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Environmental Quality, Abatement, and Urban Development

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ENVIRONMENTAL QUALITY, ABATEMENT, AND URBAN DEVELOPMENT

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ABSTRACT

The purpose of this paper is to derive a simultaneous equation model of an urban economy in which the impacts of environmental policy on the suburbanization-reurbanization process can be assessed. The variable to show these effects is a measure of the density of land-use activity. The model consists of essentially four components -- an urban land market, a market for an aggregate good, a description of a process of emission of residuals as a necessary by-product of land-use activities (production, consumption, commuting), and their accumulation and diffusion in a receiving medium (e.g., air and noise pollution).

Environmental policy is then introduced into this simultaneous equation model (emission standards and fines, ordinances to install abatement equipment, emission fees, etc.). A discussion follows on the hypothesis that a successful antipollution policy could introduce a trend toward reurbanization with the claim that clear air could induce land users to relocate closer to the center of a city.

The policies considered include fines for exceeding emission standards, inducing land users to purchase abatement devices in the market. The study attempts to derive the impacts of these measures on residuals' concentration, the roles of abatement devices, and urban density based on some *a priori* assumptions.

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ENVIRONMENTAL QUALITY, ABATEMENT, AND URBAN DEVELOPMENT

1. INTRODUCTION

Several land value studies (such as Anderson and Crocker, 1971) undertaken in highly industrialized countries show that environmental quality plays an ever increasing role in the choice of location by urban land users, particularly households and some (usually service-oriented) firms. The most important factors of environmental quality in an urban area are usually the levels of air and noise pollution, which are the effects of the levels and density of land-use activities.

Low concentration of residuals -- a stock variable -- can be found mostly in the suburban areas of a city. The flow variables determining the stock of residuals at a given location are emissions (depending on activity levels and density), and the diffusion and absorption by the environment. As residual density is usually lower in the suburbs, we expect its concentration to decrease as the distance from the city center increases.

The concentration of residuals, among other factors, has led to an accelerated suburbanization process, the results of which are ever increasing energy costs, inner city problems, and rising total pollution (v.d. Berg, et al., 1981).

Suppose now that a city authority takes steps to reduce emissions in order to improve environmental quality within the city's boundaries. (In this paper the measures applied are not differentiated by location.) If this policy turns out to be successful, one of the incentives to locate in the suburbs -- better environmental quality -- has vanished. As the locations closer to town become more accessible and transportation costs become smaller, there may be a side effect of environmental policy leading to a process of reurbanization. In a continuous process of income growth, however, where larger lots and new homes play the essential role in the location decision, it is to be suspected that the claimed effect will be over-compensated again.

In this paper an analytical model, based on *a priori* reasoning, is proposed that assesses the effects of various policy measures on population density. Since the model is basically a static one, only comparative static results are obtained (Schuber, 1979). We will restrict ourselves to the assessment of the impact of an urban environmental policy that attempts to curb the emissions of pollutants in the urban area by means of obligatory or voluntary installation of abatement devices. It should be added at this point, that the model outlined is, in principle, empirically testable. A thorough econometric analysis has so far been prevented by the lack of data, especially on policy measures. (Some preliminary first tests based on Austrian data have been carried out, but will not be reported here.)

2. A MODEL OF ENVIRONMENTAL QUALITY IN AN URBAN AREA

2.1. Basic Assumptions

Six basic assumptions are adhered to in our model.

- Following the von Thünen-Alonso (1826; 1965) analysis, we assume the model city to be situated on a homogeneous plain.
- We then postulate a "neutral" residual-receiving medium (i.e. in the case of air and noise pollution, no predominant direction of winds).
- All transactions take place in the center of town where consumers and producers have to go for their "transacting". (Strictly speaking, since all firms are not in the center of town, we assume that the "labor market" is in the central business district [CBD], and that workers commute first to this area and then to their place of work. Although this assumption may not always be realistic, it does simplify the analysis considerably.)
- To determine the activity levels of producers and consumers, we assume that households maximize utility, given an income constraint, and producers maximize profits, given a production function.
- The decisions made by the city government remain exogenous throughout the analysis.
- Our final assumption is that the land market is an example of monopolistic competition. This implies that locations are differentiated by their characteristics and that bargaining establishes their price (following Alonso's "game theoretic" approach).

2.2. Components of the Model

Concentration of residuals (R)

Environmental quality can be described by the distribution of residuals [R(r)] over the urban area (in a stationary state). The quality will depend on total emissions, which are caused by the land-use activities' consumption, production, and commuting to the center, and on the physical processes'"diffusion" and "absorption".

The urban land market (q)

The supply of land in a given zone is fixed (in a circular city: $2r\pi$). Demand for land is derived from utility and profit maximization. Among other variables it will depend on environmental quality and accessibiltiy of the CBD.

The goods market (X)

To facilitate the analysis, the total supply of consumption goods X, is assumed to be produced in the urban region (activity levels of firms = supply of X). This supply function (derived from the profit maximum conditions) depends again on accessibility and for some firms on environmental quality. It also depends on the factor market conditions, which are exogenously given. Demand depends on income (exogenous) and, indirectly, on environmental quality and accessibility (among other variables).

Emissions

All land-use activities cause emissions, which are considered as the part of waste that is "harmful" (i.e., which causes negative externalities), to be deposited in a common property resource where they accumulate. For our analysis we consider the predominant property resource to be air.

The city's administration

Urban planning in this context plays two roles: that of providing the transportation infrastructure (thus influencing accessibility) and that of applying various antipollution schemes. The activities of the administration are not space-consuming in the context of this paper, thus urban land users are only households and firms.

2.3. The Micro-economic Background of the Demand for Land and the Determination of Activity Levels

Potential land users evaluate offers on the real estate market by assessing the different characteristics of the various offers. Out of the entire range of such possible characteristics, we will only consider two "broad categories" (Richardson, 1978): accessibility and environmental quality. "Location" is seen as a good produced essentially by two agents, the city administration and all land users (Bökemann, 1977).

