

Stochastic Water Requirements for Supplementary Irrigation in Water Resource Systems

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**IIASA Research Report
November 1982**



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**STOCHASTIC WATER REQUIREMENTS FOR SUPPLEMENTARY
IRRIGATION IN WATER RESOURCE SYSTEMS**

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RR-82-34

November 1982

**INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
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International Standard Book Number 3-7045-0048-8

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FOREWORD

Water resource systems have been an important part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resource management must be developed further. This in turn requires an increase in the degree of detail and sophistication of analysis, including economic, social, and environmental evaluation of water resource development alternatives aided by application of mathematical modeling techniques, to generate inputs for planning, design, and operational decisions.

This report outlines an approach to the assessment of supplementary irrigation water requirements in semi-humid climatic zones. These water demands are seen in the broad context of input data required for long-range planning in models of water resource systems.

The stochastic character of supplementary irrigation water requirements is generally recognized, although in most cases it is not adequately reflected in the water resource systems models used in long-range planning. This report describes how mean monthly time series of supplementary irrigation water requirements may be developed, based on generally available data on rainfall, temperature, humidity, wind velocity, and amount of sunshine. The Labe River catchment in Czechoslovakia was used as a test case and illustrates the application of the proposed approach.

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STOCHASTIC WATER REQUIREMENTS FOR SUPPLEMENTARY IRRIGATION IN WATER RESOURCE SYSTEMS

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SUMMARY

In semi-humid and humid climates of the temperate zone, supplementary irrigation water requirements depend on meteorological conditions. A mathematical model is developed to assess monthly time series of irrigation water requirements, based on Penman's equation and calibrated on the basis of data obtained from irrigation systems in Czechoslovakia. In the model, monthly time series of temperature, relative humidity, sunshine, wind velocity, and precipitation are used as input data. Because of the persistence phenomena often noted in irrigation practices, the correlation between the current irrigation water requirements and those of the previous month is taken into account. The statistical properties of irrigation water requirements are analyzed as the basis for the generation of a synthetic water requirement time series. The model can be used for long-term planning of water resource systems incorporating supplementary irrigation water use, as is shown in the case of the Labe River catchment area in Czechoslovakia.

1 PROBLEM DEFINITION

In dry regions, irrigation water requirements exhibit a more or less regular cyclic form with only slight deviations from year to year, so that conventional attitudes to irrigation planning and modeling are adequate. However, in semi-humid and humid areas, supplementary irrigation is closely related to the variability of factors such as precipitation and potential evapotranspiration. This should be proved not only qualitatively but also quantitatively on the basis of data from some irrigation systems.

Meteorological data are the records of stochastic events, so the supplementary irrigation water requirements that depend on them are also stochastic. In long-term planning of water resource systems, including large-scale irrigation, the stochastic character of irrigation water requirements should be reflected on the same basis, and with the same accuracy, as other input variables. A monthly time series of flows is commonly adopted for this purpose as an appropriate input into water resource system (WRS) models. Therefore,

a monthly time series of irrigation water requirements could also be adopted as an adequate form of input to these models, bearing in mind the purpose of long-term planning of large-scale irrigation.

In keeping with this main aim of the model, a prescriptive rather than descriptive model is adequate in order to create a tool with approximately the same accuracy as the other inputs and to quantify the effects of alternative irrigation and WRS designs. Otherwise, the WRS model would be too cumbersome for engineering and planning purposes.

As the results of this study will be used for long-term WRS planning, the aggregation of some data is necessary. Therefore, the influences of the type of soil, vegetation, and agricultural production on irrigated fields are aggregated into calibration coefficients, and are not taken into account as variables. Meteorological data are the only variables used for determining monthly time series.

The monthly time series is an adequate form of input for all principal kinds of WRS models, i.e.,

- deterministic simulation models, when observed time series are used directly as inputs;
- an implicit stochastic model, where the basis of synthetic time series generation is the observed (or on the observed data calculated) time series;
- an explicit stochastic model, where the parameters of the compound probability distribution are determined on the basis of the set of input time series (observed or generated).

From this analysis, it can be concluded that the most appropriate form of irrigation requirement inputs into WRS models is the time series based on climatic data, as related to large-scale irrigation policies and methods.

2 ANALYSIS OF IRRIGATION WATER REQUIREMENTS

One of the objectives of water resource systems planning, including irrigation, may be to supply water for irrigation in such a way as to maximize the net economic return of a farm, or a whole system, or to maximize the yields of marketable products. The latter objective will be attained (e.g. Skogerboe 1977), if soil water is not the limiting factor in plant growth.

The total quantities of water affecting the soil in a month during the vegetative period can be expressed in the following water budget equation (Fleming 1975):

$$\Delta S_t = P_t - E_t - R_t - G_t - U_t \quad (1)$$

where

- ΔS_t = the change in water storage (mainly as soil water in an unsaturated zone);
- P_t = precipitation (mainly rainfall);
- E_t = evapotranspiration;
- R_t = surface run-off;
- G_t = subsurface flow;
- U_t = underflow (deep percolation);
- t = month.

The term $(R_t + G_t + U_t)$ can be used to express the unused (ineffective) part of precipitation equal to $(1 - \alpha)P_t$. Equation (1) can then be simplified to

$$\Delta S_t = \alpha_t P_t - E_t \quad (2)$$

The coefficient α is not constant and depends on many hydrological and soil conditions. However, keeping in mind the aim of the study and the aggregated character of the data, an approximation by a constant value (or determined only by precipitation) can be admitted.

The maximum value of E_t under given meteorological conditions is the potential evapotranspiration PE_t that occurs when the soil water content is not a limiting factor in evaporation and transpiration. This state can be reached by adding an amount of irrigation water, I_t . The water budget can then be expressed by

$$I_t = PE_t - \alpha_t P_t + \Delta S_t \quad (3)$$

Considering the losses in delivering irrigation water to the field (expressed by a coefficient k), the equation will be

$$I'_t = k(PE_t - \alpha_t P_t + \Delta S_t) \quad (4)$$

This equation was derived in a slightly different form by Holy (1979) for the whole vegetative period.

At the beginning of the vegetative period, the term ΔS_t can be considered as the available store of water due to winter precipitation. For planning purposes, Holy (1979) recommended the following values according to the permeability of soils:

Low: 23–55 mm

Mean: 26–45 mm

High: 12–21 mm

The depth of the active soil layer is assumed to be 0.3–0.6 m.

The individual terms in eqns. (3) and (4) will now be analyzed further, with the emphasis on the potential evapotranspiration term, as this is crucial in determining irrigation water requirements.

3 POTENTIAL EVAPOTRANSPIRATION

The best method of determining potential evapotranspiration would be its measurement under field conditions, but because this is not technically or economically feasible, sample measurements are used. Sampling may involve the measurement of soil moisture and the indirect calculation of evapotranspiration; or the lysimeter method may be used, whereby some crop (usually grass) is planted in tanks and the losses of water used to maintain satisfactory growth are measured. In general, the conditions in the tank may not closely simulate actual field conditions, and the results thus obtained may not be reliably extrapolated to a much larger area (Veihmeyer 1964). Nevertheless, the reliability of

various methods of evapotranspiration estimation on the basis of measured meteorological data is often determined by comparison with lysimetric measurements. This is one of the contradictions that this study attempts to analyze.

3.1 Evapotranspiration Estimation

The basis for the determination of evapotranspiration is the physical process of evaporation, regardless of the evaporating medium (water surface, soil, vegetation, etc.). In hydrology, the term evaporation refers to the evaporation from a water surface, and evapotranspiration refers to the evaporation from soil and vegetation (but evaporation can also include evapotranspiration from bare soil). It is commonly accepted that evaporation and evapotranspiration under conditions of abundant water supply (i.e., potential evapotranspiration) are governed by the same physical laws and can be expressed by the same, or similar, formulae. Attitudes to this process differ among authors, and the following methods have been used: energy budget approach, aerodynamic approach, eddy flux measurement, heat flow measurement of sap flux and the empirical or semi-empirical method (Rodda *et al.* 1976), water budget method, energy budget method, aerodynamic profile method, eddy correlation method, combination method, and empirical formulae (WMO 1966). This classification is not unique; other authors distinguish humidity methods (e.g., Ivanov 1954, Pýcha 1965), methods using primarily temperature (Linacre 1977), and multiple correlation methods (Christiansen 1968, Christiansen and Hargreaves 1969, Kos 1969). As the classification of methods is not the primary aspect of this study, that used here is rather arbitrary.

3.1.1 Water Budget Method

The basic water budget method requires an inflow of water to the soil profile, an outflow, and a change in storage. Determination of these relations is the basic aim of hydrological models describing the dynamics of water in soil. However, only short time intervals are required; the longest acceptable interval for these deterministic hydrological simulation models is one day.

The choice of the appropriate model for this study is very difficult, as each one has its advantages and disadvantages (e.g., US Army Corps of Engineers SSARR model; Stanford Watershed model; British Road Research model; Dawdy and O'Donnell model; Boughton model; Huggins and Monke model; Hydrocomp simulation model; Kutchment model; Hyreun model; Lichty, Dawdy, and Bergmann model; Kozak model; Mero model; USDAHL model; Institute of Hydrology model; Vemuri and Dracup model; Water Resources Board "Disprin" model; UBC watershed and flow model; Shih, Hawkins, and Chambers model; Leaf and Brink model; and Balek Dambo model). The application of deterministic hydrological simulation models is also not straightforward, and will be considered in the second phase of this study. For the estimates in this study, only a simple procedure is necessary.

3.1.2 Energy Budget Method

The energy budget method assumes that the energy received by a surface through radiation equals the energy used for evaporation and for heating the air and the soil, plus

any advective energy. For monthly balances, the energy used in heating the soil and the advective energy may be neglected (Veihmeyer 1964), and the energy balance can then be written as follows:

$$Q_s - Q_r - Q_b - Q_h - Q_e = 0 \quad (5)$$

where

- Q_s = solar radiation incident on the soil (or vegetation) surface;
- Q_r = reflected solar radiation;
- Q_b = net energy lost by a body of soil and vegetation through the exchange of long-wavelength radiation;
- Q_h = energy conducted from a body of soil and vegetation to the atmosphere as heat;
- Q_e = energy utilized for evapotranspiration.

Other authors use different terminology in the energy budget (e.g. WMO 1966), i.e.,

$$E = R_n - S - A \quad (6)$$

where

- E = energy due to evaporation;
- R_n = net radiation flux;
- S = soil heat flux;
- A = sensible heat flux,

or

$$R_n = E(1 + B) + S \quad (7)$$

where B is the Bowen ratio.

From the engineering point of view, the energy budget method cannot be used without an additional empirical approach, as there are not enough data for its application (Balek 1980).

3.1.3 Aerodynamic Profile and Eddy Correlation Methods

The classical Thornthwaite and Holzman relation (1939, 1942) gives evaporation as a function of wind speed u and the specific humidity of air q at different heights above the ground (z_1, z_2)

$$E = \frac{-k^2 \sigma (q_2 - q_1)(u_2 - u_1)}{(\log z_2/z_1)^2} \quad (8)$$

where

- E = evaporation;
- σ = density of air;
- q_1 and q_2 = specific humidities at heights z_1 and z_2 , respectively;
- u_1 and u_2 = wind speeds at heights z_1 and z_2 , respectively;
- k = Kármán's constant.

This equation is valid under strictly neutral conditions; otherwise, it gives very high results due to the breaking of the logarithmic profile law. This aerodynamic profile method, which requires precise determination of wind and water vapor profiles near the evaporating surface, is therefore suitable for short-term studies, but cannot be used as a routine method (WMO 1966). The same holds true for the eddy correlation method, which uses measurements of vertical turbulent fluxes in the atmosphere. It involves the measurement of short-period fluctuations in vertical wind velocity and water vapor at some arbitrary level.

3.1.4 Combination Methods and Empirical Formulae

From an analysis of all evapotranspiration estimation methods at monthly intervals from the standpoint of irrigation requirements determination in this study, it seems that the only adequate ones are combinations of methods and empirical formulae. As there are many of these (e.g. Seuna 1977 lists ten methods and formulae), the most commonly used will be listed in abbreviated form here, and in detail in Appendix A, and some will be discussed as to their possible application for the purpose of this study. In this listing, PE_t is the potential evapotranspiration in period t .

Penman

$$PE_t = f(\text{sunshine, temperature, relative humidity, wind velocity})$$

Linacre

$$PE_t = f(\text{temperature, relative humidity})$$

Thornthwaite

$$PE_t = f(\text{temperature})$$

Blaney and Criddle

$$PE_t = f(\text{temperature, crop coefficient})$$

Turc

$$PE_t = f(\text{temperature, solar radiation, precipitation, yield, crop coefficient})$$

$$PE_t = f(\text{temperature, solar radiation, humidity})$$

Johansson

$$PE_t = f(\text{solar radiation, wind velocity})$$

Ivanov

$$PE_t = f(\text{temperature, relative humidity})$$

Ostromecki and Alpatjev

$$PE_t = f(\text{saturation deficit, crop coefficient})$$

Pýcha

$$PE_t = f(\text{saturation deficit, crop coefficient, temperature})$$

Makking, Stephens, Jensen, Jensen and Haise

$$PE_t = f(\text{solar radiation, temperature})$$

McIlroy

$$PE_t = f(\text{atmospheric pressure, net radiation, soil heat flux, wind velocity, humidity})$$

Christiansen and Hargreaves (multicorrelation)

$$PE_t = f(\text{solar radiation, temperature, wind velocity})$$

Baier and Russelo (multicorrelation)

$$PE_t = f(\text{temperature, solar radiation, wind velocity, saturation deficit})$$

Morton

$$PE_t = f(\text{temperature, relative humidity, sunshine, areal evapotranspiration}).$$

A brief discussion of Morton's method is included in Appendix C.

3.2 Comparison of Evapotranspiration Formulae

Many authors have compared evapotranspiration values estimated by a combination of methods and empirical formulae (e.g., WMO 1966, Penman 1963, Rodda *et al.* 1976, Blaney and Criddle 1966, Christiansen 1968, Schulz 1973, Seuna 1977). Some of these comparisons were for semi-humid climatic conditions (e.g., Penman 1954, 1963), but most of them referred to arid and semi-arid zones.

Measurement of data from which the potential evapotranspiration is computed depends on local site conditions, since there is no way to measure the evapotranspiration that depends purely on meteorological conditions.

Some authors claim that the best methods are those based on net radiation, but since this is difficult to measure, it is therefore calculated from the total incoming radiation and other values, such as the amount of sunshine. In some formulae, the temperature and amount of sunshine are considered to be good indicators of radiation, and can be used for monthly intervals. According to Tanner (1967), these methods give lower values in spring and higher values in autumn since there is a time lag between radiation and temperature readings due to the storage of heat in the ground.

For the purpose of this study, the comparisons made by Johansson (1970) are important, as they were done for monthly values and in a semi-humid climate of the temperate zone (Sweden). He compared the calculations from the formulae of Penman, Thornthwaite, Blaney and Criddle, and Turc, with his own, and the results were as follows. Johansson's formula gave highest radiation values in spring and the beginning of summer. Almost as high as Johansson's values were those of Penman for May and June. Thornthwaite's formula gave highest values in August and September, while Johansson and Penman gave the lowest values. This seems to confirm the suggestion of the time lag between radiation and temperature readings.

The values calculated from the formula of Blaney and Criddle were profoundly different. Their formula was derived for arid regions and was therefore not applicable to humid and semi-humid areas.

The adequacy of evapotranspiration formulae can also be judged from the standpoint of the time and space intervals to which they are applied. The Swedish International Hydrological Decade (IHD) Commission (see Forsman 1969) recommended Penman's, McIlroy's, and Konstantinov's formulae for monthly values on the micro- and meso-scales (1 m–1 km), and Budyko's formula for annual values on the meso- and macro-scales (1–100 km).

McGuinness and Parmele (1972) investigated evapotranspiration rates in Ohio (using the US Weather Bureau method based on Penman's formula) for different periods of time (1 day to 1 month), and obtained very close correlations (coefficient of multiple correlation $R = 0.96$), taking into account only the number of months t and days d :

$$PE_{t,d} = (0.179d + 1.235t - 0.0858t^2 - 0.0082td - 3.834)/10 \quad (9)$$

For $d = 30$, this equation reduces to

$$PE_t = (1.54 + 0.989t - 0.0858t^2)/10 \text{ (ft)} \quad (10)$$

For instance, for June ($t = 6$), this formula gives 0.439 ft (134 mm), which supports the statement above that evapotranspiration and consequently irrigation requirements in arid and semi-arid areas are more or less constant and are not dependent on meteorological deviations, as are those in semi-humid zones.

In Finland, Seuna (1977) calculated evapotranspiration rates in 20 regions using the US Weather Bureau formula (based on Penman's equation). The accumulation of heat in the ground was not taken into account, but the same differences as stated above occurred.

Mustonen and McGuinness (1968) criticized the lysimeter method as a basis of measuring field evapotranspiration because it gives higher values due to advection, especially over shorter periods. This effect is more pronounced in arid regions, but it may also be noticeable during dry periods in semi-humid zones. For instance, in Arizona, evapotranspiration according to net radiation was 6.4 mm/day, but the lysimeter method gave a value 159% greater. In the UK, Penman found that lysimetric measurements over a three-day interval were 112% higher than net radiation.

Riou (1977) based his theory on Penman's equation. Using a more general thermodynamic approach, he concluded that in evapotranspiration the two main terms in Penman's equation (radiation and vapor flow) are influenced by vegetation in different ways, and he therefore used the term "apparent" saturation deficit. The same effect can be achieved using different empirical coefficients for these terms, as shown in the model described in Section 5.

Brochet and Gerbier (1977) also used Penman's equation as a basis. They suggested a correction of radiation and vapor flow terms, which then led to a correction of the regression constants in Penman's equation.

Perrier (1977) stated that some differences in methods and results were the consequences of unequal notation by different authors, so that incomparable values are then discussed. He therefore suggested a classification of evaporation phenomena, explaining different definitions of evaporation and evapotranspiration.

3.3 Penman's Equation

As a result of the comparisons made in Section 3.2, it can be stated that the choice of the "best" equation to calculate evapotranspiration is not an easy one. However, some of the equations can be excluded for semi-humid climatic conditions, some are not used as they do not use all the available information, and the results of others do not differ significantly.

According to the comprehensive evaluation of Penman's equation made by Rodda *et al.* (1976), which gives many references, and to the facts that it was derived for a semi-humid climate and, according to Jensen's statement (1973), gives best results with proper calibration, Penman's equation was taken as the basis of the irrigation water requirements model. This decision was supported by the recommendations of the WMO and the practice of the FAO/WMO agroclimatology surveys. The equation will now be described in detail; the general form is

$$PE_t = f_t E_0 \quad (11)$$

where PE_t is the potential evapotranspiration in period t (mm/month), and f_t is a factor converting potential evaporation E_0 to PE_t . For the northern hemisphere, Penman suggested the following.

t	f_t
March	0.7
April	0.7
May	0.8
June	0.8
July	0.8
August	0.8
September	0.7
October	0.7

$$E_0 = \frac{\Delta R_n + \gamma E}{\Delta + \gamma} = \text{potential evaporation (mm/month)} \quad (12)$$

where

- γ = psychrometric constant ($= 0.49 \text{ mm } ^\circ\text{C}^{-1} = 0.65 \text{ mbar } ^\circ\text{C}^{-1}$);
- Δ = slope of the saturation vapor pressure curve of air ($\text{mm } ^\circ\text{C}^{-1}$). In the model, this is approximated by $\Delta = 0.3559e^{T/18} \text{ mm } ^\circ\text{C}^{-1}$, where T is the mean monthly air temperature;
- R_n = energy budget or net radiation (mm/month);
= H/L , where H = net radiation ($\text{J cm}^{-2}/\text{month}$ or $\text{cal cm}^{-2}/\text{month}$);
- L = latent heat of evaporation ($1 \text{ mm} \approx 59 \text{ cal cm}^{-2} \approx 247 \text{ J cm}^{-2}$). The value of L at 12°C was considered; for 20°C it would be 245 J cm^{-2} . In the model, it was taken to be constant.

Because net radiation is not usually measured, it was calculated from measured data as follows:

$$H = R_c - R_b, \quad R_n = R_c/L - R_b/L \quad (13)$$

$$R_c = R_a(1 - r)(a + bn/N)$$

TABLE 1 Mean monthly intensity of solar radiation on a horizontal surface, R_a^d/L (mm/day) (after Criddle).

Latitude (°N)	M	A	M	J	J	A	S	O
70	4.3	9.1	13.6	17.0	15.8	11.4	6.8	2.4
60	6.8	11.1	14.6	16.5	15.7	12.7	8.5	4.7
50	9.1	12.7	15.4	16.7	16.1	13.9	10.5	7.1
40	11.0	13.9	15.9	16.7	16.3	14.8	12.2	9.3
30	12.7	14.8	16.0	16.5	16.2	15.3	13.5	11.3
20	13.9	15.2	15.7	15.8	15.7	15.3	14.4	12.9

R_a = maximum solar radiation ($\text{J cm}^{-2}/\text{month}$ or $\text{cal cm}^{-2}/\text{month}$). See Table 1 for R_a^d/L (mm/day); $R_a/L = R_a^d/LD$, where D is the number of days in a month;

r = surface albedo (0.05 for water);

n = duration of sunshine (h/month);

N = maximum possible duration of sunshine (h/month). See Table 2 for N^d , then $N = N^d D$, where D is the number of days in a month;

a, b = constants: $a = 0.18$; $b = 0.55$ (other values of a and b given by WMO (1974) for tropical and humid zones differ slightly; original values used in our model have been recommended by the WMO for the semi-humid temperate zone);

R_b = back-reflected radiation ($\text{J cm}^{-2}/\text{month}$ or $\text{cal cm}^{-2}/\text{month}$).

$$R_b = \sigma T_a^4 (0.56 - 0.09 \sqrt{e_d}) (0.1 + 0.9 n/N) \quad (14)$$

where

σT_a^4 = black-body radiation ($\text{J cm}^{-2}/\text{month}$ or $\text{cal cm}^{-2}/\text{month}$) at mean air temperature T_a (K);

σ_d = Stefan–Boltzmann's constant $\approx 1.17 \times 10^{-7}$ $\text{cal cm}^{-2} \text{K}^{-4}/\text{day}$;

σ = $\sigma_d D$ ($\text{cal cm}^{-2} \text{K}^{-4}/\text{month}$);

e_d = saturation vapor pressure at the dewpoint (mm);

E = vapor flow parameter (mm/month).

