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IIASA Working Paper

WP-82-132

December 1982

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ACKNOWLEDGMENTS

I wish to thank Lars Bergman, Peter Neary and Andrzej Wierzbicki for helpful discussions. I am also grateful to everybody else who contributed to the work reported here: participants in seminars at Stockholm School of Economics and at IIASA, and especially my colleagues in Hungary.

Helen Gasking edited the paper, which was typed by Susie Riley and Nora Avedisians. I thank them for their efficient work.

ABSTRACT

The treatment of foreign trade has a great influence on the results that can be obtained from multisectoral macroeconomic models. This manifests itself clearly in the problem of overspecialized solutions which arises in most of the models currently in use. This unwanted phenomenon is treated differently in the two main classes of models: programming models and general equilibrium models.

This paper discusses the theoretical and methodological problems related to this issue using a special comparative framework (*laissez-faire* equilibrium and planner's optimum). Attention is focussed on alternative export specifications and optimum tariff problems. The argument is illustrated by numerical results based on two models of the Hungarian economy.

CONTENTS

FOREIGN TRADE IN MACROECONOMIC MODELS:
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1. INTRODUCTION

Multisectoral planning or forecasting models fall roughly into three main classes: input-output models, mathematical programming models, and general equilibrium models. In this paper we consider only models typical of the second and third classes, paying particular attention to the treatment of foreign trade in these models.

The most important differences between the two modeling approaches examined here may be summarized as follows. *Mathematical programming* models are typically large-scale and linear, containing mainly real (physical) variables; most of the relations are in the form of inequalities (balances and special restrictions) and as a rule they contain quite a few individual bounds on variables. *Computable general equilibrium* models, on the other hand, are specified in terms of both real and price (value) variables; they typically take the form of an equation system and include many nonlinear terms; no explicit overall optimization is called for.

Computable general equilibrium models have many similarities to the optimal planning models used in socialist (centrally planned) economies. However, differences in terminology, and conceptual and other difficulties have led to the impression that these two schools of macroeconomic modeling diverge rather than converge. In earlier papers (see, Zalai 1980, 1981), the author has argued that computable general equilibrium models can be discussed in purely pragmatic terms as natural extensions of a certain class of programming models. Dispelling the neoclassical myth surrounding equilibrium models would have two important consequences. Firstly, it would allow central planning modelers to take advantage of some of the special features of this macroeconomic modeling approach (see the papers cited above for more details). Secondly, some of the weaknesses of computable general equilibrium models could be revealed and eliminated by examining them from a linear programming perspective.

The ideas presented in this paper can be considered as a continuation of the argument developed in the papers quoted above. Thus, the whole discussion will take place within a special comparative framework. Particular attention will be paid to the reinterpretation of some of the elements usually included in models, even though this means that we must cover some ground that will be very familiar to most readers. One should warn the more theoretically inclined reader to skip these sections which will hopefully give some new insights though for actual model builders.

This paper is basically concerned with the concepts of "equilibrium" and "optimum" in relation to export-import specification in macroeconomic models. In sections 2 and 3 we start by discussing the problem of overspecialization and possible methods of dealing with it in (linear) programming models as compared with computable general equilibrium models. The root of the problem is that most models adopt the usual definition of small open economy, which implies that it cannot influence its terms of trade. However, it is generally recognized that such exogenously fixed terms of trade tend to produce overspecialized solutions in the macroeconomic models currently in use (see, for example, Taylor 1974 and Bergman 1982). Overspecialization manifests itself in the existance of only a small number of producing and/or exporting sectors and little or no intrasectoral trade. In view of the fact that, even in the most detailed macroeconomic models, the sectors represent product groups, such overspecialized solutions cannot be defended on practical grounds. Thus, model builders must find ways of avoiding unrealistic solutions of this type.

Builders of macroeconomic models can basically use two "pure" methods to prevent overspecialized solutions. One, characteristic of linear programming models, is to make wide use of special bounds on certain groups of variables. Various opinions on this subject have been expressed in the literature, some of them rather critical of this approach. The other method, originally characteristic of computable general equilibrium models, is to use various nonlinear export-import relationships. The main aim of Sections 2 and 3 is to show that the difference between these two approaches can be viewed as one between *rigid* (fixed) and *flexible bounds*. It is argued that it would be natural and useful to include such flexible bounds in existing programming-planning models. This viewpoint has much in common with recent suggestions made by Ginsburgh and Waelbroeck (1981).

These sections also provide a basis for discussion of a number of other points. For example, we argue that it is necessary to make a clear distinction between export restrictions caused by supply, on the one hand, and export demand limitations, on the other. In the computable general equilibrium models currently in use, these two effects are not separated. A related issue is that a small response to changes in relative prices is generally modeled by very small export demand elasticities, which introduce virtually indefensible terms of trade effects into the models. These problems call for a revision of common modeling practice in this field.

Section 4 is devoted to related issues in economic theory. The theoretical definition of small economies is incompatible with the assumption of less than perfectly elastic export demand. This definition, on the other hand, is clearly unsatisfactory since, due to market and product differentiation, even small countries generally face changing terms of trade. Thus, the theoretical "small economy" is in practice a completely uninteresting case. This fact has been realized belatedly by model builders and, as a result, the use of less than perfectly elastic export as well as import demand functions is now quite common, even in models originally developed for small open economies. The theoretical justification is usually given as Armington's (1969) assumption of regional product differentiation.

It is well known in the theoretical literature on international trade that if an economy faces less than perfectly elastic export demand then the pure competitive (*laissez-faire*) equilibrium is not (Pareto) optimal*. Under special assumptions, *optimum tariffs* can be employed to produce the optimal trade pattern in an otherwise competitive setting. Strangely enough, this problem does not seem to have been considered at all in connection with computable (applied) general equilibrium models, although it arrises naturally in our comparative exercise. We argue that the modelers face a real choice here and show that basically the same model framework and solution algorithm can be used to determine both solutions. Section 5 illustrates the theory using numerical examples based on a model of the Hungarian economy, concentrating in particular on the possible magnitude of the difference between the competitive and optimal solutions. It is shown that, as might be expected, more foreign trade is not necessarily better for the home economy (although this seems to contradict some classical economic beliefs).

Going back to Section 4, we extend the classical *optimum tariff* theorem to cover *small economies*, i.e., those facing constant terms of trade. By moving away slightly from strict neoclassical assumptions, it can be shown that less than perfectly elastic export supply may also result in different equilibrium and optimum solutions. An interesting difference between the two optimum tariff situations is that in the latter case export suppliers might have to be subsidized rather than taxed in order to obtain the optimal regime (in the classical case only taxation is possible).

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*See, for example, Dixit and Norman (1980). See also Srinivasan (1982) for a theoretical discussion of this separation in a different context.

Finally, as already mentioned, Section 5 provides some numerical illustrations of the theoretical arguments and sums up the main conclusions for practical model building.

2. FOREIGN TRADE IN LINEAR MULTISECTORAL PLANNING MODELS

2.1. On the Use of Individual Bounds

In this section we will review, discuss, and illustrate the problem of overspecialization with regard to linear programming models. Whether we consider development planning models following neoclassical traditions or more pragmatic planning models based on traditional plan calculations, one of the most common means* of preventing extreme behavior is to impose upper and/or lower bounds on different variables, particularly on production, export, and import variables.

The use of individual bounds in planning models in not universally approved. One of the main criticisms is that they are *ad hoc* arbitrary restrictions, which can also distort the shadow prices (see, for example, Taylor 1975, or Ginsburgh and Waelbroeck 1981). An alternative approach favored by some model builders involves the introduction of more complicated nonlinear relationships into the model, perhaps in a piecewise linear fashion. We will come back to this possibility later.

The above criticism is, however, only partially justified. On the one hand, it is undoubtedly true that the individual constrains account for the inadequacy of the chosen model, reflecting our lack of knowledge and modeling ability. On the other hand, however, this problem, i.e., the arbitrariness of certain elements, is common to all present economic models. In some models this is quite apparent, while in others it is partially hidden behind an elegant mathematical facade. Thus, for example, the use of nonlinear relationships (rather than individual bounds) to deal with overspecialization can just be seen as introducing another type of arbitrariness into the model. Moreover, for plan coordination models at least, most of the individual bounds

*See Taylor (1975) for a more complete treatment of alternative ways of handling these problems.

are based on partial, presumably rather careful analysis of the underlying phenomena in the traditional planning process; it is doubtful that this expertise could be replaced by some simple modeling device.

To avoid this argument becoming one-sided, we must make a brief mention of some points which will be discussed in more detail in later sections. It could be argued that the real choice is not between expert judgement and individual bounds, on the one hand, and nonlinear, econometrically estimated relationships, on the other. The parameters of the nonlinear forms in question could just as well be based on expert judgement as are the individual bounds in the other solution. Both solutions are capable of providing planners with equally realistic descriptions of patterns of resource allocation.

What is more important, in our view, is the fact that the use of nonlinear relationships may result in macroeconomic models that are able to produce less distorted accounting (shadow) prices, which, in turn, may be a useful source of information for price and cost planning, or project evaluation.

In what follows we will try to show that these nonlinear functions can, in most cases, be viewed as *flexible bounds* on certain variables. The main purpose of this and the next section is to show that a large class of the multisectoral computable general equilibrium models can be seen as programming models with such flexible bounds. At the same time, through an illustrative example, we will point out some of the deficiencies of shadow prices and post-optimization analysis in the case of linear models.

2.2. A Simple Model with Bounded Export: Rigid Versus Flexible Bounds

We shall open the discussion by considering a simple example, concentrating our attention on the treatment of foreign trade. For the sake of simplicity we will use an extremely stylized, textbook type of model. We will assume that there is only one sector whose net output ($\overline{Y}$) is given (determined by available resources). The only allocation problem is to divide

$\overline{Y}$ into domestic use (C_d) and exports (Z). Exported goods will be exchanged for an imported commodity which is assumed to be a perfect substitute for the home commodity. Intermediate use will be neglected.

Following the traditional linear programming approach, export ($\overline{P}_E$) and import ($\overline{P}_M$) prices will be treated as (exogenously given) parameters of the model. Introducing M for the amount of imports purchased and C_m for the amount of imports used, our optimal resource allocation problem can be formulated in the following simple way

$$C = C_d + C_m \rightarrow \max$$

$$C_d + Z \leqq \overline{Y} \qquad (P_d)$$

$$C_m \leqq M \qquad (P_m)$$

$$\overline{P}_M M - \overline{P}_E Z \leqq 0 \qquad (V)$$

$$C_d, C_m, Z, M \geqq 0$$

where P_d, P_m, and V are the dual variables associated with the constraints, i.e., the shadow prices of domestic output, imports, and foreign currency, respectively.

The solution of the above problem obviously depends only on the relation of $\overline{P}_E$ and $\overline{P}_M$, i.e., on the terms of trade. The problem of overspecialization is illustrated here very clearly. If the terms of trade are favorable ($\overline{P}_E > \overline{P}_M$) then everything will be exported ($Z = \overline{Y}$) and only imported goods consumed ($C_d = 0$, $C_m = M = \overline{P}_E\, Z/\overline{P}_M$). However, if the terms of trade are unfavorable the optimal policy will be autarky.

Let us assume for a moment that the terms of trade are favorable at prices $\overline{P}_E$ and $\overline{P}_M$. The model builders will be aware of the fact that $\overline{P}_E$ is only an approximate value of the unit export price, and that at such a price the export markets could not absorb more than, say, an amount $\bar{Z}$ of export's. Introducing $\bar{Z}$ as an individual upper bound to Z would prevent the model producing a completely overspecialized solution. $\bar{Z}$ would clearly be binding* and the solution would be

$$Z = \bar{Z} \qquad C_d = \bar{Y} - \bar{Z} \qquad C_m = M = \bar{P}_E \, \bar{Z}/\bar{P}_M$$

It is also easy to see that the optimal values of the dual variables will be

$$P_d = P_m = V \, \bar{P}_M = 1 \quad , \quad t = V \, \bar{P}_E - P_d$$

where t is the shadow price of the individual bound, $\bar{Z}$.

We could therefore say that, in this simple situation, commodity prices are determined by the world market price of the substitute commodity; the higher export price is neutralized by an appropriate tax (t) on exports, which is determined as the shadow price of the individual export constraint.

The analysis of this hypothetical planning model should not stop here, however, for we know that $\bar{Z}$ is a constraint on export at given export prices $\bar{P}_E$. If we changed $\bar{P}_E$, would $\bar{Z}$ change too? Suppose that, at least within certain limits, the answer is yes, i.e., a decrease in the export price ($\bar{P}_E$) would increase the capacity for absorption of exports ($\bar{Z}$). In other words, the economy faces decreasing marginal export revenue or, what amounts to the same thing, less than perfectly elastic export demand. Let $D(P_E)$ be the export demand function. Instead of the rigid, fixed export bound ($\bar{Z}$) we could therefore use the following *flexible constraint:*

$$Z \leqq D(P_E)$$

simultaneously treating P_E as a variable in the balance of

*This is why we use the word "completely" in the preceding sentence. Instead of $\bar{Y}$, $\bar{Z}$ will now be the upper limit. This strong bound on Z will not qualitatively change the solution.

payments constraint. This would, however, turn our linear programming problem into a nonlinear one, which is generally more difficult to solve. To keep the linear programming framework intact we could adopt a piecewise linearization technique, as suggested, for example, by Srinivasan (1975).

As a third possibility we might try to save our linear programming model with a fixed export bound by means of appropriate *post-optimization analysis*, using the following argument. We know that $\overline{P}_E$ and $\overline{Z}$ are fixed only on the basis of some preliminary expectations concerning the volume of export and its foreign currency value. We have solved the model and found that the export constraint ($\overline{Z}$) is binding (its shadow price t is positive). This indicates that relaxing this constraint would increase the value of the objective function. We also know, however, that we can increase $\overline{Z}$ only by simultaneously decreasing $\overline{P}_E$. Thus we have to choose some other feasible combination of $\overline{P}_E$ and $\overline{Z}$, and solve the problem again. We continue to do this as long as t is positive, i.e., $\overline{Z}$ is binding.

In our simple case, it is not necessary to solve the model repeatedly, changing $\overline{P}_E$ and $\overline{Z}$ each time. It is clear that $\overline{Z}$ will remain binding as long as the terms of trade are favorable, and so the desired solution will be reached at $P_E = \overline{P}_M$. This assertion can easily be checked by analysis of the dual solution. Observe that $P_d = P_m = 1$ and $P_m = V\,\overline{P}_M$ in all solutions (independent of $\overline{P}_E$ and $\overline{Z}$). The equation $P_d = V\,\overline{P}_E - t$ implies that $t = 0$ when $\overline{P}_E = \overline{P}_M$.

The conditions fulfilled at the above solution are summarized below:

(1) $$P_d = \frac{\partial C}{\partial C_d} = 1 \qquad\qquad C_d + Z = \overline{Y} \qquad (5)$$

(2) $$P_m = \frac{\partial C}{\partial C_m} = 1 \qquad\qquad C_m = M \qquad (6)$$

(3) $$P_m = V\,\overline{P}_M \qquad\qquad P_E\,Z - \overline{P}_M\,M = 0 \qquad (7)$$

(4) $$P_d = V\,P_E \qquad\qquad Z = D(P_E) \qquad (8)$$

These eight equations in eight variables (C_d, C_m, Z, M, P_d, P_m, P_E, V) provide a formal representation of the necessary conditions for a *pure competitive* (Walrasian) *equilibrium*. Thus, our planning modeler could have reached the same solution by using a computable general equilibrium model instead of a parametric linear programming one.