Accessibility is "produced" by the urban planner, but when certain capacity thresholds are surpassed and congestion begins to be a problem, all commuters contribute to "accessi-biltiy". It will be measured, therefore, in terms of travel time from a land user's location to the city center(A).

Environmental quality is "produced" by the emissions of all urban land users, these being caused by the land-use activities. The city administration plays an indirect role by applying environmental policy measures. Environmental quality, will be measured by some aggregate index of various noxious residuals(R).

Households

The utility of a household is given by:

$u = u\ (X, q, A, R)$

its income constraint:

$y = p^X X + p^q q\ /\ TC$ $\qquad\qquad E = E\ (X, r)$

where

y .... income
X .... level of aggregate consumption good
q .... size of lot
NP .... pollution abatement devices
E .... emissions (caused by consumption of X and commuting a distance of r to the center)
p^X, p^q ..... prices
TC .... transportation costs
r .... distance from the center

Equilibrium of the household is found by maximizing u, subject to y, where A and R are exogenously given, varying with r. Setting the first order conditions equal to zero and solving for X and q yields demand functions for X and q, depending on p^X, p^q, y, TC, A and R (we postulate that the second order conditions hold):

i.e. $q^D = q^D\ (p^q,\ p^X,\ y,\ TC,\ A,\ R)$

and $X^D = X^D\ (p^X,\ p^q,\ y,\ TC,\ A,\ R)$

We expect the following "sensitivities" to changes in these variables (they cannot, as usual, be rigorously derived from the common micro-economic regularity conditions):

$$q^D_{p^q}\ ,\ q^D_{TC},\ q^D_{p^X} \quad < \quad 0$$

$$q^D_y,\quad q^D_A\ ,\ q^D_R\ > 0$$

and similarly

$$X^D_{p^X}\ ,\quad X^D_{p^q}\ ,\ X^D_{TC} \ < \ 0, \qquad X^D_y,\ X^D_A\ ,\ X^D_R \ > \ 0$$

Firms

Profits: $P = p^X X - (wL + p^q q + TC)$.

The production function is:

$$X = f\ (L,q,A,R)$$

$$E = E(X,r)$$

where:

X .... output of consumption goods

L .... non-land production factors

p .... prices

w .... wage rate

The first order conditions of a profit maximum can be solved to yield a supply function for X and demand functions for q and L. (As the factor market will be assumed to be in equilibrium, it will not be analyzed any further.)

$$X^S \quad = X\ (p^X,\ p^q,\ w,\ TC,\ A,\ R)$$

$$q^D \quad = q^D\ (p^q,\ p^X,\ w,\ TC,\ A, R)$$

where:

X^S_{pq} , X^S_w, X^S_{TC}, X^S_A > 0

$X^S_R \leq 0$

X^S_{pX} > 0

2.4. Emissions and the Distribution of Residuals over the Urban Area

There are three land-use activities causing emissions:

- Production and consumption of the "aggregate" good (activity level: X)
- Commuting to the center of the city
- Transactions at the center (which we will assume to be constant, so they will be left out of the analysis).

Activities constitute transformation processes -- "inputs" are turned into "outputs" -- some of which can be used further ("consumption goods") and waste. In terms of physical mass, the total mass of inputs is equal to the mass of outputs. Production activities use production factors (raw materials, energy, etc.) which are transformed into goods and services and waste.

Households "consume" goods -- i.e., they are being transformed into waste. Commuting takes energy -- which is also turned into waste. Some of the waste, however, is recycled or is harmless, some of it is noxious, some of it could be treated and transformed into something directly useful or better "digestible" for nature.

The noxious part of waste we will define as"emissions" (pollution, residuals).

E^X = E (X) ... emissions caused by activities production or consumption of X

$E^{commuting}$ = E^{CO} = E (r) (or, with congestion, E^{CO} = E (A ; A is traveling time)

Once residuals are emitted into a common property resource they accumulate and diffuse. Some of them are transformed by nature into harmless materials (regeneration), thus reducing the stock of pollutants in the receiving medium. Hence, we have the following stock-flow relationship:

$$\frac{dR}{dt} = \dot{R} = E^T - (c)^2 R \ , \ (c)^2 > 0, \text{ constant}$$

where:

R ... stock of residuals

E^T ... total emissions of residuals (flow)

$(c)^2$ ... rate of natural regeneration (c is squared for mathematical convenience)

t ... time

As this total stock of residuals spreads over space, we observe varying environmental quality levels at different locations. Looking at the movements of pollutant particles after emission we observe that they move randomly in all directions from the source of emission. As we have excluded any permanently disturbing factors in the medium, a certain amount, say b, of the residuals always moves to the "left", the same amount to the "right" of the source (we are talking about a one-dimensional space at the moment): $-bR_r$ moves in the "positive" and bR_r in the "negative" direction (Feller, 1980).

At a given point in time, then, the change in the stock of residuals at a given location can come from other locations (diffusion), or from emissions (at the location), or be due to regeneration (at the location). The total change in residuals within an interval of dr then becomes

$$\dot{R}dr = b\,(R_r\,(r + dr\,;\,t) - R_r\,(r,t)) + E^T\,(r,t) - (c)^2\,R\,(r + dr\,;\,t)\,dr$$

To obtain the steady state distribution of residuals over space we let $t \to \infty$, i.e. $\dot{R} = 0$ (time disappears as a variable:

$$b\,\frac{d^2R}{dr^2} + E^T\,(r) - (c)^2\,R\,(r) = 0$$

Leaving the one-dimensional space and turning to our circularly symmetric city, we can define location in terms of polar coordinates - but as the angle does not matter (symmetry !) we finally obtain (setting b = 1, without loss of generality)

$$\frac{d^2\,R(r)}{dr^2} + \frac{1}{r}\,\frac{dR(r)}{dr} - c^2\,R\,(r) = -E^T(r)$$

(for details, see Schubert, 1979). We assume $R(0)$ and $\lim_{r\to\infty} R(r)$

to be finite, and we are considering total emissions $E^{T}(r)$ to be E^{X} and E^{CO}.

