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Integrated Economic Balance of a Regional System

Issaev, B. and Umnov, A.

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INTEGRATED ECONOMIC BALANCE OF A REGIONAL SYSTEM

Boris Issaev
Alexander Umnov

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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

PREFACE

The present paper contains a description of an integrated economic flows balance adapted to the needs of analyzing dependencies between structural elements of a regional system and factors of national or international environment.

The balance reveals identity relations between main economic indicators which jointly give an exhaustive enough picture of the economic aspect of a regional system. The classification of the balance presented in the paper includes all variables at a regional level which are used in models of separate economic processes developed for the Tuscany region in Italy, and it fulfills the function of a common data base and informational linkage of models. The balance may also be interpreted as a closed, static flow model of the regional system within a national one.

In the second part of the paper the procedure is set out how to use the balance as a model which allows one to find the values corresponding to identities under different constraints on separate elements of the integrated economic balance. In this quality (capacity) the balance may be used as a tool for functional linkage of specific models describing the economic aspect of regional development.

Boris Issaev
Leader
Regional Development
Group

Laxenburg, March 1982

INTEGRATED ECONOMIC BALANCE: ADJUSTMENT PROCEDURES

B. Issaev
A. Umnov

INTRODUCTION

In the national division of labor economic regions play a role similar to that of countries in the world economy. Within a comprehensive national system, they may be considered as subsystems which contain in a framework of determined space all essential elements of the whole system. Any regional subsystem is open in respect of the national one and interacts with the latter through the exchange of some of the objective elements inherent to regional and national (including the "rest of world") systems. The open regional subsystem may be considered as closed if the "external" or national part is explicitly included into the regional system's model.

In the case of a region, for instance, Tuscany in Italy, it is highly important practically to see the region as an integrated unit, which responds to national and international influences as an economic as well as a physical system. What would be the region's balance of payments or balance of claims and liabilities; what flows of goods, labor, and capital would be affected by external and internal events or policy factors; what would be the behavior of the regional system on the basis of existing economic mechanism of its functioning within the country — all these questions may be answered only after joint consideration of all economic processes in the region and in the region's relation with the outside world. A specific tool of analysis is needed for this purpose — an integrated economic balance (IEB). The analytical function of this instrument in respect of the region as a socio-economic system would be the same as Leontiev's input-output model in the respect of a physical production system consisting of a set of product balances. The input-output table of Leontiev links together outputs and inputs of specific goods. Integrated economic balance should link "outputs" and "inputs" of all elements and activities in the regional-national system to the extent to which they enter into the economic turnover of the system.

PART I. INTEGRATED ECONOMIC BALANCE OF A REGIONAL SYSTEM

1.1. APPROACH

A regional system within a country includes in principle all elements of the national economy:

- -- all physical and value flows in the process of reproduction (output and consumption of goods and services; generation, distribution, and final use of national income; accumulation of the product; supply and use of capital; supply of money and all attributes of their functioning--prices, interest rates, wages, etc.; stocks of capital in real and value terms; accumulated wealth, etc.);
- -- all components of the natural environment (natural resources and conditions);
- -- all components of socio-demographic patterns (population and its reproduction, settlement system, public services, labor resources, and labor market, etc.);
- -- elements of country's managerial structure (decision-making centers as part of the national organization of socio-economic management.

A comprehensive model of a region as a socio-economic subsystem should thus be formed from the set of elements common for the whole national system and should reflect for these elements the actual intraregional and "extra" regional links. The problem is how to link physical values (tons, number of persons, capacities, etc.) into an integrated "economic" system, for which the unity between physical characteristics and social value of elements in the process of reproduction is essential.

Two basic principles of approach may help to resolve the problem:

1. to consider all processes and events in socio-economic reality from the point of view of money (in all forms);
2. to link flows (and stocks) of money in the model through explicit indentification of payers and receivers (or creditors and debtors), as it is usually done in the national accounting.

The first principle provides the common unit of measurement for all elements of the "economic" system and allows to integrate in value terms everything which is involved in the production of goods and services, generation and use of income, use of and reproduction of all kinds of resources. It also implies prices and other similar measures of equivalence relations between physical elements.

The second implies the interpretation of the regional economy as a multi-aspect closed system, which formally may be described by an oriented Euler graph, in which the nodes are the system's elements, the edges are flows of money related to economic acts in the process of reproduction. The following simplified schemes illustrate the above approaches (Figures 1 and 2).

Presentation of economic relations between productive sectors, households, state, and the "rest of world" through flows of values allows to construct a closed system in which there are circuits of edges which link each node with any of the others and the value of incoming edges in each node is equal to the value of outgoing ones. This could not be achieved if only physical flows were considered.

Systems of this type may be presented in the form of a set of balances linked through a double recording of each quantity as a resource of one and an expenditure of the other