If the trick has worked, the reader should by now be convinced that the above procedure is correct and that he has been given yet another example of the well-known close connection of linear programming and Walrasian competitive equilibrium.

The fact is, however, that the solution presented above is not actually the optimal solution. This can easily be checked, for example, by solving the nonlinear programming problem. Suppose the nonlinear problem is given in the same form as the original LP except that $\bar{P}_E$ is no longer a constant parameter but a function of Z [the inverse of $D(P_E)$]. The Kuhn-Tucker (necessary) conditions for the optimum will be equivalent to conditions (1)-(8), with one notable exception: instead of equation (4) we will have

$$P_d = V \left[P_E + \left(\frac{\partial P_E}{\partial Z} \right) Z \right] \tag{4'}$$

where we take the partial derivative of the Lagrangian with respect to Z. Introducing ε as the price elasticity of export demand, the above condition can be rewritten as

$$P_d = V\, P_E + V\, P_E \left(\frac{1}{\varepsilon} \right) = \left(\frac{1 + \varepsilon}{\varepsilon} \right) V\, P_E \tag{4''}$$

The difference between the two solutions can be explained plausibly in a number of ways. We will discuss one interpretation in a later section, connecting it to the optimum tariff problem and computable general equilibrium models.

Nevertheless, we can draw some useful conclusions from this simple and partly misleading exercise. First of all we have seen that traditional post-optimization analysis of shadow prices from linear programming models may give quite misleading information. In our example Z might have already been beyond its optimal level, but the shadow price of its upper bound would suggest pushing it even higher. Strangely enough, the competitive equilibrium model makes the same mistake by the very nature of its specification. We have also seen, however, that a slight modification of the competitive equilibrium framework enables us to provide solution for the nonlinear optimization problem.

Finally, the use of an export demand function in the nonlinear optimization problem can be seen as a natural way of transforming a rigid individual export bound into a variable, flexible limiting function. In the next subsection we will see that a similar flexible bound approach can also be used to treat imports.

2.3. Linear Model with Additional Constraints on Imports

As mentioned above, most linear programming models used for national resource allocation will contain individual bounds on imports as well as on exports. Typically, the ratio of imported goods used to domestic products used (m) will be forced to obey some constraint. In our original model the ratio $m = C_m/C_d$ is not constrained, and so we shall introduce m^+ and m^- as upper and lower bounds (respectively) on m. Our previous programming model will now have to be augmented by two additional constraints, which can be written jointly as

$$m^- C_d \leqq C_m \leqq m^+ C_d$$

Let t_m^- and t_m^+ denote the corresponding shadow prices. As a result of the modifications in the primal problem the dual constraints corresponding to C_d and C_m also have to be modified, as follows:

$$P_d = 1 - t_m^- m^- + t_m^+ m^+$$

$$P_m = 1 + t_m^- - t_m^+$$

Computable general euqilibrium models usually adopt a different approach (see also Section 3). There the dependence of the import share (m) is usually an explicit, continuous, smooth function of the ratio of the prices of domestic and imported commodities. In most cases, constant elasticity functions are used, such as the following:

$$m = m\left(P_d, P_m\right) = m_o \left(\frac{P_d}{P_m}\right)^{\mu}$$

In the linear programming case, observe that if the lower limit on imports is binding (neglecting degenerate solutions), then we will have $t_m^- > 0$ and $P_d < 1$, $P_m > 1$. If the upper limit is binding then $t_m^+ > 0$ and $P_d > 1$, $P_m < 1$. Otherwise $P_m = P_d$. Reversing the argument leads to the following conclusion. If the shadow price of the domestic commodity is less than that of the imported commodity, then we will not import more than the minimum required. If the shadow price of the domestic commodity is more than that of the imported commodity, we will import as much as possible. Otherwise the import volume will be determined by other considerations. We can write this formally as

$$m = m(P_d, P_m) = \begin{cases} m^- & \text{if } P_d/P_m < 1 \\ (m^-, m^+) & \text{if } P_d/P_m = 1 \\ m^+ & \text{if } P_d/P_m > 1 \end{cases}$$

Thus, the import share can formally be treated as a function of relative prices like in a computable general equilibrium model, although in this case the function is not smooth (see Figure 1).

It is worth noting here that essentially the same restrictions on imports could have been achieved by modifying the objective function rather than introducing new constraints. So far we have assumed a simple additive objective function: $C = C_m + C_d$. If, however, we introduced a piecewise linear objective function with indifference curves as illustrated in Figure 2, then we would in effect restrict the import share by the same lower (m^-) and upper (m^+) bounds as before. This type

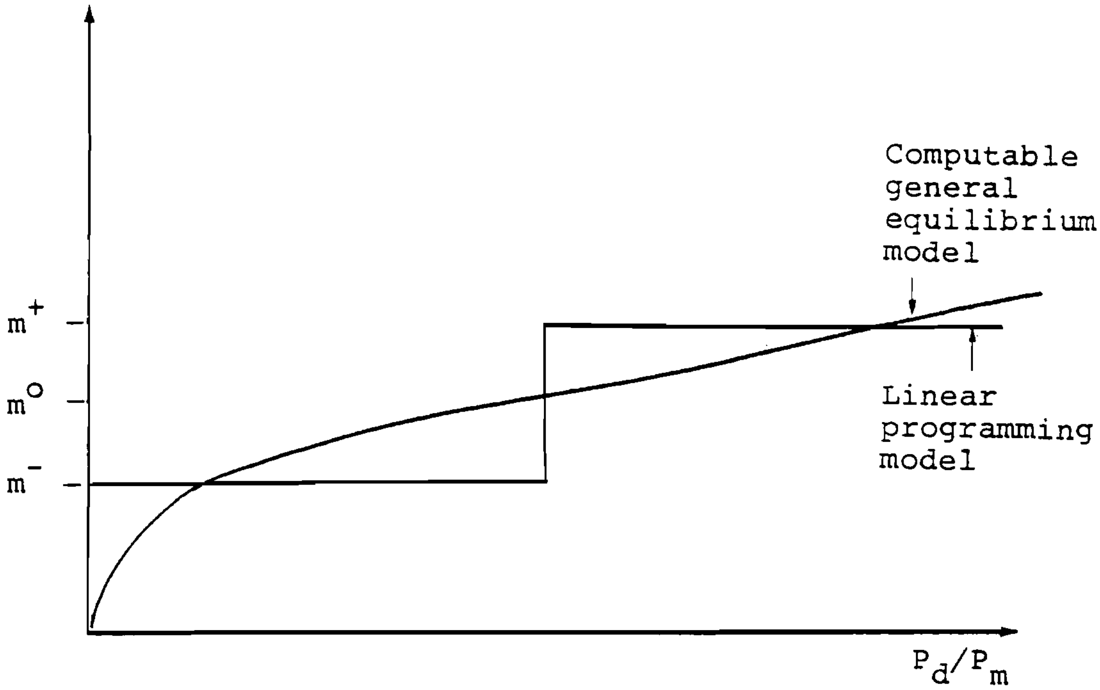

Figure 1. Import share functions.

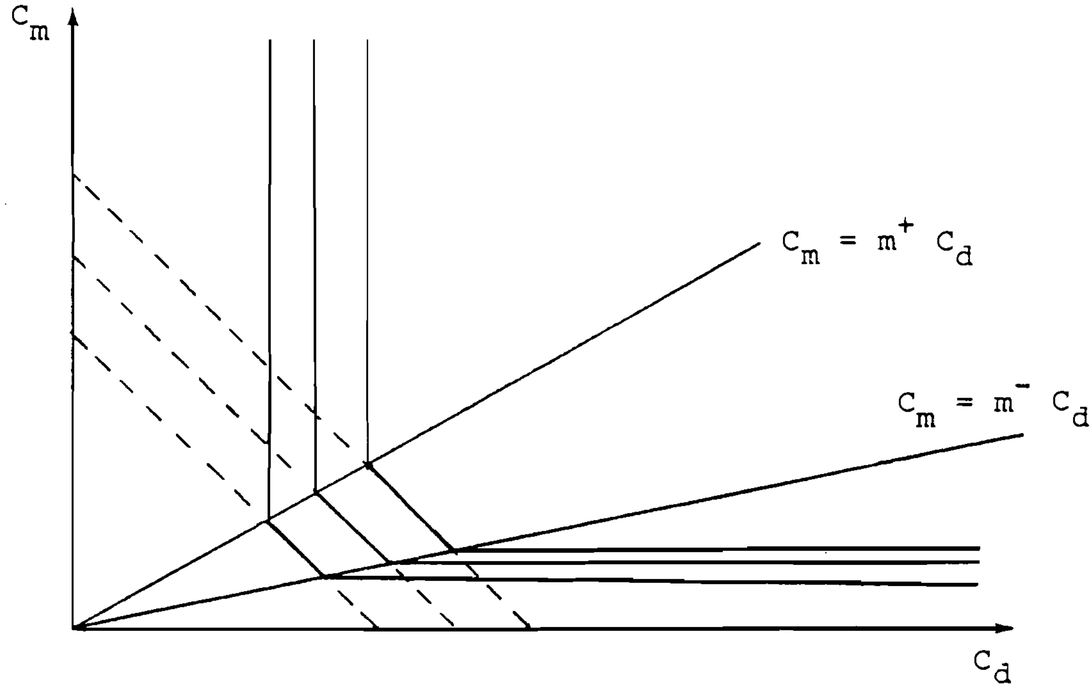

Figure 2. Import restriction built into the objective function.

of objective function could be viewed as the planners preference (utility) function with respect to the composition of total source (domestically produced versus imported goods).

The above interpretation actually seems to be even more meaningful than the competitive equilibrium interpretation. In the latter case the adoption of a relative-price-dependent import share function is usually justified on the grounds of neoclassicial utility theory. The typical argument goes as follows. Suppose consumers across all areas of use have the same CES-type utility function (preferences) with respect to domestically produced and imported variants of the same commodity. Suppose also that, when consumers make their choice, they try to minimize the cost of achieving some given level of utility. This assumed behavior would lead to the constant elasticity (relative-price-dependent) import demand function quoted earlier. (See Appendix 1 for an analytical derivation of the demand function).

We should emphasize that the difference in the treatment of import restrictions between linear programming models and computable equilibrium models can once again be seen as the difference between *fixed* (rigid) and *flexible* individual bounds. The relative-(shadow or equilibrium)-price-dependent import share implies a variable (flexible) individual bound on imports. The larger the gap between the shadow prices of the domestic and imported commodities the larger the deviation from the observed (or planned) import ratio (m_o).

In fact, allowing for a smooth variation of the import share around its proposed level in a planning model makes at least as much sense as the usual import restrictions. Smooth import share functions could be incorporated into an otherwise linear model without destroying its linear character, through the use of piecewise linearization*. In many cases, however, it might turn out to be more advantageous to transform the model into either nonlinear programming form or computable general equilibrium form.

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*Ginsburgh and Waelbroeck (1981) give examples showing how piecewise linear (nonlinear) relationships can be introduced into linear programming models and outline some applications.

To close this section on programming models, we shall examine the effect of replacing the fixed bounds in our example with flexible ones. Suppose we have a linear programming model with fixed individual bounds on both exports and import shares:

$$C = C_m + C_d \quad \max$$

$$C_d + Z \leqq \overline{Y} \qquad (P_d)$$

$$C_m \leqq M \qquad (P_m)$$

$$\overline{P}_M M - \overline{P}_E Z \leqq 0 \qquad (V)$$

$$m^- C_d \leqq C_m \leqq m^+ C_d \qquad (t_m^-, t_m^+)$$

$$Z \leqq \overline{Z} \qquad (t_e)$$

If we want to replace the fixed individual bounds by flexbiel ones, as described earlier, we should proceed in the following way. We can rewrite the above linear model in nonlinear form by replacing the objective function with one reflecting import limitations and introducing an export demand function as before. These changes yield the following model (using constant elasticity forms):

$$C = (h_m C_m^{-\eta} + h_d C_d^{-\eta})^{-1/\eta}$$

$$C_d + Z \leqq \overline{Y} \qquad (P_d)$$

$$C_m \leqq M \qquad (P_m)$$

$$\overline{P}_M M - DZ^{(1+\varepsilon)/\varepsilon} \leqq 0 \qquad (V)$$

Appendix 1 describes how the parameters h_m, h_d, and η can be determed from m_o and μ (the parameters of the import share function) and *vice versa*. Parameter D in the foreign trade balance is a constant term obtained by solving the following export demand function for P_E:

$$Z = e_o \left(\frac{P_E}{\overline{P}_{WE}}\right)^{\varepsilon}$$

where $\overline{P}_{WE}$ is the export price charged by competitors (exogenous variable) and e_o is ā scaling parameter. Solving the above equation for P_E yields

$$P_E = e_o^{-1/\varepsilon}\ \overline{P}_{WE}\ Z^{1/\varepsilon} = D\ Z^{1/\varepsilon}$$

With reasonable values for the parameters, we can expect to obtain an interior solution. By interpreting P_d, P_m, and V as Lagrangian multipliers for the corresponding constraints, the first-order necessary (Kuhn-Tucker) conditions for a maximum can be stated as follows:

$$P_d = \frac{\partial C}{\partial C_d} \qquad (1.1)$$

$$P_m = \frac{C}{C_m} \qquad (1.2)$$

$$P_m = V\ \overline{P}_m \qquad (1.3)$$

$$P_d = \left(\frac{1+\varepsilon}{\varepsilon}\right) DVZ^{1/\varepsilon} = \left(\frac{1+\varepsilon}{\varepsilon}\right) V\ P_E \qquad (1.4)$$

We can show (see Appendix 1) that conditions (1.1) and (1.2) actually yield the import share function

$$m = m_o \left(\frac{P_d}{P_m}\right)^{\mu}$$

It is also fairly easy to see that we can replace the above programming model by the following system of simultaneous equations:

$$(2.1) \quad P_m = V \bar{P}_M \qquad\qquad Z = a \left(\frac{P_E}{\bar{P}_{WE}} \right)^{\varepsilon} \quad (2.5)$$

$$(2.2) \quad P_E = \left(\frac{\varepsilon}{1 + \varepsilon} \right) \frac{P_d}{V} \qquad\qquad C_d + Z = \bar{Y} \quad (2.6)$$

$$(2.3) \quad m = m_o \left(\frac{P_d}{P_m} \right)^{\mu} \qquad\qquad C_m = M \quad (2.7)$$

$$(2.4) \quad C_m = m\, C_d \qquad\qquad \bar{P}_M M - P_E Z = 0 \quad (2.8)$$

This is already very close to a typical specification of a computable general equilibrium model. To see this more clearly we will turn our attention to computable equilibrium models in the next section and come back to the above model later.

We close this subsection with a brief discussion of the equation system derived above. Counting the variables (m, C_d, C_m, M, Z, P_m, P_d, P_E, V), we find that there is one more variable than there are equations. This might lead to problems of overdetermination. However, observe that all the equations are homogeneous of degree zero in variables P_m, P_d, and V, and thus the level of one of these variables can be chosen freely. Alternatively, if we want to reproduce the level of the Lagrangian multipliers, we could introduce an appropriate "scaling" constraint as, for example, the following one:

$$P_d C_d + P_m C_m = C_d + C_m$$

3. FOREIGN TRADE IN COMPUTABLE GENERAL EQUILIBRIUM MODELS

In this section we will first outline the argument that underlies most computable general equilibrium models, making use of essentially the same simple resource allocation problem as before. We will then deal with the choice of export function (pure demand, pure supply, or combined) and its effect on the rest of the model. The analysis of optimum and equilibrium solutions will be postponed to the next section.