Now let there be d land users in a given ring (with distance from the center r). As each land user in this ring has activity level X, and emissions E(X), to obtain total emissions in the ring we have to sum over all the land users, or to simplify, we compute: E(X) d = total emissions due to X in ring r (where d(r) is the number of land users in a given ring r, and E(X) are the average emissions/land user). The emissions rate density then is

$$\frac{E[X(r)\ d(r)]}{2r\pi}$$

as the total area in a given ring is $2r\pi$.

Each land user on his way to the center emits $E^{CO}(r)$ in each ring. How many are passing through a given ring r ? Everybody located outside r, towards the edge of town contributes to total emissions at r.

There are, say, N(r) land users passing through r (where N is the sum of all land users d outside r).

Total commuting emissions: $E^{CO}(r)\ N(r)$

Emissions rate density: $\frac{E^{CO}(r)}{2r\pi} N(r)$

Note that E(X) and $E^{CO}(r)$ are (provisionally) exogenous in this partial analysis. The solution of the differential equation yielding the partial distribution of residuals over space has to be of the form:

$$R = R' \ [E(X),\ E^{co}(r),\ d,\ N(d),\ r]$$

or $\quad R = R\ (X,s,r)$

It can be shown that the solution is of the form:

$$R = [E^{co} \int_{o}^{r} a\, N(a)\, I_o\, (ca)\, da]\ K_o\, (cr)$$

$$E(X)\ I_o\ (cr) \int_{r}^{r_m} a\, N(a)\, K_o\, (ca)\, da$$

where r_m is the "edge" of the urban region and I_o and K_o are two standard, tabulated Bessel functions, which can be roughly drawn as in Figure 1 (Abramovitz and Stegun, 1972).

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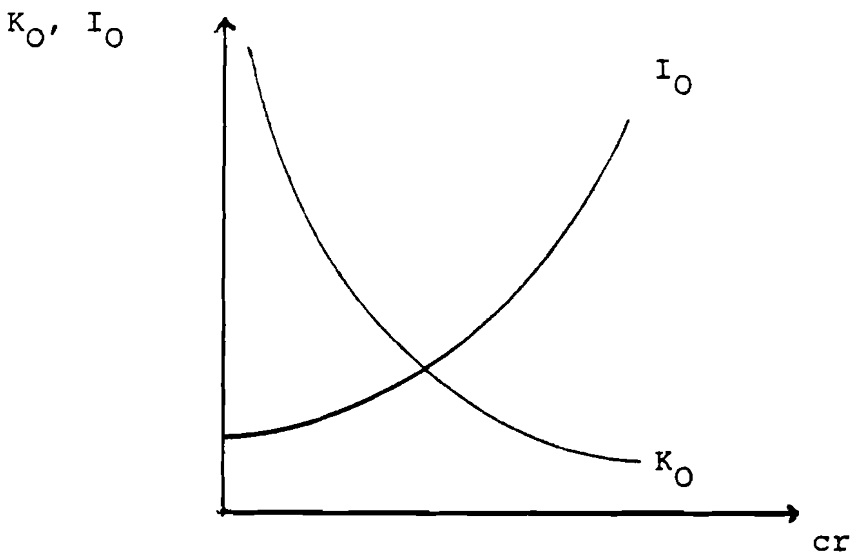

Figure 1. Components of the steady state distribution of residuals.

(Assuming emission-rate densities declining with distance from the center, the slope of R will resemble a Gaussian).
It can be shown that R_x, $R_d > 0$ and $R_r < 0$, given a decreasing emission-rate density toward the periphery of the city. As this density (given our assumptions) varies negatively with a rising r, the above assertions will hold. (For details, see Schubert, 1979).

3. MARKET EQUILIBRIUM

3.1. Density and the Land Market

The total supply of land in a given ring is $2r\pi$. Demand is equal to $q^D = q^D(y, w, p^q, p^x, A, R, TC)$, for all urban land users. Bargaining will eliminate all those potential land users that do not offer a high enough price at a given location. The equilibrium price of land will hence be the simultaneous solution of supply and demand equations - i.e. land prices p^q will be a function of all exogenous (to the land market) variables.

$$p^q = p' \ (A, R, y, w, p^x, TC, 2r\pi)$$

but as $A = A(r)$, $TC = TC(r)$, and y and w are exogenous, we can write: $p^q = p(R, r, z)$,

where: $z = (y, w, \pi, p^x)$

The equilibrium solution of the land market will also inform us about the number of land users located in a given ring

i.e.: $d = \frac{2r\pi}{q^D}$

3.2. The Goods Market

Individual demand for and supply of X depends on p^X as well as the other exogenous variables. Market equilibrium implies that the price of X, the solution of the simultaneous demand = supply system, depends on the exogenous variables defining supply and demand. Thus $p^X = p(R,r,z)$. The equilibrium quantities of X equal the activity levels causing emissions. (To make this step, we had to neglect all imports into and exports out of the urban region considered. The city's economy constitutes a closed system.) The equilibrium activity levels of urban land users hence become

$$X = X(y,\ w,\ p^q,\ p^X,\ A,\ R,\ TC)$$

where

$$X_y,\ \ 0,\ X_w,\ X_{p^q},\ X_{TC} < 0$$

X_A and X_R cannot be assessed *a priori* -- households demand more X when environment quality (accessibility) drops, firms are either not affected at all (by R) or have a decreased output.

4. A SIMULTANEOUS MODEL OF MARKETS AND RESIDUAL CONCENTRATION IN A CITY

We have been discussing a feedback system, in which some variables were taken as provisionally exogenous to derive partial equilibrium conditions and the stationary solution of the economic-environmental space/time process (where time is considered "short-run" in this formulation) (See Isard and Liossatos, 1978).

Residual concentration depends on activity levels of land users and their density -- determined in the land and goods market, which in turn depend on environmental quality.

$$R = R(X, d, r) \tag{1}$$

$$d = \frac{2r\pi}{q^D(p^q, p^X, R, z, r)} \tag{2}$$

$$p^q = p^q(R, z, r) \tag{3}$$

$$X = X(p^X, p^q, R, z, r) \tag{4}$$

$$p^X = p^X(R, z, r) \tag{5}$$

As this model constitutes a simple general equilibrium type of model (with two goods involved), we can use Walras's law, i.e., relative prices. Thus

$$p^q/p^X = p$$

where we can set $p^X = 1$, without loss of generality. We can now substitute p for p^q and p^X in all equations and drop (5).

