period $t+1$?

We consider two polar cases with regard to the assumption made about firm behavior. In one case, we assume that the firm has *no foresight*, so that it sets prices to make non-negative expected profits in every period. In the other, we assume that the firm has *perfect foresight*, in the sense that it maximizes the present discounted value of the expected profit stream over the planning horizon. We assume in each case that consumers choose the firm making the most attractive offer in the current period. In this paper, consumers do not have the foresight to consider the stream of premiums that will be charged in the current period and all future periods. This assumption therefore represents one polar case, with perfect consumer foresight models such as those of Dionne (1981), Radner (1981), and Rubinstein and Yaari (1980) at the other extreme.

We refer to the situation in which neither firm nor customers have foresight as *single- period equilibria*, since firms can change their price from one period to the next and consumers are free to stay or leave as they see fit. We refer to the situation in which firms maximize discounted expected profits but consumers choose only on the basis of current period premiums as *myopic multi- period equilibrium.*[2]

The paper is organized as follows. We first begin at the end, so to speak, by considering in Section II a static model in which the firm currently selling insurance to individuals has perfect private knowledge

[2]One of the purposes of experience rating is to cope with problems of moral hazard. This paper does not answer the question as to whether a premium adjustment process can eliminate or substantially reduce moral hazard. The paper also does not consider the possibility of requiring individuals to state their probability of loss and using experience to "punish" those who misstate (cf. Radner 1981, Rubinstein and Yaari 1980).

about each person's risk class. We show how the premiums charged are nevertheless constrained by other insurance firms. Section III develops a model in which firms use information from the claims experience of the insured in a Bayesian fashion to adjust individual premiums to experience. We show that in the single-period equilibrium model, the resulting premium schedule yields positive expected profits and monopoly distortions even if entry by new firms into the market is completely free. Profit or rate regulation would be a natural remedy if reality approximated this equilibrium. We further consider briefly the impact on the single period equilibrium of permitting customers to buy verified information on their experience. This would include purchase of data on premium classifications or claim records.

Section IV shows that in a myopic multi-period equilibrium, expected profits are zero with free entry, but price distortions remain. Premiums are generally below expected costs in the early periods, but eventually rise to exceed expected costs. The concluding section suggests applications and extensions of the analysis.

II. MARKET EQUILIBRIUM WITH ONE FULLY INFORMED FIRM

Our world consists of two types of consumers. Every consumer faces the possibility of an identical single loss (X) which is correctly estimated and which is independently distributed across individuals. Each consumer of type i has a probability of a loss Φ_i, $i=H,L$ for the high and low risk group respectively ($\Phi_H > \Phi_L$). The consumers correctly perceive these values of Φ_i. The proportion of high and low risk consumers in the population is given by N_H and N_L respectively. Type *i's* preferences are represented by a von Neumann-Morgenstern utility function, U_i, and each consumer determines the optimal amount of insurance to purchase by maximizing expected utility $E(U_i)$. The insurance industry consists of n firms, all of whom correctly estimate X and the average probability of loss correctly.

We initially assume that each consumer has been insured by the same firm for a sufficiently long period of time that the insurer has collected enough information through claim payments and other data to specify Φ_i exactly. The remaining $n-1$ firms in the industry *cannot* determine whether individuals insured by others are high or low risk people; an insured's past history does not become common knowledge.

If firms have information on the risk class of their clients they can charge differential premiums to high and low risk individuals; other firms in the industry are forced to charge the same premium to both groups because they cannot distinguish high risks from low risks. However, each firm does know how many periods the individual has been in the market, including whether he is a new customer.

Insurers' Potential Strategies

We now characterize the strategies available to insurers and consider the possibility of equilibrium. With regard to a particular client, it is useful to think of firms as either being "informed," i.e, having sold a policy to an individual in the previous period, or "uninformed," i.e., treating the client as a customer new to that firm. Each informed insurer offers a per-unit premium, P_i, to all individuals in risk group i, without specifying the amount of coverage, Q_i, which an individual may purchase, except that $0 \leq Q_i \leq X$. Insurers who are not informed about a set of individuals charge the same price to all of them. Consider first the situation of a representative uninformed firm. It knows that each consumer has the insurance demand curve:

$$Q_i^D = f(\Phi_i, P_i), \quad i = H, L \tag{1}$$

which is derived from constrained utility maximization. Since the uninformed firm cannot distinguish among risks, it will have to set $P_H = P_L = P$. In a free-entry world with firms that maximize expected profit $E(\pi)$, the breakeven premium P^* for such uninformed firms would be given by the lowest value of P such that:

$$E(\pi) = (P^* - \Phi_L)\, Q_L\, N_L + (P^* - \Phi_H)\, Q_H\, N_H = 0 \tag{2}$$

where Q_L is the total amount purchased by each L and Q_H is the total amount purchased by each H at the uniform premium P^*. If equilibrium exists, the low risk group will subsidize the high risk group and purchase partial coverage $Q_L < X$, while high risk individuals will purchase full coverage, $Q_H = X$, at subsidized rates.[3]

The informed insurer can use his exact knowledge of each present customer's Φ_i, $i = H,L$, to set rates tailored to each customer's experience. For high risk individuals, the informed firm will never charge less

[3]When the only value of P which satisfies (2) is $P^* = \Phi_H$, then $Q_L = 0$, and the market will only provide coverage to high risk individuals. This is a case of market failure, since low risk individuals cannot purchase insurance due to imperfect information by firms.

than Φ_H. For low risk individuals, the rate it will charge will depend on the premium charged by uninformed firms. The informed firm maximizes expected profits by charging low risk individuals a price P_L^I that is a little less than the uniform price offered by the uninformed firm to all purchasers of insurance. The informed firm then attracts all low risks, sells each of them Q_L units and makes profits of $(P_L^I - \Phi_L)\ Q_L^I$ per type L person.