The constants in eq. (14) may vary with latitude (Rodda *et al.* 1976), but only slightly.

TABLE 2 Maximum possible duration of bright sunshine in hours per day (N^d calculated after Veihmeyer).

Latitude (°N)	M	A	M	J	J	A	S	O
60	11.6	13.9	16.9	17.8	17.7	15.4	12.3	10.0
50	11.9	13.3	15.4	15.7	15.8	14.4	12.2	10.7
40	12.0	12.9	14.4	14.5	14.7	13.7	12.1	11.1
30	12.1	12.6	13.7	13.7	13.9	13.3	12.0	11.5
20	12.1	12.3	13.2	13.0	13.3	12.9	11.9	11.8

According to Penman's later studies

$$E = 0.35(e_a - e_d)(0.5 + 0.54w)D \quad (15)$$

where

- e_a = saturation vapor pressure at mean air temperature (mm);
- e_d = saturation vapor pressure at the dewpoint (mm);
- w = mean wind velocity 2 m above the ground (m s^{-1});
- D = number of days in a month.

Note: In SI units, e_a and e_d should be expressed in millibars (mbar), but the measured values available are in millimeters of mercury, so that the expression in mbar would require a double recalculation. For WMO recommendations concerning the use of Penman's formula, see Appendix B.

Some attempts have been made to simplify Penman's equation; for instance, Linacre (1977) suggested the formula

$$E_o = \frac{700T_m/(100 - A) + 15(T - T_d)}{80 - T} \quad (16)$$

where

- E_o = evaporation (mm/day);
- T = mean temperature ($^{\circ}\text{C}$);
- $= T_d + 0.006h$, h = elevation (m);
- A = latitude (degrees);
- T_d = mean dewpoint temperature ($^{\circ}\text{C}$).

Linacre noted that typical monthly values may differ by as much as 0.5 mm/day in the calculation of evaporation from a lake surface. In fact, Linacre's method requires only air temperature and relative humidity as input data (the dewpoint temperature T_d can be calculated from the relative humidity and vice versa; the same applies to saturation vapor pressure e_d at the dewpoint). This method is therefore only suitable for locations where these data are available for evaporation estimation.

Linacre's formula was tested on the input data used in this study and it was found that in comparison with Penman's equation, it overestimated evaporation in the late months of the vegetative period. However, when an empirically determined correction coefficient Z was introduced, the deviations of both methods in the vegetative period were less than 5% in 60% of compared pairs, and the maximal deviation was 20% in April.

The formula for potential evapotranspiration PE_t was $PE_t = E_o/Z$, where E_o is evaporation calculated from eq. (16).

t	Z
April	1.7
May	1.8
June	1.9
July	2.0
August	2.5
September	3.2

Summarizing Penman's equation, it could be expressed as F_p or F_m , i.e.,

$$PE_t = F_p(r, n/N, e_a, e_d, w, T_a) = F_m(r, n, T_a, T_b, w) \quad (17)$$

where T_b is the wet bulb temperature ($^{\circ}\text{C}$), F_p is a combination of eqs. (9)–(13), and F_m is the function of measured values n , T_a , T_b , w , and estimated albedo (r). Other values have been defined.

The sensitivity of this equation to errors or deviations in the measured values has been analyzed by Howard and Lloyd (1979), who concluded that the errors in the input parameters were found to affect the evapotranspiration estimates significantly, particularly those that were very sensitive to marginal variations in the albedo regression constants (a and b) and temperature measurements. In turn, evapotranspiration was found to be the most significant variable in the water balance. On the other hand, errors in wind speed and sunshine measurements were far less critical (this fact also supports Linacre's simplification).

4 PRECIPITATION

The second important part of the calculation of irrigation requirements is the evaluation of effective rainfall. This can be done on the basis of continuous precipitation records, or hourly, daily or monthly rainfall values. Accurate hydrological evaluation requires time intervals not longer than one hour (Balek 1980), but for preliminary planning purposes, longer intervals can be used. The effective rainfall is evaluated on the basis of average or prevailing conditions. The most common methods use a coefficient of effectiveness α (see eq. (2)), the determination of which is discussed below.

It can be taken for granted (Holy 1980) that α is closely related to the coefficient of run-off c from irrigated fields, i.e.,

$$\alpha = 1 - c - r \quad (18)$$

where r is the coefficient of evapotranspiration during the precipitation interval. This value is often neglected, mainly because of uncertainty in the determination of c . In this case $\alpha = 1 - c$ is used, and further analysis concerns the run-off coefficient c .

Härtel (1925) was one of the first scientists to deal with this problem using

$$c = n_1 n_2 n_3 n_4 \quad (19)$$

where n_1 represents the length of the field. In calculating the amount of irrigation required, the length of the field (in m) is greater than the critical value, and a constant value $n_1 = 0.55$ is used. The second term, n_2 , represents the amount of forest cover; where there is little or none, 0.95–0.9 is used for the coefficient. The slope of the field is expressed by n_3 : for hilly country, 0.8 is suggested, and for plains 0.6 (according to other authors, such as Cermak and Brenda 1971). The last term, n_4 , represents the permeability of the soil. The following table gives a summary of these terms according to various authors.

Coefficient of run-off (c).

Author	Slope (%)	Soil permeability			
		Almost impermeable	Minimum	Mean	Maximum
Härtel (1925)	Hilly	0.38	0.33	0.31	0.29
Ven Te Chow (1964)	7		0.25–0.35		0.15–0.20
	2		0.13–0.17		0.10–0.15
Kostjakov (1951)	5	0.3–0.6	0.25–0.45	0.20–0.30	0.15–0.25
	< 1	0.25–0.40	0.20–0.40	0.15–0.25	0.10–0.20
Cermak and Brenda (1971)	10	0.54	0.45	0.37	0.24
	< 5	0.38	0.32	0.26	0.18

Other authors have also investigated the run-off coefficient c , such as Hudson (1973), Němec (1972), Ogrosky and Mockus (in Ven Te Chow 1964), Rodda *et al.* (1976), and Fleming (1975). The last author evaluated the role of c in hydrological models. The coefficient can be used in simple models based on the “black-box” approach, but on the other side of the complexity scale, deterministic hydrological simulation models such as the Stanford model can be used (this approach attempts to introduce physical relevance to the equations and formulae in the model, but more detailed data on time and area are required). In the present study a compromise between the two methods was achieved by means of physically based calculations of evapotranspiration and a simple evaluation of the effective precipitation.

If the systems and sensitivity analyses of the WRS show that a more detailed investigation is necessary, a conceptual model can be used. Then, instead of a run-off coefficient, other process parameters are necessary. These can be obtained by a combination of measured data and indirect assessment in the process of model calibration. In the choice of the model, one that is readily available and relatively simple (in terms of the number of inputs and calibrated parameters) is preferred.

5 MODEL OF IRRIGATION WATER REQUIREMENTS

Having discussed all the main terms in eq. (4), the irrigation water requirements model can be formulated as follows:

$$WI_t = k_1 f_t \frac{\Delta}{\Delta + \gamma} R_{n,t} + k_2 f_t \frac{\gamma}{\Delta + \gamma} E_t + k_3 P_t + k_4 WI_{t-1} + C \quad (20)$$

where WI_t are irrigation water requirements, the first two terms express the potential evapotranspiration (corresponding to PE_t in eq. (4)), and the third term represents the precipitation P_t . The fourth term was not directly used in eq. (4), but it may have some relation to changes in the soil moisture content expressed by the last term in eq. (4). The last term is the intercept C , which can be taken to be a constant part of the effective precipitation.

In eq. (20), four coefficients k_1 , k_2 , k_3 , k_4 , and the intercept C have been used. The factor f_t converts the potential evaporation to potential evapotranspiration (see eq. (11));

$R_{n,t}$ is Penman's net radiation in period t (see eq. (12)); E_t is the vapor flow parameter in period t (see eq. (15)); P_t is the precipitation in period t ; WI_{t-1} are irrigation water requirements in the previous time period $t-1$; γ is the psychrometric constant; and Δ is the slope of saturation vapor pressure.

The coefficients k_1 , k_2 , and k_3 have been suggested *a priori* from physical and operational considerations, and these can be explained as follows:

$$k_1 = kk_e g \quad k_2 = kk_e h \quad k_3 = kk_e \alpha \quad (21)$$

where k is a coefficient (see eq. (4)) giving the losses due to transportation and distribution of water in irrigated fields (the typical value for sprinkling irrigation is $k = 1.1-1.2$), and k_e is the coefficient of exploitation, giving the degree to which the irrigation capacity has been exploited (in the WRS discussed later, this was approximately 0.2-0.4 for the present state and 0.9 for the future).

The main difference between eqs. (4) and (20) is that the evapotranspiration term has been split into two parts by using weighting coefficients g and h . If $g = h = 1$, then it is apparent that the first two terms will produce evapotranspiration PE_t calculated by Penman's equation and multiplied by the coefficient kk_e (see eq. (21)). According to the results of the model application in this study, and comments by Barton (1979), Brutsaert and Stricker (1979), and Brochet and Gerbier (1977), different values (i.e., $h \neq g$) can be used. This is due to the fact that in irrigation system management, water is supplied at a lower rate than that indicated by the requirements of potential evapotranspiration. Some crops are only partly irrigated and, at some times, potential evapotranspiration occurs. When good irrigation practices are followed, the moisture content of the soil in the most productive areas never drops significantly below the field water capacity. However, such soil surfaces cannot usually be called saturated, and some modification to the evapotranspiration formula is necessary. Barton (1979) suggested the equation

$$PE_t = \frac{\alpha \Delta}{\alpha \Delta + \gamma} R_n + \frac{\gamma}{\alpha \Delta + \gamma} E \quad (22)$$

where α is a constant. Brutsaert and Stricker (1979) used a similar equation:

$$PE_t = (2\beta - 1) \frac{\Delta}{\Delta + \gamma} R_n + \frac{\gamma}{\Delta + \gamma} E \quad (23)$$

where β is a constant. Both of these equations indicate that modified weights for the terms R_n and E might be used; in model (20) Penman's original values were modified by the weighting coefficients g and h .

The coefficient α refers to the rainfall effectiveness (see eq. (18) discussed in Section 4). The term WI_{t-1} with coefficient k_4 was used to introduce autocorrelation due to soil water storage and the persistence of weather conditions and irrigation practices. This reflects the fact that every kind of man-controlled operation is affected by human as well as physical factors. The positive and relatively high values of k_4 (see eqs. (27) and (30)) indicate the influence of long-term irrigation policies ("If the irrigation of some crop has started it will continue till the end of the vegetative period."). The initial values of WI_{t-1} can be considered to be negligible ($WI_0 = 0$).

As a second method of taking into account soil moisture storage and the persistence of irrigation practices and weather conditions, the previous irrigation index can be used, derived experimentally to be:

$$i_t = 0.75 \log(WI_{t-1} + 7.0) \quad (24)$$

Then eq. (20) can be modified to

$$WI_t = i_t \left(k_1 f_t \frac{\Delta}{\Delta + \gamma} R_{n,t} + k_2 f_t \frac{\gamma}{\Delta + \gamma} E_t \right) + k_3 P_t + C \quad (25)$$

For cases where only temperature and relative humidity had been measured (or temperature with a dry and wet bulb), Linacre's simplification with the described correction was used. The following modification to the irrigation water requirements model was then used:

$$WI_t = k_1 P E_t + k_3 P_t + k_4 WI_{t-1} + C \quad (26)$$

where $P E_t = E_0/Z$, E_0 is calculated from eq. (16), and Z was evaluated as described above.

6 APPLICATIONS OF THE MODEL

The model was applied to two irrigation subsystems in the Labe River catchment area in Czechoslovakia, namely, the Vltava III and Vltava V irrigation systems (from now on called the V-III-V system), and the Celakovice–Vsetaty irrigation system (denoted as the C-V system; see Figure 1). The technique used was sprinkling irrigation, and both systems were observed during 1970–76. In this period, no water supply deficiency was observed in either system, for the following reasons.

The V-III-V system draws water mainly from the confluence of the Labe and the Vltava Rivers. On the Vltava River there is a cascade of reservoirs, which is used for electricity generation, and serves to regulate the river flow through Prague. This low-flow augmentation is not fully utilized downstream of Prague and the withdrawal of water for irrigation is a complementary use.

The C-V system takes water from the Labe River, the flow of which is regulated by the Roskos dam. The capacity of this dam has not yet been fully utilized, and the withdrawals of water in the observed period were not limited by low flows. Therefore, both irrigation systems used in the calibration of the model were supplied with as much water as required during the calibration period, i.e., with no reduction due to deficits.

It is intended to use the model of irrigation water requirements for the Czechoslovakian general water plan for irrigation and water resource systems for the year 2000, using measurements of water withdrawals by pumping stations in the Labe River basin. The prevailing soil type is a chernozem with a silty loam texture, and typical crops grown include cereals (40%), sugar beet (8%), potatoes (10%), vegetables (10%), alfalfa (27%), and others (5%). The intensity of agriculture on irrigated fields can be demonstrated by the crop yields: wheat 0.4 kg m^{-2} , sugar beet 4.5 kg m^{-2} , potatoes 1.5 kg m^{-2} (spring), 2.3 kg m^{-2} (autumn), and alfalfa $0.8\text{--}1.0 \text{ kg m}^{-2}$ (hay). The area of cultivated land under irrigation is approximately 100 km^2 .

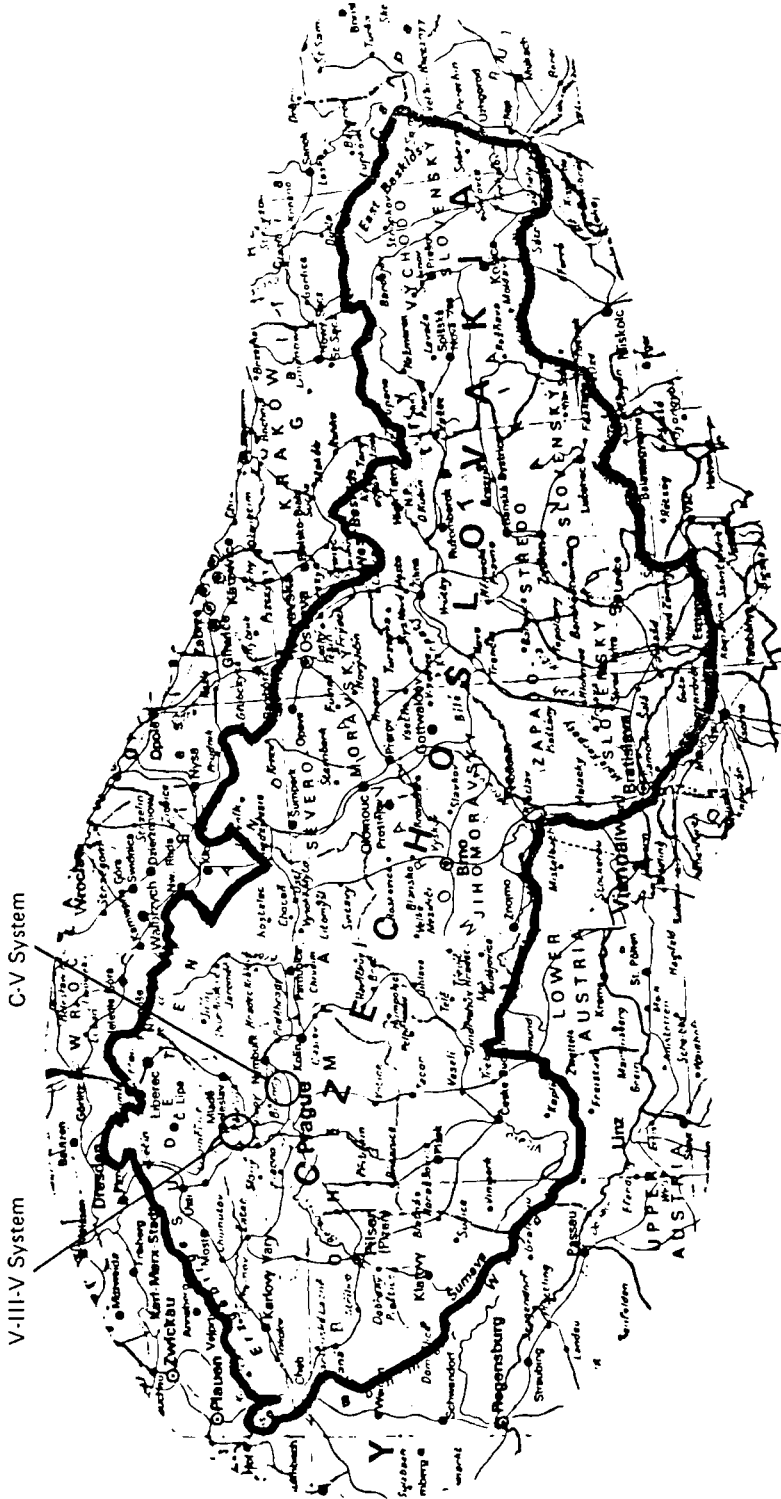


FIGURE 1. Location of irrigation systems.

TABLE 3 Input variables in regression analysis: V-III-V system (mm). $A = i_t[\Delta/(\Delta + \gamma)]R_{n,t}$; $B = i_t[\gamma/(\Delta + \gamma)]E_t$. Other variables are explained in eqs. (20) and (25).

	<i>M</i>	<i>WI</i> _{<i>t</i>-1}	<i>P</i> _{<i>t</i>}	$\frac{\Delta}{\Delta + \gamma}R_{n,t}$	$\frac{\gamma}{\Delta + \gamma}E_t$	<i>WI</i> _{<i>t</i>}	Previous irrigation index				
							<i>WI</i> _{<i>t</i>-1}	<i>P</i> _{<i>t</i>}	<i>A</i>	<i>B</i>	<i>WI</i> _{<i>t</i>}
1970	6	0.24	75.50	81.90	17.40	2.67	0.24	75.50	52.81	11.22	2.67
	7	2.67	48.90	74.70	23.10	13.61	2.67	48.90	55.20	17.07	13.61
	8	13.61	106.30	57.00	9.60	2.24	13.61	106.30	56.18	9.46	2.24
	9	2.24	23.60	28.80	11.10	0.68	2.24	23.60	20.86	8.04	0.68
1971	4	0.00	16.50	33.30	14.40	0.33	0.00	16.50	33.30	14.40	0.33
	5	0.33	124.30	57.60	13.20	1.48	0.33	124.30	37.38	8.57	1.48
	6	1.48	109.70	59.10	14.70	0.10	1.48	109.70	41.14	10.23	0.10
	7	0.10	9.30	84.30	19.80	12.47	0.10	9.30	53.83	12.64	12.47
	8	12.47	57.20	68.10	22.20	26.52	12.47	57.20	65.85	21.47	26.52
	9	26.52	37.70	26.40	9.30	7.89	26.52	37.70	30.20	10.64	7.89
1972	4	0.00	24.20	27.60	15.90	0.71	0.00	24.20	27.60	15.90	0.71
	5	0.71	76.10	53.70	17.40	1.98	0.71	76.10	35.73	11.58	1.98
	6	1.98	78.90	73.20	16.80	6.72	1.98	78.90	52.34	12.01	6.72
	7	6.72	40.70	75.30	14.10	13.78	6.72	40.70	64.24	12.03	13.78
	8	13.78	51.50	57.00	14.70	9.03	13.78	51.50	56.33	14.53	9.03
	9	9.03	37.30	24.30	6.90	0.85	9.03	37.30	21.96	6.24	0.85
1973	4	0.00	47.30	27.00	18.30	1.47	0.00	47.30	27.00	18.30	1.47
	5	1.47	54.70	62.40	18.60	3.41	1.47	54.70	43.42	12.94	3.41
	6	3.41	44.10	80.70	19.50	10.62	3.41	44.10	61.59	14.88	10.62
	7	10.62	69.00	72.90	20.70	24.72	10.62	69.00	68.13	19.35	24.72
	8	24.72	14.10	68.10	18.90	26.72	24.72	14.10	76.68	21.28	26.72
	9	26.72	9.90	29.70	17.40	14.60	26.72	9.90	34.03	19.94	14.60
1974	4	0.00	10.00	33.30	22.20	15.70	0.00	10.00	33.30	22.20	15.70
	5	15.70	70.10	54.90	18.90	3.91	15.70	70.10	55.83	19.22	3.91
	6	3.91	65.80	65.10	18.90	6.26	3.91	65.80	50.67	14.71	6.26
	7	6.26	54.30	61.20	28.20	11.88	6.26	54.30	51.53	23.74	11.88
	8	11.88	44.70	65.10	21.30	13.61	11.88	44.70	62.30	20.38	13.61
	9	13.61	38.90	30.30	13.20	6.41	13.61	38.90	29.86	13.01	6.41
1975	4	0.00	19.90	30.90	18.30	0.30	0.00	19.90	30.90	18.30	0.30
	5	0.30	65.50	55.80	15.30	2.91	0.30	65.50	36.14	9.91	2.91
	6	2.91	62.00	66.30	14.40	5.55	2.91	62.00	49.52	10.76	5.55
	7	5.55	48.50	78.30	17.40	15.58	5.55	48.50	64.52	14.34	15.58
	8	15.58	20.90	65.40	17.10	20.32	15.58	20.90	66.40	17.36	20.32
	9	20.32	20.90	33.00	8.40	9.31	20.32	20.90	35.55	9.05	9.31
1976	4	0.00	17.50	33.00	16.80	2.98	0.00	17.50	33.00	16.80	2.98
	5	2.98	55.50	63.30	26.10	16.24	2.98	55.50	47.43	19.56	16.24
	6	16.24	32.00	83.10	26.10	16.11	16.24	32.00	85.15	26.74	16.11
	7	16.11	29.50	78.00	30.30	38.39	16.11	29.50	79.78	30.99	38.39
	8	38.39	37.50	59.10	25.20	26.37	38.39	37.50	73.45	31.32	26.37
	9	26.37	29.50	24.60	9.60	11.16	26.37	29.50	28.11	10.97	11.16

The coefficients in eq. (20) were determined for the V-III-V system by linear regression analysis, using the input data shown in Table 3:

$$\begin{aligned} k_1 &= 0.176 & k_2 &= 0.669 \\ k_3 &= -0.082 & k_4 &= 0.486 & C &= -11.61 \end{aligned}$$

Then eq. (20) for the observed period becomes

$$WI_t = 0.176 f_t \frac{\Delta}{\Delta + \gamma} R_{n,t} + 0.669 f_t \frac{\gamma}{\Delta + \gamma} E_t - 0.082 P_t + 0.486 WI_{t-1} - 11.61 \quad (27)$$

A comparison between observed and calculated data is shown in Figure 2.