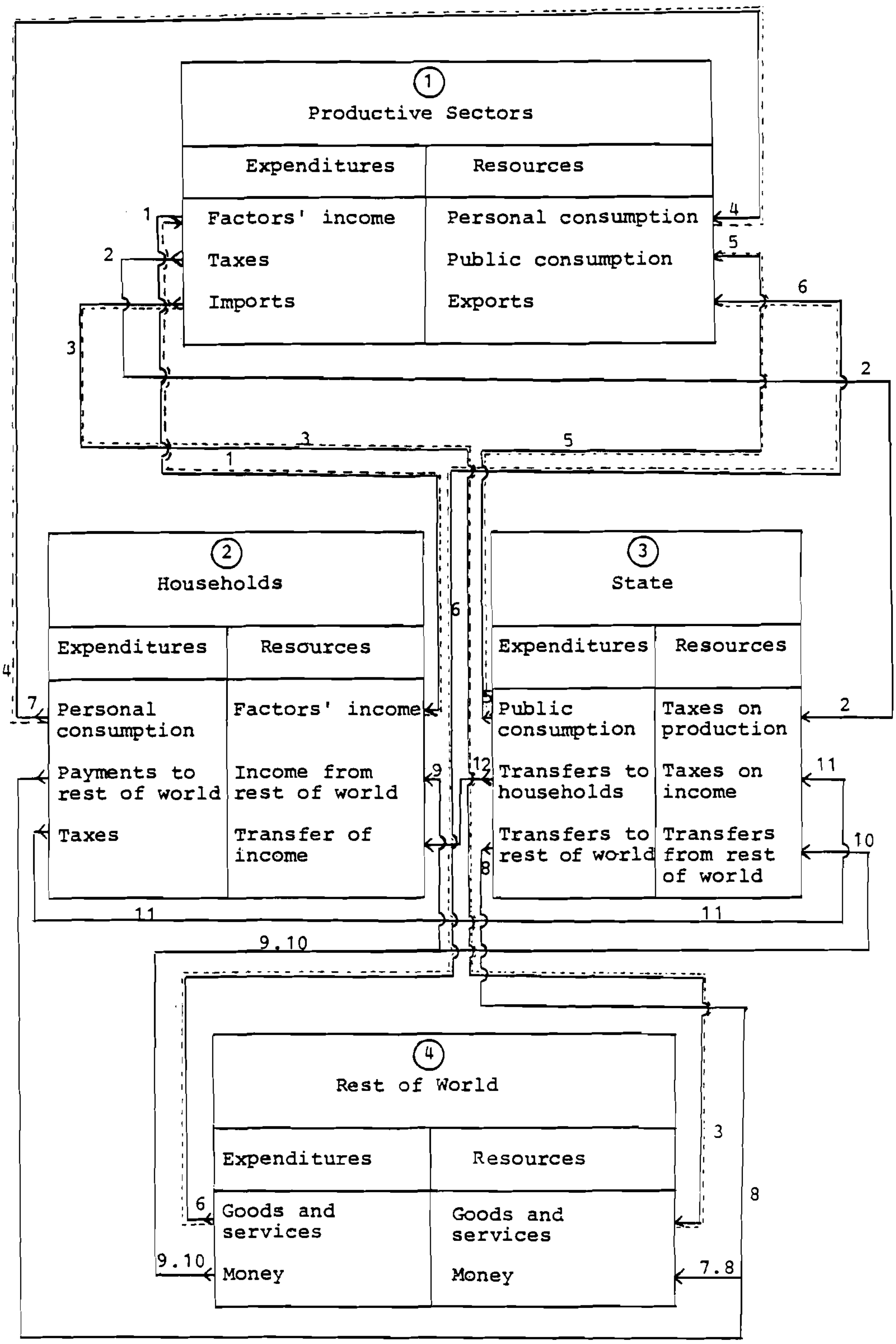
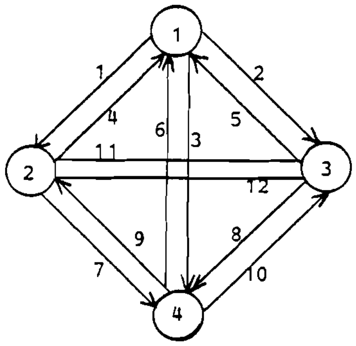
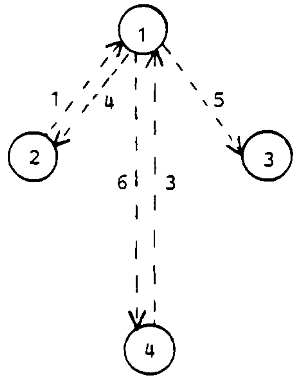


Figure 1. Integrated Economic Balance.



Flows of Values



Physical Flows

Figure 2. Graphs of economic flows in value and quantity.

balance (Figure 1) in the form of a graph in which each node corresponds to a specific balance (or account), as a system of node identities and in the form of a square matrix in which each row pair and column crossing on a main diagonal corresponds to a specific balance.

The model of integrated economic balance in this presentation is static. It reflects the principle of constant flows of values. But it may also be applied as a description of dynamics in a cinematic way and as a sequence of static balances in which the value of some nodes in each balance is determined through the value of the same or some other nodes in the preceeding (or in the sequence of preceeding) balances. In a more explicit way dynamics might be reflected if stocks of funds (in real and value terms) at the beginning and at the end of each year were included into the scheme of integrated economic balance (Figure 3). The sequence of yearly balances for a period embracing many years also may be presented in the form of an Euler graph or square matrix.

To construct an IEB for a real regional system means to determine nodes adequately reflecting the composition of the system in its different aspects and to measure the value of existing flows between the nodes. In other words a proper classification for the balance, its scheme, and its quantification constitute the actual elaboration of an IEB.