If, for example, uninformed firms are charging $P^* + \delta$, the informed firm will want to charge its low risk customers the lower of one of two rates. It will either charge P^*, or it will charge $\hat{P}_L$, the premium which would maximize profits on low-risk insureds if the firm were a monopolist. At the other extreme, if uninformed firms are charging Φ_H to everyone, then the informed firm will charge either $\Phi_H - \varepsilon$ or $\hat{P}_L$, whichever is less.

But just as the informed firm's optimal pricing strategy depends on the strategies selected by uninformed firms, so does an uninformed firm's strategy depend on what the informed firm is doing. The strategic combinations and payoffs are shown as the payoff matrix, in Figure 1, with the upper expression in each labeled cell 1...4 being the payoff (profits) to the informed firm (I), and the lower expression the payoffs to the uninformed firm (U). When one type of firm obtains no business, and all customers purchase from the other type of firm, a profit level of zero is entered. Here we are assuming that both P^* and Φ_H are less than $\hat{P}_L$.[4]

[4] If $\hat{P}_L$ is less than P^*, then the informed firm will always charge $P_H^I = \Phi_H$ and $P_L^I = \hat{P}_L$ making positive profits. Uninformed firms will not obtain any business no matter what they do.

Uninformed Firm (U) / Informed Firm (I)	$P^U = P^* + \delta < \Phi_H$	$P^U = \Phi_H$
	1	2
$P_L^I = P^*$	$N_L Q_L\ (P^* - \Phi_L) > 0$	$N_L Q_L\ (P^* - \Phi_L) > 0$
$P_H^I = \Phi_H$	$N_H Q_H\ (P^* + \delta - \Phi_H) < 0$	0
	3	4
$P_L^I = \Phi_H - \varepsilon$	0	$N_L Q_L\ (P_H - \varepsilon - \Phi_L) > 0$
$P_H^I = \Phi_H$	$(N_L Q_L + N_H Q_H)(P^* + \delta) - N_L Q_L \Phi_L - N_H Q_H \Phi_H > 0$	0

Figure 1. Payoff Matrix for Informed and Uninformed Firms.

Stability of Equilibrium

We will now show that there is no stable Nash equilibrium where there are both fully informed and uninformed firms. The argument is simple. If U (uninformed) firms chose $P^U = \Phi_H$, then I (informed) should choose $P_H^I = \Phi_H - \varepsilon$ to maximize profits. But if I chooses $\Phi_H - \varepsilon$, there exists some smaller $P^* + \delta$ at which U can make positive profits, while I gets no business and makes zero profit. But if U charges some $P^* + \delta$, I should charge a little less (e.g., P^*). Then I makes positive profits, but U suffers a loss. To prevent this loss, U must charge at least Φ_H. But then I should charge $\Phi_H - \varepsilon$, etc. If there are many players, the absence of a Nash equilibrium makes stability unlikely.[5]

[5]Note that, from the viewpoint of a single uninformed firm, the maximum value that δ can take in cell 3 in Figure 1 depends on what the firm assumes that the other uninformed firms will do. If they continue playing strategy, Φ_H, then the single uninformed firm can charge anything less than $\Phi_H - \varepsilon$ and capture all the business with a large profit. If each uninformed firm assumes the other uninformed firms will match its prices, then profits will be lower.

What other concepts of equilibrium might apply here? If both parties followed maximin strategies, the outcome would be in cell 2, with the strategy $\{P_L^I = P^*, P_H^I = \Phi_H\}$ for the informed firm, and $\{P^U = \Phi_H\}$ for the uninformed firm. In this cell, the uninformed firm is sure that it will not lose money (although it will not make profits either). The informed firm guarantees itself positive profits. Thus, in a single-play context, or with a small number of players, we might expect the outcome to be in cell 2.

Another possibility, already used in the literature on insurance markets and imperfect labor markets, is the concept of Wilson Equilibrium.[6] A given set of actions is a Wilson equilibrium if no firm can alter its behavior (i.e., propose a different premium) that will (a) earn larger positive profits immediately, and (b) continue to be more profitable after other firms have dropped all policies rendered unprofitable by the initial firm's new behavior. Is the pair $\{P_L^I = P^*, P_H^I = \Phi_H\}$ and $\{P^U = \Phi_H\}$ a Wilson equilibrium? The alternative strategy for the informed firm is to set $\{P_L^I = \Phi_H - \varepsilon, P_H^I = \Phi_H\}$. This earns it larger profits and does *not* cause the uninformed firms to lose money if they maintain their same policy as before. However, an informed firm's charging $\{P_L^I = \Phi_H - \varepsilon\ P_H^I = \Phi_H\}$ would permit uninformed firms to make positive profits by switching to $P^U = P^* + \delta$; this change reduces the informed firm's profit to zero. Thus, if we substitute the notion "rendered less profitable" for "rendered unprofitable" in part (b) of the above definition, then cell 2 does qualify as a Wilson equilibrium.