To calculate the coefficient of exploitation k_e and the weighting coefficients g and h , some assumptions have to be made since there are only two equations for the three unknowns, i.e.,

$$\begin{aligned} k_1 &= k k_e g = 0.176 \\ k_2 &= k k_e h = 0.669 \end{aligned}$$

The coefficient k was evaluated as $k = 1.11$ (i.e., efficiency 90% and $k = 1/\text{efficiency}$). The relation between g and h was based on the following.

As stated earlier, the maximum yield seems to be connected with potential evapotranspiration. If Penman's equation is used to calculate the potential evapotranspiration in the original form (eq. (12)), then the weighting coefficients in eq. (21) will be $g = h = 1$, and their sum will therefore be $g + h = 2$. In eq. (20), the condition $g = h = 1$ is not required, but a weaker condition, $g + h = 2$. With this equation, the following system can be obtained:

$$\begin{aligned} k_e g &= 0.176/1.11 = 0.158 \\ k_e h &= 0.669/1.11 = 0.603 \\ g + h &= 2 \end{aligned}$$

and the resulting values are

$$k_e = 0.381 \quad g = 0.41 \quad h = 1.59$$

If a maximum feasible coefficient of exploitation estimated by $k_e = 0.9$ has to be reached, then the regression coefficients k_1 , k_2 , k_3 , and the intercept C have to be multiplied by the ratio of actual and maximum coefficients, i.e., $d = 0.9/0.381 = 2.36$. Equation (20) then becomes:

$$WI_t = 0.415 f_t \frac{\Delta}{\Delta + \gamma} R_{n,t} + 1.579 f_t \frac{\gamma}{\Delta + \gamma} E_t - 0.194 P_t + 0.486 WI_{t-1} - 27.4 \quad (28)$$

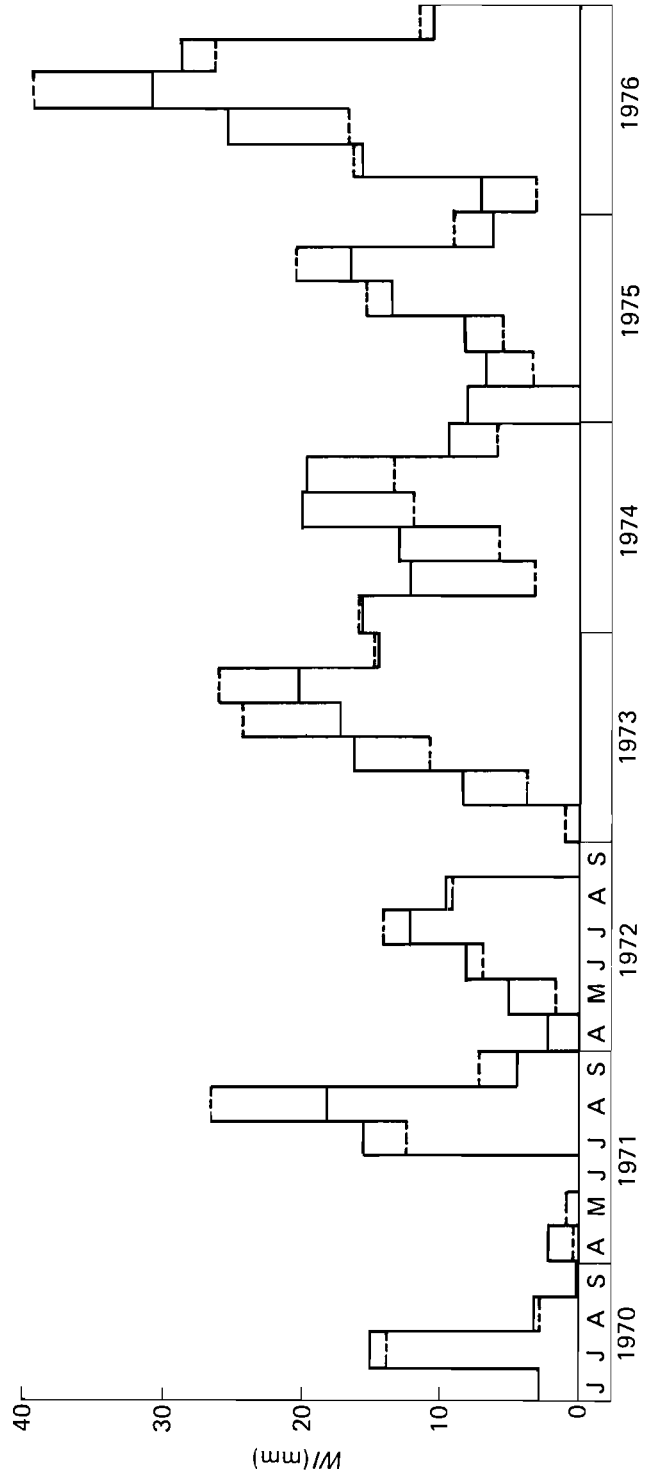


FIGURE 2 Calculated and observed WJ values for the V-III-V system (mm): —, calculated from eq. (27); - - -, observed.

The multiregression coefficient of correlation is 0.873, indicating a close correlation (further details are given below). For the same V-III-V system, eq. (25) was calibrated by regression analysis and the resulting coefficients were

$$\begin{aligned} k_1 &= 0.307 & k_2 &= 0.478 \\ k_3 &= -0.080 & C &= -8.48 \end{aligned}$$

Equation (25) then becomes:

$$WI_t = i_t \left(0.307 f_t \frac{\Delta}{\Delta + \gamma} R_{n,t} + 0.478 f_t \frac{\gamma}{\Delta + \gamma} E_t \right) - 0.080 P_t - 8.48 \quad (29)$$

The resulting multiregression coefficient of correlation is 0.846.

For the C-V system, the following results were obtained:

$$\begin{aligned} k_1 &= 0.149 & k_2 &= 0.356 \\ k_3 &= -0.119 & k_4 &= 0.438 & C &= -1.49 \end{aligned}$$

and eq. (20), based on the input data in Table 4, becomes:

$$WI_t = 0.149 f_t \frac{\Delta}{\Delta + \gamma} R_{n,t} + 0.356 f_t \frac{\gamma}{\Delta + \gamma} E_t - 0.119 P_t + 0.438 WI_{t-1} - 1.49 \quad (30)$$

The goodness-of-fit of the model is apparent from Figure 3. If the same procedure is used to calculate the coefficients k_e , g , and h , then $k_e = 0.228$, $g = 0.59$, and $h = 1.41$ will be obtained. For maximum possible utilization ($k_e = 0.9$), k_1 , k_2 , k_3 , and C can be multiplied by the ratio $d = 0.9/0.228 = 3.95$, and eq. (20) then becomes:

$$WI_t = 0.590 f_t \frac{\Delta}{\Delta + \gamma} R_{n,t} + 1.409 f_t \frac{\gamma}{\Delta + \gamma} E_t - 0.470 P_t + 0.438 WI_{t-1} - 5.90 \quad (31)$$

The relation of the individual terms in eq. (31) to irrigation water requirements can be expressed by the individual correlation coefficients $r_{i,d}$ relating the independent variable i , and the dependent variable (irrigation water requirements) d . The degree of the explained part of the relation is characterized by the multiple correlation coefficient R_i , where i denotes the number of independent variables (e.g., R_3 takes into account the first three components: the radiation term, and vapor flux term of evapotranspiration and precipitation). The reliability of the derived equation can also be tested by the F -test. The critical values of the F -test (F_{crit}) of the α value of significance ($\alpha = 0.05$) were:

	$i = 2$	3	4
F_{crit}	3.2	2.8	2.6

Because the sampling values of the F -test were much greater, the relation is highly significant. The values of $r_{i,d}$, R_i , and F_i were as follows.

TABLE 4 Input variables in regression analysis: C-V system (mm). $WI_t^1 = WI_t/2.22$, $WI_{t-1}^1 = WI_{t-1}/2.22$; $A = i_t[\Delta/(\Delta + \gamma)]R_{n,t}$; $B = i_t[\gamma/(\Delta + \gamma)]E_t$. Other variables are explained in eqs. (20) and (25).

	<i>M</i>	WI_{t-1}^1	P_t	$\frac{\Delta}{\Delta + \gamma}R_{n,t}$	$\frac{\gamma}{\Delta + \gamma}E_t$	WI_t^1	Previous irrigation index				
							WI_{t-1}^1	P_t	<i>A</i>	<i>B</i>	WI_t^1
1970	6	1.66	13.00	81.90	17.40	6.58	1.66	13.00	57.59	12.23	6.58
	7	6.58	26.00	74.70	23.10	6.11	6.58	26.00	63.48	19.63	6.11
	8	6.11	95.00	57.00	9.60	2.44	6.11	95.00	47.78	8.05	2.44
	9	2.44	21.00	28.80	11.10	3.87	2.44	21.00	21.06	8.12	3.87
1971	4	0.00	12.00	33.30	14.40	1.80	0.00	12.00	33.30	14.40	1.80
	5	1.80	98.00	57.60	13.20	2.63	1.80	98.00	40.80	9.35	2.63
	6	2.63	110.00	59.10	14.70	0.39	2.63	110.00	43.60	10.84	0.39
	7	0.39	5.00	84.30	19.80	9.86	0.39	5.00	54.91	12.90	9.86
	8	9.86	60.00	68.10	22.20	9.36	9.86	60.00	62.66	20.43	9.36
9	9.36	35.00	26.40	9.30	4.66	9.36	35.00	24.03	8.47	4.66	
1972	4	0.00	26.00	27.60	15.90	1.21	0.00	26.00	27.60	15.90	1.21
	5	1.21	94.00	53.70	17.40	1.86	1.21	94.00	36.82	11.93	1.86
	6	1.86	66.00	73.20	16.80	6.13	1.86	66.00	52.02	11.94	6.13
	7	6.13	39.00	75.30	14.10	6.23	6.13	39.00	63.16	11.83	6.23
	8	6.23	48.00	57.00	14.70	3.28	6.23	48.00	47.94	12.36	3.28
9	3.28	60.00	24.30	6.90	1.61	3.28	60.00	18.45	5.24	1.61	
1973	4	0.00	34.50	27.00	18.30	1.43	0.00	34.50	27.00	18.30	1.43
	5	1.43	48.40	62.40	18.60	4.71	1.43	48.40	43.34	12.92	4.71
	6	4.71	47.00	80.70	19.50	7.45	4.71	47.00	64.67	15.63	7.45
	7	7.45	79.10	72.90	20.70	8.26	7.45	79.10	63.42	18.01	8.26
	8	8.26	8.00	68.10	18.90	11.26	8.26	8.00	60.45	16.78	11.26
9	11.26	7.70	29.70	17.40	9.34	11.26	7.70	28.10	16.46	9.34	
1974	4	0.00	8.40	33.30	22.20	8.45	0.00	8.40	33.30	22.20	8.45
	5	8.45	80.00	54.90	18.90	3.17	8.45	80.00	48.95	16.85	3.17
	6	3.17	73.70	65.10	18.90	4.53	3.17	73.70	49.18	14.28	4.53
	7	4.53	64.60	61.20	28.20	2.69	4.53	64.60	48.73	22.45	2.69
	8	2.69	66.10	65.10	21.30	4.83	2.69	66.10	48.15	15.75	4.83
9	4.83	30.60	30.30	13.20	3.23	4.83	30.60	24.38	10.62	3.23	
1975	4	0.00	19.90	30.90	18.30	1.43	0.00	19.90	30.90	18.30	1.43
	5	1.43	74.70	55.80	15.30	1.98	1.43	74.70	38.75	10.63	1.98
	6	1.98	41.10	66.30	14.40	5.66	1.98	41.10	47.39	10.29	5.66
	7	5.66	28.80	78.30	17.40	6.43	5.66	28.80	64.75	14.39	6.43
	8	6.43	17.50	65.40	17.10	8.52	6.43	17.50	55.34	14.47	8.52
9	8.52	18.00	33.00	8.40	7.16	8.52	18.00	29.48	7.50	7.16	
1976	4	0.00	14.00	33.00	16.80	2.51	0.00	14.00	33.00	16.80	2.51
	5	2.51	48.00	63.30	26.10	7.90	2.51	48.00	46.44	19.15	7.90
	6	7.90	25.00	83.10	26.10	8.70	7.90	25.00	73.13	22.97	8.70
	7	8.70	30.00	78.00	30.30	16.77	8.70	30.00	69.96	27.18	16.77
	8	16.77	36.00	59.10	25.20	12.71	16.77	36.00	60.99	26.01	12.71
9	12.71	2.00	24.60	9.60	8.25	12.71	2.00	23.89	9.32	8.25	

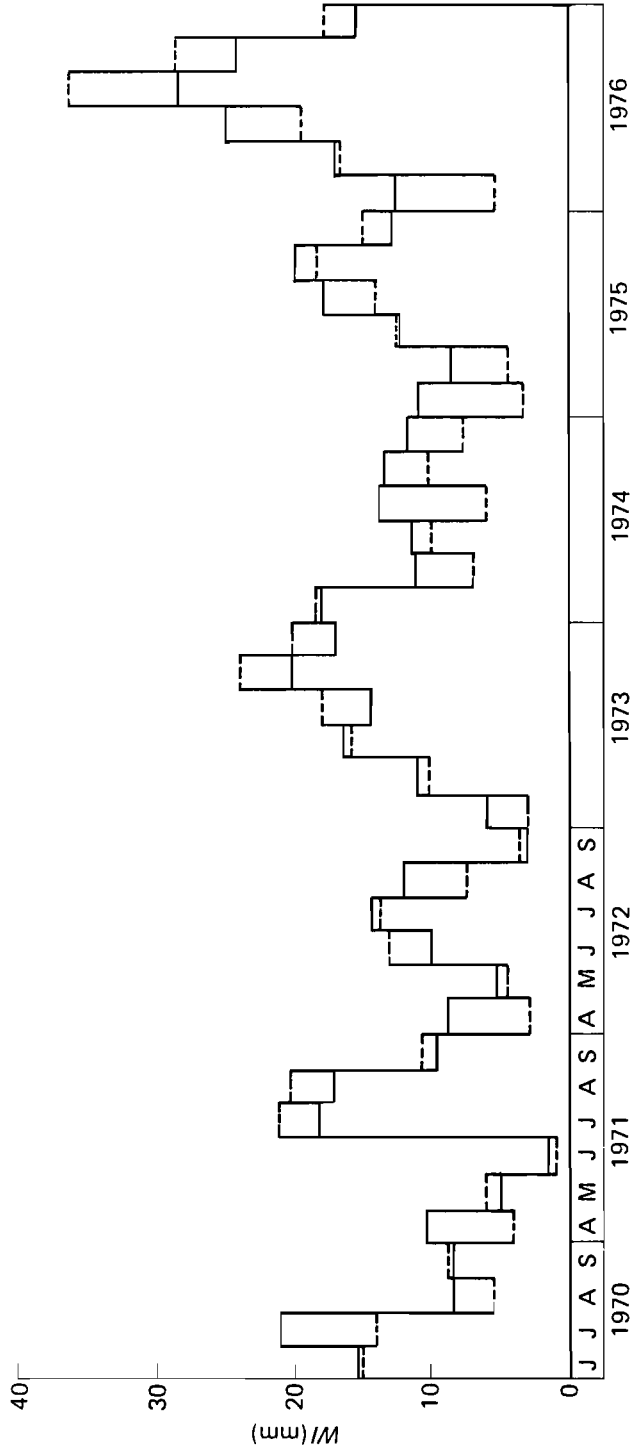


FIGURE 3 Calculated and observed W values for the C-V system (mm): —, calculated from eq. (30); - - -, observed.

For eq. (27) and the V-III-V system:

$i =$	1	2	3	4
$r_{i,d}$	0.454	0.613	-0.296	0.534
R_i		0.631	0.716	0.873
F_i		12.2	12.5	28.0

For eq. (30) and the C-V system:

$i =$	1	2	3	4
$r_{i,d}$	0.445	0.505	-0.417	0.595
R_i		0.545	0.724	0.870
F_i		7.8	13.2	27.2

For eq. (29) using the previous irrigation index and considering $i_0 = 7.0$ as an additional parameter, the values for the V-III-V system will be:

$i =$	2	3	4
$r_{i,d}$	0.734	0.725	-0.296
R_i		0.819	0.846
F_i		25.07	22.59

Very useful indicators of the significance of the regression coefficients are their standard errors and t -values; these have been computed for eqs. (27, V-III-V system) and (30, C-V system). For eq. (27) and the V-III-V system:

$i =$	1	2	3	4
k_i	0.176	0.669	-0.082	0.486
s_{k_i}	0.051	0.117	0.032	0.080
t_i	3.45	5.72	2.56	6.07

The t -values were defined as $|k_i - 0|/s_{k_i}$. When $t_i > t_{\text{crit}}$, the hypothesis that $k_i = 0$ is rejected. The value t_{crit} (level of significance $\alpha = 0.05$; $n = 40$) = 2.02. Since the relation $t_i > t_{\text{crit}}$ is fulfilled for all i , the coefficients k_i are statistically significant.

For eq. (30) and the C-V system:

$i =$	1	2	3	4
k_i	0.149	0.356	-0.119	0.438
s_{k_i}	0.042	0.149	0.024	0.076
t_i	3.55	2.39	4.96	5.76

Since $t_i > t_{\text{crit}} = 2.02$ for all i , all the regression coefficients are statistically significant.

The results of the calibration show that irrigation water requirements are more sensitive to evapotranspiration than to precipitation. As evapotranspiration has been expressed in two terms, the irrigation water requirements are more dependent on vapor flow than

on radiation, in good agreement with the observations of some authors of evaporation formulae, based on the vapor flux term only.

An interesting result is the relatively low correlation between irrigation water requirements and precipitation, which can be explained in several ways. First, the evaporation term is an index of the overall synoptic situation. High evaporation means little precipitation, and vice versa. Secondly, irrigation practices are governed more by evaporation than by precipitation. Thirdly, the intercept C can be considered to be a constant part of effective precipitation. More precisely, the effective rainfall can be considered as a linear function of precipitation:

$$P_e = \alpha P + \beta \quad (32)$$

as compared to the original equation ($P_e = \alpha' P$).

It is worth noting that there is a relatively close positive correlation between irrigation in the current month and that in the previous one, i.e., autocorrelation indicates the persistence of weather conditions and irrigation practices.

The relatively low value of α in eqs. (27)–(31) needs further discussion. According to Section 4, the expected value of α would be 0.5–0.7. At first, a fully exploited and developed irrigation system should be considered for this comparison; eqs. (28) and (31) are therefore used. Further, the intercept C is considered to be a constant part of effective precipitation. Then, for average precipitation \bar{P} , the following values are derived comparing $P_e = \alpha' \bar{P}$ with eq. (32) and considering the loss coefficient $k = 1.1$.

For the V-III-V system,

$$\alpha' = \frac{1}{k} \frac{\alpha \bar{P} + \beta}{\bar{P}} = \frac{1}{1.1} \frac{0.194 \times 47 + 27.4}{47} = 0.70 \quad (33)$$

and for the C-V system,

$$\alpha' = \frac{1}{k} \frac{\alpha \bar{P} + \beta}{\bar{P}} = \frac{1}{1.1} \frac{0.470 \times 42.78 + 5.90}{42.78} = 0.55 \quad (34)$$

The resulting values correspond closely to the expected ones, and are in accordance with the values of the run-off coefficient, c .