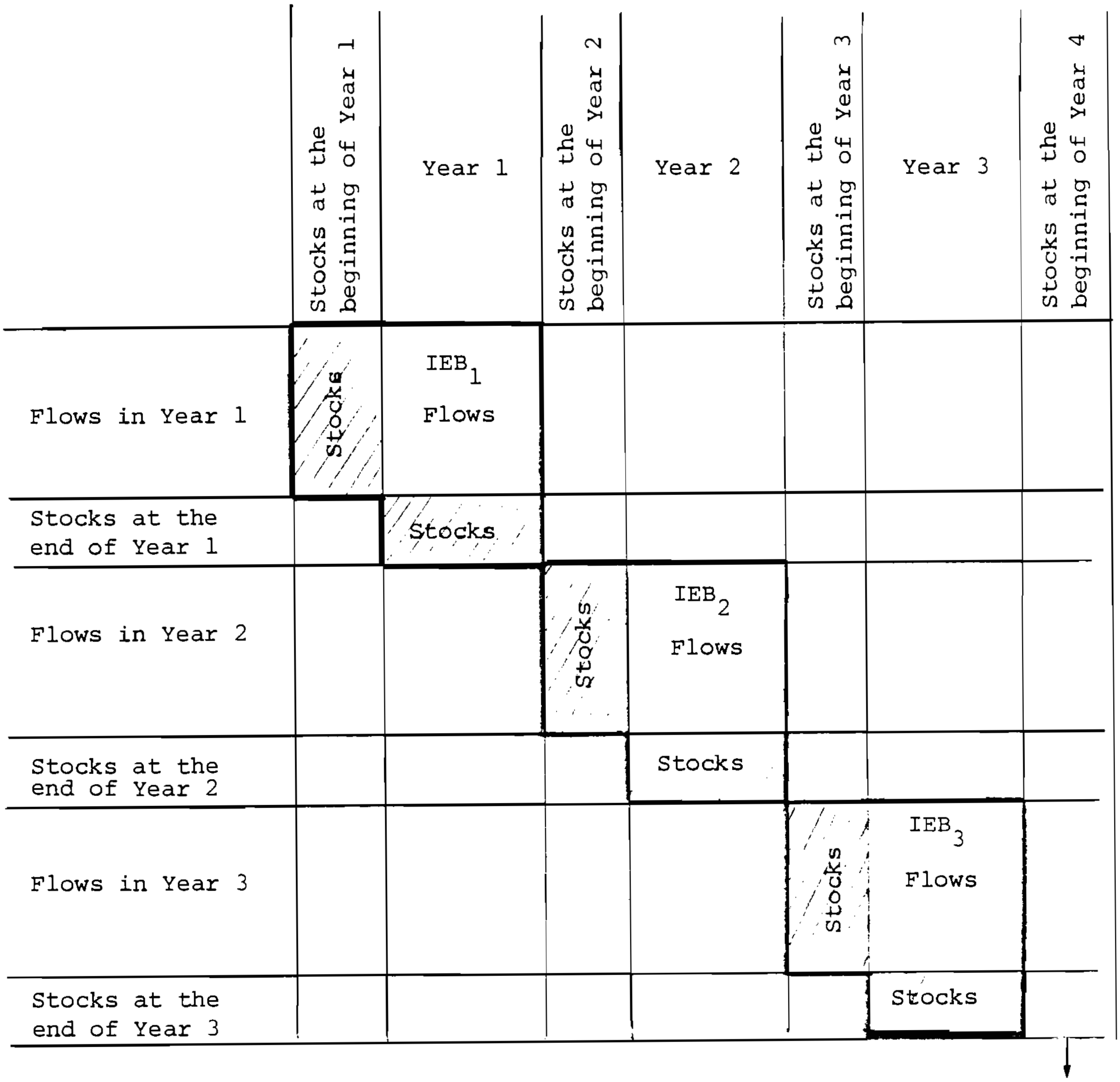


Figure 3. A time-series of integrated economic balances.

1.2. COVERAGE OF THE IEB AND ITS RELATION TO MODELS OF SPECIFIC ASPECTS OF REGIONAL ECONOMY

An integrated economic balance of a region based on methodological principles mentioned above does not describe in an exhaustive way all objective elements of a regional system. Physical elements and processes which cannot be evaluated in monetary terms could not be directly included into the scheme of the IEB (ecological processes, pollution of the environment, demography). But all elements of a regional system and all activities which constitute the economic turnover may be covered:

-- all operations with material goods and services (production, intermediate and final consumption, imports and exports, changes in stocks of inventories and fixed capital, depreciation and losses of fixed capital, factor services, wealth, etc.);

-- all operations with incomes (creation of national income, its transformation in specific types of income for primary distribution and redistribution, transfers of income, formation of final income of economic agents, its use for different purposes including savings, operations with income linking intraregional units with the rest of the country or the rest of the world, etc.);

-- all operations with financial instruments which channel purchasing power in the system from the points of surplus to those of deficits (cash, international liquidities, credits, securities, etc.);

-- all changes in the level of wealth measured as real and financial assets and liabilities in stock balances.

The physical elements of the regional system, which are not directly included into the IEB (population, labor, environment) may be reflected in the balance through their interactions with the economics:

-- population and labor resources through expenditures of sectors on wages and salaries;
-- environment — through the value of damage inflicted and the cost of protection desired or required;
-- mineral and other natural resources not in economic turnover, through their estimated values as a part of national (regional) wealth.

Not all operations with goods and services are directly associated with the opposite flow of money (use of own houses, consumption of goods produced in own household, etc.). In these cases imputed cost based on different assumptions for price should be involved.

In the case of the Tuscany region of Italy the suggested scheme of integrated economic balance is determined by the purpose of the analysis and available (or possible) information.

This scheme is based on information contained in input-output tables for Tuscany and Italy, information of regional and national accounts, ad-hoc information related to financial budget of regional authorities, information on trade of the regions with foreign countries. The scheme of the IEB links together basic economic processes, which are usually covered by national accounting:

-- production
-- distribution and redistribution of incomes
-- consumption
-- saving and capital formation
-- external economic relations.

The links of IEB for Tuscany with specific models developed for the Tuscany case study are shown in Figure 4.

Links with the regional input-output model is realized in terms of a 31 sector classification of this table, with the models related to international trade in terms of 10 sections of SITC, with an intraregional trade model in terms of regional i/o 31 sector classification, with the consolidated public sector budget model in terms of 10 financial functions of public consumption, 6 types of financial incomes and 8 types of financial outlays classified in national accounting. The