[6] It was proposed by Wilson (1977) and has been utilized by, among others, Miyasaki (1977), and Spence (1978), to characterize equilibrium.

An alternative equilibrium concept which leads to the same conclusion is based on a Stackelberg leader-follower model. It seems reasonable to suppose that the (single) informed firm will play the leadership role. We will assume that the informed firm always sets $P_H^I = \Phi_H$. The reaction function for the uninformed firm is $P^U = f\ (P_L^I)$, and the informed firm therefore maximizes its expected profit (Π^I):

$$\Pi^I = g\ \left[P_L^I, f\,(P_L^I)\right] \tag{3}$$

If the informed firm sets $P_L^I = P^*$, then $P^U = f\ (P_L^I) = \Phi_H$, and the informed firm makes positive profits of $Q_L\ (P^* - \Phi_L)$ on each type-L person. If the informed firm sets $P_L^I = \Phi_H - \varepsilon$, then $P^U = f\ (P_L^I) = P^* + \delta$, and Π^I is zero. Hence, maximization of (3) requires P_L^I to be P^*, and the Stackelberg equilibrium is given by cell 2.

To summarize, there are two conclusions based on the above discussion:[7]

(1) No single-period equilibrium exists, or

(2) A single-period equilibrium is represented by $\{P_L^I = P^*, P_H^I = \Phi_H\}$ for informed firms, $\{P^U = \Phi_H\}$ for uninformed firms, with all business going to informed firms.

In what follows, we adopt the second conclusion by assuming that the informed and uninformed firms behave in a Stackelberg fashion, with the informed firm as the leader and the uninformed firms as the followers. This equilibrium is also achieved if one assumes that either firm follows a policy that maximizes the minimum profit they could attain no matter

[7] We have not considered the possibility of mixed strategies.

what uninformed firms did, or that the modified definition of a Wilson equilibrium is appropriate.

Welfare Effects

In the no-information case, the equilibrium premium is P^* for both high and low risks. Compared to the no-information case, perfect private knowledge for just one firm leads generally to no gain in welfare for any insured person. All of the gains from information go to informed insurance firms as positive long-run profits. In the special case where low risk individuals are charged the monopoly price (i.e., $P_L^* = \hat{P}_L$), the low-risk class benefits by the amount that the premium is below P^*. Even then, the higher risk consumers are made unequivocally worse off with perfect private knowledge, since the price they pay increases from P^* to Φ_H. Moreover, the positive profits being earned by informed firms are not eroded by entry, since new firms are by definition uninformed ones.

III. INFORMED FIRMS: LEARNING OVER TIME

Nature of Equilibrium

We now turn to the more general case where firms learn over time about the characteristics of their customers through loss data. Initially each firm only knows from statistical records that the proportion of high and low risk individuals in the insured population is given by N_H and N_L respectively. It does not know whether an individual is in the H or L class but does know how many periods each potential customer has been in the market (e.g., all 20 year old males are assumed to have been driving legally since age 16).[8] Any new customer would be offered a premium, P^*, which is defined as before so that

$$E(\pi) = N_L(P^*-\Phi_L)\,Q_L + N_H\,(P^*-\Phi_H)\,Q_H = 0 \tag{4}$$

That is, the insurer prices so as to yield expected profits of zero on all new business.[9]

During each time period, we assume that an individual can suffer *at most* one loss, which will cause X dollars damage. Any time a claims payment is made, this information is recorded on the insurer's record and a new premium, which reflects his overall loss experience, is set for the next period. As before, we are assuming that informed firms do not disclose their records to other firms. Individuals who are dissatisfied with their new premium can seek insurance elsewhere. Other firms will not have access to the insured's record and hence cannot verify whether an applicant has had few or many losses under previous insurance contracts.

The informed firm uses a Bayesian updating process in readjusting its premium structure on the basis of its loss experience. Consider all customers who have been with the same insurance company for exactly t periods. They can have anywhere from 0 to t losses during this interval. The premium charged for period $t+1$ to individuals with j losses during a t period interval is $P^*_{jt}, j = 0 \cdots t$.[10] Firms with loss experience data

[8] In this sense, a firm can distinguish between new arrivals to the market and customers formerly insured by other firms.

[9] This seems to be the rule that actuaries are instructed to follow in an experience rating context. For example, the premium in any one year is supposed to be the previous year's premium plus a "bonus" G, where G is defined as:

$$G = k[(1+l)P - \xi] \qquad G \geq 0$$

and P is the expected value of losses, ξ is the actual amount of loss in the previous period, l is the "safety loading" (including normal profit) and k is a fraction less than one. In the initial period when $G = 0$, actuaries will recommend that the premium equal $(1+l)P$ (See Beard, Pentikainen, and Pesonen 1979).