The regression analysis and calibration procedure was also carried out for eq. (26) using Linacre's formula. The resulting equations were:

(a) Observed V-III-V system:

$$WI_t = 0.330 PE_t - 0.103 P_t + 0.578 WI_{t-1} - 12.76 \quad (35)$$

(b) Fully developed V-III-V system (using the transformation coefficient $d = 2.37$):

$$WI_t = 0.782 PE_t - 0.244 P_t + 0.578 WI_{t-1} - 30.24 \quad (36)$$

(c) Observed C-V system:

$$WI_t = 0.223 PE_t - 0.119 P_t + 0.502 WI_{t-1} - 2.97 \quad (37)$$

(d) Fully developed C-V system (using the transformation coefficient $d = 3.96$):

$$WI_t = 0.883 PE_t - 0.471 P_t + 0.502 WI_{t-1} - 11.76 \quad (38)$$

The statistical parameters were as shown below.

$i =$	1	2	3
V-III-V system			
$r_{i,d}$	0.473	-0.296	0.534
R_i		0.636	0.864
F_i		12.5	35.4
C-V system			
$r_{i,d}$	0.455	-0.417	0.595
R_i		0.670	0.871
F_i		15.0	37.6

6.1 Time Series Modeling

The time series of irrigation requirements were modeled using eqs. (27), (28), (30), and (31) for the period 1931–70 (for eqs. (27) and (30) in 1931–36, see Figure 4). Equation (29) was not used because it does not give significantly better results. Since data were available from meteorological station S for Penman's equation (Table 5), these were used for time series modeling. Linacre's simplification was used for comparison only; it is only useful when temperature measurements (dry and wet bulb) are available.

The soil moisture conditions at the beginning of the vegetative period were determined to be 40 mm, and this average was used for planning purposes (Holy 1979). If this stored water is not exhausted by March, the rest will be used in April. The October values were reduced by a coefficient 0.3 because only about 30% of the area is generally utilized in this month.

For time series modeling, eqs. (27), (28), (30), and (31) should contain an error term because the compiled values give averages of WI_t and the computed series will thus have lower variances than the observed series. However, it is first necessary to determine the type of probability distribution of WI_t , which was the main aim of the analysis.

The resulting time series model of irrigation water requirements was analyzed statistically. The main input time series (based on observations at station S) was also analyzed to discover the statistical properties of the results. The averages, standard deviations, and coefficients of variation of $e1_t$, $e2_t$, and P_t are shown in Table 6, where

$$e1_t = f_t \frac{\Delta}{\Delta + \gamma} R_{n,t}$$

$$e2_t = f_t \frac{\gamma}{\Delta + \gamma} E_t$$

f_t , Δ , γ , R_n , and E were defined in eqs. (11)–(15), and P_t is precipitation.

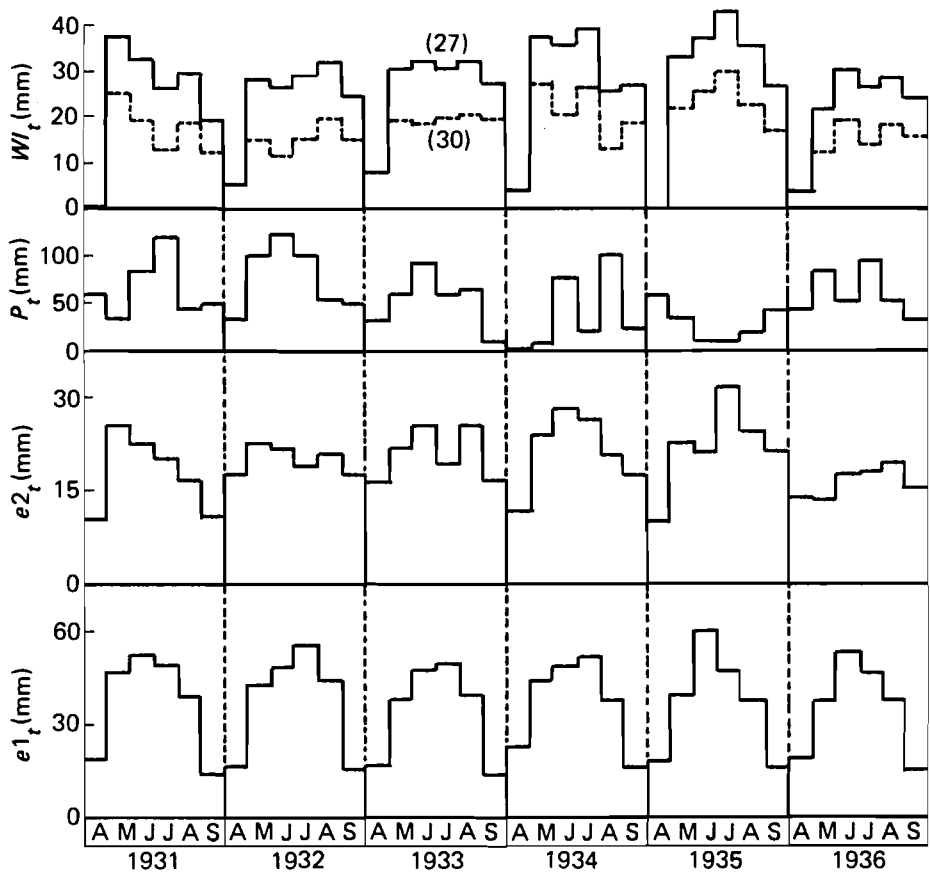


FIGURE 4 Time series of input data and irrigation water requirements of the V-III-V system (using eq. (27)), and the C-V system (using eq. (30)).

The coefficient of variation values, C_v , suggest that $e1$ is a relatively stable element ($C_v = 0.084$ on average, or 8.4%). The second evapotranspiration term expressed by vapor flux $e2$ has a higher variation ($C_v = 0.23$ on average, or 23%). Since the corresponding regression coefficients in eqs. (27)–(31) have the highest values, this term adds considerably to the final variation. Precipitation has the greatest value ($C_v = 0.52$ on average, or 52%). Therefore, in combination with a higher regression coefficient (e.g., eq. (31)), it can be an important source of variability in the resulting irrigation water requirements.

The question as to whether the differences in averages for 1931–70 and 1970–76 are statistically significant can be answered by comparing the computed t_i and t_{crit} values. Both averages and standard deviations differ, so t_{crit} values were computed by means of the formula given by Janko (1958):

$$t_{crit} = \frac{\nu_1 t_{f_1} + \nu_2 t_{f_2}}{\nu_1 + \nu_2}$$

TABLE 5 Data for Penman's equation. ($M = 2$: April; $M = 3$: May, etc.)

	M	TE	RH	W	N	R_c	R_b	E_0	PE
1970	2	7.40	0.72	3.00	3.40	3.87	1.42	2.04	1.43
	3	12.40	0.90	2.30	5.18	5.34	1.49	2.54	2.04
	4	18.10	0.72	1.70	7.72	7.15	2.02	4.13	3.31
	5	18.60	0.69	2.20	7.15	6.56	1.91	4.08	3.27
	6	17.60	0.80	1.10	5.57	5.18	1.58	2.77	2.21
	7	12.90	0.76	1.70	5.78	4.40	2.10	1.91	1.34
1971	2	9.70	0.69	2.00	5.40	4.86	1.99	2.28	1.60
	3	15.10	0.74	1.40	5.37	5.44	1.61	2.95	2.36
	4	15.20	0.77	2.00	4.31	5.25	1.32	3.08	2.46
	5	19.30	0.65	1.40	8.93	7.50	2.33	4.34	3.47
	6	19.90	0.66	1.70	7.88	6.35	2.22	3.75	3.00
	7	12.10	0.78	1.50	4.73	3.93	1.79	1.70	1.19
1972	2	8.60	0.69	2.40	3.33	3.83	1.41	2.06	1.44
	3	13.50	0.74	2.30	5.02	5.26	1.57	2.96	2.37
	4	17.10	0.72	1.70	6.51	6.47	1.80	3.76	3.00
	5	19.50	0.75	1.40	6.61	6.26	1.66	3.73	2.98
	6	17.40	0.75	1.60	5.90	5.35	1.73	2.99	2.39
	7	11.70	0.82	1.30	3.69	3.46	1.46	1.49	1.04
1973	2	6.70	0.66	2.90	3.94	4.14	1.62	2.17	1.52
	3	14.20	0.68	1.80	7.09	6.34	2.11	3.37	2.70
	4	17.30	0.70	1.90	7.94	7.26	2.13	4.18	3.34
	5	18.40	0.73	2.30	6.64	6.28	1.75	3.90	3.12
	6	18.60	0.66	1.40	8.48	6.65	2.43	3.62	2.90
	7	15.20	0.68	1.90	5.71	4.37	2.11	2.24	1.57
1974	2	9.20	0.61	2.70	5.90	5.11	2.23	2.63	1.84
	3	13.00	0.72	2.40	5.60	5.56	1.74	3.08	2.47
	4	15.40	0.74	2.40	5.48	5.90	1.60	3.50	2.80
	5	17.10	0.73	3.60	4.82	5.31	1.41	3.72	2.98
	6	19.20	0.73	2.30	7.08	5.95	1.95	3.60	2.88
	7	14.00	0.77	2.20	5.87	4.44	2.07	2.08	1.46
1975	2	8.40	0.67	2.70	4.98	4.66	1.91	2.34	1.64
	3	13.70	0.73	1.80	5.48	5.50	1.68	2.96	2.37
	4	16.50	0.74	1.50	5.34	5.82	1.54	3.36	2.69
	5	19.30	0.73	1.70	7.36	6.66	1.86	3.99	3.19
	6	19.50	0.71	1.50	7.13	5.97	1.98	3.44	2.75
	7	16.90	0.80	1.20	5.33	4.20	1.77	1.98	1.39
1976	2	8.60	0.66	2.30	5.82	5.07	2.16	2.37	1.66
	3	14.20	0.63	2.40	7.52	6.57	2.29	3.72	2.98
	4	18.20	0.61	1.90	8.52	7.59	2.38	4.55	3.64
	5	20.50	0.60	2.10	7.53	6.76	2.07	4.52	3.61
	6	17.20	0.66	2.30	6.93	5.87	2.11	3.51	2.81
	7	13.30	0.79	1.60	3.26	3.26	1.32	1.62	1.14

where

$$\nu_1 = \sigma_1^2/m (m = 40) \quad \nu_2 = \sigma_2^2/n (n = 7)$$

For the 5% level of significance, $t_{f_1} = t_{39} = 1.68$, $t_{f_2} = t_6 = 1.94$, and σ_1 and σ_2 are the standard deviations obtained from Table 6. The values t_i were

$$t_i = \frac{|\phi_1 - \phi_2|}{\sigma_d}$$

where $\sigma_d = (\nu_1 + \nu_2)^2$, and ϕ_1 and ϕ_2 are averages from Table 6. Since $t_i < t_{\text{crit}}$ in almost all cases, the hypothesis that both averages are from the same population was not rejected. The only exception was the precipitation in July, where $t_i \doteq 2.4$ and $t_{\text{crit}} \doteq 2.3$. However, the difference is very small, and for a slightly lower level of significance (e.g., $\alpha = 4\%$) the relation $t_i < t_{\text{crit}}$ will be fulfilled.

In order to investigate the serial dependence, the correlation coefficients r_i between successive months were computed. For $e1_t$ and P , the r_i values were smaller than r_{crit} ($f = n - 1 = 6$, $\alpha = 5\%$) = 0.7067 and r_{crit} ($f = n - 1 = 39$, $\alpha = 5\%$) = 0.3084, and so

TABLE 6 Statistical parameters of input variables from station S. ϕ = average (approx.); σ = standard deviation; C_v = coefficient of variation.

Value		A	M	J	J	A	S	ϕ
$e1_t$ 1931-70	ϕ_1	0.650	1.320	1.710	1.677	1.297	0.514	8.4
	σ_1	0.059	0.132	0.139	0.138	0.102	0.035	
	$C_v(\%)$	9.1	10.0	8.2	8.2	7.9	6.8	
$e2_t$ 1931-70	ϕ_1	0.467	0.630	0.624	0.628	0.590	0.434	23.3
	σ_1	0.111	0.152	0.118	0.137	0.139	0.128	
	$C_v(\%)$	23.8	24.1	18.9	21.8	23.6	29.5	
P_t 1931-70	ϕ_1	38.8	64.9	64.9	76.8	67.8	41.6	51.6
	σ_1	18.6	33.4	28.9	42.5	34.4	25.2	
	$C_v(\%)$	47.9	51.5	44.5	55.3	50.7	60.6	
$e1_t$ 1970-76	ϕ_2	0.609	1.240	1.608	1.651	1.304	0.507	
	σ_2	0.042	0.074	0.179	0.143	0.077	0.045	
	$C_v(\%)$	6.9	6.0	11.1	8.7	5.9	8.9	
$e2_t$ 1970-76	ϕ_2	0.516	0.564	0.553	0.621	0.586	0.352	
	σ_2	0.110	0.104	0.111	0.094	0.134	0.113	
	$C_v(\%)$	21.3	18.4	20.1	15.1	22.9	32.2	
P_t 1970-76	ϕ_2	34.1	72.9	69.9	49.0	51.4	31.3	
	σ_2	20.9	23.2	27.3	24.4	32.4	16.5	
	$C_v(\%)$	61.3	31.8	39.1	49.8	63.0	52.7	
$e1_t$	t_{crit}	2.338	2.295	2.407	2.388	2.347	2.406	
	t_i	2.227	2.292	1.434	0.446	0.210	0.391	
$e2_t$	t_{crit}	2.383	2.332	2.377	2.332	2.380	2.369	
	t_i	1.086	1.433	1.546	0.168	0.072	1.756	
P_t	t_{crit}	2.395	2.334	2.377	2.300	2.377	2.324	
	t_i	0.558	0.782	0.443	2.436	1.224	1.392	

these were not statistically significant. For $e2_t$ the following values for r_t were obtained:

	A-M	M-J	J-J	J-A	A-S
1931-70	-0.191	0.310	0.408	0.452	0.577
1970-76	0.041	0.675	0.257	0.035	0.222

The values for 1931-70 were statistically significant starting from May (May-June, June-July, etc.), showing a positive serial correlation.

Further analysis concerned the monthly probability distributions of $e1_{t,y}$, $e2_{t,y}$, and $P_{t,y}$, where t is the month (e.g. $t = 2$ for April, $y = \text{year} = 1, 2, \dots, 40$), and then the sums of these values for the whole vegetative period, namely:

$$E1_y = \sum_{t=1}^8 e1_{t,y}$$

$$E2_y = \sum_{t=1}^8 e2_{t,y}$$

$$PS_y = \sum_{t=1}^8 P_{t,y}$$

The cumulative frequency curves are shown in Figures 5-24. The probabilities p_i were determined by the formula $p_i = i/(m + 1)$ where i is the rank number ($i = 1, 2, \dots, m$) and m is the total number of observations ($m = 40$ in this case). In the middle part, approximately $0.2 < p_i < 0.8$, some points were not plotted because they were not important in an approximate fitting of theoretical distributions. In some figures the theoretical normal cumulative distribution function was fitted on probability paper, with p_i on the vertical axis. On this paper a normal cumulative distribution function is projected as a straight line.

The results of some of these tests of $e1$, $e2$, and P for May and July are given in Figures 5-10, and those of $E1_y$, $E2_y$, and PS_y are given in Figures 11-17. These results seem to show that the distributions of $e1$, $e2$, and P (Figures 9 and 10) can be regarded as normal with some outliers (one or two in the 40-year sequence). These outliers are probably not error measurements, but reflect the fact that in a semi-humid climate, conditions typical of a semi-dry or humid climate sometimes occur and may last for several months (the prevailing synoptic situation with persistent high or low pressure governing the air mass circulation).

The $E1$ values showed a normal distribution (Figure 11), but the probability distribution of $E2$ values was obviously not normal, and produced an S-shaped curve (Figure 12). The minimum value that caused this rather strange behavior was tested at the neighboring meteorological station B, and it was found that it occurred at both stations in 1955, so that the minimum at station B could not have been an outlier. Therefore, asymmetrical distributions were tested. At first, a log normal distribution with the transformation $w_y = \log E2_y$ was tested, but the result was unsatisfactory, so that $w_y = \log (E2_y - A)$ was used (with $A = 2$), and this was sufficient to transform the distribution to normal (Figures 13, 15).

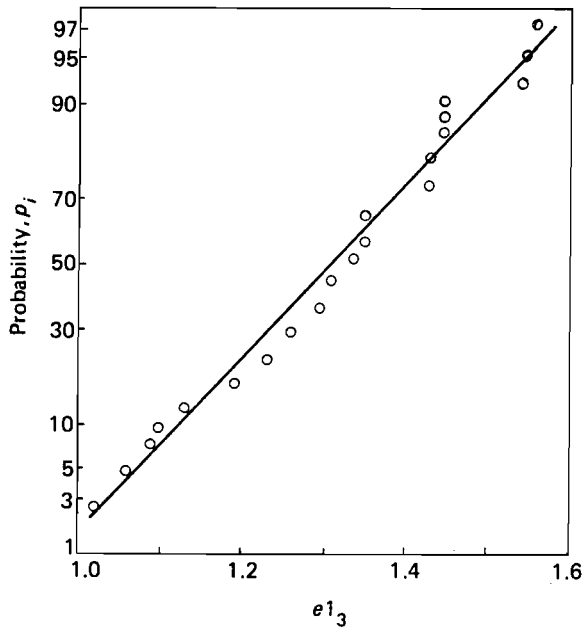


FIGURE 5 Distribution of $e1_3$ in May (daily values).

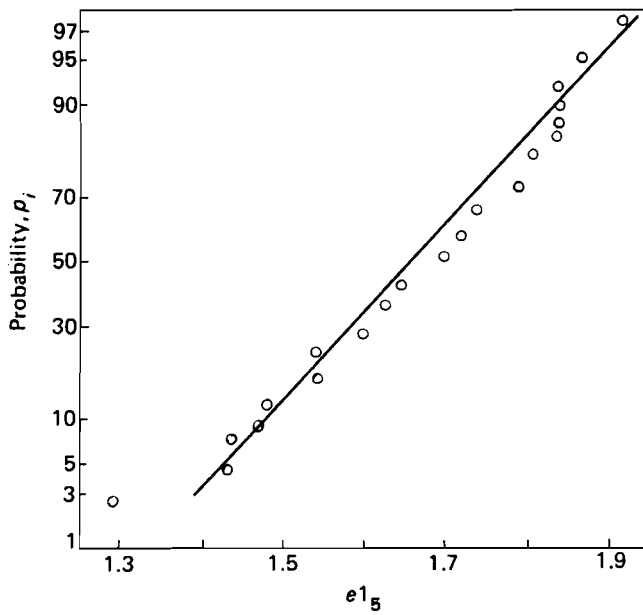


FIGURE 6 Distribution of $e1_5$ in July (daily values).

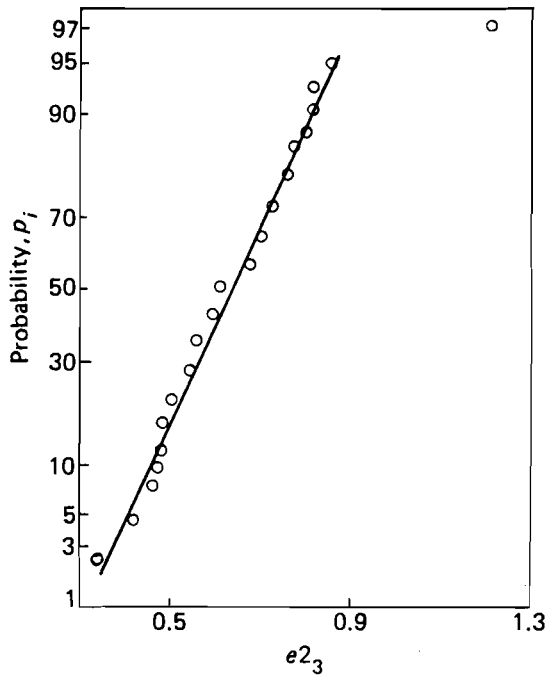


FIGURE 7 Distribution of $e2_3$ in May (daily values).

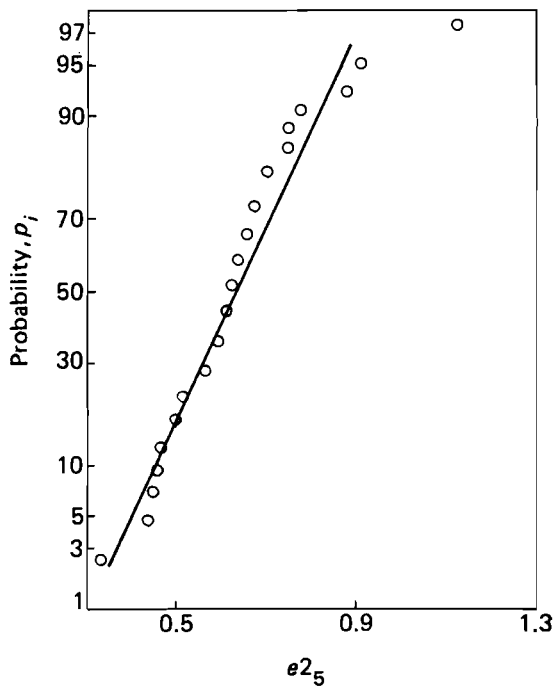


FIGURE 8 Distribution of $e2_5$ in July (daily values).