Is an equilibrium solution for this system of implicit, simultaneous equations defined; i.e., are the endogenous variables R, d, p and X defined in terms of the exogenous variables z and r? Using the "implicit function theorem", we have to postulate continuity of all implicit functions and the existence of continuous derivatives. These conditions are fulfilled by assumption. Furthermore, the Jacobian determinant $|J| \neq 0$.

Let us rewrite the equations in implicit form:

$$F_1: \quad R(X, d, r) - R = 0$$

$$F_2: \quad \frac{2r\pi}{q(R, p, z, r)} - d = 0$$

$$F_3: \quad p(R, z, r) - p = 0$$

$$F_4: \quad X(p, R, z, r) - X = 0$$

Table 1 shows [J], the derivatives of these functions with the signs in parantheses.

Table 1. The derivatives of the simultaneous, implicit form equations.

	R		d		p		X	
F^1	-1		R_d	(+)	0		R_X	(+)
F^2	$\frac{2r\pi}{q^2} q_R$	(-)	-1		$-\frac{2r\pi}{q^2} q_p$	(+)	0	
F^3	p_R	(-)	0		-1		0	
F^4	X_R	(?)	0		X_p	(-)	-1	

$$|J| = \left[1 + R_d \frac{2r\pi}{q^2} (p_R\, q_p + q_R)\right] - R_X (p_R X_p + X_R)$$

+   +   (-) (-)   + (-) (-) ~0

+   +

The term in brackets is unambiguously positive, the second negative (assuming that X_R is negligible in quantity). The strength of the positive effect depends on r, as the magnitude of the first expression varies with r. There is, hence, in general a positive and a negative branch of the Jacobian and a point where it vanishes, i.e., where changes in the exogenous variables do not affect the endogenous variables. To the left and the right of this point, effects have opposite signs (see also Figures 2 and 3).

5. THE IMPACT OF ENVIRONMENTAL POLICY MEASURES ON THE "GEOGRAPHY" OF A CITY

5.1. Assessing Density Changes in the Model

In the previous section a simultaneous model of urban land and goods markets was introduced. "Environmental quality", the result of the land-use decisions and physical processes, was introduced into the evaluation and decision calculus of land users.

Comparing the results of this feedback system to Alonso's (1964) and Muth's (1969), we observe that it is most likely that a positive valuation of environmental quality tends to "stretch" the city -- a process of suburbanization. The price and density gradients (the equilibrium solutions) tend to flatten pushing the "edge" of the city outward (see Schubert, 1979).

Suppose now that the city administration attempts to reduce pollution in the urban area. Since the deterioration of environmental quality in locations close to the city center was one of the driving forces of suburbanization, it could well be that a reversal of this process could be the result of environmental policy (v.d. Berg, et al., 1981; Edel, 1972).

To substantiate this claim, we must isolate certain effects, i.e., distinguish between the income growth effect behind suburbanization (leading to demand for bigger lots) and the environmental factor, which drives people out of town and into areas of lower residual concentration -- regardless of the size of their individual lots.

How can the effects of environmental policy on urban shape be demonstrated? We will use the "density of land-use gradient", to show the spatial effects of environmental policy. To facilitate the exposition of the claimed hypotheses we make some simplifying assumptions about the shape of d. Let d be the usual bell-shaped function as shown in Figure 2. The integral of this function is the total population of the urban area. (As our city is symmetric we can use a two dimensional curve for illustration.)

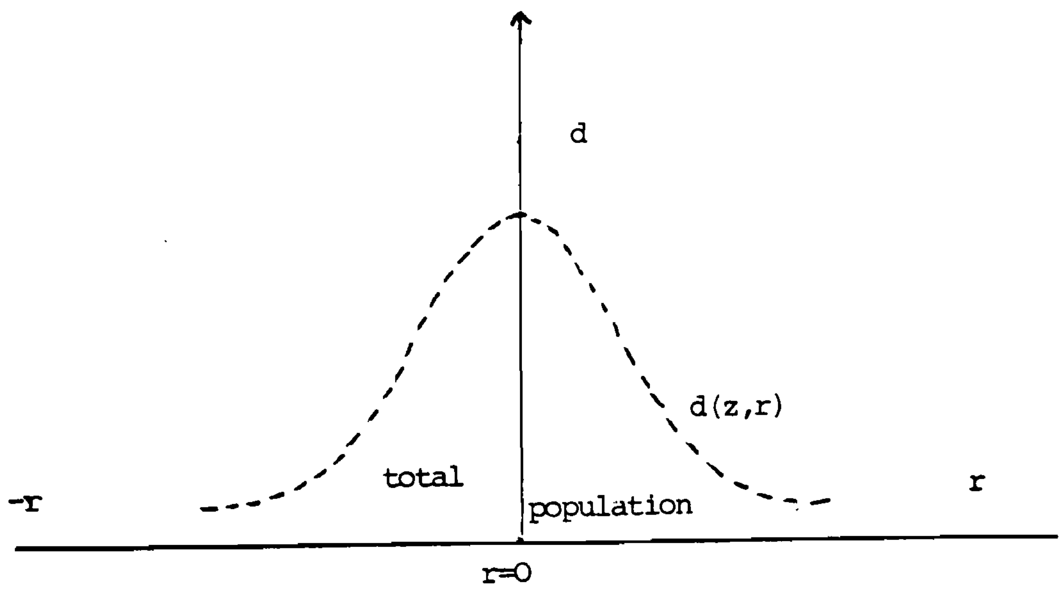

Figure 2. The distribution of population over urban space.

Comparing density profiles we can call a city "more compact", if the total population lives in a smaller area. We assume that the total population of our model city does not change. As some policy variables are altered, only the distribution over space may change.