[10] We are assuming that losses for an individual are independent of previous experience so the premium at the end of t is determined only by the number of claims.

will set each premium P^*_{jt} so that they maximize expected profits, subject to the constraint that customers remain with them. Let w^L_{oo} and w^H_{oo} be the respective probabilities that an individual is in the low and high risk class when the firm initially insures him. We can update these probabilities by using Bayes' procedure. If a customer has suffered exactly j losses in a t period interval then we define w^i_{jt}, $i = L,H$ as the probability that he is in the i^{th} risk class, where $w^H_{jt} + w^L_{jt} = 1$.[11] The premium set for each loss classification will also be determined in part by the relative values of $w^i_{jt}, i = L,H$. As j increases so does the probability that the individual is in the high risk class. Hence, $w^H_{jt} > w^H_{j-1,t}$, $j = 1, \ldots, t$.

Suppose, for example, an informed firm offers a set of premiums $\{P'_{jt}\}$, with P'_{jt} increasing as j increases.[12] An uninformed firm which charged a lower premium than P'_{jt} in any period would attract all customers with j or more losses.[13] The proportion of high and low customers in its portfolio would be given by

$$W^i_{jt} = \frac{\sum_{k=j}^{t} w^i_{kt} \, s_{kt}}{\sum_{k=j}^{t} s_{kt}}$$

where s_{kt} = probability of a person suffering exactly k losses in a t period

[11]We determine w^i_{jt} as follows. Let λ^i_{jt} = probability that an individual experiences j losses in t periods, if he is in risk class i. Specifically,

$$\lambda^i_{jt} = \frac{t!}{(t-j)!\,j!} (\Phi_i)^j \quad (1-\Phi_i)^{t-j} \quad .$$

Using Bayes formula

$$w^i_{jt} = \frac{\lambda^i_{jt} N_i}{\sum_i \lambda^i_{jt} N_i} \quad .$$

[12]We will show below that P_{jt} increases as j increases.

[13]We are assuming no transaction costs for insured individuals to switch firms.

interval. In other words, W_{jt}^{i} is a weighted average over the loss range $j \cdots t$. Since w_{jt}^{H} increases with j we know that $W_{jt}^{H} > w_{jt}^{H}$ for all $j = 1 \cdots t-1$ and $W_{tt}^{H} = w_{tt}^{H}$.

The minimum premium $(P_{jt}^{''})$ at which expected profit equals zero for uninformed firms is given by:

$$W_{jt}^{H} (P_{jt}^{''} - \Phi_H) \, Q_{jt}^{H} + W_{jt}^{L} \, (P_{jt}^{''} - \Phi_L) \, Q_{jt}^{L} = 0 \tag{5}$$

where Q_{jt}^{i} is demand for group i given a premium $P_{jt}^{''}$. We know that $P_{jt}^{''}$ increases with j since W_{jt}^{H} increases with j. Hence any new firm which sets $P = P_{jt}^{''}$ attracts only customers with j or more losses and makes zero expected profits. So (5) correctly describes the minimum level of premiums that uninformed firms can charge.

If the informed firm sets $P_{jt} = P_{jt}^{''} - \varepsilon$ for *only* those customers who have suffered exactly j losses, then these individuals will still prefer the informed firm. Its expected profits are given by

$$E(\Pi_{jt}) = w_{jt}^{H} \, (P_{jt} - \Phi_H) \, Q_H + w_{jt}^{L} \, (P_{jt} - \Phi_L) \, Q_L \geq 0 \tag{6}$$

For sufficiently small ε, expected profits in (6) are positive for $j = 1 \cdots t-1$ since w_{jt}^{H} is less than W_{jt}^{H}. For $j = t$, as $\varepsilon \to 0$, expected profits by definition will also approach zero since $w_{tt}^{i} = W_{tt}^{i}$.

To determine the premium structure, an informed firm will also have to find the monopoly premiums $\{\hat{P}_{jt}\}$ for each $j = 0 \cdots t$, which maximize $E(\Pi_{jt})$. It will maximize expected profits for each loss category if it then sets premiums (P_{jt}^{*}) as follows

$$P_{jt}^{*} = \min \{P_{jt}^{''} - \varepsilon, \hat{P}_{jt}\}, \quad j = 0 \cdots t$$

The structure of the premiums is thus similar to that in the case of perfect private knowledge outlined above; profits will be lower because firms must now use claims information to categorize their customers and hence will misclassify some of them. Aggregate expected profits for each period t are given by

$$E\ (\Pi_t) = \sum_{j=0}^{t} s_{jt}\ E\ (\Pi_{jt}) \tag{7}$$

An Illustrative Example

A two-period example using a specific utility function will help to illustrate the meaning of learning from loss experience. The appendix describes the basic form of this problem for the exponential utility function $U(y) = -e^{-cy}$. Consider the specific case where $\Phi_H = .3$, $\Phi_L = .1$, $X = 40$, $c = .04$, and $N_L = N_H = .5$. Then the equilibrium premium in the first period, obtained by solving equation (4), would be $P^* = .254$. Table 1 illustrates how one calculates the weights for determining the optimal premium structure at the end of period 1 when $j = 0$ *or* 1, and Figure 2 details the optimal rate structure at the end of period 1.