3.1. Imperfectly Elastic Export and Import Demand

Suppose that there are four collections of economic agents: suppliers and buyers in the home country and those in the rest of the world. Each set contains enough individual agents to ensure that none of them can have a significant influence on prices (they are all price takers). Suppliers of the domestically produced commodity (total available amount $\overline{Y}$) can choose whether to sell at home or abroad. They are assumed to be perfectly elastic, and thus, if at equilibrium they sell on both home and foreign markets, the prices on the two markets must be equal: $P_d = V\ P_E$.

Supplies from the rest of the world are also assumed to be perfectly elastic with no supply constraint (i.e., the home country is small). The price of the imported commodity is set exogenously at level $\overline{P}_M$. Following Armington's assumption of regionally differentiated commodities, demand in both the home country and the rest of the world is assumed to be less than perfectly elastic.

It is assumed that domestic consumers allocate their income ($P_d\overline{Y}$) between domestic and imported commodities in such a way that their aggregate utility

$$C = (h_d\ C_d^{-\eta} + h_m\ C_m^{-\eta})^{-1/\eta}$$

will be maximized. (This CES utility function is assumed to represent the regional bias in taste towards otherwise identical commodities.)

The necessary conditions for the above maximum can be expressed in many different ways (see Appendix 1). The most convenient form for our purposes is represented by the following three equations in the three variables m, C_m, and C_d:

$$m = m_o \left(\frac{P_d}{P_m}\right)^{\mu}$$

$$C_m = m\ C_d$$

$$P_m C_m + P_d C_d = P_d \bar{Y}$$

where $P_m = V\bar{P}_M$ in a pure competitive equilibrium.

Similarly, all other components being given, the demand of the rest of the world for the commodity exported by the home country will be a monotone decreasing function of the proposed export price (P_E). Following the tradition of computable general equilibrium modeling, we might specify the demand function in the following (constant price elasticity) form:

$$Z = D(P_E) = e_o \left(\frac{P_E}{\bar{P}_{WE}} \right)^{\varepsilon}$$

where ε (the export price elasticity), $\bar{P}_{WE}$ (the price offered by competitors on the world market), and e_o (a scale parameter) are all given exogenously.

We can thus summarize the conditions for competitive equilibrium as the following system of equations, in which the endogenous variables are m, C_d, C_m, M, Z, P_d, P_m, P_E, and V.

Price Identities

$$P_m = V\ \bar{P}_M \tag{3.1}$$

$$P_E = \frac{P_d}{V} \tag{3.2}$$

Demand Functions

$$m = m_o \left(\frac{P_d}{P_m} \right)^{\mu} \tag{3.3}$$

$$C_m = m_o C_d \tag{3.4}$$

$$Z = e_o \left(\frac{P_E}{\bar{P}_{WE}} \right)^{\varepsilon} \tag{3.5}$$

Market Clearing Conditions

$$C_d + Z = \overline{Y} \tag{3.6}$$

$$C_m = M \tag{3.7}$$

Current Account Balance

$$\overline{P}_M M - P_E Z = 0 \tag{3.8}$$

The above set of equations does not explicitly contain the consumers' budget constraint. This can, however, be derived from equations (3.6)-(3.8) with the help of the price identities (3.1) and (3.2) (Walras' law). It is also easy to see that all equations are homogeneous of degree zero in P_d, P_m, and V, so that one of these variables can be chosen freely. We therefore have eight equations in eight variables, which, under the usual assumptions on the parameters, will have a unique solution.

3.2. Imperfectly Elastic Export Supply

If we look at the export-import specification in typical numerical general equilibrium models for a single country, we find that demand is generally assumed to be inelastic, whereas supply is perfectly elastic*. There are only a few exceptions to this assumption. The basic reason for introducing inelastic export and import functions is to overcome the problem of over-specialization in models with linear homogeneous production relations. As mentioned earlier, the usual approach is based on Armington's (1969) assumption and typically constant (relative price) elasticities are assumed.

In most cases, and especially for small economies, it would be at least as natural to take into account limitations and

*See, for example, References 1, 3, 7, 8, 10, 16, 20.

rigidities in supply. This can be done, for example, by introducing less than perfectly elastic supply functions*, which under constant elasticity assumptions could take the following form**:

$$Z = a\left(\frac{P_d}{VP_E}\right)^{\alpha}$$

It is interesting to note that perfectly elastic supply combined with imperfectly*** elastic demand (the standard assumption) leads formally to the same export function as the opposite assumption, namely, imperfectly elastic supply with perfectly elastic demand. (It will be shown later that the same export function is obtained when both supply and demand are imperfectly elastic.)

To prove the above assertion, first observe that perfectly elastic export supply means that $P_E = P_d/V$. Substituting P_d/V for P_E in the export demand function yields:

$$Z = e_o\left(\frac{P_d}{V\bar{P}_{WE}}\right)^{\varepsilon} \qquad \text{(DF)}$$

Next, observe that perfectly elastic export demand means that $P_E = \bar{P}_{WE}$, that is, export prices are dictated by the world market. If we substitute this into our export supply function we obtain

$$Z = a\left(\frac{P_d}{V\bar{P}_{WE}}\right)^{\alpha} \qquad \text{(SF)}$$

---

*Export supply functions combined with a falling export unit price were adopted in Zalai (1980). A recent model for Sweden (Bergman and Pór, 1982) defines export supply functions as derived on the basis of neoclassical joint production models.

**Alternative forms include production (capacity) as an "explanatory variable" (see, for example, the neoclassical joint production approach referred to above), and this is particularly common in econometric estimations. See, for example, Sato (1977), Goldstein and Khan (1978).

***We will use the term imperfectly elastic in the sense of less than perfectly elastic, but not perfectly inelastic.

3.3. Equilibrium of Imperfectly Elastic Supply and Demand

If both demand and supply are imperfectly elastic, we can proceed in the following way. We may solve the demand correspondence for P_E, which gives us

$$P_E = e_o^{-1/\varepsilon} \, \bar{P}_{WE} Z^{1/\varepsilon}$$

Substitution of this expression for P_E in the export supply function and solving for Z yields

$$Z = c \left(\frac{P_d}{V\bar{P}_{WE}} \right)^{\gamma} \qquad \text{(EF)}$$

where

$$c = (a^{\varepsilon} \, e_o^{\alpha})^{\frac{1}{\alpha+\varepsilon}} = \overline{ga} \; (a, e_o)$$

$$\gamma = \frac{\alpha\varepsilon}{\alpha + \varepsilon} = \frac{1}{\frac{1}{\alpha} + \frac{1}{\varepsilon}} = \frac{1}{2} \, \overline{ha} \; (\alpha, \varepsilon)$$

Thus, the "pure export demand" (DF), "pure export supply" (SF), and "supply-demand equilibrium (EF) export functions have identical mathematical forms in our constant elasticity specification. This may imply that, in practice, it might be rather difficult to distinguish between the estimates given by the various specifications.

Note also that the equilibrium specification is in some sense an "average" of the pure supply and demand specifications (the scaling parameter is the geometric average and the elasticity is *half* of the harmonic average of the corresponding "pure" parameters). It is interesting to see that the "equilibrium elasticity" is less than either the supply or the demand elasticity, and this may partially explain why empirical estimates of the export demand elasticity tend to be rather small, even for small economies.

We should emphasize that our remarks on probable empirical findings are very hypothetical. Econometric estimates of export functions are scarce and unfortunately very unreliable, and estimates of elasticities are especially sensitive to differences in samples, estimation techniques, and model specification*. This points to the need for special care in choosing both the kind of export specification and the size of parameters. We will come back to this problem later in section 5.

Repeating our main conclusion, then, we have found that export functions determined on the basis of pure supply or pure demand or supply-demand equilibrium have the same algebraic form. Does this mean that it makes no difference which export specification is used in a general equilibrium model? Of course not. The difference will show up in the relative export earnings, i.e., in the current account balance: the income earned per unit exported (P_E) will be equal to P_d/V (endogenous) in the pure demand case and $\overline{P}_{WE}$ (exogenous) in the pure supply case. It is relatively easy to show** that, in the equilibrium case, the following relationship will hold:

$$P_E = \left(\frac{b}{a}\right)^{\frac{1}{\varepsilon+\alpha}} \left(\frac{P_d}{V}\right)^{\frac{\alpha}{\varepsilon+\alpha}} \overline{P}_{WE}^{\frac{\varepsilon}{\varepsilon+\alpha}}$$

Thus, in this case, the export price will be basically equal to the geometric average of the exogenous world market price and the domestic price divided by the exchange rate (this may be modified by a term which, in principle, should not be significantly different from 1).

The main characteristics of the different export specifications are summarized in Table 1. The table contains all possible pairs of supply-demand elasticity situations, even though some of them are not relevant (as they stand) in neoclassical general equilibrium models. It should be borne in mind that export

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*See, for example, Houthakker and Magee (1969), Hickman and Lau (1973), Sato (1977), Goldstein and Khan (1978), Stone (1979), and Browne (1982).

**First solve the demand correspondence for P_E, then substitute the supply term for Z into the resulting equation, and finally solve this new equation for P_E.

Table 1. The effect of the elasticity of export demand and supply on model specification.

Demand	Supply: Perfectly elastic[a] ($\alpha = -\infty$)	Supply: Imperfectly elastic ($-\infty > \alpha > 0$)	Supply: Perfectly inelastic ($\alpha = 0$)
Perfectly elastic[b] ($\varepsilon = -\infty$)	*Standard Model* $P_E = \frac{P_d}{V} = \overline{P}_{WE}$ Z determined by other factors (overspecialization may occur)	*Pure Supply* $P_E = \overline{P}_{WE}$ $Z = a\left(\frac{P_d}{V\,\overline{P}_{WE}}\right)^{\alpha}$	*Rigid Supply Constraint* (a) $P_E \leqq \overline{P}_{WE}$ $Z \leqq a$
Imperfectly elastic ($-\infty > \varepsilon > 0$)	*Pure Demand* $P_E = \frac{P_d}{V}$ $Z = e_o\left(\frac{P_d}{V\,\overline{P}_{WE}}\right)^{\varepsilon}$	*Supply-Demand Equilibrium* $P_E = \left[\left(\frac{a}{e_o}\right)\left(\frac{P_d}{V}\right)^{\alpha}\ \overline{P}_{WE}^{\ \varepsilon}\right]^{1/(\alpha+\varepsilon)}$ $Z = \left(a^{\varepsilon}\ \ e_o^{\alpha}\right)^{1/(\alpha+\varepsilon)}\left(\frac{P_d}{V\,\overline{P}_{WE}}\right)^{\alpha\varepsilon/(\alpha+\varepsilon)}$	*Fixed Export Supply* $P_E = \left(\frac{a}{e_o}\right)^{1/\varepsilon}\ \overline{P}_{WE}^{\ \varepsilon}$ $Z = a$
Perfectly inelastic ($\varepsilon = 0$)	*Rigid Demand Constraint* (e_o) $P_E \geqq \frac{P_d}{V}$ $Z \geqq e_o$	*Fixed Export Demand* $P_E = \left(\frac{a}{e_o}\right)^{1/\alpha}\ \frac{P_d}{V}$ $Z = e_o$	*Both Fixed* No adjustment possible

[a] Export price follows domestic price.
[b] Export price follows world market price.

functions are only discussed here as part of more complicated (multisectoral) models.

We should perhaps point out, and this is important from a computational point of view, that the usual demand-specified general equilibrium model can easily be modified to allow for alternative export specifications. All that is necessary is to replace (3.2) and (3.5) by the following equations

$$P_E = \left(\frac{a}{e_o}\right)^{\frac{1}{\varepsilon+\alpha}} \left(\frac{P_d}{V}\right)^{\frac{\alpha}{\varepsilon+\alpha}} \left(\bar{P}_{WE}\right)^{\frac{\varepsilon}{\varepsilon+\alpha}} \tag{3.2}$$

$$Z = c \left(\frac{P_d}{V\ \bar{P}_{WE}}\right)^{\gamma} \tag{3.5}$$

where c and γ are determined as above. If either α or ε decreases beyond a certain limit, our specification will reduce to the pure supply or demand case.

Figures 3 and 4, which are based on numerical simulations, summarize in geometrical form the main features of the alternative export specifications. The horizontal axis is a measure of export volume (Z) in both cases. The vertical axis represents the unit export price (P_E) in Figure 3 and the foreign currency equivalent of the domestic price (P_d/V) in Figure 4. The elasticities of supply and demand are -3 and -2, respectively, and therefore the export elasticity in the equilibrium specification will be -1.2. The figures illustrate the impact of a 10 percent change in P_d/V on the export volume in each of the three cases, and show that the amount exported increases by 37, 23, and 13 percent under supply, demand, and equilibrium specifications, respectively.

4. OPTIMUM TARIFF IN A SMALL OPEN ECONOMY

4.1. The Optimum Tariff Problem and Applied Models

In the previous two sections we have discussed some foreign trade issues as they appear in multisectoral macroeconomic models designed for numerical simulation. We have basically developed

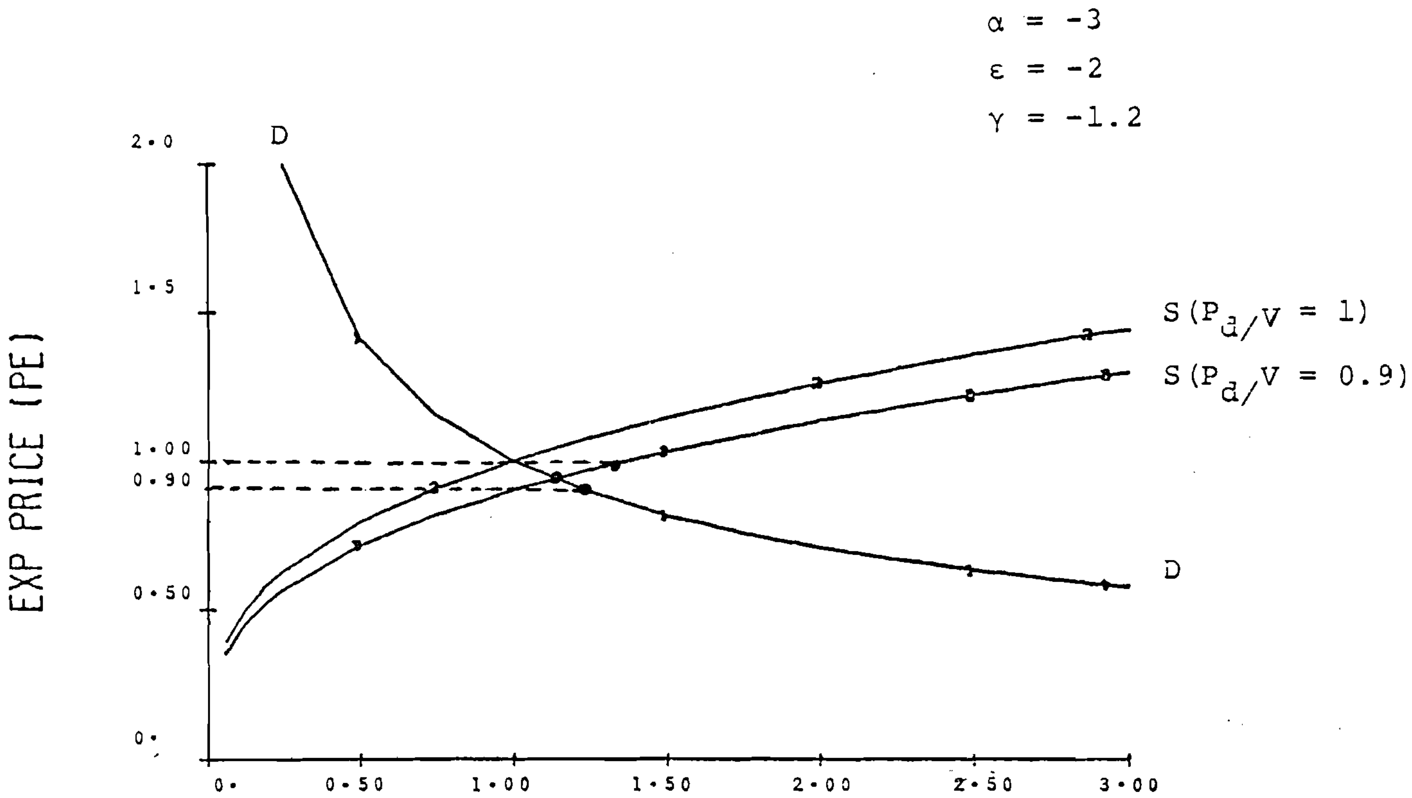

Figure 3. Export demand (D) and supply (S) as function of the export price (P_E)