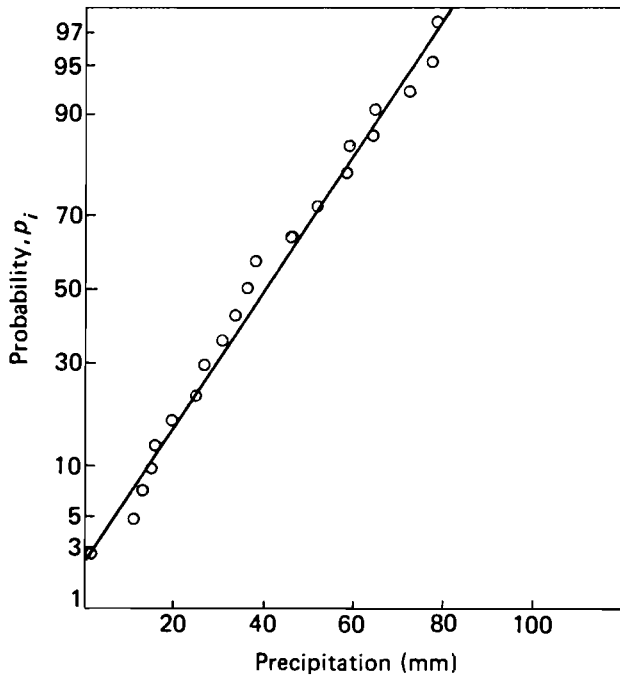


FIGURE 9 Distribution of precipitation, P_3 (May).

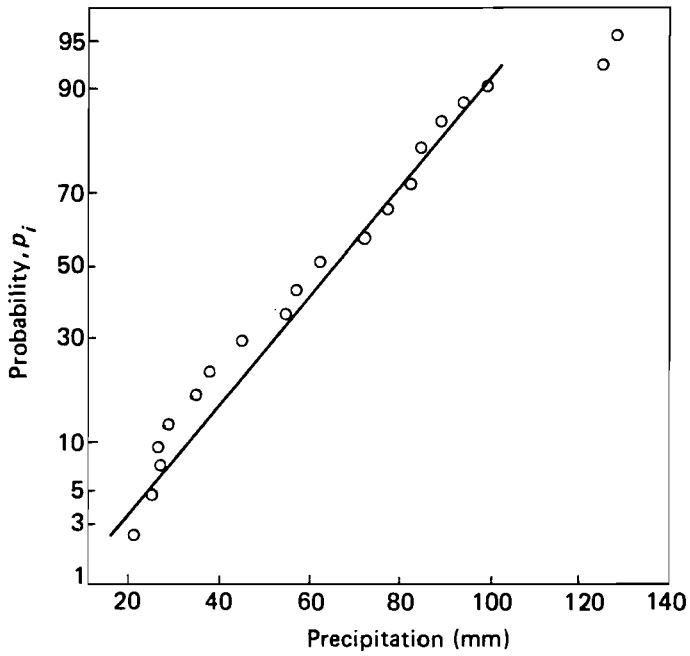


FIGURE 10 Distribution of precipitation, P_5 (July).

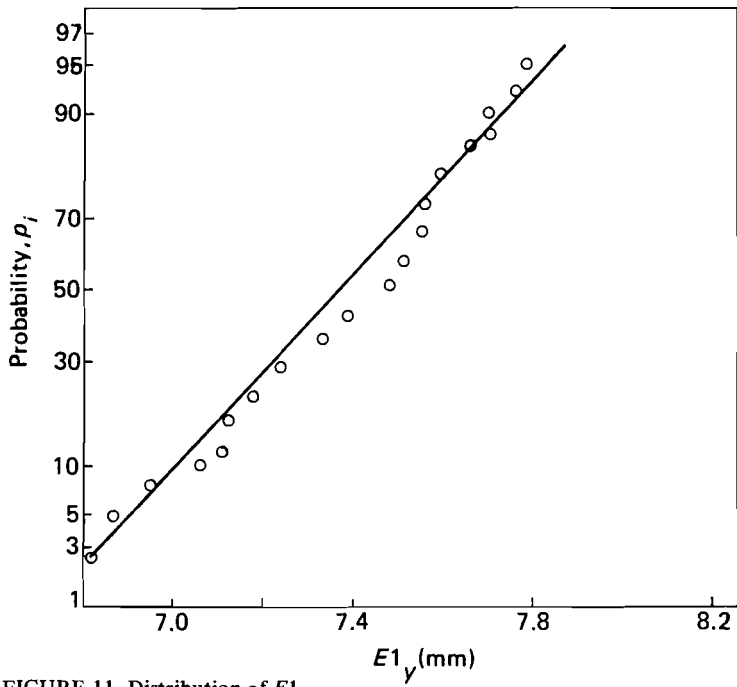


FIGURE 11 Distribution of $E1$.

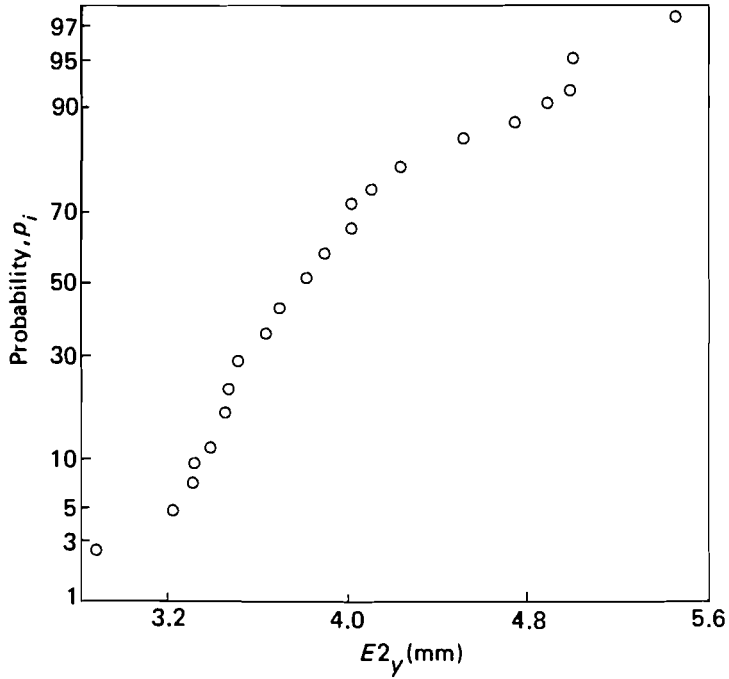


FIGURE 12 Distribution of $E2$ at station S.

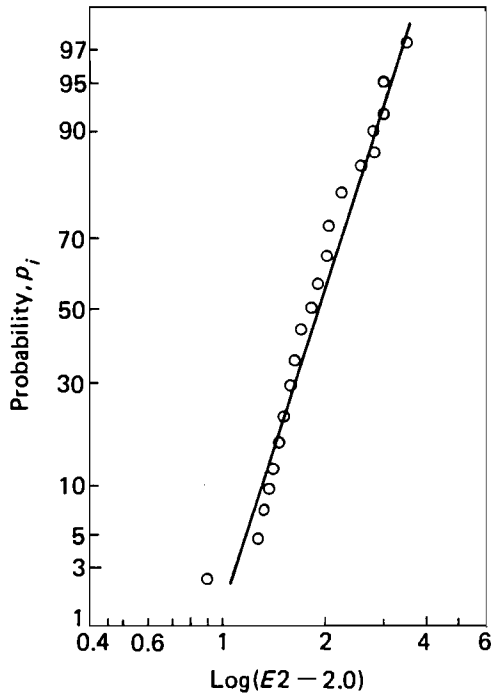


FIGURE 13 Distribution of $\log(E2 - 2.0)$ at station S.

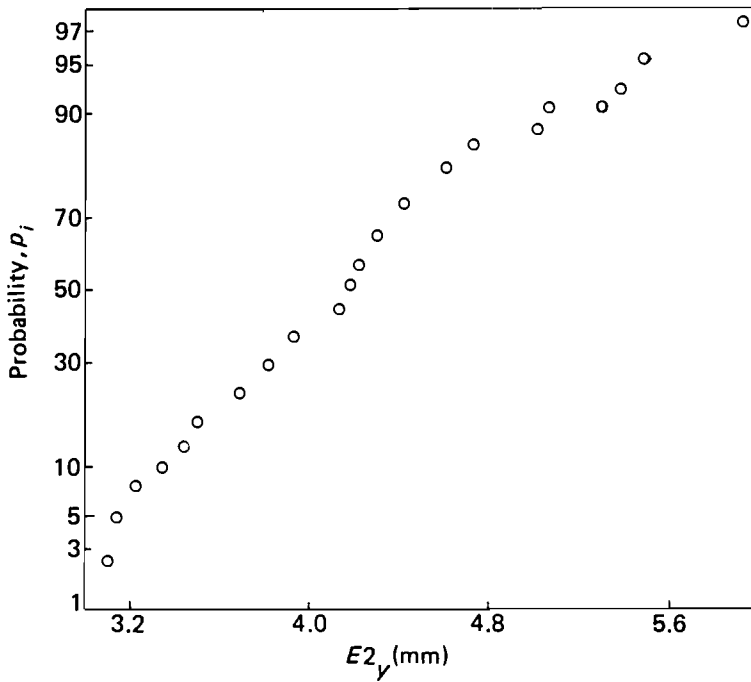


FIGURE 14 Distribution of $E2$ at station B.

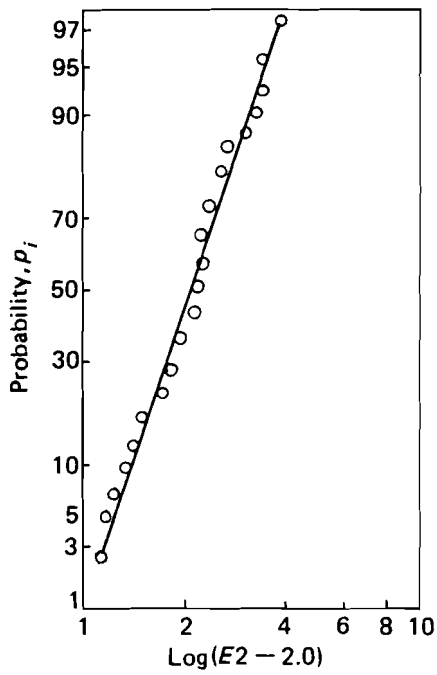


FIGURE 15 Distribution of $\log(E2 - 2.0)$ at station B.

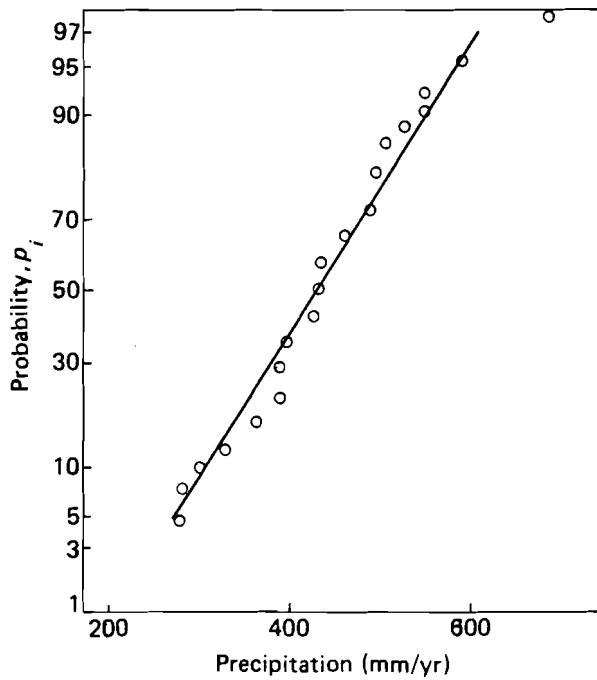


FIGURE 16 Distribution of precipitation PS at station S.

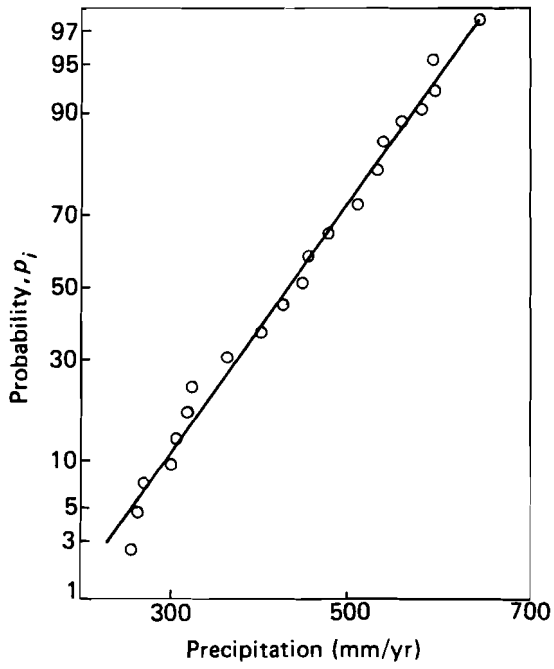


FIGURE 17 Distribution of precipitation PS at station B.

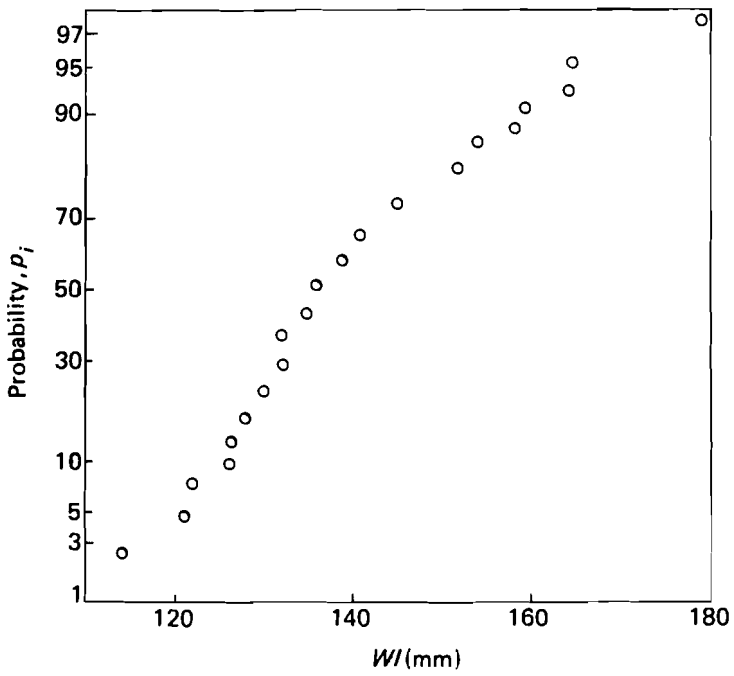


FIGURE 18 Distribution of irrigation water requirements WI , based on eq. (27).

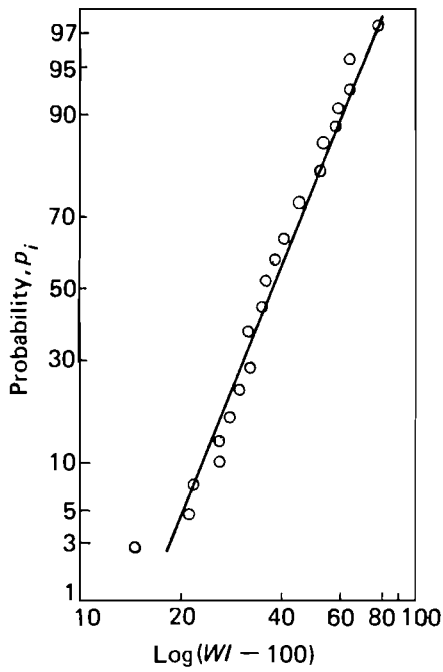


FIGURE 19 Distribution of irrigation water requirements $\log(WI - 100)$, based on eq. (27).

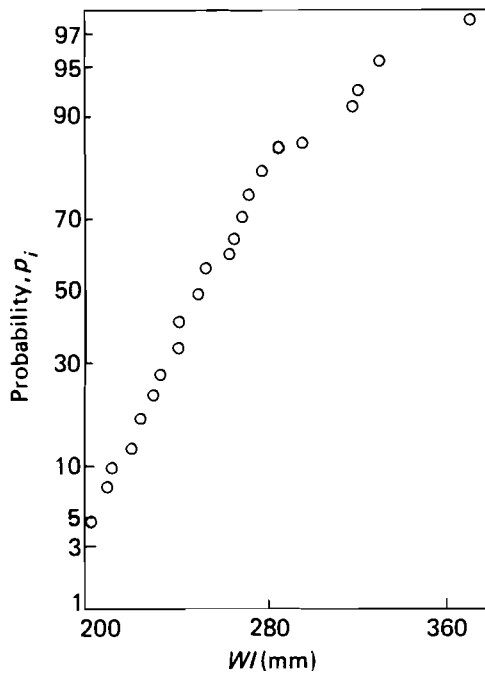


FIGURE 20 Distribution of irrigation water requirements WI , based on eq. (28).

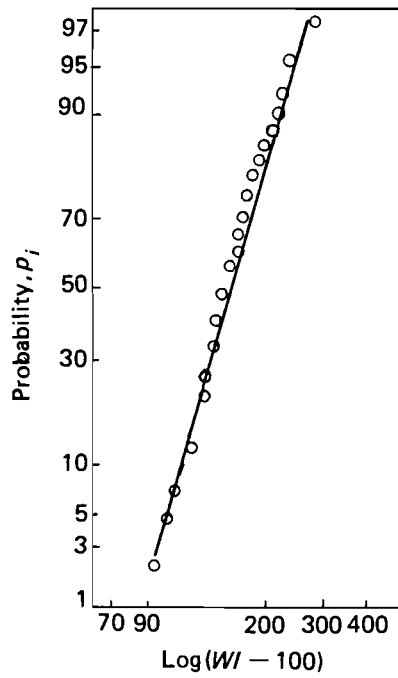


FIGURE 21 Distribution of irrigation water requirements $\log(WI - 100)$, based on eq. (28).

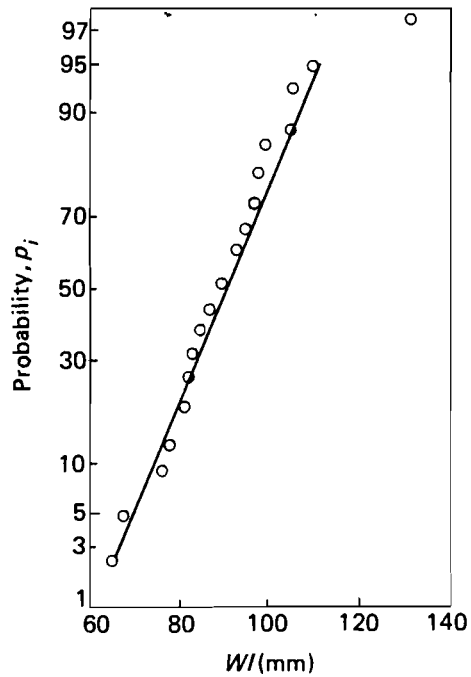


FIGURE 22 Distribution of irrigation water requirements WI , based on eq. (30).

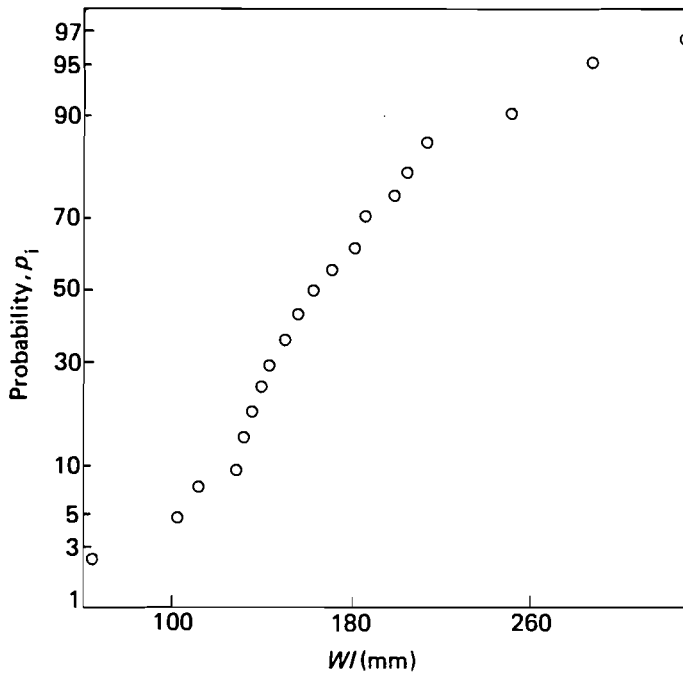


FIGURE 23 Distribution of irrigation water requirements WI , based on eq. (31).

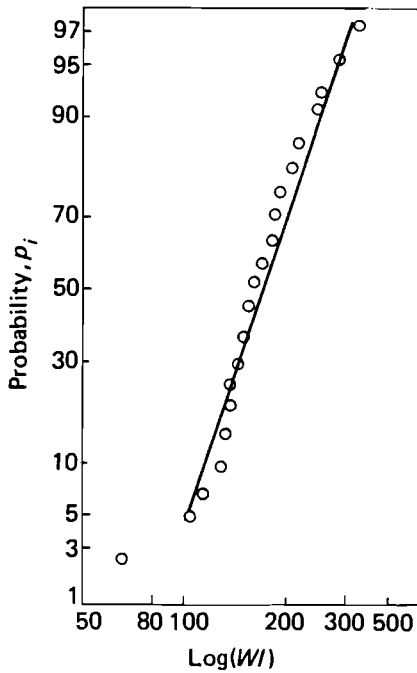


FIGURE 24 Distribution of irrigation water requirements $\log(WI)$, based on eq. (31).

The total precipitation for the vegetative period PS_y had an approximately normal distribution with an outlier on each side of the curve (minimum and maximum). This phenomenon was tested using station B values where a normal distribution fitted better (Figures 16 and 17). Because the irrigation water requirements model, WI_t , is based on a linear combination of the terms $e1_t$, $e2_t$, P_t , and WI_{t-1} , it can be expected that the distribution of WI_t will be either normal or log normal according to the prevailing component (see Figures 18–24).