The optimal premiums are $P_{01}^* = .254$ and $P_{11}^* = .288$ since $\hat{P}_{01} = \hat{P}_{11} = .495$. The premium charged to the group suffering one loss (P_{11}^*), yields $E(\Pi_{11}) = 0$ since $P_{11}^* = P_{11}''$, and $w_{11}^H = W_{11}^H$. Expected profits for the "zero loss" class is given by (6) and is

$$E(\Pi_{01}) = .5625\ (.254-.10)\ 12 + .4375\ (.254-.30)\ 40 = .23.$$

Aggregate expected profits for period 1 are given by (7) and in this case are $E\ (\Pi_1) = .8(.23) = .18$.

Table 1: Calculation of Weights w^i_{j1} and W^i_{j1} $i = L, H$ for Two Period Model.

$\Phi_L = .1$ $\Phi_H = .3$ $X = 40$ $c = .04$ $N_H = N_L = .5$

j	s_{j1}	w^H_{j1}	w^L_{j1}	W^H_{j1}	W^L_{j1}
0	.8	.4375	.5625	.5	.5
1	.2	.75	.25	.75	.25

$$s_{01} = (1 - \Phi_H)(N_H) + (1 - \Phi_L)\, N_L$$

$$s_{11} = (\Phi_H)(N_H) + \Phi_L N_L$$

$i = L, H$:

$$w^i_{01} = \frac{(1 - \Phi_i)\, N_i}{s_{01}}$$

$$w^i_{11} = \frac{\Phi_i N_i}{s_{11}}$$

$$W^i_{oo} = \frac{\sum_{k=0}^{1} w^i_{k1} s_{k1}}{\sum_{k=0}^{1} s_{k1}}$$

$$W^i_{11} = w^i_{11}$$

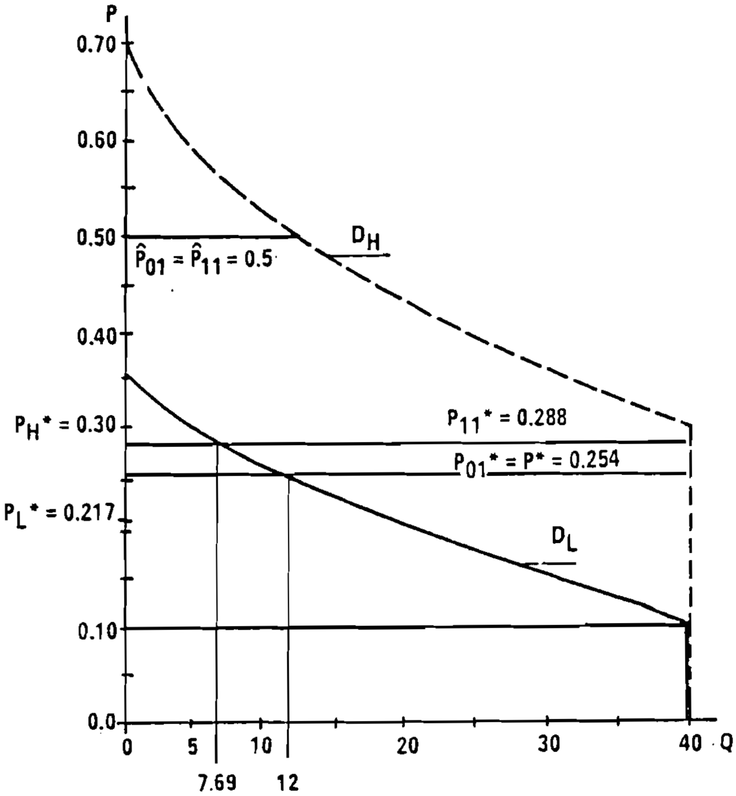

Figure 2. A Two Period Example Based on Loss Experience.

What effects do experience rating have on consumer well-being in this example? In the absence of any information, both high and low risks would have been charged .254 in each of the two periods. When information is obtained through experience, those individuals with no losses are charged the same rate as initially, .254, but the others with one loss are charged .288. Thus both high and low risk customers are made either no better off or worse off if firms can generate information. In contrast, if the firm would have charged breakeven prices, its premium would have been .237 to individuals with zero losses.

As a customer's life with the company increases, then he faces a larger number of rate classes reflecting possible outcomes. Firms make the largest profit on those insured individuals who experience the fewest losses. In the limit as $t \to \infty$, all customers will be accurately classified and we have the case of perfect private knowledge. Figure 3 graphically depicts how aggregate expected profit changes over time as a function of the proportion of low risk customers in the population. As N_L decreases then the informed firm's profit potential decreases since a larger proportion of individuals will suffer losses.