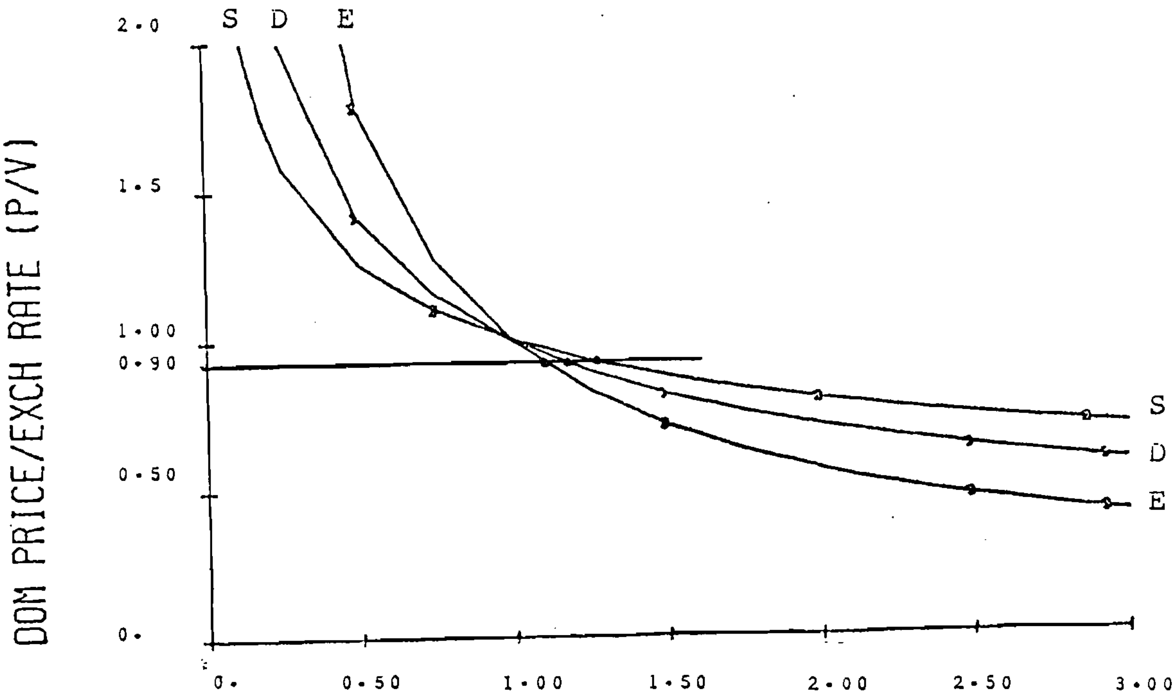

Figure 4. Demand (D), supply (S) and equilibrium (E) export functions

two simple theoretical models for comparison. One is a nonlinear programming model, obtained from its more traditional linear counterpart by introducing flexible rather than rigid individual bounds on export and import activities. The other model is an equation system representing the necessary conditions for a purely competitive (*laissez-faire*) equilibrium. We have also seen that this equation system and the first-order necessary (Kuhn-Tucker) conditions for the optimum in the programming model are almost, but not completely, identical.

The difference between the two sets of conditions is not a surprising one, in the light of the theoretical literature on international trade. This phenomenon has long been recognized as the "optimum tariff" problem (see, for example, Dixit and Norman 1981) or as the difference between the planner's optimum (welfare optimum) and the pure competitive (*laissez-faire*) equilibrium (see, for example, Srinivasan 1982). It is well known that in many situations a welfare optimum solution can only be sustained as a competitive equilibrium regulated by appropriate "optimum" taxes or subsidies, or through direct government intervention.

Although the problem is familiar and has been discussed at length in the theoretical literature, it has not been recognized as a possible source of concern in computable general equilibrium models. It is not clear why this is so; perhaps the unfortunate notion of a "small open economy" is partly responsible. (A small open economy is defined as one facing exogenously determined export-import prices.) The optimum tariff problem seems to have been discussed only in terms of "large open economies." Many of the computable models were designed for small economies and, as explained earlier, the adoption of Armington's assumption was dictated only by a pragmatic concern with overspecialization. Perhaps it was not apparent that the adoption of such an innocent assumption would change the otherwise small economy into a "large" one. Another partial explanation may lie in the ideological values associated with the concepts of pure competition and monopoly power ("it would be unfair if a country made use of its monopoly power in international trade").

The unqualified coupling of equilibrium and Pareto optimum could also have contributed to this lack of concern.

Whatever the case, it remains a fact that the optimum tariff problem is seen to distinguish multisectoral planning models of programming type from those of general equilibrium type. However, this is not actually so. In most cases it is easy to alter the general equilibrium model and its solution algorithm so as to derive the planner's optimum instead of the *laissez-faire* equilibrium (see Subsection 4.2). Thus a choice must be made. This choice is usually quite important because, as will be seen in the next section, the export specification can significantly affect the solution.

It is interesting that the optimum may be different from the *laissez-faire* equilibrium, even if the economy is "small and open" in the sense of facing exogenously given terms of trade. This side of the optimum tariff problem is not emphasized in the literature but seems to be quite important. It can be associated with short-run inflexibility in export supply, and may give rise to both taxes and subsidies (not only to taxes as in the classical optimum tariff problem). This will be discussed in Subsection 4.3. The practical lessons to be drawn from the theoretical discussion will be treated in Section 5.

4.2. Optimum and Equilibrium: Perfectly Elastic Supply

Let us examine the equation systems characterizing the optimal solution (equations 2.1 to 2.8), and the competitive equilibrium (equations 3.1 to 3.8). We see that they differ in only one pair of equations, namely, equations (2.2) and (3.2):

$$P_d = V\ P_E$$

$$P_d = (1 + t_e)\ V\ P_E = \left(\frac{1 + \varepsilon}{\varepsilon}\right) V\ P_E$$

The difference can be explained by the following familiar argument. The optimum can be achieved in an otherwise fully competitive system by introducing an *ad valorem* tax on exports. Since supply is assumed to be perfectly elastic, domestic suppliers will offer their products abroad at a price rate $[\varepsilon/(1+\varepsilon)\ P_d/V]$ (expressed in foreign currency), generating an equilibrium export demand equal to its optimal volume*.

It is also useful to look at the difference between the two solutions from a different angle. Recall that the planner's optimum can be determined by solving the following programming problem**:

$$C = (h_d\, C_d^{-\eta} + h_m\, C_m^{-\eta})^{-1/\eta} \rightarrow \max$$

$$C_d + Z \leqq \bar{Y} \qquad (P_d)$$

$$\bar{P}_M M - e_o^{-1/\varepsilon}\, \bar{P}_{WE} Z^{(1+\varepsilon)/\varepsilon} \leqq 0 \qquad (V)$$

$$C_d,\ C_m,\ Z \geqq 0$$

It is fairly easy to see that the pure competitive solution can be found by means of a parametric programming problem of the following form:

$$C = (h_d\, C_d^{-\eta} + h_m\, C_m^{-\eta})^{-1/\eta} \rightarrow \max$$

$$C_d + Z \leqq \bar{Y} \qquad (P_d)$$

$$\bar{P}_M\, C_m - \left(\frac{\varepsilon}{1+\varepsilon}\right) e_o^{-1/\varepsilon}\, \bar{P}_{WE} Z^{(1+\varepsilon)/\varepsilon} \leqq k \qquad (V)$$

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*It is interesting to note that most econometric estimates of export elasticities lie between the values -1 and -3 (see papers referred to earlier) and that such values are usually adopted in numerical general equilibrium models. Observe that $\varepsilon = -1.5$ implies a tax rate of 200 percent (i.e., two-thirds of the revenue is taxed away!); $\varepsilon = -2$ corresponds to 100 percent; $\varepsilon = -3$ to 50 percent, and so on.

**We have already shown that $C_m = M$ in the optimal solution and therefore our programming problem has only three variables and two constraints. The other variables and equations can, of course, also be derived from this model.

$C_d, C_m, Z \geqq 0$

The underlying idea is very simple. The planner's optimum model has been modified in such a way that its dual satisfies the equilibrium pricing requirements. This has been achieved simply by multiplying the export term in the foreign currency constraint by $\varepsilon/(1 + \varepsilon)$ in order to offset the "monopoly distortion" effect. This change, however, alters the meaning of the foreign currency condition, and this must be taken into account in the method of solution. This is achieved by varying the left-hand side (k) parametrically until the solution (C_m and Z, in particular) also satisfies the original current account condition*.

Figure 5 throws more light on the nature of the competitive equilibrium solution. The horizontal axis is primarily a measure of Z, but the difference between $\bar{Y}$ and Z also yields C_d. The vertical axis measures C_m. Thus, we can represent the indifference curves (involving C_m and C_d), the balance of payment condition, and the second constraint of the programming problem all on the same figure.

The curve from 0 to d = 0 represents the export-import combinations fulfilling the current account requirement. Notice that the only difference between the latter and the second constraint in the programming model at k = 0 is that the export term is multiplied by the constant $\varepsilon/(1 + \varepsilon)$, which is assumed to be greater than 1. Hence, the points satisfying this latter constraint are found on the curve from 0 to k = 0, which lies above and is steeper than the current account curve. Thus the optimal solution of the programming problem at k = 0 clearly cannot meet the current account requirement. If we change k parametrically then the optimal solutions will lie on the curve $S\bar{Y}$. The competitive equilibrium solution is found where this latter curve intersects the current account curve**.

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*Lundgren (1982) proposed an algorithm of this type for solving a special type of multisectoral equilibrium model which could incorporate nonmsooth relationships.

**See Appendix 2 for the derivation of these results.

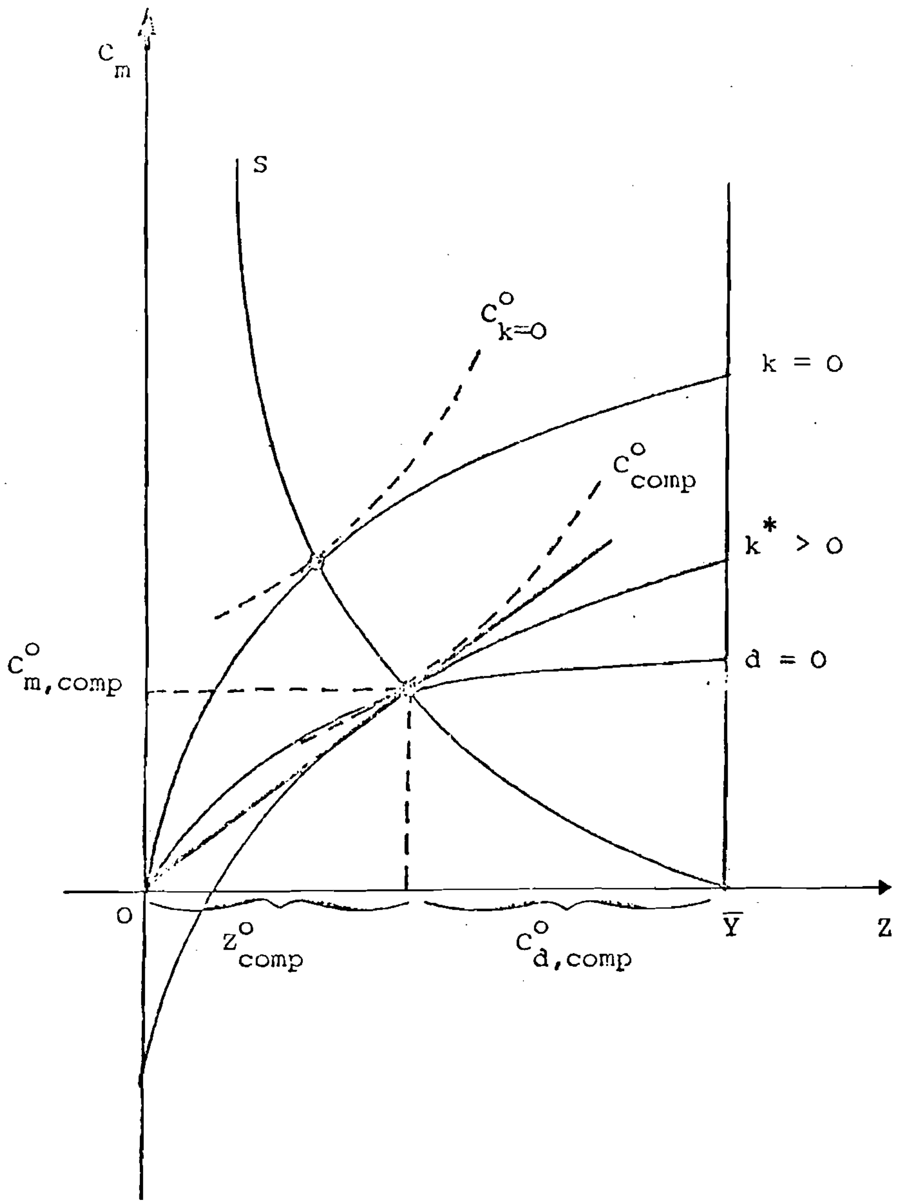

Figure 5. Finding the equilibrium solution by parametric programming.