A compact city has more people living around the center than does a "dispersed" city (see Figure 3). In equilibrium

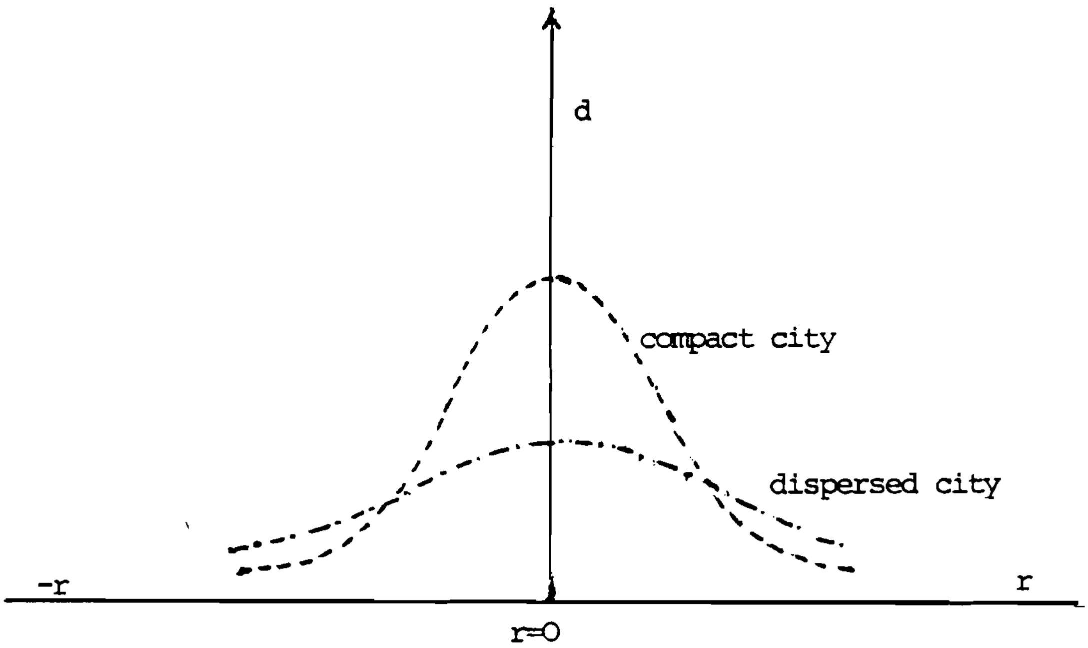

Figure 3. Density profiles of different urban forms.

d depends on r and the other exogenous variables (among them the instruments of environmental policy). Let this set of variables be represented by the vector z. We have then

$$d = d(r, z)$$

Changes in z will cause shifts in d. We claim now that, given our assumptions about d, a compact city is characterized by a "smaller" standard deviation of the population distribution, as is illustrated in Figure 4.

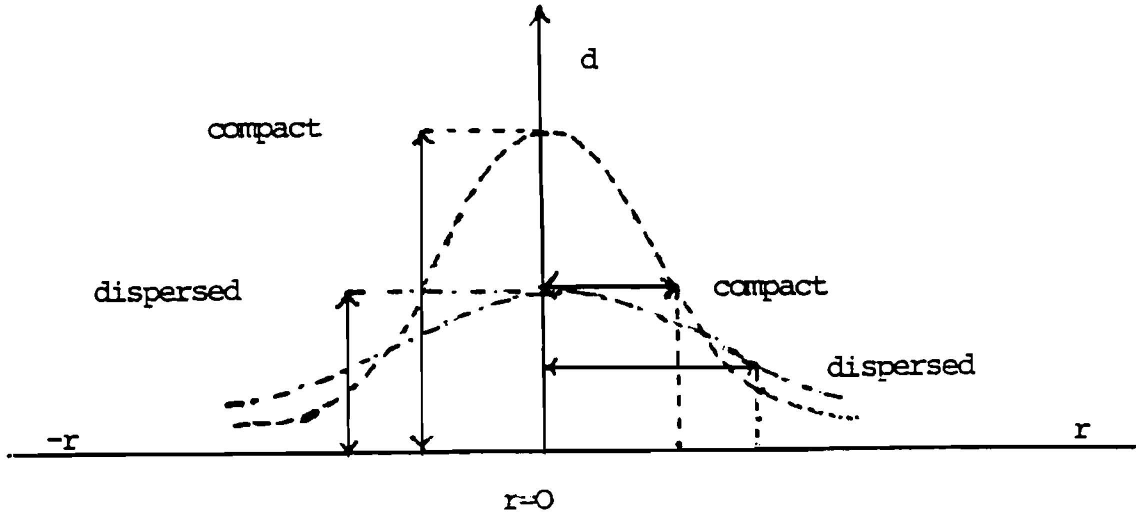

Figure 4. Urban form in terms of the standard deviation of density profiles.

Our task is to show how σ changes with z. To facilitate the derivation, "characteristic functions" will be used. Instead of using the standard deviation σ we can use the second moment m_2 of the distribution d to show the impact of changes in the policy variables z without loss of information (Fisz, 1973). To substantiate the claim of a reurbanization effect of environmental policy, we have to show that

$$\frac{\partial m_2}{\partial z} < 0$$

A characteristic function Φ of a density function is defined as (Fisz, 1973)

$$\Phi(\lambda, r, z) = \int_{-\infty}^{\infty} d(r, z)\, e^{i \lambda r}\, dr$$

where λ is an auxiliary variable, and $i = \sqrt{-1}$.

The second moment (m_2) of this function is

$$m_2 = - \frac{\partial^2 \Phi}{\partial \lambda_2} \quad (\text{at } \lambda = 0)$$

so we obtain

$$m_2 = + \int_{-\infty}^{\infty} d(r, z)\; r^2\, dr \tag{6}$$

Differentiating m_2 with respect to z we get

$$\frac{\partial m_2}{\partial z} = \int_{-\infty}^{\infty} d_z\; r^2 dr \quad (d_z = \frac{\partial d}{\partial z}) \tag{7}$$

We claim that

$$\int_{-\infty}^{\infty} d_z r^2\, dr < 0$$

Since we postulate that the population remains unchanged in size and only changes its distribution over space, we must also have

$$\int_{-\infty}^{\infty} d_z\, dr = 0$$

The integral over the urban area constitutes the population. The population difference when policy variables are changed must be equal to zero. But if the area under the curve remains the same, the density functions with or without z have to intersect (see also Figures 3 and 4). Before the intersection point, toward $r = 0$, the city is more compact if its density curve lies above the curve of the dispersed city. After the intersection point, toward the periphery, the reverse must hold.