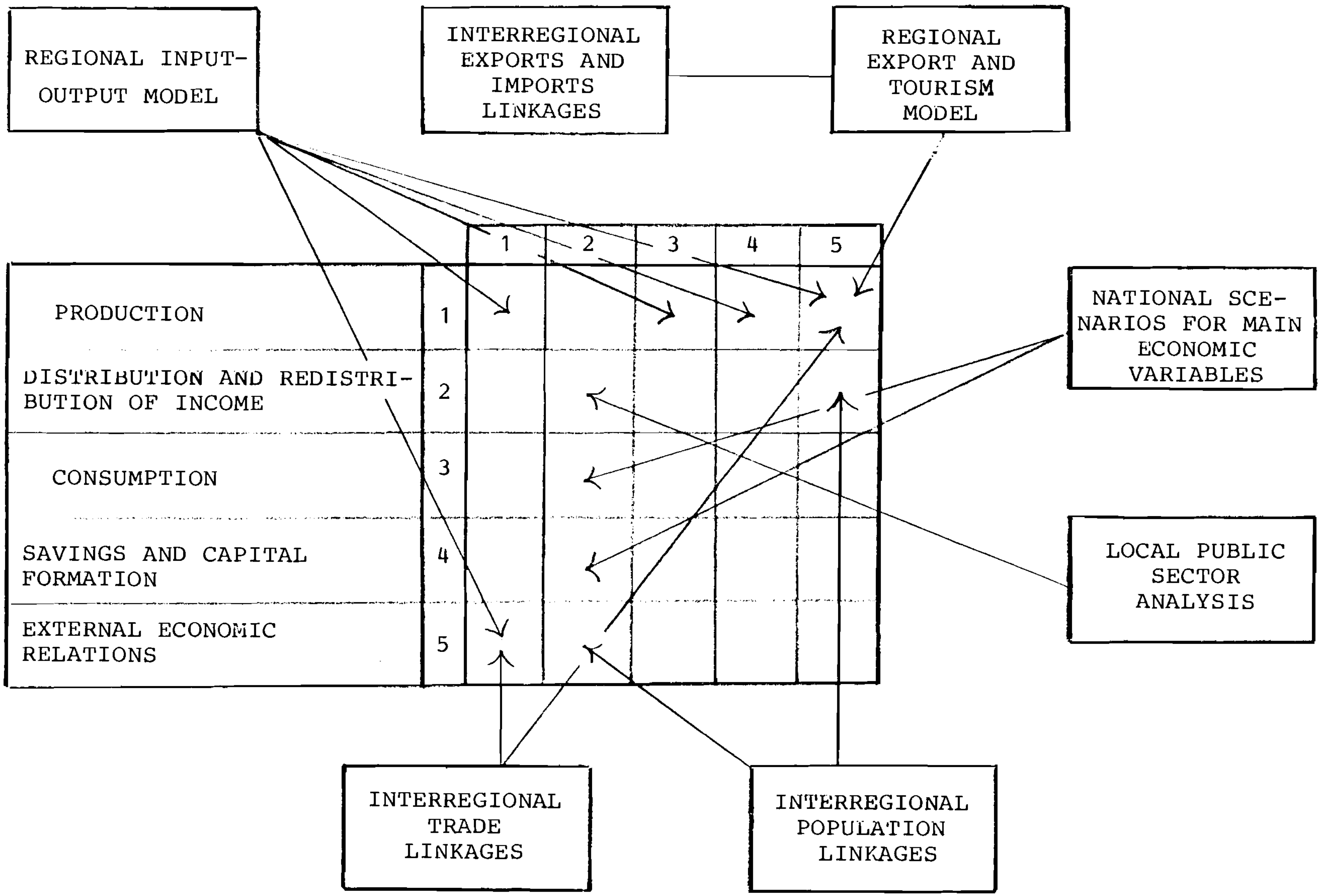


Figure 4. Links of IEB with separate models.

"independent" part of the integrated economic balance, i.e. entries which have no direct counterparts in physical-aspect models developed for the Tuscany case study, are based on data and on identities of the Tuscany regional accounting in current prices. This part is represented in the balance by 6 national accounting sectors, 3 types of income, 2 categories of administration, 9 functions of personal consumption, 2 categories of capital formation.

The link between 31 i/o sectors and 6 national accounting sectors is simplified so that each of the 6 n/a sectors is a sum of several i/o sectors.

The scheme of the IEB provides for somewhat deeper analysis of behavior of the households. For this purpose it is suggested to differentiate them by level of income and to distinguish residents and non-residents. The public sector should also be considered separately in three levels. For the analysis of foreign trade geographical classification of counterpartners by three zones is also suggested. But the problem of information in all these cases is not resolved yet.

1.3. SCHEME AND CLASSIFICATION OF THE IEB. INFORMATION REQUIRED

At the present stage IEB for Tuscany has been realized as a system of data arranged according to the scheme shown in Table 1. The shape of this scheme exactly repeats those of IEB's but entries in this table are not to be balanced because the table includes double counting. On the basis of this system of data two versions of IEB have been compiled and used for simulations. The schemes of these versions are shown in Tables 2 and 3. In these matrices the sums of elements in rows are equal to the sums of elements in respective columns. The classification of rows and columns in these tables are shown below.

Table 1. Integrated Economic Balance Data-base.

Integrated Economic Balance Data-base				Production		Distribution and Redis-tribution of Incomes						Consumption				Savings and Capital Formation				External Economic Relations			Statistical Discrepancy	TOTAL
				31	6	3	6	2	6	8	2	3	3	9	10	6	1	4	2	3	1	10	1	
				a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	u	
Production	Input-output	31	a	aa	ab	ac		ae						ak	al				ap	aq	ar	as	au	
	Regional accounting sectors	6	b	ba		bc																		
Distribution and Redis-tribution of Incomes	Types of income	3	c	ca	cb																			
	Regional accounting sectors	6	d		db	dc		de		dg														
	Government	2	e	ea				ee	ef		eh													
	Financial incomes	6	f			fc					fh													
	Financial outlays	8	g					ge																
	Households	2	h		hb	hc	hd			hg											hr			
Consumption	Personal, by types of families	3	i								ih													
	Public sector by levels	3	j							jg														
	Functions of personal consumption	9	k									ki												
	Functions of public consumption	10	l					le					lj											
Savings and Capital Formation	Regional accounting sectors	6	m				md			mg														
	Households	1	n							ng	nh													
	Monetary market	4	o							og						om	on					os		
	Capital formation	2	p	pa	pb											pm	pn							
External Economic Relations	Areas beyond Italy	3	q	qa														qo						
	Rest of Italy	1	r	ra		rc		re			rh							ro						
	Goods (SITC)	10	s	sa														so						
Statistical discrepancy		1	u	ua																				
TOTAL																								

Table 2. Integrated Economic Balance I.

Integrated Economic Balance I				Production		Distribution and Redis-tribution of Incomes						Consumption				Savings and Capital Formation				External Economic Relations			Statistical Discrepancy	TOTAL
				31 a	6 b	3 c	6 d	2 e	6 f	8 g	2 h	3 i	3 j	9 k	10 l	6 m	1 n	4 o	2 p	3 q	1 r	10 s	1 u	
Production	Input-output	31	a		ab			ae						ak	al				ap	aq	ar		au	
	Regional accounting sectors	6	b	ba																				
Distribution and Redis-tribution of Incomes	Types of income	3	c																					
	Regional accounting sectors	6	d		bd			de																
	Government	2	e	ea				ee			eh													
	Financial incomes	6	f																					
	Financial outlays	8	g																					
	Households	2	h		hb		hd														hr			
Consumption	Personal, by types of families	3	i								ih													
	Public sector by levels	3	j																					
	Functions of personal consumption	9	k									ki												
	Functions of public consumption	10	l					le																
Savings and Capital Formation	Regional accounting sectors	6	m				md																	
	Households	1	n								nh													
	Monetary market	4	o													om	on							
	Capital formation	2	p		pb											pm	pn							
External Economic Relations	Areas beyond Italy	3	q	qa														qo						
	Rest of Italy	1	r	ra				re			rh							ro						
	Goods (SITC)	10	s																					
Statistical discrepancy		1	u	ua																				
TOTAL																								

Table 3. Integrated Economic Balance II.