First of all, the normal distribution was tested for WI_t values, but it did not fit well, and the log normal distribution with the transformation $z_t = \log(WI_t)$ was not successful in all cases. With the additional parameter the transformation $z_t = \log(WI_t - A)$ fitted well with the constant $A \approx 100$ for eqs. (27) and (28). In eq. (30), a normal distribution was thought to be satisfactory if the maximum value was assumed to be an outlier; otherwise, a log normal distribution with $A = 0$ gave better results.

In eq. (31), the minimum value was assumed to be an outlier and a log normal distribution was used ($A = 0$). This decision was supported by the fact that in this study, we were interested in maximum and average, rather than in minimum values, because these influence the WRS. It was stated above that a prescription model was tested, so that it is unimportant that it did not describe the occurrence of the minimum value.

Some other probability distributions were tested (e.g., Weibull and Pearson) with no significantly better fits. If all the known distributions (e.g., Johansson 1970) were tested, a better goodness-of-fit could be found. A log normal distribution, however, has some advantage in the generation of a synthetic time series. This distribution has been carefully studied by hydrologists and is therefore recommended.

Results based on time series using Penman's equation (Table 7) were compared with those based on Linacre's simplification. For this purpose, the time series based on eqs. (35)–(38) were modeled and the results are summarized in Table 8; differences can be seen in both averages and standard deviations. The main source of these differences lies in the fact that, in irrigation water requirement models, the second term of Penman's equation is decisive, whereas in Linacre's equation both terms have the same weight. This is

TABLE 7 Statistical parameters of irrigation water requirements, WI , at station S using Penman's equation. ϕ = average (approx.); σ = standard deviation; C_v = coefficient of variation.

		A	M	J	J	A	S	Z	ϕ
WI	ϕ	3.57	28.87	30.80	29.73	27.71	22.61	143.3	
Eq. (27)	σ	3.83	5.21	3.80	5.65	4.85	4.02		
	$C_v(\%)$	107.3	18.0	12.3	19.0	17.5	17.8		19.1
WI	ϕ	20.85	48.26	52.80	50.27	45.50	33.50	251.2	
Eq. (28)	σ	11.44	12.30	8.97	13.35	11.46	9.49		
	$C_v(\%)$	54.9	25.5	17.0	26.5	25.2	28.3		26.5
WI	ϕ	--	18.04	19.72	18.19	17.17	15.13	88.2	
Eq. (30)	σ	--	5.14	4.03	6.10	4.99	3.86		
	$C_v(\%)$	--	28.5	20.4	33.5	29.1	25.5		27.3
WI	ϕ	6.81	34.07	39.90	35.47	30.50	22.16	168.9	
Eq. (31)	σ	10.05	18.17	15.65	20.66	18.06	13.57		
	$C_v(\%)$	147.6	53.3	39.2	58.2	59.2	61.2		56.9

TABLE 8 Statistical parameters of irrigation water requirements, WI , at stations S and B using Linacre's equation. ϕ_B and ϕ_S = averages at stations S and B, respectively; σ_S = standard deviation at station S; C_v = coefficient of variation at station S.

		A	M	J	J	A	S	E	ϕ
WI	ϕ_B	2.58	26.68	28.74	28.29	24.38	19.23	129.90	
Eq. (35)	ϕ_S	1.97	25.91	28.15	27.48	24.24	19.56	127.31	
	σ_S	3.00	4.62	3.76	5.31	4.33	3.11		
	$C_v(\%)$	152.2	17.8	13.4	19.3	17.9	15.9		18.9
WI	ϕ_B	11.58	39.49	44.35	43.30	34.03	21.82	194.58	
Eq. (36)	ϕ_S	9.50	37.66	42.96	41.38	33.70	22.59	187.79	
	σ_S	9.16	10.95	8.90	12.58	10.25	7.37		
	$C_v(\%)$	96.4	29.0	20.7	30.4	30.4	32.6		31.5
WI	ϕ_B	—	18.36	19.34	18.65	16.42	14.17	86.94	
Eq. (37)	ϕ_S	—	17.64	19.16	18.12	16.38	14.50	85.80	
	σ_S	—	4.65	3.88	5.62	4.56	3.31		
	$C_v(\%)$	—	26.4	20.2	31.0	27.8	22.8		25.7
WI	ϕ_B	4.03	29.40	32.43	31.43	21.79	13.51	132.59	
Eq. (38)	ϕ_S	3.29	26.47	31.53	29.16	21.77	14.18	126.40	
	σ_S	7.59	15.81	14.84	18.47	15.37	10.64		
	$C_v(\%)$	290.6	59.7	47.1	63.3	70.6	75.2		65.5

supported by a comparison of time series values for monthly evapotranspiration. With one exception in May, the values calculated by Linacre's equation for May–September were within 10% limits, as compared with those calculated by Penman's equation. In April, the values were systematically higher, so obviously a reduction by approximately 10% (e.g., a reduction by the coefficient of 0.9) was necessary. WRS are not very sensitive to April demands and, further, these are lower because of soil water storage. This difference in April was therefore not analyzed further.

As a result of these differences, Penman's equation is recommended even when the available data for, say, station X qualify for Linacre's simplification only with some station Y with "similar conditions" that has all the necessary data. These vague terms of similarity should be specified, but generally there are not enough data to do so. Then, the decision as to whether the conditions can be regarded as similar is one for meteorological and hydrological expert judgment. If conditions can be regarded as "similar", it is recommended that the missing data from station Y be used.

This problem is connected with the common question of transferability of the results from one place (such as a meteorological station) to another. In the present study, two stations (B and S) were tested, and it was found that the main difference was in precipitation, in the e_2 term (differences of up to 5%), and differences in the e_1 term were the least pronounced. The stations were in similar geographical, meteorological, and hydrological conditions, about 40 km apart. Apart from precipitation, the data were transferable from one station to the other within the error of measurement.

The irrigation water requirement values are not only dependent on meteorological conditions, but also on agricultural and irrigation practices. Equations (27) and (28) derived from the V-III-V system in Czechoslovakia reflect a relatively rigid irrigation scheme in

which the water requirements are insensitive to precipitation. This policy can be adopted where there is a relatively low degree of exploitation (low k_e). Therefore, it can be concluded that the transformation of eq. (27) to (28) does not reflect the changes that can occur where there is more effective use of irrigation water. Equations (30) and (31) derived from the C-V system reflect a better and more flexible irrigation system with more efficient use of water. Therefore, these equations are recommended for irrigation water requirement calculations as a time series for WRS modeling.

7 CONCLUSION AND DISCUSSION

Data concerning irrigation water requirements are essential in the planning of water resource systems (WRS). Used in the form of time series, they can be applied as a direct input into deterministic simulation models and as an indirect input into stochastic models for the derivation of the necessary statistical parameters. In the present study the elements that affect irrigation water requirements were analyzed, and it was found that evapotranspiration, precipitation, and irrigation in the previous time periods were the decisive factors. A model relating irrigation water requirements to these elements was derived and tested on two irrigation systems in Czechoslovakia.

The model is comprehensive enough to be used in other areas and under different conditions for irrigation systems in semi-humid climates in moderate climatic zones. However, it has to be based on observed data (monthly irrigation water requirements), since not only can the influence of individual terms change, but also the degree of exploitation and irrigation practices may differ from place to place, and this can have a profound effect on the resulting model parameters. It is not justifiable to calibrate the model in one area and then to use it in another that has different economic, agricultural, soil, vegetation, and irrigation conditions, because all of these factors must be taken into account in the calibration coefficients. Further research in this direction depends on the data available, and it is recommended that this work is carried out as soon as these are obtained.

The application of calibration coefficients makes use of prevailing irrigation practices, although their improvement is considered through the use of the coefficient of exploitation k_e . Long-term experience in Czechoslovakia has shown that changes in irrigation policy have little effect on the pattern of water requirements (distribution in the irrigation season), and therefore the difference between present and future irrigation policies can be evaluated using k_e .

The second step in the perfection of the model is connected with the effects of irrigation water requirements on the WRS, or vice versa. These can be analyzed by two basic methods. First, the model can be used (e.g., eq. (31)), and the area irrigated (with or without k_e) can be taken as the variable. This approach is called experimenting with the model. Secondly, experiments on the model can be done, i.e., the irrigated area is held constant, and the parameters and terms of the model can be analyzed as far as their influence on the WRS is concerned.

ACKNOWLEDGMENT

The author would like to express his gratitude to Dr. Janusz Kindler, Chairman of the Resources and Environment Area (on leave from the Institute of Environmental Engineering, Warsaw Polytechnic University, Warsaw, Poland), for his help in the preparation of the manuscript, and suggestions and comments concerning the model and its application.

REFERENCES

- Balek, J. (1980) State of art of modeling of the water balance processes in the agricultural field and watershed. IIASA, REN Task Force Mtg, June.
- Barton, I.J. (1979) A parametrization of the evaporation from nonsaturated surfaces. *J. Appl. Meteorol.* 18: 43–7.
- Blaney, H.F. and W.D. Criddle (1966) Determining consumptive use for water developments. *Irrigation and Drainage Specialty Conf.*, Las Vegas, Nevada.
- Brochet, P. and N. Gerbier (1977) Une méthode opérationnelle pratique de calcul de l'évapotranspiration potentielle. *La Météorologie*, VI^e Série No. 11, pp25–31.
- Brutsaert, W. and H. Stricker (1979) An advection–aridity approach to estimate actual regional evapotranspiration. *Water Resources Res.* 15: 443–50.
- Cermak, M. and I. Brenda (1971) Drainage nets and land management. 2nd Conf. CVTS (N. Smokovoc).
- Christiansen, J.E. (1968) Pan evaporation and evapotranspiration from climatic data. *J. Irrigation and Drainage Division ASCE* 94: 243–65.
- Christiansen, J.E. and G.H. Hargreaves (1969) Irrigation requirements from evaporation. *Trans. Int. Commun. on Irrigation and Drainage III*: 23.569–23.596.
- Fleming, G. (1975) *Computer simulation techniques in hydrology*. New York: Elsevier. p333.
- Forsman, A. (1969) Mät- och beräkningsmetoder för bestämning av avdunstning. Svenska kommittén för internationella hydrologiska dekaden. Statens naturvetenskapliga forskningsråd. Report 4 (30 pp).
- Härtel, O. (1925) *Die Wildbach and Lawinerverbauung*, Tübingen.
- Holy, M. (1979) *Irrigation structures*. Central Board of Irrigation and Power No. 135, New Delhi (323 pp).
- Holy, M. (1980) *Erosion and Environment*. London: Pergamon (390 pp).
- Howard, K. and J.W. Lloyd (1979) The sensitivity of parameters in the Penman evaporation equations and direct recharge balance. *J. Hydrol.* 41: 329–44.
- Hudson, N. (1973) *Soil Conservation*. London: Batsford.
- Ivanov, N.N. (1954) The determination of potential evapotranspiration. *Izvest. Vsesoyuznogo. Geograf. Obs.* p22.
- Janko, J. (1958) *Statistical Tables*. Prague: CSAV p251 (in Czech).
- Jensen, M.E. (Ed.) (1966) *Evapotranspiration and its role in water resources management*. Proc. Conf. on Evapotranspiration, Am. Soc. Agric. Engrs, Chicago, p65.
- Johansson, W. (1970) *Bevattning i fältmässig odling*. Rapport från Seminarium vid Alnarp, Sverige.
- Kos, Z. (1969) *Linear regression model and its application in hydrology*. Works and Studies No. 6 Water Courses, Prague (in Czech).
- Kostjakov, A.N. (1951) *Osnovy Melioracij*, Moscow: Selchozgiz.
- Linacre, E.T. (1977) A simple formula for estimating evaporation rates in various climates, using temperature data alone. *Agric. Meteorol.* 18: 490–24.
- McGuinness, J.L. and L.H. Parmele (1972) Maximum potential evapotranspiration frequency. East Central US. *J. Irrigation and Drainage Division ASCE* 98(IR 2), Proc. Paper 8943, pp207–14.
- Morton, F.I. (1980) On the complementary relationship between potential and areal evapotranspiration. *Nat. Hydrol. Res. Inst. Ottawa, Canada*, p80.
- Morton, F.I., R. Goard, and J. Piowar (1980) Programs REVAP and WEVAP for estimating areal evapotranspiration and lake evaporation from climatological observations. *Nat. Hydrol. Res. Inst., NHRI Paper No. 12, Ottawa, Canada*, p56.

- Mustonen, S.E. and J.L. McGuinness (1968) Estimating evapotranspiration in a humid region. US Dept of Agric. Tech. Bull. No. 1389.
- Penman, H.L. (1954) Evaporation over parts of Europe. Proc. IASH Gen. Assembly, Rome, Publ. No. 38, pp168–76.
- Penman, H.L. (1963) Vegetation and hydrology, Tech. Commun. No. 53, Commonwealth Bureau of Soils, Harpenden, UK (125 pp).
- Perrier, A. (1977) Projet de définitions concernant l'évapotranspiration en fonction de considérations théorétiques et pratiques. La Météorologie VI^e Série No. 11, pp7–16.
- Pruitt, W.O. (1960) Relation of consumptive use of water to climate. Trans. Am. Soc. Agric. Engrs 3(1): 9–13, 17.
- Pýcha, M. (1965) Research of consumptive use of water by plants. Bratislava: VÚZA (in Czech).
- Quackenbush, T.H. and J.T. Phelan (1965) Irrigation requirements of lawns. J. Irrigation and Drainage Division ASCE 91: pp11–19.
- Riou, C. (1977) Rémarques sur l'évaporation des surfaces naturelles et son évaluation. La Météorologie, VI^e Série No. 11, pp17–23.
- Rodda, J.C., R.A. Downing, and F.M. Law (1976) Systematic Hydrology. London: Newnes-Butterworth (399 pp).
- Schulz, E.F. (1973) Problems in Applied Hydrology. Fort Collins: Water Resources Publications.
- Seuna, P. (1977) On the Hydrometeorological Factors Affecting Irrigation. Helsinki: Vesihallitus (96 pp) (in Finnish).
- Skogerboe, G.C. (1977) Water measurement and water management relationships. Proc. ASCE Conf. on Water Management for Irrigation and Drainage, Reno, Nevada, pp295–304.
- Tanner, C.B. (1967) Measurements of evapotranspiration, in Irrigation of Agricultural Lands. Monogr. No. 11, Am. Soc. Agron., Madison, pp534–74.
- Thorntwaite, C.W. and B. Holzman (1939) The determination of evaporation from land and water surfaces. Monthly Weather Rev. 67: 4–11.
- Thorntwaite, C.W. and B. Holzman (1942) Measurement of evaporation from land and water surfaces. US Dept of Agric. Tech. Bull. No. 817.
- Thorntwaite, C.W. and J.R. Mather (1955) The water balance, in Climatology 8(1), Lab. of Climatic Studies, Centerton, NJ.
- Turc, L. (1954) Le bilan d'eaux des sols – relations entre les précipitations, l'évaporation et l'écoulement. Ann. Agron. 5: 491–596.
- Veihmeyer, F.J. (1964) Evapotranspiration, in Handbook of Applied Hydrology (ed. Ven Te Chow). New York: McGraw-Hill.
- Ven Te Chow (ed.) (1964) Handbook of Applied Hydrology. New York: McGraw-Hill.
- WMO (1966) Measurement and Estimation of Evaporation and Evapotranspiration. WMO-No. 201, TP 105, Tech. Note No. 83 (121 pp).

BIBLIOGRAPHY

- Allen, W.H. and J.R. Lambert (1971) Application of the principle of calculated risk to scheduling of supplemental irrigation, I: concepts. Agric. Meteorol. 8.
- Aslyng, H.C. (1965) Evaporation, evapotranspiration and water balance investigations at Copenhagen 1955–64. Acta Agric. Scandinavia, XV(3–4): 284–300.
- Aslyng, H.C. and S.E. Jensen (1965) Radiation and energy balance of Copenhagen 1955–64, R. Vet. and Agric. Coll. Yearbook 1966, Copenhagen, pp22–40.
- Baier, W. (1965) The interrelationship of meteorological factors, soil moisture and plant growth. Int. J. Biometeorol. 9: 5–20.
- Baier, W. and D.A. Russello (1968) A computer program for estimating risks of irrigation requirements from climate data. Canadian Dept of Agriculture, Ottawa, Canada, TB 58, p48.
- Balek, J. (1979) Water balance model DAMBO. IIASA, June, p23.
- Banach, K., P. Kowalik, I. Toczyłowska, and W. Wedzinski (1978) Simulation model of water management for agricultural lands. J. Hydrol. Sci. 5: 191–9.

- Beven, K. (1979) A sensitivity analysis of the Penman–Monteith actual evapotranspiration estimates. *J. Hydrol.* 44: 169–90.
- Blaney, H.F. and W.D. Criddle (1945) A method of estimating water requirements in irrigated areas from climatological data. US Dept of Agric. Soil Conservation Service.
- Blaney, H.F. and W.D. Criddle (1950) Determining water requirements in irrigated areas from climatological and irrigation data. US Dept of Agric. SCS TP 96 (48 pp).
- Blaney, H.F. and W.D. Criddle (1962) Determining consumptive use and irrigation water requirements. US Dept of Agric. Tech. Bull. 1275 (59 pp).
- Blaney, H.F. and K.V. Morin (1942) Evaporation and consumptive use of water empirical formulae. *Trans. Am. Geophys. Union* 23: 76–83.
- Bosen, J.F. (1960) A formula for approximation of saturation vapor pressure over water. *Monthly Weather Rev.* 88: 275–6.
- Brunk, H.D. (1960) *An Introduction to Mathematical Statistics*. New York: Ginn and Co. (403 pp).
- Brutsaert, W. (1965) Evaluation of some practical methods of estimating evapotranspiration in arid climates at low latitudes. *Water Resources Res.* 1: 187–91.
- Budyko, M.I. (1948) Evaporation under natural conditions. US Dept of Commerce Israel Program Sci. 751 (130 pp; translated from Russian).
- Budyko, M.I. (1956) The heat balance of the Earth's surface. US Dept of Commerce Weather Bureau PB 131692 (259 pp; translated by N.A. Stepanova, 1958).
- Budyko, M.I. (1974) *Climate and Life*. New York: Academic Press (209 pp).
- Buras, N., D. Nir, and F. Alpetovits (1973) Planning and updating farm irrigation schedules. *J. Irrigation and Drainage Division ASCE* 99 (IR 1), Proc. Paper 9586, pp43–51.
- Businger, J.A. (1956) Some remarks on Penman's equations for evapotranspiration. *Netherlands J. Agric. Sci.* 4: 77–80.
- Cowan, I.R. (1965) Transport of water in the soil–plant–atmosphere system. *J. Appl. Ecol.* 2: 221–39.
- Crawford, N.H. and R.K. Linsley (1966) Digital simulation in hydrology, Stanford Watershed Model IV, Tech. Rep. 39, Dept Civil Engng, Stanford University, Stanford, California.
- Curran, J.C. (1979) Regional mean Bowen ratios deduced from diurnal changes of temperature and humidity. *J. Hydrol.* 40: 113–21.
- David, W.P. and E.A. Hiler (1970) Predicting irrigation requirements of crops. *J. Irrigation and Drainage Division ASCE* 96 (IR 3) Proc. Paper 7507, pp241–55.
- Davies, J.A. (1967) A note on the relationship between net radiation and solar radiation. *Q.J.R. Meteorol. Soc.* 93: 109–15.
- Doorenbos, J. and W.O. Pruitt (1974) Guidelines for prediction of crop water requirements. FAO Irrigation and Drainage Paper No. 25. Rome: FAO.
- Dub, O. and J. Nemeč (1969) *Hydrologie*. TP 34. Prague: SNTL (379 pp).
- Eagleson, P.S. (1970) *Dynamic Hydrology*. New York: McGraw-Hill.
- England, C.B. (1963) Water use by several crops in a weighing lysimeter. *Agron. J.* 55: 239–41.
- Fritschen, L.J. (1965) Accuracy of evapotranspiration determinations by the Bowen ratio method. *Bull. Int. Ass. Sci. Hydrol.* 10: 38–48.
- Fritschen, L.J. (1967) Net and solar radiation relations over irrigated fields. *Agric. Meteorol.* 5: 55–62.
- Gardner, W.R. (1960) Dynamic aspects of water availability to plants. *Soil Sci.* 89: 63–73.
- Garstka, W.U. (1978) Water resources and the national welfare, Fort Collins, WRP, CO 80552, USA (638 pp).
- Gates, D.M. and R.J. Hanks (1967) Plant factors affecting evapotranspiration, in *Irrigation of Agricultural Lands* (Eds. R.M. Hagan *et al.*). Am. Soc. Agron.
- Gay, L.W. (1969) The Regression of Net Upon Solar Radiation, Presented at 9th Conf. on Agric. Meteorol. Seattle. Am. Meteorol. Soc. (20 pp).
- Hagan, R.M. and I.J. Stewart (1972) Water deficits irrigation design and programming. *J. Irrigation and Drainage Division ASCE* 98 (IR 2), Proc. Paper 8955, pp215–37.
- Haise, J.R. (1958) *Irrigation*, in *Agronomic Trends and Problems in the Great Plains*. Adv. Agron. Vol. X. New York: Academic Press pp47–56.
- Hall, W.A. and W.S. Butcher (1968) Optimal timing of irrigation. *J. Irrigation and Drainage Division ASCE* 94 (IR 2), Proc. Paper 5988, pp267–75.