For the compact city, hence, we must observe that toward the center, policy must shift d upwards, i.e., $d_z > 0$ at $r \to 0$. At the intersection point, policy measures do not affect the density curve at all, i.e., $d_z = 0$; outside the intersection points then $d_z < 0$ must hold. (For details see Schubert, 1979). What has to be demonstrated in the following then is

$$d_z \gtreqless 0$$

as r changes. We will do this by trying to show that $d_z > 0$ at $r \sim 0$. The general shape of d_z will also give some indication of the possible adjustment processes set off by environment policy in all the markets and in the residual concentration relation.

5.2. Environmental Policy and Abatement

In this paper we are dealing with abatement policies by the urban authorities. In principle, there are two kinds of abatement. Emissions can be controlled by purchasing and operating equipment (e.g., exhaust fume filters in automobiles, noise insulation on lawn mowers, etc.), or the effects of residual concentration can be mitigated (e.g., insulation in houses against outside noise, private air purification, etc.). But not all policies provide an incentive to invest in anti-pollution devices.

Without explicit policy (e.g., Ruff, 1972; Schubert,1973) individual abatement against emissions is highly unlikely. Individuals will purchase equipment to the extent that the marginal benefits from a lower concentration of residuals exceeds the marginal cost of buying and operating it. A control of emissions is hence only likely in the case of individuals whose own emissions constitute a good part of the ambient residual concentration experienced by them. (If one's own noise and smoke is the most severe environmental problem, one will do something about it -- which is hardly ever the case at higher densities of land use, where it is mostly the neighborhood effects that determine enviornmental quality. This is even more true of commuting pollution.)

We will not explicitly analyze changes in production and consumption technology (abatement sets in after the waste has been produced). In order to do so, a disaggregated model would be necessary differentiating between different kinds of technologies, inputs, goods, and transportation modes. Emissions in our model are seen to depend only on the total activity level.

In order to assess the impact of some environmental policies within our simultaneous system, the model has to be extended. First, a market for "abatement factors" (NP) has to be added; specifically we will have to analyze the impact of environmental policy on the demand for NP. The price p^{NP} will be treated as exogenous. Most of these measures have consequences on other markets as well, usually via the "income effect".

5.3. A Market for Abatement Devices

Consumers and producers can purchase devices to reduce emissions. (Again note that waste cannot be reduced due to the law of conservation of mass, but harmful residuals, defined as "emissions" in this analysis, can be reduced.) In equilibrium the total demand for abatement devices has to be equal to their supply.

Total demand: $NP^D = NP^D_{firms} + NP^D_{households}$

Total supply: NP^S, exogenous

Hence, we have $NP^D = NP^S = NP$. The price of these devices, p^{NP}, is considered exogenous, NP^D can be found by looking at the individual decisions of land users. In partial equilibrium, NP as one of the (endogenous) decision variables is a function of the exogenous variables.

In this case this implies that NP = Function (exogenous variables, parameters). We will analyze the impacts of various policies on this partial equilibrium solution first and then assess the "system-impact". There remains a problem, however. Firms supplying abatement devices also pollute. (Their activity levels are NP.) When all production takes place in the urban area the negative effect of this production could outweigh the positive effect via filtering, etc. Looking at R now, it becomes

$$R = R(X, NP, d, r)$$

But what is the effect of NP on R? The abatement effect decreases R, but the production of NP increases it. To facilitate the analysis, we will assume that $\partial R/\partial NP = R_{NP}$ has a negative net effect on residual concentration. ($R_{NP} < 0$). (The diffusion process demonstrated in Section 2 will not be explicitly changed by introducing NP, the implications of which are easily seen.)

Our simultaneous model now becomes

$$p^q/p^X = p^1, \quad p^{NP}/p^X = p^2$$

$$F^1: \quad R(X, NP, d, r) - R = 0$$

$$F^2: \quad \frac{2r\pi}{q(p,z)} - d = 0$$

$$F^3: \quad p^1(R, r, z) - p^1 = 0$$

$$F^4: \quad X(p^1, R, z) - X = 0$$

$$F^5: \quad NP(p^2, R, z) - NP = 0$$

In terms of endogenous variables as used before, the z stands for a vector of exogenous variables to be analyzed in the following sections; p is a vector of relative prices.

$$R_X,\ R_d > 0,\ R_{NP} < 0$$

$$q_p < 0,\ q_R > 0$$

$$p_R < 0$$

$$X_p < 0,\ X_R = 0$$

The micro economic background

Maximum allowable emission rates are set ($\bar{E}$). Violations of the set standards are fined. This fine rises linearly with the emission excess, i.e., total fine: $(E-\bar{E})F$, where $F > 0$, and $E^X=E(X,NP)$ or $E^{CO}=E(r,NP)$, or F is an emission fee to be paid per unit of emitted residuals. The household's decision problem is

$$\text{Max.} \quad u(X,\ q,\ a,\ R)$$

$$\text{s.t.} \quad \text{(i)}\ y - (p^X X + p^q q + TC + F(E-\bar{E}) + p^{NP} NP)$$

$$\text{(ii)}\ y - (p^X X + p^q q + TC + FE + p^{NP} NP)$$

Assuming second-order conditions to hold, we can find the first derivatives of the appropriate Lagrangian and set them equal to zero, which will yield a system of simultaneous equations.

We can solve these simultaneous equations implicitly by defining X^E, q^D, and NP^D; these solutions will be defined in terms of prices (p^X, p^q, p^{NP}), and the exogenous variables $TC, a, \bar{E}, F, 2r\pi$.