Integrated Economic Balance II				Production		Distribution and Redis- tribution of Incomes						Consumption				Savings and Capital Formation				External Economic Relations			Statistical Discrepancy	TOTAL
				31 a	6 b	3 c	6 d	2 e	6 f	8 g	2 h	3 i	3 j	9 k	10 l	6 m	1 n	4 o	2 p	3 q	1 r	10 s	1 u	
Production	Input-output	31	a	aa		ac								ak	al				ap		ar	as	au	
Production	Regional accounting sectors	6	b																					
Distribution and Redis- tribution of Incomes	Types of income	3	c	ca																				
Distribution and Redis- tribution of Incomes	Regional accounting sectors	6	d			dc				dg														
Distribution and Redis- tribution of Incomes	Government	2	e					ee	ef															
Distribution and Redis- tribution of Incomes	Financial incomes	6	f			fc					fh													
Distribution and Redis- tribution of Incomes	Financial outlays	8	g					ge																
Distribution and Redis- tribution of Incomes	Households	2	h			hc	hd			hg											hr			
Consumption	Personal, by types of families	3	i								ih													
Consumption	Public sector by levels	3	j							jg														
Consumption	Functions of personal consumption	9	k									ki												
Consumption	Functions of public consumption	10	l										lj											
Savings and Capital Formation	Regional accounting sectors	6	m				md			mg														
Savings and Capital Formation	Households	1	n							ng	nh													
Savings and Capital Formation	Monetary market	4	o							og						om	on					os		
Savings and Capital Formation	Capital formation	2	p	pa												pm	pn							
External Economic Relations	Areas beyond Italy	3	q																					
External Economic Relations	Rest of Italy	1	r	ra				re	rf		rh							ro						
External Economic Relations	Goods (SITC)	10	s	sa														so						
Statistical discrepancy		1	u	ua																				
TOTAL																								

CLASSIFICATIONS FOR INTEGRATED ECONOMIC BALANCE OF TUSCANY REGIONS FOR 1975

Account a. Input-output (see Appendix 1)

Account b., d., and m. Regional Accounting Sectors

1. Agriculture
2. Manufacturing industry
3. Energy
4. Building and construction
5. Services marketable
6. Services not marketable

Account c. Types of Income

1. Labor income
2. Net indirect taxes
3. Gross profits

Account e. Government

1. Central government
2. Regional government

Account f. Financial Incomes

1. Taxes on wealth
2. Turnover tax
3. Taxes on consumption
4. Other taxes
5. Transfers
6. Others

Account g. Financial Outlays

1. Purchase of goods and services
2. Pensions and other social expenses
3. Interest rates
4. Current subsidies to producers
5. Current transfers to public sector
6. Capital transfers to private sector
7. Capital transfers to public sector
8. Others

Account h. Households
1. Residents
2. Nonresidents

Account i. Types of Families by Level of Income
1. Rich
2. Medium
3. Poor

Account j. Public Sector by Levels
1. Regional
2. Provincial
3. Municipal

Account k. Functions of Personal Consumption
1. Food and drink
2. Tobacco
3. Clothing, etc.
4. Rent, fuel, energy
5. Household goods
6. Healthcare
7. Transport and communication
8. Recreation, education, culture
9. Other goods and services

Account l. Functions of Public Consumption
1. Services of central government
2. Services of regional goverment
3. Defense, police
4. Secondary schools
5. Primary schools
6. Housing
7. Healthcare
8. Social assistance
9. Public services
10. Maintenance of roads and other exp.

Account o. Monetary Market
1. State bonds
2. Other short-term securities
3. Other long-term securities
4. Foreign liquidity

Account p. Capital Formation
1. Fixed gross capital formation
2. Changes in stocks

Account q. Areas Beyond Italy
1. EEC countries
2. USA and Canada
3. Other countries

Account r. Rest of Italy.

Account s. Goods (SITC)
1. Food
2. Beverages and tobacco
3. Crude materials
4. Mineral fuels
5. Oils and fats
6. Chemicals and related products
7. Manufactured goods
8. Machinery and transport equipment
9. Miscellaneous articles
10. Other goods

Account u. Statistical adjustments.

In matrices of IEB inflows of monetary resources (incomes) are recorded in rows, the outflows (expenditures) in columns. The figures in the column next to the definition of rows indicate the number of items in the classification of the account. The letters in the next column are indices of the accounts. Blocks in the matrics are marked with two letters: the first indicate the row, the second the column, the intersection of which forms the respective block. The content of each block is evident from the classification of the accounts which are linked through the given block. For instance, aa reflects an intermediate consumption in the form of a 31 x 31 matrix of the first quadrant of the input-output table. Block ba contains information on composition of output of each national accounting sector in terms of 31 products of the input-output table, and so on. The complete definition of each block is given in Appendix 2.

Versions of IEB I and II have different problems and information orientation. The first version does not account for intersectoral dependencies reflected in the first quadrant of the input-output table. It deals predominantly with national accounting sectors based on the concept "enterprise" and representing integral financial units making decisions for monetary funds. Input-output sectors based on the concept "establishment" represent specific products, produced and used by n/a sectors. The first version of IEB was dictated also by consideration of data availability, although not everything is covered by the necessary data also given in this table. The second version of IEB II is more oriented towards joint analysis of intraragional productive patterns in terms of input-output sectors and foreign trade. In principle, however, the general scheme of the balance shown in Table 1, having the function of a data base, should be considered as basic. Different kinds of problems may require other than versions I and II of the balanced matrices, but all of them will be derived from the basic Table 1. Thus, the integrated economic balance of the region should be thought of as a specifically organized data base which allows compilation by computer and the analysis of matrices of real and financial flows based on principles set out in the second paragraph of this paper.

Two tables, IEB I and IEB II, are based on data for 1975. Information of the input-output table for Tuscany is directly included in the balances. The other real data have been taken from regional accounting data (Anunario di Contabilito Nazionale, Volume III 1978 tomo II Roma 1979). Actually, only these data can be considered as real. Other data have at present a semi-artificial character in the sense that their orders of magnitude have been checked with regional and national accounting aggregates but their breakdown was mainly arbitrary. Therefore, the present versions of IEB could not be used for getting reliable economic conclusions. They have been used mainly for the development of standardized software which might be applied to IEB, compiled entirely from real data.