- Hallgren, G. (1977) Studies on the influences of precipitation on crop yields in Sweden with special reference to field irrigation. *Ann. R. Agric. Coll. Sweden* 14: 175–289.
- Hanks, R.J., J.I. Stewart and J.P. Riley (1977) Four-state comparison of models used for irrigation management. *Proc. ASCE Conf. on Water Management for Irrigation and Drainage*, Reno, Nevada, pp283–94.
- Hargreaves, G.H. (1968) Consumptive use derived from evaporation pan data. *J. Irrigation and Drainage Division ASCE* 94 (IR 1), Proc. Paper 5863, pp97–105.
- Herschfield, D. (1964) Effective rainfall and irrigation water requirements. *J. Irrigation and Drainage Division ASCE* 90 (IR 2), Proc. Paper 3920, pp33–47.
- Howe, G.M. (1956) The moisture balance in England and Wales based on the concept of potential evapotranspiration. *Weather* 11: 74–82.
- Israelsen, O.W. and V.E. Hansen (1962) *Irrigation Principles and Practices*. New York: Wiley. (447 pp).
- Jaworski, J. (1978) A mathematical model of actual evapotranspiration. *J. Hydrol. Sci.* 5(3–4): 179–89.
- Jensen, M.E. (1966) Discussion of irrigation water requirements of lawns. *J. Irrigation and Drainage Division ASCE* 92: 95–100.
- Jensen, M.E. (1966) Empirical methods of estimating or predicting evapotranspiration using radiation. *Proc. Conf. on Evapotranspiration*, Am. Soc. Agric. Engrs, Chicago, pp57–61, 64.
- Jensen, M.E. (1969) Scheduling irrigation with computers. *J. Soil and Water Conservation* 24(5): 193–5.
- Jensen, M.E. and L.J. Erie (1971) Irrigation and water management, in *Advances in Sugarbeet Production* (Ed. R.T. Johnson *et al.*) Ames: Iowa State University Press, pp189–222.
- Jensen, M.E. and H.R. Haise (1963) Estimating evapotranspiration from solar radiation. *J. Irrigation and Drainage Division ASCE* (IR 4), Proc. Paper 3737, pp15–41.
- Jensen, M.E., J.L. Wright, and B.J. Pratt (1971) Estimating soil moisture depletion from climate, crop and soil data. *Trans. Am. Soc. Agric. Engrs* 14(6): 954–9.
- Jensen, M.E., J.L. Wright, and B.J. Pratt (1974) Consumptive use of water and irrigation water requirements. *Committee on Irrigation Water Requirements*, Irrigation and Drainage Division ASCE.
- Kaczmarek, Z. (1970) *Statistical methods in hydrology and meteorology*. Wydawn. Kom. i Laern. Warsaw 312 pp (in Polish).
- Keller, W. and C.W. Carlson (1967) Irrigation of principal crops: Forage crops. *Irrigation of Agricultural Lands* (Eds R.M. Hagan *et al.*). Am. Soc. Agron. pp607–21.
- Kharchenko, S.I. and D.M. Katz (1974) Methods for the study of water balance of irrigated lands and irrigation systems. *Proc. Symp. on World Water Balance*, IASH–UNESCO–WMO, Vol. II, Publ. No. 93, IASH, pp452–8.
- Kindler, J. (Ed.) (1978) *Proc. Workshop on Modeling of Water Demands*. IIASA CP-78-06 (156 pp).
- Kindler, J. (1980) The systems analysis in regional water management planning. *Proc. Workshop on Water Regional Planning in the Nordic Sea*, NORDFORSK Cooperative Orgn. for Research. Sweden: Kungälv.
- Kindler, J., K.A. Salewicz, H. Słota, and T. Terlikowski (1979) Operation of Multiple Reservoir Systems: A Case Study of the Upper Vistula System (An Introduction). IIASA CP-79-17 (35 pp).
- Kindler, J., P. Zielinski, and L. de Mare (1980) An Interactive Procedure for Multiobjective Analysis of Water Resources Allocation. IIASA WP-80-85.
- Kohler, M.A. and L.H. Parmele (1967) Generalized estimates of free-water evaporation. *Water Resources Res.* 3(4): 997–1005.
- Konstantinov, A.R. (1968) Evaporation in nature. *Gidrometidet*. Leningrad (Translation US Dept of Commerce Nat. Tech. Inf. Service, Springfield, VA 22151).
- Kos, Z. (1964) Relation of irrigation rates to water resources: Projecting and Construction, in *Water Management* pp13–16 (in Czech).
- Kos, Z. (1964) Irrigation construction planning and water resources reliability, in *Water Management* No. 3 (in Czech).
- Kos, Z. (1965) Problems of development rates of irrigation: Projecting and Construction, in *Water Management* pp14–17 (in Czech).
- Kos, Z. (1968) Economic level of irrigation withdrawals in water resource system design. PhD Thesis, Prague (in Czech).
- Kos, Z. (1969) Irrigation withdrawals in water resource systems. *Water Management* 7: 198–200 (in Czech, English summary).

- Kos, Z. (1970) Simulation models of water supply for irrigation in water resource system. *ICID Bull.*, pp47–52.
- Kos, Z. (1973) Water resource system development with catchment system optimization. *Proc. Conf. on the Biosphere*. Prague: CVUT pp232–9 (in Czech).
- Kos, Z. (1975) Chance-constrained model of water resource systems. *Proc. Int. Symp. and Workshops on Application of Mathematical Models in Hydrology and Water Resources Systems*, Bratislava (in English).
- Kos, Z. (1976) Stochastic hydrology and its application for water resources systems solution. *Water Management J.* 23: 4–5 (in Czech, English summary).
- Kos, Z. (1978, 1979) (two parts). *Stochastic Models of Water Resources Systems. Works and Studies No. 146* Prague: Water Research Institute, SZN (in Czech, English summary) p468.
- Kos, Z. (1979) Operation of water resources systems in Czechoslovakia. *IIASA Workshop on Operation of Multiple Reservoir Systems*. Poland: Jodłowy Dwor, May–June (17 pp).
- Kos, Z. (1979) Multimodeling of water resources systems. *12th European Regional Conf., ICID, Dubrovnik, Yugoslavia*.
- Kos, Z. (1979) Chance-constrained model of the water resources system in the Odra River basin. *Water Management J.* 27(1): 14–26 (in Czech, English summary).
- Kos, Z., V. Bečvář, L. Votruba, and V. Zeman (1974) Study of the important water resource system in the River Odra basin. *CWP Preliminary Study*. Prague: WRDC (in Czech).
- Kos, Z. and J. Dupačova (1979) Chance-constrained and simulation models of water resources systems. *Econ. Math. Rev.* 15(2): 178–91.
- Kos, Z., A. Medla, S. Novotný, and V. Zeman (1973) Study of the important water resource system in the River Morava basin. *CWP Preliminary Study*. Prague: WRDC (in Czech).
- Kos, Z. and D. Novák (1976) Optimization of water resources utilization in water resource systems in CSSR. *Symp. ECE to Problems of Water Resources Planning. Proc. WATER/SEM. 4/R4/CCM. 4*, Varna.
- Kos, Z., J. Pometlo, M. Sýkora, and V. Zeman (1972) Study of the important water resource system in the upper and middle part of the Labe River basin. *GWP Preliminary Study*. Prague: WRDC (in Czech).
- Kos, Z. and V. Zeman (1976) *Water Resource Systems Design in the General Water Management Plan*. Prague: SZN (in Czech, English summary) p271.
- Kruse, E.G., D.C. Kincaid, and H.R. Duke (1977) Evapotranspiration estimates with the modified Penman equation. *Proc. ASCE Conf. on Water Management for Irrigation and Drainage*, Reno, Nevada, pp245–55.
- Linacre, E.T. (1967) Climate and the evaporation from crops. *J. Irrigation and Drainage Division ASCE* 93 (IR 4), Proc. Paper 5651, pp61–79.
- Linacre, E.T. (1969) Closing discussion: climate and evaporation from crops. *J. Irrigation and Drainage Division ASCE* 95: 348–52.
- Linacre, E.T. and M.R. Till (1969) Irrigation timing and amounts. *J. Australian Inst. Agric. Sci.* 35 (3): 175–96.
- Linsley, R.K., M.A. Kohler, and Y.L.H. Paulhus (1975) *Hydrology for Engineers* (2nd edn) New York: McGraw-Hill, p482.
- Lowry, R.L. and A.F. Johnson (1942) Consumptive use of water for agriculture. *Am. Soc. Civil Engrs Trans.* 107: 1243–302.
- McCulloch, J.S.G. (1965) Tables for the rapid computation of the Penman estimate of evaporation. *East Africa Forestry J.* 30: 286–295.
- McCulloch, J.S.G. (1977) Examples of areal actual evapotranspiration by indirect methods. *Gen. Rep. on the Tech. Conf. on Assessment of Areal Evaporation*, Budapest.
- McGuinness, J.L. and E.F. Bordne (1972) A comparison of lysimeter-derived evapotranspiration with computed values, *US Dept of Agric. Tech. Bull.* 1452 (71 pp).
- McIlroy, I.C. and D.E. Angus (1964) Grass, water and soil evaporation at Aspendale. *Agric. Meteorol.* 1(3): 201–24.
- Makking, G.F. (1957) Testing the Penman formula by means of lysimeters. *J. Inst. Water Engrs* 11(3): 277–88.

- Molz, F.J. (1972) Simulation of post-irrigation moisture movement. *J. Irrigation and Drainage Division ASCE* 98(IR 4), Proc. Paper 9419, pp523–32.
- Monteith, J.L. (1965) Evaporation and the environment, in *The State and Movement of Water in Living Organisms*. Proc. 19th Soc. for Expt. Biol. Swansea. London: Cambridge University Press pp205–34.
- Olivier, H. (1961) *Irrigation and Climate*. London: Arnold (250 pp).
- Ostromecki, J. (1965) Remarks on methods of computation of water requirements of fields and grasslands. US Dept Interior TT 67-56052. (Transl. from Polish) (23 pp).
- Patil, A.C. and J.E. Christiansen (1963) Comparison of four methods of computing evaporation. University Research Project U-193, College of Engineering, Utah State University.
- Pegg, R.K. (1970) Evapotranspiration and the water balance, in *The Role of Water in Agriculture* (Ed. J.A. Taylor). University College of Wales at Aberystwyth Memo No. 12.
- Penman, H.L. (1948) Natural evaporation from open water, bare soil and grass. *Proc. R. Soc. London A*193: 120–46.
- Penman, H.L. (1949) The dependence of transpiration on weather and soil conditions. *J. Soil Sci.* 1: 74–89.
- Penman, H.L. (1950) Evaporation over the British Isles. *Q.J.R. Meteorol. Soc.* 76: 372–83.
- Penman, H.L. (1955) Components in the water balance of a catchment area. *Q.J.R. Meteorol. Soc.* 81: 280–3.
- Penman, H.L. (1956) Estimating evaporation. *Trans. Am. Geophys. Union* 37: 43–50.
- Penman, H.L. (1956) Evaporation: an introductory survey. *Netherlands J. Agric. Sci.* 1: 9–29, Discussion: 87–97, 151–3.
- Persaud, Ch. (1977) Potential evapotranspiration in different climatic regions of Guyana. McGill University CRS No. 11 (101 pp).
- Pruitt, W.O. (1966) Empirical method of estimating evapotranspiration using primarily evaporation pans. Proc. Conf. on Evapotranspiration, Am. Soc. Agric. Engrs, Chicago, pp57–61.
- Pruitt, W.O. and M.C. Jensen (1955) Determining when to irrigate. *Agric. Engr* 36: 389–93.
- Pruitt, W.O. and F.J. Lourence (1968) Correlations of climatological data with water requirements of crops. *Water Sci and Engng Paper No. 9001*, Univ. of California, Davis (59 pp).
- Pruitt, W.O., F.J. Lourence, and S. von Oettingen (1972) Water use by crops as affected by climate and plant factors, *California Agric.*, pp10–14.
- Pruitt, W.O., S. von Oettingen, and D.L. Morgan (1972) Central California evapotranspiration frequencies. *J. Irrigation and Drainage Division ASCE* 98 (IR 2), Proc. Paper 8941, pp177–84.
- Prus-Chacinski, T.M. (1963) Discussion of some aspects of the hydrology of the Thames bar. *Proc. Inst. Civil Engrs* 24: 247–87.
- Rider, N.E. (1954) Evaporation from an oat field. *Q.J.R. Meteorol. Soc.* 80: 198–211.
- Rider, N.E. (1957) Water losses from various land surfaces. *Q.J.R. Meteorol. Soc.* 83: 181–93.
- Ritchie, J.T., E. Burnett, and R.C. Henderson (1972) Dryland evaporative flux in a subhumid climate, III. Soil water influences. *Agron. J.* 64: 168–73.
- Robins, J.S. (1967) Reducing irrigation requirements, in *Irrigation of Agricultural Lands* (Eds. R.M. Hagan *et al.*). Am. Soc. Agron. pp1144–58.
- Rosenberg, N.J. (1972) Frequency of potential evapotranspiration rates in Great Central Plains. *J. Irrigation and Drainage Division ASCE* 98 (IR 2), Proc. Paper 8946, pp203–6.
- Saxton, K.E. and A.T. Lenz (1967) Antecedent retention indexes predict soil moisture. *J. Hydraulics Division ASCE* 93 (HY 4), Proc. Paper 5351, pp223–41.
- Seuna, P. (1973) A simple linear soil moisture model for irrigation purposes. Proc. 9th European Reg. Conf. ICID, Budapest.
- Shimshi, D., D. Yaron, E. Bresler, M. Weisbrod, and G. Strateener (1975) Simulation model for evapotranspiration of wheat: an empirical approach. *J. Irrigation and Drainage Division ASCE* 101 (IR 1): 1–12.
- Slatyer, R.O. and I.C. McIlroy (1961) *Practical Climatology*. Melbourne: CSIRO, and Paris: UNESCO (340 pp).
- Srivastava, M.S. and C.G. Khatri (1979) *An Introduction to Multivariate Statistics*. New York: North-Holland (350 pp).

- Stanhill, G. (1958) The accuracy of meteorological estimates of evapotranspiration. *J. Inst. Water Engrs* 12: 377–83.
- Stephens, J.C. (1965) Discussion of estimation evaporation from insulation. *J. Hydrol. Division ASCE* 91 (HY 5): 171–82.
- Stewart, J.I. and R.M. Hagan (1969) Predicting effects of water shortage on crop yield. *J. Irrigation and Drainage Division ASCE* 95 (IR 1), Proc. Paper 6443, pp91–104.
- Stewart, J.I., R.M. Hagan, and W.O. Pruitt (1974) Functions to predict optimal irrigation programs. *J. Irrigation and Drainage Division ASCE* 100 (IR 2), Proc. Paper 10626, pp179–99.
- Szeicz, G. and I.F. Long (1969) Surface resistance of crop canopies. *Water Resources Res.* 5: 622–33.
- Tanner, C.B. (1960) Energy balance approach to evapotranspiration from crops. *Proc. Soil Sci. Soc. Am.* 24: 1–9.
- Tanner, C.B. (1968) Evaporation of water from plants and soils, in *Water Deficits and Plant Growth* vol. 1 (Ed. T.T. Kozloski) New York: Academic Press, pp73–106.
- Tanner, C.B. and M. Fuchs (1968) Evaporation from unsaturated surfaces: a generalized combination method. *J. Geophys. Res.* 73: 1299–304.
- Tanner, C.B. and W.L. Pelton (1960) Potential evapotranspiration estimates by the approximate energy balance method of Penman. *J. Geophys. Res.* 65: 3391–413.
- Thornthwaite, C.W. (1944) Report of the committee on transpiration and evaporation 1943–44. *Trans. Am. Geophys. Union* 25: 5.
- Thornthwaite, C.W. (1948) An approach toward a rational classification of climate. *Geog. Rev.* 38: 55–94.
- Tomlain, J. (1979) Methods of determination of actual and potential evaporation from soil surface. *Meteorologicke zpravy* 32: 72–7 (in Czech).
- Turc, L. (1961) Evaluation des besoins en eau d'irrigation, évapotranspiration potentielle: formule climatique simplifiée et mise à jour (Estimation of irrigation water requirements, potential evapotranspiration: a simple climatic formula evolved up-to-date). *Ann. Agron.* 12: 13–49.
- USDA (1967) Irrigation water requirements. US Dept of Agric., Engng Division Soil Conservation Service, Tech. Release No. 21 (88 pp).
- van Bavel, C.H.M. (1968) Discussion of climate and evaporation of crops. *J. Irrigation and Drainage Division ASCE* 68: 533–5.
- Wilks, S.S. (1962) *Mathematical Statistics*. New York: Wiley (644 pp).
- Windsor, J.S. and Ven Te Chow (1971) Model for farm irrigation in humid areas. *J. Irrigation and Drainage Division ASCE* 97: (IR 3), Proc. Paper 8355, pp369–85.
- Wiser, E.H. (1965) Irrigation planning using climatological data. *J. Irrigation and Drainage Division ASCE* 91 (IR 4), Proc. Paper 4551, pp1–11.
- WMO (1968) *Practical Soil Moisture Problems in Agriculture*. WMO-No. 235, TP 128, Tech. Note No. 97 (69 pp).
- WMO (1974) *Guide to Hydrological Practices*. WMO-No. 168.
- WMO (1979) *Problems of Evaporation Assessment in the Water Balance* (Hounam, C.E.) Rep. No. 73, WMO-No. 285.
- Wu, I. and T. Liang (1972) Optimal irrigation quantity and frequency. *J. Irrigation and Drainage Division ASCE* 98 (IR 1), Proc. Paper 8776, pp117–33.

APPENDIX A Formulae for Evapotranspiration

Thornthwaite

$$PE_t = 16 (10T_t/I)a$$

where

$$PE_t = \text{monthly potential evapotranspiration (mm);}$$

$$T_t = \text{mean monthly air temperature (}^\circ\text{C);}$$

$$I = \text{annual heat index}$$

$$I = \sum_{t=1}^{12} \left(\frac{T_t}{5} \right)^{1.514}$$

$$a = 6.75 \times 10^{-7} I^3 - 7.71 \times 10^{-5} I^2 + 1.792 \times 10^{-2} I + 0.49239.$$

This formula is relatively simple and requires few input data, so that it has been one of the most commonly used (and misused) empirical equations in generating inaccurate estimates of potential evapotranspiration. This equation is valid only for the conditions of the east-central USA. Thornthwaite and Mather (1955) required that (1) the albedo of the evaporating surface must be a standard; (2) the rate of evapotranspiration must not be influenced by advection of moist or dry air; and (3) the ratio of energy utilized in evapotranspiration to that used in heating the air must remain essentially constant. Since it is questionable whether these conditions exist in the investigated area, this equation cannot be used.

Blaney and Criddle

$$PE_t = \sum_{t=1}^8 k_t \frac{T_t P_t}{100}$$

where

PE_t = potential evapotranspiration (in/season);

k_t = monthly crop coefficient;

T_t = mean monthly air temperature (°F);

P_t = mean monthly percentage of annual daytime hours.

(This equation could be converted to SI units (i.e., mm, °C), but with some loss of simplicity.)

This formula has been analyzed by Pruitt (1960), Quackenbush and Phelan (1965), Jensen (1966), Tanner (1967), and others, who showed that it is oversimplified and that the coefficients are influenced by radiation and humidity. Furthermore, the evapotranspiration PE (or its monthly components) is strongly dependent on the crop being irrigated, which is not convenient for long-term planning.

Turc

Turc's formula (1954) was derived from lysimetric measurements giving evapotranspiration from a cultivated field as a function of available moisture and the "evaporating power of the air":

$$PE_t = \frac{P + m + V}{\{1 + [(P + m)/L + (V/2L)]^2\}^{1/2}}$$

where

PE_t = evapotranspiration (mm/10 days);

P = precipitation (mm/10 days);

m = soil moisture available for evapotranspiration (mm) (e.g., $m = 10$ after irrigation, $m = 1$ for dry soil);

V = additional moisture available for evapotranspiration (mm).

V is determined by

$$V = 25(Mc/Z)^{1/2}$$

where

- M = final yield of dry matter (dg m^{-2} ; originally metric cents per hectare) (e.g., for wheat $M = 30$);
 c = crop constant (e.g., for cereals, carrots, $c = 1.00$; maize, beet, $c = 0.67$; potatoes, $c = 0.83$; peas, clover, $c = 1.17$; lucerne, alfalfa, meadow grass, $c = 1.33$);
 Z = length of growing season (days).

A radiation and temperature term giving the evaporating power of the air is given by

$$L = (1/16)(T + 2)\sqrt{Q}$$

where T is the mean air temperature over a ten-day period ($^{\circ}\text{C}$), and Q is the mean short-wavelength radiation over a ten-day period (cal cm^{-2}).

A simplified version (Turc 1954, Johansson 1970) for the vegetative period can be written as

$$PE_t = \frac{P + 80}{\{1 + [(P + 45)/L]^2\}^{1/2}}$$

by choosing average values of m and V . Turc (1954) published a new formula:

$$PE_t = 0.013 \frac{T}{T + 13} (Q + 50) \left(1 + \frac{50 - RH}{70}\right)$$

where T and Q have the same meanings as above, and RH is the relative humidity (%). This formula can be referred to in three forms: in one form PE_t is dependent on yield and crop coefficients, but this makes it cumbersome, so that it has been simplified using average estimates of the empirical coefficients. In this form only the stated inputs are necessary. The main advantage is that precipitation can be used as the input factor of evapotranspiration. However, the procedure is dependent on the conditions under which the input data were derived, and lacks physical sense, although under some conditions it affects the result very little (a change of 100% from 20 to 40 mm of precipitation changes the monthly evapotranspiration value by only 3%).

Johansson

$$PE_t = 0.14 + 3.7 \times 10^{-3} Q + 0.13w(e_m - e_d)$$

where

- PE_t = evapotranspiration (mm/day);
 Q = solar radiation ($\text{cal cm}^{-2}/\text{day}$);
 w = mean daily wind velocity (m s^{-1});
 $e_m - e_d$ = saturation deficit (mm Hg).