Obtaining Verified Information

The problem in achieving optimality arises, of course, because informed firms--the ones from which the consumer is currently purchasing--price so as to obtain positive long-run expected profits. A natural response of low risk consumers facing such a situation is to seek some way of providing reliable information on their status to other insurance firms. There are two ways in which such data might be

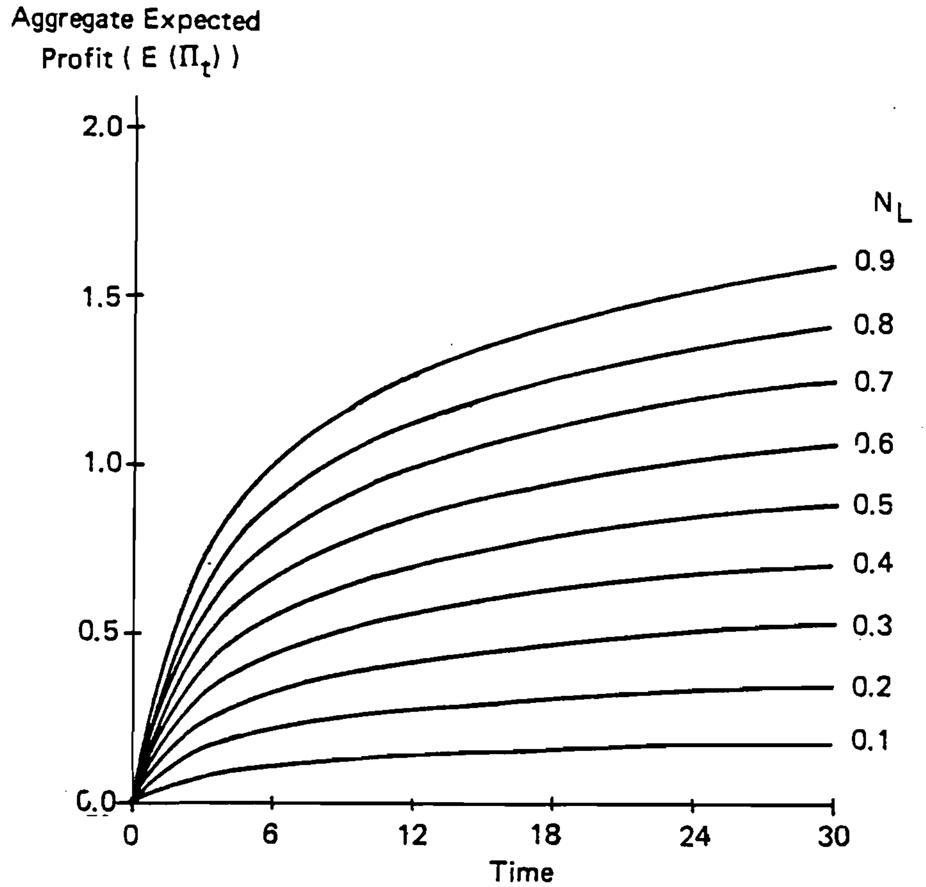

Figure 3. Aggregate Expected Profits $[E(\Pi_t)]$ as a Function of Proportion of Low Risk Customers (N_L) and Time (t).

disseminated: (1) Consumers might provide verified information on their actual number of losses (claims), and/or (2) Consumers might provide verified information on the size of their premium bill for a given level of insurance, since this is a perfect indicator of the risk class into which they are being placed by their current insurer.

We would expect that consumers will find it difficult and costly to undertake either of these actions. For one thing, the current insurer has an incentive to conceal its claims and premium data. For another thing, purchasers of insurance who have had unfavorable loss experiences may try to represent themselves as being a better risk by using techniques such as bogus invoices, or applying for insurance right after an accident but before a new bill is issued. Note that the informed firm will not discourage these actions, because such behavior makes it more difficult for customers with good experience to communicate their status reliably.

The cost of providing reliable information by insured individuals will still permit the original insurer to earn some positive profits and the above models would still be relevant in determining what rate structure could be set by informed firms. One could formally incorporate the costs of communicating verified information into a more general model of the choice processes of insurers and insured. Profits would then be limited by the alternatives available to consumers for purchasing verified information.

IV. A MYOPIC MULTI-PERIOD MODEL

We now investigate the consequences of changing the assumption that there is no insurer foresight. We consider a model in which firms look beyond current period losses to potential future profits. Firms are therefore assumed to be concerned with the present discounted value of the profit stream they expect to earn. But purchasers are still assumed to be myopic, in the sense that they choose which insurer to patronize by looking only at current period prices and selecting the firm with the lowest current premium. If the firm is willing to tolerate negative expected profits for a while in order to attract customers and observe their claims experiences, it can then use this information to make positive expected profits in the future to offset (in present value terms) the initial losses.

It is easy to see that the "single-period" premium schedule $\{P_{jt}^*\}$ may then not be an equilibrium. On the one hand, a firm that charged less than P_{00}^* in the initial period would have an expected loss in that period; on the other hand, it would have the opportunity to observe which individuals did and did not have losses during that period. If it used that information to charge the schedule P_{jt}^* in subsequent periods, the present discounted value of the profit stream associated with this pricing policy could be sufficient to offset the initial expected losses. Hence, a new schedule, with the lower P_{00}, would dominate the single-period equilibrium schedule.