As the next step in implementing IEB methodology for the Tuscany case study the following information should be obtained:

-- budget of the region in national accounting breakdown of incomes and outlays for three levels of administration--regional, provincial, and municipal;
-- data on trade in terms of SITC; link of this data with imports and exports in input-output table;
-- financial accounts of n/a sectors with linkage to incomes and outlays classification of the budget;
-- distribution of public consumption by 10 functions for regional, provincial, and municipal levels;
-- amortization of fixed capital by input-output sectors;
-- use of products in terms of 31 sectors for increase of fixed capital and for changes in inventories (separately);
-- national and regional data on monetary market.

It would be highly desirable to get information on distribution of outlays on consumption by 9 functions separately for three types of households according to their level of incomes. To test IEB as a tool for analysis and simulation of regional development, compilation of balances for later years is also necessary.

1.4. "PHILOSOPHY" OF IEB'S USE AS A MODEL

The integrated economic balance can be considered as a model describing the structure of the economic system. To the extent to which grouping of agents and interpretation of monetary flows linking them are meaningful, accounting identities and qualitative relations between the balance's entries may serve as a basis for the simulation of the balance's reaction on external disturbances introduced exogenously.

The balance determines the basic constraint on entries as variables - their equilibrium within respective accounts and their interdependencies through the system of the balance's identities.

There are two extreme cases in using the IEB as a model. One is based on the assumption that we know nothing about the interdependencies between variables but only the condition of equilibrium within each account. We may change, in the reference balance, one or several variables and find a new balance subject to some objective function. The other is based on the assumption that the accounting identities are completed with a set of functions, determining interdependencies between variables in such a way that the whole system becomes resolvable with a single solution. The most reasonable way to use the IEB as a model is between the above extremes.

The IEB accounting identities may be completed with different types of additional constraints on the variables:

-- fixing absolute values of variables;
-- limitation of the area of change for one or several variables by fixing upper and/or lower limits of change;
-- fixing ratios between values of some variables (for instance, introducing technological coefficients for variables relating to intermediate consumption);
-- fixing linear combinations of variables (for instance, introducing econometric equations for some or several variables).

To the extent to which we know the specific character of interdependence between some variables, this knowledge may take the form of a separate model. The variables included in such a model can be recorded in the IEB either directly by their absolute value after solution of this model, or in the form of relations provided by the model. If we do not know specific interdependencies, but know that some variables can change only in one direction, or cannot change at all, these limits or absolute values of entries could be introduced into the balance together with specific models. If we know that two or several variables can change only parallel to each other, we may link them through constant coefficients, and so on up to the point of exhaustion of our specific knowledge in respect of some variables or their links. But in the exhaustive system presented by IEB there will always be a set of variables for which the law of behavior is not known and the only constraint on them, derived from reality and considered as true, is their balancing within the respective account. It is logical to assume that all such variables maximally resist changes which are necessary to restore the equilibrium destroyed by exogenous interference.

Since the integrated economic balance is compiled in value terms, each entry in it relating to flows of goods, services or factors implies the existence of price (in respective form) and may be considered as a product of a quantity reflecting physical volume of the flow by a multiplier reflecting either absolute or relative price. Physical volumes of flows in IEB imply constant prices or the value of the multiplier equal to one when relative prices are considered. The IEB allows the analysis of the effect of changes introduced into the balance separately for volume and for relative price components of respective entries of the balance.

1.5 SOME PROBLEMS OF ADJUSTMENTS SOLVED FOR INTEGRATED ECONOMIC BALANCE OF TUSCANY FOR 1975

The mathematical model of IEB and software presented in Part II of this paper have been tested for the solution of several problems among which are:

- testing of IEB sensibility to changes of its particular elements;
- simulation of the balance's response to changes in some sets of foreign trade flows;
- simulation of the balance's response to essential increase in volume of production in one sector.

Test of sensibility. The response of the IEB to a change of one entry is measured by a minimal relative value of maximal deviations in both directions between the basic values of variables and new ones which restore the equilibrium in the IEB. Mathematical formulation of this LP problem with a minimax objective function is given in Part II. Many tests of the sensibility have been carried out on the detailed scheme of IEB (111 x 111) and an aggregated one (20 x 20).

The mathematical sense of all these problems was equal. However, in the case of the first problem the only constraints were balancing identities and the exogenously introduced value of one entry-variable. In all other cases, many other variables or combinations of them were introduced exogenously.

The resulting deviation in the objective function varied up to the level of relative change in the entry variable, but the latter was a rather exceptional case in which all elements of one line (or column) in the matrix had been changed. In most cases the resulting value of the objective function was essentially lower than the initial change (3-5% for the initial change of 10%).

What might be economic interpretation of the results of the IEB adjustments to exogenously given value of one or several variables? The response of the IEB to one external disturbance is objective and determined by its structure (nodes, their links, and value of edges). For the productive sector new values of intersectoral exchange may be considered as objectively necessary in terms of monetary flows. But behind these flows there are volume flows determined by technological coefficients and proportionality reflected in the basic matrix. Therefore, new values in this case may be interpreted as monetary adjustments for stable physical flows (adjustments in pricing). In cases of purely financial flows the interpretation may be direct: increased or diminished value of flow ensures the equilibrium in the new balance (more or less savings, taxes, disposable income, etc.).

When only the IEB's response to change in one variable is considered, this is simply the sensibility test. Advantages of the model are mentioned in Part II. On the basis of the solution of this type of problems all variables (especially those which may be considered as policy instruments) may be arranged according to the degree of their influence on the balance as a whole and on specific entries within the balance.

Other problems that have been tested contained additional constraints introduced into the system as scenarios. Exports of sectors 17 (textiles and clothing) and 18 (leather and leather products were increased by 10%. The measure of IEB (111 x 111) response was 5.9%. The total output of the sectors also slightly increased but value added in sector 17 diminished by 3.7% and in sector 18 increased only by 0.7%. Cost of labor in both fell down in the 17th sector by about 3% and rose in the 18th sector by 4.2%.

In conclusion one should stress that IEB should not be considered in isolation from a formalized or not formalized description of economic mechanisms determining actual flows, which are carriers of monetary values reflected in the balance. The IEB contains only balance identities. Only in combination with models, and other forms of descriptions of causal relationships between the balance's entries could interpretable results be obtained.