Ivanov

$$PE_t = 0.0018(25 + T)^2(100 - RH)$$

where

$$\begin{aligned} PE_t &= \text{evapotranspiration (mm/month);} \\ T &= \text{temperature (}^\circ\text{C);} \\ RH &= \text{relative humidity (\%).} \end{aligned}$$

Ostromecki, Alpatjev, and Pýcha

$$PE_t = k_c d_t$$

where

$$\begin{aligned} PE_t &= \text{evapotranspiration (mm/month);} \\ k_c &= \text{crop coefficient (0.2–1.1);} \\ d_t &= \text{sum of mean daily saturation deficits (mm Hg).} \end{aligned}$$

Pýcha found that k_c is dependent on the accumulated temperature:

$$\sum_{d=1}^{d+h} T_d$$

i.e., the sum of mean daily temperatures ($^\circ\text{C}$) from the beginning of the growing period of a crop.

Makking

$$PE_t = 0.61 \frac{\Delta R_s}{\Delta + \gamma 59} - 0.12$$

where

$$\begin{aligned} PE_t &= \text{evapotranspiration (mm/day);} \\ \Delta, \gamma &= \text{(see Penman's equation);} \\ R_s &= \text{solar radiation (cal cm}^{-2}\text{).} \end{aligned}$$

Stephens

$$PE_t = (0.014T - 0.37)R_s/1500$$

where

$$\begin{aligned} PE_t &= \text{evapotranspiration (in/day);} \\ T &= \text{temperature (}^\circ\text{F);} \\ R_s &= \text{solar radiation (cal cm}^{-2}\text{).} \end{aligned}$$

Jensen, Jensen and Haise

$$PE_t = C(T - T_o)R_s = 0.025(T + 3)R_s$$

where

$$\begin{aligned} PE_t &= \text{evapotranspiration (mm/day);} \\ T &= \text{mean air temperature (}^\circ\text{C);} \\ R_s &= \text{solar radiation (cal cm}^{-2}\text{).} \end{aligned}$$

Jensen later defined C as follows:

$$C = \frac{1}{C_1 + C_2 C_H}$$

where

$$\begin{aligned} C_1 &= 38 - 2M/305; \\ M &= \text{elevation above sea level (m);} \\ C_2 &= 7.6; \\ C_H &= 50/(e_n - e_d); \\ e_n \text{ and } e_d &= \text{saturation vapor pressure at mean maximum and mean minimum temperatures (mbar), respectively.} \end{aligned}$$

McIlroy

$$PE_t = \frac{s}{L(s + \gamma)} (Q_n - S) + h(D - D_o)$$

where

$$\begin{aligned} PE_t &= \text{evapotranspiration (mm/day);} \\ \gamma &= \text{(see Penman's equation);} \\ L &= \text{latent heat of vaporization (cal g}^{-1}\text{);} \\ s &= 0.63 \Delta W/p; \\ p &= \text{atmospheric pressure (mbar);} \\ \Delta W &= \text{slope of saturation vapor pressure curve at mean wet bulb temperature (mbar } ^\circ\text{C}^{-1}\text{);} \\ Q_n &= \text{net radiation flux (cal cm}^{-2}\text{);} \\ S &= \text{soil heat flux (cal cm}^{-2}\text{) (for monthly data, this can be neglected);} \\ h &= \text{wind velocity coefficient (experimentally determined)} \\ &= 0.5(1 + w), \text{ where } w \text{ is the wind velocity (m s}^{-1}\text{);} \\ D &= \text{wet bulb temperature depression (}^\circ\text{C) at height } Z \text{ (m) above the ground;} \\ D_o &= \text{wet bulb temperature depression (}^\circ\text{C) at ground level, which can be taken as } D_o = 0 \text{ (experimentally determined).} \end{aligned}$$

Christiansen and Hargreaves

$$PE_t = 0.492 R_s C_T C_w C_H$$

where

$$\begin{aligned} PE_t &= \text{evapotranspiration (mm/day);} \\ R_s &= \text{solar radiation (cal cm}^{-2}\text{);} \\ C_T &= 0.463 + 0.425T/20 + 0.112(T/20)^2; \\ T &= \text{mean temperature (}^\circ\text{C);} \\ C_w &= 0.672 + 0.406W/6.7 - 0.073(W/6.7)^2; \\ W &= \text{mean wind velocity 2 m above ground level (km h}^{-1}\text{);} \\ C_H &= 1.035 + 0.24RH/60 - 0.275(RH/60)^2, \text{ where } RH \text{ is relative humidity (\%).} \end{aligned}$$

Baier and Russelo, Baier and Robertson

$$\begin{aligned} PE_t &= 0.085[-53.39 + 0.337M + 0.531R + 0.0107Q_o + 0.0512Q_s + 0.0977W \\ &\quad + 1.77(e_w - e_s)] \end{aligned}$$

where

- PE_t = evapotranspiration (mm/day);
 M = daily maximum temperature ($^{\circ}\text{F}$);
 R = difference between daily maximum and minimum temperatures ($^{\circ}\text{F}$);
 Q_0 = solar radiation ($\text{cal cm}^{-2}/\text{day}$);
 $Q_s = Q_0 (0.261 + 0.616n/N)$, total daily solar energy on a horizontal surface ($\text{cal cm}^{-2}/\text{day}$), where n/N is the sunshine (see Penman's equation);
 W = wind velocity (miles/day);
 $e_w - e_s$ = vapor pressure deficit (mbar) from saturation vapor pressure at mean air temperature and at mean daily dewpoint temperature.

APPENDIX B FAO Modifications to Penman's Equation

According to the annex of the FAO Plant Production and Protection Paper No. 17 *Agrometeorological Crop Monitoring and Forecasting* (Rome, 1979), the following modifications to Penman's equation were recommended:

- (a) Evapotranspiration should be calculated directly using an albedo of $r = 0.25$.
 (b) A correction for elevation should be included (it is insignificant up to 150 m above sea level).
 (c) In eq. (14), a coefficient of 0.079 should be used instead of 0.9.
 (d) Equation (15) should be changed to $E = 0.26(e_a - e_d)(1 + 0.54w)D$

Other changes were not valid for the case analyzed.

With these changes and $f_t = 1$ for the V-III-V system (based on the input data shown in Table B.1), eq. (27) becomes

$$WI_t = 0.198 \frac{\Delta}{\Delta + \gamma} R_{n,t} + 0.853 \frac{\gamma}{\Delta + \gamma} E_t - 0.076P_t + 0.487WI_{t-1} - 12.08 \quad (27a)$$

and eq. (28) for $d = 1.90$ becomes

$$WI_t = 0.377 \frac{\Delta}{\Delta + \gamma} R_{n,t} + 1.621 \frac{\gamma}{\Delta + \gamma} E_t - 0.144P_t + 0.487WI_{t-1} - 22.95 \quad (28a)$$

The parameters $r_{i,d}$, R_i , and F_i become

$i =$	1	2	3	4
$r_{i,d}$	0.396	0.662	-0.296	0.534
R_i		0.662	0.718	0.875
F_i		14.46	12.78	28.59

Corresponding to eq. (23), α' becomes

$$\alpha' = \frac{1}{1.1} \frac{0.144 \times 47 + 22.95}{47} = 0.57$$

The resulting multiple correlation coefficient was nearly the same (0.873 and 0.875), the F -test values were higher by 2%, and the regression coefficients did not differ significantly. Thus the results using both methods are practically identical.

This modification was tested in the C-V system with approximately the same results concerning the significance of the resulting equation. For the C-V system, eq. (30) becomes (using the input data shown in Table B.1)

$$WI_t = 0.170 \frac{\Delta}{\Delta + \gamma} R_{n,t} + 0.499 \frac{\gamma}{\Delta + \gamma} E_t - 0.115 P_t + 0.442 WI_{t-1} - 1.72$$

and eq. (31) for $d = 2.99$ becomes

$$WI_t = 0.508 \frac{\Delta}{\Delta + \gamma} R_{n,t} + 1.492 \frac{\gamma}{\Delta + \gamma} E_t - 0.345 P_t + 0.442 WI_{t-1} - 5.14$$

The parameters $r_{i,d}$, R_i , and F_i become

$i =$	1	2	3	4
$r_{i,d}$	0.383	0.575	-0.417	0.595
R_i		0.577	0.727	0.872
F_i		9.25	13.22	27.74

$$\alpha' = \frac{\alpha \bar{P} + \beta}{\bar{P}} = \frac{0.345 \times 42.78 + 5.14}{42.78} = 0.47$$

The method of Appendix B was also used for computation in Section 6.1, but values of f_t were omitted.

This method of direct evapotranspiration determination (i.e., without the reduction coefficients f_t) was combined with the previous irrigation index. For the V-III-V system, the following modification to eq. (29) was obtained (based on the input data shown in Table B.2):

$$WI_t = i_t \left(0.365 \frac{\Delta}{\Delta + \gamma} R_{n,t} + 0.695 \frac{\gamma}{\Delta + \gamma} E_t \right) - 0.076 P_t - 8.14 \quad (29a)$$

The multiregressive coefficient was slightly lower, at 0.838, as compared with 0.846.

APPENDIX C Application of Program REVAP by Morton *et al.*

Areal evapotranspiration is an important element in the modeling of a hydrological cycle. Morton *et al.* (1980) defined it as the evapotranspiration from an area so large that the effects of evapotranspiration on the temperature and humidity of the overpassing air are fully developed.

The basic aim of Morton's investigation was to determine the complementary relationship between areal and potential evapotranspiration:

TABLE B.1 Input variables in regression analysis (method of Appendix B). $Wt_t^1 = Wt_t/2.22$; $Wt_{t-1}^1 = Wt_{t-1}/2.22$. Other variables are explained in eqs. (20) and (25).

		V-III-V system						C-V system					
	<i>M</i>	Wt_{t-1}	P_t	$\frac{\Delta}{\Delta + \gamma} R_{n,t}$	$\frac{\gamma}{\Delta + \gamma} E_t$	Wt_t	Wt_{t-1}^1	P_t	$\frac{\Delta}{\Delta + \gamma} R_{n,t}$	$\frac{\gamma}{\Delta + \gamma} E_t$	Wt_t^1		
1970	6	0.24	75.50	53.70	17.40	2.67	1.66	13.00	53.70	17.40	6.58		
	7	2.67	48.90	48.60	22.20	13.61	6.58	26.00	48.60	22.20	6.11		
	8	13.61	106.30	36.30	10.20	2.24	6.11	95.00	36.30	10.20	2.44		
	9	2.24	23.60	14.70	11.40	0.68	2.44	21.00	14.70	11.40	3.87		
1971	4	0.00	16.50	19.80	14.10	0.33	0.00	12.00	19.80	14.10	1.80		
	5	0.33	124.30	37.80	13.80	1.48	1.80	98.00	37.80	13.80	2.63		
	6	1.48	109.70	40.20	14.40	0.10	2.63	110.00	40.20	14.40	0.39		
	7	0.10	9.30	53.70	20.70	12.47	0.39	5.00	53.70	20.70	9.86		
	8	12.47	57.20	41.10	22.20	26.52	9.86	60.00	41.10	22.20	9.36		
	9	26.52	37.70	14.10	9.60	7.89	9.36	35.00	14.10	9.60	4.66		
1972	4	0.00	24.20	17.10	15.00	0.71	0.00	26.00	17.10	15.00	1.21		
	5	0.71	76.10	35.40	16.50	1.98	1.21	94.00	35.40	16.50	1.86		
	6	1.98	78.90	48.60	17.10	6.72	1.86	66.00	48.60	17.10	6.13		
	7	6.72	40.70	49.80	14.70	13.78	6.13	39.00	49.80	14.70	6.23		
	8	13.78	51.50	35.70	15.00	9.03	6.23	48.00	35.70	15.00	3.28		
	9	9.03	37.30	13.80	7.20	0.85	3.28	60.00	13.80	7.20	1.61		
1973	4	0.00	47.30	16.50	17.10	1.47	0.00	34.50	16.50	17.10	1.43		
	5	1.47	54.70	39.90	18.30	3.41	1.43	48.40	39.90	18.30	4.71		
	6	3.41	44.10	52.80	19.50	10.62	4.71	47.00	52.80	19.50	7.45		
	7	10.62	69.00	47.70	19.80	24.72	7.45	79.10	47.70	19.80	8.26		
	8	24.72	14.10	40.50	19.80	26.72	8.26	8.00	40.50	19.80	11.26		
	9	26.72	9.90	14.70	17.10	14.60	11.26	7.70	14.70	17.10	9.34		

1974	4	0.00	10.00	19.20	20.70	15.70	0.00	8.40	19.20	20.70	8.45
	5	15.70	70.10	35.70	18.00	3.91	8.45	80.00	35.70	18.00	3.17
	6	3.91	65.80	43.50	18.00	6.26	3.17	73.70	43.50	18.00	4.53
	7	6.26	54.30	40.80	25.20	11.88	4.53	64.60	40.80	25.20	2.69
	8	11.88	44.70	40.20	20.10	13.61	2.69	66.10	40.20	20.10	4.83
	9	13.61	38.90	15.60	12.90	6.41	4.83	30.60	15.60	12.90	3.23
1975	4	0.00	19.90	18.60	16.80	0.30	0.00	19.90	18.60	16.80	1.43
	5	0.30	65.50	36.30	15.30	2.91	1.43	74.70	36.30	15.30	1.98
	6	2.91	62.00	44.40	14.70	5.55	1.98	41.10	44.40	14.70	5.66
	7	5.55	48.50	51.30	17.40	15.58	5.66	28.80	51.30	17.40	6.43
	8	15.58	20.90	40.20	17.70	20.32	6.43	17.50	40.20	17.70	8.52
	9	20.32	20.90	17.70	9.00	9.31	8.52	18.00	17.70	9.00	7.16
1976	4	0.00	17.50	19.20	15.90	2.98	0.00	14.00	19.20	15.90	2.51
	5	2.98	55.50	39.90	24.90	16.24	2.51	48.00	39.90	24.90	7.90
	6	16.24	32.00	53.40	25.80	16.11	7.90	25.00	53.40	25.80	8.70
	7	16.11	29.50	50.10	29.40	38.39	8.70	30.00	50.10	29.40	16.77
	8	38.39	37.50	36.00	24.30	26.37	16.77	36.00	36.00	24.30	12.71
	9	26.37	29.50	14.10	9.60	11.16	12.71	2.00	14.10	9.60	8.25

Supplementary irrigation in water resource systems

TABLE B.2 Input variables in regression analysis (method of Appendix B, combined with previous irrigation index). $WI_t^1 = WI_t/2.22$; $WI_{t-1}^1 = WI_{t-1}/2.22$. Other variables are explained in eqs. (20) and (25).

	<i>M</i>	V-III-V system				C-V system					
		WI_{t-1}	P_t	$\frac{\Delta}{\Delta + \gamma} R_{n,t}$	$\frac{\gamma}{\Delta + \gamma} E_t$	WI_t	WI_{t-1}^1	P_t	$\frac{\Delta}{\Delta + \gamma} R_{n,t}$	$\frac{\gamma}{\Delta + \gamma} E_t$	WI_t^1
1970	6	0.24	75.50	34.63	11.22	2.67	1.66	13.00	37.76	12.23	6.58
	7	2.67	48.90	35.91	16.40	13.61	6.58	26.00	41.30	18.87	6.11
	8	13.61	106.30	35.78	10.05	2.24	6.11	95.00	30.43	8.55	2.44
	9	2.24	23.60	10.65	8.26	0.68	2.44	21.00	10.75	8.34	3.87
1971	4	0.00	16.50	19.80	14.10	0.33	0.00	12.00	19.80	14.10	1.80
	5	0.33	124.30	24.53	8.96	1.48	1.80	98.00	26.77	9.77	2.63
	6	1.48	109.70	27.98	10.02	0.10	2.63	110.00	29.65	10.62	0.39
	7	0.10	9.30	34.29	13.22	12.47	0.39	5.00	34.98	13.48	9.86
	8	12.47	57.20	39.74	21.47	26.52	9.86	60.00	37.82	20.43	9.36
9	26.52	37.70	16.13	10.98	7.89	9.36	35.00	12.84	8.74	4.66	
1972	4	0.00	24.20	17.10	15.00	0.71	0.00	26.00	17.10	15.00	1.21
	5	0.71	76.10	23.55	10.98	1.98	1.21	94.00	24.27	11.31	1.86
	6	1.98	78.90	34.75	12.23	6.72	1.86	66.00	34.54	12.15	6.13
	7	6.72	40.70	42.48	12.54	13.78	6.13	39.00	41.77	12.33	6.23
	8	13.78	51.50	35.28	14.82	9.03	6.23	48.00	30.03	12.62	3.28
9	9.03	37.30	12.47	6.51	0.85	3.28	60.00	10.48	5.47	1.61	
1973	4	0.00	47.30	16.50	17.10	1.47	0.00	34.50	16.50	17.10	1.43
	5	1.47	54.70	27.76	12.73	3.41	1.43	48.40	27.71	12.71	4.71
	6	3.41	44.10	40.29	14.88	10.62	4.71	47.00	42.31	15.63	7.45
	7	10.62	69.00	44.58	18.50	24.72	7.45	79.10	41.50	17.23	8.26
	8	24.72	14.10	45.60	22.29	26.72	8.26	8.00	35.95	17.57	11.26
9	26.72	9.90	16.84	19.59	14.60	11.26	7.70	13.91	16.18	9.34	

1974	4	0.00	10.00	19.20	20.70	15.70	0.00	8.40	19.20	20.70	8.45
	5	15.70	70.10	36.31	18.31	3.91	8.45	80.00	31.83	16.05	3.17
	6	3.91	65.80	33.86	14.01	6.26	3.17	73.70	32.86	13.60	4.53
	7	6.26	54.30	34.35	21.22	11.88	4.53	64.60	32.49	20.07	2.69
	8	11.88	44.70	38.47	19.24	13.61	2.69	66.10	29.73	14.87	4.83
	9	13.61	38.90	15.38	12.71	6.41	4.83	30.60	12.55	10.38	3.23
1975	4	0.00	19.90	18.60	16.80	0.30	0.00	19.90	18.60	16.80	1.43
	5	0.30	65.50	23.51	9.91	2.91	1.43	74.70	25.21	10.63	1.98
	6	2.91	62.00	33.16	10.98	5.55	1.98	41.10	31.74	10.51	5.66
	7	5.55	48.50	42.27	14.34	15.58	5.66	28.80	42.42	14.39	6.43
	8	15.58	20.90	40.81	17.97	20.32	6.43	17.50	34.02	14.98	8.52
	9	20.32	20.90	19.70	9.70	9.31	8.52	18.00	15.81	8.04	7.16
1976	4	0.00	17.50	19.20	15.90	2.98	0.00	14.00	19.20	15.90	2.51
	5	2.98	55.50	29.90	18.66	16.24	2.51	48.00	29.27	18.27	7.90
	6	16.24	32.00	54.72	26.44	16.11	7.90	25.00	46.90	22.70	8.70
	7	16.11	29.50	51.25	30.07	38.39	8.70	30.00	44.94	26.37	16.77
	8	38.39	37.50	44.74	30.20	26.37	16.77	36.00	37.15	25.08	12.71
	9	26.37	29.50	16.11	10.97	11.16	12.71	2.00	13.69	9.32	8.25

Supplementary irrigation in water resource systems

$$ET + ETP = 2ETW$$

where ET is the areal evapotranspiration, ETP is the potential evapotranspiration, and ETW is the wet environment evapotranspiration (evapotranspiration that occurs from a large saturated area, with water available for evapotranspiration).

Morton (1980) demonstrated that a reduction in the water available for areal evapotranspiration makes the overpassing air hotter and drier, and that this in turn increases potential evapotranspiration as computed from meteorological variables such as temperature, dewpoint temperature, and duration of sunshine.

The relationship indicates that potential evapotranspiration is more an effect than a cause of areal evapotranspiration. Morton's theoretical investigation and empirical verification showed that the average of areal and potential evapotranspiration, i.e., $(ET + ETP)/2$, is relatively stable, and he called it the wet environment evapotranspiration (ETW).

For this value, the following regression equation was proposed by Morton *et al.* (1980):

$$ETW = 14 + 1.2 \frac{\Delta}{\Delta + \gamma} R_n \text{ (mm)},$$

where Δ (the slope of the saturation vapor pressure curve) and R_n (net radiation) are at the potential evapotranspiration equilibrium temperature, and γ is the psychrometric constant. If ETP and ETW are known, ET can be computed.

Morton *et al.* (1980) published a program, REVAP, in Fortran for computation of these values, and a simplified version of this was used in this study for the data from station B. The results indicated that the potential evapotranspiration values computed by REVAP were systematically higher than those of potential evapotranspiration (PE) computed by eqs. (11)–(15) (Penman). For the period April–September, the ETP (evapotranspiration – Morton) was 1.66 times higher than the PE in 1970–76, and 1.63 times higher in 1931–70. For the period March–October, it was 1.67 times higher in 1931–70.

Similar relations were computed for areal evapotranspiration (ET) and wet environment evapotranspiration (ETW), as shown in the table below.

		April–September	March–October
ETP/PE	1931–70	1.63	1.67
	1970–76	1.66	
ET/PE	1931–70	0.83	0.83
	1970–76	0.81	
ETW/PE	1931–70	1.23	1.25
	1970–76	1.23	

According to the complementary character of Morton's relationship, it was taken for granted that it would give lower potential evapotranspiration values than Penman's formula. This disagreement between expectations and results needs further research, especially as far as the regression equation for the determination of the ETW is concerned.

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