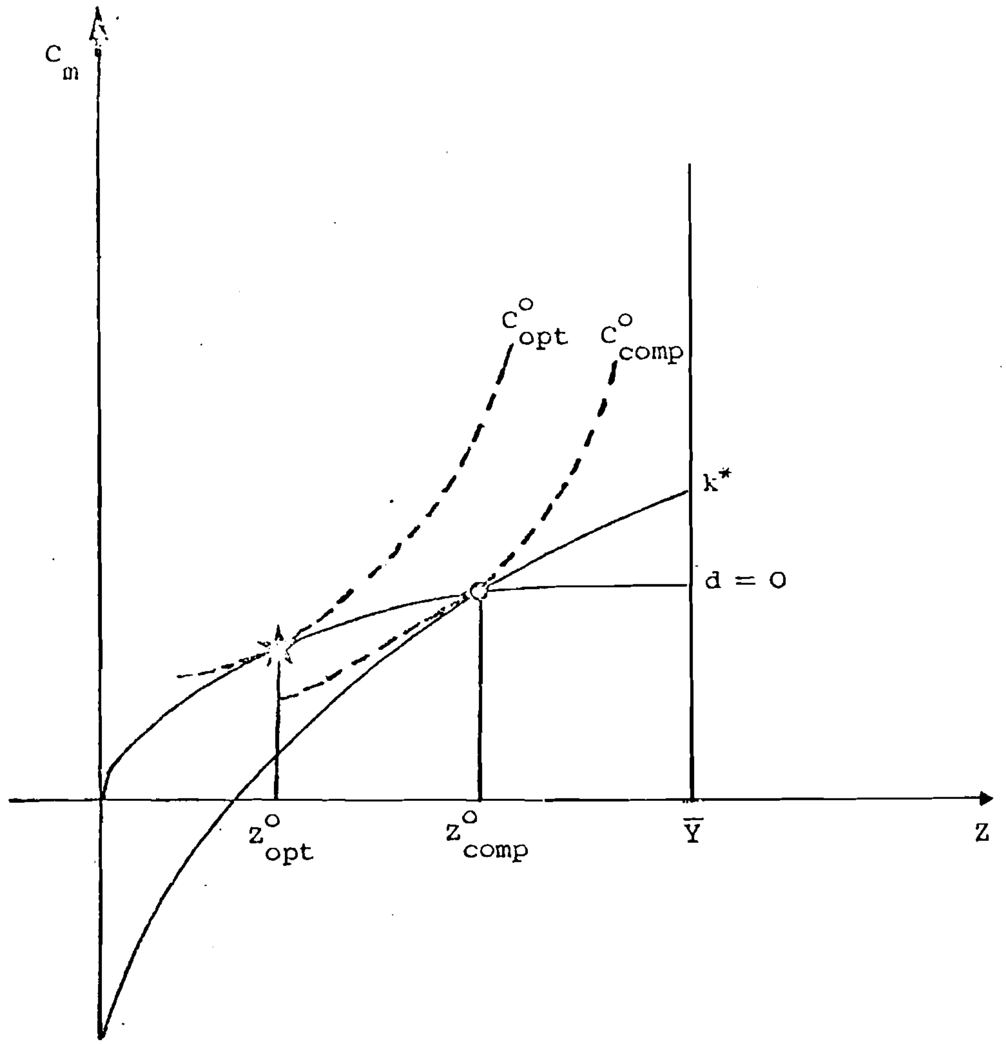

Figure 6. Planners optimum (★) and pure competitive equilibrium (●).

It is clear from Figure 5, although it is even more apparent in Figure 6, that the pure competitive equilibrium cannot be optimal. For an optimal solution the indifference curve and the current account constraint must be tangential to each other (see Figure 6). However, in the competitive equilibrium case the two curves intersect and a small movement along the current account curve toward the origin would increase the value of the objective (utility) function.

It is also interesting to note that the common tangent to the indifference curve and the transformed current account curve at the equilibrium solution is the consumers' budget line. This line will pass through the origin (no foreign trade), since this is clearly an admissible and budget-exhaustive consumption pattern at equilibrium. (This is true when the only source of income is the sale of domestic resources. Observe, however, that this is not so for an optimal plan in which taxes on exports form an additional source of income.)

The above argument has demonstrated how nonlinear programming methods can be used to compute equilibrium solutions for certain types of models. In the case of most general equilibrium models, however, the solution algorithm is tailored to the specific model and therefore will probably be more efficient than some general-purpose algorithm. Thus, it may be better to keep the equilibrium-searching algorithm. As we have shown, it is usually quite easy to alter the specification and solution algorithm of the equilibrium model (by introducing a tax on exports, for example) to obtain an optimal solution.

It is sometimes difficult to tell whether the more complicated empirical models are perfectly consistent with neoclassical competitive equilibrium theory, and thus it may happen that the introduction of tariffs will not produce the "best" solution. It may also be difficult to define a welfare function which could be used to check whether there was any improvement on introducing tariffs (when, for example, there is more than one consumer). In such cases special optimization techniques might be used to determine the "second best" solution.

4.3. Optimum Tariffs in a Small Economy: The Case of Imperfectly Elastic Export Supply

So far we have examined the usual optimum tariff argument within a special framework. The optimum tariff situation is generally associated with large economies (which have a kind of monopoly power over their export prices and potential buyers), but we have seen that it is not necessarily limited to such "large" economies, at least not in the usual sense. This claim may, however, be rejected on the grounds that it is simply a question of definition (that a small economy is defined as a price-taker on the world market!) and, as such, is a matter of taste and completely uninteresting.

Other readers may not be convinced that the optimum tariff argument always leads to taxes on exports and never to subsidies. Indeed, in practice we generally find a complicated system involving both taxes and subsidies regulating foreign trade.

For both of the above reasons it would be interesting to show that optimum tariff situations do arise in small open economies. We will demonstrate this in a case in which not only taxes but also subsidies may emerge as a means of optimal regulation. It should not be surprising that this type of situation is caused by frictions and constraints that make the export supply less than perfectly elastic (at least in the short-to-medium run).

Let us now consider a small open economy as defined in conventional (neoclassical) international trade theory, once again using an abstract theoretical model to highlight the problem. We assume that there is only one commodity involved in a pure exchange situation, that world market prices ($\overline{P}_E$ and $\overline{P}_M$) are given exogenously, and we make use of Armington's assumption only in describing demand in the home country. Figure 7 illustrates the problem to be investigated.

To add some realism to our abstract problem, let us suppose the following familiar situation. After some major deterioration in her terms of trade, the home country adopts a policy of borrowing instead of curtailing domestic consumption. This

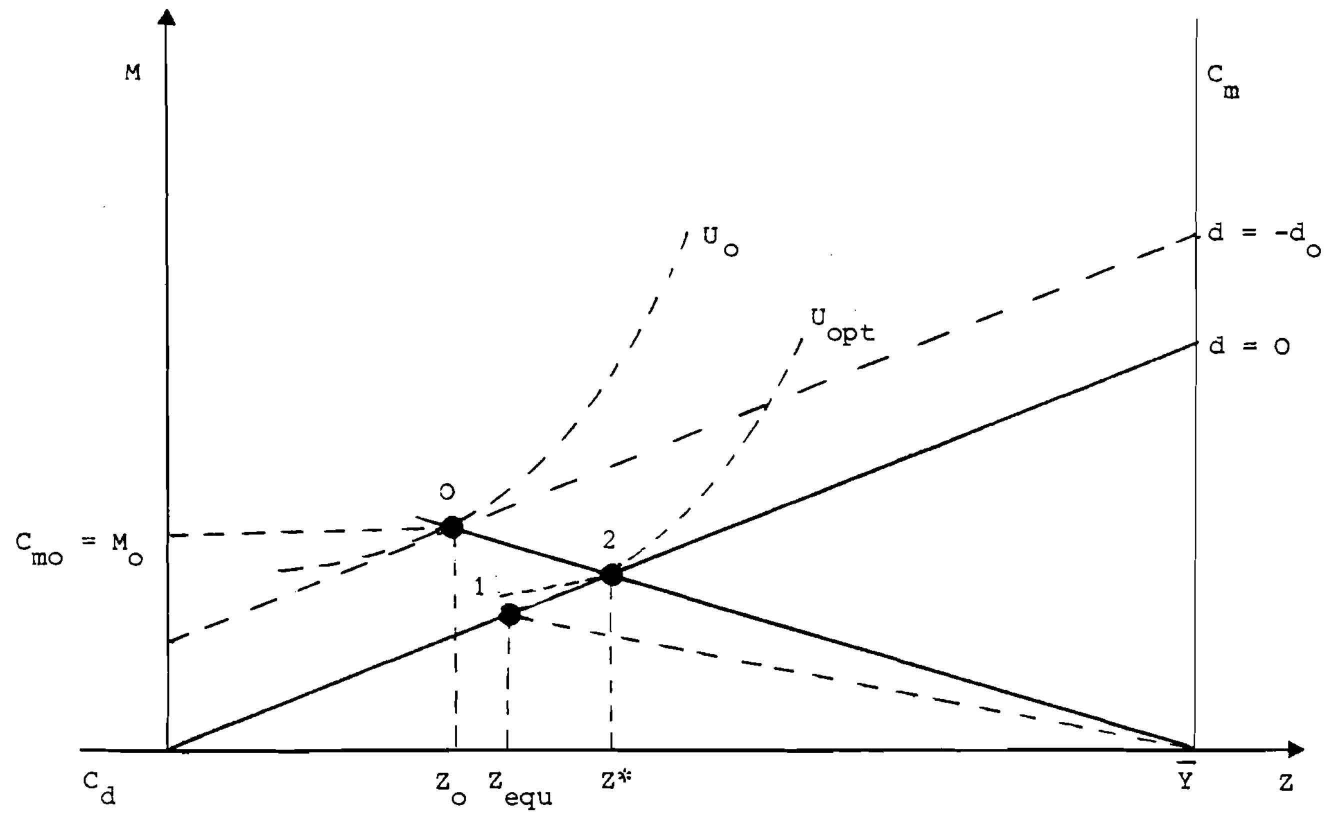

Figure 7. Base (O), laissez-faire equilibrium (1) and planners' optimum (2) in a small open economy.

leads to a (base) situation in which the current account shows a deficit (d_o), but otherwise the economy is (internally) in a state of *laissez-faire* equilibrium (parts and curves labeled with o subscripts in Figure 7). For the sake of simplicity, we also assume that this situation has already existed for sufficiently long to allow the country in question to accomodate herself fully to the new set of world market prices. Thus, the domestic price ratios are exactly the same as the world market price ratios (see equations 4.1 and 4.2).

The above assumptions imply that the following conditions are fulfilled in the base case:

$$P_{do} = V_o \bar{P}_E \tag{4.1}$$

$$P_{mo} = V_o \bar{P}_M \tag{4.2}$$

$$m_o = \bar{m} \left(\frac{P_{do}}{P_{mo}}\right)^{\mu} \tag{4.3}$$

$$C_{mo} = m_o C_{do} \tag{4.4}$$

$$C_{do} + Z_o = \bar{Y} \tag{4.5}$$

$$\bar{P}_M C_{mo} - \bar{P}_E Z_o = d_o \tag{4.6}$$

Here we have used the subscript o to refer to the base case; all other notation is the same as before. We thus have seven endogenous variables (C_d, C_m, Z, m, P_d, P_m, V) and six equations characterizing the base competitive equilibrium (as usual, relative prices are indeterminate).

One of our assumptions needs special consideration. We have assumed that long-run adjustment has brought about "equalization" of international and domestic prices, i.e., export supply is perfectly elastic in the long run. However, this does not mean that export supply is also perfectly elastic in the shorter run. It can easily be seen that these two assumptions are not contradictory. Let us assume that the short-run export supply function is given by the following constant elasticity function*

$$Z = Z_o \left(\frac{P_d}{V \bar{P}_E}\right)^{\alpha} \tag{4.7}$$

Assume now that we want to assess what would happen in the short run if the government wanted to restore external equilibrium.

*Since $P_d = V\bar{P}_E$ in the base case, the scaling constant must be equal to Z_o.

Suppose that, to achieve this, the government stops borrowing, thus cutting down on the supply of foreign currency ($d = 0$), but otherwise follows a *laissez-faire* strategy. The resulting short-run equilibrium can be calculated by solving equations (4.2)-(4.7) with a new target of zero for the current account balance.

The only structural difference between the two sets of equilibrium conditions is the replacement of equation (4.1) by (4.7). This difference is due to the assumed divergence of short- and long-run export supply adjustment: export supply is assumed to be perfectly elastic in the long run, and imperfectly elastic in the short run. (Observe that the two equations are in effect equivalent when α approaches minus infinity.)

It is easily seen that the long-run equilibrium, i.e., the solution of equations (4.1)-(4.6) for $d_o = 0$, is Pareto superior to the short-run equilibrium; it is in fact the optimal solution in the absence of friction in export supply adjustment. Under normal assumptions on the values of the parameters, the different solutions will be as shown in Figure 7. What happens is the following. Foreign currency becomes scarcer, resulting in a higher exchange rate and, as a consequence, higher domestic prices for both domestically produced and imported commodities. However, since export supply is less than perfectly elastic, the domestic price of the home produced commodity will not, in the short run, increase at the same rate as the exchange rate and the price of imports. Thus, in the short-run *laissez-faire* equilibrium the consumption of imported commodities will be reduced more than that of domestic commodities (m decreases). In the optimal case, on the other hand, because of the (assumed) linear homogeneity of the utility function, consumption of both commodities will decrease by the same proportion (as would happen in the long-run *laissez-faire* equilibrium). Of course, prices in the optimal case will also increase proportionally.

Thus, the optimal state of the economy (which is the same here as the long-run equilibrium) is different from the short-run equilibrium*. The *laissez-faire* equilibrium is less efficient than the optimum solution due to the imperfect adjustment of the export supply. This friction could, however, be overcome by appropriate export subsidies, which must be sufficient to increase the amount of goods exported to the optimal level (Z^*). Given the (short-run) supply function and optimal solution, the optimal rate of subsidy (ψ^*) can be determined by

$$\psi^* = \left(\frac{Z^*}{Z_o}\right)^{-1/\alpha}$$

which, according to our assumptions $Z^* > Z_o$, $\alpha < 0$, is indeed greater than 1. To see that ψ^* can be determined as above, first observe that $P_d^* = V^* \bar{P}_E$ if prices are set according to the optimality conditions. Thus, introducing the subsidy (ψ) into the determination of supply will result in the following relationship

$$Z = Z_o \psi^{-\alpha}$$

From this our expression for the subsidy follows immediately.

We should perhaps make a few comments concerning the above analysis. First of all, the above arrangement could only work if the government collected the money needed for the subsidy through some form of taxation. Thus, in general, this solution implies a redistribution of income which may have unwanted effects. However, this cannot be taken into account in our simplified model.

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*Observe that the distinction between long- and short-run equilibrium is not essential to our discussion. All we really need to show is that the economy would be better off if supply were perfectly elastic, and that such a state is attainable under government regulation.

A second remark concerns the possibility of generalizing our analysis. It is fairly easy to show that the above result can be extended to the case of the large open economy, i.e., an economy facing a downward-sloping demand curve. In this case, the usual optimum tariff argument and the above argument can simply be combined: this means that the optimum tariff derived from the demand relationship must be multiplied by the tariff implied by the supply function

$$t^* = \left(\frac{1+\varepsilon}{\varepsilon}\right)\left(\frac{Z^*}{a}\right)^{-1/\alpha}$$

where ε and α are the demand and supply elasticities as before, and a is the scale factor in the supply function (Z_o before). Thus, in this case, the tax implied by pure demand (frictionless supply) considerations might be reduced or even offset by the subsidy dictated by supply constraints.

Thirdly, we would like to call attention to one of our specific assumptions and point out the possibility of a supply-implied tax instead of a subsidy. This would arise if our comparative static example resulted in a decrease rather than an increase in exports (as could happen if, for example, the given country borrowed more from abroad). This is especially important in the more complex analyses involving many sectors and different types of assumed exogenous changes, where the different sectors would probably produce a variety of different combinations of taxes and/or subsidies based on export demand and supply considerations.