In the case of the Tuscany region study, the IEB is thought to be an integrated real-financial balance for analysis of responses to the results of models which constitute the substance of the study.

PART II: THE MATHEMATICAL DESCRIPTION OF THE APPROACH

2.1 DESCRIPTION OF THE MODEL

In this section we will describe mathematical tools used to analyze integrated economic balances. Generally, we will consider a mathematical model of such a system as a set of elements which are independent at a chosen level of model aggregation. For example, in the case under consideration, the role of these elements plays monetary flows from one economic agent to another. For each of these elements we construct a general quantitative characteristic, the value of which is uniquely defined. This means that we must define a function of the set of elements. Finally, we should also define a system of constraints for feasible values of the characteristics which describes the internal initial relations of the system under consideration.

As a simple example, in an integrated economic balance of a region, the model has as its elements monetary flows between economic agents of the system. A quantitative characteristic of each of the elements is the value of the flow. The domain of feasibility for the characteristics is defined by the balance conditions and the structural constraints.

The mathematical model may be described as the following. Let the model of the system consist of N elements and the state of each defined by a real value vector $x_i, i = [1,N]$. The numerical characteristic of the element f_i is a function of x_i, i.e.

$$f_i = \phi_i(x_i) \tag{1}$$

and the feasible states of the model are defined by the relations

$$F_j(f_1, f_2, \ldots, f_n) \geq 0 \;, \qquad j = [1,M] \tag{2}$$

2.2. STATEMENT OF THE PROBLEM

At the base of the model we have an opportunity to state some problems of the analysis of the integrated economic balance. One is the problem of analyzing the sensitivity of a given balance and finding some rational ways to adjust the balance

Let an initial state of the model be given $\{x_i^o, i=[1,N]\}$, characteristics of which satisfy the conditions (2). Let us suppose that a subset of the elements are changed. It is clear that the characteristics also change and may be in disagreement with the relations (2).

Now consider the problem of renewal of the balance by means of appropriate changes of other elements of the model. Having found the values of these variations, we are able to measure the level of sensitivity of the model balance. Notice that the use of any classical schemes of the theory of sensitivity is strongly limited in these problems because of the main theorem of the theory--the implicit functions theorem--is not applied in this case, except when the Jacobian of (2) is non-singular. Moreover, the classical theory considers only small input variations. But the analysis of real practical balances requires consideration of relatively big changes in the initial state of the model.

An approach is described below which gives us an opportunity to overcome the difficulties just mentioned and to investigate models like (1) - (2) of a general type. The main idea of the method consists of finding a unique valued measure of sensitivity of the model in respect of the given, generally speaking, non-small input variations of an element (or a number of elements).

Preliminarily, define a concept of a distance between two arbitrary, maybe infeasible, states A and B of the model. The definition is

if

$$A = \{x_i^A,\ i = [1,N]\} \quad \text{and} \quad B = \{x_i^B,\ i = [1,N]\}$$

then the distance ρ is

$$\rho(A,B) = \max_i \{\rho_i\{\phi_i(x_i^A),\ \phi_i(x_i^B)\},\ i = [1,N]\}$$

The question whether the space of $\{x\}$ is a metric one depends on the properties of the functions ρ_i and ϕ_i. In some cases, for example, when $\rho(a,b) = |a-b|$ and $\phi_i(x_i) = c_i x_i$ (where x is nonnegative and c is positive), ρ will have all properties of the metric distance. Now we are ready to formulate the idea of how to measure the sensitivity. Let a feasible state A of the model be given (i.e. the state satisfies the system (2))

$$A = \{x_i^A,\ i = [1,N]\}$$

which should be investigated and be given a variation of the state

$$\Delta A = \{y_i^A,\ i \in \Omega\}$$

where Ω^A is the set of those elements of the model which must be fixed to given y_i^A. After the variation of all elements belonging to Ω, we should find changes of the rest of the elements to return the model to a feasible state. Generally, it is possible to do this in many ways. Making the distance between "new" and "old" states minimal, we can use this distance as a measure of the change of state and hence, to use it as a measure of sensitivity of the state in relation to the given variation. Formally, the procedure can be written as the following mathematical programming problem.

Minimize ρ with respect to $\{f_i, i = [1,N]\}$ subject to

$$\rho = \max_i \{\rho\{f_i^A, f_i^B\}, i = [1,N]\} \quad \text{and}$$

$$F_j(f_2^B, f_2^B, \ldots, f_w^B) \geq 0, \quad j = [1,M] \qquad (3)$$

2.3. LINEAR CASE

Consider now a specific case when

-- each of the elements is described by a uniquely valued nonnegative number;

-- the characteristic is a product of the number to a positive normative coefficient;

-- system (2) is linear with respect to all f_i.

One of the important descriptive cases of a real economic system by means of the model presented is the integrated economic balance of a region. In this case the images of real elements of the object to be modeled are physical flows of goods, services, and other types of profits and payments. The positive coefficients are relative prices of units of these flows which permit us to link the real flows with their monetary evaluations. Conditions (2) are balancing constraints and structural restrictions which illustrate the usage of the model of the problem in analyzing the sensitivity of a balance table in respect to a variation of its element. Let us have a square table $||a_{ij}||$ which consists of non-negative elements. This table is balanced, i.e.

$$\sum_{i=1}^{n} a_{ki} = \sum_{j=1}^{n} a_{jk}, \quad \text{for all } k = [1,N] \qquad (4)$$

Suppose, also, that an element of the table a_{pq} receives a new fixed value y. Equations (4) would not be valid now. Try to find such corrections to all other elements Δa_{ij} so that

-- a "new" table will be balanced;
-- the distance between "old" and "new" tables are minimal in the sense of the following matrix

$$\rho = \max_{(i,j)\neq(p,q)} \left\{ \frac{|\Delta a_{ij}|}{a_{ij}} \right\} \tag{5}$$