What new set of premiums would represent an equilibrium schedule? It would be one where, for all t and j, there would be no opportunity for a previously uninformed firm to enter and earn positive expected profits. To simplify the explanation of how this schedule is derived, we assume an interest rate of zero, so as not to be concerned with discounting. We assume that the firm which has attracted a customer in period 0 will want to set its premiums for all future periods up to the end of the planning horizon T so that no firm entering the market in later periods can attract any set of its customers and make a stream of profits whose sum is positive. That is, it will want to set P_{jt}^{**} so that

$$E\left(\pi_{jt}^{**}\right) = \sum_{k=t}^{T}\left[w_{jk}^{L}\left(P_{jk}^{**} - \Phi_L\right) Q_{jk}^{L} + w_{jk}^{H}\left(P_{jk}^{**} - \Phi_H\right) Q_{jk}^{H}\right] = 0$$

for all j. Here P_{jt}^{**} is also the price that the new entrant would charge.[14]

[14]We will assume that purchasers buy all of their insurance from a single firm. Alternatively, we could have assumed that each firm receives a constant share of an insured's business in every period, and that all firms are aware of this fact.

The procedure in constructing a set of premiums $\{P_{jt}^{**}\}$ requires one to start at period T and work backwards. Any uninformed firm who enters the market at the beginning of period T must break even, because there is by definition no future period in which losses can be recouped. Hence, $P_{jT}^{**} = P_{jT}^{*}$ for all j. Now consider period $T - 1$. If a firm entered in this period it could observe the experience of its customers for one period and make profits on on all those individuals who did not have a loss during this period.

The expected profits in period T are derived using the same type of Bayesian updating procedure described in Section III. In order to prevent new entrants from coming into the market in period $T - 1$, the informed firm must set its premium in period $T - 1$ sufficiently low so that a potential new entrant would suffer a loss just a little larger than the profit he would earn in period T. As in the single period equilibrium model, there will be a different premium for each value of j. This set of policies $\{P_{j,T-1}^{**}\}$ would then be the equilibrium schedule for the fully informed firm.

The same type of reasoning is utilized to compute the equilibrium set of premiums for period $T-2$. In this case a potential entrant who attracts customers can make profits in periods $T-1$ and T by utilizing claims information on their insured population. The informed firm will then have to set $\{P_{j,T-2}^{**}\}$ at levels which erase all these potential profits of a new firm. The same process is repeated sequentially for all periods through $t = 0$.

To illustrate differences between resulting premiums in the single period equilibrium and myopic multi-period equilibrium cases we consider an example with $T = 5$. Table 2 compares the set of premiums and expected profits for the two models. In the single period equilibrium model the informed firm's premium (P^*_{00}) starts off equal to the average actuarial value ($P^* = .254$) and increases above this level for customers who experience losses. In the myopic case, the initial premium, P^{**}_{00}, is less than P^*, and increases over time whether or not the person suffers a loss.[15] As t approaches T, the premiums for the two types of equilibria converge as expected. In the single-period case, the stream of profits is positive, in all periods; in the multi-period myopic case the firm suffers losses in the early periods recouping them in later periods so that the expected stream of profits is zero.

Table 2 reveals that there is a perversity and allocative inefficiency in the multi-period myopic case. Consumers are undercharged in the early periods but will find that their premiums are raised even if they are accident free. Persons nearing the end of their risk horizons (e.g., the aged who will only be driving for a few more years) will tend to be overcharged for insurance, whereas the young will tend to be undercharged. Hence, consumers will tend to over-purchase insurance in the early periods, and under-purchase insurance in the later periods. If regulation could be used to bring premiums closer to the actuarial values, there would be a welfare gain.

[15]It is theoretically possible for consumers initially to be charged a negative premium to attract them to the insurance company so that they could be charged higher premiums as t increases. In this case, individuals could be given a free gift for taking out insurance, in an analogous fashion to the approach used by savings banks to attract new accounts.

Table 2. Comparison Between Premiums and Expected Profits for Single Period Equilibrium and Myopic Multi-Period Equilibrium Schedule for Five Period Problem.

Single Period Equilibrium

Period t / Number of losses j		0	1	2	3	4	5
0	P^*_{0t}	.254	.254	.254	.254	.254	.254
	$E(\pi^*_{0t})$	0	.18	.30	.36	.39	.39
1	P^*_{1t}		.288	.286	.285	.283	.281
	$E(\pi^*_{1t})$		0	.02	.04	.08	.12
2	P^*_{2t}			.296	.296	.295	.294
	$E(\pi^*_{2t})$			0	.00	.01	.01
3	P^*_{3t}				.299	.299	.298
	$E(\pi^*_{3t})$				0	.00	.00
4	P^*_{4t}					.30	.30
	$E(\pi^*_{4t})$					.00	.00
5	P^*_{5t}						.30
	$E(\pi^*_{5t})$						0
$E(\pi^*_t)$		0	.18	.32	.40	.48	.52

Myopic Multi Period Equilibrium

Period t / Number of losses j		0	1	2	3	4	5
0	P^{**}_{0t}	.191	.225	.245	.252	.254	.254
	$E(\pi^{**}_{0t})$	−1.21	−.15	.24	.35	.38	.39
1	P^{**}_{1t}		.250	.280	.284	.283	.281
	$E(\pi^{**}_{1t})$		−.21	−.03	.04	.08	.12
2	P^{**}_{2t}			.286	.295	.295	.294
	$E(\pi^{**}_{2t})$			−.02	− .00	.01	.01
3	P^{**}_{3t}				.299	.299	.298
	$E(\pi^{**}_{3t})$				.00	.00	.00
4	P^{**}_{4t}					.30	.30
	$E(\pi^{**}_{4t})$					.00	.00
5	P^{**}_{5t}						.30
	$E(\pi^{**}_{5t})$						0
$E(\pi^{**}_t)$		−1.21	−.36	.19	.39	.47	.52