Finally, we have to do justice to neoclassical optimum tariff theory. It is clear that our introduction of the export supply function is not strictly consistent with the usual neoclassical way of thinking and reasoning. The basis of neoclassical theory is that every action of economic agents can be explained by assuming optimizing behavior. Thus, for example, the export supply function is usually derived by assuming joint production of domestic and export commodities, and profit-maximizing producers. In such a case a supply-related optimum tariff would probably not emerge and so it is not surprising that this case is not discussed in the strictly neoclassical literature. On the other hand, however, we do not think that general equilibrium models can or should be based strictly on neoclassical theory. It is a question of personal taste whether one prefers an equilibrium model which is strictly consistent with neoclassical theory or one which is not. The export supply function, for example, can be introduced into a model in a *non*-neoclassical way simply to reflect *non*instantaneous adjustment to changing situations (frictions other than those implied by technological restrictions); this would immediately give rise to the above phenomenon.

5. ILLUSTRATIONS AND CONCLUSIONS FOR NUMERICAL MODELS

5.1 About the Models Used

As promised previously we will now present the results of some numerical simulations. Two models have been used for this illustrative purpose.

The first model is rather detailed. A complete mathematical statement of the model* is given in Appendix 3, and here we will only summarize its main characteristics.

The model distinguishes 19 sectors as follows:

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*The model is a version of the computable general equilibrium model developed for experimental purposes by the author in collaboration with experts from the Hungarian Planning Office. A more detailed description of the model can be found in Zalai (1980). The author wishes to acknowledge the valuable assistance in preparing the numerical model and its solution algorithm to Gy Boda, I. Csekő, F-né Hennel, L. László, A. Pór, S. Poviliaitis, F. Sivák, A. Tihanyi and L. Zeöld.

1. Mining
2. Electricity
3. Metallurgy
4. Machinery
5. Construction materials
6. Chemicals
7. Light industries
8. Other manufacturing
9. Food Processing
10. Construction
11. Agriculture
12. Forestry and logging
13. Transport and communication
14. Domestic trade
15. Foreign trade
16. Waterworks
17. Personal and economic service
18. Health and cultural services
19. Public administration

Commodities are distinguished according to their sectoral origin and each sectoral commodity is further classified into three categories: domestically produced, competitive and non-competitive import. In import and export activities dollar and rouble trade relations are treated separately. The share of domestic source and competitive (dollar and rouble) import changes as a function of their selective prices. Export is specified in alternative ways (pure supply, pure demand, equilibrium of supply and demand and planner's optimum) as discussed in previous sections.

Production technology is described by a Johansen-type of specification, i.e., the use of sectoral commodities is proportional to the output (Leontief technology), whereas labor and capital usage is specified by linear homogeneous (Cobb - Douglas) smooth production relationships.

Gross investment is treated as a special sectoral activity. Demand for investment is the sum of replacement and net investment (replacement rate is different from the rate of amortization!).

Production (supply) of new capital goods is represented by fixed coefficient technology.

The remainder of the final use (termed simply as consumption) is divided into a fixed and a variable part. In the runs presented here, the fixed (minimum) part is the observed 1976 (base) consumption. In order to be able to measure and compare efficiency (optimality) of various solutions easily and unambiguously the sectoral composition of the variable (excess) part of consumption is fixed, thus leaving only the level of excess consumption as variable. This treatment leads to a special demand system, formally very close to the more usual LES systems.

Price formation rules closely follow the input-output traditions. The cost of labor and capital is derived on the basis of cost minimizing assumption. Prices are formed on cost-plus-profit mark-up basis, where the exogeneous profit rates are the observed ones (one of the non-neoclassical features of the model).

The parameters and exogeneous variables of the model are evaluated on the basis of the 1976 Hungarian statistical input-output tables. The only notable exception from this rule is the subdivision of export and import figures into various subcategories (trading area, competitiveness). Because of the lack of published data the subdivision here is hypothetical and serves only for illustrative purposes. Table 2 summarizes the major features of the base solution and values of some crucial parameters.

The *second model* is in many respects a simplified and aggregated version of the first. Only 3 sectors are distinguished. The first is the aggregate of sectors 1, 2, 5, 11, 12 of the 19 sectors list ("primary sectors"), the second contains sectors 3-10 ("secondary sectors") and the third the service sectors 13-19 ("tertiary sectors"). Foreign trade is represented simply by one export and one import variable in each sector. In the various runs the volume and price of export in the service sectors is kept constant at the base level.

Table 2. Major Features of the Hungarian Economy and Some Model Parameters

Sector	I	II	III	IV	V	VI	VII	VIII
1	2.2696	2.2767	1.0444	-2.0000	-0.3000	119.1758	16.5006	0.5000
2	1.7887	1.6188	0.7998	-3.0000	-0.3000	10.3459	0.--	----
3	4.8193	30.6765	25.5257	-2.5000	-2.5000	59.6393	13.3466	0.5000
4	13.4889	42.9049	11.9199	-2.5000	-2.0000	118.2363	8.2649	0.3000
5	1.6374	11.1925	7.0836	-2.5000	-1.5000	33.9942	8.9148	0.3000
6	7.8796	19.5535	12.4953	-2.5000	-2.5000	54.7812	10.4017	0.5000
7	8.5977	22.4729	10.6944	-2.5000	-2.5000	32.1302	9.7094	1.2500
8	1.1312	8.9883	4.3683	-2.5000	-2.0000	6.3427	0.2358	1.2500
9	9.8450	17.2253	10.2227	-2.0000	-1.5000	17.4175	4.2668	1.2500
10	8.3215	0.5406	0.1588	-2.5000	-1.0000	0.--	0.--	----
11	15.9713	12.1044	6.8693	-2.0000	-0.5000	4.6249	0.5510	2.0000
12	0.6000	14.1118	14.0366	-2.5000	-0.3000	27.6386	12.5957	0.5000
13	5.3882	8.2908	5.9454	-2.5000	-1.5000	5.8374	0.0020	0.3000
14	6.1100	2.0811	1.7557	-1.2500	-2.0000	0.--	0.--	----
15	1.4234	16.4673	1.2549	-2.5000	-2.0000	33.4221	1.6089	0.3000
16	0.9102	0.8068	0.3402	-2.5000	-0.3000	0.--	0.--	----
17	3.0284	0.--	0.--	----	----	0.--	0.--	----
18	3.6801	0.--	0.--	----	----	0.--	0.--	----
19	3.1096	0.--	0.--	----	----	0.--	0.--	----
Total	100.0000	15.6786	7.5950			20.7000	4.0277	

I Share of branches in total production
II Total export/production
III Dollar export/production
IV Export demand elasticities
V Export supply elasticities
VI Total import ratio (M/(X-Z))
VII Competitive dollar import ratio
VIII (Dollar) Import elasticities

The model is made more neoclassical by treating import and domestic commodities less than perfect substitutes, according to Armington's proposition. (In the previous model the assumptions of perfect substitutability but less than perfect adjustment mechanism gave rise to basically the same import functions.) This and some other features make the smaller model similar to the ones used for simulations in Western or developing economics. Consumption of the composite (domestic and imported) commodity is, for example, determined by an LES demand structure.

The only deviation from the standard neoclassical general equilibrium specification is that the export supply functions reflect institutional rather than technological adjustment frictions. Therefore, exported and domestically sold commodities are considered perfect substitutes.

5.2. Simulation Results

Before turning our attention to the numerical results we should warn the reader to interpret them carefully. The models used here give in many aspects rather rough answers to the question of what could have happened in reality if such measures had been adopted. Further refinement of the models is under way. They are used here only as numerical illustration of the size effect of alternative export specifications.

First we will present the *results of the more aggregated* (neoclassical) *model*. In this case we have adopted a rather simple simulation framework which can be summed up as follows. The observed 1976 state of the economy was considered the *base* solution. It was assumed, as usual, that these data reflect certain partial equilibria (e.g., rational decisions under the given price regime), but they describe, in general, a distorted general equilibrium. For the sake of simplicity we assumed that the major distortions manifested themselves in the prices, or to be more precise, in the sectorally different rates of returns on the primary resources.

Thus, we have set out to analyze the effect of introducing an economically more sound (competitive) price system in terms of the corresponding relative shifts in demand and use of the produced and primary commdities. In forming the prices the amount of profit (net income) is determined according to uniform (normative) net rate of return requirement on both labor and capital.

In 1976 there was a close to 30% tax on wages and 5% tax on capital built into the Hungarian price system, and varying profit mark-ups. Thus we have chosen 0.3 and 0.05 as the base values for the net return requirements in the case of labor (wages) and capital, respectively. During the calculations we let these rates vary and set at their equilibrium value, while the profit mark-ups were abolished. The general level of prices was determined by a special scaling equation, by which we required the general consumers' price index to remain constant.

We have generated 8 solutions. They differ from each other only in the export treatment. First we calculated the results with *four* alternative export specifications: pure export demand case (Dem), pure export supply case (Sup), export supply and demand equilibrium case (Equ), and optimum tariff case (Opt). In order to illustrate the effect of the size of export elasticities we have repeated each run at larger absolute values of the elasticities, as shown below:

	Small Elasticities		Large Elasticities	
Sector	Supply	Demand	Supply	Demand
1	- 0.5	- 1.5	- 5.0	- 6.0
2	- 2.5	- 3.0	- 4.0	- 8.0

The set of smaller elasticities is representative for the numerical models used in practice. *Table 3* and *4* summarize the alternative solutions in terms of some characteristic variables. Most of the analysis can be left to the reader, since the figures speak for themselves. To amplify some conclusions we have prepared Table 5 which contains only the most relevant information.

Table 3. Major Real Variable in Various Runs (Small Model)

Sector	Dem1	Sup1	Equ1	Tar1	Dem2	Sup2	Equ2	Tar2
RELATIVE CHANGES IN EXPORT (base = 1.)								
1	0.1733	0.900	0.926	0.222	0.286	0.375	0.588	0.221
2	1.068	1.046	1.035	0.720	1.181	1.137	1.093	1.039
3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Total	1.016	1.023	1.017	0.662	1.041	1.019	1.015	0.917
RELATIVE CHANGES IN IMPORT (base = 1.)								
1	1.103	1.104	1.100	0.961	1.108	1.100	1.098	1.043
2	0.991	0.991	0.984	0.767	1.000	0.988	0.982	0.897
3	1.334	1.334	1.322	0.978	1.343	1.325	1.317	1.195
Total	1.020	1.020	1.013	0.807	1.028	1.017	1.012	0.931
EXCESS CONSUMPTION (base = 0.)								
1	566.500	272.600	183.800	2832.600	579.600	736.200	496.800	898.800
2	1753.000	844.400	568.400	8271.000	1794.100	2273.000	1533.500	2689.900
3	1549.600	745.200	503.300	8255.800	1584.800	2017.700	1362.400	2449.500
Total	3869.100	1862.200	1255.500	19359.400	3958.500	5026.800	3392.800	6038.100

Table 4. Major Price Variables in Various Runs (Small Model)

Sector	Dem1	Sup1	Equ1	Tar1	Dem2	Sup2	Equ2	Tar2
DOMESTIC PRICE INDICES								
1	1.053	1.053	1.053	1.029	1.053	1.052	1.052	1.022
2	0.837	0.837	0.837	0.843	0.837	0.837	0.837	0.831
3	1.134	1.134	1.133	1.073	1.134	1.132	1.131	1.121
EXPORT PRICE INDICES								
1	1.230	1.000	1.052	2.724	1.232	1.000	1.092	1.286
2	0.978	1.000	0.989	1.116	0.979	1.000	0.989	0.995
3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
DOMESTIC PRICE PER EXCHANGE RATE								
1	1.230	1.236	1.226	0.908	1.232	1.217	1.215	1.072
2	0.978	0.982	0.975	0.744	0.979	0.969	0.967	0.871
3	1.325	1.331	1.319	0.947	1.327	1.309	1.306	1.175

Table 5. Summary of Simulation Results with Alternative Export Specifications (Small Model) (percentage changes)

	Dem	Sup	Equ	Opt
SMALL ELASTICITIES				
Total export	+ 1.6	+ 2.3	+ 1.7	- 33.8
in sec. 1	- 27	- 10	- 8	- 78
in sec. 2	+ 7	+ 5	+ 4	- 28
Total import	+ 2	+ 2	+ 1.3	- 19.3
Total excess consumption	+ 1.1	+ 0.5	+ 0.3	+ 5.3
Term of trade	+ 0.6	0	+ 0.3	+ 18.5
Exchange rate	- 14	- 15	- 14	+ 13
LARGE ELASTICITIES				
Total export	+ 4.1	+ 1.9	+ 1.5	- 8.3
in sec. 1	- 71	- 62	- 41	- 78
in sec. 2	+ 18	+ 14	+ 9	+ 4
Total import	+ 2.8	+ 1.7	+ 1.2	- 6.9
Total excess consumption	+ 1.1	+ 1.4	+ 0.9	+ 1.6
Term of trade	- 1.0	0	+ 0.2	+ 0.5
Exchange rate	- 15	- 14	- 13	- 5

Table 5 gives some insights into the working of the general equilibrium models typically used. First of all, due to the input-output structure producers' prices are rather stable (see Table 4). Therefore the relative price dependent variables (like export, import share) will generally follow the same pattern of change in the various solutions. Only the optimal solution is an exception to this general observation, where we can see qualitatively different solutions.

It is also evident that the size of elasticities has real influence in the size order of changes. If they are relatively small the changes are larger and *vice versa*. This effect is visible even if we compare only the demand, supply and equilibrium solutions in one (small or large) class of elasticities.

As pointed out earlier, equilibrium elasticities are the smallest of all, and in this particular example we have chosen the supply elasticities smaller than the demand ones. These show up in the respective orders of change in the exports. Thus, *the larger the elasticities the larger the room for the forces of comparative advantage in structural adjustment* (allocative efficiency).

However, the above positive effects of larger elasticities are *counterbalanced by the terms of trade effects* brought in by the same demand elasticities. Thus, for example, in the pure export demand case these two effects offset each other. The increased allocative efficiency is offset by a 1.6% simultaneous deterioration of the terms of trade (from + 0.6 to - 1.0), and the increase of consumption remains the same (1.1).

The *terms of trade effects* brought in by the demand elasticities can best be seen in the case of *optimal tariff solutions* which takes them to an extreme. When the elasticities are small the optimizing logic of the solutions generates an 18.5% (!) gain in the terms of trade, and this is the real source of the outstanding welfare improvement (+ 5.3% increase in consumption). With large elasticities this effect is only marginal as compared to the allocative efficiency. This also explains why the various solutions are so close to each other in the case of larger elasticities.

It is also worth noting that the *laissez-faire* solutions and *optimal* solutions *qualitatively* differ in their economic policy suggestions. The former ones suggest a more open (in foreign trade) policy: both total exports and total imports increase in all the six solutions. The optimal solutions, on the other hand, suggest rather severe import-export restrictions.

Finally, as a matter of interest, we would like to report on some specifics of the *optimal tariff* solution. As we have discussed in the theoretical part of this paper, the optimal tariff solution works in the following way. The exchange rate will be corrected by taxes or subsidies in regulating exports. All the laissez-faire solutions suggested a 13%-15% *revaluation* of the exchange rate. (This can be explained by the cca 16%

decrease in the price of the major exporting sector, number 2.) As opposed to this, the optimal tariff solution implied a 13% *devaluation* in the case of small elasticities and only 5% revaluation in the other case. This explains why import is reduced in both cases. To discourage export, on the other hand, *export taxes* have to be introduced. Their order of *magnitude* in the first two sectors are 98% (!) and 42% when elasticities are small and 40% and 11.7% when they are high. (If supply were perfectly elastic the corresponding figures would be 67% and 33% in the first case, and 17% and 12.5% in the other. Thus, except for the last figure, the supply effect adds to that of demand.) All these results clearly question the relevance of optimal tariff argument in the case of small (constant) demand elasticities.