In this case we get a problem of linear programming:

minimize ρ subject to

$$\sum_{i=1}^{n} \Delta a_{ki} = \sum_{j=1}^{n} \Delta a_{jk}, \quad k = [1,N]$$

$$-\rho\, a_{ij} \leq \Delta a_{ij} \leq \rho\, a_{ij}, \quad \text{for all } (i,j) \neq (p,q) \text{ and}$$

$$a_{ij} + \Delta a_{ij} = y \ , \qquad \text{for } (i,j) = (p,q) \ . \tag{6}$$

2.4. CONCLUSIONS ABOUT THE USAGE OF THE APPROACH AND SOME PRACTICAL RESULTS

The suggested approach has some advantages and could be more useful in comparison with other methods of sensitivity measurement. These advantages include, first, the opportunity to solve problems with singular Jacobians, i.e. in cases when classical methods are inapplicable. Second, the use of mathematical programming enables us to obtain the unique valued characteristics of sensitivity balance tables with respect to given variations. Third, usage of the matrix (5) in linear cases permits us to build corrected balance tables with structures which are close to the initial tables. Notice that the usage of a standard euclidean matrix produces a diffusion of the initial structure which usually is undesirable from a practical point of view. On the other hand, the usage of an entropy-type matrix, i.e. functions evaluate the closeness of two characteristics f_i^A and f_i^B with the help of expression

$$f_i^A \log \frac{f_i^B}{f_i^A}$$

even in linear cases, leads to a problem of non-linear programming. This is not very convenient from a computational point of view.

A number of calculations were fulfilled on the basis of schemes (5) - (6), with the balance table having about a hundred rows and columns and 1,100 non-zero elements.

A special software library was elaborated on the VAX/11 computer. This library has subroutines to create, check, and transfer working files in format, which is required for used procedures in solving mathematical programming problems.

To solve the problems in (6) a package MINOS (linear-phase) was used. The number of iterations was between several hundreds and thousands. The transformation of the output files and the preliminary analysis were made by means of an interactive procedure. Output information is represented both in graphical and numerical forms.

APPENDIX 1: SECTORS IN TUSCANY INPUT-OUTPUT TABLE FOR 1975 (classification of the account "a" in IEB)

Sector Number	Sector Name
1	Agriculture
2	Coal and oil
3	Other energy and water
4	Minerals
5	Non-metalic minerals
6	Chemicals
7	Metal products
8	Machinery for agriculture and industry
9	Other machinery
10	Electric machinery
11	Transport equipment
12	Meat
13	Milk
14	Other food products
15	Beverages
16	Tobacco
17	Textiles
18	Footwear
19	Wood and furniture
20	Paper and paper products
21	Rubber and rubber products
22	Other manufactures
23	Construction
24	Commerce
25	Hotels
26	Transports
27	Communications
28	Credit and insurance
29	Housing
30	Other marketable services
31	Non-marketable services

APPENDIX 2: CONTENT OF SEPARATE BLOCKS IN IEB

aa (31 x 31)	intermediate consumption of 31 i/o sectors
ba (6 x 31)	sectors' outputs in terms of 31 products
ca (3 x 31)	primary incomes from 31 i/o sectros
ea (2 x 31)	taxes payed from 31 i/o sectors to central and local governments
pa (2 x 31)	fixed capital depreciation allowances in 31 i/o sectors
qa (3 x 31)	imports by 31 i/o sectors from 3 zones outside Italy
ra (1 x 31)	imports by 31 i/o sectors from the rest of Italy
sa (10 x 31)	imports in double breakdown: by 31 i/o sectors and 10 sections of SITC
ab (31 x 6)	intermediate consumption of 31 products by national accounts' sectors
ac (31 x 3)	subsidies to i/o sectors as negative taxes
ae (31 x 2)	subsidies to i/o sectors payed by governments
ak (31 x 9)	personal consumption in double breakdown: by 31 i/o products and 9 functions
al (31 x 10)	public consumption in double breakdown: by 31 i/o products and 10 functions
ap (31 x 2)	use of 31 i/o products for increase of fixed capital and for changes in stocks

aq	(31 x 3)	exports of 31 i/o products to 3 zones beyond Italy
ar	(31 x 1)	exports of 31 i/o products to the rest of Italy
as	(31 x 10)	exports in double breakdown: by 31 i/o products and 10 sections of SITC
cb	(3 x 6)	types of primary incomes by 6 n/a sectors
db	(6 x 6)	national income produced by 6 n/a sectors
hb	(2 x 6)	income of households from 6 n/a sectors
pb	(2 x 6)	fixed capital depreciation allowances by 6 n/a sectors
bc	(6 x 3)	subsidies to n/a sectors
dc	(6 x 3)	gross profits of n/a sectors
fc	(6 x 3)	distribution of sectors' payments to budgets by items of financial incomes
hc	(2 x 3)	labor and ownership income of households
rc	(1 x 3)	transfer of income to the rest of Italy
hd	(2 x 6)	income of households from 6 n/a sectors
md	(6 x 6)	savings of n/a sectors
de	(6 x 2)	subsidies to n/a sectors payed by governments
ee	(2 x 2)	transfer of funds from central to local government
ge	(8 x 2)	distribution of government's budget by types of outlays
le	(10 x 2)	public consumption broken down by 10 functions
re	(1 x 2)	transfer of governments' funds to the rest of Italy
ef	(2 x 6)	finance of public consumption from the budget
dg	(6 x 8)	subsidies to n/a sectors covered by items of financial outlays
hg	(2 x 8)	financial transfers to households
jg	(3 x 8)	finance of public consumption by 3 levels and by sources in terms of financial outlays
mg	(6 x 8)	capital transfers to n/a sectors
ng	(1 x 8)	capital transfers to households
og	(4 x 8)	financial surpluses
eh	(2 x 2)	taxes on households' incomes
fh	(6 x 2)	taxes on households' income
ih	(3 x 2)	personal consumption by types of families

nh	(1 x 2)	savings of households
rh	(1 x 2)	transfer of households' income to the rest of the world
ki	(9 x 3)	personal consumption by functions
lj	(10 x 3)	public consumption by functions
om	(4 x 6)	monetary surpluses of n/a sectors
pm	(2 x 6)	investments of n/a sectors into fixed capital and stocks
on	(4 x 1)	monetary surpluses of households
pn	(2 x 1)	investments of households into fixed capital and stocks
qo	(3 x 4)	change in the balance of financial claims and liabilities in relations with foreign countries
ro	(1 x 4)	change in the region's balance of claims and liabilities with the rest of Italy
so	(10 x 4)	imports surpluses by 10 sections of SITC
hr	(2 x 1)	transfers of money to households from the rest of Italy
os	(4 x 10)	export surpluses by 10 sections of SITC
ua and au		statistical adjustments to i/o sectors

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