How do changes in the exogenous variables affect the demand for for X, q, and NP? As some of the effects have been outlined before, we will only look at the following:

$$NP_F \text{ and } NP_{\bar{E}},\ NP_P NP,\ NP_A,\ NP_R,\ NP_{TC}$$

$P_F \geq 0$ (as the fine increases it pays to abate more)

$NP_{\bar{E}} \leq 0$, $NP_P NP < 0$, $NP_A = 0$, $NP_R \sim 0$, $NP_{TC} < 0$

As long as $E < \bar{E}$, there will be no effect of F or $\bar{E}$. The analogous problem for the urban firm is

$$\text{Max. } \pi = p^X X - (P^q q + wL + p^{NP} NP + F(E-\bar{E}) + TC)$$

$$\text{s.t. } X-f\ (L, q, a, R) = 0$$

For the resulting equations we can again determine the supply of X and the demand for L, q and NP in terms of prices and the exogenous variables. We need now

$NP_P NP < 0$	$NP_R \sim 0$
$NP_a = 0$	$NP_{TC} < 0$
$NP_F > 0$	$NP_{\bar{E}} < 0$

Let $NP^S = NP^D = NP$; then NP is the (partial) equilibrium solution of the demand=supply condition in the market. It is defined as a function of the exogenous variables, i.e.,

$$NP^D{}_H(p^X\ p^q,\ p^{NP},\ a,\ TC,\ \bar{E},\ F,\ y)$$

$$+ \quad NP^D{}_F(p^X,\ p^{NP},\ p^q,\ w,\ a,\ R,\ TC,\ F,\ \bar{E}) = NP^S$$

From this we get

$$NP = NP(p^{NP}, p^{X}, p^{q}, w, y, R, TC, F, \bar{E})$$

Effects of abatement demand in the simultaneous model

Adding the implicit equation to the simultaneous model implies the following structure (using $p^{q}/p^{X} = p$ again, setting $p^{X} = 1$):

$$F^1: \quad R(X, NP, d, r) - R = 0$$

$$F^2: \quad \frac{2r\pi}{q(y, w, p, p^{NP}, R, a, TC, F, \bar{E})} - d = 0$$

$$F^3: \quad p(y, w, p^{NP}, R, a, TC, F, \bar{E}) - p = 0$$

$$F^4: \quad X(y, w, p^{NP}, p, R, a, TC, F, \bar{E}) - X = 0$$

$$F^5: \quad NP(y, w, p^{NP}, p, R, a, TC, F, \bar{E}) - NP = 0$$

from which the following Jacobian matrix can be derived (the signs of partial derivatives are in parentheses). As in Table 1, we note that the Jacobian consists of a positive and a negative branch and a point where $|J|$ vanishes. The location of this "turning point" of the signs of effects depends on r, of which J will be positive for small values, and negative for large ones.

Table 2. The derivatives of the simultaneous, implicit form equations with a market for abatement equipment added to the model.

R		d		p		X		NP	
-1		R_d	(+)	0		R_X	(+)	R_{NP}	(-)
$\frac{-2r\pi}{q^2} q_R$	(-)	-1		$\frac{-2r\pi}{q^2} q_p$	(+)	0		0	
p_R	(-)	0		-1		0		0	
X_R	(?)	0		X_p	(-)	-1		0	
NP_R	(0)	0		NP_p	(-)	0		-1	

$$|J| = -1 - R_d \frac{2r\pi}{q^2} (p_R q_p + q_R) + R_X (X_R + p_R R_{NP} NP_p X_p)$$

$$\underbrace{-\ +\quad +\quad (-)(-)(+)}_{(<0)} \qquad \underbrace{+\sim 0\quad (-)(-)\quad (-)(-)}_{(>0)}$$

Let us first see whether there will be any demand for pollution abatement equipment in equilibrium when fees (fines) are charged on emissions; i.e., $NP_F^{\dagger} > 0$

$$NP_F^{\dagger} = \frac{|J_{NP(F)}|}{|J|} \quad \text{by Cramer's rule}$$

$$|J_{NP(F)}| = -F^2{}_F R_d \, (NP_R + NP_p p_R)$$

(+) (+) (+) (−) (−)

$$+ \; F^3{}_F \; \{ R_X \, (X_p \, NP_R + NP_p \; X_R)$$

(−) (+) (−) (+) (−) (∼0)

$$- \; (NP_p - R_d \, \frac{2r\pi}{q^2} \, q_p \; NP_R + NP_p \, \frac{2r\,\pi}{q^2} \, q_R \; R_d)\}$$

(−) (+) (−) (+) (−) (+) (+)

$$- \; F_F^4 R_X \; (NP_R + NP_p \; p_R)$$

(−) (−) (+) (+) (−) (−)

$$+ \; F^5{}_F \; R_X (X_R + X_p p_R)$$

(+) (+) (∼0) (−) (−)

$$- \; (1 + R_d \, \frac{2r\pi}{q^2} \, q_p p_R + \frac{2r\pi}{q^2} \, q_R \; R_d)$$

(+) (−)(−) (+) (+)

as $F_F^1 = R_F = 0$

$F_F^2 = - \frac{2r\pi}{q^2} \, q_F$ (+)

$F_F^3 = P_F$ (−)

$F_F^4 = X_F$ (−)

$F_F^5 = NP_F$ (+)

Let us look at the components of this expression individually.

F^2 — Collecting emission fees (fines) tends to increase density directly, as less land can be afforded by land users. The higher density deteriorates environmental quality, which makes land prices fall -- but this makes it possible to purchase more antipollution devices.

F^3 — As income has to be spent on emission fees, less can be paid for land, but this makes demand for NP go down, thus worsening environmental quality. This could affect activity levels and consequently R. Lower land prices permit the pruchase of more abatement equipment; R decreases. This leaves land users satisfied with smaller lots, which makes density go up, but at the same time increases residuals.

F^4 — Expenditures for X decrease with rising emission fees; consequently there is less emission and environmental quality rises. Land prices now go up, which leaves less income to be spent on abatement.

F^5 — As emission fees become higher, land users attempt to emit less to save and the demand for NP shifts upwards. More abatement implies better environmental quality, a possible change in activity levels, and thus the concentration of residuals. As R goes down land prices rise, decreasing activity levels, and further improving environmental quality.