V. CONCLUSIONS AND EXTENSIONS

Our results have some important implications for the notion that development of "reputations" for quality can over time alleviate the problem of agent ignorance (Akerlof 1970). In both of our models, there is an incentive to keep information about quality private, even if the explicit cost of communicating it to others is low. In our first model, the agent with private knowledge loses monoply rents by communicating this information to others. In the second model he earns no rents in the long run; however, he would impose losses on himself if he initially followed the loss-leader strategy but then communicated the information he had gained from claims experience before he had time to recoup his losses. Conversely, even if he "promised" in the initial period to communicate the truth, he would always gain from concealing or corrupting information which identifies the good (high quality) risks. These disincentives to communicate reliable information are greater if the item being transacted is bought in lumpy amounts, as in the case of a worker's services; the employer who learns which of his employees are of higher productivity will be downright reluctant to communicate that knowledge to other potential employers.

While "friends and neighbors" do sometimes communicate information about product quality (good restaurants, good doctors) and while employers do write letters of reference for good employees, our models suggest that such behavior is not likely to occur in all circumstances. Even where concealing private information ends up benefiting none and harming all, it will still be difficult for the market to break away from such an equilibrium.

The most obvious extensions of these models is to permit consumers to be less myopic. If consumers do have foresight, then they may want the insurer to agree in the initial (purchase) time period to provide accurate information on future loss experience. The mere availability of such information would, in itself, be sufficient to eliminate either monopoly profits or the zero-profit distortions in the multi-period myopic model.

Guaranteeing that such information is provided is not easy, of course, since the low risks must not only ensure that accurate information on his own experience is provided but also that accurate (unfavorable) information is provided on the experience of those high risks who are thinking of switching to another firm. That is, he must monitor the accuracy of all information provided. For example, in a labor market application of our theory, a high productivity worker must not only verify that his employer will provide him with a good and true recommendation; he must also verify that poorer quality workers are being furnished bad recommendations or references.

If the worker has foresight, it is easy to see that he will be concerned, in the initial period, with the *schedule* by which his future premiums will be adjusted as a result of his future experience. Different risk types might be expected to select different schedules and Dionne's work (1981) shows that it is possible in a monopoly context to find schedules which separate these groups *ex ante* when each risk type chooses the schedule which maximizes its utility. But as Dionne remarks, it is not obvious that these schedules will be sustainable if persons with unfavorable experience can switch from firm to firm without being compelled to provide a valid history of their experience. That is, if a high risk's history does not necessarily "follow" him from firm to firm, optimal equilibrium may not be sustainable.

There needs to be further development of the theory for such consumer-foresight models. At the same time, there needs to be further empirical verification of the degree of foresight individuals actually display. Do purchasers of automobile insurance know and fully take account of the way their premiums will vary with their claims history? Do workers know and take into account the way their future wages will vary with observed productivity? If consumers display only limited foresight, the models of market equilibrium developed in this paper will be appropriate.

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APPENDIX A *

Risk averse consumers of each type i with wealth, A_i, want to choose a value of Q_i given Φ_i and P_i which maximizes:

$$E\,[U_i(Q_i)] = \Phi_i\; U_i[A_i - X + (1-P_i)\;Q_i] + (1-\Phi_i)\;U_i\;(A_i - P_i\;Q_i) \qquad \text{(A.1)}$$

subject to

$$0 \leq Q_i \leq X \quad .$$

Let R_i be the contingency price ratio

$$R_i = \frac{P_i(1-\Phi_i)}{(1-P_i)\Phi_i}$$

and define R_i^{max} and R_i^{min} as the values of R_i where $Q_i = 0$ and $Q_i = X$ respectively when one maximizes $E\,[U_i(Q_i)]$ without any constraint on Q_i.

*A more detailed discussion of this model appears in Kunreuther (1976).

Then if

$$U_i' = \frac{dU_i}{dQ_i} > 0 \text{ and } U_i'' = \frac{d^2U_i}{dQ_i^2} < 0$$

the optimal solution to (A.1) is given by:

$$\begin{array}{lll} Q_i = 0 & \text{if} & R_i \geq R_i^{\max} \\ R_i = \dfrac{U_i'(A_i - P_i Q_i)}{U_i'(A_i - X + (1 - P_i) Q_i)} & \text{if} & R_i^{\min} < R_i < R_i^{\max} \\ Q_i = X & \text{if} & R_i \leq R_i^{\min} \end{array}$$

Whenever $P_i \leq \Phi_i$, then $Q_i = X$, since in this range the premium is either actuarially fair or subsidized. Suppose both consumer types have identical utility functions given by the exponential $U_H(Y) = U_L(Y) = -e^{-cY}$, where c is the risk aversion coefficient. Then Q_i is determined by

$$\begin{array}{lll} Q_i = 0 & \text{if} & (\ln R_i) \geq cX \\ Q_i = X - (\ln R_i)/c & \text{if} & e^{cX} > R_i > 1 \\ Q_i = X & \text{if} & R_i \leq 1 \end{array}$$