Thus, we think the small example is already convincing enough that the question of export demand specification and especially the size of demand elasticities commonly used in computable general equilibrium models must be critically reexamined. We will come back to this point in the next subsection. Before that, however, we want to present some results gained by the more complex and disaggregated model in order to show that our findings are not overexagerated by the small model.

We have already described the main features of the 19 sector model and also some crucial parameters (see Table 2). The simulation framework in this case was somewhat different. The question we asked from this model was the following. Suppose Hungary wanted to achieve a zero balance of trade in her dollar trade in 1976, what structural changes would this need? Again, we calculated four solutions differing only with respect to the export specification. Some additional specifics of the calculations should be mentioned before presenting the main results. First, the balance of trade was supposed to be restored at the cost of a more or less uniform decrease of consumption. Second, rouble trade and terms of trade were kept constant. Third, profit rates were assumed to remain the same.

The details of this model solutions are not too interesting and might also be misleading. Therefore we decided to show here some of its main indicators only (Table 6). The results are perfectly good to illustrate the results of the discussed alternative export specifications and the differences of the laissez-faire and planners' optimum solutions. The detailed prescriptions of the model are at least questionable. The figures speak for themselves and support our earlier conclusions, therefore there is no need to comment on them.

Table 6. Main Indicators (Large Model)
(base = 100)

	Dem	Sup	Equ	Opt
Total dollar export	128.18	116.51	123.90	108.74
Total dollar import	97.35	98.44	95.55	89.05
Total trade/GDP ratio*	84.81	82.90	83.57	79.45
Final consumption	92.04	95.52	92.75	94.63
Dollar terms of trade	89.89	100.00	91.27	96.92
Dollar exchange rate	111.21	108.87	125.39	188.31

* base = 80.42

5.3. Concluding Remarks

In the first part of this paper we argued that the rigid individual bounds on export and import activities, typical of programming-type macroeconomic models can be usefully replaced by flexible bounds. This replacement was, in fact, carried out using some tools borrowed from similar models of the computable general equilibrium type. We have also argued that the choice of parameters in the neoclassical export and import functions is at least as crucial as the choice of the size of individual bounds, and this is clearly demonstrated in the numerical simulations. Thus, since these parameters cannot be estimated any more reliably than the individual bounds can be determined, there is some degree of arbitrariness in both cases.

Our numerical examples also illustrate the terms-of-trade effects introduced by export demand functions. It is important to emphasize that in many cases these effects are unrealistic and unwanted. The smaller the elasticities, the larger the terms-of-trade effects. Small elasticities, however, usually arise only because the observed changes in exports are small, especially when compared to changes in relative prices. (In some cases, stragely enough, elasticities in the range (-1,0) are assumed, which would mean that the given country could increase its export earning by reducing exports!)

It is, therefore, crucial to distinguish between and possibly separate the changes in the terms-of-trade and the changes in the speed of export adjustment. The special advantage of introducing both demand and supply functions lies, in part, in this area. Small supply elasticities imply small shifts in exports (if needed), while the size of the demand elasticity can more accurately reflect the assumed changes in the terms-of-trade.

A major problem with the most commonly used export and import functions is their constant elasticity form. Even if one could rely on the econometric estimates of these elasticities, they would give an accurate representation of supply and demand behavior only in a relatively small neighborhood of the observed pattern. Another problem with constant elasticities is that the effects of increases and decreases in relative prices are treated symmetrically. It is rather unrealistic to assume that, say, a 10% increase in exports will produce a change in relative prices of the same size as a 10% decrease in exports.

One would intuitively think that the export demand would be much more elastic with respect to an increase in prices than to a decrease in prices. It would therefore seem reasonable to replace the constant elasticity forms by unsymmetric forms with variable elasticites. Since observations usually lie within a narrow range, it is extremely difficult to make econometric estimates of such functions. The only possibility seems to be the combination of econometric estimates with qualitative export judgments.

On the whole, our numerical simulations demonstrated that the treatment of foreign trade in a multisectoral macromodel has a very great influence on the final results of the model. This is not very surprising since these models operate on the basis of resource reallocation. The freedom in reallocating resources in an open economy depends greatly on the potential for foreign trade. Thus, it is very important to devise an accurate representation of this potential: it seems that the currently available techniques are not sufficiently sophisticated to handle these problems adequately.

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APPENDIX 1: SMOOTH SUBSTITUTION FUNCTIONS AND IMPLIED DEMAND FUNCTIONS: SOME BASICS

The *concept of substitutability* of commodities *in use* is a trivial and old one. It has acquired a central and much debatable role in some streams of economics, particularly in the neoclassical economies.

The concept of substitutability is closely connected with the classical concept of *use value* (or value in use). This largely forgotten concept has been in the forefront of Marx's economic analysis as well. A general and satisfying theory of use-value is still lacking. It is, however, apparent that *production functions, utility functions* and *welfare functions* try to measure the joint value in some definite use of some commodities, but of course in an extremely oversimplified manner in most cases.

In practice, there are almost insurmountable obstacles in the way of getting reliable estimates of substitution possibilities. In the statistical estimation of substitution functions a lot of arbitrary *a priori* assumptions are made about the special form of the function and the underlying substitution mechanism.

One should be very careful in distinguishing substitution possibility from the *assumed mechanism* regulating the process of substitution. The heart of the critique against the neoclassical treatment is (or should be at least) directed toward the substitution mechanism rather than the concept itself. The main assumption there is that the substitution is driven by relative price changes and that decision makers always optimize their choice of a specific commodity bundle. In short, a *perfect and rational substitution mechanism* is assumed. It is clear, however, that in reality prices along cannot explain shifts in production or consumption and, also, adjustment is never frictionless and instantaneous. Many factors influence the substitution process, most of which are neglected in estimating substitution functions on the basis of neoclassical theoretical assumptions.

In a certain limited role, nevertheless, smooth substitution functions can be fruitfully applied in macro planning (or forecasting) models. The parameters have to be chosen on the basis of available quantiative and qualitative information from planners, rather than on the usual, very unreliable econometric estimates. They should be treated as technical devices rather than theoretical constructs.

A.1. Derivation of Alternative Relationships

Suppose that two commodities (say, m = imports, d = domestic), are substitutes for a given kind of use. Any given level of "joint use-value", C_c can be achieved by various combinations of the two sources of supply, satisfying the following CES-type functional relationship:

$$C_c = (h_d C_d^{-\beta} + h_m C_m^{-\beta})^{-1/\beta} \qquad (A.1)$$

where $1/1+\beta$ is the constant elasticity of substituion, h_d and h_m are given constants, C_d and C_m are the amounts of commodities from domestic source and imports, respectively.

We look for a cost minimizing combination of individual inputs at prices P_d and P_m for fixed C_c. This requires minimizing the total cost function

$$P_d C_d + P_m C_m \qquad (A.2)$$

subject to the constraint given by (A.1).

Let us introduce P_c for the Lagrangian multiplier, which can be interpreted as the minimum (optimal) cost of achieving one unit of the joint use-value (the shadow price of the joint use-value or of the "composite commodity"). The Lagrangian will take the following form

$$L = P_d C_d + P_m C_m - P_c \left[(h_d C_d^{-\beta} + h_m C_m^{-\beta})^{-1/\beta} - C_c \right] \qquad (A.3)$$

Differentiating L with respect to C_d and C_m yields the following two (additional) necessary conditions for a minimum (after slight manipulation):

$$P_d = P_c h_d (C_c/C_d)^{1+\beta} \qquad (A.4)$$

$$P_m = P_c h_m (C_c/C_m)^{1+\beta} \qquad (A.5)$$

Let us now take equations (A.4) and (A.5) and solve them for C_d and C_m respectively:

$$C_d = h_d^{\mu} \left(\frac{P_d}{P_c} \right)^{-\mu} C_c \qquad (A.6)$$

$$C_m = h_m^{\mu} \left(\frac{P_m}{P_c} \right)^{-\mu} C_c \qquad (A.7)$$

The resulting equations determine demand for domestic and imported commodities as functions of demand for the given (joint) use-value (C_c) of it's shadow price (P_c) and of the respective individual prices (P_d or P_m). These are familiar expressions from the duality theorems of production and cost (profit) functions. They can be obtained as the first order partial derivatives of the (optimal) cost function ($P_c C_c$), which will be determined later.

Observe also that the necessary conditions (A.4) and (A.5) imply the following relationship for the ratio of the amounts of the two commodities (denoted by m):

$$m = \frac{C_m}{C_d} = \left(\frac{h_m}{h_d}\right)^{1/1+\beta} \left(\frac{P_m}{P_d}\right)^{-1/1+\beta} \qquad \text{(A.8)}$$

which is, in fact, an import demand function similar to the one used in this paper, with

$$m_o = \left(\frac{h_m}{h_d}\right)^{1/1+\beta} \qquad \text{and} \qquad \mu = \frac{1}{1+\beta}$$

Returning to the solution of the optimum problem observe now that substituting the right hand side of equation (A.6) and (A.7) for C_d and C_m in equation (A.1) respectively, after suitable rearrangement we will get the optimal unit cost function (for the joint use-value):

$$P_c = \left(h_m^{\mu} P_m^{1-\mu} + h_d^{\mu} P_d^{1-\mu}\right)^{1/1-\mu} \qquad \text{(A.9)}$$

A1.2. Base Related Forms

It is worth checking that both the optimal cost ($P_c C_c$) and the optimal amounts of the two commodities (C_m and C_d) are homogeneous functions of degree o of the parameters h_m and h_d. Thus, if P_d and P_m are price indices referring to some *base* (reference) values ($P_m^o = P_d^o = 1$) we may choose the level of h_m and h_d such that the shadow price in the base case (P_c^o) will also be 1. This implies, of course, that the measure of joint use-value must be chosen such that in the base year its level be the simple algebraic sum of the amount of the two components. This can be seen from the following chain of equations

$$C_c^o = P_c^o C_c^o = P_d^o C_d^o + P_m^o C_m^o = C_d^o + C_m^o$$

From all this it follows that in the above case the following relationships must also hold:

$$h_d = \left(\frac{1}{1 + m_o}\right)^{1/\mu} = \left(\frac{C_d^o}{C_m^o + C_d^o}\right)^{1/\mu} = s_{do}^{1/\mu} \tag{A.10}$$

and

$$h_m = \left(\frac{m_o}{1 + m_o}\right)^{1/\mu} = \left(\frac{C_m^o}{C_m^o + C_d^o}\right)^{1/\mu} = s_{mo}^{1/\mu} \tag{A.11}$$

Parameters s_{do} and $s_{mo} = 1 - s_{do}$ denote the shares of the two kinds of source in total use in the *base* year (or *base case* if we make model comparisons). Their substitution for h_d and h_m in the earlier derived correspondences will give us useful alternative forms. Let us first reformulate (A.6), the optimal demand equation for the domestic commodity (by simple analogy one can make the same transformation for equation A.7):

$$C_d = s_{do}\left(\frac{P_d}{P_c}\right)^{-\mu} C_c \tag{A.6'}$$

In some cases, total expenditure (E) is known rather than the level of the target use-value (C_c). Thus the problem is to maximize C_c subject to the budget constraint. By symmetry one can easily see that knowing E, C_c can be determined simply as E/P_c, where P_c can be calculated again in accordance with (A.9). We can also rewrite (A.9) using the base share parameters:

$$P_c = \left(s_{mo}\, P_m^{1-\mu} + s_{do}\, P_d^{1-\mu}\right)^{1/1-\mu} \tag{A.9'}$$

Equation (A.9') shows clearly that the shadow price of the "composite commodity" is nothing more but the *weighted average* of the "component" price. If $\mu = 0$, i.e., the two commodities are (strict) complements, then the shadow price is a weighted algebraic average of the component prices. If $\mu = 1$, i.e., substitution possibilites take the form of a Cobb-Douglas function, the shadow price will be a geometric average of the components (as can be expected):

$$P_c = P_d^{s_{do}} \; P_m^{s_{mo}}$$

This can be checked by taking μ to the limit 1 in (A.9'). The reader can also check the emergence of other concepts of average often used by economic statisticians.

Thus, if C_c can be expressed as E/P_c, then (A.6) can be further rewritten as

$$C_d = \frac{s_{do} \; P_d^{-\mu}}{s_{do} \; P_d^{1-\mu} + s_{mo} \; P_m^{1-\mu}} \; E \qquad (A.12)$$

This form is especially useful in specifying demand equation systems in a computable general equilibrium model. The familiar Linear Expenditure Systems (LES) can, for example, be generalized to cover cases will elasticity of substitution different from 1. The generalized form of demand for commodity i (C_i) can be written as follows

$$C_i = b_i + \frac{c_i \; P_i^{-\mu}}{\sum_{j=1}^{n} c_j \; P_j^{1-\mu}} \left(E - \sum_{j=1}^{n} P_j \; b_j \right) \qquad (A.13)$$

where b_i is the minimum (or base) consumption level of commodity i, c_i is its share from excess expenditure at orices all 1 (base share). Note that if μ = 1 then equation (A.13) is reduced to the familiar case of an LES system. At μ = 0 (lack of substitutability) the equations will result in a form that corresponds to the case of maximizing excess consumption in a fixed structure. Such treatment is characteristics for some linear planning models. To make the picture full let us see also the case of perfect substitutability, i.e., when μ goes to infinity. As can be expected, in this case the excess consumption will be zero from all commodities whose relative price is higher than the minimum (P_{min}). The rest of the commodities (in most cases one commodity only) will have their share from the excess consumption in fixed proportion (given by the corresponding values of c_i's).

APPENDIX 2: ANALYTICAL DERIVATION OF THE GRAPHICAL FIGURES 5 AND 6

The problem is to characterize the solutions of the following parametric (in k) constrained optimum problem. Maximize

$$\left(h\ C_d^{\mu} + m\ C_m^{\mu}\right)^{\frac{1}{\mu}} \tag{1*}$$

subject to

$$C_d + Z \leqq \bar{Y} \tag{2*}$$

$$\frac{1}{1+\lambda} P_e \hat{Z}^{-\lambda} Z^{1+\lambda} - P_m C_m \geqq k \tag{3*}$$

where $\lambda < 1$ ($\neq 0$), $-1 < \lambda < 0$, and k, h, m, P_e, P_m, $\hat{Z}$, and $\bar{Y}$ are all positive constants. All variables (C_d, C_m, Z) must fulfill the usual nonnegitivity constraint.

Observe that (1*) is a strictly monotonic increasing function of both C_d and C_m. Therefore in the optimal solution, both (2*) and (3*) will be fulfilled as equalities. Thus we can solve (2*) for C_d and (3*) for C_m, respectively, i.e, express them as functions of Z:

$$C_d = \bar{Y} - Z \tag{4*}$$

$$C_m = \frac{1}{1+\lambda} \frac{P_e}{P_m \hat{Z}^{\lambda}} Z^{1+\lambda} - \frac{k}{P_m} \tag{5*}$$

Replacing C_d and C_m by the resulting expressions in the objective function (1*) will reduce the problem to an unconstrained maximum (except for the sign restrictions of the variables). Observe also that instead of (1*) we can use its monotonic transformation given below

$$h\ C_d^{\mu} + m_m^{\mu} \tag{6*}$$

Thus, we can simplify our analysis and concern with the unconstrained maximum of the following function of Z:

$$h(\bar{Y} - Z)^{\mu} + m\left(\frac{1}{1+\lambda} A\ Z^{1+\lambda} - k'\right)^{\mu} \tag{7*}$$

where for notational simplicity

$$A = \frac{P_e}{P_m\ \hat{Z}^{\lambda}} \qquad \text{and} \qquad k' = \frac{k}{P_m}$$

With some manipulation the necessary first order condition for the maximum of (7*) yields the following equation:

$$h^{\alpha}\ (\bar{Y} - Z) = (ma)^{\alpha}\left(\frac{1}{1+\lambda} A\ Z^{1+\lambda} - k'\right) Z^{\lambda\alpha} \tag{8*}$$

where $\alpha = 1/\mu - 1$ and thus $\alpha < 0$.