The higher land prices also increase density, causing a deterioration of environmental quality. Density tends to increase also as R decreases because of more abatement and land users demand less land, thus improving environmental quality further. The chains of effects in F^2, F^3, and F^5 support the hypothesis that the introduction of emission fees (fines) tend to encourage the installation of anti-pollution devices; F^4 points in the opposite direction. Letting $r \to 0$ leaves us with $|J_{NF(F)}| > 0$.

As $|J|$ and $|J_{NP(F)}|$ are positive at locations close to the center, while the reverse holds for suburban locations, the introduction of fines (fees) will cause a positive demand for abatement devices. (There is a small zone where there is no demand at all.) Does this induced demand for antipollution devices tend to decrease the second moment of the residual concentration function? We will again use $d_F^{\dagger}$ for our argument.

$$d_F^{\dagger} = \frac{|d(F)|}{|J|}$$

We replace the second column of $|J|$ by

$$- \begin{bmatrix} F_F^1 \\ F_F^2 \\ F_F^3 \\ F_F^4 \\ F_F^5 \end{bmatrix} = - \begin{bmatrix} 0 & \\ -\frac{2r\pi}{q^2} q_F & (+) \\ p_F & (-) \\ X_F & (-) \\ NP_F & (+) \end{bmatrix}$$

$$
|J_{d(F)}| = (-1) \Bigg[\underset{(+)}{F_F^2} \, (\underset{(-)}{R_{NP}} \; \underset{(-)}{NP_p} \; \underset{(-)}{p_R} + \underset{(+)}{R_X} \; \underset{(\sim 0)}{X_R} + \underset{(+)}{R_X} \; \underset{(-)}{X_p} \; \underset{(-)}{p_R})
$$

$$
+ \underset{(-)}{F_F^3} \; \underset{(+)}{\frac{2r\pi}{q^2}} \, (\underset{(+)}{q_R} \; \underset{(-)}{R_{NP}} \; \underset{(-)}{NP_p} + \underset{(+)}{q_R} \; \underset{(+)}{R_X} \; \underset{(-)}{X_p} - \underset{(+)}{R_X} \; \underset{(\sim 0)}{X_R} \; \underset{(-)}{q_p} - \underset{(-)}{q_p})
$$

$$
+ \underset{(-)}{F_F^4} \; \underset{(+)}{\frac{2r\pi}{q^2}} \, (\underset{(-)}{q_R} \; \underset{(-)}{p_R} \; \underset{(+)}{R_X} + \underset{(+)}{q_R} \; \underset{(+)}{R_X})
$$

$$
+ \underset{(+)}{F_F^5} \; \underset{(+)}{\frac{2r\pi}{q^2}} \, (- \underset{(-)}{p_R} \; \underset{(-)}{R_{NP}} + \underset{(+)}{q_R} \; \underset{(-)}{R_{NP}}) \Bigg]
$$

Analyzing individual components, again we obtain

F^2 — "Voluntary" abatement points in the direction of greater density. (Higher fines increase density, allowing R to go up, and land prices to drop. This makes abatement increase and R fall.) The emission-residual concentration effect is negligible; there are two opposite signs.

F^3 — The term $(q_R R_{NP} NP_p)\frac{2r\pi}{q^2}$ makes d increase. (Higher fees make land prices fall, abatement increase, R drop and with it the size of the desired lot, and density d go up. On the other hand, the increased price p has the opposite effect again via the demand for land and goods.) The net effect again is most likely negligible in quantity.

F^4 — This term definitely increases density.

F^5 — As fees increase, the demand for NP becomes higher and R drops, which makes land prices go up (and hence d increases). Better environmental quality makes land users satisfied with smaller lots (d increases also). The net effect is positive.

Density effects in the goods (F^4) and antipollution device (F^5) markets are positive. The effects in the other components of the model (environment and land market) are ambiguous. It should be mentioned, however, that there are more terms pointing in the direction of higher densities towards the center than in the case of no market for abatement devices (see Shubert, 1979).

Summing up, it seems plausible to argue that density in the urban core regions will icrease (note that $|J|$ is positive for small r) and the density in the urban ring will decrease, a process often referred to as "reurbanization" (v. d. Berg, et al. 1981).

6. SUMMARY

The model presented demonstrates that an urban system responds to changes in (exogenous) policy variables in a very complex way. Effects are usually contradictory in direction and size, and the net outcome can only be determined by means of empirical analysis. Unhappily all the necessary data for such an enterprise are not available. It seems plausible, however, that if the environmental policy outlined in the preceding section (fines or fees on excessive emissions) were employed, there would be a tendency toward a more compact city which could outweigh the "disurbanization" effects.

A serious drawback of the model used for this analysis is its being static. Cities and systems of cities seem to follow life cycles of growth and decline and concentration and dispersion (v.d. Berg, et al. 1981). In each stage of development there appear to be characteristic constellations of the relevant variables, a fact that makes comparative statics only relevant within narrow margins.

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APPENDIX: List of Variables

A	"accessibility" (total traveling time from location to city center)
a	travelling time per zone
C	capacity of the transportation network
CT	consumption tax
d	"density" (number of land users per zone)
E, (E^X, E^{CO}) .	emissions (due to X and commuting)
E^T	total emissions
F	fee (fine) per unit of (excess) emission
[J] , |j| . . .	Jacobian matrix (determinant) of partial derivatives
L	labor
NP	pollution abatement equipment
N	total number of land users outside a zone
P	profit of an urban firm
p^{NP}, p^q, p^X .	prices of NP, q X

q .... area of an urban lot (quantity of land)

R .... concentration of residuals

r .... distance from the center

t .... time

TC .... transportation cost

u .... utility index of an urban household

w .... wage rate

X .... quantity of aggregate consumption good

y .... income of an urban household

z .... vector of exogenous variables

Partial derivatives are denoted by lower case letters (e.g. $\frac{\partial R}{\partial r} = R_r$), second partials are indicated by a superscript 2 (e.g. $\frac{\partial^2 R}{\partial r^2} = R^2{}_r$).

Time derivatives are indicated by a dot (e.g. $\frac{\partial R}{\partial t} = \dot{R}$).

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