On the basis of condition (8*) we first establish that the optimal value of Z is a monotonic increasing function of parameter k. To show this, we will treat (8*) as an implicit function of Z and k' and take its derivative with respect to k', which yields:

$$-h^{\alpha} Z'_k = (ma)^{\alpha} \lambda\alpha \underbrace{\left(\frac{1}{1+\lambda} A Z^{1+\lambda} - k'\right)}_{C_m} Z^{\lambda\alpha-1} Z'_k$$

$$+ (ma)^{\alpha} Z^{\lambda\alpha} A Z^{\lambda} Z'_k - 1$$

From this we can express Z'_k, the derivative of the optimal value of Z with respect to k', in the following way

$$A'_k = \frac{\overset{+}{(ma)^{\alpha}} \overset{+}{Z^{\lambda\alpha}}}{\underset{+}{(ma)^{\alpha}} \underset{+}{(\lambda\alpha)} \underset{+}{C_m} \underset{+}{Z^{\lambda\alpha-1}} + \underset{+}{(ma)^{\alpha}} \underset{+}{A} \underset{+}{Z^{(1+\alpha)\lambda}} + \underset{+}{h^{\alpha}}} \qquad (9^*)$$

Assuming that $0 < Z < \bar{Y}$ and $C_m \geqq 0$ it can easily be checked that $Z'_k > 0$ as postulated. This means that as k increases, say, from level 0 (i.e., the constraint curve in Figure 5 shifts downwards), the optimal amount exported will increase and *vice versa*.

There exist, however, upper and lower limits on export, $\bar{Y}$ and 0, respectively. Taking these limits into consideration, we need to find out under what circumstances Z will approach these limits and what happens to the other variables at the same time.

It is easy to see that there is a critical value of k such that constraint (3*) can be satisfied only if $Z = \bar{Y}$ and $C_m = 0$. If this is the case, then C_d must clearly be 0 in this single feasible solution. Depending on the value of μ it may or may not be in the domain of the objective function, thus an optimal solution will approach $Z = \bar{Y}$, $C_m = C_d = 0$.

Next we look at the other limit for Z, i.e., Z = 0, which is approached if k decreases beyond any limit. It is easy to see from (8*) that Z cannot assume zero value in an optimal solution with finite k (because in this case the RHS would be 0, while the LHS would be $h^{\alpha}\bar{Y}$). Thus we can conclude that with k decreasing beyond any limit, Z will approximate 0 and C_m goes to infinity.

These considerations imply that the locus of optimal solutions of the parametric programming problem discussed, i.e., the $S\bar{Y}$ curve is downward sloping and assymptotic to the vertical axes as k approaches minus infinity (i.e., Z to zero). Also, $S\bar{Y}$ approaches point $\bar{Y}$ on the Z axis when k tends to its upper critical value. The homogeneity of the objective function implies that $S\bar{Y}$ will be convex from below as shown in Figure 5.

This analysis shows us that there will always be such a value of k, at which the optimal solution lies on the zero balance of payment curve, and that such a solution can be sought by means of simple iteration.

APPENDIX 3: FORMAL STATEMENT OF THE MODELS USED IN THE NUMERICAL SIMULATIONS

Endogenous Variables

X_j	gross output in sector $j = 1,2,\dots,n$
M_{ir}, M_{id}	competitive rouble and dollar import of commodity $i = 1,2,\dots,n$
X_{ij}	use of domestic-import composite commodity $i = 1,2,\dots,n$ in sector $j = 1,2,\dots,n,n+1$
Z_i, Z_{ir}, Z_{id}	total, rouble and dollar export of commodity i
X_{n+1}	total gross investments
I	total net investments at base price level
$\overline{M}_i, \overline{M}_{ir}, \overline{M}_{id}$	total, rouble and dollar noncompetitive import of commodity $i = 1,2,\dots,n$
$\overline{M}_{ij}$	use of noncompetitive import commodity $i = 1,2,\dots,n$ in sector $j = 1,2,\dots,n,n+1$
$\overline{C}_i$	total private and public consumption of noncompetitive import commodity $i = 1,2,\dots,n$
K_j	capital used in sector $j = 1,2,\dots,n$
L_j	labor employed in sector $j = 1,2,\dots,n$

S_j (optimal) user cost of labor and capital per unit of output in sector j = 1,2,...,n

W_j user cost of labor in sector j = 1,2,...,n

W net rate of return requirement (tax) on labor

Q_j user cost of capital in sector j = 1,2,...,n

R net rate of return requirement (tax) on capital

$\bar{m}_i$ share of rouble import in total noncompetitive import of commodity i = 1,2,...,n

m_{ir}, m_{id} proportions of competitive rouble and dollar imports of commodity i = 1,2,...,n

P_j domestic seller price of commodity j = 1,2,...,n produced

P_{jd}^E dollar export price of commodity j = 1,2,...,n

V_r, V_d exchange rage of roubles and dollars

$\bar{P}_i^{DI}$ average domestic price of noncompetitive import of commodity i = 1,2,...,n

P_i^D average price of domestic-import composite commodity i = 1,2,...,n

E total consumption expenditure

EE excess expenditure level

C total consumption at base price level

Exogeneous Variables and Parameters

s_j capital replacement rate in sector j = 1,2,...,n

δ_j depreciation rate in sector j = 1,2,...,n

K total capital stock

L total labor

Z_{id}^o, Z_{ir}^o, $\varepsilon_{ir}, \varepsilon_{id}$ parameters in the export functions

λ_i, ∂_i	dollar export supply and demand elasticities in sector i = 1,2,...,n
$P_{id}^{WE}, P_{ir}^{WE}, P_{id}^{WI}, P_{ir}^{WI}$, $\bar{P}_{id}^{WI}, \bar{P}_{ir}^{WI}$	world market export and import prices of commodity i (rouble-dollar, competitive-noncompetitive import)
D_d, D_r	target surplus or deficit on dollar and rouble foreign trade balance
a_{ij}	input coefficient of domestic-import composite commodity i = 1,2,...,n in sector j = 1,2,...,n,n+1
$\bar{m}_i^o, \rho_i$	parameters in the determination of the area composition of the noncompetitive import of commodity i = 1,2,...,n
m_{ir}^o, m_{id}^o, μ_{ir}, μ_{id}	parameters in the import functions, i = 1,2,...,n
$b_i, \bar{b}_i$	fixed (base) amount of total consumption of commodity i = 1,2,...,n
$c_i, \bar{c}_i$	fixed structure of excess consumption of commodity i = 1,2,...,n
σ	real consumption-net investment ratio
w_j	wage coefficient in sector j = 1,2,...,n

Balancing Equations

Intermediate Commodities

$$X_i + M_{ir} + M_{id} = \sum_{j=1}^{n+1} X_{ij} + C_i + Z_{ir} + Z_{id} \qquad (1)$$

$$i = 1,2,...,n$$

$$X_{n+1} = \sum_{j=1}^{n} s_j K_j + I \qquad (2)$$

Noncompetitive Imports

$$\bar{M}_i = \sum_{j=1}^{n+1} \bar{M}_{ij} + \bar{C}_i \qquad i = 1,2,\ldots,n \qquad (3)$$

Primary Factors

$$K = \sum_{j=1}^{n} K_j \qquad (4)$$

$$L = \sum_{j=1}^{n} L_j \qquad (5)$$

Trade Balances

$$\sum_{i=1}^{n} P_{id}^{E} Z_{id} - \sum_{i=1}^{n} P_{id}^{WI} M_{id} - \sum_{i=1}^{n} \bar{P}_{id}^{WI} \bar{M}_{id} = D_d \qquad (6)$$

$$\sum_{i=1}^{n} P_{ir}^{WE} Z_{ir} - \sum_{i=1}^{n} P_{ir}^{WI} M_{ir} - \sum_{i=1}^{n} \bar{P}_{ir}^{WI} \bar{M}_{ir} = D_r \qquad (7)$$

Technological Choice

$$X_j = F_j\,(L_j, K_j) = \zeta_j\, L_j^{\xi_j}\, K_j^{1-\xi_j} \qquad j = 1,2,\ldots,n \qquad (8)$$

$$X_{ij} = a_{ij} X_j \qquad i = 1,2,\ldots,n;\; j = 1,2,\ldots,n,n+1 \qquad (9)$$

$$\bar{M}_{ij} = \bar{m}_{ij} X_j \qquad i = 1,2,\ldots,n;\; j = 1,2,\ldots,n,n+1 \qquad (10)$$

$$S_j \frac{\partial F_j}{\partial L_j} = W_j \qquad j = 1,2,\ldots,n \qquad (11)$$

$$S_j \frac{\partial F_j}{\partial K_j} = Q_j \qquad j = 1,2,\ldots,n \qquad (12)$$

Import and Export Functions

Noncompetitive Imports

$$\bar{m}_i = \bar{m}_i^o \left(\frac{V_d \ \bar{P}_{id}^{WI}}{V_r \ \bar{P}_{ir}^{WI}} \right)^{\rho_i} \qquad i = 1,2,\ldots,n \qquad (13)$$

$$\bar{M}_{ir} = \bar{m}_i \ \bar{M}_i \qquad i = 1,2,\ldots,n \qquad (14)$$

$$\bar{M}_{id} = (1 - \bar{m}_i) \ \bar{M}_i \qquad i = 1,2,\ldots,n \qquad (15)$$

Competitive Imports

$$m_{ir} = m_{ir}^o \left(\frac{P_i}{V_r \ P_{ir}^{WI}} \right)^{\mu_{ir}} \qquad i = 1,2,\ldots,n \qquad (16)$$

$$m_{id} = m_{id}^o \left(\frac{P_i}{V_d \ P_{id}^{WI}} \right)^{\mu_{id}} \qquad i = 1,2,\ldots,n \qquad (17)$$

$$M_{ir} = m_{ir} \ (X_i - Z_i) \qquad i = 1,2,\ldots,n \qquad (18)$$

$$M_{id} = m_{id} \ (X_i - Z_i) \qquad i = 1,2,\ldots,n \qquad (19)$$

Exports

$$Z_i = Z_{ir} + Z_{id} \qquad i = 1,2,\ldots,n \qquad (20)$$

$$Z_{ir} = Z_{ir}^o \left(\frac{P_i}{V_r \ P_{ir}^{WE}} \right)^{\varepsilon_{ir}} \qquad i = 1,2,\ldots,n \qquad (21)$$

$$Z_{id} = Z_{id}^o \left(\frac{P_i}{V_d \ P_{id}^{WE}} \right)^{\varepsilon_{id}} \qquad i = 1,2,\ldots,n \qquad (22)$$

where

$$\varepsilon_{id} = \begin{cases} \lambda_i & \text{if export supply function} \\ \partial_i & \text{if export demand function} \\ \lambda_i \; \partial_i / (\lambda_i + \partial_i) & \text{if export equilibrium function} \end{cases}$$

Final Demand Equations

$$C_i = b_i + \frac{c_i}{\sum_{i=1}^{n} P_i^D \, c_i} \; EE \qquad i = 1,2,\ldots,n \qquad (23)$$

$$\bar{C}_i = \bar{b}_i + \frac{\bar{c}_i}{\sum_{i=1}^{n} \bar{P}_i^{DI} \, \bar{c}_i} \; EE \qquad i = 1,2,\ldots,n \qquad (24)$$

$$EE = E - \sum_{j=1}^{n} \left(P_j^D \, b_j + \bar{P}_j^{DI} \, \bar{b}_j \right) \qquad (25)$$

$$C = \sum_{i=1}^{n} C_i + \sum_{i=1}^{n} \bar{C}_i \qquad (26)$$

$$C - \sigma \cdot I = 0 \qquad \text{or} \qquad I = \bar{I} \qquad (27)$$

Prices and Costs

$$W_j = (1 + W) \; w_j \qquad j = 1,2,\ldots,n \qquad (28)$$

$$Q_j = (\delta_j + R) \; P_{n+1} \qquad j = 1,2,\ldots,n \qquad (29)$$

$$P_{n+1} = \sum_{i=1}^{n} P_i^D \, a_{i,n+1} + \sum_{i=1}^{n} \bar{P}_i^{DI} \, \bar{m}_{i,n+1} \qquad (30)$$

$$\bar{P}_i^{DI} = \bar{m}_i V_r \bar{P}_{ir}^{WI} + (1 - \bar{m}_i) V_d \bar{P}_{id}^{WI} \qquad i = 1, 2, \ldots, n \qquad (31)$$

$$P_j = \sum_{i=1}^{n} P_i^D a_{ij} + \sum_{i=1}^{n} \bar{P}_i^{DI} \bar{m}_{ij} + S_j \qquad j = 1, 2, \ldots, n \qquad (32)$$

$$P_i^D = \frac{1}{1 + m_{id} + m_{ir}} P_i + \frac{m_{id}}{1 + m_{id} + m_{ir}} V_d P_{id}^{WI}$$

$$+ \frac{m_{ir}}{1 + m_{id} + m_{ir}} V_r P_{ir}^{WI} \qquad i = 1,2,\ldots,n \qquad (33)$$

$$P_{id}^E = \begin{cases} P_{id}^{WE} & \text{of export supply specification} \\ \left(\dfrac{Z_{id}}{Z_{id}^o}\right)^{1/\partial_i} P_{id}^{WE} & \text{otherwise} \end{cases} \qquad (34)$$

$$i = 1,2,\ldots,n$$

Price normalization rule

$$\sum_{i=1}^{n} P_i^D C_i + \sum \bar{P}_i^{DI} \bar{C}_i = C \qquad (35)$$

The Specifics of the Small Model

As mentioned in section 5 the small model is partly simpler, partly different from the more detailed one. There are only three sectors, one foreign trade area, all imports as treated as competitive. These simplified assumptions indicate some plausible changes in the above model specifications.

One of the more important differences is that we treat home produced and imported commodities as imperfect substitutes. Therefore instead of equations 1, 16-19, and 33 we have to use the following ones.

$$X_i = s_{id} \left(\sum_{j=1}^{n+1} X_{ij} + C_i \right) + Z_i$$

$$M_i = s_{im} \left(\sum_{j=1}^{n+1} X_{ij} + C_i \right)$$

$$s_{id} = s_{id}^o \left(\frac{P_i}{P_i^D} \right)^{-\mu_i}$$

$$s_{im} = s_{im}^o \left(\frac{V\ P_i^{WI}}{P_i^D} \right)^{-\mu_i}$$

$$P_i^D = s_{id}\ P_i + s_{im}\ V\ P_i^{WI}$$

Where s_{id} and s_{im} are the relative shares of home produced and imported sources available for domestic use, s_{id}^o and s_{im}^o their base values, respectively (see Appendix 1 for explanation).

The other real difference stems from the assumed substitutability of commodities in consumption. We have used an LES type of consumption demand system. Therefore, equation 23 will be in this case as follows:

$$C_i = b_i + \frac{c_i}{P_i^D}\ EE$$